Wu, Shaomin (2017) ***Three models for the failure process of a repairable system.*** **In: The 10th international conference on mathematical methods in reliability, July 3-6, 2017, Grenoble, France.**

THREE MODELS FOR THE FAILURE PROCESS OF A REPAIRABLE SYSTEM

SHAOMIN WU
University of Kent
Canterbury CT2 7FS, UK
email: s.m.wu@kent.ac.uk

Development of models for the failure process of a repairable system has been an interesting research topic for decades. There have been many models developed in the literature. However, more research is still needed to deal with various difficulties raised in the practical applications, which includes the cases that most real systems are composed of more than one component and that the failure process may not be stochastically monotone. For such cases, we have recently developed three models, which are briefly discussed in this paper.

Keywords: Non-homogeneous Poisson process, renewal process, repair, doubly geometric process, semi-geometric process.

1. Introduction

Modelling the failure process of a repairable system is needed in many applications such as optimisation of maintenance policy and lifecycle costing. For this purpose, there are many models that have been developed in the literature. If we denote times-between-failures of a repairable system as a sequence of random variables $\{X_k, k = 1, 2, \dots\}$, models in the existing literature can be categorised into two classes, as shown in Figure 1. Type I models include the renewal process, the geometric process (GP) and its extensions [1,2,3,4]; type II models include the non-homogeneous Poisson process (NHPP) and its various extensions such as the virtual age model [5], the reduction of intensity model [6] and the hybrid intensity models [7]. Both type I and type II models have limitations: most of those models are designed for the failure process of a single component system and type I models have restrictive assumptions. However, most real systems are composed of more than one component and the failure process may not need the restrictive assumptions.

While those models developed in pre-2017 have been widely studied, this paper mainly focuses on three models we have developed and published in

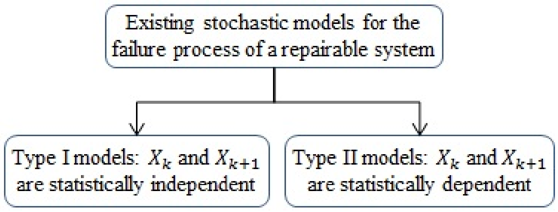

Figure 1. A taxonomy of models for the failure process of a repairable system

2017. Those three models are: two extensions of the GP [4,3] and a hybrid-intensity model [7] and briefly discussion them.

2. Two extensions of the GP

The definition of the GP is given by Lam [1].

Definition 2.1. [1] Given a sequence of non-negative random variables $\{X_k, k = 1, 2, \ldots\}$, if they are independent and the cdf of X_k is given by $F(a^{k-1}x)$ for $k = 1, 2, \ldots$, where a is a positive constant, then $\{X_k, k = 1, 2, \cdots\}$ is called a geometric process (GP).

The GP has been widely applied in optimisation of maintenance policy since its introduction in 1988, which may be mainly due to its elegant expression of the time-to-failure distribution after a repair and little consideration has been devoted onto the real-world application of this model. As can be seen, the GP has two restrictive assumptions: (1) $\{X_k, k = 1, 2, \cdots\}$ are independent and their distributions and (2) X_k is stochastically monotone. Those assumptions may restrict it from wide applications. To relax those two assumptions, we introduce the following two definitions, respectively.

Definition 2.2. [3] Given a sequence of non-negative random variables $\{X_k, k = 1, 2, \ldots\}$, if $P\{X_k < x | X_{k-1} = x_{k-1}, \ldots, X_1 = x_1\} = P\{X_k < x | X_{k-1} = x_{k-1}\}$ and the marginal distribution of X_k is given by $P\{X_k < x\} = F_k(x) (\equiv F(a^{k-1}x))$, where a is a positive constant, then $\{X_k, k = 1, 2, \ldots\}$ is called a semi-geometric process (SGP).

Definition 2.3. [4] Given a sequence of non-negative random variables $\{X_k, k = 1, 2, \ldots\}$, if they are independent and the cdf of X_k is given by $F(a^{k-1}x^{h(k)})$ for $k = 1, 2, \ldots$, where a is a positive constant, $h(k)$ is a function of k and the likelihood of the parameters in $h(k)$ has a known closed form, and $h(k) > 0$ for $k \in \mathbb{N}$, then $\{X_k, k = 1, 2, \cdots\}$ is called a doubly geometric process (DGP).

As can be seen, the SGP removes the independence assumption of the GP and the DGP can model the cases where X_k is stochastically monotone or non-monotone.

[3,4] discuss the probabilistic properties of the SGP and the DGP, respectively and also describe parameter estimation methods. Numerical examples on real-world data show that the SGP and DGP outperform the GP and other models.

3. A hybrid intensity model

[7] proposed two models for the failure process of a repairable series multi-component system. Assume a replacement is carried out upon system failure. Then the failure process can be regarded as a process resulted from the maintenance activities of two subsystems: one subsystem that is maintained with minimal repair and one subsystem, part of which is replaced upon system failure. [7] then proposes two models, Model I and Model II. On the basis of numerical examples, Model II outperforms model I. Denote $\lambda(t|\mathscr{H}_{t-})$ as the intensity function of the system. Then Model II gives the following definition.

$$\lambda(t|\mathscr{H}_{t-}) = \begin{cases} \lambda_1(t) + \lambda_2(t), & \text{if } N_t = 0, \\ \lambda_1(t) + \dfrac{1}{m}\left(\displaystyle\sum_{k=0}^{N_t-1} \lambda_2(t - T_{N_t-k}) + (m - N_t)\lambda_2(t)\right), & \text{if } 1 \le N_t < m, \\ \lambda_1(t) + \dfrac{1}{m}\displaystyle\sum_{k=0}^{m-1} \lambda_2(t - T_{N_t-k}), & \text{if } N_t \ge m. \end{cases}$$

where $t \in (0, +\infty)$, $\lambda_1(t)$ and $\lambda_2(t)$ are two intensity functions.

The above model regards the failure process of a series system equivalent to that of a virtual system composed of two subsystems: subsystems 1 and 2. Subsystem 1 contains one virtual component and subsystem 2 is composed of a pre-specified number, m, of virtual components. Broadly speaking, whenever the real system fails, minimal repair is assumed to be conducted on subsystem 1 and the oldest virtual component in subsystem 2 is assumed to be replaced with a new identical virtual component.

It is shown that Model II outperforms the four models (i.e, RP, GP NHPP, the virtual age model) on six artificially generated data and on some real data [7].

4. Conclusion

The three models discussed above may be useful in different applications. Our future work will be focused on investigation of their statistical properties.

Acknowledgements

The author would like thank Professor Phil Scarf and Dr Guanjun Wang for joint work.

References

[1] Y, Lam. Geometric processes and replacement problem. *Acta Mathematicae Applicatae Sinica.* 4, 366-377, 1988.

[2] L. Bordes. and S. Mercier. Extended geometric processes: semiparametric estimation and application to reliability imperfect repair, Markov renewal equation, replacement policy. *Journal of the Iranian Statistical Society*, 12(1), 1-34, 2013.

[3] S. Wu, and G. Wang. The semi-geometric process and some properties. *IMA Journal of Management Mathematics.* DOI: 10.1093/imaman/dpx002, 2017.

[4] S. Wu. Doubly geometric processes and applications. *Journal of the Operational Research Society.* DOI: 10.1057/s41274-017-0217-4, 2017.

[5] M. Kijima, H. Morimura and Y. Periodical replacement problem without assuming minimal repair. *European Journal of Operational Research* 37(2), 194-203, 1988.

[6] L. Doyen. and G. Olivier. Classes of imperfect repair models based on reduction of failure intensity or virtual age. *Reliability Engineering and System Safety* 84(1),45-56, 2004.

[7] S. Wu, and P. Scarf. Two new stochastic models of the failure process of a series system. *European Journal of Operational Research* 257(3), 763-772, 2017.