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Decentralised Sliding Mode Control for Nonlinear Interconnected Systems in the Generalised Regular Form

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Abstract: In this paper, a decentralised control strategy based on sliding mode techniques is proposed for a class of nonlinear interconnected systems in generalised regular form. All the isolated subsystems and interconnections are fully nonlinear. It is not required that the nominal isolated subsystems are either linearizable or partially linearizable. The uncertainties are nonlinear and bounded by known functions. Under mild conditions, sliding mode controllers for each subsystem are designed by only employing local information. Sufficient conditions are developed under which information on the interconnections is employed for decentralised controller design to reduce the effects of the interconnections on the entire systems. The bounds on the uncertainties have more general forms compared with previous work. A simulation example is used to demonstrate the effectiveness of the proposed method.

Keywords: Interconnected system, decentralised control, sliding mode control, generalised regular form

1. INTRODUCTION

In real world, large scale systems are often modelled as a collection of subsystems with interconnections, e.g. multi-machine power systems (e.g. see Fusco and Russo (2013); Singh and Pal (2016)). Due to the complex dynamics caused by nonlinearity in the interconnections and subsystems, it is difficult to control such systems by using classical methods. Moreover, as mentioned by Yan et al. (2017), such a class of systems is usually distributed in space, resulting in difficulty in implementation of a centralised strategy. Therefore, the development of decentralised control strategies in which each local controller of subsystem is working independently is essential. Moreover, uncertainties such as modelling errors, external disturbances and parameter variation also widely exist in the real world which may also greatly affect the control system performance. Therefore, robust control theory has received much attention in the past few decades (see e.g. Yan et al. (2005); Cheng and Chang (2008); Mi et al. (2013); Labibi and Alavi (2014); Zhang et al. (2016)). Multi-area power systems with decentralised sliding mode control are designed by Mi et al. (2013). However, the uncertainties are assumed to be matched. Cheng and Chang (2008) applied adaptive techniques to estimate an upper bound on mismatched uncertainty, which is used to counteract the effects of uncertainty in control design. In most previous work, the nominal part of the system is usually assumed to be linear, which limits the application of the obtained results. Labibi and Alavi (2014) proposed a decentralised quantitative feedback control for a class of large-scale systems in the presence of uncertainties in the state-space matrices, and the designed controllers have been implemented on a Selective Compliance Assembly Robot Arm system.

Sliding mode control has been recognised as a powerful approach in dealing with nonlinear systems with uncertainties owing to its reduced-order sliding mode and complete robustness against matched uncertainties (e.g. see Edwards and Spurgeon (1998); Utkin et al. (2009)). It is shown by Yan et al. (2005) that the sliding mode approach can be used to deal with systems in the presence of unmatched uncertainty. A sliding mode control scheme proposed by Ginoya et al. (2014) shows a way for linear system to reject the effect of the mismatched uncertainties by using disturbance observer. However, the structure of the system is restricted, which makes the method difficult to be implemented. Zhang et al. (2016) also proposed a sliding mode scheme applied with disturbance observer techniques to reject mismatched uncertainties. Although the structure and the upper bounds of the uncertainties are known, the proposed method can be applied to a wide class of linear systems. To reduce the effect of uncertainties on the whole system in nonlinear interconnected systems is very challenging using decentralised control strategy (see Yan et al. (2004)). Moreover, in most existing sliding mode control design, the system is usually required to be in regular form or to be transferred into such a form for analysis (see Edwards and Spurgeon (1998)). Although such a transformation matrix can be easily obtained by basic matrix theory for linear systems, it is very difficult to find a diffeomorphism to transfer a nonlinear system into the traditional regular form even though the existence of such a diffeomorphism is guaranteed. Moreover, associated conditions may be too strong to be applied for most nonlinear systems (see e.g. Yan et al. (2014)). These have motivated the decentralised control design for a class of nonlinear interconnected systems in generalised regular form in this paper. Since the generalised regular form includes the traditional regular form as a special case as mentioned in Mu et al. (2015a), the developed results can be applied to a wide class of nonlinear systems.

In this paper, a nonlinear decentralised control strategy for a class of nonlinear interconnected systems in generalised regular form is proposed based on a sliding mode control paradigm. The interconnected system is fully nonlinear, and the form of the system is more general than the system with the classical regular form considered in Mu et al. (2015b). Moreover, the uncertainties are assumed to be bounded by known functions which are employed in the control design to counteract the effects of the uncertainties on the controlled interconnected system. The bounds on the uncertainties take more general forms when compared with existing work. It is also shown that if the uncertainties/interconnections possess a superposition property, a decentralised control scheme may be designed to counteract the effect of the uncertainty. A numerical example with simulation results is presented to show the effectiveness of the approach proposed.

2. SYSTEM DESCRIPTION

Consider a class of nonlinear large-scale interconnected systems composed of N subsystems where the i-th subsystem can be transformed or described by

$$\dot{X}_i^a = F_i^a(t, X_i) + G_i^a(t, X_i)(U_i + \Phi_i(t, X_i)) + \sum_{j=1}^{N} H_{ij}^a(t, X_j) \tag{1}$$

$$\dot{X}_i^b = F_i^b(t, X_i) + G_i^b(t, X_i)(U_i + +\Phi_i(t, X_i)) + \sum_{j=1}^{N} H_{ij}^b(t, X_j) \tag{2}$$

where $X_i := \mathrm{col}(X_i^a, X_i^b) \in \Omega_i \subset \mathcal{R}^{n_i}$ are the state variables of the i-th subsystem with $X_i^a \in \mathcal{R}^{n_i - m_i}$, $X_i^b \in \mathcal{R}^{m_i}$. The functions $F_i^a(\cdot)$, $F_i^b(\cdot)$ with $F_i^a(t, 0) = 0$ and $F_i^b(t, 0) = 0$ and the function matrix $G_i^a(\cdot)$ and $G_i^b(\cdot)$ are continuous with appropriate dimensions. $U_i \in \mathcal{R}^{m_i}$ denote inputs of the i-th subsystem respectively for $i = 1, 2, \ldots, N$. Matched uncertainty is denoted by $\Phi_i(\cdot)$ The nonlinear functions $H_{ij}^a(\cdot) \in \mathcal{R}^{n_i - m_i}$ and $H_{ij}^b(\cdot) \in \mathcal{R}^{m_i}$ represent the uncertain interconnections. It is assumed that all the nonlinear functions are sufficiently smooth such that the unforced system has a unique continuous solution.

Choose the sliding function $\sigma_i(X_i)$ as follows:

$$\sigma_i(X_i) = X_i^b + \varphi_i(X_i^a), \quad i = 1, 2, \ldots, N. \tag{3}$$

where $\varphi_i(\cdot)$ is a known Frechet-differentiable function with $\varphi_i(0) = 0$ satisfy

$$M_{\varphi_i}(\xi) \left(M_{\varphi_i}(\xi)\right)^{\tau} \leq \beta_i I_m \quad \forall \xi \in \mathcal{R}^{n_i - m_i} \tag{4}$$

where $M_{\varphi_i}(\cdot) \in \mathcal{R}^{m_i \times (n_i - m_i)}$ represent the Jacobian matrix of function $\varphi_i(\cdot)$, and β_i is a positive constant.

For the sliding functions in (3), the sliding surface is described by

$$\mathcal{S}_i = \{X_i \in \mathcal{R}^{n_i} | \quad \sigma_i(X_i) = 0\}, \quad i = 1, 2, \ldots, N. \tag{5}$$

Assumption 1. Function $G_i^a(\cdot)$ in system(1) satisfies

$$G_i^a(t, X_i)|_{X_i \in \mathcal{S}_i} = 0, \quad i = 1, 2, \ldots, N. \tag{6}$$

where $\mathcal{S}_i$ is defined in (5).

Define function matrix $\Gamma_i(t, X_i)$ as

$$\Gamma_i(t, X_i) := G_i^b(t, X_i) + M_{\varphi_i}(X_i^a) G_i^a(t, X_i) \tag{7}$$

where $G_i^a(\cdot)$ and $G_i^b(\cdot)$ are defined in systems (1)-(2) and $\varphi_i(\cdot)$ are defined in (3).

Assumption 2. There exist known continuous nondecreasing functions $\Xi_{ij}^a(\cdot)$ in $\mathcal{R}^+$ with $\Xi_{ij}^a(t, 0) = 0$, and known continuous functions $\Xi_i^b(\cdot)$ and $\rho_i(\cdot)$ such that

(i) $\left\|H_{ij}^a(t, X_j)\right\| \leq \Xi_{ij}^a(\|X_j\|)$ (8)

(ii) $\left\|H_{ij}^b(t, X_j)\right\| \leq \Xi_{ij}^b(\|X_j\|)$ (9)

(iii) $\|\Phi_i(t, X_i)\| \leq \rho_i(t, X_i)$ (10)

for all $t \in \mathcal{R}^+$, $X_i \in \Omega_i$.

3. STABILITY ANALYSIS OF THE SLIDING MODE

Choose the composite sliding surface for the interconnected system (1)-(2) as follows

$$\sigma(X) = 0 \tag{11}$$

where $\sigma(X) \equiv: \mathrm{col}\left(\sigma_1(X_1), \sigma_2(X_2), \ldots, \sigma_N(X_N)\right)$ and $X := \mathrm{col}(X_1, X_2, \ldots, X_N)$ with $\sigma_i(\cdot)$ defined in (3). During sliding motion, $\sigma_i(X_i) = 0$ for $i = 1, 2, \ldots, N$, under Assumption 1, $G_i^a = 0$ and from (3), $X_i^b = -\varphi_i(X_i^a)$ for $i = 1, 2, \ldots, N$. Then, the sliding mode dynamics for the system (1)-(2) associated with the designed sliding surface (5) can be described by

$$\dot{X}_i^a = F_i^s(t, X_i^a) + \sum_{j=1}^{N} H_{ij}^s(t, X_j^a) \tag{12}$$

where

$$F_i^s(t, X_i^a) := F_i^a(t, X_i^a, -\varphi_i(X_i^a)) \tag{13}$$

and

$$H_{ij}^s(t, X_j^a) := H_{ij}^a(t, X_j^a, -\varphi_j(X_j^a)) \tag{14}$$

for $i, j = 1, 2, \ldots, N$ with $H_{ij}^a(t, X_j)$ defined in (1).

Lemma 1. For terms $H_{ij}^s(t, X_j^a)$ in system (12), if inequality (8) in Assumption 2 holds, then

$$\|H_{ij}^s(t, X_j^a)\| \leq \Xi_{ij}^s(\|X_j^a\|) \tag{15}$$

where

$$\Xi_{ij}^s(\|X_j^a\|) = \Xi_{ij}^a(\sqrt{1 + \beta_i}\|X_j^a\|)$$

where $\Xi_{ij}^a(\cdot)$ are defined in (8).

Proof. From the definition of $H_{ij}^s(\cdot)$ in (14), it follows that

$$H_{ij}^s(t, X_j^a) = H_{ij}^a(t, X_j^a, -\varphi_i(X_j^a)) \tag{16}$$

From (4), it is straightforward to see that

$$\|M_{\varphi_i}(\xi)\| \le \sqrt{\beta_i} \tag{17}$$

Then from the mean value theorem,

$$\|\varphi_i(h) - \varphi_i(0)\| = \|\varphi_i(h)\| \le \sqrt{\beta_i}\|h\| \tag{18}$$

When the system is on the sliding surface

$$\begin{aligned}\|X_i\| =& \sqrt{(X_i^a)^\tau X_i^a + (X_i^b)^\tau X_i^b} \\ =& \sqrt{(X_i^a)^\tau X_i^a + \varphi_i^\tau(X_i^a)\varphi_i(X_i^a)} \\ \le& \sqrt{\|X_i^a\|^2 + \beta_i\|X_i^a\|^2} \\ =& \sqrt{1+\beta_i}\|X_i^a\| \end{aligned} \tag{19}$$

From (16), (8) and (19), it follows that

$$\begin{aligned}\|H_{ij}^s(t, X_j^a)\| \le& \Xi_{ij}^a(\|X_j\|) \\ \le& \Xi_{ij}^a(\sqrt{1+\beta_i}\|X_j^a\|) \\ =& \Xi_{ij}^s(\|X_j^a\|) \end{aligned} \tag{20}$$

Hence the result follows. ■

Assumption 3. There exist continuous C^1 function $V_i : \mathcal{R}^+ \times \mathcal{R}^{n_i - m_i} \to \mathcal{R}^+$ and functions $\varsigma_{i1}(\cdot)$, $\varsigma_{i2}(\cdot)$, $\varsigma_{i3}(\cdot)$ and $\varsigma_{i4}(\cdot)$ of class $\mathcal{K}$ such that for all $X_i \in \Omega_i$ and $t \in \mathcal{R}^+$

(i) $\varsigma_{i1}(\|X_i^a\|) \le V_i(t, X_i^a) \le \varsigma_{i2}(\|X_i^a\|)$

(ii) $\dfrac{\partial V_i(t, X_i^a)}{\partial t} + \dfrac{\partial V_i(t, X_i^a)}{\partial X_i^a} F_i^s(t, X_i^a) \le -\varsigma_{i3}^2(\|X_i^a\|)$

(iii) $\left\| \dfrac{\partial V_i(t, X_i^a)}{\partial X_i^a} \right\| \le \varsigma_{i4}(\|X_i^a\|)$

where

$$\frac{\partial V_i(t, X_i^a)}{\partial X_i^a} = \left(\frac{\partial V_i(t, X_i^a)}{\partial X_1^a}, \frac{\partial V_i(t, X_i^a)}{\partial X_2^a} \cdots \frac{\partial V_i(t, X_i^a)}{\partial X_n^a} \right)$$

Theorem 1. Under Assumptions 1, 2 and 3, the sliding modes (12) of the systems (1)-(2) for $i = 1, 2, \ldots, N$ associated with the sliding surface (11) are asymptotically stable if there exists a domain Ω_{X^a} of the origin in $X^a = \mathrm{col}(X_1^a, X_2^a, \ldots, X_N^a) \in \mathcal{R}^{\sum_{i=1}^N (n_i - m_i)}$ such that

$$(W(t, X))^\tau + W(t, X) > 0$$

in domain $\Omega_{X^a} \backslash \{0\}$ with $W(t, X) = (w_{ij}(t, X_i, X_j))_{N \times N}$ and for $i, j = 1, 2, \ldots, N$

$$\begin{aligned} & w_{ij}(t, X_i, X_j) \\ &= \begin{cases} \mu_{i3}(\|X_i^a\|) - \mu_{i4}(\|X_i^a\|)\gamma_{ii}(\|X_i^a\|), & i = j \\ -\mu_{i4}(\|X_i^a\|)\gamma_{ij}(\|X_j^a\|), & i \ne j \end{cases} \end{aligned}$$

where $\mu_{i3}(\cdot)$, $\mu_{i4}(\cdot)$ and $\gamma_{ij}(\cdot)$ are defined respectively by

$$\mu_{i3}(\xi) = \int_0^1 \frac{\partial \varsigma_{i3}(\xi h)}{\partial h} \mathrm{d}h \tag{21}$$

$$\mu_{i4}(\xi) = \int_0^1 \frac{\partial \varsigma_{i4}(\xi h)}{\partial h} \mathrm{d}h \tag{22}$$

$$\gamma_{ij}(\xi) = \int_0^1 \frac{\partial \Xi_{ij}^s(\xi h)}{\partial h} \mathrm{d}h \tag{23}$$

Proof. From (21)-(23), it can be observed that

$$\varsigma_{i3}(\|X_i^a\|) = \mu_{i3}(\|X_i^a\|)\|X_i^a\| \tag{24}$$

$$\varsigma_{i4}(\|X_i^a\|) = \mu_{i4}(\|X_i^a\|)\|X_i^a\| \tag{25}$$

$$\Xi_{ij}^s(\|X_i^a\|) = \gamma_{ij}(\|X_i^a\|)\|X_i^a\| \tag{26}$$

From the analysis above, it is seen that system (12) represents the sliding mode dynamics of the system (1)-(2) corresponding to the sliding surface (11).

For system (12), consider the Lyapunov function candidate

$$V(t, X^a) = \sum_{i=1}^N V_i(t, X_i^a) \tag{27}$$

where $V_i(t, X_i^a)$ is given in Assumption 3. Then, the time derivative of $V(t, X_i^a)$ along equation (12) is given by

$$\begin{aligned} \dot{V} =& \sum_{i=1}^N \Big\{ \frac{\partial V_i(t, X_i^a)}{\partial t} + \frac{\partial V_i(t, X_i^a)}{\partial X_i^a} F_i^s(t, X_i^a) \\ & + \frac{\partial V_i(t, X_i^a)}{\partial X_i^a} \sum_{j=1}^N H_{ij}^a(t, X) \Big\} \\ \le& \sum_{i=1}^N \Big\{ -\varsigma_{i3}^2(\|X_i^a\|) + \varsigma_{i4}(\|X_i^a\|) \sum_{j=1}^N \Xi_{ij}^s(\|X_j^a\|) \Big\} \\ & - \sum_{i=1}^N \mu_{i3}^2(\|X_i^a\|)\|X_i^a\|^2 \\ & + \sum_{i=1}^N \sum_{j=1}^N \mu_{i4}(\|X_i^a\|)\gamma_{ij}(\|X_j^a\|)\|X_i^a\|\|X_j^a\| \\ =& -\frac{1}{2}(\|X_1^a\|, \|X_2^a\|, \ldots, \|X_N^a\|)\,(W^\tau + W) \begin{pmatrix} \|X_1^a\| \\ \|X_2^a\| \\ \vdots \\ \|X_N^a\| \end{pmatrix} \end{aligned}$$

Since the matrix function $W^\tau + W$ in $\Omega_{X^a} \backslash \{0\}$ is positive definite, it follows that V is negative definite in domain Ω_{X^a}. Hence, the results follow. ■

4. DECENTRALISED CONTROL DESIGN

For the nonlinear interconnected system (1)-(2), the corresponding reachability condition is described by (e.g. see Hsu (1997); Yan et al. (2004))

$$\sum_{i=1}^N \frac{\sigma_i^\tau(X_i)\dot{\sigma}_i(X_i)}{\|\sigma_i(X_i)\|} < 0 \tag{28}$$

where $\sigma_i(X_i)$ is defined in (3).

Consider the decentralised control

$$\begin{aligned} U_i =& -\Gamma_i^{-1}(t, X_i)\Big\{ M_{\varphi_i}(X_i^a)F_i^a(t, X_i) + F_i^b(t, X_i) \Big\} \\ & - \Gamma_i^{-1}(t, X_i)\mathrm{sgn}(\sigma_i(X_i))\Big\{ \sum_{j=1}^N \varepsilon_j^{-1}\left(\Xi_{ji}^a(\|X_i\|)\right)^2 \\ & + \sum_{j=1}^N \Xi_{ji}^b(t, X_i) + N\varepsilon_i\beta_i + \|\Gamma_i(t, X_i)\|\rho_i(t, X_i) \\ & + \zeta_i \Big\} \end{aligned} \tag{29}$$

where $\Xi_{ji}^a(\cdot)$, $\Xi_{ji}^b(\cdot)$ and $\rho_i(t, X_i)$ are defined in Assumption 2, ζ_i and ε_j are positive constants which can be considered as design parameters.

Theorem 2. Consider the nonlinear interconnected systems (1) and (2) for $i = 1, 2, \ldots, N$. Under Assumptions 1-2, the closed-loop systems (1)-(2) with the decentralised controls (29) are convergent to the composite sliding surface (11) and maintain a sliding motion on it thereafter.

Proof. From the analysis above, all that needs to be proved is that the composite reachability condition (28) is satisfied. From (3), for $i = 1, 2, \ldots, N$

$$\begin{aligned}\dot{\sigma}_i(X_i) =& M_{\varphi_i}(X_i^a)F_i^a(t, X_i) + F_i^b(t, X_i) \\ &+ M_{\varphi_i}(X_i^a)\sum_{j=1}^{N} H_{ij}^a(t, X_j) + \sum_{j=1}^{N} H_{ij}^b(t, X_j) \\ &+ \Gamma_i(t, X_i)(U_i + \Phi_i(t, X_i)) \end{aligned} \tag{30}$$

Substituting (29) into (30),

$$\begin{aligned}&\sum_{i=1}^{N}\frac{\sigma_i^\tau(X_i)\dot{\sigma}_i(X_i)}{\|\sigma_i(X_i)\|} \\ &=\sum_{i=1}^{N}\Big\{\frac{\sigma_i^\tau(X_i)}{\|\sigma_i(X_i)\|}\Big\{\sum_{j=1}^{N} H_{ij}^b(t, X_j)+M_{\varphi_i}(X_i^a)\sum_{j=1}^{N} H_{ij}^a(t, X_j) \\ &\quad + \Gamma_i(t, X_i)\Phi_i(t, X_i)\Big\} - \|\Gamma_i(t, X_i)\|\rho_i(t, X_i) \\ &\quad -\sum_{j=1}^{N}\varepsilon_j^{-1}\left(\Xi_{ji}^a(\|X_j\|)\right)^2-\sum_{j=1}^{N}\Xi_{ji}^b(t, X_i)-N\varepsilon_i\beta_i-\zeta_i\Big\} \\ &\le \sum_{i=1}^{N}\|\Gamma_i(t, X_i)\Phi_i(t, X_i)\| + \sum_{i=1}^{N}\sum_{j=1}^{N}\|H_{ij}^b(t, X_j)\| \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\sigma_i^\tau(X_i)}{\|\sigma_i(X_i)\|}M_{\varphi_i}(X_i^a)H_{ij}^a(t, X_j) \\ &\quad - \sum_{i=1}^{N}\|\Gamma_i(t, X_i)\|\rho_i(t, X_i) - \sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_j^{-1}\left(\Xi_{ji}^a(\|X_i\|)\right)^2 \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{N}\Xi_{ji}^b(t, X_i) - N\sum_{i=1}^{N}\varepsilon_i\beta_i - \sum_{i=1}^{N}\zeta_i \end{aligned} \tag{31}$$

From the fact that for any positive constant ε (e.g. see Yan et al. (2012)),

$$2W^\tau Z \le \varepsilon W^\tau W + \varepsilon^{-1}Z^\tau Z, \quad \forall W, Z \in \mathcal{R}^l \tag{32}$$

Then, from (9) and (4), it is straightforward to obtain that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\sigma_i^\tau(X_i)}{\|\sigma_i(X_i)\|}M_{\varphi_i}(X_i^a)H_{ij}^a(t, X_j) \\ &\le N\sum_{i=1}^{N}\varepsilon_i\frac{\sigma_i^\tau(X_i)}{\|\sigma_i(X_i)\|}M_{\varphi_i}(X_i^a)\left(M_{\varphi_i}(X_i^a)\right)^\tau\frac{\sigma_i(X_i)}{\|\sigma_i(X_i)\|} \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i^{-1}\left(H_{ij}^a(t, X_j)\right)^\tau H_{ij}^a(t, X_j) \\ &\le N\sum_{i=1}^{N}\varepsilon_i\beta_i\frac{\sigma_i^\tau(X_i)\sigma_i(X_i)}{\|\sigma_i(X_i)\|^2}+\sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i^{-1}\left\|H_{ij}^a(t, X_j)\right\|^2 \\ &= N\sum_{i=1}^{N}\varepsilon_i\beta_i + \sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i^{-1}\left\|H_{ij}^a(t, X_j)\right\|^2 \end{aligned} \tag{33}$$

where ε_i is a positive constant.

Then, from Assumption 2 and identity

$$\sum_{i=1}\sum_{j=1} a_{ij} \equiv \sum_{i=1}\sum_{j=1} a_{ji} \tag{34}$$

it is straightforward to see that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i^{-1}\|H_{ij}^a(t, X_j)\|^2 \\ &\le \sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_i^{-1}\left(\Xi_{ij}^a(\|X_j\|)\right)^2 \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\varepsilon_j^{-1}\left(\Xi_{ji}^a(\|X_i\|)\right)^2 \end{aligned} \tag{35}$$

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}\|H_{ij}^b(t, X_j)\| \\ &\le \sum_{i=1}^{N}\sum_{j=1}^{N}\Xi_{ij}^b(t, X_j) \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\Xi_{ji}^b(t, X_i) \end{aligned} \tag{36}$$

$$\begin{aligned}\|\Gamma_i(t, X_i)\Phi_i(t, X_i)\| &\le \|\Gamma_i(t, X_i)\|\|\Phi_i(t, X_i)\| \\ &\le \|\Gamma_i(t, X_i)\|\rho_i(t, X_i) \end{aligned} \tag{37}$$

Then from inequalities (33)-(37), it is straightforward to verify that

$$\sum_{i=1}^{N}\frac{\sigma_i(X_i)^\tau\dot{\sigma}_i(X_i)}{\|\sigma_i(X_i)\|} \le -\sum_{i=1}^{N}\zeta_i < 0 \tag{38}$$

Hence, the result follows. ■

5. NUMERICAL SIMULATION

Consider the following nonlinear interconnected system composed of three subsystems described by

$$\begin{aligned}\dot{X}_1^a =& \underbrace{-0.6X_{11}\cos(X_{12}) + X_{13}}_{F_1^a(t,X_1)} + \sum_{j=1}^{2} H_{1j}^a(t, X_j) \\ &+ \underbrace{\begin{bmatrix} 0 & \sin(X_{12}) \end{bmatrix}}_{G_1^a(t,X_1)}(U_1 + \Phi_1(t, X_1)) \end{aligned} \tag{39}$$

$$\begin{aligned}\dot{X}_1^b =& \underbrace{\begin{bmatrix} 0.2X_{11}X_{13} \\ -X_{11} + 0.8X_{13}\cos(X_{12}) \end{bmatrix}}_{F_1^b(t,X_1)} + \sum_{j=1}^{2} H_{1j}^b(t, X_j) \\ &+ \underbrace{\begin{bmatrix} 1 & X_{13} \\ 0 & 1 \end{bmatrix}}_{G_1^b(t,X_1)}(U_1 + \Phi_1(t, X_1)) \end{aligned} \tag{40}$$

$$\dot{X}_2^a = \underbrace{-0.5X_{21}\cos(X_{22}) + X_{23}}_{F_2^a(t,X_2)} + \sum_{j=1}^{2} H_{2j}^a(t, X_j)$$

$$+\underbrace{\left[\,0\ \sin(X_{22})\,\right]}_{G_2^a(t,X_2)}(U_2+\Phi_2(t,X_2)) \tag{41}$$

$$\dot{X}_2^b=\underbrace{\begin{bmatrix}0.4X_{21}X_{23}\\ -0.8X_{21}+0.8X_{23}\cos(X_{22})\end{bmatrix}}_{F_2^b(t,X_2)}+\sum_{j=1}^{2}H_{2j}^b(t,X_j)$$

$$+\underbrace{\begin{bmatrix}1 & X_{23}\\ 0 & 1\end{bmatrix}}_{G_2^b(t,X_2)}(U_2+\Phi_2(t,X_2)) \tag{42}$$

$$\dot{X}_3^a=\underbrace{-0.7X_{31}\cos(X_{32})+X_{33}}_{F_3^a(t,X_3)}+\sum_{j=1}^{2}H_{3j}^a(t,X_j)$$

$$+\underbrace{\left[\,0\ \sin(X_{32})\,\right]}_{G_3^a(t,X_3)}(U_3+\Phi_3(t,X_3)) \tag{43}$$

$$\dot{X}_3^b=\underbrace{\begin{bmatrix}0.2X_{31}X_{33}\\ -X_{31}+0.9X_{33}\cos(X_{32})\end{bmatrix}}_{F_3^b(t,X_3)}+\sum_{j=1}^{2}H_{3j}^b(t,X_j)$$

$$+\underbrace{\begin{bmatrix}1 & X_{33}\\ 0 & 1\end{bmatrix}}_{G_3^b(t,X_3)}(U_3+\Phi_3(t,X_3)) \tag{44}$$

where $X_{i1} := X_i^a$, $\mathrm{col}(X_{i2}, X_{i3}) := X_i^b$ for $i = 1, 2, 3$. Assume the matched uncertainties satisfy

$$\|\Phi_1(t,X_1)\|\le\underbrace{0.24\sqrt{X_{13}^2+1}}_{\rho_1(t,X_1)}$$

$$\|\Phi_2(t,X_2)\|\le\underbrace{0.16\sqrt{X_{23}^2+1}}_{\rho_2(t,X_2)}$$

$$\|\Phi_3(t,X_3)\|\le\underbrace{0.18\sqrt{X_{33}^2+1}}_{\rho_2(t,X_3)}$$

Assume the bounds of the interconnections satisfy

$$\sum_{j=1}^{2}\|H_{1j}^a(t,X_j)\|\le\underbrace{0.72|\cos(X_{12})|\|X_1\|}_{\Xi_{11}^a(\|X_1\|)}+\underbrace{0.5\|X_2\|}_{\Xi_{12}^a(\|X_2\|)}+\underbrace{0.64\|X_3\|}_{\Xi_{13}^a(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{1j}^b(t,X_j)\|\le\underbrace{0.83\|X_1\|}_{\Xi_{11}^b(\|X_1\|)}+\underbrace{1.01\|X_2\|}_{\Xi_{12}^b(\|X_2\|)}+\underbrace{0.68\|X_3\|}_{\Xi_{13}^b(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{2j}^a(t,X_j)\|\le\underbrace{0.5\|X_1\|}_{\Xi_{21}^a(\|X_1\|)}+\underbrace{0.78|\cos(X_{22})|\|X_2\|}_{\Xi_{22}^a(\|X_2\|)}+\underbrace{0.58\|X_3\|}_{\Xi_{23}^a(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{2j}^b(t,X_j)\|\le\underbrace{0.63\|X_1\|}_{\Xi_{21}^b(\|X_1\|)}+\underbrace{0.63\|X_2\|}_{\Xi_{22}^b(\|X_2\|)}+\underbrace{0.42\|X_3\|}_{\Xi_{23}^b(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{3j}^a(t,X_j)\|\le\underbrace{0.64\|X_1\|}_{\Xi_{31}^a(\|X_1\|)}+\underbrace{0.78\|X_2\|}_{\Xi_{32}^a(\|X_2\|)}+\underbrace{0.64|\cos(X_{33})|\|X_3\|}_{\Xi_{32}^a(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{3j}^b(t,X_j)\|\le\underbrace{0.64\|X_1\|}_{\Xi_{31}^b(\|X_1\|)}+\underbrace{0.65\|X_2\|}_{\Xi_{32}^b(\|X_2\|)}+\underbrace{0.73\|X_3\|}_{\Xi_{33}^b(\|X_3\|)}$$

Now define the sliding function in the form of (3) with

$$\varphi_i(X_i^a)=\begin{bmatrix}0\\ \frac{\sqrt{\beta_i c_i}X_{11}}{\sqrt{c_i+X_{11}^2}}\end{bmatrix}\quad i=1,2,3$$

where the design parameters β_i and c_i are chosen as $\beta_i = 1$ and $c_i = 0.25$. Then it is straightforward to verify that

$$M_{\varphi_i}(X_i^a)M_{\varphi_i}(X_i^a)^\tau=\begin{bmatrix}0 & 0\\ 0 & \frac{\beta_i c_i^3}{(c_i+X_{11}^2)^3}\end{bmatrix}\le\beta_i I_2$$

From Lemma 1, when the sliding motion takes place,

$$\sum_{j=1}^{2}\|H_{1j}^s(t,X_j^a)\|\le\underbrace{0.51\|X_1^a\|}_{\Xi_{11}^s(\|X_1\|)}+\underbrace{0.35\|X_2^a\|}_{\Xi_{12}^s(\|X_2\|)}+\underbrace{0.45\|X_3^a\|}_{\Xi_{13}^s(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{2j}^s(t,X_j^a)\|\le\underbrace{0.35\|X_1^a\|}_{\Xi_{21}^s(\|X_1\|)}+\underbrace{0.55\|X_2^a\|}_{\Xi_{22}^s(\|X_2\|)}+\underbrace{0.41\|X_3^a\|}_{\Xi_{23}^s(\|X_3\|)}$$

$$\sum_{j=1}^{2}\|H_{3j}^s(t,X_j^a)\|\le\underbrace{0.45\|X_1^a\|}_{\Xi_{31}^s(\|X_1\|)}+\underbrace{0.55\|X_2^a\|}_{\Xi_{32}^s(\|X_2\|)}+\underbrace{0.45\|X_3^a\|}_{\Xi_{33}^s(\|X_3\|)}$$

Choose the Lyapunov function candidate

$$V=\sum_{i=1}^{3}V_i \tag{45}$$

where

$$V_i=\frac{1}{2}(X_i^a)^\tau X_i^a,\quad i=1,2,3$$

Then,

$$\underbrace{0.4\|X_i^a\|^2}_{\varsigma_{i1}}\le V_i(t,X_i^a)\le 0.6\underbrace{\|X_i^a\|^2}_{\varsigma_{i2}}$$

Define $\varsigma_{i3}(\cdot)$ for $i = 1, 2, 3$ as

$$\varsigma_{13}(r)=\underbrace{0.6}_{\mu_{13}}r,\quad \varsigma_{23}(r)=\underbrace{0.5}_{\mu_{23}}r,\quad \varsigma_{33}(r)=\underbrace{0.7}_{\mu_{33}}r$$

and $\varsigma_{i4}(\cdot)$ as

$$\varsigma_{i4}(r)=\underbrace{1}_{\mu_{i4}}\cdot r,\quad i=1,2,3$$

By direct computation, it is straightforward to verify that

$$W(t,X)+(W(t,X))^\tau>0$$

with

$$\begin{aligned}&\gamma_{11}(\|X_1\|)=0.51,\ \gamma_{12}(\|X_2\|)=0.35,\ \gamma_{13}(\|X_3\|)=0.45\\ &\gamma_{21}(\|X_1\|)=0.35,\ \gamma_{22}(\|X_2\|)=0.55,\ \gamma_{23}(\|X_3\|)=0.41\\ &\gamma_{31}(\|X_1\|)=0.45,\ \gamma_{32}(\|X_2\|)=0.55,\ \gamma_{33}(\|X_3\|)=0.45\end{aligned}$$

Thus the designed sliding modes are asymptotically stable.

From (29), the controllers U_i are well defined with $\zeta_i = 1$ and $\varepsilon_i = 0.5$ for $i = 1, 2, 3$, which guarantee that the condition (28) is satisfied for $X_i \in \mathcal{R}^3$, $i = 1, 2, 3$. Thus systems (39)-(42) for $i = 1, 2, 3$ can be stabilised by the designed controls U_i proposed in (29).

The time response of the system states is shown in Fig.1. The simulation results show that the proposed approach is effective. It should be noted that in the simulation, a boundary layer is used to remove the chattering.

6. CONCLUSION

This paper has proposed a robust decentralised sliding mode control design approach for a class of nonlinear systems in generalised regular form with uncertain interconnections. The bounds on the uncertainties are assumed to be known functions which have been used to enhance robustness to uncertainties. Sliding mode controllers are designed to reduce the effects of the interconnections on the entire system. The developed results can be applied to the interconnected systems which can be transformed to the generalised regular form described in (1)-(2). A numerical example is given to show how to use the sliding mode technique to stabilise a system with uncertainty interconnections. Simulations have been presented to demonstrate the effectiveness of the approach.

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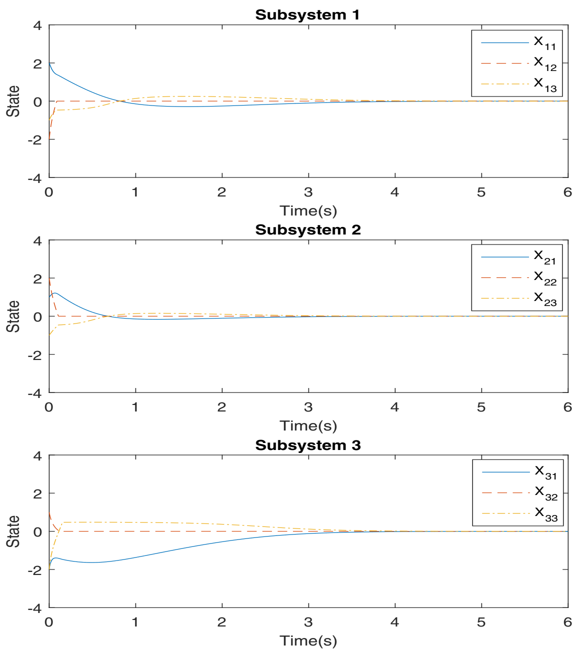

Fig. 1. Time response of the state variables of system (39)-(44).

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