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Study of the Banking System's Stability Using Control Theory

A Thesis Submitted to the
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By

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Abstract

The 2007-2009 financial meltdown reflected the failure of the regulators to address financial fragility and it has clearly showed that regulating banks on an individual basis was an ineffective approach to prevent financial crises. Before the crisis, financial regulation was primarily focused on managing the risk of individual banks by requiring them to keep sufficient reserves to safeguard themselves from the inherent risk of their own investments. Since they ignored the risks that are generated by links between the banks, i.e. interbank borrowing and lending, a failure in a small number of banks could spread to other banks, and cause the paralysis of the whole banking system. Therefore, there is the need to give special emphasis to systemic risk, rather than consider the risk at an individual level. From an academic research point of view, the 2007-2009 financial crisis renewed the interest in finding new ways of studying financial systems. More specifically, since then new modelling frameworks have been proposed that incorporate the interconnected nature of the banking system. Network models have been used to investigate the stability of the banking system under different conditions, e.g. different banks' size and connectivity. This thesis proposes a new dynamic network model based on ordinary differential equations, which represents the banking system and seeks to interface the network model approach with control engineering. Control theory is an interdisciplinary branch of engineering, which is used to study the behaviour of dynamical systems, and how their behaviour can be modified by feedback mechanisms to achieve a desirable performance. In this work control theory is applied for the first time to analyse a model of the banking system and to propose feedback mechanisms, which preserve the stability of the system and that can ultimately inform financial regulators.

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Nomenclature

ODE	Ordinary Differential Equations
D_i	Deposit of $bank_i$
C_i	Cash of $bank_i$
I_i	Investment of $bank_i$
B_{ij}	Borrowing $bank_i$ borrows from $bank_j$
L_{ij}	Lending $bank_i$ lends to $bank_j$
opp_i	Investment opportunity of $bank_i$
g	Deposit interest rate
p	Return rate of the investment
w	Proportion of the total investment that matured
v	Proportion of the total investment that failed
h_{ij}	Lending interest rate $bank_i$ claims from $bank_j$
k_{ij}	Borrowing interest rate $bank_i$ pays to $bank_j$
α_{ij}	Proportion of the total borrowing that $bank_i$ repays to $bank_j$
h_0	Basic lending interest rate
a	Premium of the lending interest rate
z	The speed of interest rate transition between the states $h_{ij} = h_0 + \frac{a}{2}$ and $h_{ij} = h_0 + a$
y	Threshold used to quantify the 'health' of the borrowing bank
r	Reserve ratio
l_r	Link rate
$\boldsymbol{x}$	State variable
$\boldsymbol{u}$	Control variable
$\boldsymbol{d}$	Disturbance
$\boldsymbol{A}$	State matrix of the linear model
$\boldsymbol{B}$	Control input distribution
γ	Feedback gain
r_e	Reference input

Chapter 1

Introduction

1.1 Background and motivation

The financial crisis that occurred in 2007-2009 has driven financial regulators (e.g. central banks) and researchers to revisit ways to understand and regulate the banking system [1–4]. Traditionally, financial regulation was primarily focused on managing the risk of individual banks by requiring them to keep sufficient reserves to safeguard themselves from the inherent risk of their own investments. Since the systemic risks due to links between the banks (e.g. interbank borrowing and lending) are ignored, a failure in a small number of banks can spread to other banks, and cause the paralysis of the whole banking system. Thus, it has been understood that to improve financial stability through regulation, more attention should be given to systemic risk [5], rather than only to individual institutions [6-8]. The necessity for managing systemic risk has persuaded financial operators to think about new regulatory approaches that recognize the interconnected nature of the banking system.

The motivation of this thesis is to try to find new ways to model and analysis the dynamics of the banking system. The network model proposed in this thesis is based on ordinary differential equations and it incorporates the interconnected nature of the system; the model also facilitates the application of engineering control theory, which can suggest ways to reduce the occurrence of systemic failure. This thesis presents a novel and interdisciplinary research at the interface between economics and control engineering, which aims to provide tools to better understand the banking system's dynamics and the corresponding systemic risk.

A network is widely understood as a collection of nodes connected by links. Systems taking the form of networks abound in the world, such as the Internet, social networks, and biological networks. Researchers have developed a variety of techniques and models to help us understand and predict the behaviour of these systems [9]. Network models have been applied in different areas such as the Internet, epidemiology, ecosystems and financial markets [10]. The banking system exhibits a high degree of interdependence, with connections between different banks stemming from both the asset and the liability sides of their balance sheets. As such, the banking system can be modelled as a network where nodes are individual banks and edges are the loans between any two banks. The 2007-2009 financial crisis has drawn a sharply increase in the academic research which has used network models to study financial systems and to propose ways to preserve financial stability [11–14]. To the best of our knowledge, no research work in the literature has proposed network models of the banking system based on ordinary differential equations, and importantly, none of them has applied engineering control theory on their models.

Control theory is an interdisciplinary field of applied mathematics and engineering [15], a major application of which is in the engineering discipline known as control engineering, which deals with the design of control systems for industry. "Controlling a system" means to influence the behaviour of the system in order to achieve a desired goal. Feedback is a key concept in control theory, and a feedback process is the one in which the state of the system determines the way the control has to be exerted at any time. Nowadays, as the understanding of the dynamics of business, social, and political systems increases, control engineering is not limited to only engineering discipline but is equally applicable to these systems. As the general theory of feedback systems, control theory is useful wherever feedback occurs [16]. Therefore, in this thesis, we propose to apply control theory to a model of the banking system, with the aim to study its stability and try to control it around equilibrium.

This thesis develops a new dynamical network model, in which the banking system is represented as a network where the nodes are individual banks and the links between any two banks consist of interbank loans and borrowing. The dynamic structure of the model is represented as a set of ordinary differential equations consisting of balance sheet dynamics. This dynamic structure not only allows us to analyse systemic risk but also to incorporate an analysis of control mechanisms.

This thesis also provides details of the implementation of the dynamic network model in MATLAB Simulink, which is a powerful tool for simulating and analysing efficiently the solutions of complicated systems modelled with differential equations; moreover, Simulink is widely used in control theory for simulation and design. In the simulations performed in this thesis, shocks are introduced into the system via deposit fluctuations of the banks in the system. In order to study the stability of the banking system, the number of survival banks at the end of the simulation period is calculated and compared in different scenarios characterised by different values for the rate of connectivity, reserve ratio, the amplitude of the shock and the heterogeneity in bank's size. In this work, a measure of contagion is also proposed to study how the failure of a bank can affect the failure of other banks under different scenarios; this thesis shows interesting nonlinear effects on contagion due to the rate of connectivity and reserve ratio.

This thesis presents also for the first time the application of control mechanisms on the model of the banking system. Classical control theory is used to study the stability of a system and subsequently an output feedback control is designed to improve the stability of the system. In order to achieve this, an equilibrium point analysis is performed on the mathematical model representing the system, to gain an insight of how different parameter values affect the model's stability. Proper control mechanisms are designed according to the different system dynamics to achieve desired objectives. This work proposes feedback mechanisms in which single banks sell their assets to avoid failure; the novelty of the proposed approach is to sell assets according to rigorous control laws, which allow the bank to regain and maintain a stable condition.

1.2 Contributions

This thesis contributes to the knowledge and research in both network models and control theory applied to the banking system. The novelty of the approach consists in developing a model of the banking system based on differential equations and in applying control analysis to study its stability.

The thesis contributes the following three main results:

1. Development of a dynamical network model of the banking system: a new dynamic model based on a system of ordinary differential equations is developed to describe the banking system as a network where the nodes are the individual banks. This model has been used to study how different parameters (e.g. reserve ratio, connectivity, bank's size) affect the stability of the banking system. In particular, this work has found interesting nonlinear effects of the reserve ratio and connectivity on the spread of failure within the banking system.

2. Development of Simulink block diagrams of the dynamic network model: the proposed dynamical network model is implemented in the simulation environment MATLAB Simulink which simulates and analyses dynamical systems. Simulink was chosen because of its block structure, which allows adding complexity to the system in a visual and modular way; moreover, it facilitates the application of control analysis, given that Simulink is the software mostly used by the control theory community.

3. Application of control theory analysis: Control theory is applied for the first time to assess and to preserve the stability of the proposed dynamic model representing the banking system. Output feedback control mechanisms are designed in which single banks sell their assets to prevent bankruptcy; the novelty of the approach presented in this thesis lies in the way banks sell their assets; the sale of assets is prescribed by specific control mechanisms, which allow the bank to resume and maintain a stable condition.

The work in this thesis has produced the following articles and presentations:

Papers

- 21st International Conference on Computing in Economics and Finance
  June 20-22, 2015, Taipei, Taiwan
  Paper entitle: A dynamic network model of banking system stability
- 5th International Conference of the Financial Engineering and Banking Society
  June 11- 13, 2015, Nantes, France
  Paper entitle: Study of banking system stability using differential equations

This paper received the Best Paper Award, offered by LabEx ReFi - a European research facility dedicated to the evaluation of financial policies regulation. More than 240 papers were submitted to the conference.

- Paper entitle: Study of the Banking System's Stability Using Control Theory
  This paper is in preparation for the Journal of Financial Stability

Talks/Presentations

- Presentation at the Research Group Seminar 2013.
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- Presentation at the Research Group Seminar 2014 at University of Kent
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Courses Attended

- 2014 Summer Workshop Banking Stability: Lessons from the Global Financial Crisis 2 July 2014 in London.

1.3 Organization of the thesis

The thesis is structured as follows:

Chapter 2 provides a summary of the existing literature on the study of stability of the banking system. An introduction to financial systems is provided and work on how financial crises occur and spread within the banking system are reviewed. Next, the chapter introduces some network models developed by academic researchers to describe the banking system and assess the systemic risk within it. The chapter also introduces the control theory as well as the existing work in the literature that applies control theory to financial problems.

Chapter 3 introduces the dynamic network model of the banking system which has been developed in this Ph.D. project. The banking system is represented as a network where nodes are individual banks and the links between any two banks consist of interbank loans and borrowing; the dynamic nature of the system is prescribed by a set of ordinary differential equations representing the balance sheet dynamics of the banks. The dynamic model is presented in the chapter step by step, from a one-bank model to a two-bank model and finally to a multi-bank model.

Chapter 4 presents the numerical simulation results of the dynamical network model which is introduced in Chapter 3. These results are generated using MATLAB Simulink; details of the implementation are provided in the first subsection in this chapter, followed by three subsections that show the numerical simulation results of the one-bank model, the two-bank model and the multi-bank model respectively.

Chapter 5 presents the details of the application and analysis of control mechanisms on the one-bank model. Classical control theory is used to study the stability of the dynamic model and subsequently output feedback control is designed to improve the stability of the model.

Chapter 6 summarises the main conclusions and provides the discussion for future work.

Chapter 2

Literature Review

This chapter provides a summary of the existing literature related to this Ph.D. project. Due to the interdisciplinarity of the topic, the literature is reviewed from three perspectives. Firstly, a general overview of financial crises and contagion is provided. Secondly, network models are introduced. Finally, control theory is introduced and discussed. This chapter is organised as follows: Section 2.1 presents an introduction of financial systems and the contagion that characterised the 2007-2009 financial crisis. This section reviews the function of financial systems and the way the financial crisis evolved in a systemic risk crisis, which shows how the interconnections within financial systems facilitate risk sharing but also are the vehicle for transmission of the systemic risk; thus studying the role of the interconnectedness within financial systems is therefore becoming more and more important. Section 2.2 reviews the literature about network models which focus on studying financial contagion within the banking systems and how the network structure of the banking system affects and responds to crisis. Section 2.2 shows that the network model can be instrumental in capturing and analysing systemic risk. Section 2.3 introduces the concept of feedback in control engineering and reviews some of its applications to economic issues, which show that the feedback control can be a possible tool to analyse and control the stability of the banking system. Finally, Section 2.4 concludes the chapter.

2.1 Financial systems and contagion

The following subsection 2.1.1 introduces some general concepts about financial systems and then describes, specifically, how the banking system works; this helps in building the foundations for developing the structure of the dynamic model proposed in this project. Subsection 2.1.2 reports some historical examples of financial crises and the concept of systemic risk to illustrate contagion mechanisms that acted in the banking system. Those historical examples provide ideas on how to interrogate the proposed model.

2.1.1 Financial systems

A financial system can be described as a structural interconnected network of financial markets, financial intermediaries and financial instruments [17]. The key role of the financial system is to channel the funds from units who have surplus of funds (called lender-savers e.g. household) to units who have a shortage of funds (called borrower-savers e.g. firms and government) [18]. Well-functioning financial systems can improve the efficiency of the circulation of funds in the economy and help the economy growing sustainably and stably [19, 20].

Figure 2.1 schematically shows how the funds are channelled. The lender-savers are shown at the left of figure 2.1 and the borrower-savers are shown at the right. Funds can be transferred through two routes. One is the direct route (the route at the bottom of figure 2.1), in which the borrowers borrow funds directly from lenders in financial markets by selling lenders financial instruments. Financial instruments (which can also be called securities) are monetary contracts between parties that can be traded in the financial market [21]. It represents the claims on the borrower's future income or assets. Financial instruments can be treated as assets for the person who buys them, but as liabilities for the individual or firm that sells (issues) them [22].For example, if a firm need to borrow funds to expand its business, it might borrow the funds from

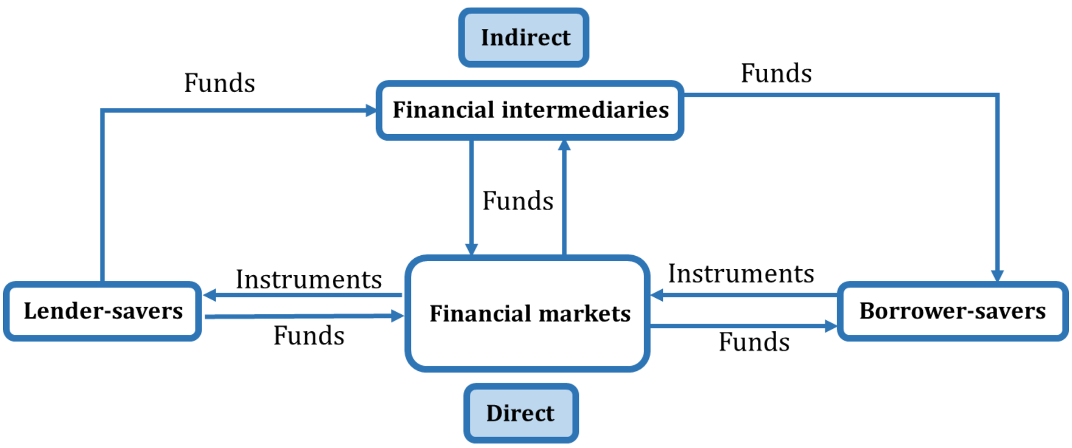

Figure 2.1 Direct and indirect route of funds flow from lender-savers to borrower-savers in a financial system [23].

households by selling them a bond which is a debt security that promises to make non-contingent payments to the households periodically for a specified period of time; or by selling a stock which is a security that entitles the households to a share of the company's profits and assets (and hence is contingent as it depends on the state of the world).

Another route to transfer funds is the indirect route (at the top of figure 2.1) in which funds are transferred through financial intermediaries. Financial intermediaries are institutions who specialise in the activities of buying and selling financial instruments (i.e. bonds and stocks) in order to help individuals and firms to transfer funds. These institutions include banks (which is the major component) as well as other institutions such as building societies, credit unions, insurance companies and so on; these institutions may be named differently in different countries. In the United Kingdom, these institutions form the banking system which mainly comprises commercial banks, investment banks and building societies. They provide different types of services shown as follows.

A *commercial bank* attracts deposits by paying the depositors interest and then lends those deposits out to individuals and firms with a charge that is greater than the interest the bank pays to the depositors. In this way, the bank can not only earn profits to support its activities but also, more importantly, facilitate the transfer of resources to places where it is needed. An *investment bank* helps governments or firms raise financial capital by issuing securities [23]. First, it advises the firms on which type of securities to issue (stocks or bonds); then it helps sell (also called underwrite) the securities by purchasing them from the firms at a predetermined price and reselling them in the market. The investment bank then bears the risk that they are not able to resell the entire issue in which case it will hold the unsold securities by itself. In return for managing this risk, the investment company receives an underwriting fee from the issuing firms. Investment banks also act as deal makers and earn enormous fees by helping firms acquire other firms through mergers or acquisitions. In the UK, banks are allowed to engage in both commercial banking and investment banking, and such banks are called universal banks[1]. A *building society* is a financial institution that was

[1] Four big clearing banks (call 'big four') currently dominate commercial banking in UK: Barclays, Royal Bank of Scotland (RBS), HSBC, and Lloyds. They are essentially universal banks as they also help firms to issue securities.

originally constrained in their activities to provide residential mortgage loan (which will be introduced later) to individuals by acquiring funds primarily through deposits [23]. Over time, these restrictions have been loosened so that they expand their activities into traditional banking; as a result, the distinction between building societies and commercial banks has blurred and building societies are now competitors of the universal banks.

Not only the households and firms lend or borrow funds through financial market, financial institutions such as banks also lend or borrow funds in the financial market shown in figure 2.1. The market in which banks exchange short-term loans to one another is called interbank market [24]. Sometimes a commercial bank cannot meet the reserve ratio set by the central bank, while other commercial banks have excess cash above the reserve requirement. These banks will lend money in the interbank market, receiving interest from the borrowing bank. Most interbank loans have maturities of one week or less; the majority being overnight. The rate of interest charged on the loans between banks is called interbank rate. The published interbank rate in the UK is the LIBOR (London Inter Bank Offered Rate) [25], which is the average of interest rates estimated by each of the leading banks in London.

Given the functions of the financial market and financial intermediates, the banks exist because of asymmetric information in financial markets [26]. Banks specialise in monitoring and screening the information, which gives them a comparative advantage in helping individuals to reduce the transaction costs; this allows small savers and borrowers to benefit from the existence of financial markets. Moreover, banks can help agents when they need liquidity. As explained by Diamond and Dybvig [27], in fact, banks are institutions that facilitate risk sharing, by pooling deposits from a large customer base and using these in a diversified portfolio of investment projects. Importantly, deposits are liquid, in the sense that they can be easily withdrawn. Because consumption needs are uncertain, customers prefer liquid deposits instead of directly financing businesses. Business investment projects are illiquid, because they cannot be withdrawn at any point without a loss. Banks thus transform illiquid investment opportunities into liquid deposits that provide insurance against unexpected future events.

Moreover, risk-transfer between banks also has strong economic motivation since banks seek to transfer risk as part of their day-to-day business. For example, two banks might have, respectively, surplus and deficit liquidity; these banks might agree an unsecured loan or a repurchase agreement. These interbank activities cause a series of bilateral transactions, through which banks are highly interconnected. Interbank market facilitates a growing volume of bilateral transactions, as individual banks constantly optimise their own risk exposures, but at the same time, it provides more channels for default to spread. The next section reviews the contagion of defaults in the banking system.

2.1.2 Financial crisis and contagion

In normal times, banking systems can help the economic growth by funding investment opportunities and to reduce volatility in the financial markets by facilitating risk sharing. However, during financial crises, the banking system itself can become a vehicle for amplifying and spread financial shocks [28]. The 2007-2009 financial crisis showed the fragility of the banking system and its origins were many and varied; one of the most important was the underestimation of the true aggregate risk on banks' balance sheet positions. In the following subsections, the subprime mortgage crisis which contributes to the crisis of 2007-2009 is introduced in more detail.

The subprime mortgage crisis

The immediate cause of the crisis in 2007-2009 was the large portion of the increased mortgage loan defaults which are also referred to as 'sub-prime' loans [29]. Traditionally, banks provided loans to people for house purchase via mortgages, as shown in figure 2.2.

Figure 2.2 Funding chain showing the funding flow from the household savers to households who obtain a mortgage to buy a house. The arrows indicate the direction of payments due.

Since house prices were increasing before the crisis, such mortgage loans were thought to be secured against saleable real-estate and can make good profits for the banks. Therefore, these mortgage loans had good liquidity in the mortgage market due to their 'high safety level'; to make more profit the banks sold these mortgage loans at a higher price to other banks and financial institutions through derivatives. Thus many derivatives were generated to trade the mortgage loans; one example is the mortgage-backed security, which is secured by a mortgage or collection of mortgages.

Figure 2.3 shows one possible funding chain (adapted from Glasserman and Young [30]) representing how the mortgage-backed security is generated and traded in the financial market.

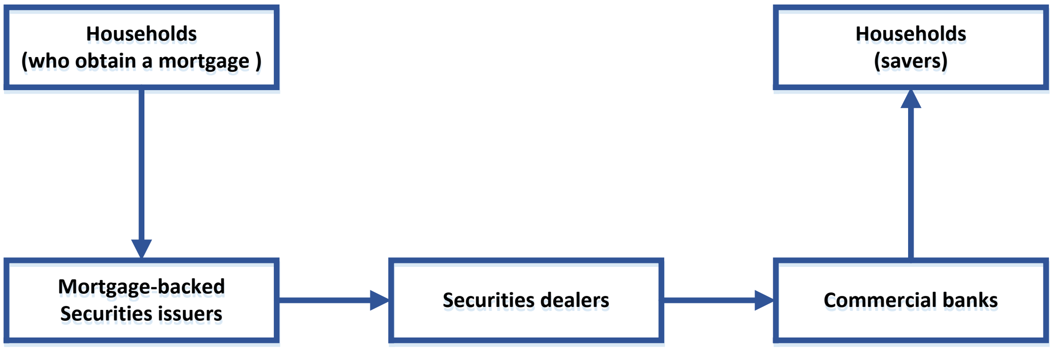

Figure 2.3 Funding chain showing the funding flow from the ultimate creditors (household savers) to the ultimate debtors (households who obtain a mortgage to buy a house). The arrows indicate the direction of payments due, funding flows clockwise through the chain. Adapt from Glasserman and Young [30] and Shin 2010 [31].

Starting from the left side of figure 2.3, households that want to buy houses take on mortgage debts from banks or financial institutions (called mortgage-backed securities issuer). This kind of banks or institutions issue and sell the mortgage-backed securities to securities dealers (usually these are investment banks) in order to get funding for the household. The dealers (investment banks) then pledge the securities as collateral to borrow from commercial banks to get funding for the mortgage-backed securities issuer. Commercial banks fund themselves by taking deposits from household savers. Before the financial crisis, banks generated huge profits by selling the mortgage-backed securities at a higher price to the next buyer in the chain. Therefore, the banks decided to expand their lending by making easy access loans (sub-prime mortgages) for borrowers who have a poor credit record. In this way, banks attracted more mortgage loans to issue more mortgage-backed securities.

However, this lending expansion generated high risk. At each step in the chain, there is a potential loss of information about the quality of the underlying debt. Banks increase their lending to households with poor credits while the supply of real assets from the household savers remains relatively fixed. From the perspective of an individual bank, the risk from the households to default was sold together with the mortgage-backed securities to the next bank or financial institution in the chain. As a result, the individual bank's risk correspondingly reduced while the risk to the whole banking system remained the same. This is because the total value of assets multiplied, whilst the collateral backing these securities remained relatively fixed. This collateral scarcity increased the risk of a sudden house price drop and consequent mortgage defaults. Ultimately, a large portion of the mortgage loan and mortgage-backed securities holders (banks) defaulted because of the poor credit of the borrowers (householders).

As shown in figure 2.3, defaults in mortgage loans can affect commercial banks through the chain. Defaults in mortgage loan repayments can cause significant losses in the commercial banks' assets that can drive the banks to fail. The idiosyncratic default of a bank on its interbank liabilities can spread and cause losses among other banks. The losses in other banks may result in further defaults. Besides the direct loss caused by the repayment defaults of the household, the value of the mortgage-backed securities decreases and the interbank interest rate increases. These changes in the prices affect the banks' strategy to allocate resources. The banks may change the composition of their assets and liabilities in response to economic stress, in order to safeguard themselves from the losses due to stress. For example, to secure its own assets from loss, banks may cut lending in the interbank market, but this may result in further banks' failure due to lack of funding supply which may further increase funding cutting in the interbank market. This is known as *liquidity hoarding*. Due to liquidity hoarding, the bank may face liquidity shortage. To pay its obligations a bank may sell its illiquid assets at heavily discounted prices, which is known as *fire sales*. This causes the decrease in asset prices and mark-to-market losses (a loss generated through an accounting entry rather than the actual sale of a security) for other banks which hold the same assets; the affected banks may face a liquidity shortage due to the mark-to-market losses and need to sell their illiquid assets to pay their obligations, which cause further decrease in asset prices. Finally, a bank may default due to the big loss in its assets. It can be seen that these banks' behavioural responses might be rational and favourable for the individual bank, but together they increase the instability of the banking system and therefore they may trigger and amplify systemic crises. The banking system as a whole becomes distressed and unable to perform its intermediation and insurance functions. In this scenario, the risk generated in an individual bank can be amplified through the connectivity between banks thus to become systemic risk. Next section presents relevant literature, which focuses on the use of network models to study how the banking system's stability is affected by the structure of its interconnection and the banks' dynamical behaviours (such as liquidity hoarding, fire sale or both[2]).

This thesis mainly focuses on the commercial activities of the banks and, specifically, on their interbank borrowing and lending. The dynamic model developed in this Ph.D. project describes a banking system rather than the entire financial system; the model contains only banks in it and the links between the banks are formed of interbank borrowing and lending. The behaviour of depositors is implemented in a simplistic manner in our model by using a stochastic signal which represents the amount of money deposited into or withdrawn from a bank at a given time. The financial markets, as well, are modelled in a simplistic manner by a stochastic signal called investment opportunity that provides the amount of money that a bank can invest in a given time (see Chapter 3 for details).

2.2 Network models of the banking system

The complexity and the instability characterising financial systems played a significant role in the 2007-2009 financial crisis; the recent focus on macro-prudential regulation[3] is a direct response to the inherent instability of the complex financial systems. New analytical methods for studying the effect of the interconnectedness in financial systems have been developed by academics [39]. This thesis is focused specifically on network model analysis; existing work using network models to study financial systemic risk is quite diverse and also fast-growing [40]. Two seminal papers by Allen and Gale [41] and Freixas et al. [42] evaluate the potential for contagion following a common or idiosyncratic liquidity or solvency shock, showing that to be able to assess systemic stability, it is important to understand the financial systems' structure. This section presents the literature that mainly focuses on studying the

[2] The model developed in this Ph.D. project doesn't consider the case of fire sale, but an appropriate redefinition of the equations can be introduced to account for this behaviour.
[3] There is a rich literature about the macro-prudential regulation. See [32-38] for details.

systemic risk on the interbank market[4]. Papers on this topic usually carry out the network model analysis using one of the following two distinct approaches; the first approach is called static network analysis and the second approach is called dynamic network analysis [43]. Static network analysis uses topological indicators to describe the network structure (i.e. degree, strength, density); this kind of analysis does not include a mechanism by which shocks are transmitted. For this reason, it is referred to as static network analysis and papers that used this analysis are introduced in subsection 2.2.1. The second approach, introduced in subsection 2.2.2, focuses, instead, on modelling how different network structures react to shocks, in order to identify the key factors that affect the resilience of the network. This approach usually involves a dynamic simulation to model how the default spread in the network, thus it referred as dynamic network analysis.

2.2.1 Static network analysis

The topology of a network affects its functionality and stability, therefore general-network-theory methods [44] can be applied to analysis the network representing a financial system. The stability of the interbank market is important for the proper functioning of modern financial systems and the market structure may play an important role in determining the risk of contagion; by studying the network metrics, the information about the stability of the network structure can be gained which helps analysts to identify central nodes which are more likely to propagate shocks.

One line of the static network research falls in capturing and analysing the interconnectedness in the interbank market based on empirical data. Some common network metrics such as degree, strength, density, centrality and clustering can be used to analyse the interbank market, which helps to identify the characteristics of the interbank markets. The findings show that the network of interbank market has the complex characteristics of small-world network and scale-free network[5] [45]; this means there are often a small number of highly connected large nodes(banks) in the

[4] Network models are also widely used to study the different kinds of risks (such as credit risk, systemic risk and liquidity risk) on different financial market (such as banking market, interbank market and CDS market), see the paper [14] for a review.

[5] A small-world network is a network in which most nodes are not neighbours of one another, but the neighbours of any given node are likely to be neighbours of each other and most nodes can be reached from every other node by a small number of hops or steps. A scale-free network is a network whose degree distribution follows a power law [44].

network (interbank market) that connect to a large number of small nodes with few links. Therefore, the networks of the interbank market exhibit a 'core-periphery' structure for most countries [46-51][6] as well as for global financial networks[52]. The 'core-periphery' structure is defined as a connected network that has two tiers, a core and a periphery, the core forming a fully connected clique, whereas peripheral banks are only connected to the core. The cores of the networks, composed of the most connected banks, are often much more important than nodes in the periphery. Due to the difference in the nature of transactions in the banking system, different markets can have different topological properties [53]. The topological analysis in [54] shows that the structure of the Danish money market is different from the structure of the payments network and the banks in the core of the money market are of more equal size. Langfield et al. [55] find that the strength of the core-periphery structure varies significantly by asset class in the UK interbank market: the observed interbank network fits the core-periphery model more strongly for derivatives and marketable securities than for unsecured lending and repurchase agreements.

The other line of the static network analysis focuses on looking into the changes in the network topology of the financial system over the last decades [56-59]and also the changes due to financial crises, to study the effects of crises on dynamic link formation. Some studies focused on one-country cases and they found that the core-periphery structure tends to be stable over time but the number of core banks and the aggregate level of interbank activity may vary over time. Puhr et al. [58] show that financial crises decreased the network density between 2008 and 2010, with central nodes becoming more important. While in the studies by Fricke and Lux [59], core banks tend to rely on the liquidity of periphery banks during crises, whereas in normal times they tend to be net providers of liquidity to the system. Studies on global financial system find that structure of global banking networks varies over time. They become more connected [52, 60, 61] and respond to economic and financial shocks. Kubelec and Sá [52] show that there has been a remarkable increase in interconnectivity over the past two decades; financial links have become larger and more frequent and countries have become more open. Minoiu and Reyes [62] show that the 2007–2009 global financial crisis stands out as an unusually large perturbation to the cross-border banking network. Connectivity tends to fall during and after systemic banking crises

[6] [51] analyses the network structure of CDS market.

and sovereign debt crises. Hale [63] also finds that the global financial crisis of 2007–2009 had a large negative impact on the formation of new relationships in the global banking network, especially for large banks, which were previously immune to effects of banking crises and recessions. A recent study by Minoiu [64] shows that financial interconnectedness has early warning potential, especially for the 2007-2010 wave of systemic banking crisis.

A third line of the research draws in the research work which developed algorisms to identify different interbank activities, such as the transactions of interbank loans on overnight market [65] or with maturities of up to 1 year [66, 67], the presence of intraday lead-lag relationships between financial assets [68], and the presence of lending relationships [69] and preferential trading [70] in interbank market. These works uncover some characteristics of the interbank network structure which can help to give a better understanding of the dynamics of the interbank market.

The existing static network analysis considers the overall structure of the network. The results contribute to the study on systemic risk in the interbank market, which also provides a stronger basis for the assessment of contagion risk using dynamical simulations [40].

2.2.2 Dynamic network analysis

Computational techniques have been adopted on network analysis to assess the possible extent of contagion via interbank liabilities, which refers to the dynamic network analysis. This often involves computer simulation that is used to explore the resilience of a network in certain stress scenarios. The literature regarding network models with dynamic network analysis can be reviewed following three strands.

1. The first strand of the dynamic network analysis is to evaluate the resilience of the network to different shocks based on real interbank data. First, the network structure is constructed using the real data from banks' balance sheet. Then an external shock is applied to this constructed network, which propagates through the system affecting the balance sheets of individual institutions, thus the contagion effects caused by one or more bank bankruptcy can be studied. These studies [71-76] generally found that significant contagion effects are potential but a substantial weakening of the whole banking sector is unlikely to happen. Due to

the difficulty in obtaining the balance sheet data between banks, a *maximum entropy method* has been used in many works [77- 80] to make an estimation of bank assets and liabilities positions. However, maximum entropy approach is found to overrate the scope for contagion in the work by Mistrulli [81]. Anand et al. [82] propose an efficient alternative that combines maximum entropy with a *minimum-density solution* to define a useful range that bounds the cost of contagion in the true interbank network when counterparty exposures are unknown.

2. The second strand of the research focuses on modelling and studying the different kinds of contagion propagation mechanisms, which usually consists of two steps: firstly, develop a mathematical model of the banking system; these dynamic models use mathematical equations to describe contagion mechanisms between banks. Secondly, parameters values and initial conditions are used to run simulations of the model, which are derived from real data collected from the banks' balance sheet or virtual data generated for testing. The data used should exhibit scenarios that represent the banking system under stress (such as default on liabilities or falls in asset price). Then the behaviour of the networks is investigated to understand the different kinds of propagation mechanisms under different types of applied shocks. The shock used in the simulations can be a systemic shock that affects all the banks in the system or just an idiosyncratic one that affects one single bank. Further, the resilience of the network can be investigated too. Two types of shock propagation are usually studied: mechanical propagation and behavioural dynamics, which are described as follows.

   Mechanical propagation

   As introduced in subsection 2.1.2, with this type of propagation mechanism the bank does not take behavioural reactions (e.g. liquidity hoarding and fire sale), when there is a shock, instead it changes its balance sheet according to the loss. This may affect other banks' balance sheet if the loss includes liabilities from those banks. The default spreads through interbank loans. Studies focusing on this propagation mechanism usually run simulations with different network structures with different bank sizes, connectivity and concentration, and ultimately they study how the network structure affects the system stability. Georg

[83] shows that money-centre networks (where a small number of large banks is very highly interconnected and a large number of banks is very little interconnected) are typically more stable than random networks. Sachs [84], by contrast, finds that a money centre model with asset concentration among core banks is less stable than a random graph with banks of homogeneous size.

Robust-yet-fragile characteristic of the banking system has been investigated [12]. Battiston et al. [85] study how network density (the number of connecting links) relates to systemic risk in a model of the economy as a credit network (in which nodes represent agents and links represent credit relationships); they found that connections between banks improve risk sharing, but connectivity also leads to trend reinforcement. When an economic agent suffers a negative shock, trade partners react by making conditions even worse. Studies in the literature found that contagion stemming through mechanical propagation has a limited effect [72, 75], while the effect caused by the banks' behavioural dynamics (e.g. liquidity hoarding and fire sale) is more important than the direct solvency contagion. The likelihood of contagion through mechanical propagation is very small due to the robust properties of financial networks.

Behavioural dynamics

As introduced in subsection 2.1.2, banks will show behavioural dynamics if the economy is under stress, which includes liquidity hoarding and fire sales[7]. The following subsection explains these phenomena in more detail.

Liquidity hoarding: this happens when a shock appears in the banking system and the subsequent credit losses in banks may weaken investor confidence, resulting in a general reduction in the bank funding supply. Then the banks prefer to hold its liquidity rather than invest or lend to other banks, therefore this results in *liquidity hoarding* within the network. Studies on liquidity hoarding based on network models mainly focus on two phenomena, the first one is the general reduction in the bank funding supply [86, 87]. The other phenomenon is how banks cut lending in the interbank market within the network; banks may withdraw lending from a specific infected bank or run on all banks

[7] Another behavioural dynamic is reinvestment, which it's not introduced here.

indiscriminately [88] or banks may cut lending according to their own health and confidence in the system [89] and withdrawing deposits held at other banks [90]. The general conclusion from these studies is that liquidity is very important for systemic stability; it can impose a negative externality on the entire financial system and amplify other sources of risk. The model in our work assumes that a bank lends money depending on its own health and the health of the borrowing bank.

Fire sales: Fire sales happen when banks face liquidity shortage, forcing them to sell illiquid assets which ultimately causes the assets' prices to decrease. Mark-to-market losses result on other banks which hold the same assets. Papers are written by Plantin et al. [91]and Allen and Carletti [92] discussed the potentially destabilizing effects of mark-to-market accounting; these reactions may have repercussions on other financial institutions and to some extent exacerbate contagion. Fire sales cannot only happen in the course of liquidation or resolution after a bank default [69, 87, 89], but also happen as a defensive action in a bid to prevent or defer failure. In the model of Gauthier et al. [93], it assumes that the bank needs to reduce their size and leverage because a pre-specified minimum capital ratio is breached and finds that the effect from the fire sales accelerates bank defaults as bank capitalization decreases.

While the mechanical or behavioural shock propagation often operate simultaneously in a real banking system [94], network models studies generally find, through simulations, that the behavioural dynamics has a more significant effect on the stability of the system than the mechanical propagation mechanism. Therefore, Gauthier et al. [86] conclude that comprehensive bank regulation should be based on a set of requirements related to capital, liquid asset holdings and short-term liabilities.

3. The third strand of the research is a group of recent and growing work aiming to consider uncertainty within the modelling [13], [95-102]. As the behaviour of the bank is very complex, the network structure changes with time and these changes interact with the behaviour of the banks, which generates the uncertainty in the network. Therefore, researchers recently applied game-theoretical tools on the network model trying to model how the network structure is affected by the banks' behaviour.

Related work

The model of the banking system proposed in this Ph.D. project belongs to the dynamic network analysis which involves simulations. There are three papers which are closely related to this Ph.D. project, need to be introduced in details in order to better understand the proposed model. Iori et al. [103][8] studied the performance of the interbank market in its role as a safety net by simulating interbank lending. The work shows that some characteristics, such as size and connectivity, of a market's constituents and the nature of their interconnectedness affect the potential for contagion. When banks are homogeneous, interbank lending plays an insurance role to stabilize the system, while when the banks are heterogeneous, contagion effects may arise and systematically increase with connectivity. It should be stressed here that the initial differential equation model developed in our work was inspired by the difference equation model developed by Iori et al.

Work by May, Arinaminpathy [105] and Haldane, May [106] shows an interesting perspective to study the banking system which draws analogies with the dynamics of ecological food webs and with networks within which infectious diseases spread. In these papers, banks are nodes in the network and bank activities have been classified into four categories: deposit, external assets, borrowing and lending. The borrowing and lending are the links between the banks. This structure was extended and used in our work.

From the existing literature, it can be seen that network models can be a natural way to model the complex dynamics by simulating different network structures in different degrees of size, connectivity, concentration and so on. Though this can address the issue of having uncertainty and the change of connectivity in the system, existing network models do not include mechanisms that can control and affect behaviours emerging from the dynamics of the units which make the entire systems. In the next section, control system engineering will be introduced as a tool to monitor and control a dynamical/interconnected system characterised by uncertainty.

[8] A previous contribution of this work can be found in [104].

2.3 Control theory and its applications in finance

Control theory is an interdisciplinary field of applied mathematics and engineering that deals with the basic principles underlying the analysis and design of control systems. “Controlling a system” means to influence the behaviour of the system in order to achieve a desired goal. Control theory deals with the use of a controller to achieve this purpose.

Subsection 2.3.1 first gives a brief introduction to the history and development of the control theory, then followed by a detailed introduction of feedback, which is a key concept in control theory. Essentially, a feedback process is the one in which the state of the system determines the way the control has to be exerted at any time. Control theory has its roots in the use of feedback as a means to regulate physical processes and mediate the effect of modelling uncertainty and noise [107, 108]. Subsection 2.3.2 reviews some applications of feedback control in the economic area.

2.3.1 Development of the control theory and feedback control system

Development of the control theory

The development of the control theory can be divided conveniently into four main periods shown in figure 2.4 [109, 110]. Control systems of various types date back to antiquity, a more formal analysis of the field began with a dynamics analysis of the centrifugal governor, conducted by the physicist James Clerk Maxwell in 1868 [111]. He described and analysed the phenomenon of self-oscillation, in which lags in the system may lead to overcompensation and unstable behaviour. The invention of the flyball centrifugal governor enabled effective speed control of the steam turbine and thereby shares credit for the industrial revolution. Ever since, control has played a key role as an “enabling technology” in applications ranging from autopilots, navigation and telecommunications, to manufacturing, and power systems.

A Physical system can be modelled in the "time domain", where the response of a given system is a function of time, the various inputs and the previous system values. As time progresses, the state of the system and its response change. However, time-domain models for systems are frequently modelled using high-order differential equations which can become impossibly difficult for humans to solve and some of which can even become impossible for modern computer systems to solve efficiently.

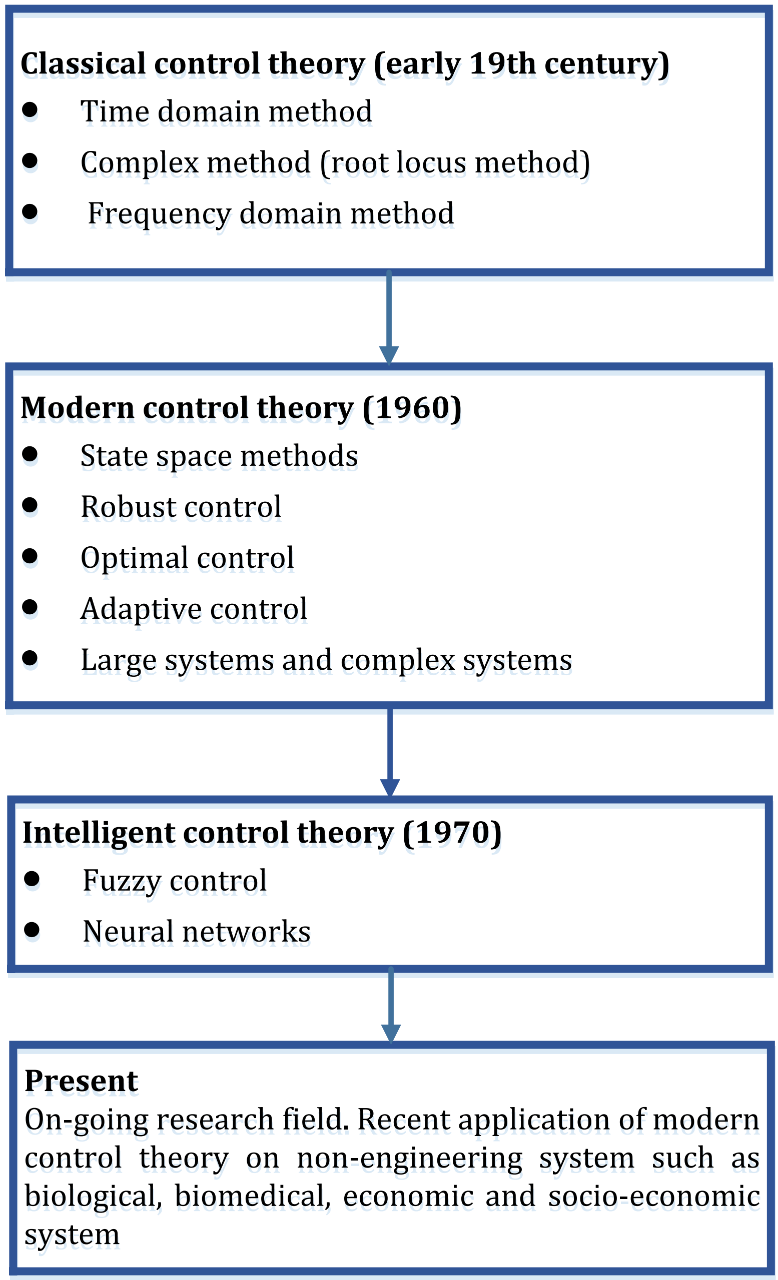

Figure 2.4 Flow chart shows the development of the control theory.

To counteract this problem, *classical control theory* uses the Laplace transform [112] to change an Ordinary Differential Equation (ODE) in the time domain into a regular algebraic polynomial in the transform domain. Once a given system has been converted into the transform domain where it can be manipulated with greater ease. *Modern control theory*, instead of changing domains to avoid the complexities of time-domain ODE mathematics, converts the differential equations into a system of lower-order time domain equations called state equations, which can then be manipulated using techniques from linear algebra. The closing of the 20th century saw a rapid development of the mathematics of systems, control and optimization with a focus placed on understanding the benefits and limitations of feedback. *Intelligent control* starts around the 1970s due to remarkable developments in computing, communications, and sensing technologies. The scope of control theory is rapidly evolving to encompass hybrid, hierarchical, data-driven, decision-making networks where their connectivity at various scales affects functionality.

Feedback control system

A major application of control theory in the engineering discipline is known as control engineering, which deals with the design of control systems for industry. It applies control theory to design systems with desired performance. Such designed systems are called control systems. There are basically two types of control systems: the open loop system and the closed loop system. As shown in figure 2.5, system in which the output quantity has no effect upon the input to the control process is called an open-loop control system.

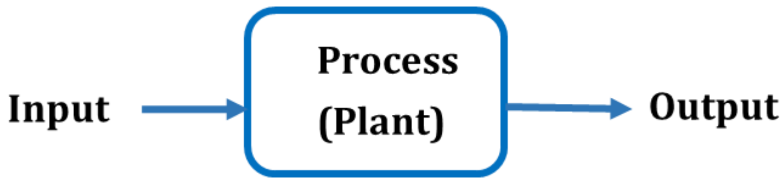

Figure 2.5 A block diagram of an open-loop control system.

The closed-loop system, also known as a feedback control system, uses the concept of an open loop system as its forward path but has one or more feedback loops or paths connecting the output and the input. Feedback is the foundation for control system analysis and design. A simple feedback control system is shown in figure 2.6. In a feedback control system, the usual objective is to control a system, often called the plant, so that its output follows a desired signal, called the reference, which may be a fixed or changing value. To do this a controller is designed (generating a control input on the planet). The difference between the actual and desired output, called the error signal, is applied as a feedback to inform the input of the system, to bring the actual output closer to the reference. In any real-time control system, there is always some amount of external noise which is called disturbance. With feedback, the controller is able to use the output to shape the input of the system. In this way, it reduces the effect of the disturbances on the system, which is called disturbance rejection in feedback control.

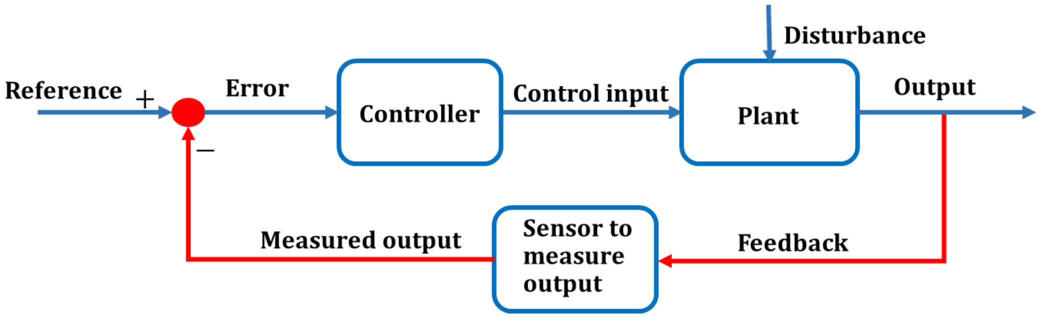

Figure 2.6 A block diagram of a negative feedback control system. It illustrates the concept of using a feedback loop to control the behaviour of a system by comparing its output with a desired value (called the reference input), and applying the difference as an error signal to dynamically change the output so it is closer to the desired behaviour.

A typical example of a feedback control system is a person steering an automobile (see figure 2.7). The driver can control the car to drive on the desired path through the steering wheel. The automobile is the plant and the driver is the controller. The driver will adjust the error which is the difference between actual course of travel and the desired course of travel using the steering wheel.

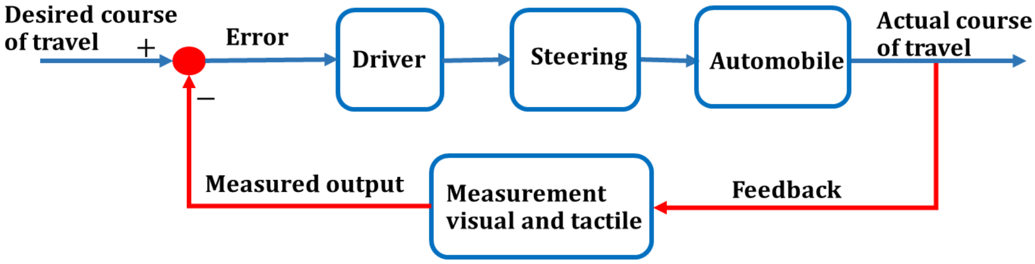

Figure 2.7 A block diagram of an automobile steering control system. The error is the difference between the actual course of travel and the desired course of travel. The driver adjusts the error by using the steering wheel to control the car so that it travels on the desired path.

It can be seen from the feedback control system that control engineering has the ability to deal with uncertainty as the system can reduce the error without the knowledge of why the error occurs. For instance, the driver may not know why the car has deviated from the desired path but s/he can still bring it back to the desired path. Therefore, the initial motivation to design and apply a control approach to model the banking system is to expect the control approach to bring the banking system back to stable when the causes of instability in the banking system are unknown.

2.3.2 Feedback control application in finance

Nowadays, as the understanding of the dynamics of business, social, and political systems increases, control engineering is not limited to engineering discipline but is equally applicable to systems above. As the general theory of feedback systems, control theory is useful wherever feedback occurs. Many applications have been successful in the area such as ecosystems, physiology, climate modelling, and neural networks as well as in finance. The existing literature shows that control theory has been used to analyse financial problems.

Wingrove and Davis [113, 114] show some results based on the application of classical linear control to the analysis of economic system dynamics. The linear control analysis is applied as an aid in understanding the fluctuations of business cycles in the past, and to examine monetary policies that might improve stabilization. The results confirm that to improve stabilization of the business cycle, a general rule is that any movements in the growth of money supply should be countercyclical with respect to the growth of real GNP. Novotna [115] studied the finance system with the distributed time delay and indicated that the complex dynamic behaviour in such a finance system can be controlled under appropriate strength feedback and delay times, as well as that the feedbacks either suppress or enhance the dynamic behaviour. Barmish et al. [116] provide an overview of basics of simulation and performance evaluation associated with stock trading via feedback control methods; from this it shows the feedback control has been widely studied in the stock trading strategies.

It can be seen from the literature that the feedback control applications are mainly focused on macro-economic models to study how business cycles affect economic stability. In this thesis, feedback control is applied for the first time on a banking system model to study its stability as well as design proper controllers to keep the bank in a stable state.

2.4 Conclusion

This chapter presents a brief introduction of financial systems and provides an extensive literature review of the 2007-2009 financial crisis as well as the network models used to study of financial systems. The chapter also introduces control theory used in engineering and its application to financial problems. While network models have provided useful insight in the understanding of financial crises, they have never been combined with control theory to study the stability of the banking system. The following chapter, Chapter 3, shows the details of the new dynamic network model of the banking system that has been developed in this Ph.D. project; the links with existing network models are also highlighted. Chapter 4 compares the results of the proposed model with the findings in the literature. Chapter 5 presents the novel combination of the proposed model of the banking system with control theory.

Chapter 3

The Dynamic Network Model

In this chapter, the dynamic network model developed in this Ph.D. project is introduced. The banking system is represented as a network where the nodes are individual banks, while the links between any two banks are interbank loans and borrowing. The dynamic structure of the model is represented as a set of ordinary differential equations consisting of balance sheet dynamics. The choice of differential equations not only allows the analysis of systemic risk but also allows applying control theory. In this chapter, the dynamic model is introduced step by step, from a one-bank model to a two-bank model and finally to a multi-bank model.

Section 3.1 provides details of how the basic structure of a bank has been designed and how the banks are connected; some assumptions are also explained. In Section 3.2 the one-bank model and the corresponding differential equations are introduced; subsequently, the two-bank model and the multi-bank model are presented. Specifically, Section 3.2 explains how the differential equations are used to model the banking system have been generated. Section 3.3 concludes the chapter.

3.1 Description of the banking system model

This section describes the basic structure of each individual bank as well as how banks are connected with each other. As explained in the literature review, the real banking system is very complex in nature. Banks perform many activities which cannot all be modelled in detail, so some assumptions and simplifications are made in the proposed work. The goal is to develop a model which, on one hand, retains the most important characteristics of the banking system and, on the other, allows us to develop an analysis of its stability which has implications for the real system. In the proposed work, the following main banking activities are considered:

1) Collection of deposits to accumulate cash, and payment of interests to depositors

2) Investment of cash to generate profit through receipt of returns

3) Lending money to other banks, and receipt of interests

4) Borrowing money from other banks, and payment of interests

The primary purpose of a bank in performing these activities is to generate profit, to maintain (and increase) a positive net worth and cash, so to avoid defaulting and failure. A bank needs to keep its cash above a threshold (reserve ratio requirement): more specifically, the cash of a bank, at any given time, has to be equal or bigger than the total deposits multiplied by a positive factor (<1). This factor is called reserve ratio and is usually set by the Central Bank. By preserving this cash, the banks safeguard themselves from shocks such as bank runs, debtors' defaults and investment failures. There may be situations in which a given bank may not be able to meet the reserve ratio requirement due, for example, to investment failure or cash withdraws by depositors. In these cases, the bank has to find cash in the interbank borrowing/lending market. In the interbank market, the bank lacking of cash looks for opportunities to borrow money from banks with cash above the reserve ratio requirement.

In the proposed model, each bank is characterised by five activities which produce 5 different variables, as shown in figure 3.1: accumulation of deposits - variable D, interbank borrowings – variable B, interbank lending – variable L, investment - variable I and accumulation of cash – variable C, see figure 3.1. The net-worth, N, is given by the formula:

$$N = I + C + L - D - B.$$

In this model, the deposit, D, represents the funds deposited by creditors, which cannot be controlled by the bank. Therefore, D is modelled as an exogenous signal. When new funds are deposited in a bank, this automatically increases the cash: more specifically, any change in the deposit, ΔD, of a bank corresponds to a change in its cash, $\Delta C = \Delta D$. Equivalently, when a bank changes its investments, ΔI, its cash will change accordingly, $\Delta C = -\Delta I$. The negative sign represents the fact that an increase in the investment corresponds to a reduction of the cash, and vice versa. Similar consideration apply for changes in borrowing, ΔB, and lending, ΔL.

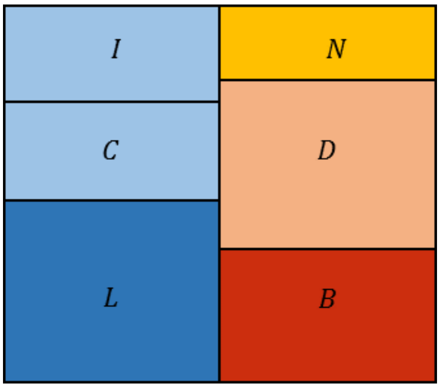

Figure 3.1 Bank's components: N =net-worth, D =deposits, B =interbank borrowings, L =interbank loans, C =cash and I =investment.

The borrowing and lending activities make the linkages between banks which ultimately form the network; in this network, the banks are the nodes and borrowing and lending between any two banks (L_{ij} and B_{ij}[9]) are the connections (see figure 3.2). These links, which determine the network structure of the system, 'appear' within the differential equations of the dynamical model. Next section introduces the ordinary differential equations developed in our work to implement the network model we have just described.

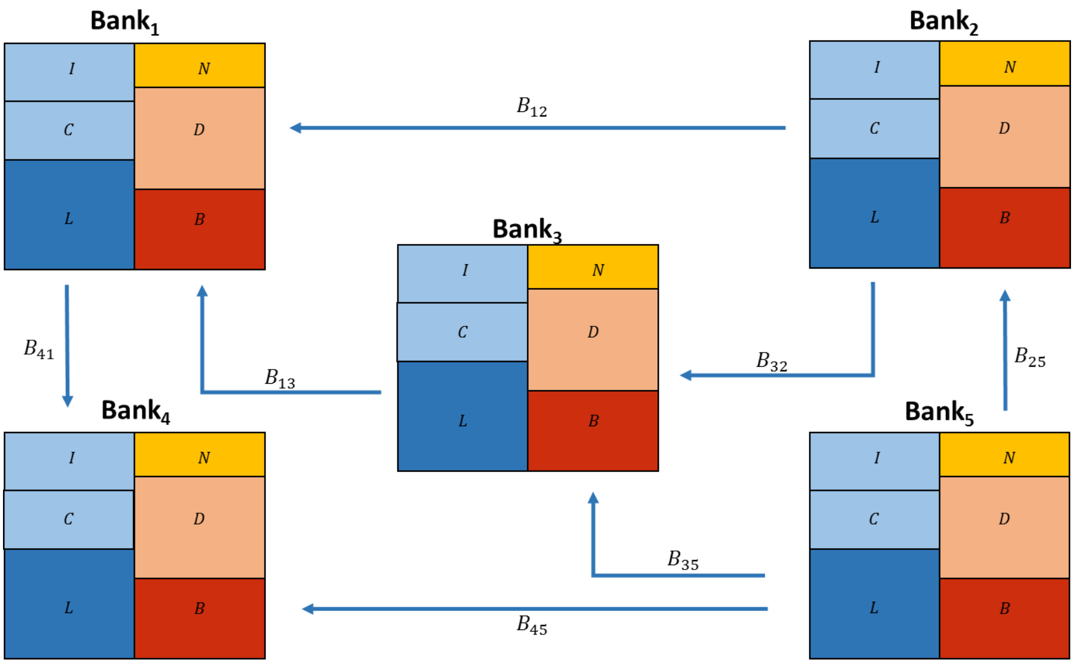

Figure 3.2 Linkages between banks: the network links between banks are made by the interbank lending/borrowings.

[9] L_{ij} is the total lending that bank$_i$ lends to bank$_j$. B_{ij} is the total borrowing that bank$_i$ borrows from bank$_j$

3.2 Ordinary differential equations of the banking system model

In this section, the development of the ordinary differential equations is introduced in three stages. In the first stage, a one-bank model is presented, which is characterised by differential equations describing the cash, deposit and investment only. This model can be used to elucidate the basic activities within one bank. In the second stage, a two-bank model is presented by adding in the differential equations the borrowing and lending activities between the two banks. This two-bank model allows studying in a simple way the effect of interbank connections on the system. In the last subsection, the number of banks in the system is increased to any value larger than 2. In this multi-bank model, the number of connections between banks can be increased arbitrarily, which increases the complexity of the system. In particular, new algorithms need to be developed to deal with the more sophisticated way banks exchange cash with each other; modifications of the differential equations are introduced to describe the borrowing and lending behaviours as well as the corresponding interest payments in the multi-bank model.

The differential equations characterising the banking system model contain the time derivatives of the quantities C, I, L and B, which govern the changes of these quantities, i.e. ΔC, ΔI, ΔL and ΔB, as function of time. These differential equations prescribe the dynamics of the model, and allow us to apply control theory tools in a straightforward way, as described in Chapter 5.

3.2.1 Differential equations for one-bank model

The one-bank model contains only three quantities: cash, deposit and investment. The bank is independent from other banks, and the cash can only be affected by the deposit and the investment. The one-bank model allows one to easily appreciate how the banking system has been modelled. Moreover, it is easier to investigate the effect of the exogenous signal (deposit) on the bank's cash and investment since there are no interbank borrowing and lending effects. Importantly, the one-bank model allows a simple and analytical implementation of the stability analysis, which can be used also for the more complex multi-bank model.

Deposits

In this one-bank model, D_1 is used to represents the total deposits of the bank$_1$. D_1 is assumed to be assigned by an exogenous signal. This means the deposit cannot be controlled by the bank and the fluctuations in the deposit introduce shocks in the bank by affecting its cash and exposing the bank to the risk of failure. In this thesis, the deposit signal is generated numerically. Different kinds of deposit signals have been generated for different purposes. For example, constant deposit is used for testing and verifying the model. To simulate cases with interesting dynamics, the deposit signal is set as the following random function of time, t, (see also Iori et al. [117]):

$$D_1 = |\overline{D} + \overline{D}\sigma_D\varepsilon_t| \tag{3.1}$$

Equation (3.1) models the case in which fluctuations (shocks) in the deposit are caused by random but mutually uncorrelated payments/withdrawals of deposits. $\overline{D}$ represents the average size of the deposits; $\sigma_D (> 0)$ represents the amplitude of the shocks, while ε_t is a random variable ($\varepsilon_t \sim N(0,1)$). When ε_t is positive an increase (payment) in the deposit is made while when ε_t is negative a decrease (withdraw) of the deposit is experienced by the bank.

Investments

The investment behaviour of the bank is described in equation (3.2) below;

$$\frac{dI_1}{dt} = \min[(C_1 - rD_1)^+, opp_1] - w_1 I_1 - v_1 I_1 \tag{3.2}$$

$$opp_1 = \left|\overline{opp} + \overline{opp}\,\sigma_{opp}\eta_t\right| \tag{3.3}$$

each bank invests at time t depending on two factors: one is the availability of cash, the other one is the stochastic investment opportunity. In equation (3.2), $\frac{dI_1}{dt}$ is the change of the investment of the bank over the time dt; C_1 is the total cash of the bank, D_1 is the total deposit of the bank and r is the reserve ratio. The reserve ratio is the proportion of the total deposit that the bank must have on hand as cash. Therefore, the availability of cash is represented by, $(C_1 - rD_1)^+$, where $(x)^+$ stands for $\max\{x, 0\}$. This means the bank can only invest using the cash which is above the value required by the reserve ratio, rD_1. If the $C_1 < rD_1$, $(C_1 - rD_1)^+$ is equal 0, which means that there is no available cash for investment.

The term opp_1 in equation (3.2) is the stochastic investment opportunity at time t; as for the deposit, opp_1 is also an exogenous signal and it is described in equation (3.3). opp_1 fluctuates randomly around an average value, $\overline{opp}$, where $\overline{opp} = \delta\overline{D}$ (with $0 < \delta < 1$ and $\eta_t \sim N(0,1)$), which means that the average size of the investment opportunity is affected by the size of the bank (see also Iori et al. [117]). Therefore, taking these two factors into consideration, $(C_1 - rD_1)^+$ and opp_1, the amount of resources invested per unit of time is the minimum value of these two terms. This means that a bank invests only when it has sufficient cash and investment opportunity. Strictly speaking the term, $\min[(C_1 - rD_1)^+, opp_1]$, in equation 3.2 should be multiplied by a factor with dimensions, time^{-1}; for simplicity, we assume that this factor is equal to one.

Besides adding new investment, investments made in the previous time mature after some time. Moreover, there will be some failed investment that cannot be recovered. In equation (3.2), $-w_1$ represents the proportion of total investment, per unit time, that has matured. While v_1 represents the proportion of total investment that has been lost per unit time. For simplicity w_1 is constant in the proposed model. In future work, to account for more realistic scenarios future work, w_1 can be different for each bank and time dependent; w_1 can also be introduced as an exogenous signal based on real data. This would add further nonlinearity to the dynamic model.

Cash

The changes of the deposit, $\frac{dD_1}{dt}$, and of the investment, $\frac{dI_1}{dt}$, can affect the change in cash, $\frac{dC_1}{dt}$. The differential equation governing the change over time of the cash, $\frac{dC_1}{dt}$, of bank$_1$ is given by:

$$\frac{dC_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1 \tag{3.4}$$

In equation (3.4), $\frac{dD_1}{dt}$ represents the change of the deposit and it has a positive sign because the change of the deposit will affect the cash in the same direction. As more deposits are saved into the bank, the cash increases and as deposits are withdrawn from the bank, the cash decreases. Similarly, $\frac{dI_1}{dt}$ represents the change of the investment. It has a negative sign because any increase in the investment means that the bank uses its cash to make new investments, so the cash decreases. It must be stressed that matured investments, instead, increase the cash.

Besides the change of deposit and investment, the cash of the bank can also be affected by paying interest to depositors and getting returns from the investment. In equation (3.4), g_1 is the deposit interest. Therefore, $-g_1 D_1$ represents the reduction in cash over time due to the payment of interest to depositors. Similarly, p_1 is the return rate of the investment and $p_1 I_1$ is the increase in cash due to the receipt of returns from investments. $v_1 I_1$ represents the proportion of total investments that has been lost, per unit of time, due to defaults. This term also appears in equation (3.2), since the lost investment causes reduction in investment, but it doesn't increase the cash, therefore, in equation (3.4), $v_1 I_1$ has to be subtracted from the cash so that it does not increase the cash.

The cash needs to be above zero to keep the bank functioning. If the cash of a bank falls below zero, that bank is labelled as a failed and all its activities are stopped.

Net-worth

Another important variable is the net-worth of the bank, N_1, which is the difference between asset and liabilities. The asset is the sum of the cash, C_1, and investment, I_1, while the liabilities are the deposit, D_1. Therefore, N_1 is represented by the equation (3.5).

$$N_1 = C_1 + I_1 - D_1 \tag{3.5}$$

A positive net-worth means that the bank is managing its cash and investment well so to make a profit.

3.2.2 Differential equations for two-bank model

In this subsection the development of the two-bank model is presented. Compared to the one-bank model, two new activities are introduced; interbank borrowing and lending. Due to fluctuations in deposits, the cash of any given bank may fall below the required amount dictated by the reserve ratio. In this case that bank needs to borrow from the other bank. The interbank borrowing and lending activities make the interconnection between the two banks and affects the cash of both banks.

Deposits

In the two-bank model, the total deposit is modelled in the same way as in the one-bank model. As shown in equation (3.6), the total deposit of bank$_1$, D_1, and bank$_2$, D_2, at time t are all exogenous signals, which fluctuate randomly around the average size $\bar{D}$.

$$D_1 = |\bar{D} + \bar{D}\sigma_D \varepsilon_t|$$

$$D_2 = |\bar{D} + \bar{D}\sigma_D \varepsilon_t| \quad (3.6)$$

Interbank borrowing and lending

The interbank borrowing and lending activities in the two-bank model are represented in equation (3.7).

$$\frac{dB_{12}}{dt} = \frac{dL_{21}}{dt} = min[\,(rD_1 - C_1)^+,\ (C_2 - rD_2)^+] - \alpha_{12}B_{12} \quad (3.7)$$

$\frac{dB_{12}}{dt}$ represents the amount of cash borrowed by bank$_1$ from bank$_2$, over time dt, which is equal to, $\frac{dL_{21}}{dt}$, the amount of cash lent by bank$_2$ to bank$_1$, over time dt. It is assumed that in this two-bank model, only bank$_1$'s cash falls below the reserve ratio requirement and bank$_2$'s cash is above the reserve ratio requirement, so the borrowing only happens for bank$_1$ and lending only happens for bank$_2$.

The right-hand side of equation (3.7) shows how the total borrowing (lending) is updated. Bank$_1$ borrows just enough to meet the reserve ratio requirements, which is the amount represented by the term, $(rD_1 - C_1)^+$, where $(x)^+$ stands for max$\{x, 0\}$. Bank$_2$ only lends cash that is above its required reserve, which is represented by $(C_2 - rD_2)^+$. Therefore, the actual borrowing (lending) is the minimum of these two factors. Also in this case the term, $min[\,(rD_1 - C_1)^+,\ (C_2 - rD_2)^+]$, in equation 3.7 should be multiplied by a factor with dimensions, time^{-1}; for simplicity, we assume that this factor is equal to one.

Bank$_1$ needs to repay its borrowing to bank$_2$ after some time; this repayment behaviour is represented by the term, $\alpha_{12}B_{12}$, where α_{12} is the proportion of the total borrowing, B_{12}, repaid, per unit time, by bank$_1$ to bank$_2$ during the current period of time.

Investments

The investment in the two-bank model is slightly different from the one-bank model.Bank$_1$ is unable to make any investment when its cash falls below the reserve ratio requirement; therefore the change of the investment of bank$_1$, $\frac{dI_1}{dt}$, is represented as in equation (3.8), where the newly added investment is equal to 0. It is assumed that the investment happens after the interbank borrowing and lending. The change in the investment for bank$_2$, is given by the cash above the required reserve, $C_2 - rD_2$, minus the lending to bank$_1$, $\frac{dL_{21}}{dt}$, (see equation (3.9)). The investment opportunity is the same as in the one-bank model, which is an exogenous signal represented by equation (3.10).

$$\frac{dI_1}{dt} = 0 - w_1 I_1 - v_1 I_1 \tag{3.8}$$

$$\frac{dI_2}{dt} = \min\left[\left(C_2 - rD_2 - \frac{L_{12}}{dt}\right)^+, opp_2\right] - w_2 I_2 - v_2 I_2 \tag{3.9}$$

$$opp_2 = \left|\overline{opp} + \overline{opp}\,\sigma_{opp}\eta_t\right| \tag{3.10}$$

Cash

The differential equation governing the change in time of the cash, $\frac{dC_1}{dt}$ and $\frac{dC_2}{dt}$, of bank$_1$ and bank$_2$ is given by:

$$\frac{dC_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1 + \frac{dB_{12}}{dt} - B_{12}h_{12} \tag{3.11}$$

$$\frac{dC_2}{dt} = \frac{dD_2}{dt} - \frac{dI_2}{dt} - g_2 D_2 + p_2 I_2 - v_2 I_2 - \frac{dL_{21}}{dt} + L_{21}k_{21} \tag{3.12}$$

In equations (3.11) and (3.12), the deposit and investment affect the cash in the same way as for the one-bank model. Two more terms are added to each equation, which represent the effect of the borrowing (lending) on the cash. For bank$_1$, any increase in the change of borrowing, $\frac{dB_{12}}{dt}$, means that the bank borrows money to increase its cash, while any decrease in $\frac{dB_{12}}{dt}$ means that the bank repays money using its cash. So in equation (3.11), $\frac{dB_{12}}{dt}$ has a positive sign. Similarly, in equation (3.12), $\frac{dL_{21}}{dt}$ has a negative sign because that the lending will decrease bank$_2$'s cash and the repayment of the lending increases bank$_2$'s cash. Moreover, bank$_1$ needs to pay the interest to bank$_2$ according to the total borrowing, B_{12}. In equation (3.11), h_{12} is the interest rate bank$_1$ pays to bank$_2$, while $k_{21}$is the interest rate bank$_2$ receives from bank$_1$; therefore, $h_{12} = k_{21}$. The interest rate is considered as a constant in the two-bank model for simplicity. The interest for borrowing (lending) is paid using the cash, so bank$_1$'s cash decreases (represented by the term, $-B_{12}h_{12}$) and bank$_2$'s cash increases (represented by the term, $L_{21}k_{21}$) due to the payment of interest.

Net-worth

Borrowing is part the bank's liabilities of the bank, while the lending is part of the assets. So the net-worth of bank$_1$ and bank$_2$, N_1 andN_2, are given by equations (3.13) and (3.14), respectively.

$$N_1 = C_1 + I_1 - D_1 - B_1 \tag{3.13}$$

$$N_2 = C_2 + I_2 + L_2 - D_2 \tag{3.14}$$

3.2.3 Differential equations for the multi-bank model

This subsection presents the development of the multi-bank model has n ($n \geq 2$) banks. The multi-bank model is more complex than the two-bank model in the following reasons:

banks can be connected in many different patterns, for example, a bank can be connected to some banks but not some others or some banks can be connected to more banks than others. So a new parameter σ_{ij} is introduced to represent the connections between any two banks i and j ($, j = 1,2, \dots n$ and $i \neq j$). σ_{ij} can be 0, which means that there is no link between the two banks, or 1, which means that the two banks are connected. All the values for σ_{ij} are generated at the beginning of the simulation according to the choice of the link rate, l_r, which can take values from 0 to 1. l_r can be seen as the probability that $\sigma_{ij} = 1$. Therefore, l_r is a variable that represents the degree of connectivity of the system; the closer the link rate is to 1, the more connected the system will be.

The network model representing the banking system is characterised by a given pattern of the connections, and the order by which a bank borrows from its links can differ. Therefore, an algorithm has been developed to determine the order of the interbank borrowing and lending of each bank. The details of this algorithm are introduced later in the *borrowing and lending* subsection.

As there are more banks in the system, the default of one bank may affect other banks as well as the pattern of the connections, so a default liquidation procedure is introduced in the system to deal with the remaining assets and liabilities of any failed bank. The details of this liquidation procedure are reported later in this section. Also, a parameter called s_i is introduced to represent the 'mode' of a bank$_i$; when bank$_i$ is still functioning, $s_i = 1$, while $s_i = 0$ when the bank$_i$ has failed.

In this section, the differential equations of the multi-bank model are introduced with the characteristics above.

Deposits

The deposits of bank$_i$, D_i, has the same characteristics as in the previous models. In the multi-bank model, two cases are simulated: homogeneous case and heterogeneous case. The heterogeneous case is characterised by different average size of the deposits. In the homogeneous case, each bank has the same average size, $\overline{D}$, as shown in equation (3.15):

$$D_i = |\overline{D} + \overline{D}\sigma_D\varepsilon_t| \tag{3.15}$$

While in the heterogeneous case, the average size of the deposit of any bank$_i$ is assigned by sampling from a Gaussian distribution with the mean, μ_s, and variance, ${\sigma_s}^2$:

$$\overline{D}_i \sim |N(\mu_s, {\sigma_s}^2)| \, .$$

Therefore, the deposit signal for a bank$_i$ at time t in the heterogeneous case is given by:

$$D_i = |\overline{D}_i + \overline{D}_i\sigma_D\varepsilon_t| \tag{3.16}$$

Interbank borrowing and lending

Equation (3.17) shows how the total borrowing bank$_i$ is updated.

$$\frac{dB_{ij}}{dt} = min\left[(rD_i - C_i)^+,\ \left(C_j - rD_j\right)^+ \right] \sigma_{ij} - s_i\, s_j\, \alpha_{ij} B_{ij} \qquad (3.17)$$

The first term in the right-hand-side equation (3.17) is the amount borrowed by bank$_i$ from bank$_j$, over time dt. As in the two-bank model, this amount should be the minimum value between the required cash of bank$_i$, $(rD_i - C_i)^+$, and the available cash of bank$_j$, $\left(C_j - rD_j\right)^+$. The difference with the two-bank model is that, this minimum value needs to be multiplied by the parameter, σ_{ij}, which represents the connection between bank$_i$ and bank$_j$. This is to ensure that the borrowing only happens between the two banks that are connected. If the two banks are not connected ($\sigma_{ij} = 0$), then there is no borrowing.

The second term in equation 3.17 is the proportion, α_{ij}, of the total borrowing repaid, per unit time, by bank$_i$ to bank$_j$ during the current period of time. When a bank fails, it cannot repay its borrowing to the other banks and it cannot receive previous lending back from other banks. Therefore, to make sure the repayment only works for the survival banks, the repayment term, $\alpha_{ij} B_{ij}$, needs to be multiplied by the parameters, s_i and s_j, which represent the mode of the bank. If one of the two banks failed ($s_i = 0$ or $s_j = 0$), then there is no repayment, ($s_i\, s_j\, \alpha_{ij} B_{ij} = 0$).

The total lending bank$_i$ gives to bank$_j$ is updated in a similar way as shown in equation (3.18):

$$\frac{dL_{ij}}{dt} = min\left[\left(rD_j - C_j\right)^+,\ (C_i - rD_i)^+ \right] \sigma_{ij} - s_i\, s_j\, \alpha_{ji} L_{ij} \qquad (3.18)$$

The total borrowing and lending of bank$_i$, B_i and L_i, are represented in equation (3.19) and equation (3.20):

$$B_i = \sum_{j \neq i} B_{ij} \qquad (3.19)$$

$$L_i = \sum_{j \neq i} L_{ij} \qquad (3.20)$$

Interest rates

In the multi-bank model the interest rates for borrowing and lending are not constant; h_{ij} and k_{ji}, in fact, change with time according to equation (3.21).

$$h_{ij} = \; k_{ji} = h_0 + \frac{a}{e^{\left(y - \frac{B_{ij}}{C_j}\right)z} + 1} \tag{3.21}$$

In equation (3.21), h_0 is the basic interest rate applied for lending and borrowing. This can be thought of as the base interest rate determined by an exogenous monetary authority.

The term, $\frac{a}{e^{\left(y - \frac{B_{ij}}{C_j}\right)z} + 1}$, is the premium charged depending on the health of both borrowing and lending banks, which is measure with $\frac{B_{ij}}{C_j}$; B_{ij} is the total borrowing of bank$_i$ from bank$_j$ and C_j is the total cash of bank$_j$. The larger ratio, $\frac{B_{ij}}{C_j}$, the more risk the borrowing brings to the lending bank. Figure 3.3 illustrates the behaviour of h_{ij} for different values of $\frac{B_{ij}}{C_j}$. When $\frac{B_{ij}}{C_j} = 0$, the interest rate is close to h_0 . When $\frac{B_{ij}}{C_j} = y$, the rate becomes $h_{ij} = h_0 + \frac{a}{2}$. When $\frac{B_{ij}}{C_j} \rightarrow +$ *infinity*, the rate becomes $h_{ij} = h_0 + a$, which is the maximum value possible. Therefore, as the $\frac{B_{ij}}{C_j}$ gets larger, the lending bank faces more risk, thus it needs to charge higher interest rates. z represents the speed of transition between the states $h_{ij} = h_0 + \frac{a}{2}$ and $h_{ij} = h_0 + a$. The larger the values of z, the faster the interest rate switches. The values of the parameters, a, y, z, used in our simulations are reported in the following chapter.

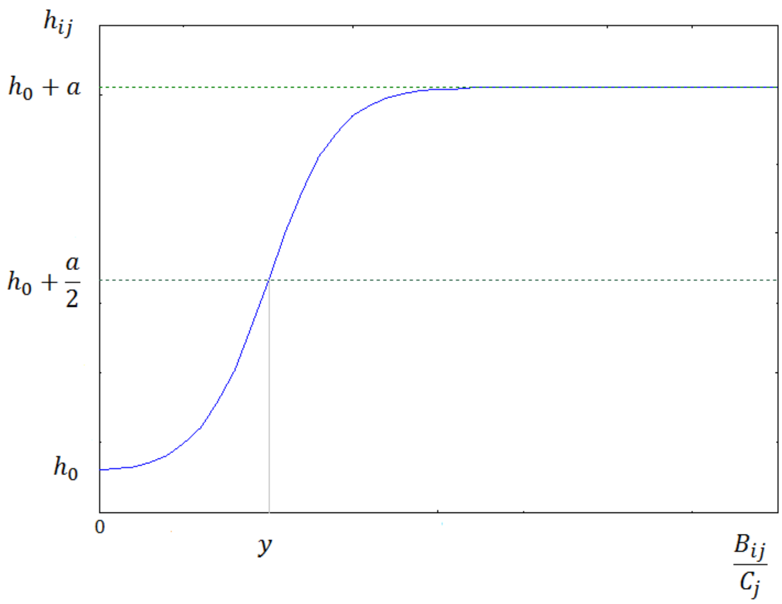

Figure 3.3 Behaviour of h_{ij} as function of $\frac{B_{ij}}{C_j}$.

Thus, interest rates are endogenous to the fluctuations in balance sheets, although in a simple mechanical way; our approach, in fact, abstracts from a more complex modelling of asset prices based on optimising decisions and taking risk into account. This helps us to keep the model simple in our investigation of the stability of the system.

Interbank borrowing and lending process

Once the interest rates are set, the interbank borrowing and lending process works as follows: the bank with greatest net worth can first choose the bank to borrow money from. The borrowing bank will choose the bank with the lowest lending interest rate and, if the available borrowing is not enough, it will move to the bank with the second lowest lending rate. When the first bank has finished borrowing, the bank with the second greatest net worth starts to borrow according to the same rule. Again, this sequential form of borrowing and lending is a simplification of the simultaneous trading structure in interbank markets, but reflects the fact that certain banks will find it easier to finance their liquidity needs in the interbank market than others. A more complex structure could be introduced where banks post their financing needs simultaneously and are matched according to their financing capacities and prevailing market rates.

Investments

As it in the two-bank model, equation (3.22) below describes the investment behaviour of bank$_i$; each bank makes its investment at time t depending on two factors: one is the availability of cash above the value required by the reserve ratio after interbank lending, $\left(C_i - rD_i - \sum_{j\neq i}\frac{dL_{ij}}{dt}\right)^+$; where $(x)^+$ stands for $\max\{x, 0\}$. The second factor is the stochastic investment opportunity at time t, opp_i; this is described in equation (3.23) where $\overline{opp} = \delta\overline{D}$ (with $0 < \delta < 1$ and $\eta_t \sim N(0,1)$), which means that the investment opportunity is affected by the size of the bank. Therefore, taking these two factors into consideration, a bank invests only when it has sufficient cash and investment opportunity. In equation (3.22), $-w_i I_i$ represents the proportion of total investment, per unit time, that has matured. And $v_i I_i$ represents the proportion of total investment that has been lost, per unit time.

$$\frac{dI_i}{dt} = \min\left[\left(C_i - rD_i - \sum_{j\neq i}\frac{dB_{ij}}{dt}\right)^+, opp_i\right] - w_i I_i - v_i I_i \tag{3.22}$$

$$opp_i = \left|\overline{opp} + \overline{opp}\,\sigma_{opp}\eta_t\right| \tag{3.23}$$

Cash

The differential equation governing the change in time of the cash, $\frac{dC_i}{dt}$, of bank$_i$ is reported in equation (3.24).

$$\begin{aligned}\frac{dC_i}{dt} = \frac{dD_i}{dt} - \frac{dI_i}{dt} - g_i D_i + p_i I_i - v_i I_i + \sum_{j\neq i}\frac{dB_{ij}}{dt} - \sum_{j\neq i} h_{ij}B_{ij} \\ - \sum_{j\neq i}\frac{dB_{ji}}{dt} + \sum_{j\neq i} h_{ji}B_{ji}\end{aligned} \tag{3.24}$$

In equation (3.24), $\frac{dD_i}{dt}$ represents the change cash due to changes in the deposit, while $-\,g_i D_i$ represents the reduction in cash over time due to the payment of interest to depositors. Similarly, $-\,\frac{dI_i}{dt}$ represents the reduction in cash due to new investments and $p_i I_i$ is the increase in cash due to the receipt of interest from investments, while $-v_i I_i$ represents the proportion of total investments that has been lost, per unit of time, due to defaults. $\sum_{j\neq i}\frac{dB_{ij}}{dt}$ represents the increase in cash accrued from interbank borrowing and $-\sum_{i\neq j} h_{ij}B_{ij}$ is the reduction in cash due to the payment of interests to lending banks. $-\sum_{j\neq i}\frac{dB_{ji}}{dt}$ represents the reduction in cash due to loans to other banks and $\sum_{j \neq i} h_{ji} B_{ji}$ is the increase of cash due to the receipt of interest from borrowing banks. g_i, p_i, h_{ij} and k_{ij} represent interest rates. Importantly, when C_i becomes non-positive, bank_i fails and is removed from the system.

Net-worth

The net-worth of bank_i, N_i, is the sum of the total cash, investment and lending, minus the sum of the total deposit and borrowing, as shown in equation (3.25):

$$N_i = C_i + L_i + I_i - D_i - B_i \tag{3.25}$$

Simulation of the model

Figure 3.4 shows the flowchart of the model illustrating how the banks' activities take place during each step of the computer simulation. At the beginning of each step, the banks' cash changes due to interest payments to depositors and changes in deposits due to stochastic shocks. If the cash of a bank falls below the value required by the reserve ratio that bank has to borrow from other banks. After this step, each bank repays creditors in cash. Those banks that cannot meet the repayment obligations will need to borrow from other banks. Banks that still have extra cash will invest. Those banks that are left with negative cash, as they could not borrow enough cash, are deemed to be in default; these banks are removed from the system and their remaining assets are distributed to depositors and to lending banks. After any default liquidation, a new simulation step will start. At the end of the simulation, i.e. after a chosen number of steps, the banks that survived are counted and other relevant quantities are calculated.

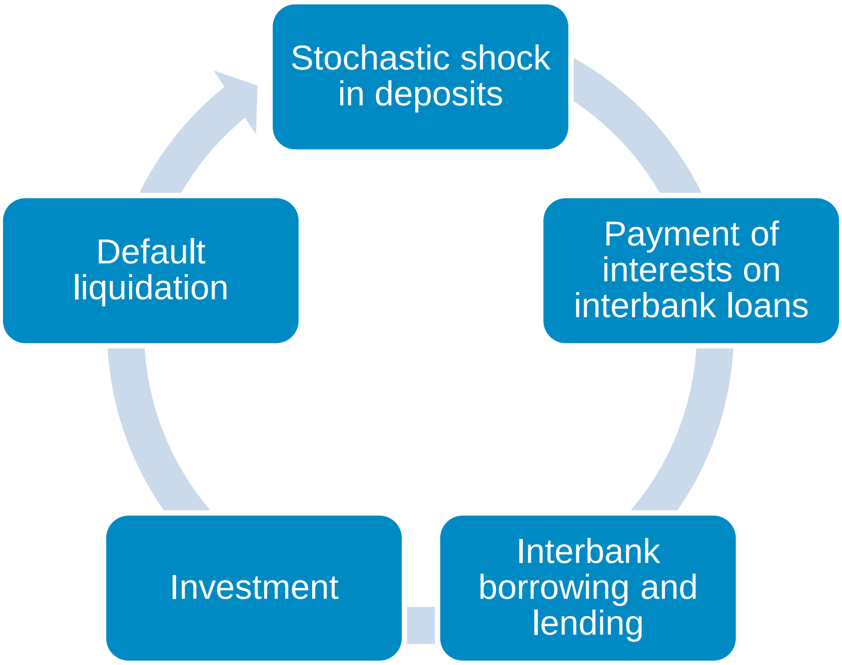

Figure 3.4 Flowchart showing activities taking place during one simulation step.

Units of the variables and parameters used in the simulation

This subsection reports the units for the values of the variables and parameters of the model used in the simulations presented in Chapter 4. Since the interest rates, i.e. g, p, v, h, k, alpha are expressed as daily rates, the time unit used in the simulations is day. The values of variables representing money, i.e. D, C, I, B, L and opp, are expressed in arbitrary units of money. Table 3.1 reports the units for all the variables and parameters used in the simulation. Unless stated otherwise, the units of the values of the variables and parameters are not reported in the remaining of the thesis.

Table 3.1 Units for all the variables and parameters used in the simulation.

Variable/parameter	Unit
D	arbitrary unit of money
C	arbitrary unit of money
I	arbitrary unit of money
B	arbitrary unit of money
L	arbitrary unit of money
opp	arbitrary unit of money
g	day^{-1}
p	day^{-1}
v	day^{-1}
h	day^{-1}

k	day^{-1}
α	day^{-1}
a	day^{-1}
r	pure number (between 0 and 1)
l_r	pure number (between 0 and 1)
z	pure number
y	pure number

3.3 Conclusion

A new dynamic network model of the banking system has been presented in this chapter. The dynamic structure of the banking system is modelled using ordinary differential equations, which prescribe how bank activities (e.g. cash, investment and borrowing) change with time. The proposed model not only allows carrying out simulations that describe how banks behave, but also allows incorporating feedback mechanisms typical of control theory. Ordinary differential equations are more suitable to facilitate the application of control theory tools compared to discrete models [112]. In the next chapter, Chapter 4, the differential equations are solved using MATLAB Simulink in the form of computer simulations, which show how banks receive deposits, invest, exchange money with each other and occasionally fail. In Chapter 5, control mechanisms are applied to the model to prevent banks from failing.

Chapter 4

Simulation Results of the Dynamic Network Model

This chapter presents the results of numerical simulations for the dynamic models described in Chapter 3. The results are generated using MATLAB Simulink implementation of the models; details of the implementation are provided in this chapter.

This chapter is structured as follows: Section 4.1 introduces the software packages MATLAB and Simulink. Section 4.2 introduces the implementation of the one-bank model in Simulink and its numerical simulations results with different initial conditions and parameter values; Section 4.3 shows the implementation and results of the two-bank model while Section 4.4 presents the multi-bank model which is made of 50 banks. Finally, Section 4.5 concludes the chapter.

4.1 Introduction of MATLAB and Simulink

MATLAB (Matrix Laboratory) is a proprietary programming language developed by MathWorks, which allows matrix manipulations, plotting of functions and data, implementation of algorithms, and interfacing with programs written in other languages, including C, C++, C#, Java [118]. Simulink is a commercial tool also developed by MathWorks for modelling, simulating and analysing dynamical systems [119], which can interface with the rest of the MATLAB environment. Simulink has integrated solvers that can numerically approximate the solutions of the differential equations which represent dynamical systems; differential equations are expressed graphically in Simulink as block diagrams. The reason to choose Simulink for the simulations in this Ph.D. project is that it allows to analyse efficiently the solutions of complicated systems modelled with differential equations that may be difficult or impossible to deal with analytically; moreover, Simulink is widely used in control theory for simulation and design.

4.1.1 General process of the simulation

Figure 4.1 is a flowchart showing the general process used to run the simulations. Different MATLAB scripts are written to generate the initial conditions (for cash, investment, borrowing and lending), exogenous signals (such as deposit, D, and investment opportunity, opp) and parameters values (such as w, v, p and g), which can be used into the Simulink model. The Simulink implementation solves the differential equations presented in Chapter 3 with given initial conditions and parameter values (details are explained later in this section), and the results are outputted in the MTALAB workspace. Finally, other MATALAB scripts are used to process and plot automatically the results generated by Simulink; this facilitates the analysis of the results.

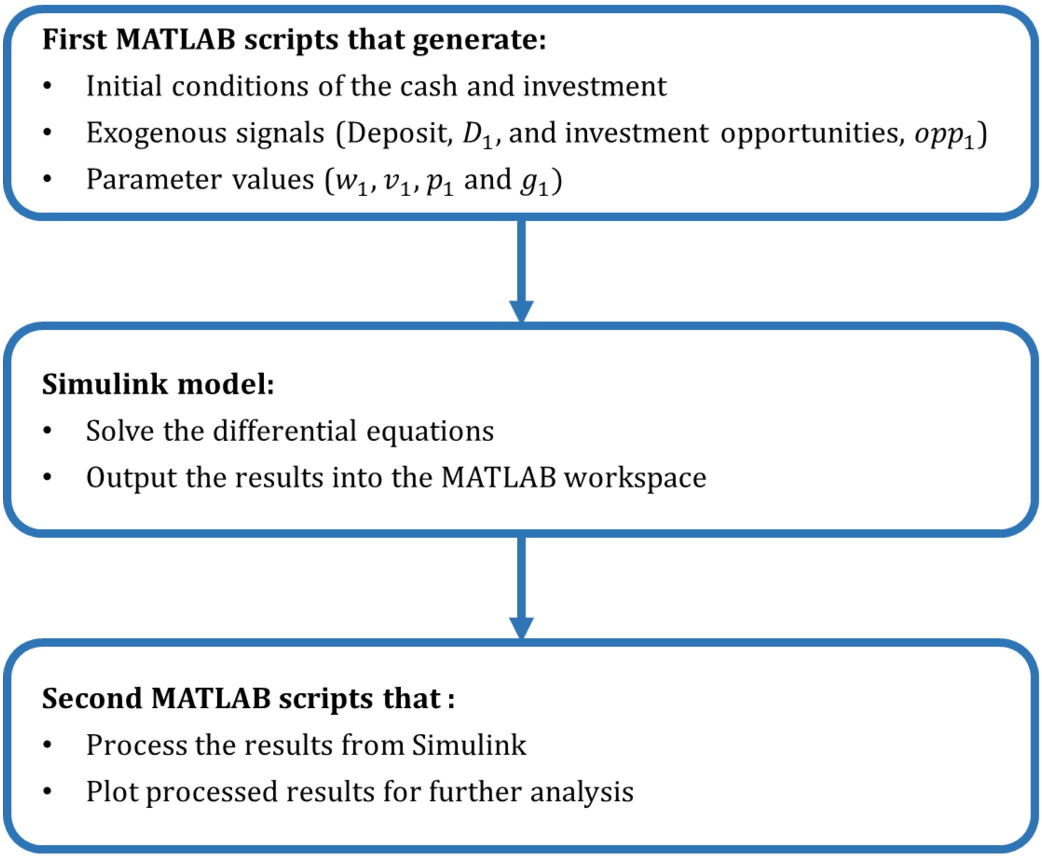

Figure 4.1 Flowchart showing the general processes used to run a simulation.

4.1.2 Numerical solver of differential equations

A dynamical system is simulated by computing its states (i.e. the values of the variables characterising the system) at successive time steps over a specified time span. This is done by numerically solving the differential equations representing the system; the calculations of the solutions are performed by a chosen solver in Simulink. The flow chart in figure 4.2 shows the steps used to select a solver.

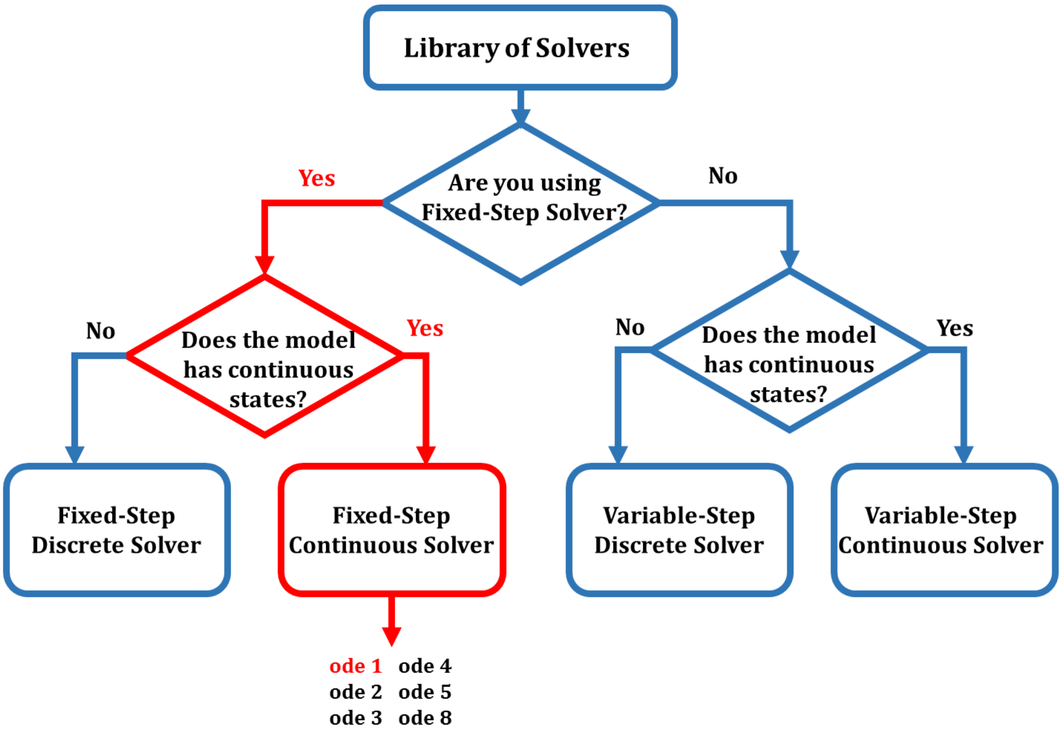

Figure 4.2 Flow chart showing how to choose the solver. Red route indicates the selected solver.

The step size of the time steps used to compute the states of the system can be constant or can vary during the simulation; accordingly, fixed-step solvers or variable-step solvers can be used respectively [120]. Fixed-step solvers compute the states of the system at constant time intervals from the beginning to the end of the simulation. Variable-step solvers can reduce the step size to increase accuracy when the states of the system change rapidly; when the states of the system change slowly, instead, they can decrease the step size to save computational time. From preliminary simulations, it has been concluded that using a fixed time step of 0.01 day allows the solution of the differential equations in the model with sufficient accuracy; smaller time steps, in fact, produce nearly identical results. The choice of the time step is 0.1 day, which also allows running the simulation within practical periods of time (the longest simulation took 5 hours of CPU time on a commercial PC). Besides the step size, the solvers are also classified as continuous or discrete. As the proposed model of the banking system is a continuous system, a continuous solver has been chosen for the simulations. Finally, the fixed-step continuous solvers 'ode1', which uses Euler's method [121] to solve ordinary differential equations, has been chosen as it uses less computational time compared to other solvers, while its accuracy is not significantly different from other more-time-consuming solvers (as found out from preliminary tests).

4.2 One-bank model

In this section, the implementation of the one-bank model is explained in detail and the corresponding results are presented. The first subsection explains the Simulink block diagram of the one-bank model while the second subsection reports the results and related discussion.

4.2.1 Implementation of the one-bank model in Simulink

The Simulink block diagram of the one-bank model is shown in figure 4.3. Two exogenous signals, total deposit (D_1) and the investment opportunity (opp_1), are two functions of time and are generated using MATLAB codes; they are imported into the Simulink block diagram through the red blocks in figure 4.3. These red blocks import the values of the variables from MATLAB workspace where all generated values are saved. The light blue blocks output the simulation results that are generated from Simulink. Parameters such as w_1, v_1, p_1 and g_1 (proportion of matured investment, proportion of failed investment, return rate of investment and the inetrest rate of deposits respectevely) are used in the yellow blocks, which can multiply the input (any signal imported from the left side of the block) by a constant value (here the values of w_1, v_1, p_1 and g_1). The differential equations are solved by the integrator in Simulink which is the green block in figure 4.3. There are two differential equations in the one-bank model, one is for cash, equation (3.4), and another one is for investment equation (3.2); therefore, there are two green blocks in the Simulink block diagram. Details of how the integrator works are explained in the following paragraph.

The block named 'Integrator 1' in the figure 4.3 integrates the time derivative of the investment, which is represented by equation (3.2). The signal imported in the integrator is the change of the investment $\frac{dI_1}{dt}$, while the signal exported is the total investment I_1. There are three parts in equation (3.2), $-w_1 I_1$, $-v_1 I_1$ and $\min[(C_1 - rD_1)^+, opp_1]$, so the block 'subtract 1' in figure 4.3 gathers these three inputs and imports them into the integrator. The first two inputs, $-w_1 I_1$ and $-v_1 I_1$, are generated by using the total investment, which is exported from the 'Integrator 1' and are multiplied by w_1 and v_1. The third input, $\min[(C_1 - rD_1)^+, opp_1]$, is generated as follows: first using the total cash (which is exported after the integrator 2) minus the product of the two blocks named 'Deposit 1' and' r'; this results into $C_1 - rD_1$, which is imported into a 'max' block to be compared with 0. The 'max' block outputs the maximum value of these two, i.e. $(C_1 - rD_1)^+$; this signal is then imported into a 'min' block, $\min[(C_1 - rD_1)^+, opp_1]$, which compares it with opp_1. Similar implementation is done for equation (3.4) for the time derivative of the cash. Five inputs are imported into the 'subtract 2' block, which correspond to the five terms in equation (3.4), and then they are inputted into the 'Integrator 2',

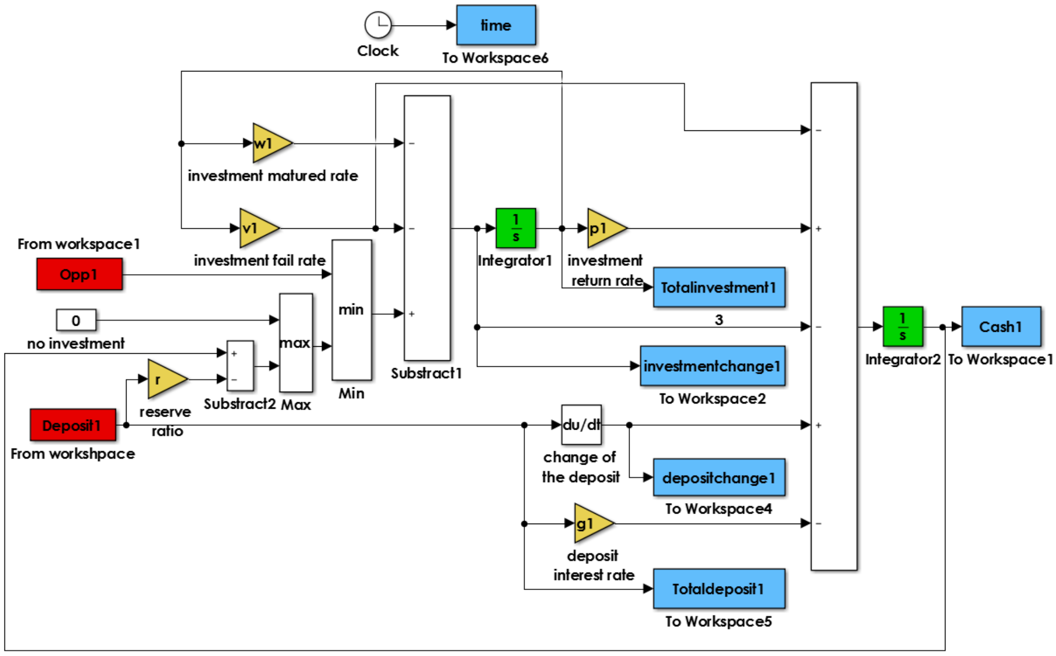

Figure 4.3 The Simulink block diagram of the one-bank model.

4.2.2 Simulation results of one bank model

In this section, the results of the one-bank model are presented and analysed, in order to show how a single bank is affected by the different parameters of the model. This analysis facilitates the understanding of the more complex models where more than one bank is present. The simulation results of one-bank model are generated using the Simulink block diagram described in the previous section. In the simulations, the unit time is one day and the total simulation period is 10 days, which is a sufficient time period to show the simple dynamics of the one-bank model. The step size is set to 0.1, which means that the bank does all its daily activities in 10 time steps (we run some test simulations with smaller time steps and noticed that the results did not change significantly). The parameters, w_1, v_1, p_1 and g_1, have the dimension of time^{-1}, so their values are expressed as day^{-1}. Results from six different simulations are presented in this section to illustrate how the bank's cash and investment are affected by a shock in the deposit under different conditions: 1) all the parameters, w_1, v_1, p_1 and g_1 equal to zero, 2) interest rate of the deposit bigger than zero ($g_1 > 0$), 3) the return rate of the investment bigger than zero ($p_1 > 0$), 4) the proportion of matured and lost investment bigger than zero ($w_1 > 0$ and $v_1 > 0$) and 5) investment opportunity (opp_1) is smaller than available cash, 6) larger shock in the deposit that makes the bank to fail.

Simulation results 1: effect of the shock in deposit

This first type of simulation is used to show the bank's behaviour following a negative shock in the deposit. The details of initial conditions and parameter settings are shown in table 4.1. To study the effect of the deposit only, the parameter values of g_1, p_1, w_1 and v_1 are set to 0 to disregard the effect of deposit interest payments, investment returns, matured investments and lost investments. The reserve ratio r is set to 0.2; the effect of different values of the reserve ratio will be analysed with more complex models. In table 4.1, the values of cash (C_1), deposit (D_1), investment (I_1) and investment opportunity (opp_1) represent money and they are reported in arbitrary units. The value of the cash in table 4.1 is set to 0.25 in order to be close to the reserve ratio requirement (rD_1=0.2×1=0.2). The values in table 4.1 for the investment and the investment opportunity are chosen large enough to allow for investment activities.

Table 4.1 Initial conditions and parameter values used in simulation 1.

Initial conditions	Parameter value
$C_1 = 0.25$	$g_1 = 0\,, p_1 = 0$
$I_1 = 0.75$	$w_1 = 0\,,\ v_1 = 0$
$D_1 = 1,\ opp_1 = 1$	$r = 0.2$

Figure 4.4 shows the behaviour of the bank's total deposit (a), total investment (b), added investment (c) and cash (d). It can be seen from figure 4.4(a) that at the beginning the deposit is set to 1, then the deposit decreases linearly to 0.94 at day 4 as the result of the shock, and subsequently it goes back to 1 linearly at day 6. According to the behaviour of the deposit, the dynamics of the system can be analysed in the following three periods.

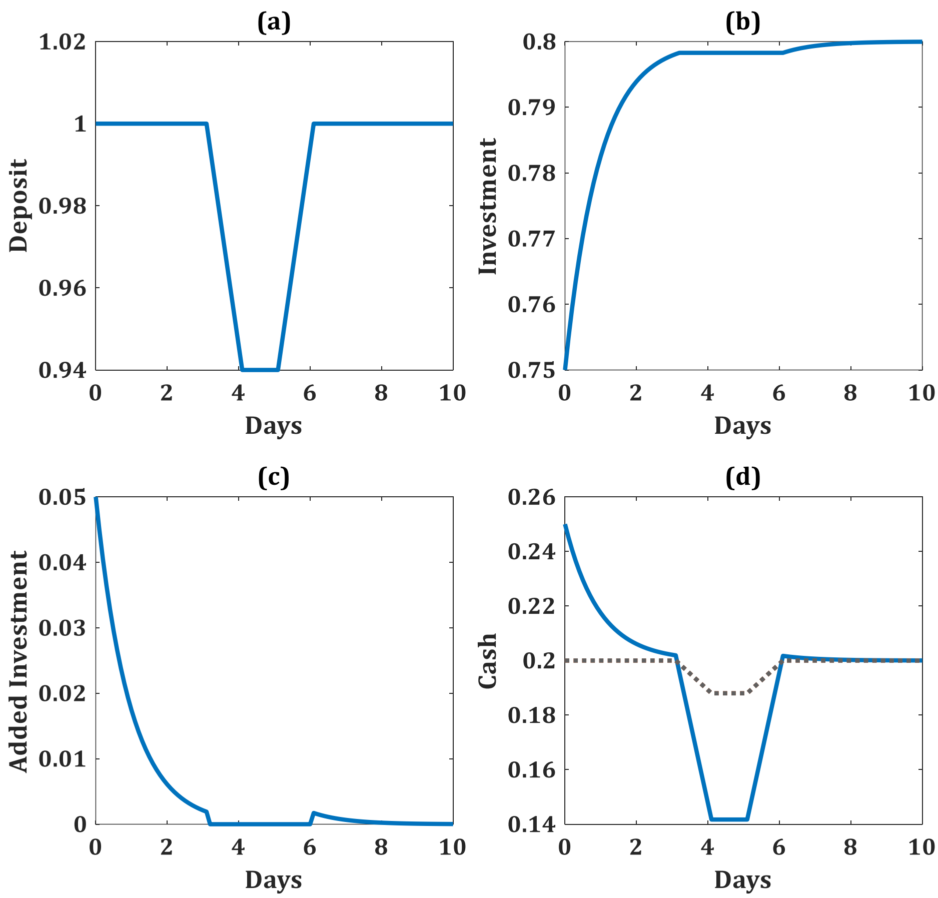

Figure 4.4 Dynamics of the bank's deposit (a), total investment (b), added investment (c), cash (blue line in (d)) and the reserve ratio requirement (grey dash line in (d)), when $g_1 = 0\,, p_1 = 0, w_1 = 0\,,\ v_1 = 0$ and $r = 0.2$.

Period 1 (day 1- day3): when the deposit is at 1, the cash in figure 4.4(d) (blue line) is above the reserve ratio requirement (grey dash line). Therefore, in this period, the bank has extra cash to invest. This can be seen both from figure 4.4(b) where the total investment increases and from figure 4.4(c) where the added investment is above zero. As the bank keeps increasing the investment, the bank's cash decreases, which causes the added investment to decrease and total investment to increase more slowly.

Period 2 (day 3- day5): when the deposit drops linearly from 1 to 0.94, the cash also drops and becomes lower than the reserve ratio requirement (the blue line goes under the grey dash line in figure 4.4(d)). During this period, the bank stops investing because of the lack of cash. Therefore, the added investment becomes zero (figure 4.4(c)) and the total investment stays constant (figure 4.4(b)).

Period 3 (day 5- day10): when the deposit starts to go back to 1, the cash starts to increase as well and after some time becomes higher than the reserve ratio requirement (at day 6 in figure 4.4 (d) - the blues line goes above the grey dash line). The bank starts to invest again at this period, therefore both the added investment in figure 4.4(c) and the total investment in figure 4.4(b) start to increase on day 6.

From the analysis of the three periods, it can be concluded that, the cash changes at the same rate as the deposit and that the negative shock in the deposit can force the cash to decrease which causes the bank's added investment to decrease down to zero if the shock is large enough.

Simulation results 2: effect of paying deposit interest

In this simulation, the same deposit signal as in simulation 1 is used. To study the effect of g_1, the interest rate of the deposit, all the initial conditions and other parameters values are kept as in table 4.1, but g_1 is set to 0.5/360 - this value may not be realistic as 50% annual interest rate would be too high in a real scenario, but it is used for testing purpose only to get an observable difference in the results. Figure 4.5 shows the dynamics of the bank when $g_1 = 0.5/360$ (in red dashed lines) compared to the dynamics when $g_1 = 0$ (blue lines). Looking at figure 4.5(d), the payment of interest decreases the cash as the red line is always below the blue line. This causes the decrease in add investment as it can be seen in figure 4.5 (c) - the red line goes to zero after some time, which means that due to the interest payments there is no available cash for investment. Figure 4.5(b) shows that the total investment (red line) doesn't increase after some time because there is no new added investment.

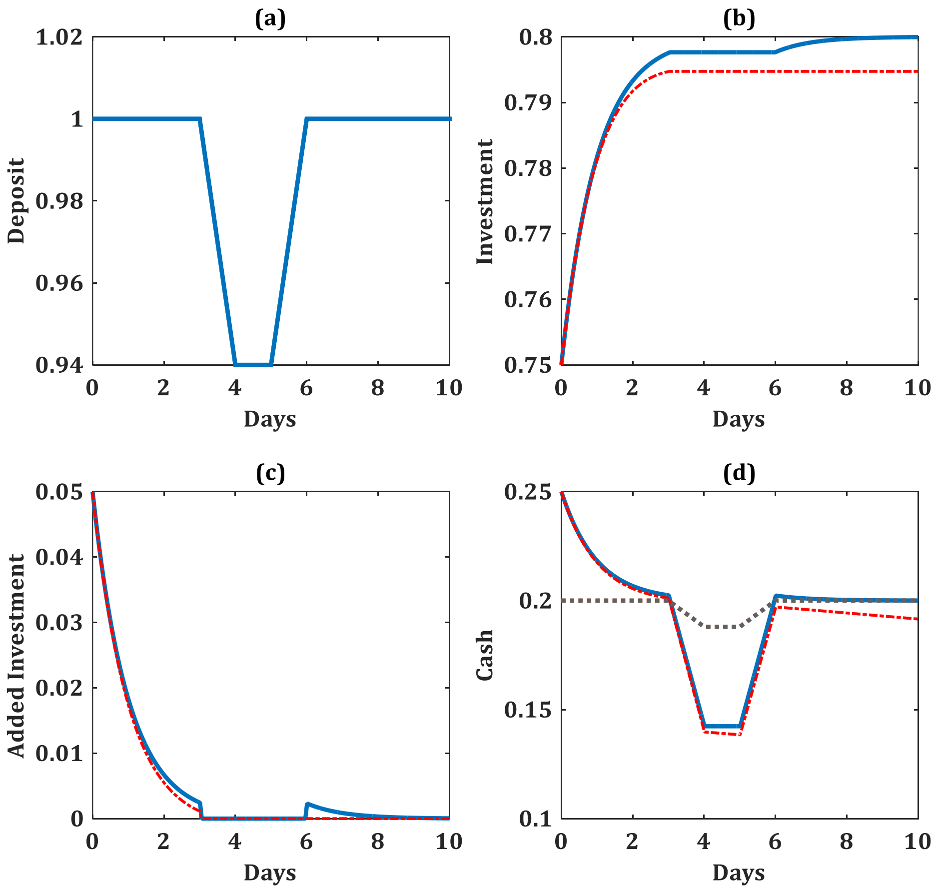

Figure 4.5 Dynamics of bank's deposit (a), total investment (b), added investment (c) and cash (d) for two different cases: $g_1 = 0$ (blue lines) and $g_1 = 0.5/360$(red dash dot lines). The grey dash line in (d) is the reserve ratio requirement; $p_1 = 0$, $w_1 = 0$, $v_1 = 0$ and $r = 0.2$.

Simulation results 3: effect of investment returns

To study the effect of investment returns, $p_1 I_1$, a similar simulation as the previous ones is performed, in which the investment return rate is set to a non-zero value ($p_1 = 0.5/360$) while the other values are kept as in table 4.1. In this situation the bank can earns returns (as cash) from its investment. Figure 4.6 shows the dynamics of the bank with the investment returns (red lines) compared to the dynamics without the returns (blue lines). Obviously, the investment returns increase the cash and therefore increases the added investment, so in figures 4.6(b), (c) and (d) the red lines are always above the blue lines.

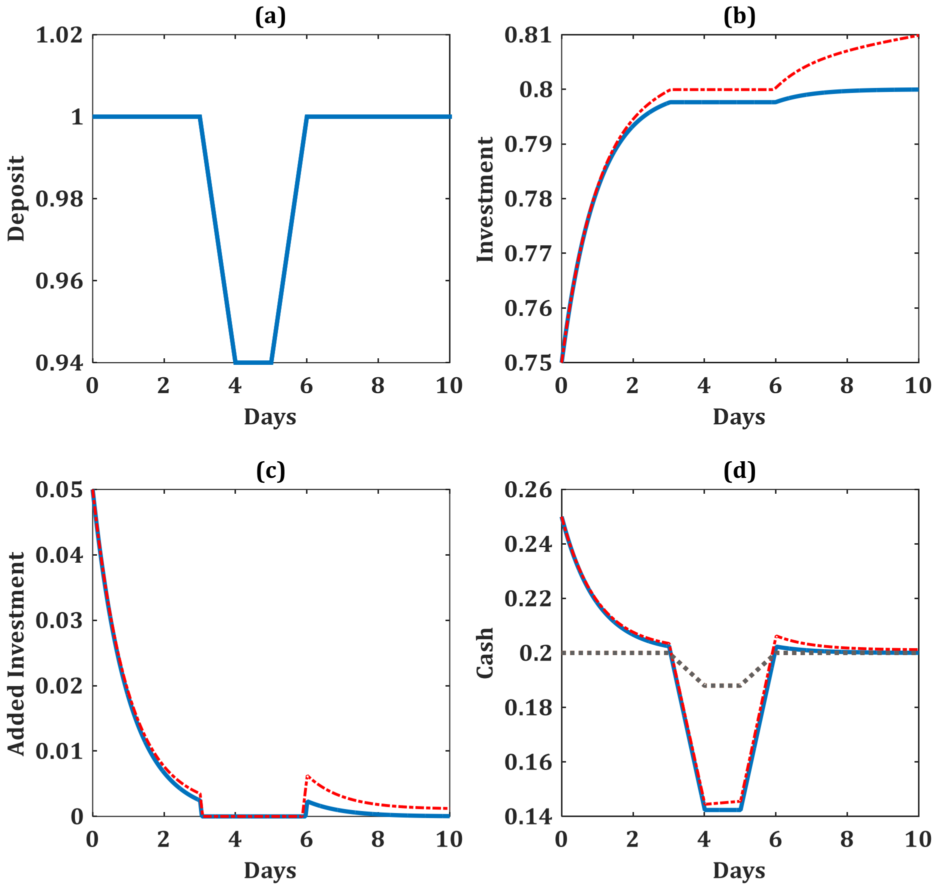

Figure 4.6 Dynamics of the bank's deposit (a), total investment (b), added investment (c), cash (d) for two different cases: $p_1 = 0$ (blue lines) and $p_1 = 0.5/360$ (red dash dot lines). The grey dash line in (d) is the reserve ratio requirement ; $g_1 = 0$, $w_1 = 0$, $v_1 = 0$ and $r = 0.2$.

Simulation results 4: effect of matured and failed investments

In this simulation, the effects of matured and failed investments are studied by manipulating the values of the parameter w_1 and v_1, which represent the proportion of the total investment that matured and failed, respectively. All the other values are the same as in table 4.1. Figure 4.7 represents the dynamics of the bank for three different cases: the first case is when $w_1 = 0$ and $v_1 = 0$, which means there is no matured and failed investment - results of this case are shown in blue lines; the red line in figure 4.7 represents the results when $w_1 = 1/360$ and $v_1 = 0$, i.e. 100% annual matured investment rate and no failed investment; the green line, instead, represents the results when $w_1 = 1/360$ and $v_1 = 0.2/360$, i.e. 100% annual matured investment rate and 20% failed investment rate.

Figures 4.7(d) and (c) show that the red lines are above the blue lines, which means that the matured investment increases the cash and then increases the added investment, as the bank gets back the investment as cash. Nevertheless, as the matured investment is subtracted from the total investment, the total investment decreases (see figure 4.7(b)). The green lines overlap the red lines in figure 4.7(c) and (d), which means that the failed investment doesn't affect significantly the added investment and cash, in the case analysed here. The only effect failed investment brings is the decrease in the total investment, in fact, in figure 4.7(b) the green line is below the red line. This is because the failed investment is subtracted from the total investment. Moreover, the failed investment doesn't have a direct relationship with the added investment, so the added investment doesn't change.

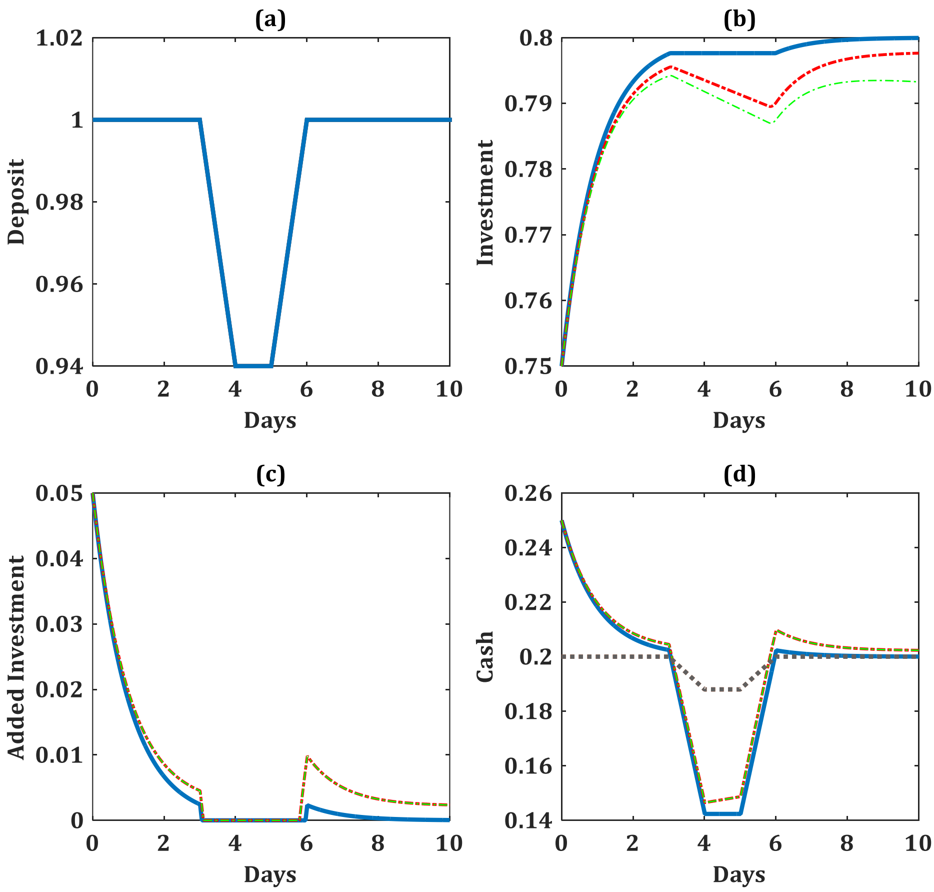

Figure 4.7 Dynamics of the bank's deposit (a), total investment (b), added investment (c), cash (d) for three different cases: $w_1 = 0$ and $v_1 = 0$ (blue lines), $w_1 = 1/360$ and $v_1 = 0$ (red dash dot lines), $w_1 = 1/360$ and $v_1 = 0.2/360$. The grey dash line in (d) is the reserve ratio requirement; $g_1 = 0$, $p_1 = 0$ and $r = 0.2$ are the same for the three cases.

Simulation results 5: effect of investment opportunities

Besides the deposit, the investment opportunity is another exogenous signal that can affect the bank as it represents an upper boundary for the added investment; in fact, the added investment is the minimum value of the investment opportunity, opp_1, and the available cash, $(C_1 - rD_1)^+$, see equation (3.2) in Chapter 3. Figure 4.8 shows the behaviour of the added investment (blue line) when the investment opportunity (red dash line) is set to 0.01 during the first two days and then increases to 0.2; the green line represents the amount of available cash $(C_1 - rD_1)^+$.

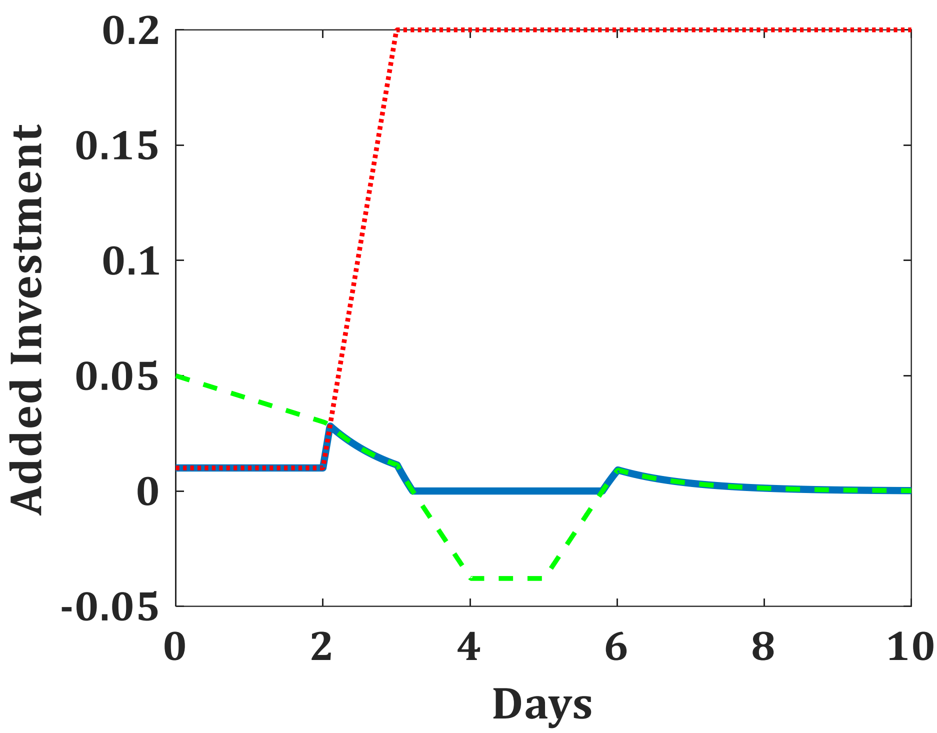

Figure 4.8 Dynamics of added investment when $g_1 = 0$, $p_1 = 0$, $w_1 = 0$, $v_1 = 0$, $r = 0.2$ and opp_1 follows the curve represented by the red dot line. The green dash line represents the amount of available cash, $(C_1 - rD_1)^+$.

In this simulation, the deposit undergoes a shock as in the previous cases. It can be seen that the blue line first follows the red, then follows the green line, then becomes zero and finally follows the green line again. This is because during the first two days the investment opportunity is smaller than the available cash, so the added investment is limited by the investment opportunity. After two days, the opportunity increases and exceeds the available cash, so the added investment is limited by the available cash. Because of the shock in the deposit, the cash drops below the reserve requirement and the bank is unable to invest, therefore there is a period of time in which the added investment is 0.

Figure 4.9 reports further results of simulation 5, to see the effects of the investment opportunity on other activities; more specifically, figure 4.9 compares the case (blue lines) in which there is plenty of opportunities ($opp_1 = 1$) with the case (red lines) in which the investment opportunity changes with time, as in figure 4.8. As shown in figure 4.9(c), during the first two days the red line (non-constant opportunity) is below the blues line ($opp_1 = 1$) because the added investment is limited by the investment opportunity and fewer investments are made. This causes the total investment to increase more slowly and the cash to decrease more slowly for the non-constant opportunity case, so in figure 4.9(c), the red line is below the blue line while in figure 4.9(d) the red line is above the blue line.

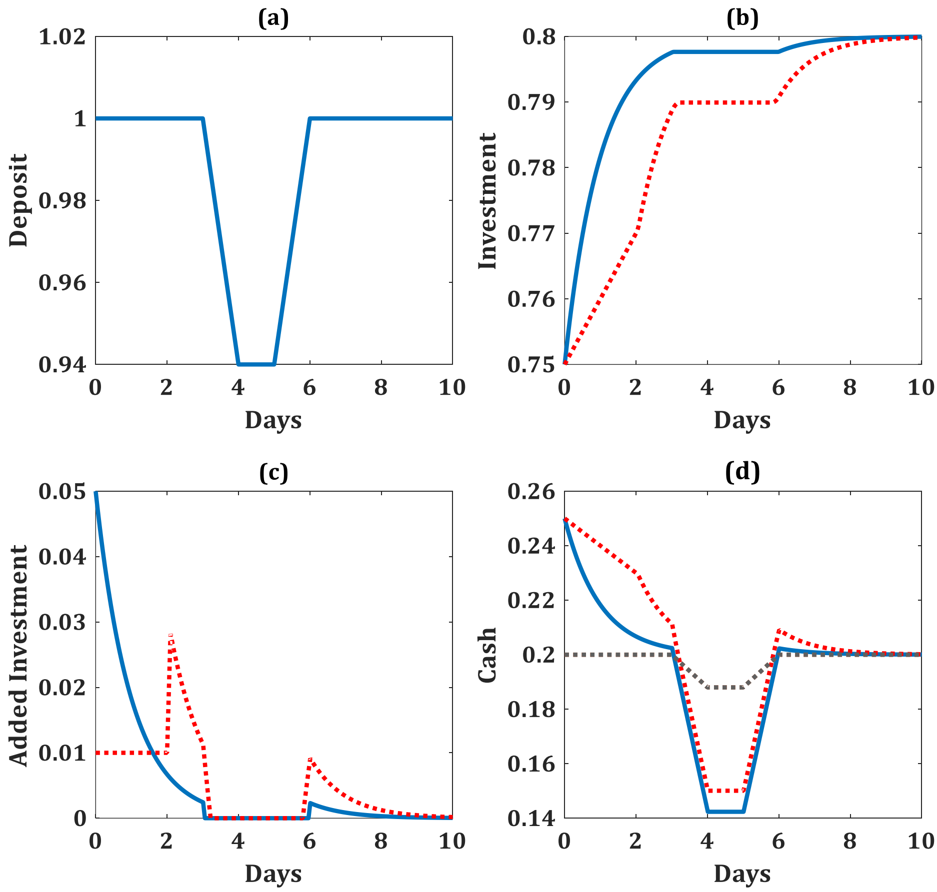

Figure 4.9 Dynamics of the bank's deposit (a), total investment (b), added investment (c), cash (d) for two different cases: (1) $opp_1 = 0.01$ *when* $0 \le t \le 2$ *and* $opp_1 = 1$ *when* $2 \le t \le 10$ (blue lines). (2) $opp_1 = 1$ *when* $0 \le t \le 10$ (red dash dot lines). The grey dash line in (d) is the reserve ratio requirement; $g_1 = 0$, $p_1 = 0$, $w_1 = 0$, $v_1 = 0$ and $r = 0.2$.

Simulation results 6: large shock in the deposit

This subsection presents the case in which the bank fails; figure 4.10 reports the results of the simulation in which a shock in the deposit is large enough to make the cash negative and consequently the bank to fail. The deposit in figure 4.10(a) drops at day 3 to a level that causes the cash (shown in figure 4.1(d)) to decrease to a negative value and as result of this, the bank fails at day 4. This is a situation in which the shock in the deposit is too large for the bank to buffer it by itself. In the next sections, we introduce cases with more than one bank in the system to see whether borrowing between banks can reduce the occurrence of bank failure. Furthermore, in Chapter 5 we analyse the case in which a control mechanism is developed and implemented in the one-bank model to make the bank more resilient to shocks.

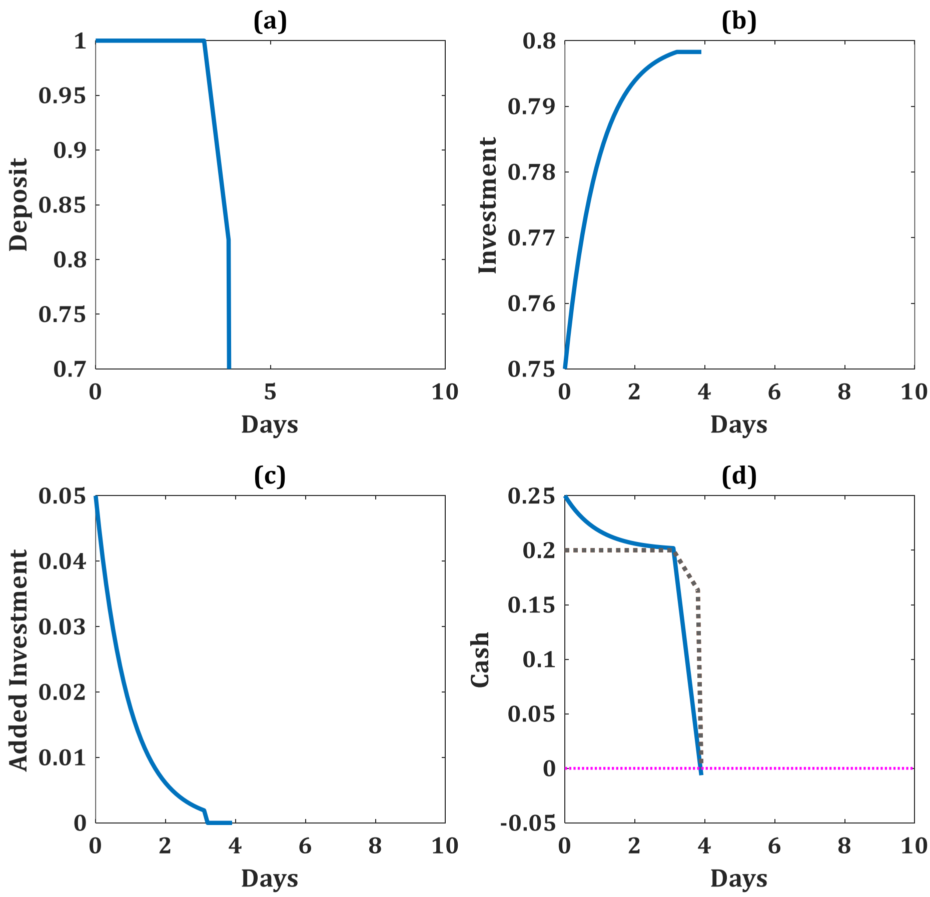

Figure 4.10 Dynamics of the bank's deposit (a), total investment (b), added investment (c), cash (blue line in (d)) and the reserve ratio requirement (grey dash line in (d)), when $g_1 = 0$, $p_1 = 0$, $w_1 = 0$, $v_1 = 0$ and $r = 0.2$. The deposit undergoes a large shock causing the cash to fall below zero and the bank to fail at day 4.

4.3 Two-bank model

In this section the implementation and simulation results of the two-bank model are presented in detail. The first subsection explains the Simulink block diagram of the two-bank model. The second subsection shows the simulation results to illustrate the effect of borrowing on the banks' behaviour.

4.3.1 Implementation of the model in Simulink

Figure 4.11 shows the Simulink block diagram of the two-bank model. The deposit and investment opportunity signals for each bank are imported using the same blocks as in the one-bank model (red blocks).

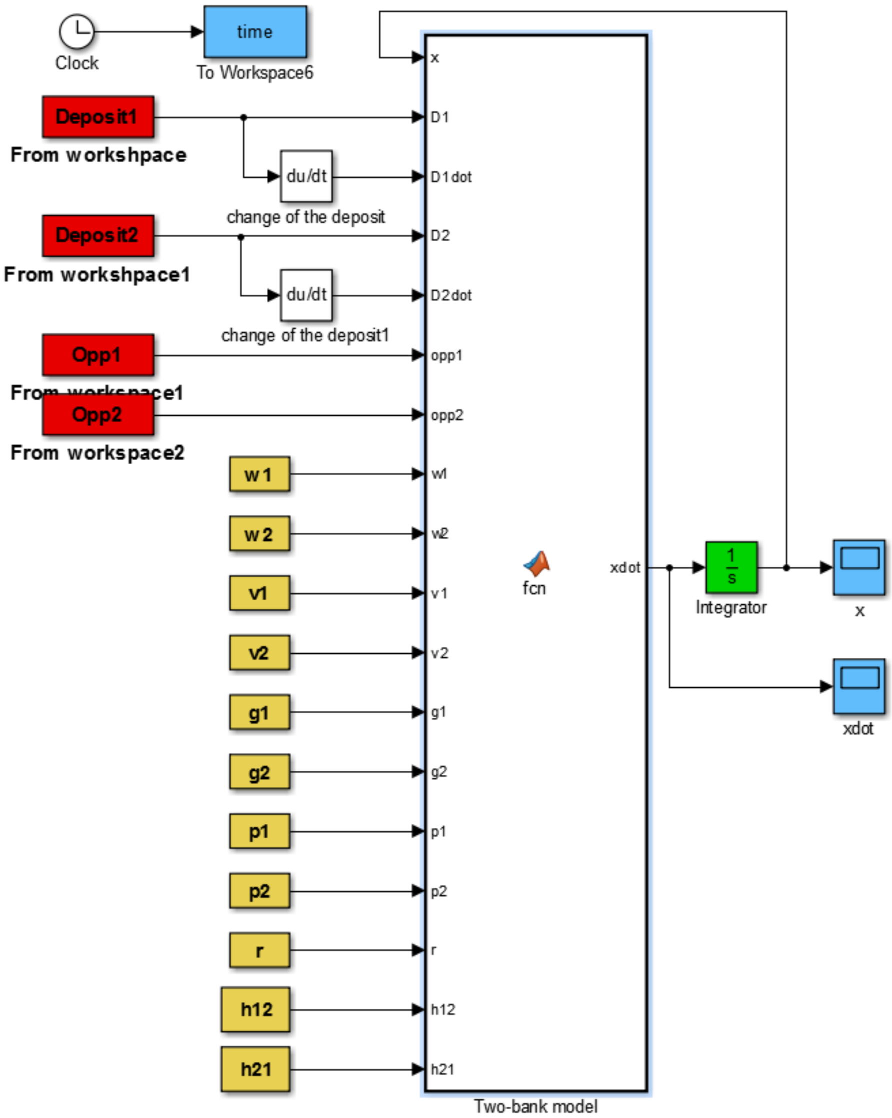

Figure 4.11 Two-bank model in Simulink.

Differently from the one-bank model, all the calculations are put into a single block which is the large block named 'Two-bank model'; this block allows the user to write codes and generate their own functions and is used for simplifying the implementation. With the codes defined by the users, the two-bank model block can do all the desired calculations without using further blocks (such as 'subtract' and 'min'); it uses all the imported information to calculate the derivatives of the cash and investment of each bank (the two derivatives are gathered into one vector called 'xdot' in figure 4.11) and, subsequently, it passes them to the integrator. The integrator calculates the values of cash and investment and outputs them to the workspace through the light blue block named 'x' in figure 4.11.

4.3.2 Simulation Results

This subsection shows the simulation results of the two-bank model which are generated from the Simulink block diagram that is described in figure 4.11; these results allow us to analyse how two banks interact with each other through borrowing and lending. This analysis facilitates the understanding of the more complex models where more than two banks are present. As in the simulations of the one-bank model, the unit time is one day and the total simulation period is 10 days. The step size is set to 0.1. Since the bank either needs to borrow or to lend money, the borrowing and lending cannot happen in one bank at the same time; therefore, it is assumed that $bank_1$ acts as the borrowing bank and $bank_2$ acts as the lending bank. Results from two different simulations are presented: 1) in the first case we study the effect of borrowing on bank 1 which experiences a shock on its deposit; 2) in the second case we apply a shock on $bank_1$ and another shock on $bank_2$ to study the indirect effect of lending on $bank_2$.

Simulation results 1: the effect of the borrowing

In this simulation, the initial conditions and parameter values of the two banks are shown in table 4.2. $Bank_1$ experiences a large shock which makes the deposit, D_1, to decrease from 1 to 0.76 at day 4. $Bank_1$ needs to borrow money from $bank_2$ which does not experience any shock in the deposit ($D_2 = 1$ all the time). Furthermore, $bank_2$ has cash above the reserve requirement ($C_2 = 0.3 > rD_2 = 0.2$) that can be lent to $bank_1$. To avoid that $bank_2$ uses cash to invest, the investment opportunity in $bank_2$ is set to zero ($opp_2 = 0$). We consider two subcases: in the first one, α_{12}, the proportion of the total borrowing that is repaid, is set to 0, so that bank 1 doesn't repay the principal of the loans that have been borrowed; in the second subcase $\alpha_{12} > 0$.

Table 4.2 Initial conditions and parameter values used in simulation 6.

Initial conditions	
$C_1 = 0.25$	$C_2 = 0.3$
$I_1 = 0.75$	$I_2 = 0.5$
$D_1 = 1$ when $0 \leq t \leq 3$	$D_2 = 1$
$D_1 = 0.76$ when $4 \leq t \leq 5$	when $0 \leq t \leq 10$
$D_1 = 1$ when $6 \leq t \leq 10$	$opp_2 = 0$
$opp_1 = 1$ when $0 \leq t \leq 10$	when $0 \leq t \leq 10$
Parameter value	
$g_1 = 0$, $p_1 = 0$	$g_2 = 0$, $p_2 = 0$
$w_1 = 0$, $v_1 = 0$	$w_2 = 0$, $v_2 = 0$
$r = 0.2$	$r = 0.2$
$h_{12} = 0$	$h_{21} = 0$
$\alpha_{12} = 0$	$\alpha_{21} = 0$

Results in figure 4.12 are reported to study the behaviour of equation (3.7) when $\alpha_{12} = 0$, in which case the added borrowing, $\frac{dB_{12}}{dt}$, is only given by the minimum value between the cash demand of $bank_1$ and the available cash of $bank_2$. The blue line in figure 4.12 shows the money that $bank_1$ borrows from $bank_2$ at each time step. The red dash line represents the money $bank_1$ needs to borrow, $(rD_1 - C_1)^+$. The green dash dot line represents the money $bank_2$ can lend $(C_2 - rD_2)^+$. It can be seen the blue line follows the red line up to day 4, which means that $bank_2$ can lend all the money $bank_1$ needs; then the blue line follows the green up to around day 5, which means that $bank_1$ requires more money than $bank_2$ can provide; after around day 5, the blue line follows the reed one as $bank_2$ can provide all the money needed by $bank_1$.

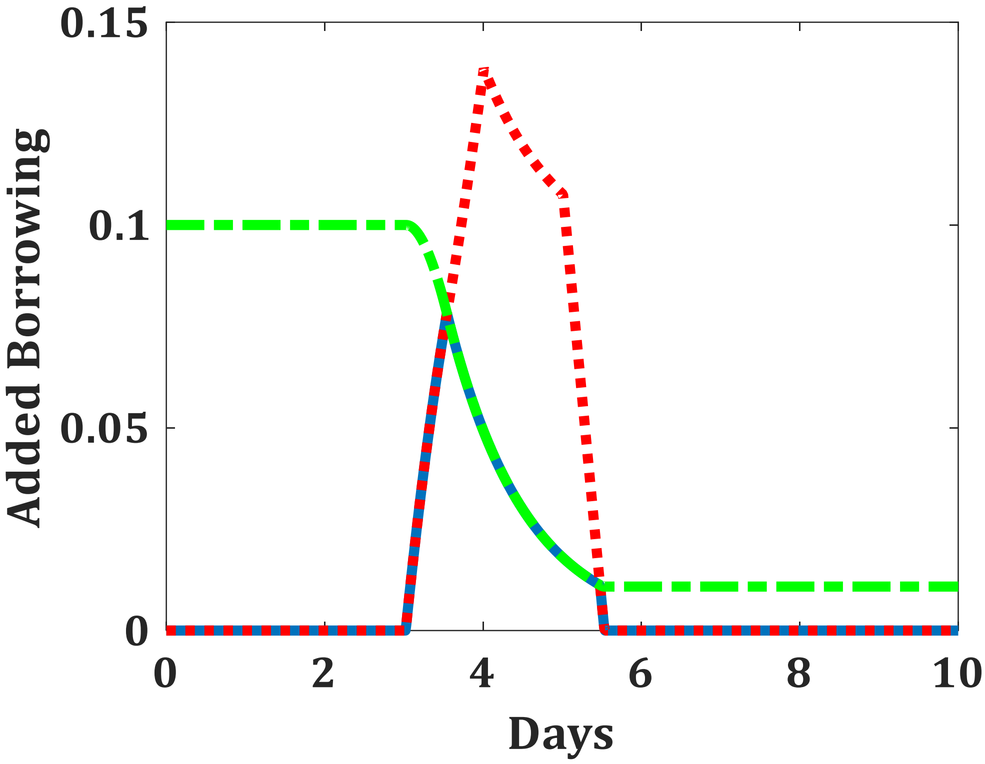

Figure 4.12 Dynamics of added borrowing of bank1 when $g_1 = g_2 = 0$, $p_1 = p_2 = 0$, $w_1 = w_2 = 0$, $v_1 = v_2 = 0$, $r = 0.2$ and $opp_1 = 1, opp_2 = 0$. The red dash line represents the money bank$_1$ needs to borrow $(rD_1 - C_1)^+$. The green dash dot line represents the money bank$_2$ can lend $(C_2 - rD_2)^+$. The blue line represents the added borrowing.

The results with $\alpha_{12} = 0$ are then compared with the results with $\alpha_{12} = 1$. The comparison is made in figure 4.13, where the blue lines represent the results with $\alpha_{12} = 0$, while the red lines represent the results with $\alpha_{12} = 1$. From figures 4.13(b1) and (c1), it can be seen that the red line is always below the blue line, which means that repaying the borrowing causes a decrease of bank$_1$'s investment, because extra cash has to be used to make the repayment instead of the investment. In figure 4.13(d1) the red line and blue line overlap, which means that the bank$_1$'s cash with or without the repayment of the borrowing doesn't change. In figure 4.13(d2), bank$_2$'s cash increases at around day 5 because bank$_2$ receives borrowing repayments from bank$_1$. Since bank$_2$ doesn't have any investment opportunities, its investment stays at the same value (see figures 4.13(b2) and (c2)).

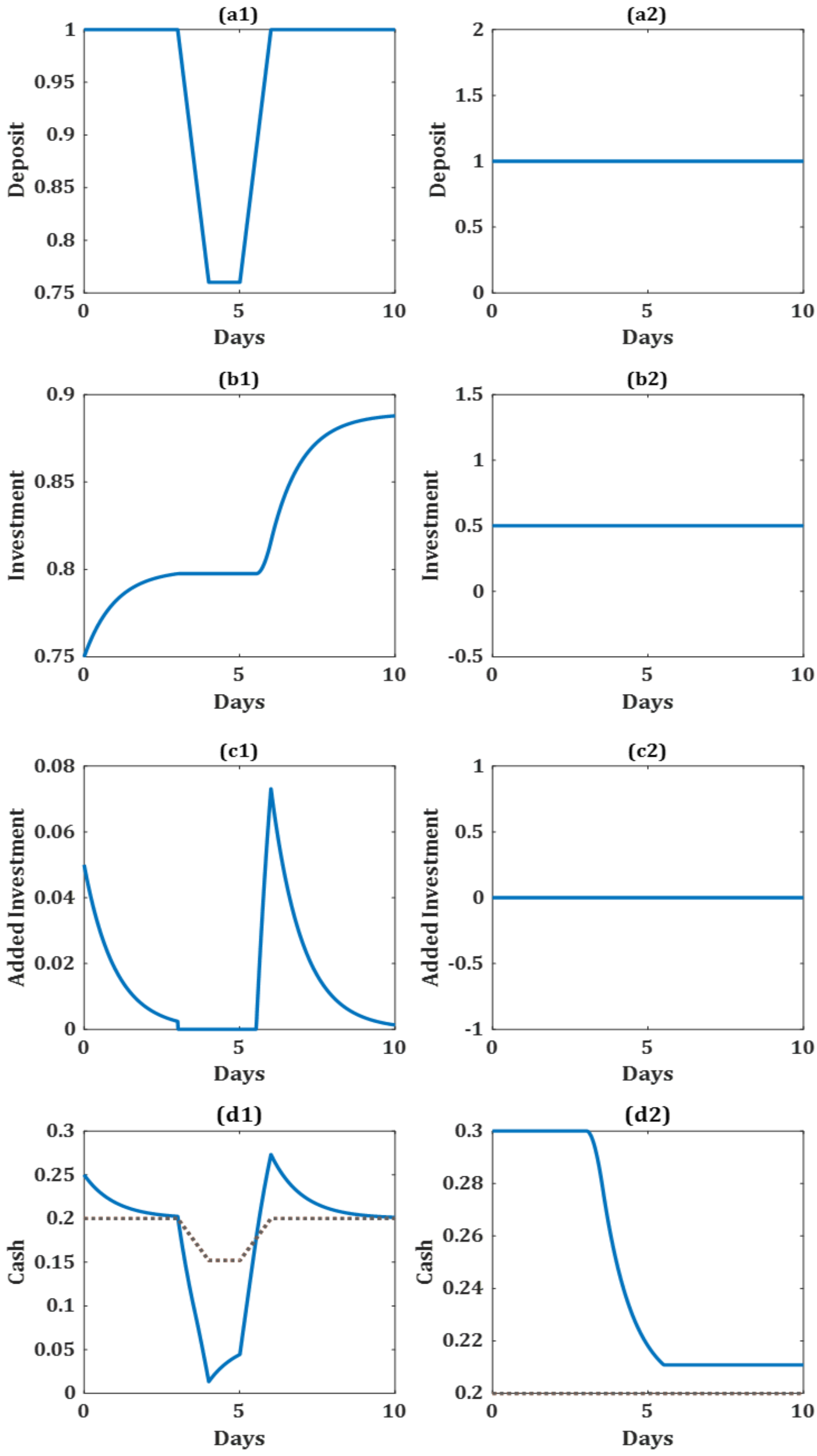

Figure 4.13 Dynamics of the two banks without borrowing repayment (blue lines) and with borrowing repayment (red lines). (a1) deposit, (b1) total investment, (c1) added investment, (d1) cash for bank1 - reserve ratio requirement in grey dash line in (d1). (a2) deposit, (b2) total investment, (c2) added investment, (d2) cash for bank1 - reserve ratio requirement in grey dash line in (d2). $g_1 = 0$, $p_1 = 0$, $w_1 = 0$, $v_1 = 0$, $r = 0.2$ and $opp_1 = 1$. $g_2 = 0$, $p_2 = 0$, $w_2 = 0$, $v_2 = 0$, $r = 0.2$ and $opp_2 = 0$.

Simulation result 2: indirect effect of the borrowing on the lending bank

This subsection shows a case in which borrowing has a negative effect on the lending bank. In order to let this behaviour to emerge, a shock in the deposit is applied on $bank_1$, which is followed by a shock on $bank_2$. Figure 4.14 reports the results of two different cases: blues lines represent the results when borrowing between the two banks is present (case 1) while red lines represent the results without borrowing (case 2). The initial conditions and parameter values used in these two simulations are shown in table 4.3.

Table 4.3 Initial conditions and parameter values used in simulation 2.

Initial conditions	
$C_1 = 0.25$	$C_2 = 0.3$
$I_1 = 0.75$	$I_2 = 0.5$
$D_1 = 1$ when $0 \le t \le 3$	$D_2 = 1$ when $0 \le t \le 5$
$D_1 = 0.76$ when $3 \le t \le 5$	$D_2 = 0.76$ when $5 \le t \le 7$
$D_1 = 0.75$ when $5 \le t \le 8$	$D_2 = 1$ when $7 \le t \le 10$
$D_1 = 1$ when $8 \le t \le 10$	
$Opp_1 = 1$	$Opp_2 = 0.01$
Parameter value	
$g_1 = 0 , p_1 = 0$	$g_2 = 0 , p_2 = 0$
$w_1 = 0 ,\ v_1 = 0$	$w_2 = 0 ,\ v_2 = 0$
$r = 0.2$	$r = 0.2$
$h_{12} = 0.01/360$	$h_{21} = 0.01/360$
$\alpha_{12} = 1$	$\alpha_{21} = 1$

The deposit signals of both banks are shown in figures 4.14(a1) and (a2). The shock in $bank_1$ causes the deposit to decrease to 0.76 at day 4 and then to further decrease to 0.75 at day 6. At the same time (day 6) a shock in $bank_2$ happens causing $bank_2$'s deposit to decrease to 0.76. $Bank_1$'s cash drops below the reserve ratio due to the shock (see figure 4.14(d1)); with the borrowing from $bank_2$, $bank_1$'s cash decreases more slowly and it even increases during day 3 to day 5 (blue line), while without the borrowing from $bank_2$, $bank_1$ fails at day 4 (red line). Due to the lending to $bank_1$, $bank_2$'s cash decreases more quickly during day 3 to day 5 compared to the results when no money is lent to $bank_1$ (see figure 4.14(d2) where the blue line is below the

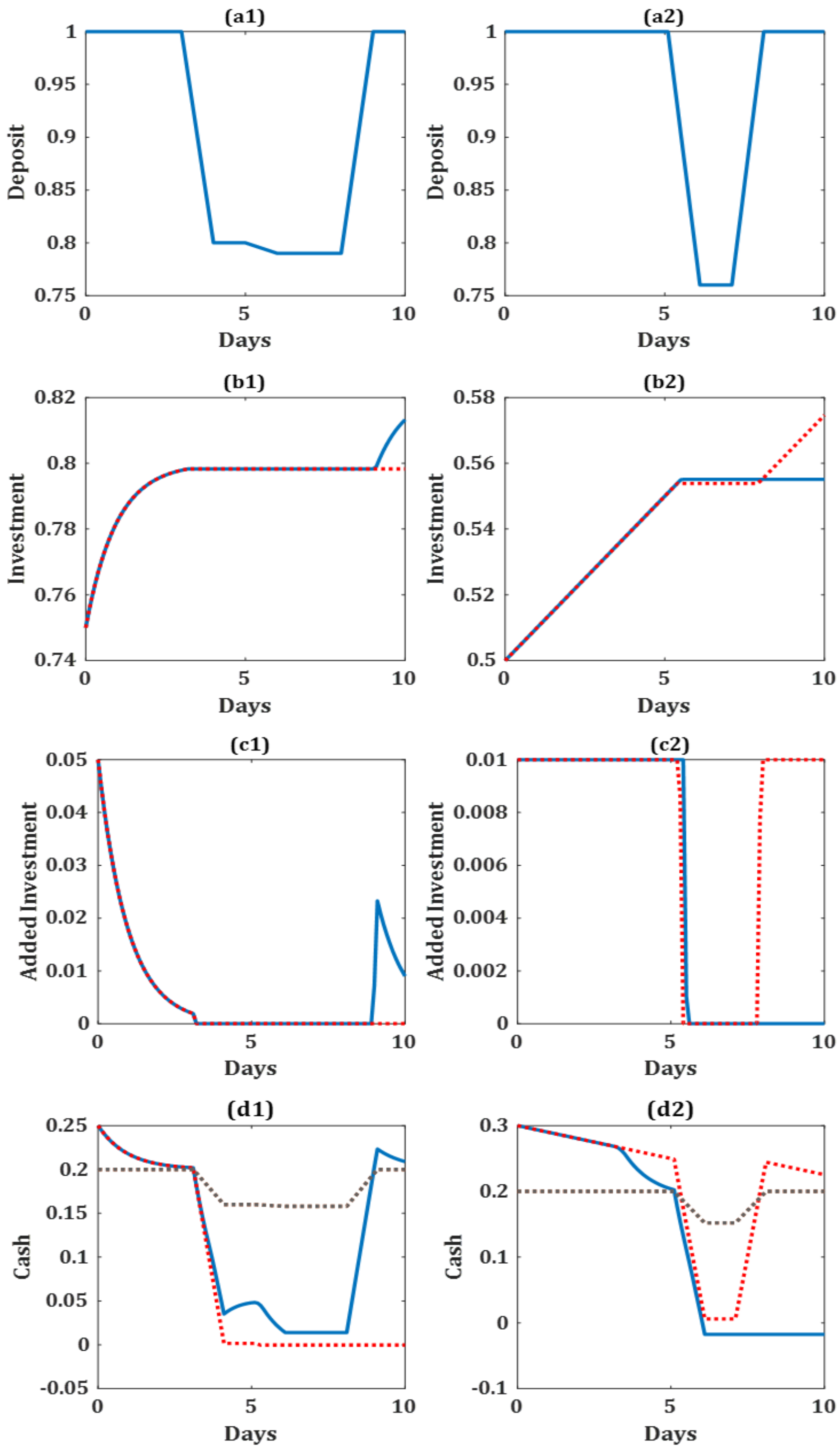

Figure 4.14 Dynamics of the two-bank model without borrowing (blue lines) and with borrowing (red lines). (a1) deposit, (b1) total investment, (c1) added investment, (d1) cash for bank1 - reserve ratio requirement in grey dash line in (d1). (a2) deposit, (b2) total investment, (c2) added investment, (d2) cash for bank2 - reserve ratio requirement in grey dash line in (d2). $g_1 = 0$, $p_1 = 0$, $w_1 = 0$, $v_1 = 0$, $r = 0.2$ and $opp_1 = 1$, $g_2 = 0$, $p_2 = 0$, $w_2 = 0$, $v_2 = 0$, $r = 0.2$ and $opp_2 = 0.1$.

red line during day 3 to day 5). At day 5 a shock is experienced by $bank_2$. As the cash of both banks is below the reserve ratio requirement, there is no borrowing/lending between the two banks during the second shock. $Bank_2$ fails during this shock, while $bank_1$ survives at the end of the simulation period. The indirect effect of the borrowing on the lending bank can be seen in this case: $bank_2$ (lending bank) could survive during the second shock if it didn't lend money to $bank_1$ (see red line in figure 4.14(d2)). The borrowing doesn't cause the bank to fail directly, it only weakens the bank's ability to buffer a possible following shock. However, the borrowing has a positive effect for $bank_1$, since it saves the bank from the shock. More complex cases will be studied in the multi-bank model in the next section, in which there are 50 banks in the system.

4.4 Multi-bank model

This section shows the implementation and the simulation results of the multi-bank model which is described in Chapter 3. The first subsection shows the Simulink block diagram of the multi-bank model while the second subsection presents the simulation results. In the simulations, shocks are introduced into the system via deposit. To study the stability of the banking system, the number of survival banks at the end of the simulation period are calculated and compared in different scenarios characterised by different values of the link rate, reserve ratio, the amplitude of the shock and the heterogeneity in bank's size. Moreover, the contagion effect is characterised and studied as a function of link rate, reserve ratio and heterogeneity under different amplitudes of the shock.

4.4.1 Implementation of multi-bank model in Simulink

The Simulink block diagram of the multi-bank model, shown in figure 4.15, is similar to the one used for the two-bank model, but with two main improvements:

1. The new Simulink block diagram can accommodate any number of banks in the system. The input signals (red blocks in figure 4.15) are represented by multi-column matrixes (rather than just two-column matrixes) that contain the data for all banks. For example, the deposit signal in the two-bank Simulink block diagram has two columns: time and the values of the bank's deposit at each time; in the new block diagram the deposit signal has one column representing the time and other N (number of banks in the system) columns representing the deposit values for

the N banks. All the other parameters (such as w_i, v_i, p_i and g_i) and initial conditions used in Simulink (yellow blocks) are vectors that represent the information for all the banks.

2. A new MATLAB script has been developed to perform the simulations, save and plot the results automatically with parameter values and initial conditions provided by the user. The script can also repeat the simulation a few times with the same parameter settings and calculate the average results automatically. More details about the MATLAB script are included in Appendix 3.

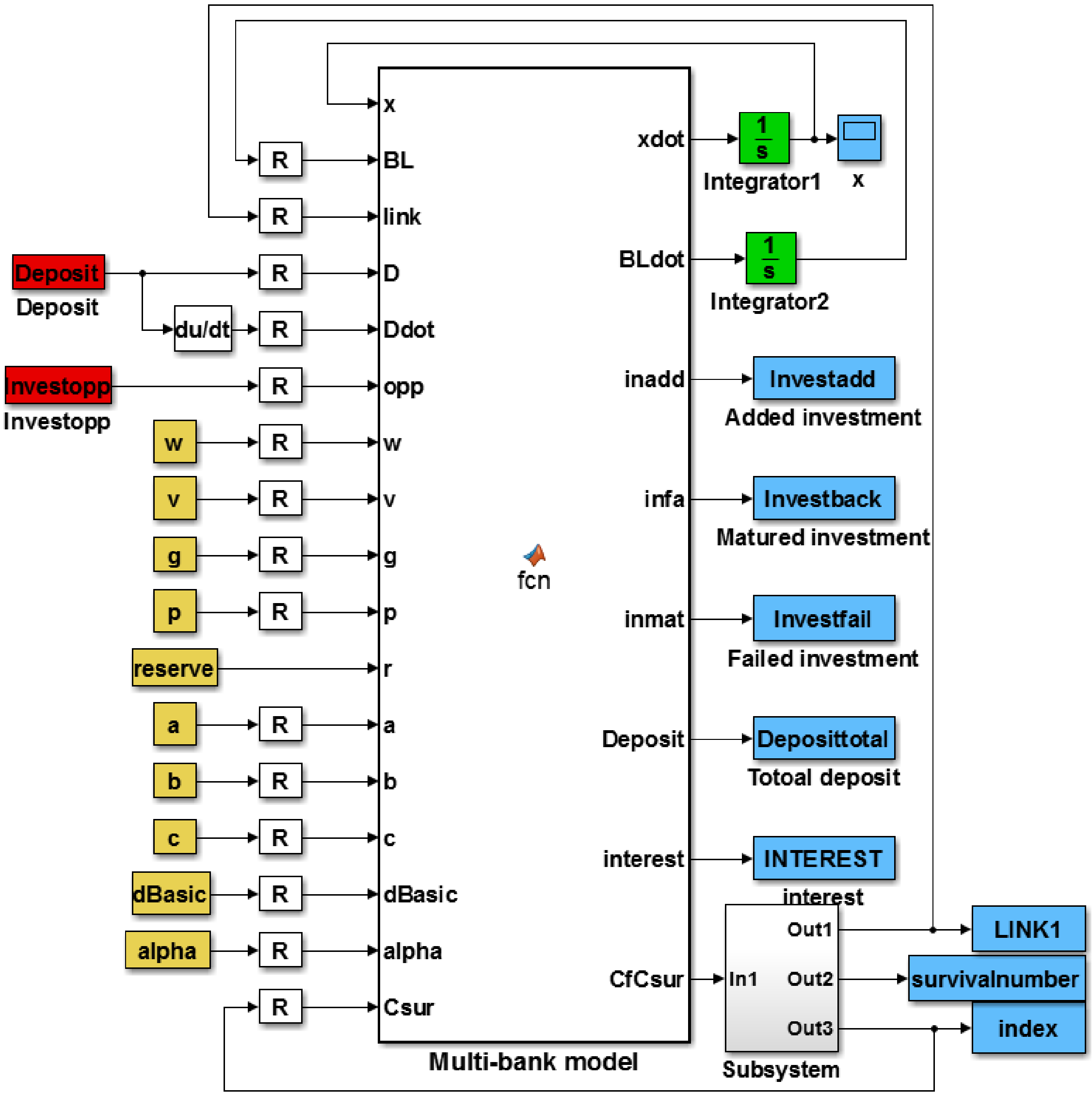

Figure 4.15 Simulink block diagram of the multi-bank model.

4.4.2 Simulation results of multi-bank model

This subsection presents the results of simulations for two different cases: homogenous case (all banks have similar deposit size) and heterogeneous case (banks have deposit sizes that follow a Gaussian distribution) - see equations (3.15) and (3.16) in Chapter 3. In both cases, the following parameters are the same for all the simulations that are presented in this subsection: the number of banks in the system at the beginning of simulations is 50 - this number of banks is sufficient to provide a rich dynamics of the system. The unit time is one day and the total simulation time is 300 days, which is a period of time long enough to observe the relevant dynamics.

Homogeneous case simulation results

This subsection shows the simulation results in the homogenous case, in which banks have similar size; the shocks is introduced into the system via deposit fluctuations. The values for the parameters in equation (3.15), which provide the deposit signal, are set as follow: $\overline{D}$ =1000 (average size of each bank) and σ_D= 0.3 or σ_D= 0.7 (representing low and high amplitude of the shocks), which are similar to the values used in Iori's work [103]. The initial conditions of each bank are set as follows: the cash is set to $C_i = 200$ and the investment is I_i=800, so to have a net-worth close to zero; both borrowing, B_i, and lending, L_i, are set to 0.

The values used for the link rate, l_r, are 0, 0.03, 0.15, 0.3, 0.5, 0.75 and 1. The values used for the reserve ratio, r, are 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7. Thus, there are 7x7= 49 different combinations of link rate and reserve ratio, corresponding to 49 different scenarios. Each scenario is simulated 20 times – this number of simulations provide averages of the results with small standard deviations, for the given random shocks used in the simulations. All the results shown in this subsection are the averages of the 20 simulations.

Simulation results 1: effect of the reserve ratio on the number of survival banks

Figure 4.16 shows how the number of survival banks is affected by different reserve ratios when the link rate, l_r, is fixed, and σ_D= 0.3. Figure 4.16(a) reports the results corresponding to l_r= 0; this figure shows that when there is no interbank lending, the reserve ratio definitely plays a positive role to preserve the stability of the system. In fact, as the reserve ratio increases, more banks survive at the end of the simulation period. It should be noticed that around 6 banks fail very quickly, within 1 or 2 days, and then the rate of failure decreases. This can be explained as follows: at the beginning of the simulation, all banks have cash equals to 200, investment equals to 800 and they can get 240 from matured investments every day. Therefore, if they experience a shock which is larger than 440 (=200+240), they fail because there is no interbank lending (l_r= 0). According to equation (3.15), the shock on a bank at time t is given by $\overline{D}\sigma_D\varepsilon_t$, where $\overline{D} = 1000$, $\sigma_D = 0.3$ and $\varepsilon_t \sim N(0,1)$; therefore, $\overline{D}\sigma_D\varepsilon_t \sim N(0, 300^2)$. The probability, p, that $\overline{D}\sigma_D\varepsilon_t$ is smaller than -440 (i.e. the shock is larger than the sum of cash and matured investment) can be calculated as follows: first transform $\overline{D}\sigma_D\varepsilon_t$ into normal distributed variable, $z = \frac{\overline{D}\sigma_D\varepsilon_t - 0}{300}$; then, it can get that $p(\overline{D}\sigma_D\varepsilon_t < -440) = p\left(\frac{\overline{D}\sigma_D\varepsilon_t - 0}{300} < -\frac{440}{300}\right) = p(z < -1.4) = 0.08$. Therefore, about 4 banks, (50×0.08) are likely to fail at day 1, which is close to the value in figure 4.16(a). In the following days, the survival banks have more cash from matured investments and from potential positive shocks, so the failure rates decreases.

The effect of the reserve ratio, however, is different when the link rate increases as shown in figures 4.16(b) (c) and (d); the figures show that some banks fail around day 1 and the number of failed banks is higher if the reserve ratio is higher - the remaining banks survive until the end of the simulation. This is because high reserve ratios discourage banks from lending, which is a problem when some banks experience negative shocks in their deposits. When the reserve ratio is low, for example r=0.3, banks have more available cash for lending; with lending/borrowing, fewer banks fail. As reserve ratio increases the banks have less available cash for lending, because they need to keep the cash to meet their reserve requirement; therefore, more banks fail because they cannot borrow enough money to buffer the shock.

There two reasons why the number of survival banks stays constant after the first a few days: when the reserve requirement is higher than the shock's standard deviation, (300 as $\overline{D}\sigma_D\varepsilon_t \sim N(0, 300^2)$), the banks have enough cash to resist the shocks without the help of interbank lending. When the reserve ratio, r, ranges between 0.1 and 0.3, the reserve requirement is lower than the shock's standard deviation and therefore banks survive thanks to the help from other banks' lending.

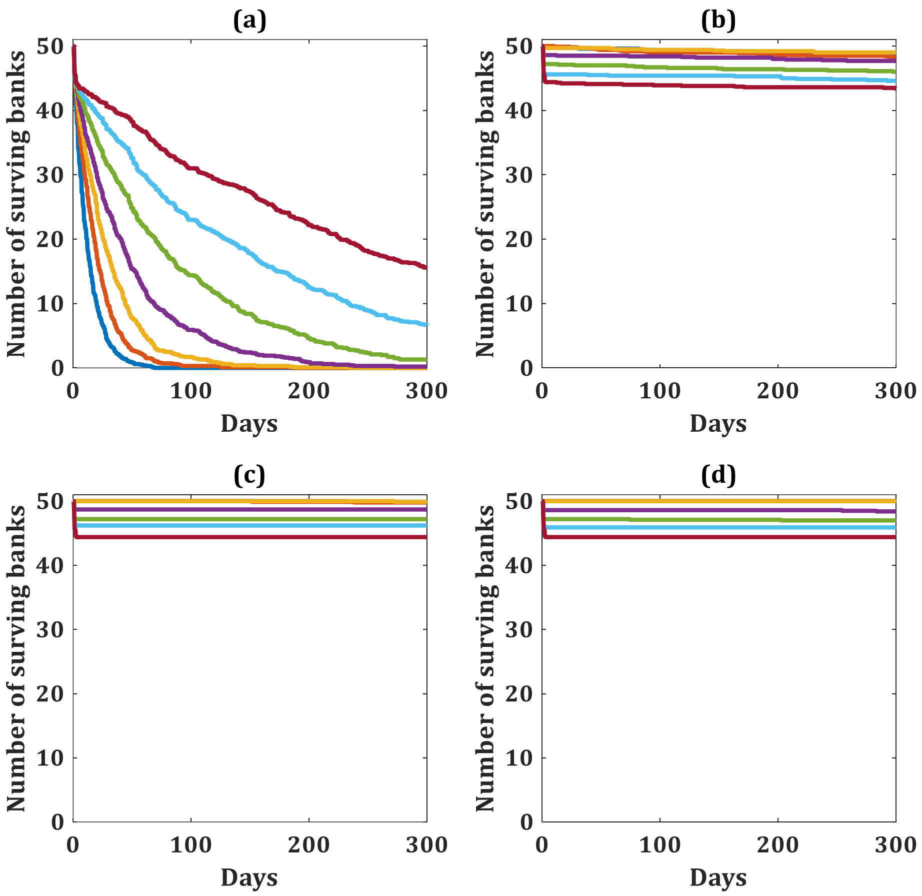

Figure 4.16 Number of survival banks in the homogeneous case with $\sigma_D = 0.3$, $\sigma_{opp} = 0.5$ with different reserve ratios, r =0.1 (dark blue line), r =0.2(light red line), r =0.3 (yellow line), r =0.4 (purple line), r =0.5 (green line), r =0.6 (light blue line), r =0.7 (dark red line), and under different link rates, l_r = 0 (a), l_r = 0.15 (b), l_r = 0.5 (c), l_r= 1 (d).

From these results, it can be concluded that it is safe for the isolated bank to keep a high reserve ratio to preserve itself from the large shocks, but this is not always beneficial for the whole system, in fact, the dark red line (r=0.7) in figure 4.16(a) is lower than the corresponding dark red lines in figures 4.16(b), (c) and (d) which have higher link rate; this indicates that the effect of the reserve ratio should be analysed together with the effect of the link rate. Moreover, the link rate seems to have a more powerful effect on the number of survival banks.

Simulation results 2: effect of the link rate on the number of survival banks

Figure 4.17 shows the effect of the link rate on the number of surviving banks, when the reserve ratio is fixed. Figures 4.17(a), (b), (c) and (d) show that more banks survive as the link rate increases; looking at the dark blue line, (l_r = 0), and the light red line, (l_r = 0.03), across the 4 figures, the higher the reserve ratio the more banks survive. With higher link rate (from 0.3 to 1) most banks survive and not significant differences exist for the different cases. This indicates that, when exceeding a threshold value, the increase in the link rate does not help much in improving any further the stability of the system. These results show that the banks don't need to be fully connected to keep the system stable, when the shock is not very larger.

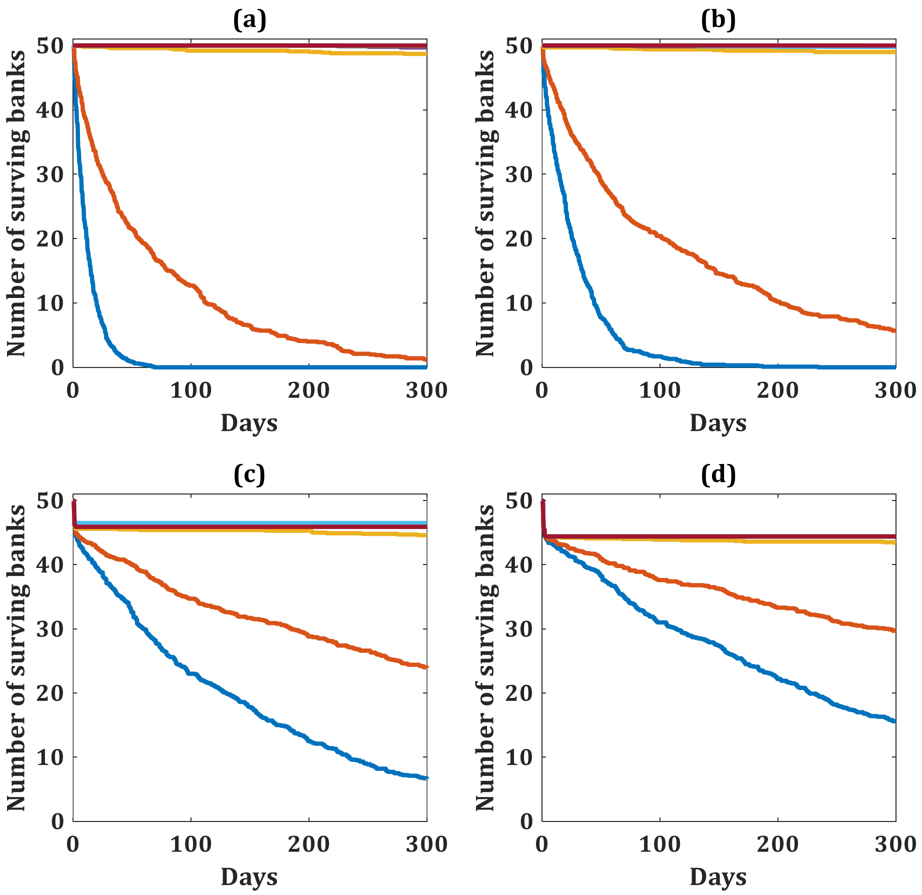

Figure 4.17 Number of survival banks in the homogeneous case with $\sigma_D = 0.3, \sigma_{opp} = 0.5$, with different link rates, l_r = 0 (dark blue line), l_r = 0.03 (light red line), l_r = 0.15 (yellow line), l_r = 0.3 (purple line), l_r = 0. 5 (green line), l_r = 0.75 (light blue line), l_r = 1 (dark red line), and under different reserve ratios, r =0.1(a), r =0.3(b), r =0.6 (c), r =0.7 (d).

Simulation results 3: effect of the shock amplitude on number of survival banks

This subsection shows the simulation results when shocks have a higher amplitude. The amplitude of the shocks, σ_D, is increased to 0.7; the results are compared with the ones with σ_D= 0.3, reported in simulations 1 and 2, to see if the effects of the link rate and reserve ratio change when the system is under large shocks.

Figure 4.18 shows how the number of survival banks is affected by different reserve ratios when the link rate, l_r, is fixed. Figure 4.18(a) shows that banks fail more quickly (at day 40 all banks fail) than in simulation 1 due to larger shocks. When the link rate is fixed, similar effect of the reserve ratio is found as in simulation1. In figure 4.18(a), in which $l_r = 0$, a higher reserve ratio makes bank survive longer while, when the link

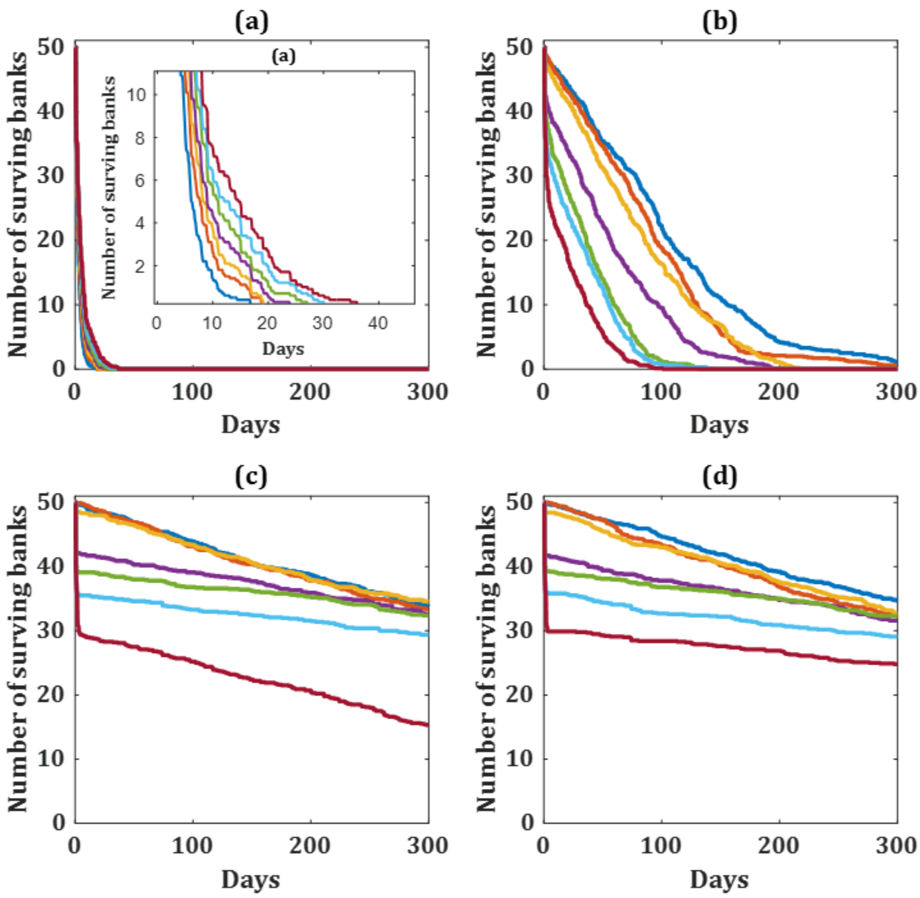

Figure 4.18 Number of survival banks in the homogeneous case with σ_D = 0.7, σ_{opp} = 0.5 with different reserve ratios: r = 0.1 (dark blue line), r = 0.2(light red line), r = 0.3 (yellow line), r = 0.4 (purple line), r = 0.5 (green line), r = 0.6 (light blue line), r = 0.7 (dark red line) under different link rates, l_r= 0 (a), l_r = 0.15 (b), l_r = 0.5 (c), l_r = 1 (d).

rate increases to 0.15, 0.5 and 1 as in figures 4.18(b), (c) and (d), higher reserve ratio works in an opposite way, causing more banks to fail because it discourages banks from lending. The negative effect of the high reserve ratio is more obvious in figures 4.18(b), (c) and (d) when compared to figures 4.16(b), (c) and (d).

Figure 4.19 shows the effect of the link rate on the number of surviving banks, when the reserve ratio is fixed. Comparing the results in figure 4.19 with the results from simulation 2, the same effect of the link rate is found; as the link rate increases more banks survive. In figures 4.19(a) and (b), when the link rate is equal or bigger than 0.3, the increase in the link rate does not make a noticeable difference. In figure 4.19(c) this happens when the link rate is equal or bigger than 0.5, while in figure 4.19(d) this happen when the link rate is equal or bigger than 0.75. This indicates that high reserve

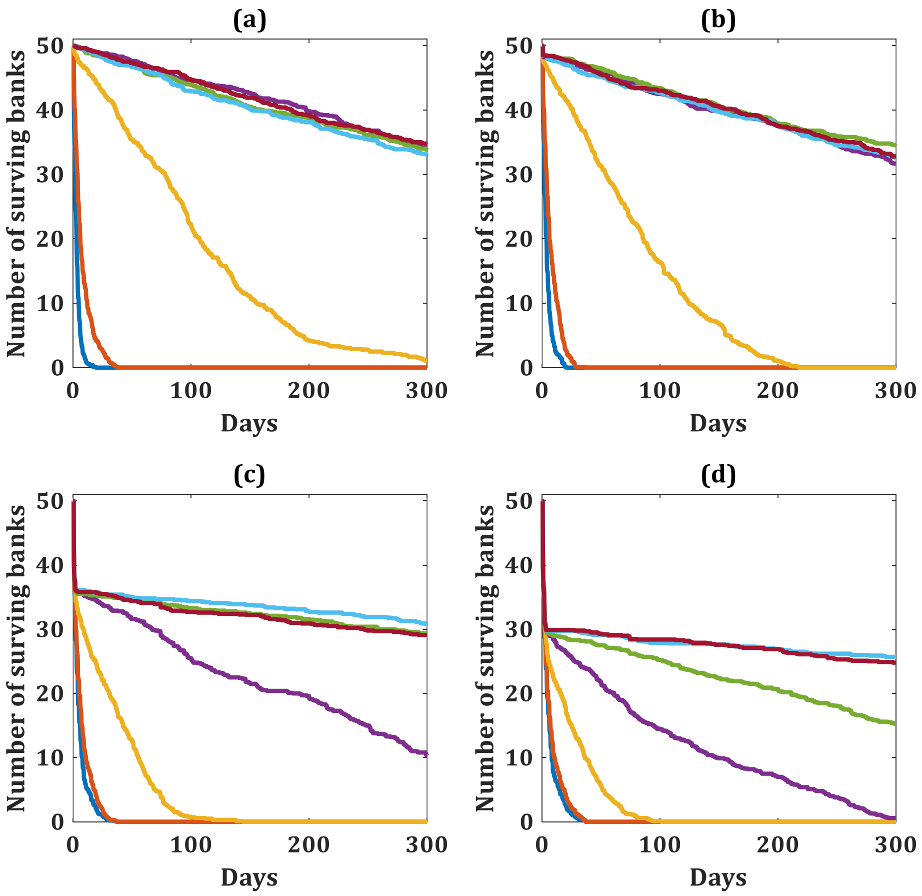

Figure 4.19 Number of survival banks in the homogeneous case with $\sigma_D = 0.7, \sigma_{opp} = 0.5$, with different link rates, l_r = 0 (dark blue line), l_r = 0.03 (light red line), l_r = 0.15 (yellow line), l_r = 0.3 (purple line), l_r = 0. 5 (green line), l_r = 0.75 (light blue line), l_r = 1 (dark red line), and under different reserve ratios, r = 0.1 (a), r = 0.3 (b), r = 0.6 (c), r = 0.7 (d).

ratios limit banks from lending; it is harder for banks to find available cash to borrow as reserve ratio increases, therefore, banks need to be more connected to have more chance to borrow and survive.

Simulation results 4: joint effect of the link rate, reserve ratio and shock amplitude on the number of survival banks

The trends showed in simulation 1, 2 and 3 can be observed in figures 4.20, which reports the waterfall plot of the number of surviving banks at day 300 (the end of any simulation) as function of both link rate and reserve ratio, when $\sigma_D = 0.3$ or $\sigma_D = 0.7$. Figures 4.20(a) and (c) show the results for lower and higher amplitude of the shock, respectively; a larger amplitude of the shock causes more banks to fail, as expected, but the link rate has always a positive effect on the system. Reserve ratio acts positively when the link rate is low (see figure 4.20(b) for clearer evidence) and acts in the opposite way when link rate is high (see figure 4.20(d)).

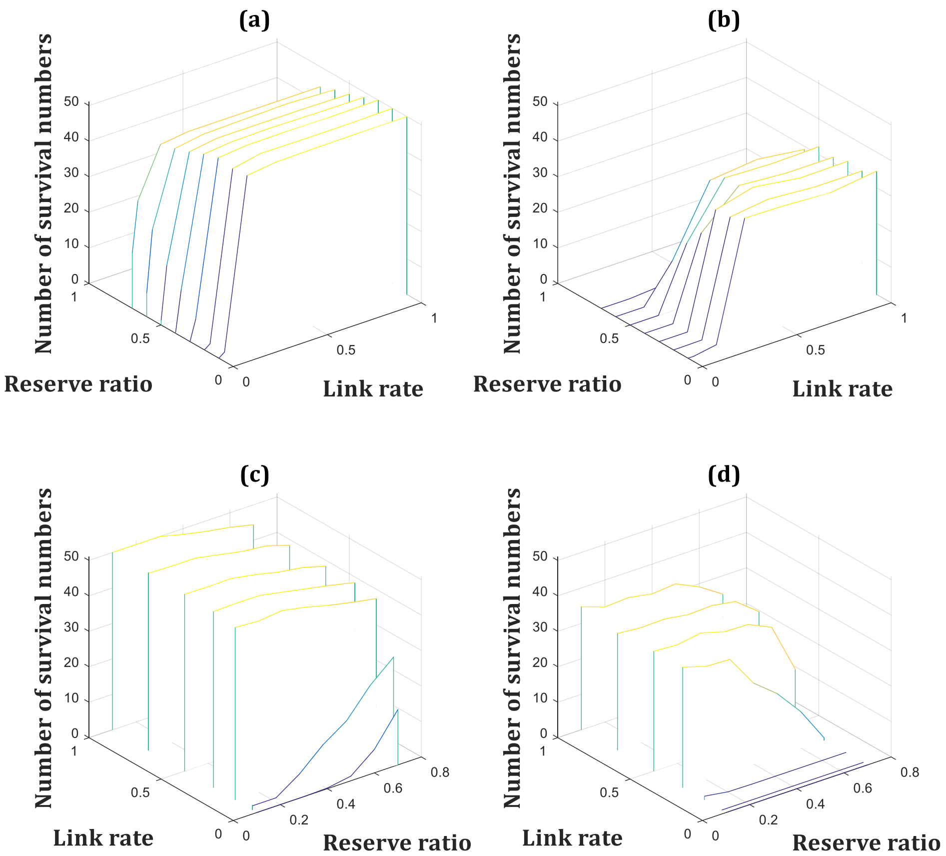

Figure 4.20 Waterfall plots showing the number of survival banks in the homogeneous case at day 300 as function of both link rate and reserve ratio; (a) and (b) $\sigma_D = 0.3$; (c) and (d) $\sigma_D = 0.7$.

Simulation results 5: contagion

Another interesting aspect of the model is the analysis of contagion of failure due to the interbank borrowing and lending. A measure of contagion is defined as follows: when a bank fails, the algorithm checks whether that bank has unpaid loans from those others that failed earlier. The total of the unpaid loans is judged to be 'significant' in the case in which the bank would have survived if it did not have those unpaid loans: therefore, the total of unpaid loans is considered to be significant if it is bigger than the difference between the shock and the bank's cash. At the end of the simulation, the proportion of failed banks with significant unpaid loans, as compared to the total number of failed banks, is calculated; the higher this proportion, the higher the contagion. Figure 4.21 reports the proportion of failed banks with significant unpaid loans as a function of both link rate and reserve ratio when $\sigma_D = 0.3$ (a) and $\sigma_D = 0.7$ (b). Figure 4.21(a) shows that the ratios are all quiet small (maximum is 0.1, see the colour bar) when $\sigma_D = 0.3$, which means the contagion is small when the shock is small. This is also because most banks did not fail at the end of the simulation (see figure 4.20(a)). When the shocks are larger, contagion increases as in figure 4.21(b); up to 60% of the failed banks are affected by contagion. Therefore, the following analysis is focused on the high-shock case, to study how the link rate and reserve ratio affect contagion.

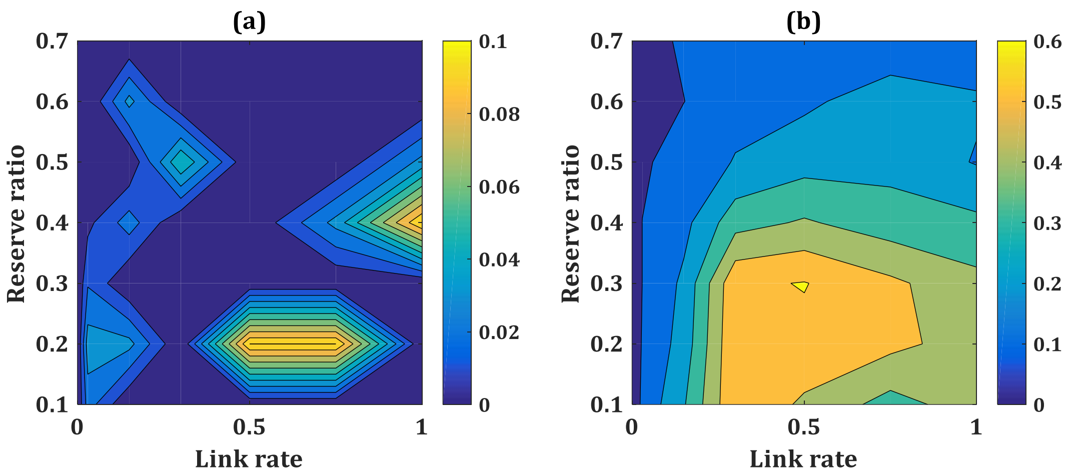

Figure 4.21 Contagion for the homogeneous case as function of link rate (l_r = 0 to 1) and reserve ratios (r = 0.1 to 0.7), when $\sigma_D = 0.3$ (a) and $\sigma_D = 0.7$ (b).

Figure 4.22 is a larger version of figure 4.21(b), and is divided into 4 areas: Area1 (r from 0.1 to 0.35 and l_r from 0 to 0.5), Area2 (r from 0.35 to 0.7 and l_r from 0 to 0.5), Area3 (r from 0.35 to 0.7 and l_r from 0.5 to 1) and Area4 (r from 0.1 to 0.35 and l_r from 0.5 to 1).

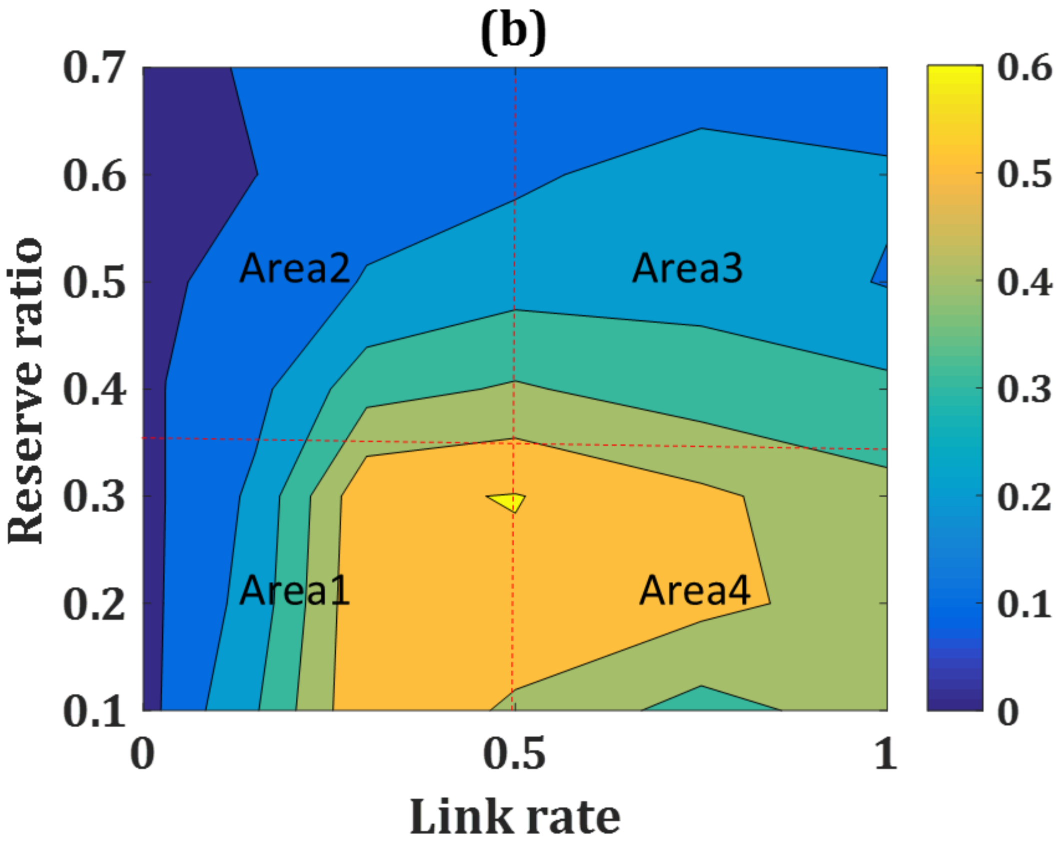

Figure 4.22 Contagion for the homogeneous case as function of link rate (l_r= 0 to 1) and reserve ratios(r = 0.1 to 0.7), when σ_D = 0.7 .

Looking at Area1 and Area2, when link rate is fixed and less than 0.2, an increase in the reserve ratio does not change the contagion significantly, as banks are less connected and failure has fewer channels to spread; when link rate is fixed and between 0.2 and 0.5, an increase in the reserve ratio decreases the contagion, as a high reserve ratio discourages the banks from lending to others, thus decreases the channels for failure to spread.

Area4 and Area3 show how contagion is affected by the reserve ratio when the link rate is high and fixed; an increase in the reserve ratio has a nonlinear effect on contagion, in fact, contagion increases as the reserve ratio increases from 0.1 to 0.35 (Area4), and it decreases as the reserve ratio increases from 0.35 to 0.7 (Area3). The reason for the decrease in contagion in Area3 is the same as explain in the previous paragraph: high reserve ratio discourages the banks from lending to others. To explain the increase in contagion in Area4, figure 4.23 is reported, which plots the mean and variance of all the unpaid loans as a function of the link rate. The different colour of the lines represents the different reserve ratios. It can be seen that when link rate ranges from 0.5 to 1, the mean of the unpaid loans increase as r increases from 0.1 (light red line) to 0.3 (yellow line). This indicates that banks borrow more money (the mean increases), because they may need to satisfy an increasing reserve requirement. Therefore, the unpaid loans may increase in size, resulting in overall increase in contagion.

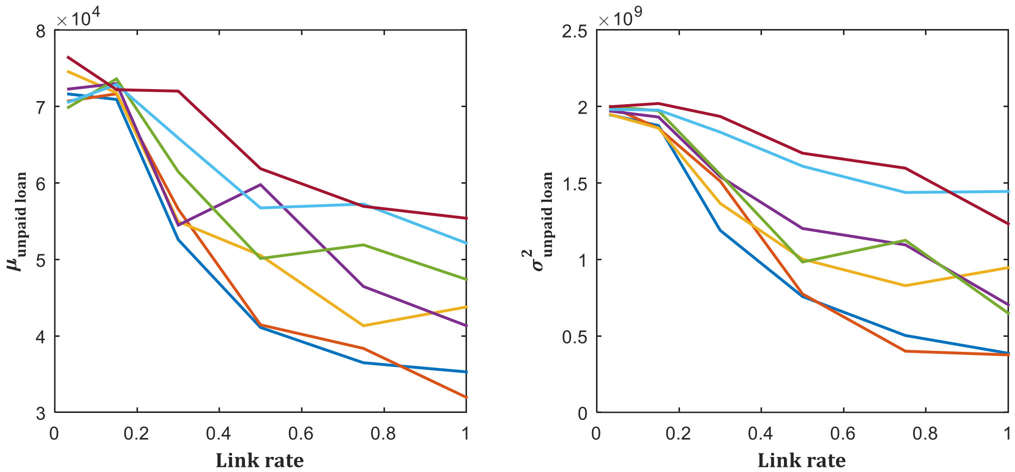

Figure 4.23 Mean and variance of all the unpaid loans as a function of link rate, when $\sigma_D = 0.7$, $\sigma_{opp} = 0.5$ and for different reserve ratios: r = 0.1 (dark blue line), r = 0.2 (light red line), r = 0.3 (yellow line), r = 0.4 (purple line), r = 0.5 (green line), r = 0.6 (light blue line), r = 0.7 (dark red line).

Looking at Area1 and Area4 in figure 4.22, it can be seen how contagion is affected by the link rate when the reserve ratio is low and fixed. For a fixed value of the reserve ratio, the link rate shows a nonlinear effect on contagion; when the link rate increase from 0 to 0.5 (Area1), the contagion increases - banks are more connected thus they have more channels to spread failure, while when the link rate increases from 0.5 to 1 (area 2), contagion decreases. This can be explained looking at figure 4.23; for low values of r (from 0.1 to 0.3, corresponding to light red line, dark blue line and yellow line respectively), as l_r increases from 0.5 to 1, the mean and variance of the unpaid loans decrease. This indicates that as link rate increases, the amount of borrowings between banks become smaller in size and more uniformly distributed. Therefore, failed banks have smaller loans thus it is unlikely they are significant enough to cause the contagion.

Area2 and Area3 show how contagion is affected by the link rate when reserve ratio is high and fixed. The contagion increases as link rate increases from 0 to 0.25 because banks are more connected. As link rate increases from 0.25 to 1, the contagion does not change much. This is because when reserve ratio is high, an increase in the link rate does not increase the borrowing, as banks need to keep cash as reserves. Therefore, the channels for spreading failure do not increase significantly.

Heterogeneous case simulation results

This section shows the simulation results in the heterogeneous case, in which banks vary in size. Deposit data for each bank are generated using equation (3.16), in which $\overline{D}_i \sim |N(\mu_s, \sigma_s{}^2)|$, is assigned to the banks by sampling from a Gaussian distribution with the mean, μ_s, and variance, $\sigma_s{}^2$. The values of σ_s , representing the degree of the heterogeneity, are set to 100, 200, 300, 400, 500, 600 and 700. The initial conditions of bank$_i$'s cash (C_i), investment (I_i), borrowing (B_i), lending (L_i) and net-worth (N_i) are $0.2 \times D_i$, $0.8 \times D_i$, 0, 0 and 0 respectively. The reserve ratio, r, is set 0.2. This reserve ratio does not limit the banks much from lending, so that the effect of the link rate can be observed.

The values of the link rate, l_r, are 0, 0.03, 0.15, 0.3, 0.5, 0.75 and 1 and given the 7 different values of the degree of the heterogeneity, σ_s , there are 49 different combinations in total, corresponding to 49 different scenarios. Each simulation case is repeated 20 times, to make sure that the main trends in the results are not significantly affected by random noise. All the results shown in this subsection are the averages of the 20 times. The simulation results show how the number of survival banks is affected by heterogeneity, link rate and the amplitude of the shock.

Simulations results 6: effect of the heterogeneity on number of survival banks

Figure 4.24 reports the waterfall plots of the number of surviving banks at day 300 as function of both link rate and heterogeneity when $\sigma_D = 0.3$ and $\sigma_D = 0.7$. Looking at figure 4.24(a), when the shock amplitude is low, $\sigma_D = 0.3$, the effect of the link rate seems not to be significant, in fact, as link rate increases from 0.03 to 1, the lines in the figure 4.24(a) stay flat and almost all the banks survive at the end of the simulation. In figure 4.24(c), instead, the effect of heterogeneity seems not be significant, in fact, for all different link rate cases, the lines stay flat. It should be pointed out that with $l_r = 0$, only a few banks survive. Therefore, the following analysis is focused on the case in which a high shock amplitude, $\sigma_{\mathrm{D}} = 0.7$, is applied.

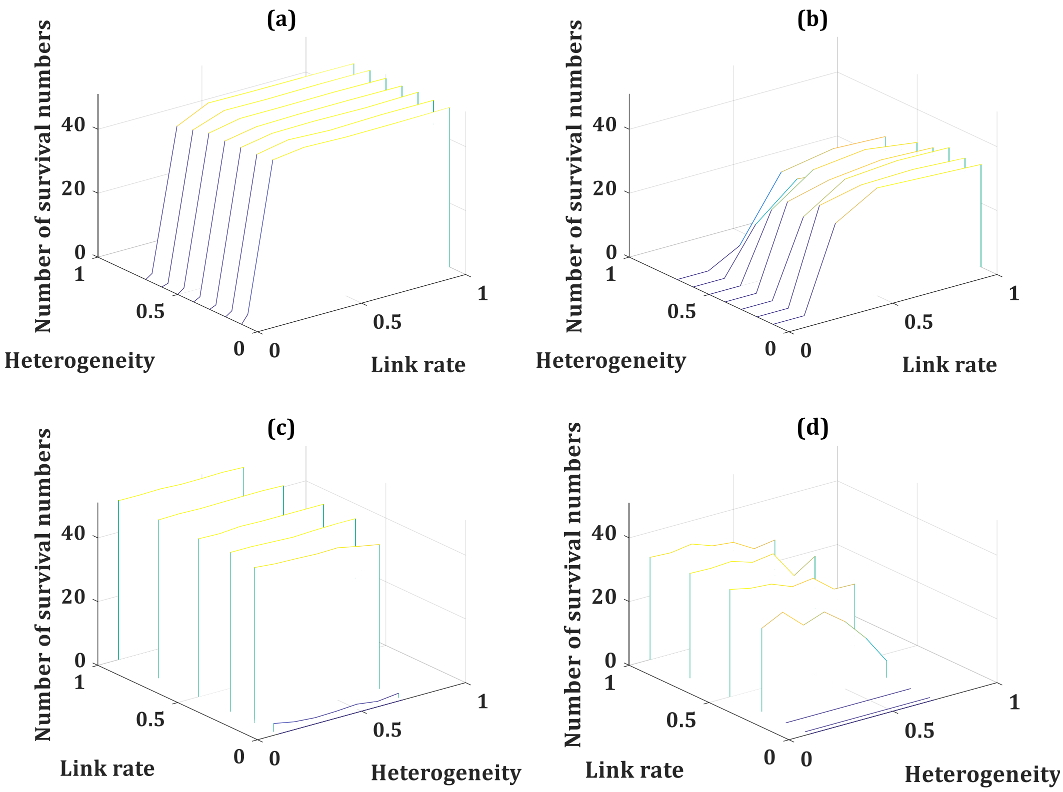

Figure 4.24 Waterfall plots showing the number of survival banks in the heterogeneity case at day 300, as function of both heterogeneity and link rate, when $\sigma_D = 0.3$ (a) and (c), and $\sigma_D = 0.7$ (b) and (d).

Figure 4.25 shows the effect of the link rate on the number of surviving banks, when the link rate is fixed. Figure 4.25(a) shows that when l_r= 0, there is little difference in number of survival banks as the heterogeneity changes. Since banks are isolated and are not affected by the interbank borrowing and lending, changes in banks' size do not affect the results. As the link rate increases, banks are connected and start to affect each other. Link rate has a positive effect as shown in figure 4.25(b), (c) and (d), in which the link rate is equal to 0.15, 0.5 and 1, respectively. This can also be observed in figure 4.25(b), in which more banks survive as the link rate increases. However, high heterogeneity tends to destabilise the system as shown in figures 4.25(b), (c) and (d); more banks fail when l_r= 0.75 (light blue line) and l_r= 1 (dark red line), compared to other lines with lower link rates.

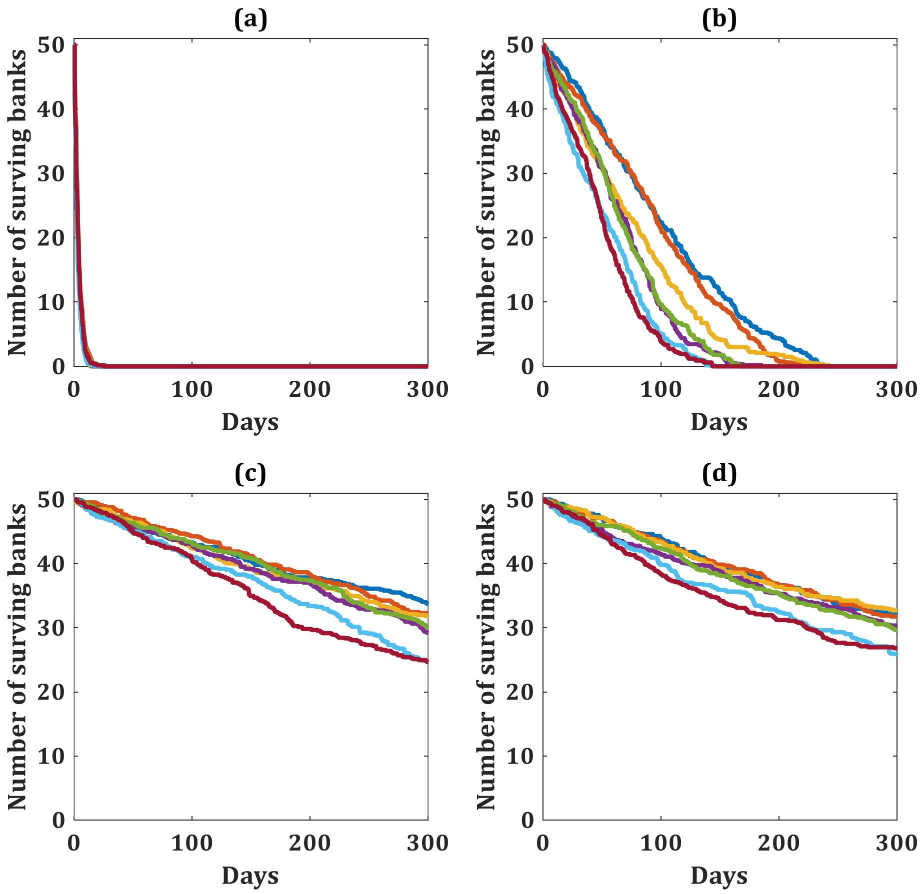

Figure 4.25 Number of survival banks in heterogeneous case with σ_D = 0.7, σ_{opp} = 0.5, and heterogeneities, σ_s = 100 (dark blue line), σ_s = 200 (light red line), σ_s = 300 (yellow line), σ_s = 400 (purple line), σ_s = 500 (green line), σ_s = 600 (light blue line), σ_s = 700 (dark red line), and under different link rates l_r= 0 (a), l_r= 0.15 (b), l_r= 0.5 (c), l_r=1 (d).

Simulation results 7: contagion effect on heterogeneous banks

The same definition is used in this subsection to quantify contagion as in the homogeneous case. Figure 4.26 shows the contagion effect as function of the interbank link rate and the heterogeneity, when $\sigma_D = 0.3$ (a) and $\sigma_D = 0.7$ (b). The contagion effect is very small when the shock amplitude is low, see figure 4.26(a), compared to 4.26(b) when shock amplitude is high.

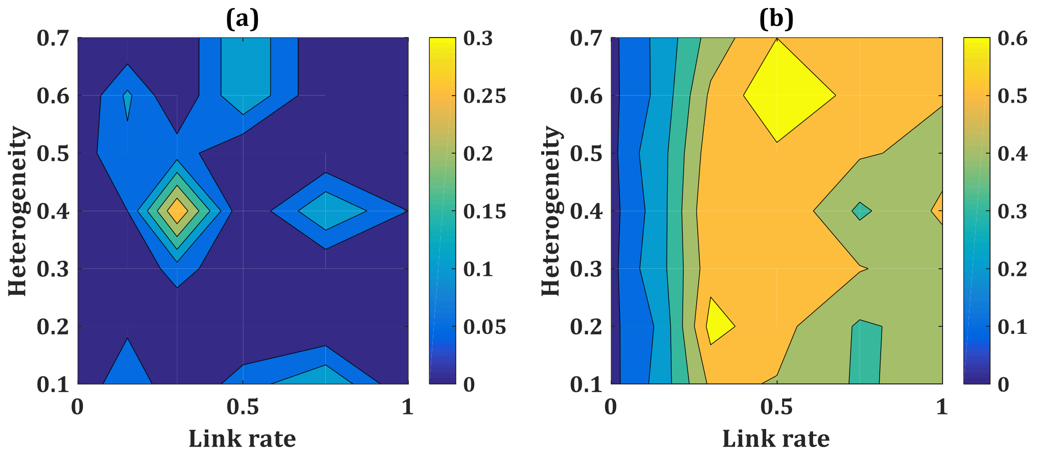

Figure 4.26 Contagion in heterogeneous banks as function of interbank link rate (l_r= 0 to 1) and heterogeneity (σ_s =100 to 700), when σ_D = 0.3 (a) and σ_D = 0.7 (b).

4.5 Conclusions

This chapter shows the MATLAB Simulink implementation of the dynamic models described in Chapter 3 as well as the results of numerical simulations. It is useful to highlight how results presented in this chapter compare with results in the literature, more specifically results presented by Iori et al. [103].

In Iori et al.'s work, it is found that more banks survive when the degree of linkage is high. Same conclusion about the link rate can be obtained from the proposed dynamic model, which shows that the link rate always contributes positively to the number of survival banks. Increasing the reserve ratio has the effect of decreasing the incidence of bank failures when the link rate is zero. These findings can be found both in Iori et al.'s work and in this work, which indicate that reserve ratio adds stability to the individual banks. However, with an interbank market (link rate larger than zero), the effect of the reserve ratio is less clear in Iori et al.'s model; increasing the reserve ratio initially leads to an increase in the incidence of bank failures, but when reserve ratio crosses a critical level, it results in fewer banks' failures. In this work, no such change is found; increasing the reserve ratio always causes more banks to fail. This may be because in this work larger shocks have been applied; in fact, the amplitude of the shocks, σ_D, is set to 0.3 and 0.7, which are larger than the value used in Iori et al.'s work, (0.25), thus, the reserve ratio is not sufficient for the individual banks to withstand the shock. Therefore, high reserve ratio cannot make more banks survive from the large shock. A further simulation with σ_D= 0.25 have been run, and similar results to Iori et al.'s ones have been found (see figure A.1 in Appendix 4).

The methods to qualify the contagion of banks' failure in Iori et al.'s model are different from the method proposed in this chapter, but two same conclusions can be drawn: the first one is that increasing the link rate is likely to increase contagion. In Iori et al.' work, the size of the avalanches (i.e. the number of failing banks at one point of time) gets larger when link rate gets higher; the results of contagion in this chapter also show that the contagion increases as link rate increase. The second conclusion is that the heterogeneity can contribute to contagion; in Iori et al.'s work, more contagion is found in the heterogeneous case compared to the homogenous case - the same trend is found in the proposed model.

The results presented in this chapter show that increasing the reserve ratio to preserve the banking system's stability may not have always a positive effect. A high reserve ratio, in fact, may be advantageous only when the shocks in the deposit is not very large and the banks are sparsely connected; when the shocks are large, a high reserve ratio may be detrimental to the survival of the banks. Encouraging the interbank activities (large link rate), instead, has always a positive effect on the number of survival banks in the system. More results of the effect of the reserve ratio and the link rate on the interest rate and net-worth of the banks can be found in Appendix 4 (figures A.2 and A.3).

Furthermore, the results presented in this chapter show that both link rate and reserve ratio have nonlinear effects on the contagion of banks' failure. These results show the importance of modelling the banking system when simultaneous changes in different parameters may have a non-intuitive effect; it is the belief of the authors of the proposed work that the findings presented in this chapter can ultimately help financial regulators in implementing new policies to preserve the banking system's stability.

Finally, it is important to stress that there are situations in which some or all banks in the system fail; this suggests that the introduction of control mechanisms to prevent banks from failing would be desirable. The next chapter shows the application of control mechanisms on the one-bank model to study and improve the stability of each individual bank.

Chapter 5

Control Analysis

This chapter presents the application and analysis of control mechanisms on the model of the nonlinear banking system introduced in Chapter 3. Classical control theory is used to study the stability of the dynamic model and subsequently output feedback control is designed to improve the stability of the system. It is well known that common analytical techniques in the classical control theory are more powerful to linear models rather than nonlinear models. Moreover, a nonlinear system is very complex and there is no generic way to deal with a nonlinear control system. Therefore, linearization is performed on the original model at first. The designed feedback control mechanisms are then applied to both the nonlinear model and linearized model for testing and comparison.

This chapter is organized as follows. Section 5.1 shows the general process of control system design which can also be used to design controllers for the banking system. Section 5.2 explains some basic methodologies which are used in the analysis including introduction of state-space models, linearization around the equilibrium point and analysis of the stability, observability and controllability of a system. Section 5.3 presents an equilibrium point analysis for the one-bank model, while Section 5.4 shows how to perform linearization around these equilibrium points. Section 5.5 gives details of the control design according to the analysis in Sections 5.3 and 5.4 as well as presenting some simulation results with the designed controller applied. Section 5.6 presents the conclusions.

5.1 Control System Design Process

The design of a control system is a specific example of engineering design. The aim of control design is to obtain the system configuration, set the performance specification, and manipulate the key parameters so that the proposed system performs as desired. Though this process is used in the engineering industry, it can be transferred to the banking system. Figure 5.1 shows the general process of control system design. It consists of seven blocks which are arranged into the three groups explained as follows.

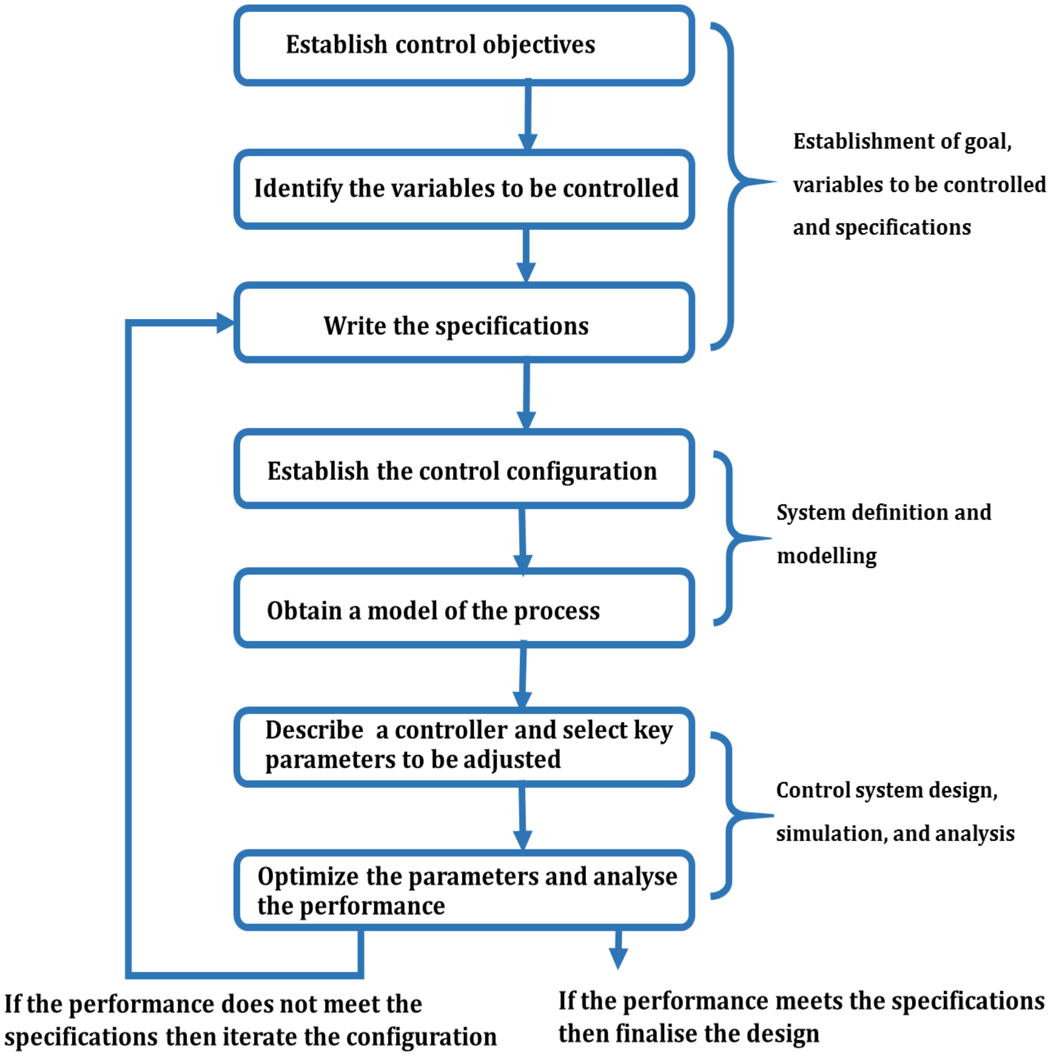

Figure 5.1 Control system design process, adapted from [15].

Establishment of goals, variables to be controlled and specifications

The first step of a control design is to establish the control objectives. For the banking system, the general objective is to keep the system stable and ensure that banks do not fail. The objectives can vary according to the different problems that the study focuses on. For example, in the one-bank model the objective is simply to keep the cash positive.

Based on the defined overall control objectives, the variables to be controlled and the performance specification are decided upon. The variables to be controlled should be the variables that can be manipulated. For example, in the banking system, what the bank can control are the lending and investment amongst other things, and these can be considered as control variables. The bank cannot control its deposit, so this cannot be used as control variables. The performance specifications describe how the closed-loop system should perform. Usually, it requires the closed-loop system to be stable and has a desirable response. The desired response should meet the requirements in rise time (the time needed by the control system to reach the desired value after a perturbation), peak overshoot (the highest value reached by the response before reaching the desired value) and settling time (the time system needs to be stable).

System definition and modelling

With the given control objectives and specifications, a control configuration (as shown in figure 2.6) can be identified. Then a suitable mathematical model should be built for the control analysis and design. Part of this modelling has been performed in chapter 3 where three models of the banking system have been generated. As the chosen design techniques can only be applied to a linear model, linearization of the nonlinear banking system models is needed. The linearization involves creating a linear approximation of a nonlinear system that is valid in a small region around the equilibrium point, a steady-state condition in which all model states are constant. A linear model can be obtained by linearization. The analysis and designs are done based on this linear model.

Control system design, simulation, and analysis

Based on the control configuration and the valid linear model, a controller can be designed. The most important process in control design is to adjust the parameter values of the controller in order to meet the objectives and performance specifications. Then the controller is tested and verified in computer simulations using the full model.

The design can be finalized and can be proceed to document the results if the desired performance is achieved. If the performance is not stable, an improved system configuration needs to be established. Then the design process will be repeated until the specifications are met, or sometimes the specifications are considered to be too demanding and need to be relaxed.

5.2 Methodologies in control theory

This section introduces some basic concepts of control theory which form the methodology of the control analysis for the banking system. These include how to use the state-space form to represent the model, linearize the model around the equilibrium point and study the stability, controllability and observability of the linear model.

5.2.1 State-space representation of the model

In control engineering, a state-space model is a mathematical model of a system whereby the input, output and state variables are related by ordinary differential equations. The idea of state-space modelling comes from the state-variable method of describing differential equations. In this method, the differential equations describing a dynamic system are organized as a set of ordinary differential equations in the vector-valued state of the system, and the solution is visualized as a trajectory of this state vector in space.

A general state-space model is shown in equation (5.1),

$$\dot{\boldsymbol{x}} = f_1(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}, t)$$

$$\mathbf{y} = h_1(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{d}, t) \tag{5.1}$$

where $\boldsymbol{x}$ is a column vector that represents the state variables. It contains n elements for an n^{th}-order system. The state variables are the smallest possible subset of system variables that can represent the entire state of the system at any given time. The minimum number of state variables required to represent a given system, n, is usually equals to the order of the system's defining differential equation. $\dot{\boldsymbol{x}}$ represents the time derivative of each state variable which is also a column vector that has the same size as $\boldsymbol{x}$. $\boldsymbol{u}$ represents the control inputs variable and $\boldsymbol{d}$ is the disturbance. Both $\boldsymbol{u}$ and $\boldsymbol{d}$ are row vectors. The numbers of elements in $\boldsymbol{u}$ and $\boldsymbol{d}$ are the numbers of the control inputs and disturbances. In the one-bank model the cash C_1 and investment I_1 are the state variables, deposit D_1 and investment opportunities Opp_1 are the disturbance and output is the cash C_1.

Figure 5.2 recalls the block diagram shows the feedback control with state-space representation. From the figure it can be seen that the disturbance is an exogenous signal that the system cannot control.

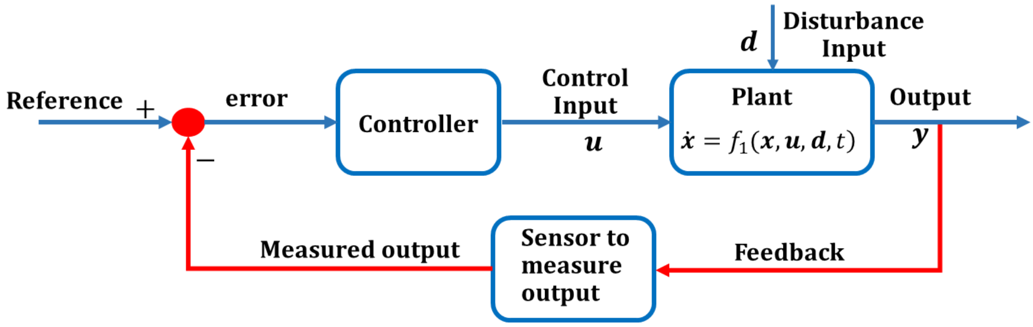

Figure 5.2 A block diagram of a negative feedback control system represented in a state-space form. The plant of the system now is described by $\dot{\boldsymbol{x}}$ which is the derivative of the state variable $\boldsymbol{x}$. $\boldsymbol{y}$ represents the output of the system. Both $\dot{\boldsymbol{x}}$ and $\boldsymbol{y}$ are functions of the state variable $\boldsymbol{x}$, control input variable $\boldsymbol{u}$, external disturbances $\boldsymbol{d}$ and time t.

A state-space model is used because it can deal with multi-input and multi-output systems, i.e. systems that have more than one control input or more than one sensed output. Though in the analysis of the one-bank model, the system is a single-input single-output system (SISO), this method still can be used and it can build up a good foundation for the future analysis of the two-bank model as well as the multi-bank model. Another reason to use the state-space model is that it is particularly well suited to the use of computer techniques, which enhance our ability to take the advantage of the computational efficiency of MATLAB.

In equation (5.1), when f_1 and h_1 are nonlinear functions of $\boldsymbol{x}$ and $\boldsymbol{u}$, it can be said that the system is nonlinear, while when f_1 and h_1 are linear with respect to both $\boldsymbol{x}$ and $\boldsymbol{u}$, the system is linear system. Additionally, if the dynamical system is linear and there is no disturbance input, the SISO system can be expressed in matrix form shown as follows:

$$\boldsymbol{x}(\dot{\boldsymbol{t}}) = \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t})$$

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{C}\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}\boldsymbol{u}(\boldsymbol{t}) \tag{5.2}$$

where $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$, $\boldsymbol{B} \in \boldsymbol{R}^{n\times 1}$, $\boldsymbol{C} \in \boldsymbol{R}^{1\times n}$, and $\boldsymbol{D} \in \boldsymbol{R}$. Sometimes, real systems are described by nonlinear models such as (5.1), and the tools in classic control theory usually cannot be employed to design controllers for nonlinear systems. Therefore, linearization technique is used to find the corresponding linear models which can approximate the original nonlinear system thus it is possible to study the behaviour of the nonlinear system with the classic control tools by considering the linearized models.

5.2.2 Equilibrium point and linearization

Frequently behaviour of the nonlinear model within a certain operating range of an equilibrium point can be reasonably approximated by that of a linear model. One reason for approximating the nonlinear system by a linear model is that, by doing so, one can apply rather simple and systematic linear control design techniques such as root locus analysis. The behaviour of the solutions of the linear system are expected to be the same as the nonlinear ones, so that a controller designed based on the linear model will perform well also on the nonlinear model. However, it needs to be stressed that a linearized model is valid only when the system operates in a sufficiently small range around an equilibrium point. To take into account the presence of nonlinearities, more sophisticated tools are needed which are beyond the scope of simple linear analysis.

Consider a system in state-space representation in (5.1). When $\boldsymbol{u}$ is set to be a constant value $\boldsymbol{u}^*$and there is no disturbance, if $\dot{\boldsymbol{x}} = f_1(\boldsymbol{x}^*, \boldsymbol{u}^*) = [\boldsymbol{0}\ \boldsymbol{0} \dots \boldsymbol{0}]^{\boldsymbol{T}}$**,** then $(\boldsymbol{x}^*, \boldsymbol{u}^*)$ is said to be an equilibrium point of system (5.2). Let $\boldsymbol{y}^* = h_1(\boldsymbol{x}^*, \boldsymbol{u}^*)$**,** which is the output value at the equilibrium point. The aim of the linearization is to find a linear system when $(\boldsymbol{x}, \boldsymbol{y})$ is close to $(\boldsymbol{x}^*, \boldsymbol{y}^*)$. In order to do that, it is necessary to approximate the functions $f_1(\boldsymbol{x}, \boldsymbol{y})$ and $h_1(\boldsymbol{x}, \boldsymbol{y})$ when $(\boldsymbol{x}, \boldsymbol{y})$ is close to $(\boldsymbol{x}^*, \boldsymbol{y}^*)$**.**

Since the linearization is done around the equilibrium point, a coordinate transformation is defined as follows. Denote $\Delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}^*$, $\Delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}^*$, and $\Delta\boldsymbol{y} = \boldsymbol{y} - \boldsymbol{y}^*$. The new coordinates $\Delta\boldsymbol{x}$, $\Delta\boldsymbol{u}$, and $\Delta\boldsymbol{y}$ represent the variations of $\boldsymbol{x}$, $\boldsymbol{u}$, and $\boldsymbol{y}$ from their equilibrium values. The linearization of (5.1) at $\boldsymbol{x}^*$ is given by

$$\Delta\dot{x} = A\Delta x + B\Delta u$$

$$\Delta y = C\Delta x + D\Delta u \tag{5.3}$$

Where $\quad A = \left[\frac{\partial f_1}{\partial \mathrm{x}}\right]_{x^*,u^*} \; B = \left[\frac{\partial f_1}{\partial \mathrm{u}}\right]_{x^*,u^*} \; C = \left[\frac{\partial h_1}{\partial \mathrm{x}}\right]_{x^*,u^*} \; D = \left[\frac{\partial h_1}{\partial \mathrm{u}}\right]_{x^*,u^*}$

From the above linearization process, the relationship between the linear and nonlinear systems are shown as in figure 5.3. If the nonlinear system is linearized at the equilibrium point (x^*, u^*, y^*), then the dynamics of the linear model can approximate the dynamics of nonlinear model when the same inputs and initial conditions are applied around this equilibrium point.

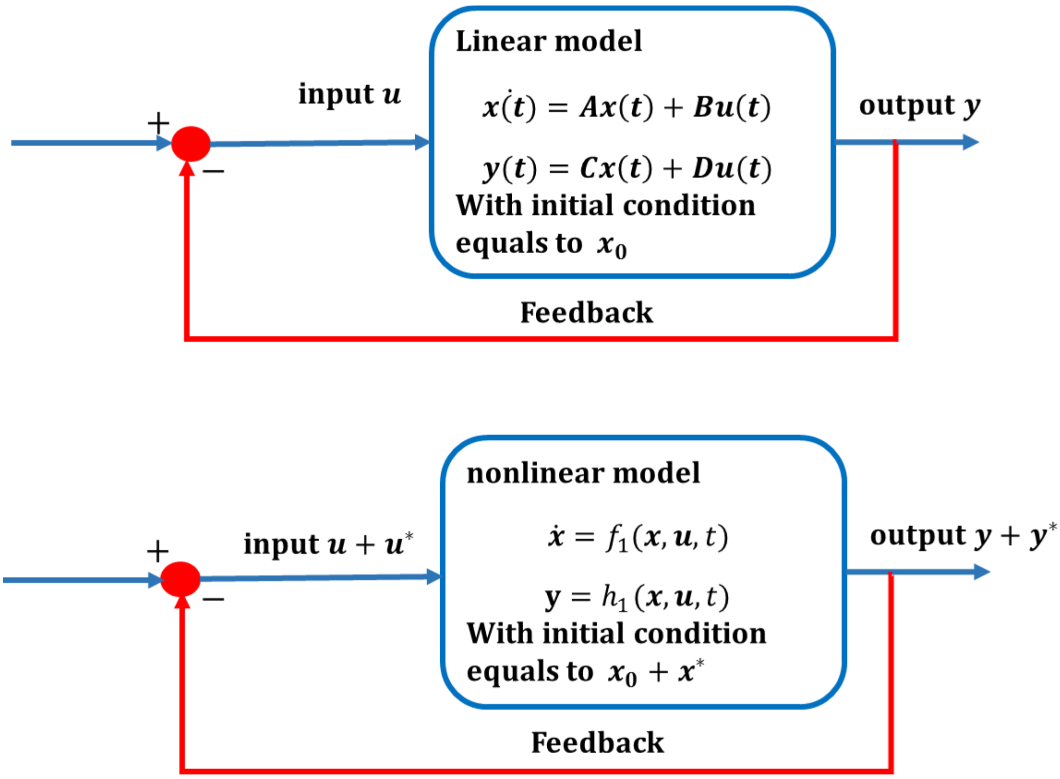

Figure 5.3 Block diagrams showing the relationship between linear and nonlinear models. Any input u applied to linear model represents an input of $u + u^*$ applied to the nonlinear model. Any initial condition x_0 in the linear model is equivalent to initial condition of $x_0 + x^*$in the nonlinear model. Any output y in the linear model is expected to be the same as the output in nonlinear model when pluses by y^*.

For convenience, the notation Δx, Δu and Δy in equation (5.3) is replaced by x, u and y respectively in the rest of this chapter. Therefore, if an input $u = 1$ is applied to the linear system, the corresponding input for the nonlinear system should be $u^* + 1$. An output of 1 in the linear model is corresponding to the output of $\boldsymbol{y}^* + 1$ in the nonlinear model. An initial condition of $\boldsymbol{x} = 1$ in the linear model represents an initial condition of $\boldsymbol{x}^* + 1$ in the nonlinear model.

5.2.3 Stability, observability and controllability of the linear system

This section shows some analysis done based on the linear model to study the stability, observability and controllability of the linear system to facilitate the subsequent control design. There are many concepts of stability, and many different definitions are possible. Here the eigenvalues of the matrix $\boldsymbol{A}$ in the linear model is used to investigate the stability of the linear system. Before deciding the control, observability and controllability of a system must be considered, since they can tell whether is possible to observed or control (stabilize) the system and if the measured output can be used to represent the behaviour of the system.

Stability

Considering the linear system (5.2), it can be stated that the linear system is

(a) *stable* if all of the eigenvalues of the matrix $\boldsymbol{A}$ have negative-real values, i.e. the real part of each eigenvalue must be less than zero. Practically speaking, for continuous time system, stability requires that the complex eigenvalues reside in the open left half of the complex plane.

(b) *marginally stable* if at least one eigenvalue has a zero real part and other eigenvalues have negative real parts. In the continuous time case, if the eigenvalue with a zero real part and a zero imaginary part, the system response neither decays nor grows over time, while when imaginary part is not equal to zero, oscillations can be expected

(c) *unstable* if any eigenvalues of matrix $\boldsymbol{A}$ have a positive real part.

Controllability and Observability

Controllability is related to the possibility of steering the states of a system from any initial value to any final value by using an appropriate control signal within some finite time window. If a state is not controllable, then there is no signal that will ever be able to control the state.

A system is controllable if and only if the controllability matrix $\overline{\boldsymbol{C}}$ is full rank.

$$\overline{\boldsymbol{C}} = [\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A}^2\boldsymbol{B} \quad \dots \quad \boldsymbol{A}^{n-1}\boldsymbol{B}]$$

Observability instead is related to the possibility of observing the state of a system through output measurements. If a state is not observable, it can be stable from the controller if the unobservable structure is stable. However, if the unobservable structure is not stable, the output feedback controller is not able to use it to stabilize the system since the controller cannot determine the behaviour of an unobservable state.

A system is observable if and only if the observability matrix $\overline{\boldsymbol{O}}$ is full rank.

$$\overline{\boldsymbol{O}} = [\boldsymbol{C} \quad \boldsymbol{CA} \quad \boldsymbol{CA}^2 \quad \dots \quad \boldsymbol{CA}^{n-1}]$$

It is important and necessary to check the controllability of a system before designing the controller. Since if one of the eigenvalues of the system is neither controllable nor observable, then this part of the dynamics will remain untouched in the closed-loop system. If such an eigenvalue is not stable, then the dynamics of this eigenvalue that present in the closed-loop system will also be unstable. Therefore, for all the states of each variable of the system to be controlled, every "bad" state of these variables must be controllable and observable to ensure a good behaviour in the closed-loop system.

In the remaining of the chapter, the control design and the control methodologies are applied only on the one-bank model. The control objective of the one-bank model is to keep the cash of the bank at a desired positive value. A suitable equilibrium point is identified and linearization is done around this equilibrium point. After the stability and observability of the linear model are checked, a controller can be designed based on linear control theory. The performance is evaluated by testing the controller using both linear and nonlinear models. In the following sections these procedures are explained in detail.

5.3 Equilibrium point analysis of the one-bank model

The equilibrium analysis for the one-bank model consists, firstly, of finding the equilibrium points. More specifically the values of the state variables, I_1, and C_1, which render equations (3.2) and (3.4) zero, for given values of the input variables, D_1 and opp_1. By setting the derivatives to zero, the equilibrium points can be identified：

$$\min[(C_1^* - rD_1^*)^+, opp_1^*] - w_1 I_1^* - v_1 I_1^* = 0 \tag{5.4}$$

$$-g_1 D_1^* + p_1 I_1^* - v_1 I_1^* = 0 \tag{5.5}$$

where C_1^*, I_1^*, D_1^* and opp_1^* represent the value of the bank's cash, investment, deposit and investment opportunity at the equilibrium point.

In this analysis, it is assumed that D_1^* and opp_1^*, i.e. the disturbance inputs variables, are kept constant in a given simulation; the equilibrium points are then found from equations (5.4) and (5.5) which can be solved under 3 different cases for the value of the cash, C_1^*, with respect to rD_1^* and opp_1^*:

Case 1: $0 \le rD_1^* \le C_1^*$ and $0 \le opp_1^* \le C_1^* - rD_1^*$ (bank's cash above reserve and limited investment opportunity).

Equations (5.4) and (5.5) become:

$$opp_1^* - w_1 I_1^* - v_1 I_1^* = 0 \tag{5.6}$$

$$-g_1 D_1^* + p_1 I_1^* - v_1 I_1^* = 0 \tag{5.7}$$

From equation (5.6) it can be derived:

$$I_1^* = \frac{opp_1^*}{w_1 + v_1} \tag{5.8}$$

From equation (5.7) the value for I_1^* can be derived:

$$I_1^* = \frac{g_1 D_1^*}{p_1 - v_1} \tag{5.9}$$

From (5.8) and (5.9), it can be derived that:

$$\frac{opp_1^*}{w_1+v_1} = \frac{g_1 D_1^*}{p_1-v_1} \tag{5.10}$$

Moreover, there is an infinite number of equilibria characterised by positive and constant values for C_1^*.

<u>Case 2: $0 \leq rD_1^* \leq C_1^*$ and $opp_1^* > C_1^* - rD_1^*$ (bank's cash above reserve and investment opportunity is larger than available cash).</u>

Equations (5.4) and (5.5) become:

$$C_1^* - rD_1^* - w_1 I_1^* - v_1 I_1^* = 0 \tag{5.11}$$

$$-g_1 D_1^* + p_1 I_1^* - v_1 I_1^* = 0 \tag{5.12}$$

From equation (5.11) it can be derived:

$$I_1^* = \frac{C_1 - rD_1^*}{w_1+v_1} \tag{5.13}$$

From equation (5.12) it can be obtained:

$$I_1^* = \frac{g_1 D_1^*}{p_1-v_1} \tag{5.14}$$

Substitute (5.14) into (5.13) and rearrange, the following expression for the cash can be derived:

$$C_1^* = D_1^* \left[\frac{g_1(w_1+v_1)}{p_1-v_1} + r\right] \tag{5.15}$$

<u>Case 3: $0 \leq C_1^* \leq rD_1$ (bank's cash below reserve).</u>

In this case opp_1^* cannot affect the equilibrium since the cash is below the reserve so the bank cannot invest.

Equations (5.4) and (5.5) become:

$$0 - w_1 I_1^* - v_1 I_1^* = 0 \tag{5.16}$$

$$-g_1 D_1^* + p_1 I_1^* - v_1 I_1^* = 0 \tag{5.17}$$

From equation (5.16) the following value of the investment can be obtained, $I_1^* = 0$.

From equation (5.17) the following value of the deposit can be derived, $D_1^* = 0$.

As $0 \le C_1^* \le rD_1^*$ and $D_1^* = 0$, so this means that $C_1^* = 0$.

Table 5.1 reports a summary of the three equilibrium points. Equilibrium point 1 is found in case 1, in which the bank is in a condition where it has enough cash but it can only add investment determined by investment opportunity availability. In case 2, where equilibrium point 2 is found, the bank has again enough cash and will invest as much as it can since the investment opportunity is larger than the available cash. In cases 1 and 2, non-zero equilibrium states for C_1^* and I_1^* exist. In case 3, the bank doesn't have enough cash to invest and the only equilibrium state consists of zero values for C_1^* and I_1^*.

Table 5.1 Summary of the equilibrium points for different cases.

Equilibrium point 1	Equilibrium point 2	Equilibrium point 3
With a given D_1^*, $opp_1^* = \frac{g_1 D_1(w_1+v_1)}{p_1-v_1}$ $C_1^* > D_1^* * r$ $I_1^* = \frac{opp_1^*}{w_1+v_1}$	With a given D_1^*, $opp_1^* > C_1^* - r * D_1^*$ $C_1^* = D_1^* \left(\frac{g_1(w_1+v_1)}{p_1-v_1} + r\right)$ $I_1^* = \frac{g_1 D_1^*}{p_1-v_1}$	$D_1^* = 0$, $opp_1^* = 0$ $C_1^* = 0$ $I_1^* = 0$

It is interesting to see from table 5.1 how changes in the parameter values, g_1, p_1, w_1 and v_1, can affect C_1^* and I_1^*. For example in equation (5.14), the corresponding value of I_1^* depends on the value of g_1, p_1 and v_1. So if p_1 gets larger, then I_1^* becomes smaller. This means that as the return rate for the investment increases, then less investment is required to keep the system in the equilibrium state. While when v_1 (investment failure rate) and g_1 (deposit interest rate) are large then I_1^* becomes large, which means a high level of investment is needed to keep the bank in equilibrium. Similar effects can be found in equation (5.15), as the proportion of the matured investment, w_1, and return rate for the investment, p_1, get higher, less cash is needed to keep the system in equilibrium state; v_1 and g_1 have the opposite effect on C_1^*. So from the above analysis, parameters that increase the net-worth of the bank, such as p_1, decrease the state variables equilibrium values, C_1^* and I_1^*, while those parameters, such as v_1 and g_1, that decrease the net-worth, increase the equilibrium values, C_1^* and I_1^*. These effects of the parameters can be evidenced by the simulation results that are represented in subsection 4.2.2.

5.4 Model linearization and analysis of the one-bank model

This section represents how the linearization is performed using MATLAB and Simulink. Three linear models are obtained around the three equilibrium points (shown in the previous section). Stability analysis is carried out for the three linear models separately.

Linearization using MATLAB and Simulink

Linearization of the one-bank model is performed around the three equilibrium points in table 5.1. The MATLAB command 'linmond' is used to do the linearization. This command is in a format of '[A, B, C,D]=linmod ('SYS',X,U,Y)', which can obtain the state-space linear model of the system from the ordinary differential equations described in the Simulink block diagram called 'SYS', when the state vector, X, input, U, and output Y are specified given the value at the equilibrium point. The Simulink diagram of the one-bank model, 'SYS', is reported in figure 5.4, which is a slight modification of the diagram reported in figure 4.3 to identify the input, U (red blocks) and output, Y (light blue block linked to 'Intergrator2').

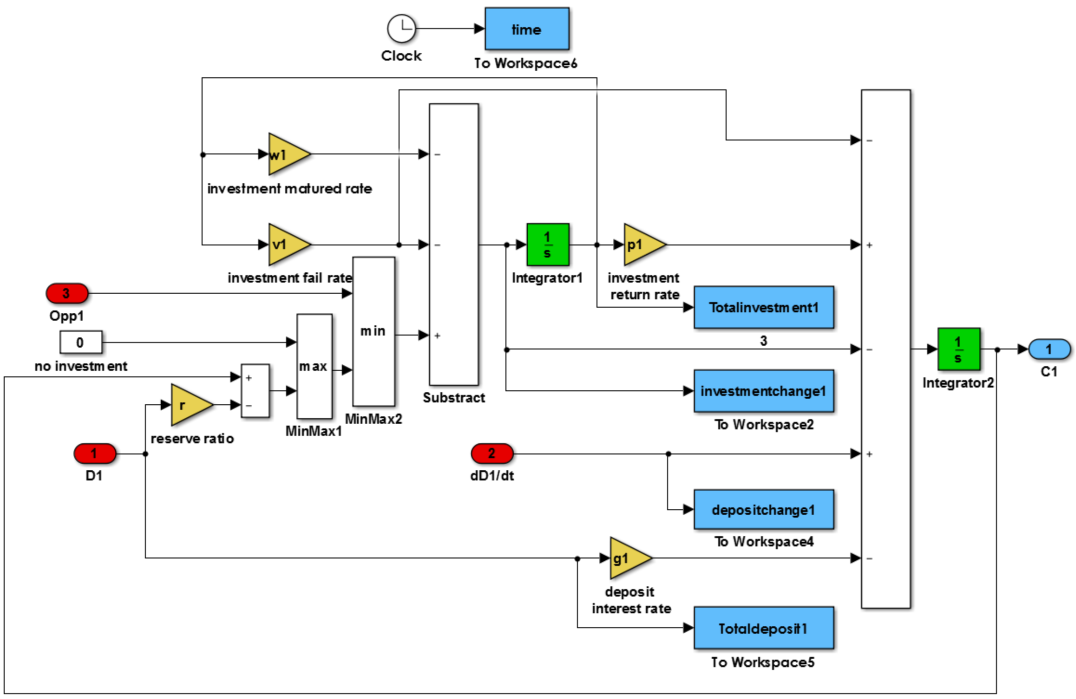

Figure 5.4 The one-bank model in Simulink used for the linearization, named 'SYS'.

Since at this stage there is no controller applied, the linear model only describes the dynamics of the open-loop system which doesn't have the control input variables, u, but only the disturbance inputs, i.e. the deposit and investment opportunity. Therefore, in the Simulink model in figure 5.4 the blocks shown in red and the block linked with the integrator 2 shown in blue identify the input variables, D_1 and opp_1, and the output variable, C_1, for the linearization command. The state vector, X, in the command is given the value of C_1^* and I_1^* that are calculated by the equilibrium equations and the input, U, is given the value of D_1^* and opp_1^* that are calculated by the equilibrium equations. Then the command will output the $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ that describe the linear model as shown in equations (5.2).

Three linear models

Three linear models are derived using the equilibrium points and parameter values shown in table 5.2. The three linear models are represented in the following equations, where $\boldsymbol{x} = \begin{bmatrix} C_1 \\ I_1 \end{bmatrix}$ are the state variables and $\boldsymbol{d} = \begin{bmatrix} D_1 \\ dD_1 \\ opp_1 \end{bmatrix}$ is the disturbance.

<u>Linear model 1</u>

$$\dot{\mathrm{x}} = \boldsymbol{A_1 x} + \boldsymbol{D_{1d} d} \tag{5.18}$$

$$y = \boldsymbol{C_1 x} \tag{5.19}$$

where $\boldsymbol{A_1} = \begin{bmatrix} 0 & w_1 + p_1 \\ 0 & -w_1 - v_1 \end{bmatrix}$, $\boldsymbol{D_{1d}} = \begin{bmatrix} -g_1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, $\boldsymbol{C_1} = [1 \quad 0]$.

<u>Linear model 2</u>

$$\dot{\mathrm{x}} = \boldsymbol{A_2 x} + \boldsymbol{D_{2d} d} \tag{5.20}$$

$$y = \boldsymbol{C_2 x} \tag{5.21}$$

where $\boldsymbol{A_2} = \begin{bmatrix} -1 & w_1 + p_1 \\ 1 & -w_1 - v_1 \end{bmatrix}$, $\boldsymbol{D_{2d}} = \begin{bmatrix} r - g_1 & 1 & 0 \\ -r & 0 & 0 \end{bmatrix}$, $\boldsymbol{C_2} = [1 \quad 0]$.

<u>Linear model 3</u>

$$\dot{\mathrm{x}} = \boldsymbol{A_3 x} + \boldsymbol{D_{3d} d} \tag{5.22}$$

$$y = \boldsymbol{C_3 x} \tag{5.23}$$

where $\boldsymbol{A_3} = \begin{bmatrix} 0 & w_1 + p_1 \\ 0 & -w_1 - v_1 \end{bmatrix}$, $\boldsymbol{D_{3d}} = \begin{bmatrix} -g_1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{C_3} = [1 \quad 0]$.

The eigenvalues of $\boldsymbol{A_1}$, $\boldsymbol{A_2}$ and $\boldsymbol{A_3}$ are calculated using the parameter values shown in table 5.2. Also the ranks of the observability matrix ($\overline{\boldsymbol{O}} = [\boldsymbol{C} \quad \boldsymbol{CA}]$) are calculated to check whether the three linear models are observable. From the results shown in table 5.2 it can be seen that for all the three models the ranks of observability matrix ($\overline{\boldsymbol{O}} = [\boldsymbol{C} \quad \boldsymbol{CA}]$) are all equal to 2, that is full rank. Therefore, all the states are observable across the three models and all the states if available for measurement can be used as the feedback in the control design.

Table 5.2 The three equilibrium points used for linearization and the eigenvalues of the A matrix, observability matrix of the three linear models. The parameter values used for calculating the eigenvalues are: $g_1 = 0.02/360$, $p_1 = 0.05/360$, $w_1 = 0.05/360$, $v_1 = 0.01/360$ and $r = 0.2$.

Model	Equilibrium point	Eigenvalues of matrix $\boldsymbol{A}$	Rank[C;CA]
1	D_1^* with a given value $opp_1^* = \frac{g_1 D_1^*(w_1+v_1)}{p_1-v_1}$ $C_1^* = D_1^* * r$ $I_1^* = \frac{opp_1^*}{w_1+v_1}$	$\lambda_1 = \frac{-(w_1+v_1)+\sqrt{(w_1+v_1)^2}}{2} = 0$ $\lambda_2 = \frac{-(w_1+v_1)-\sqrt{(w_1+v_1)^2}}{2}$ $= -0.1667 * 1.0e - 03$	2
2	D_1^* with a given value $C_1^* = D_1\left(\frac{g_1(w_1+v_1)}{p_1-v_1} + r\right)$ $I_1^* = \frac{g_1 D_1}{p_1-v_1}$, $opp_1^* = C_1^* - r * D_1^*$ $N_1 = C_1^* + I_1^* - D_1^*$	$\lambda_1 =$ $\frac{-(w_1+v_1+1)-\sqrt{(w_1+v_1+1)^2-4(v_1-p_1)}}{2}$ $= -1.0003$ $\lambda_2 =$ $\frac{-(w_1+v_1+1)+\sqrt{(w_1+v_1+1)^2-4(v_1-p_1)}}{2}$ $= 0.0001$	2
3	$D_1^* = 0$, $opp_1^* = 0$ $C_1^* = 0$ $I_1^* = 0$	$\lambda_1 = \frac{-(w_1+v_1)+\sqrt{(w_1+v_1)^2}}{2} = 0$ $\lambda_2 = \frac{-(w_1+v_1)-\sqrt{(w_1+v_1)^2}}{2}$ $= -0.1667 * 1.0e - 03$	2

The eigenvalues of $\boldsymbol{A_1}$, $\boldsymbol{A_2}$ and $\boldsymbol{A_3}$ allow the analysis of the stability of the linear models. In this case, stability means the linear model will stay around the corresponding equilibrium point. It must be stressed that all the system's behaviour obtained from the stability analysis applied only in the linear model. In the nonlinear model, this behaviour only happens when all the states are close to the equilibrium point; if the states value move too far away from the equilibrium, then the current linear model is not able to represent nonlinear model behaviour anymore, so all the stability analysis based on the currently linear model is no longer applicable. For this reason, the dynamics of the non-linear model (original one-bank model) needs to be represented by all three linear models.

Stability analysis of linear model 1

For linear model 1, one of the eigenvalues of $\boldsymbol{A_1}$ is zero and the other one is always negative (see table 5.2). So linear model 1 is marginally stable, which means when there is no disturbance, wherever the initial condition starts, the system will always move back to zero (zero in the linear model is equivalent to the equilibrium point in the nonlinear model). Linear model 1 represents the dynamic of the non-linear model when all state variables satisfy the conditions shown in Section 5.3 case 1. In this dynamic, assuming that the initial conditions for cash and investment are C_1^0 and I_1^0, with a given value of the deposit D_1^*, as long as $opp_1^* = \frac{g_1 D_1^*(w_1+v_1)}{p_1-v_1}$ and $C_1^0 \geq D_1^* * r$, no matter what the value of I_1^0 is at the beginning, the total investment I_1 will always come to the value equals to $I_1^* = \frac{opp_1^*}{w_1+v_1}$.

Stability analysis of linear model 2

Linear model 2 represents the dynamic of the nonlinear model when all variables satisfy the conditions shown in Section 5.3 case 2. One of the eigenvalues of $\boldsymbol{A_2}$ is positive when $p_1 > v_1$, while the other eigenvalue is negative (see table 5.2). So linear model 2 is unstable and if the initial conditions, C_1^0 and I_1^0, do not start at the equilibrium point in the nonlinear model, the system will not be able to come back to the equilibrium point.

Stability analysis of linear model 3

The eigenvalues of $\boldsymbol{A_3}$ are the same as $\boldsymbol{A_1}$, so linear model 3 is also marginally stable. The difference between linear model 3 and linear model 1 is that the equilibrium points are different. Therefore, if the nonlinear system is in the dynamic that represented by linear model 3 (when all variables satisfy the conditions shown in Section 5.3 case 3), all the states will come to the equilibrium point.

5.5 Controller design and testing results of the one-bank model

After carrying out the stability analysis of the linear models presented in the previous section, this section introduces how to design the controller to reach the control objectives. Three different control objectives are set in the subsection 5.5.1 according to different dynamics of the model. Two output feedback controllers are designed in subsection 5.5.2 to meet these objectives. The controllers are tested in different dynamic characteristics in subsections 5.5.3, 5.5.4 and 5.5.5. In the last subsection, simulation results show how the controller switches when the system enters different dynamics.

5.5.1 Possible dynamics and objectives of the control design

In the control design for the one-bank model, the objectives are established according to the level of the cash the bank holds at a given time. Three zones are defined according to the level of cash the bank has, as shown in figure 5.5.

The cash of the bank changes with time and the bank can enter different dynamics that are characterised by different linear models; therefore, according to the equilibrium and the linearization analysis, different linear models are used to represent the dynamics in each zone.

Regime 1: in this regime, the cash is above the reserve ratio requirement, which satisfies the condition shown in Section 5.3 cases 1 and 2, so this dynamic can be described by linear models 1 and 2. The bank is considered to stay in a 'safe zone' under this dynamic in the absence of deposit shocks. Since the bank has extra cash to invest in this dynamics, the aim of the control is not merely to keep the cash above zero, but also to keep the cash at a steady desired level that allows the bank to generate more profit.

Regime 2: in this regime, the cash is below reserve ratio requirement but still positive, and this satisfies the conditions shown in Section 5.3 case 3. Therefore, this dynamic can be described by linear model 3. Under this dynamics, the bank is considered to be in a 'dangerous zone'. Since it cannot meet the reserve ratio requirement. Furthermore, because of the lack of cash, it stops adding new investment. Therefore, in this dynamics, the aim of the control design is to try to adjust the cash to meet the reserve ratio requirement.

Regime 3: In this regime, the cash at the bank becomes negative and thus the bank stops all activities. This can happen when a shock in the deposit is big enough to make the cash negative. It is assumed that the bank faces a serious funding gap and if the bank does not take action to get more cash, it will fail. The aim of the control is to try to bring the cash from negative to positive within a very short time.

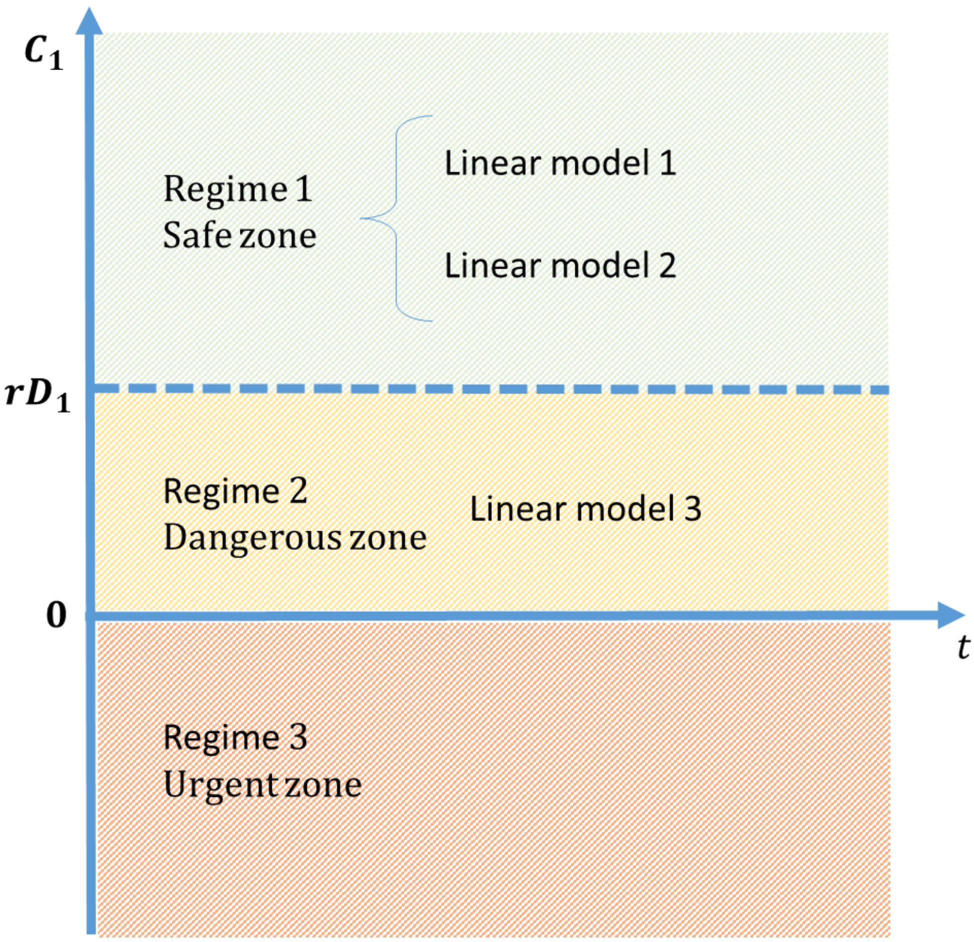

Figure 5.5 Plot shows the different dynamics when C_1 has the different values.

The case considered in dynamics 3 can only happen within the one-bank model if the shock in the deposit is big enough to make the cash negative. Once the cash becomes negative the bank cannot recover back to positive cash unless a controller is implemented (i.e. within the equations of the model) to bring the cash back to positive and avoid failure. The design and the implementation of the controller will be explained in the following sections.

5.5.2 Design of output feedback controllers

Before designing the controller, there is the need to establish the control configuration, in which the control input variables and method to measure the output are identified. It is proposed that the bank manipulates its investment when its cash is lower or higher than the equilibrium value to adjust its cash to the equilibrium value. The amount of investment to sell or to add depends on the value of the current cash compared with the equilibrium value of the cash. In this control design, the cash of the bank is chosen to be the measured output and the control input, $u = \gamma(C_1 - C^*)$, is the difference between actual, C_1, and equilibrium, C^* , cash multiplied with a properly selected feedback gain, γ. The control configuration is shown in figure 5.6.

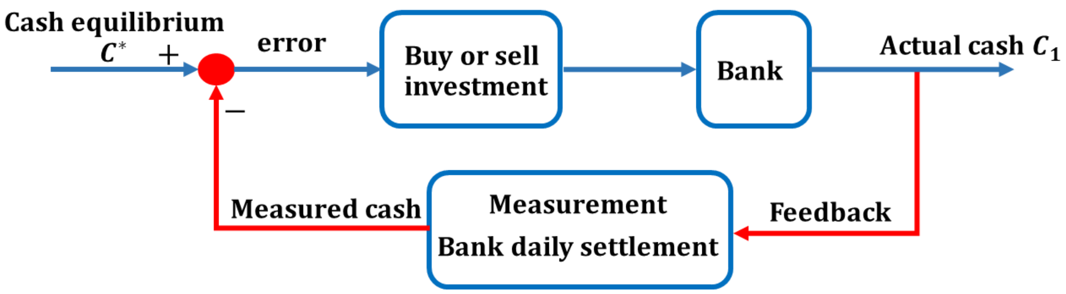

Figure 5.6 The configuration of the feedback control design for the one-bank model. The bank manipulates its investment to adjust its cash to the equilibrium value. The error signal is the difference between the equilibrium cash value and the actual cash value which is obtained through the bank daily settlement.

The actual cash of the bank, C_1, is measured through the bank daily settlement and compared to the equilibrium value of the cash. The bank will then sell or buy assets (investment) according to this difference, $(C_1 - C^*)$, to adjust its cash. At the first stage the controller brings the cash to the equilibrium value C^* and theoretically the, controller is able to bring the cash to any expected value, C_{re}, by adding an extra part in the controller called the reference input. Details are introduced in the later part of this section.

The output feedback control law u in the linear model can be designed as shown in equation (5.24). u represents the amount of the investment sold or bought. The γ is the feedback gain which will be multiplied by the difference between the actual cash C_1 and equilibrium point C^*. This difference is the output y of linear model, and it is also the error signal $(C_1 - C^*)$ in the nonlinear system.

$$u = \gamma y \tag{5.24}$$

In equation (5.24) $\gamma > 0$. Depending on the strategy used by the bank to adjust the cash, two controllers are developed as shown in the following.

Controller 1

This controller always tries to bring the cash to the equilibrium value by selling the bank's investment when its cash is higher than the equilibrium point and adding investment while its cash is lower than the equilibrium. The linear system in the state-space representation now becomes as shown in equation (5.25) where $\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. When y is positive, this means the bank sells the investment, while when y is negative the bank adds investment.

$$\dot{\mathbf{x}} = \boldsymbol{Ax} + \boldsymbol{B_1}u + \boldsymbol{D_d d}$$

$$y = \boldsymbol{Cx}$$

$$u = \gamma y \tag{5.25}$$

The non-linear system of the one-bank model is described by differential equations reported in equation (5.26). When $\gamma(C_1 - C^*)$ is positive, it means the bank sells the investment, while when $\gamma(C_1 - C^*)$ is negative the bank adds investment.

$$\frac{dI_1}{dt} = \min[(C_1 - rD_1)^+, opp_1] - w_1 I_1 - v_1 I_1 - \gamma(C_1 - C^*)$$

$$\frac{dC_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1 \tag{5.26}$$

Controller 2

Under this control strategy, the bank sells its investment when its cash is lower than the equilibrium point and adds investment while its cash is higher than the equilibrium. The linear system in the state-space representation is the same as shown in equation (5.27), but now with $\boldsymbol{B_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. When y is positive, this means the bank adds the investment while when y is negative the bank sells investment.

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{B_2}u + \boldsymbol{D_d d}$$

$$y = \boldsymbol{Cx}$$

$$u = \gamma y \tag{5.27}$$

The nonlinear system described by differential equations of the one-bank model is now shown in equations (5.28) and (5.29). When $\gamma(C_1 - C^*)$ is positive, it means the bank adds the investment while when $\gamma(C_1 - C^*)$ is negative the bank sells investment.

$$\frac{dI_1}{dt} = \min[(C_1 - rD_1)^+, opp_1] - w_1 I_1 - v_1 I_1 + \gamma(C_1 - C^*) \tag{5.28}$$

$$\frac{dC_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1 \tag{5.29}$$

the controllability of the three linear models is checked. The ranks of the controllability matrices are shown in table 5.3. All the controllability matrixes have full rank, so the linear models are controllable when applied the two distributions.

Table 5.3 Rank of the controllability matrixes $\overline{\boldsymbol{C}} = [\boldsymbol{B} \quad \boldsymbol{AB}]$ for controller 1 and controller 2 applied on linear model 1, 2 and 3. The parameter values used for calculating the ranks are: $g_1 = 0.02/360$, $p_1 = 0.05/360$ $w_1 = 0.05/360$, $v_1 = 0.01/360$ and $r = 0.2$.

	Controller 1	Controller 2
Linear model 1	Rank($[\boldsymbol{B_1} \quad \boldsymbol{A_1 B_1}]$)=2	Rank($[\boldsymbol{B_2} \quad \boldsymbol{A_1 B_2}]$)=2
Linear model 2	Rank($[\boldsymbol{B_1} \quad \boldsymbol{A_2 B_1}]$)=2	Rank($[\boldsymbol{B_2} \quad \boldsymbol{A_2 B_2}]$)=2
Linear model 3	Rank($[\boldsymbol{B_1} \quad \boldsymbol{A_3 B_1}]$)=2	Rank($[\boldsymbol{B_2} \quad \boldsymbol{A_3 B_2}]$)=2

Root locus analysis to find the value of $\boldsymbol{\gamma}$

After the controller u is set, the next step is to find a proper value of the feedback gain γ to make the closed-loop system stable. In control theory, the performance of a feedback system can be described in terms of the location of the roots of the characteristic equation in the s-plane. A technique called root locus analysis was developed by Walter R. Evans [122] for designing and analysing the stability of the feedback system. The root locus plot is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain within a feedback system.

Root locus analysis has been performed on the three linear models (linear model 1, 2 and 3 got in Section 5.4) with two control distributions ($\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\boldsymbol{B_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$) on them (so 6 cases in total) to study the trajectories of eigenvalues of the matrix $\boldsymbol{A} + \boldsymbol{B}\gamma$ in the complex s-plane as a function of the feedback gain γ. Matlab function 'rlocus' is used to sketch the root locus plots for the 6 cases. The results are shown in figure 5.7.

In each subplot, the green line and blue line represent the two trajectories of the two poles. From the figure 5.7 (a1), (a2) and (a3), it can be seen that there is always a situation when two poles are both in the left half plane. This means that with the control distribution $\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, there always exits a feedback gain γ that can make the eigenvalues of the matrix $\boldsymbol{A} + \boldsymbol{B}\gamma$ negative, thus can make the system stable. While in the figure 5.7 (b1), (b2) and (b3), the green line never comes to left side of the s-plane, which means with the control distribution $\boldsymbol{B_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, it cannot find a γ to make both eigenvalues negative. Since there will always be a non-negative eigenvalue which makes the system unstable. In the next section, the control term, $\boldsymbol{B_1 u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \boldsymbol{u}$, will be added to the three linear models first.

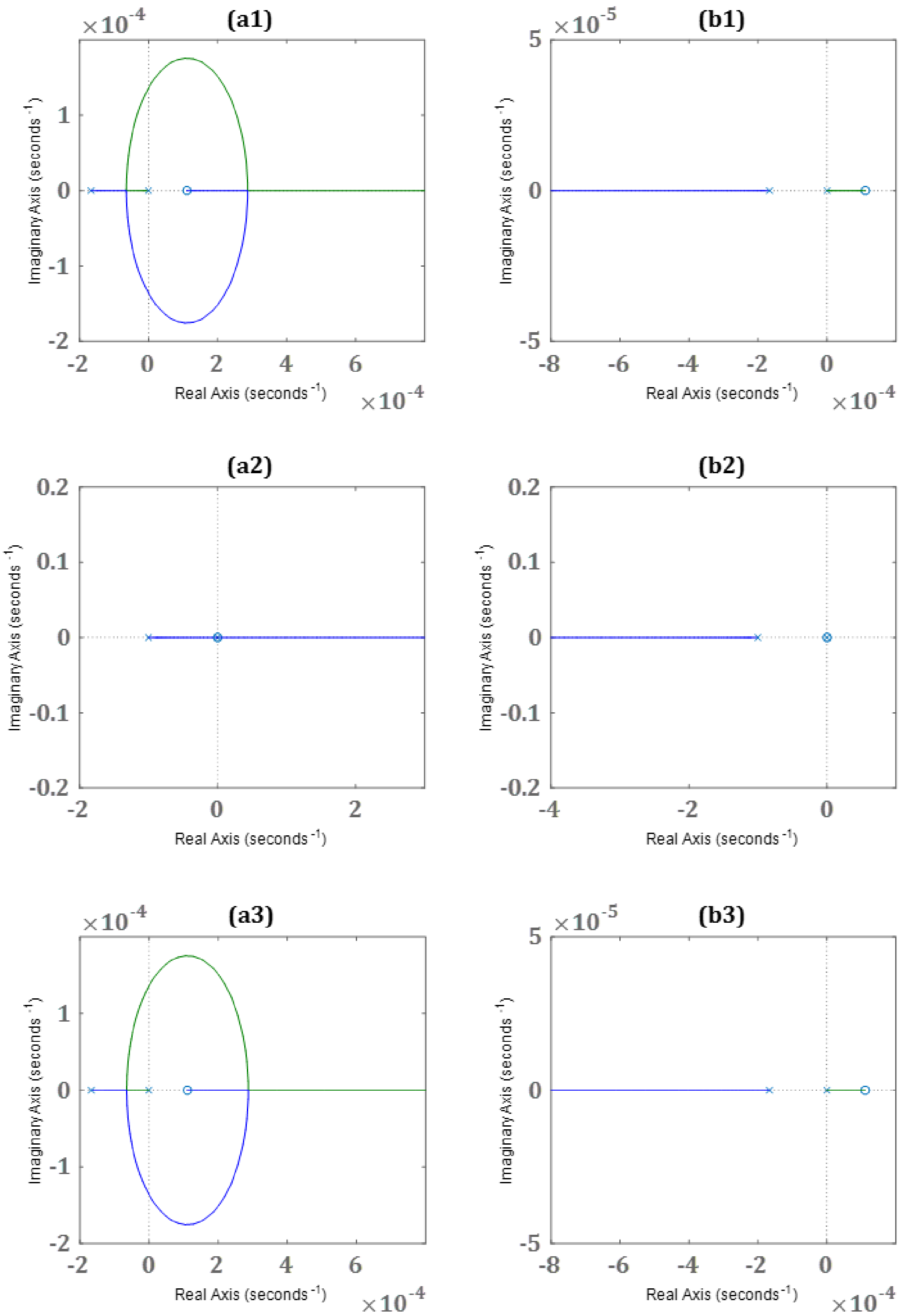

Figure 5.7 Plot of root locus of linear models 1, 2 and 3 (1st, 2nd and 3rd rows correspondingly) with two distributions $\boldsymbol{B}^1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (1st column) and $\boldsymbol{B}^2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ (2nd column).

The command '[R,K] = rlocus (SYS)' is used to find out the exact value of γ . This command returns the matrix ***R*** which includes the roots of the close-loop system SYS, and the matrix ***K*** which includes the corresponding gains for each pair of root. From output values of ***R*** and ***K***, the suitable feedback gain in ***K*** can be found, and the corresponding roots in ***R*** have negative values in the real axis. Different γ values are selected from ***K*** for different models, testing results are shown in the following subsections.

Reference input

The controller developed in the last section is derived to drive the output in the linear model to 0, and to the equilibrium in the nonlinear system. Very often in applications, however, it is desired the output to track a reference input, i.e. if $r(t)$ is the reference input, it would like the output $y(t) \rightarrow r(t)$ as $t \rightarrow \infty$. To make the controller that can bring the output (cash) to a desired value which is different from the equilibrium, one more term is added to the controller as shown in equation (5.30). γ is the feedback gain that can make the close-loop system stable, r_e is the reference input that applied in the linear model. In the nonlinear model, the desired value of the output is expected to reach, C_{r_e} , equals to the equilibrium, C^*,plus r_e.

$$u_1 = \gamma y + \overline{N_1}\, r_e \tag{5.30}$$

r_e is multiplied with an element $\overline{N_1}$ which can make the derivative of state variable in the steady state $\dot{x_{ss}} = 0$ and output variable in steady state $y_{ss} = r_e$. The following shows the procedure to calculate the value of $\overline{N_1}$.

In the steady states, it can be obtained that

$$\dot{x_{ss}} = 0 \tag{5.31}$$

$$y_{ss} = Cx_{ss} = r_e \tag{5.32}$$

From equation (5.31), it can be derived:

$$Ax_{ss} + Bu_1 = 0 \tag{5.33}$$

Substitute equation (5.30) in equation (5.33):

$$Ax_{ss} + B\gamma y + B\overline{N_1}r_e = 0$$

$$Ax_{ss} + B\gamma Cx + B\overline{N_1}r_e = 0$$

$$(A + B\gamma C)x_{ss} + B\overline{N_1}r_e = 0$$

$$x_{ss} = -(A + B\gamma C)^{-1}B\overline{N_1}r_e$$

From equation (5.32), it can be derived:

$$Cx_{ss} = r_e$$

$$-C(A + B\gamma C)^{-1}(B\overline{N_1}re) = r_e$$

$$-C(A + B\gamma C)^{-1}B\overline{N_1}re = r_e$$

Therefore $$-C(A + B\gamma C)^{-1}B\overline{N_1} = 1$$

Thus

$$\overline{N_1} = (-C(A + B\gamma C)^{-1}B)^{-1} \tag{5.34}$$

Implementation of the models in Simulink for testing

Each controller is tested on both the linear and nonlinear models. The implementation of the controller in the linear and nonlinear models is shown in figures 5.8 and 5.9. As shown in figure 5.8, the linear model is built using a block called 'State-space', which is a commonly used Simulink block to generate the linear model in a state-space format. The elements of matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and the initial conditions of the state variable are all defined in this block. Since here the closed-loop system is tested, the value of the matrix $\boldsymbol{A}$ is changed to $\boldsymbol{A} + \boldsymbol{B}\gamma\boldsymbol{C}$. The red block imports the input variables including the disturbance inputs as well as control inputs.

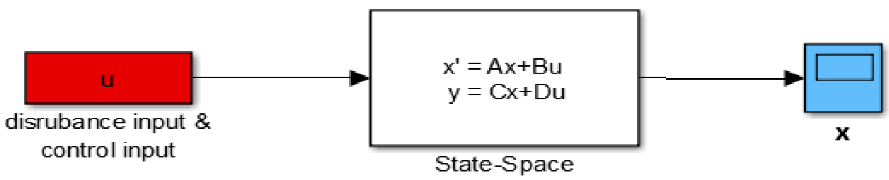

Figure 5.8 Block diagram of the linear closed-loop system with the applied controller in Simulink.

Figure 5.9 shows the Simulink diagram when a controller is implemented in the nonlinear model. It can be seen that there is one more input named 'u' in the main function of the one-bank model, which represents the control input. It consists of two parts, one is the feedback gain times the difference between the actual and the expected cash value, the other one is the control input at the equilibrium point (shown as 'u01' in the figure). It is the value of 'U' used in command '[A, B,C,D] = linmod ('SYS', X, U)' when doing the linearization. It means the amount of investment added or sold at the equilibrium point.

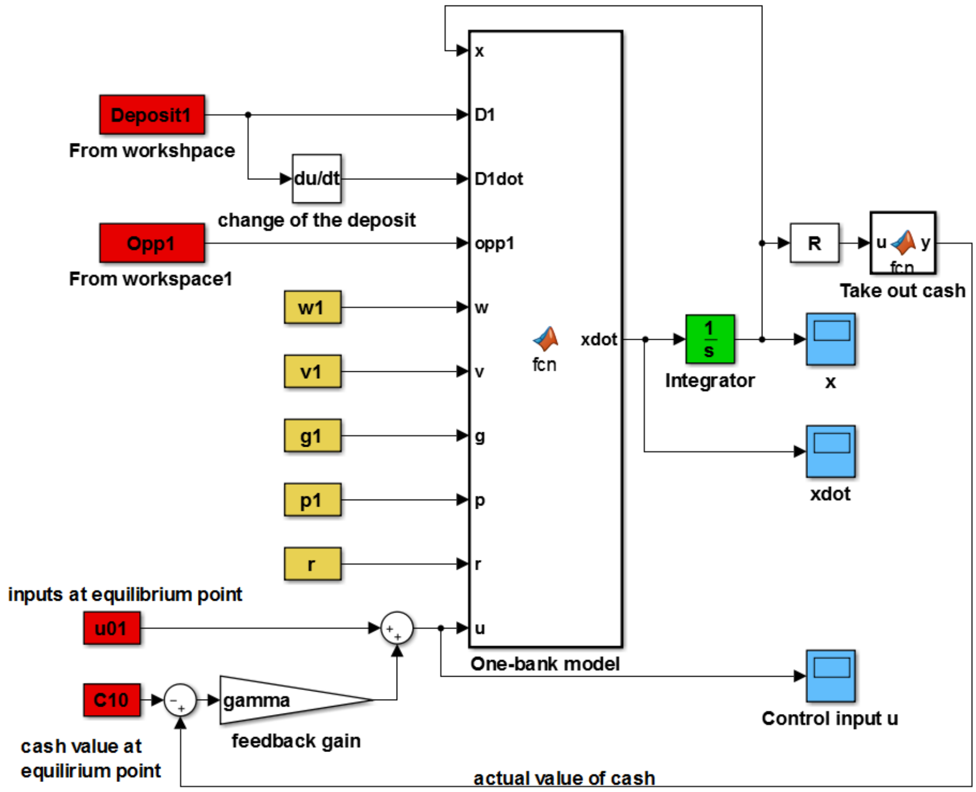

Figure 5.9 Block diagram of the nonlinear close-loop system with the applied controller in Simulink.

Here it is necessary to recall the relationship between the linear and nonlinear model. As introduced in Section 5.2.2, the linear model is based on the equilibrium point where linearization is done. To make sure the linear and nonlinear model are tested under the same situation, any inputs, and initial conditions that are applied on the linear model should be adjusted to reflect the equilibrium values when applied to the nonlinear model.

5.5.3 Testing results for regime 1

Ad shown in figure 5.5, the nonlinear model in regime 1 can be described by linear model 1 and linear model 2 (depending on the different investment opportunity). This section shows the simulation results when controller 1 is applied on linear models 1 and 2.

Controller 1 applied to linear model 1

To find a proper value for the feedback gain γ, the command '[R,K] = rlocus(SYS)' is used. The SYS used in the command is the system represented by equation (5.26), in which the input distribution $\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is applied on linear model 1. From the output $\boldsymbol{K}$, , a series of γ values can be found that can make the close-loop system having two negative poles.

For testing purpose, a γ value which equals to 3.75×10^5 is selected and the controller 1 now becomes:

$$u_1 = 3.75 \times 10^5\, y$$

Figure 5.10 shows the simulation results[10] when u_1 is applied to the nonlinear model under two situations starting with different initial conditions. In the first situation the initial value of cash $C_1^0 = 25$ and the initial value of investment $I_1^0 = 50$. While in the second situation, $C_1^0 = 21$ and $I_1^0 = 50$. With a given value of D^* equals to 100, the equilibrium point lays at:

$$D^* = 100$$

$$Opp_1^* = \frac{\mathrm{g}_1 D^*(w_1 + v_1)}{p_1 - v_1} = 0.0083$$

$$C^* = D^* r + 3 = 23$$

$$I^* = \frac{Opp_1^*}{w_1 + v_1} = 50$$

10 The simulation are carried out in both linear and nonlinear model, since they all get the same results, only the results from the nonlinear model will be presented.

The controller u_1 aims to bring the cash to the equilibrium point where $C^* = 23$. In figure 5.10 (a), it can be seen that the deposit is a constant so that there is no shock.

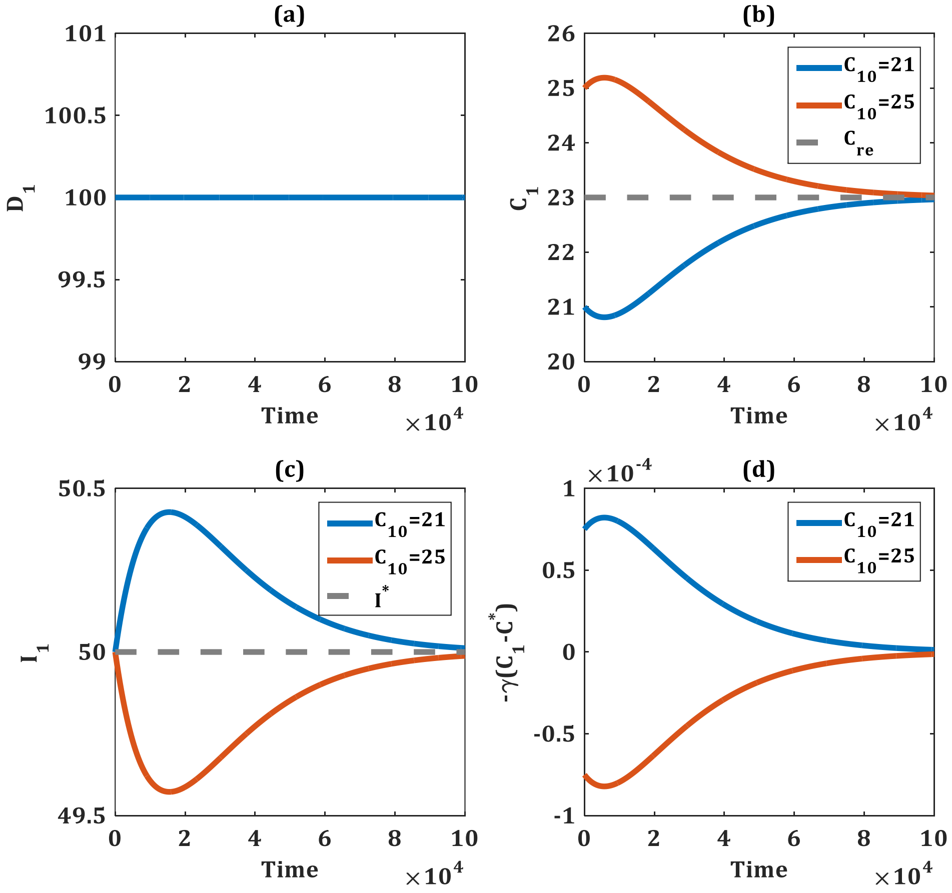

Figure 5.10 Dynamic behaviour of the bank's deposit (a), cash(b), total investment(c) and add/sold investment (d) with different initial conditions of cash: $C_1^0 = 25$ (red lines) and $C_1^0 = 21$ (blue lines). The grey dash lines in (b) and (c) represent the equilibrium value of the cash and investment accordingly. The positive value in (d) means adding new investment while negative value means selling investment. The feedback gain in controller applied in this case is $\gamma = 3.75 \times 10^5$.

In situation 1 (results are shown in red lines) when initial cash $C_1^0 = 25$, which is 2 unit above the equilibrium point $C^* = 23$. To bring the cash down, the bank sells investment; as shown in figure 5.10 (d), the red line starts with a negative value and finally reaches to zero when the cash comes to the equilibrium value. The total investment in figure 5.10 (c) decreases at the beginning, and then starts to increase after some time. The reason for this behaviour is explained as follows.

According to the differential equations describing the investment behaviour ($\frac{dI_1}{dt}$ in equation (5.26)), the change of the investment, $\frac{dI_1}{dt}$, is affected by four elements: the added investment, $\min[(C_1 - rD_1)^+, opp_1]$, matured investment $-w_1 I_1$, failed investment, $-v_1 I_1$, and sold investment $-\gamma(C_1 - C^*)$. The sum of these four elements is the change of the investment as shown in figure 5.11 in blue line. It can be seen that the change of the investment moves from negative to positive and then to zero, which is the reason that the total investment decreases first and then increases.

The derivative of the cash, as shown in red line in figure 5.11, moves from positive value to negative and then to zero. This is why the red line in figure 5.10 (b) first increases and then decreases to the equilibrium.

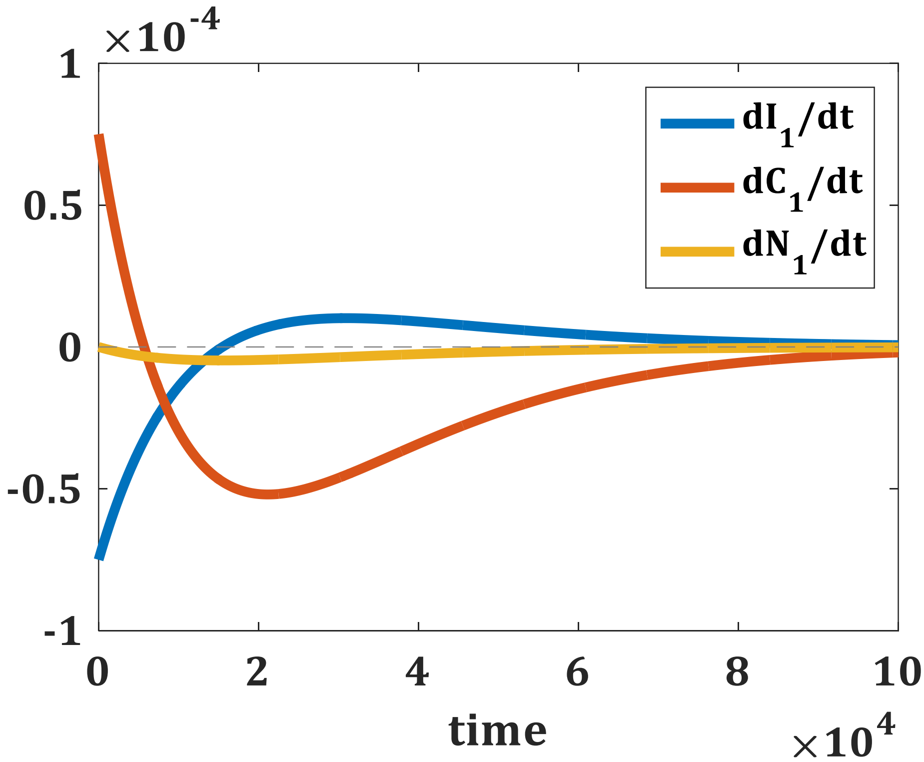

Figure 5.11 The derivatives of the investment (blue line), cash (red line) and net-worth (yellow line) in situation 1 when $C_1^0 = 25$ and $I_1^0 = 50$.

According to equation (3.4)11, as the deposit is constant, $\frac{dD_1}{dt} = 0$. Though $\frac{dI_1}{dt}$ changes during the test period, it becomes zero finally, thus actually it cannot contribute to the

11 Remind equation (3.4): $\frac{dC_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1$

change of the cash. The parts that actually contribute to the change of the cash are $p_1 I_1 - v_1 I_1$. What the controller 1 aims to do is to affect the cash by manipulating with the investment, I_1, so that the value of $p_1 I_1 - v_1 I_1$ changes. In this situation, as $p_1 > v_1$, $p_1 I_1 - v_1 I_1 = (p_1 - v_1) I_1$ is monotonically increasing. When the bank sells investment, I_1 decreases thus causing $p_1 I_1 - v_1 I_1$ decreases. This causes $\frac{dC_1}{dt}$ to decrease, as shown in figure 5.11 red line; the derivative of the cash keeps dropping and after sometime becomes negative. As $\frac{dC_1}{dt}$ becomes negative, the cash C_1 starts to decrease. As C_1 gets closer to the equilibrium value, the sold investment becomes less. The total investment stops dropping and starts to move back to equilibrium point. As I_1 increases, $p_1 I_1 - v_1 I_1$ increases causing $\frac{dC_1}{dt}$ to increase and move closer to zero, thus C_1 drops slower and finally stays at the equilibrium.

Since the aim of the controller is to bring C_1 2 units down to the equilibrium and to keep the investment at the same level at the equilibrium, the net-worth at the end should be 2 units lower than before. According to the equation that represents the net-worth in equation (3.5)[12].

The derivative of the net-worth should be

$$\frac{dN_1}{dt} = \frac{dC_1}{dt} + \frac{dI_1}{dt} - \frac{dD_1}{dt} \tag{5.35}$$

Substitute the equation (3.4) in (5.35), the expression for the net-worth derivative in another version can be get as shown in equation (5.36).

$$\frac{dN_1}{dt} = \frac{dD_1}{dt} - \frac{dI_1}{dt} - g_1 D_1 + p_1 I_1 - v_1 I_1 + \frac{dI_1}{dt} - \frac{dD_1}{dt} = -g_1 D_1 + p_1 I_1 - v_1 I_1 \tag{5.36}$$

This equation shows that the change of the net-worth is only affected by three elements: the interest paid to the depositor, the return of the investment and the failed investment. Since g_1, p_1, v_1 and D_1 are assumed to be constant in the simulation, the only part that can affect the net-worth is $p_1 I_1 - v_1 I_1$. Therefore, by manipulating the investment, the net-worth can also be affected.

---

12 Remind equation (3.5) $N_1 = C_1 + I_1 - D_1$

Given an overview of the whole dynamic, the bank sells investment (or invest less) to lose some return that could be earned from the investment. In that way, the cash can be brought down. However, this is a theoretical situation for testing. In the reality, the bank usually will not face such a situation that it wants to bring down its cash. Instead, the bank may want to bring their cash to a higher level to safeguard itself from the shock, which corresponds to situation 2. With the initial cash $C_1^0 = 21$, the controller tries to bring the cash to the equilibrium point $C^* = 23$. To bring the cash up, the bank adds investment (shown in figure 5.10 (d) blue line) in order to earn more return from the investment. In this way, the cash increases as shown in figure 5.10(b) blue line.

Controller 1 applied to linear model 2

When the input distribution $\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is applied to linear model 2, the output of the command '[R,K] = rlocus (SYS)' shows that, only one pair of roots in $\boldsymbol{R}$ that have negative values in the real axis. The corresponding γ in $\boldsymbol{K}$ is 1.

It can be seen that sometimes the choice of γ can be very limited. An analysis has been done to see how the value of the parameters can affect the range of the available γ values. It has been found that when the value of the parameter w_1 changes from 0.05/360 to 0.3, there are more values in $\boldsymbol{K}$ that have the corresponding negative roots in $\boldsymbol{R}$. This indicates that with a larger value of w_1 (which means every day there will be

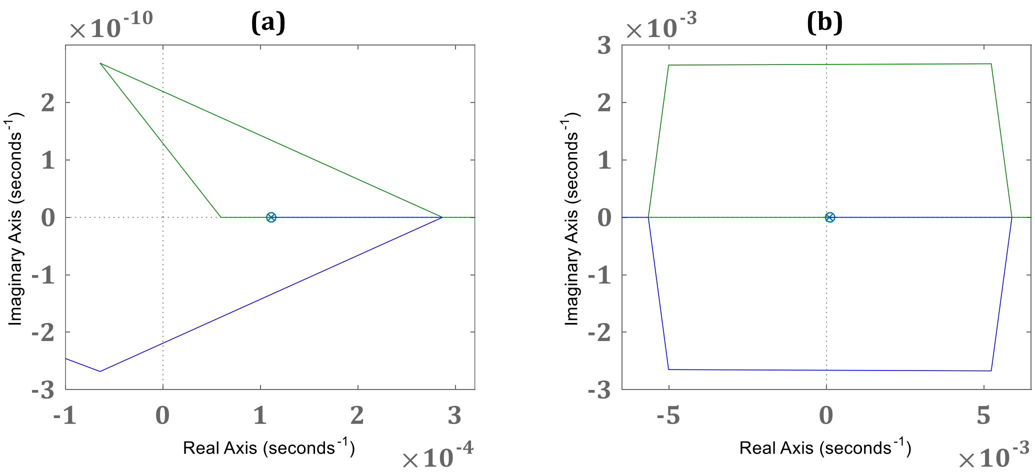

Figure 5.12 The root locus plots of linear model 1 when applied control feedback $\boldsymbol{B^1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ with different values of parameter w_1 equals to 0.05/360 in (a) and 0.3 in (b).

more matured investment), the bank has more flexibility to manage its investment to keep its cash at a desired level. The above effect can be verified from figure 5.12, which is the plot of root locus when the value of parameter w_1 is different in linear model 2. More green trajectory exits at the left side of the s-plane in figure 5.12 (b) when $w_1 = 0.3$ than in figure 5.12 (a) when $w_1 = 0.05/360$. Therefore, a larger range of γ can be found when $w_1 = 0.3$ to keep the roots of the close-loop system negative.

Figure 5.13 shows the simulation results when $w_1 = 0.3$. The controller $u_1$with three different feedback gains are tested. In this simulation, the controller follows the form $u_1 = \gamma y + \overline{N_1}\, r_e$ and aims to bring the cash to a desired value C_{r_e}.

The equilibrium point is calculated by the equations shown in table 5.2:

$$D^* = 100$$

$$C^* = D^*\left(\frac{g_1(w_1 + v_1)}{p_1 - v_1} + r\right) = 35$$

$$I^* = \frac{g_1 D^*}{p_1 - v_1} = 50$$

$$Opp^* = C_1 - r * D_1 + 100 = 105$$

The desired value C_{re} is assumed to be 41, which is 6 units higher than the equilibrium $C^* = 35$. Therefore, $re = \ C_{re} - C^* = 6$. The value of $\overline{N_1}$ is calculated using equation (5.34). In figure 5.13 (b), as the γ increases, the system takes shorter time to reach to the desired cash value.

The controller 1 has the following limitations: it only works when $\boldsymbol{C_1 \geq rD_1}$; when the disturbance (shock) is too large to drag the cash down below the reserve ratio ($\boldsymbol{0 \leq C_1 \leq rD_1}$) or to negative ($\boldsymbol{C_1 \leq 0}$), this controller will not be able to pull the cash back to the level that above the reserve ratio, since it only works when system are in regime 1. Therefore, a cases when $\boldsymbol{C_1 \leq 0}$ and $\boldsymbol{0 \leq C_1 \leq rD_1}$ must be considered in the following sections to see if possible controller can be found.

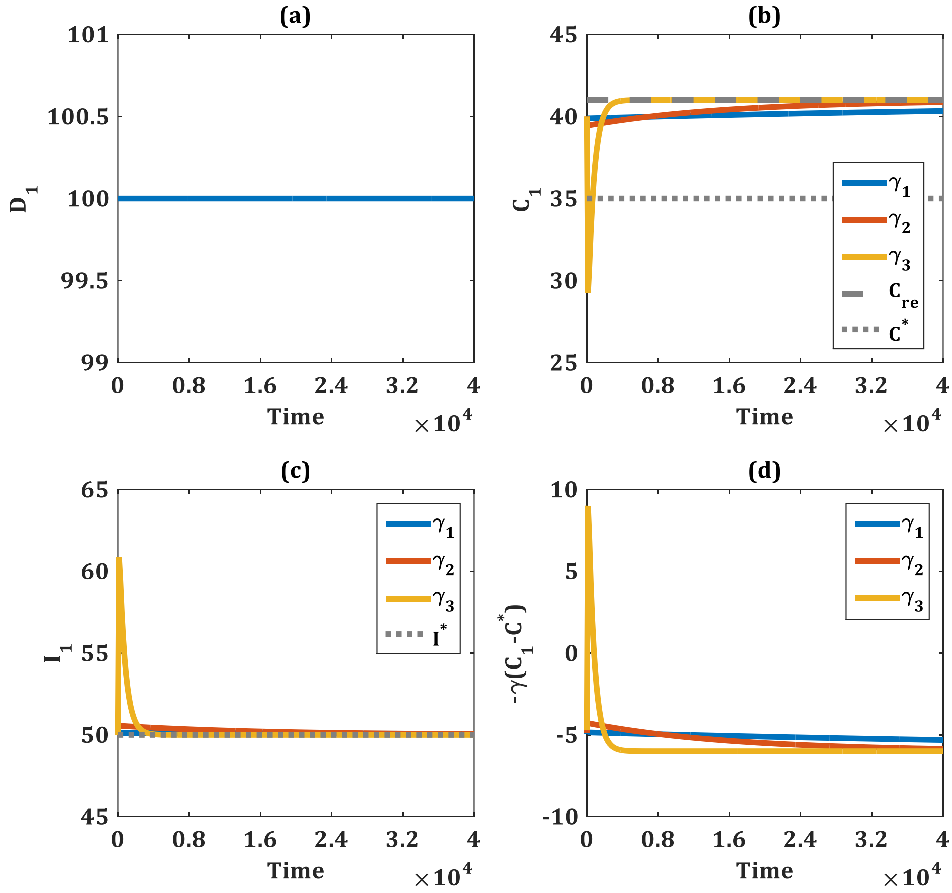

Figure 5.13 Dynamic behaviour of the bank's deposit (a), cash(b), total investment(c) and add/sold investment (d) with initial conditions: $C_1^0 = 25$ and $I_1^0 = 50$ with different feedback gains equal to $\gamma_1 = 1.0313$ (blue line), $\gamma_2 = 1.1076$ (red line) and $\gamma_3 = 1.2776$ (yellow line). The positive value in (d) means adding new investment while negative value means selling investment.

5.5.4 Testing results for regime 2

Controller 1 applied to linear model 3

As shown in the linearization section, when $\mathbf{0} \leq \boldsymbol{C_1} \leq \boldsymbol{rD_1}$ the nonlinear model can be linearized to linear model 3. In linear model 3, the open-loop system is marginally stable. Thus with no disturbance, the system always comes back to the equilibrium, where $C^* = 0$ and $I^* = 0$. However, this is not a steady state where the bank wants to be. Therefore, in this dynamic, the aim of the control is to bring the cash to a desired value which is different from the equilibrium. The desired value is chosen to be $\boldsymbol{rD_1}$, since this is the minimum value to meet the reserve ratio requirement. The same method will be used as introduced in the previous section to find out a γ for the corresponding feedback controller.

The input distribution $\boldsymbol{B_1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is considered first. Root locus analysis has been done when $\boldsymbol{B_1}$ is applied to linear model 3. As shown in figure 5.7 (a3), there always exists a γ that can make both eigenvalues negative. Thus for testing purpose, γ=0.2725 is selected. The controller now becomes:

$$u_1 = \gamma y + \overline{N_1}\, r_e = 0.2725\, y + (-0.27) \times\ 20$$

The results are shown in figure 5.14. The equilibrium point is: $C^* = 0$ and $I^* = 0$, and the desired value which controller aims to bring to is $C_{re} = 20$. It can be

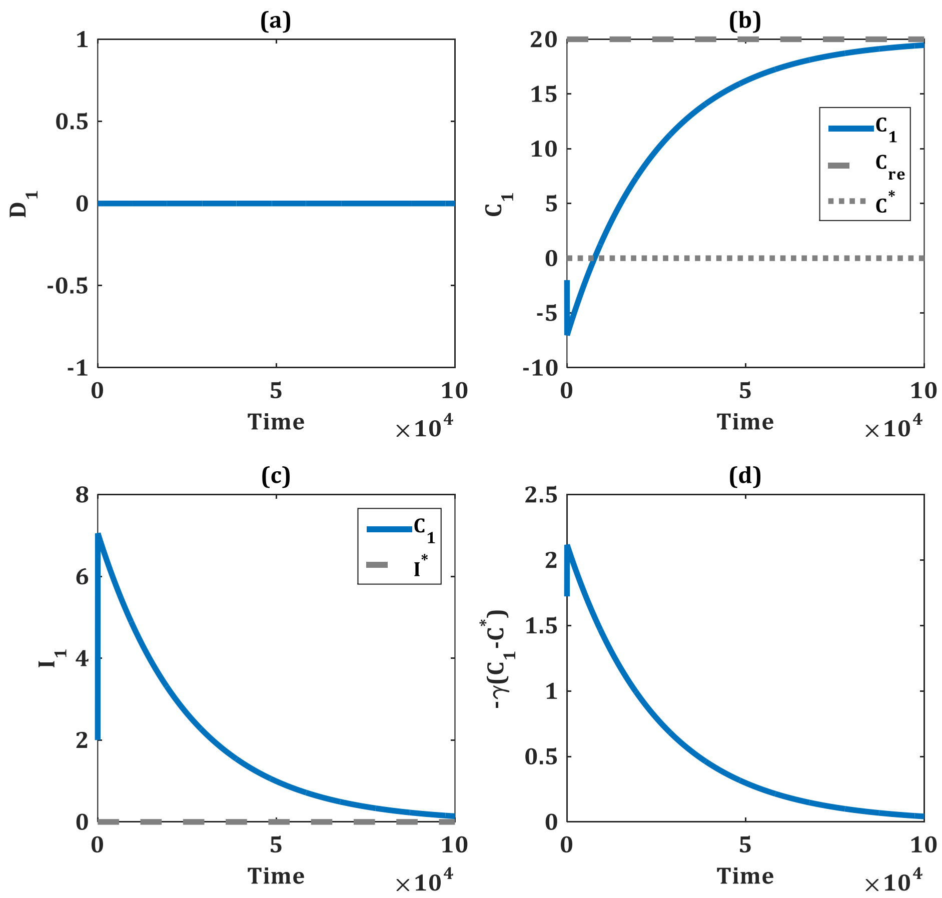

Figure 5.14 Dynamic behaviour of the bank's deposit (a), cash(b), total investment(c) and add/sold investment (d) with initial conditions of cash: $C_1^0 = -2$. The positive value in (b) means adding new investment while negative value means selling investment. The feedback gain in controller applied in this case is $\gamma = 0.2725$.

seen from the figure 5.14 (d) that the first term $\gamma y = -\gamma(C_1 - C^*)$ in u_1 is always above zero, which means the bank is adding the investment all the time. This causes the cash to decrease to a negative value at a very early time as shown in figure 5.14(b). Since the model allows bank to failure, the simulation keeps running and after some time the cash comes back up above zero and finally reaches the control reference value $C_{r_e} = 20$.

Theoretically controller 1 can bring the output to the reference value, but in practice, the added investment should be limited by the cash, the bank can only invest when cash is positive. However, in this simulation, as there is no such limit, the bank still adds investment while there is no available cash. This is the reason that the cash goes to negative in the simulation. This means in the real situation, the bank does not have enough cash to keep adding the investment which the controller is needed. The results of using controller 1 do not have physical meanings in the real banking system. Thus, the other input distribution $\boldsymbol{B_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is considered.

Controller 2 applied to linear model 3

Figure 5.7 (b3) shows the root locus of the system, in which the input distribution $\boldsymbol{B_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is applied on linear model 3. From the figure, it shows that there always exists a positive eigenvalue, as the green trajectory starts at zero and goes to the right hand side of the s-plane. This means it is impossible to find out a suitable γ that can make both of the eigenvalues negative. However, it should be noticed that the scale of the eigenvalues is very small (at 10^{-5}). Moreover, if taking a further look at the outputs from the command '[R,K] = rlocus (SYS)', the roots values in $\boldsymbol{R}$ are all small numbers. This indicates that this one-bank system is a very slow system; even it is not stable, its states will not go to infinity within a short time. Therefore, the control mechanism can be considered to use in the following way: the controller 2 is applied when cash is below the expected value ($C_1 < C_{r_e}$), and once the cash reaches the target ($C_1 = C_{r_e} = 20$), the controller is switched off. Though the system is not at the steady state, its cash now meets the reserve requirement and the system is in regime 1. As explained in section 5.5.3, in regime 1, the controller 1 can then be applied to bring the system to a steady state.

For the testing purpose, $\gamma = 1$ is used in controller 2 and the controller now becomes:

$$u_2 = \gamma y + \overline{N_1}\, re = y + (-1) \times\ 20$$

The simulation results using controller 2 are shown in figure 5.15 (blue line) compared to the simulation results without using the controller (red line). The deposit is constant and equals to 100 as shown in figure 5.15 (a). The initial value of the cash is 10, which is 10 units below the reserve requirement (reserve ratio is 0.2 so reserve requirement is 100×0.2=20).

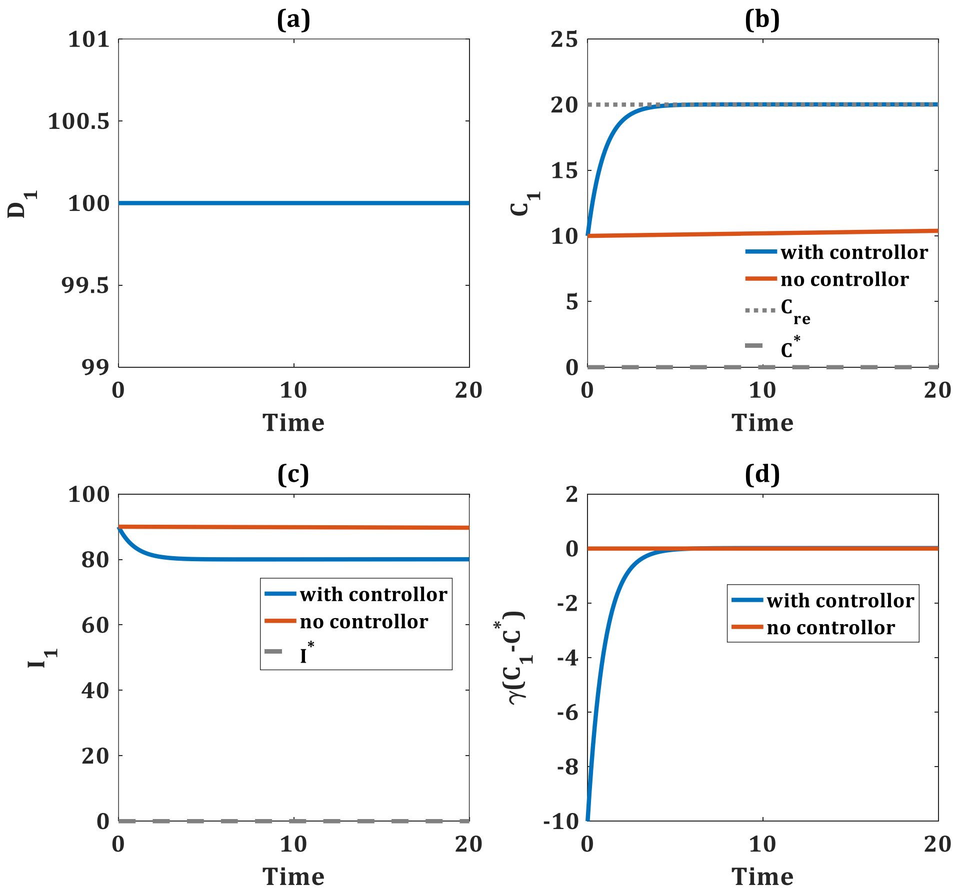

Figure 5.15 Dynamic behaviour of the bank's deposit (a), add/sold investment (b), total investment(c) and cash (d) with initial conditions: $C_1^0 = 10$ and $I_1^0 = 90$ in two cases: without controller (red lines) and with controller (blue lines). The positive value in (b) means adding new investment while negative value means selling investment. The feedback gain in controller applied in this case is $\gamma = 1$.

The red line shows the dynamic when no controller is applied. It can be seen that the bank does not sell any investment (red line in figure 5.15(b) equals to zero). The cash increases slowly as bank receives its matured investment, $w_1 I_1$, and the return of the investment, $p_1 I_1$. As these two parts have very small values, it is difficult to see the increase of the cash in the figure 5.15(b). furthermore, it will take a long time for the bank to get enough cash to meet the reserve requirement. Letting the cash level stay below the reserve requirement is very dangerous for the bank, as the bank has less cash to buffer the shock. Therefore, the bank needs to get its cash back to the reserve requirement as soon as possible.

The blue line shows the dynamic when the controller is applied. At the beginning, the bank sells 10 units of the investment as shown in figure 5.15 (d) blue line (the value of $\gamma(C_1 - C^*)$ equals to 10 at the beginning). By selling the investment, the bank transfers its investment to cash. Therefore, the total investment, I_1, in figure 5.15 (c) decreases while the cash , C_1, in figure 5.15(b) increases. After a short time the cash reaches to the reserve requirement 20 so the controller is switched off. Therefore, no more investment is sold in (d) and no more decrease in total investment in (c).

It is interesting to take a further look into the net-worth, which is shown in figure 5.16. the figure shows the values of net-worth under two cases, one with the controller and the other one without the controller. In both cases, the net-worth is increasing. The net-worth increases more slowly when the controller applied. The speed at which the net-worth increases, depends on the value of $\frac{dN_1}{dt}$. The smaller the $\frac{dN_1}{dt}$ is the more slowly net-worth increases. According to the equation (5.36), the change of the net-worth $\frac{dN_1}{dt} = -\, g_1 D_1 + \, p_1 I_1 - v_1 I_1$. When there is no controller applied, the total investment, I_1, is decreasing (Since every day there are some matured investment), which causes the $p_1 I_1 - v_1 I_1$ becomes smaller, while the interest paid to the depositors, $g_1 D_1$, is the same as before. Therefore, $\frac{dN_1}{dt}$ becomes smaller. When the controller is applied, the bank sells investment, which makes the total investment decrease more quickly, thus causes $\frac{dN_1}{dt}$ to decrease more quickly. The value of $\frac{dN_1}{dt}$ is always smaller in the case with the controller than in the case without the controller. This indicates that, when the bank is in the dangerous zone, it has to sell some of the investment to get enough reserve to safeguard itself from the shock, however, the bank will lose some profit because of selling the investment.

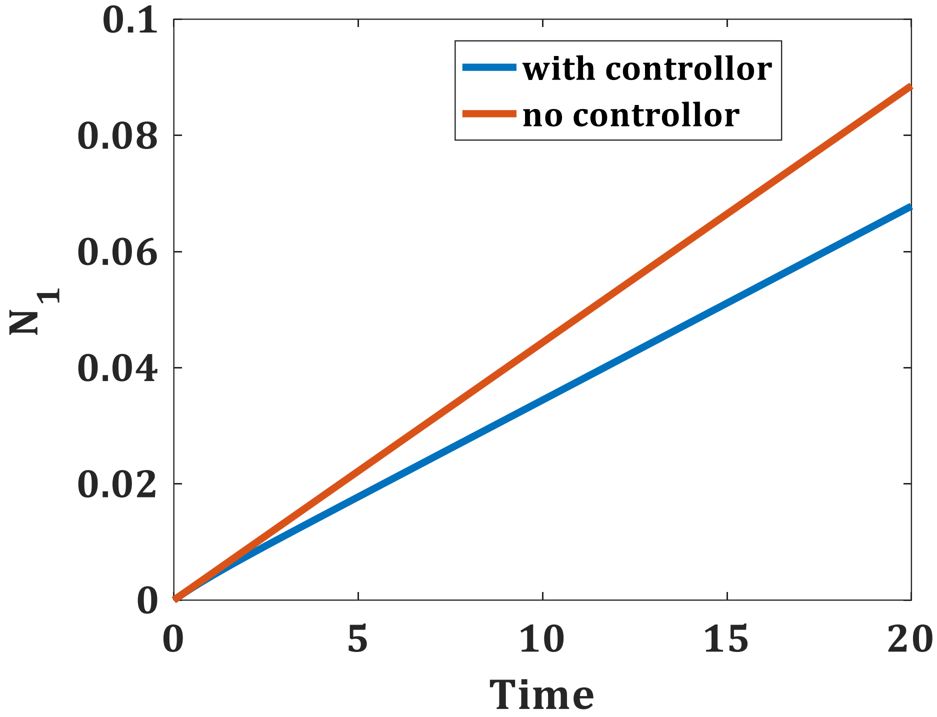

Figure 5.16 Dynamic behaviour of the bank's net-worth when $C_1^0 = 10$ and $I_1^0 = 90$ for two case: without controller (red lines) and with controller (blue lines).

5.5.5 Testing results for regime 3

In regime 3, the bank is in a state that it is about to fail. In this state, the cash is negative because of the shock in the deposit is larger than the current cash. If the bank does not take any action at the current time step, the bank will fail in the next time step. Therefore, there is a need to apply a possible controller to bring the cash back to positive. From the analysis in the last section, controller 1 cannot be used, as at this state the bank does not have cash to add more investment. However, controller 2 can be used, as at this state the bank still has investment to sell.

In this state the bank does not add new investment and neither pays the interest to depositors, since it is lack of cash. However, it can still get the matured investment and the return of the investment back. Therefore, the matrix A in the linear model is the same as in the linear model 3. When controller 2 is applied, the root locus plot is the same as shown in figure 5.7 (b3). Thus in regime 3, it is impossible to find a γ that can make the system stable. As explained in the last section, the scale of the eigenvalues is very small (at 10^{-5}) and the roots values in $\boldsymbol{R}$ are all small numbers. So that the system is a very slow system and even it is not stable, its states will not go to infinity in a very short time. Therefore, controller 2 can be applied to get the cash back to zero and then switched off. Again for testing, a $\gamma = 4$ is used in the controller 2. The simulation results are shown in figure 5.17. By selling the investment, the bank brings its cash from -2 back to 0 within one-day time.

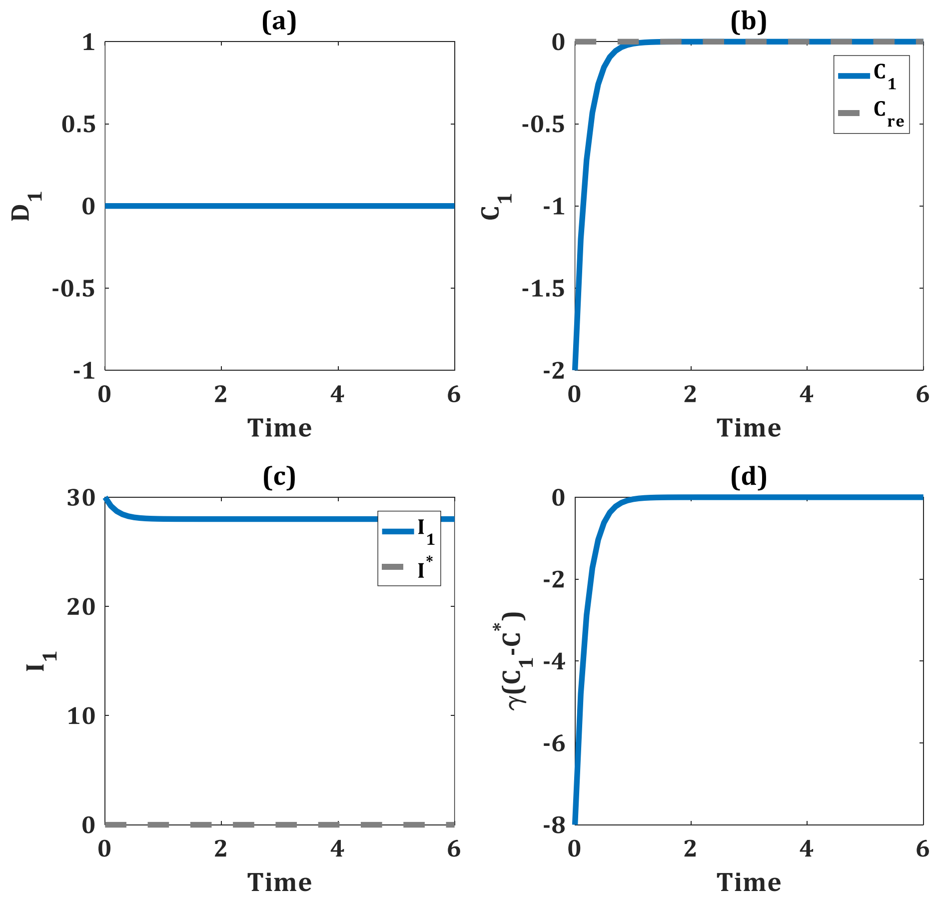

Figure 5.17 Dynamic behaviour of the bank's deposit (a), add/sold investment (b), total investment(c) and cash (d) with initial conditions: $C_1^0 = -2$ and $I_1^0 = 30$ with controller. The positive value in (b) means adding new investment while negative value means selling investment. The feedback gain in controller applied in this case is $\gamma = 4$.

5.5.6 A switched control paradigm

From the previous sections, it can be seen that the proper feedback gain can be selected according to the different system dynamics to make the cash to reach a desired value. Figures 5.18 and 5.19 show the simulation results in a case that as the cash of the bank changes, the nonlinear system switches between different regimes and the controller switches accordingly.

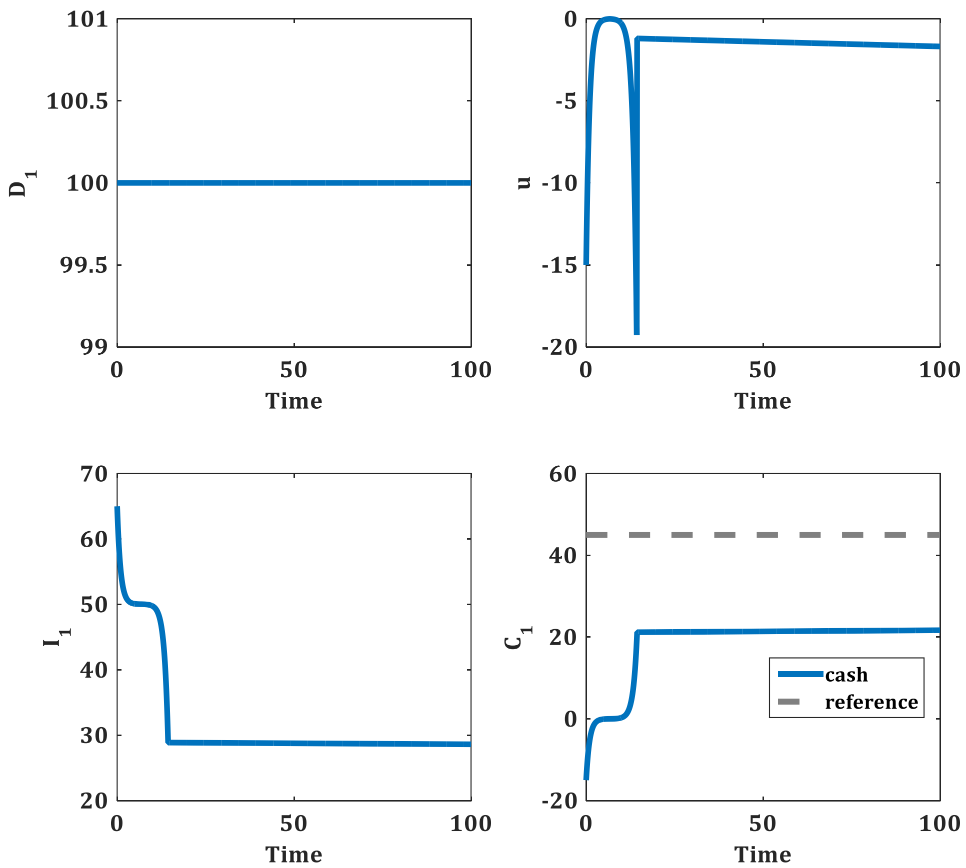

Figure 5.18 Dynamic behaviour in first 100 days of the bank's deposit (a), add/sold investment (b), total investment(c) and cash (d) with initial conditions: $C_1^0 = -15$ and $I_1^0 = 65$ where the controller switches when the system enters different regimes.

The initial conditions of the simulation are: $C_1^0 = -15$ and $I_1^0 = 65$. The aim of the control is to bring the cash to a desired value 45. The system starts in regime 3 and the controller applied is the one introduced in section 5.5.5; the bank's cash is brought from -15 to 0 within a short time as shown in figure 5.18 (d). Then the system moves into regime 2, therefore the controller is switched to the one introduced in section 5.5.4 which is $u_2 = \gamma y + \overline{N_1}\, r_e = y + (-1) \times\ 20$. By continuing selling investment, the bank's cash is brought to the reserve requirement 20. Then the system moves to regime 1, in which the controller introduced in section 5.5.3 can be applied. Figure

5.19 shows it takes a long time to finally get the system to a steady state with the desired cash value.

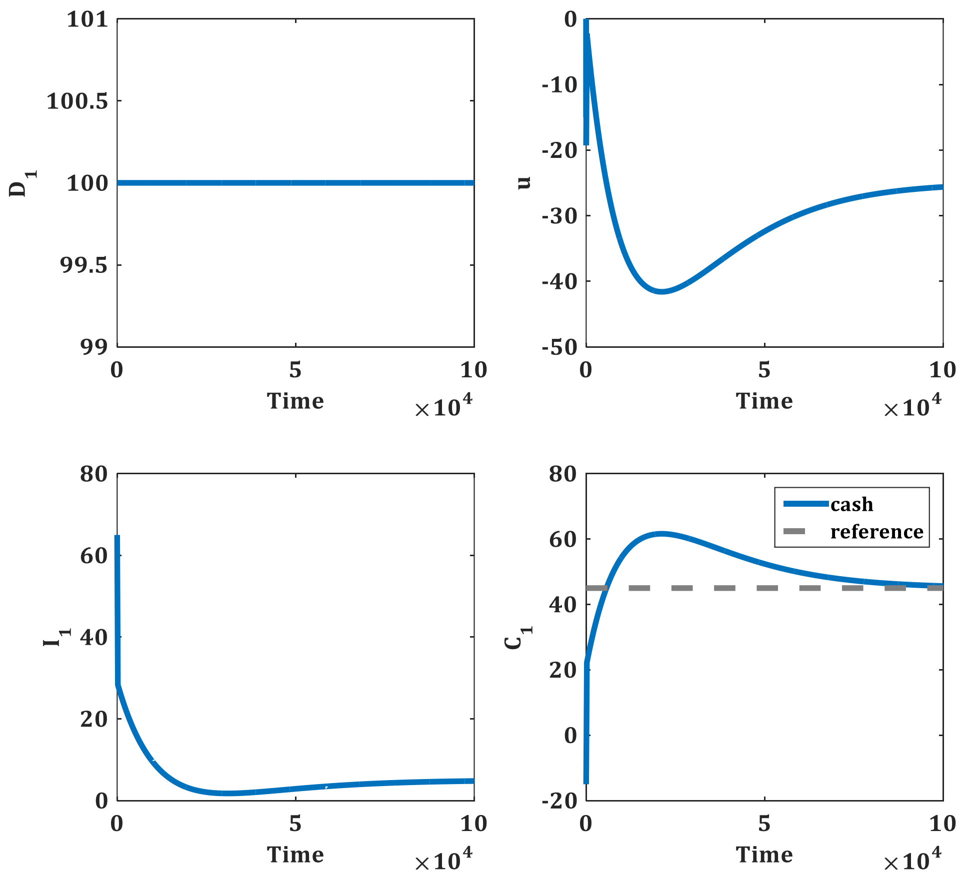

Figure 5.19 The dynamic behaviour in 10^5 days of the bank's deposit (a), add/sold investment (b), total investment(c) and cash (d) with initial conditions: $C_1^0 = -15$ and $I_1^0 = 65$ where the controller switches when the system enter different regimes.

5.6 Conclusion

This chapter performs the application of the control theory on the one-bank model. The following conclusions can be drawn.

The equilibrium point analysis has been performed on the one-bank model and three equilibrium points are found. The nonlinear model has been linearized around the equilibrium points and three linear models are obtained to describe the dynamics of the one-bank model. Through the analysis of the eigenvalues of the state matrices in the linear models, it can be seen that one equilibrium point is not stable and the other two are marginally stable. Therefore, there is the need to design a controller to keep the one-bank model stable. The design of output feedback controllers has been proposed in which the proper feedback gain can be selected according to the different system dynamics to keep the cash of the bank at a desired value.

Moreover, the controllers can be switched according to the dynamics of the system. As different feedback gains are needed for different system dynamics, therefore, when system switches between different dynamics, a single feedback control mechanism will not work in all different dynamics. Thus a switched control mechanism is designed, which can examine the current system's dynamic first. Then it can be switched to the proper feedback control mechanism for the current dynamic. Furthermore, the equilibrium point analysis can give insight from the control perspective of how the parameter values g, w, v and p affect the system's dynamic behaviour, which can be evidenced by the simulation results that are represented in subsection 4.4.2. Parameters that increase the net-worth of the bank, such as p_1, decrease the state variables equilibrium values, C_1^* and I_1^*, which is beneficial for the bank since it does not need to have high cash and investment to stay at the equilibrium point. Those parameters, such as v_1 and g_1, that decrease the net-worth, instead, increase the equilibrium values, C_1^* and I_1^*, which is detrimental for the bank.

The control design and control analysis proposed in this chapter show how a bank can sell its assets (investment) to keep itself stable if needed; the proposed analysis show the exact amount of assets needed to be sold, according to control laws. This procedure can drive the bank back to a steady state after selling the assets properly. The next chapter draws conclusions of the whole work presented in this thesis and presents a discussion on future research directions.

Chapter 6

Conclusions and Future Work

This chapter firstly summarises the main results and achievements presented in this thesis and then proposes ideas for future work.

6.1 Summary

This thesis presents an interdisciplinary research at the interface between economics and control engineering which proposes an innovative model and analysis approach to study the dynamics of the banking system. The novelty of this Ph.D. project lies in the combination of two research methodologies, one from network modelling and the other from control theory. The results contribute to new knowledge regarding the understanding of the dynamics of the banking system, which can be ultimately used to inform financial regulators and operators.

The following paragraphs in this section summarise all the contributions of the Ph.D. project.

The first contribution of this thesis is the development of a new network model describing the banking system. In this dynamic model, the banking system is represented as a network where the nodes are individual banks and the links between any two banks consist of interbank loans and borrowings. Ordinary differential equations are used to describe the dynamic structure of the banking system. To the best of our knowledge, it is the first time that ordinary differential equations are used to describe the balance sheet dynamics of banks' activities (e.g. lending, borrowing, investment) and the behaviours of other related quantities (e.g. deposits, investment opportunities, interest rates). The proposed model has been implemented in a way that allows carrying out numerical simulations and accommodating feedback mechanisms typical of control theory.

The second contribution is the development of Simulink block diagram of the dynamic network model. The Simulink implementation facilitates the application of control analysis tools, and it allows adding complexity to the system in a visual and modular way. MATLAB scripts have been developed to perform the simulations, save and plot the results automatically and in a user-friendly manner.

The third contribution is the insight that the results of numerical simulations provide regarding the dynamics of the banking system. On one hand, the simulation results confirm findings that have been obtained by network models reported in the literature. These findings regard the role played by the reserve ratio and link rate on the failure of banks in the system. Specifically, results presented in this thesis show that the reserve ratio helps individual banks to survive when there is no interbank lending, but when the banks can lend and borrow money from each other, a high reserve ratio has a negative effect on the ability of banks to borrow money and therefore to survive; the link rate always contributes positively to the survival of banks, but there are conditions under which it increases contagion. On the other hand, in this thesis, a new approach to quantify contagion of banks' failure has been proposed, and simulation results illustrate quantitatively, for the first time, the nonlinear effect of link rate and reserve ratio on contagion. The nonlinear effect of the link rate emerges when the reserve ratio is low, namely the increase of the link rate increases, at first, contagion and then it decreases contagion after crossing a critical level. The nonlinear effect of reserve ratio is shown when the link rate is high, in fact, contagion, at first, increases and then decreases as the reserve ratio goes from low to high values. It can be suggested that these findings can ultimately help financial regulators in implementing new policies to preserve the banking system's stability.

The fourth contribution of this thesis is the application of control theory, in fact, it is the first time, to the best of our knowledge, that control theory has been applied to assess and preserve the stability of a dynamic model representing the banking system. The equilibrium point analysis has been implemented on the one-bank model and three equilibrium points have been found. The original nonlinear model has been linearized around the equilibrium points and three linear models have been obtained. The equilibrium-point analysis gives insight from a control perspective of how many different dynamics can emerge in the original nonlinear model. The stability of each dynamics has been studied by analysing the eigenvalues of the state matrix of the linear models. Output feedback control mechanisms have been designed in which single banks sell their assets to prevent bankruptcy. The novelty of the approach lies in the way banks sell their assets; the sale of assets is prescribed by specific control mechanisms, which allow a bank to resume and maintain a stable condition. Moreover, a switched control mechanism has been proposed to preserve the bank from bankruptcy. The switched control mechanism can examine the dynamics of the system and it can be switched to a specific feedback gain which is suitable to control the dynamics so to avoid failure.

This thesis shows how the proposed network model based on the ordinary differential equations can be used to simulate the dynamics of a complex system such as the banking system. The implementation of control theory presented in this work shows how feedback control can be used to design an appropriate control mechanism to stabilise the dynamics of a bank. This thesis represents a first, but yet important, step towards the combination of network models of the banking system with control theory; it should be now plausible and desirable to apply control theory to better understand the stability of financial systems.

The work proposed in this Ph.D. project has the following limitations:

1. The lack of real data and information regarding banking activities. The proposed work gives a theoretical framework to model the banking system. The signals used in the simulation such as the random shocks in the deposit and the investment opportunities may not be realistic in their amplitude, duration or frequency. Furthermore, information about the realistic timing of bank activities such as investment, borrowing and lending would be beneficial for the model. The knowledge of realistic data regarding the size of banks and the interconnection between them would be very useful as well.
2. The proposed approach does not model all the real bank's activities, e.g. dividend to shareholders are not considered, and more realistic ways of modelling how investments are made and mature would be beneficial.
3. Another limitation is that the appropriate values for the feedback gains are highly dependent on parameter values, such as g, w, v and p , therefore the knowledge of realistic values of those parameters may help in choosing the feedback gains and arguably design more appropriate control mechanisms.

The next section presents some ideas that may tackle some of the limitations highlighted above.

6.2 Future research directions

1. One direction for future work is to look for and use real data characterising banks' activities for modelling and simulating. The real data could not be used in this work because often values reported in the public domain are mainly aggregated data [123], which cannot be used for modelling and simulating a system based on a network of individual banks. To overcome this problem, one solution is to use a *maximum entropy method* and a *minimum-density solution* to make an estimation of bank assets and liabilities positions. These two method can help to fill in blanks of the network structure by using the available information on each bank's total interbank lending. The maximum entropy method assumes that banks diversify their exposures by spreading their lending and borrowing across all other active banks, while the minimum-density methods assumes that interbank linkages are costly to add and maintain, therefore, it aims to determine a pattern of linkages for allocating interbank positions that is efficient in the sense of minimizing these costs. The future study can follow the work of Anand et al. [82], in which the maximum entropy method are combined with a minimum density method in order to define a useful range that bounds the cost of contagion in the true interbank network when counterparty exposures are unknown. Another way to solve the issue of using real data is to seek the possibility of a joint project with researchers of central banks to have the detailed and disaggregated balance sheet data of individual banks from the central bank. With these data, simulations can be performed by considering a realistic number of banks and realistic level of deposit, investment and interbank borrowing of each bank in order to better validate the findings of the proposed approach. Moreover, another problem of using the aggregated data to validate the proposed model is that the data is often monthly or quarterly based, while the proposed model is daily based. To overcome this problem statistical methods can be used for interpolating data that are not available in a daily frequency. A low-frequency series can be considered as a partially observed high-frequency variable. For example, in the empirical application, quarterly variables can be treated as

monthly series observed only in the third month of each quarter, i.e. with missing data in the first and second month of each quarter. Interpolation techniques [124 –127] can be applied to obtain expectations for the "missing" days conditional on the information in the monthly series.

2. A more systematic study of the effect of the initial conditions on the behaviour of the proposed model should be conducted. For example, initial conditions could be chosen so that the interbank borrowing and lending have a value different from zero at time zero, as for the real situations. Furthermore, different patterns of connections, rather than random, can be used, to see how the network structure can affect the system's dynamic.
3. The proposed dynamic model can be extended to account for other behaviours of banks such as liquidity hoarding and fire sales. To include such behaviours, the proposed model can be linked to an agent-based model of the security market. An agent based model [128] is a computational model which can be used to study financial/economic systems [129] as a whole through simulating the actions and interactions of autonomous units, known as agents. Following the work by Lux and Marchesi [130], an agent based model of the security market can be connected with the proposed model, as shown in figure 6.1.

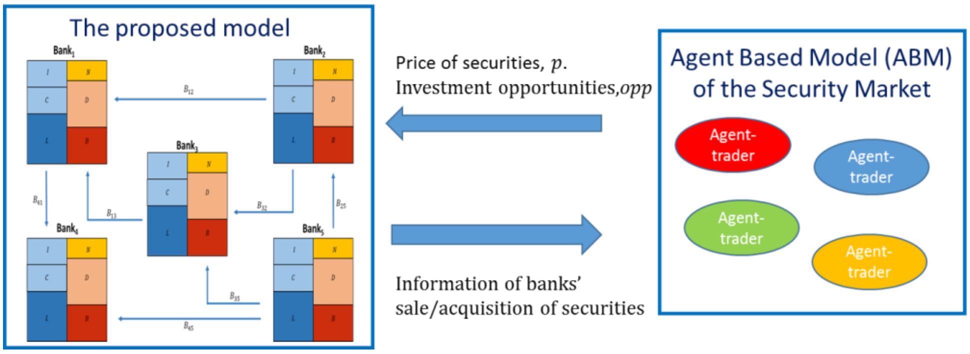

Figure 6-1 the proposed dynamic model with the agent based model.

The agent based model of the security market would take the information of banks' sale/acquisition of securities (investment), then the agents-traders in the security market would compete with each other to maximise their wealth by trading securities. The agents' behaviour would affect the price of securities, and the investment opportunities, *opp*, in the market. The information of the security prices and investment opportunities would feedback into the banking system and the banks' investments and sale of securities would be affected by the market's behaviour. When the market becomes volatile, the agents may change their trading strategies, causing the change of the price of securities and opp, which will feedback into the proposed model. This loop between banks and market could ultimately cause liquidity hoarding and fire sales.

4. Another extension of the model could be to add a central bank into the network as a node that is linked to all the other banks in the system. New differential equations can be proposed to model the central bank's activities. For example, the central bank could accumulate deposits from the banks that have extra cash and then loan them to the banks that are in need of cash. The central bank could change the reserve ratio, r, and the basic lending interest rate, h_0, according to the current dynamics of the system. This modelling framework could be a useful tool test the effects of the implementations of different regulatory policies by the central bank.
5. More advanced control theory tools could be applied to the proposed dynamic model in the future. There are many financial problems [13] that have been studied using stochastic optimal control [131– 136], which aims to design the controllers to complete the desired control task with minimum cost. This could be applied as a mechanism used by a bank in the model, acting as a central bank, to find the optimal policies to stabilise the system. As the banking system is highly nonlinear, the control tools applied should also be able to deal with nonlinear systems. Two types of methods of solving nonlinear optimal control problems can be found in the literature: the first type contains direct methods, converts the problem into a nonlinear programming by using the discretization or parameterization techniques [137], while the second one contains indirect methods and leads to the Hamilton–Jacobi–Bellman (HJB) equation, on the basis of dynamic programming [138, 139], or nonlinear two-point boundary value problem (TPBVP), on the basis of the PontryaginŠs maximum principle [140]. Results in many recent works [141 – 143] show the control algorithms are quite efficient and is well suited for solving nonlinear optimal control problems, which can be considered to be applied in the future work.

[13] Problems such as portfolio allocation, quadratic hedging of options and optimal selling of an asset.

6. In the real world, the network structure of the banking system can change with time in an unpredictable way [144]; this can generate uncertainties in the number of links between the banks, which should be taken into consideration when modelling the system. Other uncertainties can be introduced by the lack of precise values of parameters in the model. Moreover, the dynamics of large scale interconnected systems (e.g. the banking system) are usually highly nonlinear. It is not only the structure of the system which produces complexity but also the nonlinearity of the dynamics. The study of a network with a simple linear dynamics does not permit the existence of the multiple states observed in real networks and does not accommodate global properties of the system. Therefore, the nonlinear systems control theory is needed for the study of the uncertainty and the nonlinearity of the interconnected banking system.

   Control tools, such as decentralized feedback control and sliding mode control, have received much attention in the literature due to their capacity to deal with uncertainties, in nonlinear scenarios[14]. Work by Yan et al. [153 – 157] proposed constructive frameworks to implement decentralised output feedback control strategies based on sliding mode techniques; this work encompasses nonlinear system representations, uncertainty and unknown perturbations as well as limited available information in the framework. Moreover, it is important to note that delays usually exist in the banking system due to information transfer [158]; most recent works in the area of control for time-delay interconnected systems in [159 – 164] could be applied to deal with the time-delay issues in the banking system. In the work by Yan et al. [162], a class of nonlinear interconnected systems with time-varying delays is considered, where the time delay appears not only in isolated subsystems, but also in the interconnections. A decentralised static output feedback control strategy is proposed in Yan et al. [162] to drive the system to exhibit desirable dynamics, which could also be applied to the model of the banking system to study and control its stability. This could be an interesting topic for future work.

[14] Works of decentralized output feedback control can be found in [145 – 147] ; works of sliding mode control can be found in [148 – 152].

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Appendix 1: Script for one-bank model simulation

All the codes and scripts for the different models and the application of control analysis can be found in:

https://drive.google.com/open?id=0BzCTFEKfyl8tSzhOckZMNTJCOUE

The following script assigns the initial conditions, parameter values and other information to run the simulations for the one-bank model.

```
%*****************************************************************
clear all;close all;
%Parameter settings of one_bank model simulation
p1=0/100/360; %investment return rate
w1=0/100/360; %proportion of the total investment that matured
v1=0/100/360; %proportion of the total investment that failed
g1=0/100/360; %deposit interest rate
r=0.2;        %reserve ratio

t=10;tt=0:1:t;%simulation time
step=0.1;     %step size

%Generate exogenous signals
%Deposit signal
D10=1;
Deposit1=[tt' [D10*ones(4,1);(D10-0.26)*ones(2,1);D10*ones(t-
5,1)]];
%Investment oppotunity signal
opp10=1;
Opp1=[tt' opp10*ones(t+1,1)];
%Initial conditions
C10=0.25;     %total cash
I10=0.75;     %total investment
x0=[C10;I10];
sim('onebank')%run the simulation in the Simulink model
%*****************************************************************
```

Appendix 2: Script and function for two-bank model simulation

The following script assigns the initial conditions, parameter values and other information to run the simulations for the two-bank model.

```
%***********************************************************
clear all;close all;
%Parameter settings of two_bank model simulation
p1=0/100/360; %investment return rate of bank1
p2=0/100/360; %investment return rate of bank2
w1=0/100/360; %proportion of the total investment that matured of
bank1
w2=0/100/360; %proportion of the total investment that matured
v1=0/100/360; %proportion of the total investment that failed of
bank1
v2=0/100/360; %proportion of the total investment that failed
r=0.2;        %reserve ratio
g1=0/100/360; %deposit interest rate of bank1
g2=0/100/360; %deposit interest rate
alpha1=1;     %proportion of the borrowing bank1 repays to bank2
alpha2=1;     %proportion of the borrowing bank2 repays to bank1
h12=1/100/360;%interest rate of borrowing bank1 repays to bank2
h21=1/100/360;%interest rate of borrowing bank2 repays to bank1

t=10; tt=0:1:t;%simulation time
step=0.1; %step size

%Generate exogenous signal
D10=1;D20=1;
Deposit1=[tt' [D10*ones(4,1);(D10-0.2)*ones(2,1);(D10-
0.21)*ones(3,1);D10*ones(t-8,1)]];% Deposit of bank1
Deposit2=[tt' [D20*ones(6,1);(D20-0.24)*ones(2,1);D20*ones(t-
7,1)]];% Deposit of bank2

opp10=1;opp20=0.01;
Opp1=[tt' opp10*ones(t+1,1)];%Investment opportunity of bank1
Opp2=[tt' opp20*ones(t+1,1)];%Investment opportunity of bank1

C10=0.25;C20=0.3;%Initial cash of bank1 and bank2
I10=0.75;I20=0.5;%Initial investment of bank1 and bank2
B10=0;B20=0;     %Initial borrowing of bank1 and bank2

x0=[C10;I10;B10;B20;C20;I20];%summary of initial conditions
sim('twobank_repay') %run the simulation in the Simulink model
%****************************************************************
```

User defined function used in the two-bank model Simulink Block

```
function
[xdot,inadd,infa,inmat,Deposit,boadd,borepay,Cfb,Cfre,CBB]=
fcn(x,D1,D1dot,D2,D2dot,opp1,opp2,w1,w2,v1,v2,g1,g2,p1,p2,r,alpha1
,alpha2,h12,h21,Csur)
% in this model input u is the threshold for borrow, lend and
invest
C1=x(1);
I1=x(2);
B1=x(3);
B2=x(4);
C2=x(5);
I2=x(6);
Csur1=Csur(1);
Csur2=Csur(2);
%repay stage
if C1>=0 && Csur1>0 && C2>=0 && Csur2>0;
B1repay=min(max(C1,0),alpha1*B1);
else
B1repay=0;
end

if C2>=0 && Csur2>0 && C1>=0 && Csur1>0;
B2repay=min(max(C2,0),alpha2*B2);
else
B2repay=0;
end

%borrowing stage
if Csur1>0
B1dot=min(max(r*D1-(C1-B1repay+B2repay),0),max(C2+B1repay-B2repay-
r*D2,0))-B1repay;
else
    B1dot=0;
end
if Csur2>0;
B2dot=min(max(r*D2-(C2-B2repay+B1repay)-
D2dot,0),max(C1+B2repay+B1repay-r*D1,0))-B2repay;
else
    B2dot=0;
end
D1=D1*Csur1;D2=D2*Csur2;
D1dot=D1dot*Csur1;D2dot=D2dot*Csur2;
I1=I1*Csur1;I2=I2*Csur2;
B1=B1*Csur1;B2=B2*Csur2;

I1dot=min(max((C1-B1repay+B2repay)+B1dot-D1*r-B2dot,0),opp1)-
w1*I1-v1*I1;
C1dot=D1dot-I1dot+B1dot-B2dot-g1*D1-v1*I1+p1*I1-h12*B1+h21*B2;

I2dot=min(max((C2-B2repay+B1repay)+B2dot-D2*r-B1dot,0),opp2)-
w2*I2-v2*I2;
C2dot=D2dot-I2dot-B1dot+B2dot-g2*D2-v2*I2+p2*I2+h12*B1-h21*B2;

xdot=[C1dot;I1dot;B1dot;B2dot;C2dot;I2dot];
```

```
inadd=[min(max((C1-B1repay+B2repay)+B1dot-D1*r-
B2dot,0),opp1)*Csur1;min(max((C2-B2repay+B1repay)+B2dot-D2*r-
B1dot,0),opp2)*Csur2];
infa=[v1*I1*Csur1;v2*I2*Csur2];
inmat=[w1*I1*Csur1;w2*I2*Csur2];
Deposit=[D1*Csur1;D2*Csur2];
boadd=[min(max(r*D1-(C1-B1repay+B2repay),0),max(C2+B1repay-
B2repay-r*D2,0));min(max(r*D2-(C2-B2repay+B1repay)-
D2dot,0),max(C1+B2repay+B1repay-r*D1,0))];
borepay=[B1repay;B2repay];
Cfb=[max(r*D1-(C1-B1repay+B2repay),0);max(C2+B1repay-B2repay-
r*D2,0)];
Cfre=[max(C1,0);alpha1*B1];
CBB=[C1+B1dot-B2dot;C2+B2dot-B1dot];
```

Appendix 3: Scripts and function for multi-bank model simulation

User defined function used in the multi-bank model Simulink Block

```
function
[xdot,BLdot,inadd,infa,inmat,interest,Deposit,B_detail,Bre_detail,
CfCsur]=
fcn(x,BL,link,D,Ddot,opp,w,v,g,p,r,a,b,c,dBasic,alpha,Csur)
%function solve the differencial equations
%input: the total number of the deposit, cash,
investment,state(survive or failed)
%borrow(matrix&vector), lend(matrix&vector), investment opp, link
%w,v,g,p,r,a,b,c,basic interest rate, alpha

%output: the change of the cash,investment,borrow(matrix&vector),
lend(matrix&vector),
%link,state(survive or failed)

%split the x to total cash,investment,borrowing,lenidng, net worth
C=x(1,:);%C is a 1*N
I=x(2,:);%I is a 1*N
B=x(3,:);
Net=x(5,:);
Bto_detail=BL;% BL is a N*N
N=length(C);%N is the number of the banks

%Bank experience shock, pay depositor interest, receive investment
return
C1= C+Ddot-g.*D+p.*I;%total cash
%==================================================================
========
%interest rate of lending
frate=zeros(N,N);
interest=zeros(N,N);
for j=1:N;%borrowing bank
    for i=1:N;%lending bank
        if Csur(1,i)==0 || Csur(1,j)==0 || i==j;
            frate(i,j)=0;
            interest(i,j)=dBasic(1,j);
        else
            frate(i,j)= a(1,i)*1/((exp((B(1,j)/C1(1,i)-
c(1,i))*b(1,i))+1));
            interest(i,j)=frate(i,j)+dBasic(1,i);
        end
    end
end
%Bank repay their borrowing
Bre_detail=zeros(N,N);
Breinterest=zeros(N,N);
C2=C1;
for jj=1:N;%borrowing bank
    for ii=1:N;%lending bank
```

```
    Bre_detail(ii,jj)=min(max(C2(ii)-
D(ii)*r,0),alpha(ii)*Bto_detail(ii,jj));
    %Bto_detail(ii,jj) represent total borrowing bank i borrows
form bank j
    Breinterest(ii,jj)=interest(ii,jj).*Bto_detail(ii,jj);
    C2(ii)=C2(ii)+Bre_detail(ii,jj)+Breinterest(ii,jj);%update
borrowing bank's cash
    C2(jj)=C2(jj)-Bre_detail(ii,jj)-Breinterest(ii,jj);%update
lending bank's cash
    end
end
%==================================================================
========


%borrowing process
%calculate the protencial borrowing and lending
pborrows=max(r.*(D-Ddot)-C2,0);%1*N
plends=max(C2-r.*(D-Ddot),0);%1*N
%calculate the lending and borrowing betwwen every two banks
NN=linspace(1,N,N);
number=NN';%the index of the banks (N*1)

%prepare the matrixs
pborrow = [number pborrows' Net'];%borrowing matrix with networth
information
plend = [number plends' interest']; %add interest information to
the leniding matrix
%find the size for borrow and lend banks
LL=sortrows(plend,-2);
BB=sortrows(pborrow,-2);
nl=length(nonzeros(LL(:,2)));%number of lending banks
nb=length(nonzeros(BB(:,2)));%unmber of borrowing banks
Pborrow=sortrows(BB(1:nb,:),-3);%borrowing banks with net-worth
PPlend=sortrows(LL(1:nl,:),1); %lending banks
Pborrow2=Pborrow;%borrowing banks with net-worth
PPlend2=PPlend; %lending banks

%start to calculate
B_detail=zeros(N,N);%create an empty matrix for borrowing value
 for j=1:nb;%borrowing bank
      for i=1:nl;%lending bank
      PPlend2=sortrows(PPlend2,(2+Pborrow2(j,1)));%lending banks
with interest rates
      %calculate the borrowing for each bank
       if link(Pborrow2(j,1),PPlend2(i,1))==0 %||
Pborrow(j,2)>sum(PPlend(:,2));
       B_detail(PPlend2(i,1),Pborrow2(j,1))=0;
       else

B_detail(PPlend2(i,1),Pborrow2(j,1))=min(Pborrow2(j,2),PPlend2(i,2
));
       end
       %after borrowing decide whether the bank is failed
       if sum(B_detail(:,j))<sum(Pborrow2(j,2))%if bank doesnt get
enough
           B_detail(:,j)=zeros(N,1);%then borrowing not happen
           Bre_detail(:,j)=zeros(N,1);%repayment not happen
```

```
            Breinterest(:,j)=zeros(N,1);
            C2(ii)=C(ii);%update borrowing bank's cash
            C2(jj)=C(jj);%update lending bank's cash
            PPlend2(i,2)=PPlend(i,2);%lending bank's cash update
            Pborrow2(j,2)=Pborrow(j,2);
        else
            PPlend2(i,2)=PPlend2(i,2)-
B_detail(PPlend2(i,1),Pborrow2(j,1));
            Pborrow2(j,2)=Pborrow2(j,2)-
B_detail(PPlend2(i,1),Pborrow2(j,1));
        end
      end
 end
%create the LINTEREST,BINTEREST
Badd_sum=sum(B_detail);%added borrowing details
Ladd_sum=sum((B_detail'));
Bre_sum=sum(Bre_detail);%repay borrowing details
Lre_sum=sum((Bre_detail'));
Brein_sum=sum(Breinterest);%repya interest
Lrein_sum=sum((Breinterest'));
Bdot=Badd_sum-Bre_sum;%change of borrowing details
Ldot=Ladd_sum-Lre_sum;
C3=C2+Bdot-Ldot;%updated cash
%==================================================================
========
survive_num=length(find(C3>0));
index=zeros(1,N);
inv_liqui1=0;
left_invest=I-D;
for nn=1:N
    if C3(1,nn)>0
       index(1,nn)=1;
       inv_liqui1=inv_liqui1+0;
    else
        index(1,nn)=0;
        inv_liqui1=inv_liqui1+max(left_invest(1,nn),0);
    end
end
%liquidation
Z=zeros(size(C));
D=D.*Csur;
Ddot=Ddot.*Csur;%set deposit of previous failed banks to zeros

if survive_num~=0
I=I+index*(inv_liqui1/survive_num);%update investment value
inv_liqui=index*(inv_liqui1/survive_num);
else
inv_liqui=zeros(1,N);
end
%calculationg the derevation
Idot=(min(max(C3-D.*r,Z),opp)-w.*I-v.*I+inv_liqui).*index;
Cdot=Ddot.*index+(-Idot-g.*D-v.*I+inv_liqui+p.*I+Bdot-Ldot-
Brein_sum+Lrein_sum).*index;
Ndot=Cdot+Idot+Ldot-Bdot-Ddot;
xdot=[Cdot;Idot;Bdot;Ldot;Ndot];
BLdot=B_detail-Bre_detail;%change of borrowing N*N
```

```
inadd=(min(max(C-D.*r+Badd_sum-Ladd_sum,Z),opp)-w.*I-v.*I).*index;
infa=v.*I.*index;
inmat=w.*I.*index;
Deposit=index;
%create the index of survial and failed banks
CfCsur=C3-Idot-v.*I+inv_liqui;
```

The codes of running the simulation

TEST.m

```
%This is the file that run the simulation, it includes many sub
srcipts
clear all;close all;
set_testing_parameters;%this is the file in which parameter values
are set
main_simulation;%this is the file run and save all the simulations
reults
clear all;close all;
set_testing_parameters;
take_average;%this is the file calculate the average of
simulations reults
plot_results;%this is the file plots all the results
```

set_testing_parameters.m

```
%This is the file that set the simulation time, repeat
time,testing period, number of banks,link rate
%values and reserve ratios
T=10;%the repeated time of the simulation
%the value of reserve ratio used for testing
reserve_m_1={'01', '02', '03','04','05','06','07'};
reserve_m=[0.1 0.2 0.3 0.4 0.5 0.6 0.7];
R_length=length(reserve_m);
%the values of link rate used for testing
link_m_1={'0','015','035','05','065','085','1'};
link_m=[0 0.15 0.35 0.5 0.65 0.85 1];
L_length=length(link_m);
N=50;%Number of the bank
t=300;%The simulation time period daily based
step=0.1;%step size
```

main_simulation.m

```
%This is the file run all the simulations
for i=1:T;% repeat time
    T_1=num2str(i);
    NameT=strcat('time',T_1);
    mkdir(NameT);
    Dvan=0.7;%the shock amplitude
    bankdata;%generate other parameters and intial conditions
    for j=1:R_length;%different reserve ratio
        for jj=1:L_length;%different link rate
```

```
Name=strcat('Multibank_borrow_repay','reserve',reserve_m_1{j},'lin
k',link_m_1{jj});
        reserve=reserve_m(:,j);
        linkrate=link_m(:,jj);
        createlink;%create the link matrix
        sim('Multibank_borrow_repay');
        ii;%take out the needed results
        save(Name);%save the results
        Name_1=strcat(Name,'.mat');
        ppr;%summary the results of different links but same
reserve ratio
        movefile(Name_1,NameT);
        end
        NameRE=strcat('re',reserve_m_1{:,j},'_',T_1);
        save(NameRE);
    end
end
```

bankdata.m

```
%This is the file generate extenal signals,set initial conditions
and other
%parameters values.
tt=linspace(1,t,t);
%The initial condition for Depsoit, Cash, Investment, Borrowing
and Lending
%Homogeneous case
%Deposit
Dbar=1000;
TT=[0;tt'];
RandD=normrnd(0,1,t+1,N);
deposit_c=zeros(t,N);
deposit=Dbar*ones(t,N);
for jjjj=1:t;
    for iiii=1:N;
     deposit_c(jjjj,iiii) = Dvan*Dbar*RandD(jjjj,iiii);%Model B
     deposit(jjjj,iiii)=deposit(jjjj,iiii)+deposit_c(jjjj,iiii);
    end
end
deposit(deposit<0)=0;
Deposit=[TT [Dbar*ones(1,N);deposit]];
%Investment oppotunity
rho=0.3; %0<rho<1, maximun propertion of the depsoit
Ibar = rho*Dbar;
Ivan = 0.3;
Rand1=normrnd(0,1,t,N);
investopp=zeros(t,N);
for jjjj=1:t;
   for iiii=1:N;
investopp(jjjj,iiii) = abs(Ibar +Ivan*Ibar*Rand1(jjjj,iiii));
   end
end
Investopp=[0 Ibar*ones(1,N);tt' investopp];
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
w=[TT 0.18*ones(t+1,N)];%proportion of the matured investment
```

```
dBasic = [TT 1/100/360*ones(t+1,N)];%lending/borrowing interesr
rate
p = [TT 5/100/360*ones(t+1,N)];%investment return rate
v=[TT 1*p(:,2:N+1)-0*p(:,2:N+1)];%investment faluire rate
g = [TT 2/100/360*ones(t+1,N)];%deposit interest rate
%Other initial conditions(size 1*N)
N0=0*ones(1,N);%networth
I0=800*ones(1,N);%investment
B0=zeros(1,N);%borrowing
L0=zeros(1,N);%lending
D0=Deposit(1,2:N+1);%deposit
C0=N0-I0+D0;%cash
x0=[C0;I0;B0;L0;N0];
Bd0=zeros(N,N);%borrowing
Ld0=zeros(N,N);%lending
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%Flucration of the lending interest rate function parameters
a = [TT 0.5/100/360*ones(t+1,N)];
b = [TT -50*ones(t+1,N)];
c = [TT 0.3*ones(t+1,N)];
%Repay proportion
alpha=1*ones(1,N);
```

creatlink.m

```
%Define the link matrix
link=zeros(N,N); x=rand(N,N);
for iii =1:N;
    for jjj=iii:N;
        if x(iii,jjj)<=linkrate && iii~=jjj;
            link(iii,jjj) = 1;
        else link(iii,jjj) = 0;
        end
        link(jjj,iii)=link(iii,jjj);
    end
end
```

ppr.m

```
%This is the file saves all the needed results
sre0(:,jj)=survivalnumber;
sree0(:,jj)=failnumber;
snet_sum0(:,jj)=net_sum;
snet_ave0(:,jj)=net_ave;
scash_sum0(:,jj)=cash_sum;
scash_ave0(:,jj)=cash_ave;
sinv_sum0(:,jj)=inv_sum;
sinv_ave0(:,jj)=inv_ave;
sinvt_sum0(:,jj)=invt_sum;
sinvt_ave0(:,jj)=invt_ave;
slend_sum0(:,jj)=lend_sum;
slend_ave0(:,jj)=lend_ave;
slendt_sum0(:,jj)=lendt_sum;
slendt_ave0(:,jj)=lendt_ave;
sborrow_sum0(:,jj)=borrow_sum;
sborrow_ave0(:,jj)=borrow_ave;
```

```
sborrowt_sum0(:,jj)=borrowt_sum;
sborrowt_ave0(:,jj)=borrowt_ave;
sdep_sum0(:,jj)=dep_sum;
sdep_ave0(:,jj)=dep_ave;
sinterest_ave0(:,jj)=interest_ave1;
sPLE(1,jj)=ple;
sPLE1(1,jj)=ple1;
sELE(1,jj)=ele;
sPFB(1,jj)=pfb;
sAVERAGERATIO(1,jj)=averageratio;
sFIRSTF(1,jj)=firstfailtime;
sSECONDF(1,jj)=secondfailtime;
```

ii.m

```
%This is the file rearrange and process the results
tt = length(time);
INDEX=zeros(tt,N);
Cash1=zeros(tt,N);
Networth1=zeros(tt,N);
Deposittotal1=zeros(tt,N);
Depositchange1=zeros(tt,N);
Investtotal1=zeros(tt,N);
Investadd1=zeros(tt,N);
Investback1=zeros(tt,N);
Investfail1=zeros(tt,N);
Borrowtotal1=zeros(tt,N);
Lendtotal1=zeros(tt,N);
Borrowadd1=zeros(tt,N);
Lendadd1=zeros(tt,N);
Borrowrepay1=zeros(tt,N);
Lendrepay1=zeros(tt,N);
LENDADD=zeros(1,N);
LENDREPAY=zeros(1,N);
%Index for the bank
for i=1:N;
%Transfer the results to matrix
for x =1:tt;
    INDEX(x,i)=index(1,i,x);
    Cash1(x,i) = Cashtotal1(1,i,x);
    Networth1(x,i) = Networth(1,i,x);
    Deposittotal1(x,i)=Deposittotal(1,i,x);
    Investtotal1(x,i) = Investtotal(1,i,x);
    Investadd1(x,i) = Investadd(1,i,x);
    Investback1(x,i) = Investback(1,i,x);
    Investfail1(x,i) = Investfail(1,i,x);
    Borrowtotal1(x,i) = Borrowtotal(1,i,x);
    Lendtotal1(x,i) = Lendtotal(1,i,x);
    Borrowadd1(x,i) = Borrowadd(1,i,x);
    Lendadd1(x,i) = Lendadd(1,i,x);
    Borrowrepay1(x,i) = Borrowrepay(1,i,x);
    Lendrepay1(x,i) = Lendrepay(1,i,x);
end
LENDADD(1,i)=sum(Lendadd1(:,i));LENDREPAY(1,i)=sum(Lendrepay1(:,i)
);
end
INDEX1=zeros(t/step,N);%find when is the bank fails
```

```
for i=1:N;
    for ji=2:t/step+1;
        if INDEX(ji,i)-INDEX(ji-1,i)==-1;
            INDEX1(ji,i)=1;
        else INDEX1(ji,i)=0;
        end
    end
end
ind = find(INDEX1==1);
[Time,INDEX2] = ind2sub(size(INDEX1),ind);
INDEX3= sortrows([Time INDEX2],1);%find the order that banks
failed
A=LENDADD-LENDREPAY; %at final date the unpaid loan
AA=Lendadd1-Lendrepay1;%upaid loan at each time point
aaa = AA;
aaa(aaa<1e-10)=0;%upaid loan at each time point
INDEX4=aaa;
INDEX4(aaa>0)=1;
%=====================================================
%calculate the percentage that failed banks has upaid loans
le=zeros(length(Time),1);
for ji=1:length(Time)
    if aaa(t,INDEX3(ji,2))>0
        le(ji,1)=1;
    else
        le(ji,1)=0;
    end
end
ple=sum(le)/length(Time);
ple(isnan(ple))=0;ple(isinf(ple))=0;
%=========================================
%calulate the extent of the effect from upaid loans
%calculate the ratio= unpaidloan/(-cash) to measure contagion
ratio=aaa./-Cash1;
loanvscash=ratio(1:t/step,:).*INDEX1(2:t/step+1,:);
loanvscash(isnan(loanvscash))=0;
loanvscash1=zeros(t/step,N);
for i=1:t/step;
    for ji=1:N;
if loanvscash(i,ji)<1
    loanvscash1(i,ji)=0;
else loanvscash1(i,ji)=1;
end
    end
end
ple1=sum(sum(loanvscash1))/length(Time);
ele=sum(sum(loanvscash))/length(Time);
ele(isnan(ele))=0;ele(isinf(ele))=0;
%==========================================
%calculate the ratio betwwen number of failed bank/total number of
banks
pfb= length(Time)/N;
%==========================================
%try to see the protencial contange
ratio1=Borrowtotal1./Cash1.*INDEX;
averageratio=sum(sum(ratio1(1:t,:)))/sum(sum(INDEX(1:t,:)));
averageratio(isnan(averageratio))=0;
%==========================================
```

```
%Networth1=Cash1+Investtotal1+Lendtotal1-Borrowtotal1-
Deposittotal1;
net=Networth1.*INDEX;
net_sum=sum(net,2);
net_ave=net_sum./survivalnumber;
net_ave(isnan(net_ave))=0;
net_ave(isinf(net_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
cash=Cash1.*INDEX;
cash_sum=sum(cash,2);
cash_ave=cash_sum./survivalnumber;
cash_ave(isnan(cash_ave))=0;
cash_ave(isinf(cash_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
inv_sum=sum(Investadd1,2);
inv_ave=inv_sum./survivalnumber;
inv_ave(isnan(inv_ave))=0;
inv_ave(isinf(inv_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Invest=Investtotal1.*INDEX;
invt_sum=sum(Invest,2);
invt_ave=invt_sum./survivalnumber;
invt_ave(isnan(invt_ave))=0;
invt_ave(isinf(invt_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
depo=Deposittotal1.*INDEX;
dep_sum=sum(depo,2);
dep_ave=dep_sum./survivalnumber;
dep_ave(isnan(dep_ave))=0;
dep_ave(isinf(dep_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
lend_sum=sum(Lendadd1,2);
lend_ave=lend_sum./survivalnumber;
lend_ave(isnan(lend_ave))=0;
lend_ave(isinf(lend_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
lendt_sum=sum(Lendtotal1,2);
lendt_ave=lendt_sum./survivalnumber;
lendt_ave(isnan(lendt_ave))=0;
lendt_ave(isinf(lendt_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
borrow_sum=sum(Borrowadd1,2);
borrow_ave=borrow_sum./survivalnumber;
borrow_ave(isnan(borrow_ave))=0;
borrow_ave(isinf(borrow_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
Borrowt=Borrowtotal1.*INDEX;
borrowt_sum=sum(Borrowt,2);
borrowt_ave=borrowt_sum./survivalnumber;
borrowt_ave(isnan(borrowt_ave))=0;
borrowt_ave(isinf(borrowt_ave))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
LINK1=zeros(N,N,tt);
for x=1:tt;
    for ji=1:N;
        for i=1:N;
        if INDEX(x,i)==1 && INDEX(x,ji)==1 && i~=ji;
            LINK1(i,ji,x)=1;
```

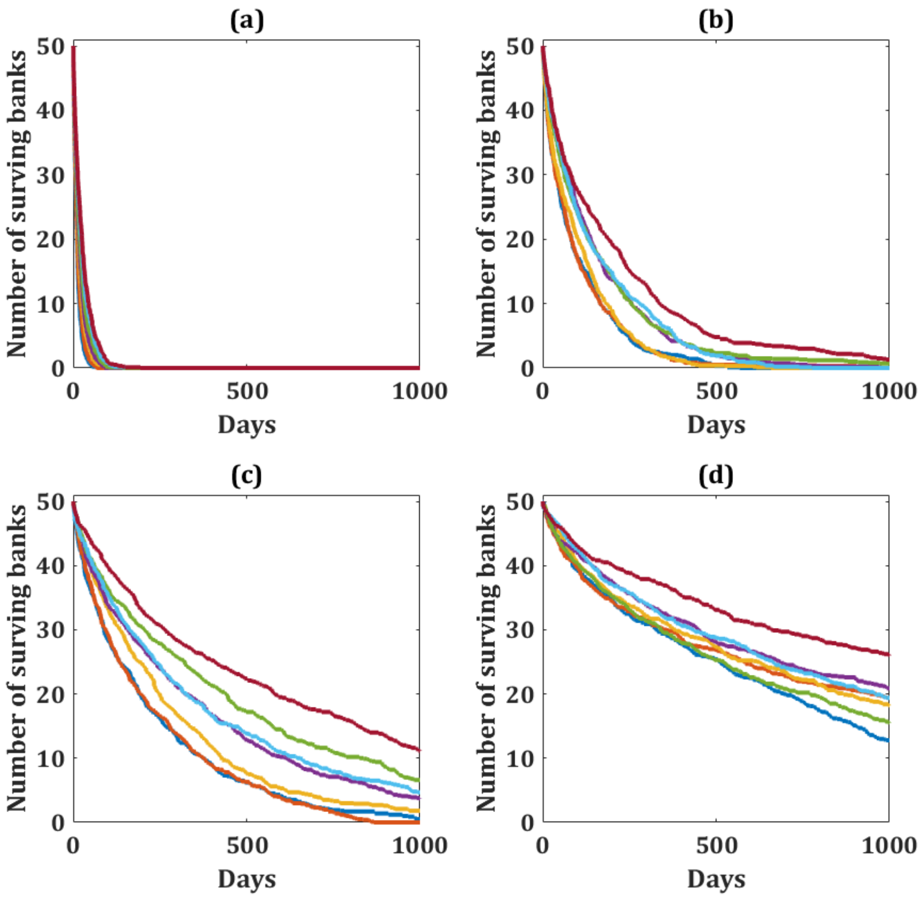
```
        else LINK1(i,ji,x)=0;
        end
        end
    end
 end
interest=INTEREST.*LINK1;
interestave=sum(sum(interest))./sum(sum(LINK1));
interest_ave1=zeros(tt,1);
for x =1:tt;
    interest_ave1(x,1) = interestave(1,1,x);
end
interest_ave1(isnan(interest_ave1))=0;
%~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%calculate the number of failed banks at each time step
failnumber=zeros(t/step,1);
for i=2:t/step
    failnumber(i,:)= survivalnumber(i-1,:)-survivalnumber(i,:);
end
%calculate the first defalut time
[row,col] = find(failnumber);
if numel(row)==0;
    firstfailtime=t/step;
    secondfailtime=t/step;
elseif numel(row)==1;
    firstfailtime=row(1,1);
    secondfailtime=t/step;
else
    firstfailtime=row(1,1);
    secondfailtime=row(2,1);
end
```

Appendix 4: Additional Simulation results

This Appendix shows additional simulation results that support the conclusions in Chapter 4.

1. Effect of reserve ratio and link rate on number of survival banks

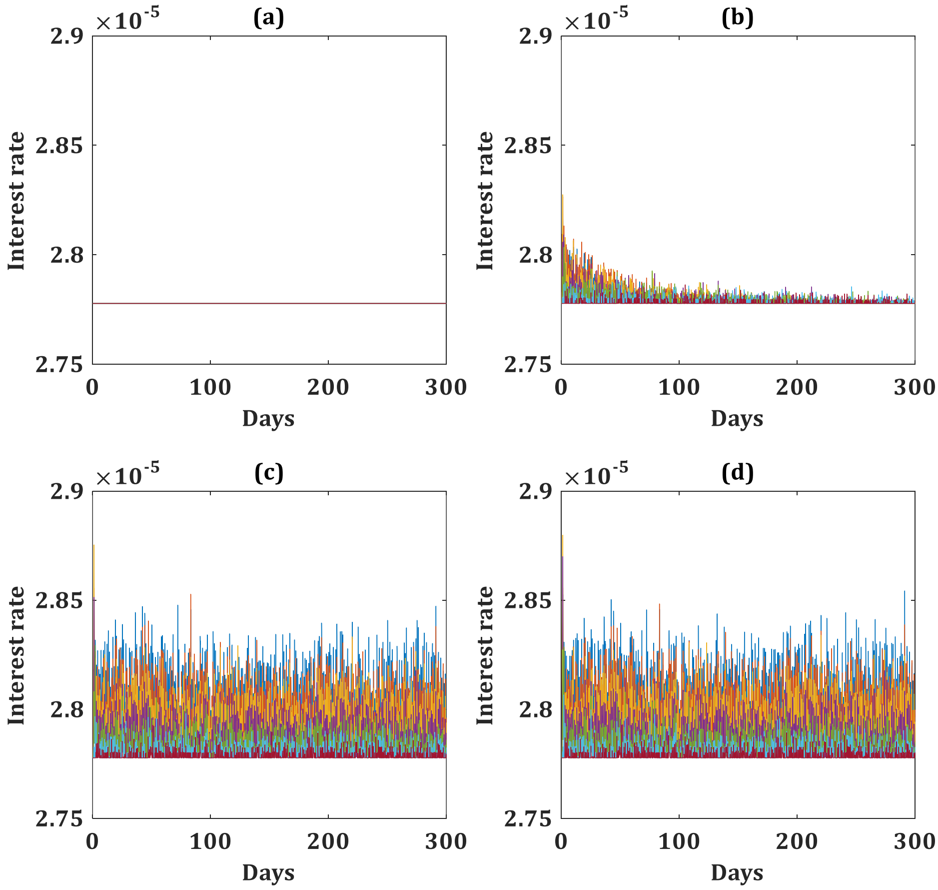
Figure A.1 reports how the number of survival banks changes with a low aptitude, $\sigma_D = 0.25$, of shocks in the deposits.



A. 1 Number of survival banks in the homogeneous case with $\sigma_D = 0.25$, $\sigma_{opp} = 0.5$ with different reserve ratios, r =0.1 (dark blue line), r =0.13(light red line), r =0.17 (yellow line), r =0.2 (purple line), r =0.23 (green line), r =0.3 (light blue line), r =0.4 (dark red line), and under different link rates, l_r =0 (a), l_r =0.04 (b), l_r =0.06 (c), l_r= 0.08 (d).

2. Lending interest rate as a function of link rate and reserve ratio

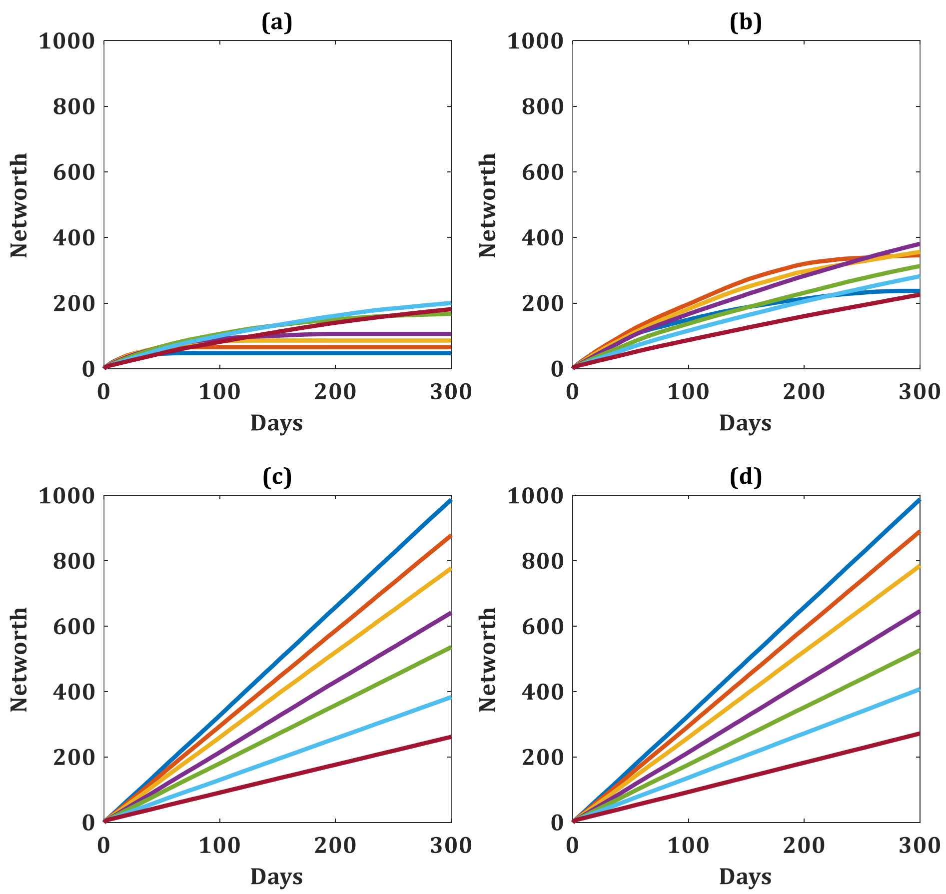
Figure A.2 reports how the average lending interest rate of all survival banks is affected by different reserve ratios and link rates when σ_D= 0.3. Figure A.2(a) reports the results corresponding to l_r= 0; since there is no interbank lending, the average interest rate stays at the basic rate level. As the link rate increases to 0.15,0.5 and 1, as shown in figure A.2(b), (c) and (d) respectively, the average lending interest rate increases due to more borrowing and lending happen between banks. As the reserve ratio increases, the average interest rate decreases, which indicates that high reserve ratio prevent the banks from lending, thus the banks become less active in the interbank market.



A. 2 Average lending interest rate of all survival banks in the homogeneous case with σ_D = 0.3, σ_{opp} = 0.5 with different reserve ratios, r =0.1 (dark blue line), r =0.2(light red line), r =0.3 (yellow line), r =0.4 (purple line), r =0.5 (green line), r =0.6 (light blue line), r =0.7 (dark red line), and under different link rates, l_r= 0 (a), l_r= 0.15 (b), l_r = 0.5 (c), l_r= 1 (d).

3. Net-worth as a function of link rate and reserve ratio

Figure A.3 shows how the average lending interest rate of all survival banks is affected by different reserve ratios and link rates when σ_D= 0.3. Figure A.3(a) reports the results when l_r= 0; this figure shows that when there is no interbank lending, as the reserve ratio increases, more net-worth is generated at the end of the simulation period. However, when the link rate increases as shown in figures A.3 (b) (c) and (d), an increase in the reserve ratio causes a decrease in the total net-worth. This result shows similarities in the effect of the reserve ratio and link rate on the average lending interest rate as in simulation results 1 in Section 4.4.2.



A. 3 Total net-worth of all banks in the homogeneous case with σ_D = 0.3, σ_{opp} = 0.5 with different reserve ratios, r =0.1 (dark blue line), r =0.2(light red line), r =0.3 (yellow line), r =0.4 (purple line), r =0.5 (green line), r =0.6 (light blue line), r =0.7 (dark red line), and under different link rates, l_r= 0 (a), l_r= 0.15 (b), l_r = 0.5 (c), l_r= 1 (d).