An importance measure for multistate systems with external factors*

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Abstract: Many technical systems are operated under the impact of external factors that may cause the systems to fail. For such systems, an interesting question is how those external factors and their impacts on the system can be identified at an earlier stage. Importance measures in reliability engineering are used to prioritise weak components (or states) of a system. Component failures and the impact of external factors in the real world may be statistically dependent as external factors may affect system performance. This paper proposes a new importance measure for analysing the impact of external factors on system performance. The measure can evaluate the degree of the impact of external factors on the system and can therefore help engineers to identify the factors with the strong impact on the system performance. A real-world case study is used to illustrate its applicability.

Keywords: Reliability; importance measure; external factor; system performance; multistate system

Acronyms

BN	Bayesian network
MDD	Multistate decision diagram
HUD	Head-up display

Notations

n	number of components

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N	number of external factors
i	index of component *i*, $i = 1, 2, \ldots, n$
X_i	state of component *i*, $X_i = 0, 1, 2, \ldots, M_i$
k	index of external factor *k*, *k*=1, 2,…, *N*
Y_k	state of external factor *k*, $Y_k = 0, 1, \ldots, S_k$
a_j	performance level corresponding to state *j* of the system
U	expected performance of a system
X	$(X_1, X_2, \ldots, X_n)$: state vector of the components
Y	$(Y_1, Y_2, \ldots, Y_N)$: state vector of the external factors
$\Phi(X)$	system structure function, $\Phi(X) = \Phi(X_1, X_2, \ldots, X_n)$
$(\cdot_i, X)$	$(X_1, \ldots, X_{i-1}, \cdot, X_{i+1}, \ldots, X_n)$
P_{im}	$\Pr\{X_i = m\}, m = 0, 1, 2, \ldots, M_i$
ρ_{im}	$\rho_{im} = \Pr\{X_i \geq m\} = P_{im} + P_{i(m+1)} + \cdots + P_{iM_i}$
n_{Path}	number of the MDD paths
l	index of MDD paths l, l =1,2,…, n_{Path}
$Path_l$	MDD paths l
n_{dPath}	number of the divided MDD paths
l_d	index of divided MDD paths l_d, l_d =1,2,…, n_{dPath}
$dPath_{l_d}$	divided MDD paths l_d

1. Introduction

Importance measures are widely used to identify the weakest component of a system and to support system improvement activities in reliability engineering. Kuo and Zhu [1-3] summarise the concepts of importance measures in reliability and their applications in a wide spectrum of different areas. These measures can also provide valuable information that facilitates the safety and efficient operation of systems at different phases. For example, identifying the weakness of a system and understanding how the failure of each individual component affects the reliability of the system are crucial at the design phase. Engineers may then allocate resources for important components during the system operation stage and maintain the reliability of a system at a certain level. Importance measures are also used at the system maintenance phase to help engineers minimise maintenance cost and prolong the life of the system.

In binary systems, Birnbaum [4] originally defines the component importance, which evaluates the effect of changing the reliability of a component on the reliability of the system. Since then, many importance measures of binary systems are proposed from different perspectives [5-9]. For example, Wu and Coolen [10] introduce a cost-based importance, which extends the well-known Birnbaum importance. Borgonovo et al. [11, 12] propose differential importance measure and time-independent reliability importance measure.

Many real-world systems have multiple states, ranging from a perfectly functioning state to one of complete failure. Several studies have been conducted to evaluate the reliability and performance of multistate systems [13-16]. To explore multistate systems, authors frequently use importance measures to identify the most critical components that facilitate the improvement and prioritization of system performance. For instance, Griffith [17] formalises the concept of system performance through expected utility and studies the effect of component improvement on system performance by generalizing Birnbaum importance. Zio and Podofillini [18] generalise the measure of Birnbaum importance with the performance level of multistate systems in contrast to binary systems that utilise Monte Carlo simulation. Wu and Chan [19] define a new utility importance of components of multistate systems to measure the importance of states. Ramirez–Marquez and Coit [20, 21] present composite importance measures to identify and rank multistate components based on their impact on the reliability behavior of multistate systems. Ramirez–Marquez et al. [22] propose a multistate redundancy importance measure that provides information on the potential of components for improvement. Levitin et al. [23] consider the commonly used importance measures in multistate systems. Peng et al. [24] study the component reliability and importance of criticality to systems with degrading components. Tyrväinen [25] presents new risk importance measures applicable to a dynamic reliability analysis approach with multi-state components. Si and Dui et al. [26-28] propose an integrated importance measure to evaluate the effects of transition of components on system performance.

Many technical systems operate under the impact of external factors, such as intentional attacks, accidents, environmental factors, or natural disasters, these factors may cause damage of system components [29]. External factors such as fires, storms, earthquakes, high and low temperatures are often considered in probabilistic safety assessment (PSA), which is a widely used risk assessment tool in many industries such as the nuclear power industry, in which abnormal events, or external factors, may affect the normal operation of the facility in a firm [30]. A well-known example of the impact of external factors on a technical system is the Fukushima Daiichi nuclear disaster in Japan, which was initiated primarily by the tsunami following the Tōhoku earthquake on 11 March 2011 and caused several hydrogen-air chemical explosions. Another example is: the reliability of water mains is affected by environmental factors such as soil properties and temperature. Existing importance measures [31-34] are mainly concerned about system performance that resulted from changes in component reliability in terms of random failures, common cause failures and human errors.

A vital problem in engineering is to identify the factors with the strong impact on the system performance. Importance measure that can evaluate the degree of the impact of external factors on the system should therefore be developed to help engineers to protect the system from damage and further to improve the performance of the system. However, existing relevant research mainly analyses the protection of external factors on the system and the optimal defense based on different algorithms. The research in this area includes, for example, Levitin et al [35-39] estimate the protection for the impact of external factors on the system's survivability based on the universal generating function method. Zhang and Ramirez-Marquez [40] develop optimal protection strategies for critical infrastructures against intentional attacks. Shin and Kim [41] analyse the flight envelope protection systems to prevent an aircraft from exceeding structure limits. Considering mutually exclusive events and common cause failures, Vaurio [42-46] develops importance measures and their applications in fault tree techniques, multi-phase missions and non-coherent systems for the reliability and risk analysis.

It can be seen from the above examples that measuring the importance of external factors and then identifying possible hazards are vitally important. In practice, component failures and the impact of external factors may be statistically dependent as external factors may affect system performance. This paper proposes a new importance measure to evaluate the impact of external factors on a system performance.

The rest of this study is organised as follows. Section 2 introduces the importance measure of external factors. Section 3 evaluates the system performance based on the importance. Section 4 provides the method for evaluating the importance measure of external factors. Section 5 uses a case study to illustrate the applicability of the proposed measure. Section 6 concludes the paper.

Assumptions

1) The state space of component *i* is $\{0,1,2,\ldots,M_i\}$ and the state space of the system is $\{0,1,2,\ldots,M\}$, where 0 represents the completely failed state of the system/components and M_i (M) is the perfectly functioning state of component *i* (system). The performance of component *i* (the system) deteriorates from M_i (M) to 0.
2) The state space of external factor *k* is $\{0,1,\ldots,S_k\}$, where 0 represents that the external factor can cause the complete failure of the system. S_k represents that the external factors has no impact on the system. The severity decreases from 0 to S_k.
3) All external factors (states) are statistically independent.
4) The states of component *i* is impacted by external factors. All external factors and their states are known.

2. Importance measure with external factors

External factors may have impacts on system/component reliability. The state of an external factor represents the impact severity of the external factor. For example, state 0 of the external factor represents that the external factor can cause the complete failure of the system. With the change of impact severity of external factor, the external factors may change from one state to another state, and cause the system partial failure or complete failure. For example, in a system of water mains, when the temperature rises to 65 degrees Celsius, the pipe may fail.

We assume there are *N* external factors, which affect system performance. The change of an external factor from one state to another may affect the states of components in a multistate system. Therefore, using the total probability formula and Assumption 3), the probability of component *i* being at state *m* is given as:

$$\begin{aligned}
P_{im} &= \Pr(X_i = m) \\
&= \sum_{b_1=0}^{S_1}\sum_{b_2=0}^{S_2}\cdots\sum_{b_N=0}^{S_N}(\Pr(Y_1{=}b_1, Y_2{=}b_2,\dots,Y_N{=}b_N)\Pr(X_i = m \mid Y_1{=}b_1, Y_2{=}b_2,\dots,Y_N{=}b_N)) \\
&= \sum_{b_1=0}^{S_1}\sum_{b_2=0}^{S_2}\cdots\sum_{b_N=0}^{S_N}(\Pr(Y_1{=}b_1)\Pr(Y_2{=}b_2)\dots\Pr(Y_N{=}b_N)\Pr(X_i = m \mid Y_1{=}b_1, Y_2{=}b_2,\dots,Y_N{=}b_N)) \\
&= \sum_{b_k=0}^{S_k}\Pr(Y_k = b_k)\sum_{b_1=0}^{S_1}\cdots\sum_{b_{k-1}=0}^{S_{k-1}}\sum_{b_{k+1}=0}^{S_{k+1}}\cdots\sum_{b_N=0}^{S_N}((\prod_{r=1,r\neq k}^{N}\Pr(Y_r{=}b_r))\Pr(X_i = m \mid Y_1 = b_1,\dots,Y_N = b_N)).
\end{aligned}$$

Denote

$$f_{Y_k=b_k}(X_i = m) = \sum_{b_1=0}^{S_1}\cdots\sum_{b_{k-1}=0}^{S_{k-1}}\sum_{b_{k+1}=0}^{S_{k+1}}\cdots\sum_{b_N=0}^{S_N}((\prod_{r=1,r\neq k}^{N}\Pr(Y_r{=}b_r))\Pr(X_i = m \mid Y_1 = b_1,\dots,Y_N = b_N)).$$

For convenience, let $b_k = b$. We then obtain

$$P_{im} = \Pr(X_i = m) = \sum_{b=0}^{S_k}(\Pr(Y_k = b) f_{Y_k=b}(X_i = m)), \tag{1}$$

where $\sum_{m=0}^{M_i} P_{im} = 1$ and $\sum_{b=0}^{S_k}\Pr(Y_k = b) = 1$.

State 0 of the external factor is the perfectly functioning state that can cause component *i* to fail. Thus, $\Pr\{X_i = 0 \mid Y_k = 0\}{=}1$ and

$$\Pr\{X_i = m \mid Y_k = 0\}{=}0, m = 1,2,\dots,M_i. \tag{2}$$

The expected performance of the system is $U = \sum_{j=0}^{M} a_j \Pr(\Phi(X) = j)$, $0 \le a_0 \le a_1 \le \dots \le a_M$ [17]. When considering the impact of the external factor on the system performance, based on reference [17], for any *i*, we obtain

$$U = \sum_{j=1}^{M}(a_j - a_{j-1})\Pr\left[\Phi(0_i, X) \ge j\right] + \left(\frac{\partial U}{\partial \rho_{i1}},\dots,\frac{\partial U}{\partial \rho_{iM_i}}\right)\cdot \rho_i^{\mathrm{T}}, \tag{3}$$

where $\rho_i = (\rho_{i1}, \rho_{i2},\dots,\rho_{iM_i})$, $\rho_{im} = P_{im} + P_{i(m+1)} + \dots + P_{iM_i}$ and

$$\frac{\partial U}{\partial \rho_{im}} = \sum_{j=1}^{M}(a_j - a_{j-1})\left[\Pr(\Phi(m_i, X) \geq j) - \Pr(\Phi((m-1)_i, X) \geq j)\right]. \tag{4}$$

According to (1), we obtain

$$\rho_{im} = \sum_{c=m}^{M_i}\sum_{b=0}^{S_k}\Pr(Y_k = b) f_{Y_k=b}(X_i = c) = \sum_{b=0}^{S_k}[\Pr(Y_k = b)\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)]. \tag{5}$$

Hence, the last term at the right side of (3) can be converted to

$$\begin{aligned} &\left(\frac{\partial U}{\partial \rho_{i1}}, \ldots, \frac{\partial U}{\partial \rho_{iM_i}}\right) \cdot \rho_i^{\mathrm{T}} = \sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\rho_{im}) \\ &= \sum_{m=1}^{M_i}\frac{\partial U}{\partial \rho_{im}}\sum_{b=0}^{S_k}(\Pr(Y_k = b)\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)) = \sum_{b=0}^{S_k}\Pr(Y_k = b)\sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)) \end{aligned} \tag{6}$$

Given that $\Pr(Y_k = 0) = 1 - \sum_{b=1}^{S_k}\Pr(Y_k = b)$, let vector $(\Pr(Y_k = 1), \Pr(Y_k = 2), \ldots, \Pr(Y_k = S_k))$ describe the state distribution of Y_k. According to (3), (4), and (6), the importance of the impact of state *b* of external factor *k* on system performance is given by

$$I_b(k) = \frac{\partial U}{\partial \Pr(Y_k = b)} = \sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}(\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c) - \sum_{c=m}^{M_i} f_{Y_k=0}(X_i = c))), b \geq 1. \tag{7}$$

Based on (2), $\sum_{c=m}^{M_i} f_{Y_k=0}(X_i = c) = 0$. Hence, one can define the following measure.

Definition 1: The importance of the state of external factor *k* on the system is defined as

$$\begin{aligned} I_b(k) &= \sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)) \\ &= \sum_{m=1}^{M_i}\left[\sum_{j=1}^{M}(a_j - a_{j-1})\left[\Pr(\Phi(m_i, X) \geq j) - \Pr(\Phi((m-1)_i, X) \geq j)\right] \cdot \sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)\right], b \geq 1. \end{aligned} \tag{8}$$

$I_b(k)$ in (8) describes the importance of the state of the external factor on the system performance. Equation (8) is complicated in computation. A computation method is proposed in Section 4.

With $I_b(k)$, the importance of external factor *k* on the system can be ranked, on which proper actions may then be taken. In practical application, importance ranking can be used to identify the most important external factor within a system and protect the system from damage of external factors.

Any system change at the design or operation phase may alter the impact of external factors. For example, flooding is one of the external factors that may cause a water pumping station to fail. If needed, a wall may add to protect a pumping station from possible damage. Engineers may also identify possible hazards (i.e., external factors) at the operation and optimization phase.

In practice any change in an external factor usually leads to changes in groups of its state probabilities. Therefore, managers are more interested to consider groups of external factor states. In order to evaluate the overall importance of external factors on system performance, the importance of external factor is equal to the sum of the ones of external factor states, so we use $I(k)=\sum_{b=1}^{S_k} I_b(k)$.

3. Evaluation of system performance based on the importance

When the external factors change, the system performance also changes. In the following, we discuss how the system performance changes based on the importance measure with external factors.

When one state of an external factor changes, based on (3) and (6),

$$U=\sum_{j=1}^{M}(a_j-a_{j-1})\Pr\left[\Phi(0_i,X)\geq j\right]+\sum_{b=0}^{S_k}[\Pr(Y_k=b)\sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\sum_{c=m}^{M_i}f_{Y_k=b}(X_i=c))].$$

Denote U^* the system performance after the change of state *b* of external factor *k*. Hence, the change of the system performance is given as

$$\begin{aligned}U^*-U&=(\Pr{}^*(Y_k=b)-\Pr(Y_k=b))\cdot\sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\sum_{c=m}^{M_i}f_{Y_k=b}(X_i=c))\\&\quad-(\Pr(Y_k=0)-\Pr{}^*(Y_k=0))\cdot\sum_{m=1}^{M_i}(\frac{\partial U}{\partial \rho_{im}}\sum_{c=m}^{M_i}f_{Y_k=0}(X_i=c)).\end{aligned}$$

Based on (2), $U^* - U = \Delta_{kb} \sum_{m=1}^{M_i} (\frac{\partial U}{\partial \rho_{im}} \sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c)).$

According to (8), we can obtain $U^* - U = \Delta_{kb} I_b(k)$.

Then for state *b* of external factor *k*, the different states b_1 and b_2 of external factor *k*, and the different states b_1 and b_2 of different external factors k_1 and k_2, we have the following inferences, respectively.

(1) If the probability of severity impacts of external factor *k* increases from probability $\Pr(Y_k = b)$ to probability $\Pr^*(Y_k = b)$, where $\Pr^*(Y_k = b) - \Pr(Y_k = b) = \Delta_{kb}$, then the change of the system performance is $\Delta_{kb} I_b(k)$.

(2) Assume that the improvements of probabilities of the states of external factor *k* are $\Delta_{kb_1} = \Delta_{kb_2}$, where $b_1 \neq b_2$. If $I_{b_1}(k) > I_{b_2}(k)$, then, system performance undergoes a larger increase when an improvement on state b_1 of external factor *k* is carried out than when an improvement on state b_2 of external factor *k* is carried out.

(3) Assume that the improvements of probabilities of the states of external factors k_1 and k_2 are $\Delta_{k_1 b_1} = \Delta_{k_2 b_2}$, where $k_1 \neq k_2$. If $I_{b_1}(k_1) > I_{b_2}(k_2)$, the increase in system performance will then be higher when an improvement is carried on external factor k_1 than on external factor k_2.

Furthermore, when all states of an external factor change, the change of the system performance is given as

$$\begin{aligned} U^* - U &= \sum_{b=1}^{S_k} [(\Pr^*(Y_k = b) - \Pr(Y_k = b)) \sum_{m=1}^{M_i} (\frac{\partial U}{\partial \rho_{im}} \sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c))] \\ &\quad - (\Pr(Y_k = 0) - \Pr^*(Y_k = 0)) \sum_{m=1}^{M_i} (\frac{\partial U}{\partial \rho_{im}} \sum_{c=m}^{M_i} f_{Y_k=0}(X_i = c)) \\ &= \sum_{b=1}^{S_k} [\Delta_k \sum_{m=1}^{M_i} (\frac{\partial U}{\partial \rho_{im}} \sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c))] = \Delta_k \sum_{b=1}^{S_k} \frac{\partial U}{\partial \Pr(Y_k = b)}. \end{aligned}$$

According to (8), we can obtain $U^{*}-U=\Delta_{k}\sum_{b=1}^{S_k} I_{b}(k)=\Delta_{k} I(k)$.

So if the probability of severe impacts of external factor *k* increases from probability $\Pr(Y_k=b)$ to probability $\Pr^*(Y_k=b)$ with $\Pr^*(Y_k=b)\geq \Pr(Y_k=b)$ for all nonzero states *b*, $\Pr^*(Y_k=1)-\Pr(Y_k=1)=\cdots=\Pr^*(Y_k=S_k)-\Pr(Y_k=S_k)=\Delta_k$ and the sum of the increased probabilities remains less than 1, then the corresponding change in system performance is $\Delta_k I(k)$.

4. Calculations of the importance measure

In this section, we discuss the calculation methods for scenarios: series systems and parallel systems, and a more general case.

4.1 Importance of external factors on a series system

For a multistate series system, the structure function is $\Phi(X)=min\{X_1,X_2,\ldots,X_n\}$. The importance measure of external factors on a series system is discussed in the following.

Based on (8),

$$I_b(k)=\sum_{m=1}^{M}\sum_{j=1}^{M}(a_j-a_{j-1})\left[\Pr(\Phi(m_i,X)\geq j)-\Pr(\Phi((m-1)_i,X)\geq j)\right]\sum_{c=m}^{M_i} f_{Y_k=b}(X_i=c).$$

In a multistate series system, $\Phi(X)=min\{X_1,X_2,\ldots,X_n\}$. Thus, we can obtain $\Pr(\Phi(m_i,X)\geq j)=0, j=m+1,\ldots,M$. Then, we obtain:

$$
\begin{aligned}
&\sum_{j=1}^{M}(a_j-a_{j-1})\Pr(\Phi(m_i,X)\geq j)\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\Pr(\Phi(m_i,X)\geq j)+\sum_{j=m+1}^{M}(a_j-a_{j-1})\Pr(\Phi(m_i,X)\geq j)\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\Pr(\Phi(m_i,X)\geq j)\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\Pr\left(\min\{X_1,\ldots,X_{i-1},m,X_{i+1},\ldots,X_n\}\geq j\right)\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\Pr\left\{X_1\geq j,\ldots,X_{i-1}\geq j,m\geq j,X_{i+1}\geq j,\ldots,X_n\geq j\right\}\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\prod_{l=1,l\neq i}^{n}\Pr(X_l\geq j).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&\sum_{j=1}^{M}(a_j-a_{j-1})\left[\Pr(\Phi(m_i,X)\geq j)-\Pr(\Phi((m-1)_i,X)\geq j)\right]\\
&=\sum_{j=1}^{m}(a_j-a_{j-1})\prod_{l=1,l\neq i}^{n}\Pr(X_l\geq j)-\sum_{j=1}^{m-1}(a_j-a_{j-1})\prod_{l=1,l\neq i}^{n}\Pr(X_l\geq j)\\
&=(a_m-a_{m-1})\prod_{l=1,l\neq i}^{n}\Pr(X_l\geq m)=(a_m-a_{m-1})\prod_{l=1,l\neq i}^{n}\rho_{lm}.
\end{aligned}
$$

According to (5), we obtain

$$
(a_m-a_{m-1})\prod_{l=1,l\neq i}^{n}\rho_{lm}=(a_m-a_{m-1})\prod_{l=1,l\neq i}^{n}\sum_{b=0}^{S_k}\Pr(Y_k=b)\sum_{c=m}^{M_l}f_{Y_k=b}(X_l=c).
$$

Finally, we can obtain

$$
I_b(k)=\sum_{m=1}^{M}(a_m-a_{m-1})\prod_{l=1,l\neq i}^{n}\sum_{b=0}^{S_k}\Pr(Y_k=b)\sum_{c=m}^{M_l}(f_{Y_k=b}(X_l=c)\sum_{c=m}^{M_i}f_{Y_k=b}(X_i=c)).
$$

4.2 Importance of external factors on a parallel system

For a multistate parallel system, the structure function is $\Phi(X)=max\{X_1,X_2,\ldots,X_n\}$. Then we have the following results. According to (8),

$$
\begin{aligned}
I_b(k)&=\sum_{m=1}^{M}\sum_{j=1}^{M}(a_j-a_{j-1})\left[\Pr(\Phi(m_i,X)\geq j)-\Pr(\Phi((m-1)_i,X)\geq j)\right]\sum_{c=m}^{M_i}f_{Y_k=b}(X_i=c)\\
&=\sum_{m=1}^{M}\sum_{j=1}^{M}(a_j-a_{j-1})\left[\Pr(\Phi(m-1_i,X)<j)-\Pr(\Phi(m_i,X)<j)\right]\sum_{c=m}^{M_i}f_{Y_k=b}(X_i=c).
\end{aligned}
$$

For a multistate parallel system, $\Pr(\Phi(m_i,X)<j)=0, j=1,2,\ldots,m$. Therefore,

$$
\begin{aligned}
&\sum_{j=1}^{M}(a_j - a_{j-1})\Pr(\Phi(m_i, X) < j) \\
&= \sum_{j=1}^{m}(a_j - a_{j-1})\Pr(\Phi(m_i, X) < j) + \sum_{j=m+1}^{M}(a_j - a_{j-1})\Pr(\Phi(m_i, X) < j) \\
&= \sum_{j=m+1}^{M}(a_j - a_{j-1})\Pr(\Phi(m_i, X) < j) \\
&= \sum_{j=m+1}^{M}(a_j - a_{j-1})\prod_{l=1, l\neq i}^{n}\Pr(X_l < j).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\sum_{j=1}^{M}(a_j - a_{j-1})\left[\Pr(\Phi(m_i, X) \geq j) - \Pr(\Phi((m-1)_i, X) \geq j)\right] \\
&= \sum_{j=1}^{M}(a_j - a_{j-1})\ 1 - \Pr(\Phi(m_i, X) < j) - \left[1 - \Pr(\Phi(m-1_i, X) < j)\right] \\
&= \sum_{j=1}^{M}(a_j - a_{j-1})\left[\Pr(\Phi(m-1_i, X) < j) - \Pr(\Phi(m_i, X) < j)\right] \\
&= \sum_{j=m}^{M}(a_j - a_{j-1})\prod_{l=1, l\neq i}^{n}\Pr(X_l < j) - \sum_{j=m+1}^{M}(a_j - a_{j-1})\prod_{l=1, l\neq i}^{n}\Pr(X_l < j) \\
&= (a_m - a_{m-1})\prod_{l=1, l\neq i}^{n}\Pr(X_l < m) = (a_m - a_{m-1})\prod_{l=1, l\neq i}^{n}(1 - \rho_{lm}).
\end{aligned}
$$

According to (5), we can obtain

$$
I_b(k) = \sum_{m=1}^{M}(a_m - a_{m-1})\prod_{l=1, l\neq i}^{n}(1 - \sum_{b=0}^{S_k}\Pr(Y_k = b)\sum_{c=m}^{M_l} f_{Y_k=b}(X_l = c))\sum_{c=m}^{M_i} f_{Y_k=b}(X_i = c).
$$

4.3 Importance of external factors on a more complex system

For more complex systems, we can use a method based on multistate decision diagram (MDD) to calculate the importance.

First, the Bayesian network (BN) for the multistate system with external factors is modeled. Second, system-level MDD is generated from the system's BN model. Lastly, the proposed importance measure is evaluated based on MDD. The detailed steps are as follows.

Step 1: The system BN model is built with external factors.

Nodes in the BN of a multistate system with the presence of external factors represent the components, system, and external factors. An edge of the BN represents the conditional dependency between an external factor and a component. For example, the BN for a multistate system with two external factors is shown in Fig. 1. System X_S consists of components X_1 and X_2, which are impacted by external factors 1 and 2.

<Insert Fig 1>

Fig. 1. An example of BN with external factors

Step 2: System level MDD is generated from the system's BN model.

A multistate system with external factors has two types of nodes in an MDD: (1) non-sink nodes that represent the states of components or external factor and (2) sink nodes that represent system states. The outgoing edges of each non-sink node represent the states of components or external factors.

Each multi-valued variable that correspond to each external factor is first assigned a different order or index. The MDD of impact severity *k* is illustrated in Fig. 2.

<Insert Fig 2>

Fig. 2. MDD of impact severity *k*

The mathematical rules of logic operation on two sub-MDDs can be described by (9), where G and H are the case formats for logic expressions of two sub-MDDs: $G = case(x, G_0, \ldots, G_{S_x})$ and $H = case(y, H_0, \ldots, H_{S_x})$ [47-49]. Operator $\Diamond$ represents either logic AND or OR operation. The same rules can be used for logic operation between sub-expressions until one of them becomes a constant expression "0" or "1."

$$G \Diamond H = case(x, G_0, \ldots, G_{S_x}) \Diamond case(y, H_1, \ldots, H_{S_y}) \\ = \begin{cases} case(x, G_0 \Diamond H_0, \ldots, G_{S_x} \Diamond H_{S_y}) & index(x) = index(y) \\ case(x, G_0 \Diamond H, \ldots, G_{S_x} \Diamond H) & index(x) < index(y) \\ case(y, G \Diamond H_0, \ldots, G \Diamond H_{S_y}) & index(x) > index(y) \end{cases} . \quad (9)$$

Step 3: The probability that the system is at state *j* is evaluated by the probability of all paths from root to sink node "*j*" in the generated MDD.

$$\Pr(\Phi(X)=j)=\sum_{l=0}^{n_{Path}}\Pr(\Phi(X)=j \mid Path_l)\Pr(Path_l), \tag{10}$$

where n_{Path} represents the number of the MDD paths and $\Pr(Path_l)=\prod_{k=0}^{N}\text{ite}\ Y_k \in Path_l, \text{case}\left[Y_k, \Pr(Y_k=0), \Pr(Y_k=1), \cdots, \Pr(Y_k=S_k)\right], 1$. The ite{} operator is a Boolean expression encoded in the if-then-else (ite) format.

Step 4: Importance measure is calculated with external factors.

The importance measure with external factors is given by (8).

In the MDD generated in Step 2, the conditional probability that component i is at state m for $Path_l$ is denoted by $\Pr(X_i=m \mid Path_l)$. The conditional probability that the state of the system is not less than state *j* when component *i* is at state *m* is shown in (11).

$$\begin{aligned}\Pr(\Phi(m_i,X)\geq j) &= \frac{\Pr(\Phi(X)\geq j, X_i=m)}{\Pr(X_i=m)} \\ &= \frac{\sum_{l=1}^{n_{Path}}\left[\Pr(\Phi(X)\geq j \ \&\ X_i=m \mid Path_l)\cdot \Pr(Path_l)\right]}{\sum_{b=0}^{S_k}\Pr(Y_k=b) f_{Y_k=b}(X_i=m)}.\end{aligned} \tag{11}$$

Thus, the importance measure with external factors can be obtained.

In Eq. (11), the practical states of external factors can be derived by expert elicitation/judgment. For example, in practice, when it is hard to determine the exact states of some external factor variables, the expert elicitation/judgment may used to estimate the probability distributions of the states of external factors.

When the most important external factor is determined, the components with the strongest impact on system performance should be identified under the impact of external factors.

Step 5: The importance measure for component is then evaluated.

Under the expected performance of the system $U=\sum_{j=0}^{M}a_j \Pr(\Phi(X)=j)$, the importance measure of state *m* of component *i* is given by:

$$I_m^C(i) = \frac{\partial U}{\partial \Pr(X_i = m)} = \sum_{j=0}^{M} a_j \frac{\partial \Pr(\Phi(X) = j)}{\partial \Pr(X_i = m)}.$$

The probability that the system is at state *j* can be converted is given by

$$\Pr(\Phi(X) = j) = \sum_{c=0}^{M_i} \Pr(X_i = c) \Pr(\Phi(c_i, X) = j),$$

Hence, we obtain

$$I_m^C(i) = \sum_{j=0}^{M} a_j \left[\Pr(\Phi(m_i, X) = j) - \Pr(\Phi(0_i, X) = j) \right]. \tag{12}$$

When calculating the importance measure of an external factor, MDD paths should be divided until each component state is known. We use $dPath_{l_d}$ to denote divided MDD path l_d.

The probability of each divided path for component *i* is shown in (13).

$$\Pr(dPath_{l_d}) = \prod_{k=0}^{N} \text{ite}\left(Y_k \in dPath_{l_d}, \text{case}\left[Y_k, \Pr(Y_k = 0), \Pr(Y_k = 1), \cdots, \Pr(Y_k = S_k)\right], 1 \right). \tag{13}$$

In a divided MDD path, the conditional probability that component i is at state m for $dPath_{l_d}$ is denoted by $\Pr(X_i = m \mid dPath_{l_d})$. Then, we can obtain

$$\Pr(\Phi(m_i, X)=j) = \frac{\Pr(\Phi(X)=j, X_i = m)}{\Pr(X_i = m)}$$
$$= \frac{\sum_{l_d=1}^{n_{dPath}} \left[\Pr(\Phi(X)=j \,\&\, X_i = m \mid dPath_{l_d}) \Pr(dPath_{l_d}) \right]}{\sum_{b=0}^{S_k} \Pr(Y_k = b) f_{Y_k=b}(X_i = m)}. \tag{14}$$

Thus, substituting (14) into (12), the component importance can be obtained.

5. Application to a head-up display (HUD) system

This section uses an actual HUD system to illustrate the applicability of the proposed importance measure. The proposed importance can of course be used to identify the most important external factor on the system and protect the HUD system from damage of temperature, pressure, and vibration.

A HUD system, which is studied in [50], is shown in Fig. 3. The component states of the HUD system are presented in Table 1.

<Insert Fig 3>

Fig. 3. The structure of HUD system [50].

Table 1. Component states of the HUD system

<Insert Table 1>

Temperature, vibration, and pressure are the three external factors that can affect the HUD system and components. These factors are denoted as Y_1, Y_2, and Y_3, respectively. Given this information and the structure of the HUD system, Fig. 4 shows the BN of the system, which considers the impact of external factors.

<Insert Fig 4>

Fig. 4. The BN of HUD system.

Table 2 shows the value range and state probabilities of each external factor.

Table 2. Value range and state probabilities of each external factor

<Insert Table 2>

The conditional probability distributions of the impact of external factors on the states of the components are shown in Table 3. Based on (2), when the state of an external factor is 0, $\Pr\{X_i = 0 \mid Y_k = 0\}=1$ and $\Pr\{X_i = m \mid Y_k = 0\}=0, m = 1,2,\ldots,M_i$. Thus, in Table 3, we discuss conditional probability distributions when the states of external factors are 0, 1 or 2, which correspond to the state classification in Table 1.

Table 3. Conditional probability distributions of the impact of external factors on the states of components

<Insert Table 3>

The conditional probability distributions of the impact of components on the states of the systems are presented in Table 4. The table also shows the state probability distributions of variables, which are listed based on the different state vectors of their father variables. The state probability distributions of variables without father variables, such as $\{X_2, X_4, X_5, X_6, X_7, X_8\}$, are the percentages of corresponding states appearing in the entire dataset.

Table 4. Conditional probability distributions in HUD system

<Insert Table 4>

By applying the traditional MDD manipulation rules based on Fig. 4, we obtain the MDD of components in Fig. 5.

<Insert Fig 5>

Fig. 5. MDD for the BN

The MDD of external factors for the BN model is created for each external factor, as shown in Fig. 6.

<Insert Fig 6>

Fig. 6. MDD of external factors

The probability of a system is at a given state is evaluated by (10). Thus, we obtain $\Pr(\Phi(X)=0)=0.3688, \Pr(\Phi(X)=1)=0.1467, \Pr(\Phi(X)=2)=0.4845$.

We assume that $a_0 = 0, a_1 = 1, a_2 = 2$ to obtain the importance value of external factors as shown in Table 5.

Table 5. Importance values of external factors

<Insert Table 5>

Table 5 shows that the importance of state 2 for each external factor is larger than that of state 1. This finding agrees with its state probability distribution. States 1 and 2 of temperature have the largest importance of external factors. Thus, temperature has the largest impact on the performance of HUD system. Table 5 shows that the importance ranking can be used to identify the most important external factor within the system and protect the performance of the HUD system from damage of temperature, pressure, and vibration.

The MDD in Fig. 6 shows that Component 5 may be at state 1 or 0 for path (Y_1=1; Y_2=1; Y_3=1, 2). To evaluate the conditional probability that the system state is at state *j* when component *i* is at state *m*, we divide this path. Fig. 7 shows the divided MDD.

<Insert Fig 7>

Fig. 7. Divided MDD

The importance of components under the impact of external factors can be obtained by (12). The results are shown in Table 6.

Table 6. Importance values of components

<Insert Table 6>

Table 6 shows that States 1 and 2 of Components 4 and 6 have the same importance value, whereas Components 5 and 7 have the same importance value. Table 3 shows that Components 4 and 6 are similar in the system structure, whereas external factors have similar impacts on the components. Thus, Components 4 and 6 have the same importance value. The impacts of external factors on Components 5 and 7 are similar as shown in Table 3. Thus, Components 5 and 7 also have the same importance value. Table 3 shows that Component 8 is always at state 1 regardless of changes in external factors (temperature, pressure, and vibration). Component 8 is not impacted by external factors. Thus, the optical module (Component 8) in state 1 has the largest importance value.

6. Conclusions

This study proposed a new importance measure to determine the impact of external factors on system performance when we assume that such factors may cause simultaneous damage on a system. If the probability of component state changes with the presence of external factors, system performance may be affected.

The proposed importance measure describes the impact of external factors on system performance and can be used to prioritise the most important external factors. It can provide guidance to engineers in protecting the system from damage of external factors.

It is understood that the probability distribution of the states of external factors may be more complicated than that assumed in this paper. As such, one of our future research topics is to collect real-world data, find the distribution of the states of external factors, and then investigate the applicability of the proposed importance measures.

Acknowledgment

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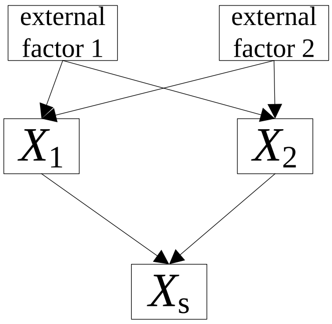

Fig. 1. An example of BN with external factors

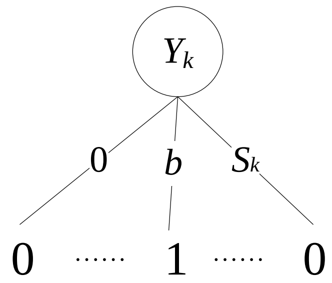

Fig. 2. MDD of impact severity k

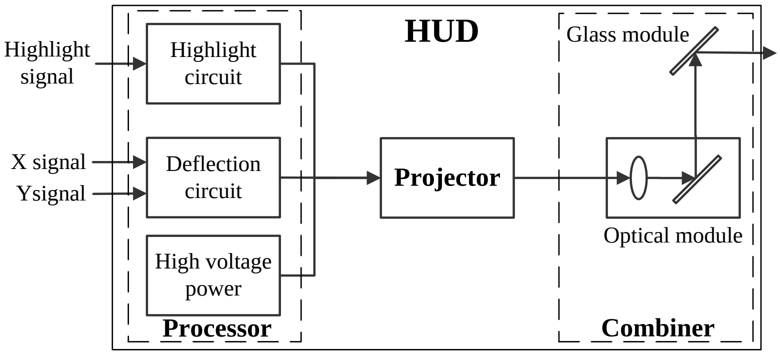

Fig. 3. The structure of HUD system [33].

Table 1. Component states of the HUD system

name	variable	state
HUD	X_s	No image (0), Unclear (1), Normal (2)
Processor	X_1	No signal (0), Fuzzy signal (1), Normal (2)
Projector	X_2	Malfunction (0), Normal (1)
Combiner	X_3	Malfunction (0), Normal (1)
Highlight circuit	X_4	No signal (0), Fuzzy signal (1), Normal (2)
Deflection circuit	X_5	Fuzzy signal (0), Normal (1)
High voltage power	X_6	No power (0), Uncontrolled power (1), Normal (2)
Glass module	X_7	Malfunction (0), Normal (1)
Optical module	X_8	Malfunction (0), Normal (1)

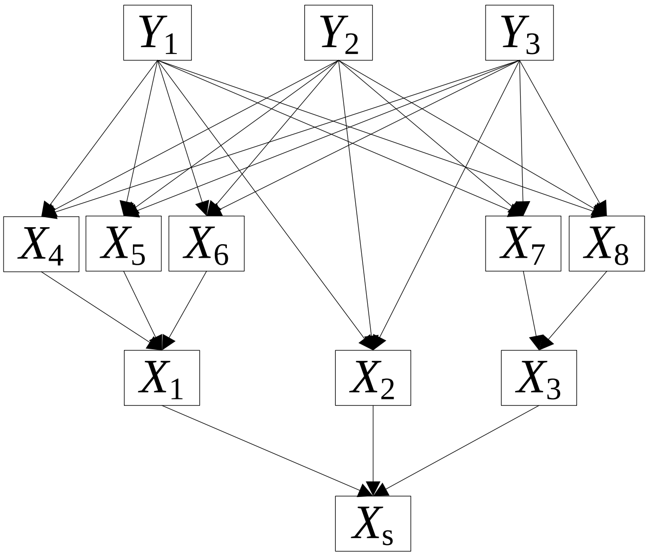

Fig. 4. The BN of HUD system.

Table 2. Value range and state probabilities of each external factor

External factor	General description	State	Probability
temperature	30 °C – 100 °C	2	0.4636
	-40 °C – 30 °C	1	0.3409
	<-40 °C	0	0.1955
vibration	1000 HZ – 2000 HZ	2	0.5568
	10 HZ – 1000 HZ	1	0.3360
	>2000 HZ	0	0.1072
pressure	45 kPa – 106 kPa	2	0.6364
	86 Pa – 45 kPa	1	0.2422
	>106 kPa	0	0.1214

Table 3. Conditional probability distributions of the impact of external factors on the states of components

Component	External factor			$\Pr(X_i = m \mid Y_1 = b_1, Y_2 = b_2, Y_3 = b_3)$		
				State 0	State 1	State 2
X_4	Y_1	Y_2	Y_3			
	1	1	1	1	0	0
	1	1	2	0	1	0
	1	2	1	0	1	0
	1	2	2	0	1	0
	2	1	1	0	0	1
	2	1	2	0	0	1
	2	2	1	0	0	1
	2	2	2	0	0	1
X_5	Y_1	Y_2	Y_3			
	1	1	1	1	0	
	1	1	2	1	0	
	1	2	1	0	1	
	1	2	2	0	1	
	2	1	1	0	1	
	2	1	2	0	1	
	2	2	1	0	1	
	2	2	2	0	1	
X_6	Y_1	Y_2	Y_3			
	1	1	1	1	0	0
	1	1	2	1	0	0
	1	2	1	0	1	0
	1	2	2	0	1	0
	2	1	1	0	0	1
	2	1	2	0	0	1
	2	2	1	0	0	1
	2	2	2	0	0	1
X_2	Y_1	Y_2	Y_3			
	1	1	1	1	0	
	1	1	2	1	0	
	1	2	1	1	0	
	1	2	2	0	1	
	2	1	1	0	1	
	2	1	2	0	1	
	2	2	1	0	1	
	2	2	2	0	1	
X_7	Y_1	Y_2	Y_3			
	1	1	1	1	0	
	1	1	2	1	0	
	1	2	1	0	1	
	1	2	2	0	1	
	2	1	1	0	1	
	2	1	2	0	1	
	2	2	1	0	1	
	2	2	2	0	1	
X_8	Y_1	Y_2	Y_3			
	1	1	1	0	1	
	1	1	2	0	1	
	1	2	1	0	1	
	1	2	2	0	1	
	2	1	1	0	1	
	2	1	2	0	1	
	2	2	1	0	1	
	2	2	2	0	1	

Table 4. Conditional probability distributions in HUD system

Component /System	Father variables			Probability distributions		
				State 0	State 1	state2
X_s	X_1	X_2	X_3			
	0	0	0	1	0	0
	0	0	1	0	1	0
	0	1	0	0	1	0
	0	1	1	0	1	0
	1	0	0	0	1	0
	1	0	1	0	1	0
	1	1	0	0	0	1
	1	1	1	0	0	1
	2	0	0	0	0	1
	2	0	1	0	0	1
	2	1	0	0	0	1
	2	1	1	0	0	1
X_1	X_4	X_5	X_6			
	0	0	0	1	0	0
	0	0	1	0	1	0
	0	0	2	0	0	1
	0	1	0	0	1	0
	0	1	1	0	1	0
	0	1	2	0	0	1
	1	0	0	0	1	0
	1	0	1	0	1	0
	1	0	2	0	0	1
	1	1	0	0	1	0
	1	1	1	0	1	0
	1	1	2	0	0	1
	2	0	0	0	0	1
	2	0	1	0	0	1
	2	0	2	0	0	1
	2	1	0	0	0	1
	2	1	1	0	0	1
	2	1	2	0	0	1
X_3	X_7	X_8				
	0	0		1	0	
	0	1		0	1	
	1	0		0	1	
	1	1		0	1	

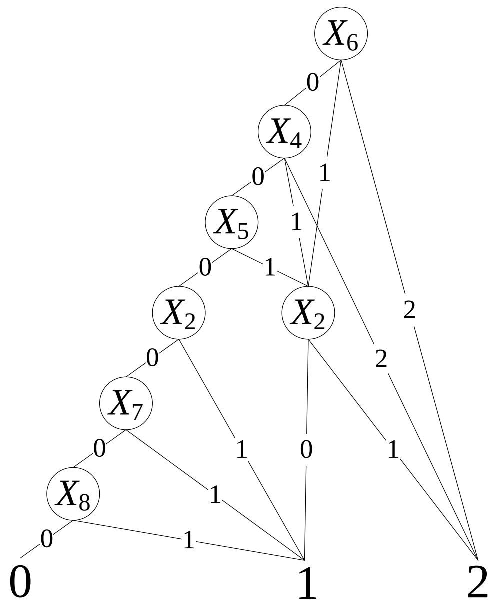

Fig. 5. MDD for the BN

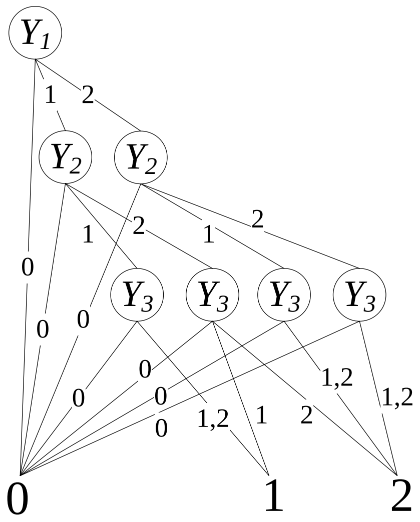

Fig. 6. MDD of external factors

Table 5. Importance values of external factors

External factor	$I_1(k)$	Ranking	$I_2(k)$	Ranking	$I(k)$	Ranking
temperature	1.1387	1	1.5688	1	2.7075	1
vibration	1.1141	3	1.3312	2	2.4453	3
pressure	1.1322	2	1.3220	3	2.4542	2

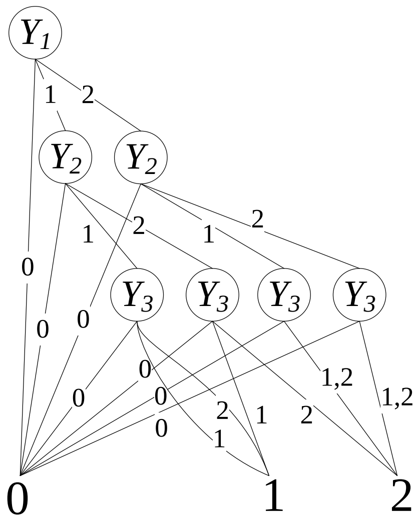

Fig. 7. Divided MDD

Table 6. Importance values of components

Component	$I_1^C(i)$	Ranking	$I_2^C(i)$	Ranking
X_2	1.7154	2		
X_4	1.5097	5	1.7855	1
X_5	1.6988	3		
X_6	1.5097	5	1.7855	1
X_7	1.6988	3		
X_8	1.7676	1		