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The Integrated Aircraft Routing and Crew Pairing Problem: ILP based formulations

Nurul Farihan Mohamed[a*], Zaitul Marlizawati Zainuddin[a,b], Said Salhi[c], Nurul Akmal Mohamed[d]

[a]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[b]UTM Centre for Industrial and Applied Mathematics (UTM-CIAM), Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[c]Centre for Logistics and Heuristic Optimisation, Kent Business School, University of Kent, Canterbury CT2 7PE, UK
[d]Mathematics Department, Faculty of Science & Mathematics, 35900 Universiti Pendidikan Sultan Idris, Proton City, Tanjung Malim, Perak, Malaysia

Graphical abstract
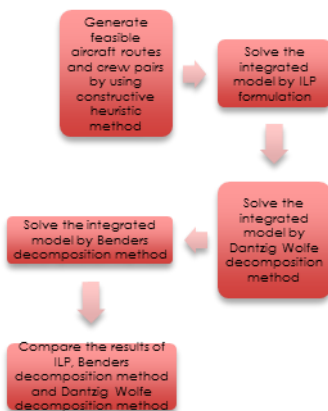
Abstract

Minimization of cost is very important in airline as great profit is an important objective for any airline system. One way to minimize the costs in airline is by developing an integrated planning process. Airline planning consists of many difficult operational decision problems including aircraft routing and crew pairing problems. These two sub-problems, though interrelated in practice, are usually solved sequentially leading to suboptimal solutions. We propose an integrated aircraft routing and crew pairing problem model, one approach to generate the feasible aircraft routes and crew pairs, followed by three approaches to solve the integrated model. The integrated aircraft routing and crew scheduling problem is to determine a minimum cost aircraft routes and crew schedules while each flight leg is covered by one aircraft and one crew. The first approach is an integer programming solution method, the second formulation is developed in a way to lend itself to be used efficiently by Dantzig Wolfe decomposition whereas the third one is formulated as a Benders decomposition method. Encouraging results are obtained when tested on four types of aircraft based on local flights in Malaysia for one week flight cycle.

Keywords: Aircraft routing problem, crew pairing problem, integer linear programming, constructive heuristic method

1.0 INTRODUCTION

The aircraft routing problem is to seek the minimal cost that yields the sequence of routing each type of aircraft such that each flight leg is covered exactly once. In addition, the routing adopted must satisfy the appropriate aircraft maintenance restrictions. The crew pairing problem is then solved based on the given aircraft routes which are to allocate a crew for each flight leg while minimizing the cost and satisfying some rules.

There are several approaches proposed for the individual aircraft routing and crew pairing problem. For instance, [1] provide a general review whereas [2], and [3] present some recent work in this area. Other studies that include uncertainty are covered by [4], [5], and [6]. [7] were the first to address the integrated aircraft routing and crew pairing problem. The authors introduced some

linking constraints to their model and solve the problem using Benders decomposition. Time window and plane count constraints were also introduced to integrate aircraft routing and crew scheduling problem by [8]. The paper by [9] presented the integrated model of crew scheduling and maintenance routing decisions. They introduce the extended crew pairing model and the aircraft solution that does not have any short connection. [10] solved the integrated model of aircraft routing and crew pairing problem using Benders decomposition. A Pareto optimal cut has been introduced to detect the speed of convergence of such a decomposition method. An integrated model of fleet assignment, aircraft maintenance routing and the crew scheduling problems has been proposed by [11] using Benders decomposition enhanced by column generation. Recently, [12] proposed an iterative approach for solving the integrated aircraft routing and crew scheduling problem.

[13] integrates aircraft routing, crew pairing and flight retiming problem that incorporate time windows for the departure time of the legs and solve it by using Benders decomposition. The latest work is [14] which they integrate aircraft routing, crew pairing and re-timing and introduced the heuristic method in the two algorithms that are able to re-timing any aircraft and crew schedule so the costs of delay flights are decreased.

The contributions of our study include (i) a formulation based on the constructive method in generating aircraft routes and crew pairs, (ii) a new ILP formulation for the integrated problem, (iii) a formulation based on Dantzig–Wolfe decomposition, (iv) a formulation based on Benders decomposition, and (v) an experimental testing to demonstrate the efficiency of the proposed approaches. In our integer linear programming (ILP) formulation, few constraints are deducted from [10]. Also, our first problem decomposition differs from previous work as it is based on the Dantzig Wolfe decomposition method while [10], [11] and [13] used Benders decomposition method instead, whereas [12] adopted an iterative approach. For comparison purposes, we also provide the solution based on Benders decomposition.

The next section provides a description of the problem, the constructive method in generating the aircraft routes and crew pairs and the three problem formulations for the integrated model which are the ILP formulation, the Dantzig Wolfe decomposition and the Benders decomposition. This is followed by a section on the experimental results and the summary of our findings.

2.0 PROBLEM FORMULATIONS

In this section, a brief on both the aircraft routing and the crew pairing problems are given followed by a mathematical model that is decomposable into three problems for Dantzig Wolfe decomposition and same as for Benders decomposition method. In this study, we solve the problems for seven days operating flight legs and only the domestic flights are being regarded. The maintenance check for all aircraft is performed every

night after the last operation for that day is completed since the domestic flights end before midnight.

2.1 Aircraft Routing Problem

The aircraft routing problem is solved for each aircraft type to determine the sequence of flight legs to be flown by each type of aircraft at a minimum cost to ensure that each leg is covered exactly once.

Let $N^A = (D^A, R^A)$ be a network of aircraft routes where D^A is a set of nodes while R^A is a set of arcs. For each aircraft type, we describe a set of flight legs as F. In the network, each node $i \in D^A$ represents the departure time or the arrival time of a flight leg $f_i \in F$. Let m be the starting stations for routes with $m \in M$ being the maintenance stations for the aircrafts. Let p_m^A and q_m^A be the source and sink nodes, as the start and the end of a route respectively. The source nodes p_m^A represent the starting nodes of the legs at a certain maintenance station, m while the sink nodes q_m^A represent the last nodes of the legs that end at a certain maintenance station, m, at any time of the day.

2.2 Crew Pairing Problem

A crew pairing is a sequence of duty and rest periods for crews. The objective of the crew pairing problem is to find a minimum cost for the set of pairings in order to assign a qualified crew. As stated by [10], a pairing is a sequence of flight legs that begin and end at a crew base which can be the city where the crew is stationed. The limits in the number of duty periods for crews do not exceed five in any pairing.

Let $N^C = (D^C, R^C)$ be a network of crew pairing where D^C is a set of nodes while R^C is a set of arcs. In the network, each node $i \in D^C$ stands for the departure time or the arrival time of a flight leg $f_i \in F$. Let B be the set of crew bases. Let p_b^C and q_b^C be the source and sink nodes of a pairing for each crew base $b \in B$. The source nodes p_b^C represent the starting node of a pairing that starts at the crew base b while the sink nodes q_b^C represent the end node of a pairing that ends at the crew base b.

2.3 Possible Aircraft Routes and Crew Pairs by The Constructive Heuristic Method

The aircraft routes and crew pairs are needed in solving integrated model of aircraft routing and crew pairing problems. In order to produce the aircraft routes and crew pairs, the constructive heuristic method is proposed as shown in Figure 1, Figure 2 and Table 1 represented the notation.

Phase 1
Step 1: Define the number of aircraft route g , the number of flight legs k , the flight leg number defined in the flight schedule $f = 1,2,3,...,n$, the departure node of flight leg f, $p = 1,2,3,...,t$, the arrival node of flight leg f, $q = t+1, t+2, t+3,...,t+u$, the maintenance station m, the departure time of flight leg f as v, the arrival time of flight leg f as w, the departure station of flight leg f as r, the arrival station of flight leg f as s.
Step 2: Choose a maintenance station, m, $g = 1, k = 0$.

Phase 2
Step 1: Choose a flight leg f that has a departure station at the maintenance station m.
Step 2: Add it to the current route, $k = k+1$. Delete f from the flight schedule.
Step 3: Choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. The arrival station of f equals to m, add the pairing into the current route. Retrieve all flight legs f except for flight leg f in Step 1, Phase 2 into the list. $g = g+1$.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 2 until there is no more route obtained.

Phase 3
Step 1: Use f in the Step 1 in Phase 2, go to Step 2 in Phase 2.
Step 2: Choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. Add it to the current route, $k = k+1$. Delete f from the flight schedule.
Step 3: Choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. If the arrival station of f equals to m, stop. Add the flight leg into the current route. Retrieve all the flight legs f into the list. $g = g+1$.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 3 until there is no more route obtained.

Phase 4
Step 1: Choose other flight leg f that has a departure station at the home base m but exclude f that had been used. Go to Phase 2 and 3. Stop when all f that has a departure station at the home base m is used.

Figure 1 The steps in generating aircraft routes

time, $dpst$.
Step 2: $h = 1, k = 0, dp = 0, st = 0$. Choose an initial crew base b.

Phase 2
Step 1: Choose a flight leg f that has a departure station at the crew base b.
Step 2: Add it to the current pairing, $k = k+1$ and $dp = st + ft$. Record the duty period of f, amount of sit time of f, number of flight leg of f and $dpst$. Delete f from the flight schedule.
Step 3: If $dpst \leq 480$, choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. The arrival station of f equals to b, add pairing into the current pairing. Retrieve all the flight legs f except for flight leg f in Step 1 of Phase 2 into the list. $h = h+1$. Record the duty period of f, amount of sit time of f, $dpst$ in minutes and number of flight leg f.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 2 until there is no more pair obtained.

Phase 3
Step 1: Use f in the Step 1 in Phase 2, go to Step 2 in Phase 2.
Step 2: If $dpst \leq 480$, choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. Add it to the current pairing, $k = k+1$. Delete f from the flight schedule. Record the duty period of f, amount of sit time of f, $dpst$ in minutes and number of flight leg of f.
Step 3: If $dpst \leq 480$, choose a flight leg f that has these criteria:
　i. The r of k is the same with s of $k-1$.
　ii. The v of k minus the w of $k-1$ is bigger or equal to 20 minutes.
　iii. If the arrival station of k equals to b, stop. Add the pairing into the current pairing. Retrieve all of the flight legs f except for flight leg f in Step 1 of Phase 2 into the list. $h = h+1$.
Step 4: Repeat the Step 1, Step 2 and Step 3 of Phase 3 until there is no more pair obtained.

Phase 4
Step 1: Choose other flight leg f that has a departure station at the home base b but exclude f that had been used. Go to Phase 2 and 3. Stop when all the f that has a departure station at the crew base b is used and the $dpst > 480$.

Figure 2 The steps in generating crew pairs

2.4 Model Formulation

The minimum time of a sit time is 20 minutes and the connection between two flight legs $f_i, f_j \in F$ is considered to be a restricted connection if the sit time for the crews is between 60 and 90 minutes. A short connection happens when the sit time for the crews is

Phase 1
Step 1: Define the number of crew pair h, the number of flight legs k, the flight leg number defined in the flight schedule $f = 1,2,3,...,n$, the departure node of flight leg f, $p = 1,2,3,...,t$, the arrival node of flight leg f, $q = t+1, t+2, t+3,...,t+u$, the initial crew base b, the departure time of flight leg f as v, the arrival time of flight leg f as w, the departure station of flight leg f as r, the arrival station of flight leg f as s, length of duty period, dp, the sit time, st, the initial crew base x and the total amount time of duty period and sit

between 20 and 59 minutes. For simplicity, we provide a summary of the notation in Table 1.

Table 1 The notation

Notation	Description
M	Set of maintenance stations
B	Set of crew bases
R	Set of two flight legs that has a restricted connection
S	Set of two flight legs that has a short connection
p_m^A	The source nodes for aircraft paths
q_m^A	The sink nodes for aircraft paths
p_b^C	The source nodes for crew paths
q_b^C	The sink nodes for crew paths
α^m	Set of possible paths from the source node p_m^A to a sink node q_m^A in N_m^A
α^b	Set of possible paths from the source node p_b^C to the sink node q_b^C in N_b^A
w_μ^f	Equal to 1 if leg f belongs to path μ, and 0 otherwise
η_μ	Binary constant that represents the flow on the crew path μ
ξ_μ	Binary constants that represents the flow on the aircraft path μ
R_{ij}	Binary constant will be 1 if connection $(f_i, f_j) \in R$ is operated by the same crew but not same aircraft, 0 otherwise
c_μ	The cost of using the path μ
n_μ^{ij}	Equal to 1 if leg i and j are operated sequentially in path μ, and 0 otherwise
l_μ	The number of required aircrafts in the path μ
ω^A	The number of available aircrafts
ω^B	The number of duty periods allowed in a crew pairing
ω^C	The number of short connections allowed in one path
v_μ	The number of duties in path μ
s_μ	The number of short connections in path μ
z_{ij}	Penalty cost associated with $(f_i, f_j) \in R$

2.4.1 The Integrated Model

The integrated model of aircraft routing and crew pairing problem that we denote by (M1) is as follows:

$$\text{Minimize} \sum_{b \in B} \sum_{\mu \in \alpha^b} c_\mu \eta_\mu + \sum_{m \in M} \sum_{\mu \in \alpha^m} c_\mu \xi_\mu + \sum_{(f_i, f_j) \in R} z_{ij} R_{ij} \quad (1)$$

subject to
$$\sum_{m \in M} \sum_{\mu \in \alpha^m} w_\mu^f \xi_\mu = 1 \quad (f \in F) \quad (2)$$

$$\sum_{b \in B} \sum_{\mu \in \alpha^b} w_\mu^f \eta_\mu = 1 \quad (f \in F) \quad (3)$$

$$\sum_{m \in M} \sum_{\mu \in \alpha^m} l_\mu \xi_\mu \le \omega^A \quad (4)$$

$$\sum_{m \in M} \sum_{\mu \in \alpha^m} s_\mu \xi_\mu \le \omega^C \quad (5)$$

$$\sum_{b \in B} \sum_{\mu \in \alpha^b} v_\mu \eta_\mu \le \omega^B \quad (6)$$

$$\sum_{b \in B} \sum_{\mu \in \alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m \in M} \sum_{\mu \in \alpha^m} n_\mu^{ij} \xi_\mu \le 0 \quad ((f_i, f_j) \in S) \quad (7)$$

$$\sum_{b \in B} \sum_{\mu \in \alpha^b} n_\mu^{ij} \eta_\mu - \sum_{m \in M} \sum_{\mu \in \alpha^m} n_\mu^{ij} \xi_\mu - R_{ij} \le 0 \quad ((f_i, f_j) \in R) \quad (8)$$

$$R_{ij} \in \{0,1\} \quad ((f_i, f_j) \in R) \quad (9)$$

$$\xi_\mu \in \{0,1\} \quad (m \in M; \mu \in \alpha^m) \quad (10)$$

$$\eta_\mu \in \{0,1\} \quad (b \in B; \mu \in \alpha^b). \quad (11)$$

The objective function (1) is to minimize the cost of the aircraft routing and crew pairing problems and penalty costs. Equation (2) and (3) ensure that each flight leg use one aircraft and one crew pair only. Equation (4) imposed that all flight legs operated at one time do not exceed the available aircraft. Equation (5) guarantees that the short connection in the path μ is only limited to ω^C. Equation (6) ensures that each crew pairing does not exceed the number of duty periods allowed. Equation (7) ensures that a crew does not change the aircraft when the connection is too short. Equation (8) is to impose penalty costs if the second flight leg uses the same crew but not the same aircraft. Equations (9), (10) and (11) are the binary decision variables.

2.4.2 The Dantzig Wolfe Decomposition Method

Dantzig Wolfe decomposition method which we refer to as (M2) can be used to reformulate the integrated model (1)-(11) using three problems as follows:

(a) The master problem

Minimize $\sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu V_\mu^P \lambda + \sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu V_\mu^A \pi + \sum_{(f_i,f_j)\in R} z_{ij}R_{ij}$ (12)

subject to $1^T\lambda = 1 \qquad \rightarrow \delta^P$ (13)

$1^T\pi = 1 \qquad \rightarrow \delta^A$ (14)

$\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}V_\mu^P\lambda - \sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij}V_\mu^A\pi - R_{ij} \le 0 \ ((f_i,f_j)\in R).$

$\rightarrow \delta$ (15)

(b) The crew pairing problem (sub-problem 1)

Minimize $\sum_{b\in B}\sum_{\mu\in\alpha^b}(c_\mu - \delta^P n_\mu^{ij})\eta_\mu$ (16)

subject to $\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f\eta_\mu = 1 \quad (f\in F)$ (17)

$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu\eta_\mu \le \omega^B$ (18)

$\eta_\mu\in\{0,1\} \qquad (b\in B; \mu\in\alpha^b).$ (19)

(c) The aircraft routing problem (sub-problem 2)

Minimize $\sum_{m\in M}\sum_{\mu\in\alpha^m}(c_\mu + \delta^A n_\mu^{ij})\xi_\mu$ (20)

subject to $\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f\xi_\mu = 1 \ (f\in F)$ (21)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu\xi_\mu \le \omega^A$ (22)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu\xi_\mu \le \omega^C$ (23)

$\xi_\mu\in\{0,1\} \quad (m\in M; \mu\in\alpha^m),$ (24)

The steps of the approach are as in Figure 3.

Step 1- Solve the master problem (eqs 12-15).
Step 2- Solve the 2 sub-problems .
2a: Solve the crew pairing problem (eqs 16-19)The two sub-problems are solved.
Step 3- If the reduced costs are negative in the generated columns from steps 2a and 2b then augment the set V_μ^P and V_μ^A of the master problem with the generated columns and solve the master problem. Otherwise (i.e., there is no column with negative reduced cost) stop.

Figure 3 The Dantzig-Wolfe decomposition approach

2.4.3 The Benders Decomposition Method

The model of (1)-(11) can be reformulated as Benders decomposition method and referred to as (M3) that consists of three problems as follows:

(a) The primal sub-problem

Minimize $\sum_{m\in M}\sum_{\mu\in\alpha^m} c_\mu\xi_\mu + \sum_{(f_i,f_j)\in R} b_{ij}R_{ij}$ (25)

subject to $\sum_{m\in M}\sum_{\mu\in\alpha^m} w_\mu^f\xi_\mu = 1 \ (f\in F)$ (26)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} l_\mu\xi_\mu \le \omega^A$ (27)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} s_\mu\xi_\mu \le \omega^C$ (28)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij}\xi_\mu \ge \sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\bar\eta_\mu \quad ((f_i,f_j)\in S)$ (29)

$\sum_{m\in M}\sum_{\mu\in\alpha^m} n_\mu^{ij}\xi_\mu + R_{ij} \ge \sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\bar\eta_\mu \quad ((f_i,f_j)\in R)$ (30)

$R_{ij} \ge 0 \qquad ((f_i,f_j)\in R)$ (31)

$\xi_\mu \ge 0 \qquad (m\in M; \mu\in\alpha^m)$ (32)

(b) The dual sub-problem
The dual subproblem is as follows:
Maximize

$\sum_{f\in F}\beta_f + \omega^A\gamma + \omega^C\delta + \sum_{(f_i,f_j)\in S}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\eta_\mu\vartheta_{ij} + \sum_{(f_i,f_j)\in R}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\eta_\mu\chi_{ij}$

(33)

Subject

to $\sum_{f\in F} w_\mu^f\beta_f + l_\mu\gamma + s_\mu\delta + \sum_{(f_i,f_j)\in S} n_\mu^{ij}\vartheta_{ij} + \sum_{(f_i,f_j)\in R} n_\mu^{ij}\chi_{ij} \le c_\mu$

$(m\in M; \mu\in\alpha^m)$ (34)

$\chi_{ij} \le z_{ij} \qquad ((f_i,f_j)\in R)$ (35)

$\gamma,\delta \le 0$ (36)

$\vartheta_{ij} \ge 0 \qquad ((f_i,f_j)\in S)$ (37)

$\chi_{ij} \ge 0 \qquad ((f_i,f_j)\in R)$ (38)

(c) The master problem

Minimize $\sum_{b\in B}\sum_{\mu\in\alpha^b} c_\mu\eta_\mu + y_0$ (39)

subject to

$y_0 + \sum_{(f_i,f_j)\in S}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\eta_\mu\vartheta_{ij} + \sum_{(f_i,f_j)\in R}\sum_{b\in B}\sum_{\mu\in\alpha^b} n_\mu^{ij}\eta_\mu\chi_{ij} \ge \sum_{f\in F}\beta_f + \omega^A\gamma + \omega^C\delta$

$((\beta,\gamma,\delta,\vartheta,\chi)\in P_\Delta)$ (40)

$\sum_{b\in B}\sum_{\mu\in\alpha^b} w_\mu^f\eta_\mu = 1 \quad (f\in F)$ (41)

$\sum_{b\in B}\sum_{\mu\in\alpha^b} v_\mu\eta_\mu \le \omega^B$ (42)

$\eta_\mu\in\{0,1\} \ (b\in B; \mu\in\alpha^b)$ (43)

The solution approach is summarized in Figure 4.

Phase I

1. Remove integrality constraints on all variables (relax variables).

2. Set $\rho = 1$ and $P_\Delta^1 = \varphi$.

3. Solve relaxed master problem by branch and bound.

(a) If the solution obtained is infeasible, then the problem is infeasible and stop.

(b) Otherwise, let $(\bar{\eta}^\rho, y_0^\rho)$ be an optimal solution.

Phase II

1. Reintroduce integrality constraints on the master problem and return to Step 3 in Phase I.
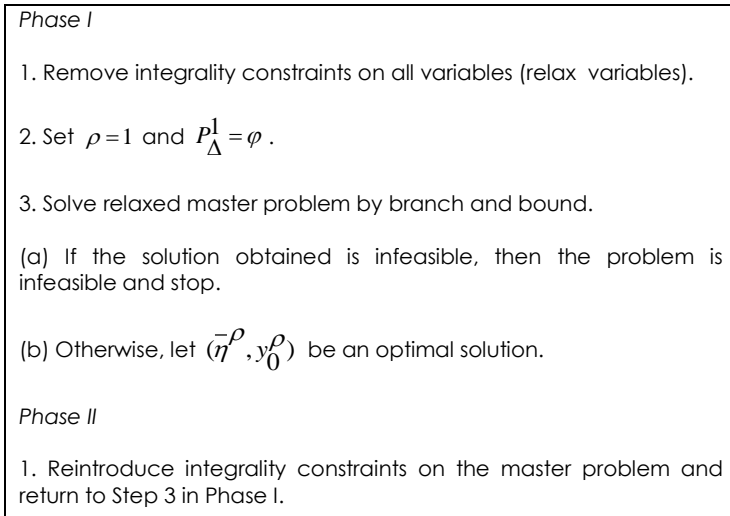
Figure 4 The Benders decomposition approach

3.0 EXPERIMENTAL RESULTS

All approaches namely the ILP, the Dantzig Wolfe decomposition and the Benders decomposition method were solved on an Intel Core Duo processor running at 2.10 GHz using modelling languages, Microsoft Visual Studio C++ interface with ILOG CPLEX Callable Library 2.4.

3.1 Data Sets

Table 2 shows the data sets involve one week operating by an airline in Malaysia for local flights only for four types of aircraft illustrates.

Table 2 Number of aircraft routes and crew pairs

Aircraft	Number of aircraft	Number of crew
EQV	77	70
738	112	112
AT7	882	672
734	1099	854

3.2 Solution Approaches

Table 3 presents the solution quality for models (M1), (M2) and (M3) that include the obtained cost and the respective CPU time for the four instances. The cost of the solution is given in Ringgit Malaysia (RM).

Table 3 Summary results for the three proposed models (M1, M2 & M3)

Aircraft type (#legs,#nodes)	Model 1 (M1) Cost (RM)	Time (secs)	Model 2 (M2) Cost (RM)	Time (secs)	Model 3 (M3) Cost (RM)	Time (secs)
EQV (126,252)	147746	0.00	147745	0.01	147764	0.00
738 (70,140)	149719	1.12	149719	1.22	149725	1.18
AT7 (364,728)	328030	720.6	328030	1122.5	328274	934.3
734 (546,1092)	817604	1651.8	817604	2357.6	817941	2011.4

According to the results from Table 2, all legs are operated and all pairings are used. The total computing time of Dantzig Wolfe decomposition method (M2) is found to be larger than the time required by (M1) and (M3) while the cost of Benders decomposition approach (M3) is higher than the cost obtained by (M1) and (M2).

From the Table 2 above, we can say that the computing time grows drastically with the increase number of legs, number of nodes, number of aircraft routes and number of crew pairs. The computational times of solution for the EQV and 738 aircraft types that involved small numbers of those three attributes require a negligible amount of effort compared to the larger problems AT7 and 734 aircraft types that use larger values of these attributes.

4.0 CONCLUSIONS

The solutions obtained in this paper are encouraging though the integrated formulation appears to be relatively faster on those tested instances. One possible research avenue is to test relatively much larger instances which are likely to exist for bigger worldwide airline companies. This could be academically challenging and practically useful for such larger instances.

Acknowledgment

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