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Interval Sliding Mode Observer Based Incipient Fault Detection with Application to a High-Speed Railway Traction Device

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***Abstract*—In this paper, a novel interval sliding mode observer is designed to detect incipient faults for a class of non-Lipschitz nonlinear systems with mismatched uncertainties. The interval estimation concept is introduced to design interval estimator for the nonlinear subsystem with uncertainties bounded by known intervals. Then novel injection functions are designed to ensure that the sliding motion takes place and maintains thereafter. At last, new residual generators and adaptive threshold generators are designed, and the corresponding fault detectability is studied. Case study on a traction device in CRH (China Railway High-Speed) is presented to demonstrate the effectiveness of proposed incipient fault detection scheme.**

I. Introduction

Model-based fault detection relies on the use of dynamic models, residual generation and evaluation [1], [2]. One of the central schemes in this area is the so-called observer-based fault diagnosis technique, where fault diagnosis observers provide the estimated value of dynamic models. In residual generation and evaluation scheme, the difference between the estimated value of models and the real value measured by sensors, known as residual, will be compared with a threshold value (zero in the ideal case). When the residual is bigger than the threshold, it is determined that there is a fault in the system. Otherwise, it is considered that the system is working properly. However, when building a model of a dynamic process to monitor its behavior, there is always a mismatch between the modeled and real behavior. This is because some effects are neglected in the model, some nonlinearities are linearized in order to simplify the model, some parameters have tolerances when they are compared between several units of the same component, some errors in parameters or in the structure of the model are introduced in the model estimation process, etc. These modeling errors introduce uncertainties in the model and interfere with the fault detection.

The problem of observer-based fault diagnosis, especially incipient fault diagnosis for nonlinear systems, have been extensively studied in the literature, see e.g. [2], [3], [4] and [5]. Robust fault diagnosis observer design against modeling uncertainties and external disturbances is the most important IFD (incipient fault detection) step in observer-based fault diagnosis. However, it is impossible to estimate nonlinear systems with observer mismatched uncertainties exactly only by input-output signals due to that they do not satisfy the necessary relative degree one condition. This condition is actually a structure requirement on the uncertainties , which is used in most papers of this field (see e.g. [2] and [6]). Nevertheless, the interval estimation technique appears to estimate the dynamic models with uncertainties represented by interval models, and has been used for uncertain biological systems in [7], also for fault diagnosis field (see e.g. [8] and [9]). Fortunately, interval observers have no structural requirements on uncertainties, which provide an effective way to improve the robustness against modeling uncertainties and disturbances. Unfortunately, most exist results of interval observers are for linear systems. In the nonlinear case, the basic idea is to replace the nonlinear complexity of the original nonlinear system by an enlarged parametric variation in the LPV representation, which simplify the observer design. There are several approaches to design observers for LPV systems, see for instance [8] and [10]. Of course, there is paper [11] to study the interval observer design for Lipschitz nonlinear systems, which motivates us to deal with nonlinear complexity by relax Lipschitz condition.

On the other hand, during the past decades, sliding mode technique has used for observer based FDI (fault detection and isolation) widely. The sliding mode observer based FDI has been extensively studied in [6], [12], [13] and [14]. In [12], a sliding mode observer is proposed to detect faults by considering the disruption of sliding motion, which motivates much research in this area. In [6] and [13], the "equivalent output injection" concept is used to explicitly reconstruct fault signals to detect and isolate sensor faults and actuator faults. Using sliding mode observer, another idea is developed for actuator FDI by generating residuals instead of reconstructing fault signals in [15]. This methodology is extended to sensor FDI scheme in [16]. Therefore, interval estimation technique in combination with sliding mode technique is a pertinent solution to improve robustness of IFD against modeling uncertainties and disturbances.

In this paper, an interval sliding mode observer for a more general class of nonlinear systems without Lipschitz condition is designed as incipient fault detection estimator (IFDE). More specifically, the known nonlinearity under consideration is modeled as a general nonlinear function of the system inputs and state variables. It is a challenging problem to construct observer for this nonlinear system by only input-output signals. A new approach, the Min-Max approach (see [17]), is used in this paper to design interval sliding mode observers with the interval width being guaranteed. Then residual generators and the corresponding adaptive threshold generators are proposed based on designed IFDE, and the incipient fault detectability is studied. The main contribution of this paper is that a novel interval sliding mode observer as IFDE is designed for a class of non-Lipschitz nonlinear systems with mismatched uncertainties. Based on novel designed IFDE, a sequence of proper adaptive threshold generators are proposed to evaluate the proposed residuals. The incipient fault detectability is studied as well.

II. Preliminaries

From [18], the nonlinear system with incipient sensor faults can be represented by an augmented system, which can be transformed to

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + g_1(x_1, x_2, u) + \eta_1(x, u, \omega, t), \quad (1)$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + g_2(x_1, x_2, u) + \eta_2(x, u, \omega, t) + D_2\xi(x, u, t), \quad (2)$$

$$y = C_2x_2, \quad (3)$$

where $x_1 \in \mathcal{R}^{n_1}$ and $x_2 \in \mathcal{R}^{p}$ with $n_1 + p = n$ are state vectors, $u \in \mathcal{R}^m$ is control, $\omega \in \mathcal{R}$ represents perturbation parameter and assumed to belong to a known compact set Θ i.e. $\omega \in \Theta$. $g_1(\cdot)$ and $g_2(\cdot)$ are known nonlinear smooth vector fields, $\eta_1(\cdot)$ and $\eta_2(\cdot)$ represent lumped uncertainties. It should be pointed out that $\eta_1(\cdot)$ is observer mismatched uncertainties. The function ξ is a continuous and small amplitude vector to drive incipient faults. The matrices A_{11}, A_{12}, A_{21}, A_{22}, $C_2 \in \mathcal{R}^{p\times p}$ (non-singular), $D_2 \in \mathcal{R}^{p\times q}$ all can be obtained based on [6]. In addition, from [18], if the minimum phase condition is satisfied, the matrix A_{11} is Hurwitz. It is assumed throughout this paper that C_2 is identity matrix, $n_1 = 1$, $p \geq q$, and A_{11} is a negative scalar.

Based on [6], the matrix D_2 has structure that $D_2 = \mathrm{col}(0, D_{22})$ with $D_{22} \in \mathcal{R}^{q\times q}$ being non-singular. Then, Eqs. (2) and (3) can be rewritten as

$$\dot{x}_{21} = A_{21}^1 x_1 + A_{22}^{11}x_{21} + A_{22}^{12}x_{22} + g_2^1(\cdot) + \eta_2^1(\cdot), \quad (4)$$

$$\dot{x}_{22} = A_{21}^2 x_1 + A_{22}^{21}x_{21} + A_{22}^{22}x_{22} + g_2^2(\cdot) + \eta_2^2(\cdot) + D_{22}\xi(\cdot), \quad (5)$$

$$y = C_{21}x_{21} + C_{22}x_{22}, \quad (6)$$

where $x_2 := \mathrm{col}(x_{21}, x_{22}) = C_2^{-1}y$ with $x_{21} = [I_{p-q}, 0]C_2^{-1}y \in \mathcal{R}^{p-q}$ and $x_{22} = [0, I_q]C_2^{-1}y \in \mathcal{R}^q$.

For certain practical system, there is an inherent operation region determined by physics which is not influenced by work condition. By considering this inherent operation region, there is an known interval such that before and after faults occur, $x_1 \in \Omega = [x_{1\min},\ x_{1\max}]$. However, the interval Ω is not tight enough for incipient fault detection. Therefore, it is necessary to redesign a new interval based on Ω such that the new interval is tighter than Ω. Suppose that there exist $\hat{\Omega} = [\underline{x}_{1\min}, \bar{x}_{1\max}]$ with $\underline{x}_1,\ \bar{x}_1 \in \mathcal{R}^{n_1}$ such that $\bar{x}_{1\max} \geq x_{1\max}$, $\bar{x}_{1\max} > 0$, $\underline{x}_{1\min} \leq x_{1\min}$, $\underline{x}_{1\min} < 0$.

Some general notation to be used in interval estimator design is shown as follows. For two vectors $x_1,\ x_2 \in \mathcal{R}^n$ or matrices $A_1,\ A_2 \in \mathcal{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are defined in element wise, respectively. Given a matrix $A \in \mathcal{R}^{m\times n}$ or a vector $x \in \mathcal{R}^n$, defining $A^+ = \max\{0, A\}$, $A^- = A^+ - A$ and $x^+ = \max\{0, x\}$, $x^- = x^+ - x$, respectively, then $A^+,\ A^-,\ x^+,\ x^-$ are nonnegative.

Lemma 1: [10] Let $x,\ \underline{x},\ \bar{x} \in \mathcal{R}^n$ satisfy that $\underline{x} \leq x \leq \bar{x}$. Then, for any matrix A with appropriate dimensions, $A^+\underline{x} - A^-\bar{x} \leq Ax \leq A^+\bar{x} - A^-\underline{x}$.

Assumption 1: There exist functions $\underline{\eta}_1(\underline{x}_1, \bar{x}_1, \cdot)$, $\bar{\eta}_1(\underline{x}_1, \bar{x}_1, \cdot)$ such that $\eta_1(\cdot) \in [\underline{\eta}_1(\cdot),\ \bar{\eta}_1(\cdot)]$. Moreover, there exist functions $\bar{\eta}_2^1(y, u, t)$ such that $\|\eta_2^1(\cdot)\| \leq \bar{\eta}_2^1(\cdot)$, and functions $\bar{\eta}_2^2(y, u, t)$ and $\underline{\eta}_2^2(y, u, t)$ such that $\eta_2^2(\cdot) \leq [\underline{\eta}_2^2(\cdot),\ \bar{\eta}_2^2(\cdot)]$.

Remark 1: According to Assumption 1, the uncertainty $\eta_1(\cdot)$ is enclosed in the interval $[\bar{\eta}_1(\cdot), \underline{\eta}_1(\cdot)]$. In addition, the bound on $\eta_2^1(\cdot)$ is used to guarantee the sliding motion takes place in finite time and maintains thereafter [6] [13], and the interval bound $[\underline{\eta}_2^2(\cdot), \bar{\eta}_2^2(\cdot)]$ is to distinguish the effect between faults and uncertainties [2], [4]. ∇

III. Fault Detection Estimator Design

In this section, an interval sliding mode observer as FDE for general nonlinear systems will be designed to detect incipient faults.

Consider following systems

$$\dot{\bar{x}}_1 = A_{11}\bar{x}_1 + A_{12}C_2^{-1}y + \bar{\varphi}(\cdot) + \bar{\eta}_1(\cdot) + F_1(\cdot)(\bar{x}_1 - \underline{x}_1) - \bar{\Upsilon}(\cdot), \quad (7)$$

$$\dot{\underline{x}}_1 = A_{11}\underline{x}_1 + A_{12}C_2^{-1}y + \underline{\varphi}(\cdot) + \underline{\eta}_1(\cdot) - F_2(\cdot)(\bar{x}_1 - \underline{x}_1) - \underline{\Upsilon}(\cdot) \quad (8)$$

where $\bar{\varphi}(\cdot)$ and $\underline{\varphi}(\cdot)$ are defined in (40) and (41) respectively (See Appendix), $F_1(\cdot)$ and $F_2(\cdot)$ are nonnegative scalar functions.

Based on the fact that $\Omega \subseteq \hat{\Omega}$, functions $\bar{\Upsilon}(\cdot)$ and $\underline{\Upsilon}(\cdot)$ are designed as

$$\bar{\Upsilon}(\cdot) = \begin{cases} A_{12}C_2^{-1}y + \bar{\varphi}(\cdot) + \bar{\eta}_1(\cdot) + F_1(\cdot)(\bar{x}_1 - \underline{x}_1), & \text{if } \bar{x}_1 \geq \bar{x}_{1\max}, \\ 0, & \text{elsewise.} \end{cases} \quad (9)$$

$$\underline{\Upsilon}(\cdot) = \begin{cases} A_{12}C_2^{-1}y + \underline{\varphi}(\cdot) + \underline{\eta}_1(\cdot) - F_2(\cdot)(\bar{x}_1 - \underline{x}_1), & \text{if } \underline{x}_1 \leq \underline{x}_{1\min} \\ 0, & \text{elsewise.} \end{cases} \quad (10)$$

Denote $e_1 = \mathrm{col}(\bar{e}_1, \underline{e}_1)$ where $\bar{e}_1 = \bar{x}_1 - x_1$ and $\underline{e}_1 = x_1 - \underline{x}_1$. By comparing (7) and (8) with (1), the error dynamic is obtained by

$$\dot{e}_1 = \hat{A}e_1 + \phi(\bar{x}_1, \underline{x}_1, x_1, \psi) \quad (11)$$

where: $\hat{A} = \begin{bmatrix} A_{11}+F_1(\cdot) & F_1(\cdot) \\ F_2(\cdot) & A_{11}+F_2(\cdot) \end{bmatrix}$ and $\phi(\cdot) = \begin{bmatrix} \phi_1(\cdot) \\ \phi_2(\cdot) \end{bmatrix} = \begin{bmatrix} \bar{\varphi}(\cdot) - g_1(\cdot) + \bar{\eta}_1(\cdot) - \eta_1(\cdot) - \bar{\Upsilon}(\cdot) \\ g_1(\cdot) - \underline{\varphi}(\cdot) + \eta_1(\cdot) - \underline{\eta}_1(\cdot) + \underline{\Upsilon}(\cdot) \end{bmatrix}$.

Proposition 1: *Under Assumption 1, if there exist nonnegative scalar functions $F_1(\cdot)$ and $F_2(\cdot)$ such that the matrix $\hat{A}$ in (11) is Metzler, and the pair ((7), (8)) is initialized by $\underline{x}_1(0)$ and $\bar{x}_1(0)$ satisfying that $\underline{x}_{1\min} < \underline{x}_1(0) \leq x_1(0) \leq \bar{x}_1(0) < \bar{x}_{1\max}$, then $\forall t \geq 0$, $\underline{x}_{1\min} \leq \underline{x}_1(t)$, $\bar{x}_1(t) \leq \bar{x}_{1\max}$, moreover, the pair ((7), (8)) is a framer (defined in [19]) of system (1), (i.e., $\forall t \geq 0$, $\underline{x}_{1\min} \leq \underline{x}_1(t) \leq x_1(t) \leq \bar{x}_1(t) \leq \bar{x}_{1\max}$).*

Proof: From the limitation to initial conditions, it is straightforward to see that $0 \leq e_1(0)$, $\underline{x}_{1\min} < \underline{x}_1(0)$ and $\bar{x}_1(0) < \bar{x}_{1\max}$. Assume that there is time constant t_0 at which $\bar{x}_1$ increases to $\bar{x}_{1\max}$. Under the function $\bar{\Upsilon}(\cdot)$ given in (9), it has that $\dot{\bar{x}}_1(t_0) = A_{11}\bar{x}_{1\max}$. From the fact that A_{11} is negative constant and $\bar{x}_{1\max} > 0$, $\dot{\bar{x}}_1(t_0) < 0$. As a consequence, $\bar{x}_1$ will be smaller than $\bar{x}_{1\max}$ and $\bar{x}_1(t) \leq \bar{x}_{1\max}$ for all time $t \geq 0$. Also, using the same analysis, under function $\underline{\Upsilon}(\cdot)$ given in (10), $\underline{x}_1(t) \geq \underline{x}_{1\min}$ for all time $t \geq 0$.

Considering the first time constant t_1 when $\bar{e}_1$ of vector e_1 is equal to zero, it has that

$$\dot{\bar{e}}_1(t_1) = (A_{11} + F_1(\cdot))\bar{e}_1(t_1) + F_1\underline{e}_1(t_1) + \phi_1(\cdot)|_{t=t_1}. \quad (12)$$

Note that $\bar{e}_1(t_1) = 0$, $\bar{x}_1(t_1) = x_1(t_1)$, which implies that $\bar{x}_1(t_1) < \bar{x}_{1\max}$, and $\bar{\Upsilon}(\cdot)|_{t=t_1} = 0$ in (11). From (39) and Assumption 1, it can infer that $\phi_1(\cdot)|_{t=t_1} \geq 0$. Since $\underline{e}_1(t_1) > 0$, it can be concluded that $\dot{\bar{e}}_1(t_1) \geq 0$. As a consequence $\bar{e}_1$ will stay nonnegative and finally $\bar{e}_1$ remains nonnegative for any time $t \geq 0$. Using the same analysis, the same result for $\underline{e}_1$ can be got, that is $\bar{e}_1$ remains nonnegative for any time $t \geq 0$. Therefore, $e_1(t) \geq 0$ for any time $t \geq 0$.

Hence, the result follows. □

Let $e_1^*(t) = \mathrm{col}(\bar{e}_1^*(t), \underline{e}_1^*(t))$, and $e_1^*(t)$ satisfies that $0 \leq e_1(t) \leq e_1^*(t), \forall t \geq 0$. From (40)-(43) in Appendix, it follows that

$$0 \leq \bar{\varphi}(\cdot) - g_1(\cdot) \leq (\underline{a}_1 + \bar{a}_1)(\cdot) + \left(\underline{w}_1^- + \bar{w}_1^+\right)(\cdot)\bar{e}_1^* + \bar{w}_1^+(\cdot)\underline{e}_1^*, \quad (13)$$

$$0 \leq g_1(\cdot) - \underline{\varphi}(\cdot) \leq (\underline{a}_1 + \bar{a}_1)(\cdot) + (\bar{w}_1^- + \bar{w}_1^+)(\cdot)\underline{e}_1^* + \bar{w}_1^+(\cdot)\bar{e}_1^*. \quad (14)$$

According to [17], it has that $\bar{a}_1(\cdot)$, $\underline{a}_1(\cdot)$ and $\bar{w}_1(\cdot)$ and $\underline{w}_1(\cdot)$ obtained from Lemma 2 in Appendix are piecewise continuous functions with respect to $\bar{x}_1$, $\underline{x}_1$. Based on Proposition 2, it has that $\underline{x}_{1\min} \leq \underline{x}_1 \leq x_1 \leq \bar{x}_1 \leq \bar{x}_{1\max}$. Therefore, it can be deduced that there exist functions $\Lambda_1(\psi)$, $\Lambda_2(\psi)$, $\Lambda_3(\psi)$, $\Lambda_4(\psi)$, $\Lambda_5(\psi)$ and $\Lambda_6(\psi)$ such that $(\underline{a}_1 + \bar{a}_1)(\bar{x}_1, \psi) \leq \Lambda_1(\cdot)$, $\left(\underline{w}_{11}^- + \bar{w}_1^+\right)(\bar{x}_1, \psi) \leq \Lambda_2(\cdot)$, $\bar{w}_1^+(\bar{x}_1, \psi) \leq \Lambda_3(\cdot)$, $(\underline{a}_1 + \bar{a}_1)(\underline{x}_1, \psi) \leq \Lambda_4(\cdot)$, $(\bar{w}_1^- + \bar{w}_1^+)(\underline{x}_1, \psi) \leq \Lambda_5(\cdot)$ and $\bar{w}_1^+(\underline{x}_1, \psi) \leq \Lambda_6(\cdot)$.

Suppose that $\bar{\eta}(\cdot) - \eta(\cdot) < \bar{\Delta}_1(\cdot)$ and $\eta(\cdot) - \underline{\eta}(\cdot) < \underline{\Delta}_1(\cdot)$. Consider the system

$$\dot{e}_1^* = \hat{A}e_1^* + \bar{\phi}(\cdot) \quad (15)$$

where $\bar{\phi}(\cdot) = \begin{bmatrix} \Lambda_1(\cdot)+\Lambda_2(\cdot)\bar{e}_1^*+\Lambda_3(\cdot)\underline{e}_1^*+\bar{\Delta}_1(\cdot) \\ \Lambda_4(\cdot)+\Lambda_5(\cdot)\underline{e}_1^*+\Lambda_6(\cdot)\bar{e}_1^*+\underline{\Delta}_1(\cdot) \end{bmatrix}$. Denoting $\breve{e}_1 = e_1^* - e_1$, then $\dot{\breve{e}}_1 = \hat{A}\breve{e}_1 + \breve{\phi}(\cdot)$ where $\breve{\phi}(\cdot) = \bar{\phi}(\cdot) - \phi(\cdot)$. Under the initial condition that $0 \leq e_1(0) \leq \bar{e}_1^*(0)$, i.e., $\breve{e}_1(0) \geq 0$, for the first time t_3 when the kth component $\breve{e}_{1k}$ of vector $\breve{e}_1$ is equal to zero, it has that $\dot{\breve{e}}_{1k}(t_3) = \sum_{i=1}^{2n_1} \hat{a}_{ki}\breve{e}_{1i}(t_3) + \breve{\phi}_k(\cdot)|_{t=t_3}$, where $\hat{a}_{ki}$ is the kth row and jth column of $\hat{A}$. From (13) and (14), $\breve{\phi}(\cdot)|_{t=t_3} \geq 0$, then $\dot{\breve{e}}_{1k}(t_3) \geq 0$, which means that $\breve{e}_{1k}$ will stay nonnegative and $\breve{e}_{1k}$ will remain nonnegative for all time $t \geq 0$. Then it can be concluded that $0 \leq e_1(t) \leq e_1^*(t)$, $\forall t \geq 0$.

By putting $\Lambda_2(\cdot)\bar{e}_1^*$ and $\Lambda_5(\cdot)\underline{e}_1^*$ out of $\bar{\phi}(\cdot)$, system (15) can be rewritten as

$$\dot{e}_1^* = \tilde{A}e_1^* + H \quad (16)$$

where $\tilde{A} = \begin{bmatrix} A_{11}+F_1(\cdot)+\Lambda_2(\cdot) & F_1(\cdot)+\Lambda_3(\cdot) \\ F_2(\cdot)+\Lambda_6(\cdot) & A_{11}+F_2(\cdot)+\Lambda_5(\cdot) \end{bmatrix}$, $H = \begin{bmatrix} \Lambda_1(\cdot)+\bar{\Delta}_1(\cdot) \\ \Lambda_4(\cdot)+\underline{\Delta}_1(\cdot) \end{bmatrix}$.

Then, a proposition is presented as follows.

Proposition 2: *Under Assumptions 1, if the nonnegative scalar functions $F_1(\cdot)$, $F_2(\cdot)$, and the initial condition $\bar{e}_1(0)$ and $\underline{e}_1(0)$ are chosen such that the matrix $\tilde{A}$ given in (16) is Hurwitz, and the conditions in Proposition 1 are satisfied, respectively, then the pair ((7), (8)) is an interval observer (defined in [20]) of subsystem (1).*

Proof: Because $\tilde{A}$ is Hurwitz, system (16) is ISS and e_1^* in system (16) will asymptotically converge to a bounded region associated to $\tilde{A}$ and H. Then, it can be got e_1 in (11) will also asymptotically converge to a guaranteed bound of e_1^*. Hence, the result follows. □

Denote $\hat{x}_{21}$ as estimation of x_{21}, and $\hat{x}_1 \in [\bar{x}_1, \underline{x}_1] \subseteq \hat{\Omega}$ as the estimation of x_1 where the dynamics $\bar{x}_1$ and $\underline{x}_1$ are given in (7) and (8) respectively. Consider the following system

$$\begin{aligned} \dot{\hat{x}}_{21} =& A_{21}^1\hat{x}_1 + A_{22}^{11}\hat{x}_{21} + A_{22}^{12}[0, I_q]C_2^{-1}y + g_2^1(\hat{x}_1, \psi) \\ &+ (A_{22}^{11} - \hat{A}_{22}^{11})\left([I_{p-q}, 0]C_2^{-1}y - \hat{x}_{21}\right) + \nu_1 + \nu_2 \end{aligned} \quad (17)$$

where $\hat{A}_{22}^{11}$ is symmetric negative definite. The functions ν_1 and ν_2 are defined by

$$\begin{aligned} \nu_1 =& m_1(\cdot)\mathrm{sgn}([I_{p-q}, 0]C_2^{-1}y - \hat{x}_{21}), \\ \nu_2 =& M_2(\cdot)\mathrm{sgn}([I_{p-q}, 0]C_2^{-1}y - \hat{x}_{21}) \end{aligned} \quad (18)$$

where $m_1(\cdot)$ is a positive scalar function, and $M_2(\cdot)$ is a diagonal matrix function, which are both determined later.

Let $e_{21} = x_{21} - \hat{x}_{21}$ and $\mathbb{e}_1 = x_1 - \hat{x}_1$. For nonlinear function $g_2^1(\cdot)$, based on Lemma 2 with respect to $\hat{x}_1 \in \hat{\Omega}$ in Appendix, there exist functions $w_{2i}^1(\hat{x}_1, \psi)$ and $a_{2i}^1(\hat{x}_1, \psi)$, $i = 1, 2, \cdots, p-q$ such that $\Delta_i \leq 0$ where

$$\Delta_i = \mathrm{sgn}(e_{21i})(g_{2i}^1(\cdot) - g_{2i}^1(\hat{x}_1, \psi) + w_{2i}^1(\cdot)\mathbb{e}_1) - a_{2i}^1(\cdot). \quad (19)$$

Denote $W_2^1(\cdot) = \mathrm{col}(w_{21}^1(\cdot), \cdots, w_{2(p-q)}^1(\cdot))$. Then comparing (4) with (17), the error dynamic is obtained by

$$\begin{aligned} \dot{e}_{21} =& (A_{21}^1 - W_2^1(\cdot))\mathbb{e}_1 + \hat{A}_{22}^{11}e_{21} + g_2^1(x_1, x_2, u) \\ &- g_2^1(\hat{x}_1, \psi) + W_2^1(\cdot)\mathbb{e}_1 + \eta_2^1(x, u, \omega, t) - \nu_1 - \nu_2. \end{aligned} \quad (20)$$

Consider a sliding surface $\mathscr{L} = \{(\mathbb{e}_1, e_{21}) | e_{21} = 0\}$. Then, the following conclusion is ready to presented.

Proposition 3: *Under Assumptions 1, the error system is driven to the sliding surface $\mathscr{L}$ in finite time and remains on it thereafter if the gains $m_1(y, u, t)$ and $M_2(\hat{x}_1, \psi)$ satisfy that*

$$m_1(\cdot) \geq \|A_{21}^1 - W_2^1(\cdot)\| \max\{\underline{e}_1^*, \bar{e}_1^*\} + \bar{\eta}_2^1(y, u, t) + \kappa, \quad (21)$$

$$M_2(\cdot) = A_2^1(\cdot) \quad (22)$$

where $\underline{e}_1^*$ and $\bar{e}_1^*$ are defined in (16), $\bar{\eta}_2^1(y, u, t)$ satisfies Assumption 1, κ is a positive constant and $A_2^1(\cdot) =$ $\mathrm{diag}\{a_{21}^1(\cdot), \cdots, a_{2(p-q)}^1(\cdot)\}$.

Proof: Let $V = e_{21}^T e_{21}$. It follows from (20) that

$$\dot{V} = e_{21}^T \left(\hat{A}_{22}^{11} + \hat{A}_{22}^{11T}\right) e_{21} + e_{21}^T (A_{21}^1 - W_2^1(\cdot)) \mathbb{e}_1 + e_{21}^T (\eta_2^1(\cdot) - \nu_1 + g_2^1(x_1, x_2, u) - g_2^1(\hat{x}_1, \psi) + W_2^1(\cdot)\mathbb{e}_1 - \nu_2). \quad (23)$$

Since $\hat{A}_{22}^{11}$ is symmetric negative definite, $e_{21}^T(\hat{A}_{22}^{11} + (\hat{A}_{22}^{11})^T)e_{21} \leq 0$. From (21), it follows that $\|e_{21}\| \left(\|(A_{21}^1 - W_2^1(\cdot))\| \, |\mathbb{e}_1| + \|\eta_2^1(\cdot)\|\right) \leq -\kappa\|e_{21}\|$. Since $\hat{x}_1$ in (17) satisfies that $\hat{x}_1 \in [\bar{x}_1, \underline{x}_1]$, then $-\bar{e}_1 \leq \mathbb{e}_1 \leq \underline{e}_1$. Thus from the fact that $0 \leq e_1 \leq e_1^*$, it has that $|\mathbb{e}_1| \leq \max\{\bar{e}_1, \underline{e}_1\} \leq \max\{\bar{e}_1^*, \underline{e}_1^*\}$. In addition, it follows from (19) that $e_{21}^T \left(\left(g_2^1(\cdot) - g_2^1(\hat{x}_1, \psi) + W_2^1(\cdot)\mathbb{e}_1\right) - M_2(\cdot)\mathrm{sgn}\,(e_{21})\right) = \sum_{i=1}^{p-q} |e_{21i}| \Delta_i \leq 0$. Therefore, $\dot{V} \leq -\kappa\|e_{21}\| \leq -\kappa V^{1/2}$, which means that the reachable condition is satisfied.

Hence, the conclusion follows. □

IV. Incipient Fault Detection Schemes

A. Residual Generators and threshold generator Design

For subsystem (5), an interval estimator is designed as

$$\dot{\bar{x}}_{22} = A_{21}^{2+}\bar{x}_1 - A_{21}^{2-}\underline{x}_1 + A_{22}^{21}[I_{p-q}, 0]C_2^{-1}y + A_{22}^{22}\bar{x}_{22} + g_2^2(\bar{x}_1, \psi) + \bar{\eta}_2^2(\cdot) + K_{22}\left([0, I_q]C_2^{-1}y - \bar{x}_{22}\right), \quad (24)$$

$$\dot{\underline{x}}_{22} = A_{21}^{2+}\underline{x}_1 - A_{21}^{2-}\bar{x}_1 + A_{22}^{21}[I_{p-q}, 0]C_2^{-1}y + A_{22}^{22}\underline{x}_{22} + g_2^2(\underline{x}_1, \psi) + \underline{\eta}_2^2(\cdot) + K_{22}\left([0, I_q]C_2^{-1}y - \underline{x}_{22}\right) \quad (25)$$

where K_{22} is chosen as $A_{22}^{22} - \hat{A}_{22}^{22}$ with $\hat{A}_{22}^{22}$ being Hurwitz and Metzler.

Denote $\bar{e}_{22} = \bar{x}_{22} - x_{22}$ and $\underline{e}_{22} = x_{22} - \underline{x}_{22}$ as residual generators to detect incipient fault. Before incipient faults occur, by comparing (24) and (25) with (5), the error dynamics are obtained by

$$\dot{\bar{e}}_{22} = A_{21}^{2+}\bar{e}_1 + A_{21}^{2-}\underline{e}_1 + \hat{A}_{22}^{22}\bar{e}_{22} + g_2^2(\bar{x}_2, \psi) - g_2^2(\cdot) + \bar{\eta}_2^2(\cdot) - \eta_2^2(\cdot), \quad (26)$$

$$\dot{\underline{e}}_{22} = A_{21}^{2+}\underline{e}_1 + A_{21}^{2-}\bar{e}_1 + \hat{A}_{22}^{22}\underline{e}_{22} + g_2^2(\cdot) - g_2^2(\underline{x}_2, \psi) + \eta_2^2(\cdot) - \underline{\eta}_2^2(\cdot). \quad (27)$$

Based on Lemma 2, for nonlinear function $g_2^2(\cdot)$, when $\alpha < 0$, there exist column vectors $\underline{W}_2^2(\bar{x}_1, \psi) = \mathrm{col}(\underline{w}_{21}^2(\cdot), \cdots, \underline{w}_{2q}^2(\cdot))$, and $\underline{A}_2^2(\bar{x}_1, \psi) = \mathrm{col}(\underline{a}_{21}^2(\cdot), \cdots, \underline{a}_{2q}^2(\cdot))$ such that $g_2^2(\bar{x}_1, \psi) - g_2^2(\cdot) + \underline{W}_2^2(\cdot)(\bar{x}_1 - x_1) - \underline{A}_2^2(\cdot) \leq 0$. When $\alpha > 0$, there exist column vectors $\bar{W}_2^2(\underline{x}_1, \psi) = \mathrm{col}(\bar{w}_{21}^2(\cdot), \cdots, \bar{w}_{2q}^2(\cdot))$ and $\bar{A}_2^2(\underline{x}_1, \psi) = \mathrm{col}(\bar{a}_{21}^2(\cdot), \cdots, \bar{a}_{2q}^2(\cdot))$ such that $g_2^2(\cdot) - g_2^2(\underline{x}_1, \psi) + \bar{W}_2^2(\cdot)(x_1 - \underline{x}_1) - \bar{A}_2^2(\cdot) \leq 0$.

Suppose that $\bar{\eta}_2^2 - \eta_2^2 \leq \bar{\Delta}_2^2$ and $\eta_2^2 - \underline{\eta}_2^2 \leq \underline{\Delta}_2^2$. Based on positive system theory, under the initial condition that $\bar{e}_{22}(0) = \bar{\delta}(0) = 0$ and $\underline{e}_{22}(0) = \underline{\delta}(0) = 0$, it can be obtained that $\bar{e}_{22}(t) \leq \bar{\delta}(t)$ and $\underline{e}_{22}(t) \leq \underline{\delta}(t)$, where $\bar{\delta}$ and $\underline{\delta}$ are given by

$$\dot{\bar{\delta}} = A_{21}^{2+}\bar{e}_1 + A_{21}^{2-}\underline{e}_1 + \hat{A}_{22}^{22}\bar{\delta} - \underline{W}_2^2(\bar{x}_1, \psi)\bar{e}_1 + \underline{A}_2(\bar{x}_1, \psi) + \bar{\Delta}_2^2, \quad (28)$$

$$\dot{\underline{\delta}} = A_{21}^{2+}\bar{e}_1 + A_{21}^{2-}\underline{e}_1 + \hat{A}_{22}^{22}\underline{\delta} - \bar{W}_2^2(\underline{x}_1, \psi)\underline{e}_1 + \bar{A}_2(\underline{x}_1, \psi) + \underline{\Delta}_2^2 \quad (29)$$

where $\bar{\delta}(0) = \underline{\delta}(0) = 0$.

Before incipient faults occur, $\bar{e}_{22}(t) \leq \bar{\delta}(t)$ and $\underline{e}_{22}(t) \leq \underline{\delta}(t)$. Thus, according to that threshold selection principle, the dynamic function (28) and (29) is selected as threshold generator.

From (28), it follows that $\bar{\delta} = \int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}(A_{21}^{2+}\bar{e}_1 + A_{21}^{2-}\underline{e}_1 - \underline{W}_2^2(\cdot)\bar{e}_1 + \underline{A}_2^2(\cdot) + \bar{\Delta}_2^2)d\tau$. For the Metzler matrix $\hat{A}_{22}^{22}$, it has that $e^{\hat{A}_{22}^{22}t} > 0$, $\forall t \geq 0$. From $\underline{W}_2^2(\cdot) = \underline{W}_2^{2+}(\cdot) - \underline{W}_2^{2-}(\cdot)$ and $\underline{A}_2^2(\cdot) = \underline{A}_2^{2+}(\cdot) - \underline{A}_2^{2-}(\cdot)$, and $0 < \bar{e}_1 \leq \bar{e}_1^*$, $0 < \underline{e}_1 \leq \underline{e}_1^*$ in Proposition 2, it follows that

$$\bar{\delta} \leq \int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}((A_{21}^{2+} + \underline{W}_2^{2-}(\bar{x}_1, \psi))\bar{e}_1^* + A_{21}^{2-}\underline{e}_1^* + \underline{A}_2^{2+}(\bar{x}_1, \psi) + \bar{\Delta}_2^2)d\tau - \int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}\underline{A}_2^{2-}(\bar{x}_1, \psi)d\tau. \quad (30)$$

Using the same analysis as for (29), it can obtain that

$$\underline{\delta} \leq \int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}((A_{21}^{2+} + \bar{W}_2^{2-}(\underline{x}_1, \psi))\underline{e}_1^* + A_{21}^{2-}\underline{e}_1^* + \bar{A}_2^{2+}(\underline{x}_1, \psi) + \underline{\Delta}_2^2)d\tau - \int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}\bar{A}_2^{2-}(\underline{x}_1, \psi)d\tau. \quad (31)$$

In the sequel, the fault detection decision scheme and detectability will be studied.

B. Incipient Fault Detection Decision Scheme

The decision scheme on incipient faults (continuous and small amplitude faults) is derived as follows. The decision on the occurrence of an incipient fault (detection) is made if residuals $\bar{e}_{22j}$ in (26) and $\underline{e}_{22j}$ in (27), $j = 1, \cdots, q$ are continuous all the time, and there exists at least one j such that either $\bar{e}_{22j}$ exceeds adaptive threshold $\bar{\delta}_j$ (jth row of $\bar{\delta}$ in (30)), or $\underline{e}_{22j}(t)$ exceeds adaptive threshold $\underline{\delta}_j$ (jth row of $\underline{\delta}$ in (31)).

The above design and analysis is summarized in the following theorem.

Theorem 1: *Under Assumption 1, for nonlinear systems (1)-(3), the fault detection decision scheme, characterized by FDEs (7), (8) and (17), residual generators (28), (29) and corresponding adaptive thresholds $\bar{\delta}$ in (30) and $\underline{\delta}$ in (31), guarantees that there will be no false alarms before incipient faults occur.*

C. Incipient Fault Detectability Schemes

After incipient faults occur, the new error dynamics represented by (26) adds $-D_{22}\xi(\cdot)$ and (27) adds $D_{22}\xi(\cdot)$. Only the one case that $g_2^2(\cdot)$ is a monotone increasing function vector and the fault vector $D_{22}\xi(\cdot) \geq 0$ is studied. Then the fact that $\hat{A}_{22}^{22}$ is Metzler implies that $\bar{e}_{22}$ decreases and $\underline{e}_{22}$ increases. Therefore, the increase $\underline{e}_{22}$ in (27) and $\underline{\delta}$ in (31) are chosen as residual generator and adaptive threshold respectively to detect this incipient fault.

From Lemma 2 in Appendix, there exist column vectors $\underline{A}_2^2(\underline{x}_1, \psi) = \mathrm{col}(\underline{a}_{21}^2(\cdot), \cdots, \underline{a}_{2q}^2(\cdot))$ and $\underline{W}_2^2(\underline{x}_1, \psi) = \mathrm{col}(\underline{w}_{21}^2(\cdot), \cdots, \underline{w}_{2q}^2(\cdot))$ such that when $\alpha < 0$, $g_2^2(\cdot) - g_2^2(\underline{x}_1, \psi) \geq -\underline{A}_2^2(\underline{x}_1, \psi) - \underline{W}_2^2(\underline{x}_1, \psi)\underline{e}_1$. Since $\underline{W}_2^2(\cdot) = \underline{W}_2^{2+}(\cdot) - \underline{W}_2^{2-}(\cdot)$ with $\underline{W}_2^{2+}(\cdot)$ and $\underline{W}_2^{2-}(\cdot)$ being nonnegative, and $0 \leq e_1 \leq e_1^*$, it can be obtained that

$$g_2^2(\cdot) - g_2^2(\underline{x}_1, \psi) \geq -\underline{A}_2(\underline{x}_1, \psi) - \underline{W}_2^{2+}(\underline{x}_1, \psi)\underline{e}_1^*. \quad (32)$$

Then it follows from (27) that

$$\underline{e}_{22}(t) \geq -\int_0^t e^{\hat{A}_{22}^{22}(t-\tau)}\left(\underline{A}_2(\cdot) + \underline{W}_2^{2+}(\cdot)\underline{e}_1^* + D_{22}\xi(\cdot)\right)d\tau. \quad (33)$$

Assuming that the incipient fault is detected at time $T_d > T_0$ where T_0 is the fault occurrence time, there exists a $\underline{e}_{22j}$, $j = 1, \cdots, q$ such that $\underline{e}_{22j}(T_d) \geq \underline{\delta}_j(T_d)$, which requires that

$$\left(\int_{T_0}^{T_d} e^{\hat{A}_{22}^{22}(t-\tau)} D_{22}\xi(\cdot)d\tau\right)_j \geq \left(\int_{T_0}^{T_d} e^{\hat{A}_{22}^{22}(t-\tau)}(A_{21}^{2+} + A_{21}^{2-})\underline{e}_1^* + (\bar{W}_2^{2-} + \underline{W}_2^{2+})(\cdot)\underline{e}_1^* + \bar{A}_2^{2+}(\cdot) + \underline{A}_2(\cdot) + \underline{\Delta}_2^2 - \bar{A}_2^{2-}(\cdot)d\tau\right)_j \quad (34)$$

where $(\cdot)_j$ represents the jth row.

Therefore, it can be concluded that if the incipient fault signals satisfy (34), then the faults will be detected at time instant T_d. Of course, the incipient fault detectability schemes in other cases (such as $g_2^2(\cdot)$ is monotone decreasing function vector or that the fault vector is non-positive, i.e., $D_{22}\xi(\cdot) \leq 0$) can be studied based on the same methodology to get (34).

V. Case Study: Application To A Traction Device

In this section, an application to an single phase rectifier of traction system in CRH is presented. The single phase PWM boost rectifier is considered. The differential equations of grid side current i_n and dc-link voltage u_{dc} are given by

$$\begin{bmatrix} \frac{di_n}{dt} \\ \frac{du_{dc}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_N}{L_N} & \frac{S}{L_N} \\ -\frac{S}{C_f} & 0 \end{bmatrix} \begin{bmatrix} i_n \\ u_{dc} \end{bmatrix} + \begin{bmatrix} -\frac{1}{L_N} \\ 0 \end{bmatrix} i_L + \begin{bmatrix} 0 \\ -\frac{1}{C_f} \end{bmatrix} u_n \quad (35)$$

where the parameters R_N, L_N, R_L, C_f are same as in [21], the load current i_L and grid voltage u_n are considered as control inputs. Based on [22], the sensor incipient fault is expressed by $\dot{f} = A_f f + f^2 + \xi(f, x, u, t)$. Through augmentation of (35) and f, and linear transformation with coordinate transformation matrix given in [6], the incipient faulty system is obtained as follows:

$$\begin{aligned} \dot{x} &= Ax + g(x, y, u) + Bu + \eta(t, x, y, u, \omega) + D\xi(t, x, y, u), \\ y &= Cx \end{aligned} \quad (36)$$

where matrices A, B, C, D are given by

$$A_{S=0} = \begin{bmatrix} -10.0000 & -0.7071 & -7.0711 \\ 176.7767 & -75.0000 & -125.0000 \\ -14.1421 & 1.0000 & -10.0000 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1.4142 \\ -250 & 0 \\ 0 & -1.0000 \end{bmatrix},$$

$$A_{S=1} = \begin{bmatrix} -10.0000 & 0.7071 & -7.0711 \\ -176.7767 & -75.0000 & 125.0000 \\ -14.1421 & -1.0000 & -10.0000 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

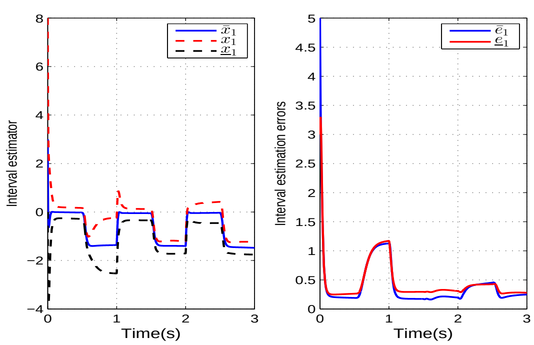

Fig. 1. The interval estimations $\bar{x}_1$ and $\underline{x}_1$, and estimation errors $\bar{e}_1$ and $\underline{e}_1$ of state $x(1)$.

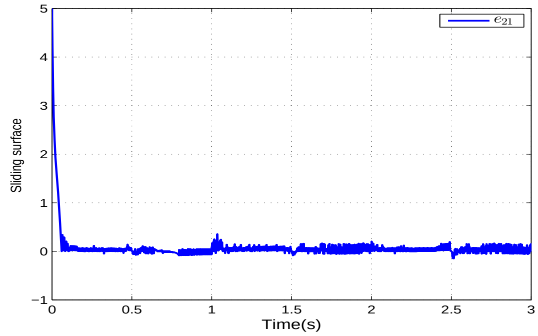

Fig. 2. The dynamic of e_{21}.

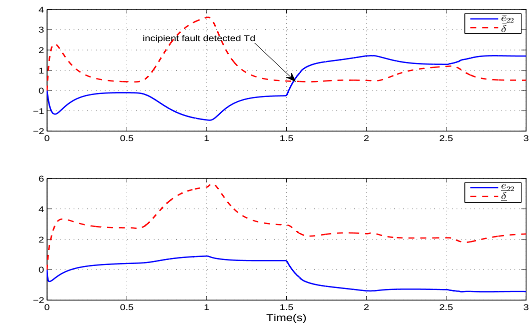

Fig. 3. Residuals $\bar{e}_{22}$ and $\underline{e}_{22}$, adaptive thresholds $\bar{\delta}$ and $\underline{\delta}$.

The nonlinear terms is expressed by $g(x, y, u) = \mathrm{col}\left(x_1^2 x_3^2, x_1^2 x_2^2 + 100sin(x_2), x_1^2 x_3^2\right)$, and $\eta(t, x, y, u, \omega)$ represents uncertainity, given by $\eta = 2\sin(x_1 x_3) + \omega$ with $\omega = 2\sin(t)$. The incipient fault signal $\xi(\cdot)$ is given by

$$\xi(\cdot) = \begin{cases} 0, t < 10s, \\ -20 + 20e^{0.2t-10} + 2\sin(5x_1(t-10)), t \geq 10s. \end{cases} \quad (37)$$

The simulation results are shown in Figs. 1-3.

It can be seen from Fig. 1 that the designed interval estimations $\hat{x}_1$ and $\underline{x}_1$ of x_1 guarantee that $\underline{x}_1 < x_1 < \bar{x}_1$, and the estimation errors $\underline{e}_1$ and $\bar{e}_1$ converge to a region of the origin. Fig. 2 shows that e_{21} is driven to the sliding surface and remains on it thereafter. From Fig.3, it shows that residual $\bar{e}_{22}$ (blue and solid lines) exceeds adaptive threshold $\bar{\delta}$ (red and dash lines) at time instant T_d, which means that the incipient fault can be detected at this time instant.

VI. Conclusions

This paper has proposed new residuals and adaptive thresholds based on the novel designed interval sliding mode observer for a general class of nonlinear systems with mismatched uncertainties. For nonlinear subsystem with mismatched uncertainties, a novel interval estimator is designed. The injection functions are novel designed to ensure that the sliding motion takes place in finite time. Furthermore, the fault detectability is studied. At last, a sensor incipient fault of rectifier in CRH is detected based on the proposed fault detection method to demonstrate the effectiveness of the proposed incipient fault detection scheme.

Appendix

In this paper, the Min-Max approach is used for general nonlinear functions to design sliding mode observer, which is shown in the following lemma.

Lemma 2: For any continuous scalar function $g(x, y, u)$ with $x \in \Sigma = [x_{\min}, x_{\max}]$ and bounded with respect to its arguments, and for any $\hat{x} \in \Sigma$, there exist functions $w(\hat{x}, y, u)$ and $a(\hat{x}, y, u)$ such that

$$J(w(\cdot), x) - a(\cdot) \leq 0, \tag{38}$$

where $J(w, x) = \text{sgn}(\alpha)(\text{g}(x, y, u) - \text{g}(\hat{x}, y, u) + w(\cdot)(x - \hat{x}))$ with α being known quantity.

Proof: See [17]. □

Remark 2: In fact, the functions $w(\cdot)$ and $a(\cdot)$ may be obtained from solving the optimization problem

$$a = \min_{\|y\| \leq Y, \|u\| \leq U} \max_{x \in \Sigma} J(w, x), w = \arg \min_{\|y\| \leq Y, \|u\| \leq U} \max_{x \in \Sigma} J(w, x).$$

The method to solve $w(\cdot)$ and $a(\cdot)$ is available in [17]. ∇

For $g_1(x_1, x_2, u)$ in (1), $x_1 \in \Omega \subseteq \hat{\Omega}$, it follows that there exist functions $a_1(\cdot)$ and $w_1(\cdot)$ are calculated based on Lemma 2 with $x_1 \in \hat{\Omega}$.

Let ψ represents measurable signals y and u. Supposing that $\hat{x}$ as the estimation of x, $a_1(\cdot)$ and $w_1(\cdot)$ are written as when $\alpha > 0$, $a_1(\cdot) = \bar{a}_1(\hat{x}_1, \psi)$, $w_1(\cdot) = \bar{w}_1(\hat{x}_1, \psi)$, when $\alpha < 0$ $a_1(\cdot) = \underline{a}_1(\hat{x}_1, \psi)$, $w_1(\cdot) = \underline{w}_1(\hat{x}_1, \psi)$. Now, denote $\bar{x}_1$ and $\underline{x}_1$ as the estimation of upper bound and low bound of x_1, respectively. Then it follows from Lemma 2 that when $\alpha > 0$, $g_1(\cdot) \leq g_1(\bar{x}_1, \psi) + \bar{w}_1(\bar{x}_1, \psi)\bar{x}_1 - \bar{w}_1(\bar{x}_1, \psi)x_1 + \bar{a}_1(\bar{x}_1, \psi)$, and when $\alpha < 0$, $g_1(\cdot) \geq g_1\left(\underline{x}_1, \psi\right) + \underline{w}_1\left(\underline{x}_1, \psi\right)\underline{x}_1 - \underline{w}_1\left(\underline{x}_1, \psi\right)x_1 - \underline{a}_1\left(\underline{x}_1, \psi\right)$. By applying Lemma 1 to $-\bar{w}_1(\bar{x}_1, \psi)x_1$ and $-\underline{w}_1(\underline{x}_1, \psi)x_1$ under the condition that $\underline{x}_1 \leq x_1 \leq \bar{x}_1$, it follows that

$$\underline{\varphi}\left(\bar{x}_1, \underline{x}_1, \psi\right) \leq g_1(\cdot) \leq \bar{\varphi}\left(\bar{x}_1, \underline{x}_1, \psi\right) \tag{39}$$

where

$$\begin{aligned} \bar{\varphi}\left(\bar{x}_1, \underline{x}_1, \psi\right) &= g_1(\bar{x}_1, \psi) + \bar{w}_1(\bar{x}_1, \psi)\bar{x}_1 \\ &+ \bar{w}_1^-(\bar{x}_1, \psi)\bar{x}_1 - \bar{w}_1^+(\bar{x}_1, \psi)\underline{x}_1 + \bar{a}_1(\bar{x}_1, \psi), \end{aligned} \tag{40}$$

$$\begin{aligned} \underline{\varphi}\left(\bar{x}_1, \underline{x}_1, \psi\right) &= g_1\left(\underline{x}_1, \psi\right) + \underline{w}_1\left(\underline{x}_1, \psi\right)\underline{x}_1 \\ &+ \underline{w}_1^-\left(\underline{x}_1, \psi\right)\underline{x}_1 - \underline{w}_1^+\left(\underline{x}_1, \psi\right)\bar{x}_1 - \underline{a}_1\left(\underline{x}_1, \psi\right). \end{aligned} \tag{41}$$

Moreover, from Lemma 2, when $\alpha > 0$, $g_1(\cdot) - g_1\left(\underline{x}_1, \psi\right) \leq \bar{a}_1(\underline{x}_1, \psi) - \bar{w}_1(\underline{x}_1, \psi)\left(x_1 - \underline{x}_1\right)$, and when $\alpha < 0$, $g_1(\bar{x}_1, \psi) - g_1(\cdot) \leq \underline{a}_1(\bar{x}_1, \psi) - \bar{w}_1(\bar{x}_1, \psi)(\bar{x}_1 - x_1)$. Then, it follows that

$$g(\bar{x}_1, \psi) - g(\cdot) \leq \underline{a}_1(\bar{x}_1, \psi) - \underline{w}_1(\bar{x}_1, \psi)\bar{e}_1, \tag{42}$$

$$g(\cdot) - g\left(\underline{x}_1, \psi\right) \leq \bar{a}_1(\underline{x}_1, \psi) - \bar{w}_1(\underline{x}_1, \psi)\underline{e}_1 \tag{43}$$

where $\bar{e}_1 = \bar{x}_1 - x_1$ and $\underline{e}_1 = x_1 - \underline{x}_1$.

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