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Entrepreneurial Finance with Equity-for-Guarantee Swap and Idiosyncratic Risk

Huamao Wang[1], Zhaojun Yang[2], Hai Zhang[3]

Abstract

We consider a risk-averse entrepreneur who invests in a project with idiosyncratic risk. In contrast to the literature, we assume the entrepreneur is unable to get a loan from a bank directly because of the low creditability of the entrepreneur and so an innovative financial contract, named equity-for-guarantee swap, is signed among a bank, an insurer, and the entrepreneur. It is shown that the new swap leads to higher leverage, which brings more diversification and tax benefits. The new swap not only solves the problems of financing constraints, but also significantly improves the welfare level of the entrepreneur. The growth of welfare level increases dramatically with risk aversion index and the volatility of idiosyncratic risk.

Keywords: Finance, Borrowing Constraints, Equity-for-Guarantee Swap, Capital Structure, Cash-out Option

JEL: G11, G31

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1. Introduction

One of fundamental characteristics of entrepreneurship is lack of diversification. Specifically, the revenue of an entrepreneurial firm (private firm) suffers systematic and idiosyncratic risks. Entrepreneurs can trade risk-free bonds and the diversified market portfolio to diversify the systematic business risk but not the idiosyncratic risk. Therefore, the diversification benefit of risky debt is important to entrepreneurs in addition to the standard trade-off between tax benefits and costs of financial distress, see Chen et al. (2010) among others.

In addition, there are many small and medium enterprises (SMEs) and fresh graduates every year who are hungry for money to start a new business. Such investment is generally extremely high-risk, and to compensate for such risk, the entrepreneur comes with the potential for high returns. However, due to low credibility and lack of guarantee, many entrepreneurs, let alone fresh graduates, are unable to get a bank loan or get other debt financing cheaply. Under such situation, traditional financial theory on optimal capital structure is not reasonable since the entrepreneur has no other choice beyond starting her business with her own money only or simply giving up the business.

To overcome borrowing constraints, some insurers and entrepreneurs in China have developed an innovative financial product, called equity-for-guarantee swap (EGS). This is an agreement between a lender (bank), an insurer, and a borrower (entrepreneur), where the bank lends at a given in-

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terest rate to the entrepreneur and if the entrepreneur defaults on the debt, the insurer will make a compensatory payment to the creditor so that the creditor will always be paid up-to a certain guarantee level. In return for the guarantee, the firm needs to allocate a percentage of the firm's equity to the insurer. This contract was first signed in 2002 in China and it has become increasingly popular in the country.[4]

In this article, we extend the model established by Chen et al. (2010) to take into account both idiosyncratic risk and the EGS. This paper relates to Yang and Zhang (2013), who provide the first formal study on the swap. However, Yang and Zhang (2013) merely discuss traditional capital structure issues in the classic framework of Leland (1994). Our model examines this contract in a more general context with idiosyncratic risk and cash-out option.

The main results in Chen et al. (2010) are based on the assumption that the entrepreneur has "deep pockets", i.e. she can issue debt with the coupon rate being higher than the project's revenue since she can inject cash into the firm to pay coupons. However, this assumption is not feasible for many entrepreneurs, not to mention fresh graduates. Actually, Chen et al. (2010) point out that entrepreneurs may be liquidity-constrained, i.e. no external funds are available to cover the firm's debt service, and hence an earlier liquidation will be forced by the creditor. We argue that the assumption becomes practical thanks to the EGS. In fact, under the swap, the entrepreneur is equivalent to the one who has deep pockets and the default

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[4]The guarantee in our model shares a few similarities with that in Ju and Sohn (2014), where the contract is based on a technology credit scoring model.

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threshold can be lower than the coupon level because the claim owned by the creditor is guaranteed by the insurer. In exchange for the guarantee, the entrepreneur needs to pay the insurer a proportion of equity of the firm. In addition, since the insurer guarantees the debt, the creditor under the swap does not demand a protective covenant.

We consider a risk-averse entrepreneur having access to standard financial investment opportunities with a chance to invest in a project. The objective of the entrepreneur is to maximize her expected lifetime utility over intertemporal consumption. We choose the exponential utility primarily for analytical tractability. While constant absolute risk aversion (CARA) utility does not capture wealth effects, it reduces the dimension, especially for the double-barrier boundary problem, see Henderson (2002), Miao and Wang (2007), Ewald and Yang (2008), and Yang and Yang (2012) among others.[5]

The main results of the paper are as follows. First, our setting improves a generalized model of capital structure trade-off among borrowing constraints, tax, diversification benefits, and costs of financial distress. Second, the EGS fundamentally raises the entrepreneur's borrowing capacity and therefore the entrepreneur optimally issues more debt and takes higher leverage than that without the swap. Higher leverage leads to larger tax shields and diversification benefits because the entrepreneur faces less equity exposure to the

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[5]As argued by Miao and Wang (2007) and Chen et al. (2010), it is believed that our model and insights apply to regular utility functions, since the precautionary savings effect, which is captured by utility functions with convex marginal utility like CARA, is the driving force. For this reason and mathematical convenience, we only consider CARA utility in the paper.

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project and thus her portfolio is less risky. Third, the entrepreneur with the swap receives more welfare increments and has more investment opportunities because of being more willing to invest. Higher risk-averse entrepreneurs under higher nondiversifiable idiosyncratic risk gain more benefits resulting from the swap.

This paper is organized as follows. Section 2 presents the model. Section 3 solves the model. Section 4 discusses the numerical results. Section 5 concludes. Appendices provide equilibrium valuation of corporate securities.

2. Model setup

2.1. Investment Opportunities

We consider an infinitely-lived risk-averse entrepreneur who has an option to invest in a take-it-or-leave-it project at present time 0, which requires a one-time investment cost I. All sources of uncertainty arise from two independent standard Brownian motions B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$.

In addition to the project opportunity, the entrepreneur has access to standard financial investment opportunities, see Merton (1971). Let W denote the entrepreneur's liquid wealth process. The entrepreneur invests an amount of π_t in a diversified market portfolio and the remaining amount $W_t - \pi_t$ in the risk-free asset with a constant interest rate r. The return of the diversified market portfolio is denoted by R which satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \tag{1}$$

where μ_M and $\sigma_M > 0$ are constants, and $\eta \equiv (\mu_M - r)/\sigma_M$ is the Sharpe ratio of the market portfolio.

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We assume the project generates a stochastic revenue process $\{y_t : t \geq 0\}$ that follows a geometric Brownian motion (GBM):

$$dy_t = \mu_y y_t dt + \rho \sigma y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given}, \tag{2}$$

where μ_y is the expected growth rate, σ is the total volatility and $\rho \in [-1, 1]$ is the correlation coefficient between the project payoff and the return on the market portfolio given by (1). A higher absolute value $|\rho|$ of the correlation coefficient implies that the systematic volatility has a larger weight, *ceteris paribus*. The parameters $\omega \equiv \rho\sigma$ and $\epsilon \equiv \sqrt{1 - \rho^2}\sigma$ are respectively the systematic and idiosyncratic volatility of the revenue growth.

2.2. Entrepreneurial Financing with Equity-for-Guarantee Swap

We assume that the entrepreneur runs the project by setting up a limited liability entity, such as a limited liability company (LLC) or an S corporation, which allows her to face single-layer taxation for her business income and makes the debt nonrecourse. We follow the simple tax system in Chen et al. (2010). Entrepreneurial business profits incur taxes at a rate τ_e. A public firm is subject to a double taxation which is captured by an effective marginal tax rate τ_m. The capital gains upon cash-out are taxed at a rate τ_g.

The entrepreneur finances the initial one-time lump-sum cost I via her own funds and external financing. We assume that the main source of external financing is debt, e.g. bank loans. Due to the high default probability and relative lack of collateral, it is much more difficult for the entrepreneurial firm to take debt financing than for a large company. Unlike Chen et al. (2010) who do not consider borrowing constraints, we study the entrepreneur who is constrained in borrowing due to protected covenants demanded by the

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lenders. This financing constraint is alleviated by introducing the EGS supported by a commercial guarantee company or insurer. Unlike the traditional credit hypothecation, though, the entrepreneurial firm in the new credit guarantee scheme must pay to the guarantee company a portion (φ) of equity as guarantee costs instead of regular guarantee fees.

Under the guarantee, the entrepreneur chooses to issue an interest-only consol with coupon b and par value $F_0 = F(y_0)$ at time 0 and remains unchanged until the entrepreneur exits from the project, see (A.9) and (A.11). After the debt is in place, at any time $t \geq 0$, the entrepreneur has three choices: (1) She runs the firm and receives a fraction $(1-\varphi)$ of cash payments from the firm; (2) She defaults once the default threshold y_d of the revenue process is reached and then the insurer must make a compensatory payment to the creditor so that the creditor is paid up-to a certain guarantee level; (3) She cashes out by selling the firm to a diversified buyer at the cash-out threshold y_u, which incurs a fixed transaction cost K.

Once the entrepreneur defaults, the debt holders (lenders) take control and liquidate/sell the firm. Bankruptcy ex post is costly and the bankruptcy loss can be interpreted in different ways, such as loss from selling real assets, asset fire-sale losses, legal fees, etc. We assume that $\kappa \equiv 1 - \alpha$ is the bankruptcy loss rate, i.e. α is the recovery rate. Then the remaining liquidation/sale value of the firm is equal to $\alpha A(y_d)$, where $A(y_d)$ is the equilibrium value of an unlevered (all-equity without debt) public firm given by (A.1). Moreover, the debt holders will gain the compensatory payment from the guarantee company so that under the arrangement of equity-for-guarantee contract the debt holders gain $\phi b/r$ once the entrepreneur defaults instead of

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the remaining value $\alpha A(y_d)$ only, where ϕ is the guarantee level. Therefore, the value, denoted by P_{guar}, of the compensatory payment is given by

$$P_{guar} = (\frac{\phi b}{r} - \alpha A(y_d))\underline{q}(y), \qquad (3)$$

where $\underline{q}(y)$ is the value of a security that claims one unit of account at the default time. It is (A.2) for the default option only and (A.7) for the case with the cash-out option as well.

While selling the firm to cash out, the entrepreneur needs to retire the firm's debt obligation at par F_0 given by (A.11). Similar to Chen et al. (2010), we assume that the buyer is well diversified who will optimally relever the firm, see Leland (1994). The value of the firm after sale is the value of an optimally levered firm in the complete market, i.e. $V^*(y_u)$ given by (A.6).

Let $\mathcal{T}$ be the set of $\{\mathcal{F}_t : t \geq 0\}$-stopping times. An investor is characterized by her initial wealth W_0, a time-discount rate β and her preference $U(\cdot)$. She seeks to choose the default timing $T_d \in \mathcal{T}$ and cash-out timing $T_u \in \mathcal{T}$, a bond coupon b and a consumption process c so as to maximize her expected lifetime time-additive utility of consumption:

$$\mathbb{E}\left[\int_0^\infty \exp\left(-\beta s\right) U(c_s) ds\right]. \qquad (4)$$

In the following text, thanks to the justification argued in the last section, we assume the utility function in (4) is a CARA utility, i.e.

$$U(c) = -\exp(-\gamma c)/\gamma, \quad c \in \Re,$$

where $\gamma > 0$ is the absolute risk aversion parameter.

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3. Model Solution

In this section, we analyze the entrepreneur's optimal decisions under borrowing constraints and the EGS. We note that the entrepreneur's problem is significantly different from that considered by Chen et al. (2010). In fact, the problem here is much more challenging than Chen et al. (2010) since the guarantee cost depends on both the default threshold and the cash-out threshold, which conversely depend on the guarantee cost at the same time.

Firstly, we solve the standard Merton consumption and portfolio choice problem faced by the entrepreneur, see Merton (1971), after she exits from her business via either cashing out or defaulting. Secondly, we solve a mixture of optimal control and optimal stopping problem. We derive the guarantee cost, which is a function of the default threshold y_d and the cash-out threshold y_u. Then, for any given y_d and y_u, we find the maximum for

$$\mathbb{E}\left[\int_0^{\tau_{\mathcal{D}}} \exp\left(-\beta s\right) U(c_s) ds + \exp(-\tau_{\mathcal{D}}) J^e(W_{\tau_{\mathcal{D}}})\right], \quad y_d \leq y_0 \leq y_u, \quad (5)$$

where $\mathcal{D} = \{(w, y) \in \Re^2 | y_d \leq y \leq y_u\}$, $\tau_{\mathcal{D}} \equiv \inf\{t \geq 0 | (W_t, y_t) \notin \mathcal{D}\} = \min\{T_d, T_u\}$, and the function $J^e(\cdot)$ is derived by solving the Merton problem given by (7) below. The maximum of (5) is also a function of the default threshold y_d and the cash-out threshold y_u. Therefore, we need to solve a constrained nonlinear programming problem to obtain the optimal default threshold y_d^* and the cash-out threshold y_u^*. Finally, we determine the entrepreneur's initial investment and optimal capital structure.

3.1. Guarantee Costs and Equity-for-Guarantee Swaps

Unlike the traditional credit hypothecation, though, the entrepreneurial firm in the new credit guarantee scheme must pay to the guarantee company

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a portion (φ) of equity instead of regular guarantee fees. Thus, we call φ the guarantee cost of the EGS.

Generally, a guarantee company is usually a diversified investor who signs such contracts with a large number of firms and therefore the idiosyncratic risk of a entrepreneurship firm is well-diversified. This means that, in addition to (3), the value P_{guar} of compensatory payment must be equal to the market value of the equity allocated to the insurer. That is

$$P_{guar} = \varphi E_0(y; y_d, y_u), \tag{6}$$

where $E_0(y; y_d, y_u)$ is the market value of the outside equity held by the insurer and it is given by (A.4) if the cash-out option is prohibited and given by (A.10) if the cash-out option is admissible.

Therefore, combining (3) and (6) gives the guarantee cost φ as follows

$$
\begin{aligned}
\varphi \equiv \varphi(y; y_d, y_u) &= \left(\tfrac{\phi b}{r} - \alpha A(y_d)\right) \underline{q}(y) / E_0(y; y_d, y_u) \\
&= \frac{\left(\tfrac{\phi b}{r} - \alpha(1-\tau_m)\tfrac{y_d}{r-\mu}\right)\underline{q}(y)}{(1-\tau_e)\left(\tfrac{y}{r-\mu}-\tfrac{b}{r}\right)-(1-\tau_e)\left(\tfrac{y_d}{r-\mu}-\tfrac{b}{r}\right)\underline{q}(y)+\left[V^*(y_u)-(1-\tau_e)\left(\tfrac{y_d}{r-\mu}-\tfrac{b}{r}\right)\right]\bar{q}(y)},
\end{aligned}
$$

where $V^*(y)$, $\underline{q}(y)$ and $\bar{q}(y)$ are defined in (A.6), (A.7) and (A.8).

Remark 1. *The guarantee cost here is fundamentally different from that given by Yang and Zhang (2013), which does not take into account that the entrepreneur is a risk-averse individual and has the option to cash out. For this reason, thanks to game theory, the equilibrium value of equity must be related to the entrepreneur's decisions on the cash-out option and a default threshold, which is clearly different from that in Yang and Zhang (2013) based on a risk-neutral world.*

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3.2. Consumption and Portfolio Choice after Exit

After exiting from her business, the entrepreneur lives on her own financial wealth and faces the standard consumption and portfolio choice problem in Merton (1971). The maximum of the expected lifetime time-additive utility of consumption is given by

$$J^e(w) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(w + \frac{\eta^2}{2\gamma r^2} + \frac{\beta - r}{\gamma r^2}\right)\right]. \tag{7}$$

3.3. Entrepreneur's Decisions and Utility Indifference Prices

Before exit, the entrepreneur's financial wealth evolves as follows,

$$dW_t = (rW_t + \pi_t(\mu_M - r) + (1 - \tau_e)(1 - \varphi)(y - b) - c_t)dt + \pi_t\sigma_M dB_t, \quad 0 < t < \tau_{\mathcal{D}}. \tag{8}$$

Compared to the exogenously given fraction of equity retained by the entrepreneur in Chen et al. (2010), the fraction $1 - \varphi$ here is endogenously determined by the EGS, which depends on the entrepreneur's decisions on default and cash-out. For this reason, we first solve the optimization problem (5) for any given exit threshold pair (y_d, y_u). Under this case, the entrepreneur's value function $J^s(w, y)$ satisfies the following Hamilton-Jacobi-Bellman equation according to Bellman's principle of optimality:

$$\sup_{c \geq 0, \pi} \{U(c) + (rw + \pi(\mu_M - r) + (1 - \tau_e)(1 - \varphi)(y - b) - c)J^s_w(w, y)$$
$$+ \frac{(\pi\sigma_M)^2}{2}J^s_{ww}(w, y) + \pi\sigma_M\sigma\rho y J^s_{wy}(w, y) + \mu_y y J^s_y(w, y) + \frac{\sigma^2 y^2}{2}J^s_{yy}(w, y) \tag{9}$$
$$- \beta J^s(w, y)\} = 0,$$

with the value-matching conditions

$$J^s(w, y_d) = J^e(w), \tag{10}$$

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$$J^s(w, y_u) = J^e(w + (1 - \varphi)V^*(y_u) - F_0 - K - \tau_g((1 - \varphi)V^*(y_u) - K - I)). \quad (11)$$

According to utility indifference pricing principle, for the current revenue $y_d < y < y_u$, the utility indifference price (also called subjective value)[6] of equity owned by the entrepreneur, denoted by $G(y)$, satisfies

$$J^s(w, y) = J^e(w + G(y)) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(w + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\beta - r}{\gamma r^2}\right)\right]. \quad (12)$$

Substituting (7) and (12) into (9) - (11), according to the first-order conditions for the optimal consumption and portfolio choice, we obtain the following theorem immediately:

Theorem 3.1. *The entrepreneur exits from her business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever comes first. For any given exit threshold pair (y_d, y_u), liquid wealth level w and revenue $y_d < y < y_u$, the optimal consumption and portfolio rule is given by*

$$c^*(w, y) = r\left[w + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\beta - r}{\gamma r^2}\right], \quad (13)$$

$$\pi^*(w, y) = \frac{\eta}{\gamma r \sigma_M} - \frac{\rho \sigma}{\sigma_M} y G'(y), \quad (14)$$

where $G(y)$ is the utility indifference price of equity owned by the entrepreneur and is a solution of the following ordinary differential equation:

$$\begin{aligned} rG(y) &= (1 - \tau_e)(1 - \varphi)(y - b) + (\mu_y - \rho \sigma \eta)yG'(y) \\ &\quad + \frac{\sigma^2 y^2}{2}G''(y) - \frac{\gamma r(1 - \rho^2)\sigma^2 y^2}{2}G'(y)^2, \end{aligned} \quad (15)$$

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[6]Thanks to the exponential utility assumption, the utility indifference price is independent of the wealth level of the entrepreneur.

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subject to the boundary conditions:

$$G(y_d) = 0, \tag{16}$$

$$G(y_u) = (1-\varphi)V^*(y_u) - F_0 - K - \tau_g((1-\varphi)V^*(y_u) - K - I). \tag{17}$$

In order to complete the computation of the entrepreneur's optimization problem, we now need to derive the optimal default threshold y_d^* and optimal cash-out threshold y_u^*, such that the value function $J^s(w, y)$ is maximized. Equivalently, we need only to find the maximum point (y_d^*, y_u^*) of the function $G(y; y_d, y_u)$ for any given revenue level[7]. The constrained nonlinear programming problem is solved by numerical methods.

Remark 2. *At first sight, this theorem is similar to* Chen et al. (2010), *but the fraction* $1 - \varphi$ *of equity owned by the entrepreneur here is endogenously determined under the newly invented EGS. As a result, one of the distinctions from* Chen et al. (2010) *is that the state transition equation (8) itself depends on the optimal stopping times. Therefore, to derive the optimal default threshold and cash-out threshold more effectively, we solve a nonlinear programming problem instead of utilizing the smooth-pasting conditions as done by* Chen et al. (2010).

Similar to Chen et al. (2010), Equation (15) implies that the systematic and idiosyncratic risk premium denoted by $\xi^s(y)$ and $\xi^i(y)$ respectively, are

$$\xi^s(y) = \rho\sigma\eta y \frac{G'(y)}{G(y)}, \tag{18}$$

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[7]Obviously, the maximum point will not depend on the current revenue level in our model. Thus we find the maximum point at $y_0 = 1$, and we set the constraint $(0, y_0)' \leq (y_d, y_u)' \leq (y_0, \bar{y})'$, where $\bar{y}$ is a sufficiently large boundary to include y_u^*.

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$$\xi^i(y) = \frac{\gamma r}{2} \frac{(\sqrt{1-\rho^2}\sigma y G'(y))^2}{G(y)}. \tag{19}$$

3.4. Capital Structure under Equity-for-Guarantee Swap

Now we turn to the entrepreneur's initial decision on the optimal debt borrowed from the lender for investing in the project in order to maximize the entrepreneur's net profit under the EGS.

The entrepreneur's initial optimal capital structure to start the project is to find optimal coupon level b^*, which maximizes the entrepreneur's net profit (target function)

$$P(y_0; b, I) \equiv G(y_0) + [F(y_0) - I]. \tag{20}$$

We identify a break-even investment cost I^* by solving a root-finding problem $P^*(y_0; b^*, I^*) = 0$.

Denote by $\hat{P}^*(y_0)$ the maximum net profit obtained by an entrepreneur without signing an EGS contract, and by $W^s(y_0)$ the welfare loss of this entrepreneur because the swap is not signed, and then we have

$$W^s(y_0) \equiv P^*(y_0) - \hat{P}^*(y_0).$$

After the entrepreneur invests in the project, a natural measure of leverage for the entrepreneur is the private leverage ratio $L(y_0) \equiv F(y_0)/S(y_0)$, where $S(y_0)$ is the total subjective value of the entrepreneurial firm. In contrast to Chen et al. (2010), the total subjective value in our model is given by $S(y_0) = G(y_0) + F(y_0)$, which excludes the value φE_0 of the outside equity held by the insurer because the value of the outside debt held by the lender is already taken into account.

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4. Discussion of Entrepreneurial Finance

To make an effective comparison, similar to Chen et al. (2010), our baseline parameter values are carefully selected. For example, we select the parameter values that ensure $y_d < y_0 < y_u$. I.e., we excludes two uninteresting cases: One corresponds to a sufficiently large asset recovery rate α, together with a sufficiently high guarantee level ϕ and a large risk aversion γ, which might lead to an immediate default; The other corresponds to a sufficiently small cash-out cost K, which might make the entrepreneur sell the firm immediately.

Our numerical simulations are based on the following annualized baseline parameter values: interest rate $r = 3\%$, expected growth rate $\mu_y = 4\%$, the Sharpe ratio of market $\eta = 0.4$, asset recovery rate $\alpha = 0.6$. The entrepreneur's rate of time preference is $\beta = 3\%$ and the initial level of the project revenue is $y_0 = 1$. The initial investment cost for the project is $I = 10$ and the cash-out cost is $K = 27$. The effective marginal tax rate τ_m is 11.29%. The tax rate τ_e of entrepreneurial firm is set to equal τ_m for emphasizing the entrepreneur's nondiversifiable idiosyncratic risk. The effective capital gains tax rate is $\tau_g = 10\%$. In addition, we set the guarantee level $\phi = 50\%$, the systematic volatility of growth rate $\omega = 10\%$, and idiosyncratic volatility $\epsilon = 10\%$. Hence, the total volatility of the project $\sigma = 0.02^{0.5}$, correlation coefficient $\rho = 0.5^{0.5}$. In fact, except for ϕ, these baseline values are also assumed in Chen et al. (2010), who justify why the values are so selected.

At the very beginning, a sensitivity analysis of $G(y)$ and $G'(y)$ is provided in Table 1.

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Table 1: The table presents the sensitivity analysis of the subjective value $G(y)$ and its derivative $G'(y)$ to the changes of guarantee level ϕ, idiosyncratic volatility ϵ, and cash-out cost K for risk aversion $\gamma = 1$ and the current project revenue $y_0 = 1$.

ϕ	0.40	0.45	0.50	0.55	0.60	0.65	0.70
$G(y)$	12.96	10.60	9.76	3.40	1.38	0.87	0.50
$G'(y)$	21.86	20.59	19.81	11.17	5.69	3.68	2.95
ϵ	0.00	0.05	0.10	0.15	0.20	0.25	0.30
$G(y)$	18.16	16.37	9.76	3.92	3.63	3.20	3.16
$G'(y)$	28.72	26.20	19.81	11.16	9.44	7.83	7.17
K	10.00	20.00	30.00	40.00	50.00	60.00	70.00
$G(y)$	10.02	9.80	9.75	9.72	9.71	9.70	9.70
$G'(y)$	20.27	19.90	19.78	19.73	19.70	19.69	19.68

4.1. Equity Value with Equity-for-Guarantee Swap

We apply Figure 1 to illustrate the properties of the utility indifference price G of equity, i.e. the value of the cash flow $(1-\tau_e)(1-\varphi)(y-b)$, received by the entrepreneur with the EGS. Figure 1 shows the results under the case of default option only and under the case with both default option and cash-out option respectively. After that, we present the risk premiums demanded by the entrepreneur in Figure 2. Here we let $\tau_e = 0$ to exclude the effects of tax.

The results in Figure 1 under the EGS are similar to Chen et al. (2010) if we think the amount of equity allocated to the insurer in our model as the outside equity in Chen et al. (2010), who do not take into account the
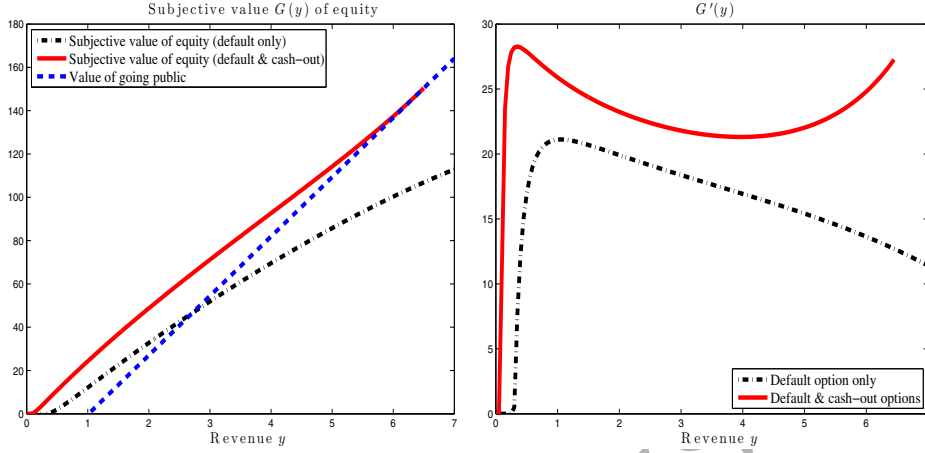
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Figure 1: The figure depicts the utility indifference price $G(y)$, its derivatives $G'(y)$ and the value of going public under EGS when risk aversion $\gamma = 1$, firm tax $\tau_e = 0$, interest rate $r = 3\%$, growth rate $\mu_y = 4\%$, Sharpe ratio $\eta = 0.4$, recovery rate $\alpha = 0.6$, time preference $\beta = 3\%$, initial revenue $y_0 = 1$, investment cost $I = 10$, cash-out cost $K = 27$, guarantee level $\phi = 50\%$, systematic volatility $\omega = 10\%$, and idiosyncratic volatility $\epsilon = 10\%$.

swap. The subjective values $G(y)$ of equity are convex functions of revenue y when it is sufficiently low, i.e. the default option is deep in the money, or it is sufficiently high, i.e. the cash-out option is deep in the money. Under other situations, the subjective values are concave because the precautionary saving demand dominates the impact of the two options.

The properties of risk premium demanded by the entrepreneur shown in Figure 2 are similar to Chen et al. (2010) as well. On the one hand, when the revenue y approaches default threshold y_d, the systematic risk premium $\xi^s(y)$ diverges to infinity as seen in (18) because of the significant leverage effect. It rises when the revenue y approaches the cash-out threshold y_u because the cash-out option makes the value G more sensitive to cash flow shocks.
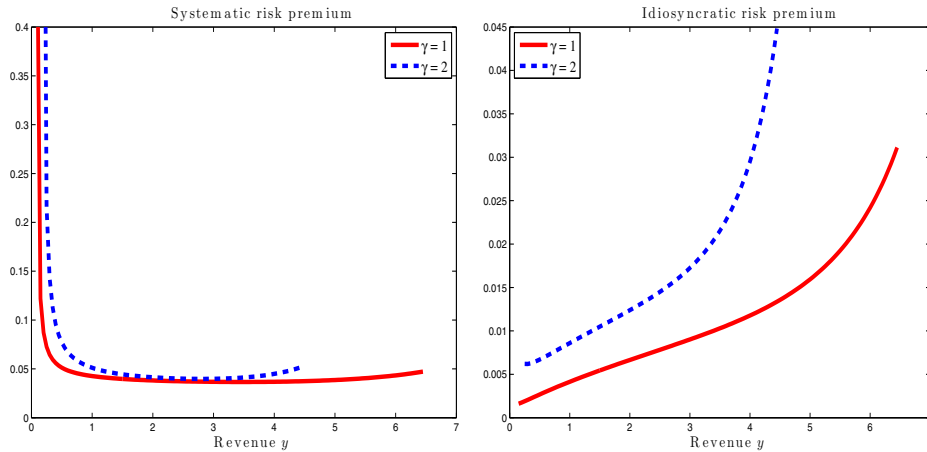
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Figure 2: This figure shows the systematic and idiosyncratic risk premium under EGS when firm tax $\tau_e = 0$, interest rate $r = 3\%$, growth rate $\mu_y = 4\%$, Sharpe ratio $\eta = 0.4$, recovery rate $\alpha = 0.6$, time preference $\beta = 3\%$, initial revenue $y_0 = 1$, investment cost $I = 10$, cash-out cost $K = 27$, guarantee level $\phi = 50\%$, systematic volatility $\omega = 10\%$, and idiosyncratic volatility $\epsilon = 10\%$.

On the other hand, the idiosyncratic risk premium $\xi^i(y)$ is small when y is close to y_d. When revenue y is large, its growth leads to a fast growth of $\xi^i(y)$ because the conditional idiosyncratic variance $(\sqrt{1 - \rho^2}\sigma y G'(y))^2$ rises faster than $G(y)$, as seen in (19).

4.2. Comparison of Capital Structures

We compare three capital structures under different financing arrangements in Table 2 without taxes $\tau_e = 0$ and in Table 3 with taxes $\tau_e = \tau_m = 11.29\%$ respectively. In Tables 2 and 3, Panel A represents the unlevered entrepreneurial firm; Panel B is the entrepreneur who is able to choose optimal risky debt but is unable to determine optimal default due to the protective covenant $y_d = b$; Panel C exhibits the entrepreneurial firm with optimal

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leverage and optimal default supported by the EGS. Table 2 shows the special case where entrepreneurs acquire only diversification benefits without tax benefits from risky debt.

Similar conclusions to Chen et al. (2010) are found from the two tables. First, the model is equivalent to the complete-market benchmark when the risk aversion $\gamma \to 0$ and $\tau_e = 0$, and hence the firm is valued at the market value 33.33. Second, the subjective value for a risk-averse entrepreneur decreases because they discount the nontradable equity due to nondiversifiable idiosyncratic business risks. Third, more risk-averse entrepreneurs issue more debt in order to achieve greater diversification benefits, and as a result, it leads to a less subjective value of equity held by the entrepreneur.

More importantly, our results reveal large increased benefits for the entrepreneur through introducing the EGS. Under this swap, the entrepreneur is not forced to default by the protective covenant even when the revenue y is lower than the coupon b. Accordingly, the entrepreneur is able to choose the endogenous optimal default threshold y_d that is generally lower than b. In other words, thanks to the swap, the entrepreneur has deep pockets now without liquidity constraints, which is an assumption widely applied by Chen et al. (2010). For this reason, we argue that the EGS in our model makes the important assumption in Chen et al. (2010) feasible.

Admittedly, the entrepreneur with the swap faces higher credit spreads and default probabilities due to higher optimal leverage if the debt is guaranteed by only $\phi = 50\%$ level, i.e. the insurer only ensures that the debt holder receives 50% of the value b/r. Certainly, credit spreads and default probabilities can be reduced by raising the guarantee level ϕ if it is agreed

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Table 2: The table gives guarantee cost φ^*, optimal coupon b, debt value F_0, equilibrium value φE_0 of equity held by the insurer, subjective value G_0 of equity held by the entrepreneur, optimal leverage L_0, credit spread CS, 10 years default probability $p_d(10)$, 10 years cash-out probability $p_u(10)$, and welfare loss W^s under the tax rate $\tau_e = 0$ for risk aversion $\gamma = 0, 1, 2$ respectively.

γ	φ^* (%)	b	F_0	φE_0	G_0	L_0 (%)	CS (bp)	$p_d(10)$ (%)	$p_u(10)$ (%)	W^s
Panel A: without guarantee, no leverage										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	28.43	0.00	0.00	0.00	0.07	0.34
2.00	0.00	0.00	0.00	0.00	26.10	0.00	0.00	0.00	6.09	0.54
Panel B: without guarantee, optimal leverage and $y_d = b$										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.05	1.65	0.00	27.05	5.75	2.89	0.00	0.04	0.08
2.00	0.00	0.11	3.57	0.00	22.92	13.46	8.35	0.00	0.60	0.15
Panel C: with guarantee, optimal leverage and optimal default										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.05	0.13	4.48	0.01	24.30	15.57	1.25	0.00	0.04	0.00
2.00	0.73	0.29	9.11	0.18	17.53	34.19	16.81	0.00	0.60	0.00

by the three parties. However, a higher guarantee level ϕ demands that the entrepreneur should pay a higher portion of equity to the insurer in exchange for the guarantee. If the portion is greater than or equal to one, then it means that no insurer wants to sign the swap contract with the entrepreneur.

From Tables 2 and 3, thanks to the EGS, the entrepreneur with the guarantee achieves a considerable welfare increment, i.e. the welfare losses

Table 3: This table gives guarantee cost φ^*, optimal coupon b, debt value F_0, equilibrium value φE_0 of equity held by the insurer, subjective value G_0 of equity held by the entrepreneur, optimal leverage L_0, credit spread CS, 10 years default probability $p_d(10)$, 10 years cash-out probability $p_u(10)$, and welfare loss W^s under the tax rate $\tau_e = \tau_m = 11.29\%$ for risk aversion $\gamma = 0, 1, 2$ respectively.

γ	φ^* (%)	b	F_0	φE_0	G_0	L_0 (%)	CS (bp)	$p_d(10)$ (%)	$p_u(10)$ (%)	W^s
Panel A: without guarantee, no leverage										
0.00	0.00	0.00	0.00	0.00	29.51	0.00	0.00	0.00	0.00	0.84
1.00	0.00	0.00	0.00	0.00	25.77	0.00	0.00	0.00	0.10	1.52
2.00	0.00	0.00	0.00	0.00	23.93	0.00	0.00	0.00	0.88	2.43
Panel B: without guarantee, optimal leverage and $y_d = b$										
0.00	0.00	0.18	5.85	0.00	24.07	19.54	16.43	0.00	0.00	0.44
1.00	0.00	0.20	6.42	0.00	19.79	24.49	19.32	0.00	0.21	1.08
2.00	0.00	0.24	7.53	0.00	16.87	30.88	25.21	0.02	1.28	1.97
Panel C: with guarantee, optimal leverage and optimal default										
0.00	3.14	0.46	14.02	0.53	16.33	46.21	30.79	0.05	0.00	0.00
1.00	7.17	0.61	17.53	0.99	9.76	64.24	49.46	0.55	0.10	0.00
2.00	21.10	0.84	21.72	1.99	4.64	82.39	84.53	4.93	0.21	0.00

W^s of the entrepreneurs without the guarantee are significant. The welfare increment is enlarged by a higher tax rate and a higher risk-aversion index of the entrepreneur. In particular, the welfare increment gets even more relative to the case of no leverage, which is particularly common in China among others, since a large number of entrepreneurs of SMEs are fundamentally unable to issue debt directly.

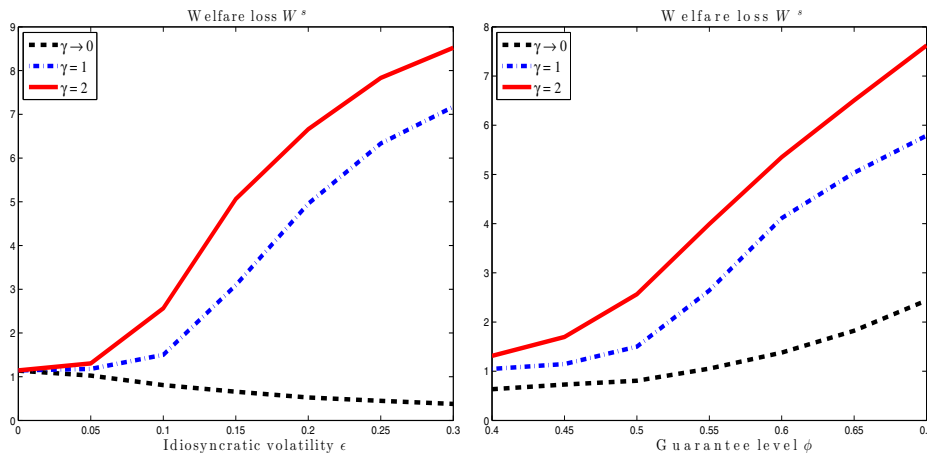## 4.3. Analysis of Welfare Loss

Figure 3: The figure reports how welfare loss W^s changes with idiosyncratic volatilities ϵ and guarantee levels ϕ when firm tax $\tau_e = 11.29\%$, interest rate $r = 3\%$, growth rate $\mu_y = 4\%$, Sharpe ratio $\eta = 0.4$, recovery rate $\alpha = 0.6$, time preference $\beta = 3\%$, initial revenue $y_0 = 1$, investment cost $I = 10$, cash-out cost $K = 27$, and systematic volatility $\omega = 10\%$.

To demonstrate the increased benefits resulting from the EGS for the entrepreneur further, we present comparative statics on welfare loss W^s for different parameter values in Figures 3 and 4. We only show the results of welfare loss W^s incurred by the entrepreneur without leverage due to the pervasive low borrowing capacity of an SME. The findings on W^s of the entrepreneur with the protective covenant are similar.

Figures 3 and 4 highlight large benefits resulting from the EGS. Compared with the unlevered firm, the swap raises net profit, and the welfare increment ascends for more risk-averse entrepreneurs who optimally take higher leverage. First, for a risk-averse entrepreneur, the welfare loss W^s
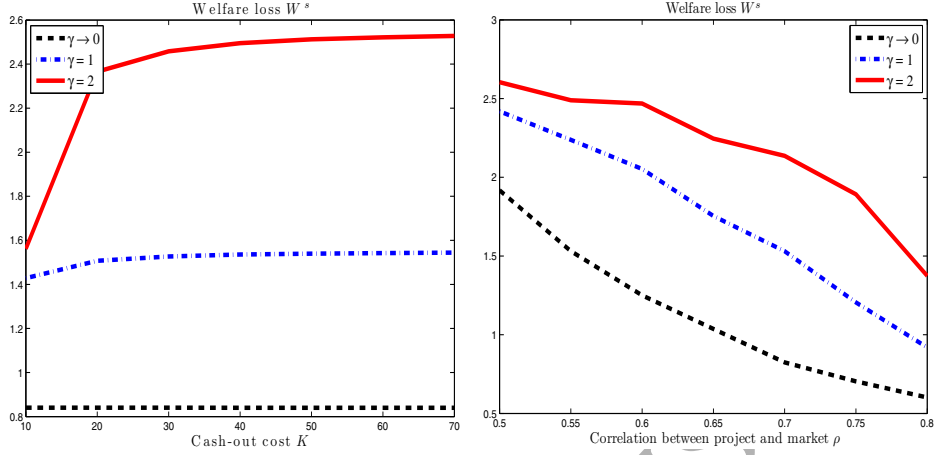
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Figure 4: This figure presents how welfare loss W^s changes with the cash-out costs K and correlation ρ when firm tax $\tau_e = 11.29\%$, interest rate $r = 3\%$, growth rate $\mu_y = 4\%$, Sharpe ratio $\eta = 0.4$, recovery rate $\alpha = 0.6$, time preference $\beta = 3\%$, initial revenue $y_0 = 1$, investment cost $I = 10$, guarantee level $\phi = 50\%$, systematic volatility $\omega = 10\%$, and idiosyncratic volatility $\epsilon = 10\%$.

increases substantially with the idiosyncratic volatility ϵ and the guarantee level ϕ. Second, if the cash-out cost K is small, the welfare loss W^s increases quickly with K for a sufficiently risk-averse entrepreneur, and then keep unchanged if K is sufficiently large. It also shows that the welfare increment increases quickly with the risk-aversion index and does not depend on the cash-out cost if the cost is sufficiently large. Finally, the welfare loss W^s decreases with the correlation coefficient ρ since a large absolute value of ρ means a less idiosyncratic risk, keeping parameter σ unchanged.

4.4. Break-even Investment Cost

The previous subsections explore the benefits of the EGS in financing decisions. Now we focus on the effects of the swap on an entrepreneur's

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investment decisions. Chen et al. (2010) examine the effects of idiosyncratic volatility $\epsilon \in \{0.15, 0.20, 0.25\}$ on the break-even investment cost I^*. Here we provide more detailed analysis on the cut-off rule I^* and emphasize the benefits of the swap.

We compare the break-even cost I^* under three different capital structures, i.e. no leverage without the swap, optimal leverage with bankruptcy protection but without the EGS, and the optimal capital structure with the swap. As reported in Table 4, generally the break-even investment cost I^* decreases for a more risk-averse entrepreneur and/or a higher idiosyncratic volatility of the revenue. I.e., a high risk aversion reduces the likelihood to invest, which is consistent with the observations of Chonopoulos et al. (2011). For instance, if $\epsilon = 0.90$, the entrepreneur with $\gamma = 0$ under the swap (Panel C) will invest in the project even the investment cost is as much as 27.26. By contrast, if the entrepreneur is risk-averse enough, say $\gamma = 2$, she will give up the project once the investment cost is greater than 21.55.

However, the trend of I^* does not hold true all the time. In fact, we notice that in Panels B and C of Table 4, as idiosyncratic volatility ϵ grows further, break-even investment costs fluctuate slightly. This is because the subjective values of equity held by the entrepreneur include the values of the default option and the cash-out option, both of which increase with a growth of the volatility of idiosyncratic risk although for a risk-averse investor, the subjective value of larger risk asset will be less in general.

In addition, we find that in Panel B under risk aversion $\gamma = 2$, the break-even investment cost is higher than that under $\gamma = 1$ if the volatility of idiosyncratic risk is large enough, say $\epsilon \geq 0.4$. This is because a more risk-

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averse entrepreneur would borrow more money from the bank and get more diversified benefits and tax shields under bankruptcy protection $y_d = b$.

Table 4: This table gives break-even investment cost I^* under tax rate $\tau_e = \tau_m = 11.29\%$ for different volatility ϵ of idiosyncratic risk and risk aversion $\gamma = 0, 1, 2$ respectively.

$\epsilon =$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Panel A: without guarantee, no leverage									
0.00	29.51	29.17	28.59	28.00	27.52	27.15	26.87	26.66	26.50
1.00	25.78	20.99	17.83	15.92	14.73	13.98	13.48	13.12	12.86
2.00	23.94	18.44	15.18	13.30	12.14	11.37	10.92	10.60	10.37
Panel B: without guarantee, optimal leverage and $y_d = b$									
0.00	29.97	29.39	28.78	28.14	27.63	27.24	26.96	26.74	26.58
1.00	26.29	21.31	18.04	17.61	17.78	17.73	17.70	17.68	17.66
2.00	24.49	18.68	17.71	17.84	17.81	17.80	17.79	17.78	17.77
Panel C: with guarantee, optimal leverage and optimal default									
0.00	30.35	29.69	28.97	28.51	28.20	27.55	27.51	27.35	27.26
1.00	27.29	25.42	24.43	23.69	23.65	22.56	22.92	22.93	22.69
2.00	26.36	24.90	23.10	22.78	22.24	21.97	21.58	21.52	21.55

Actually, under the EGS, the break-even investment cost of a more risk-averse entrepreneur decreases gradually and it is considerably greater than the corresponding break-even investment cost if the swap contract is not signed. These results imply that an entrepreneur armed with the swap is more willing to invest, since the swap provides the entrepreneur with the eligibility to issue debt and to freely choose the time to default.

More importantly, a high break-even investment cost means a large net

profit obtained by the entrepreneur after investing in the project. By comparing the three financing schemes in Table 4, we find that a risk-averse entrepreneur obtains considerable welfare increments resulting from the EGS, although the welfare increments are very limited if the entrepreneur's risk aversion $\gamma = 0$.

5. Conclusion

Usually, it is difficult or even impossible for SMEs to get bank loan to start a new business because of low credibility and lack of guarantee, even if the business is substantially profitable. There are a large number of SMEs encountering such obstacles. Moreover, a great number of students graduate from college every year (e.g. record-high 6.99 million in China in 2013), and with the increasingly serious employment situation, many governments encourage them to start their own businesses. Clearly all of them need funds, which in general cannot be borrowed directly from a bank. In order to overcome such financing constraints, a financial product, called equity-for-guarantee swap (EGS), which was invented in China since 2002, is becoming more and more popular. However, to the best of our knowledge, there is no quantitative study on such swaps apart from Yang and Zhang (2013), who consider only the equilibrium pricing problem for a firm with the swap.

This paper provides a dynamic incomplete-market framework that models the impact and interactions of the two frictions: borrowing constraints and lack of diversification, on entrepreneurial investment, interdependent consumption, portfolio allocation, financing, and business exit decisions. We show that the EGS brings great benefits to the entrepreneur. The swap

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improves the capital structure trade-off among tax, diversification benefit and financial distress costs. Hence, issuing covered risky debt generates substantial diversification benefits and tax benefits. The more risk-averse the entrepreneur, or the higher the idiosyncratic risk, the greater the benefit. Naturally, the most important advantage of the newly invented swap is to effectively overcome borrowing constraints on an entrepreneur, who are unable to invest in a project without the swap, even though the project is valuable.

Appendices

Appendix A   An Equilibrium Valuation of Corporate Securities

Denote by $\nu \equiv \mu_y - \rho\sigma\eta$ the risk-adjusted drift rate of the project, and $B^{\mathbb{Q}}$ a standard Brownian motion under an equivalent risk-neutral probability measure $\mathbb{Q}$, satisfying $dB_t^{\mathbb{Q}} = dB_t + \eta dt$. Under $\mathbb{Q}$, the dynamics of the revenue in (2) can be rewritten as[8]

$$dy_t = \nu y_t dt + \rho\sigma y_t dB_t^{\mathbb{Q}} + \epsilon y_t dZ_t, \quad y_0 \text{ given.}$$

Then one can derive the following equilibrium price (value)

$$V^f(y) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^\infty \exp\left(-r(s-t)\right) f(y_s) ds \,|y_t = y\right]$$

for any time-independent claim underlying the revenue y of the project with a payment flow $f(y_s)$ to the claimant. Therefore, the value $A(y)$ of the

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[8]Noting that Z is independent of B, we conclude from the equilibrium pricing theory and the well-known Girsanov theorem, that Z is a standard Brownian motion under both $\mathbb{Q}$ and the physical probability measure.

unlevered firm is immediately given by

$$A(y) = \mathbb{E}^{\mathbb{Q}} \left[\int_t^\infty \exp\left(-r(s-t)\right)(1 - \tau_m) y_s ds \,|\, y_t = y \right] = (1 - \tau_m) \frac{y}{r - \nu}.$$
(A.1)

Denote by θ_1 and θ_2 the two roots of the quadratic equation $\frac{1}{2}\sigma^2 \theta(\theta - 1) + \nu\theta - r = 0$, then we get

$$\theta_{1,2} = \frac{-(\nu - \sigma^2/2) \mp \sqrt{(\nu - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}.$$

Obviously, we have $\theta_1 < 0$ and $\theta_2 > 1$.

Define $\tau_d = \inf\{t \geq 0 : y_t \leq y_d\}$ and denote by $\underline{q}_d(y)$ the value of a security that claims one unit of account at the default time without cash-out option, then

$$\underline{q}_d(y) = \mathbb{E}^{\mathbb{Q}}[e^{-r(\tau_d - t)}|y_t = y] = \left(\frac{y}{y_d}\right)^{\theta_1}.$$
(A.2)

Thus, the equilibrium value of equity of a levered public firm is given by

$$
\begin{aligned}
E(y) &= \mathbb{E}^{\mathbb{Q}} \left[\int_t^{\tau_d} e^{-r(s-t)}(1 - \tau_m)(y_s - b)ds \,|\, y_t = y \right] \\
&= (1 - \tau_m)\left(\frac{y}{r - \nu} - \frac{b}{r}\right) - (1 - \tau_m)\left(\frac{y_d}{r - \nu} - \frac{b}{r}\right)\underline{q}_d(y).
\end{aligned}
$$
(A.3)

Naturally, the market value of the outside equity held by the insurer without cash-out option is given by

$$E_0(y; y_d) = (1 - \tau_e)\left(\frac{y}{r - \nu} - \frac{b}{r}\right) - (1 - \tau_e)\left(\frac{y_d}{r - \nu} - \frac{b}{r}\right)\underline{q}_d(y).$$
(A.4)

The default threshold y_d^p of a levered public firm is given by

$$y_d^p = \frac{\theta_1}{\theta_1 - 1}\frac{(r - \nu)b}{r}.$$

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The value of the debt $D(y)$ of the public firm is given by

$$
\begin{aligned}
D(y) &= \mathbb{E}^{\mathbb{Q}} \left[\int_t^{\tau_d} e^{-r(s-t)} b \, ds + \int_{\tau_d}^{\infty} e^{-r(s-t)} \alpha(1-\tau_m) y_s \, ds \, | \, y_t = y \right] \\
&= \frac{b}{r} \left(1 - \left(\frac{y}{y_d} \right)^{\theta_1} \right) + \alpha A(y_d) \left(\frac{y}{y_d} \right)^{\theta_1}.
\end{aligned}
\tag{A.5}
$$

Then, the sum of $E(y)$ in (A.3) and $D(y)$ in (A.5) gives the firm value $V(y)$. The optimal coupon b^* is the solution of $b^* = \arg \max_b V(y)$. Then, the public firm value under the optimal capital structure is given from Chen et al. (2010) by

$$
V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{\kappa(1-\tau_m)\theta_1}{\tau_m} \right)^{1/\theta_1} \right] \frac{y}{r-\nu},
\tag{A.6}
$$

which is the value of the firm when the entrepreneur exercises cash-out option.

Finally, we summarize the market values of outside debt $F(y)$ and equity $E_0(y; y_d, y_u)$ of the entrepreneurial firm with cash-out option. Denote by $\underline{q}(y)$ (resp. $\bar{q}(y)$) the market value of a security that claims one unit of account at the default time before cash-out (resp. when cash-out occurs before default). They are given by

$$
\underline{q}(y) = \frac{y^{\theta_2} y_u^{\theta_1} - y^{\theta_1} y_u^{\theta_2}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}},
\tag{A.7}
$$

$$
\bar{q}(y) = \frac{y^{\theta_1} y_d^{\theta_2} - y^{\theta_2} y_d^{\theta_1}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}.
\tag{A.8}
$$

At the default trigger y_d, debt was guaranteed with level ϕ, in that $F(y_d) = \phi\frac{b}{r}$, and the shareholder gets $E_0(y_d) = 0$. At the cash-out trigger y_u, debt is retired and recovers to $F(y_u) = F_0$, and the equity value $E_0(y_u) = V^*(y_u)$. We have

$$
F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r} \right) \bar{q}(y) + (\phi - 1)\frac{b}{r}\underline{q}(y),
\tag{A.9}
$$

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$$
\begin{aligned}
E_0(y; y_d, y_u) &= (1 - \tau_e)\left(\tfrac{y}{r-\nu} - \tfrac{b}{r}\right) - (1 - \tau_e)\left(\tfrac{y_d}{r-\nu} - \tfrac{b}{r}\right)\underline{q}(y) \\
&+ \left[V^*(y_u) - (1 - \tau_e)\left(\tfrac{y_d}{r-\nu} - \tfrac{b}{r}\right)\right]\bar{q}(y).
\end{aligned} \tag{A.10}
$$

Then, solving the equation $F_0 = F(y_0)$ gives the following value of the initial outside debt:

$$
F_0 = \frac{b}{r} + (\phi - 1)\frac{b}{r}\frac{\underline{q}(y_0)}{1 - \bar{q}(y_0)}. \tag{A.11}
$$

References

Chen, H., Miao, J., Wang, N., 2010. Entrepreneurial finance and nondiversifiable risk. Review of Financial Studies, 23(12), 4348-4388.

Chonopoulos, M., De Reyck, B., Siddiqui, A., 2011. Optimal investment under operational flexibility, risk aversion, and uncertainty. European Journal of Operational Research 213(1), 221-237.

Ewald, C., and Yang, Z., 2008. Utility based pricing and exercising of real options under geometric mean reversion and risk aversion toward tdiosyncratic risk. Mathematical Methods of Operations Research, 68(1), 97-123.

Henderson, V., 2002. Valuation of claims on non-tradable asset using utility maximization. Mathematical Finance, 12(4), 351-373.

Ju, Y. H., Sohn, S. Y., 2014. Updating a credit-scoring model based on new attributes without realization of actual data. European Journal of Operational Research 234(1), 119-126.

Leland, H., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance, 49(4), 1213-1252.

Merton, R., 1971. Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory, 3(4), 373-414.

Miao, J., Wang, N., 2007. Investment, Consumption and Hedging under Incomplete Markets. Journal of Financial Economics, 86(3), 608-642.

Yang, J. and Yang, Z., 2012. Consumption Utility-Based Pricing and Timing of the Option to Invest with Partial Information, Computational Economics, 39(2): 195-217.

Yang, Z. and Zhang, H., 2013. Optimal Capital Structure with an Equity-for-Guarantee Swap. Economics Letters, 118(2): 355-359.