Military aid, direct intervention and counterterrorism

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Abstract

We analyze the choice often faced by countries of whether to directly intervene to counter an external terrorist threat or to subsidize a foreign government to do it. We present a model which analyzes this policy choice where two countries, home and foreign, face a terrorist threat based in the foreign country. The home country chooses how much to invest in defending itself and in reducing terrorist resources either indirectly by subsidising the foreign country or by directly by intervening itself and risking destabilizing the foreign country. Using a calibrated model, we are able to show that direct intervention is only an equilibrium if foreign and home efforts are not good substitutes in the technology used to reduce the resources of the terrorist group. A higher relative military efficiency by the home country makes intervention more likely.

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1 Introduction

In military conflicts, a recurrent policy choice is the extent to which one should fight oneself, by direct intervention, or subsidize allies to fight for you. To help understand this issue, we provide a model in which a home country can invest in: defending itself against the enemy; direct intervention by attacking the enemy; or indirect intervention by subsidising a foreign ally to fight the enemy. As we shall show, this choice will depend on both the relative costs and the degree of substitutability between direct and indirect intervention.

For example, in recent years there have been debates in the US and Europe about intervention in Libya and Syria, in France about intervention in Mali, and in Kenya about intervention in Somalia. Russia has faced the choice between just supporting proxies or also committing its own troops as it did in Georgia in 2008, Crimea in 2014 and Syria in 2015. After the 9/11 attacks in 2001, the US invaded Afghanistan and, with the help of the Northern Alliance, displaced the Taliban government. After the invasion, the US had the choice of fighting the Taliban directly or just indirectly by providing military aid to foreign "allied" governments in Afghanistan or Pakistan, to encourage their efforts against the Taliban. The direct attacks on the Taliban could be done with boots on the ground or using unmanned aerial vehicles, UAVs, usually known as drones. The political economy of the US use of drones is discussed in Hall and Coyne (2014). While we focus on coercive interventions, there may also be non-coercive interventions designed to win "hearts and minds".

A historical example is British policy during the 18th and 19th centuries. Not only did it defend itself against invasion by France, but it repeatedly subsidized allies to fight France either alongside Britain or instead of Britain. Britain could afford to support allies, having an effective tax system and good credit which allowed her to borrow. Ferguson (2001) argues that the combination of a Parliament, tax bureaucracy, national debt and central bank gave Britain a decisive military advantage over its main rival France: finance as much as firepower decided the fate of nations. To reflect this, in our model, we will assume that the home country is a relatively rich hegemonic power, which gives it a credible first mover advantage. The British strategic choices also reflected the degree of substitution between British and allied forces and the relative military effectiveness of each against the French. British naval power and the landbased power of its allies were poor substitutes and defeating the French required fighting on land in Europe, something its allies could do more effectively than Britain. The model will also reflect that there are risks for foreign allies. The British allies who provided the armies often did so at the cost of being overthrown, after defeat by France for instance.

In our model, a home country can invest in defensive effort, direct intervention effort or a subsidy to an allied country's military effort. Depending on circumstances, one observes countries investing in one, two or all three of these alternatives. Thus allowing for corner solutions, zero investment in one or more of the alternatives, is important. The model is a three stage complete information game.[1] We will label the enemy terrorists, and think of them being located in the allied country, though the enemy could be insurgents in a civil war enemy or another country. We assume that the home government acts first, committing to the amounts invested in defense, direct intervention and the subsidy to the foreign ally. The foreign ally then commits to counter-terrorist effort. Finally the terrorists choose the effort they devote to attacking the home and foreign countries. These attacks then succeed with some probability. Our model also allows for the fact that the direct intervention may destabilise the foreign ally through unintended, though anticipated, consequences.

Our main purpose is to examine what determines which alternatives appear in the Subgame Perfect Nash Equilibrium (SPNE) strategy of the home country and how the chosen alternatives depend on the technology of the interaction between direct and indirect intervention. We begin with general functional forms, which allows us to highlight the different strategic effects present in the model. Some results are as one might expect. For instance, an increase in either direct intervention or allied effort will reduce the resources available to the terrorists and their ability to attack both the home and foreign country. An increase in defensive investment by home causes the terrorists to switch their target from the home country to the foreign country. However, the analysis also brings out that these effects often counteract each other, producing ambiguous results. For instance, the impact of the home government's defense and direct intervention on the ally's effort is ambiguous, depending on the size of the counteracting effects. This is to be expected; the nature of such

[1] We discuss the complete information assumption together with the other assumptions in Section 2.

strategic interactions is likely to be sensitive to the particular circumstances of the conflict. These circumstances will differ from case to case, thus there are no "typical" values of the parameters. In particular, direct intervention can take many different forms: cyber-warfare, drones, manned aircraft, covert special forces, intelligence operatives, standard land forces, etc.

The full equilibrium of the model is analytically intractable and we must therefore rely on numerical solutions. However, some partial equilibria results are possible, but even there signs are often ambiguous: actions have offsetting effects, a characteristic of conflict. Numerical computation involves assuming specific functional forms, simulating calibrated models, to allow us to obtain comparative statics results for the full set of equilibrium strategies presented in the game. We impose ranges for some parameters, but calibrate others to target outcomes which at least in principle are observable. Then, we investigate the effect of parameter variations within these ranges on the likelihood of non-zero investments in each of the three alternatives in specific circumstances. We are not trying to match a particular set of empirical data, conflicts are too varied. Rather we investigate a reasonable range of imposed parameters and target outcomes. We then relate particular parameterisations to examples of conflicts which would fit such circumstances.

We are particularly interested in the extent to which the degree of substitution between home and foreign military efforts and the relative outcomes effectiveness of the two countries' efforts determine whether direct intervention is an equilibrium outcome. Given this, a constant elasticity of substitution production function is a natural way to parameterize the aggregation of home and allied forces.

Our analysis indicates that, if the forces of the home country and the foreign ally are fairly close substitutes, subsidising the ally has a positive effect on the ally's efforts, making an investment in subsidy more likely, while direct intervention may crowd out allied effort, making intervention less likely. However, if the efforts are not close substitutes, such as air power provided by the home country and "boots on the ground" provided by the ally, investment in direct intervention becomes part of an equilibrium strategy. In many circumstances, the degree of substitution may be low if the allies have local knowledge and expertise and if foreign occupying forces are seen as illegitimate by the local population (see Salehyan (2010)). The balance between subsidy and direct intervention will also depend on the relative military effectiveness of the forces of the two countries as well as the degree of substitutability. Using comparative statics around our equilibrium non-intervention state, we show that direct military intervention becomes more likely as its relative military effectiveness increases. Higher military effectiveness needs not correspond to more technologically advanced, for instance the foreign ally, e.g. Pakistan, may be more effective at infiltrating terrorist networks as they may have a shared identity through language, religion or culture.

The rest of the paper is structured as follows. Section 2 discusses the relation of our model to the literature. Section 3 formally describes the game, sets out the equilibrium and provides some useful analytical results. Section 4 provides the quantitative analysis of the equilibrium calibrated to fit the outcomes to stylized facts. Section 5 concludes.

2 Features of the model

Before conducting the formal analysis, this section discusses some features of our model and relates it to the literature. The model broadly follows the framework used by Bandyopadhyay *et al.* (2011), hereafter BSY, which explains general aid rather than direct intervention, the focus of our analysis. Our model differs from BSY in the central choice variables and the methodology used to analyse the comparative statics. In their paper, the counter-terrorist strategy of the home country has three elements. Their first two choice variables, defensive effort and military subsidy, are the same as us, but their third element is a general subsidy that acts as a lump sum transfer to the resources of the foreign country. They analyze the interaction between the different decision variables along the backward induction solution of the model. They present a comparative statics result of a change in the utility weight that the terrorist group puts on the home country's interests on the equilibrium strategies. However, they note on page 440 that comparative statics that involve all three choice variables are analytically intractable, thus they analyse a reduced form model which ignores regime instability and where general aid is not a choice variable for the home government. In contrast, our quantitative methodological approach allows us to conduct a set of comparative statics on the full counter-terrorist strategies for the home country. We also do not need to make the assumption that third order derivatives are zero that they require.

Like BSY, we consider a complete information game. In practice, the home or foreign country may not be able to monitor the effort or type of the other country from their observed actions. Brauer and van Tuyl (2008, chapter 3) examine the principal-agent problem faced by Renaissance Italian Cities who hired Condottieri to fight for them. In this case, there were significant problems of moral hazard and adverse selection and the Condottieri often found it more profitable to attack the city that hired them, rather than that city's enemies. There is also a literature on asymmetric information games with terrorists (see e.g., Lapan and Sandler (1993) and Arce and Sandler (2007), Arce and Sandler (2010)). Salehyan (2010) uses a principal agent framework, though not a formal model, to examine the choice between attacking an enemy directly and indirectly through the support of insurgents operating in your enemy's country. He emphasises the importance of both divergent interests and informational asymmetries. We focus on the divergent interests of the home and foreign countries rather than incomplete information, since the former is likely to be more important in practice. In addition, introducing asymmetric information would add to the complexity of what is already a complex model.

We assume that the terrorists are the only enemies that the home and foreign government face. In practice, each may have multiple enemies. In particular, the foreign government may face a regional rival and divert the military aid, which the home government intended to be used against the terrorists, to use against the rival. In such circumstances, Boutton (2014) argues that the foreign government may have an incentive not to disarm terrorist groups, but rather to play up the threat from terrorism in order to continue receiving aid. For instance, he argues that Pakistan, seeing India as a greater threat than the Taliban, diverted a substantial portion of the US military aid, intended for use against the Taliban, to boost its military capability to fight India. He finds that while US foreign aid can help decrease terrorist activity in non-rivalrous states, the opposite is true in states with at least one rival. A similar argument is also made in Bapat (2011), there a stylized game theoretical framework is presented where military aid, rather than eliminating the terrorist threat, simply keeps host states from negotiating with the terrorists. Boutton (2014) is primarily an empirical paper and does not provide a formal model of the process. Neither our model nor that of BSY allows for such an effect explicitly, but it could be implicitly allowed for by adjustments to the equation determining allied efforts against the terrorists as a function of home country subsidy.

Our concept of direct intervention effort can be linked to the idea of pre-emptive effort present in a large closely related literature. Counter-terrorist efforts tend to be described in two basic types: defensive effort and pre-emptive or pro-active effort. The basic difference between the two is that, whereas defensive effort makes it harder for terrorists to attack specific targets (see e.g., Sandler and Lapan (1988)), pre-emptive effort has a direct negative impact on the success probability of attack or damage on any target (see e.g., Sandler and Siqueira (2006) and Bandyopadhyay and Sandler (2011)). A key element of pre-emptive effort is that a country can commit to it before the terrorists' group decides the attack efforts.[2] In our paper, direct intervention effort is committed to before the terrorists' choices and it affects the resources that the terrorists have at their disposal to organize terrorist attacks. However, direct intervention effort also affects the counter-terrorist effort of the foreign country, where terrorists are assumed to be based.

We assume that direct intervention and foreign effort interact to determine the resources available to the terrorists. We use a technology for this interaction which allows for different levels of substitution between home and foreign efforts. Our modelling can be related to Bandyopadhyay and Sandler (2014). In their paper, terrorism is set in the context of immigration policy. Pre-emptive efforts lower the incentives that individuals have to become terrorists. They endogeneize the production technology for terrorist damage and assume that it is different for attacks on developing or developed countries, with skilled terrorists being required for attacks on developed countries. They also assume that the pre-emptive effort of the two target countries can affect different inputs of the production technology. In their paper, the pre-emptive effort is the only choice variable that the developed country has at its disposal, there is no trade off across policy options. Their focus is the analysis of the comparative statics of the impact of exogenous immigration policies on target country efforts and the comparison of different modeling frameworks in relation to the timing of decisions of target countries (simultaneous versus sequential) or the choice variable of the developed country (pre-emptive or defensive effort).

[2] Das and Chowdhury (2014) analyse the choice of attack or defense when a number of countries have a common terrorist enemy and also argue that some of the assumptions used in the reduced form approach of Sandler and Siqueira (2006) may not be robust to explicit modelling of terrorist behaviour. In particular, they question the assumption that an increase in defence (security-deterrence) by one country induces the terrorist organisation to focus more on other target countries.

We assume that direct intervention by the home government could have "blowback" effects that help the terrorists. For instance, the continued US drone strikes in Pakistan, despite repeated protests by the Pakistani government, has made the Pakistani government seem ineffective and unable to maintain its sovereignty. This resulted in the Pakistani government reducing co-operation with the US in anti-Taliban activities. In addition, Boyle (2013), discussing the blowback from the US use of drones argues "that drone strikes corrode the stability and legitimacy of local governments, deepen anti-American sentiment and create new recruits for Islamist networks aiming to overthrow these governments." If such direct intervention undermines the legitimacy of the foreign government, this may cause the population to be less supportive of the regime, thus less likely to provide the regime with information about the terrorists. In either case, the probability of a successful terrorist attack, either on the foreign country or the home country, is increased. For convenience, we follow BSY and label this "regime change". In their case, the probability of regime change increases with military subsidy and decreases with general aid, in our framework we focus on the destabilizing impact of direct intervention. Rosendorff and Sandler (2004) examine backlash or blowback effects of attacks on terrorists, on their recruitment and on general grievance (see also Arce and Sandler (2010)), Bloom (2010) and Jacobson and Kaplan (2007)). There is a substantial controversy over whether counter-insurgency warfare is best prosecuted by military means or trying to win "heart and minds" in order to lower population support for the terrorist group.[3] Dear (2014) examines the effectiveness of one form of direct military action: targeting the leaders of the terrorists. He gives a number of examples where even killing terrorist leaders can be counter-productive. For instance, the killing of a relatively moderate leader can lead to their replacement by a much more violent leadership as happened with Boko Haram after the Nigerian government killed Mohammad Yusuf in 2009.

This blowback that we label regime change is an anticipated, but undesired, by-product of the direct intervention. Of course, foreign intervention may be designed to effect regime change as in Aidt and Albornoz (2010) where foreign direct investments provide a motive for the investing country to intervene in order to induce regime transitions or regime

[3] In another related paper, Dunne *et al.* (2006) prove that a defensive type of conflict can arise as an equilibrium result of the conflict between an incumbent and a contestant group to avoid military confrontation, which may encourage asymmetric conflict.

consolidation. Bonfatti (2011) has a related model in which the rationale for foreign intervention is to facilitate the rule of groups that are more dependent on foreign economic policy. Bove *et al.* (2015) develop a three party model of the decision by a third party to intervene in a civil war, emphasising the role of oil in motivating third party intervention. Albornoz and Hauk (2010), Enders and Sandler (2011) and Hoffman (1999) provide additional discussion of how conflict may lead to regime change.

3 The game

3.1 Elements of the game

In this section, we introduce the main elements of our model. There are three players, the home country H who decides on defensive effort e^H to counter terrorist attacks at home, direct intervention (pro-active counter-terrorist effort), e^{HF}, and indirect intervention, a military aid package to a foreign ally F, the second player, to assist its efforts against the third player, the terrorists.[4]

The military aid package is a subsidy to allied effort αe^F where e^F is the effort chosen by the foreign ally.

The third player is a terrorist organization who chooses attack effort aimed at the home country, a^H and the foreign country a^F subject to their resource constraint

$$a^H + a^F = M(e^F, e^{HF}); \quad M_1, M_2 < 0, \; M_{11}, M_{22} > 0. \tag{1}$$

The counter-terrorist efforts of the home and foreign countries reduce the resources available to the terrorists. Thus the choice variables are e^H, e^{HF} and α for the home country, e^F for the foreign recipient country and, a^H and a^F for the terrorists.[5]

[4] Given our interest in corner solutions, an alternative to making investment e^{HF} a continuous variable, would be to make it a discrete binary variable: either directly intervene or not. Sometimes the decision is binary, bomb Syria or not; but often there is both a discrete and a continuous dimension: decide whether to put troops into Afghanistan and if so how many. In addition, from a modelling perspective making e^{HF} continuous treats it symmetrically with the investments in the other two alternatives.

[5] Regarding partial and full derivatives, the following notation is adopted. Consider a function of two variable $f(x, y)$. Then $f_1 \equiv \frac{\partial f}{\partial x}$, $f_2 \equiv \frac{\partial f}{\partial y}$, $f_{11} \equiv \frac{\partial^2 f}{\partial x^2}$, $f_{22} \equiv \frac{\partial^2 f}{\partial y^2}$ and $f_{12} = f_{21} \equiv \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ in the usual way. For conciseness for a function of one variable, $f(x)$ we define $f_1 \equiv \frac{df}{dx}$ and $f_{11} \equiv \frac{d^2 f}{dx^2}$. To completely characterize the equilibrium we will need higher derivatives of the form $f_{111} \equiv \frac{\partial^3 f}{\partial x^3}$, $f_{112} \equiv \frac{\partial^3 f}{\partial x^2 \partial y}$ etc. Again for conciseness for a function of one variable, $f(x)$, we denote $f_{111} \equiv \frac{d^3 f}{dx^3}$.

As discussed above, direct intervention may cause a regime change that increases terrorist effectiveness. The probability of a regime change is given by

$$p^F = p^F(e^{HF}); \quad p_1^F > 0. \tag{2}$$

The relative effectiveness of the efforts of the home and foreign countries in reducing the resources of the terrorists, $M(\cdot)$ in Eq. (1), will play a central role in the model. Also of importance will be the probabilities of a successful terrorist attack on country H and country F and how these are affected by a regime change, the probability of which is given by Eq. (2).

The probability of a successful terrorist attack on the home country H, in the absence of regime change in the foreign country F, is determined by the defensive effort of H and the effort terrorists devote to attacking H:

$$\sigma^H = \sigma^H(e^H, a^H); \quad \sigma_1^H < 0,\, \sigma_2^H > 0,\, \sigma_{21}^H < 0,\, \sigma_{11}^H > 0,\, \sigma_{22}^H < 0. \tag{3}$$

Regime change increases the probability of a successful terrorist attach to

$$\tilde{\sigma}^H = (1+\eta)\sigma^H(e^H, a^H). \tag{4}$$

The probability of a successful terrorist attack on the foreign country F just depends on the effort terrorists devote to attacking F:

$$\sigma^F = \sigma^F(a^F); \quad \sigma_1^F > 0, \quad \sigma_{11}^F < 0. \tag{5}$$

The counter-terrorist efforts of H and F will influence the probability of a successful terrorist attack on F, through their impact on a^F through Eq. (1). Again, if regime change takes place, this is scaled up to

$$\tilde{\sigma}^F = (1+\eta)\sigma^F(a^F). \tag{6}$$

We can now write down the resource constraints and the payoffs of the players. Country H suffers not only from a successful terrorist attack at home, with a cost T^H, but also from a successful terrorist attack on country F, with a cost T^{HF}. This cost could arise from various forms of interdependence, for instance because H has foreign direct investment in country F, which can be damaged by terrorist attacks or because terrorist attacks disrupt oil supplies from F to H. We define the potential national income of country H as $\bar{Y}^H$, substitute for $\tilde{\sigma}^H$ and $\tilde{\sigma}^H$ from Eqs. (4) and (6) and define

$$\gamma(e^{HF}, \eta) \equiv 1 + p^F(e^{HF})\eta. \tag{7}$$

Then, the expected income of country H can be written

$$\begin{aligned} Y^H &= \bar{Y}^H - \underbrace{(1-p^F)(\sigma^H T^H + \sigma^F T^{HF})}_{\text{E(costs) without regime change}} - \underbrace{p^F(\tilde{\sigma}^H T^H + \tilde{\sigma}^F T^{HF})}_{\text{E(costs) with regime change}} - \underbrace{(e^H + e^{HF} + \alpha e^F)}_{\text{military-aid expend.}} \\ &= \bar{Y}^H - \gamma(e^{HF}, \eta)[T^H \sigma^H(e^H, a^H) + T^{HF}\sigma^F(a^F)] - (e^H + e^{HF} + \alpha e^F) \\ &= U^H(e^H, e^{HF}, \alpha, e^F, a^H, a^F) \end{aligned} \tag{8}$$

The payoff of the H country can be written in terms of the choice variables as $Y^H = U^H(e^H, e^{HF}, \alpha, e^F, a^H, a^F)$ where the direct, indirect efforts and the subsidy rate (e^H, e^{HF}, α) are the choices of country H, the foreign country's counter-terrorist effort e^F is the choice of country F, and the efforts devoted to attacking H and F, (a^H, a^F), are the choices of the terrorists.

Similarly, we have the expected national income for country F

$$\begin{aligned} Y^F &= \bar{Y}^F - \gamma(e^{HF}, \eta)\sigma^F(a^F)T^F - e^F(1-\alpha) \\ &= U^F(e^F, e^{HF}, \alpha, a^F), \end{aligned} \tag{9}$$

where T^F is the cost a successful terrorist attack inflicts on country F.

Finally, the aim of the terrorists is to inflict damage on countries H and F with weights ϕ^H and $\phi^F = 1 - \phi^H$ respectively. Thus their payoff is

$$\begin{aligned} U^T &= \gamma(e^{HF}, \eta)[\phi^H\{T^H \sigma^H(e^H, a^H) + T^{HF}\sigma^F(a^F)\} + \phi^F T^F \sigma^F(a^F)] \qquad (10) \\ &= U^T(e^H, e^{HF}, e^F, a^H, a^F). \end{aligned}$$

We have now described the military technology, resource constraints and payoffs for each of the three players and can now consider the equilibrium.

3.2 Equilibrium

The equilibrium concept is a complete information backward induction outcome with country H as first mover, country F as second mover and the terrorists as third mover. Thus country H is able to commit with respect to the moves of country F and the terrorists, and country F can commit with respect to the terrorists. The backward solution can be described in the following way:

- **Stage 3**: Terrorists maximize $U^T(e^H, e^{HF}, e^F, a^H, a^F)$ with respect to $a^H \geq 0$ and $a^F \geq 0$ given their resource constraint Eq. (1) and given actions undertaken at stages 2 and 1, e^F, e^{HF}, e^H and α,

- **Stage 2**: The F country maximizes $U^F(e^F,\, e^{HF},\, \alpha,\, a^F)$ with respect to $e^F \geq 0$ given the reaction function $a^F(e^H, e^{HF}, e^F)$ from stage 1 and given e^H, e^{HF}, α,

- **Stage 1**: The H country maximizes $U^H(e^H,\, e^{HF},\, \alpha,\, e^F,\, a^H,\, a^F)$ with respect to $e^H \geq 0$, $e^{HF} \geq 0$, $\alpha \in [0, 1]$ given reaction functions $a^F(e^H, e^{HF}, e^F)$, $a^H(e^H,\, e^{HF},\, e^F)$ and $e^F(e^H, e^{HF}, \alpha)$.

We now proceed to describe the backward induction solution using general functions. This will allow us to provide economic intuition of the strategic interactions between the different decision makers. By focusing on the interior solutions, the use of the Implicit Function Theorem will enable us to discuss the impact of the decisions of the home and foreign countries on the decisions of the terrorists at stage 3 of the game. We will also be able to discuss the impact of the decisions of the home government on the decisions of the foreign country. The details of the first order conditions (FOCs) are given in Appendix A. In Appendix B, we provide details of the application of the Implicit Function Theorem at stages 2 and 3 in the game.

3.2.1 Stage 3

To maximize Eq. (10) with respect to $a^F \geq 0$ and $a^H \geq 0$ given the constraint $a^H + a^F \leq M(e^F, e^{HF})$ and previous actions e^H, e^{HF}, e^F, define the Lagrangian

$$\mathcal{L} = \sigma^H(e^H, a^H) + \tau\sigma^F(a^F) + \lambda^M(M(e^F, e^{HF}) - a^H - a^F) + \lambda^{a^H} a^H + \lambda^{a^F} a^F, \quad (11)$$

where $\tau \equiv \frac{\phi^H T^{HF} + \phi^F T^F}{\phi^H T^H}$ and $\lambda^M,\ \lambda^{a^H},\ \lambda^{a^F} \geq 0$ are multipliers. Details of the first order conditions, FOCs, are in Appendix A. Solving them gives the *reaction functions of the terrorists*

$$a^H = a^H(e^H, e^{HF}, e^F), \quad (12)$$

$$a^F = a^F(e^H, e^{HF}, e^F). \quad (13)$$

Equations (12) and (13) constitute the stage 3 equilibrium given previous actions e^H, e^{HF}, e^F.

Using the Implicit Function Theorem, it is straightforward to prove the following results for interior solutions (see Appendix B for details).

$$a^i_j(e^H, e^{HF}, e^F) < 0 \text{ except } a^F_1(e^H, e^{HF}, e^F) > 0. \quad (14)$$

That is, an increase in either direct intervention effort or foreign government effort will reduce the resources available to the terrorists and this will unambiguously reduce their attack effort both at home and abroad. An increase in defensive home effort however will discourage attack effort at home but it will encourage attack effort abroad.

It is also possible to prove (see Appendix B) that changes to the weights that the terrorists put on attack on home country have an ambiguous effect on their attack efforts (the signs of $\frac{\partial a^H}{\partial \phi^H}$ and $\frac{\partial a^F}{\partial \phi^H}$ are ambiguous). However, an increase in T^H, the cost inflicted on the home country by a successful attack, increases the attack effort that the terrorists devote to the home country and reduces the effort devoted to the foreign country ($\frac{\partial a^H}{\partial T^H} > 0$ and $\frac{\partial a^F}{\partial T^H} < 0$). These results are in line with BSY.

3.2.2 Stage 2

At the second stage, country F maximizes its objective function Eq. (9) with respect to its effort e^F, given the reaction functions Eqs. (12) and (13) and previous actions by country H, e^H, e^{HF} and α. We define the Lagrangian

$$\mathcal{L} = \bar{Y}^F - \gamma(e^{HF}, \eta)\sigma^F(a^F)T^F - e^F(1-\alpha) + \lambda^{e^F} e^F, \tag{15}$$

where $\lambda^{e^F} \geq 0$ is the multiplier.

The FOCs (see Appendix A for details) lead to the *reaction function of country F* which can be written as

$$e^F = e^F(e^H, e^{HF}, \alpha). \tag{16}$$

Focusing on the interior solutions, Appendix B establishes that in general, the sign of the impact of defensive effort or direct intervention on the effort of the foreign country ($e_1^F(e^H, e^{HF}, \alpha)$ and $e_2^F(e^H, e^{HF}, \alpha)$) are ambiguous. However, an increase to the military subsidy encourages foreign effort ($e_3^F(e^H, e^{HF}, \alpha) > 0$).

Using the Implicit Function Theorem, we can obtain the expression which will determine the sign of the impact of defensive effort on foreign effort:

$$e_1^F = \frac{\gamma T^F \left(\sigma_{11}^F a_1^F a_3^F + \sigma_1^F a_{31}^F\right)}{Y_{e^F e^F}^F}.$$

Note that, for the second order conditions, SOCs, to hold we require concavity in the objective function of the foreign government, $Y_{e^F e^F}^F < 0$ (the conditions for this to hold are discussed in Appendix B). If, following BSY, we assume third order derivatives of the probability function are zero, then $a_{31}^F = 0$ and e_1^F would have a negative sign. However, in general, the expression has an ambiguous sign. An increase in home defensive effort would encourage terrorist attack effort at foreign ($a_1^F > 0$) and therefore, lower the incentive for foreign effort since $\sigma_{11}^F < 0$. However, the increase in home defensive effort will also affect the impact of foreign effort on attack effort of the terrorists in the foreign country a_{31}^F, and the sign of this will be in general ambiguous.

The impact of direct intervention on foreign effort is given by

$$e_2^F = \frac{T^F a_3^F \left(\gamma_1 \sigma_1^F + \gamma \sigma_{11}^F a_2^F\right) + \gamma T^F \sigma_1^F a_{32}^F}{Y_{e^F e^F}^F}.$$

Even if we assume that third order derivatives are zero, we would still have an ambiguous sign for e_2^F. This is due to the fact that, as further illustrated in Appendix B, the sign of a_{32}^F (how direct intervention in the foreign country affects the impact that foreign effort has on the attack effort of the terrorists in the foreign country) will depend on M_{12}, how the direct intervention and foreign effort interact in the reduction of the terrorists' resources. Thus, the sign of M_{12} is a crucial element in our analysis. For instance, if $M_{12} > 0$, then increasing the effort of one party (home or foreign) reduces the negative impact of the other party's effort on terrorist capacity M.

Finally, we have an expression for the impact of the military subsidy on foreign effort

$$e_3^F = \frac{1}{-Y_{e^F e^F}^F} > 0.$$

Clearly, an increase in the foreign effort subsidy, α, encourages foreign effort. This is in line with BSY.

What we learn from this section is that, even if we make the same assumption about third derivatives as BSY, we cannot sign e_2^F: the impact of direct intervention on foreign effort. This is realistic, the impact of US direct intervention in World War II on British efforts to fight Germany was very different from the impact of US direct intervention in South Vietnam on the South Vietnamese efforts at fighting the Viet Cong. As with BSY, our results in stage 2 indicate that it is not possible to produce analytical results for the overall SPNE in our model. We proceed however, to describe the first stage of the game as it will provide a basis for the intuition of the results in our calibrated model.

3.2.3 Stage 1

We now proceed to stage 1 of the game where the home government chooses its counter-terrorist strategy. In this section, we discuss the intuition of the FOCs that define the internal solutions. In our calibration however, we will allow for corner solutions as well. Maximizing Eq. (8) with respect to e^H, e^{HF} and α respectively, given the reaction functions Eqs. (12), (13) and (16) gives the three FOCs for an internal solution e^H, e^{HF},

$\alpha > 0$:

$$\gamma(e^{HF},\eta)[T^H(\sigma_1^H(e^H,a^H) + \sigma_2^H(e^H,a^H)\left(a_1^H(e^H,e^{HF},e^F) + a_3^H e_1^F\right)$$
$$+ \quad T^{HF}\sigma_1^F(a^F)\left(a_1^F(e^H,e^{HF},e^F) + a_3^F e_1^F\right)] + \left(1 + \alpha e_1^F\right) = 0. \tag{17}$$

The first element of the FOC for home defensive effort in country H above represents the positive direct impact of increasing home defensive effort as it directly reduces the probability of a successful attack at home $\sigma_1^H < 0$. However, the increase in home defensive effort e^H encourages terrorists foreign attack effort, $a_1^F > 0$, which in turn increases the probability of a successful attack on home country interests in the foreign country, $\sigma_1^F > 0$. In addition, an increase in home defensive effort affects the foreign country effort in an ambiguous way, this change will affect both the cost of the subsidy given to the foreign government (αe_1^F) and, indirectly, the attack efforts of the terrorists on countries H and F, $(a_3^H e_1^F$ and $a_3^F e_1^F)$. For the case when $e_1^F < 0$, $a_3^F e_1^F > 0$, hence causing an increase in the expected damage on national interests at foreign (i.e., the second term above is positive).

$$p_1^F(e^{HF})\eta[T^H\sigma^H(e^H,a^H) + T^{HF}\sigma^F(a^F)] + \gamma(e^{HF},\eta)[T^H\sigma_2^H(e^H,a^H)\left(a_2^H(e^H,e^{HF},e^F) + a_3^H e_2^F\right)$$
$$+T^{HF}\sigma_1^F(a^F)\left(a_2^F(e^H,e^{HF},e^F) + a_3^F e_2^F\right)] + (1 + \alpha e_2^F) = 0. \tag{18}$$

The first element on the left hand side, LHS, of the FOC for direct intervention effort above is positive, it represents the positive impact that direct intervention has on the probability of successful attack through its increase in the likelihood of regime change. The third element in the FOC represents the impact on the cost of a change in the direct intervention effort. The second element represents the impact of direct intervention on the terrorists attack effort. An increase in direct intervention effort decreases this attack effort directly ($a_2^H < 0$ and $a_2^F < 0$), but, it has an ambiguous indirect effect ($a_3^H e_2^F$, $a_3^F e_2^F$) whose sign depends on the sign of e_2^F. If home direct effort discourages the foreign government's effort, $e_2^F < 0$ the indirect effect will undermine the direct effect. As already discussed, the sign of e_2^F will be determined by how the home and foreign effort interact on the reduction of the terrorist resources.

$$\begin{aligned}\gamma(e^{HF},\eta)[T^{H}\sigma_2^{H}(e^{H},a^{H})a_3^{H}(e^{H},e^{HF},e^{F}) &+ T^{HF}\sigma_1^{F}(a^{F})a_3^{F}(e^{H},e^{HF},e^{F})]e_3^{F}(e^{H},e^{HF},\alpha)] \\ +(e^{F}+\alpha e_3^{F}) &= 0. \qquad (19)\end{aligned}$$

Given our results in stages 2 and 3, we know that the first element in the FOC for military subsidy above is negative. It represents the negative impact that the military subsidy to the foreign government, α, has on the expected damage on national interests by the terrorists. The subsidy induces an increase in military effort by the foreign government $\left(e_3^F > 0\right)$ and this reduces the terrorists attack effort both at foreign $\left(a_3^F < 0\right)$ and home $\left(a_3^H < 0\right)$ countries.

It seems intuitive to argue that anything that enhances the impact of direct effort on the probability of regime change should discourage such effort. Also the incentive to undertake direct effort, will be determined by the relative effectiveness of the military efforts of the foreign, e^F, and the direct intervention by the home government, e^{HF}, in reducing terrorist resources: M_1 and M_2. Another important element will be the ease with which a military subsidy is able to induce foreign effort, T^F and ϕ^Fwill be important parameters to consider in this respect. If regime change caused a re-weighting of targets on the objective function of the terrorist towards home, a lower ϕ^F, this would decrease the incentive for direct action as it would reduce the incentive of the foreign country to invest in effort.

4 Quantitative analysis

So far, we have considered general functional forms. This has allowed us to highlight the different strategic effects present in our model and how these effects can counteract each other producing ambiguous results. In particular, the impact of defence and direct intervention by home on foreign effort is ambiguous. This is to be expected; the nature of such strategic interactions is likely to be sensitive to the particular circumstances of the conflict. These circumstances will differ from case to case, thus there are no ”typical” values of the parameters. In particular, direct intervention can take many different forms: boots on the ground, manned aircraft, drones, special forces, intelligence operatives, cyber-warfare etc. Each of these will interact differently with the counter-terrorist efforts of the foreign country. In the present section, we introduce specific functional forms and calibrate the model using parameters that could describe particular conflicts in order to clarify the likely nature of the strategic effects at play in our framework. It will also allow us to obtain comparative statics results for the full set of equilibrium strategies presented in the game. The calibration is particularly useful in illustrating the importance of the degree of substitution between home and foreign military efforts and the relative effectiveness of the two countries' efforts at reducing the terrorist resources in determining whether direct intervention might turn out to be an equilibrium outcome. Since, direct intervention is a controversial choice we choose our parameters to focus on whether intervention happens or not. We present our results using figures which we will interpret in the light of our general model.

4.1 Choice of functional forms

There are three sets of functional forms to choose in order to conduct numerical solutions: the probability of regime change, the success probabilities and the terrorist capacity function. We consider these in turn:

Probability of regime change: $p^F = p^F(e^{HF})$; $(p^F)' > 0$. Given this is a probability, $p^F \in (0, 1)$, a logit functional form is a natural choice:

$$\log \frac{p^F}{1 - p^F} = \alpha_p + \beta_p e^{HF}, \tag{20}$$

which can be written as

$$p^F = \frac{\exp(\alpha_p + \beta_p e^{HF})}{1 + \exp(\alpha_p + \beta_p e^{HF})}. \tag{21}$$

Terrorists' success probability of an attack on F: $\sigma^F = \sigma^F(a^F)$; $\sigma_1^F > 0$; $\sigma_{11}^F < 0$ chosen so that $\sigma^F \in [0, 1)$ and $\sigma^F(0) = 0$.

By analogy with Eq. (21) we choose

$$\sigma^F(a^F) = \frac{\exp(a^F) - 1}{1 + \exp(a^F)}, \tag{22}$$

then

$$\begin{aligned}\sigma_1^F(a^F) &= \frac{2\exp(a^F)}{(1+\exp(a^F))^2} > 0,\\ \sigma_{11}^F(a^F) &= \frac{2\exp(a^F)(1-\exp(a^F))}{(1+\exp(a^F))^3} < 0 \text{ if } a_F > 0.\end{aligned}$$

Therefore, we have that one unit of terrorist capacity results in a success probability of $\frac{\exp(1)-1}{1+\exp(1)} = 0.4621$ in the F country and (from below) in the H country if no counterterrorist effort is expended. In other words, a terrorist unit (or cell) results in $1/0.4621 = 2.164$ successful attacks.

Terrorists' success probability of an attack on H: $\sigma^H = \sigma^H(e^H, a^H)$; $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{21}^H < 0$, $\sigma_{11}^H > 0, \sigma_{22}^H < 0$.

We choose a contest success function of the general form

$$\sigma^H(e^H, a^H) = \frac{f(a^H)}{f(a^H) + f(\beta_\sigma e^H)},$$

where $f(\cdot)$ is an increasing function of normalized effort. A contest success function of this form fulfills the five axioms of Skaperdas (1996) for any $n \geq 2$ player contest. Hwang (2012) discusses the choice of $f(.)$.

In addition, we impose the conditions

$$\begin{aligned}\sigma^H(e^H, 0) &= 0,\\ \sigma^H(e^H, a^H) &\to 0 \text{ as } e^H \to \infty,\\ \sigma^H(e^H, a^H) &\to 1 \text{ as } a^H \to \infty,\end{aligned}$$

for any e^H, $a^H \geq 0$. The choice $f(x) = \exp(x) - 1$ so that

$$\sigma^H(e^H, a^H) = \frac{\exp(a^H) - 1}{\exp(a^H) + \exp(\beta_\sigma e^H) - 2},$$

satisfies all these conditions.

With these functional forms we have the following first and second partial derivatives

$$\begin{aligned}
\sigma_1^H(e^H,a^H) &= -\frac{\beta_\sigma \exp(\beta_\sigma e^H)(\exp(a^H)-1)}{(\exp(a^H)+\exp(\beta_\sigma e^H)-2)^2} < 0 \text{ for all } a^H > 0, \\
\sigma_2^H(e^H,a^H) &= \frac{\exp(a^H)(\exp(\beta_\sigma e^H)-1)}{(\exp(a^H)+\exp(\beta_\sigma e^H)-2)^2} > 0 \text{ for all } e^H > 0, \\
\sigma_{11}^H(e^H,a^H) &= \frac{{\beta_\sigma}^2 \exp(\beta_\sigma e^H)(\exp(a^H)-1)(2+\exp(\beta_\sigma e^H)-\exp(a^H))}{(\exp(a^H)+\exp(\beta_\sigma e^H)-2)^3} \\
&\geq 0 \text{ iff } 2 \geq \exp(a^H)-\exp(\beta_\sigma e^H), \\
\sigma_{22}^H(e^H,a^H) &= -\frac{\exp(a^H)(\exp(\beta_\sigma e^H)-1)(2-\exp(\beta_\sigma e^H)+\exp(a^H))}{(\exp(a^H)+\exp(\beta_\sigma e^H)-2)^3} \\
&\leq 0 \text{ iff } 2 \geq \exp(\beta_\sigma e^H)-\exp(a^H), \\
\sigma_{12}^H(e^H,a^H) &= -\frac{\beta_\sigma \exp(\beta_\sigma e^H + a^H)(\exp(\beta_\sigma e^H)-\exp(a^H))}{(\exp(a^H)+\exp(\beta_\sigma e^H)-2)^3} \\
\leq 0 & \qquad \text{for all } a^H, e^H \geq 0 \text{ if } \exp(\beta_\sigma e^H)-\exp(a^H) > 0.
\end{aligned}$$

It follows that all the conditions $\sigma^H = \sigma^H(e^H, a^H)$; $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{21}^H < 0$, $\sigma_{11}^H > 0$, $\sigma_{22}^H < 0$ are satisfied iff a^H, $e^H \geq 0$, $\exp(\beta_\sigma e^H) > \exp(a^H)$ and

$$|\exp(\beta_\sigma e^H) - \exp(a^H)| < 2.$$

That is, iff normalized efforts by the H country and the terrorist in that country are not too far apart in equilibrium. These conditions impose the following bounds on σ^H

$$\frac{1}{2} - \frac{1}{2\exp(a^H)} < \sigma^H < \frac{1}{2}.$$

Terrorists' Capacity: $M(e^F, e^{HF})$; M_1, $M_2 < 0$, M_{11}, $M_{22} > 0$.

The $M(e^F, e^{HF})$ function aggregates the military effectiveness of foreign, e^F, and home, e^{HF}, efforts against enemy resources. In the literature on alliances, surveyed in Murdoch (1995), considerable attention is paid to how the military efforts of allies should be aggregated. The aggregation technology may be that strength depends on the simple sum of military expenditures, the best shot in the alliance or the weakest link. Since we want to examine the effects of differences in the degree of substitution between the efforts of H and F, a natural choice for the aggregation technology is a constant elasticity of substitution, CES, production function. We assume:

$$M(e^F, e^{HF}) = \bar{M} \exp(-E)$$

where E is CES:

$$E = \left((\beta_{MF} e^F)^\epsilon + (\beta_{MHF} e^{HF})^\epsilon\right)^{\frac{1}{\epsilon}} \qquad -\infty < \epsilon < 1.$$

The elasticity of substitution between the two forms of effort, e^F and e^{HF} is $(1-\epsilon)^{-1}$. For $\epsilon = 1$ we have the case of perfect substitutes, $\epsilon = 0$ corresponds to the Cobb-Douglas case with a unit elasticity of substitution, whilst as $\epsilon \to -\infty$ we approach the Leontief case, where they are not substitutes, both are required and overall effectiveness is given by the minimum of the two terms. This would be the case, for instance, if both foreign's land and home's air forces were required.[6]

We noted above that the relative military effectiveness of home and foreign forces is likely to be a major determinant of whether the home country undertakes direct military intervention. The parameters representing military effectiveness are β_{MHF} and β_{MF}. These will reflect the technologies available to the two governments. For instance β_{MHF} would be large relative to β_{MF} if the home government has access to drone technology not available to the foreign government.

With this functional form we have that

$$\begin{aligned}
M_1 &= -\bar{M}\exp(-E)\frac{\partial E}{\partial e^F} < 0, \\
M_2 &= -\bar{M}\exp(-E)\frac{\partial E}{\partial e^{HF}} < 0, \\
M_{11} &= \bar{M}\exp(-E)\left(\left(\frac{\partial E}{\partial e^F}\right)^2 - \frac{\partial^2 E}{\partial (e^F)^2}\right) > 0, \\
M_{22} &= \bar{M}\exp(-E)\left(\left(\frac{\partial E}{\partial e^{HF}}\right)^2 - \frac{\partial^2 E}{\partial (e^{HF})^2}\right) > 0, \\
M_{12} &= M_{21} = \bar{M}\exp(-E)\left(\frac{\partial E}{\partial e^F}\frac{\partial E}{\partial e^{HF}} - \frac{\partial^2 E}{\partial e^F \partial e^{HF}}\right).
\end{aligned}$$

[6] In principle there could be negative effects at this stage of the game: direct intervention by H reduces the effectiveness of F's efforts. But then, there would be no trade-off between the negative effects through the increase in the probability of regime change, accounted for in Stage 1, and the positive effects at Stage 3. In our rational agent model, this would not be an interesting case since then H would simply never directly intervene (but may still subsidise foreign effort).

Thus $M_1,\ M_2 < 0$ and $M_{11},\ M_{22} > 0$, but the sign of M_{12} is ambiguous since

$$M_{12} = \bar{M}E\exp(-E)\frac{\beta_{MF}^{\epsilon}\beta_{MHF}^{\epsilon}(e^F)^{\epsilon-1}(e^{HF})^{\epsilon-1}}{((\beta_{MF}e^F)^{\epsilon} + (\beta_{MHF}e^{HF})^{\epsilon})^2}(E - 1 + \epsilon).$$

Since $M_1 < 0$, $M_{12} > 0$ means that the higher the effort of one party (home or foreign) the lower the negative impact of the other party's effort on terrorist capacity M. For the case of perfect substitutes, $\epsilon = 1$ and $M_{12} > 0$. But for $\epsilon < 1$ there is a high level of capacity relative to its maximum at which $E = \log\frac{\bar{M}}{M} < 1 - \epsilon$ and $M_{12} < 0$. This condition can be written

$$\frac{M}{\bar{M}} > \exp(-(1-\epsilon)).$$

At that point higher effort on one party actually increases the other party's negative impact on terrorist capacity.[7]

4.2 Calibration strategy

To calibrate the parameters, we assume an observed baseline equilibrium for the variables and solve for model parameters consistent with those observations. This baseline equilibrium can be described in terms of a vector of outcomes $\underline{X} = f(\underline{\theta})$ where $\underline{\theta}$ is a vector of parameters. The calibration strategy is to choose a subset $\underline{X}_1$ of n observed outcomes to calibrate a subset $\underline{\theta}_1$ of n parameters. Partition $\underline{X} = [\underline{X}_1, \underline{X}_2]$ and $\underline{\theta} = [\underline{\theta}_1, \underline{\theta}_2]$. Then $\underline{\theta}_1$ is then found by solving

$$[\underline{X}_1, \underline{X}_2] = f([\underline{\theta}_1, \underline{\theta}_2]), \tag{23}$$

for $\underline{X}_2$ and $\underline{\theta}_1$, given $\underline{X}_1$ and $\underline{\theta}_2$. If such a solution exists for economically meaningful parameter values (usually real positive numbers) $\underline{\theta}_1$, then a successful calibration has been achieved.

To implement this calibration strategy in the model we have seven parameters to be set associated with success probabilities and terrorist capacity: α_p, β_p, η, β_σ, $\bar{M}$, β_{MF} and β_{MHF}. Further parameters associated with costs of successful attacks are T^H, T^{HF}, T^F and ϕ^H. These are the parameters $\underline{\theta}$ that determine the actions of the players e^H, e^{HF}, α for the H country, e^F for the F country and a^H, a^F for the terrorists. Outcomes

[7] But note that this result depends on the exponential form of the function M. If instead we choose a power function $M = \bar{M}E^{-1}$, then the sign of M_{12} is the same as $1 - \epsilon$ so for $\epsilon < 1$ we have that $M_{12} > 0$ unambiguously.

from these actions, also determined by $\underline{\theta}$, are the probabilities σ^H, $\tilde{\sigma}^H$, σ^F, $\tilde{\sigma}^F$, p^F and the capability M.

We can first pin down the maximum terrorist capacity $\bar{M}$ as follows. Consider a scenario in the F country where there is no counterterrorist effort ($e^F = e^{HF} = 0$). Then a *maximum success probability*, $(\sigma^F)^{max}$ is reached given by

$$(\sigma^F)^{max} = \frac{\exp(\bar{M}) - 1}{1 + \exp(\bar{M})} \Rightarrow \bar{M} = \log\left(\frac{1 + (\sigma^F)^{max}}{1 - (\sigma^F)^{max}}\right).$$

If we can observe $(\sigma^F)^{max}$, this then determines $\bar{M}$.

Second we impose $\phi^H = 1 - \phi^F$ and consider variations as different scenarios. For example $\phi^H = 0$ ($\phi^H = 1$) is the case where terrorists only target the F (H) country.

Third we construct fear factor outcomes from the equilibrium as follows. Consider a worst-case scenario where attacks in both countries are successful. Then, the costs incurred are $T^H + T^{HF}$ for the home country and T^F for the foreign country which compares with expenditures $e^H + e^{HF} + \alpha e^F$ for the home country and $e^F(1 - \alpha)$ in the foreign country. Then define '*fear factor*' parameters as the ratios of these costs

$$\begin{aligned} ff^H &= \frac{T^H + T^{HF}}{e^H + e^{HF} + \alpha e^F}, \\ ff^F &= \frac{T^F}{e^F(1 - \alpha)}, \end{aligned}$$

for the home and foreign countries respectively. Thus if we impose the ratio $\frac{T^{HF}}{T^H}$ by observing (or just targeting) these fear factors we can pin down T^H and T^F from any equilibrium of e^H, e^{HF} and α.

The three parameters in the terrorist capacity function, which describe how H and F's counter-terrorist efforts interact ϵ, β_{MF} and β_{MHF}, together with β_σ in σ^H, which determines the terrorists success probability are crucial for determining the choice of effort by all parties in creating and reducing terrorist activity. We impose the elasticity ϵ and consider variations as scenarios. For our baseline $\epsilon = 1$, we then solve for parameters β_σ and β_{MF} to achieve target probabilities σ^H and σ^F. This leaves β_{MHF} which we assume is equal to β_{MF}.

This leaves parameters determining the probability of regime change α_p and $\beta_p e^{HF}$ in Eq. (20) and η determining the effect of regime change on $\tilde{\sigma}^H$ in Eq. (4). We impose

β_p and η and then calibrate α_p to achieve a target for p^F. This completes the calibration strategy. Table 1 summarizes the procedure.

Variable	Target Outcome
β_σ	Home Success Probability σ^H
β_{MF}	Foreign Success Probability σ^F
α_p	Probability of Regime Change p^F
Inflicted Costs T^H	Home Fear Factor ff^H
Inflicted Costs T^F	Foreign Fear Factor ff^F
Inflicted Costs Abroad T^{HF}	Assume $T^{HF} = T^H$
Direct Intervention Effect on Capacity β_{MHF}	Assume $\beta_{MHF} = \beta_{MF}$
Max Military Capacity $\bar{M}$	Max of probability σ^F

Table 1: Parameters to Calibrate and the Target Outcomes

4.3 Equilibrium computation

We now present results for the following choice of imposed parameters values summarized in Table 1: $\phi^H = \phi^F = 0.5$, $\eta = 0.5$, $(\sigma^F)^{max} = 0.75$, $\beta_p = 0.1$ and $\epsilon = 1$. To calibrate the remaining parameters, we choose the following target outcomes: $\sigma^H = 0.1$, $\sigma^F = 0.2$, $p^F = 0.25$ and $ff^H = ff^F = 5$. With these targets we compute the parameters implied by the equilibrium as set out in Table 1. The results for the equilibrium and actual outcomes are set out in the first column of Table 2. The calibrated parameters turned out as: $\beta_\sigma = \beta_{MF} = \beta_{MHF} = 2.2$, $T^H = T^F = 1.5$. As can be seen from the Table, we were not able to hit the targets exactly but we came close.[8]

With these parameter values, we find a Stage 1 equilibrium with $e^H = 0.38$, $\alpha = 0.24$ and $e^{HF} = 0$ and $e^F = 0.53$ at Stage 2 of the game.[9] In this equilibrium, success probabilities are $\sigma^H = 0.10$ and $\sigma^F = 0.23$. Thus the Home country chooses not to intervene directly and 10% of attacks are successful in the home country and 23% of attacks in the foreign country. In Figure 1 and 2 variations in α around this equilibrium are plotted. In Figures 3 and 4, we plot variations in e^H and in Figures 5 and 6 variations in e^{HF} for the case of $\epsilon = 1$.

[8] One cannot assume that a solution to Eq. (23) exists for all equilibrium outcomes.

[9] We searched over a grid with intervals 0.005 in the space of stage 1 decisions (e^H, α, e^{HF}). We report the equilibrium corresponding to the *global* minimum of the welfare loss for country H equal to minus the utility given by Eq. (8). The grid was chosen to be consistent with the outcomes used in the calibration.

The plots in Figures 1 and 2 confirm the results from the general model. An increase in the foreign military subsidy α encourages foreign effort and this in turn will decrease the incentive that the terrorists have to invest in attack effort both home and foreign. As a result, the terrorist attack success probabilities decrease. Note that the Home welfare loss function is minimized at the baseline equilibrium value of $\alpha = 0.24$.

Figures 3 and 4 clarify the ambiguous effect that changes in home defensive effort has on foreign effort. Our plot indicates an initially positive and then declining impact of defensive effort on foreign effort. Note that for low values of home defensive effort, the plot for terrorist attack effort in country H, a^H, has a positive slope with the slope becoming negative later (this is also the case for the impact of e^H on a^F, which is illustrated by the slope turning from negative to positive for higher levels of defensive effort). Our Appendix demonstrates that the sign of the impact of home defensive effort on the home attack effort and foreign attack effort is reversed when $\sigma_{21}^H > 0$. This will happen if $e^H < \frac{a^H}{\beta_\sigma}$ or $e^H < \frac{a^H}{2.2}$. Note that the Home welfare loss function is minimized at the baseline equilibrium value of $e^H = 0.52$.

In Figures 3 and 4, we also see the indirect impact that defensive effort has on the attack efforts of the terrorist through its impact on foreign effort. As the Figures show, this impact is ambiguous as well, for low values of defensive effort increases in this effort will encourage foreign effort, a clear crowding out effect develops for higher levels of defensive effort. The indirect effect reinforces the impact that defensive effort has on foreign attack effort a^F, however, it generates a counteracting force for the direct impact of e^H on a^H. The sign of direct effect however prevails as described in our previous paragraph. Although our setting is different from BSY, they also get a crowding out effect of defensive effort on foreign effort under a $\sigma_{21}^H < 0$ assumption.

Figures 5 and 6 clarify the ambiguous effect of direct intervention of foreign effort. For the case where direct intervention and foreign effort interact as perfect substitutes in the lowering of terrorist resources, we have that direct intervention crowds out foreign effort (see Figure 6). As seen in the theoretical framework, the impact of direct intervention of attack efforts was negative for both foreign and home attack effort as it reduced the resources available to the terrorists, however, the crowding out of foreign effort counteracts the first effect as this in turn increases the terrorist resources. As the plots show, the impact that these effects together have on attack efforts and therefore attack success rates is negligible. Note that in this case we have a corner solution as the welfare loss function is minimized at $e^{HF} = 0$.

So far, under the assumption that the efforts of the home and foreign country are perfect substitutes, $\epsilon = 1$, the Stage 1 equilibrium involves no direct military intervention ($e^{HF} = 0$). We have seen from the analysis that the sign of $\frac{\partial e^F}{\partial e^{HF}}$ is ambiguous. In fact with our parameter setting we see that $\frac{\partial e^F}{\partial e^{HF}} < 0$ so military intervention by the home country crowds out anti-terrorist effort by the foreign country and is counterproductive. However, we are now in a position to examine what happens if we reduce the degree of substitution between e^F and e^{HF} by lowering ϵ. With $\epsilon = 0.5$, Figures 7 and 8 show this now produces a Stage 1 equilibrium with some military intervention with $e^{HF} = 0.025$ (where the welfare loss function is now minimized). Therefore, the nature of the technology by which H and F influence terrorist capacity is crucial for the choice of direct intervention. Figure 9 presents a 3-dimensional plot of the equilibrium for the $\epsilon = 1$ case.

Next we explore the corner solution at which direct military intervention is welfare-reducing for the home country by constructing a measure of the home versus foreign relative military efficiency defined by $\beta \equiv \frac{\beta_{MHF}}{\beta_{MF}}$. Up to now, we have set $\beta = 1$. Figure 10 then plots β against the threshold value of ϵ at which the corner solution to the equilibrium, $e^{HF} = 0$, occurs. We see that as β increases, with low substitution between home and foreign effort, direct intervention emerges as a possible equilibrium.

We see that around the equilibrium where direct intervention does not happen, increases in relative military efficiency will increase the threshold level of the elasticity of substitution below which direct intervention becomes a possibility.

Columns 2–4 of Table 2 set out the full equilibrium for the case of imperfect substitution between e^F and e^{HF} with $\epsilon = 0.75$, 0.5, and 0.25. We see that, in these equilibria, there is steady reduction of military aid to 0 and with some substitution by the country H towards more combined expenditure on both defensive effort and direct intervention. The former disincentivises and the latter crowds out counter-terrorist effort e^F by the F country. Terrorism ceases owing to the reduction of their capacity and the success probability falls to zero in the country H. Eventually, for $\epsilon = 0.25$, the success probability falls substantially in the F country as well.

Table 2 shows that the impact of a decrease in the elasticity of substitution between foreign effort and direct intervention effort has a clear negative impact on the subsidy to the foreign country. However, the impact of lowering the elasticity on the different effort levels is non linear. We start at a point at which efforts are perfect substitutes and, for our set of parameters assumptions, direct intervention does not take place. From that point, a decrease in the elasticity of substitution to 0.75 leads to an initial increase in all efforts, including direct intervention. This leads to a reduction to zero in the success probability of terrorists attacks on H. Terrorists shift their effort to attacking F thereby increasing the foreign success probability. However, a further decrease in the elasticity to 0.5 leads to a further increase in direct intervention effort, this time coupled with a reduction in home defence effort: a lower defence effort still maintains a zero chance of successful attacks at home. Foreign effort falls as higher direct intervention effort takes over the task of reducing the foreign success probability of the terrorists. As the elasticity of substitution falls to 0.25 lower levels of all types of effort are now enough to maintain very low success probabilities.

Note that the decrease in elasticity reduces the success probabilities at all levels. For constant levels of effort by H and F, as their efforts become poorer substitutes, the total effect of their efforts on the terrorists increase. Military doctrine emphasises how joint operations between different types of forces, (land, sea or air) are more effective than operations which must rely on a single type of force. Skill at combined arms fighting is a characteristic of a good commander.

Historically, the crucial parameters (for relative military effectiveness,β, and substitution ϵ) have been determined not just by technology but also by social and institutional factors. An example of social factors is that troops from rich countries, like the US, are less willing to suffer the privations than those from poor countries, like the Viet Cong and Taliban, were willing to suffer. An example of institutional constraints is given by Allen and Leeson (2015). For over a century, the longbow was the most effective missile weapon but only English Kings used it in conjunction with their other forces. The Scots and French did not adopt longbows, but relied on their cavalry, mounted Knights, which were vulnerable to arrows, despite being repeatedly beaten by the English. This was because they faced a trade-off between internal threats, from their nobles, and external threats.

England alone was sufficiently stable to risk training its troops to use the longbow. A more recent example of the impact of institutional constraints on effectiveness is given by Garicano and Posner (2005). The US FBI was less effective in anti-terrorist activities as a result of combining those activities with crime-fighting, which required an organisational design and incentives unsuited to the counter-intelligence and counter-terrorist mission. They argue that organisations, like the UK Security Service, MI5, which had a dedicated counter-espionage role were more effective.

Variable	Value	Value	Value	Value
Elasticity ϵ	1.0	0.75	0.5	0.25
Home expenditure e^H	0.380	0.410	0.405	0.390
Military Aid α	0.240	0.110	0.005	0
Expenditure on Direct Intervention e^{HF}	0	0.010	0.025	0.020
Foreign Expenditure e^F	0.53	0.58	0.43	0.20
Home Success Probability σ^H	0.10	0.00	0.00	0.00
Foreign Success Probability σ^F	0.23	0.25	0.22	0.07
Home Fear Factor ff^H	5.38	5.73	6.56	6.81
Home Expected Cost to Expenditure $\sigma^H ff^H$	0.54	0	0	0
Foreign Fear Factor ff^F	3.72	2.91	3.51	7.51
Foreign Expected Cost to Expenditure $\sigma^F ff^F$	0.86	0.72	0.79	0.55
Probability of Regime Change p^F	0.250	0.250	0.251	0.250

Table 2: Stage 1 Computed Equilibrium: e^F and e^{HF} perfect substitutes and imperfect ($\epsilon \in [0, 1]$). $\phi^H = 0.5$, $\eta = 0.5$, $(\sigma^F)^{max} = 0.75$

5 Conclusions

This paper presents a model of a conflict in which two countries, home and foreign, under threat of terrorist attack, interact non-cooperatively with the objective of limiting the expected damage done by the terrorists. Whereas the terrorists follow an offensive strategy, with the objective of causing damage to both countries, the two countries follow a defensive strategy, with the objective of limiting the expected damage. The two countries face different types of threat. The *foreign* country can only be damaged by terrorist attacks in their own territory. The *home* country, has national interests in both countries which can be damaged by the terrorists.

The two countries have different policy instruments. The foreign country just decides the level of effort it devotes to limiting the resources available to the terrorists to carry out their attacks. The home country decides its effort on defence to protect its national territory, its military subsidy to encourage its foreign ally to attack terrorists assets and its own direct intervention against the terrorists.

We model the interaction between the countries and the terrorist group as a multiple stage game where the home country first commits to their policy decisions, then the foreign government does, finally, the terrorist group decides how much effort to put into terrorist actions against the home or foreign country. We solve the game using backward induction.

The objective of our analysis is to identify the elements in the interaction among the different players which will explain the circumstances under which direct intervention will be part of an equilibrium. Our theoretical model shows different effects at play and these often counteract each other, a characteristic feature of many conflicts. Our modeling strategy expands the BSY framework in a number of ways, the main difference is that we allow for direct intervention and investigate its interaction with foreign effort. As we find the solution to the model, unlike BSY we do not restrict ourselves by presuming that the third order derivatives of probability functions are zero.

Our backward induction method allows us to show that an increase in either direct intervention effort by the home government or the foreign government effort will unambiguously reduce terrorist attacks both on the home and the foreign country. An increase in defensive home effort however will discourage terrorist attack effort at home but will encourage attack effort abroad. As we proceed to the second stage we find that whereas the military subsidy to the foreign government has a clear positive impact on foreign effort. The impact of both defensive and direct intervention efforts are ambiguous. However, our use of calibration allows us to resolve the sign of these effects and find the overall equilibrium.

Our calibration results confirm the positive impact of the military subsidy on foreign effort and identify a negative impact of direct intervention on foreign effort. The closer the degree of substitution between direct intervention and foreign effort, the stronger this crowding out effect on foreign effort.

Calibration of the model also allows us to find the overall backward induction solution to the model. For the chosen parameters, we are able to show that direct intervention will only be part of the equilibrium if the foreign and home effort are poor substitutes in the technology used to reduce the resources of the terrorist group. In addition we show that direct intervention will become more likely as the effectiveness of the home country at reducing the terrorists resources relative to that of the foreign country increases.

Within the framework of this game, there is scope to examine the effect of a number of exogenous factors that change the incentive for the home country to intervene directly, including the probability of blowback which strengthens the terrorists, for instance through regime change.

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A Appendix

As explained in the main text, we solve the game using backwards induction.

First, using the third stage Lagrangian equation we find the first order conditions for the optimization problem of terrorists:

$$\begin{aligned}
\sigma_2^H(e^H, a^H) - \lambda^M + \lambda^{a^H} &= 0, && \text{(A.1)}\\
\tau\sigma_1^F(a^F) - \lambda^M + \lambda^{a^F} &= 0, && \text{(A.2)}\\
\lambda^M(M(e^F, e^{HF}) - a^H - a^F) &= 0, && \text{(A.3)}\\
\lambda^{a^H} a^H &= 0, && \text{(A.4)}\\
\lambda^{a^F} a^F &= 0, && \text{(A.5)}\\
a^H,\ a^F,\ M - a^H - a^F,\ \lambda^M,\ \lambda^{a^H},\ \lambda^{a^F} &\geq 0. && \text{(A.6)}
\end{aligned}$$

Equations (A.1)–(A.6) constitute the stage 3 equilibrium given previous actions e^H, e^{HF}, e^F. Clearly the capacity constraint must bind at the optimum so $\lambda^M = 0$. For an internal solution $a^H, a^F > 0$ we must also have that $\lambda^{a^H} = \lambda^{a^F} = 0$ so that

$$\sigma_2^H(e^H, a^H) = \tau\sigma_1^F(a^F) = \tau\sigma_1^F(M - a^H), \tag{A.7}$$

which equates the marginal utility from effort by the terrorist in countries H and F. The second order condition for the internal solution is

$$\sigma_{22}^H(e^H, a^H) + \tau\sigma_{11}^F(M - a^H) < 0, \tag{A.8}$$

which is guaranteed by the conditions σ_{22}^H, $\sigma_{11}^F < 0$.

Second, using the second stage Lagrangian equation we find the first order conditions for the optimization problem of the foreign government:

$$\begin{aligned} \gamma(e^{HF}, \eta)\sigma_1^F(a^F)T^F a_3^F(e^H, e^{HF}, e^F) + 1 - \alpha - \lambda^{e^F} &= 0, & \text{(A.9)} \\ \lambda^{e^F} e^F &= 0, & \text{(A.10)} \end{aligned}$$

where to compute $a_3^F(e^H, e^{HF}, e^F)$, we differentiate the FOCs from stage 3 to obtain four additional equations to compute a_3^F, a_3^H, $\lambda_3^{a^F}$ and $\lambda_3^{a^H}$

$$\begin{aligned} \tau\sigma_{11}^F(a^F) - \sigma_{22}^H(e^H, a^H)a_3^H - \lambda_3^{a^H} + \lambda_3^{a^F} &= 0, & \text{(A.11)} \\ \lambda_3^{a^H} a^H + \lambda^{a^H} a_3^H &= 0, & \text{(A.12)} \\ \lambda_3^{a^F} a^F + \lambda^{a^F} a_3^F &= 0, & \text{(A.13)} \\ M_1 - a_3^H - a_3^F &= 0. & \text{(A.14)} \end{aligned}$$

The FOCs and Second Order Conditions (SOCs) for an internal solution $e^F > 0$, $a^F > 0$, $a^H > 0$, $\lambda^{e^F} = \lambda^{a^H} = \lambda^{a^F} = \lambda_3^{a^H} = \lambda_3^{a^F} = 0$ are

$$\begin{aligned} \gamma(e^{HF}, \eta)\sigma_1(a^F)T^F a_3^F(e^H, e^{HF}, e^F) + 1 - \alpha &= 0, & \text{(A.15)} \\ -\gamma(e^{HF}, \eta)T^F(\sigma_{11}(a^F)(a_3^F(e^H, e^{HF}, e^F))^2 + \sigma_1(a^F)a_{33}^F(e^H, e^{HF}, e^F)) &< 0. & \text{(A.16)} \end{aligned}$$

To complete the equilibrium we require expressions for $a_i^H(e^H, e^{HF}, e^F)$, $a_i^F(e^H, e^{HF}, e^F)$ and $e_i^F(e^H, e^{HF}, \alpha;\ i = 1, 3)$. a_3^F and a_3^H have already been obtained at Stage 2. Now, differentiating stage 3 condition (A.7) with respect to e^H, e^{HF} and e^F respectively gives

$$
\begin{aligned}
\sigma^H_{21}(e^H,a^H)+\sigma^H_{22}(e^H,a^H)\,a^H_1(e^H,e^{HF}e^F) &= \tau\,\sigma^F_{11}(a^F)\,a^F_1(e^H,e^{HF},e^F), &\text{(A.17)}\\
a^H_1(e^H,e^{HF},e^F)+a^F_1(e^H,e^{HF},e^F) &= 0, &\text{(A.18)}\\
\sigma^H_{22}(e^H,a^H)\,a^H_2(e^H,e^{HF}e^F) &= \tau\,\sigma^F_{11}(a^F)\,a^F_2(e^H,e^{HF},e^F), &\text{(A.19)}\\
a^H_2(e^H,e^{HF},e^F)+a^F_2(e^H,e^{HF},e^F) &= M_2(e^F,e^{HF}), &\text{(A.20)}\\
\sigma^H_{22}(e^H,a^H)\,a^H_3(e^H,e^{HF}e^F) &= \tau\,\sigma^F_{11}(a^F)\,a^F_3(e^H,e^{HF},e^F), &\text{(A.21)}\\
a^H_3(e^H,e^{HF},e^F)+a^F_3(e^H,e^{HF},e^F) &= M_1(e^F,e^{HF}). &\text{(A.22)}
\end{aligned}
$$

Given the functional forms for the probabilities p^F, σ^H, σ^F and M considered in the quantitative analysis section, we have so far 12 equations in 6 choice variables e^H, e^{HF}, α for country H, e^F for country F, a^H and a^F for terrorists; and 7 reaction function derivatives a^H_1, a^F_1, a^H_2, a^F_2, a^H_3, a^F_3 and e^F_3.

It remains to find expressions for e^F_i, $i=1,3$. To do this first differentiate (A.15) with respect to e^H, e^{HF} and α to obtain respectively:

$$
\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a^F_3(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a^F_{33}(e^H,e^{HF},e^F)e^F_1(e^H,e^{HF},\alpha)\right] = 0, \quad \text{(A.23)}
$$

$$
\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a^F_3(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a^F_{33}(e^H,e^{HF},e^F)e^F_2(e^H,e^{HF},\alpha)\right] = 0, \quad \text{(A.24)}
$$

$$
\gamma(e^{HF},\eta)T^F\left[\sigma_{11}(a^F)(a^F_3(e^H,e^{HF},e^F))^2+\sigma_1(a^F)a^F_{33}(e^H,e^{HF},e^F)e^F_3(e^H,e^{HF},\alpha)\right]-1 = 0. \quad \text{(A.25)}
$$

Finally, differentiating (A.14) and (A.15) with respect to e^F, we have

$$
\begin{aligned}
\sigma^H_{222}(e^H,a^H)\,(a^H_3(e^H,e^{HF},e^F))^2 &+ \sigma^H_{22}(e^H,a^H)\,a^H_{33}(e^H,e^{HF}e^F))\\
&= \tau[\sigma^F_{111}(a^F)\,(a^F_3(e^H,e^{HF},e^F))^2+\sigma^F_{11}a^F_{33}(e^H,e^{HF}e^F))], \quad \text{(A.26)}\\
a^H_{33}(e^H,e^{HF},e^F)+a^F_{33}(e^H,e^{HF},e^F) &= M_{11}(e^F,e^{HF}). \quad \text{(A.27)}
\end{aligned}
$$

Equations (A.23)–(A.27) provide five additional equations for a_{33}^H, a_{33}^F and e_i^F, $i = 1,3$ completing the equilibrium.

B Appendix

Stage 3

As in BSY, terrorists choose their attack effort distribution across their base country, foreign and home, a^H and a^F that maximize their objective function subject to their resource constraint. For the interior solution, we find the tangency condition:

$$\phi^H T^H \sigma_2^H(e^H, a^H) = \left(\phi^H T^{HF} + \phi^F T^F\right) \sigma_1^F(a^F).$$

Substituting budget constraint

$$a^F = M(e^F, e^{HF}) - a^H,$$

we get:

$$\phi^H T^H \sigma_2^H(e^H, a^H) - [\phi^H T^{HF} + \phi^F T^F] \frac{\partial \sigma^F(M(e^F, e^{HF}) - a^H)}{\partial a^F} = 0.$$

The following results can be established using the Implicit Function Theorem and the assumptions over the properties of the probabilities of successful attack $\sigma_1^H < 0$, $\sigma_2^H > 0$, $\sigma_{11}^H > 0$, $\sigma_{22}^H < 0$, $\sigma_1^F > 0$, $\sigma_{11}^F < 0$.

Following BSY, we define

$$D = -\left[\phi^H T^H \sigma_{22}^H(e^H, a^H) + [\phi^H T^{HF} + \phi^F T^F] \sigma_{11}^F(a^F)\right] > 0.$$

Now, using the IFT, we obtain the following expressions for the impact of the different country efforts on the attack efforts of the terrorists:

$$a_1^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{21}^H(e^H, a^H)}{D} = \frac{\phi^H T^H \sigma_{21}^H}{D} < 0 \Leftrightarrow \sigma_{21}^H < 0,$$

$$a_1^F(e^H, e^{HF}, e^F) = -a_1^H(e^H, e^{HF}, e^F) = -a_1^H > 0 \Leftrightarrow \sigma_{21}^H < 0,$$

$$a_2^H(e^H, e^{HF}, e^F) = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F(a^F)M_2(e^F, e^{HF})}{-D} = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F M_2}{-D} < 0,$$

$$a_2^F(e^H, e^{HF}, e^F) = M_2(e^F, e^{HF}) - a_2^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{22}^H(e^H, a^H)M_2(e^F, e^{HF})}{-D} = \frac{\phi^H T^H \sigma_{22}^H M_2}{-D} < 0,$$

$$a_3^H(e^H, e^{HF}, e^F) = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F(a^F)M_1(e^F, e^{HF})}{-D} = \frac{[\phi^H T^{HF} + \phi^F T^F]\sigma_{11}^F M_1}{-D} < 0,$$

$$a_3^F(e^H, e^{HF}, e^F) = M_1(e^F, e^{HF}) - a_3^H(e^H, e^{HF}, e^F) = \frac{\phi^H T^H \sigma_{22}^H(e^H, a^H)M_1(e^F, e^{HF})}{-D} = \frac{\phi^H T^H \sigma_{22}^H M_1}{-D} < 0.$$

In addition,

$$\phi^H T^H \sigma_2^H(e^H, a^H) - [\phi^H T^{HF} + \phi^F T^F]\frac{\partial \sigma^F(M(e^F, e^{HF}) - a^H)}{\partial a^F} = 0,$$

$$\frac{\partial a_3^H}{\partial \phi^H} = \frac{T^H \sigma_2^H - T^{HF}\sigma_1^F}{D},$$

$$\frac{\partial a_3^F}{\partial \phi^H} = -\frac{T^H \sigma_2^H - T^{HF}\sigma_1^F}{D}.$$

Note that, $\sigma_2^H > 0$ and $\sigma_1^F > 0$, hence, the above will depend on parameters. Finally, we have

$$\frac{\partial a_3^H}{\partial T^H} = \frac{\phi^H \sigma_2^H}{D} > 0,$$

$$\frac{\partial a_3^F}{\partial T^H} = -\frac{\phi^H \sigma_2^H}{D} < 0.$$

Stage 2

The FOC for an internal solution $e^F > 0$ can be written as

$$-\gamma(e^{HF}, \eta)\sigma_1^F(a^F)a_3^F(e^H, e^{HF}, e^F)T^F - 1 + \alpha = 0,$$

which leads to the *reaction function of country F* as

$$e^F = e^F(e^H, e^{HF}, \alpha).$$

The second order condition implies

$$Y^F_{e^Fe^F} = -\gamma(e^{HF}, \eta)T^F \left[\sigma^F_{11}(a^F)\left(a^F_3\right)^2 + \sigma^F_1(a^F)a^F_{33}\right] = -\gamma T^F \left[\sigma^F_{11}\left(a^F_3\right)^2 + \sigma^F_1 a^F_{33}\right] < 0.$$

Note that

$$a^H_{33} = \frac{\tau\sigma^F_{111}(-a^H_3)^2 - \sigma^H_{222}\,(a^H_3)^2 + \tau\sigma^F_{11}M_{11}}{\sigma^H_{22} + \tau\sigma^F_{11}},$$

$$a^F_{33}(e^H, e^{HF}, e^F) = M_{11} - \frac{\tau\sigma^F_{111}(-a^H_3)^2 - \sigma^H_{222}\,(a^H_3)^2 + \tau\sigma^F_{11}M_{11}}{\sigma^H_{22} + \tau\sigma^F_{11}}$$

or

$$a^F_{33}(e^H, e^{HF}, e^F) = \frac{\sigma^H_{22}\,M_{11} + (-a^H_3)^2\left(\sigma^H_{222} - \tau\sigma^F_{111}\right)}{\sigma^H_{22} + \tau\sigma^F_{11}}.$$

Now, we know that, for the second order condition to hold, we need

$$Y^F_{e^Fe^F} = -\gamma T^F \left[\sigma^F_{11}\left(a^F_3\right)^2 + \sigma^F_1 a^F_{33}\right] = -\gamma T^F \left[\sigma^F_{11}\left(a^F_3\right)^2 + \sigma^F_1 a^F_{33}\right] < 0.$$

For the above to hold, we need $a^F_{33} >> 0$. For that, we need $M_{11} >> 0$, even if we assumed third order derivatives to be equal to zero.

For the comparative statics results we first need to find explicit expressions for a^F_{31} and a^F_{32}. For that, we differentiate (A.21) and (A.22) with respect to e^H and e^{HF} and substitute into each other:

First, a^F_{31}

$$\begin{aligned}\sigma^H_{22}(e^H, a^H)\left(M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right) &= \tau\sigma^F_{11}(a^F)a^F_3(e^H, e^{HF}, e^F)\\ a^H_3(e^H, e^{HF}, e^F) &= M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\end{aligned}$$

$$\left(\sigma^H_{221}(e^H, a^H) + \sigma^H_{222}(e^H, a^H)a^H_1\right)\left(M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right) + \sigma^H_{22}(e^H, a^H)\left(-a^F_{31}(e^H, e^{HF}, e^F)\right)$$

$= \tau\sigma^F_{111}(a^F)a^F_1\, a^F_3(e^H, e^{HF}, e^F) + \tau\sigma^F_{11}(a^F)a^F_{31}(e^H, e^{HF}, e^F)$

$$a^F_{31} = \frac{\left(\sigma^H_{221} + \sigma^H_{222}a^H_1\right)a^H_3 - \tau\,\sigma^F_{111}a^F_1\,a^F_3}{\tau\,\sigma^F_{11} + \sigma^H_{22}}.$$

In general, the sign of the above is ambiguous. However, if we assumed third order derivatives are zero, as BSY do, it would be zero.

Second, a^F_{32}

$\sigma^H_{22}(e^H, a^H)\left[M_{12}(e^F, e^{HF}) - a^F_{32}(e^H, e^{HF}, e^F)\right] + \sigma^H_{222}(e^H, a^H)a^H_2\left[M_1(e^F, e^{HF}) - a^F_3(e^H, e^{HF}, e^F)\right]$

$= \tau\sigma^F_{111}(a^F)a^F_2\, a^F_3(e^H, e^{HF}, e^F) + \tau\,\sigma^F_{11}(a^F)a^F_{32}(e^H, e^{HF}, e^F).$

We rewrite to get

$$a^F_{32}(e^H, e^{HF}, e^F) = \frac{\sigma^H_{22}M_{12} + \sigma^H_{222}a^H_2\left[M_1 - a^F_3\right] - \tau\,\sigma^F_{111}a^F_2\,a^F_3}{\sigma^H_{22} + \tau\,\sigma^F_{11}}.$$

Once more, the sign of the above is ambiguous, but in this case, even if we assume that third order derivatives are zero, we are still left with

$$a^F_{32}(e^H, e^{HF}, e^F) = \frac{\sigma^H_{22}M_{12}}{\sigma^H_{22} + \tau\,\sigma^F_{11}}.$$

The sign of the above will be positive as long as $M_{12} > 0$, since all the other terms are negative.

Now we can proceed to comparative statics result. Using the IFT and results from third stage, $a^H(e^H, e^{HF}, e^F)$ and $a^F(e^H, e^{HF}, e^F)$,we get:

First,

$\frac{\partial e^F}{\partial e^H} = e^F_1 = \frac{Y^F_{e^F e^H}}{-Y^F_{e^F e^F}}$

$= \frac{-\gamma(e^{HF},\eta)T^F\sigma^F_{11}(a^F)a^F_1 a^F_3(e^H,e^{HF},e^F) - \gamma(e^{HF},\eta)T^F\sigma^F_1 a^F_{31}}{-Y^F_{e^F e^F}}$

$= \frac{\gamma T^F\left(\sigma^F_{11}a^F_1 a^F_3 + \sigma^F_1 a^F_{31}\right)}{Y^F_{e^F e^F}}.$

If we assumed third order derivatives are zero as BSY do, then $a^F_{31} = 0$ and the above would have a clear negative sign. Otherwise the sign will be ambiguous.

Second,

$$\frac{\partial e^F}{\partial \alpha} = e_3^F = \frac{Y^F_{e^F\alpha}}{-Y^F_{e^F e^F}} = \frac{1}{-Y^F_{e^F e^F}} > 0.$$

Third,

$$\frac{\partial e^F}{\partial e^{HF}} = e_2^F = \frac{Y^F_{e^F e^{HF}}}{-Y^F_{e^F e^F}}$$

$$= \frac{-\gamma_1(e^{HF},\eta)T^F\sigma_1^F(a^F)a_3^F(e^H,e^{HF},e^F)-\gamma(e^{HF},\eta)T^F\sigma_{11}^F(a^F)a_2^F a_3^F(e^H,e^{HF},e^F)-\gamma T^F\sigma_1^F a_{32}^F}{-Y^F_{e^F e^F}}$$

$$= \frac{T^F a_3^F\left(\gamma_1\sigma_1^F+\gamma\sigma_{11}^F a_2^F\right)+\gamma T^F\sigma_1^F a_{32}^F}{Y^F_{e^F e^F}} > 0.$$

As discussed in the main text, even if we assume that third order derivatives are zero, we would still have an ambiguous sign for a_{32}^F which would depend on the sign of M_{12}. As $\sigma_1^F > 0$, we would need $a_{32}^F < 0$ to not get an ambiguous sign above, for zero third order derivatives this would happen if $M_{12} < 0$.

Description of Figures

Figure 1: a^H is the terrorist organization attack effort aimed at the home country; σ^H is the probability of a successful terrorist attack on the home country, in the absence of regime change in the foreign country. The welfare loss consists of the direct and indirect military costs including aid to the foreign country. The latter equals αe^F where e^F is the military effort chosen by the foreign ally. e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value $e^H = 0.38$ and e^{HF}, the military effort of the home country devoted to direct military intervention, is set at zero. Note that for our calibrated values and choice of contest function, for $a^H << 1$ we have that $\sigma^H \approx a^H$ and the welfare loss function is minimized at the baseline equilibrium value of $\alpha = 0.24$.

Figure 2: a^F is the terrorist organization attack effort aimed at the foreign country; σ^F is the probability of a successful terrorist attack on the foreign country, in the absence of regime change in that country; e^F is the military effort chosen by the foreign ally. e^H, the defensive military effort chosen by the home country is set at the baseline equilibrium

value $e^H = 0.38$ and e^{HF}, the military effort of the home country devoted to direct military intervention, is set at zero.

Figure 3: a^H is the terrorist organization attack effort aimed at the home country; σ^H is the probability of a successful terrorist attack on the home country, in the absence of regime change in the foreign country F. The welfare loss consists of the direct and indirect military costs including aid to the foreign ally. The latter equals αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 4: a^F is the terrorist organization attack effort aimed at the foreign country; σ^F is the probability of a successful terrorist attack on the foreign country, in the absence of regime change in that country; aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 5: a^H is the terrorist organization attack effort aimed at the home country; σ^H is the probability of a successful terrorist attack on the home country, in the absence of regime change in the foreign country F. The welfare loss consists of the direct and indirect military costs including aid to the foreign ally. e^{HF} is the military effort of the home country devoted to direct military intervention. e^H, the defensive military effort chosen by the home country, and e^{HF} are perfect substitutes (the elasticity of substitution $\epsilon = 1$); aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 6: a^F is the terrorist organization attack effort aimed at the foreign country; σ^F is the probability of a successful terrorist attack on the foreign country, in the absence of regime change in that country; e^F is the military effort chosen by the foreign ally. e^{HF}

is the military effort of the home country devoted to direct military intervention. e^F and e^{HF} are perfect substitutes (the elasticity of substitution $\epsilon = 1$); aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 7: a^H is the terrorist organization attack effort aimed at the home country; σ^H is the probability of a successful terrorist attack on the home country, in the absence of regime change in the foreign country. The welfare loss consists of the direct and indirect military costs including aid to the foreign costs. e^{HF} is the military effort of the home country devoted to direct military intervention. e^F, the military effort chosen by the ally, and e^{HF} are imperfect substitutes (the elasticity of substitution $\epsilon = 0.5$); aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 8: a^F is the terrorist organization attack effort aimed at the foreign country; σ^F is the probability of a successful terrorist attack on the foreign country, in the absence of regime change in that country; e^F is the military effort chosen by the foreign ally. e^{HF} is the military effort of the home country devoted to direct military intervention. e^F and e^{HF} are imperfect substitutes (the elasticity of substitution $\epsilon = 0.5$); aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

Figure 9: The welfare loss consists of the direct and indirect military costs including aid to the foreign country. The latter equals αe^F where e^F is the military effort chosen by the foreign ally; e^H, the defensive military effort chosen by the home country; e^F and e^{HF} are perfect substitutes (the elasticity of substitution $\epsilon = 1.0$).

Figure 10: ϵ is the elasticity of substitution between e^F, the military effort chosen by the

foreign ally and e^{HF}. β is a measure of the home versus foreign relative military efficiency at combatting terrorism; aid to the foreign ally is αe^F where e^F is the military effort chosen by the foreign ally and α is set at the baseline equilibrium value of $\alpha = 0.24$; e^H, the defensive military effort chosen by the home country, is set at the baseline equilibrium value of $e^H = 0.38$.

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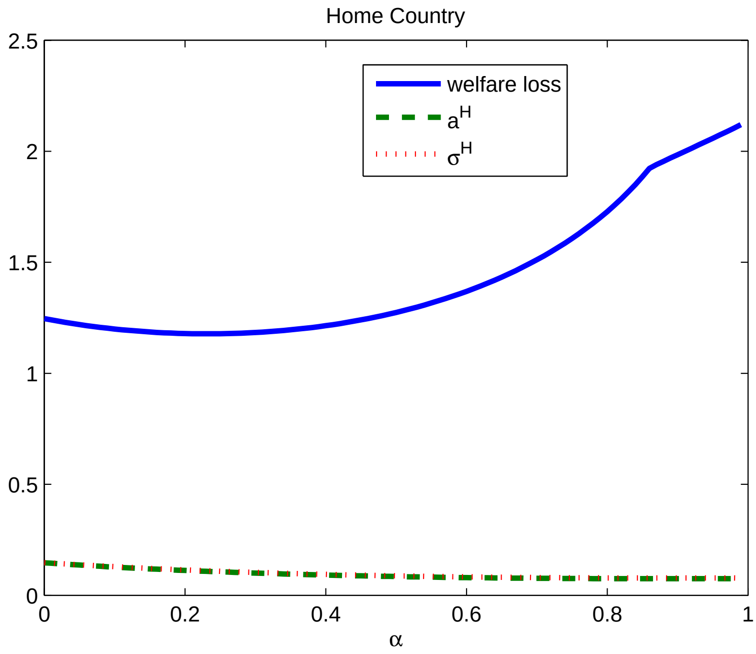

Figure 1: **Stage 2: Response to α for Home Country.** $e^H = 0.38$, $e^{HF} = 0$.

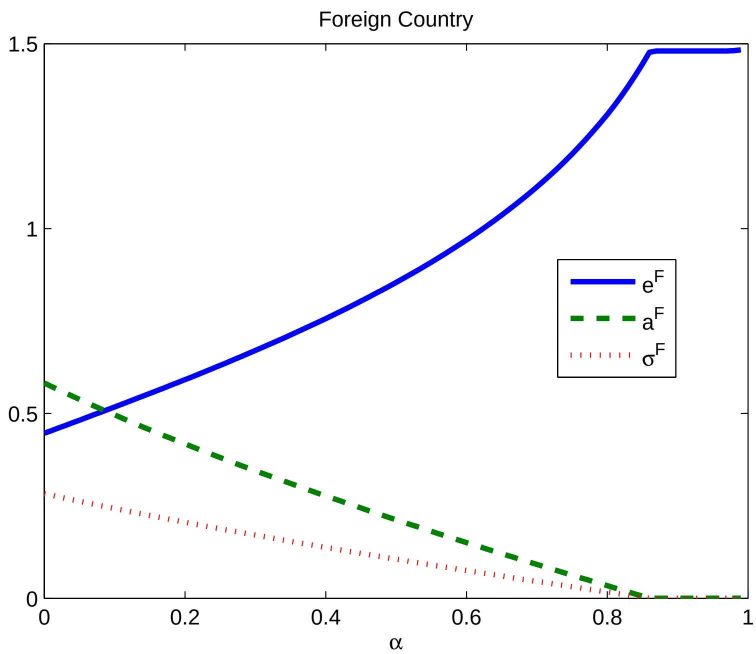

Figure 2: **Stage 2: Response to α for Foreign Country.** $e^H = 0.38$, $e^{HF} = 0$

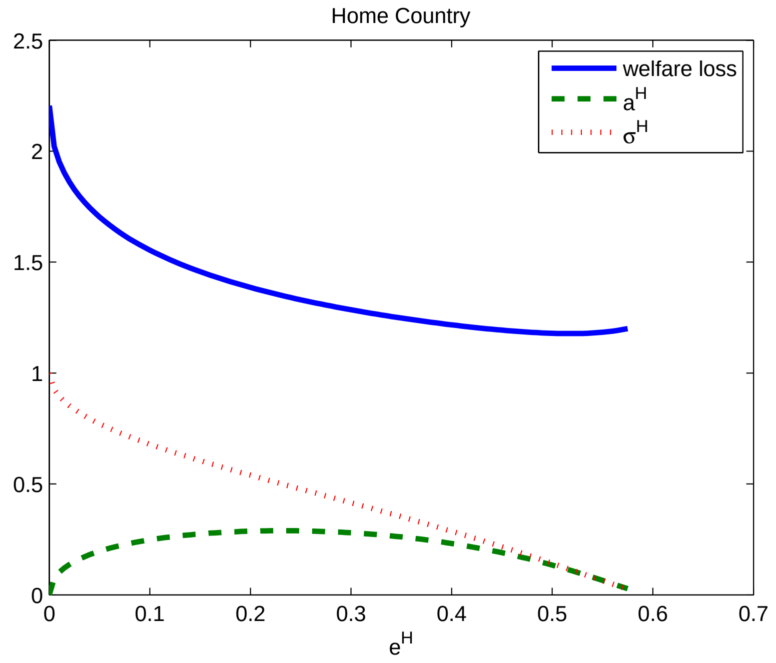

Figure 3: **Stage 2: Response to e^H for Home Country.** $\alpha = 0.24$, $e^{HF} = 0$

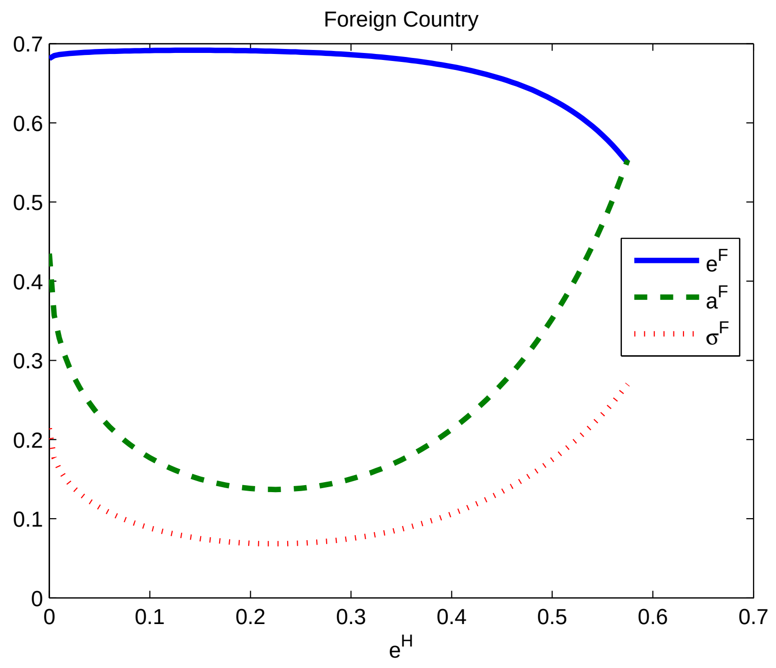

Figure 4: **Stage 2: Response to e^H for Foreign Country.** $\alpha = 0.24$, $e^{HF} = 0$

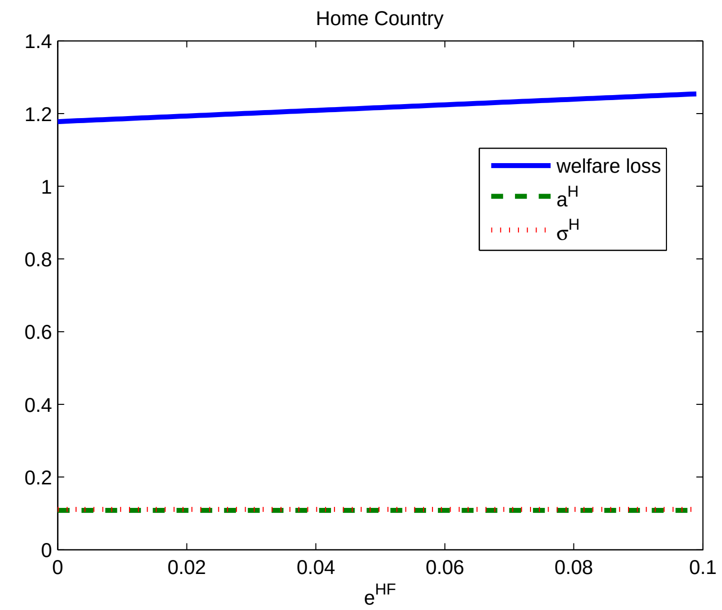

Figure 5: **Stage 2: Response to e^{HF} for Home Country. e^H, e^{HF} Perfect Substitutes.** $\alpha = 0.24$, $e^H = 0.38$

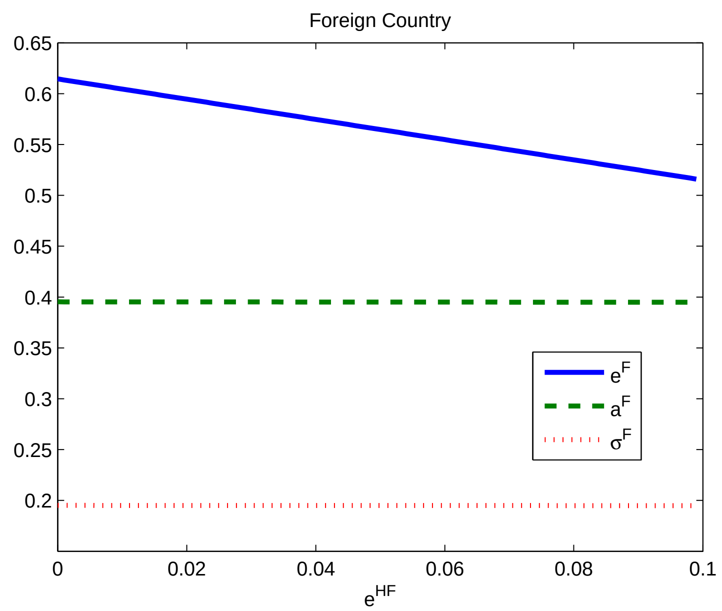

Figure 6: **Stage 2: Response to e^{HF} for Foreign Country. e^F, e^{HF} Perfect Substitutes.** $\alpha = 0.24$, $e^H = 0.38$

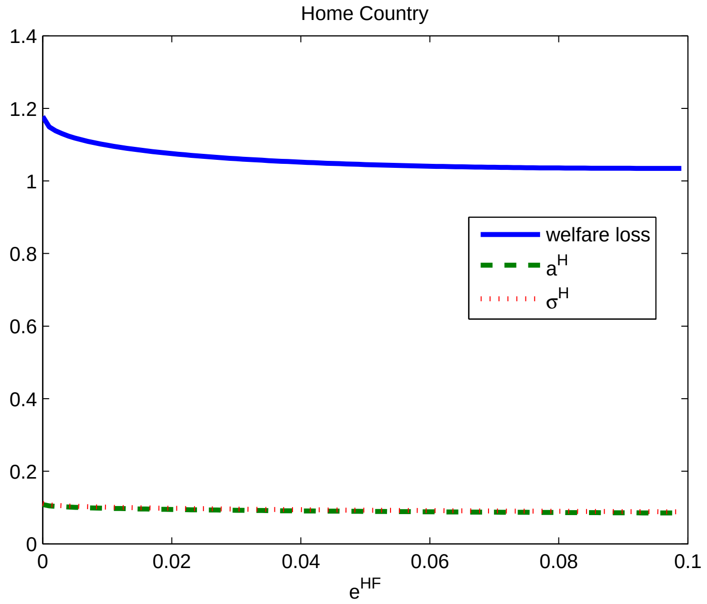

Figure 7: **Stage 2: Response to e^{HF} for Home Country. e^F, e^{HF} Imperfect Substitutes ($\epsilon = 0.5$).** $\alpha = 0.24$, $e^H = 0.38$

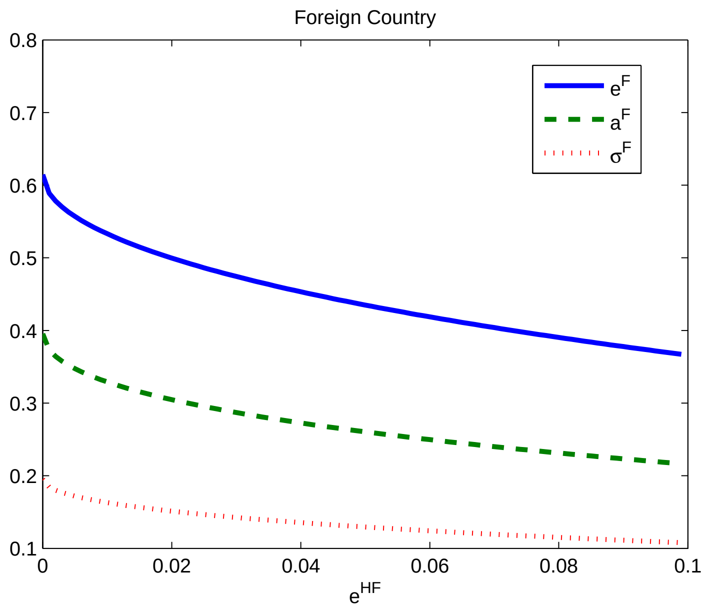

Figure 8: **Stage 2: Response to e^{HF} for Foreign Country. e^F, e^{HF} Imperfect Substitutes ($\epsilon = 0.5$).** $\alpha = 0.24$, $e^H = 0.38$

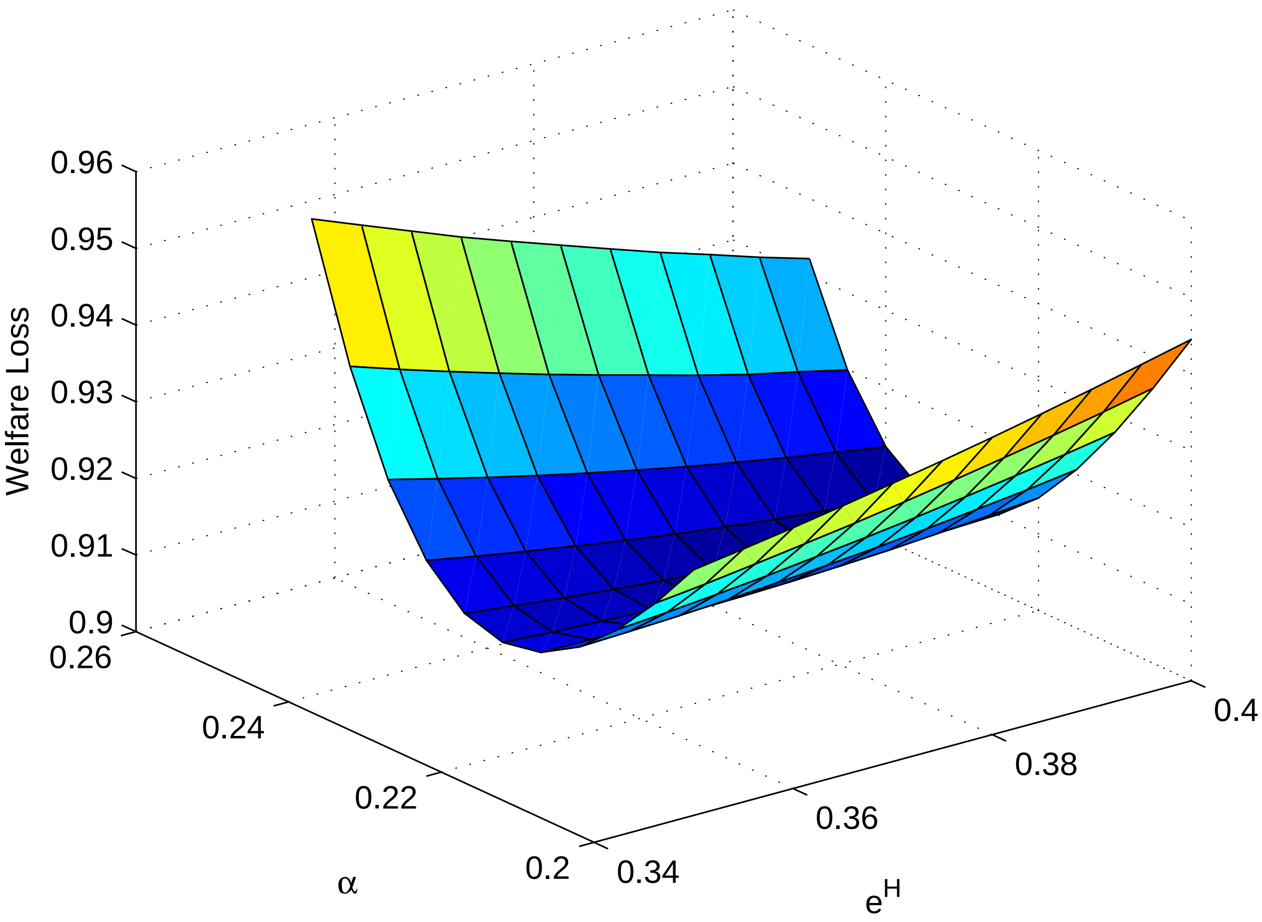

Figure 9: **Stage 1: Optimal Choice of e^H and α with $e^{HF} = 0$ (e^H and e^{HF} Perfect Substitutes).**

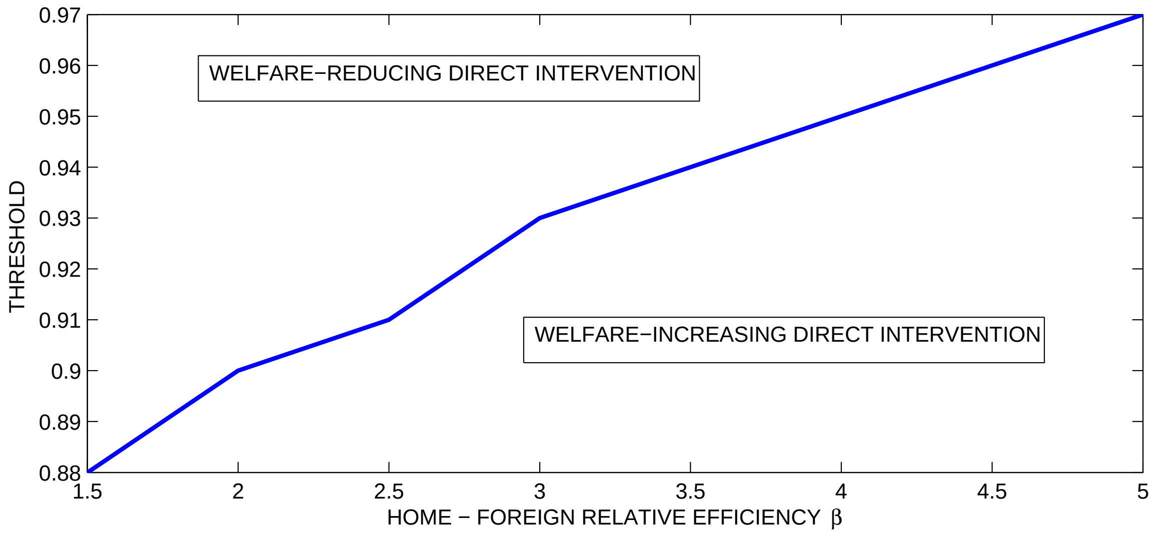

Figure 10: **Stage 2: Threshold of ϵ as Relative Home/Foreign Military Efficiency Increases.** $\alpha = 0.24$, $e^H = 0.38$