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Sliding Mode Observer Based Incipient Sensor Fault Detection with Application to High-Speed Railway Traction Device

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Abstract

This paper considers incipient sensor fault development detection issue for a class of nonlinear systems with “observer unmatched” uncertainties. A particular FD (fault detection) sliding mode observer is designed for the augmented system formed by the original system and incipient sensor faults. The parameters are obtained using LMI and line filter techniques to guarantee that the generated residuals are robust to uncertainties and that sliding motion is not destroyed by faults. Then, three levels of novel adaptive thresholds (incipient sensor fault thresholds, sensor fault thresholds and sensor failure thresholds) are proposed based on the reduced order sliding mode dynamics, which effectively improve the incipient sensor fault development detectability. Case study of on the traction system in CRH (China Railway High-speed) is presented to demonstrate the effectiveness of the proposed incipient sensor fault development and senor faults detection schemes.

Keywords: Incipient sensor fault, sliding mode observer, adaptive threshold, fault development detection.

I. Introduction

Modern control systems have become more complex in order to meet the increasing requirement for high levels of performance. Control engineers are faced with increasingly complex systems for which both the reliability and safety are very important. However, component

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incipient faults, such as electrolyte loss effectiveness of electrolytic capacitor, mechanical wears and bears etc., may induce drastically changes and result in undesirable performance degradation, even instability. These are life-critical for safety and actuate critical systems such as aircrafts, spacecrafts, nuclear power plants, chemical plants processing hazardous materials and high-speed railways. Therefore, incipient fault detection and development detection techniques are of practical significance. And, the most important issue of reliable system operation is to detect and isolate incipient faults as early as possible, which can give operators enough information and time to take proper measures to prevent any serious consequences on systems.

Typically, abrupt faults affect safety-relevant systems, which have to be detected early enough so that catastrophic consequences can be avoided by early system reconfiguration. Such faults normally have larger effect on detection residuals than that of modeling uncertainties, which can be detected by choosing appropriate thresholds. At the other end, incipient faults are closely related to maintenance problems and early detection of worn equipment is necessary. In this case, the amplitude of incipient faults are typically small. Thus the detection presents challenges to model-based FDI techniques due to the inseparable mixture between incipient fault and modeling uncertainty. Therefore, it is important to improve the residual robustness to system uncertainties and select more proper thresholds to improve the detectability of fault detection mechanism.

There are many methods proposed in last few decades to enhance the robustness in observer based fault detection, such as perfect unknown input decoupling [1], [2], [3], [4], optimal $\mathcal{H}_2$, $\mathcal{H}_\infty$ schemes [5], [6], [7], [8], total measurable fault information residual [9], and projection method [10]. Fault detection schemes for switching systems [11], [12] and semiconductor manufacturing processes [13] have also been proposed. It has been recognized from general existence condition in [2] that, for a residual generator perfectly decoupled from unknown input, it is only possible when enough output signals are available. Different from perfect decoupling approach, the robust residual generators are designed in the context of a trade-off between robustness against disturbances and sensitivity to faults [5]. When perfect decoupling is not possible, the decision functions determined by residuals will be corrupted by unknown inputs. The common practice to evaluate the decision functions is to define appropriate thresholds, with which the decision functions are compared [1]. Therefore, the robustness residuals and proper selected thresholds are two important factors to improve detectability of incipient fault detection mechanism.

During the past decades, sliding mode observers have been used for FDI extensively [14], [15], [16], [17], [18], [19], [20], [21], [22]. The reference [14] uses a sliding mode observer to detect faults by disruption of sliding motion which is a difficult problem and motivate much research in the area. In [15], [16], [17], [18] and [19], the “equivalent output injection” concept is used to explicitly construct fault signals to detect and isolate the faults, including sensor faults and actuator faults. In [18], uncertainties and disturbances are considered, which need the so called “matched uncertainty” in [23] assumption on the distribution matrices of the modeling uncertainties and disturbances. Also, [17] studies the so called “unmatched uncertainty” case based on the robust $\mathcal{H}_\infty$ to enhance the robustness. Based on different structure of distribution matrices of faults and uncertainties, [20] and [22] combine the Luenberger observer with sliding mode observer to detect faults, which needs perfect decoupling between faults and uncertainties. Therefore, sliding mode observer based FDI framework in [17] and [21] mainly focus on robust residual generator design to get a trade-off between robustness against disturbances and sensitivity to faults. In reality, fault detectability can also be improved by selecting proper thresholds and the adaptive threshold is intuitive (see, e.g. [24]). However, adaptive threshold design based on sliding mode observers has not been available.

In this paper, a nonlinear sliding mode observer with novel designed sliding surface is proposed for incipient sensor fault detection. The parameters of the observer are particular designed relying on $\mathcal{L}_2$ gain, guaranteeing residual robustness to uncertainties. At the same time, proper adaptive thresholds are obtained based on the reduced order sliding motion, which effectively improves incipient sensor fault detectability. Furthermore, different levels of detection decision schemes for incipient sensor fault development are proposed. The main contribution of this paper is as follows:

1) a novel FD sliding mode observer framework is proposed to get proper adaptive thresholds to improve incipient fault detectability.
2) incipient sensor fault development detection schemes are studied and levels of detection decisions are proposed.

The remainder of this paper is organized as follows. In Section II, preliminaries and assumptions are presented. In Section III, the FDE sliding mode observer is proposed with parameters of observer being designed based on LMI and linear filter techniques. In Section IV, the sensor fault adaptive thresholds (for incipient fault, fault and failure) are designed and the continuous and piecewise continuous incipient sensor fault development detection decisions are made. In section V, case study of an application to the traction system in CRH (China Railway High-speed) is presented to demonstrate the obtained results. Section VI concludes this paper.

II. Problems Formulation

A. System Description and incipient sensor fault Modeling

Consider a class of linear systems with sensor faults described by

$$\begin{aligned} \dot{x} &= Ax + g(x,u) + \eta(x,u,\omega,t), \\ y &= Cx + Ff(x,u,t), \end{aligned} \tag{1}$$

where $x \in \mathcal{R}^n$ is state vector, $u \in \mathcal{R}^m$ is control, $\omega \in \mathcal{R}^h$ represent external disturbance vector, $f : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R} \to \mathcal{R}^q$ is a nonlinear smooth vector representing the incipient sensor faults. $g(x,u) : \mathcal{R}^n \times \mathcal{R}^m \to \mathcal{R}^n$ is a known nonlinear smooth vector and $\eta(x,u,\omega,t) : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}^h \times \mathcal{R} \to \mathcal{R}^n$ is a nonlinear smooth vector representing the lumped disturbance, which is a generalized concept, possibly including external disturbances, un-modelled dynamics, parameter variations, and complex nonlinear dynamics. Matrices $A \in \mathcal{R}^{n\times n}$, $C \in \mathcal{R}^{p\times n}$ and $F \in \mathcal{R}^{p\times q}$ are known with C being full row rank and F full column rank.

Assuming that $n \geq p > q$. Without loss of generality, it is assumed that the outputs of the system (1) have been reordered (and scaled if necessary) so that the matrix F has the structure

$$F = \begin{bmatrix} 0 \\ I_q \end{bmatrix}. \tag{2}$$

A lemma for piecewise continuous signals to establish differential dynamic model is given as follows:

Lemma 1. [29] For any piecewise continuous vector function $f : \mathcal{R}^+ \to \mathcal{R}^q$, and a stable $q \times q$ matrix A_f, there will exists an input vector $\xi \in \mathcal{R}^q$ such that $\dot{f} = A_f f + \xi$.

Based on the continuous developing way of incipient faults analyzed in [26] and [28], this paper considers the incipient sensor fault $f(t)$ which is modeled by

$$\dot{f} = A_f f + \xi(x,u,t),\ \ f(0) = 0, \tag{3}$$

where A_f is a stable matrix with appropriate dimensions and $\xi = [\xi_1^T, \cdots, \xi_q^T]^T \in \mathcal{R}^q$ is unknown vector. Taking the Laplace transformation of Eq.(3), it is clear to see that in the frequency domain, $f(s) = (sI - A_f)^{-1}\xi$, which shows that the fault signal f is determined by $\xi(x,u,t)$ completely. It should be noted that A_f is not a designed parameter. Such a class of incipient faults has been studied in [26] and [28].

Generally speaking, the amplitudes of the incipient faults are small. With time going on, the incipient faults may continuously develop to faults, and their amplitudes are bigger than that

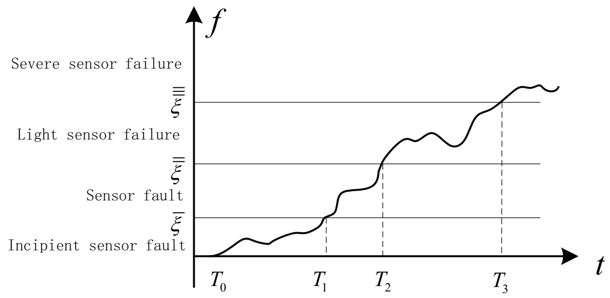


Fig. 1. Incipient sensor faults develop process.

of incipient faults. If no actions is taken, faults may continuously evolve into failures, which means that output signals are meaningless. The incipient sensor fault develops in a continuous way shown as Fig.1. For the considered continuous developing fault signals f in system (1), it can be divided into three stages: incipient sensor fault, sensor fault and sensor failure. As seen from Fig.1, the following terms can be given: $0 < \|\xi(x,u,t)\| < \bar{\xi}$, called "incipient sensor fault"; $\bar{\xi} \leq \|\xi(x,u,t)\| < \bar{\bar{\xi}}$, called "sensor fault"; and $\bar{\bar{\xi}} \leq \|\xi(x,u,t)\| < +\infty$ called "sensor failure". The "sensor failure" can be further divided into "light sensor failure" and "severe sensor failure" by the bound $\bar{\bar{\bar{\xi}}}$, that is $\bar{\bar{\xi}} \leq \|\xi(x,u,t)\| < \bar{\bar{\bar{\xi}}}$ called "light sensor failure" and $\bar{\bar{\xi}} \leq \|\xi(x,u,t)\| < +\infty$ called "light sensor failure". In addition, four time instants T_0, T_1, T_2 and T_3 are defined, which represent incipient sensor fault occurrence time, incipient sensor fault developing to sensor fault time (i.e., the time when ξ surpassing $\bar{\xi}$), incipient sensor fault developing to sensor failure time (i.e., the time when ξ surpassing $\bar{\bar{\xi}}$) respectively.

Remark 1. For mechanical components such as bears, wears and electrolytic capacitors, $\bar{\xi}, \bar{\bar{\xi}}$ and $\bar{\bar{\bar{\xi}}}$ represent differnet damage levels which can be obtained by real experiences and/or statistical data. To some extend, $\bar{\xi}, \bar{\bar{\xi}}$ and $\bar{\bar{\bar{\xi}}}$ are determined by the requirement of system performance level. An example of linear state feedback closed-loop system with the only pole at $\delta = a$ is given in Fig.2 to illustrate how to choose these bounds. Assuming that after incipient sensor faults occur, the linear system performance will degrade and the placed pole will go to right direction in S plane. As shown in Fig.2, when the linear system performance degrade to a level where the pole $\delta = b$, the value of $\xi(\cdot) = \bar{\xi}$. Also the linear system performance degrade to a level where the pole $\delta = d$, the value of $\xi(\cdot) = \bar{\bar{\xi}}$. Moreover, when the linear system is marginal stable, that is the pole $\delta = 0$, the value of $\xi(\cdot) = \bar{\bar{\bar{\xi}}}$.

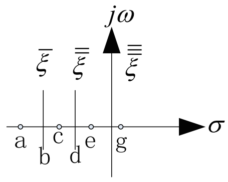


Fig. 2. The sketch for the selection of $\bar{\xi}$, $\bar{\bar{\xi}}$ and $\bar{\bar{\bar{\xi}}}$

B. Preliminaries and Assumptions

Consider system (1) with the output y partitioned as

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Cx + Ff(x,u,t), \quad C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ I_q \end{bmatrix}, \tag{4}$$

where $C_1 \in \mathcal{R}^{(p-q)\times n}$ and $C_2 \in \mathcal{R}^{q\times n}$.

From (4), the system (1) and incipient sensor faults (3) can be represented in an augmented form as follows:

$$\begin{aligned} \underbrace{\begin{bmatrix} \dot{x} \\ \dot{f} \end{bmatrix}}_{\dot{x}_a} &= \underbrace{\begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}}_{A_a} \underbrace{\begin{bmatrix} x \\ f \end{bmatrix}}_{x_a} + \underbrace{\begin{bmatrix} g(x,u,t) \\ 0 \end{bmatrix}}_{g_a(x_a,u,t)} + \underbrace{\begin{bmatrix} \eta(x,u,\omega,t) \\ 0 \end{bmatrix}}_{\eta_a(x_a,u,\omega,t)} + \underbrace{\begin{bmatrix} 0 \\ I_q \end{bmatrix}}_{D_a} \xi_a(x_a,u,t), \\ y &= \underbrace{[C, F]}_{C_a} \begin{bmatrix} x \\ f \end{bmatrix}, \end{aligned} \tag{5}$$

where $x_a := col(x, f)$, $A_a \in \mathcal{R}^{(n+q)\times(n+q)}$, $C_a \in \mathcal{R}^{p\times(n+q)}$ and $D_a \in \mathcal{R}^{(n+q)\times q}$ with C_a being full row rank and D_a being full column rank. Notice that the triple (A_a, D_a, C_a) is inherently relative degree one since $C_a D_a = I_q$ and rank$(D_a) = q$. From [31] and relative degree one fact, there exists a coordinate transformation T_1 such that, without loss of generality that system (5) is transformed into the following form

$$\begin{aligned} \dot{x}_1 &= A_{a11}x_1 + A_{a12}x_2 + g_{a1}(x_a,u,t) + \eta_{a1}(x_a,u,\omega,t), \\ \dot{x}_2 &= A_{a21}x_1 + A_{a22}x_2 + g_{a2}(x_a,u,t) + \eta_{a2}(x_a,u,\omega,t) + D_{a2}\xi(x,u,t), \\ y &= C_{a2}x_2, \end{aligned} \tag{6}$$

where $x_a = col(x_1, x_2)$, $x_1 \in \mathcal{R}^{n+q-p}$, $x_2 \in \mathcal{R}^p$, A_{a11}, A_{a12}, A_{a21}, A_{a22}, D_{a2}, C_{a2}, $g_{a1}(\cdot)$, $g_{a2}(\cdot)$ $\eta_{a1}(\cdot)$ and $\eta_{a2}(\cdot)$ can be got based on [31]. Moreover, C_{a2} is nonsingular.

Assumption 1. The triple (A_a, D_a, C_a) is minimum phase (The invariant zeros (if any) of the triple (A_a, D_a, C_a) lie in the left half plane).

Remark 2. Assumption 1 is necessary for the sliding mode observer design for systems with unknown inputs [15], [17], [31]. It has proved in [36] that the unobservable modes of the pair (A, C) are the invariant zeros of the triple (A_a, D_a, C_a). Therefore, in order to check Assumption 1, it is only required to find the unobservable modes of the pair (A, C) and check whether all the unobservable modes lie in the left half plane. ∇

III. FDE SLIDING MODE OBSERVER DESIGN

In this section, the sliding mode observer with designed sliding surface as FDE (fault detection estimator) will be designed to guarantee that the $\mathcal{L}_2$ gain from uncertainties to output estimation errors are minimized. Both the healthy and faulty systems enter into the sliding surface before the incipient sensor fault developing to severe sensor failure (i.e., $\xi > \bar{\bar{\bar{\xi}}}$).

From [18], there exist another linear transformation T described by

$$T = \begin{bmatrix} I_{n+q-p} & L \\ 0 & I_q \end{bmatrix} \tag{7}$$

with $L = [L_1, 0]$ with $L_1 \in \mathcal{R}^{(n+q-p)\times(p-q)}$ such that $\hat{A}_{a11} = A_{a11} + LA_{a21}$ is stable, and $\hat{A}_{a12} = (A_{a11} + LA_{a21})L + (A_{a12} + LA_{a22})$, $\hat{g}_{a1} = g_{a1} + Lg_{a2} = [I_{n+q-p}, L]g_a$, $\hat{\eta}_{a1} = \eta_{a1} + L\eta_{a2} = [I_{n+q-p}, L]\eta_a$. Therefore, in the new coordinates $z = Tx_a$, system (6) can be described by

$$\begin{aligned}
&\dot{z}_1 = \hat{A}_{a11}z_1 + \hat{A}_{a12}z_2 + \hat{g}_{a1}(T^{-1}z, u, t) + \hat{\eta}_{a1}(T^{-1}z, u, \omega, t),\\
&\dot{z}_{21} = A^1_{a21}z_1 + A^{11}_{a22}z_{21} + A^{12}_{a22}z_{22} + g^1_{a2}(T^{-1}z, u, t) + \eta^1_{a2}(T^{-1}z, u, \omega, t),\\
&\dot{z}_{22} = A^2_{a21}z_1 + A^{21}_{a22}z_{21} + A^{22}_{a22}z_{22} + g^2_{a2}(T^{-1}z, u, t) + \eta^2_{a2}(T^{-1}z, u, \omega, t) + D_{a22}\xi\left(T^{-1}z, u, t\right),\\
&y = C_{a21}z_{21} + C_{a22}z_{22},
\end{aligned} \tag{8}$$

where $z = col(z_1, z_2)$ with $z_1 \in \mathcal{R}^{n+q-p}$, $z_2 \in \mathcal{R}^p$, and $z_2 := col(z_{21}, z_{22}) = C^{-1}_{a2}y$ with $z_{21} \in \mathcal{R}^{p-q}$ and $z_{22} \in \mathcal{R}^q$. Moreover, $z_{21} = [I_{p-q}, 0]C^{-1}_{a2}y$ and $z_{22} = [0, I_q]C^{-1}_{a2}y$.

Assumption 2. The modeling uncertainties, represented by $\eta_a(\cdot)$ in (5), $\eta_{a1}(\cdot)$ and $\eta_{a2}(\cdot)$ in (6), satisfy that $\forall(x_a, y, u, \omega) \in \mathcal{X}_a \times \mathcal{Y} \times \mathcal{U} \times \mathcal{W}$, $\forall t > 0$,

$$\|\eta_a(x_a, u, \omega, t)\| \le \bar{\eta}, \|\eta_{a1}(x_a, u, \omega, t)\| \le \bar{\eta}_1(y, u, t), \|\eta_{a2}(x_a, u, \omega, t)\| \le \bar{\eta}_2(y, u, t) \tag{9}$$

where $\bar{\eta}$ is known constant, $\bar{\eta}_1(\cdot)$ and $\bar{\eta}_2(\cdot)$ are known functions, and $\mathcal{X}_a \subset \mathcal{R}^{n+q}, \mathcal{W} \subset \mathcal{R}^h, \mathcal{U} \subset \mathcal{R}^m$ and $\mathcal{Y} \subset \mathcal{R}^p$ are compact sets.

Assumption 3. The known nonlinear terms $g_{a1}(x_a, u, t)$ and $g_{a2}(x_a, u, t)$ in (6) are uniformly Lipschitz in $u \in \mathcal{U}$, i.e., $x_a, \hat{x}_a \in \mathcal{X}_a$,

$$\begin{aligned} \|g_{a1}(x_a, u, t) - g_{a1}(\hat{x}_a, u, t)\| &\leq \mathscr{L}_1 \|x_a - \hat{x}_a\|, \\ \|g_{a2}(x_a, u, t) - g_{a2}(\hat{x}_a, u, t)\| &\leq \mathscr{L}_2 \|x_a - \hat{x}_a\| \end{aligned} \tag{10}$$

where $\mathscr{L}_1$ and $\mathscr{L}_2$ are the known Lipschitz constants for $g_{a1}(x_a, u, t)$ and $g_{a2}(x_a, u, t)$, respectively.

Remark 3. Assumption 2 requires that bounds on uncertainties in (5) and (6) are known, which is important to obtain the proper adaptive thresholds [34], [35]. In this paper, there is no constraint on the distribution matrices of uncertainties and faults. However, in some sliding mode observer based fault diagnosis papers [20] and [22], additional conditions on the distribution matrices are necessary to completely decouple faults and uncertainties. ∇

Since z_2 is known, then z_2 can be used to construct observers. Denoting $\bar{\hat{z}} = col(\hat{z}_1, C_{a2}^{-1}y)$, then the sliding mode observer for system (8) is chosen as

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{A}_{a11}\hat{z}_1 + \hat{A}_{a12}C_{a2}^{-1}y + \hat{g}_{a1}(T^{-1}\bar{\hat{z}}, u, t), \\ \dot{\hat{z}}_{21} &= A_{a21}^1\hat{z}_1 + A_{a22}^{11}\hat{z}_{21} + A_{a22}^{12}\hat{z}_{22} + g_{a2}^1(T^{-1}\bar{\hat{z}}, u, t) + K_{11}\left([I_{p-q}, 0]C_{a2}^{-1}y - \hat{z}_{21}\right) \\ &\qquad + K_{12}\left([0, I_q]C_{a2}^{-1}y - \hat{z}_{22}\right) + \nu, \\ \dot{\hat{z}}_{22} &= A_{a21}^2\hat{z}_1 + A_{a22}^{21}\hat{z}_{21} + A_{a22}^{22}\hat{z}_{22} + g_{a2}^2(T^{-1}\bar{\hat{z}}, u, t) + K_{21}\left([I_{p-q}, 0]C_{a2}^{-1}y - \hat{z}_{21}\right) \\ &\qquad + K_{22}\left([0, I_q]C_{a2}^{-1}y - \hat{z}_{22}\right), \\ \hat{y} &= C_{a21}\hat{z}_{21} + C_{a22}\hat{z}_{22} \end{aligned} \tag{11}$$

where K_{11} and K_{22} are chosen such that $A_{a22}^{11} - K_{11}$ and $A_{a22}^{22} - K_{22}$ are stable, and from [18], K_{21} will not effect the observer stability and can be any matrix with appropriate dimension. The function ν is defined by

$$\nu = M(\cdot)sgn([I_{p-q}, 0]C_{a2}^{-1}y - \hat{z}_{21}) \tag{12}$$

where $M(\cdot)$ is a positive scalar function to be determined.

Let $e_1 = z_1 - \hat{z}_1$, $e_{21} = z_{21} - \hat{z}_{21}$ and $e_{22} = z_{22} - \hat{z}_{22}$. Then from (8) and (11), before incipient sensor faults occur (i.e., for $t < T_0$), the state estimation error dynamics are described by

$$\begin{aligned}
\dot{e}_1 &= \hat{A}_{a11}e_1 + \hat{g}_{a1}(T^{-1}z, u, t) - \hat{g}_{a1}(T^{-1}\bar{\hat{z}}, u, t) + \hat{\eta}_{a1}(T^{-1}z, u, \omega, t), && (13)\\
\dot{e}_{21} &= A^1_{a21}e_1 + \left(A^{11}_{a22} - K_{11}\right)e_{21} + (A^{12}_{a22} - K_{12})e_{22} \\
&\quad + g^1_{a2}(T^{-1}z, u, t) - g^1_{a2}(T^{-1}\bar{\hat{z}}, u, t) + \eta^1_{a2}(T^{-1}z, u, \omega, t) - \nu, && (14)\\
\dot{e}_{22} &= A^2_{a21}e_1 + (A^{21}_{a22} - K_{21})e_{21} + \left(A^{22}_{a22} - K_{22}\right)e_{22} \\
&\quad + g^2_{a2}(T^{-1}z, u, t) - g^2_{a2}(T^{-1}\bar{\hat{z}}, u, t) + \eta^2_{a2}(T^{-1}z, u, \omega, t), && (15)\\
e_y &= C_{a21}e_{21} + C_{a22}e_{22}. && (16)
\end{aligned}$$

Note that

$$T^{-1}z - T^{-1}\bar{\hat{z}} = \begin{bmatrix} I_{n+q-p} & L \\ 0 & I_q \end{bmatrix} \begin{bmatrix} z_1 - \hat{z}_1 \\ z_2 - C^{-1}_{a2}y \end{bmatrix} = \begin{bmatrix} e_1 \\ 0 \end{bmatrix}. \tag{17}$$

For error dynamics (13)-(16), the sliding surface is chosen as

$$\mathscr{S} = \{(e_1, e_{21}, e_{22}) : e_{21} = 0\}. \tag{18}$$

Remark 4. In [34] and [35], the output estimation errors e_y (including e_{21} and e_{22}) are chosen as residuals. However, from error dynamics (13)-(16), it can be seen that e_{22} reflects fault information directly, e_1 and e_{21} reflect fault information through e_{22} indirectly. Therefore, only e_{22} is chosen as residual can arrive the same results comparing with that choosing e_y as residual in [34] and [35]. Furthermore, choosing e_{22} as residual facilitates to design more proper adaptive threshold to improve detectability. ∇

Remark 5. In [15], [16], [18] and [30], the hyperplane $e_y = 0$ is chosen as sliding surface, in which faults are completely rejected by "equivalent output rejection function". In this paper, based on the chosen sliding surface (18), the faults will not be rejected by designed discontinuous rejection function ν in (12), which facilitates to generate residuals to detect faults. Moreover, the designed adaptive thresholds are more proper than the adaptive thresholds in [34] and [35] because of the reduced order sliding motion. ∇

Then the following conclusion is ready to presented.

Proposition 1. *Under Assumptions 1-3, the sliding motion of system (13)-(16) without lumped uncertainties $\hat{\eta}_{a1}$ and η^2_{a2} associated with the surface (18) is asymptotically stable if $K_{21} = A^{21}_{a22}$*

and there exist SPD matrices P_1 and P_2, L defined in (7) and K_{22} such that for the given positive constants ε_1, ε_2, ε_3, γ, $\mathscr{L}_1$ and $\mathscr{L}_2$ (Lipchitz constants for $g_{a1}(x_a, u, t)$ and $g_{a2}(x_a, u, t)$ with respect to x_a) such that the matrix inequalities

$$\begin{aligned}\left(\bar{P}_1\bar{A}_1 + \bar{A}_1^T\bar{P}_1^T\right) + \frac{1}{\varepsilon_1}\bar{P}_1\bar{P}_1^T + \varepsilon_1(\mathscr{L}_a)^2 I_{n+q-p} + \varepsilon_2(\mathscr{L}_2)^2 I_{n+q-p} & \\ + \frac{1}{\gamma^2}P_1P_1 + \frac{1}{\gamma^2}P_1LL^TP_1 + \varepsilon_3 A_{a21}^{2T}A_{a21}^2 & < 0, \end{aligned} \tag{19}$$

$$\left(P_2\bar{A}_2 + \bar{A}_2^T P_2^T\right) + \left(\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \frac{1}{\gamma^2}\right)P_2^2 + C_{a22}^T C_{a22} < 0 \tag{20}$$

with $\mathscr{L}_a = \mathscr{L}_1 + \mathscr{L}_2$ is solvable, where

$$\bar{P}_1 := P_1\left[I_{n+q-p}, L\right], \bar{A}_1 := \begin{bmatrix} A_{a11} \\ A_{a21} \end{bmatrix}, \bar{A}_2 := A_{a22}^{22} - K_{22}. \tag{21}$$

Furthermore, with lumped uncertainties $\hat{\eta}_{a1}$ and η_{a2}^2, under Assumption 2, the error systems (13) and (15) are ISS (input-to-state stable), and the $\mathcal{L}_2$ gain from $\eta_{a1}(\cdot)$ and $\eta_{a2}(\cdot)$ to e_{22} satisfies that

$$\int_0^t e_{22}^T C_{a22}^T C_{a22} e_{22} d\tau \le \gamma^2 \int_0^t \left(\eta_{a1}^T\eta_{a1} + 2\eta_{a2}^{2T}\eta_{a2}^2\right) d\tau + \epsilon \tag{22}$$

where ϵ is defined later.

Proof: Consider a Lyapunov candidate function

$$V = e_1^T P_1 e_1 + e_{22}^T P_2 e_{22}. \tag{23}$$

If $K_{21} = A_{a22}^{21}$, the time derivative of V along the trajectories of the systems (13) and (15) is given by

$$\begin{aligned}\dot{V} = & e_1^T\left(P_1\left(A_{a11} + LA_{a21}\right) + \left(A_{a11} + LA_{a21}\right)^T P_1\right)e_1 \\ & + 2e_1^T P_1\left[I_{n+q-p}, L\right]\left(g_a\left(T^{-1}z, u, t\right) - g_a\left(T^{-1}\bar{\hat{z}}, u, t\right)\right) + 2e_1^T P_1\eta_{a1}(\cdot) + 2e_1^T P_1 L\eta_{a2}(\cdot) \\ & + e_{22}^T\left(P_2\left(A_{a22}^{22} - K_{22}\right) + \left(A_{a22}^{22} - K_{22}\right)^T P_2\right)e_{22} + 2e_{22}^T P_2 A_{a21}^2 e_1 \\ & + 2e_{22}^T P_2\left(g_{a2}^2\left(T^{-1}z, u, t\right) - g_{a2}^2\left(T^{-1}\bar{\hat{z}}, u, t\right)\right) + 2e_{22}^T P_2\eta_{a2}^2(\cdot).\end{aligned}$$

Note that, from $\mathscr{L}_a = \mathscr{L}_1 + \mathscr{L}_2$, it can be obtained that $\left\|g_a\left(T^{-1}z, u, t\right) - g_a\left(T^{-1}\bar{\hat{z}}, u, t\right)\right\| \le \left\|g_{a1}\left(T^{-1}z, u, t\right) - g_{a1}\left(T^{-1}\bar{\hat{z}}, u, t\right)\right\| + \left\|g_{a2}\left(T^{-1}z, u, t\right) - g_{a2}\left(T^{-1}\bar{\hat{z}}, u, t\right)\right\| \le \mathscr{L}_1\left\|T^{-1}z - T^{-1}\bar{\hat{z}}\right\| +$

$\mathscr{L}_2 \left\| T^{-1}z - T^{-1}\bar{\hat{z}} \right\| = \mathscr{L}_a \left\| T^{-1}z - T^{-1}\bar{\hat{z}} \right\| = \mathscr{L}_a \|e_1\|$, then from the well-known inequality $2X^TY \leq \frac{1}{\varepsilon}X^TX + \varepsilon Y^TY$ for any scalar $\varepsilon > 0$, it follows that

$$\begin{aligned}
&\dot{V} + e_{22}^T C_{a22}^T C_{a22} e_{22} - \gamma^2 \left(\eta_{a1}^T \eta_{a1} + 2\eta_{a2}^{2T} \eta_{a2}^2 \right) \\
&= e_1^T \left(\bar{P}_1 \bar{A}_1 + \bar{A}_1^T \bar{P}_1^T \right) e_1 + \frac{1}{\varepsilon_1} e_1^T \bar{P}_1 \bar{P}_1^T e_1 + \varepsilon_1 (\mathscr{L}_a)^2 e_1^T e_1 + \varepsilon_2 (\mathscr{L}_2)^2 e_1^T e_1 \\
&+ \frac{1}{\gamma^2} e_1^T P_1 P_1 e_1 - \left(\gamma \eta_{a1} - \frac{1}{\gamma} e_1^T P_1 \right) \left(\gamma \eta_{a1} - \frac{1}{\gamma} e_1^T P_1 \right)^T \\
&+ \frac{1}{\gamma^2} e_1^T P_1 L L^T P_1 e_1 - \left(\gamma \eta_{a2} - \frac{1}{\gamma} e_1^T P_1 L \right) \left(\gamma \eta_{a2} - \frac{1}{\gamma} e_1^T P_1 L \right)^T \\
&+ e_{22}^T \left(P_2 \bar{A}_2 + \bar{A}_2^T P_2^T \right) e_{22} + \frac{1}{\varepsilon_2} e_{22}^T P_2 P_2 e_{22} + \frac{1}{\varepsilon_3} e_{22}^T P_2 P_2 e_{22} + \varepsilon_3 e_1^T A_{a21}^{2T} A_{a21}^2 e_1 \\
&+ e_{22}^T C_{a22}^T C_{a22} e_{22} + \frac{1}{\gamma^2} e_{22}^T P_2 P_2 e_{22} - \left(\gamma \eta_{a22}^2 - \frac{1}{\gamma} e_{22}^T P_2 \right) \left(\gamma \eta_{a22}^2 - \frac{1}{\gamma} e_{22}^T P_2 \right)^T.
\end{aligned} \tag{24}$$

Then

$$\begin{aligned}
&\dot{V} + e_{22}^T C_{a22}^T C_{a22} e_{22} - \gamma^2 \left(\eta_{a1}^T \eta_{a1} + 2\eta_{a2}^{2T} \eta_{a2}^2 \right) \\
&\leq e_1^T \left(\left(\bar{P}_1 \bar{A}_1 + \bar{A}_1^T \bar{P}_1^T \right) + \tfrac{1}{\varepsilon_1} \bar{P}_1 \bar{P}_1^T + \varepsilon_1 (\mathscr{L}_a)^2 I_{n+q-p} + \varepsilon_2 (\mathscr{L}_2)^2 I_{n+q-p} + \tfrac{1}{\gamma^2} P_1 P_1 \right) e_1 \\
&+ e_1^T \varepsilon_3 A_{a21}^{2T} A_{a21}^2 e_1 + \tfrac{1}{\gamma^2} e_1^T P_1 L L^T P_1 e_1 \\
&+ e_{22}^T \left(\left(P_2 \bar{A}_2 + \bar{A}_2^T P_2^T \right) + \left(\tfrac{1}{\varepsilon_2} + \tfrac{1}{\varepsilon_3} + \tfrac{1}{\gamma^2} \right) P_2^2 + C_{a22}^T C_{a22} \right) e_{22} \leq 0.
\end{aligned} \tag{25}$$

Thus the inequality (22) is satisfied with $\epsilon = V(0) = e_1^T(0) P_1 e_1(0) + e_{22}^T(0) P_2 e_{22}(0)$, which only depends on the initial estimation error $e_1(0)$ and $e_{22}(0)$.

Hence the result follows. ■

Note that inequalities (19) and (20) can be transformed into the following LMI problem: for the given positive constants ε_1, ε_2, ε_3, γ, $\mathscr{L}_1$ and $\mathscr{L}_2$, solving P_1, P_2, Y_1, Y_2 such that

$$\begin{bmatrix}
\Xi_1(P_1, Y_1) & P_1 & Y_1 & P_1 & Y_1 & (A_{a21}^2)^T \\
* & -\varepsilon_1 I_{n+q-p} & 0 & 0 & 0 & 0 \\
* & * & -\varepsilon_1 I_p & 0 & 0 & 0 \\
* & * & * & -\gamma^2 I_{n+q-p} & 0 & 0 \\
* & * & * & * & -\gamma^2 I_p & 0 \\
* & * & * & * & * & -\varepsilon_3 I_q
\end{bmatrix} < 0, \tag{26}$$

$$\begin{bmatrix} \Xi_2(P_2, Y_2) & P_2 & P_2 & P_2 & C_{a22}^T \\ * & -\varepsilon_2 I_q & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I_q & 0 & 0 \\ * & * & * & -\gamma^2 I_q & 0 \\ * & * & * & * & -I_p \end{bmatrix} < 0, \tag{27}$$

where $\Xi_1(P_1, Y_1) = P_1 A_{a11} + A_{a11}^T P_1 + Y_1 A_{a21} + A_{a21}^T Y_1^T + \varepsilon_1 (\mathscr{L}_a)^2 I_{n+q-p} + \varepsilon_2 (\mathscr{L}_2)^2 I_{n+q-p}$, $Y_1 = P_1 L$ with $P_1 > 0$, $\Xi_2(P_2, Y_2) = P_2 A_{a22}^{22} + (A_{a22}^{22})^T P_2 - Y_2 - Y_2^T$, $Y_2 = P_2 K_{22}$ with $P_2 > 0$.

The estimation error e_{22} is the residual used to detect the fault occurrence. The objective here is to choose the gain L and K_{22} such that minimizing the effect of the lumped disturbances $\hat{\eta}_{a1}(\cdot)$ and $\eta_{a2}^2(\cdot)$ on e_{22}, that is, to minimize the $\mathcal{L}_2$ gain $\gamma > 0$. Therefore an optimization problem can be posed with regard to P_1, P_2, Y_1, Y_2 and γ^2, i.e., Minimize γ^2 s.t. (26) and (27) with $P_1 > 0$ and $P_2 > 0$.

Remark 6. From Proposition 1, it can be seen that the Lyapunov matrix in (23) of the error dynamics (13) and (15) is block diagonal matrix, which implies that $\hat{A}_{a11}$ and $\hat{A}_{a22}^{22}$ are stable and hence the sliding motion (13) and (15) associated with sliding surface (18) is ISS with the lumped uncertainties $\hat{\eta}_{a1}(\cdot)$ and $\eta_{a2}^2(\cdot)$. ∇

To design gain $M(\cdot)$ in (12), the bound of e_1 in (13) with Lipchitz nonlinear term should be calculated. Therefore, the following lemmas are introduced.

Lemma 2. (Bellman-Gronwall Lemma [33]). Let t_0, c_0, c_1 and c_2 be nonnegative constants, and $\kappa(t)$ be a nonnegative piecewise continuous function. If $h(t)$ satisfies the inequality

$$h(t) \le c_0 e^{-\lambda(t-t_0)} + c_1 + c_2 \int_{t_0}^{t} e^{-\lambda(t-\tau)} \kappa(\tau) h(\tau) d\tau, \forall t \ge t_0,$$

then

$$h(t) \le (c_0 + c_1) e^{-\lambda(t-t_0)} e^{c_2 \int_{t_0}^{t} \kappa(s) ds} + c_1 \lambda \int_{t_0}^{t} e^{-\lambda(t-\tau)} e^{c_2 \int_{\tau}^{t} \kappa(s) ds} d\tau, \forall t \ge t_0.$$

Lemma 3. Consider the error dynamic system described by (13) with $\hat{A}_{a11}$ being stable. Let k_0 and λ_0 be positive constants such that $\left\| e^{\hat{A}_{a11} t} \right\| \le k_0 e^{-\lambda_0 t}$. Assume that $\lambda_0 > k_0 (1 + \|L\|) \mathscr{L}_a$, where $\mathscr{L}_a$ is given in Proposition 1. Then the state estimation error $e_1(t)$ satisfies:

$$\|e_1(t)\| \le \chi(t). \tag{28}$$

where $\chi(t) \triangleq \frac{k_0\bar{\eta}}{\lambda_0 - k_0(1+\|L\|)\mathscr{L}_a} + \left(k_0\omega_1 - \frac{k_0\bar{\eta}}{\lambda_0 - k_0(1+\|L\|)\mathscr{L}_a}\right) e^{-(\lambda_0 - k_0(1+\|L\|)\mathscr{L}_a)t}$ and ω_1 is a constant bound for $\|z_1(0)\|$.

Proof: From (13), it is obtained that

$$e_1 = e^{\hat{A}_{a11}t} e_1(0) + \int_0^t e^{\hat{A}_{a11}(t-\tau)} \left([I_{n+q-p}, L] \left(g_a\left(T^{-1}z, u, t\right) - g_a\left(T^{-1}\bar{\bar{z}}, u, t\right) + \eta_a \right) \right) d\tau. \quad (29)$$

By using (9), (10) and (17) and applying the triangle inequality, it is got that

$$\|e_1\| \leq \frac{k_0\bar{\eta}}{\lambda_0} + (1 + \|L\|)\mathscr{L}_a \int_0^t e^{-\lambda_0(t-\tau)} \|e_1\| d\tau + k_0\left(\omega_1 - \frac{\bar{\eta}}{\lambda_0}\right) e^{-\lambda_0 t}, \quad (30)$$

where k_0 and λ_0 are positive constants satisfying that $\left\| e^{\hat{A}_{a11}t} \right\| \leq k_0 e^{-\lambda_0 t}$, and ω_1 is a (possibly conservative) constant bound for $z_1(0)$, such that $\|e_1(0)\| = \|z_1(0)\| \leq \omega_1$, which always exist as in [34].

Now, by applying Lemma 2 to (30) with $c_0 = k_0\left(\omega_1 - \frac{\bar{\eta}}{\lambda_0}\right)$, $c_1 = \frac{k_0\bar{\eta}}{\lambda_0}$, $c_2 = k_0(1 + \|L\|)\mathscr{L}_a$ and $\kappa(t) = 1$. The inequality (28) follows. ■

Proposition 2. *Under Assumptions 1-3, before sensor fault develop to sever sensor failure, i.e., $\xi \leq \bar{\bar{\bar{\xi}}}$, the error dynamics (13)-(16) are driven to the sliding surface $\mathscr{S}$ given in (18) in finite time and remain on it if K_{11} and K_{12} in (14) are chosen as $K_{11} = A_{a22}^{11} - \hat{A}_{a22}^{11}$ with $\hat{A}_{a22}^{11}$ being stable and $K_{12} = A_{a22}^{12}$ respectively, and the gain $M(\cdot)$ in (12) satisfies*

$$M(\cdot) \geq \left(\left\|A_{a21}^1\right\| + \mathscr{L}_1\right)\chi(t) + \bar{\eta}_2(\cdot) + \|D_{a22}\bar{\bar{\bar{\xi}}}\| + \varpi, \quad (31)$$

where ϖ is a positive constant, $\chi(t)$ is defined in Lemma 3.

Proof: Let $V = e_{21}^T e_{21}$. From the expression of (14) and $K_{11} = A_{a22}^{11} - \hat{A}_{a22}^{11}$ where $\hat{A}_{a22}^{11}$ is stable, and $K_{12} = A_{a22}^{12}$, it follows after faults occur that

$$\begin{aligned}\dot{V} =& e_{21}^T \left(\hat{A}_{a22}^{11} + (\hat{A}_{a22}^{11})^T\right) e_{21} + 2e_{21}^T D_{a22}\xi(\cdot) \\ &+ 2e_{21}^T \left(A_{a21}^1 e_1 + g_{a2}^1(T^{-1}z, u, t) - g_{a2}^1(T^{-1}\bar{\bar{z}}, u, t) + \eta_{a2}^1(T^{-1}z, u, \omega, t)\right) - 2e_{21}^T\nu.\end{aligned} \quad (32)$$

Since $\hat{A}_{a22}^{11}$ is symmetric negative definite by designing appropriate K_{11}, it follows that $\hat{A}_{a22}^{11} + (\hat{A}_{a22}^{11})^T < 0$. Then by applying (12),

$$\dot{V} \leq 2\|e_{21}\| \left(\left(\left\|A_{a21}^1\right\| + \mathscr{L}_1\right)\|e_1\| + \bar{\eta}_2(\cdot) + \|D_{a22}\xi(\cdot)\|\right) - 2M(\cdot)\|e_{21}\|. \quad (33)$$

From (31) and (33), it follows that $\dot{V} \leq -2\varpi\|e_{21}\| \leq -2\varpi V^{1/2}$, which means that a reachability condition is satisfied. Hence the conclusion follows. ■

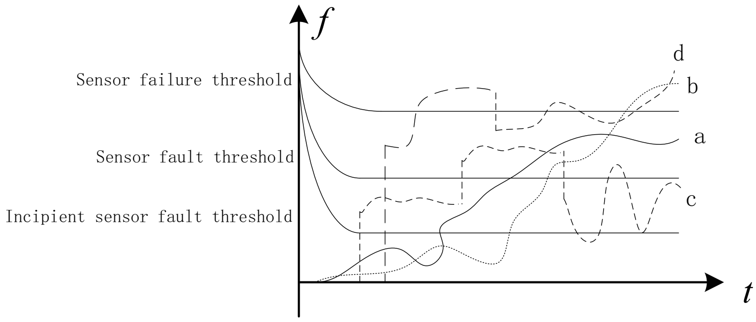


Fig. 3. Lines a, b: continuous incipient fault developments; Lines c, d: piecewise continuous faults.

Remark 7. Propositions 1 and 2 show that the error dynamical systems (13)-(15) are asymptotically stable. It should be noted that this paper mainly focuses on fault detection by designing proper thresholds. The observer designed here may not be directly used to estimate/reconstruct fault as in [15]- [18]. ∇

IV. Sensor Fault Detection Decision Schemes

In this paper, the faults considered are generated by differential equation (3), which represents two types of faults: continuous faults and piecewise continuous faults, shown as Fig 3. Therefore, the general sensor fault detection decision schemes, proposed in this paper, are divided into two types:

1) incipient sensor fault development detection decision scheme, which is used to decide what time the incipient sensor faults are developed into sensor faults and what time the incipient faults are developed into sensor failures.
2) fault detection decision scheme, which is used to detect the incipient faults, faults and failures occurrence.

Decision Principles: Corresponding to above two types fault detection schemes and from Fig.3, there are also two decision principles:

1) **Incipient Sensor Fault Development Detection Decision Principle:** For incipient sensor fault developing to sensor fault detection, if the estimation errors e_y are continuous, and there is a time instant such that there is at least one of estimation errors e_y surpassing incipient fault threshold and another time instant surpassing fault threshold. Then the development is considered as completed, such as the curve a in Fig.3; For incipient sensor fault developing to sensor failure detection, the estimation errors e_y are also required to

be continuous, excepting above two time instants, there is another time instant to surpass sensor failure threshold, such as the curve b in Fig.3.

2) **Fault Detection Decision Principle:** If there is at least one of estimation error e_y surpasses incipient fault threshold, the incipient sensor fault is considered occurrence. The detections on sensor fault and failure are the same with incipient sensor fault detection. Curves c and d in Fig.3 have provided two examples.

Remark 8. It should be pointed out that Principle 2) is for traditional fault detection scheme, which has been discussed in, [25], [27], [28], [34], and Principle 1) is a novel development detection decision scheme, which is mainly used to detect and decide the development of continuous incipient fault.

A. Fault Detection Decision Schemes

1) ***Incipient Sensor Fault Developing Detection Decision Schemes****:* When sliding motion takes place and maintains on $\mathscr{S}$ given by (18), $e_{21} = \dot{e}_{21} = 0$. Therefore, each component of the output estimation error $e_{yj}(t), j = 1, 2, \cdots, p$ can be expressed as $e_{yj}(t) \triangleq C_{a22j}e_{22}$ where C_{a22j} is the jth row vector of matrix C_{a22}.

a) ***Incipient Sensor Fault developing to Sensor Fault Decision Scheme****:* By applying (14), it is obtained that

$$|e_{yj}| \leq k_j \int_0^t e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right) \|e_1\| + \bar{\eta}_2\right] d\tau + k_j\omega_2 e^{-\lambda_j t} \tag{34}$$

where k_j and λ_j are positive constants satisfying $\left|C_{a22j}e^{\hat{A}_{a22}^{22}t}\right| \leq k_j e^{-\lambda_j t}$ and ω_2 is a bound on $\|z_{22}(0)\|$, that is $\|e_{22}(0)\| = \|z_{22}(0)\| \leq \omega_2$ (note that $\hat{z}_{22}(0) = 0$).

Based on (28) and (34), it follows that

$$|e_{yj}| \leq k_j \int_0^t e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right) \chi(\tau) + \bar{\eta}_2\right] d\tau + k_j\omega_2 e^{-\lambda_j t} \tag{35}$$

where $\chi(t)$ is defined in Lemma 3. Then incipient sensor fault threshold δ_{1j} is given by

$$\delta_{1j}(t) \triangleq k_j \int_0^t e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right) \chi(\tau) + \bar{\eta}_2\right] d\tau + k_j\omega_2 e^{-\lambda_j t}. \tag{36}$$

Based on Proposition 2, before incipient sensor fault is developed to severe sensor failure, (i.e., $\|\xi\| \leq \bar{\bar{\bar{\xi}}}$), the sliding motion maintains on $\mathscr{S}$. In presence of incipient sensor faults, by using similar reasoning as in (36), the sensor fault threshold δ_{2j} is given by

$$\delta_{2j}(t) \triangleq k_j \int_0^t e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right) \chi(\tau) + \bar{\eta}_2 + \|D_{a22}\|\bar{\xi}\right] d\tau + k_j\omega_2 e^{-\lambda_j t}. \tag{37}$$

According to (36) and (37), the decision scheme on incipient sensor fault developing to sensor fault is derived as follows:

I If output estimation errors e_y are continuous all the time and there exists at least one j with $j \in \{1, 2, \cdots, p\}$ such that e_{yj} exceeds incipient sensor fault threshold δ_{1j} given by (36), then the decision that there exists at least one incipient sensor fault developing to sensor fault is made at the time when e_{yj} exceeds sensor fault threshold δ_{2j} given in (37).

The detection time instant T_{ditft} is defined as the first time instant such that $|e_{yj}(T_{ditft})| > \delta_{2j}(T_{ditft})$ for some $T_{ditft} > T_0$ and some $j \in \{1, 2, \cdots, p\}$, that is,

$$T_{ditft} \triangleq \inf \bigcup_{j=1}^{p} \{t \geq T_0 \,||e_{yj}(t)| > \delta_{2j}(t)\}. \tag{38}$$

*b) **Incipient Sensor Fault Developing to Sensor Failure Decision Scheme**:* After sensor failures occur (i.e., $t > T_2$), the sliding motion on sliding surface $\mathscr{S}$ may be disrupted. Based on Proposition 2, there exists a bound $\bar{\bar{\bar{\xi}}}$ such that when $\bar{\bar{\xi}} < \xi \leq \bar{\bar{\bar{\xi}}}$, the sliding motion maintains on sliding surface. In addition, when $\xi > \bar{\bar{\bar{\xi}}}$, the sliding motion is destroyed, which is easy to decide sensor failures occurrence [14].

When the sensor failure signals are not big enough to disrupted the sliding motion, that is $\bar{\bar{\xi}} < \xi \leq \bar{\bar{\bar{\xi}}}$, in presence of sensor faults, by using similar reasoning as in (37), the sensor failure threshold δ_{3j} is given by

$$\delta_{3j}(t) \triangleq k_j \int_0^t e^{-\lambda_j(t-\tau)} \left[\left(\left\| A_{a21}^2 \right\| + \mathscr{L}_2 \right) \chi(\tau) + \bar{\eta}_2 + \|D_{a22}\| \bar{\bar{\xi}} \right] d\tau + k_j \omega_2 e^{-\lambda_j t}. \tag{39}$$

According to (37) and (36), the decision scheme on incipient sensor fault developing to sensor failure is as follows:

II If output estimation errors e_y are continuous and there exists at least one j with $j \in \{1, 2, \cdots, p\}$ such that e_{yj} exceeds incipient sensor fault threshold δ_{1j} given by (36) and sensor fault threshold δ_{2j} given by (37), then the decision that there exists at least one incipient sensor fault developing to sensor failure is made when e_{yj} exceeds sensor failure threshold δ_{3j} given in (39).

It is emphasized that the sensor failure detection time instant T_{ditfe} should be the first time instant such that $|e_{yj}(T_{ditfe})| > \delta_{3j}(T_{ditfe})$ for some $T_{ditfe} > T_0$ and some $j \in \{1, 2, \cdots, p\}$. Therefore,

$$T_{ditfe} \triangleq \inf \bigcup_{j=1}^{p} \{t > T_0 \,||e_{yj}(t)| > \delta_{3j}(t)\}. \tag{40}$$

*2) **Fault Detection Decision Scheme**:*

*a) **incipient sensor fault Detection Decision Scheme**:* According to (35), the decision scheme on incipient sensor fault detection is derived as follows:

III The decision on the occurrence of an incipient sensor fault is made when the modulus of at least one component of the output estimation errors (i.e., e_{yj}) exceeds incipient sensor fault threshold δ_{1j} given by (36). The incipient sensor fault detection time T_{di} is given by

$$T_{di} \triangleq \inf \bigcup_{j=1}^{p} \left\{ t > T_0 \,||e_{yj}(t)| > \delta_{1j}(t) \right\}. \tag{41}$$

*b) **Sensor Fault Detection Decision Scheme**:* Based on Proposition 2, after a sensor fault occurrence and before developing to sensor severe failure, the sliding motion maintains on $\mathscr{S}$. According to (37), sensor fault detection decision scheme is given as follows:

IV The decision on the occurrence of a sensor fault is made when the modulus of at least one component of the output estimation errors (i.e., e_{yj}) exceeds sensor fault threshold δ_{2j} given by (37). The sensor fault detection time instant T_{dft} is given by

$$T_{dft} \triangleq \inf \bigcup_{j=1}^{p} \left\{ t > T_1 \,||e_{yj}(t)| > \delta_{2j}(t) \right\}. \tag{42}$$

*c) **Sensor Failure Decision Scheme**:* If the sensor failure signals are not big enough to destroy the sliding motion, that is $\bar{\bar{\xi}} < \xi \leq \bar{\bar{\bar{\xi}}}$, the decision on sensor failure is given as follows:

V The decision on the occurrence of a sensor failure is made when the modulus of at least one component of the output estimation errors (i.e., e_{yj}) exceeds sensor failure threshold δ_{3j} given by (39). The sensor failure detection time T_{dfe} is given by

$$T_{dfe} \triangleq \inf \bigcup_{j=1}^{p} \left\{ t > T_2 \,||e_{yj}(t)| > \delta_{3j}(t) \right\}. \tag{43}$$

Remark 9. When the sliding motion of observer (11) on sliding surface $\mathscr{S}$ is disrupted by failure signals, the sensor failure occurrence decision can also be made [14]. However, for incipient sensor fault developing to sensor failure, the detection time T_{dfes}, where T_{dfes} is the time instant that the sliding motion of observer (11) is destroyed, is bigger than T_{dfe} given by (43) since the period of the continuous sensor failure $\bar{\bar{\xi}} < \xi < \bar{\bar{\bar{\xi}}}$ is ahead of the period that $\bar{\bar{\bar{\xi}}} < \xi$.

Therefore, the following theorem about fault detection is got:

Theorem 1. *For the nonlinear system (8), the fault detection decision schemes (I), (II) with adaptive thresholds (36), (37) and (39), guarantee that there is no false alarms before incipient sensor fault developing to sensor fault and sensor failure respectively. Furthermore, the fault* *detection decision schemes (III), (IV), (V) characterized by adaptive thresholds (36), (37) and (39) guarantee that there is no false alarms before incipient sensor fault, sensor fault and sensor failure occurrence respectively.*

Remark 10. It should be pointed out that, all the detection decisions (I-V) are made after that $e_{21} = 0$, that is after sliding motion takes place, which means that these decisions require that sliding motion takes place earlier than that faults occur. However, compared with the abrupt fault, an incipient fault (for example, the fault caused by mechanical wear) usually takes long time to cause system failure. Moreover, the reachability constant can be adjusted to guarantee that the sliding motion occurs at the very initial stage. Therefore, the developed results can be applied to a majority of cases in reality. ∇

B. Fault Detectability Schemes

In presence of incipient sensor fault and sensor fault (i.e., $T_0 < t < T_2$), based on Proposition 2, the sliding motion of error dynamics (13)-(16) maintains on $\mathscr{S}$ defined in (18), and each component e_{yj} of the output estimation error is given by

$$\begin{aligned} e_{yj}(t) = & \int_{T_0}^{t} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} \left[g_{a2}^{2}(T^{-1}z,u,\tau) - g_{a2}^{2}(T^{-1}\bar{\hat{z}},u,\tau) \right] d\tau \\ & + \int_{T_0}^{t} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} \left[A_{a21}^{2} e_1 + \eta_{a2}^{2}(T^{-1}z,u,\omega,\tau) \right] d\tau \\ & + \int_{T_0}^{t} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} D_{a22} \xi \left(T^{-1}z,u,\tau \right) d\tau + C_{a22j} e^{\hat{A}_{a22}^{22}(t-T_0)} e_{22}\left(T_0 \right). \end{aligned} \tag{44}$$

By applying the triangular inequality, it follows that

$$\begin{aligned} & \left| e_{yj} \right| \geq \left| \int_{T_0}^{t} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} D_{a22} \xi \left(T^{-1}z,u,\tau \right) d\tau \right| - k_j \left| e_{22}\left(T_0 \right) \right| e^{-\lambda_j (t-T_0)} \\ & -k_j \int_{T_0}^{T} e^{-\lambda_j (t-\tau)} \left[\left(\left\| A_{a21}^{2} \right\| + \mathscr{L}_2 \right) \chi\left(\tau \right) + \bar{\eta}_2 \right] d\tau. \end{aligned} \tag{45}$$

Corresponding to I-V fault detection decision schemes, there are five fault detectability schemes.

1) ***Incipient Sensor Fault Developing to Sensor Fault Detectability Scheme****:* The incipient sensor fault threshold $\delta_{1j}(t)$ given by (35) for $t > T_0$ can be written as

$$\delta_{1j}\left(t \right) \triangleq k_j \int_{T_0}^{t} e^{-\lambda_j (t-\tau)} \left[\left(\left\| A_{a21}^{2} \right\| + \mathscr{L}_2 \right) \chi(\tau) + \bar{\eta}_2 \right] d\tau + \delta_{1j}\left(T_0 \right) e^{-\lambda_j (t-T_0)}. \tag{46}$$

Therefore, based on (45) and (46), if there exist $T_0 < T_{di} < T_1$ such that

$$\begin{aligned} & \left| \int_{T_0}^{T_{di}} C_{a22j} e^{\hat{A}_{a22}^{22}(T_{di}-\tau)} D_{a22} \xi \left(T^{-1}z,u,\tau \right) d\tau \right| \geq 2k_j \int_{T_0}^{T_{di}} e^{-\lambda_j (T_{di}-\tau)} \left[\left(\left\| A_{a21}^{2} \right\| + \mathscr{L}_2 \right) \chi\left(\tau \right) + \bar{\eta}_{a2}^{2} \right] d\tau \\ & + \left[k_j \left| e_{22}\left(T_0 \right) \right| + \delta_j\left(T_0 \right) \right] e^{-\lambda_j (T_{di}-T_0)}, \end{aligned} \tag{47}$$

then $|e_{yj}| \geq \delta_{1j}$, and the incipient sensor fault will be detected at time $t = T_{di}$, i.e., $|e_{yj}(T_{di})| > \delta_{1j}(T_{di})$ before it develops to sensor fault.

Using the similar reasoning as in (46), the sensor fault adaptive threshold δ_{2j} for $t > T_1$ can also be written as

$$\delta_{2j}(t) \triangleq k_j \int_{T_1}^{t} e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right)\chi(\tau) + \bar{\eta}_2 + \|D_{a22}\|\bar{\xi}\right] d\tau + \delta_{2j}(T_1)\, e^{-\lambda_j(t-T_1)}. \quad (48)$$

Based on (45) and (48), if there exists $T_1 \leq T_{ditft} < T_2$ such that

$$\begin{aligned} &\left|\int_{T_1}^{T_{ditft}} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} D_{a22}\xi\left(T^{-1}z, u, \tau\right) d\tau\right| \geq \\ &k_j \int_{T_1}^{T_{ditft}} e^{-\lambda_j\left(T_{ditft}-\tau\right)} \left[2\left(\left(\|A_{a21}^2\| + \mathscr{L}_2\right)\chi(\tau) + \bar{\eta}_{a2}^2\right) + \|D_{a22}\bar{\xi}\|\right] d\tau \\ &+ \left[k_j\left|e_{22}(T_1)\right| + \delta_j(T_1)\right] e^{-\lambda_j\left(T_{ditft}-T_1\right)}, \end{aligned} \quad (49)$$

then $|e_{yj}| \geq \delta_{2j}$, and if incipient sensor fault has been detected at time T_{di}, incipient sensor fault developing to sensor fault is detected at time $t = T_{ditft}$, i.e.,$|e_{yj}(T_{ditft})| > \delta_{2j}(T_{ditft})$ before it develops to sensor failure.

Therefore, the following theorem is got:

Theorem 2. *For the nonlinear system (8) with the fault decision scheme I, defined by the fault detection estimator (11) and adaptive thresholds (36), (37), if there exist some time instants* $T_0 < T_{di} < T_1$ *and* $T_1 < T_{ditft} < T_2$*, and some* $j \in \{1, 2, \cdots, q\}$*, such that the unknown input function* $\xi(T^{-1}z, u, t)$ *satisfies (47) and (49), then incipient sensor fault developing to sensor fault will be detected at time* T_{ditft}.

2) ***Incipient Sensor Fault Developing to Sensor Failure Detectability Scheme****:* The sensor failure adaptive threshold δ_{3j} for $t > T_2$ can be written as

$$\delta_{3j}(t) \triangleq k_j \int_{T_2}^{t} e^{-\lambda_j(t-\tau)} \left[\left(\left\|A_{a21}^2\right\| + \mathscr{L}_2\right)\chi(\tau) + \bar{\eta}_2 + \|D_{a22}\|\bar{\bar{\xi}}\right] d\tau + \delta_{3j}(T_2)\, e^{-\lambda_j(t-T_2)}. \quad (50)$$

Based on (45) and (50), if there exists $T_{ditfe} > T_2$ such that

$$\begin{aligned} &\left|\int_{T_2}^{T_{ditfe}} C_{a22j} e^{\hat{A}_{a22}^{22}(t-\tau)} D_{a22}\xi\left(T^{-1}z, u, \tau\right) d\tau\right| \geq \\ &k_j \int_{T_2}^{T_{ditfe}} e^{-\lambda_j(t-\tau)} \left[2\left(\left(\|A_{a21}^2\| + \mathscr{L}_2\right)\chi(\tau) + \bar{\eta}_{a2}^2\right) + \|D_{a22}\|\bar{\bar{\xi}}\right] d\tau \\ &+ \left[k_j\left|e_{22}(T_2)\right| + \delta_j(T_2)\right] e^{-\lambda_j\left(T_{ditfe}-T_2\right)}, \end{aligned} \quad (51)$$

then $|e_{yj}| \geq \delta_{3j}$, and if incipient sensor fault developing to sensor fault at time T_{ditft}, the incipient sensor fault developing to sensor failure is detected at time $t = T_{ditfe}$, i.e.,$|e_{yj}(T_{ditfe})| > \delta_{3j}(T_{ditfe})$.

In addition, the sensor failure can also be detected if the sensor failure signals are big enough (i.e., $\xi > \bar{\bar{\bar{\xi}}}$) to destroy the sliding motion of observer (11) on the sliding surface $\mathscr{S}$. Comparing with (51), the detectability is weaker than using the adaptive threshold method since it is not require $\xi > \bar{\bar{\bar{\xi}}}$.

Theorem 3. *For the nonlinear system (8) with the fault decision scheme II, defined by the fault detection estimator (11) and adaptive thresholds (36), (37) and (39), if there exist some time instants $T_1 < T_{ditft} < T_2$ and $T_{ditfe} > T_2$, and some $j \in \{1, 2, \cdots, q\}$, such that the unknown input function $\xi\left(T^{-1}z, u, t\right)$ satisfies (47), (49) and (51), then the incipient sensor fault developing to sensor failure will be detected at time $t = T_{ditfe}$.*

Fault detection decision schemes III-V are traditional adaptive threshold decision schemes for piecewise continuous faults. For sensor fault detection scheme, if there exists $T_1 < T_{dft}$ such that (49) holds, with T_{ditft} replaced by T_{dft}, then the sensor fault is detected at time $t = T_{dft}$. Also, for sensor failure detection scheme, if there exists $T_2 < T_{dfe}$ such that (51) holds, with T_{ditfe} replaced by T_{dfe}, then the sensor failure is detected at time $t = T_{dfe}$. Therefore the following result is ready to presented.

Theorem 4. *For the nonlinear system (8) with the fault decision scheme III-V, defined by the fault detection estimator (11) and adaptive thresholds (36), (37) and (39), if there exist some time instants $T_0 < T_{di}$, $T_1 < T_{dft}$ and $T_2 < T_{dfe}$, and some $j \in \{1, 2, \cdots, q\}$, such that the unknown input function $\xi\left(T^{-1}z, u, t\right)$ satisfies (47), (49) with T_{ditft} replaced by T_{dft} and (51) with T_{ditfe} replaced by T_{dfe}, then the incipient sensor fault, sensor fault and sensor failure will be detected at time $t = T_{di}$, $t = T_{ditft}$ and $t = T_{ditfe}$ respectively.*

Remark 11. It can be seen when sliding mode takes place, $e_{21} = 0$ and $e_{yj} = C_{a22j}e_{22}, j = 1, \cdots, p$, which means that the fault detection detectability of proposed FD mechanism is improved. However, in [34] and [35], e_{21} will never be zero. Therefore, the proposed adaptive thresholds (36), (37) and (39) are more proper than these in [34] and [35]. ∇

V. Case study: application to traction system

A typical ac/dc/ac power system, with a single phase PWM boost rectifier and a three phase PWM inverter, used for electrical traction drives is shown in Fig.4. The topology structure of three phase PWM voltage source inverter is shown in Fig.5. Based on the Kirchoff current and

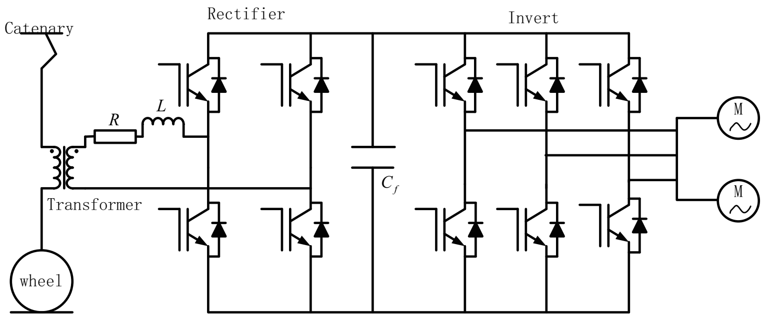


Fig. 4. Railway Traction circuit schematic diagram

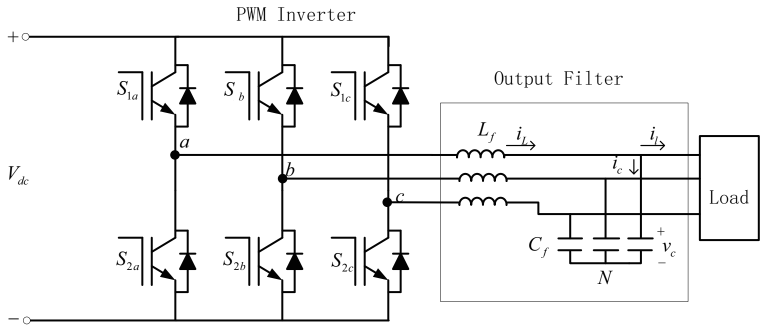


Fig. 5. Three phase PWM inverter topology

voltage lemma, it can be got that

$$\dot{v}_{cd} = \frac{1}{C_f} i_d + \omega_0 v_{cq} - \frac{1}{C_f} i_{ld}, \tag{52}$$

$$\dot{v}_{cq} = \frac{1}{C_f} i_q - \omega_0 v_{cd} - \frac{1}{C_f} i_{lq}, \tag{53}$$

$$\dot{i}_d = \frac{1}{L_f} v_d + \omega_0 i_q - \frac{1}{L_f} v_{cd}, \tag{54}$$

$$\dot{i}_q = \frac{1}{L_f} v_d - \omega_0 i_q - \frac{1}{L_f} v_{cq}, \tag{55}$$

where L_f and C_f are filter inductor, capacitor respectively, v_d and v_q are $d-q$-axis inverter output voltages, v_{cd} and v_{cq} are $d-q$-axis capacitor voltages, i_d and i_q are $d-q$-axis inverter output currents, i_{ld} and i_{lq} are $d-q$-axis load currents, and ω_0 is operation source angle frequency.

Furthermore, an instantaneous power balance between the input and output terminals of LC filters, which can improve the dynamical response [32], is introduced as follows:

$$\begin{aligned} i_{ld} &= \tfrac{p_f v_{cd} + q_f v_{cq}}{\left(v_{cd}^2 + v_{cq}^2\right)} + \omega_0 C_f v_{cq} - \tfrac{\omega_0 L_f \left(i_d^2 + i_q^2\right)}{\left(v_{cd}^2 + v_{cq}^2\right)} v_{cq}, \\ i_{lq} &= \tfrac{p_f v_{cq} - q_f v_{cd}}{\left(v_{cd}^2 + v_{cq}^2\right)} - \omega_0 C_f v_{cd} + \tfrac{\omega_0 L_f \left(i_d^2 + i_q^2\right)}{\left(v_{cd}^2 + v_{cq}^2\right)} v_{cd}, \end{aligned} \tag{56}$$

where p_f and q_f are calculated with measured voltages and currents.

Considering the measurement noises of voltages and currents, which leads to the lumped uncertainties $\eta(\cdot)$ given in (1), then Eqs. (52)-(55) can be described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Ei_l(x,u) + \eta(x,u,\omega,t), \\ y &= Cx + Ff, \end{aligned} \tag{57}$$

where $x = [v_{cd}, v_{cq}, i_d, i_q]^T$, $u = [v_d, v_q]^T$, $i_l = [i_{ld}, i_{lq}]^T$ with i_{ld} and i_{lq} given in (56),

$$A = \begin{bmatrix} 0 & \omega_0 & \frac{1}{C_f} & 0 \\ -\omega_0 & 0 & 0 & \frac{1}{C_f} \\ -\frac{1}{L_f} & 0 & -\frac{r}{L_f} & \omega_0 \\ 0 & -\frac{1}{L_f} & -\omega_0 & -\frac{r}{L_f} \end{bmatrix}, \ B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{V_{dc}}{L_f} & 0 \\ 0 & \frac{V_{dc}}{L_f} \end{bmatrix}, \ E = \begin{bmatrix} -\frac{1}{C_f} & 0 \\ 0 & -\frac{1}{C_f} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } \eta\left(x,u,\omega,t\right) = \begin{bmatrix} 20\sin\left(v_{cd}v_{cq}\right) + [2,5]u \\ \cos\left(v_{cd}i_d\right) \\ 0.2\sin\left(10v_{cd}i_q\right) \\ 2\cos\left(i_q^2\right) \\ 0 \end{bmatrix}.$$

Assuming that the sensor fault occurs in the measured voltage of v_{oq}, then $F = [0,1]^T$. The incipient sensor fault considered is generated by (3) as $\dot{f} = -1000f + \xi(x,u,t)$, $f(0) = 0$. There are many different fault modes depended on $\xi(x,u,t)$ to detect. In this simulation, three fault modes will be considered.

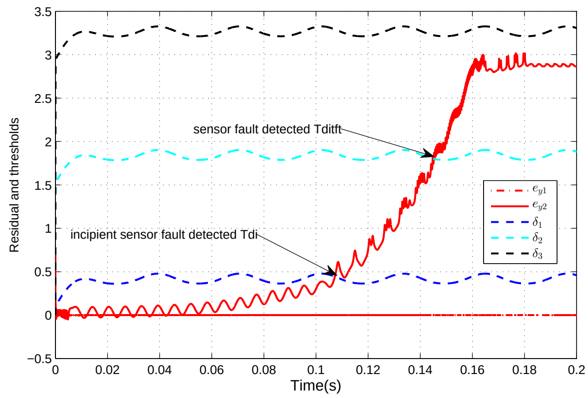


Fig. 6. Incipient sensor fault developing to sensor fault detection in $d-q$-axis.

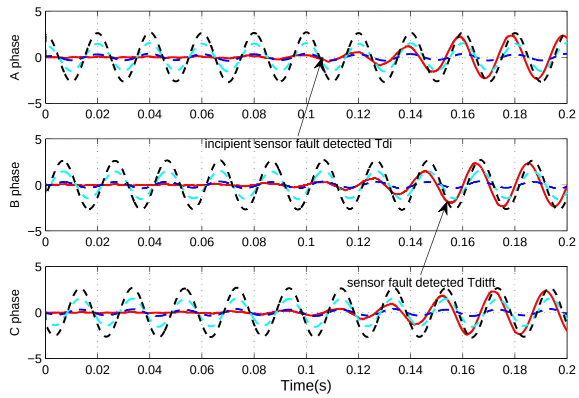


Fig. 7. Incipient sensor fault developing to sensor fault detection in $A-B-C$-axis.

A. Continuous incipient sensor fault developing to sensor fault detection

In first case, the $\xi(x,u,t)$ is given by

$$\xi(x,u,t)=\begin{cases} e^{(60te^{-2t})}+20sin(20x(1))+3cos(10sin(x(3)))+ \\ 20sin(1000t)+20cos(10x(5)x(3))+[0.2,20]u, t<0.16; \\ e^{60se^{-2s}}+20sin(20x(1))+3cos(10sin(x(3))) \\ +20sin(2000t)+20cos(10x(5)x(3))+[0.2,20]u, s=0.16, 0.16<t<0.2. \end{cases} \tag{58}$$

which is continuous. Fig.6 and Fig.7 show the continuous residual (solid and red line) and adaptive thresholds (including incipient sensor fault threshold δ_1 (dash and blue line), sensor fault threshold δ_2 (dash and cyan line) and sensor failure threshold δ_3 (dash and black line)). It can be seen that the incipient sensor fault is detected at time instant T_{di}, and its development to sensor fault is detected at time instant T_{ditft}.

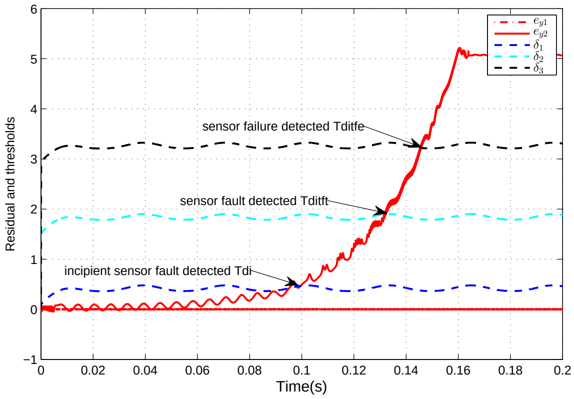


Fig. 8. Piecewise continuous fault developing detection in $d-q$-axis.

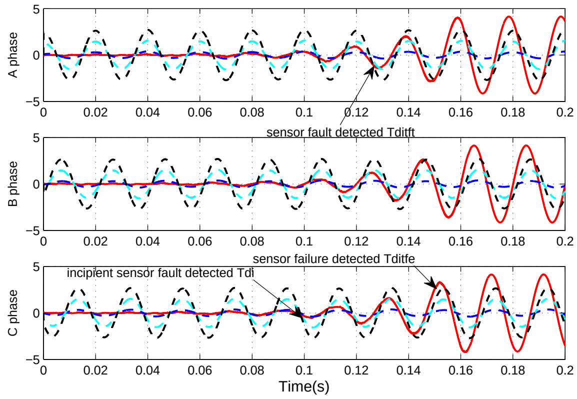


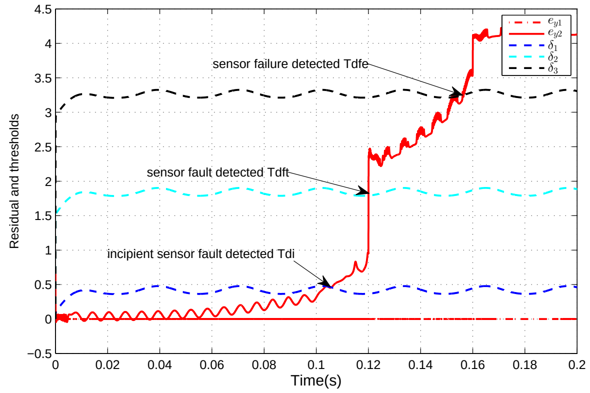
Fig. 9. Piecewise continuous fault developing detection in $A-B-C$-axis.

B. *Continuous incipient sensor fault developing to sensor failure detection*

In this case, $\xi(x,u,t)$ is given by

$$\xi(x,u,t)=\begin{cases} e^{(65te^{-2t})}+20sin(20x(1))+3cos(10sin(x(3)))+ \\ 20sin(1000t)+20cos(10x(5)x(3))+[0.2,20]u, t<0.16; \\ e^{65se^{-2s}}+20sin(20x(1))+3cos(10sin(x(3))) \\ +20sin(2000t)+20cos(10x(5)x(3))+[0.2,20]u, s=0.16, 0.16<t<0.2. \end{cases} \tag{59}$$

which is also continuous. Comparing with the first case, the incipient fault with input signal (59) develops faster than the fault drove by (56) demonstrated by Fig.8 and Fig.9. As can be seen, the incipient sensor fault develops to sensor failure at time instant T_{ditfe}.



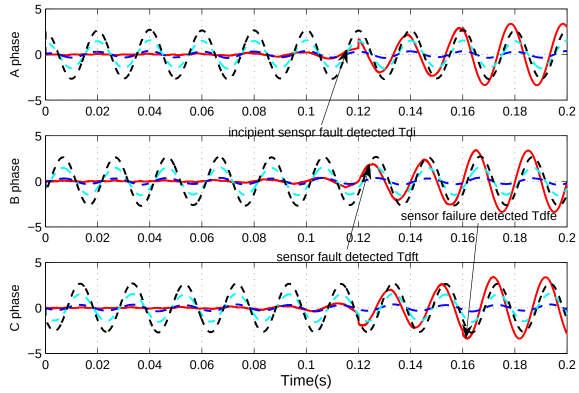


Fig. 10. Piecewise continuous fault developing detection in $d-q$-axis.

Fig. 11. Piecewise continuous fault developing detection in $A-B-C$-axis.

C. Piecewise continuous sensor fault detection

In this case, the sensor fault also expressed as $\dot{f} = -1000f + \xi(x, u, t), f(0) = 0$ where

$$\xi(x,u,t) = \begin{cases} e^{45t} + e^{40t} + 20sin(20x(1)) + 3cos(10sin(x(3))) + 20sin(1000t) \\ +20cos(10x(5)x(3)) + [0.2, 20]u, t < 0.12; \\ e^{55s} + exp(40t) + 20sin(20x(1)) + 3cos(10sin(x(3))) + 30sin(1000t) \\ +20cos(10x(5)x(3)) + [0.2, 20]u, s = 0.12, 0.12 < t < 0.16; \\ e^{60s} + 200 + 20sin(20x(1)) + 3cos(10sin(x(3))) + 20sin(1000t) \\ +20cos(10x(5)x(3)) + [0.2, 20]u, s = 0.12, 0.16 < t < 0.2. \end{cases} \quad (60)$$

which is piecewise continuous and has jumps at time instants $t = 0.12s$ and $t = 0.16s$. As shown in Fig.10 and Fig.11, incipient sensor fault is detected at time instant T_{di}. After first jump at time $t = 0.12s$, the incipient sensor fault develops to sensor fault which is detected at this time instant $T_{dft} = 0.12s$. Then the sensor fault develops to sensor failure and is detected at time instant T_{dfe}.

VI. Conclusion

This paper has proposed a sliding mode observer based FDE, which is used to generate levels of residuals for the Lipchitz nonlinear systems and obtain levels of proper adaptive thresholds. As shown in the paper, the levels of proper adaptive thresholds are effectively improve incipient fault detectability. Furthermore, the incipient sensor fault detection decision schemes have been studied, including continuous incipient sensor faults developing to sensor fault, continuous incipient sensor faults developing to sensor failures and piecewise continuous sensor fault detection. At last, an application example to the traction system in CRH (China Railway High-speed) example is presented to demonstrate the effectiveness of proposed incipient sensor fault development detection schemes.

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