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Essays on the Endogeneity of the Natural Rate of Growth in Latin American Countries

by

Ivan Irmin Mendieta Muñoz

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Abstract

This Thesis contains four original essays that have been devoted to the study of different elements of the hypothesis of endogeneity of the natural rate of growth. The theoretical framework of the Thesis is presented in Chapter 1. In it, we explore various elements that are of utmost importance in order to understand the hypothesis of endogeneity and that have been generally overlooked by the literature.

The four empirical essays presented in Chapters 2 to 5 explore different aspects of the endogeneity of the natural rate of growth in a sample of thirteen Latin American countries during the period 1981-2011. The first two empirical essays test the hypothesis of endogeneity using new specifications and various econometric techniques. The results indicate that the natural rate of growth is endogenous to the actual rate of growth, so that the long-run economic growth rate presents sensitivity both in the upward and downward directions in the majority of countries of study. We also find evidence that suggests that expansions are more important than recessions in the sample of Latin American countries.

Chapter 4 tries to: 1) estimate a time-varying natural rate of growth; and 2) measure the sensitivity of the latter with respect to its individual components: the rate of growth of labour productivity and the rate of growth of labour force. The results show that the natural rate of growth is more sensitive to labour force growth in the sample of Latin American countries.

Finally, the fifth essay studies the interactions between the individual components of the natural rate of growth and the individual components of the rate of growth of aggregate demand. The empirical results show that the rate of growth of labour productivity is more sensitive to the different components of the rate of growth of aggregate demand. However, we find mixed evidence regarding which component of the rate of growth of aggregate demand is more relevant, so that it is not possible to derive a single conclusion that encompasses all the Latin American countries of study.

All in all, the present research finds both theoretical elements and empirical evidence that support the hypothesis of endogeneity of the natural rate of growth in Latin America during the period 1981-2011.

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Extended abstract

The hypothesis of endogeneity of the natural rate of growth considers that the potential rate of growth in an economy reacts endogenously to the actual growth rate. The latter means that short-run business cycle fluctuations influence the long-run growth rate or, in other words, that economic growth is influenced by aggregate demand fluctuations.

This Thesis contains four original essays that have been devoted to the study of different elements of the hypothesis of endogeneity of the natural rate of growth. The theoretical framework of the Thesis is presented in Chapter 1. In it, we explore various elements that are of utmost importance in order to understand the hypothesis of endogeneity and that have been generally overlooked by the literature. Specifically, we: 1) offer an historical review of the concept; 2) relate the hypothesis of endogeneity of the natural rate of growth to the concept of hysteresis and to the study of the interactions between business cycle fluctuations and long-run growth; and 3) review the main empirical findings of the literature that has explicitly tested the hypothesis of endogeneity.

The four empirical essays presented in Chapters 2 to 5 explore different aspects of the endogeneity of the natural rate of growth in a sample of thirteen Latin American countries during the period 1981-2011. The first two empirical essays test the hypothesis of endogeneity using new specifications and various econometric techniques. The results indicate that the natural rate of growth is endogenous to the actual rate of growth, so that the long-run economic growth rate presents sensitivity both in the upward and downward directions in the majority of countries. We also find evidence that suggests that expansions are more important than recessions in the majority of countries of study.

Chapter 4 tries to: 1) estimate a time-varying natural rate of growth; and 2) measure the sensitivity of the latter with respect to its individual components: the rate of growth of labour productivity and the rate of growth of labour force. The results show that the natural rate of growth is more sensitive to labour force growth in the sample of Latin American countries.

Finally, the fifth essay studies the interactions between the individual components of the natural rate of growth and the individual components of the rate of growth of aggregate demand. The empirical results show that the rate of growth of labour productivity is more sensitive to the different components of the rate of growth of aggregate demand. However, we find mixed evidence regarding which component of the rate of growth of aggregate demand is more relevant, so that it is not possible to derive a single conclusion that encompasses all the Latin American countries of study.

All in all, the present research finds both theoretical elements and empirical evidence that support the hypothesis of endogeneity of the natural rate of growth in Latin America during the period 1981-2011.

CHAPTER 1

Theoretical Considerations on the Hypothesis of Endogeneity of the Natural Rate of Growth

1.1 Introduction

This Chapter tries to offer a description of the theoretical underpinnings of the hypothesis of endogeneity of the natural rate of growth. Firstly, we offer an historical review of the concept, so that we revisit Harrod's model, the neoclassical versus post-Keynesian growth debates that emerged in the 1950s, and the "new" or "endogenous" neoclassical growth theory.

Secondly, we explore the concepts of path dependency and hysteresis, and try to relate the latter to the hypothesis of endogeneity of the natural rate of growth (henceforth g_n) proposed by León-Ledesma and Thirlwall (2002a; 2002b) (henceforth LLT) since both frameworks are closely related.

Thirdly, we present the main empirical findings of the literature that has followed the LLT approach, stressing the results for Latin American countries since the latter are the object of study of the present Thesis.

The rest of the chapter is organized as follows. Section 1.2 deals with: a) Harrod's model, emphasizing his views about g_n (Section 1.2.1); b) the growth theories that emerged in the 1950s as a response to the long-run secular problem of potential differences between the warranted rate of growth and the g_n (Section 1.2.2); and c) the "endogenous" neoclassical growth theory (Section 1.2.3). Section 1.3 studies the concepts of path dependency and hysteresis and tries to relate them to the hypothesis of endogeneity of g_n proposed by LLT;

Section 1.4 presents the main empirical findings of the literature that has followed the LLT framework; and finally Section 1.5 presents the main conclusions.

1.2 The natural rate of growth in historical perspective

1.2.1 Roy Harrod's model

It goes without saying that, once the capital accumulation-led growth theory of the Classical surplus approach faded into oblivion with the advent of the marginalist revolution in the late 19th century, Roy Harrod's 1939 paper brought forth renewed concerns as regards the requisites to guarantee both the full-employment of labour and the full-capacity utilization of capital in a growing economy through time. The main goal of Harrod's *Essay in Dynamic Theory* was to develop a dynamic theory that entailed "a marriage of the 'acceleration principle' and the 'multiplier'" (Harrod, 1939, p. 14) since Keynes' *General Theory* was essentially a static equilibrium model in the Marshallian short-period tradition in which, amongst other things, the stock of capital goods was assumed to be fixed (Kriesler and Nevile, 2012).[1]

As Besomi (1999) has already pointed out, the *Essay* was not meant to provide a model of economic growth: Harrod never discussed the determinants of g_n. Hence, it might be more appropriate to think of Harrod's work as a foray into a dynamic theory of the trade –or business– cycle around an unexplained trend and not as a proper economic growth theory (León-Ledesma and Thirlwall, 2002b). To provide a better understanding of why this is the

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[1] However, it is noteworthy to mention that John Maynard Keynes alluded to problems stemming from the divergences between the warranted (explained below) and the natural rates of growth in a lecture to the Eugenics Society in 1937 (see León-Ledesma and Thirlwall, 2002b).

case, let us briefly describe Harrod's dynamic theory and the role that g_n fulfils within his theoretical framework.

In an economic system without government intervention and without foreign trade the condition of equilibrium in the goods' market is represented by the equality between savings and investment decisions. Following Keynes' ideas, Harrod assumed that the latter is not generated by the former or, in other words, that savings and investment decisions are independent of one another. With respect to the savings' equation, Harrod's formal presentation considered the propensity to save as given[2], so that we have the traditional Keynesian saving function:

$$s = \frac{S}{Y} \quad \ldots\ldots\ldots\ldots \quad (1.1)$$

where in equation (1.1) S denotes total saving; Y is total income or output; and s is the average propensity to save or the saving-output ratio.

On the other hand, planned or *ex-ante* investment depends on the acceleration principle and on the degree of utilization of capital equipment:

$$i = \frac{I}{Y} = v^* g_w \quad \ldots\ldots\ldots\ldots \quad (1.2)$$

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[2] It is important to mention, however, that Harrod also made some reference to the influence of the interest rate on the propensity to save (Commendatore et al., 2003). In his followings writings he also recalls the possibility of using Frank Ramsey's intertemporal approach to the savings rate in order to develop this part of his analysis (see particularly Harrod, 1960).

where in equation (1.2) i is the average propensity to invest or the investment-output ratio; I is total planned or *ex-ante* investment; $v^* = (\frac{K}{Y})^*$ is the steady state capital-output ratio[3]; and $g_w = \left(\frac{\Delta Y}{Y}\right)_w$ is the warranted rate of growth, so that:

$$s = i = v^* g_w \quad \dots \dots \dots \quad (1.2')$$

$$g_w = \frac{s}{v^*} \quad \dots \dots \dots \quad (1.3)$$

Consequently, g_w is the "entrepreneurial equilibrium"; that is, the rate of growth "which, if achieved, will satisfy profit takers that they have done the right thing" (Harrod, 1948, p. 87) "in the sense that they have produced neither more nor less than the right amount" (Harrod, 1939, p. 16). Thereby, g_w describes the full-capacity utilization growth rate (Hagemann, 2009), and it refers to the capital accumulation equilibrium growth rate in which there is saving-investment equilibrium so there is neither under- nor over-utilisation of capital and equipment or production capacity.

Harrod regarded the g_w as intrinsically unstable (Hagemann, 2009), representing a "moving equilibrium" (Harrod, 1939, p. 22): "[i]ndeed, there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity" (Harrod, 1939, p. 30). There is no reason for g_w to be associated with the full-employment of labour since the labour market has not been integrated.

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[3] Harrod's original definition used what he called the required incremental capital coefficient $c_r = (\frac{\Delta K}{\Delta Y})_r$: "the new capital [investment] required to sustain the output which will satisfy the demands for consumption arising out of consumers' marginal addition to income" (Harrod, 1948, p. 83). In the steady state $c_r = v^*$ since the various quantities in the economy grow at a constant rate. Harrod (1960, p. 278) also argued that that the value of v^* "may be somewhat dependent on the rate of interest", which in equilibrium corresponds to the rate of profit on capital in a single commodity model.

Harrod envisaged a secular problem that arises in an unfettered capitalist economy, which comprises a short-run and a long-run dimension. The short-run divergence problem stems from the divergence between the actual rate of growth $g = \frac{\Delta Y}{Y} = \frac{s}{v}$[4] and g_w, and therefore it raises the question if it is possible to achieve full-capacity growth where $g = g_w$.

Since g is simply a "truistic equation which must be satisfied whatever advance or recession takes place" (Harrod, 1948, p. 85), it can be considered as the short-run rate of growth that reflects the business cycle caused by demand and/or monetary fluctuations/shocks in the economy. In boom periods when $g > g_w$, inventories are being reduced, entrepreneurs will increase their investment decisions, pushing g further above the g_w and widening the gap between g and g_w. On the other hand, in recession periods when $g < g_w$, stocks are being accumulated, entrepreneurs will reduce their investment decisions, pushing g further below the g_w and increasing the gap even more. Thereby, centrifugal forces come to the fore in both cases and the initial disequilibrium situations will tend to be exacerbated by the investment decisions of the entrepreneurs.

Harrod related the aforementioned instability problem with the labour market by introducing the concept of g_n, that is, the "economic optimum growth rate" (Harrod, 1970, p. 737) or the "welfare optimum, in which resources are fully employed and the best available technology used" (Harrod, 1960, p. 279):

$$g_n = \tau^* + l^* \quad \dots\dots\dots\dots \quad (1.4)$$

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[4] Again, Harrod's original formulation used the concept of incremental capital coefficient $c = \frac{\Delta K}{\Delta Y}$; that is, the incremental capital-output ratio that actually occurs. Hence, c includes changes in stocks or inventories.

where in equation (1.4) τ^* and l^* respectively refer to the maximum or long-run rate of growth of labour productivity or technical progress[5] and to the maximum or long-run rate the rate of growth of labour force or labour supply.

Therefore, Harrod was the first to formally introduce the concept of g_n as "the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work leisure preference schedule, supposing that there is always full employment in some sense" (Harrod, 1939, p. 30). In other words, Harrod's g_n corresponds to the concept of long-run or trend or potential rate of growth determined by supply side factors; that is, the full-employment growth rate that is "concerned with the growth of an economy's capacity to produce independent of the pressure of demand upon resources" (Thirlwall, 1969, p. 87). Thus, by definition, g_n "excludes the possibility of 'involuntary' unemployment" (Harrod, 1948, p. 87), and it refers to the long-run rate of growth or rate of growth of potential output in which monetary fluctuations play no role.

Additionally, according to Harrod (1960):

$$g_n^{pc} = g_n - l^* = \tau^* = g_n^{pc}(r_n), \qquad \frac{dg_n^{pc}}{dr} ? \, 0 \quad \dots\dots\dots\dots \quad (1.5)$$

where in equation (1.5) g_n^{pc} is the natural rate of growth per capita which can be an increasing or decreasing function of the natural rate of interest r_n (Harrod, 1960).[6]

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[5] Hence, by definition τ^* is anything which increases labour productivity including a rise in the capital-labour ratio (Thirlwall, 1969) or a reduction in the capital-output ratio. Technical progress should be neutral in Harrod's sense or labour augmenting in order to generate a steady state situation; this means that technical progress should leave the capital-output ratio unchanged, so the increase in labour productivity is caused by an increase in the capital-labour ratio.

[6] In Harrod's view, the natural rate of interest corresponds to that "rate of interest that is necessary to have if the economy is to advance at the optimum rate in accordance with its potential growth, is determined by the prospective growth of income and the elasticity of the community income utility function" (Harrod, 1960: 192).

The effect of r_n on g_n is unclear since Harrod asserted that

> "[t]here is an inclination to suppose that [g_n] would be greater, the lower the rate of interest. As a generalisation, this is fallacious, and due to a confusion of dimensions, which it should be the first task to a 'dynamic economics' to prevent. (…) It is quite an open question whether [g_n] will be higher or lower with a lower rate of interest. *All depends on the nature of technological innovations*. If these are concentrated on substitute modes of production or substitute products where the yields on the modes and products for which they are substitutes are low, then the low-interest-rate economy will show a higher growth rate than the high-interest one. It will have the opportunity of taking advantage of a number of innovations which the high-interest economy has simply to ignore because, for it, they are outside the range of paying propositions. But if the innovations are such that the substituted processes or products were previously a long way inside the margin of substitution, the reverse may well be true". (Harrod, 1960, p. 283; emphasis added).

Hence, the long-run divergence problem stems from the difference between g_w and g_n, and it posits the problem to achieve a Keynesian full-employment path of growth equilibrium where $g_w = \frac{s}{v^*} = g_n = \tau^* + l^*$. It is important to emphasize that, contrary to the common belief, Harrod's writings show that he deemed that there was no inherent tendency for these two rates to coincide despite the introduction of the rate of profit on capital (henceforth r), so that the allowance of substituability of factors of production was not a solution for the fundamental difference between g_w and g_n.[7]

Thus, since g_w depends on the phase of the trade cycle reflected in g and because g_n represents a fixed value, when $g_w > g_n$ the economy will experience a chronic tendency to depression or permanent stagnation; and when $g_w < g_n$ there will be a chronic tendency to

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[7] See for example Harrod (1960; 1970). Therefore, as Hahn and Matthews (1964) and Commendatore, et al. (2003) stress, the widespread views that simply consider that Harrod built up his analysis assuming fixed technical coefficients or a Leontief technology –principally related to the current neoclassical textbooks– neglect an essential component of Harrod's economic thought since he did not maintain that v^* was unalterable for technological reasons, but he considered that, if r is monetarily determined, then the entrepreneurs can choose a v^* that is other than the one required for a steady growth with full-employment (Hahn and Matthews, 1964).

inflation accompanied by a growing unemployment of the structural variety (Thirlwall, 1972).

In consequence, the foremost importance of g_n in Harrod's model is palpable: it represents the fixed "ceiling" that limits both the maximum expansion attainable by an economy and the divergence between g and g_w. The fact that Harrod never discussed explicitly the nature of the determinants of g_n means that Harrod's analysis took the latter as an exogenously determined variable or, to use Joan Robinson's expression, he considered that g_n was "given independently by God and the engineers" (Robinson, 1970, p. 732). This is the main reason why both the long-run rate of growth of the economy and its determinants remained unexplained in his model.

Nevertheless, there is a slight trace in Harrod's writings that point towards the possibility that he never totally overlooked that g_n depends (reacts) on (to) the prevailing economic conditions in an economy. As regards τ^*, he mentions that

> "[t]he concept of natural growth embodies not only technological progress *but also the increase of personnel well adapted to enterprise and business management, and the increase of know-how, whether natural or artificially stimulated"* (Harrod, 1960, p. 289; emphasis added).

Furthermore, he even seems to foreshadow the concept of path dependency –explained in Section 1.3 of the present Chapter– in g_n^{pc}:

> "For natural growth per caput occurs through the cumulative accretion of experience and know-how and the improvement of personnel. *Lost years cannot be regained in full*. The very essence of growth (*per caput*) is education by practice and the gradual drawing out of the latent potentialities of personnel. *Vires acquirit eundo* [We gather strength as we go] (Harrod, 1960, p. 291; emphasis in the original).

The quotations above suggest the idea that, in Harrod's view, g_n may experience a certain degree of endogeneity.

1.2.2 "Old" growth theories: post-Keynesian and neoclassical schools

The dire conclusion that a capitalist economy was unlikely to maintain a steady state of growth at the g_n brought about the contentious growth and distribution debates in the 1950s between the neoclassical (Cambridge, Massachusetts, USA; represented by people like Paul Samuelson, Robert Solow, Franco Modigliani, etc.) and the Keynesian/post-Keynesian (Cambridge, England; represented by people like Nicholas Kaldor, Joan Robinson, Luigi Pasinetti, Richard Khan, etc.) schools.

By and large, the growth and distribution debates in the 1950s and 1960s between the two competing schools of thought focused on showing the possible solutions to Harrod's long-run secular problem, thus providing a relationship between the growth rate on one hand and income distribution on the other (Kurz and Salvadori, 2003). In this sense, instead of treating s, v^*, τ^* and l^* as exogenous variables, the developments in growth theory in the 1950s relied on the observation that the rate of profit $r = \frac{P}{K}$ (where P denotes profits and K is the capital stock) influences both s and v^*. Throughout this debate, however, the g_n and therefore τ^* and l^* were taken as given since the post-Keynesian and the neoclassical schools respectively endogenized s and v^* in order to show the possibility of convergence between g_w and g_n.

Let us firstly explore the post-Keynesian framework. With given technical conditions and the degree of monopoly, Kaldor (1955-1956) distinguished between savings out of profits s_p and savings out of wages s_w. With $1 \geq s_p \geq s_w \geq 0$ he demonstrated that, within certain limits related to the wage and profit shares, there is always a distribution of income (subject to a minimum r) at which the system produces the required amount of savings in order to generate the convergence between g_w and g_n.

Some years later, Pasinetti (1962) generalized the post-Keynesian solution by noting that workers could earn both wages and profits, and that, in order to correct this situation, it was necessary to distinguish between workers on the one hand and capitalists on the other, the latter earning only profits. Therefore, assuming that the propensities to save remain constant over time, if $g_w > g_n$ there will be a fall in r that will cause a fall in s, generating a reduction in g_w and the subsequent convergence of the latter towards g_n; whereas if $g_w < g_n$ there will be an increase in r, s, and g_w, achieving the convergence of the latter with g_n.

This result depends on the assumption that v^* is invariant to the profit share (Kaldor 1955-1956), which in turn does not imply

> "that there are fixed coefficients as between capital equipment and labour –only that technical innovations (which are also assumed to be 'neutral' in their effects) are far more influential on the chosen [v^*] than price relationships" (Kaldor, 1955-1956, p. 98).

To put it differently, the chosen v^*

> "is far more dependent on the prevailing prices of different types of capital goods and on the price of labour in terms of commodities generally (…) than on the prevailing rate of profit or the prevailing interest rates." (Kaldor, 1957, p. 682)

Thus,

> "[i]f an entrepreneur in an advanced economy employs bulldozers for making roads, whilst his opposite number in an under-developed country employs only shovels, this is not, to any significant degree, the consequence of differences in the prevailing rates of profit (or of the rates of interest on loans) in the two communities, but simply of the fact that the price of bulldozers in terms of shovels is much lower in the advanced economy than in the primitive community." (Kaldor 1957, p. 682).

Hence, the post-Keynesian approach demonstrated that g_w and g_n are not independent variables since, if profit margins are flexible, g_w will adjust itself to g_n via a consequential change in the profit share (Kaldor, 1955-1956). It is in this sense that it is possible to assert that, following the Classical idea of a certain connection between distribution of income and capital accumulation, the main result achieved by the post-Keynesian framework in the 1950s was to establish an equilibrium relation connecting r and the distribution of income with economic growth (Pasinetti, 1962).

Despite the post-Keynesian authors treating g_n as an exogenous variable, there seems to be some evidence that indicates that they regarded a certain degree of endogeneity of the components of potential output with respect to the prevailing economic conditions of an economy. Nicholas Kaldor is particularly clear when he says that:

> "So far, so good. But is this situation, from an intellectual or analytical point of view, wholly satisfactory? *The trend itself is not 'explained'; it is introduced as a datum. There can be no pretence, therefore, of these theories for providing the basis for a theory of economic growth*. Yet the very fact that different human societies experience such very different rates of growth –in fact, differences in rates of growth in different ages or in different parts of the world in the same age are one of the most striking facts of history– in itself provides powerful support for the view that *technical invention and population growth, the two factors underlying the trend, are not like the weather or the movement of seasons, that go on quite independently of human action, but are very much the outcome of social processes. The growth in population, in particular, is as much the consequence of economic growth as the condition of it.* (…) *The same is true of technical invention or innovation. Through new ideas, looked at in isolation, are the spontaneous product of the workings of human brain, the kind of ideas that come forth, and their frequency, is very much a matter of environment.*" (Kaldor, 1954, pp. 65-66; emphasis added).

Moreover, he admits that there is an impact of g_w on g_n, and therefore an interaction between the business cycle (manifested in g) on g_n since g_w depends on g:

> "The conclusion which emerges from this is that so far from the trend of growth determining the strength or duration of booms, *it is the strength and duration of booms which shapes the trend rate of growth*.
>
> (…) This is not to suggest, of course, that the long-term trend of growth is simply a matter of degree of recklessness of society's entrepreneurs. The external 'conditioning factors' are still there –in the sense that there probably always is a *maximum* attainable rate of saving, a *maximum* attainable rate of population growth or a *maximum* attainable flow of ideas. But the point is that the actual values of these variables, in any given society and at any given age, are not determined by their theoretical maximum values, but are capable of being slowed down or accelerated in accordance with the push or pull exerted by entrepreneurial behaviour. (Perhaps this idea could best be expressed in Mr. Harrod's terms by saying that while the 'warranted rate' of growth and the 'natural rate of growth' are two different things, they are not independent of each other, since the more the 'warranted rate' tends to exceed the 'natural rate', the more it will *bend* the 'natural rate' in its own direction)." (Kaldor, 1954, pp. 68-69; first emphasis added).

Let us now explore the neoclassical tradition as outlined by the Solow-Swan-Meade-Tobin models, from which Solow (1956) can be regarded as the quintessential case. In this view, the main problem of divergence between g_w and g_n stems from

> "the crucial assumption that production takes place under conditions of fixed proportions.[8] There is no possibility of substituting labour for capital in production. If this assumption is abandoned, the knife-edge notion[9] of unstable balance seems to go with it." (Solow, 1956, p. 65).

Thus, in order to generate the adjustment between g_w and g_n, the neoclassical theory in the 1950s rested on two crucial assumptions (Hagemann, 2009): 1) substitution between capital and labour, and 2) flexibility of factor prices. The adjustment between the two rates is as follows: if $g_w > g_n$ then capital is abundant with respect to labour so the r will fall relative

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[8] As we have previously mentioned, "to narrow Harrod's argument exclusively to a fixed-coefficient production function misses the essential feature of Harrod's analysis" (Hagemann, 2009, p. 83).

[9] Comendatore, et al. (2003) and Hageman (2009) have pointed out that the well-known knife-edge metaphor was a concept that utterly irritated Harrod since he would have preferred to replace it by a corridor concept instead. For example, with respect to Robinson (1970) he wrote: "[i]n this article she uses the word 'knife-edge' in relation to my theory, as she has done in previous writings. This seems to be quite unwarranted (…) I hope that we shall hear no more of the 'Harrod knife-edge'" (Harrod, 1970, pp. 740-741).

to the real wage rate, firms will adopt more capital intensive techniques increasing the capital-labour ratio (henceforth k) and, because of the assumption of diminishing returns to capital, the v^* will also increase, reducing the g_w until the point it equals g_n. In turn, if $g_w < g_n$ then labour is abundant with respect to capital so the price of labour will fall relative to the price of capital, firms will adopt more labour intensive techniques (or less capital intensive techniques) so the k will decrease, generating a decrease in v^* that will increase the g_w until it equals g_n.[10]

Two remarks have to be made with respect to the canonical neoclassical model. In the first place, the assumption of diminishing returns to capital meant that g per capita was never explained by the neoclassical tradition. This is the reason why the neoclassical model resorted to an exogenous change in technical progress or total factor productivity or v^* (Solow, 1957) in order to explain increases in g per capita, and also the reason why neoclassical growth models *à la* Solow-Swan-Meade-Tobin were dubbed exogenous. The latter was also considered intellectually unsatisfactory for the neoclassical authors. For example, Robert Solow mentioned that

> "[t]he first [reason for wanting to extend the neoclassical growth model] is that *it is intellectually unsatisfactory to have the growth rate exogenous*. *The actual long-run growth rate of an economy is a very important characteristic and to say that is exogenous is not satisfactory*. (…) *You should also keep in mind that everyone, so to speak, has always known that there is an endogenous side of technical progress. Part of the growth of technology is endogenous.* But unless you have a good theory, a reasonable and productive theory of endogenous technological progress, a theory of innovations in other words, it is not worthwhile spending any time on it. We also take the rate of population growth as exogenous. *We all know that population growth is partially endogenous, that has been known since Malthus and no doubt before that.* But it would be pointless for me or for Lucas or Domar or anyone to say that the rate of population growth is endogenous unless I have something to

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[10] As Kaldor (1955-1956; 1957) explained, the extreme sensitivity of v^* to changes in the share of profits was a consequence of accepting the marginal productivity theory as the basic principle in the explanation of the pricing process and the determination of distributive shares. It is in this sense that the choice between more or less capital intensive techniques, which respectively generate higher or lower k and v^*, depends on r.

> say about it. Not having something to say about it, or not having something very interesting or new to say about it, one can just take it as given." (Solow, 1992, p. 17; emphasis added).

In the same vein, Keneth Arrow asserted that

> "[t]hough doubtless no economist would ever have denied the role of technical change in economic growth, its overwhelming importance relative to capital formation has perhaps only been fully realized with the important empirical studies of Abramovitz and Solow. (…) *Nevertheless a view of economic growth that depends so heavily on an exogenous variable, let alone one so difficult to measure as the quantity of knowledge, is hardly intellectually satisfactory*. From a quantitative, empirical point of view we are left with time as an explanatory variable. *Now trend projections, however necessary they may be in practice, are basically a confession of ignorance, and, what is worse from a practical viewpoint, are not policy variables.*" (Arrow, 1962, p. 155; emphasis added).

Secondly, one of the chief problems with the paradigmatic neoclassical growth theory is that it starts from the assumption that *ex ante* savings determine investment, which means that the core of the Keynesian revolution was never recognised by the neoclassical canonical model (Hagemann, 2009 and Kurz and Salvadori, 2003). In other words, i is not determined independently of s, which wipes out the existence of motivational and behavioural patterns such as investors' expectations of entrepreneurs or 'animal spirits', and also the access to credit. Because of this situation the model does not allow any possibilities of demand constraints: it embodies a dynamic version of Say's law whereby all output growth is willingly demanded, so growth of demand expands *pari passu* with supply (Palley, 1996). Hence, in contrast to Harrod and the post-Keynesian school in the 1950s, the neoclassical exogenous growth theory clearly separated the study of the long-run trend and trade cycle components, and focused exclusively on the analysis of the steady-state (Hagemann, 2009).

Indeed, with respect to the high degree of substitutability between factors of production (or technological flexibility) Robert Solow asserted that

> "[t]here was one bad by-product of this focus on the description of technology. *I think I paid too little attention to the problems of effective demand. To put it differently: a theory of equilibrium growth badly needed –and still needs– a theory of deviations from the equilibrium growth path.* I can honestly say that I realized the need at the time. There is a brief section at the end of my 1956 article that deals in a perfunctory way with the implications of real-wage rigidity and with the possibility of a liquidity trap. That was just a lick and a promise." (Solow, 1988, p. 309; emphasis added).

The latter is true since nowadays it has been practically forgotten that in the final section of his 1956 article, "VII. Qualifications", Robert Solow mentioned that:

> "Everything above is the neoclassical side of the coin. Most especially it is full employment economics (…). *All the difficulties and rigidities which go into modern Keynesian income analysis have been shunted aside. It is not my contention that these problems don't exist, nor that they are of no significance in the long-run. My purpose was to examine what might be called the tightrope view of economic growth and to see where more flexible assumptions about production would lead a simple model.*" (Solow, 1956, p. 91; emphasis added).

In the same vein, in recent times Robert Solow's recent comments have been breathing a sort of Harrodian/post-Keynesian spirit (Hagemann, 2009). For example:

> "But if one looks at substantial more-than-quarterly departures from equilibrium growth, as suggested for instance by the history of the large European economies since 1979, *it is impossible to believe that the equilibrium growth path itself is unaffected by the short- to medium-run experience.* In particular the amount and direction of capital formation is bound to be affected by the business cycle. (…) *So a simultaneous analysis of the trend and fluctuations really does involve an integration of long run and short run equilibrium and disequilibrium.*" (Solow, 1988, pp. 311-312; emphasis added).

Additionally:

> "One major weakness in the core of macroeconomics as I have represented it is the lack of real coupling between the short-run picture and the long-run picture. *Since the long run and*

the short run merge into one another, one feels they cannot be completely independent. There are some obvious, perfunctory connections: every year's realized investment gets incorporated in the long-run model. That is obvious. *A more interesting question is whether a major episode in the growth of potential output can be driven from the demand side. Can demand create its own supply?* (…) *The demand-driven growth story sounds quite implausible to me under current conditions; but it is an example of the kind of question that needs to be asked*." (Solow, 1997, pp. 231-232; emphasis added).

And more recently:

"'Growth' means growth of potential output. The idea is to try to isolate relatively smooth trend-like growth, dominated by supply side factors, from economic fluctuations or business cycles, usually driven by the demand side. *There is no implication that either sort of path ever occurs in its pure form in actual economies. (It may be worth mentioning that the three modern founders of neoclassical growth theory – R.M. Solow, T. W. Swan and J. Tobin – were all 'Keynesian in their approach to short-run macroeconomics.)* This analytical intention takes the form of supposing the available supply of labor always to be fully employed and the existing stock of productive capital goods always to be fully utilized ('Fully' could be replaced by 'normally' or by any other constant degree of utilization.) This assumption of full utilization could be better be made explicit by introducing a government that makes (useless) expenditures and levies (lump-sum) taxes simply in order to preserve full utilization; but this is rarely done, presumably because the financial complications would obscure the essential supply orientation of the model. Full employment/utilization is usually just assumed. In other words, saving and investment turn out to be equal at that level of employment and utilization, although the mechanism that brings them into equality is left unspecified.

This is a choice of consequences. It is possible that economic growth and fluctuations are so closely bound together that any attempt to separate them must inevitably omit essential factors governing the growth of potential output. One can imagine a Schumpeterian making such a claim, though its truth is not self-evident. The neoclassical model allows in one important respect for the interaction between fluctuations and growth: fluctuations will surely perturb the rate of investment and that will necessarily affect the path of potential output. *There are no doubt other interactions, but the neoclassical model ignores them.*" (Solow, 2000, pp. 349-350; emphasis added)

Moreover, when revisiting the reasons behind both the development of the growth and distribution theories in the 1950s and 1960s and the decisions that led to the endogenization of s and v^*, he mentions that, in principle, both τ^* and l^* are not exogenous:

> "*In principle there is no reason to exclude the endogeneity of [labour productivity growth] and [labour force growth]*. But induced population growth, although an important matter in economic development seemed not to figure essentially in the rich countries for which these models were devised. The idea of endogenous technological progress was never far below the surface. In those days it would have seemed rash to conjure up some simple connection between the allocation of resources and the *rate of growth* of productivity." (Solow, 1994, p. 47; first emphasis added).

All these elements suggest that, in Robert Solow's view, the problem of combining short, medium and long-run macroeconomics has not been solved.

1.2.3. Some remarks on "new" or "endogenous" neoclassical growth theory

The main prediction of neoclassical exogenous growth theory was that, given identical saving-investment behaviour and technology across countries and due to the assumption of diminishing returns to capital, poor countries with a lower capital-output ratio and higher rate of profit-real wage ratio should grow faster than rich countries with a larger amount of capital per head and lower rate of profit-real wage proportion, so per capita incomes across the world should converge. Nevertheless, the fact that different empirical studies could not find evidence of cross-country unconditional convergence (see Romer, 1994 and Thirlwall, 1972 for reviews on this issue) can be considered one of the major triggers behind the development of the "new" or "endogenous" growth theory (Romer, 1986; 1990a; 1990b; 1994; Lucas, 1988; Barro and Sala-i-Martin, 2004).

"New" or "endogenous" Growth Theory (hereafter NGT) relaxed the assumption of diminishing returns to capital by means of broadening the definition of capital included in the aggregate production function and using instead a composite measure of capital (that is, physical capital plus other types of reproducible capital), and tried to show that, with constant or increasing returns, there is no presumption of the convergence of per capita growth rates across the world economy (Thirlwall, 1972). In consequence, it is possible to say that the inclusion of non-convexities or non-diminishing returns to capital led neoclassical authors to drop the assumption of exogenous technical progress or exogenous change in the capital-output ratio, thus endogenizing the steady-state rate of growth per capita.

There are certain comments that are required in order to provide a better understanding of the original NGT:

1) Taking into account the existence of non-diminishing returns *per se* can hardly be considered something new since such ideas were already crystal-clear in many other authors: in Adam Smith's example of the pin factory in the *Wealth of Nations* where "the key to the growth of labour productivity is the division of labour which in turn depends on the extent of the market" (Kurz and Salvadori, 2003: 3-4); in Alfred Marshall's distinction of increasing returns that are external to a firm but internal to an industry –that is, in his concept of external economies to scale[11]; and in Allyn Young's (1928) article. It may be relevant to recall some of Allyn Young's words:

> "I shall venture, nevertheless, to put further stress upon two points, which may be among those which have a familiar ring, but which appear sometimes to be in danger of being forgotten. (…) *The first point is that the principal economies which manifest themselves in increasing returns are the economies of capitalist or roundabout methods of production.*[12] *These economies, again, are largely identical with the economies of the division of labour in its most important modern forms.* (…) The second point is that the economies of roundabout methods, even more than the economies of other forms of the division of labour, depend upon the extent of the market –and that, of course, is why we discuss them under the head of increasing returns." (Young, 1928, p. 531; emphasis added).

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[11] As Thirlwall (2003) explains, this concept was Alfred Marshall's attempt to reconcile the price-taking equilibrium model of the firm (preserving the U-shaped cost curve and the notion of competitive equilibrium) with the existence of increasing returns by treating the latter as externalities.

[12] The degree of roundaboutness of production is the capital-labour ratio (McCombie et al., 2002).

Hence, as Romer (1986) has mentioned, strictly speaking the adjective "new" under the NGT headline means that old ideas are modelled in a more rigorous way for the specific purpose of rehabilitating the neoclassical model to make it compatible with the observation that the convergence of living standards has not been taking place (Thirlwall, 2003).[13]

2) The main implication of the NGT is that without diminishing returns to capital the saving-investment ratio is important for actual growth because of the positive externalities associated with human capital formation, research and development expenditure, learning by doing, embodied technical progress, technological spillovers from trade, foreign direct investment, etc., which taken together prevent the rise (fall) in the capital-output ratio (marginal product of capital) when the saving-investment ratio rises (Thirlwall, 1972). This means that growth is "endogenous" under the NGT because the saving-investment ratio and therefore capital accumulation are important for economic growth (Hussein and Thirlwall, 2000; Thirlwall, 2003).

Cesaratto (2010) speaks about an article published in 1962 by Marving Frenkel entitled "The production function in allocation and growth: a synthesis", where he first observed that in the Harrod and Domar models the actual rate of growth depends precisely on the saving-investment ratio.[14] In the same vein and more recently, Hussein and Thirlwall (2000) have

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[13] One of the most probable reasons why an endogenous theory of technical change or accumulation of knowledge was not subsumed before by the neoclassical framework is that the standard assumptions of the perfectly-competitive-Pareto-optimal world cannot be maintained (Romer, 1994; Barro and Sala-i-Martin, 2004). Specifically, since technical change involves the creation of new ideas which are partially non-rival, increasing returns to scale appear if this non-rival ideas are included as factors of production in the production function, hence the compensation of the non-rival old ideas in accordance to its marginal cost of production (which is zero) will not provide the appropriate reward for the research effort that underlies the creation of new ideas. In other words, if non-rival ideas are included in the production function, the marginal remuneration of factors of production will not correspond to its marginal cost of production.

[14] It should be pointed out, however, that most NGT models do not assume given saving rates but present the story of infinitely-lived consumers (that is, one representative consumer or dynasty is considered) that maximize their present discounted utility level (via the interest rate) over their lifetime under the assumption of perfect

shown that the AK model (which is the simplest NGT model) is nothing but the Harrod-Domar growth equation with a fixed capital-output ratio.

In the same vein, Robert Solow mentions that:

> "There is a third device [in which growth theory has tried to get beyond an exogenous growth theory] that I want to mention as well and give you an example, and that is to drop one or more of the standard assumptions of the neoclassical growth model. The one that is usually dropped is diminishing returns to capital. *As you will see, without diminishing returns to capital one is back to Domar, actually. It is rather amazing. I will try to give you an example of this, namely, of the fact that the modern literature is in part just a very complicated way of disguising the fact that is going back to Domar and, as with Domar, the rate of growth becomes endogenous*". (Solow, 1992, p. 18; emphasis added).

Finally, since the models developed by the NGT consider that the capital-output ratio remains broadly unchanged, such models implicitly embody Kaldor's (1957, 1961) technical progress function, which was originally used as an attempt to remove what Kaldor considered an artificial distinction between the "movement along a 'production function' with a given state of knowledge, and a shift in the 'production function' caused by a change in the state of knowledge" (Kaldor, 1957, p. 596). Kaldor's technical progress function posits a positive relationship between the growth of the capital-labour ratio and the growth of labour productivity, which means that in spite of the massive accumulation of capital manifested by the increase in the former variable, there will be no increase in the capital-output ratio.[15]

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foresight. Robert Solow has repeatedly criticized this practice (see Dutt, 2003). For example: "Maybe I reveal myself merely as old-fashioned, but I see no redeeming social value in using this construction [the intertemporal optimizing representative agent], which Ramsey intended as a representation of the decision-making of an idealized policy-maker, as if it were a descriptive model of an industrial capitalist economy. It adds little or nothing to the story anyway, while encumbering it [the growth model] with unnecessary implausibilities and complexities" (Solow, 1994, p. 49).

[15] Using Kaldor's (1961, pp. 207-208) words, the "technical progress function postulates a relationship between the rate of increase of capital and the rate of increase in output which embodies the effect of constantly improving knowledge and know-how, as well as the effect of increasing capital per man, without any attempt to isolate the one from the other".

Because of this it is not an exaggeration to affirm that the true progenitor of NGT is precisely Nicholas Kaldor (Palley, 1996; Thirlwall, 1972).

3) Steedman (2003) has pointed out that most of the models developed by the NGT authors resort to the concept of accumulated knowledge (that is, the stock of knowledge or stock of ideas) as a factor of production with a surprising lack of clarification. For the stock of knowledge to become an essential factor of production together with capital and labour, such a stock of knowledge needs to be cardinally measurable. In this sense, the assertion that a production function with technical change or stock of knowledge as one of its arguments does (or does not) exhibit constant returns to scale has significance only if there exists some sort of criterion for measuring it. Alas, in most NGT models it is common to treat the stock of knowledge as if it were a single magnitude with a cardinal measure without any justification being given for this dubious assumption. This contrasts, for example, with Arrow's (1962) paper where the central variable of analysis is not the stock of knowledge but the cumulative gross investment, which is the main determinant of the acquisition of knowledge in his model.

4) Most importantly, NGT has never examined issues relating to unemployment caused by the lack of effective demand (Palley, 1996; Dutt, 2003; León-Ledesma and Thirlwall, 2002b; Thirlwall, 2003), so that the NGT has not fully integrated short and medium-run macroeconomic phenomenon with long-run issues. In this sense, the original NGT has typically considered g_n as an exogenous variable independent of g. Both τ^* and l^* are themselves determined by exogenous variables, namely the preferences and decisions for the accumulation of human capital of forward-looking, profit maximizing agents or by changes in government policy (Palley, 1996; McCombie et al., 2002; Vogel, 2009), so that g_n is

typically regarded as invariant to the economy's actual experience of growth manifested in g.

Nevertheless, in recent times there seems to be an increasing tendency to try to incorporate elements of the NGT as a channel between the short-, medium-, and long-run, allowing the possibility of explaining g_n in a causal relationship with cyclical fluctuations since the departures from the long-run growth trend associated with cyclical disturbances play a role in the determination of the trend itself, thus establishing a link between business cycles and economic growth (see Saint-Paul, 1997 and Gaggl and Steindl, 2007 for surveys on this). Specifically, the following approaches consider that downturns can generate detrimental long-lasting effects in the economy, thus affecting its future long-run performance (Christopoulos and León-Ledesma, 2014): the learning-by-doing approach, which highlights the pro-cyclicality of productivity growth, so that expansion phases of the business cycle are associated with faster technical progress (hence, this literature follows very closely the idea of Arrow, 1962); and the "opportunity cost" and the "cleansing effects" literature, where recessions clean industries from its inefficient units (hence, this literature is linked to Schumpeter's idea of creative destruction).

Moreover, empirical evidence has explicitly tackled the interaction of economic fluctuations on growth (see for example the literature mentioned in Steindl and Tichy, 2009). All these elements point out the relevance of concepts such as path dependency and hysteresis effects in g_n.

1.3 Path dependency and hysteresis effects in the natural rate of growth

DeLong and Summers (2012) have recently studied the efficacy of fiscal policy in depressed economies, drawing attention back to the concept of hysteresis because of their findings that recessions provoke deleterious effects even after they end and that high pressure economies have continuing benefits:

> "Whereas many economists have assumed that the path of potential output is invariant to even a deep and prolonged downturn, *the available evidence raises a strong fear that hysteresis is indeed a factor*." (DeLong and Summers, 2012, p. 233; emphasis added).

Moreover, they mention that

> *"[i]t would indeed be surprising if economic downturns did not cast a shadow over future levels of economic activity*. A host of mechanisms have been suggested, including reduced labor force attachment on the part of the long term unemployed, scarring effects on young workers who have trouble beginning their careers, reductions in government physical and human capital investments as social insurance expenditures make prior claims on limited public financial resources, reduced investment in both in research and development and in physical capital, reduced experimentation with business models and informational spillovers, and changes in managerial attitudes." (DeLong and Summers, 2012, p. 254; emphasis added)

The term hysteresis originates from physics. According to Setterfield (1995; 2009; 2010b), hysteresis refers to a particular type of (rather than a synonym for) path dependency.[16] In short, it is possible to say that the outcomes of path dependent systems (including anything that can be constructed as a long-run or final outcome) are affected by the path (that is, the prior sequence of adjustments and associated outcomes) that led up to them, so that the earlier

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[16] Other concepts of path dependency are cumulative causation and lock-in (for an extensive revision see Setterfield 1995; 2009). Cumulative causation involves a circular interaction between variables, so that the behaviour of the variable of interest is self-reinforcing and successive changes in this variable are positively correlated. This is the reason why systems that display cumulative causation depend crucially on initial conditions. In turn, lock-in occurs when the current behaviour of a decision making unit is conditional on either its own past practices, or the current behaviour of other agents in the system, so that "repetition is self-reinforcing" (Setterfield, 1995, p. 19). Essentially, it can be considered similar to cumulative causation in its emphasis on the dominant effect of initial conditions.

states of the system affect its final or long-run outcome. In this way, a path dependent system is a process that has a memory of past shocks (Lang and Peretti, 2009). Once the possibility of path dependency is recognized, all equilibrium states that are postulated as describing the actual outcomes of the economic systems need to be regarded as temporary or “conditional” equilibria, that is to say, a state of rest brought about by a temporary suspension of the forces of endogenous change in the system; whereas traditional equilibrium configurations can be considered as path independent or ahistorical systems since the long-run results are unaffected by events in the past.

There is only one rigorous mathematical definition of hysteresis, which corresponds to a path dependent process that possesses two key properties (Lang and Peretti, 2009): remanence, which occurs when the application of two successive shocks of the same magnitude but opposite signs does not bring the system back to its initial position; and selective, erasable memory, which means that only the non-dominated extremum values of the past shocks that have hit the system remain in its memory. Thus, hysteresis arises from properties of the adjustment dynamics of path dependent systems, and it is closely associated with non-linearities and structural change along such dynamic adjustment paths (Setterfield, 1995; 2009; 2010b).

One of the key features of hysteric systems is the propensity for even transitory causes to have permanent effects; however, this cannot be considered a defining feature since it is also a property of other concepts of path dependence. In this sense, the central characteristic of hysteresis is that it causes the long-run or final outcome to depend on its previous outcomes.[17]

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[17] Cumulative causation and lock-in reduce the importance of the history of the system to the impact of initial conditions on long-run outcomes. Therefore, both concepts possess a sort of “dynamic determinism”

In economics, however, several conceptualizations of hysteresis have arisen. According to Lang and Peretti (2009), the multiple other uses of the term hysteresis are inappropriate since at least one of the properties of genuine hysteresis is violated, and these inappropriate uses can be found in economics only. This is the reason why the definition of hysteresis as defined above has been called "true" or "genuine" or "strong" hysteresis in economics. Setterfield (2009, 2010b) mentions other two conceptualizations that have appeared: the unit (zero) root approach in the context of linear difference (differential) equations, and the concept of hysteresis as a product of historical time. The former is the most common interpretation of hysteresis in economics and, despite its problems[18], it may provide a useful approximation to tackle macrodynamics mainly because models following this approach are easy to construct, and to compare and to contrast with traditional equilibrium systems; whereas the latter tries to ground the concept in what are understood to be dynamical properties of specifically social systems. However, the properties of these three conceptualizations of hysteresis are by no means mutually exclusive (Setterfield; 2009; 2010b).

With the elements mentioned above and following Setterfield (2009) it is possible to illustrate the idea of hysteresis in the natural rate of growth at the theoretical level:

$$\tau^* = \alpha + \beta g_{-1} \quad \dots\dots\dots\dots \quad (1.6)$$

$$l^* = \gamma + \delta g_{-1} \quad \dots\dots\dots\dots \quad (1.7)$$

(Setterfield, 1995, p. 19) since, apart from the initial conditions, no other part of the historical trajectory of the system exerts an independent influence on its long-run or final outcome. By contrast, hysteric systems only present the property that the value of a variable today can influence its value in the future (see Setterfield, 1995).

[18] This conceptualization of hysteresis refers only to linear dynamical systems. In other words, no consideration is given to the possibility of non-linearities.

where equations (1.6) and (1.7) show that both long-run labour productivity and labour supply growth rates are endogenous to the actual rate of growth experienced in the recent past (g_{-1}). Specifically, equation (1.6) can be considered as a version of the Kaldor-Verdoorn law (see Chapter 3).

Substituting (1.6) and (1.7) into (1.4) we obtain:

$$g_n = \alpha + \beta g_{-1} + \gamma + \delta g_{-1} = \eta + \theta g_{-1} \quad \dots\dots\dots\dots \quad (1.8)$$

where $\eta = \alpha + \gamma$ and $\theta = \beta + \delta$.

According to equation (1.8), the natural rate of growth is endogenous to the actual economic growth experienced, that is, it reacts to the particular economic conditions experienced during the recent past that are reflected in g_{-1}: the welfare optimum rate of growth is now path dependent.

In other words, equation (1.8) tries to capture the idea that the parameters of g_n, that is, τ^* and l^*, react endogenously to the actual rate of growth g. Consequently, under this framework the g_n only sets a maximum value of the growth rate at any point in time that is directly influenced by g, rather than acting as an exogenously given full-employment ceiling. This creates a form of path dependence in the model in the sense that the natural rate of growth will depend on the actual growth history of an economy. According to Setterfield (2010a), path dependence in the natural rate of growth can be "weak" if it is sensitive to initial conditions, or "strong" if the experience of a particular equilibrium or disequilibrium growth trajectory can induce discrete structural change associated with the economy's technology and/or institutions, as a result of which the economy will evolve through a series of discrete regimes or episodes of growth.

As an analogy with the literature that has analysed hysteresis effects in the natural rate of unemployment and in the NAIRU (see DeLong and Summers, 2012 for a survey), it is possible to say that if hysteresis and path dependency effects are also present in the natural rate of growth, then the latter should be interpreted not as the long-run equilibrium value but as a short-term growth barrier that shifts with economic activity. It is in this sense that "there is nothing 'natural' about the natural rate of growth" (León-Ledesma and Thirlwall, 2002b: 435). Thus, like the warranted growth rate in Harrod's model, the natural rate of growth would be a moving equilibrium with the actual rate of growth as attractor.[19]

Finally, it is also necessary to note that the endogenous reaction of g_n with respect to g complicates even more the adjustment to equilibrium in the context of Harrod's model (León-Ledesma and Thirlwall, 2000; 2002b; Vogel, 2009). In the short-run divergence problem between g and g_w, when $g > g_w$ in boom periods g_n will also rise following g, which in turn increases the possibility that the cyclical upturn is not brought to an end by the fixed full-employment ceiling but by demand constraints that can be associated with inflation (and balance of payments problems if we consider an open economy) due to bottlenecks in the system. The latter might explain why cyclical peaks are often accompanied by excess capacity. With respect to the long-run divergence problem between g_w and g_n, if $g_w > g_n$ the economy is in recession and therefore g_n will fall even more as employment falls and

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[19] It is important to emphasize the fact that the view that the g_n reacts endogenously to g does not imply that supply simply adjusts passively to demand or monetary shocks in a manner of "Say's law in reverse" (Vogel, 2009, p. 50), but rather it recognizes a mutual interaction between supply and demand, so that demand and/or monetary conditions, within limits, can create its own supply. If the latter is true, then this situation has to be modelled explicitly since the notion of a full-employment production frontier –as conceived in neoclassical exogenous growth and in the standard NGT frameworks– is no longer tenable (León-Ledesma and Thirlwall, 2002b).

productivity slows, whereas if $g_w < g_n$ the economy is in expansion and therefore g_n will increase as employment rises and productivity accelerates.

1.4 Empirical evidence on the endogeneity of the natural rate of growth

The elements described above offer a broader perspective to the estimation procedure developed by LLT, which is one of the building blocks of the present thesis. This approach has pointed towards the possibility that cyclical variations in output may have a direct impact on the long-run potential output by presenting evidence of the sensitivity of a statistically estimated g_n with respect to g. The view that g has an impact on g_n implies that some large and persistent shocks during the transition towards equilibrium can move the equilibrium itself, establishing an empirical connection between short-run fluctuations and long-run growth. In this sense, the econometric method presented by LLT has tried to show that g_n is an endogenous result of g in the sense that the former variable presents flexibility in the downward and upward directions.

Nevertheless, it is highly important to underline that the econometric method of LLT is not a theory of the g_n *per se* (León-Ledesma and Thirlwall, 2002a, Lanzafame and León-Ledesma, 2010). In other words, the statistical specifications presented following this approach cannot be considered a theoretical model of the endogeneity of g_n, but merely a statistical device to obtain average estimates of the g_n associated with high and low growth regimes. Therefore, the LLT approach should be interpreted as an econometric specification that tries to test for hysteresis effects on a statistically calculated g_n.[20]

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[20] This means that theories of endogeneity of g_n under the post-Keynesian framework would need to be based on demand-growth models that encompass different elements of path dependency; for example, cumulative causation models that incorporate Kaldor-Verdoorn effects. The model presented in Setterfield (2010a) shows

Table 1.1 below summarizes the studies that, to our knowledge, have applied the standard LLT estimation approach. From this Table it is possible to see that: 1) the empirical evidence supports the view of the endogeneity of g_n for OECD, Latin-American and Asian countries[21]; and 2) on average, the natural rate of growth in Latin American countries is more sensitive to actual economic growth than in Asian or OECD countries.

The relatively higher sensitivity of the natural rate of growth in Latin American countries can be explained by means of the following effects (Libânio, 2009; Vogel, 2009). In the first place, in Latin American countries there is a large proportion of the labour force employed in the informal sector or in the subsistence economy which can easily move into formal employment in boom periods; therefore, labour force employed in the informal sector or in the subsistence economy functions as a reserve of labour that can be used in periods of expansion.

In second place, industries in less developed countries are generally more labour-intensive than those in industrialized countries, which might further explain the comparably large decrease in the unemployment rate in periods of high growth.

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that both the actual and natural rates of growth are path dependent because the latter is sensitive to the former, precisely via the operation of Verdoorn's law.

[21] However, empirical evidence on African, Asian and developed countries remains scarce.

Table 1.1. Empirical evidence on the endogeneity of the natural rate of growth*

Study	Period	Econometric techniques employed**	Countries or regions	Average elasticity between the natural rate in boom periods and the original estimate of the natural rate[a]
León-Ledesma and Thirlwall (2002b)[b,c,d]	1961-1995	OLS	15 OECD countries	52%[f] or 40%[g]
Perrotini-Hernández and Tlatelpa-Pizá (2003)	1970-2000	OLS	Canada, US and Mexico	41%[f]
Libânio (2009)[b,c]	1981-2003	OLS	10 Latin American countries	103%[f] or 73%[g]
Vogel (2009)[b,e]	1986-2003	SUR	11 Latin American countries	64%[f]
Acikgoz and Mert (2010)[b,d]	1980-2008	OLS	Turkey	36%
Lanzafame (2010)	1977-2003	LSDV and SUR	20 Italian regions	42% using the LSDV technique and 85% following the SUR estimation[f]
Dray and Thirlwall (2011)[b,e]	1982-2005	OLS	9 Asian countries	30%[g]
Oreiro et al. (2012)[b]	1990-2005 (quarterly data)	OLS	Brazil	220%[f] or 108%[g] (annualized rate)

*Daria Ciriaci's study entitled "Tasso di crescita naturale e crescita cumulativa nelle regioni italiane" was left out of this survey. For a discussion of her results see León-Ledesma and Lanzafame (2010).

**Acronyms employed: OLS: Ordinary Least Squares; SUR: Seemingly Unrelated Regressions; and LSDV: Fixed-effects Least Square Dummy Variables.

[a] These results are with respect to the estimated natural rate of growth following Thirlwall's (1969) reversal.

[b] Studies that corrected for autocorrelation of the errors if necessary.

[c] Studies that tested for the possibility of biased coefficients due to "abnormal" observations using Maddala's test. León-Ledesma and Thirlwall (2002b) found that only for the case of Italy (for the period of 1961 to 1995) the "abnormal" observations may cause a significant bias in the results obtained; whereas Libânio (2009) found that only the cases of Colombia and Uruguay (both of them during the period of 1981 to 2003) seem to be affected by the existence of "abnormal" observations.

[d] Studies that also performed Instrumental Variable estimation.

[e] Studies that introduced dummies to take into account outliers.

[f] If the dummy variable takes the value of 1 for those years for which the actual rate of growth is higher than the estimated natural rate of growth.

[g] If the dummy variable takes the value of 1 for those years in which a three to five year moving average of the growth of output is above the estimated natural rate of growth.

Last but not least, via the Kaldor-Verdoorn law (see Chapter 3), the lower the level of development reached in an economy, the easier to gain increases in productivity with relative small increases in investment.

There are two studies that are particularly relevant in the context of the present thesis. Firstly, Libânio (2009) finds that the Latin American countries that present the highest elasticities of the natural rate of growth are Argentina, Peru and Uruguay; whereas the countries that present the lowest elasticities are Chile, Colombia and Costa Rica. Secondly, Vogel (2009) finds that the Latin American countries that present the highest elasticities are Venezuela, Argentina and Nicaragua; whereas the ones with the lowest elasticities are Chile, Colombia and Costa Rica.

Therefore, it is possible to conclude that both Libânio (2009) and Vogel (2009) have found that Chile, Colombia and Costa Rica are countries that present low sensitivity of the natural rate of growth; whereas the natural rate of growth in Argentina seems to present high sensitivity.[22]

1.5 Concluding remarks

The present Chapter has tried to offer the theoretical framework of the Thesis. Firstly, we have offered an historical review of the concept of the natural rate of growth, so that we have revisited Harrod's model, the neoclassical versus post-Keynesian growth debates that emerged in the 1950s, and the "new" or "endogenous" neoclassical growth theory. We have found that, nevertheless it is true that "old" post-war growth theorists considered that the natural rate of growth was an exogenous variable, a closer inspection of some of the original works reveals that both Roy Harrod and Nicholas Kaldor considered that the natural rate of growth presented a certain degree of endogeneity with respect to the prevailing economic conditions in an economy. In the same vein, Robert Solow's recent comments that have been

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[22] In Chapters 2 and 3 we compare our results with the ones presented by Libânio (2009) and Vogel (2009).

breathing a sort of Harrodian/post-Keynesian spirit –in the sense that he seems to believe that the fundamental problem of combining short, medium and long-run macroeconomics has not been solved– also indicate that he does not consider that the natural rate of growth is exclusively an exogenous phenomenon.

Secondly, we have tried to relate the concept of hysteresis to the hypothesis of endogeneity of the natural rate of growth that was inaugurated at the empirical level by the papers of León-Ledesma and Thirlwall (2002a; 2002b) since both approaches are closely related.

Finally, we have also presented the main empirical findings of the literature that has followed the León-Ledesma and Thirlwall approach, stressing the results for Latin American countries since the latter are the object of study of the present thesis. The results obtained by Libânio (2009) and Vogel (2009) show that Chile, Colombia and Costa Rica are countries that present low sensitivity of the natural rate of growth in boom periods; whereas the natural rate of growth in Argentina presents high sensitivity in the upward direction.

Nevertheless, it is important to emphasize the following points with respect to the different studies summarised in Table 1.1:

1) The literature has only focused on the results of the elasticity of the natural rate of growth in the upward direction –that is, in boom periods. As a matter of fact, the different studies that have applied the LLT estimation procedure have not mentioned that the natural rate of growth can also experience movements in the downward direction as a consequence of its interaction with the actual rate of growth. We develop further this point in Section 3.3 of Chapter 3.

2) None of these studies has looked at the time-varying nature of the natural rate of growth, or offered a decomposition analysis of the latter. We try to develop this new approach in Chapter 4.

3) None of these studies has looked at the sensitivity of the individual components of the natural rate of growth with respect to the different sources of aggregate demand variation. We try to study this research question in Chapter 5.

CHAPTER 2

The Natural Rate of Growth in Latin American Countries, 1981-2011

2.1 Introduction

The present Chapter tries to estimate the natural rate of growth (henceforth g_n) in 13 Latin American countries for the period 1981-2011 and to test the robustness of these estimates by using different techniques. Firstly, following Thirlwall's (1969) reversal estimation procedure based on the simple difference version of Okun's law, the g_n is estimated using: 1) Ordinary Least Squares (henceforth OLS); 2) panel estimators with general multifactor error structures that take into account parameter heterogeneity and cross section dependence (Bond and Eberhardt, 2013; Eberhardt and Teal, 2010; Eberhardt, 2012; Pesaran, 2006); and 3) a penalized regression spline modeling approach which allows us to take into account the possibility of time-varying effects in the Okun coefficient (Zanin and Marra, 2012).

Secondly, the g_n is also calculated from an adaptation of the dynamic specification of the first difference version of Okun's law proposed by Knotek (2007) following the general-to-specific modeling approach using both OLS and Seemingly Unrelated Regressions (henceforth SUR).

Finally, the present Chapter also tries to deal with the endogeneity bias that may exist in the standard procedure based on Thirlwall's (1969) reversal using various Instrumental Variable (henceforth IV) estimations.

Thereby, this Chapter makes new contributions to the literature. In the first place, it applies new econometric techniques to the study of g_n, namely: panel estimators with multifactor structures and a penalized regression spline approach. Secondly, it is the first time that a dynamic specification of the first difference version of Okun's law is used to obtain estimates of g_n. Thirdly, this is also the first time that the endogeneity bias that may exist in the OLS estimation results of g_n is explicitly tackled using IV estimation for the case of Latin American countries.

The rest of the Chapter is organized as follows. Section 2.2 briefly presents Thirlwall's (1969) estimation procedure and introduces the dynamic version of Okun's law; Section 2.3 presents a brief overview of the econometric techniques; Section 2.4 presents the estimates of g_n for the sample of Latin American countries; and finally Section 2.5 presents the main conclusions of the Chapter.

2.2 Thirlwall's (1969) reversal estimation procedure and the dynamic version of Okun's law

Following the first difference version of Okun's (1962) law, Thirlwall (1969) presented a way to estimate g_n by noting that if the actual rate of growth (henceforth g_t) falls below the g_n, the unemployment rate will rise; and vice versa, if the g_t rises above the g_n, the unemployment rate will fall. Hence, the g_n under this framework is defined as the rate of growth that keeps the unemployment rate constant (Thirlwall, 1969).[23]

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[23] It is interesting to note that very recently Knotek (2007) and the International Monetary Fund (2010) have used the same definition of Thirlwall's (1969) natural rate of growth in order to define the rate of output growth needed for a stable unemployment rate without any reference to Thirlwall's paper.

However, in order to avoid estimation biases caused by labour hoarding –that is, the average number of hours worked by each worker, Thirlwall (1969) suggested reversing the dependent and independent variables, so that:

$$g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1 \qquad \dots\dots\dots\dots\dots\dots \qquad (2.1)$$

where in equation (2.1) $\Delta\%U_t$ is the change in the percentage level of unemployment rate and ε_1 is the stochastic disturbance term that satisfies the standard statistical properties.

From equation (2.1) it can be seen that if $\Delta\%U_t = 0$ then $g_n^A = \alpha$, where g_n^A is the average natural rate of growth in the estimation period. This method of estimating g_n^A has been dubbed as "Thirlwall's reversal" by recent literature (León-Ledesma and Thirlwall, 2002b; Libânio, 2009; Vogel, 2009).

More recently and in the context of the U. S. economy, Knotek (2007) has mentioned that the phenomenon of "jobless recoveries" –that is, periods following the end of recessions when output growth resumes but employment does not grow– "is symptomatic of a fundamental change in the timing of the relationship between output and the labour market that the simple difference version of Okun's law is not able to capture" (Knotek, 2007, p. 87). Based on the latter, he proposes the use of a dynamic version of Okun's law. It may be possible to employ this notion in the context of Thirlwall's (1969) reversal, so that we assume that current output growth can be affected by past output growth, and by both current and past changes in the unemployment rate[24]:

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[24] However, it should be pointed out that both Knotek (2007) and the International Monetary Fund (2010) use quarterly data, whereas we are working with annual data.

$$g_t = \alpha_0 + \alpha_1(g_{t-1}) + \alpha_2(g_{t-2}) + \beta_0(\Delta\%U_t) + \beta_1(\Delta\%U_{t-1}) + \beta_2(\Delta\%U_{t-2}) + \varepsilon_2 \ldots (2.2)$$

where ε_2 denotes the error term.

There are advantages and disadvantages of using this dynamic version. The main drawback is that, since it no longer only captures the contemporaneous effect of changes in the unemployment rate and output growth, the relationship does not have the simple interpretation as the original difference version of Okun's law. In our context, the latter means that if, for example, both α_1 and α_2 are found to be statistically significant, then the estimated average g_n has to be retrieved from equation (2.2) as follows:

$$g_n^{A^*} = \frac{\alpha_0}{1 - \alpha_1 - \alpha_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.3)$$

Furthermore, the use of lags of the dependent variable (g_t) in a model like equation (2.2) introduces further complications in a time-series setting since these variables are only weakly exogenous, and therefore their inclusion violates the exogeneity assumption of OLS (see also Section 2.4.3).

On the other hand, the main advantage of the dynamic version of Okun's law is that it is not as restrictive in terms of the timing connection between output growth and changes in unemployment.

2.3 Econometric techniques: a brief overview

In addition to the well-known OLS estimation, we have employed SUR estimation, panel estimators with multifactor structures that take into account parameter heterogeneity and cross section dependence, and a penalized regression spline modeling approach. This section tries to offer a description of the latter two approaches.

2.3.1 Seemingly unrelated regressions and panel estimators with multifactor structures

In brief, SUR estimation (Zellner, 1962) consists in estimating an N-equation system of “unrelated” equations by Generalized Least Squares (henceforth GLS) techniques, assuming that the error terms are correlated across equations. Thus, the N equations are “unrelated” in the sense that any variable, dependent or independent, appears in only one equation, so that the systems have no common variables. As Baum (2006) explains, the SUR estimator can be considered a multiple time-series estimator since it is based on the large-sample properties of large time series dimension (henceforth T) and small cross-section dimension (henceforth N) in which $T \rightarrow \infty$, unlike the Fixed Effects (henceforth FE) and the Random Effects estimators whose large-sample justification is based on small T and large N datasets in which $N \rightarrow \infty$.

There are some advantages of estimating the equations jointly in the present context, namely: it is possible to gain efficiency since, in contrast to OLS, SUR-GLS takes into account potential cross-country residuals correlation due, for example, to Latin American common shocks. However, one problem with the SUR estimation is that it assumes a very particular form of correlation between the countries –that is, cross-country residuals correlation. Indeed, as Pesaran et al. (1999) have indicated, it may be that the cause of non-zero error covariances is due to omitted common effects that impact all countries, which in turn would indicate model misspecification rather than error correlation.

Additionally, the typical case in which an unrestricted SUR-GLS approach is employed is the case of panel data models where $N < 10$ and T is large since its application to large N and T panels involve nuisance parameters that increase at a quadratic rate as the N of the panel is allowed to rise (Pesaran, 2006).

These are the main reasons why the present Chapter has also employed panel data estimators that take into account parameter heterogeneity and cross-section dependence (Bond and Eberhardt, 2013; Eberhardt and Teal, 2010; Pesaran and Smith, 1995; Pesaran, 2006; and see also Lanzafame, 2014 for a summary of these estimators) in order to estimate g_n.

In the context of panel models, and provided that both N and T are sufficiently large, different estimations can be performed using many alternative approaches, which differ according to the degree of parameter heterogeneity allowed for. It is well-known that the pooled estimator imposes full homogeneity of both slope and intercepts coefficients, whereas the FE estimator allows only the intercept to differ across groups. However, if the coefficients are in fact heterogeneous, both estimators will yield inconsistent and misleading results. As a solution to the problem of coefficient heterogeneity, Pesaran and Smith (1995) proposed the Mean Group (henceforth MG) estimator, which estimates N separate regressions –one for each group– and then averages the coefficients over groups. Hence, the MG estimator can be regarded as a fully heterogeneous-coefficient model since it imposes no cross-group parameter restrictions –that is to say, all coefficients are fitted separately for each group– that assumes cross-section independence.[25]

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[25] Between the pooled and the MG estimators we can find the Pooled Mean Group (PMG) estimator developed by Pesaran et al. (1999). This approach combines both pooling and averaging since it constrains long-run coefficients to be identical but allows short-run coefficients, the intercept, and error variances to differ across groups. When this hypothesis is correct, the PGM estimator turns out to be more efficient than the MG estimator

In turn, if cross-section dependence is present in the data then the MG estimator will produce inconsistent and biased results. There are several available tests of cross-sectional dependence that have been developed, and most of them are typically based on estimates of pair-wise error correlations (henceforth ρ_{ij}). An early test of this type is the Breusch-Pagan Lagrange multiplier (henceforth LM) test, which is based on the squares of the sample estimate of the ρ_{ij} and tests the null hypothesis that $\rho_{ij} = 0$ for all $i \neq j$. However, the latter test tends to exhibit substantial size distortions in the case of panels with relatively large N (Pesaran, 2004; Chudik and Pesaran, 2013). As an alternative to the Breusch-Pagan LM test, Pesaran (2004) proposed the following Cross-section Dependence (henceforth CD) test when N is large:

$$CD = \sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right) \quad \dots\dots\dots\dots \quad (2.4)$$

Hence, unlike the Breusch-Pagan LM test, Pesaran's (2004) CD test is based on the sample estimate of the ρ_{ij} ($\hat{\rho}_{ij}$) rather than on their squares (as is done in the Breusch-Pagan LM test).

Under the null of cross-section independence we have that $CD \sim N(0,1)$. The latter holds for fixed values of T and N under a wide class of panel data models –including heterogeneous dynamic models subject to multiple breaks in their slope coefficients and error variances, so long as the unconditional means of both independent and dependent variables are time-invariant and their innovations are symmetrically distributed (see Pesaran, 2004). However,

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(Pesaran et al., 1999). Since we are only dealing with the simple difference version of Okun's law (that is to say, we are not including any long-run slope coefficients), the PMG estimator was not performed in the present chapter.

the CD test is likely to over-reject in the case of panel models with weakly exogenous regressors if N is much larger than T (Chudick and Pesaran, 2013).

As a solution to the problem of CD in panel data models, Pesaran (2006) developed an estimation procedure named Common Correlated Effects (henceforth CCE), which provides consistent results in panel data models with a general multifactor error structure. The basic idea behind the CCE estimation procedure consists in approximating the unobservable common factors via the cross-sectional averages of the observable variables (Lanzafame, 2014). In other words, the CCE estimator filters the individual-specific regressors by means of cross-section aggregates such that, as $N \rightarrow \infty$, the differential effects of unobserved common factors are eliminated (Pesaran, 2006).

Under the CCE estimation approach it is possible to find the Common Correlated Effects Mean Group (henceforth CCEMG) estimator and the Common Correlated Effects Pooled (henceforth CCEP) estimator (Pesaran, 2006). The former –that is, the CCEMG estimator– produces consistent estimates of the model parameters as simple averages of the country-specific estimates; whereas the latter –that is, the CCEP estimator– is obtained from the standard pooled version of the CCE estimator. The CCEP estimator: 1) is a more efficient estimator in small samples and assumes, possibly incorrectly, that the individual slope coefficients are the same across N –although the Monte Carlo simulations presented by Pesaran (2006) show that this assumption does not affect its performance; and 2) can be considered a generalization of the FE estimator that allows for the possibility of error CD (Pesaran, 2006).

Bond and Eberhardt (2013), Eberhardt and Teal (2010) and Eberhardt (2012) have developed an alternative method to the CCEMG with production function estimation in mind: the Augmented Mean Group (henceforth AMG) estimator. The AMG estimator accounts for CD by including a "common dynamic process" in the country regression, which in turn represents an estimated cross-group average of the evolution of the "unobservable effects" over time. According to Bond and Eberhardt (2013), Eberhardt and Teal (2010) and Eberhardt (2012), it is possible to provide a simple but economically meaningful interpretation of the common dynamic process in the context of cross country growth models: it represents common total factor productivity evolution over time, whereby "common" is defined either in the literal sense or as the sample mean country-specific total factor productivity evolution. However, it is important to bear in mind that the AMG estimator was developed in the context of a production function analysis, controlling both for capital and for labour force growth. The latter is not the case in the simple difference version of Okun's law, and therefore the common dynamic process cannot be interpreted as common total factor productivity evolution in the estimates of equation (2.1).

Following Eberhardt (2012), it is possible to offer a summary of the differences between all the MG estimators that have been used in the context of Thirlwall's (1969) reversal estimation procedure:

$$g_{it} = \beta_i(\Delta\%U_{it}) + u_{it} \qquad \dots\dots.\dots\dots\dots\dots \qquad (2.5)$$

$$u_{it} = a_i^1 + \mu_i(f_t) + e_{it}^1 \qquad \dots\dots\dots\dots\dots\dots\dots\dots \qquad (2.6)$$

$$\Delta\%U_{it} = a_i^2 + \mu_i(f_t) + \gamma_i(j_t) + e_{it}^2 \quad \dots\dots\dots\dots \quad (2.7)$$

where in equations (2.5) to (2.7) we have that, in addition to the previously defined variables, u_{it} depicts the unobservable common factors; a_i^1 and a_i^2 are standard group-specific fixed effects which capture time-invariant heterogeneity across groups; f_t and j_t are unobserved common factors with heterogeneous factor loadings μ_i and γ_i –which in turn can capture time-variant heterogeneity and CD[26]; and e_{it}^1 and e_{it}^2 are the error terms.

All the MG estimators here employed follow the same basic methodology, namely they estimate N group specific OLS regressions and then average the estimated coefficients across groups (Eberhardt, 2012). In the first place, the MG estimator does not pay attention to CD and assumes away $\mu_i(f_t)$ –or at best models these unobservables components with a linear trend. Hence, in this case equation (2.5) is estimated for each country i including an intercept to capture fixed effects –which in our framework can be associated with the different g_ns, and also including a linear trend to capture time-variant unobservable common factors. Then the estimated coefficients β_i are subsequently averaged across panel members, and in our case we have attributed less weight to outliers using Hamilton's (1991) procedure.

Pesaran's (2006) CCEMG estimator allows for CD, time-variant unobservables with heterogeneous impact across panel members, and problems of identification since β_i is unidentified if $\Delta\%U_{it}$ contains f_t. The CCEMG solves this problem by augmenting the group-specific regression equation: apart from $\Delta\%U_{it}$, equation (2.5) includes the cross-section averages of both g_{it} and $\Delta\%U_{it}$ as additional regressors. The combination of the cross-section averages of both g_{it} and $\Delta\%U_{it}$ can account for the unobserved common factor f_t; and, given the group-specific estimation, the heterogeneous impact λ_i will also be given.

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[26] According to Eberhardt (2012), the factors f_t and j_t are not limited to linear evolution over time since they can be nonlinear and nonstationary.

However, in empirical application the estimated coefficients on the cross-section-averaged variables and their average estimates are not interpretable in a meaningful way since they exist only to correct for the bias caused by the unobservable common factor (Eberhardt, 2012). In turn, the coefficients β_i are averaged across panel members, and we have applied again the procedure developed by Hamilton (1991) in order to attribute less weight to outliers.

Finally, the AMG procedure is implemented in three steps in the present context:

$$\Delta g_{it} = -\beta' \Delta\Delta\% U_{it} + \sum_{t=2}^{T} c_t \Delta D_t + e_{it} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.8)$$

$$\Rightarrow \hat{c}_t = \hat{\mu}_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.9)$$

$$g_{it} = \alpha_i' - \beta_i'(\Delta\% U_{it}) + d_i \hat{\mu}_t + \varepsilon_{it} \quad \ldots\ldots\ldots\ldots \quad (2.10a)$$

$$g_{it} - \hat{\mu}_t = \alpha_i'' - \beta_i''(\Delta\% U_{it}) + \varepsilon_{it}' \quad \ldots\ldots\ldots\ldots \quad (2.10b)$$

where in equations (2.8) to (2.10b) we have that Δ is the first difference operator; c_t are the coefficients on the $T-1$ year dummies D_t in first differences, so that c_t represents the common dynamic process or the estimated cross-group average of the evolution of unobservable common factors; α_i', β_i', d_i, α_i'' and β_i'' are parameters to be estimated; and e_{it}, ε_{it} and ε_{it}' are error terms.

Hence, in the first stage –that is, equation (2.8), a pooled OLS regression in first differences augmented with year dummy variables is estimated and the coefficients on the (differenced) year dummies are collected. These coefficients ($\hat{c}_t$) are then relabelled as $\hat{\mu}_t$ in equation (2.9).[27] In the second stage –that is, equations (2.10a) and (2.10b), the group-specific

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[27] The $\hat{c}_t$ coefficients are extracted from the pooled regression in first differences since nonstationary variables and unobservable common factors are believed to bias the estimates in the pooled levels regressions. Hence, in

regression model is augmented with $\hat{\mu}_t$. The latter can be done either including $\hat{\mu}_t$ as an explicit variable as depicted in equation (2.10a) or imposing $\hat{\mu}_t$ on each group member with unit coefficient by subtracting the estimated process from the dependent variable as depicted in equation (2.10b).[28] Finally, in the third stage and as in the MG and CCEMG estimators, the group-specific model parameters are then averaged across the panel, so that $\hat{\beta}_{AMG} = N^{-1}\sum_{i=1}^{N}\hat{\beta}_i$.[29]

2.3.2 Penalized regression spline approach[30]

As discussed in Chapter 1, there exists substantial literature that shows that the effect of economic growth on unemployment is asymmetric and higher during recessions than during expansions. The latter means that nonlinear effects may be present in Okun's law and, therefore, it points out a possible source of bias in equation (2.1). In other words, since Thirlwall's (1969) reversal estimation procedure used to estimate g_n assumes that the Okun coefficient is constant over time then the results obtained from this estimation might be neglecting potential nonlinearities.

In order to deal with this possible source of bias, we have tried to consider the possibility that Okun's coefficient for different time points might be dissimilar. Therefore, following Zanin

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principle, both the CCE and AMG methods may help to deal with the concern raised by Attfield and Silverstone (1997) with respect to the first difference version of Okun's law: if both output and unemployment are *I(1)* variables as well as co-integrated, then the first difference form of Okun's law will be misspecified. Since we are trying to follow Thirlwall's (1969) original specification the issue of cointegration between output and unemployment is not tackled in the present chapter.

[28] We have also estimated equations (2.10a) and (2.10b) including country-specific time trends that try to capture omitted idiosyncratic processes evolving in a linear fashion over time (Bond and Eberhardt, 2013; Eberhardt and Teal, 2010).

[29] The Monte Carlo simulations presented by Bond and Eberhardt (2013) show that the AMG and CCEMG estimators performed similarly well in terms of bias or root mean squared error in panels with nonstationary variables (cointegrated or not) and multifactor error terms or cross-section dependence.

[30] This section relies heavily upon Marra and Radice (2010), Wood (2006) and Zanin and Marra (2012).

and Marra (2012), we accommodate time-varying features in Thirlwall's (1969) estimation procedure as follows:

$$g_t = \alpha^* - \beta_t(\Delta\%U_t) + \varepsilon_4 \quad \dots\dots\dots\dots \quad (2.11)$$

where in equation (2.11) we have that the effect of $\Delta\%U_t$ on g_t on time t is represented by β_t.

The estimated g_n obtained from equation (2.11), that is α^*, can be considered as the natural rate of growth that takes into account the possibility of a time-varying Okun coefficient.

The approach here adopted considers that the coefficient associated with $\Delta\%U_t$ is an unknown smooth function s of time t, with parameter vector $\boldsymbol{\delta}$ –that is, subject to centering constraints. Thus:

$$\beta_t = s(t, \boldsymbol{\delta}) \qquad \dots\dots\dots\dots\dots\dots \qquad (2.12)$$

Therefore, under this approach, the vector of $\Delta\%U_t$ effects $\boldsymbol{\beta} = (\beta_1, \dots\dots, \beta_T)_{TX1}$ is modelled as $s(t, \boldsymbol{\delta})$.

As Zanin and Marra (2012) explain, the model depicted in equation (2.11) is a time-varying coefficient model, which in turn is special case of a varying-coefficient model (see Hastie and Tibshirani, 1993) for which the effect modifier is t. Thus,

> "[t]he advantage of this approach is that $\boldsymbol{\beta}$ is completely smooth, with its shape determined from the data and not from the parametric form specified by the investigator.
>
> It is important to point out that the use of [s] in [2.11] is crucial since the functional shape of any relationship is not typically known *a priori*, hence it does not make sense to impose any structure on it (e.g., linear or quadratic) but rather we should let the data determine whether this relationship is linear or non-linear and for which countries" (Zanin and Marra, 2012, p. 94).

Thereby, the use of s allows for flexible specification of the dependence of the response of g_t on $\Delta\%U_t$, and the model in equation (2.11) can flexibly determine the functional shape of the relationship between g_t and $\Delta\%U_t$, thus avoiding some of the drawbacks of modelling data using parametric relationships.

However, this flexibility comes at the cost of two new theoretical problems. First, it is necessary to represent s in some way; and second, it is necessary to choose the "degree of smoothness". As regards the first problem, s can be represented using regression splines (see Marra and Radice, 2010 and Wood, 2006). In our case, the regression spline of t is made up of a linear combination of known basis functions[31] and unknown regression parameters:

$$s(t, \boldsymbol{\delta}) = \sum_{k=1}^{q} \delta_k b_k(t) \quad \dots\dots\dots\dots \quad (2.13)$$

where in equation (2.13) we have that δ_k represents the unknown regression parameters; $b_k(t)$ are the known basis functions; and q is the number of basis functions.

At this point, it is important to mention two things. In the first place, in order to identify model (2.11), s is subject to the following constraint: $s(t, \boldsymbol{\delta}) = 0$.

In second place, the different $b_k(t)$ have to be chosen in order to come up with an estimate for $s(t, \boldsymbol{\delta})$ (for example, a 3rd order polynomial). The latter means that q determines the maximum possible flexibility allowed for a smooth term: as q increases, the polynomial bases

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[31] In mathematics, a basis function is an element of a particular basis for a given function space. In other words, a basis function is an element of a set of linearly independent vectors that, in a linear combination, can represent every continuous function in a set of functions of a given kind (see Ito, 1993).

become increasingly linear, which in turn means that overfitting is likely to occur if q is too large. If this is the case then the parameter estimators will be highly correlated, which in turn leads to high estimator variance and numerical problems (Marra and Radice, 2010). In consequence, adding several polynomial terms does not represent a valid solution to capture nonlinear relationships.

In order to ensure that the $b_k(t)$ have convenient mathematical properties and good numerical stability it may be possible to use thin plate regression splines[32] with a penalized approach (Wood, 2003). The penalized approach here adopted keeps the number of q fixed at a 10 since this ensures good flexibility in the estimation of the model and therefore controls the trade-off between the goodness of fit and roughness of s by the smoothing parameter (henceforth λ) (see Wood, 2003).

Hence, the model depicted in equation (2.11) is fitted as follows:

$$min \ \parallel \boldsymbol{g} - \boldsymbol{X\delta} \parallel^2 + \lambda \int [s^d(t, \boldsymbol{\delta})]^2 dt, \ \ w.r.t. \ \ \boldsymbol{\delta} \ \ \dots\dots\dots\dots \quad (2.14)$$

where, in addition to the previously defined variables, in equation (2.14) we have that $\boldsymbol{g}$ is the vector that contains the annual rates of growth; $\parallel . \parallel$ denotes the Euclidean norm; $\boldsymbol{X}$ is the model matrix containing $b_k(t)$ –that is, the basis functions for the time-varying components– interacted with their corresponding $\Delta\%U_t$; $\boldsymbol{\delta}$ is the spline parameter vector; the integral measures the roughness of the smooth term to be used in the fitting process; and d –which

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[32] Thin plate regression splines are low rank isotropic smoothers since they approximate well the behaviour of a full rank thin plate spline, and its use possesses some specific advantages such as convenient mathematical properties, reasonably well computationally efficiency, and avoid having to choose knot locations (Marra and Radice, 2010; Wood, 2003; Zanin and Marra, 2012).

usually is set to 2 in order to study the possibility of nonlinearities– indicates the order of the derivative for the smooth term to be used in the fitting process.

Since regression splines are linear in their model parameters we have that:

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \int \left[\frac{\partial^2 s(t,\boldsymbol{\delta})}{\partial(t,\boldsymbol{\delta})^2}\right]^2 dt \quad \ldots\ldots\ldots..\ldots\ldots.\ldots\ldots\ldots \quad (2.15)$$

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \int \left[\frac{\partial^2 \sum_{k=1}^{q} \delta_k b_k(t)}{\partial(t,\boldsymbol{\delta})^2}\right]^2 dt \quad \ldots\ldots...\ldots..\ldots \quad (2.16)$$

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \int [\boldsymbol{\delta^T b(t)}]^2\, dt \quad \ldots\ldots\ldots\ldots\ldots.\ldots\ldots.\ldots.\ldots \quad (2.17)$$

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \int [\boldsymbol{\delta^T b(t) b(t)^T \delta}]\, dt \quad \ldots.\ldots\ldots\ldots..\ldots.\ldots \quad (2.18)$$

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \boldsymbol{\delta^T} \left(\int [\boldsymbol{b(t)b(t)^T}]\, dt\right) \boldsymbol{\delta} \quad \ldots\ldots\ldots.\ldots..\ldots \quad (2.19)$$

$$\int [s^d(t,\boldsymbol{\delta})]^2 dt = \boldsymbol{\delta^T S \delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots \quad (2.20)$$

In the equations above $\boldsymbol{b(t)}$ is a vector containing the second derivatives of the basis function for the smooth term with respect to t and $\boldsymbol{S}$ is the known coefficient penalty matrix.

Substituting (2.20) into (2.14) we obtain:

$$min \;\; \| \boldsymbol{g - X\delta} \|^2 + \lambda \boldsymbol{\delta^T S \delta}, \qquad w.r.t.\;\; \boldsymbol{\delta} \quad \ldots\ldots\ldots\ldots\ldots \quad (2.21)$$

It turns out that the penalized least squares estimator of $\boldsymbol{\delta}$ is:

$$\boldsymbol{\hat{\delta}} = (\boldsymbol{X^T X} + \boldsymbol{\lambda S})^{-1} \boldsymbol{X^T g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2.22)$$

Equation (2.22) shows that if the vector of smoothing parameters $\boldsymbol{\lambda}$ is too high then the smooth function will be over-smooth, and if $\boldsymbol{\lambda}$ is too low then the component will be under-smoothed. Wood (2006) showed that $\boldsymbol{\lambda}$ can be effectively estimated by minimization of a prediction error estimate such as the Generalized Cross Validation (henceforth GCV) score, so that:

$$GCV(\boldsymbol{\lambda}) = \frac{n \parallel \boldsymbol{g} - \widehat{\boldsymbol{\psi}} \parallel^2}{[n - \mathrm{tr}(\boldsymbol{A})]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.23)$$

where n in equation (2.23) denotes the number of observations and $\mathrm{tr}(\boldsymbol{A})$ represents the trace of the matrix $\boldsymbol{A}$.

Thus, the vector $\boldsymbol{\lambda}$ enters the GCV score via:

$$\boldsymbol{A} = \boldsymbol{X}(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X} + \boldsymbol{\lambda}\boldsymbol{S})^{-1}\boldsymbol{X}^{\boldsymbol{T}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.24)$$

$$\widehat{\boldsymbol{\psi}} = \boldsymbol{A}\boldsymbol{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.25)$$

In practice, as Zanin and Marra (2012) explain, the amount of smoothing to choose for the smooth term in (2.11) has to be settled, and the fast computational procedure developed by Wood (2006) –in which the smoothing parameters are automatically estimated– represents a good solution to this problem. Specifically, once q and d have been set –usually $q = 10$ and $d = 2$ as was mentioned before, Wood's (2006) numerical procedure will select λ so that the part of smooth term complexity which has no support from the data will be suppressed. Therefore, this approach can produce smooth and reliable curve estimates.

Finally, if we are interested in testing smooth terms for equality to zero such that $H_o\colon \beta_t = 0$ in equation (2.11), *p*-values calculations can be based on the following result:

$$\frac{\hat{\boldsymbol{\delta}}^{\boldsymbol{T}}\, \boldsymbol{V}_{\hat{\boldsymbol{\delta}}}^{\boldsymbol{r-}}\hat{\boldsymbol{\delta}}}{\hat{\sigma}^2}\left[\frac{\sigma^2}{r}\right] = \frac{\hat{\boldsymbol{\delta}}^{\boldsymbol{T}}\, \boldsymbol{V}_{\hat{\boldsymbol{\delta}}}^{\boldsymbol{r-}}\hat{\boldsymbol{\delta}}}{r} \sim F_{r,n-edf} \quad \dots\dots\dots\dots\dots \quad (2.26)$$

$$\boldsymbol{V}_{\hat{\boldsymbol{\delta}}} = (\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X} + \boldsymbol{S})^{-1}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X} + \boldsymbol{S})^{-1}\sigma^2 \quad \dots\dots\dots\dots \quad (2.27)$$

where in equations (2.26) and (2.27) $\hat{\boldsymbol{\delta}}$ contains the estimated coefficients for the smooth term; $\boldsymbol{V}_{\hat{\boldsymbol{\delta}}}$ is the covariance matrix of $\hat{\boldsymbol{\delta}}$ –which has to be employed in order to overcome possible matrix rank deficiencies due to the fact that the smoothing penalty may suppress some dimensions of the parameter space; and $\boldsymbol{V}_{\hat{\boldsymbol{\delta}}}^{\boldsymbol{r-}}$ is the rank r pseudo-inverse of $\boldsymbol{V}_{\hat{\boldsymbol{\delta}}}$.

In equation (2.26) the estimated variance ($\hat{\sigma}^2$) can be calculated by the usual residual sum of squares divided by the residual degrees of freedom:

$$\hat{\sigma}^2 = \frac{\parallel \boldsymbol{g} - \hat{\boldsymbol{\psi}} \parallel^2}{[n - \mathrm{tr}(\boldsymbol{A})]} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (2.28)$$

Finally, it is important to note that $\mathrm{tr}(\boldsymbol{A})$ represents the estimated degrees of freedom (henceforth edf) or number of parameters of the fitted model. If the edf turn out to be statistically significant above 1 then it is possible to say that the coefficients are statistically time-varying at the 5% level of significance.

2.4 New empirical evidence for Latin American countries

This section presents the results of the estimation of g_n obtained from equations (2.1) and (2.2) for a sample of 13 Latin American countries during the period 1981-2011 using annual data. Series for g_t and $\Delta\%U_t$ were extracted from the World Bank electronic database for all countries. However, the World Bank electronic database presents some missing observations for the $\Delta\%U_t$ series, so that the necessary observations were extracted from the ECLAC and the IMF electronic databases.

The g_n obtained from equation (2.1) was calculated using OLS, the panel model econometric techniques and the penalized regression spline approach described in Section 2.3. In turn, the estimated g_n obtained from equation (2.2) was retrieved adopting the general-to-specific modeling approach using OLS and SUR. In principle, it could also be possible to employ both the panel model econometric techniques and the penalized regression spline approach to calculate g_n from equation (2.2). However, this was not carried out in the present Chapter because data constraints impede certain statistical computations.[33]

Finally, this section also tries to take into account the endogeneity bias in the procedure here employed to estimate g_n that was pointed out by León-Ledesma and Thirlwall (2002b): since $\Delta\%U_t$ should really be regarded as an endogenous variable –that is to say, since $\Delta\%U_t$ is a function of g_t– then the estimated coefficients obtained from Thirlwall's (1969) estimation procedure will be inconsistent. Therefore, equations (2.1) and (2.2) have also been estimated using IV methods.

2.4.1 Simple difference version of Okun's law

OLS estimation of equation (2.1) is shown in Table 2.1:

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[33] Specifically, data constraints impede the statistical computation of equation (2.11) in the software environment **R** if the lags of g_t and $\Delta\%U_t$ are included in the estimation.

Table 2.1. Equation (2.1) using OLS:

$$\boldsymbol{g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1}$$

Country	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	**Correct specification tests**[a,b]	**Adjusted R^2**
Argentina	3.084***	1.651***	Autocorrelation=0.02; Heteroskedasticity=0.30; Normality=0.46; Ramsey RESET test=0.20	0.24
Bolivia[c]	3.062**	0.137	/	0.02
Brazil	2.925***	2.702***	Autocorrelation=0.05; Heteroskedasticity=0.12; Normality=0.51; Ramsey RESET test=0.73	0.54
Chile	4.697***	1.039***	Autocorrelation=0.06; Heteroskedasticity=0.01; Normality=0.00; Ramsey RESET test=0.00	0.25
Colombia	3.575***	0.630***	Autocorrelation=0.11; Heteroskedasticity=0.06; Normality=0.82; Ramsey RESET test=0.02	0.23
Costa Rica	4.178***	1.669***	Autocorrelation=0.09; Heteroskedasticity=0.91; Normality=0.00; Ramsey RESET test=0.33	0.33
Ecuador	3.004***	0.259	Autocorrelation=0.58; Heteroskedasticity=0.91; Normality=0.26; Ramsey RESET test=0.55	-0.01
Mexico	2.508***	2.502***	Autocorrelation=0.99; Heteroskedasticity=0.62; Normality=0.80; Ramsey RESET test=0.39	0.41
Nicaragua[d]	2.301***	0.792***	Autocorrelation=0.11; Heteroskedasticity=0.48; Normality=0.21; Ramsey RESET test=0.03	0.62
Paraguay	3.049***	1.748***	Autocorrelation=0.30; Heteroskedasticity=0.26; Normality=0.06; Ramsey RESET test=0.13	0.40
Peru	3.262***	2.025***	Autocorrelation=0.10; Heteroskedasticity=0.12; Normality=0.17; Ramsey RESET test=0.25	0.27
Uruguay	2.259***	2.145***	Autocorrelation=0.24; Heteroskedasticity=0.76; Normality=0.75; Ramsey RESET test=0.26	0.51
Venezuela	2.456***	2.607***	Autocorrelation=0.47; Heteroskedasticity=0.60; Normality=0.04; Ramsey RESET test=0.91	0.62

[a] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[b] Only *p*-values are reported for each correct specification test.

[c] We do not report the standard correct specification tests since we used the Cochrane-Orcutt estimator to deal with autocorrelation problems.

[d] A dummy variable was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

From Table 2.1 it is possible to say that, with the exceptions of Chile that presents problems of normality and incorrect functional form and of Costa Rica that presents problems of normality, equation (2.1) satisfies the correct specification tests –no autocorrelation, no heteroskedasticity, normality and correct functional form– at the 10% level of significance in all countries. However, it was necessary to employ the Cochrane-Orcutt estimator to deal with autocorrelation problems in the case of Bolivia.[34]

Thus, the estimated g_n for the different countries can be retrieved from the intercept terms in Table 2.1 since the latter was found to be statistically significant in all countries. In turn, the coefficient β seems to be statistically significant in all countries except for the cases of Bolivia and Ecuador.

As regards the estimation of g_n using the panel model econometric techniques described in Section 2.3.1, we first implemented the standard MG estimator (see Table 2A.1 in the Appendix) with and without country-specific time trends. We followed Hamilton's (1991) procedure in order to calculate the standard errors and parameter estimates, so that the MG estimation attributes less weight to outliers. It is important to note that, with the exception of Bolivia, these results correspond to the OLS estimation presented in Table 2.1; however, the MG estimation seems to present autocorrelation problems (see Table 2A.1 in the Appendix for a description of the results).

Furthermore, the existence of 5 out of 13 significant country-specific time trends (found in Bolivia, Costa Rica, Ecuador, Nicaragua and Peru) may indicate the presence of common factors and therefore of CD. Indeed, the CD statistic depicted in equation (2.4) associated with the MG estimation without the country-specific time trends is 10.93 (p-value=0.00); whereas the one for the MG estimation with country-specific time trends is 8.99 (p-value=0.00). The latter means that the CD test strongly rejects the null hypothesis of cross-

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[34] It should be pointed out that although it may be possible to use Feasible Generalized Least Squares (FGLS) –such as the Cochrane-Orcutt estimator– to deal with autocorrelation problems, this diagnosis may reflect misspecification of the model's dynamics or omission of one or more key factors from the model (Baum, 2006).

section independence, and that we require the implementation of panel estimators robust to the presence of CD such as the CCEMG and the AMG estimators.

The CCEMG and the CCEP estimators are respectively presented in Tables 2A.2 and 2A.3 of the Appendix.[35] Again, standard errors and parameter estimates for the case of the CCEMG estimator were computed via the outlier-robust method proposed by Hamilton (1991); whereas for the CCEP estimation we used bootstrapped standard errors (with 2000 replications). The results of both estimators do not seem to present autocorrelation problems at the 10% level of significance.

From Tables 2A.2 and 2A.3 it is possible to observe that the constant term –that is, the estimated g_n– turns out to be statistically non-significant in the CCEP estimation (see Table 2A.3); whereas it is statistically significant in only 5 out of 13 countries (Brazil, Colombia, Costa Rica, Ecuador and Uruguay) for the case of the CCEMG estimation (see Table 2A.2). The introduction of a country-specific time trend does not change these results since it turned out to be statistically non-significant for the majority of the countries (see Tables 2A.2 and 2A.3 for a brief description). One possible explanation of why the estimated g_n turns out to be statistically non-significant when the CCE methodology is employed may be that the latter approach uses a high number of degrees of freedom since for q regressors requires $q+1$ cross-sectional averages on the right-hand-side. Indeed, as Eberhardt (2012) has mentioned, both the CCEMG and the AMG estimators have been designed for "moderate-T, moderate-N" macro panels, where "moderate" means about 15 time-series/cross-section observations. Thus, since in our case we have a relatively short sample, *a priori* we can expect that the

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[35] In theory, CCEP estimation should yield biased results if there exists slope heterogeneity, but Pesaran (2006) has shown that the latter does not affect the performance of the CCEP estimator in small samples.

CCEMG and CCEP estimators are less efficient compared to the AMG estimator, which in turn means that the former estimators generate fewer significant estimates.

The results of the AMG estimation can be found in Tables 2.2 and 2.3. Once again, the parameter estimates and standard errors were computed following Hamilton's (1991) outlier-robust methodology and we estimated the models with and without country-specific time trends. The latter turned out to be statistically significant in only 3 out of 13 countries and therefore we only report the AMG results without the country specific time trend (although see footnote b in Tables 2.2 and 2.3 for a brief description of the results obtained using country specific time trends).

Table 2.2 presents the results of the estimation of equation (2.10a) –that is, including the estimated common dynamic process as an additional regressor; whereas Table 2.3 presents the results of the estimation of equation (2.10b) –that is, imposing the common dynamic process with unit coefficient:

Table 2.2. Equation (2.10a) using AMG estimation:

$$\boldsymbol{g_{it} = \alpha_i{}' - \beta_i{}'(\Delta\%U_{it}) + d_i\hat{\mu}_t + \varepsilon_{it}}^{\text{a,b,c}}$$

Country	$\boldsymbol{\alpha_i{}'}$	$\boldsymbol{\beta_i{}'}$	$\boldsymbol{d_i}$
Argentina	2.333***	1.164***	1.788***
Bolivia	2.376***	0.279**	0.701***
Brazil	2.691***	2.448***	0.543***
Chile	4.327***	0.776***	1.056***
Colombia	3.364***	0.551***	0.550***
Costa Rica	3.918***	1.389***	0.645***
Ecuador	2.752***	0.158	0.680***
Mexico	2.303***	2.317***	0.534***
Nicaragua	1.632***	0.764***	0.527*
Paraguay	2.719***	1.147***	0.798***
Peru	2.671***	1.443***	1.532***
Uruguay	1.728***	0.852**	1.552***
Venezuela	2.168***	2.144***	0.669*
Average	2.564***	1.123***	0.663***

[a] All the parameter estimates and the standard errors were computed using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but since it turned out to be significant in only 3 out of 13 cases (Bolivia, Chile and Nicaragua at the 10% level of significance) the results obtained from this specification are not reported.

[c] The *p*-values associated with the correct specification tests in this case are: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1)=0.03; and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality)=0.00. We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 2.3. Equation (2.10b) using AMG estimation:
$\boldsymbol{g_{it} - \hat{\mu}_t = \alpha_i'' - \beta_i''(\Delta\%U_{it}) + \varepsilon_{it}'}$ [a,b,c]

Country	α_i''	β_i''
Argentina	2.664***	1.379***
Bolivia	2.274***	0.206
Brazil	2.495***	2.235***
Chile	4.346***	0.790***
Colombia	3.191***	0.487***
Costa Rica	3.775***	1.235***
Ecuador	2.634***	0.110
Mexico	2.125***	2.156***
Nicaragua	1.453**	0.722**
Paraguay	2.635***	0.996***
Peru	2.876***	1.645***
Uruguay	1.917***	1.312***
Venezuela	2.025***	1.915***
Average	2.591***	1.166***

[a] All the parameter estimates and the standard errors were computed computing using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but since it turned out to be significant in only 4 out of 13 cases (Bolivia, Brazil, Chile, Colombia at the 10% level of significance) the results obtained from this specification are not reported.

[c] The *p*-values associated with the correct specification tests in this case are: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1)=0.01; and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality)=0.00. We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

The results presented in Tables 2.2 and 2.3 do not present problems of autocorrelation at the 10% level of significance. In both cases the intercept term –or estimated g_n– turns out to be statistically significant at the 1% level of significance for all countries; whereas the coefficients on $\Delta\%U_{it}$ are found to be statistically significant for all countries except in Ecuador when the common dynamic process is used as additional regressor (Table 2.2) and in Bolivia and Ecuador when the common dynamic process is imposed with unit coefficient (Table 2.3). These results corroborate the idea previously mentioned before: the AMG estimator is more parsimonious than the CCE estimation since the former procedure uses up fewer degrees of freedom than the latter methodology.

Finally, Table 2.4 presents the estimation of g_n for our sample of Latin American countries using the penalized regression spline approach described in Section 2.3.2. This estimation was carried out using the **mgcv** package of the (public domain) statistical software environment **R** with default settings. The results in Table 2.4 satisfy all the correct specification tests at the 10% level of significance with the exceptions of Chile and Costa Rica that present problems of normality; and of Chile, Colombia and Nicaragua that present problems of incorrect functional form according to the RESET test. It is also possible to see that the estimated g_n for all countries is statistically significant at the 5% level of significance and that the edf of the smooth terms are statistically significant above 1 in all cases except for Costa Rica, Ecuador and Uruguay, which in turn means that the parameter β_t is statistically time-variant in all countries except in the latter three.

The results of the penalized regression approach shown in Table 2.4 also allow us to calculate the time-varying evolution of β_t –that is, Okun’s coefficient– for each of the 13 Latin American countries. The time-varying Okun coefficients are shown in Figure 2.1 below, where is possible to observe that the countries that exhibit a higher volatility of β_t are Paraguay, Bolivia, Argentina and Brazil.

Table 2.4. Equation (2.11) using the penalized regression spline approach:
$$\boldsymbol{g_t = \alpha^* - \beta_t(\Delta\%U_t) + \varepsilon_4}$$

Country	$\boldsymbol{\alpha^*}$	$\boldsymbol{\beta_t}$[a]	**Correct specification tests**[b,c]	**Adjusted** $\mathbf{R^2}$
Argentina	2.706**	3.339***	Autocorrelation=0.03; Heteroskedasticity=0.77; Normality=0.21; RESET test=0.61	0.36
Bolivia	3.082***	3.553***	Autocorrelation=0.04; Heteroskedasticity=0.01; Normality=0.23; RESET test=0.16	0.43
Brazil	2.926***	2.746***	Autocorrelation=0.05; Heteroskedasticity=0.02; Normality=0.50; RESET test=0.66	0.57
Chile	4.681***	2***	Autocorrelation=0.06; Heteroskedasticity=0.05; Normality=0.00; RESET test=0.00	0.23
Colombia	3.628***	2.482**	Autocorrelation=0.11; Heteroskedasticity=0.05; Normality=0.90; RESET test=0.00	0.23
Costa Rica	4.186***	1	Autocorrelation=0.31; Heteroskedasticity=0.42; Normality=0.00; RESET test=0.24	0.30
Ecuador	2.832***	2	Autocorrelation=0.31; Heteroskedasticity=0.86; Normality=0.69; RESET test=0.66	0.01
Mexico	2.581***	2.094***	Autocorrelation=0.77; Heteroskedasticity=0.34; Normality=0.99; RESET test=0.52	0.41
Nicaragua[d]	2.348***	2***	Autocorrelation=0.69; Heteroskedasticity=0.26; Normality=0.23; RESET test=0.00	0.63
Paraguay	3.104***	6.535***	Autocorrelation=0.36; Heteroskedasticity=0.08; Normality=0.53; RESET test=0.20	0.72
Peru	3.312***	2***	Autocorrelation=0.47; Heteroskedasticity=0.07; Normality=0.23; RESET test=0.49	0.28
Uruguay	2.271***	1.229	Autocorrelation=0.26; Heteroskedasticity=0.91; Normality=0.74; RESET test=0.20	0.50
Venezuela	2.191***	2.098***	Autocorrelation=0.47; Heteroskedasticity=0.75; Normality=0.11; RESET test=0.91	0.64

[a] The estimated degrees of freedom (edf) of the smooth terms are shown.

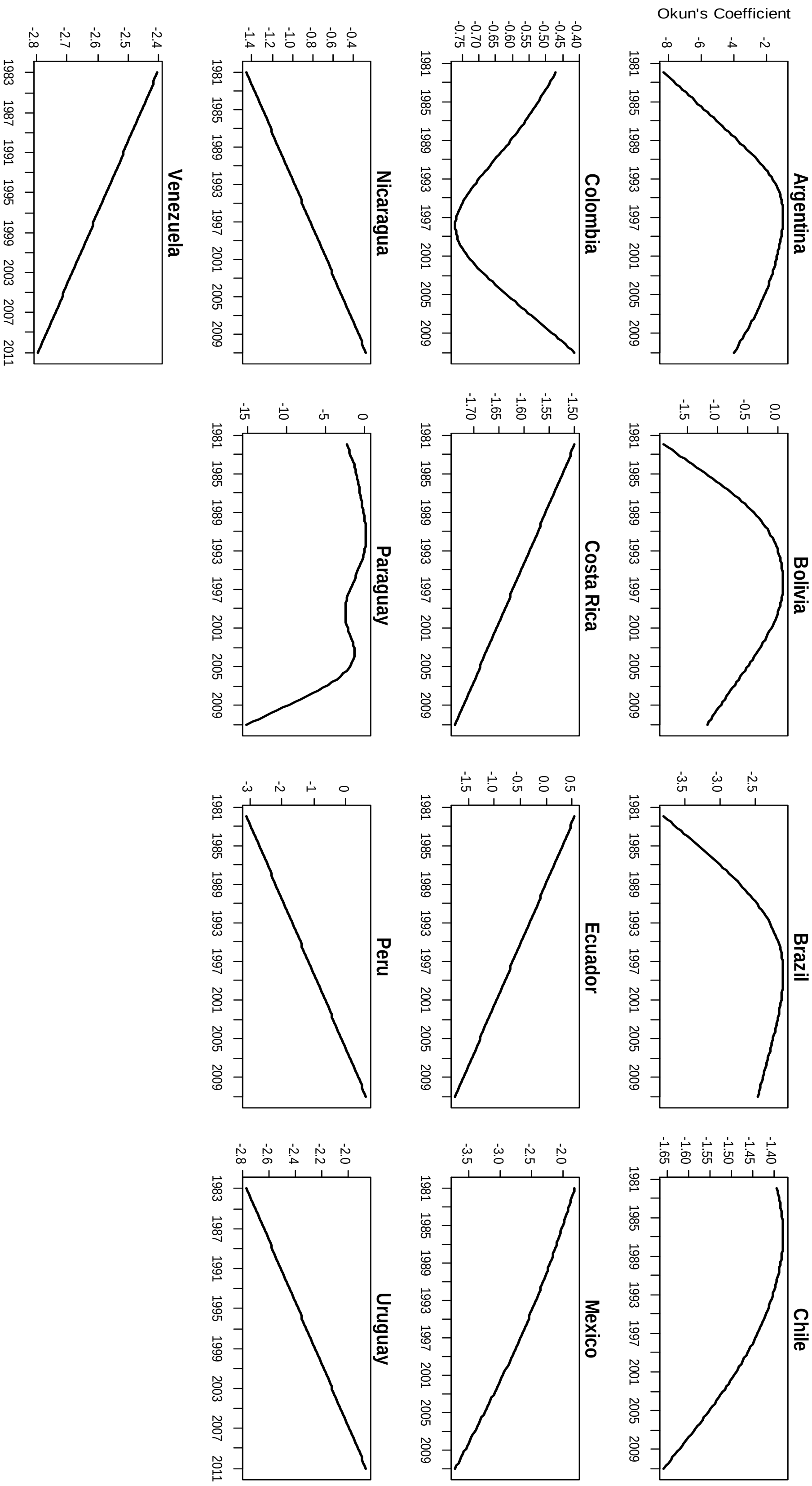
[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation of order 1 (Ho: no autocorrelation); b) Breusch-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera test for normality (Ho: residuals are normally distributed). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[c] Only *p*-values are reported for each correct specification test.

[d] A dummy variable was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Figure 2.1. Time-varying Okun coefficients of Latin American countries, 1981-2011



2.4.2 Dynamic difference version of Okun's law

OLS and SUR estimations of g_n can be found respectively in Tables 2.5 and 2.6. For both cases we started off with equation (2.2) as the initial general model and then we reduced it in complexity by eliminating statistically non-significant variables, so that we explicitly adopted the general-to-specific modeling approach in order to calculate g_n.[36] The correct specification tests and the R^{2}s of the OLS and the SUR-GLS estimations are presented in Table 2.7.

Table 2.5. Final models derived from equation (2.2) using OLS:

$$\boldsymbol{g_t = \alpha_0 + \alpha_1(g_{t-1}) + \alpha_2(g_{t-2}) + \beta_0(\Delta\%U_t) + \beta_1(\Delta\%U_{t-1}) + \beta_2(\Delta\%U_{t-2}) + \varepsilon_2}^{\mathrm{a}}$$

Country	$\boldsymbol{\alpha_0}$	$\boldsymbol{\alpha_1}$	$\boldsymbol{\alpha_2}$	$\boldsymbol{\beta_0}$	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$
Argentina	2.205**	0.402***	/	-1.686***	/	/
Bolivia	0.891*	0.720***	/	/	/	/
Brazil	2.484***	0.203*	/	-2.540***	/	/
Chile	2.777***	0.376**	/	-1.356***	-0.668**	/
Colombia	2.548***	0.317**	/	-0.689***	/	/
Costa Rica	2.798***	0.354**	/	-1.546***	/	/
Ecuador	2.958***	/	/	-0.490*	-0.754**	/
Mexico	2.508***	/	/	-2.502***	/	/
Nicaragua[b]	1.757***	0.263**	/	-0.607***	/	/
Paraguay	3.049***	/	/	-1.748***	/	/
Peru	2.179**	0.312**	/	-1.833***	/	/
Uruguay	1.673**	/	0.328**	-2.357***	/	/
Venezuela	2.456***	/	/	-2.607***	/	/

[a]For each country we only show the coefficients that were found to be statistically significant at the conventional levels.

[b]A dummy variable was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

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[36] As Campos et al. (2005: 3) explain, "[i]n general-to-specific modeling, empirical analysis starts with a general statistical model that captures the essential characteristics of the underlying dataset, *i.e.*, that the model is congruent. Then, that general model is reduced in complexity by eliminating statistically insignificant variables, checking the validity of the reductions at every stage to ensure congruence of the finally selected model." There are many reasons for adopting a general-to-specific approach. Following Campos et al. (2005) we can mention two: the fact that general-to-specific modeling implements the theory of reduction in an empirical context and that general-to-specific modeling has excellent characteristics for model selection as documented in Monte Carlo studies of automatic general-to-specific modeling algorithms.

Table 2.6. Final models derived from equation (2.2) using SUR-GLS:

$\boldsymbol{g_t = \alpha_0 + \alpha_1(g_{t-1}) + \alpha_2(g_{t-2}) + \beta_0(\Delta\%U_t) + \beta_1(\Delta\%U_{t-1}) + \beta_2(\Delta\%U_{t-2}) + \varepsilon_2}$[a]

Country	α_0	α_1	α_2	β_0	β_1	β_2
Argentina	2.599***	0.314***	/	-1.519***	/	/
Bolivia	1.257***	0.623***	/	-0.200**	/	/
Brazil	2.863***	/	0.120*	-2.830***	/	/
Chile	3.008***	0.383***	/	-0.984***	-0.657***	/
Colombia	3.748***	/	/	-0.810***	/	-0.272***
Costa Rica	4.159***	0.288***	-0.191**	-1.193***	/	/
Ecuador	3.073***	/	/	-0.355*	-0.863***	/
Mexico	2.441***	/	/	-2.523***	-0.606*	/
Nicaragua[b]	1.736***	0.353***	/	-0.540***	/	
Paraguay	3.121***	/	/	-1.978***	-0.630**	-0.514**
Peru	2.419***	0.299***	/	-1.746***	/	0.780**
Uruguay	2.044***	/	0.233***	-1.467***	-0.818***	/
Venezuela	2.452***	/	/	-2.634***	/	0.906***

[a] For each country we only show the coefficients that were found to be statistically significant at the conventional levels.

[b] A dummy variable was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 2.7. Correct specification tests and R^2 obtained from equation (2.2) using OLS and SUR-GLS[a]

	OLS estimation (Table 2.5)		SUR-GLS estimation (Table 2.6)	
Country	**Correct specification tests[b]**	**Adjusted R^2**	**Correct specification tests[c]**	**“R^2”**
Argentina	Autocorrelation=0.87; Heteroskedasticity=0.25; Normality=0.77; Ramsey RESET test=0.26	0.37	Autocorrelation=0.63; Heteroskedasticity=0.20; Normality=0.67	0.39
Bolivia	Autocorrelation=0.25; Heteroskedasticity=0.11; Normality=0.04; Ramsey RESET test=0.10	0.50	Autocorrelation=0.62; Heteroskedasticity=0.13; Normality=0.07	0.57
Brazil	Autocorrelation=0.59; Heteroskedasticity=0.30; Normality=0.09; Ramsey RESET test=0.67	0.52	Autocorrelation=0.29; Heteroskedasticity=0.37; Normality=0.67	0.55
Chile	Autocorrelation=0.43; Heteroskedasticity=0.57; Normality=0.91; Ramsey RESET test=0.06	0.59	Autocorrelation=0.92; Heteroskedasticity=0.99; Normality=0.88	0.43
Colombia	Autocorrelation=0.21; Heteroskedasticity=0.52; Normality=0.92; Ramsey RESET test=0.02	0.32	Autocorrelation=0.55; Heteroskedasticity=0.25; Normality=0.82	0.30
Costa Rica	Autocorrelation=0.51; Heteroskedasticity=0.46; Normality=0.03; Ramsey RESET test=0.02	0.38	Autocorrelation=0.68; Heteroskedasticity=0.14; Normality=0.75	0.42
Ecuador	Autocorrelation=0.56; Heteroskedasticity=0.26; Normality=0.42; Ramsey RESET test=0.59	0.17	Autocorrelation=0.50; Heteroskedasticity=0.07; Normality=0.85	0.21
Mexico	Autocorrelation=0.99; Heteroskedasticity=0.62; Normality=0.80; Ramsey RESET test=0.39	0.41	Autocorrelation=0.52; Heteroskedasticity=0.16; Normality=0.53	0.45
Nicaragua	Autocorrelation=0.30; Heteroskedasticity=0.28; Normality=0.07; Ramsey RESET test=0.29	0.67	Autocorrelation=0.60; Heteroskedasticity=0.44; Normality=0.14	0.73
Paraguay	Autocorrelation=0.30; Heteroskedasticity=0.26; Normality=0.06; Ramsey RESET test=0.13	0.40	Autocorrelation=0.70; Heteroskedasticity=0.91; Normality=0.00	0.41
Peru	Autocorrelation=0.50; Heteroskedasticity=0.02; Normality=0.72; Ramsey RESET test=0.84	0.34	Autocorrelation=0.99; Heteroskedasticity=0.01; Normality=0.91	0.44
Uruguay	Autocorrelation=0.80; Heteroskedasticity=0.63; Normality=0.15; Ramsey RESET test=0.10	0.51	Autocorrelation=0.56; Heteroskedasticity=0.56; Normality=0.58	0.55
Venezuela	Autocorrelation=0.47; Heteroskedasticity=0.60; Normality=0.04; Ramsey RESET test=0.91	0.62	Autocorrelation=0.83; Heteroskedasticity=0.33; Normality=0.75	0.62

[a] Only *p*-values are reported for each correct specification test.

[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[c] The following tests were used: a) Harvey LM test for autocorrelation (Ho: no autocorrelation); b) Hall-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera LM test of multivariate normality (Ho: normality). Breusch-Pagan test of independence of residuals: $\chi^2(78)=145.27$; *p*-value=0.00.

Regarding the OLS results (Table 2.5) we can see that the respective final models derived from equation (2.2) satisfy the correct specification tests at the 10% level of significance in all countries and that, in general, the final specifications present higher levels of adjusted R^2 compared to the OLS results of equation (2.1) (see Table 2.7). It is interesting to note that for all Latin American countries the coefficient β_2 in equation (2.2) was found to be statistically non-significant; whereas the coefficient β_1 is statistically significant for the cases of Chile and Ecuador, which indicates that the first lag of the change in the unemployment rate contains relevant information for these two countries. In turn, the parameter β_0 was found to be statistically significant in all countries except for the case of Bolivia.

Finally, it should also be pointed out that for the cases of Mexico, Paraguay and Venezuela the application of the general-to-specific modelling approach yields the model specified in equation (2.1). Thus, for these countries the estimated average g_n that is retrieved from equations (2.1) and (2.2) is exactly the same in both cases.

With respect to the SUR-GLS estimation, a 13-equation system is formed by stacking the equations associated with each of the 13 countries. Regarding the final models (Table 2.6), it is possible to see that according to the Breusch-Pagan LM statistic test the estimation of equation (2.2) via SUR yielded a significant gain in efficiency since we reject the null hypothesis of independence of the residual series at the 1% level of significance (see footnote c in Table 2.7). Moreover, from Table 2.7 it is possible to see that these estimations satisfy the correct specification tests of autocorrelation, heteroskedasticity and normality –at the 10% level of significance, except Paraguay that presents problems of normality. It is also possible to observe that, compared to the respective final models obtained via OLS, the use of the general-to-specific modeling approach in the SUR estimation yields the same model for the cases of Argentina, Chile, Ecuador and Nicaragua. In contrast to the OLS estimation, the SUR-GLS estimator seems to find that the coefficient β_0 is a relevant parameter in Bolivia; that the second lag of the output growth rate contains relevant information for the cases of Brazil and Costa Rica; that the parameter associated with the second lag of the change in unemployment rate (that is, β_2 in equation (2.2)) is statistically significant for the cases of Colombia, Paraguay, Peru and Venezuela; and that the parameter β_1 is statistically significant for the cases of Mexico, Paraguay and Uruguay.

2.4.3 Instrumental variable estimation

León-Ledesma and Thirlwall (2002b) point out that one problem with the estimation of g_n following the method suggested by Thirlwall (1969) is that $\Delta\%U_t$ should really be regarded as an endogenous variable, that is to say, $\Delta\%U_t$ is a function of g_t. The fact that the regressor $\Delta\%U_t$ is contemporaneously correlated with the error term (that is, a violation of the zero-covariance condition: $Cov(\Delta\%U_t, \varepsilon_1) \neq 0$ in equation (2.1) and $Cov(\Delta\%U_t, \varepsilon_2) \neq 0$ in equation (2.2)) means that the zero-conditional mean assumption is not satisfied (that is, $E[\varepsilon_1|\Delta\%U_t] \neq 0$ and $E[\varepsilon_2|\Delta\%U_t] \neq 0$). If the zero-conditional mean assumption is violated then the coefficient estimates will be inconsistent.

In the present chapter we have dealt with the endogeneity bias of $\Delta\%U_t$ only for the case of the OLS estimation results.[37] This means that we have assumed, possibly incorrectly, that the lags of g_t that were incorporated in equation (2.2) are exogenous regressors.

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[37] Given that both the panel estimators and the penalized regression spline estimator employed are very recent econometric techniques, the use of IV methods in these estimators has not been developed.

IV estimation methods can produce consistent estimators in a situation in which a regressor is contemporaneously correlated with the error term, assuming that the instruments satisfy simultaneously the conditions of relevance –that is, instruments are correlated with the original endogenous regressor– and exogeneity –that is, instruments are uncorrelated with the disturbance term. However, notwithstanding that the IV method generates estimates that are consistent, these are always less efficient compared to the ones generated using OLS. In other words, the property of consistency of the IV estimator is achieved at the cost of a loss in efficiency since the asymptotic variance of the latter is always larger than the asymptotic variance of the OLS estimator (Baum, 2006). The loss of efficiency is a price worth paying if the OLS estimator is biased and inconsistent, but it is important to keep in mind that turning to IV estimation for the sake of consistency must always be balanced against the inevitable loss in efficiency (Baum, 2006).[38]

Thus, the IV estimation method was performed as follows. For each individual country we re-estimated both equation (2.1) and the respective final specifications that resulted from the application of the general-to-specific modelling approach to equation (2.2) (shown in Table 2.5) using the **ivreg2** command in Stata 13 with different combinations of the lags of $\Delta\%U_t$ as instruments[39], and using the **endog** option which allows us perform a C-statistic type test of endogeneity (Hayashi, 2000).[40] Under conditional homoskedasticity this test is

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[38] Baum (2006) mentions that if the zero-conditional-mean assumption cannot be refuted we should use OLS rather than IV, especially in small samples.

[39] It may be possible to use as instruments the lagged values of the independent variable in question since they are usually correlated with the original independent variable and, because they are lagged, they are not contemporaneously correlated with the disturbance term (Kennedy, 2003).

[40] Like the C-statistic, the endogeneity test in Stata 13 is defined as the difference of two Sargan-Hansen statistics: one for the equation with the smaller set of instruments, where $\Delta\%U_t$ is treated as endogenous, and one for the equation with the larger set of instruments, where $\Delta\%U_t$ is treated as exogenous (Baum, 2006).

numerically equal to a Hausman (1978) test statistic (see Hayashi, 2000)[41], and therefore it can be used to determine if the regressor in the model ($\Delta\%U_t$) is in fact exogenous since, as discussed in Baum (2006), the Hausman test can be considered as a test of the appropriateness of OLS and the necessity to resort to IV.[42]

With respect to the simple difference version of Okun's law –that is, equation (2.1)– the null hypothesis of the endogeneity test was rejected for the cases of Bolivia, Chile, Colombia, Ecuador and Paraguay; whereas for the case of the dynamic version of Okun's law –that is, equation (2.2)– the null hypothesis was rejected only for the cases of Chile and Paraguay.[43,44] The latter means that only for these countries it would be more appropriate to estimate the g_n from the IV coefficient estimates instead of the OLS results.

However, all our IV results using different combinations of the lags of $\Delta\%U_t$ are subject to the problem of weak identification.[45] If instruments are only marginally relevant or "weak", then the first-order asymptotics can be a poor guide to the actual sampling distributions of conventional Two-stage Least Squares (henceforth 2SLS) regression statistics (Stock and Yogo, 2005).

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[41] Like the Hausman test, this C-test of endogeneity is formed by choosing OLS as the efficient estimator and the IV estimator as the inefficient but consistent estimator. Thus, "[t]he test is perhaps best interpreted not as a test for the endogeneity or exogeneity of regressors per se but rather as a test of the consequence of using different estimation methods on the same equation. Under the null hypothesis that OLS is an appropriate estimation technique, only efficiency should be lost by turning to IV" (Baum, 2006, p. 212).

[42] However, one needs to be aware of the power of the Hausman test since the latter implicitly assumes that the instruments are valid. If instruments are weak (which seems to be the case), then the test statistic could also be misleading.

[43] These results are not reported here in order to present only the most relevant results.

[44] The case of Paraguay was obvious since the application of the general-to-specific modelling approach to equation (2) led us to the conclusion that the relevant model was precisely the simple difference version of Okun's law.

[45] Weak identification arises when the excluded instruments are correlated with the endogenous regressors, but only "weakly" (see Stock and Yogo, 2005).

Furthermore, estimators can perform poorly when instruments are “weak”, and different estimators have different properties when this situation occurs (Stock and Yogo, 2005). Specifically, according to Stock and Yogo (2005) the limited-information maximum likelihood (henceforth LIML) estimator is superior to 2SLS when the researcher has weak instruments –at least from the perspective of coverage rates; and, similarly, the Fuller-*k* estimator is more robust to weak instruments than IV/2SLS –when viewed from the perspective of bias. Finally, it also seems to be that Monte Carlo simulations report substantial reductions in bias and mean squared error using Fuller-*k* estimators relative to 2SLS and LIML (Stock et al., 2002).

Because of the latter, the estimated g_n for all countries was obtained from the Fuller-*k* coefficient estimates shown in Tables 2.8 and 2.9. The former shows the IV estimation of the relevant cases as regards the simple difference version of Okun’s law; whereas the latter presents the results for Chile regarding the dynamic difference version of Okun’s law:

Table 2.8. Equation (2.1) using the Fuller-k estimator for the relevant cases:
$\boldsymbol{g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1}$[a]

Country	α	β	Correct specification tests[b,c]	Overidentification test of all instruments[c,d]	Cragg-Donald Wald F-statistic[e]
Bolivia[f]	2.820***	0.918*	Autocorrelation=0.65; Heteroskedasticity=0.96; Normality=0.02; RESET test=0.61	0.72	1.53
Chile[g]	6.367***	-2.403	Autocorrelation=0.35; Heteroskedasticity=0.92; Normality=0.00; RESET test=0.09	0.81	1.47
Colombia[f]	3.725***	1.832	Autocorrelation=0.49; Heteroskedasticity=0.70; Normality=0.87; RESET test=0.89	0.77	0.40
Ecuador[g]	3.219***	-1.258	Autocorrelation=0.55; Heteroskedasticity=0.61; Normality=0.19; RESET test=0.14	0.72	1.49
Paraguay[g]	3.002***	-0.165	Autocorrelation=0.72; Heteroskedasticity=0.69; Normality=0.42; RESET test=0.60	0.72	1.19

[a] All results were obtained using the Fuller-k estimator setting 1 as the Fuller parameter since this Fuller-k estimator is best unbiased to second order (Stock et al., 2002).

[b] The following tests were used: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); b) Pagan-Hall heteroskedasticity test (Ho: no heteroskedasticity); c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality); and d) Ramsey/Pesaran-Taylor RESET test (Ho: there are no neglected nonlinearities).

[c] Only *p*-values are reported for each correct specification test.

[d] Anderson-Rubin statistic (Ho: instruments are exogenous).

[e] For the case of a single endogenous regressor the Cragg-Donald F-statistic is simply the first-stage F-statistic (see Stock and Yogo, 2005). When the Cragg-Donald F-statistics for the respective countries are compared with the Stock and Yogo (2005) weak identification critical values it is not possible to reject the null hypothesis that instruments are weak for all cases. Indeed, as a rule of thumb, for the case of one endogenous regressor the first-stage F-statistic needs to exceed 10 for IV inference to be reliable (see Stock et al., 2002 and Baum, 2006).

[f] The first and third lags of the change in the percentage level of unemployment were used as instruments since they seem to provide useful information according to the LM test of redundancy of specified instruments. However, we fail to reject the respective null hypothesis of the Anderson canonical correlation LM statistic (*p*-value=0.22 for the case of Bolivia and *p*-value=0.65 for the case of Colombia), suggesting that, although we have more instruments than coefficients, these instruments may be inadequate to identify the equation.

[g] The first and second lags of the change in the percentage level of unemployment were used as instruments since they seem to provide useful information according to the LM test of redundancy of specified instruments. However, we fail to reject the null hypothesis of the Anderson canonical correlation LM statistic (*p*-value=0.23 in the cases of Chile and Ecuador and *p*-value=0.30 in the case of Paraguay), suggesting that, although we have more instruments than coefficients, these instruments may be inadequate to identify the equation.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 2.9. Chile: Fuller-*k* estimator of the final model obtained from the application of the general-to-specific modelling approach to equation (2.2) using OLS:

$$\boldsymbol{g_t = \alpha_0 + \alpha_1(g_{t-1}) + \beta_0(\Delta\% U_t) + \beta_1(\Delta\% U_{t-1}) + e^{a}}$$

Country	α_0	α_1	β_0	β_1	Correct specification tests[b,c]	Over-identification test of all instruments[c,d]	Cragg-Donald Wald F-statistic[e]
Chile[f]	3.323***	0.405*	-0.058	0.218	Autocorrelation=0.94; Heteroskedasticity=0.75; Normality=0.00; Ramsey RESET test=0.86	0.60	3.37

[a] Results obtained using the Fuller-*k* estimator setting 1 as the Fuller parameter since this Fuller-*k* estimator is best unbiased to second order (Stock et al., 2002).

[b] The following tests were used: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); b) Pagan-Hall heteroskedasticity test (Ho: no heteroskedasticity); c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality); and d) Ramsey/Pesaran-Taylor RESET test (Ho: there are no neglected nonlinearities).

[c] Only *p*-values are reported for each correct specification test.

[d] Anderson-Rubin statistic (Ho: instruments are exogenous).

[e] For the case of a single endogenous regressor the Cragg-Donald F-statistic is simply the first-stage F-statistic (see Stock and Yogo, 2005). When the latter is compared with the Stock and Yogo (2005) weak identification critical values it is not possible to reject the null hypothesis that instruments are weak for all cases. Indeed, as a rule of thumb, for the case of one endogenous regressor the first-stage F-statistic needs to exceed 10 for IV inference to be reliable (see Stock et al., 2002 and Baum, 2006).

[f] The second and fourth lags of the change in the percentage level of unemployment were used as instruments since they seem to provide useful information according to the LM test of redundancy of specified instruments. Moreover, we reject the null hypothesis of the Anderson canonical correlation LM statistic at the 5% level of significance (*p*-value=0.04), suggesting that the instruments may be relevant to identify the equation.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Tables 2.8 and 2.9 show that the IV estimations satisfy all the standard statistical properties at the 10% level of significance, except the case of Chile that presents problems of normality in the estimation of both the simple and the dynamic difference versions of Okun’s law.

2.4.4 Summary of results

The estimated g_n obtained from the simple difference version of Okun’s law is straightforward since the intercept was found to be statistically significant in all countries using OLS, AMG estimation, and the penalized spline regression approach (see Tables 2.1, 2.2, 2.3 and 2.4). However, as explained in the previous section, for the cases of Bolivia, Chile, Colombia, Ecuador and Paraguay the estimated g_n was retrieved from the IV estimation instead of the OLS results (see Table 2.8).

As regards the estimated g_n retrieved from the dynamic version of Okun's law using OLS (Table 2.5) we can see that for the cases of Ecuador, Mexico, Paraguay and Venezuela the $g_n^{A^*}$ –calculated as shown in equation (2.3)– can be retrieved using solely the intercept term α_0 obtained from equation (2.2) since neither α_1 nor α_2 are statistically significant in these countries. For all the other countries it seems to be that the lags of output growth contain relevant information, and therefore the $g_n^{A^*}$ has to be calculated taking into account α_1 for the cases of Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Nicaragua, and Peru (since α_2 is statistically non-significant in these countries) and using α_2 for the case of Uruguay (since α_1 is statistically non-significant in this case). However, as explained in Section 2.4.3, for the cases of Chile and Paraguay the g_n was obtained using IV estimation.

In turn, SUR-GLS estimation (Table 2.6) also indicates that in the cases of Argentina, Bolivia, Chile, Nicaragua, and Peru the first lags of the rate of growth of output contain relevant information in order to calculate $g_n^{A^*}$; whereas the second lag of the output growth rate is significant for the cases of Brazil and Uruguay, and in Costa Rica both lags seem to be relevant variables.

Table 2.10 presents the average rate of growth and the estimated natural rates of growth obtained from the simple difference version of Okun's law and from its dynamic specification for all Latin American countries using the different estimators:

Table 2.10. Latin American countries: average rate of growth and natural rate of growth estimates, 1981-2011*

		Simple difference version of Okun's law				Dynamic version of Okun's law	
Country	$\overline{g_t}$	**OLS[a] (Table 2.1)**	**AMG (Table 2.2)**	**AMG with the common dynamic process imposed with unit coefficient (Table 2.3)**	**Penalized regression spline (Table 2.4)**	**OLS[b] (Table 2.5)**	**SUR (Table 2.6)**
Argentina	2.82	3.08	2.33	2.66	2.71	3.69	3.79
Bolivia	2.68	2.82	2.38	2.27	3.08	3.17	3.33
Brazil	2.62	2.93	2.69	2.50	2.93	3.12	3.25
Chile	4.81	6.37	4.33	4.35	4.68	5.59	4.88
Colombia	3.54	3.73	3.36	3.19	3.63	3.73	3.75
Costa Rica	4.08	4.18	3.92	3.78	4.19	4.33	4.61
Ecuador	3.02	3.22	2.75	2.63	2.83	2.96	3.07
Mexico	2.47	2.51	2.30	2.13	2.58	2.51	2.44
Nicaragua	1.82	2.30	1.63	1.45	2.35	2.38	2.68
Paraguay	2.96	3.00	2.72	2.64	3.10	3.00	3.12
Peru	3.22	3.26	2.67	2.88	3.31	3.17	3.45
Uruguay	2.35	2.26	1.73	1.92	2.27	2.49	2.67
Venezuela	2.25	2.46	2.17	2.03	2.19	2.46	2.45

*Acronyms employed: g_n: Natural rate of growth (average of the different estimates); $\overline{g_t}$: Average actual rate of growth; OLS: Ordinary Least Squares; AMG: Augmented Mean Group; and SUR: Seemingly Unrelated Regressions.

[a] Except for the cases of Bolivia, Chile, Colombia, Ecuador and Paraguay, the natural rate of growth in all countries was retrieved from the OLS estimation results (Table 2.1). The natural rate of growth in these 5 countries was calculated via the Fuller-*k* estimator setting 1 as the Fuller parameter (Table 2.8).

[b] Except for the cases of Chile and Paraguay, the natural rate of growth in all countries was retrieved from the OLS estimation results (Table 2.5). For these 2 countries the respective natural rates of growth were calculated via the Fuller-*k* coefficient estimates setting 1 as the Fuller parameter. However, since for the case of Paraguay the application of the general-to-specific modelling approach to equation (2.2) retrieved the same model depicted in equation (2.1), then the estimated natural rate of growth in this case corresponds to the one shown in Table 2.8. In turn, the natural rate of growth in Chile was retrieved from the estimation shown in Table 2.9.

From Table 2.10 it can be seen that both AMG estimations show g_ns that are below the ones obtained via OLS, SUR-GLS and the penalized regression spline approach. However, the main result obtained from the estimation of both the simple difference version of Okun's law and its dynamic specification is fairly clear: countries that have experienced the highest rates of growth during the period of study (Chile, Costa Rica and Colombia) present the highest natural rates of growth estimated via the different econometric techniques (although the estimated g_n in Argentina is slightly above the one in Colombia according to the SUR-GLS estimator); whereas countries that have experienced the lowest rates of growth (Nicaragua, Venezuela and Uruguay) present the lowest natural rates of growth estimated with the different techniques (although the estimated g_n in Mexico is the lowest one according to the SUR-GLS estimation).

Finally, in Table 2.11 we present the average natural rate of growth obtained from the different models shown in Table 2.10, together with the natural rate of growth estimates presented by Libânio (2009) and Vogel (2009). From Table 2.11 it is possible to conclude that the estimated g_ns are similar for the majority of countries.

Table 2.11. Natural rates of growth in Latin American countries: a comparison

Country	Libânio (2009)*	Vogel (2009)**	Average natural rate of growth calculated from the estimates shown in Table 2.10
Argentina	2.25	3.03	3.04
Bolivia	-[a]	3.03	2.84
Brazil	2.15	3.03	2.90
Chile	4.42	6.12	5.03
Colombia	3.34	3.82	3.57
Costa Rica	3.76	4.77	4.17
Ecuador	2.38	-[a]	2.91
Mexico	2.57	2.64	2.41
Nicaragua	-[a]	2.64	2.13
Paraguay	-[a]	2.64	2.93
Peru	2.13	5.13	3.12
Uruguay	1.81	-[a]	2.22
Venezuela	2.36	1.78	2.29

*Period: 1980-2004. Argentina and Brazil: 1980-2002.

**Period: 1986-2003 for the majority of countries. Colombia: 1979-2004 and Bolivia: 1990-2003.

[a] Countries not included in the respective studies.

2.5 Conclusions

The present Chapter has tried to estimate the natural rate of growth in 13 Latin American countries for the period 1981-2011 following the original Thirlwall's (1969) reversal estimation procedure based on the simple difference version of Okun's law and an adaptation of a dynamic specification of Okun's law proposed by Knotek (2007) that tries to capture the possibility of jobless recoveries.

As for the natural rates of growth obtained using the simple difference version of Okun's law, the current chapter has employed the following econometric techniques: Ordinary Least Squares, panel estimators that take into account parameter heterogeneity and cross-section dependence, and a penalized regression spline approach that allows us to take into account the possibility of time-varying effects in the Okun coefficient. The use of the latter approach has also allowed us to provide figures of the time-varying structure of the Okun coefficient for the sample of Latin American countries, which show that Paraguay, Bolivia, Argentina and Brazil are the countries that exhibit a higher volatility in the Okun coefficient.

On the other hand, we have employed both Ordinary Least Squares and Seemingly Unrelated Regressions in order to estimate the natural rates of growth via the dynamic version of Okun's law.

Finally, we have also tried to deal with the endogeneity bias that may exist in the OLS estimates of the natural rate of growth using different Instrumental Variable estimations.

Thus, the current chapter has retrieved 6 different estimates of the natural rate of growth for 13 Latin American countries that seem to offer a fairly homogeneous picture: during the period of 1981-2011, countries that have experienced high (low) rates of GDP growth have presented high (low) natural rates of growth. This stylized fact points towards the hypothesis of endogeneity of the natural rate of growth, which is tackled in the following Chapter.

Appendix CHAPTER 2

Table 2A.1. Equation (2.1) using Pesaran and Smith’s (1995) MG estimation:
$\boldsymbol{g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1}$[a,b,c,d]

Country	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$
Argentina	3.084***	1.651***
Bolivia	2.616**	0.450***
Brazil	2.925***	2.702***
Chile	4.697***	1.039***
Colombia	3.575***	0.630***
Costa Rica	4.178***	1.669***
Ecuador	3.004***	0.259
Mexico	2.508***	2.502***
Nicaragua	2.301***	0.792***
Paraguay	3.049***	1.748***
Peru	3.262***	2.025***
Uruguay	2.259***	2.145***
Venezuela	2.456***	2.607***
Average	2.980***	1.562***

[a] All the parameter estimates and the standard errors were computed using outlier-robust means (Hamilton, 1991). Therefore, the MG estimations here reported attribute less weight to outliers in their computation.

[b] The cross-section dependence (CD) statistic of Pesaran (2004) is 10.93, with an associated p-value of 0.00. Hence, we reject the null hypothesis of cross-section independence.

[c] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but since it turned out to be significant in only 5 out of 13 cases (Bolivia, Costa Rica, Ecuador, Nicaragua and Peru at the 10% level of significance) the results obtained from this specification are not reported. Moreover, the inclusion of the country-specific time trend does not change the main results since the cross-section dependence (CD) statistic when the time trend is included is 8.99, with an associated p-value of 0.00. Hence, we reject again the null hypothesis of cross-section independence.

[d] The *p*-values associated with the correct specification tests in this case are: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1)=0.00; and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality)=0.00. We used a robust form of the Cumby-Huizinga test since it was not possible to test for heteroskedasticity after the use of the MG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 2A.2. Equation (2.1) using Pesaran's (2006) CCEMG estimation:
$\boldsymbol{g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1}$[a,b,c]

Country	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$
Argentina	-1.809	1.070***
Bolivia	-0.027	0.334**
Brazil	1.393*	2.419***
Chile	0.908	0.892***
Colombia	2.152***	0.498***
Costa Rica	2.117**	1.412***
Ecuador	2.290**	0.070
Mexico	0.855	2.320***
Nicaragua	-1.676	0.794***
Paraguay	0.113	1.304***
Peru	-2.430	1.560***
Uruguay	-2.332**	0.857**
Venezuela	0.444	2.133***
Average	0.176	1.177***

[a] All the parameter estimates and the standard errors were computed using outlier-robust means (Hamilton, 1991). Therefore, the MG estimations here reported attribute less weight to outliers in their computation.

[b] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but since it turned out to be significant in only 3 out of 13 cases (Bolivia, Chile and Nicaragua at the 10% level of significance) the results obtained from this specification are not reported.

[c] The *p*-values associated with the correct specification tests in this case are: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1)=0.04; and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality)=0.00. We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the CCEMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 2A.3. Equation (2.1) using Pesaran's (2006) CCEP estimation:
$\boldsymbol{g_t = \alpha - \beta(\Delta\%U_t) + \varepsilon_1}$[a,b]

Parameters	CCEP coefficients
α[c]	0.000 (0.384)
β[c]	-0.874*** (0.132)

[a] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but since it turned out to statistically non-significant the results obtained from this specification are not reported.

[b] The *p*-values associated with the correct specification tests in this case are: a) Wooldridge test for autocorrelation in panel data (Ho: no first-order autocorrelation)=0.09; b) Modified Wald test for group-wise heteroskedasticity in fixed effect regression model (Ho: constant variance)=0.00; and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality)=0.00.

[c] The standard errors (shown in parenthesis) used to evaluate the statistical significance were obtained via bootstrapping (2000 replications).

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

CHAPTER 3

The Endogeneity of the Natural Rate of Growth in Latin American Countries, 1981-2011

3.1 Introduction

Following the approach developed by León-Ledesma and Thirlwall (2000; 2002a; 2002b) (henceforth LLT), the purpose of the present chapter is to test the hypothesis of endogeneity of the natural rate of growth (henceforth g_n) for the sample of 13 Latin American countries during the period 1981-2011 and to check the robustness of these estimates. In order to achieve this, we estimate both the simple and the dynamic versions of Okun's law using the four different econometric techniques that were used in the previous Chapter –Ordinary Least Squares (henceforth OLS), Augmented Mean Group (henceforth AMG) estimation, the penalized regression spline, and Seemingly Unrelated Regressions (henceforth SUR). Hence, since the LLT estimation approach proposes 2 independent tests of endogeneity, we have retrieved 12 different estimates of the g_n for each country that correspond to the low growth regime and 12 different estimates of the g_n that correspond to the high growth regime.

It may be possible to find three main contributions to the literature in the current Chapter. Firstly, this is the first time that both the AMG and a penalized regression spline modeling approach –both introduced in Chapter 2– are used to test the specifications proposed by LLT. Secondly, it is the first time that the LLT estimation approach has been tested using a dynamic specification. Finally, with respect to Latin American countries, this is also the first time that the modification to the original LLT specification proposed by Lanzafame (2010) has been used.

The rest of the chapter is organized as follows. Section 3.2 presents some theoretical reasons that try to provide an explanation of the endogeneity of the g_n, emphasizing some empirical findings for Latin American countries; Section 3.3 presents the original LLT estimation procedure along with some refinements that can be made to this approach, and introduces the models that were estimated; Section 3.4 presents the empirical results; and finally Section 3.5 presents the main conclusions.

3.2 Reasons for the endogeneity of the natural rate of growth

León-Ledesma and Thirlwall (2002a; 2002b), Lanzafame and León-Ledesma (2010) and Thirlwall (1972) explain the endogeneity of the g_n on the basis of the pro-cyclicality of labour productivity growth (henceforth τ) and labour force growth (henceforth l) with respect to the actual rate of growth (henceforth g_t).

In the first place, l is extremely elastic to trade/demand/business cycles due to: 1) the encouraged-worker effect; and 2) labour immigration. The encouraged-worker effect explains that workers in the secondary labour market –that is, the labour market consisting of high-turnover, low-pay, and usually part time and/or temporary jobs– have a tendency to move in and out of the labour force in response to the business cycle. Hence, workers have a tendency to look for jobs when they are available and to give up job search during recessions, so that when demand for labour is strong –that is, in boom periods– hours worked increase mainly because part-time workers become full-time workers and because overtime work and participation rates also increase –particularly amongst married women[46], and young and retired people.

In turn, labour immigration towards booming labour markets takes place because migration is to a great extent determined by the availability of job vacancies and wages, which in turn are highly pro-cyclical. As regards the latter, both John Cornwall's *Modern Capitalism* and Charles Kindleberger's *Europe's Postwar Growth* document the important role that immigrant labour played in Europe during the so-called "golden age" of economic growth between 1950 and 1973, in which immigration of labour from Portugal, Spain, Greece and Turkey into Germany, France, Switzerland and northern Italy took place not as a casual movement, but fuelled by an excess demand for labour in the receiving countries because the growth of demand for output was strong. Another more recent example is that in 2012 Germany experienced its highest levels of immigration since 1995 according to the German Federal Statistical Office, mainly because of the immigration from Poland, Greece, Spain and Portugal.

On the other hand, τ reacts to g_t mainly because of the different mechanisms that play a role in the Kaldor-Verdoorn law (Kaldor, 1966), which posits a positive structural or long-run relationship –*i.e.* not simply a "stylised fact"– between the rate of growth of labour

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[46] Another important determinant of the increased labour force participation by women is the added-worker effect (see, for example, Parker and Skoufias, 2004 for a study on Mexico). The latter refers to women's labour market entry in response to husband's unemployment, explaining that women's labour force participation is a countercyclical variable (see Sabarwal et al., 2010, 2011 for surveys on this). However, evidence also suggests that both labour market entry (added workers) and exit (discouraged workers) during crisis may operate simultaneously, affecting different groups of women differently. Thus, increasing labour force participation and exiting labour force do not necessarily represent competing hypotheses since they do not apply to the same sections of the population (Sabarwal et al., 2010, 2011): entry into the labour force (that is, the added-worker effect) appears to be strongest for low-income households, among women with low education, and among older women; whereas the discouraged-worker effect appears to be strongest for the more educated, younger women in the labour force.

productivity and actual growth, so that long-run labour productivity growth is a positive function of actual output growth. The Kaldor-Verdoorn law can be explained because of the existence of: 1) static and dynamic returns to scale associated with increases in the volume of output and the technical progress embodied in capital accumulation: as the size and the scope of the market increases plants become more productive through the exploitation of internal and external economies to scale; 2) macro-increasing returns in the Young (1928) sense: an initial demand expansion leads to a series of changes that propagate themselves in a cumulative way; and 3) the learning-by-doing process, which means that labour productivity is a function of cumulative output: the more output produced, the more adept labour becomes at producing it.

The Kaldor-Verdoorn law can be regarded as the key element in models of circular and cumulative causation, and the model presented by Dixon and Thirlwall (1975) has come to be regarded as the standard model within this approach (McCombie, 2002; Roberts, 2002). The Dixon-Thirlwall model considers an export-led growth economy that competes via prices –determined in turn via the application of a mark-up on unit labour costs– on international (or inter-regional) markets in the sale of a diversified variety of goods. Thus, as described by Roberts (2002) and Libânio and Moro (2011), in this model an initial growth in output induces higher productivity that allows for reductions of unit labour costs, which –given the mark-up pricing rule– generate a fall in prices, increasing the competitiveness of the country (or region). These gains, in turn, allow for further output expansion through increasing exports, which reinitiate the cycle. Therefore, once the country (or region) acquires a growth advantage, it will tend to maintain it through the process of increasing returns and the consequent competitive gains that growth itself induces.[47]

To the best of our knowledge, there are very few empirical studies that have dealt with the pro-cyclicality of l and τ in Latin American countries. Regarding the pro-cyclicality of l, Galli and Kucera (2003) found evidence of a countercyclical pattern of informal employment shares in 14 Latin American countries in the 1990s, and that employment in small firms and self-employment has acted as a reservoir for employment in large firms.

Likewise, Orrenius and Zovodny (2009) have shown that Latin American immigrants display the greatest sensitivity to the business cycle compared to Asian immigrants or Western European or Canadian immigrants. Moreover, it seems to be that fluctuations in Latin American's employment and unemployment rates are more closely tied to the business cycle than those of Asian or Western immigrants. The relatively higher vulnerability to the business cycle of Latin American immigrants has to do with its relatively lower levels of education and with unauthorized immigration[48] since the latter tends to increase the cyclicality of Latin American immigrants' employment and unemployment rates because many unauthorized immigrants enter only when they can find work.

Finally, as regards the pro-cyclicality of τ in Latin American countries, Libânio and Moro (2011) have found evidence that confirms the existence of increasing returns in the manufacturing sector and the possibility of cumulative growth cycles for the seven largest

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[47] However, there are nuances with respect to the identification of the theoretical structure underlying the Kaldor-Verdoorn's law that escape the purposes of this Chapter. For a survey and advances on this and on the empirical evidence of the Kaldor-Verdoorn law see the different works in McCombie et al. (2002).

[48] As mentioned by Orrenius and Zovodny (2009), over half of Mexican immigrants are in the United States illegally (Pew Hispanic Center, 2009), and the number of workers illegally crossing the U.S.-Mexico border changes quickly in response to shifts in employment conditions in the United States (Papademetriou and Terrazas, 2009).

economies in Latin American during 1980-2006. They employ four different panel econometric techniques (pooled OLS, fixed effects and random-effects, and the Arelano-Bond dynamic estimation), finding that the regression that considers the growth of capital stock as an exogenous variable yields estimates of returns to scale between 2.7% and 3.1%; whereas the regression that considers the capital stock as endogenous to output yields estimates of the Verdoorn coefficient between 0.42% to 0.48%, and the degree of returns to scale is around 2.3% in this case.

3.3 The León-Ledesma-Thirlwall approach and the hypothesis of endogeneity of the natural rate of growth

LLT have developed an econometric specification aimed at showing that g_n is an endogenous result of g_t in the sense that the former variable presents flexibility both in the downward and upward directions. This approach consists of two alternative econometric procedures used to calculate the sensitivity of the estimated g_n, which differ according to the way used to identify the boom periods in each economy.

Let us use the simplest case in order to describe these two procedures:

$$g_t = a_1 - c_1(\Delta\%U_t) + \varepsilon_1 \quad \dots\dots\dots\dots\dots \quad (3.1)$$

where, in addition to the variables defined in the previous chapter, in equation (3.1) we now have that $a_1 = \alpha$ and $c_1 = \beta$ (note that both α and β were the parameters employed in equation (2.1) of Chapter 2).

The first endogeneity test consists in introducing a dummy variable ($D = 1$) for the periods of growth buoyancy when $g_t > g_n = g_n^A = a_1$ and zero otherwise; whereas the second endogeneity test consists in introducing a dummy variable ($D' = 1$) that identifies the booming periods when a constructed moving average of g_t is above g_t. The latter procedure has not received enough attention in the literature; however, it is of utmost importance because: a) it is a test independent of the estimation of g_n obtained using Thirlwall's (1969) reversal and, thus, it may help to deal with the issue of second stage regressions with generated regressors pointed out by Pagan (1984); and b) it may help to capture longer-run effects associated with increasing returns that may be neglected by the first procedure.

Hence, after the introduction of the respective dummy variables, equation (3.1) is re-estimated as follows:

$$g_t = a_2 + b_2(D) - c_2(\Delta\%U_t) + \varepsilon_{1a} \quad \dots\dots\dots\dots\dots\dots \quad (3.2)$$

$$g_t = a_2' + b_2'(D') - c_2'(\Delta\%U_t) + \varepsilon_{1b} \quad \dots\dots\dots\dots\dots\dots \quad (3.3)$$

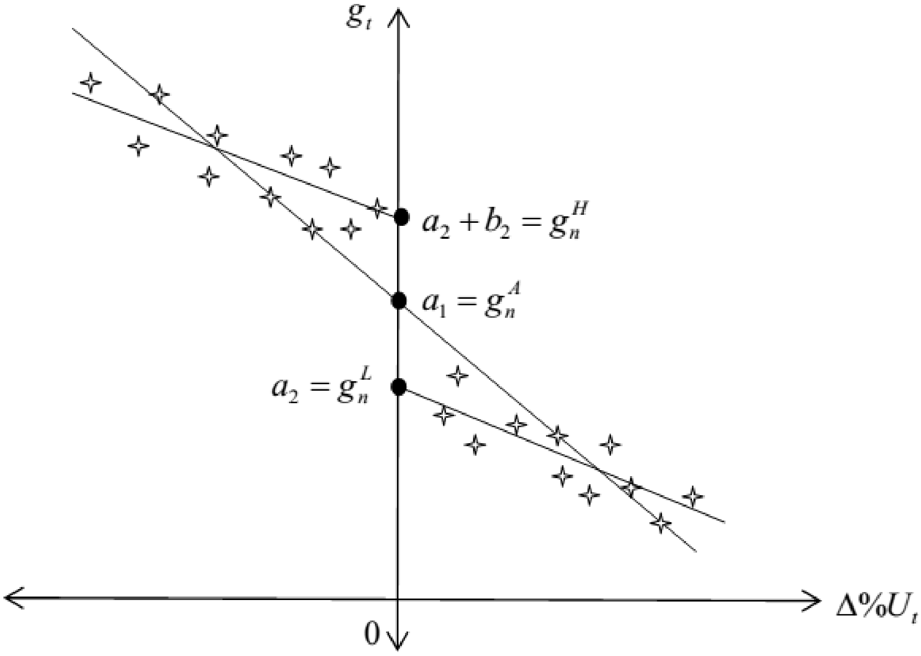
where in equations (3.2) and (3.3) ε_{1a} and ε_{1b} represent the respective stochastic disturbances.

Hence, with equations (3.2) and (3.3) it is possible to define different g_ns associated with different growth regimes due to increased/reduced labour productivity and labour force growth. Specifically, it is possible to define one g_n associated with a high growth regime (henceforth g_n^H) and one g_n associated with a low growth regime (henceforth g_n^L). According to the specification presented in equation (3.2), the former corresponds to the intercept term (a_2) plus the coefficient on the dummy (b_2), so that $g_n^H = a_2 + b_2$; whereas the latter corresponds only to the constant (a_2), so that $g_n^L = a_2$. In turn, according to the specification presented in equation (3.3), we will have that $g_n^{H\prime} = a_2' + b_2'$ and $g_n^{L\prime} = a_2'$.

If $g_n^H = a_2 + b_2$ and $g_n^{H'} = a_2' + b_2'$ are statistically significantly higher than the original $g_n^A = a_1$ in equation (3.1), then it means that the g_t raised the estimated natural rate of growth during the boom periods. In turn, if $g_n^L = a_2$ and $g_n^{L'} = a_2'$ are statistically significantly lower than the original $g_n^A = a_1$, then g_t must have pulled down the estimated g_n during the slump periods.

Figure 3.1 below tries to illustrate these ideas using the parameters in equation (3.2):

Figure 3.1. Natural rates of growth estimated via the Leon-Ledesma and Thirlwall aproach



Source: Own elaboration based on León-Ledesma and Thirlwall (2002a: 230; 2002b: 443)

Figure 3.1 slightly differs from the one presented by León-Ledesma and Thirlwall (2002b: 443) in the sense that it tries to correct a mistake regarding the continuous horizontal line associated with $g_n^A = a_1$ in that Figure, which would imply that there are changes in unemployment when the natural rate of growth remains fixed.[49] In Figure 3.1 we have that $\Delta\%U_t$ is measured on the horizontal axis and that g_t is measured on the vertical axis. Thus, the average natural rate of growth $g_n^A = a_1$ estimated via equation (3.1) is defined where $\Delta\%U_t = 0$; and the natural rates of growth in boom and depression –both estimated via equation (3.2)– are respectively defined by $g_n^H = a_2 + b_2$ and $g_n^L = a_2$.

Lanzafame (2010) has drawn attention to a possible source of bias in equations (3.2) and (3.3) due to the presence of asymmetries in Okun's law. In a nutshell, one problem of equations (3.2) and (3.3) is that both assume that in the switch between low and high growth regimes the slope coefficients (c_2 and c_2') remain unaffected, which contradicts the bulk of empirical evidence that shows the presence of an asymmetric Okun coefficient over the business cycle. Thus, Lanzafame (2010) suggests that the estimated $g_n^A = a_1$ and the moving average of g_t should also be used to construct both intercept and slope dummy variables, so that equations (3.2) and (3.3) need to be modified as follows:

$$g_t = a_3 + b_3(D) - c_3(\Delta\%U_t) + \theta_1(D * \Delta\%U_t) + \varepsilon_{1c} \quad \dots\dots\dots\dots\dots\dots \quad (3.4)$$

$$g_t = a_3{}' + b_3{}'(D') - c_3{}'(\Delta\%U_t) + \theta_1{}'(D' * \Delta\%U_t) + \varepsilon_{1d} \quad \dots\dots\dots\dots\dots\dots \quad (3.5)$$

In equations (3.4) and (3.5) we have that ε_{1c} and ε_{1d} are the error terms, and that the terms $(D * \Delta\%U_t)$ and $(D' * \Delta\%U_t)$ are the respective slope dummies on the percentage change in unemployment. If the null hypothesis that $\theta_1 = 0$ and $\theta_1' = 0$ are rejected, then this indicates the presence of a significant asymmetric Okun coefficient. However, the respective g_ns associated with the high and low growth regimes are measured as before, namely $g_n^{H*} =$

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[49] Tony Thirlwall has mentioned to me that this was a publisher's mistake.

a_3+b_3 and $g_n^{L*} = a_3$ in equation (3.4), and $g_n^{H*\prime} = a_3{}' + b_3{}'$ and $g_n^{L*\prime} = a_3{}'$ in equation (3.5).

Using the AMG estimator that was described in Chapter 2, we will have the following modifications to equations (2.10a) and (2.10b) –see Chapter 2– with respect to the first test of endogeneity of the LLT approach:

$$g_{it} = \alpha_i'' + b_{3i}'(D) - \beta_i''(\Delta\%U_{it}) + \theta_1''(D * \Delta\%U_{it}) + d_i'\hat{\mu}_t' + \varepsilon_{it}' \quad \dots\dots\dots\dots\dots \quad (3.4a)$$

$$g_{it} - \hat{\mu}_t' = \alpha_i''' + b_{3i}''(D') - \beta_i'''(\Delta\%U_{it}) + \theta_1'''(D' * \Delta\%U_{it}) + \varepsilon_{it}'' \quad \dots\dots\dots\dots\dots \quad (3.4b)$$

where equation (3.4a) is the AMG estimation that includes the common dynamic process as an additional regressor, equation (3.4b) is the AMG estimation that imposes the estimated common dynamic process with unit coefficient, and $\varepsilon_{it}{}'$ and $\varepsilon_{it}{}''$ are the error terms.

In this case the respective g_n^Hs and g_n^Ls can be retrieved as follows: $g_{n,i}^H = \alpha_i'' + b_{3i}'$ and $g_{n,i}^L = \alpha_i''$ from equation (3.4a), and $g_{n,i}^{H\prime} = \alpha_i''' + b_{3i}''$ and $g_{n,i}^{L\prime} = \alpha_i'''$ from equation (3.4b).

Regarding the second test of endogeneity using the AMG estimator we have the following equations:

$$g_{it} = \alpha_i'''' + b_{3i}'''(D) - \beta_i''''(\Delta\%U_{it}) + \theta_1''''(D * \Delta\%U_{it}) + d_i''\hat{\mu}_t'' + \varepsilon_{it}''' \quad \dots\dots\dots \quad (3.5a)$$

$$g_{it} - \hat{\mu}_t'' = \alpha_i''''' + b_{3i}''''(D') - \beta_i'''''(\Delta\%U_{it}) + \theta_1'''''(D' * \Delta\%U_{it}) + \varepsilon_{it}'''' \quad \dots\dots\dots \quad (3.5b)$$

where equations (3.5a) and (3.5b), respectively, depict the AMG estimation that includes the common dynamic process as an additional regressor and the AMG estimation that imposes the common dynamic process with unit coefficient. In this case ε_{it}''' and ε_{it}'''' are the respective error terms and the different g_n^Hs and g_n^Ls can be retrieved as follows: $g_{n,i}^{H\prime\prime} = \alpha_i'''' + b_{3i}'''$ and

$g_{n,i}^{L''} = \alpha_i''''$ from equation (3.4b), and $g_{n,i}^{H'''} = \alpha_i''''' + b_{3i}''''$ and $g_{n,i}^{L'''} = \alpha_i'''''$ from equation (3.5b).

On the other hand, the penalized regression approach described in Section 2.3.2 of Chapter 2 may also help to tackle the presence of asymmetries in Okun's law since, as explained in Chapter 2, the model depicted in equation (2.11) tries to take into account the possibility of a time-varying Okun coefficient. Thus, after the introduction of the intercept dummy variables, equation (2.11) can be re-estimated as follows:

$$g_t = \alpha^{*\prime} - \beta_t'(\Delta\%U_t) + \xi(D) + \varepsilon_{1e} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.4c)$$

$$g_t = \alpha^{*\prime\prime} - \beta_t''(\Delta\%U_t) + \xi'(D') + \varepsilon_{1f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.5c)$$

where equations (3.4c) and (3.5c) depict the first and second test of endogeneity according to the LLT approach, respectively. Likewise, the g_ns associated with the high and low growth regimes in equations (3.4c) and (3.5c) are respectively $g_n^{H1} = \alpha^{*\prime} + \xi$ and $g_n^{L1} = \alpha^{*\prime}$, and $g_n^{H1\prime} = \alpha^{*\prime\prime} + \xi'$ and $g_n^{L1\prime} = \xi'$.

Finally, following the idea of Lanzafame (2010), it may also be possible to test the hypothesis of endogeneity of g_n using the dynamic specification of Thirlwall's reversal depicted in equation (2.2) (see Chapter 2). Hence, regarding the first endogeneity test proposed by LLT, $g_n^{A^*}$ is used to build both intercept and slope dummy variables for the current and lagged values of the independent variables. Thereby, in order to test the hypothesis of endogeneity following the first method proposed by LLT, equation (2.2) can be modified as follows:

$$g_t = a_4 + a_5(g_{t-1}) + a_6(g_{t-2}) + b_4(D) + b_5(D * g_{t-1}) + b_6(D * g_{t-2}) +$$

$$c_4(\Delta\%U_t) + c_5(\Delta\%U_{t-1}) + c_6(\Delta\%U_{t-2}) + \theta_2(D * \Delta\%U_t) +$$

$$\theta_3(D * \Delta\%U_{t-1}) + \theta_4(D * \Delta\%U_{t-2}) + \varepsilon_{2a} \quad \dots\dots\dots\dots \quad (3.6a)$$

Similarly, if the moving average of g_t is used to build the dummy variables, the second endogeneity test proposed by the LLT estimation method in a dynamic context would be:

$$g_t = a_4{}' + a_5{}'(g_{t-1}) + a_6{}'(g_{t-2}) + b_4{}'(D') + b_5{}'(D' * g_{t-1}) + b_6{}'(D' * g_{t-2}) +$$

$$c_4{}'(\Delta\%U_t) + c_5{}'(\Delta\%U_{t-1}) + c_6{}'(\Delta\%U_{t-2}) + \theta_2{}'(D' * \Delta\%U_t) +$$

$$\theta_3{}'(D' * \Delta\%U_{t-1}) + \theta_4{}'(D' * \Delta\%U_{t-2}) + \varepsilon_{2b} \quad \dots\dots\dots\dots \quad (3.6b)$$

In equations (3.6a) and (3.6b) the parameters θ_2, θ_3, θ_4, $\theta_2{}'$, $\theta_3{}'$, and $\theta_4{}'$ measure the possibility of a statistically significant asymmetric Okun coefficient; whereas the terms $(D * g_{t-1})$, $(D * g_{t-2})$, $(D' * g_{t-1})$, and $(D' * g_{t-2})$ are the intercept dummies on the lagged values of the output growth rates.

Therefore, assuming that the respective parameters are found to be statistically significant, the g_ns related to the high and low growth regimes can be retrieved from equations (3.6a) and (3.6b) as follows:

$$g_n^{H*} = \frac{a_4 + b_4 + b_5 + b_6}{1 - a_5 - a_6} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (3.7a)$$

$$g_n^{L*} = \frac{a_4}{1 - a_5 - a_6} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (3.7b)$$

$$g_n^{H*\prime} = \frac{a_4{}' + b_4{}' + b_5{}' + b_6{}'}{1 - a_5{}' - a_6{}'} \qquad \dots\dots\dots\dots\dots\dots\dots\dots \qquad (3.8a)$$

$$g_n^{L*\prime} = \frac{a_4{}'}{1 - a_5{}' - a_6{}'} \qquad \dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots \qquad (3.8b)$$

where in equations (3.7a) and (3.8a) we have that g_n^{H*} and $g_n^{H*\prime}$ are the natural rates of growth associated with the high growth regime; whereas in equations (3.7b) and (3.8b) we have that g_n^{L*} and $g_n^{L*\prime}$ are the natural rates of growth associated with the low growth regime.

3.4 New empirical evidence for Latin American countries

This section presents the results of both tests of endogeneity proposed by LLT using the same econometric techniques that were employed in Chapter 2 in order to estimate g_n. Thus, the hypothesis of endogeneity of the g_n using the simple difference version of Okun's law was estimated via OLS, AMG estimation, and the penalized regression spline approach. In turn, the hypothesis of endogeneity of g_n using the dynamic difference version of Okun's law was estimated via OLS and SUR following the general-to-specific modelling approach.

The first and second tests of endogeneity of the LLT approach are respectively presented in Sections 3.4.1 and 3.4.2. Each of these Sections contains the results of the simple difference version of Okun's law (Sections 3.4.1.1 and 3.4.2.1); the results of the dynamic difference version of Okun's law (Sections 3.4.1.2 and 3.4.2.2); and a summary of results (Sections 3.4.1.3 and 3.4.2.3) in which both the different estimates of g_n –associated with the low and high growth regimes– and the elasticities with respect to the original estimates of g_n –shown in Chapter 2– are presented.

We have employed the same data set that was employed in Chapter 2.

3.4.1 First test of endogeneity: using the estimated natural rate of growth to build the dummy variables

We first tested the endogeneity of g_n using a dummy variable that adopted the value of one for years in which the actual rate of growth is above the estimated g_n and zero otherwise. The g_ns used to build the dummy variables for the different techniques can be found in the third, sixth, seventh and eighth columns of Table 2.10 in Chapter 2. For the case of the AMG estimations we used the estimated average g_n obtained for each panel: 2.56 when the common dynamic process was included as additional regressor (see Table 2.2 in Chapter 2) and 2.59 when the common dynamic process is imposed with a unit coefficient (see Table 2.3 in Chapter 2).

3.4.1.1 Simple difference version of Okun's law

OLS and AMG results of equations (3.4), (3.4a) and (3.5a) can be respectively found in Tables 3.1, 3.2 and 3.3; whereas the first test of endogeneity of the LLT approach using the penalized regression spline approach is presented in Table 3.4.

Table 3.1. Equation (3.4) using OLS:

$$\boldsymbol{g_t = a_3 + b_3(D) - c_3(\Delta\%U_t) + \theta_1(D * \Delta\%U_t) + \varepsilon_{1c}}^{\bar{a}}$$

Country	a_3	b_3	c_3	θ_1	Correct specification tests[a,b]	Adjusted R^2
Argentina	-3.213***	10.931***	0.109	-0.128	Aut=0.75; Het=0.08; Nor=0.52; RESET=0.87	0.75
Bolivia[c]	1.064*	3.388***	0.855**	0.791**	Aut=0.56; Nor=0.21; RESET=0.00	0.74
Brazil	1.023*	3.561***	1.662***	0.273	Aut=0.10; Het=0.08; Nor=0.17; RESET=0.45	0.74
Chile[d]	3.880***	4.104***	1.451***	0.927	Aut=0.49; Het=0.74; Nor=0.44; RESET=0.11	0.82
Colombia[c]	2.374***	3.174***	0.595*	0.956**	Aut=0.61; Nor=0.11; RESET=0.00	0.73
Costa Rica[e]	2.950***	3.381***	1.261***	1.069*	Aut=0.81; Het=0.26; Nor=0.33; RESET=0.22	0.81
Ecuador	0.545	4.761***	0.185	-0.210	Aut=0.11; Het=0.11; Nor=0.10; RESET=0.75	0.54
Mexico	0.228	4.439***	2.340***	2.289***	Aut=0.52; Het=0.76; Nor=0.06; RESET=0.86	0.79
Nicaragua[f]	-1.477***	5.939***	-0.303	-0.276	Aut=0.52; Het=0.46; Nor=0.44; RESET=0.71	0.90
Paraguay[g]	0.263	4.422***	1.078***	0.330	Aut=0.98; Het=0.84; Nor=0.48; RESET=0.76	0.81
Peru[c]	-0.859	7.923***	1.642*	0.881	Aut=0.26; Nor=0.11; RESET=0.06	0.70
Uruguay	-0.448	6.490***	1.944***	1.842***	Aut=0.13; Het=0.89; Nor=0.07; RESET=0.21	0.84
Venezuela	-0.579	6.035***	1.505**	0.257	Aut=0.99; Het=0.37; Nor=0.01; RESET=0.92	0.71

[ā] Acronyms employed: Aut: Autocorrelation; Het: Heteroskedasticity; Nor: Normality; and RESET: Ramsey RESET.

[a] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[b] Only p-values are reported for each correct specification test.

[c] The Huber-White-sandwich estimator was used in order to deal with heteroskedasticity problems; therefore in these cases we report both robust standard errors (in parenthesis) and the standard R^2, and we do not report any heteroskedasticity test. The autocorrelation test reported in these cases is the Cumby-Huizinga test (Ho: disturbance is a moving average of known order 1).

[d] A dummy in 1983 was included in order to deal with normality problems.

[e] A dummy in 1982 was included in order to deal with normality problems.

[f] A dummy in 1988 was included in order to deal with normality problems.

[g] A dummy in 2010 was included in order to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.2. Equation (3.4a) using AMG estimation:

$$\boldsymbol{g_{it} = \alpha_i'' + b_{3i}'(D) - \beta_i''(\Delta\%U_{it}) + \theta_1''(D * \Delta\%U_{it}) + d_i{}'\hat{\mu}_t' + \varepsilon_{it}{}'^{a,b,c}}$$

Country	α_i''	b_{3i}'	β_i''	θ_1''	$d_i{}'$
Argentina	-1.553	8.396***	0.416	0.087	1.507***
Bolivia	1.120**	3.198***	0.656***	0.612**	0.523**
Brazil	1.882**	2.435***	2.241***	0.915	0.824***
Chile	-0.030	5.882***	0.569**	-0.170	0.845*
Colombia	2.017***	2.772***	0.615***	0.575**	0. 331*
Costa Rica	0.642	4.747***	0.663	-0.083	0.826***
Ecuador	0.284	4.722***	0.032	-0.059	0.665**
Mexico	0.113	4.599***	2.385***	2.410***	-0.151
Nicaragua	-1.277	5.263***	-0.089	-0.573	0.482
Paraguay	0.339	4.652***	0.749*	0.032	0.680*
Peru	-0.333	6.882***	1.320***	0.681	1.828***
Uruguay	0.316	5.584***	1.797***	1.721***	0.582
Venezuela	-0.523	6.079***	1.039	0.030	0.993*
Average	0.217	4.983***	0.840***	0.363*	0.677***

[a] All the parameter estimates and the standard errors were computed using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also included a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but it turned out to be non-significant for all countries; hence, the results obtained from this specification are not reported.

[c] The *p*-values associated to the correct specification tests in this case are: a) Autocorrelation=0.74; and b) Normality=0.00. The test used were the following: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.3. Equation (3.4b) using AMG estimation:

$$\boldsymbol{g_{it} - \widehat{\mu}'_t = \alpha'''_i + b''_{3i}(D) - \beta'''_i(\Delta\%U_{it}) + \theta_1'''(D' * \Delta\%U_{it}) + \varepsilon_{it}''}^{\text{a,b,c}}$$

Country	$\boldsymbol{\alpha'''_i}$	$\boldsymbol{b''_{3i}}$	$\boldsymbol{\beta'''_i}$	$\boldsymbol{\theta_1'''}$
Argentina	-2.306**	9.318***	0.288	-0.063
Bolivia	1.374***	2.917***	0.515**	0.502*
Brazil	2.166***	2.149***	2.451***	1.126
Chile	0.307	5.521***	0.579**	-0.185
Colombia	2.486***	2.272***	0.583***	0. 465
Costa Rica	0. 813	4.601***	0. 695	-0.039
Ecuador	0.456	4.595***	-0.000	-0.026
Mexico	0.984	3.375***	2.041***	1.495
Nicaragua	-0.949	4.630***	-0.107	-0.837
Paraguay	0.481	4.512***	0.635*	-0.034
Peru	-0.914	7.490***	1.466***	0.706
Uruguay	0. 722	4.862***	1.653***	1.536**
Venezuela	-0.523	6.080***	1.036	0.028
Average	0.433	4.623***	0.820***	0.330

[a] All the parameter estimates and the standard errors were computed using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also included a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but it turned out to be non-significant for all countries; hence, the results obtained from this specification are not reported.

[c] The *p*-values associated to the correct specification tests in this case are: a) Autocorrelation=0.80; and b) Normality=0.00. The test used were the following: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.4. Equation (3.4c) using the penalized regression spline approach:
$\boldsymbol{g_t = \alpha^{*\prime} - \beta_t^{\prime}(\Delta\%U_t) + \xi(D) + \varepsilon_{1e}}^{\bar{a}}$

Country	$\boldsymbol{\alpha^{*\prime}}$	$\boldsymbol{\beta_t^{\prime}}^{\mathbf{a}}$	$\boldsymbol{\xi}$	**Correct specification tests**[b,c]	**Adjusted** $\mathbf{R^2}$
Argentina	-3.231***	2.488	10.901***	Aut=0.41; Het=0.34; Nor=0.52; RESET=0.93	0.78
Bolivia	1.145**	3.521**	3.324***	Aut=0.31; Het=0.02; Nor=0.02; RESET =0.02	0.73
Brazil[d]	1.097***	3.658***	3.398***	Aut=0.07; Het=0.09; Nor=0.56; RESET=0.68	0.88
Chile[e]	3.091***	2***	3.508***	Aut=0.04; Het=0.42; Nor=0.20; RESET=0.00	0.78
Colombia[f]	2.348***	2.511	3.048***	Aut=0.91; Het=0.26; Nor=0.85; RESET=0.09	0.84
Costa Rica[g]	2.447***	3.089**	3.736***	Aut=0.84; Het=0.50; Nor=0.49; RESET=0.08	0.81
Ecuador	0.331	2	4.686***	Aut=0.13; Het=0.42; Nor=0.55; RESET=0.98	0.56
Mexico	-0.183	2***	4.451***	Aut=0.98; Het=0.48; Nor=0.92; RESET=0.02	0.72
Nicaragua[h]	-1.164***	2	5.788***	Aut=0.31; Het=0.94; Nor=0.84; RESET=0.10	0.90
Paraguay	1.265***	7.217***	3.543***	Aut=0.70; Het=0.34; Nor=0.48; RESET=0.91	0.88
Peru	-1.028	2**	7.841***	Aut=0.40; Het=0.04; Nor=0.14; RESET=0.40	0.67
Uruguay	-1.198*	2***	6.899***	Aut=0.26; Het=0.72; Nor=0.24; RESET=0.01	0.81
Venezuela[i]	-1.179	2***	5.781***	Aut=0.03; Het=0.86; Nor=0.35; RESET=0.48	0.85

[ā] Acronyms employed: Aut: Autocorrelation; Het: Heteroskedasticity; Nor: Normality; and RESET: Ramsey RESET.

[a] The estimated degrees of freedom (edf) of the smooth terms are shown.

[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation of order 1 (Ho: no autocorrelation); b) Breusch-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera test for normality (Ho: residuals are normally distributed). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[c] Only *p*-values are reported for each correct specification test.

[d] A dummy in 1990 was included in order to deal with normality problems.

[e] A dummy in 1983 was included in order to deal with normality problems.

[f] A dummy in 1999 was included in order to deal with normality problems.

[g] A dummy in 1982 was included in order to deal with normality problems.

[h] A dummy in 1988 was included to order to deal with normality problems.

[i] A dummy in 2004 was included in order to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

With respect to the OLS results of equation (3.4) (Table 3.1) we can see that, with the exceptions of Bolivia and Colombia that present problems of correct functional form according to the Ramsey RESET test, all countries satisfy the standard correct specification tests at the 10% level of significance (although it was necessary to employ the Huber-White-sandwich estimator in order to deal with heteroskedasticity problems for the cases of Bolivia, Colombia and Peru). On the other hand, both AMG estimations (Tables 3.2 and 3.3) do not present problems of autocorrelation (see footnote c in both Tables); whereas the results obtained from the penalized regression spline approach (Table 3.4) satisfy the correct specification tests in all countries at the 10% level of significance, except in the case of Chile where problems of functional form seem to be present.

Both OLS and AMG results show that an asymmetric Okun coefficient may be present in the cases of Bolivia, Colombia, Mexico and Uruguay; and a statistically significant asymmetric Okun coefficient was also found in Costa Rica when OLS were employed.

Finally, Table 3.4 also shows that a time-varying Okun coefficient was found to be statistically significant in all cases except in Argentina, Colombia, Ecuador and Nicaragua.

3.4.1.2 Dynamic difference version of Okun's law

The different estimates of the g_ns used for the OLS and SUR estimations of the first endogeneity test in the dynamic context are shown in the seventh and eighth columns of Table 2.10 in Chapter 2, respectively.

The final models obtained after the application of the general-to-specific modelling approach to equation (3.6a) using OLS and SUR are presented in Tables 3.5 and 3.6, respectively. In turn, Table 3.7 presents the correct specification tests and the R^2 of both estimations.

As regards the OLS estimation results (Tables 3.5 and 3.7) it is possible to see that, with the exception of Colombia that present problems of incorrect functional form, the final models satisfy all the conventional specification tests (although it was necessary to employ the Huber-White-sandwich estimator for the cases of Bolivia and Colombia). These results also show that Bolivia, Chile, Colombia, Costa Rica, Mexico, Peru, and Uruguay may present an asymmetric Okun coefficient.

In turn, SUR estimation results (Tables 3.6 and 3.7) show that the final models satisfy the correct specification tests –except Brazil and Ecuador that present problems of normality; that once again this estimation gains in efficiency according to the Breusch-Pagan LM statistic test (we reject the null hypothesis of independence of the residual series at the 5% level of significance); and that an asymmetric Okun coefficient may exist in all cases of our sample of 13 Latin American countries except in Brazil.

Table 3.5. Final model derived from equation (3.6a) using OLS:

$$g_t = a_4 + a_5(g_{t-1}) + a_6(g_{t-2}) + b_4(D) + b_5(D * g_{t-1}) + b_6(D * g_{t-2}) + c_4(\Delta\%U_t) + c_5(\Delta\%U_{t-1}) + c_6(\Delta\%U_{t-2}) + \theta_2(D * \Delta\%U_t) + \theta_3(D * \Delta\%U_{t-1}) + \theta_4(D * \Delta\%U_{t-2}) + \varepsilon_{2a}{}^{a}$$

Country	a_4	a_5	a_6	b_4	b_5	b_6	c_4
Argentina	-3.315***	0.493***	-0.176*	11.350***	-0.401*	/	/
Bolivia[b]	1.024**			3.513***	/	/	/
Brazil	0.937*	/	/	3.595***	/	/	-1.527***
Chile	3.151***	/	/	3.714***	/	/	-1.109***
Colombia[b]	2.374***	/	/	3.174***	/	/	-0.595*
Costa Rica	3.088***	0.178**	-0.167**	3.600***	/	/	-0.878**
Ecuador[c]	0.553	/	/	4.772***	/	/	/
Mexico	1.247**	-0.224**	/	3.795***	/	/	-3.108***
Nicaragua[d]	-1.018***	/	/	5.464***	/	/	/
Paraguay	0.227	/	/	4.800***	/	/	-1.036***
Peru	-1.101	0.414**	/	8.170***	-0.388*	/	-2.214***
Uruguay	-0.086	/	/	6.273***	/	/	-1.830***
Venezuela	-3.041***	/	/	9.946***	/	/	/

Table 3.5. Continuation

Country	c_5	c_6	θ_2	θ_3	θ_4
Argentina	/	/	/	/	/
Bolivia[b]	-0.832***	/	0.752***	/	/
Brazil	/	/	/	/	/
Chile	-1.270***		/	1.389***	/
Colombia[b]	/	/	0.956**	/	/
Costa Rica	/	/	1.119**	/	/
Ecuador[c]	/	/	/	/	/
Mexico	2.781***	/	3.139***	-3.600***	/
Nicaragua[d]	/	0.220**	/	/	/
Paraguay	/	/	/	/	/
Peru	-1.874**	1.720***	1.570**	1.540*	-1.476**
Uruguay	-0.560**	/	2.007***	/	/
Venezuela	/	0.712**	/	/	/

[a] With the exception of the intercept terms, for each country we only show the coefficients that are statistically significant at the conventional levels.

[b] The Huber-White-sandwich estimator was used in order to deal with heterosckedasticity problems; therefore in these cases we the standard R^2, and we do not report any heteroskedasticity test. The autocorrelation test reported in these cases is the Cumby-Huizinga test (Ho: disturbance is a moving average of known order 1).

[c] It was not possible to perform Ramsey's RESET because all explanatory variables that were significant are indicator variables.

[d] A dummy in 1988 was included in order to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.6. Final model derived from equation (3.6a) using SUR-GLS:

$$\boldsymbol{g_t = a_4 + a_5(g_{t-1}) + a_6(g_{t-2}) + b_4(D) + b_5(D * g_{t-1}) + b_6(D * g_{t-2}) + c_4(\Delta\%U_t) + c_5(\Delta\%U_{t-1}) + c_6(\Delta\%U_{t-2}) + \theta_2(D * \Delta\%U_t) + \theta_3(D * \Delta\%U_{t-1}) + \theta_4(D * \Delta\%U_{t-2}) + \varepsilon_{2a}}^{a}$$

Country	a_4	a_5	a_6	b_4	b_5	b_6	c_4
Argentina	-3.023***	0.679***	-0.493***	11.717***	-0.691***	0.300**	/
Bolivia	1.049***	/	/	3.574***	/	/	-1.055***
Brazil	1.014**	0.144*	/	4.516***	/	-0.376**	-1.036***
Chile	2.663***	/	/	5.090***	/	/	-0.775**
Colombia	1.563***	0.185**	/	3.825***	/	/	-0.589***
Costa Rica	2.186***	0.157***	/	3.934***	/	/	-0.628**
Ecuador	0.689	/	/	4.346***	/	/	/
Mexico	0.914***	-0.198***	0.157***	3.709***	/	/	-3.267***
Nicaragua[b]	-0.853***	0.106**	/	5.032***	0.282***	-0.411***	0.309***
Paraguay	0.508	/	/	5.650***	/	/	-1.012***
Peru	-1.406***	0.436***	/	8.720***	-0.455***	/	-2.025***
Uruguay	0.233	/	/	6.042***	/	/	-2.385***
Venezuela	-2.561***	/	/	7.690***	/	/	1.302***

Table 3.5. Continuation

Country	c_5	c_6	θ_2	θ_3	θ_4		
Argentina	/	-1.154**	/	/	0.978**		
Bolivia	/	/	0.909***	/	/		
Brazil	/	-0.862**	/	/	/		
Chile	-0.840***	/	0.864*	0.916***	/		
Colombia	0.295***	-0.231**	1.243***	/	0.608***		
Costa Rica	/	0.483***	0.767**	/	/		
Ecuador	/	/	-0.958***	-0.362*	/		
Mexico	2.736***	0.538**	3.477***	-3.591***	/		
Nicaragua[b]	/	/	-0.536***	/	/		
Paraguay	/	/	/	1.478***	/		
Peru	-1.476***	1.544***	1.240***	1.241**	-1.067***		
Uruguay	/	-0.566***	1.993***	/	0.678**		
Venezuela	/	3.465***	-1.168**	/	-0.928*		

[a] With the exception of the intercept terms, for each country we only show the coefficients that are statistically significant at the conventional levels.

[d] A dummy in 1988 was included in order to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.7. Correct specification tests and R^2 obtained from equation (3.6a) using OLS and SUR-GLS[*,a]

	OLS estimation (Table 3.5)		SUR-GLS estimation (Table 3.6)	
Country	**Correct specification tests**[b]	**Adjusted R^2**	**Correct specification tests**[c]	**"R^2"**
Argentina	Aut=0.65; Het=0.06; Nor=0.71; RESET=0.20	0.80	Aut=0.70; Het=0.60; Nor=0.81	0.84
Bolivia	Aut=0.73; Nor=0.13; RESET=0.02	0.75	Aut=0.29; Het=0.01; Nor=0.83	0.80
Brazil	Aut=0.10; Het=0.09; Nor=0.09; RESET=0.51	0.75	Aut=0.80; Het=0.15; Nor=0.00	0.77
Chile	Aut=0.19; Het=0.97; Nor=0.17; RESET=0.02	0.74	Aut=0.54; Het=0.39; Nor=0.59	0.69
Colombia	Aut=0.61; Nor=0.11; RESET=0.00	0.73	Aut=0.92; Het=0.02; Nor=0.01	0.75
Costa Rica	Aut=0.45; Het=0.63; Nor=0.39; RESET=0.75	0.75	Aut=0.35; Het=0.83; Nor=0.44	0.78
Ecuador	Aut=0.14; Het=0.19; Nor=0.07	0.54	Aut=0.15; Het=0.28; Nor=0.00	0.57
Mexico	Aut=0.98; Het=0.16; Nor=0.91; RESET=0.01	0.89	Aut=0.76; Het=0.39; Nor=0.56	0.92
Nicaragua	Aut=0.39; Het=0.63; Nor=0.26; RESET=0.56	0.91	Aut=0.78; Het=0.54; Nor=0.94	0.92
Paraguay	Aut=0.85; Het=0.10; Nor=0.01; RESET=0.99	0.66	Aut=0.96; Het=0.37; Nor=0.60	0.70
Peru	Aut=0.56; Het=0.35; Nor=0.12; RESET=0.21	0.86	Aut=0.35; Het=0.94; Nor=0.93	0.90
Uruguay	Aut=0.14; Het=0.84; Nor=0.95; RESET=0.76	0.88	Aut=0.37; Het=0.07; Nor=0.34	0.89
Venezuela	Aut=0.22; Het=0.25; Nor=0.55; RESET=0.23	0.69	Aut=0.56; Het=0.26; Nor=0.07	0.75

[*] Acronyms employed: Aut: Autocorrelation; Het: Heteroskedasticity; Nor: Normality; and RESET: Ramsey RESET.

[a] Only *p*-values are reported for each correct specification test.

[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[c] The following tests were used: a) Harvey LM test for autocorrelation (Ho: no autocorrelation); b) Hall-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera LM test of multivariate normality (Ho: normality). Breusch-Pagan test of independence of residuals: $\chi^2(78)=101.86$; *p*-value= 0.04.

3.4.1.3 Summary of results

Tables 3.8 and 3.9 below present the different estimates of the g_ns that correspond to the low and high growth regimes (using the different econometric techniques for the simple and dynamic versions of Okun's law) and the percentage variation of the natural rate of growth in low and high growth periods with respect to the original natural rates of growth presented in Table 2.10 of Chapter 2, respectively.

From Table 3.8 it is possible to see that for all countries the g_ns that correspond to the high growth regimes are statistically significantly higher than the original g_ns that were calculated. Specifically, Table 3.9 shows that Argentina, Venezuela, Peru and Uruguay are the countries that present the highest elasticities; whereas Chile, Costa Rica, Colombia and Bolivia present the lowest elasticities in the upward direction.

On the other hand, with respect to the different g_ns that correspond to the low growth regimes, the results seem to show that the latter was found to be statistically non-significant in some countries.[50] Nevertheless, for all other countries in which the different g_ns associated with the low growth regime were found to be statistically significant it is possible to see that these g_ns are statistically significantly lower than the original g_ns. Thus, Table 3.9 shows that Argentina, Nicaragua and Bolivia present the highest sensitivity in the downward direction; Ecuador, Peru, Uruguay and Paraguay are countries that present null elasticity of their respective g_ns in the downward direction; and Costa Rica, Chile and Colombia are countries that present low elasticity in the downward direction.

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[50] Specifically, Table 3.8 shows that this g_n is statistically non-significant in Ecuador; whereas for the cases of Peru and of Paraguay and Uruguay this g_n was found to be statistically significant only when SUR and the penalized regression spline approach were used. Moreover, the g_n related to the low growth regime was statistically significant only when dynamic effects were introduced for the cases of Mexico and Venezuela.

Table 3.8. Latin American countries: natural rate of growth in low and high growth periods using the first endogeneity test of the León-Ledesma and Thirlwall (2002b) approach, 1981-2011*

Natural rate of growth in low growth periods

	Simple difference version of Okun's law				**Dynamic version of Okun's law**	
Country	**OLS (Table 3.1)**	**AMG (Table 3.2)**	**AMG with the common dynamic process imposed with unit coefficient (Table 3.3)**	**Penalized regression spline approach (Table 3.4)**	**OLS (Table 3.5)**	**SUR (Table 3.6)**
Argentina	-3.21	/[a]	-2.31	-3.23	-4.85	-3.71
Bolivia	1.06	1.12	1.37	1.15	1.02	1.05
Brazil	1.02	1.88	2.17	1.10	0.94	1.19
Chile	3.88	/[a]	/[a]	3.09	3.15	2.66
Colombia	2.37	2.02	2.49	2.35	2.37	1.92
Costa Rica	2.95	/[a]	/[a]	2.45	3.12	2.59
Ecuador	/[a]	/[a]	/[a]	/[a]	/[a]	/[a]
Mexico	/[a]	/[a]	/[a]	/[a]	1.02	0.88
Nicaragua	-1.48	/[a]	/[a]	-1.16	-1.02	-0.95
Paraguay	/[a]	/[a]	/[a]	1.27	/[a]	/[a]
Peru	/[a]	/[a]	/[a]	/[a]	/[a]	-2.49
Uruguay	/[a]	/[a]	/[a]	-1.20	/[a]	/[a]
Venezuela	/[a]	/[a]	/[a]	/[a]	-3.04	-2.56

Natural rate of growth in high growth periods

	Simple difference version of Okun's law				**Dynamic version of Okun's law**	
Country	**OLS (Table 3.1)**	**AMG (Table 3.2)**	**AMG with the common dynamic process imposed with unit coefficient (Table 3.3)**	**Penalized regression spline approach (Table 3.4)**	**OLS (Table 3.5)**	**SUR (Table 3.6)**
Argentina	7.72	8.40	7.01	7.67	11.18	10.20
Bolivia	4.45	4.32	4.29	4.47	4.54	4.62
Brazil	4.58	4.32	4.31	4.50	4.53	6.02
Chile	7.98	5.88	5.52	6.60	6.87	7.75
Colombia	5.55	4.79	4.76	5.40	5.55	6.61
Costa Rica	6.33	4.75	4.60	6.18	6.76	7.26
Ecuador	4.76	4.72	4.60	4.69	4.77	4.35
Mexico	4.44	4.60	3.38	4.45	4.12	4.44
Nicaragua	4.46	5.26	4.63	4.62	4.45	4.53
Paraguay	4.42	4.65	4.51	4.81	4.80	5.65
Peru	7.92	6.88	7.49	7.84	13.28	12.16
Uruguay	6.49	5.58	4.86	5.70	6.27	6.04
Venezuela	6.04	6.08	6.08	5.78	6.91	5.13

*Acronyms employed: OLS: Ordinary Least Squares; AMG: Augmented Mean Group; and SUR: Seemingly Unrelated Regressions.

[a] These natural rates of growth were found to be statistically non-significant in the regressions and therefore are not reported.

Table 3.9. Latin American countries: percentage variation of the natural rate of growth in low and high growth periods, 1981-2011*

Natural rate of growth in low growth periods

	Simple difference version of Okun's law				Dynamic version of Okun's law	
Country	**OLS**	**AMG**	**AMG with the common dynamic process imposed with unit coefficient**	**Penalized regression spline approach**	**OLS**	**SUR**
Argentina	-204.22	-[a]	-186.84	-219.19	-231.44	-197.89
Bolivia	-62.41	-52.94	-39.65	-62.66	-67.82	-68.47
Brazil	-65.19	-30.11	-13.20	-62.46	-69.87	-63.38
Chile	-39.09	-[a]	-[a]	-33.97	-43.65	-45.49
Colombia	-36.46	-39.88	-21.94	-35.26	-36.46	-48.80
Costa Rica	-29.43	-[a]	-[a]	-41.53	-27.94	-43.82
Ecuador	-[a]	-[a]	-[a]	-[a]	-[a]	-[a]
Mexico	-[a]	-[a]	-[a]	-[a]	-59.36	-63.93
Nicaragua	-164.35	-[a]	-[a]	-149.36	-142.86	-135.45
Paraguay	-[a]	-[a]	-[a]	-59.03	-[a]	-[a]
Peru	-[a]	-[a]	-[a]	-[a]	-[a]	-172.17
Uruguay	-[a]	-[a]	-[a]	-152.86	-[a]	-[a]
Venezuela	-[a]	-[a]	-[a]	-[a]	-223.58	-204.49

Natural rate of growth in high growth periods

	Simple difference version of Okun's law				Dynamic version of Okun's law	
Country	**OLS**	**AMG**	**AMG with the common dynamic process imposed with unit coefficient**	**Penalized regression spline approach**	**OLS**	**SUR**
Argentina	150.65	260.52	163.53	183.03	202.98	169.13
Bolivia	57.80	81.51	88.99	45.13	43.22	38.74
Brazil	56.31	60.59	72.40	53.58	45.19	85.23
Chile	25.27	35.80	26.90	41.03	22.90	58.81
Colombia	48.79	42.56	49.22	48.76	48.79	76.27
Costa Rica	51.44	21.17	21.69	47.49	56.12	57.48
Ecuador	47.83	71.64	74.90	65.72	61.15	41.69
Mexico	76.89	100.00	58.69	72.48	64.14	81.97
Nicaragua	93.91	222.70	219.31	96.60	86.97	69.03
Paraguay	47.33	70.96	70.83	55.16	60.00	81.09
Peru	142.94	157.68	160.07	136.86	318.93	252.46
Uruguay	187.17	222.54	153.13	151.10	151.81	126.22
Venezuela	145.53	180.18	199.51	163.93	180.89	109.39

*Acronyms employed: OLS: Ordinary Least Squares; AMG: Augmented Mean Group; and SUR: Seemingly Unrelated Regressions.

[a] Not calculated since the natural rate of growth for these cases was found to be statistically non-significant (see Table 3.8).

3.4.2 Second test of endogeneity: using a three year moving average of the actual growth rate to build the dummy variables

As for the second test of endogeneity, we used a three year moving average of the respective g_ts to build the dummy variables for each estimator. Thereby, both intercepts and dummy variables in each case adopted the value of 1 when a three year moving average of g_t was above the average g_t in each country. The average g_t in each country can be found in the first column of Table 2.10 in Chapter 2; whereas for the case of the AMG estimations we used the (unweighted) average g_t of the 13 countries: 2.97.

3.4.2.1 Simple difference version of Okun's law

Tables 3.10, 3.11 and 3.12 respectively show the results obtained using OLS, the standard AMG estimation, and the AMG estimation in which the common dynamic process is imposed with unit coefficient.

From Table 3.10 it is possible to observe that the estimation of equation (3.5) via OLS required the use of the Huber-White-sandwich estimator for the case of Bolivia in order to deal with heteroskedasticity problems; and that, with the exceptions of Bolivia and Chile that present problems of correct functional form according to the Ramsey RESET test, the respective equations in all countries satisfy the correct specification tests at the 10% significance level. On the other hand, both AMG estimations (Tables 3.11 and 3.12) do not present problems of autocorrelation (see footnote c in both Tables).[51]

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[51] Both OLS and AMG results show that an asymmetric Okun coefficient may be present in Bolivia; whereas a statistically significant asymmetric Okun coefficient was found in Argentina and Uruguay when OLS was used; and in Argentina, Brazil and Ecuador when the two different specifications of the AMG estimator were employed.

Table 3.10. Equation (3.5) using OLS:

$$\boldsymbol{g_t = a_3{}' + b_3{}'(D') - c_3{}'(\Delta\%U_t) + \theta_1{}'(D' * \Delta\%U_t) + \varepsilon_{1d}}\ ^{\bar{a}}$$

Country	$a_3{}'$	$b_3{}'$	$c_3{}'$	$\theta_1{}'$	Correct specification tests[a,b]	Adjusted R^2
Argentina	-0.176	6.971***	2.629***	1.673*	Aut=0.49; Het=0.47; Nor=0.10; RESET=0.28	0.62
Bolivia[c]	0.861	3.351***	0.820**	0.743*	Aut=0.39; Nor=0.33; RESET=0.00	0.72
Brazil	1.324**	2.549***	1.954***	-0.311	Aut=0.87; Het=0.06; Nor=0.81; RESET=0.64	0.64
Chile	2.434***	4.481***	0.906***	0.584	Aut=0.53; Het=0.87; Nor=0.01; RESET=0.00	0.47
Colombia	2.227***	2.303***	0.727***	0.563	Aut=0.06; Het=0.30; Nor=0.57; RESET=0.12	0.50
Costa Rica[d]	2.768***	2.791***	1.131***	-0.460	Aut=0.63; Het=0.37; Nor=0.73; RESET=0.21	0.76
Ecuador	2.173***	2.045*	0.348	0.640	Aut=0.03; Het=0.21; Nor=0.46; RESET=0.28	0.05
Mexico	0.923	3.500***	2.249***	1.172	Aut=0.05; Het=0.07; Nor=0.05; RESET=0.73	0.62
Nicaragua[e]	0.077	3.601***	0.507	0.128	Aut=0.39; Het=0.40; Nor=0.23; RESET=0.01	0.78
Paraguay[f]	1.223	3.158***	1.404***	-0.477	Aut=0.69; Het=0.17; Nor=0.02; RESET=0.97	0.52
Peru	0.651	5.567***	1.964**	0.998	Aut=0.58; Het=0.01; Nor=0.63; RESET=0.09	0.45
Uruguay	-0.702	5.583***	1.889***	1.338**	Aut=0.06; Het=0.52; Nor=0.15; RESET=0.48	0.74
Venezuela	1.043	3.887**	2.188***	0.284	Aut=0.22; Het=0.48; Nor=0.05; RESET=0.91	0.66

[ā] Acronyms employed: Aut: Autocorrelation; Het: Heteroskedasticity; Nor: Normality; and RESET: Ramsey RESET.

[a] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[b] Only *p*-values are reported for each correct specification test.

[c] The Huber-White-sandwich estimator was used in order to deal with heterosckedasticity problems; therefore in these cases we report both robust standard errors (in parenthesis) and the standard R^2, and we do not report any heteroskedasticity test. The autocorrelation test reported in these cases is the Cumby-Huizinga test (Ho: disturbance is a moving average of known order 1).

[d] A dummy in 1982 was included to deal with normality problems.

[e] A dummy in 1988 was included to deal with normality problems.

[f] A dummy in 2010 was included to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.11. Equation (3.5a) using AMG estimation:

$$\boldsymbol{g_{it} = \alpha_i'''' + b_{3i}'''(D') - \beta_i''''(\Delta\%U_{it}) + \theta_1''''(D' * \Delta\%U_{it}) + d_i'''\hat{\mu}_t'' + \varepsilon_{it}'''}\ ^{\mathbf{a,b,c}}$$

Country	$\boldsymbol{\alpha_i''''}$	$\boldsymbol{b_3'''}$	$\boldsymbol{\beta_i''''}$	$\boldsymbol{\theta_1''''}$	$\boldsymbol{d_i'''}$
Argentina	2.048**	3.963**	1.451***	1.662**	1.721***
Bolivia	1.322**	2.692***	0.797***	0.758**	0.345**
Brazil	2.359***	1.335	2.687***	1.051	0.544**
Chile	1.896	3.740**	0.578**	-0.026	0.798***
Colombia	2.653***	1.524**	0.667***	0.481	0.409***
Costa Rica	1.905	2.886**	1.272*	0.298	0.473*
Ecuador	2.501***	1.688*	0.395	1.036*	0.801***
Mexico	1.395**	2.562***	1.999***	0.158	0.474**
Nicaragua	0.572	2.646*	0.475	-0.804	0.417
Paraguay	1.726**	2.407**	1.134***	-0.093	0.594*
Peru	0.854	4.741***	1.610***	1.022	1.370***
Uruguay	1.099	2.741**	1.019**	0.752	1.365***
Venezuela	1.642*	2.891	1.931***	0.277	0.604
Average	1.701***	2.592***	1.194***	0.514***	0.626***

[a] All the parameter estimates and the standard errors were computed using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but it turned out to be non-significant for all countries; hence, the results obtained from this specification are not reported.

[c] The *p*-values associated to the correct specification tests in this case are: a) Autocorrelation=0.28; and b) Normality=0.00. The test used were the following: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.12. Equation (3.5b) using AMG estimation:

$$\boldsymbol{g_{it} - \hat{\mu}_t'' = \alpha_i''''' + b_{3i}''''(D') - \beta_i'''''(\Delta\%U_{it}) + \theta_1'''''(D' * \Delta\%U_{it}) + \varepsilon_{it}''''}^{\text{a,b,c}}$$

Country	α_i'''''	b_{3i}''''	β_i'''''	θ_1'''''
Argentina	1.165	5.541***	1.308***	1.203
Bolivia	2.198***	1.441*	0.755**	0.785**
Brazil	3.198***	0.224	3.292***	2.114***
Chile	2.251**	3.281**	0.558**	-0.066
Colombia	3.400***	0.422	0.591**	0.258
Costa Rica	2.807**	1.817	1.207	0.344
Ecuador	2.583***	1.600	0.406	1.134**
Mexico	1.637***	1.877*	1.829***	-0.692
Nicaragua	0.829	2.177	0.462	-0.697
Paraguay	2.070***	1.893*	0.950***	0.170
Peru	0.504	5.273***	1.651***	0.958
Uruguay	0.855	3.181***	1.270***	0.877
Venezuela	1.818**	2.277	1.746***	0.200
Average	1.949***	1.876***	1.148***	0.493**

[a] All the parameter estimates and the standard errors were computed using outlier-robust means instead of unweighted means as suggested by Hamilton (1991). Therefore, the AMG estimations here reported attribute less weight to outliers in their computation.

[b] We also estimated the specification including a country-specific time trend that can be introduced to control for possibly idiosyncratic time effects, but it only turned out to be significant in 2 out of 13 cases (Brazil and Colombia at the 10% level of confidence); hence, the results obtained from this specification are not reported.

[c] The *p*-values associated to the correct specification tests in this case are: a) Autocorrelation=0.77; and b) Normality=0.00. The test used were the following: a) Cumby-Huizinga test for autocorrelation (Ho: disturbance is a moving average of known order 1); and b) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). We used a robust form of the Cumby-Huizinga test for autocorrelation since it was not possible to test for heteroskedasticity after the use of the AMG estimator in Stata 13, so that these estimates take into account that the error term may exhibit conditional heteroskedasticity.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

On the other hand, Table 3.13 shows the result of the penalized regression spline approach following the second test of endogeneity of the LLT approach. With the exception of Chile –that presents problems of incorrect functional form– the correct specification tests are satisfied in all cases. A time-varying Okun coefficient was found to be statistically significant in all countries except in Bolivia and Ecuador.

Table 3.13. Equation (3.5c) using the penalized regression spline approach:
$$\boldsymbol{g_t = \alpha^{*\prime\prime} - \beta_t^{\prime\prime}(\Delta\% U_t) + \xi^{\prime}(D^{\prime}) + \varepsilon_{4b}}$$

Country	$\boldsymbol{\alpha^{*\prime\prime}}$	$\boldsymbol{\beta_t^{\prime\prime}}$[a]	$\boldsymbol{\xi^{\prime}}$	Correct specification tests[b,c]	Adjusted R^2
Argentina	-0.585	2.684**	7.344***	Autocorrelation=0.46; Heteroskedasticity=0.15; Normality=0.65; Ramsey RESET test=0.10	0.60
Bolivia	0.533	2.974	3.579***	Autocorrelation=0.85; Heteroskedasticity=0.02; Normality=0.32; Ramsey RESET test=0.17	0.67
Brazil	1.427**	2.93***	2.402***	Autocorrelation=0.79; Heteroskedasticity=0.10; Normality=0.69; Ramsey RESET test=0.56	0.69
Chile[d]	3.669***	2***	2.672***	Autocorrelation=0.38; Heteroskedasticity=0.13; Normality=0.84; Ramsey RESET test=0.00	0.73
Colombia	2.118***	2.889**	2.419***	Autocorrelation=0.05; Heteroskedasticity=0.18; Normality=0.72; Ramsey RESET test=0.04	0.50
Costa Rica[e]	2.809***	2.407***	2.853***	Autocorrelation=0.33; Heteroskedasticity=0.68; Normality=0.53; Ramsey RESET test=0.93	0.76
Ecuador	1.836**	2.172	2.076*	Autocorrelation=0.04; Heteroskedasticity=0.94; Normality=0.75; Ramsey RESET test=0.63	0.09
Mexico	0.989*	2.357***	3.255***	Autocorrelation=0.03; Heteroskedasticity=0.62; Normality=0.17; Ramsey RESET test=0.59	0.62
Nicaragua[f]	-0.043	2.518**	3.706***	Autocorrelation=0.30; Heteroskedasticity=0.03; Normality=0.27; Ramsey RESET test=0.03	0.90
Paraguay	1.411***	8.092***	3.195***	Autocorrelation=0.60; Heteroskedasticity=0.26; Normality=0.76; Ramsey RESET test=0.86	0.91
Peru	0.351	2**	5.712***	Autocorrelation=0.46; Heteroskedasticity=0.12; Normality=0.66; Ramsey RESET test=0.39	0.48
Uruguay	-1.437	2***	6.082***	Autocorrelation=0.04; Heteroskedasticity=0.37; Normality=0.37; Ramsey RESET test=0.12	0.72
Venezuela	0.792	2***	3.589**	Autocorrelation=0.22; Heteroskedasticity=0.89; Normality=0.25; Ramsey RESET test=0.83	0.70

[a] The estimated degrees of freedom (edf) of the smooth terms are shown.

[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation of order 1 (Ho: no autocorrelation); b) Breusch-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera test for normality (Ho: residuals are normally distributed). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables.

[c] Only *p*-values are reported for each correct specification test.

[d] A dummy in 1983 was included in order to deal with normality problems.

[e] A dummy in 1982 was included in order to deal with normality problems.

[f] A dummy in 1988 was included in order to deal with normality problems.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

3.4.2.2 Dynamic difference version of Okun’s law

Regarding the second test of endogeneity of the LLT approach we present the final models in Tables 3.14 and 3.15. These two Tables respectively present the final results of the application of the general-to-specific approach to equation (3.6b) using both OLS and SUR. We also present the correct specification tests in Table 3.16, which shows that the null hypothesis of the Breusch-Pagan test of independence of residuals is strongly rejected, so that it is possible to conclude that the SUR estimation provided gains in efficiency.

Both OLS and SUR results respectively presented in Tables 3.14 and 3.15 satisfy the correct specification tests at the 10% level of significance, except in the case of Mexico where problems of correct functional form are present when using OLS (see Table 3.16). The OLS results (Table 3.14) show that a statistically significant Okun coefficient is present in Argentina, Bolivia, Chile, Costa Rica, Mexico, Nicaragua and Paraguay; whereas significant Okun coefficients were found in all countries except in Brazil when using SUR (Table 3.15).

Table 3.14. Equation (3.6b) using OLS:

$$\boldsymbol{g_t = a_4{}' + a_5{}'(g_{t-1}) + a_6{}'(g_{t-2}) + b_4{}'(D') + b_5{}'(D' * g_{t-1}) + b_6{}'(D' * g_{t-2}) + c_4{}'(\Delta\%U_t) + c_5{}'(\Delta\%U_{t-1}) + c_6{}'(\Delta\%U_{t-2}) + \theta_2{}'(D' * \Delta\%U_t) + \theta_3{}'(D' * \Delta\%U_{t-1}) + \theta_4{}'(D' * \Delta\%U_{t-2}) + \varepsilon_{2b}}^{a}$$

Country	$a_4{}'$	$a_5{}'$	$a_6{}'$	$b_4{}'$	$b_5{}'$	$b_6{}'$	$c_4{}'$
Argentina	0.980	/	/	5.801***	/	/	-2.931***
Bolivia	3.727***	/	-0.456***	/	/	0.664***	-1.716***
Brazil	1.417**	/	/	2.485***	/	/	-2.123***
Chile	3.025***	/	/	3.775***	/	/	-1.134***
Colombia	2.148***	/	/	2.320***	/	/	-0.531***
Costa Rica	3.954***	/	-0.272***	/	0.539***	/	/
Ecuador	2.958***	/	/	/	/	/	-0.490*
Mexico[b]	0.151	/	/	3.565***	/	/	-2.308***
Nicaragua[c]	-0.360	/	/	4.912***	/	-0.307**	-0.603***
Paraguay	0.738	/	/	6.039***	/	-0.510*	-1.135***
Peru	0.320	/	/	5.673***	/	/	-1.406**
Uruguay	-1.262	/	/	6.453***	-0.366*	0.279*	-1.492***
Venezuela	1.096	/	/	4.782***	-0.288**	/	-2.321***

Table 3.14. Continuation

Country	$c_5{}'$	$c_6{}'$	$\theta_2{}'$	$\theta_3{}'$	$\theta_4{}'$		
Argentina	-1.826**	/	2.034**	1.661*	/		
Bolivia	-1.278***	/	1.645***	1.491***	/		
Brazil	/	/	/	/	/		
Chile	-1.112***	/	/	1.646***	/		
Colombia	/	/	/	/	/		
Costa Rica	/	/	-1.528***	0.979**	/		
Ecuador	-0.754**	/	/	/	/		
Mexico[b]	1.082***	/	2.581**	-2.915***	/		
Nicaragua[c]	0.715**	/	/	-1.087***	/		
Paraguay	/	/	/	1.522*	/		
Peru	/	/	/	/	/		
Uruguay	/	/	/	/	/		
Venezuela	/	/	/	/	/		

[a] With the exception of the intercept terms, for each country we only show the coefficients that are statistically significant at the conventional levels.

[b] A dummy variable for 1994 was included to deal with normality problems.

[c] A dummy was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.15. Equation (3.6b) using SUR-GLS:

$$\boldsymbol{g_t = a_4' + a_5'(g_{t-1}) + a_6'(g_{t-2}) + b_4'(D') + b_5'(D' * g_{t-1}) + b_6'(D' * g_{t-2}) + c_4'(\Delta\%U_t) + c_5'(\Delta\%U_{t-1}) + c_6'(\Delta\%U_{t-2}) + \theta_2'(D' * \Delta\%U_t) + \theta_3'(D' * \Delta\%U_{t-1}) + \theta_4'(D' * \Delta\%U_{t-2}) + \varepsilon_{2b}}^{a}$$

Country	a_4	a_5	a_6	b_4	b_5	b_6	c_4
Argentina	0.716	/	/	6.148***	/	/	-2.502***
Bolivia	3.681***	/	-0.431***	/	/	0.658***	-1.665***
Brazil	1.602***	-0.429***	/	/	1.042***	/	-1.971***
Chile	3.099***	/	/	4.489***	/	/	-0.553***
Colombia	2.418***	/	/	1.900***	/	/	-0.752***
Costa Rica	2.670***	/	/	2.656***	0.343**	-0.271**	-0.893***
Ecuador	2.756***	/	0.249**	/	/	/	-0.548***
Mexico[b]	0.553*	-0.327***	/	4.616***	/	/	-2.059***
Nicaragua[c]	0.056	/	/	3.327***	0.353***	-0.197**	-0.883***
Paraguay	0.601	/	/	5.903***	/	-0.425**	-0.744***
Peru	1.033	/	/	4.861***	/	/	-2.319***
Uruguay	0.944	/	/	4.404***	-0.274**	0.253**	-2.823***
Venezuela	0.431	0.659**	-0.190**	6.366***	-1.084***	/	-2.170***

Table 3.15. Continuation

Country	c_5'	c_6'	θ_2'	θ_3'	θ_4'		
Argentina	-1.544***	/	1.980***	1.262*	/		
Bolivia	-1.277***	/	1.598***	1.515***	/		
Brazil	1.730***	/	/	/	/		
Chile	-0.877***	/	1.142***	0.667***	/		
Colombia	/	/	0.618***	-0.540***	/		
Costa Rica	/	/	/	0.944***	/		
Ecuador	-0.917***	/	0.843**	0.739*	/		
Mexico[b]	/	/	2.536***	-2.079***	/		
Nicaragua[c]	0.646***	0.211**	0.644***	-0.643***	/		
Paraguay	/	/	/	1.248**	/		
Peru	-1.262***	/	1.016*	1.633***	/		
Uruguay	/	-1.583***	1.901***	/	1.800***		
Venezuela	1.577**	/	/	-2.669***	/		

[a] With the exception of the intercept terms, for each country we only show the coefficients that are statistically significant at the conventional levels.

[b] A dummy for 1994 was included in order to deal with normality problems.

[c] A dummy was included to correct for the outlier in 1988.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 3.16. Correct specification tests and R^2 obtained from equation (3.6b) using OLS and SUR-GLS[*,a]

	OLS estimation (Table 3.14)		SUR-GLS estimation (Table 3.15)	
Country	**Correct specification tests[b]**	**Adjusted R^2**	**Correct specification tests[c]**	**"R^2"**
Argentina	Aut=0.92; Het=0.10; Nor=0.99; RESET=0.07	0.64	Aut=0.69; Het=0.22; Nor=0.78	0.67
Bolivia	Aut=0.06; Het=0.35; Nor=0.99; RESET=0.19	0.92	Aut=0.04; Het=0.29; Nor=0.86	0.94
Brazil	Aut=0.76; Het=0.04; Nor=0.89; RESET=0.59	0.65	Aut=0.69; Het=0.05; Nor=0.23	0.68
Chile	Aut=0.69; Het=0.67; Nor=0.50; RESET=0.18	0.70	Aut=0.12; Het=0.65; Nor=0.54	0.61
Colombia	Aut=0.05; Het=0.20; Nor=0.88; RESET=0.03	0.47	Aut=0.75; Het=0.40; Nor=0.54	0.57
Costa Rica	Aut=0.04; Het=0.78; Nor=0.23; RESET=0.96	0.62	Aut=0.08; Het=0.28; Nor=0.60	0.68
Ecuador	Aut=0.56; Het=0.26; Nor=0.42; RESET=0.59	0.17	Aut=0.48; Het=0.07; Nor=0.93	0.19
Mexico	Aut=0.05; Het=0.10; Nor=0.13; RESET=0.00	0.83	Aut=0.16; Het=0.04; Nor=0.15	0.86
Nicaragua	Aut=0.57; Het=0.57; Nor=0.27; RESET=0.24	0.88	Aut=0.50; Het=0.48; Nor=0.45	0.92
Paraguay	Aut=0.38; Het=0.20; Nor=0.10; RESET=0.66	0.55	Aut=0.27; Het=0.26; Nor=0.06	0.59
Peru	Aut=0.55; Het=0.01; Nor=0.51; RESET=0.67	0.45	Aut=0.59; Het=0.13; Nor=0.81	0.56
Uruguay	Aut=0.03; Het=0.73; Nor=0.35; RESET=0.43	0.68	Aut=0.10; Het=0.26; Nor=0.52	0.76
Venezuela	Aut=0.68; Het=0.70; Nor=0.64; RESET=0.75	0.71	Aut=0.53; Het=0.27; Nor=0.45	0.76

[*] Acronyms employed: Aut: Autocorrelation; Het: Heteroskedasticity; Nor: Normality; and RESET: Ramsey RESET.

[a] Only *p*-values are reported for each correct specification test.

[b] The following tests were used: a) Breusch-Godfrey LM test for autocorrelation (Ho: no autocorrelation); b) Breusch-Pagan/Cook-Weisberg test for heteroskedasticity (Ho: constant variance); and c) Doornik–Hansen test of multivariate normality (Ho: multivariate normality). It is also important to bear in mind that the RESET test (Ho: no misspecification or no incorrect functional form) has very low power since the powers of the fitted values are only proxies for the potentially omitted variables. It was not possible to perform Ramsey's RESET because all explanatory variables that were significant are indicator variables.

[c] The following tests were used: a) Harvey LM test for autocorrelation (Ho: no autocorrelation); b) Hall-Pagan LM test for heteroskedasticity (Ho: constant variance); and c) Jarque-Bera LM test of multivariate normality (Ho: normality). Breusch-Pagan test of independence of residuals: χ^2(78)=121.75; *p*-value= 0.00.

3.4.2.3 Summary of results

The different estimated g_ns for each country associated with the low and high growth regimes following the second test of endogeneity of the LLT approach can be found in Table 3.17 below. The latter shows that: 1) the estimated g_n associated with the low growth periods is statistically non-significant for the cases of Peru and Uruguay; whereas it was found to be statistically significant in Argentina and Venezuela only when the AMG estimator was used; and 2) the estimated g_n associated with the high growth regime was found to be statistically significant in almost all cases; but for the case of Ecuador the latter was found to be statistically non-significant when the dynamic version of Okun's law was estimated.

On the other hand, Table 3.18 presents the percentage variation of the estimated g_ns presented in Table 3.17 with respect to the original g_ns presented in Table 2.10 of Chapter 2. This Table shows that, in general, all Latin American countries present sensitivity of the g_n in the upward direction; but these are lower compared to the ones obtained using the standard dummy approach. However, some inconsistent results were found for the cases of Bolivia, Brazil and Costa Rica when dynamic effects were included, and for the cases of Chile and Costa Rica when the standard AMG estimator was utilized since in these cases the g_n associated with the high growth regime is below the original g_n.

Table 3.17. Latin American countries: natural rate of growth in low and high growth periods using the second endogeneity test of the León-Ledesma and Thirlwall (2002b) approach, 1981-2011*

Natural rate of growth in low growth periods

	Simple difference version of Okun's law				**Dynamic version of Okun's law**	
Country	**OLS (Table 3.10)**	**AMG (Table 3.11)**	**AMG with the common dynamic process imposed with unit coefficient (Table 3.12)**	**Penalized regression spline approach (Table 3.13)**	**OLS (Table 3.14)**	**SUR (Table 3.15)**
Argentina	/[a]	2.05	/[a]	/[a]	/[a]	/[a]
Bolivia	/[a]	1.32	2.20	/[a]	2.56	2.57
Brazil	1.32	2.36	3.20	1.43	1.42	1.12
Chile	2.43	/[a]	2.25	3.67	3.03	3.10
Colombia	2.28	2.65	3.40	2.12	2.15	2.42
Costa Rica	2.77	/[a]	2.81	2.81	3.11	2.67
Ecuador	2.17	2.50	2.58	1.84	2.96	3.67
Mexico	/[a]	1.40	1.64	0.99	/[a]	0.42
Nicaragua	/[a]	/[a]	/[a]	/[a]	/[a]	/[a]
Paraguay	/[a]	1.73	2.07	1.41	/[a]	/[a]
Peru	/[a]	/[a]	/[a]	/[a]	/[a]	/[a]
Uruguay	/[a]	/[a]	/[a]	/[a]	/[a]	/[a]
Venezuela	/[a]	1.64	1.82	/[a]	/[a]	/[a]

Natural rate of growth in high growth periods

	Simple difference version of Okun's law				**Dynamic version of Okun's law**	
Country	**OLS (Table 3.10)**	**AMG (Table 3.11)**	**AMG with the common dynamic process imposed with unit coefficient (Table 3.12)**	**Penalized regression spline approach (Table 3.13)**	**OLS (Table 3.14)**	**SUR (Table 3.15)**
Argentina	6.97	6.01	5.54	7.34	5.80	6.15
Bolivia	3.35	4.01	3.64	3.58	3.02	3.03
Brazil	3.87	/[a]	/[a]	3.83	3.90	1.85
Chile	6.92	3.74	5.53	6.34	6.80	7.59
Colombia	4.53	4.18	/[a]	4.54	4.47	4.32
Costa Rica	5.56	2.89	/[a]	5.66	3.53	5.40
Ecuador	4.22	4.19	/[a]	3.91	/[a]	/[a]
Mexico	3.50	3.96	3.51	4.24	3.57	3.90
Nicaragua	3.60	2.65	/[a]	3.71	4.61	3.48
Paraguay	3.16	4.13	3.96	4.61	5.53	5.48
Peru	5.57	4.74	5.27	5.71	5.67	4.86
Uruguay	5.58	2.74	3.18	6.08	6.37	4.38
Venezuela	3.89	/[a]	/[a]	3.59	4.49	9.95

[*] Acronyms employed: OLS: Ordinary Least Squares; AMG: Augmented Mean Group; and SUR: Seemingly Unrelated Regressions.

[a] These natural rates of growth were found to be statistically non-significant in the regressions and therefore are not reported.

Table 3.18. Latin American countries: percentage variation of the natural rate of growth in low and high growth periods according to the 3 year moving average dummy variable, 1981-2011*

Natural rate of growth in low growth periods

	Simple difference version of Okun's law				Dynamic version of Okun's law	
Country	**OLS**	**AMG**	**AMG with the common dynamic process imposed with unit coefficient**	**Penalized regression spline approach**	**OLS**	**SUR**
Argentina	-[a]	-12.02	-[a]	-[a]	-[a]	-[a]
Bolivia	-[a]	-44.54	-3.08	-[a]	-19.24	-22.82
Brazil	-54.95	-12.27	28.00	-51.19	-54.49	-65.54
Chile	-61.85	-[a]	-48.28	-21.58	-45.80	-36.48
Colombia	-38.87	-21.13	6.58	-41.60	-42.36	-35.47
Costa Rica	-33.73	-[a]	-25.66	-32.94	-28.18	-42.08
Ecuador	-32.61	-9.09	-1.90	-34.98	0.00	19.54
Mexico	-[a]	-39.13	-23.00	-61.63	-[a]	-82.79
Nicaragua	-[a]	-[a]	-[a]	-[a]	-[a]	-[a]
Paraguay	-[a]	-36.40	-21.59	-54.52	-[a]	-[a]
Peru	-[a]	-[a]	-[a]	-[a]	-[a]	-[a]
Uruguay	-[a]	-[a]	-[a]	-[a]	-[a]	-[a]
Venezuela	-[a]	-24.42	-10.34	-[a]	-[a]	-[a]

Natural rate of growth in high growth periods

	Simple difference version of Okun's law				Dynamic version of Okun's law	
Country	**OLS**	**AMG**	**AMG with the common dynamic process imposed with unit coefficient**	**Penalized regression spline approach**	**OLS**	**SUR**
Argentina	126.30	157.94	108.27	170.85	57.18	62.27
Bolivia	18.79	68.49	60.35	16.23	-4.73	-9.01
Brazil	32.08	-[a]	-[a]	30.72	25.00	-43.08
Chile	8.63	-13.63	27.13	35.47	21.65	55.53
Colombia	21.45	24.40	-[a]	25.07	19.84	15.20
Costa Rica	33.01	-26.28	-[a]	35.08	-18.48	17.14
Ecuador	31.06	52.36	-[a]	38.16	-[a]	-[a]
Mexico	39.44	72.17	64.79	64.34	42.23	59.84
Nicaragua	56.52	62.58	-[a]	57.87	93.70	29.85
Paraguay	5.33	51.84	50.00	48.71	84.33	75.64
Peru	70.86	77.53	82.99	72.51	78.86	40.87
Uruguay	146.90	58.38	65.63	167.84	155.82	64.04
Venezuela	58.13	-[a]	-[a]	63.93	82.52	306.12

*Acronyms employed: OLS: Ordinary Least Squares; AMG: Augmented Mean Group; and SUR: Seemingly Unrelated Regressions.

[a] Not calculated since the natural rate of growth for these cases was found to be statistically insignificant (see Table 3.15).

It may be possible to provide an explanation for the cases of Chile and Costa Rica when the AMG estimator was used since these two countries are the ones that present the highest actual rates of growth in our sample (4.81 in Chile and 4.08 in Costa Rica). Thus, since for the second test of endogeneity of the LLT approach estimated via AMG we used a dummy variable that adopted the value of 1 when a three year moving average of g_t was above the (unweighted) average g_t of the 13 countries (2.97), then this result would indicate that the g_n in Chile and Costa Rica does not react in the upward direction if the respective g_ts are above 2.97.

Regarding the sensitivity of the g_n in the downward direction, countries like Peru, Uruguay and Nicaragua do not seem to present sensitivity in this direction; whereas the results in Argentina and Venezuela need to be taken with caution since a downward sensitivity of the g_n in these countries was found only when the AMG estimation was used. Likewise, some inconsistent results were found for the cases of Brazil and Colombia when the AMG estimation was used, and for Ecuador when dynamic effects were included in the estimation since in these cases the respective g_ns associated with the low growth regimes are above the original estimate of g_n.

According to the results presented in Table 3.18 it may be possible to conclude the following: 1) countries that present high sensitivity of the g_n in the upward direction are Argentina, Uruguay, Peru and Venezuela; 2) countries that present low sensitivity of the g_n in the upward direction are Colombia, Chile and Costa Rica; 3) countries that present high sensitivity of the g_n in the downward direction are Chile, Brazil and Costa Rica; and 4) countries that do not seem to present sensitivity of the g_n in the downward direction are

Paraguay, Peru, Nicaragua; whereas countries like Ecuador, Costa Rica, Colombia and Chile are countries with a low sensitivity in the downward direction.

3.5 Conclusions

The present Chapter has tried to test the hypothesis of endogeneity of the natural rate of growth following the two econometric procedures proposed by the seminal paper of León-Ledesma and Thirlwall (2002b) for a sample of 13 Latin American countries during the period 1981-2011. Thus, for all countries we have identified expansion periods as follows: 1) when the actual rate of growth is above the original estimates of the natural rate of growth (presented in Table 2.10 of Chapter 2); and 2) when a three year moving average of the actual rate of growth is above the average rate of growth.

The main contributions to the literature are the following. Firstly, we have underlined some important features of the León-Ledesma-Thirlwall approach that have not been highlighted previously, namely that with this approach it is possible to calculate natural rates of growth associated with low and high growth regimes. Secondly, the chapter has also proposed a dynamic specification that follows the dynamic version of Okun’s law –presented in Chapter 2– in order to retrieve estimates of the natural rates of growth in low and high growth periods. Thirdly, this is the first time that the use of the AMG estimator and the penalized regression spline approach –both described in Chapter 2– have been used; and this is also the first time that the possibility of an asymmetric Okun coefficient following the specification proposed by Lanzafame (2010) has been explored for the case of Latin American countries.

Thus, we have retrieved 6 different estimates of the natural rate of growth associated with the different growth regimes (low and high) for each one of the two tests of endogeneity of the León-Ledesma-Thirlwall approach. The results obtained seem to offer a relatively homogeneous picture of our sample of 13 Latin American economies. On the one hand, and with respect to the sensitivity of the natural rate of growth in the upward direction, it is possible to say that all countries present sensitivity of the natural rate of growth. Specifically, Argentina, Peru, Uruguay and Venezuela are countries that present high sensitivity of the natural rate of growth in this direction; whereas Chile, Costa Rica and Colombia are countries that present low sensitivity.

On the other hand, and with respect to the sensitivity of the natural rate of growth in the downward direction, it is possible to say that Chile, Colombia, Costa Rica, Ecuador, Peru, and Uruguay are countries that either do not present or present low sensitivity. However, as regards the countries that present high sensitivity in the downward direction, the results are not so homogeneous since the first test of endogeneity of the León-Ledesma-Thirlwall approach shows that countries like Argentina and Nicaragua present high sensitivity of the natural rate of growth, whereas the second test shows that Brazil is a country with high sensitivity.

Our new results are similar to the previous empirical studies for Latin American countries (Libânio, 2009 and Vogel, 2009). In Table 3A.1 of the Appendix we offer a summary of the main results obtained by these studies. From this Table it is possible to observe that:

1) Chile, Colombia and Costa Rica are countries that present low sensitivity of the natural rate of growth in boom periods; whereas the natural rate of growth in Argentina presents high sensitivity in the upward direction.

2) Chile, Colombia and Peru are countries that present sensitivity of the natural rate of growth in the downward direction.

Finally, it is interesting to note that the countries that present a relatively low sensitivity of the natural rate of growth both in the upward and downward directions (Chile, Colombia and Costa Rica) are those that have experienced the highest rates of growth during the period of study (4.81, 4.08 and 3.54, respectively), so that it is possible to say that these countries have experienced less scope for sensitivity in the natural rate of growth. However, the opposite – that low growth countries presented a relatively high sensitivity of the natural rate of growth both in the upward and downward directions– does not seem to hold in our sample of Latin American countries since, for example, countries like Uruguay do not present sensitivity in the downward direction.

Appendix CHAPTER 3

Table 3A.1. Percentage variation of the natural rate of growth in low and high growth periods: results obtained by Libânio (2009) and Vogel (2009)

Country	**Libânio (2009)***		**Vogel (2009)****	
Natural rate of growth in low growth periods				
	First test of endogeneity	**Second test of endogeneity**	**First test of endogeneity**	**Second test of endogeneity**
Argentina	-[a]	-[a]	-67.0	-[a]
Bolivia	-[a]	-[a]	-60.73	-[a]
Brazil	-[a]	-[a]	-67.0	-[a]
Chile	-45.93	-46.83	-47.71	-[a]
Colombia	-43.11	-43.71	-10.64	-[a]
Costa Rica	-[a]	-[a]	-73.82	-[a]
Ecuador	-[a]	-[a]	-[a]	-[a]
Mexico	-[a]	-[a]	-[a]	-[a]
Nicaragua	-[a]	-[a]	-[a]	-[a]
Paraguay	-[a]	-[a]	-[a]	-[a]
Peru	-247.89	-[a]	-80.51	-[a]
Uruguay	-179.56	-[a]	-[a]	-[a]
Venezuela	-[a]	-[a]	-214.61	-[a]
Natural rate of growth in high growth periods				
	First test of endogeneity	**Second test of endogeneity**	**First test of endogeneity**	**Second test of endogeneity**
Argentina	177.09	144.25	137.62	-[a]
Bolivia	-[a]	-[a]	64.36	-[a]
Brazil	97.86	112.79	45.87	-[a]
Chile	49.60	23.70	29.25	-[a]
Colombia	30.41	29.34	36.39	-[a]
Costa Rica	52.07	29.16	42.77	-[a]
Ecuador	82.48	59.64	-[a]	-[a]
Mexico	71.27	70.37	76.52	-[a]
Nicaragua	-[a]	-[a]	89.39	-[a]
Paraguay	-[a]	-[a]	71.97	-[a]
Peru	139.68	119.45	55.17	-[a]
Uruguay	188.88	109.66	-[a]	-[a]
Venezuela	140.98	31.60	159.55	-[a]

*Period: 1980-2004. Argentina and Brazil: 1980-2002.

**Period: 1986-2003 for the majority of countries. Colombia: 1979-2004 and Bolivia: 1990-2003.

[a] Results not presented because these countries were not included in the respective studies or because the respective estimates of the natural rate of growth were found to be statistically non-significant.

CHAPTER 4

A Decomposition Analysis of the Natural Rate of Growth in Latin American Countries, 1981-2011

4.1 Introduction

How can we measure the degree of sensitivity of the natural rate of growth (henceforth g_n) with respect to its individual components? The present chapter tries to estimate the sensitivity of g_n with respect to the rate of growth of the labour force and the rate of growth of labour productivity in Latin American countries. Using annual data for the period 1981-2011, we estimate the coefficients associated with these variables using Maximum Likelihood Estimation (henceforth MLE) via the Kalman (1960) filter. As a robustness test, we have also employed the rolling regressions technique to estimate a time-varying natural rate of growth in order to compare these results with the ones obtained via the Kalman filter.

Therefore, this Chapter makes the following contributions to the literature. Firstly, we estimate various time-varying natural rates of growth by using: a) time-varying parameter models estimated via the Kalman filter; and b) the rolling regressions technique. Secondly, we employ two different state-space models (see Section 4.2 below) in order to estimate the sensitivity of the unobserved time-varying g_n with respect to the rate of growth of the labour force and to the rate of growth of labour productivity.

The rest of the Chapter is organized as follows. In Section 4.2 we offer a succinct description of state-space models and of the Kalman filter. Section 4.3 describes the empirical strategy followed to generate a time-varying g_n and to estimate its sensitivity with respect to its individual components. In Section 4.4 we present the empirical results obtained using both rolling regressions and the Kalman filter, and we discuss the main findings. Finally, the main conclusions are presented in Section 4.5.

4.2 State-space models and the Kalman filter

State-space models typically deal with dynamic time series models that involve unobservable variables (Kim and Nelson, 1999). Such models consist of a measurement equation –which describes the relationship between observed variables and unobserved state variables– and a transition (or state) equation –which describes the dynamics of the state variables and that has the form of a first-order difference equation in the state vector.

A linear state-space representation of the dynamics of a $nx1$ vector of variables observed at time t ($\mathbf{y_t}$) is given by the following system of equations (Kim and Nelson, 1999):

$$\mathbf{y_t} = \mathbf{H_t B_t} + \mathbf{A z_t} + \mathbf{e_t} \quad \dots\dots\dots\dots \quad (4.1)$$

$$\mathbf{B_t} = \bar{\boldsymbol{\mu}} + \mathbf{F B_{t-1}} + \mathbf{v_t} \quad \dots\dots\dots\dots \quad (4.2)$$

$$\mathbf{e_t} \sim \text{i. i. d.} \, N(0, \mathbf{R}) \quad \dots\dots\dots\dots\dots\dots \quad (4.3)$$

$$\mathbf{v_t} \sim \text{i. i. d.} \, N(0, \mathbf{Q}) \quad \dots\dots\dots\dots\dots\dots \quad (4.4)$$

$$E(\mathbf{e_t v_s'}) = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (4.5)$$

where $\mathbf{B_t}$ is a $kx1$ vector of unobserved state variables; $\mathbf{H_t}$ is an nxk matrix that links the observed $\mathbf{y_t}$ vector and the unobserved $\mathbf{B_t}$ (elements of which can be either data on exogenous variables or constant parameters); $\mathbf{A}$ is an nxr matrix; $\mathbf{z_t}$ is an $rx1$ vector of exogenous or predetermined observed variables; $\bar{\boldsymbol{\mu}}$ is a $kx1$ vector; and $\mathbf{F}$ is kxk. Thus, equations (4.1) and

(4.2), respectively, represent the measurement and the transition equations of the general state-space model.

Once a dynamic time series model is written in state-space form, it is possible to use the Kalman filter to compute the optimal estimate of the unobserved state vector $\mathbf{B_t}$[52], assuming that $\bar{\boldsymbol{\mu}}$, $\mathbf{F}$, $\mathbf{R}$, and $\mathbf{Q}$ are known. The Kalman filter is a recursive procedure that provides a minimum mean square error estimate of $\mathbf{B_t}$, given the appropriate information set. Depending upon the information set used, it is possible to find the basic filter and smoothing filter: the former refers to an estimate of $\mathbf{B_t}$ based on information available up to time t; whereas the latter refers to an estimate of $\mathbf{B_t}$ based on all the available information in the sample through time T (Kim and Nelson, 1999).

The basic Kalman filter consists of two steps: the prediction step and the updating step. These can be represented by the following sets of recursive equations, where equations (4.6) to (4.9) depict the prediction step, and equations (4.10) and (4.11) depict the updating step:

$$\mathbf{B_{t|t-1}} = \bar{\boldsymbol{\mu}} + \mathbf{F}\mathbf{B_{t-1|t-1}} \quad \dots\dots\dots\dots \quad (4.6)$$

$$\mathbf{P_{t|t-1}} = \mathbf{F}\mathbf{P_{t-1|t-1}}\mathbf{F}' + \mathbf{Q} \quad \dots\dots\dots\dots \quad (4.7)$$

$$\boldsymbol{\eta}_{\mathbf{t|t-1}} = \mathbf{y_t} - \mathbf{y_{t|t-1}} \quad \dots\dots\dots\dots\dots \quad (4.8)$$

$$\mathbf{f_{t|t-1}} = \mathbf{H_t}\mathbf{P_{t|t-1}}\mathbf{H_t'} + \mathbf{R} \quad \dots\dots\dots\dots \quad (4.9)$$

$$\mathbf{B_{t|t}} = \mathbf{B_{t|t-1}} + \mathbf{K_t}\boldsymbol{\eta}_{\mathbf{t|t-1}} \quad \dots\dots\dots\dots \quad (4.10)$$

---

[52] It might also be possible to employ Generalized Least Squares regressions. However, this method may be extremely inefficient in terms of its computational burden (Kim and Nelson, 1999).

$$\mathbf{P}_{\mathbf{t|t}} = \mathbf{P}_{\mathbf{t|t-1}} - \mathbf{K}_{\mathbf{t}}\mathbf{H}_{\mathbf{t}}\mathbf{P}_{\mathbf{t|t-1}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4.11)$$

where $\mathbf{B}_{\mathbf{t|t-1}}$ and $\mathbf{B}_{\mathbf{t|t}}$ are the expectations (estimates) of $\mathbf{B}_{\mathbf{t}}$ conditional on information up to $t-1$ and t, respectively; $\mathbf{P}_{\mathbf{t|t-1}}$ and $\mathbf{P}_{\mathbf{t|t}}$ are the covariance matrices of $\mathbf{B}_{\mathbf{t}}$ conditional on information up to $t-1$ and t, respectively; $\boldsymbol{\eta}_{\mathbf{t|t-1}}$ is the prediction error; $\mathbf{y}_{\mathbf{t|t-1}}$ is the forecast of $\mathbf{y}_{\mathbf{t}}$ given information up to $t-1$; $\mathbf{f}_{\mathbf{t|t-1}}$ is the conditional variance of the prediction error; and $\mathbf{K}_{\mathbf{t}} = \mathbf{P}_{\mathbf{t|t-1}}\mathbf{H}'_{\mathbf{t}}\mathbf{f}^{-1}_{\mathbf{t|t-1}}$ is the Kalman filter gain, which determines the weight assigned to new information about $\mathbf{B}_{\mathbf{t}}$ contained in $\boldsymbol{\eta}_{\mathbf{t|t-1}}$.

On the other hand, the smoothing filter is described by the following equations:

$$\mathbf{B}_{\mathbf{t|T}} = \mathbf{B}_{\mathbf{t|t}} + \mathbf{P}_{\mathbf{t|t}}\mathbf{F}'\mathbf{P}^{-1}_{\mathbf{t+1|t}}\left(\mathbf{B}_{\mathbf{t+1|T}} - \mathbf{F}\mathbf{B}_{\mathbf{t|t}} - \bar{\boldsymbol{\mu}}\right) \qquad \ldots\ldots\ldots\ldots\ldots \qquad (4.12)$$

$$\mathbf{P}_{\mathbf{t|T}} = \mathbf{P}_{\mathbf{t|t}} + \mathbf{P}_{\mathbf{t|t}}\mathbf{F}'\mathbf{P}^{-1}_{\mathbf{t+1|t}}\left(\mathbf{P}_{\mathbf{t+1|T}} - \mathbf{P}_{\mathbf{t+1|t}}\right){\mathbf{P}^{-1}_{\mathbf{t+1|t}}}'\mathbf{F}\mathbf{P}'_{\mathbf{t|t}} \qquad \ldots\ldots\ldots \qquad (4.13)$$

where $\mathbf{B}_{\mathbf{t|T}}$ and $\mathbf{P}_{\mathbf{t|T}}$, the initial values for the smoothing, are obtained from the last iteration of the basic filter.

Finally, it is possible to use the sample log likelihood function based on the prediction error decomposition in order to estimate the model’s parameters when some of the latter are unknown:

$$\ln L = -\frac{1}{2}\sum_{t=1}^{T}\ln\left(2\pi\mathbf{f}_{\mathbf{t|t-1}}\right) - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\eta}'_{\mathbf{t|t-1}}\mathbf{f}^{-1}_{\mathbf{t|t-1}}\boldsymbol{\eta}_{\mathbf{t|t-1}} \qquad \ldots\ldots\ldots\ldots \qquad (4.14)$$

The likelihood values can be maximized with respect to the unknown parameters of the model. In this Chapter we have used the Marquadt algorithm, which is a first derivative method (so that only the first derivatives of the objective function at the parameter values are required)[53] that modifies the traditional Gauss-Newton algorithm by adding a ridge factor (correction matrix) to the Hessian approximation.[54]

4.3 Empirical strategy

Following Thirlwall's (1969) estimation procedure used to retrieve an estimate of g_n, which was introduced in Chapter 2, we allow for time-varying coefficients in the simple difference version of Okun's law in order to reflect changes in g_n and cyclical variations:

$$g_t = \beta_{1,t} - \beta_{2,t}(\Delta\%U_t) + \varepsilon_{1,t} \quad \dots\dots\dots\dots\dots \quad (4.15)$$

where, in addition to the variables defined in the previous Chapters, $\varepsilon_{1,t}$ is the error term in equation (4.15).

Equation (4.15) depicts a time-varying parameter model (henceforth TVPM) composed of the observed (data) variables $\Delta\%U_t$ and g_t, and the unobserved parameters $\beta_{1,t}$ and $\beta_{2,t}$. As Kim and Nelson (1999) explain, the TVPM is a special case of state-space models in which $\mathbf{H_t}$ in equation (4.1) is replaced by a matrix of exogenous or predetermined variables.

The estimation of equation (4.15) allows us to generate a time-varying g_n (henceforth $g_{n,t}$), which in this formulation corresponds to the time-varying intercept $\beta_{1,t}$. However, it is also possible to employ the state-space formulation in order to estimate the sensitivity of this unobserved time-varying parameter with respect to the rates of growth of labour productivity and labour force. Thereby, we have used the following stochastic formulation:

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[53] Second derivative methods may be computationally costly since it is necessary to evaluate the $k(k+1)/2$ (where k is the number of parameters) elements of the second derivative matrix at every iteration (EViews, 2014).

[54] The ridge correction handles numerical problems when the outer product is singular and may improve the convergence rate (see EViews (2014)).

$$g_{n,t} = \beta_{1,t} = \gamma_0(l_t) + \gamma_1(\tau_t) + \varepsilon_{2,t} \qquad \dots\dots\dots\dots\dots\dots\dots \qquad (4.16)$$

where in equation (4.16) l_t is the rate of growth of the labour force; τ_t is the rate of growth of labour productivity; γ_0 and γ_1 are parameters to be estimated that measure the degree of sensitivity of $g_{n,t}$ with respect to l_t and τ_t; and $\varepsilon_{2,t}$ is the error term.

Given that τ_t is measured as the rate of growth of output per person employed, then by definition we also have that:

$$\tau_t \equiv r_t + h_t \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots\dots\dots\dots\dots\dots \qquad (4.17)$$

where r_t denotes the rate of growth of output per number of hours worked –which can be considered as another measure of labour productivity– and h_t is the rate of growth of hours worked per person employed.

Hence, it is also possible to estimate the following model:

$$g_{n,t}{}' = \beta_{1,t}{}' = \gamma_2(l_t) + \gamma_3(r_t) + \gamma_4(h_t) + \varepsilon_{3,t} \quad \dots\dots\dots\dots\dots \quad (4.18)$$

where in equation (4.18) $\varepsilon_{3,t}$ is the error term; and γ_2,γ_3, γ_3 are parameters to be estimated that measure the degree of sensitivity of $g_{n,t}{}'$ with respect to l_t, r_t, and h_t, respectively.[55]

In this sense, it is also possible to provide an alternative interpretation of the parameters γ_0 and γ_1 shown in equation (4.16) and of the parameters γ_2, γ_3, and γ_4 shown in equation (4.18). The coefficients γ_0 and γ_1 measure the sensitivity of the actual rate of growth g_t with respect to l_t and τ_t, respectively, assuming that the unemployment rate remains constant.

---

[55] The estimation of equation (4.18) was only possible for countries in which the series for the number of hours worked are available (see below).

Likewise, the coefficients γ_2, γ_3, and γ_4 measure the elasticity of g_t associated with l_t, r_t, and h_t, respectively, assuming that there are no changes in the unemployment rate –that is, when $\Delta\%U_t = 0$.

We have estimated equations (4.15) and (4.16) simultaneously using MLE via the Kalman filter. Hence, the measurement and transition equations in this case are:

$$\mathbf{y_t} = \mathbf{H_t B_t} + \boldsymbol{\varepsilon}_\mathbf{t} \quad \dots\dots\dots\dots\dots \quad (4.19)$$

$$\mathbf{B_t} = \mathbf{B_{t-1}} + \mathbf{v_t} \quad \dots\dots\dots\dots\dots \quad (4.20)$$

where $\mathbf{y_t} = [g_t\ ]$, $\mathbf{H_t} = [1 \quad \Delta\%U_t\ ]$; $\mathbf{B_t} = \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \end{bmatrix}$; $\boldsymbol{\varepsilon}_\mathbf{t} = \left[\varepsilon_{1,t}\ \right]$; $\mathbf{B_{t-1}} = \begin{bmatrix} \gamma_0(l_t) + \gamma_1(\tau_t) \\ \beta_{2,t-1} \end{bmatrix}$; $\mathbf{v_t} = \begin{bmatrix} \varepsilon_{2,t} \\ v_{2,t} \end{bmatrix}$.

In the same vein, using equations (4.15) and (4.18), we have the following state-space model in matrix form:

$$\mathbf{y_t} = \mathbf{H_t B_t}' + \boldsymbol{\varepsilon}_\mathbf{t}' \quad \dots\dots\dots\dots\dots \quad (4.21)$$

$$\mathbf{B_t}' = \mathbf{B_{t-1}}' + \mathbf{v_t}' \quad \dots\dots\dots\dots\dots \quad (4.22)$$

where $\mathbf{B_t}' = \begin{bmatrix} \beta_{1t}' \\ \beta_{2t} \end{bmatrix}$; $\boldsymbol{\varepsilon}_\mathbf{t}' = \left[\varepsilon_{1,t}\ '\right]$; $\mathbf{B_{t-1}}' = \begin{bmatrix} \gamma_2(l_t) + \gamma_3(r_t) + \gamma_4(h_t) \\ \beta_{2,t-1} \end{bmatrix}$; $\mathbf{v_t}' = \begin{bmatrix} \varepsilon_{3,t} \\ v_{2,t} \end{bmatrix}$; and $\varepsilon_{1,t}\ '$ is the error term associated with this estimation.

The transition equations (4.20) and (4.22) also show the dynamics of the time-varying Okun coefficient on unemployment (that is, $\beta_{2,t}$). Following the standard approach to estimating

TVPMs (Kim and Nelson, 1999), we have assumed that the respective β_{2t} are random walks, so that $\beta_{2t} = \beta_{2,t-1} + v_{2,t}$.[56]

4.4 Empirical results

This Section shows: 1) the time-varying natural rates of growth obtained using the rolling regressions technique (Section 4.4.1) and the Kalman filter (Section 4.4.2); and 2) the estimation results obtained from the decomposition analysis of the different $g_{n,t}$s, which are presented in Section 4.4.2.

We have employed the same dataset that was employed in Chapters 2 and 3. The series for l_t were obtained from the World Bank electronic database; whereas the series for r_t and h_t were extracted from The Conference Board Total Economy Database of the Groningen Growth and Development Centre (henceforth TED).

Equations (4.19) and (4.20) were estimated for the 13 Latin American countries considered in the two previous Chapters. On the other hand, the estimation of equations (4.21) and (4.22) has only been possible for the following countries: Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela. This was so because the series for number of hours worked per person employed are available only for these 7 countries via the TED.

4.4.1 Time-varying natural rates of growth using rolling regressions

We first estimated equation (4.15) using rolling regressions. The rolling regression technique consists in estimating an equation in several overlapped sub-periods of equal size. The idea

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[56] For the case of Venezuela we assumed that $\beta_{2t} = \chi$ in both transition equations (equations (4.20) and (4.22)), so that the Okun coefficient on unemployment was assumed to be a constant parameter. This was necessary in order to achieve convergence of the MLE and to generate unique covariance coefficients.

behind this methodology is to estimate the parameter of interest –that is, $\beta_{1,t}$ in our case– by using equation (4.15) across different sampling periods that have identical temporal dimensions or window sizes. In this chapter we have selected 15 as the window size.[57]

The results for the 13 Latin American countries obtained using rolling regressions are presented in Figures 4A.1 to 4A.13 in the Appendix. The estimated $g_{n,t}$s do not present statistically significant trends in Argentina, Brazil, Uruguay and Venezuela; whereas they show a significant downward trend in Chile, Colombia, Paraguay, and a significant upward trend in Bolivia, Costa Rica, Ecuador, Mexico, Nicaragua, and Peru.

However, rolling regressions present some problems. As Zanin and Marra (2012) have mentioned, window size choice is one of the main drawbacks of rolling regressions because: a) it can heavily affect the behaviour of the estimates over time; b) it does not allow to obtain parameter estimates for the whole period of observation; and c) although rolling regressions are typically employed to obtain time-varying coefficients, the estimates within the chosen samples are assumed to be constant.

For these reasons it is more appropriate to use the Kalman filter in order to retrieve estimates of the different $g_{n,t}$s for the period of study. These are presented in the following section.

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[57] This is in line with the duration of one business cycle according to the Kuznets (1930) investment cycle or Kuznets swing, which asserts that the duration of a typical business cycle lasts between 15 and 25 years. According to Kuznets (1930), the waves are connected with demographic processes (such as immigrant inflows/outflows, and the changes in construction intensity that the latter cause). Empirical evidence (Schnabel, 2002; Knotek, 2007) has found that selecting a different length for the window size (such as 10 or 13 years) has minimal impact on the results. It seems to be that what matters for the results is the number of quarters that the economy is in recession within each window size. Since we are using annual data, this issue has not been addressed here.

4.4.2 A decomposition analysis of the natural rate of growth using the Kalman filter

We followed the strategy implemented by Kim and Nelson (1989, 1999) in order to obtain the Kalman filter estimates of the models described in equations (4.19) to (4.22). Thus, we first run the Kalman filter in order to obtain the respective innovation variances and the initial values of the parameters to be estimated in equations (4.19) and (4.20) and in equations (4.21) and (4.22). In the subsequent step, the Kalman filter was run again with the preceding estimates of $\boldsymbol{\varepsilon}_\mathbf{t}$ and $\mathbf{v}_\mathbf{t}$ (or $\boldsymbol{\varepsilon}_\mathbf{t}{}'$ and $\mathbf{v}_\mathbf{t}{}'$), the initial values of the parameters and their respective variance-covariance matrices in order to obtain the evolutionary coefficients of the models.

However, the aforementioned procedure yielded non-unique covariance coefficients for the majority of cases (the only exceptions were Ecuador, Venezuela, and the estimation of equations (4.19) and (4.20) for Argentina). Therefore, for the great majority of cases it was necessary to first specify the initial values of the innovation variances[58] and then run again the Kalman filter given these estimates.

The different $g_{n,t}$s obtained and the respective g_ts are plotted in the Figures below, where gt denotes the actual rate of growth, $gnt\ 1 = g_{n,t}$, and $gnt\ 2 = g'_{n,t}$.[59]

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[58] As explained in EViews (2014), there are no general rules that can be followed in order to specify the initial conditions and, therefore, their choice is to some degree arbitrary. We selected the highest fractional number that generated unique covariance coefficients in the MLE. For the estimation of equations (4.19) and (4.20), the values selected were: $4.54X10^{-5}$ for Bolivia, Brazil, Colombia, Mexico, Nicaragua, Uruguay; 0.14 for Costa Rica and Paraguay; and $9.12X10^{-4}$ for Chile and Peru. On the other hand, for the estimation of equations (4.21) and (4.22) the initial values were: $4.54X10^{-5}$ for Argentina, Brazil, Colombia, Mexico, Peru; and 0.14 for Chile. We also used different initial conditions (using, for example, higher fraction numbers), but the results obtained were fairly similar to the ones here presented.

[59] We show the filtered estimates of the $g_{n,t}$s since this filter takes into account only the information available up to t, which is more consistent with the idea that the values of g_n are influenced by previous values of g_t (and not by values of g_t in the whole sample, as considered by the smoothing filter). Moreover, the results obtained from the smoothing filter are very similar for the majority of countries.

Figures 4.1. to 4.13. Actual rates of growth and time-varying natural rates of growth in Latin American countries

Figure 4.1. Argentina

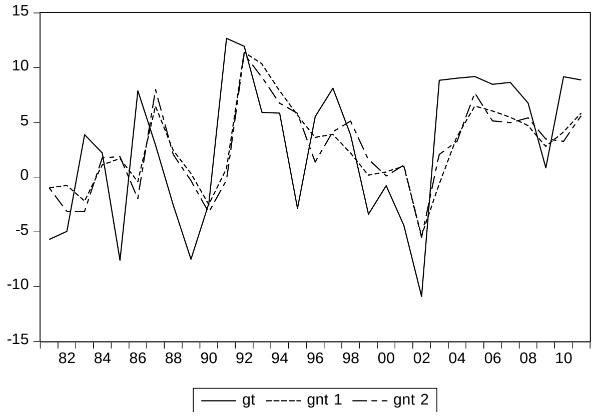


Figure 4.2. Bolivia

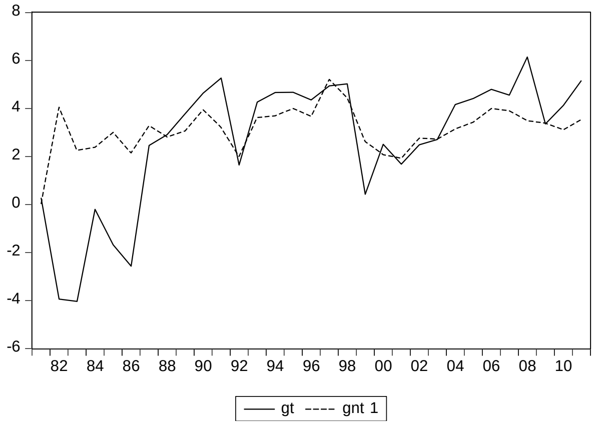


Figure 4.3. Brazil

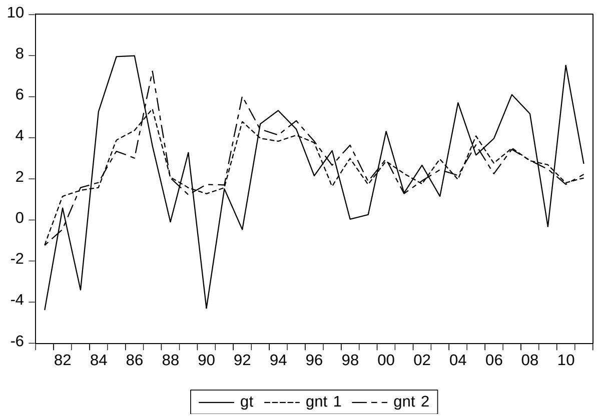


Figure 4.4. Chile

Figure 4.5. Colombia

Figure 4.6. Costa Rica

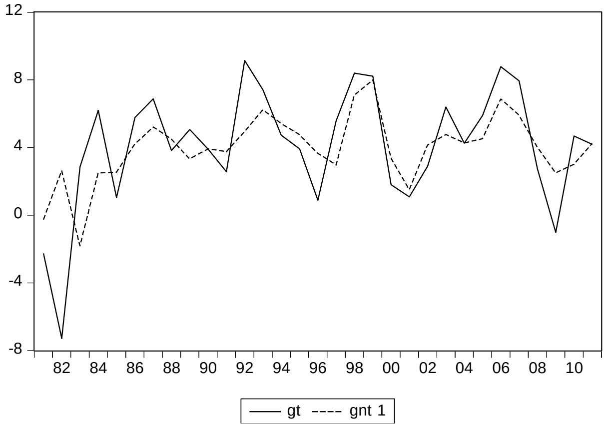


Figure 4.7. Ecuador

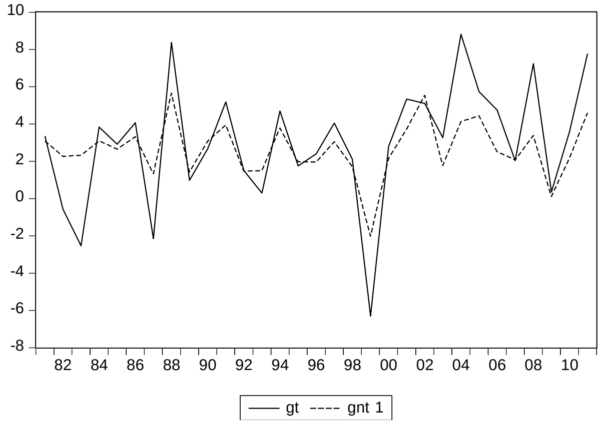


Figure 4.10. Paraguay

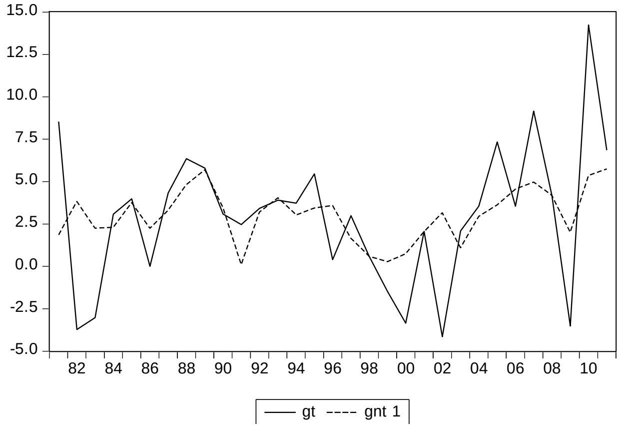


Figure 4.8. Mexico

Figure 4.11. Peru

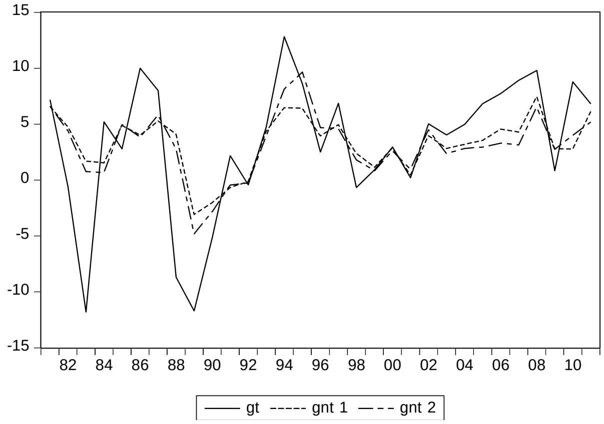


Figure 4.9. Nicaragua

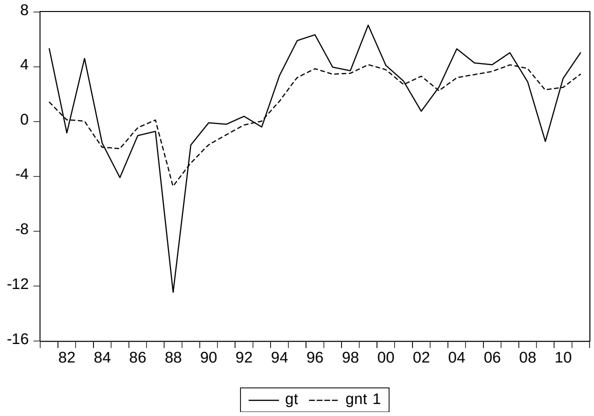


Figure 4.12. Uruguay

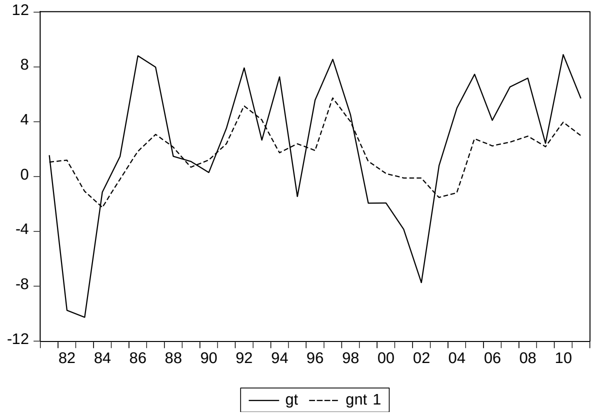
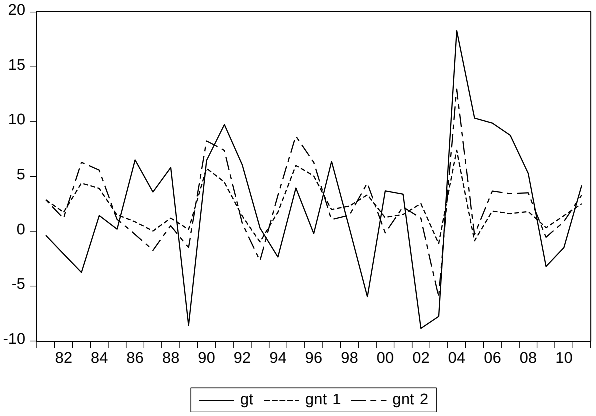


Figure 4.13. Venezuela



In Table 4.1 we show the first two sample moments of all series presented in the Figures above. It can be seen that the standard deviation of the estimated $g_{n,t}$s is lower than the standard deviation of the respective g_ts in all cases, which is consistent with the idea that the g_n should experience a lower amount of variation around the mean than the g_t.

Table 4.1. Mean and standard deviation of the actual rate of growth (g_t) and the estimated time-varying natural rates of growth measuring labour productivity as output per person employed ($g_{n,t}$) and as output per hours worked ($g_{n,t}'$)

Country	g_t	$g_{n,t}$	$g_{n,t}'$
Argentina	Mean=2.82; Std. deviation=6.50	Mean=2.77; Std. deviation=3.77	Mean=2.66; Std. deviation=3.95
Bolivia	Mean=2.68; Std. deviation=2.75	Mean=3.13; Std. deviation=0.96	-[a]
Brazil	Mean=2.62; Std. deviation=3.26	Mean=2.63; Std. deviation=1.36	Mean=2.67; Std. deviation=1.66
Chile	Mean=4.81; Std. deviation=4.33	Mean=4.38; Std. deviation=1.77	Mean=4.32; Std. deviation=2.57
Colombia	Mean=3.54; Std. deviation=2.25	Mean=3.39; Std. deviation=0.93	Mean=3.43; Std. deviation=1.02
Costa Rica	Mean=4.08; Std. deviation=3.53	Mean=3.96; Std. deviation=1.99	-[a]
Ecuador	Mean=3.02; Std. deviation=3.25	Mean=2.65; Std. deviation=1.53	-[a]
Mexico	Mean=2.47; Std. deviation=3.59	Mean=2.27; Std. deviation=1.50	Mean=2.28; Std. deviation=1.49
Nicaragua	Mean=1.82; Std. deviation=3.92	Mean=1.46; Std. deviation=2.43	-[a]
Paraguay	Mean=2.96; Std. deviation=4.16	Mean=3.03; Std. deviation=1.55	-[a]
Peru	Mean=3.22; Std. deviation=6.08	Mean=3.27; Std. deviation=2.51	Mean=3.10; Std. deviation=2.96
Uruguay	Mean=2.35; Std. deviation=5.28	Mean=1.71; Std. deviation=1.91	-[a]
Venezuela	Mean=2.25; Std. deviation=6.15	Mean=2.19; Std. deviation=0.65	Mean=2.48; Std. deviation=0.52

[a] Not estimated since the series for number of hours worked per person employed are not available in these cases.

Figures A.1 to A.13 in the Appendix present the average of the estimates of $g_{n,t}$ in order to compare the results obtained via the Kalman filter with the ones obtained from the rolling regressions technique. From these Figures it is possible to observe that both techniques offer similar results in the majority of countries –the only exceptions being Bolivia in the first third of the period shown in Graph 4A.2; Ecuador, Mexico and Nicaragua in the last two-thirds of the period shown in Graphs 4A.7, 4A.8 and 4A.9, respectively; and Uruguay during the whole period shown in Graph 4A.12 (since in this country the Kalman filter estimates seem to be smoother than the ones obtained using the rolling regressions technique). Thereby, we can be confident that the results obtained are robust.

In Table 4.2 we present the coefficient estimates obtained via the MLE of equations (4.16) and (4.18). Following Engle and Watson (1981) and Kim and Nelson (1989, 1999), we have also corroborated the appropriateness of the specified models checking for the lack of serial correlation and of heteroskedasticity in the standardized one-period-ahead-forecast errors. These results are presented in Table 4.3, which show that: 1) the null hypothesis of no serial correlation (up to order 2) of the Ljung-Box test is not rejected at the 10% level of significance; and 2) the standardized prediction errors do not exhibit autoregressive conditional heteroscedastic (ARCH) effects since the null hypothesis of no conditional heteroscedasticity of the ARCH test is not rejected at the 10% level of significance. This means that the results suggest no evidence of model misspecification for all countries.

Table 4.2. A decomposition analysis of the natural rate of growth in Latin American countries

Country	**Equation (4.16)** $g_{n,t} = \gamma_0(l_t) + \gamma_1(\tau_t) + \varepsilon_{2,t}$		**Equation (4.18)** $g_{n,t}{}' = \gamma_2(l_t) + \gamma_3(r_t) + \gamma_4(h_t) + \varepsilon_{3,t}$		
	$\boldsymbol{\gamma_0}$	$\boldsymbol{\gamma_1}$	$\boldsymbol{\gamma_2}$	$\boldsymbol{\gamma_3}$	$\boldsymbol{\gamma_4}$
Argentina	1.173*	0.528***	1.135***	0.669***	0.251***
Bolivia	1.115***	0.114***	-[a]	-[a]	-[a]
Brazil	0.992***	0.418***	0.918***	0.469***	0.010***
Chile	1.201***	0.517***	1.206***	0.534***	0.269*
Colombia	0.910***	0.573***	0.865***	0.608***	0.491***
Costa Rica	1.007***	0.535***	-[a]	-[a]	-[a]
Ecuador	0.618**	-0.035	-[a]	-[a]	-[a]
Mexico	0.677***	0.087***	0.676***	0.107***	0.037***
Nicaragua	0.750***	0.259***	-[a]	-[a]	-[a]
Paraguay	0.986***	0.332**	-[a]	-[a]	-[a]
Peru	0.891***	0.452***	0.698***	0.121***	0.439***
Uruguay	0.910***	0.420***	-[a]	-[a]	-[a]
Venezuela	0.531**	-0.131	0.471*	-0.142	-0.351

[a]Not estimated since the series for number of hours worked per person employed are not available in these cases.

*, **, *** Respectively denote statistical significance at the 10%, 5% and 1% level.

Table 4.3. Correct specification tests on the one-period-ahead forecast errors obtained from the different state-space models

	Autocorrelation tests[a]				ARCH tests[b]	
	Using equation (4.16)		Using equation (4.18)		Using equation (4.16)	Using equation (4.18)
	Up to order 1	Up to order 2	Up to order 1	Up to order 2		
Argentina	0.88 (0.35)	1.98 (0.37)	0 (0.99)	2.94 (0.23)	3.57 (0.07)	0.19 (0.66)
Bolivia	5.20 (0.02)	7.56 (0.02)	-[c]	-[c]	1.89 (0.18)	-[c]
Brazil	2.47 (0.12)	3.37 (0.19)	2.43 (0.12)	2.43 (0.30)	0 (0.98)	1.58 (0.22)
Chile	0.29 (0.59)	1.77 (0.41)	0.29 (0.59)	2.25 (0.33)	0.42 (0.52)	0.22 (0.64)
Colombia	0.44 (0.51)	0.99 (0.61)	0.55 (0.46)	0.90 (0.64)	0.12 (0.73)	0 (0.99)
Costa Rica	0 (0.99)	0.81 (0.67)	-[c]	-[c]	0.05 (0.82)	-[c]
Ecuador	0.28 (0.60)	0.59 (0.75)	-[c]	-[c]	0.05 (0.83)	-[c]
Mexico	1.11 (0.29)	1.12 (0.57)	1.35 (0.25)	1.36 (0.51)	0.02 (0.90)	0.02 (0.90)
Nicaragua	0.09 (0.71)	0.12 (0.94)	-[c]	-[c]	0.01 (0.91)	-[c]
Paraguay	0.07 (0.79)	0.44 (0.80)	-[c]	-[c]	0.08 (0.78)	-[c]
Peru	0.09 (0.76)	0.95 (0.62)	0.09 (0.76)	0.97 (0.62)	0.04 (0.85)	0.02 (0.88)
Uruguay	5.09 (0.02)	5.84 (0.05)	-[c]	-[c]	0.95 (0.34)	-[c]
Venezuela	0.09 (0.76)	1.12 (0.57)	0.22 (0.64)	2.42 (0.30)	5.65 (0.02)	5.69 (0.02)

[a] Ljung-Box statistic for serial autocorrelation (Ho: no autocorrelation). *P*-values are shown in parenthesis.

[b] F-statistics are shown (Ho: no autoregressive conditional heteroscedastic effects using 1 lag). *P*-values are shown in parenthesis.

[c] The estimation of these equations in the respective countries was not possible because the series for number of hours worked per person employed are not available in these cases (see also Section 4.3).

Given that the estimation results seem to be robust and that there is no evidence of misspecification in the models, we finally proceed to analyse the coefficient estimates obtained via the Kalman filter that are shown in Table 4.2. Firstly, the results obtained show that the coefficients associated with the different components of the $g_{n,t}$s –that is, l_t, τ_t (r_t), and h_t– are positive and statistically significant in the great majority of countries. The only exceptions are Ecuador and Venezuela, which present a negative coefficient associated with τ_t (and both r_t and h_t for the case of Venezuela) that is statistically non-significant.

Secondly, the coefficient on l_t is greater than the one associated with τ_t (r_t) and h_t in all countries. This means that the natural rate of growth in the sample of Latin American countries is more sensitive to labour force growth. The countries that present the highest elasticities of g_n with respect to l_t are Chile, Argentina and Bolivia; whereas Venezuela, Ecuador, and Mexico are the countries with the lowest elasticities.

Thirdly, the coefficients associated with labour productivity are fairly similar irrespective of the way in which the latter is measured (either as τ_t or r_t). The only exception is Peru, where the coefficient associated with τ_t is considerably lower than the one associated with r_t. In this country it is also possible to observe that the coefficient on h_t is greater than the respective coefficient on r_t.

In this sense, with respect to the coefficients associated with τ_t (or r_t), it is possible to observe that Colombia, Costa Rica, Argentina and Chile are the countries that present the highest elasticities of g_n with respect to labour productivity; whereas Mexico, Bolivia, and Nicaragua are the countries that present relatively lower elasticities. In the same vein, Colombia, Peru, and Argentina are countries in which the elasticity of g_n with respect to the rate of growth of hours worked per person employed is high; whereas Mexico and Brazil present low elasticities.

Finally, we have tried to identify some relationships between the results obtained and other variables in the different countries. However, there seems to be no evidence of a clear pattern since, for example, countries that present high (Chile, Costa Rica and Colombia) or low

(Nicaragua, Venezuela and Uruguay) rates of output growth during the period of study are not directly related to the countries in which the highest or lowest elasticities of the components of the natural rate of growth were found –although Colombia and Costa Rica are countries that present high elasticity of labour productivity growth, and Chile presents high elasticity of labour force growth. Likewise, there does not seem to be any relationship with respect to the countries with high (Colombia, Uruguay and Venezuela) or low unemployment rates (Costa Rica, Paraguay and Mexico), nor with the countries that present higher (Argentina, Brazil and Venezuela) or lower (Bolivia, Chile and Ecuador) changes in the percentage level of unemployment.[60]

The latter means that plausible explanations of the results here found may be associated with the composition of the labour force and/or labour participation rates in the different countries; and with labour productivity levels in the different sectors of the economies. We leave these topics for future research.

4.5 Conclusions

The present chapter has tried to: 1) estimate a time-varying natural rate of growth using rolling regressions and the Kalman filer; and 2) decompose the sensitivity of the natural rate of growth with respect to its individual components: the rate of growth of the labour force and the rate of growth of labour productivity in a sample of 13 Latin American countries

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[60] As explained in Chapter 2, most of the unemployment rate series for Latin American countries were extracted from the World Development Indicators database, which in turn were extracted from the International Labour Organization database. Thus, it is important to bear in mind that national definitions of unemployment may differ from the recommended international standard definition. The national definitions used vary from one country to another as regards inter alia age limits, reference periods, criteria for seeking work, treatment of persons temporarily laid off and of persons seeking work for the first time. Specifically, differences between countries with regard to the treatment of unemployed persons with respect to classification by status in employment are particularly pronounced (see International Labour Organization, 1982).

during the period of 1981-2011. We have estimated a time-varying parameter model using rolling regressions and the Kalman filter, and we have employed maximum likelihood estimation in order to compute the coefficients associated with the components of the time-varying natural rate of growth. We have also separated the components of labour productivity growth –the rate of growth of output per number of hours worked and the rate of growth of hours worked per person employed– for 7 countries for which it has been possible to find data for the number of hours worked.

In the first place, the estimated time-varying natural rates of growth obtained via the Kalman filter and rolling regressions offer a similar picture, so that the results seem to be robust for the majority of countries of study. If we consider the variation of the natural rate of growth over consecutive periods of fifteen years, then both the Kalman filter and the rolling regressions show that the natural rate of growth: 1) has remained constant in Argentina, Uruguay and Venezuela; 2) has decreased in Chile, Colombia and Paraguay; and 3) has increased in Bolivia, Costa Rica, Nicaragua and Peru. The results for Brazil, Ecuador and Mexico are inconclusive since both econometric techniques offer different results regarding the trend of the natural rate of growth in these countries.

Secondly, with respect to the decomposition analysis of the natural rate of growth, the most important finding is that the results show that the natural rates of growth in Latin American countries are more sensitive to labour force growth: Chile, Argentina and Bolivia are the countries that present the highest elasticities of the actual rate of growth that keeps the unemployment rate constant (that is, the natural rate of growth) with respect to the rate of growth of labour force; whereas Venezuela, Ecuador and Mexico are the countries with the lowest elasticities.

Thirdly, the elasticity of the natural rate of growth with respect to productivity growth (measured either as output per worker or output per hour worked) is relatively high in Colombia, Costa Rica, Argentina and Chile; whereas it is relatively low in Mexico, Bolivia and Nicaragua. The natural rate of growth does not seem to be sensitive with respect to labour productivity growth in Ecuador and Venezuela; and in Peru the elasticity associated with the rate of growth of number of hours worked is greater than the one related to labour productivity growth during the period of study.

Appendix CHAPTER 4

Figures 4A.1 to 4A.13. Time-varying natural rate of growth estimates using Rolling Regressions (RR); and 15-year averages of the results obtained via the Kalman filter measuring labour productivity as output per person employed (KF 1) and as output per hours worked (KF 2). (Years in the x-axis denote the last year included in the estimation).

Figure 4A.1. Argentina

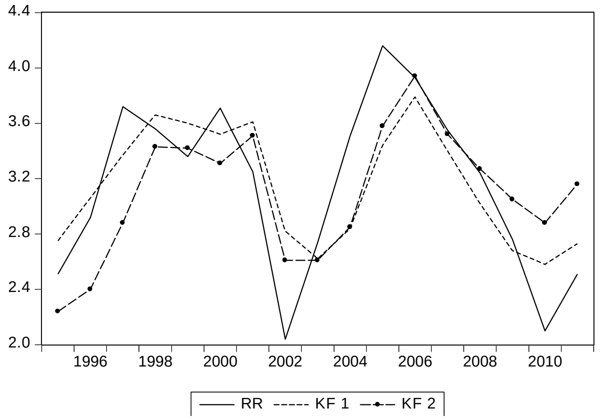


Figure 4A.3. Brazil

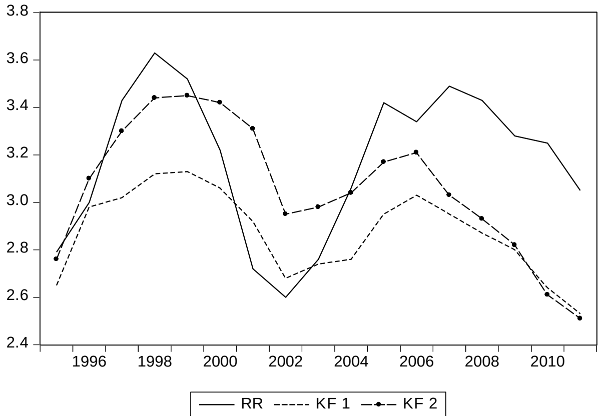


Figure 4A.2. Bolivia

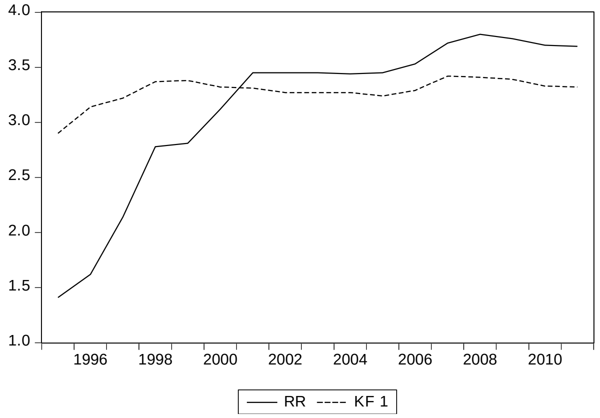


Figure 4A.4. Chile

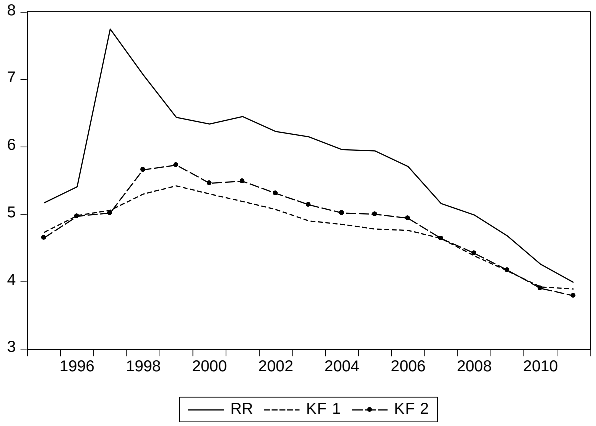


Figure 4A.5. Colombia

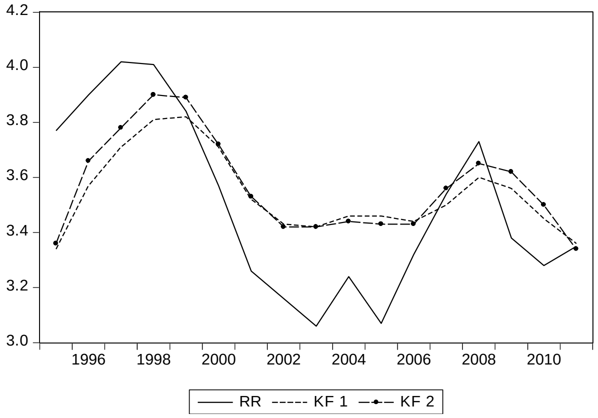


Figure 4A.6. Costa Rica

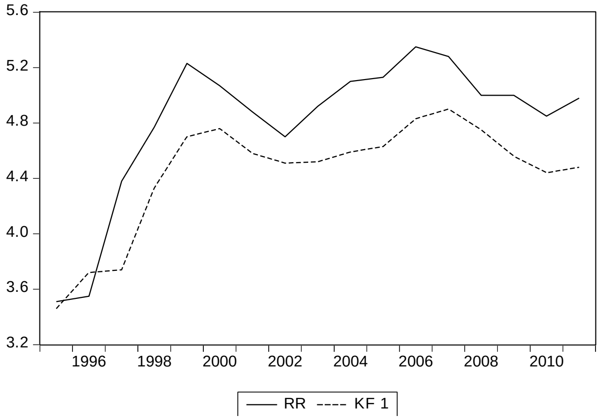


Figure 4A.7. Ecuador

Figure 4A.8. Mexico

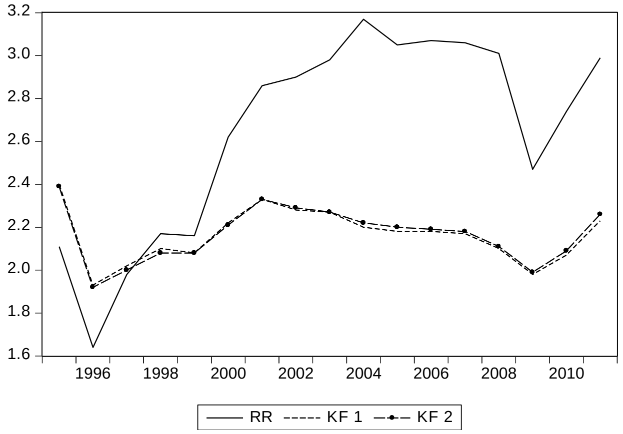


Figure 4A.9. Nicaragua

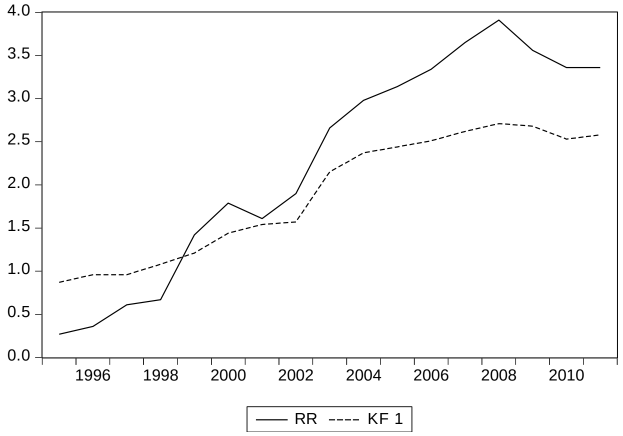


Figure 4A.10. Paraguay

Figure 4A.11. Peru

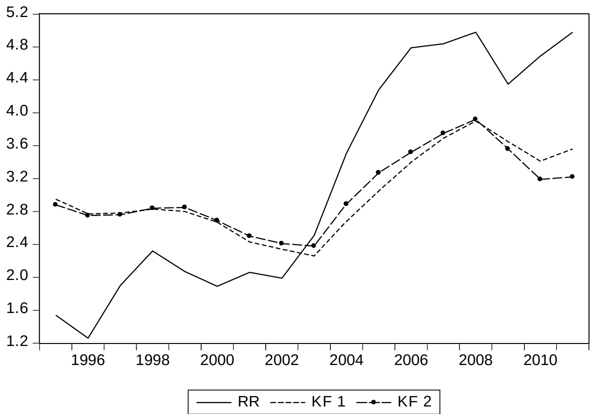


Figure 4A.12. Uruguay

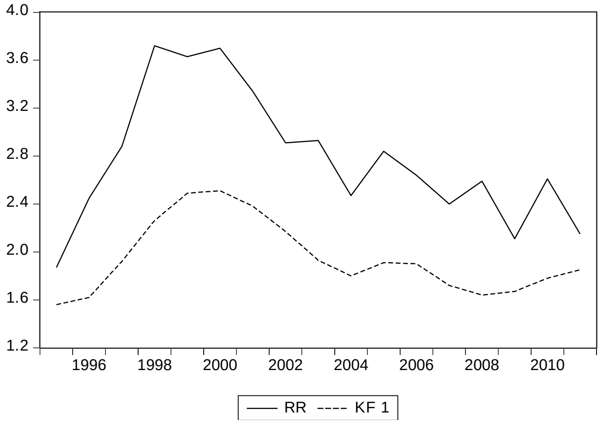
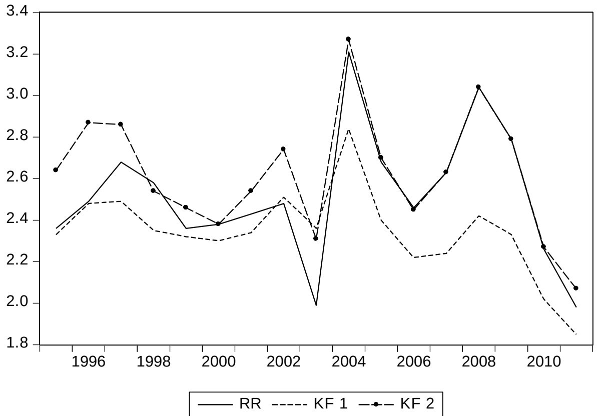


Figure 4A.13. Venezuela



CHAPTER 5

Do the Components of the Natural Rate of Growth Differ according to the Sources of Demand? An Analysis for Latin American Countries, 1981-2011

5.1 Introduction

The hypothesis of endogeneity of the natural rate of growth considers that the individual components of the latter react endogenously to the rate of growth of aggregate demand. In this Chapter we are interested in studying the interactions between the components of the natural rate of growth (henceforth g_n) –that is, the rate of growth of labour force (henceforth l_t) and the rate of growth of labour productivity (henceforth r_t)– and the components of the rate of growth of aggregate demand –that is, the rate of growth of exports (henceforth x_t), the rate of growth of government expenditure (henceforth gov_t), the rate of growth of investment (henceforth i_t), and the rate of growth of household consumption expenditure (henceforth c_t)– in Latin American countries during the period 1981-2011.

This is the first time that an analysis of the interactions between the components of g_n and the components of the rate of growth of aggregate demand has been carried out. In this sense, the main contribution of the current Chapter is in using two different econometric approaches that try to unveil which of the components of the rate of growth of aggregate demand are more important in explaining fluctuations in l_t and r_t in Latin American countries.

Firstly, we use the Vector Autoregression (henceforth VAR) methodology as a tool to provide a description of the statistical relationships between the components of aggregate demand and the components of g_n. This econometric technique has been used in the macroeconomics literature as a systematic way to capture rich dynamics in multiple time series (Stock and Watson, 2001) since it offers a very general framework to analyse co-movements of, and dynamic interactions amongst, a group of variables with few *a priori* restrictions (Basu et al., 2013). Specifically, we compute: 1) Granger-causality statistics in order to examine whether the lagged values of x_t, gov_t, i_t and c_t help to predict l_t and r_t; and 2) the responses of both l_t and r_t to shocks in the different components of aggregate demand by using impulse-response analysis.

Secondly, following Harding and Pagan (2002), we compute a measure of pro-cyclicality between the different components of the rate of growth of aggregate demand with respect to the components of g_n. This measure takes into account the fraction of time that the series of interest are simultaneously in the same state of expansion or contraction. This enables us to describe which components of the rate of growth of aggregate demand are more strongly associated with the individual components of the g_n, considering both expansions and recessions. In order to apply this methodology, it is necessary to: 1) locate turning points – that is, expansions and recessions– in the series of interest (x_t, gov_t, i_t, c_t, l_t and r_t); and 2) compute an index of concordance between the individual components of the rate of growth of aggregate demand and the l_t and r_t series.

The remainder of this Chapter is structured as follows. Section 5.2 provides a discussion of the topic under study and specifies some a priori predictions. In Section 5.3 we describe the two different econometric methodologies employed. Section 5.3.1 describes the Vector Autoregression models, Granger non-causality tests and impulse-response analysis; whereas Section 5.3.2 describes the methodology employed to compute the indexes of concordance. Section 5.4 presents the results of both econometric techniques. Finally, in Section 5.5 we summarise the main findings and present the conclusions.

5.2 A brief preliminary discussion

As mentioned before, this is the first time that an analysis of the interactions between the components of g_n and the components of the rate of growth of aggregate demand has been carried out.[61] We believe that the specification of a priori theoretical hypotheses regarding which component of g_n is more sensitive to the different components of the rate of growth of aggregate demand is not straightforward. However, the theoretical elements that underlie the hypothesis of endogeneity of g_n provide us some insights that are useful in order to discuss the possible reactions of the rate of growth of labour productivity.

As discussed in Chapter 3, labour productivity reacts to increases/decreases in the actual rate of growth via the Kaldor-Verdoorn law. Different models of circular and cumulative causation explain that the rate of growth of exports plays a major role in order to reinitiate the cycle that generates increases in output growth, higher productivity growth, reductions of unit labour costs, reductions in prices, and further increases in the competitiveness of the country or region. Thereby, a priori we expect to find that the rate of growth of labour productivity is more sensitive to increases in the rate of growth of exports than to the other components of the rate of growth of aggregate demand.

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[61] Other studies have tried to analyse the linkages between: 1) actual growth rates and the allocation of resources amongst sectors of the economy (Acevedo et al., 2009); and 2) actual growth rates and income distribution (Nishi, 2011). Acevedo et al. (2009) estimate a panel of 18 Latin American counties over the period 1950-2006, thus providing some evidence of the sectors that can be considered as “drivers of growth”. They find: 1) that besides the manufacturing sector, certain groups of services can also play the role of “leading sectors”; and 2) a consistently low or negligible relationship between primary resource sectors and economic growth.
On the other hand, Nishi (2011) studies income distribution and the demand formation pattern of the Japanese economy by estimating a VAR model with the following variables: wage share, capital accumulation rate, and the growth rates of exports, consumption, GDP, and government expenditure. His results indicate that during the period of study (1985-2008) the Japanese economy has experienced a profit-led demand formation pattern.

5.3 Econometric methodologies: an overview

5.3.1 Vector Autoregression models, Granger causality tests and generalized impulse response functions[62]

A univariate autoregression is a single-equation, single-variable linear model in which the current value of a variable is explained by its own lagged values. A VAR model is an n-variable linear model in which each variable is explained by its own lagged values, plus current and past values of the remaining $n-1$ variables. As Stock and Watson (2001) explain, VAR models can be used to describe and summarize macroeconomic data, make macroeconomic forecasts, quantify the known (or unknown) facts regarding the true structure of the macroeconomy, and to provide policy analysis.

VARs come in three varieties: reduced form, recursive and structural. A reduced form VAR expresses each variable as a linear function of its own past values, the past values of all other variables being considered and a serially uncorrelated error term. On the other hand, a recursive VAR constructs the error term in each regression equation to be uncorrelated with the error term in the preceding equations by judiciously including some contemporaneous values as regressors. Finally, a structural VAR uses economic theory to sort out the contemporaneous links amongst the variables in order to differentiate between correlation and causation.[63]

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[62] This section relies heavily upon Stock and Watson (2001), Enders (2010), Stata (2011) and Basu et al. (2013).

[63] This is known as the "identification problem", which allows correlations to be interpreted causally (Stock and Watson, 2001).

In this chapter we have only employed a reduced form VAR and recursive VARs in order to describe and summarize the co-movements of the variables of interest: x_t, gov_t, i_t, c_t, l_t and r_t.[64] Thus, collecting the aforementioned six variables in the (6X1) vector $\mathbf{Y_t}$,

$$\mathbf{Y_t} = \begin{bmatrix} x_t \\ gov_t \\ i_t \\ c_t \\ l_t \\ r_t \end{bmatrix} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5.1)$$

the empirical model becomes

$$\mathbf{Y_t} = \mathbf{A_0} + \mathbf{A_1}\mathbf{Y_{t-1}} + \cdots + \mathbf{A_p}\mathbf{Y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t} \quad \dots\dots\dots\dots \quad (5.2)$$

where $\mathbf{Y_t}$ is the (6X1) vector appearing in (5.1); $\mathbf{A_0}, \mathbf{A_1}, \dots, \mathbf{A_p}$ are (1X6) coefficient matrices, and $\boldsymbol{\varepsilon_t}$ is the (6X1) vector of errors with $\mathbf{E}(\boldsymbol{\varepsilon_t}) = \mathbf{0}$, $\mathbf{E}(\boldsymbol{\varepsilon_t}\boldsymbol{\varepsilon}'_s) = \mathbf{0}$ for $s \neq t$, and $\boldsymbol{E}(\boldsymbol{\varepsilon_t}\boldsymbol{\varepsilon}'_t) = \Sigma_{\boldsymbol{\varepsilon}}$, where $\Sigma_{\boldsymbol{\varepsilon}}$ is a positive definite covariance matrix.

Firstly, we examine whether the lagged values of x_t, gov_t, i_t and c_t help to predict l_t and r_t using Granger non-causality tests.[65] Specifically, in the context of the VAR model shown in equation (5.2), we employ a multivariate generalization of the Granger causality test called block-causality test.[66] The latter restricts all lags of one variable (for example, x_t, gov_t, i_t or c_t) in the equation of interest (that is, in the l_t or r_t equations) to be equal to zero. This cross-

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[64] Section 5.4.1 presents a description of the series employed.

[65] It is important to mention that Granger causality is something different from a test for exogeneity. For a variable –say, z_t– to be exogenous, we would require that it not be affected by the contemporaneous value of another variable –say, y_t. However, Granger causality refers only to the effects of past values of the sequence $\{y_t\}$ on the current value of z_t. In other words, Granger causality actually measures whether current and past values of the sequence $\{y_t\}$ help to forecast future values of sequence $\{z_t\}$.

[66] As Enders (2010) explains, given the distinction between exogeneity and Granger causality, a block-exogeneity test should actually be called block-causality test.

equation restriction is then properly tested using the following Likelihood Ratio (henceforth LR) test:

$$(T-c)(\ln|\Sigma_r| - \ln|\Sigma_u|) \qquad \dots\dots\dots\dots\dots\dots. \qquad (5.3)$$

where T is the number of observations; $c = 6p + 1$, where p is the lag length of the VAR model; and Σ_r and Σ_u are the variance/covariance matrices of the restricted and unrestricted systems, respectively. This statistic has a χ^2 distribution with degrees of freedom equal to p.

Therefore, the block-causality test presented in equation (5.3) is estimated as follows: 1) the l_t and r_t equations are estimated using p lagged values of the sequences $\{x_t\}$, $\{gov_t\}$, $\{i_t\}$, $\{c_t\}$, $\{l_t\}$ and $\{r_t\}$ in order to calculate Σ_u; 2) the l_t and r_t equations are re-estimated excluding the lagged values of the variable of interest (that is, $\{x_t\}$, $\{gov_t\}$, $\{i_t\}$ or $\{c_t\}$) in order to calculate Σ_r.

Secondly, we estimate the responses of both l_t and r_t to shocks in the different components of the rate of growth of aggregate demand by using impulse-response analysis. Following the standard practice in VAR analysis, we have considered only the interactions amongst the innovations to capture contemporaneous relationships rather than the interaction between the endogenous variables.[67]

Let us explain the notion behind the impulse-response analysis. If the VAR system presented in equation (5.2) is stable (stationary) –that is, if all roots have modulus less than one and lie

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[67] The reduced form VAR depicted in equation (5.2) implies that there are no direct contemporaneous relationships amongst the six endogenous variables. Contemporaneous interactions can be captured either by interactions amongst the endogenous variables (which requires the estimation of a structural VAR) or interactions amongst the innovations in the error terms (contained in the vector $\boldsymbol{\varepsilon_t}$), or both.

inside the unit circle– then it is possible to re-write the model as a Vector Moving Average (VMA):

$$\mathbf{Y_t} = \boldsymbol{\mu} + \sum_{\boldsymbol{i=0}}^{\infty} \boldsymbol{\varphi_i \varepsilon_{t-i}} \quad \dots\dots\dots\dots \quad (5.4)$$

where $\boldsymbol{\mu}$ is the (6X1) time-invariant mean of the vector $\mathbf{Y_t}$. This formulation is important since it allows us to trace out the time path of the various shocks on the variables contained in the VAR system, so that equation (5.4) is a useful tool to describe how the innovations to one variable affect another variable after a given number of periods.

The coefficients of $\boldsymbol{\varphi_i}$ are called Impulse-Response Functions (henceforth IRFs) and can be used to generate the effects of $\boldsymbol{\varepsilon_t}$ shocks on the entire time paths of the $\{\mathbf{Y_t}\}$ sequences in order to examine the dynamic interactions amongst the variables in the VAR. Specifically, the coefficients $\varphi_{jk}(i)$ represent the impact of a one-unit increase in the k-th element of the $\boldsymbol{\varepsilon_t}$ vector on the j-th element of $\mathbf{Y_t}$ after i periods, holding everything else constant.[68] Thus, plotting the IRFs (that is, plotting the coefficients of $\varphi_{jk}(i)$ against i) is a practical way to visually represent the behaviour of the series of interest in response to the various shocks.

However, the crucial assumption that everything else is held constant is not satisfied in a reduced form VAR model since the $\boldsymbol{\varepsilon_t}$ are contemporaneously correlated, so that a shock to one variable is likely to be accompanied by shocks to some of the other variables. Therefore, it is necessary to impose an additional restriction on the VAR system in order to identify the

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[68] In this sense, the elements $\varphi_{jk}(0)$ represent impact multipliers since they show the instantaneous impact of a one-unit increase in the k-th element of $\boldsymbol{\varepsilon_t}$ on the j-th element of $\mathbf{Y_t}$. In the same way, the elements $\varphi_{jk}(1)$ are the one-period responses of unit changes in the k-th element of $\boldsymbol{\varepsilon_{t-1}}$ ($\boldsymbol{\varepsilon_t}$) on the j-th element of $\mathbf{Y_t}$ ($\mathbf{Y_{t+1}}$), holding everything else constant.

IRFs. Sims (1980) proposed to use the Choleski decomposition of $\Sigma_{\boldsymbol{\varepsilon}}$, which yields the Orthogonalized IRFs (henceforth OIRFs). The latter is equivalent to imposing a recursive structure for the corresponding dynamic structural equation model in which the ordering of the recursive structure is the same as the ordering imposed in the Cholesky decomposition. This decomposition forces a potentially important asymmetry on the system since the order imposed is arbitrary, and different orderings assumed in the Cholesky decomposition may generate different OIRFs.[69]

One solution to the problem of ordering dependence of the OIRFs is to use the Generalized Impulse Response Functions (henceforth GIRFs) proposed by Koop et al. (1996) and Pesaran and Shin (1998). This technique does not require orthogonalization of shocks, so that these IRFs are invariant to the ordering of the variables in the VAR.

According to Pesaran and Shin (1998), it is possible to view the IRF as the outcome of a conceptual experiment in which the time profile of the effect of a hypothetical vector or shock $\boldsymbol{\delta}$, say, hitting the economy at time t is compared with a base-line profile at time $t+i$, given the economy's history $\boldsymbol{\Omega_{t-1}}$.[70] Thereby, the GIRFs of $\mathbf{Y_t}$ ($\mathbf{GI_{Y_t}}$) at horizon i are:

$$\mathbf{GI_{Y_t}}(i, \boldsymbol{\delta}, \boldsymbol{\Omega_{t-1}}) = \mathbf{E}(\mathbf{Y_{t+i}} | \boldsymbol{\varepsilon_t} = \boldsymbol{\delta}, \boldsymbol{\Omega_{t-1}}) - \mathbf{E}(\mathbf{Y_{t+i}} | \boldsymbol{\Omega_{t-1}}) \quad \dots\dots\dots\dots \quad (5.5)$$

---

[69] In other words, the OIRFs depend on the order of the variables in the VAR. For example, let us assume that we impose the following order of the variables in the VAR model: $x_t \rightarrow gov_t \rightarrow i_t \rightarrow c_t \rightarrow l_t \rightarrow r_t$. This particular order means that the error term in the r_t equation has no contemporaneous effect on the errors in the x_t, gov_t, i_t, c_t and l_t equations; that the error in the l_t equation has no effect on the x_t, gov_t, i_t and c_t equations but impacts the error in the r_t equation, contemporaneously; that the error in the c_t equation has no impact on the x_t, gov_t and i_t equations but impacts the l_t and r_t equations, contemporaneously; and so on and so forth.

[70] In our case, $\boldsymbol{\delta}$ represents a (6X1) vector of shocks that show the hypothetical shocks to each of the endogenous variables in the VAR.

However, instead of shocking all the elements of $\boldsymbol{\varepsilon}_{\mathbf{t}}$, Pesaran and Shin (1998) show that it is possible to shock only one element, say its j-th element, and integrate out the effect of other shocks using an assumed (or the historically observed) distribution of the errors:

$$\mathbf{GI}_{\mathbf{Y_t}}\left(i, \delta_j, \mathbf{\Omega_{t-1}}\right) = \mathbf{E}\left(\mathbf{Y_{t+i}} | \varepsilon_{\mathrm{j,t}} = \delta_j, \mathbf{\Omega_{t-1}}\right) - \mathbf{E}(\mathbf{Y_{t+i}} | \mathbf{\Omega_{t-1}}) \quad \dots\dots\dots\dots \quad (5.6)$$

And:

$$\mathbf{E}\left(\boldsymbol{\varepsilon}_{\mathbf{t}} | \varepsilon_{\mathrm{j,t}} = \delta_j\right) = \sum \boldsymbol{e}_j \, \boldsymbol{\sigma}_{jj}^{-1} \delta_j \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5.7)$$

where $\sum \boldsymbol{e}_j = \left(\sigma_{1,j}, \sigma_{2,j}, \dots, \sigma_{6,j}\right)'$.

Finally, by setting $\delta_j = \sqrt{\sigma_{jj}}$ we obtain the different scaled GIRFs at horizon i ($\psi_j^g(i)$) as follows:

$$\psi_j^g(i) = \boldsymbol{\sigma}_{jj}^{-1/2} \boldsymbol{\varphi}_{\mathbf{i}} \sum \boldsymbol{e}_j \, , \qquad i = 0,1,2, \dots \qquad (5.8)$$

In this Chapter we have employed the GIRFs in order to study the responses of both l_t and r_t to shocks in x_t, gov_t, i_t and c_t for the different Latin American countries.

5.3.2 Measuring the degree of concordance between the components of the natural rate of growth and the components of the rate of growth of aggregate demand

Harding and Pagan (2002) have developed an algorithm to locate turning points –that is, peaks and troughs– together with a new measure of pro-cyclicality. They explain that the detection and description of any cycle is accomplished by first isolating turning points in the series, after which those dates are used to mark off periods of expansions and contractions. At a minimum such an algorithm needs to perform three tasks:

1) Determination of a potential set of turning points, that is, peaks and troughs in a series.

2) A procedure for ensuring that peaks and troughs alternate.

3) A set of rules that re-combine the turning points established after steps one and two in order to satisfy pre-determined criteria concerning the duration and amplitudes of phases and complete cycles.

Thus, following the work of the National Bureau of Economic Research (NBER) and set out in Bry and Boschan (henceforth BB) (1971), a candidate peak in a series –say z– is an observation pe for which:

$$z_{pe-h}, \dots, z_{pe-1} < z_{pe} > z_{pe+1}, \dots, z_{pe+h} \qquad \dots\dots\dots\dots\dots.\dots\dots\dots\dots \qquad (5.9)$$

Likewise, a candidate trough is an observation tr for which:

$$z_{tr-h}, \dots, z_{tr-1} > z_{tr} < z_{tr+1}, \dots, z_{tr+h} \qquad \dots\dots\dots\dots\dots.\dots\dots\dots\dots \qquad (5.10)$$

where in equations (5.9) and (5.10) h represents the number of observations on both sides over which local minima and maxima are computed.

Therefore, this method for locating a turning point can be thought of as defining the latter as an event to which probabilities can be attached, and recognition of that fact enables a formal statistical analysis to be performed. In this sense, as Harding and Pagan (2002) explain, it is in the process of understanding cycles rather than in their definition that a need for studying "co-movement" arises. They propose that co-movement be measured by the degree of concordance between the specific cycle for a variable $z_{d,t}$ –where t denotes time– and the reference cycle –based on (say) the variable $z_{o,t}$, and that this be quantified by an index $I_{d,o}$

that measures the fraction of time both series are simultaneously in the same state of expansion ($S_t = 1$) or contraction ($S_t = 0$):

$$I_{d,o} = T^{-1}\left[\#\left\{S_{d,t} = 1, S_{o,t} = 1\right\}\right] + T^{-1}\left[\#\left\{S_{d,t} = 0, S_{o,t} = 0\right\}\right] \quad \ldots\ldots\ldots\ldots \quad (5.11)$$

$$I_{d,o} = T^{-1}\left\{\sum S_{d,t}S_{o,t} + (1 - S_{d,t})(1 - S_{o,t})\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.12)$$

where, in our case, the variable $z_{d,t}$ (and, therefore, the estimated turning points $S_{d,t}$ and $(1 - S_{d,t})$) represents x_t, gov_t, i_t or c_t; whereas the variable $z_{o,t}$ (and, therefore, the estimated turning points $S_{o,t}$ and $(1 - S_{o,t})$) represents either l_t or r_t.

Thereby, we have calculated the indexes of concordance shown in equation (5.12) between the different components of the rate of growth of aggregate demand relative to each of the components of the g_n. The different $I_{d,o}$ can provide information regarding the pro-cyclicality of the components of the rate of growth of aggregate demand with respect to each of the components of g_n, considering both peaks and troughs: if the variable $z_{d,t}$ is exactly pro-cyclical (counter-cyclical) with respect to the variable $z_{o,t}$, then $I_{d,o} = 1$ ($I_{d,o} = 0$).

To summarise, we applied this methodology as follows:

1) We identified the dates of peaks and troughs for each of the components of the rate of growth of aggregate demand (x_t, g_t, i_t, and c_t) and the components of g_n (l_t and r_t) as

shown in equations (5.9) and (5.10). Following Harding (2002) and Berge and Jordà (2013), we have used $h = 1$ since we are working with annual data.[71,72]

2) We compute the different $I_{d,o}$ as shown in equation (5.12) between x_t, gov_t, i_t and c_t relative to l_t and r_t.

5.4 Empirical results

5.4.1 Data

Regarding the components of the natural rate of growth, we have used the same series for l_t and r_t that were employed in the previous Chapter. Therefore, r_t represents the rate of growth of labour productivity measured as output per number of hours worked.

With respect to the components of the rate of growth of aggregate demand, we have employed the following variables: x_t corresponds to the rate of growth of exports of goods and services; gov_t is the rate of growth of general government final consumption expenditure; i_t is the rate of growth of gross fixed capital formation; and c_t corresponds to the rate of growth of household final consumption expenditure. With the exception of the latter, these series were extracted from the World Bank electronic database; whereas the series for household final consumption expenditure was constructed as follows: final

---

[71] Harding and Pagan (2002) explain that for monthly observations h is generally set to five; whereas an analogue would be to put $h = 2$ when the data is measured at the quarterly frequency.

[72] Bracke (2011) and Berge and Jordà (2013) explain that, in business cycle analysis, a phase is an expansion or a contraction; whereas a cycle is the period between two peaks or two troughs. At the quarterly frequency it is common to impose the following restrictions: that every phase is at least 2 quarters long; and that the minimum length of a complete recession-expansion cycle is at least 4 quarters long. Since we are using annual data we have used 1 both as the minimum phase length and as the minimum cycle length.

consumption expenditure (also extracted from the World Bank database) minus general government final consumption expenditure.[73]

The series for x_t, gov_t, i_t, and c_t are available only for seven Latin American countries during the period 1981-2011: Brazil, Chile, Colombia, Mexico, Peru, Uruguay, and Venezuela. Hence, the different empirical analyses have been carried out only for these countries.

5.4.2 Granger non-causality tests and GIRFs

We constructed the VAR system shown in equation (5.2) for each individual country. The lag length (p) for the different VAR models was initially set to two; and the optimal lag order (p^*) was selected according to a sequential modified LR test statistic (each test was carried out at the 5% level). This test indicates that $p^* = 2$ in Brazil, Mexico, Peru, and Venezuela; and that $p^* = 1$ in Chile, Colombia, and Uruguay.

We then proceeded to analyse if the different VAR models satisfy the standard specification tests. Firstly, the stability condition in all VAR models is satisfied since all roots have modulus less than 1 and lie inside the unit circle (see Graphs 5.A1 to 5.A7 in the Appendix).

Secondly, we corroborated that the residuals obtained from the VAR models satisfy the standard correct specification tests: no serial correlation, no heteroskedasticity, and normality. The results obtained are presented in Table 5.1 below:

---

[73] Table 5.A1 in the Appendix shows the different variables extracted from the World Bank database together with their respective definitions.

Table 5.1. Correct specification tests of the VAR models[a]				
Country	**Autocorrelation[b]**		**Heteroskedasticity[c]**	**Normality[d]**
	Lag order: 1	**Lag order: 2**		
Brazil	0.23	0.28	0.40	0.30
Chile	0.06	0.42	0.74	0.00
Colombia	0.74	0.11	0.42	0.00
Mexico	0.87	0.36	0.36	0.00
Peru	0.06	0.49	0.43	0.68
Uruguay	0.94	0.89	0.58	0.00
Venezuela	0.57	0.01	0.32	0.15
[a]Only p-values are reported for each correct specification test.				
[b]LM test for autocorrelation (Ho: no autocorrelation).				
[c]White heteroskedasticity test (Ho: no heteroskedasticity).				
[d]Orthogonalization: Cholesky (Lütkepohl) of covariance test (Ho: residuals are multivariate normal).				

From this Table it is possible to observe that, with the exceptions of Chile, Colombia, Mexico and Uruguay that presented problems of normality, the results obtained satisfy all the standard diagnostic tests. According to Lütkepohl (2011), normality problems associated with the residuals in VAR models do not affect either the Granger causality tests or the IRFs, so that it is possible to perform the techniques described in Section 5.3.1.

We now present the results obtained from the block-causality tests/Granger non-causality tests in Table 5.2.[74]

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[74] We only present the results for the l_t and r_t equations since we are interested in finding the components of the rate of growth of aggregate demand that help to forecast future values of the components of the natural rate of growth.

Table 5.2. Block-causality tests/Granger non-causality tests of the VAR models[a,b]

Variable	l_t	r_t	l_t	r_t	l_t	r_t	l_t	r_t	l_t	r_t	l_t	r_t	l_t	r_t
	Brazil		Chile		Colombia		Mexico		Peru		Uruguay		Venezuela	
x_t	0.60	0.42	0.19	0***	0.73	0.95	0.79	0.45	0.90	0.89	0.16	0.82	0.07*	0.09*
gov_t	0.59	0.44	0.04**	0.54	0.74	0.03**	0.27	0***	0.72	0.09*	0.16	0.98	0.35	0.23
i_t	0.83	0.61	0.69	0.61	0.51	0.38	0.48	0.09*	0.57	0.62	0.54	0.45	0.43	0***
c_t	0.40	0.02**	0.19	0.50	0.36	0.52	0.26	0.59	0.75	0.33	0.60	0.37	0.83	0.05*

[a] Only p-values are reported.

[b] Ho: lags of the respective variables (x_t, gov_t, i_t or c_t) are equal to zero in the equations of interest (l_t or r_t equations). In other words, the Ho states that the lags of the components of the rate of growth of aggregate demand do no Granger-cause the components of the natural rate of growth.

*, **, *** Respectively denote rejection of the null hypothesis at the 10%, 5% and 1% level.

From Table 5.2 it is possible to conclude the following:

1) Lagged values of x_t, gov_t, i_t, and c_t help to forecast values of r_t in Brazil, Chile, Colombia, Mexico, Peru and Venezuela during the period of study. Specifically, it is possible to observe that x_t is important in Chile and Venezuela; gov_t in Colombia, Mexico and Peru; i_t in Mexico and Venezuela; and c_t in Brazil and Venezuela.

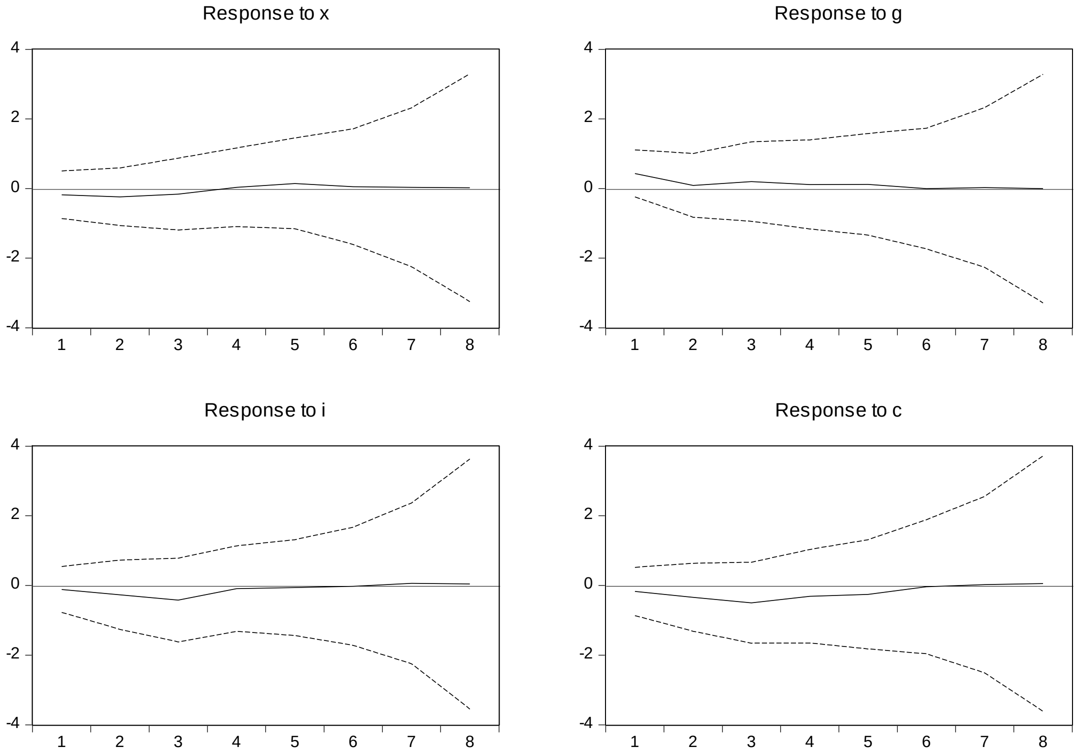
2) Chile and Venezuela are the only two countries in which the components of the rate of growth of aggregate demand help to forecast future values of l_t. Specifically, lagged values of gov_t and of x_t are important to forecast values of l_t in Chile and Venezuela, respectively.

3) Uruguay is the only country in which none of the components of aggregate demand help to forecast values of l_t and r_t.

On the other hand, we plot the GIRFs described in Section 5.3.1 in Figures 5.1 to 5.14. In these graphs we trace out the individual response of current and future values of l_t and r_t to a one standard deviation increase in the current value of one of the errors of interest in the VAR model (that is, the errors in the x_t, gov_t, i_t and c_t equations), assuming that this error returns to zero in subsequent periods and that all other errors are equal to zero. The responses of l_t for the different countries are presented in Figures 5.1 to 5.7; whereas the responses of r_t to the different shocks in each respective country are presented in Graphs 5.8 to 5.14.

Furthermore, in Figures 5.1 to 5.14 we have the following: 1) the x-axis shows the years after the initial shock; 2) shocks to the errors in the x_t, gov_t, i_t and c_t equations are denoted by x, g, i, and c, respectively; and 3) continuous lines represent the mean of the responses of l_t and r_t; whereas dotted lines represent the ± 2 standard error bands –which yield approximately 95 percent confidence interval for each of the impulse responses.

Figure 5.1. Brazil: l_t responses to the different demand shocks

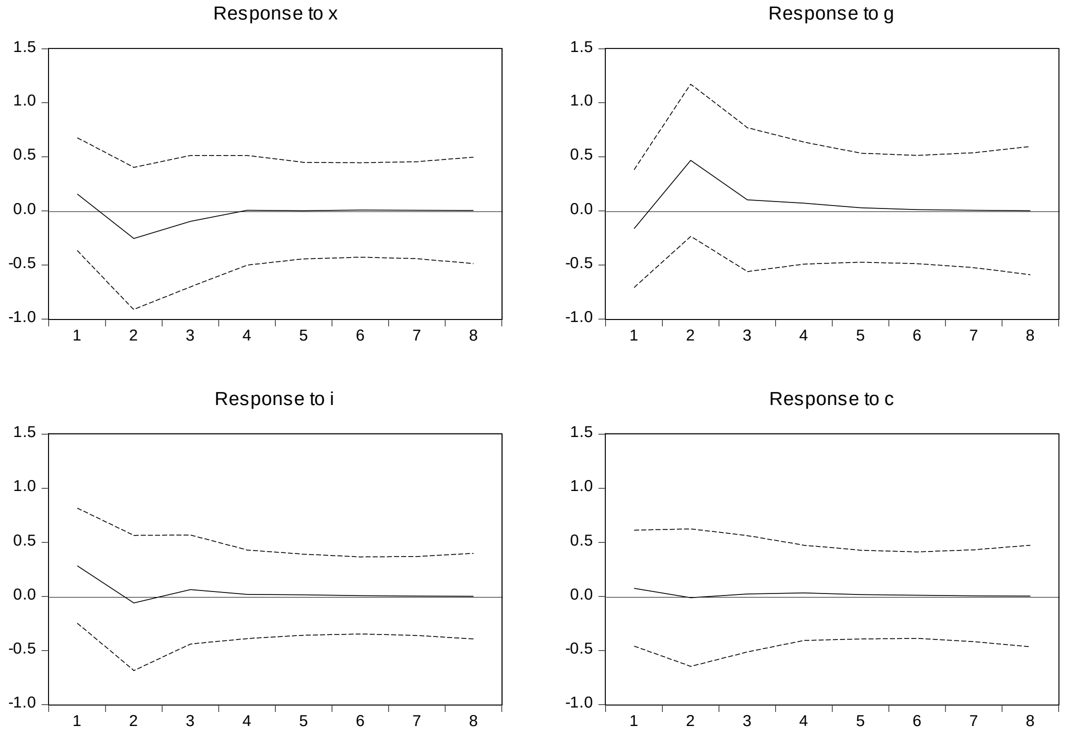

Response to x

Response to g

Response to i

Response to c

Figure 5.2. Chile: l_t responses to the different demand shocks


Response to x

Response to g

Response to i

Response to c

Figure 5.3. Colombia: l_t responses to the different demand shocks

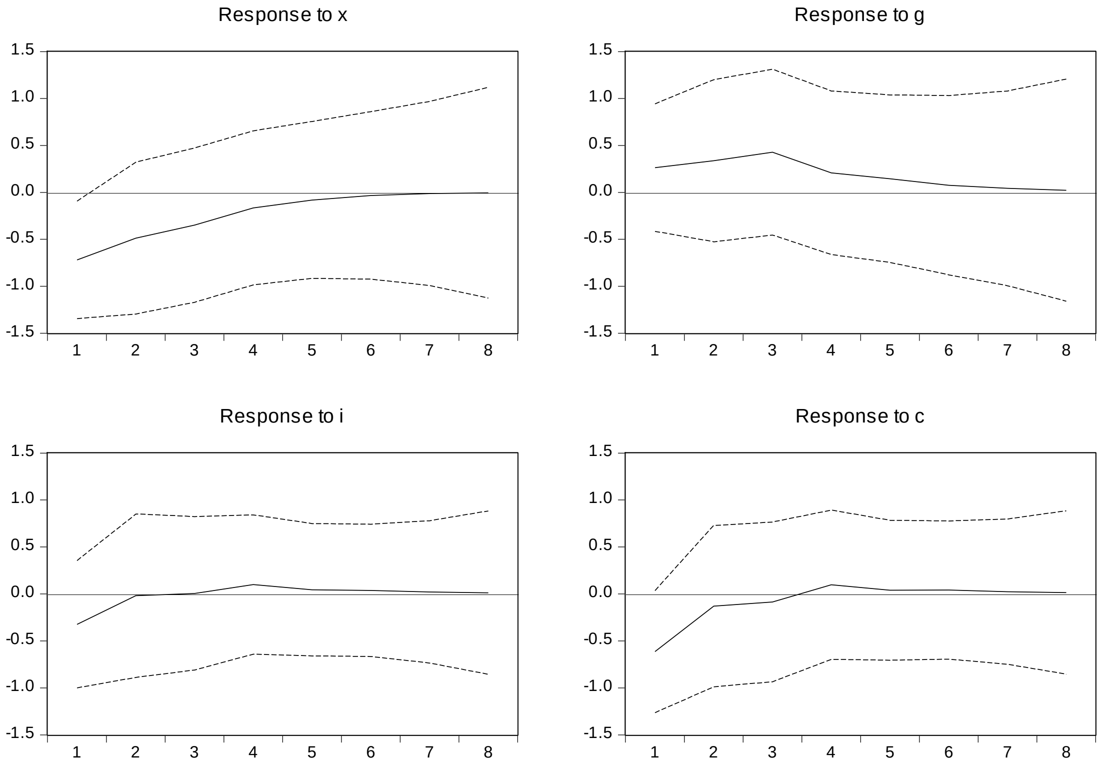


Figure 5.4. Mexico: l_t responses to the different demand shocks

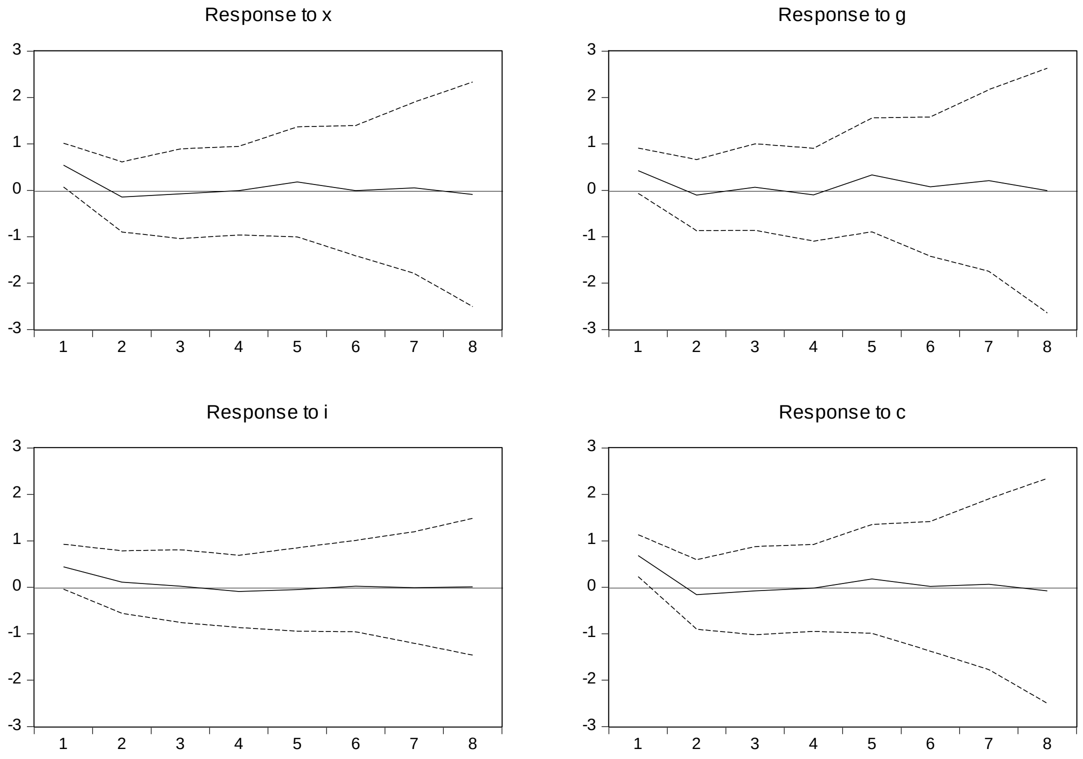


Figure 5.5. Peru: l_t responses to the different demand shocks

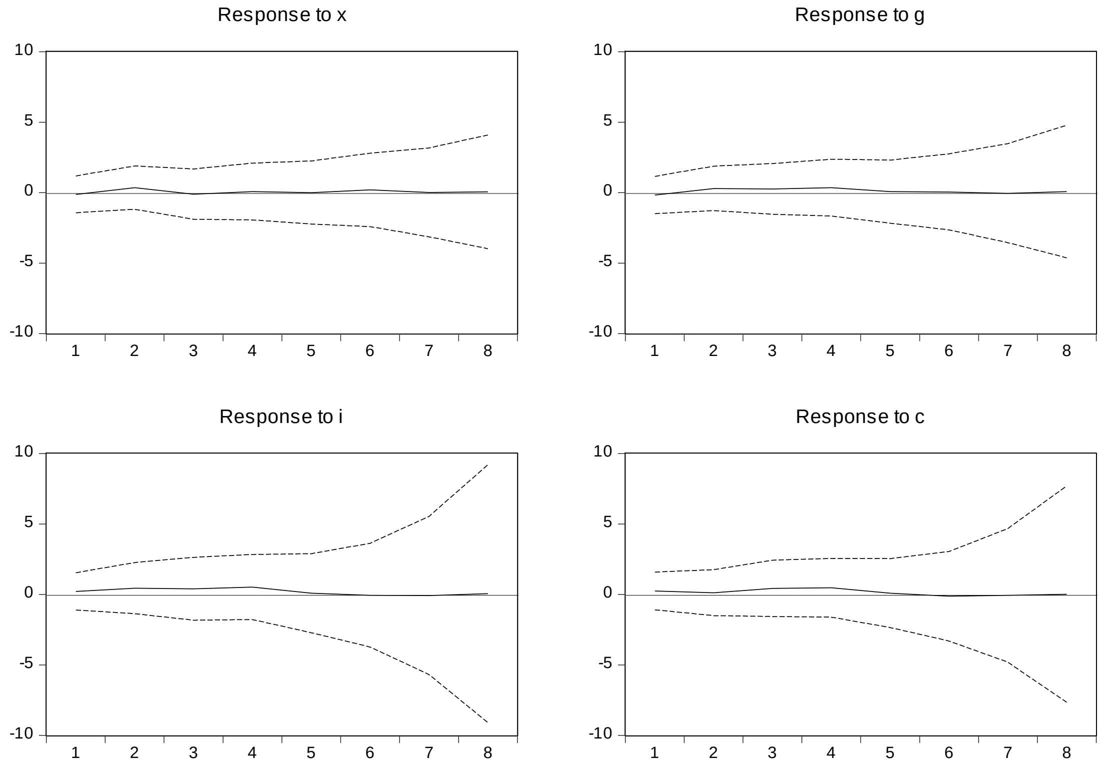


Figure 5.6. Uruguay: l_t responses to the different demand shocks

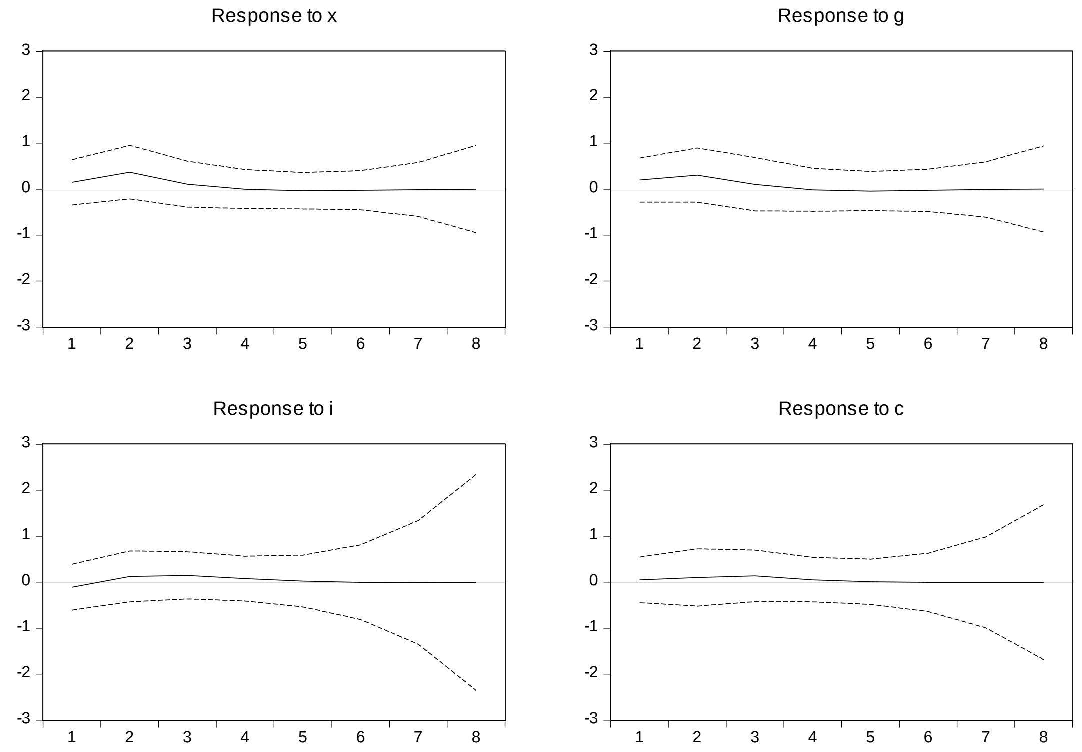


Figure 5.7. Venezuela: l_t responses to the different demand shocks

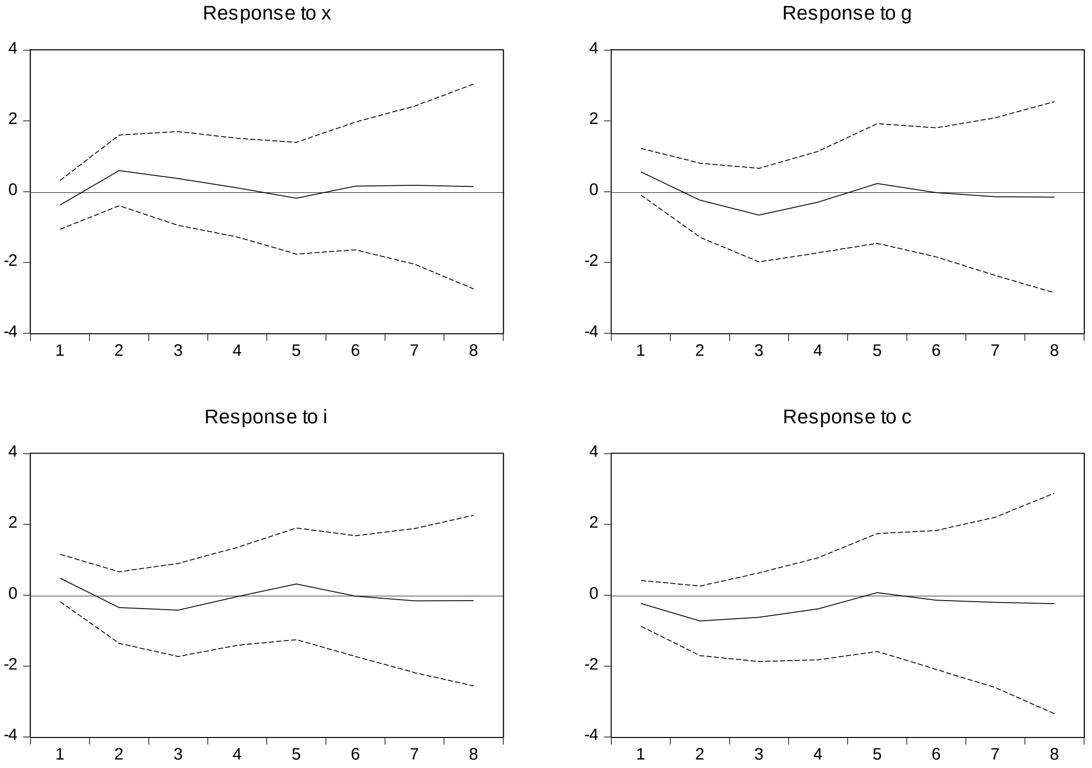


Figure 5.8. Brazil: r_t responses to the different demand shocks

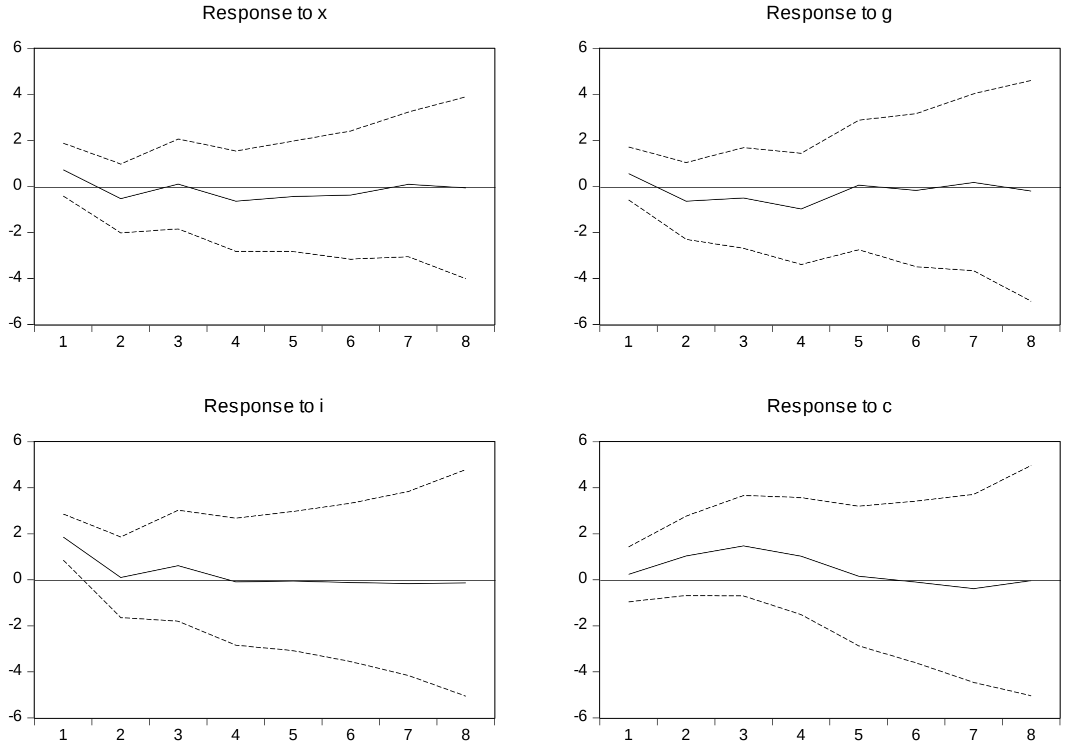


Figure 5.9. Chile: r_t responses to the different demand shocks

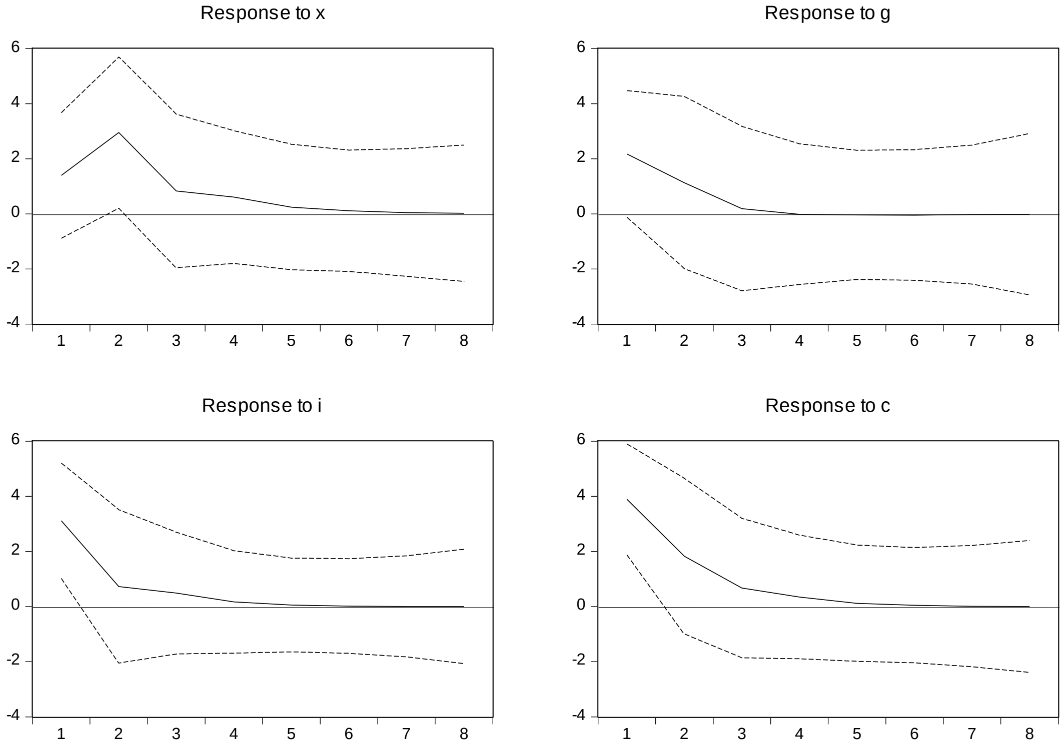


Figure 5.10. Colombia: r_t responses to the different demand shocks

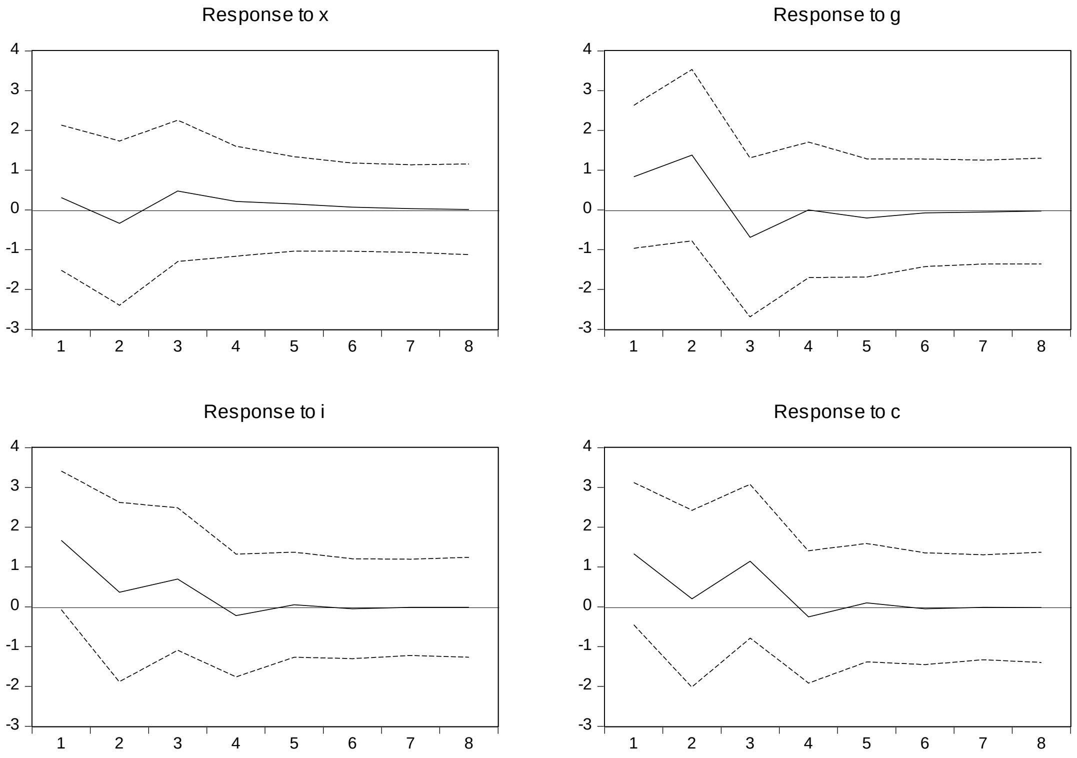


Figure 5.11. Mexico: r_t responses to the different demand shocks

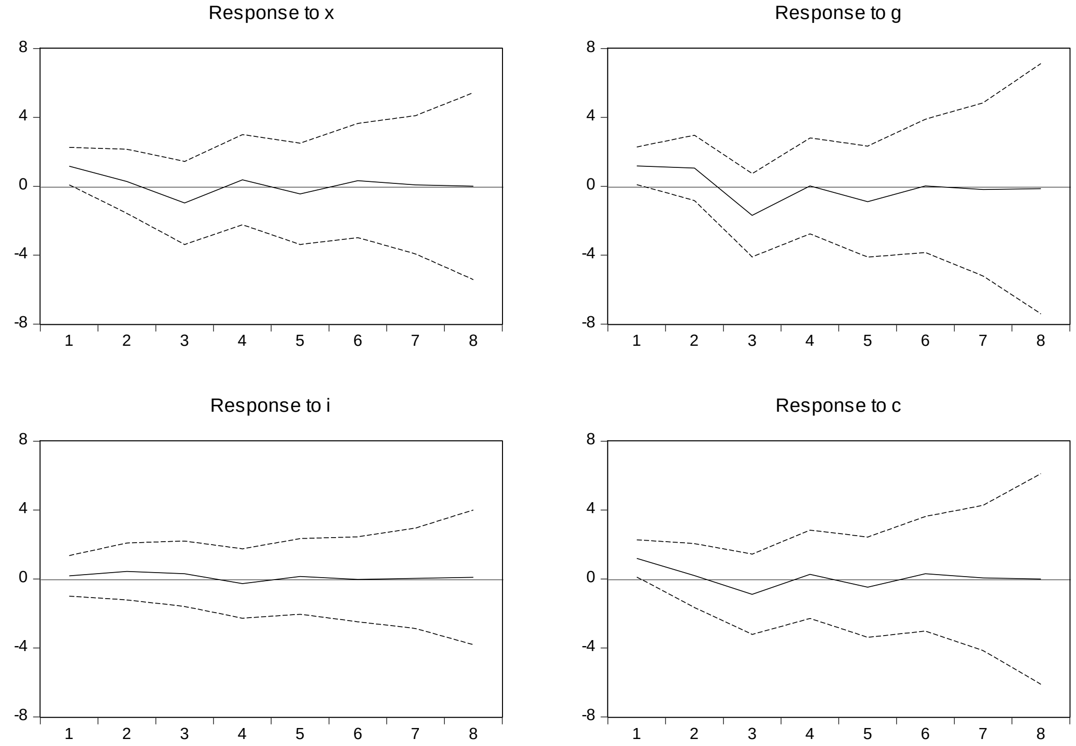


Figure 5.12. Peru: r_t responses to the different demand shocks

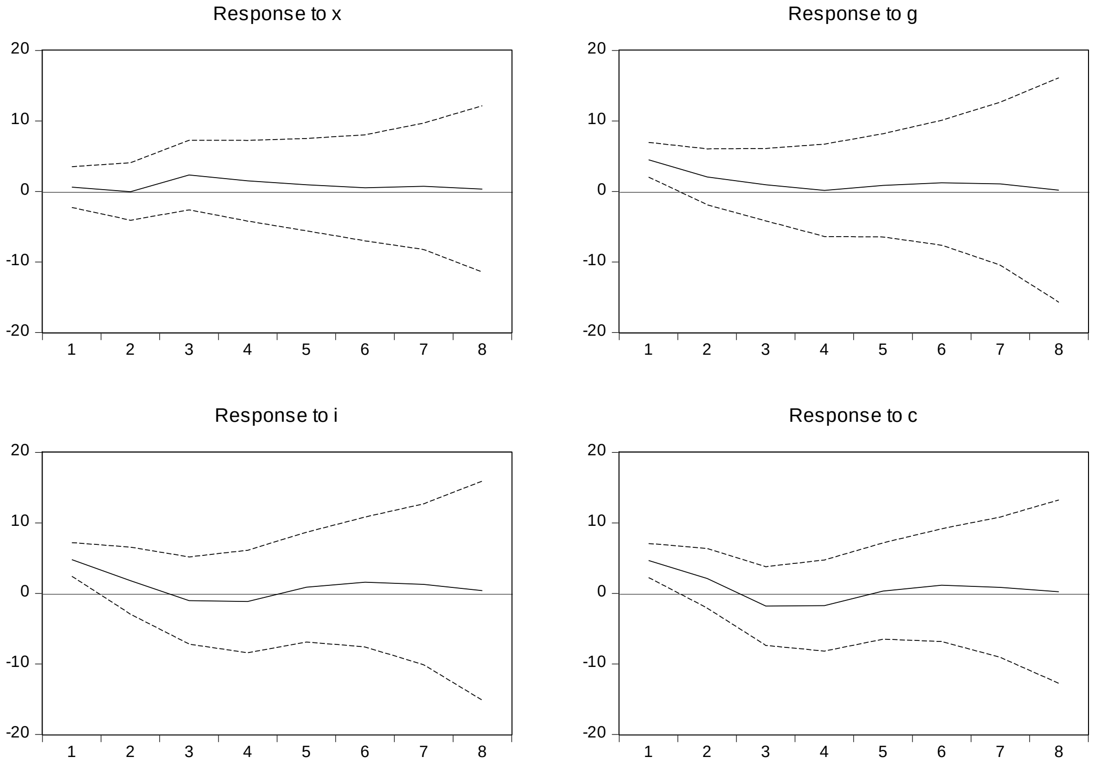


Figure 5.13. Uruguay: r_t responses to the different demand shocks

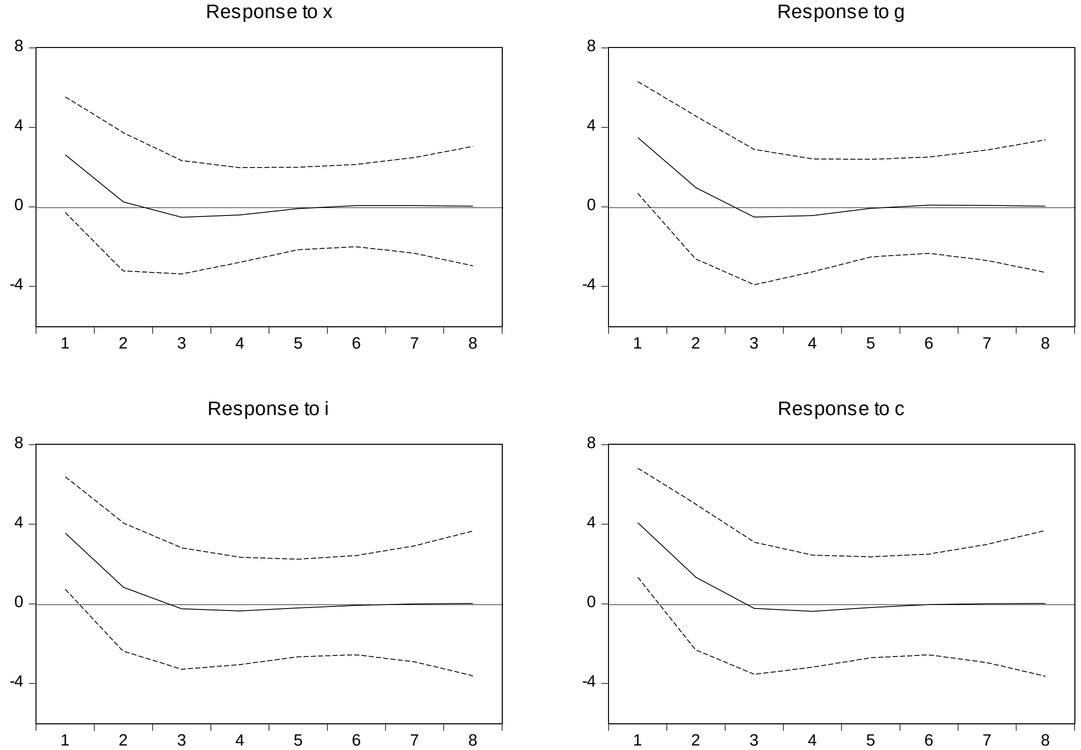
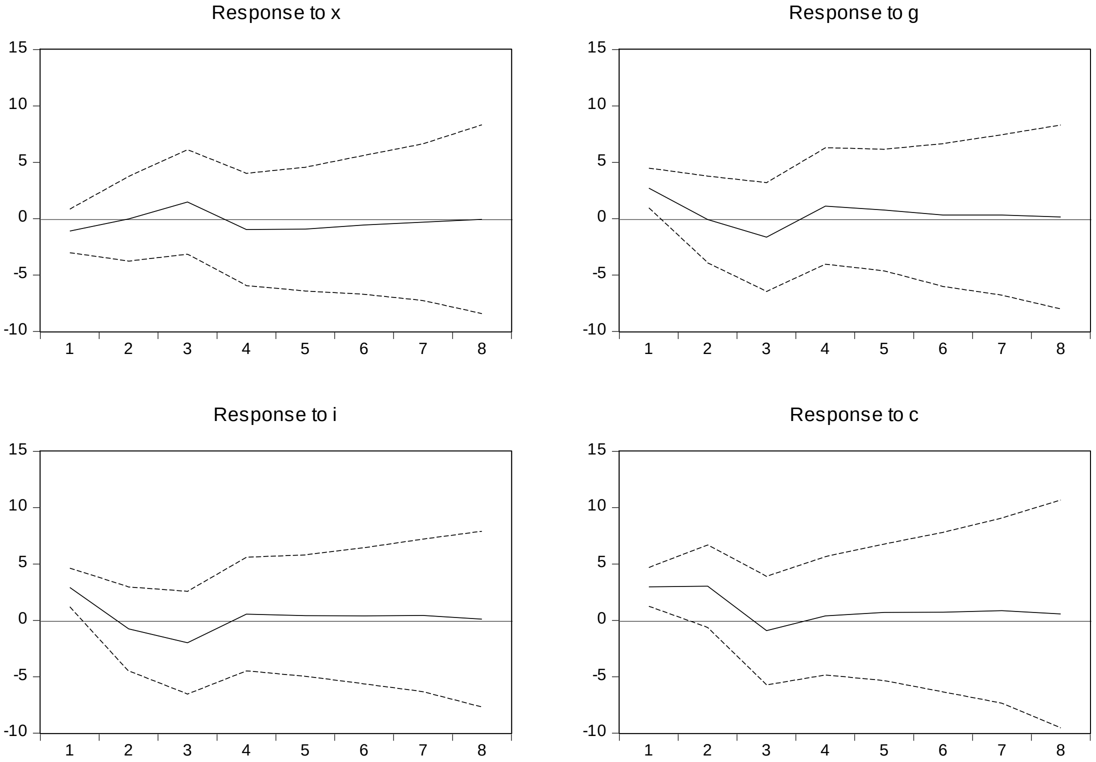


Figure 5.14. Venezuela: r_t responses to the different demand shocks



From the Figures above it is possible to observe that:

1) The r_t series show consistently stronger fluctuations to the impulses in the errors in the x_t, gov_t, i_t and c_t equations compared with the fluctuations in the l_t series –the only exception being Venezuela, where both r_t and l_t seem to experience similar fluctuations. The latter suggests that shocks to the components of the rate of growth of aggregate demand generate larger fluctuations in the rate of growth of labour productivity than in the rate of growth of labour force.

2) With respect to the responses of the l_t series to shocks in the x_t, gov_t, i_t and c_t series, it is possible to say that Venezuela seems to be the only country that experiences relatively stronger fluctuations. The l_t series in Chile also experiences fluctuations in response to shocks to x_t, gov_t and i_t; whereas shocks to x_t, gov_t and c_t also generate responses in the l_t series in Mexico.

3) Regarding the responses of the r_t series, it is possible to classify the countries into two categories: countries in which shocks in the x_t, gov_t, i_t and c_t generate stronger fluctuations in the r_t series: Brazil, Colombia, Mexico and Venezuela; and countries in which the response of r_t slowly fades away after some years: Chile, Peru and Uruguay.

With respect to the former countries, the r_t series in Venezuela seems to experience similar fluctuations to shocks in the different components of the rate of growth of aggregate demand; whereas it responds relatively more to: a) shocks to the x_t, gov_t and c_t series in Brazil and in Mexico; b) shocks to gov_t, i_t and c_t in Colombia.

On the other hand, in those countries in which the response of r_t slowly fades away after some years, it is possible to mention that in Uruguay the results of all shocks are similar. Finally, shocks to x_t seem to generate larger effects on r_t in Chile; whereas shocks to i_t, gov_t and c_t seem to generate larger effects in Peru.

Hence, the main conclusions that we can derive from the different empirical analysis using both Granger non-causality tests and GIRFs in the context of VAR models are the following:

1) With respect to the components of g_n, r_t seems to be more sensitive to the different components of the rate of growth of aggregate demand than l_t in the majority of Latin American countries.

2) Chile and Venezuela are the only two countries in which l_t seems to be sensitive to some of the components of the rate of growth of aggregate demand, according to the two different econometric techniques employed. Specifically, x_t is the most important aggregate demand component in Venezuela; whereas gov_t is the most important aggregate demand component for Chile.

3) Regarding the sensitivity of r_t with respect to the different components of the rate of growth of aggregate demand, it is possible to say that x_t is a relevant variable in Chile and Venezuela; gov_t in Colombia, Mexico and Peru; i_t in Venezuela; and c_t in Brazil and Venezuela.

4) The evidence found for Uruguay is inconclusive since the results obtained from the Granger non-causality tests and from the GIRFs do not offer a clear picture of the sensitivity of the components of g_n with respect to the components of the rate of growth of aggregate demand.

5.4.3 Indexes of concordance

The dates of peaks and troughs obtained from the Harding and Pagan (2002) algorithm described in Section 5.3.2 for the seven Latin American countries are presented in Table 5.3:

Table 5.3. Dates of peaks (P) and troughs (T) obtained from the Harding and Pagan (2002) algorithm in Latin American countries, 1981-2011

l_t		r_t		x_t		gov_t		i_t		c_t	
P	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**
Brazil											
1991	1990	1984	1985	1984	1982	1982	1984	1982	1983	1984	1985
1995	1993	1986	1988	1987	1986	1985	1988	1986	1988	1986	1987
1997	1996	1989	1990	1992	1990	1989	1990	1989	1990	1988	1989
1999	1998	1991	1992	1997	1995	1992	1996	1991	1992	1991	1992
2002	2001	1994	1995	2000	1998	1998	2000	1994	1996	1995	1998
2004	2003	1996	2000	2004	2002	2002	2003	1997	1999	2000	2001
2008	2007	2001	2002	2007	2006	2004	2006	2000	2002	2002	2003
		2004	2005	2010	2009	2007	2009	2004	2005	2007	2009
		2007	2009			2010		2007	2009	2010	
		2010						2010			
Chile											
1983	1984	1984	1982	1982	1983	1982	1983	1985	1982	1984	1982
1985	1986	1989	1985	1986	1987	1984	1985	1987	1986	1987	1985
1990	1991	1992	1990	1989	1990	1986	1987	1989	1988	1989	1988
1993	1995	1995	1994	1992	1993	1989	1990	1992	1991	1992	1990
1998	2001	2000	1999	1994	1995	1992	1994	1995	1994	1995	1993
2004	2005	2004	2001	1996	1998	1995	1996	1997	1996	2000	1999
2006	2007	2007	2006	1999	2000	1997	1998	2000	1999	2005	2002
2008	2009		2010	2001	2002	1999	2003	2005	2002	2007	2006
2010				2004	2005	2004	2005	2008	2006	2010	2009
				2006	2009	2007	2008		2009		
						2009					
Colombia											
1995	1992	1985	1988	1986	1988	1982	1983	1984	1983	1984	1983
1999	1998	1989	1990	1990	1992	1985	1986	1986	1985	1987	1985
2001	2000	1992	1994	1993	1994	1988	1990	1988	1987	1993	1991
2005	2004	1995	1996	1996	1997	1992	1993	1990	1989	1995	1994
2009	2006	1997	1999	1998	2000	1994	1995	1993	1991	1997	1996
		2000	2001	2001	2002	1996	1998	1997	1996	2000	1999
		2002	2003	2004	2005	2000	2002	2000	1999	2007	2001
		2004	2005	2006	2009	2004	2005	2006	2001		2009
		2006	2009			2006	2008		2009		
						2009					

Table 5.3. Continuation

l_t		r_t		x_t		gov_t		i_t		c_t	
P	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**	**P**	**T**
Mexico											
1992 1997 2000 2002 2004 2006 2008 2010	1991 1994 1999 2001 2003 2005 2007 2009	1984 1987 1990 1992 1994 1996 1998 2000 2003 2006 2010	1983 1986 1988 1991 1993 1995 1997 1999 2002 2005 2009	1982 1987 1993 1995 2000 2002 2004 2006 2010	1985 1992 1994 1999 2001 2003 2005 2009	1984 1986 1991 1993 1997 2000 2006 2008	1982 1985 1987 1992 1996 1998 2001 2007 2010	1985 1990 1994 1997 2000 2004 2006	1983 1986 1993 1995 1999 2001 2005 2009	1985 1989 1993 1997 2000 2004 2006 2010	1983 1986 1991 1995 1999 2003 2005 2009
Peru											
1983 1987 1990 1992 1994 1997 2001 2006 2009	1982 1986 1989 1991 1993 1996 2000 2003 2008 2010	1984 1986 1994 1996 1998 2000 2008	1985 1989 1997 2001 2002 2006 2009	1983 1986 1990 1992 1994 1997 2000 2004 2008 2010	1982 1984 1987 1991 1993 1995 1999 2002 2007 2009	1986 1990 1992 1994 1996 1998 2004 2008	1983 1988 1991 1993 1995 1997 2002 2005 2010	1987 1991 1996 2001 2004 2008 2010	1983 1990 1995 2000 2002 2007 2009	1987 1989 1992 1994 1996 1998 2004 2008 2010	1983 1988 1990 1993 1995 1997 2002 2005 2009
Uruguay											
1983 1987 1990 1992 1994 1997 2001 2006 2009	1982 1986 1989 1991 1993 1996 2000 2003 2008 2010	1984 1986 1994 1996 1998 2000 2008	1985 1989 1997 2001 2002 2006 2009	1983 1986 1990 1992 1994 1997 2000 2004 2008 2010	1982 1984 1987 1991 1993 1995 1999 2002 2007 2009	1986 1990 1992 1994 1996 1998 2004 2008	1983 1988 1991 1993 1995 1997 2002 2005 2010	1987 1991 1996 2001 2004 2008 2010	1983 1990 1995 2000 2002 2007 2009	1987 1989 1992 1994 1996 1998 2004 2008 2010	1983 1988 1990 1993 1995 1997 2002 2005 2009
Venezuela											
1991 1995 1997 2001 2009	1986 1993 1996 2000 2005	1984 1988 1991 1995 1997 2001 2004	1982 1985 1989 1994 1996 1999 2003 2009	1984 1986 1990 1993 1996 1998 2000 2004 2008	1982 1985 1987 1992 1995 1997 1999 2003 2007 2009	1984 1986 1988 1991 1995 1997 2001 2004 2007	1983 1985 1987 1989 1994 1996 1999 2002 2006 2009	1986 1988 1991 1995 1997 2001 2004	1983 1987 1989 1994 1996 1999 2003 2009	1988 1992 1995 1997 2001 2005 2007	1983 1989 1994 1996 1999 2002 2006 2009

From the Table above it is possible to observe that: 1) with respect to the components of g_n, the Harding and Pagan (2002) algorithm identifies more turning points in r_t than in l_t in Brazil, Colombia, Mexico and Venezuela; whereas more turning points were found in the l_t series than in the r_t series in Chile, Peru and Uruguay; and 2) with respect to the components of the rate of growth of aggregate demand, more turning points were found in the x_t series in Mexico, Peru, Uruguay and Venezuela; in the gov_t series in Chile, Colombia, Mexico and Venezuela; and in the i_t series in Brazil.

In Table 5.4 below we now present the different indexes of concordance of x_t, g_t, i_t, and c_t with respect to l_t and r_t as shown in equation (5.12). The results seem to indicate the following:

1) In general terms, the indexes of concordance between the individual components of the g_n and the individual components of the rate of growth of aggregate demand are low. As a matter of fact, the latter are closer to 0 than to 1, which seems to indicate that the components of g_n and the components of the rate of growth of aggregate demand are counter-cyclical. This would contradict the idea of endogeneity of the natural rate of growth, which posits a pro-cyclical relationship between the components of g_n and the components of the rate of growth of aggregate demand.

One possible explanation of why the indexes of concordance are low may be associated with the relatively short annual sample that we have for each country. Nevertheless, the different components of the rate of growth of aggregate demand generate higher indexes of concordance with respect to r_t than with respect to l_t, the only exceptions being x_t in Chile, Mexico and Uruguay and gov_t in Mexico.

Table 5.4. Indexes of concordance of the components of the rate of growth of aggregate demand with respect to the components of the natural rate of growth

Variable	l_t	r_t
Brazil		
x_t	0.129	0.258
gov_t	0.161	0.258
i_t	0.161	0.419
c_t	0.161	0.258
Chile		
x_t	0.161	0.129
gov_t	0.065	0.258
i_t	0.161	0.258
c_t	0.065	0.355
Colombia		
x_t	0.065	0.194
gov_t	0.065	0.194
i_t	0	0.226
c_t	0.032	0.258
Mexico		
x_t	0.355	0.226
gov_t	0.194	0.097
i_t	0.258	0.355
c_t	0.323	0.323
Peru		
x_t	0.097	0.355
gov_t	0.097	0.258
i_t	0	0.355
c_t	0.065	0.452
Uruguay		
x_t	0.258	0.194
gov_t	0.194	0.226
i_t	0.097	0.129
c_t	0.129	0.226
Venezuela		
x_t	0	0.226
gov_t	0.161	0.419
i_t	0.161	0.387
c_t	0.129	0.290

2) The highest indexes of concordance of the components of the rate of growth of aggregate demand with respect to l_t per country are the following: x_t in Chile, Colombia, Mexico, Peru and Uruguay; gov_t in Brazil, Colombia, Peru and Venezuela; i_t in Brazil, Chile and Venezuela; and c_t in Brazil.

3) The highest indexes of concordance of the components of the rate of growth of aggregate demand with respect to r_t per country are the following: gov_t in Uruguay and Venezuela; i_t in Brazil and Mexico; and c_t in Chile, Colombia, Peru and Uruguay.

5.5 Conclusions

The present Chapter has tried to study whether the individual components of the rate of growth of aggregate demand generate different effects on the two individual components of the natural rate of growth –the rate of growth of labour force and the rate of growth of labour productivity– in seven Latin American countries during the period 1981-2011.

We have employed both the VAR methodology and a measure of pro-cyclicality in order to study the relationships between the different components of the rate of growth of aggregate demand and the components of the natural rate of growth. The results indicate that the rate of growth of labour productivity is both more sensitive and more pro-cyclically related to the different components of the rate of growth of aggregate demand than the rate of growth of labour force.

However, the results obtained from the two different econometric methodologies offer inconclusive results regarding which components of the rate of growth of aggregate demand are more important for the rate of growth of labour productivity and for the rate of growth of labour force in the different countries.

Firstly, regarding the rate of growth of labour productivity, the VAR methodology indicates that that rate of growth of exports is important in Chile and Venezuela; that the rate of growth of government expenditure is important in Colombia, Mexico and Peru; and that the rate of growth of household consumption expenditure is a relevant variable in Brazil and Venezuela.

On the other hand, the components of the rate of growth of aggregate demand that presented the highest indexes of concordance associated with the rate of growth of labour productivity were the following: the rate of growth of government expenditure in Uruguay and Venezuela; the rate of growth of investment in Brazil and Mexico; and the rate of growth of consumption expenditure in Chile, Colombia, Peru and Uruguay.

Hence, these results mean that the a priori hypothesis described in Section 5.2 of the present Chapter regarding the relatively higher sensitivity of labour productivity growth with respect to exports growth is corroborated only in Chile and Venezuela.

Secondly, with respect to the rate of growth of the labour force, the VAR methodology shows that the rate of growth of exports is the most important aggregate demand component in Venezuela and that the rate of growth of government expenditure is the most important component in Chile.

On the contrary, the components of the rate of growth of aggregate demand that presented the highest indexes of concordance associated with the rate of growth of labour force in Chile and Venezuela are export growth and the rate of growth of government expenditure, respectively.

Appendix CHAPTER 5

Table 5.A1. Components of aggregated demand: data obtained from the World Bank and constructed series	
Variable[1]	**Definition**
Exports of goods and services	Value of all goods and other market services provided to the rest of the world. They include the value of merchandise, freight, insurance, transport, travel, royalties, license fees, and other services, such as communication, construction, financial, information, business, personal, and government services. They exclude compensation of employees and investment income (formerly called factor services) and transfer payments.
General government final consumption expenditure	Formerly general government consumption. It includes all government current expenditures for purchases of goods and services (including compensation of employees). It also includes most expenditures on national defence and security, but excludes government military expenditures that are part of government capital formation.
Gross fixed capital formation	Formerly gross domestic fixed investment. It includes land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. Net acquisitions of valuables are also considered capital formation.
Final consumption expenditure	Formerly total consumption. It represents the sum of household final consumption expenditure (formerly private consumption) and general government final consumption expenditure (formerly general government consumption).
Household final consumption expenditure	Constructed as: Final consumption expenditure minus General government final consumption expenditure
[1]All variables are measured in constant 2005 U.S. dollars.	

Figures 5.A1 to 5.A7. Inverse roots of the characteristic autoregressive polynomial of the VAR models for Latin American countries

Figure 5A.1. Brazil

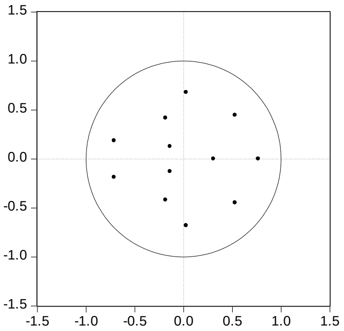


Figure 5A.2. Chile

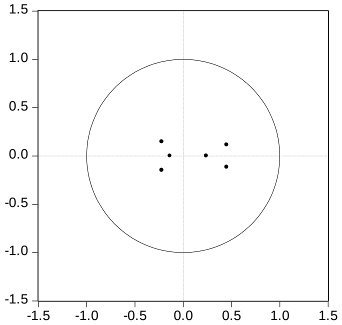


Figure 5A.3. Colombia

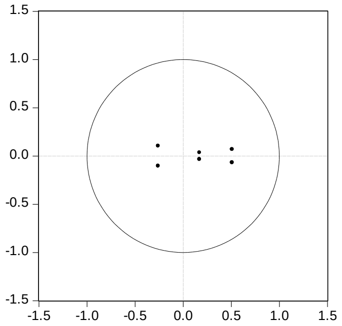


Figure 5A.4. Mexico

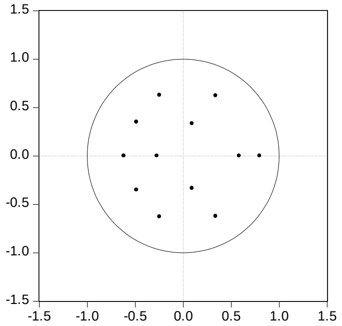


Figure 5A.5. Peru

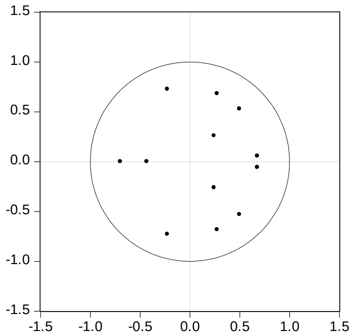


Figure 5A.6. Uruguay

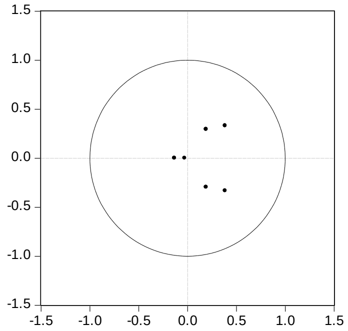
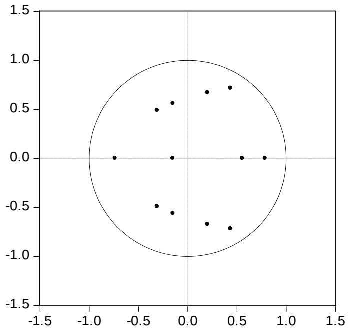


Figure 5A.7. Venezuela



CHAPTER 6

Concluding Remarks and Future Research

This Thesis presents four original Chapters that have been devoted to the study of different aspects of the hypothesis of endogeneity of the natural rate of growth –that is to say, to the hypothesis that business cycle/aggregate demand fluctuations influence the long-run/potential growth rate. The theoretical framework of the Thesis has been presented in Chapter 1; whereas new empirical evidence for Latin American countries during the period 1981-2011 is presented in Chapters 2 to 5.

Chapter 1 has tried to complement the hypothesis of endogeneity of the natural rate of growth originally proposed by León-Ledesma and Thirlwall (2000; 2002a; 2002b) by considering various elements that have been generally overlooked by the literature. Firstly, we have revisited the concept of the natural rate of growth in Harrod’s model and in the neoclassical versus post-Keynesian growth debates that took place in the 1950s. A closer inspection of some of the original works of “old” post-war growth theorists reveals that, for example, Roy Harrod and Nicholas Kaldor considered that the natural rate of growth presented a certain degree of endogeneity with respect to the prevailing economic conditions in an economy. Likewise, Robert Solow’s recent comments may also indicate that he does not consider that the natural rate of growth is exclusively an exogenous phenomenon.

Secondly, we have related the hypothesis of endogeneity of the natural rate of growth to the concept of hysteresis and to the study of the interactions between business cycle fluctuations and long-run growth. In this sense, it is possible to say that the hypothesis of endogeneity of the natural rate of growth that was inaugurated at the empirical level by the papers of León-

Ledesma and Thirlwall (2002a; 2002b) can be regarded as a particular empirical setup aimed at testing hysteresis effects relating to the potential rate of growth.

Thirdly, we have reviewed the main empirical findings of the literature that has tested the hypothesis of endogeneity of the natural rate of growth in Latin American countries (Libânio, 2009 and Vogel, 2009). These studies reveal that Chile, Colombia and Costa Rica are countries that present low sensitivity of the natural rate of growth in the upward direction; whereas Argentina seems to present high sensitivity of the natural rate of growth in boom periods.

Chapters 2 and 3 test the hypothesis of endogeneity of the natural rate of growth following the methodology proposed by León-Ledesma and Thirlwall (2002b). We have also: 1) used the new specification proposed by Lanzafame (2010), which considers the possibility of an asymmetric Okun coefficient over the business cycle; 2) adopted a dynamic version of Okun's law proposed by Knotek (2007), which considers the possibility of jobless recoveries; and 3) implemented diverse econometric techniques: Ordinary Least Squares and Instrumental Variable estimations, panel estimators that take into account parameter heterogeneity and cross-section dependence, a penalized regression spline approach, and Seemingly Unrelated Regressions.

Our results allow us to derive robust conclusions in the majority of cases. Firstly, it is possible to say that the natural rate of growth in all countries presents sensitivity in the upward direction. Specifically, Argentina, Peru, Uruguay and Venezuela are countries that present high sensitivity of the natural rate of growth in boom periods; whereas Chile, Costa Rica and

Colombia are countries that present low sensitivity. Thereby, these results are in line with the previous findings of the empirical literature for Latin American countries.

Secondly, as regards the sensitivity of the natural rate of growth in the downward direction, Chile, Colombia, Costa Rica, Ecuador, Peru, and Uruguay are countries that either do not present or present low sensitivity. However, the results are not so homogeneous regarding the countries that present high sensitivity of the natural rate of growth in low growth periods since some tests show that countries like Argentina and Nicaragua present high sensitivity of the natural rate of growth; whereas other tests show that Brazil is a country with high sensitivity.

Hence, the main conclusion that we can derive from the tests of endogeneity of the natural rate of growth in Latin American countries during the period of 1981-2011 is the following: countries that have presented a relatively low sensitivity of the natural rate of growth both in the upward and downward directions –Chile, Colombia and Costa Rica– are those that have experienced the highest rates of growth. Nevertheless, the opposite –that low growth countries have presented a relatively high sensitivity of the natural rate of growth both in the upward and downward directions– does not seem to hold in our sample of Latin American countries since, for example, a country like Uruguay (which has experienced low growth rates over the period of study) does not present sensitivity of the natural rate of growth in the downward direction.

Chapter 4 has tried to: 1) estimate a time-varying natural rate of growth using rolling regressions and the Kalman filer; and 2) decompose the sensitivity of the natural rate of growth with respect to its individual components: the rate of growth of the labour force and the rate of growth of labour productivity.

Firstly, the estimated time-varying natural rates of growth obtained via the Kalman filter and rolling regressions offer a similar picture in most countries. We can conclude that the natural rate of growth: 1) has remained constant in Argentina, Uruguay and Venezuela; 2) has decreased in Chile, Colombia and Paraguay; and 3) has increased in Bolivia, Costa Rica, Nicaragua and Peru.

Secondly, the natural rate of growth in Latin American countries is more sensitive to labour force growth than to labour productivity growth. Chile, Argentina and Bolivia are the countries that present the highest elasticities; whereas Venezuela, Ecuador and Mexico are the countries with the lowest elasticities.

Thirdly, the elasticity of the natural rate of growth with respect to productivity growth is relatively high in Colombia, Costa Rica, Argentina and Chile; whereas it is relatively low in Mexico, Bolivia and Nicaragua. Both Ecuador and Venezuela are countries in which the natural rate of growth does not seem to react to the rate of growth of labour productivity; whereas Peru is the only country in which the elasticity of the natural rate of growth associated with the rate of growth of number of hours worked is greater than the one related to labour productivity growth.

Chapter 5 tries to study the interactions between the individual components of the natural rate of growth –that is, the rate of growth of labour productivity and the rate of growth of labour force– and the individual components of the rate of growth of aggregate demand –that is, the rate of growth of exports, the rate of growth of government expenditure, the rate of growth of investment, and the rate of growth of consumption expenditure– in seven Latin American countries.

Both the VAR methodology and a new measure of pro-cyclicality (which takes into account the fraction of time that the series of interest are simultaneously in the same state of expansion or contraction) show that the rate of growth of labour productivity is more sensitive and more pro-cyclically related to the different components of the rate of growth of aggregate demand.

However, the results obtained from the two different econometric methodologies also show mixed results with respect to which aggregate demand component is more important for each individual component of the natural rate of growth. The fact that both results differ may be explained by the following. The VAR methodology offers the possibility to study the co-movements –and the dynamic interactions amongst– the components of the rate of growth of aggregate demand and the natural rate of growth; whereas the indexes of concordance only take into account the fraction of time that the series of interest are simultaneously in the same state of expansion or contraction, thus simply measuring the correlation between the variables of interest over the business cycle.

In this sense, further research is needed in order to identify the sources of aggregate demand that are more important for each individual component of the natural rate of growth in each country. In other words, our research does not allow us to provide a single recommendation useful for all Latin American countries. One possibility could be to estimate different structural VAR models, which may be useful to capture causal interactions among the endogenous variables. Another possibility could be to include variables relating to income distribution in order to explore the possibility of different demand formation patterns in Latin America. We leave these topics for future research.

Finally, we believe that our research allows us to derive the following policy recommendations for Latin American countries:

1) The natural rate of growth seems to be more sensitive to expansions –boom periods– than to recessions since all the countries in the sample present sensitivity of the natural rate of growth in the upward direction; whereas not all countries present sensitivity of the natural rate of growth in the downward direction. Therefore, expansionary economic policies are important for long-run economic growth in all Latin American countries; whereas economic policies that deal with the recessionary phase of the business cycle are more important in some countries (Argentina, Bolivia, Brazil, Mexico, Nicaragua, Paraguay and Venezuela) than in others (Chile, Colombia, Costa Rica, Ecuador, Peru and Uruguay).

2) At the individual level, the rate of growth of labour productivity seems to be more sensitive to different components of the rate of growth of aggregate demand than the rate of growth of labour force. Thus, economic policies that increase the components of the rate of growth of aggregate demand will affect the rate of growth of labour productivity.

3) The natural rate of growth seems to be more sensitive to the rate of growth of labour force. Hence, policies that stimulate aggregate demand in order to increase labour force growth could be particularly beneficial to economic growth. However, the fact that we have also found that the rate of growth of labour productivity is more sensitive to the individual components of the rate of growth of aggregate demand than the rate of growth of labour force means that the latter is not exclusively determined by aggregate demand fluctuations, so that

exogenous and/or supply factors are also relevant in order to determine the natural rate of growth. In this sense, it may also be possible to: a) take advantage of the labour force employed in the informal sector in order to increase the different labour force participation rates; and b) reduce the large number of unauthorized Latin American immigrants by increasing formal employment in their respective homelands.

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