Adding a Dimension to Logic Diagramming

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Abstract. A blind learner needs some method other than Venn diagrams to test syllogisms for validity. I present here a sketch of a three-dimensional apparatus, Sylloid, invented to fill this need and to inculcate deep learning rather than the mere ability to get answers right. What one learns in the design process is then used in designing a successor, Son of Sylloid, for sighted users that is pedagogically superior to Venn diagrams. This dog-legged approach to materials design is of wide application: first design apparatus for a user, real or imagined, weak in one or more of Howard Gardner's 'intelligences', then create a successor design for the non-deficient.

Keywords: Gardner intelligences Venn syllogism blind

1 Introduction: Gardner-inspired Design of Teaching Materials.

The aim of this paper is to fundamentally re-think the design of teaching materials in the light of what is now known about cognitive deficits and about what the psychologist Howard Gardner has termed 'multiple intelligences', and to construct more effective, more attractive teaching materials as a result. What emerges is a recipe for innovatory design that promotes deep learning, and engages the different 'intelligences' of the learner. My case study here is the design of a sequence of aids for the learning of syllogistic logic.

The springboard for the present project was a request to construct specialized equipment for teaching elementary logic to blind students. The logic in question was the theory of syllogisms, and the standard method for testing syllogistic arguments for validity is to use diagrams named after their inventor, the nineteenth century logician John Venn. Obviously, the Venn-diagrammatic technique is unavailable to a blind student — someone who, in Gardnerian terms is deficient in the visual dimension of spatial intelligence — so a substitute apparatus needs to be invented that taps into a different dimension of intelligence of the blind person . And then the question arises as to whether this apparatus, or some successor of it that re-engages with the visual, ill provide a richer learning experience for non-impaired users. If it does then one can reasonably expect this design process to generalize to other areas of learning.

Let us elaborate just a little. The problem of designing teaching equipment tailored to the needs of a group of individuals whose access to certain modes of learning is restricted e.g. because of impairment or weakness in one or other ‘intelligence’, ought to confront designers with the challenge fundamentally and creatively to rethink questions such as ‘What is the real nature of what is being taught (and why is it important, if it is, that it be taught at all)?’ and ‘What type of learning experience will best promote real, deep understanding of the subject matter?’. Such questions, when addressed seriously, inform the design of the new equipment and so benefit the target users. But the pedagogically important consideration is this: If the piece of equipment is a sophisticated solution to educational questions that have not previously been raised, then it will supply a superior means of learning not just for the group for whom it was designed, but for all students. And this will be true right through the age spectrum. The equipment will, however, typically need to be reconfigured so that a learner who is not restricted is not disadvantaged. For example, if braille letters are used in a device designed for blind subjects, then the braille would be replaced by standard letters or some other visible substitute in versions of the device to be used by the non-blind. And the reconfigured apparatus could also, for example, make effective use of colour. In summary, the dog-legged design process is this:

1. Identify some part of the syllabus that is taught by some traditional means, e.g. by book learning, where you feel the traditional teaching methods to be stodgy or ineffective.
2. Construct learning material X (it may be a piece of apparatus, a competitive or collaborative activity for two or more students, an interactive computer game, etc.) for a target group of students that suffers some real or imaginary cognitive deficit. The use of X will engage a range of intelligences different from that invoked by the traditional teaching method.
3. Construct a new apparatus, son-of-X that preserves all the pedagogical advantages of X but which also features elements that enhance the learning experience of students who do not suffer the cognitive deficit mentioned in 2.
4. Test the effectiveness of the new apparatus against traditional methods of teaching. Effectiveness is measured not just by the speed at which the student solves various problems, but by the depth of the knowledge imparted. (Testing for depth of learning is a by no means trivial task.)

The area of learning under review is syllogistic logic but it must be emphasized that this is for illustrative purposes only. The dog-legged design process, if effective, can be employed in any area of teaching to deliver deep learning.

2 Design for the Blind.

In this section, I shall recount the evolution of the design of some equipment for teaching elementary logic. Logic is a subject that features in almost every tertiary philosophy curriculum. Syllogisms are a particularly simple type of argument originally investigated by Aristotle, and the theory of syllogisms was the core of logic right up to the last quarter of the nineteenth century. It is still regarded as an important and rather beautiful part of logic, and in recent times has been developed, most notably by Fred Sommers. A syllogism consists of two premises and a conclusion, with three noun phrases (or 'terms') each occurring twice over, and each sentence in the syllogism has to be of one of just four allowable types. In the following example, the first premise is of type I, the second premise is of type E and the conclusion is of type O. Type A sentences are of the form 'All Xs are Ys', for example 'All ducks are elephants'.

Some tax cheats are parliamentarians.
No blue-eyed people are tax cheats.
Therefore
Some parliamentarians are not blue-eyed.

This particular example elicits conflicting verdicts from people asked to say whether it is valid or invalid. This alone shows the usefulness of an objective method for determining the correct answer. The validity or invalidity of any syllogism is usually established via algebraic (Boolean) equations or graphically via Venn or Euler diagrams and make use of the notion of a set (or class), e.g. the set of parliamentarians corresponding to the common noun 'parliamentarians'. Neither technique is readily available to the visually impaired student. It is easy enough to design software such that a visually impaired student could input a coding of the premises and the conclusion of a syllogism, hit a button and receive instantaneously a correct verdict on that argument's validity. But, of course, the intellectual/educational value to a user of such a device would be close to zero. Hence the need, identified above, to raise the question of just what it is about the nature of syllogisms that makes them a fascinating object of study, questions about the nature of classes (sets) and the relations between them, questions about what it is to inculcate an understanding of these and of entailment and validity. Such questions are important if one cares about deep learning.

A Venn diagram consists of three intersecting circles, each (if your imagination is sufficiently vivid) to be thought of as containing all of the objects, if any, corresponding to a noun-phrase occurring in the syllogism (e.g. if we were representing the above argument Venn-diagrammatically, one circle would 'contain' all the tax cheats, another all the parliamentarians so that the intersection of these circles contains all the parliamentarians who are tax cheats, if there are indeed any. With three circles intersecting, there are seven distinct areas. There are two basic operations when representing premises on a Venn diagram: (i) shading areas to show that they are empty (contain no objects) and (ii) using a heavy short line (a 'bar') to show the presence of objects where the bar lies. (Interestingly, the invention of the bar was due not to Venn but to Charles Sanders Peirce.) Venn's two dimensional diagrams are a much simpler way of testing syllogisms for validity than any of the earlier methods. But why not go up a dimension and produce a physical, highly tactile three dimensional model? The apparatus (called 'Sylloid') designed by the author for the use of blind students exchanges the seven areas of a Venn diagram for seven solid tetrahedra, and the counterpart to shading an area (Venn) is to remove a tetrahedron from the core by pulling it off. The counterpart to drawing a bar between two areas

(Venn) is slapping a hinge in the valley between two tetrahedra. If, in the course of representing another premise, one of the tetrahedra on which the hinge is resting is removed, the hinge is folded back, revealing a differently textured surface and remains attached to the tetrahedron that has not been removed. This corresponds, in a regular Venn diagram, to part of a bar being eclipsed when one of the areas in which it lies is declared empty (so the objects that the bar represents as existing must lie in the area where the bar remains, uneclipsed). Blind students who have used this apparatus get the hang of it remarkably quickly.

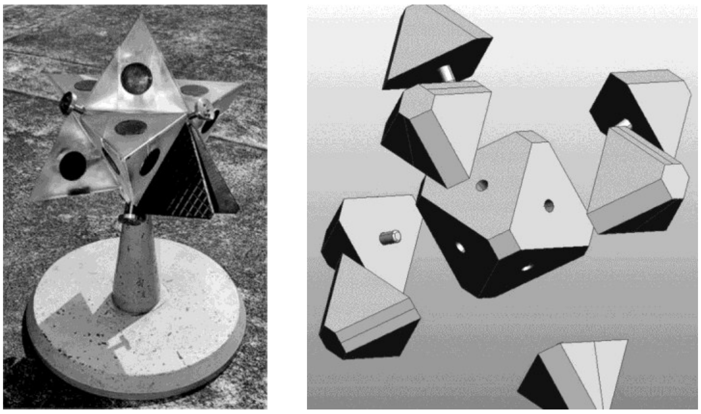

Fig. 1. SYLLOID Note that, just visible in the picture, the steel buttons are embossed with the braille equivalent of 'X', 'Y' and 'Z'. Note also the textured metal hinge. The device consists essentially of seven tetrahedra that plug into a central core – see the computer deconstruction on the right. Small round magnets are embedded in the exposed faces of the tetrahedra. Sylloid is supplied with an audiotape containing instructions for use.

3 Sight Restored.

The task of designing an apparatus for the sight–impaired presented the opportunity not just to produce a device by means of which blind people can test syllogisms for validity or invalidity but to think about what a deep understanding of syllogism involves, and to ensure that it is this kind of understanding that will be delivered to the user working with the apparatus. Sylloid was a response to this opportunity, and among its pedagogical advantages are the following:

1. A solid tetrahedron is a slightly more intuitive representation of a set of objects than is a two-dimensional shape.
2. Each tetrahedron in Sylloid visibly represents a discrete set. In Venn, the intersection (lens) between sets is clearly depicted on the diagram, but the lens has two parts and it is by no means clear what set each part represents. (It is a pedagogically useful, exercise to explain to students using Venn just which sets each of the seven areas represents.)

3. Physically removing a tetrahedron from Sylloid is a much more natural and attractive way of demonstrating the absence of the relevant set of objects than is shading an area in Venn.
4. The smooth side of the hinge represents the possibility of the presence of objects, the textured side (revealed when one side of the hinge is folded back on the other) represents their actual presence. Neither Venn nor Son of Sylloid sport any counterpart to this useful feature.
5. There is a strong element of play in Sylloid — pulling out blocks, slapping on hinges etc; Venn is not quite so much fun.
6. Sylloid is a beautifully crafted object; one can thus take advantage of the Thorndike effect.

An intriguing possibility now presents itself. Because of the advantages just listed, and because of the fundamental re-thinking that went into its design, Sylloid is probably a better learning tool than Venn, and, if we modified it slightly (e.g. by replacing the brailled letters with regular letters) then it could be used to advantage by sighted students. But why not go one step further and produce a radically new design for the sighted, a Son of Sylloid, that incorporates all the virtues of Sylloid and of Venn and that makes maximum use of the visual sense? A useful first step in this process is to take a hard look at the defects of Sylloid to ensure that they are not transmitted to its heir. These defects are:

1. It is difficult to get a firm grip on the sloping sides of a tetrahedron made of perspex, especially with clammy hands. Dropping one of the pieces on the floor is obviously a nightmare for a blind user. (This hazard would have been apparent, at the outset, to a competent designer, but was not to yours truly.)
2. There is no representation of class intersection in Sylloid. This was pointed out to me by Jon Williamson at a workshop, and it is a major strike against Sylloid, since part of the deep learning of syllogistic is understanding the connection (which so excited George Boole, when he discovered it) between the four types of Aristotelian sentence and their counterpart class relations, that can be captured in algebraic equations.
3. The tetrahedra are of equal size, and this may create the false impression that the classes they represent are equinumerous.
4. This piece of equipment, the prototype of which was constructed at the Ho Tung Engineering workshop, University of Hong Kong, needs to be built to fine tolerances and is therefore expensive to produce.
5. It is also very heavy since it has not to topple over when in use. (An alternative would be to screw it down to the workbench.)

Son of Sylloid, exploits colour, and the user represents the premises of a syllogism by removing bits of the jigsaw (corresponding to shading the area in Venn) or shows existence by using a bridging piece (corresponding to the bar in Venn). When a the bridging piece straddles two areas then, if one of those areas is declared empty in the next premise and the corresponding piece removed, the bar moves to the

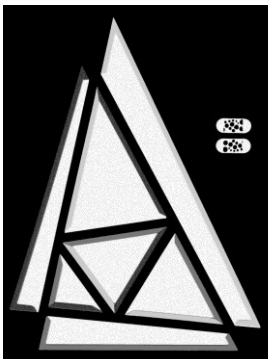

Fig. 2. SON OF SYLLOID Each of the seven coloured pieces can be removed from the black housing, though typically, when representing the premise of a syllogism, only one or two pieces are removed. The bridging bars (top right) are used when representing existential premises.

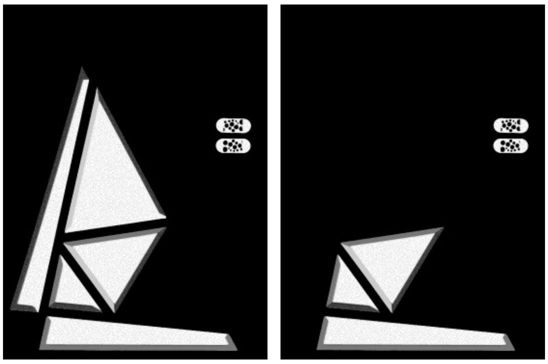

Fig. 3. All Y(ellow)s are B(lue)s FOLLOWED BY All B(lue)s are R(ed)s

This illustrates testing for validity the syllogism
All humans are mortals
All mortals are arachnophobes
Therefore all humans are arachnophobes.

Use the variables 'Y', 'B' and 'R' as stand-ins for, respectively, the nouns 'human', 'mortal' and 'arachnophobe'. The form of the first premise is 'All Ys are Bs'. Read this 'All Yellows are Blues', then simply remove from the apparatus each piece that has a yellow edge but no blue edge. For the second premise, of the form 'All Bs are Rs', remove any of the remaining pieces that have a blue edge but no red edge. By inspection of the mutilated apparatus that survives, one notes that the only piece with a yellow edge also has a red edge, hence the conclusion 'All Ys are Rs'. In the course of representing the two premises, all those pieces with yellow edges but no red edge were removed. In representing the premises, we eo ipso represented the conclusion, hence the argument is valid. Somewhat metaphorically, a deductively valid argument is often characterised as an argument the conclusion of which is contained in the premises. This notion of containment is made vivid in all three methods of testing described in this paper, but not in Aristotle's original deductions nor their mediaeval refinements nor in Boolean algebra. The invalidity of a syllogistic argument can be immediately read off a diagrammatic representation, for the conclusion is visibly not contained in the premises. Note, for example, that the conclusion 'All arachnophobes are humans' (All Rs are Ys) does NOT follow from the original two premises, for, in its final configuration (right hand figure, above), the mutilated apparatus contains red edged pieces that are not also edged in yellow.

I leave it as an exercise for the reader to imagine how testing our tax-cheating parliamentarians example above would be performed using Son of Sylloid..

4 Advantages of Son of Sylloid over both Sylloid and Venn.

1. The apparatus is visually attractive (Piet Mondriaan) and its operation makes essential use of colour.
2. The three main classes and the seven subclasses are represented by pieces of different shapes and sizes, reflecting the differences between the associated classes.
3. As in Sylloid, the emptiness of a class is signalled by physically removing the relevant piece, but this operation is easy, since hollows have been gouged out to create space for prising fingertips. Also, the operation of showing the presence of objects by the use of a bridging bar, is dead simple.
4. Coloured ridges round the sides of the 'jigsaw' pieces in Son of Sylloid give an immediate indication of the class represented by that piece. Thus a piece edged only in yellow indicates the class that contains objects, if any, that are just Y (i.e. they are not also R or also B); a piece edged in red, yellow and blue indicates a class of objects that are R, Y and B.
5. The jigsaw pieces are not contiguous; the world of parliamentarians and tax cheats and blue-eyed people also contains goats, planets, prime numbers etc..
6. When a bridging bar traverses two pieces, the subsequent removal of one of those pieces removes the support for one side of the bridge and there is just one place for it to go – onto the remaining piece. This is a more natural operation than the eclipsing of a bar by a shaded area, as in Venn.
7. When representing in Venn a type A sentence of the form 'All Xs are Ys', one first has to mentally paraphrase this as 'There is nothing that is X that is not Y' and, accordingly shade as empty the area of the X-circle lying outside the Y-circle. A lot of students find this mental manipulation difficult, and get it wrong. With Son of Sylloid, no such mental manipulation is required (see the preceding description of representing a type A sentence). Arguably, understanding why 'All Xs are Ys' is equivalent to 'There is nothing that is X that is not Y' is part of deep learning and would need to be taught as an extra to users of Son of Sylloid.
8. Tthe process of testing in Son of Sylloid is quicker, easier and more fun than in either Venn or Sylloid, BUT it is so only once you have got used to it. In his original thinking, the author badly underestimated the time it would take a student

new to the subject to learn about syllogisms and how to test them for validity. It was simply unrealistic to suppose that this could be done in one hour with no prior knowledge of Venn diagrams.
9. Son of Sylloid, like Venn but unlike Sylloid, visibly represents class intersection. In Venn, the intersecting circles need to be labelled; in Son of Sylloid, intersecting quadrilaterals are identified by the colours of their edges.
10. Unlike Venn, Son of Sylloid is re-usable and is cheap and easy to produce.

5 Conclusion.

The idea of constructing apparatus for reasoning is not new. A distinguished precursor is Raimundus Lullus, or Ramon Llull (ca. 1232-1316). Lull's chief invention was a so-called Ars Magna of encoded, inter-rotating wheels developed in the latter decades of the thirteenth century and articulated in a treatise called the Ars Generalis Ultima'. See [1]. The idea of switching from traditional media has also been anticipated by Barwise and Etchemendy in their logic software 'Turing's World' and 'Tarski's World', which makes use of 3-D graphic techniques for inference. They write: '[T]here is no principled distinction between inference formalisms that use text and those that use diagrams.' [2], p.214. True, but are diagrams or solid models better as learning devices? The design methodology outlined here is applicable to the construction of all kinds of teaching material for students of all ages, and is, at root just a way of calculatingly tapping into the rich range of intelligences that learners possess. I have used, merely as an illustration, the teaching of a very narrow aspect of logic. I also run workshops in which teachers are invited to adopt a similar approach to the design of innovatory materials in their own areas of expertise, based on thinking about students weak in one or other of the Gardnerian intelligences,.

Acknowledgements. I thank Kevin Smith, Kent School of Architecture workshop for the CAD analysis of Sylloid and for building six Sons of Sylloid. For help with the photographs, I am grateful to Charles Young and, for helping to set up the instruction/testing sessions at Simon Langton School for Boys, Charlie Artingstoll.

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