Location-routing: Issues, models and methods[*]

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Abstract*:* This paper is a survey of *location-routing*: a relatively new branch of locational analysis that takes into account vehicle routing aspects. We propose a classification scheme and look at a number of problem variants. Both exact and heuristic algorithms are investigated. Finally, some suggestions for future research are presented.

Keywords: combinatorial optimisation, heuristics, location, logistics, routing

1. Introduction

The aim of this paper is to survey the *state of the art* in *location-routing*. The *location-routing problem* (LRP) is a research area within locational analysis, with the distinguishing property of paying special attention to underlying issues of vehicle routing. Although there are a large number of surveys on various aspects of location theory (see EWGLA (2003)), the LRP received little attention, with previous surveys by Balakrishnan, Ward and Wong (1987), Laporte (1988, 1989), Berman, Jaillet and Simchi-Levi (1995) and Min, Jayaraman and Srivastava (1998). Furthermore, research has moved on considerably since these works – about a third of the papers we report on appeared since the last review. Thus, we felt that a new state-of-the-art survey was timely and desirable.

We intend to make this paper both a comprehensive review of location-routing and an accessible introduction for those working in other areas of location theory. We particularly hope it will be a useful guide to doctoral students who wish to begin their research career in this area. We wish to include all the journal papers on location-routing and make references to related research areas. For those who are new to the wider research field of location, an extensive list of introductory textbooks and survey papers is given in EWGLA (2003). For vehicle routing, we can recommend Christofides *et al.* (1979), Laporte (2000) and Toth and Vigo (2002a, 2002b).

1.1. Definition

The phrase "location-routing problem" is misleading, as location-routing is not a single well-defined problem like the Weber problem or the travelling salesman problem. It can be thought of as a *set of problems* within location theory. However, we prefer to think of the LRP as an *approach* to modelling and solving locational problems. Thus, we define location-routing, following Bruns (1998), as "*location planning with tour planning aspects taken into account*" ("Standortplanung unter

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Berücksichtigung von Tourenplanungsaspekten"). This is in line with Balakrishnan *et al.* (1987, p.56.), who observe that "*location/routing problems are essentially strategic decisions concerning ... facility location.*" Our definition stems from a *hierarchical* viewpoint, whereby our aim is to solve a facility location problem (the "master problem"), but in order to achieve this we simultaneously need to solve a vehicle routing problem (the "subproblem"). This also implies an *integrated solution approach*, *i.e.* we do not classify as belonging to the LRP an approach that deals with both location and routing aspects of a problem but does not address their inter-relation. Another important characteristic of our definition is the requirement for the existence of *tour planning*, *i.e.* the existence of multiple stops on routes. This occurs if customer demands are less than a full truckload. Other authors coin different definitions of the LRP, some including a wider range of problem versions than we do.

1.2. Location, routing and location-routing

It is well known that facility location and vehicle routing are interrelated areas. Maranzana (1964, p.261.) points out that "*the location of factories, warehouses and supply points in general ... is often influenced by transport costs.*" (Some consider this paper as the first publication on the LRP, although strictly speaking it incorporates shortest-path, rather than vehicle-routing, problems into a locational problem.) Further to the above, Rand (1976, p.248.) observes that "*many practitioners are aware of the danger of suboptimizing by separating depot location and vehicle routing.*" However, both academics and practitioners often ignore this interrelation, and solve locational problems without paying attention to underlying routing considerations. We list below three possible reasons for this.

(1) There are many practical situations when locational problems do not have a routing aspect. In these cases, the location-routing approach is clearly not an appropriate one.

(2) Some researchers object to location-routing on the basis of a perceived inconsistency. They point out that location is a *strategic*, while *routing* is a tactical problem: routes can be re-calculated and re-drawn frequently (even daily), depot locations are normally for a much longer period. Thus, they claim that it is inappropriate to combine location and routing in the same planning framework due to their different planning horizons. This criticism led the authors to investigate this issue: it was found that the use of location-routing could decrease costs over a long planning horizon, within which routes are allowed to change. (For a more detailed discussion on this issue, see Salhi and Nagy (1999)).

(3) The LRP is conceptually more difficult than the classical location problem. Berman *et al.* (1995, p.431.) observe that in the LRP, "*the facility ... must be "central" relative to the* ensemble *of the demand points, as ordered by the (yet unknown) tour through all of them. By contrast, in the classical problems the facility ... must be located by considering distances to* individual *demand points, thus making the problem more tractable.*" This may have also contributed to slow progress on the LRP.

Location-routing problems are clearly related to both the classical location problem and the vehicle routing problem. In fact, both of the latter problems can be viewed as special cases of the LRP. If we require all customers to be directly linked to a depot, the LRP becomes a standard location problem. If, on the other hand, we fix the depot locations, the LRP reduces to a VRP. From a practical viewpoint, location-routing forms part of *distribution management*, while from a mathematical point of view, it can usually be modelled as a *combinatorial optimisation* problem. We note that this is an NP-hard problem, as it encompasses two NP-hard problems (facility location and vehicle routing). Since a number of problem versions exist, we cannot reproduce all the formulations here. In the first instance, the reader is referred to Laporte (1988) for an excellent review of various formulations. Table 1 presents a summary of formulations for a variety of LRP versions developed since the publication of the above review.

Problem type	Paper	Section
Stochastic LRP	Laporte *et al.* (1989)	5.2
Dynamic LRP	Laporte and Dejax (1989)	5.3
Hamiltonian *p*-median	Branco and Coelho (1990)	4.2
Road-train routing	Semet (1995)	6.4
Vehicle routing–allocation (VRAP)	Nascimento and Beasley (1996)	6.3
Many-to-many LRP	Nagy and Salhi (1998)	6.2
Eulerian location	Ghiani and Laporte (1999)	3
LRP with mixed fleet	Wu *et al.* (2002)	4.3
Location-routing-inventory	Liu and Lee (2003)	5.2
Plant cycle location	Labbé *et al.* (2004)	3
Many-to-many LRP	Wasner and Zäpfel (2004)	6.2
Multi-level location-routing-inventory	Ambrosino and Scutellà (2005)	6.4
Deterministic LRP	Albareda-Sambola *et al.* (2005)	4.4
VRAP (median cycle problem)	Labbé *et al.* (2005)	6.3
LRP with non-linear costs	Melechovský *et al.* (2005)	4.4
Planar LRP (single-depot)	Schwardt and Dethloff (2005)	4.2
Restricted VRAP	Gunnarsson *et al.* (2006)	6.3
Planar LRP (multi-depot)	Salhi and Nagy (2007)	4.3

Table 1. ILP formulations for various LRP problems

Finally, we wish to point out that an integrated approach to solving distribution management problems is not restricted only to location-routing. Integrated approaches are becoming more popular and both the location and the routing problems have been studied in conjunction with other logistical problems. Although such problems are not within the remit of this article, readers interested in combined logistics problems in general may find the following summary useful. Location aspects are present in:

(a) the queueing–location problem (reviewed by Berman and Krass (2001) and Boffey *et al.* (2007)),

(b) the location–assignment problem (Maze and Khasnabis (1985)),

(c) the location–capacity-acquisition problem (Verter and Dincer (1995)),

(d) the location–network-design problem (Melkote and Daskin (2001), Lee *et al.* (2003)),

(e) the inventory–location problem (Daskin *et al.* (2002), Shen *et al.* (2003), Drezner *et al.* (2003)) and

(f) the location–scheduling problem (Hennes and Hamacher (2006)).

Routing forms part of the following combined logistics problems:

(g) the inventory–routing problem (reviewed by Baita *et al.* (1998) and Moin and Salhi (2007)),

(h) the routing–scheduling problem (Metters (1996), Averbakh and Berman (1999)) and

(i) the routing–packing problem (Türkay and Emel (2007), Ferrer, Nagy and Wassan (2007)).

However, very few authors investigate integrating location-routing with other aspects of distribution management. Murty and Djang (1999) (see 6.3) look at a combined location–routing–scheduling problem. Both Liu and Lee (2003) (see 5.2) and Ambrosino and Scutellà (2005) (see 6.4) include inventory aspects in the LRP.

1.3. Applications of location-routing

Operational Research is primarily an applications-oriented discipline. Therefore, we thought it important to highlight practical applications of location-routing. Table 2 summarises the main characteristics of papers describing practical applications, giving reference to the section where they will be discussed in detail. It also shows the size of the largest instances solved, in terms of the number of potential facilities and number of customers.

Paper	Section	Application area	Country	Facilities	Customers
Watson-Gandy and Dohrn (1973)	4.4	Food and drink distribution	United Kingdom	40	300
Bednar and Strohmeier (1979)	4.2	Consumer goods distribution	Austria	3	50
Or and Pierskalla (1979)	4.4	Blood bank location	United States	3	117
Jacobsen and Madsen (1980)	6.4	Newspaper distribution	Denmark	42	4510
Nambiar *et al.* (1981)	4.1/4.4	Rubber plant location	Malaysia	15	300
Perl and Daskin (1984, 1985)	4.3	Goods distribution	United States	4	318
Labbé and Laporte (1986)	6.3	Postbox location	Belgium	Not given	Not given
Nambiar *et al.* (1989)	5.3	Rubber plant location	Malaysia	10	47
Semet and Taillard (1993)	6.4	Grocery distribution	Switzerland	9	90
Kulcar (1996)	6.1	Waste collection	Belgium	13	260
Murty and Djang (1999)	6.3	Military equipment location	United States	29	331
Bruns *et al.* (2000)	6.2	Parcel delivery	Switzerland	200	3200
Chan *et al.* (2001)	5.3	Medical evacuation	United States	9	52
Lin *et al.* (2002)	4.4	Bill delivery	Hong Kong	4	27
Lee *et al.* (2003)	3	Optical network design	Korea	50	50
Wasner and Zäpfel (2004)	6.2	Parcel delivery	Austria	10	2042
Billionnet *et al.* (2005)	3	Telecom network design	France	6	70
Gunnarsson *et al.* (2006)	6.3	Shipping industry	Europe	24	300
Lischak and Triesch (2007)	6.2	Parcel delivery	Poland	22	750

Table 2. A summary of LRP applications

We can see that practical problems with hundreds of possible depot locations and thousands of customers can be solved. These papers show an enormous variety. Although most of them focus on distribution of consumer goods or parcels, there are also some applications in health, military and communications. Operational Research is all too often applied only in the affluent countries of Western Europe and North America, thus it is pleasing to see that LRP has also been applied in developing countries. We also note that application-oriented papers account for about a fifth of the LRP literature. The above observations show that LRP is really applicable in practice and is not just a purely academic construct.

1.4. Selection criteria and organisation of the paper

Even though location-routing is a fairly small research area, some selection was needed to keep the paper within reasonable lengths. Thus, we decided to omit working papers, conference publications and book chapters from our review and focus only on journal articles. (However, we included a few papers that are currently in the "publication pipeline".) Nor do we discuss dissertations within this article, however we do feel that it is encouraging to see a steady stream of theses devoted to this topic, such as Perl (1983), Reece (1985), Srivastava (1986), Salhi (1987), Simchi-Levi (1987), Nagy (1996), Berger (1997), Bruns (1998), Lischak (2001), Albareda-Sambola (2003), Çetiner (2003), Souid (2003), Barreto (2004) and Tham (2005). (The theses of Melkote (1996) and Rodríguez-Martín (2000) address related topics.) To show the development of ideas, we usually follow a chronological order, but sometimes deviate from it to group similar problem versions or methods together. (Note that the chronological order relates to publication date, unless a paper published earlier is a follow-up to one published later). We have aimed to be as comprehensive as possible, but we apologise if we have inadvertently omitted any articles.

Some papers and areas will be reviewed in more details than others. To some extent, this is determined by our personal taste and judgement about their importance. We wish to focus on the models and variants rather than technical improvements and results. Areas that are related to but not strictly within LRP are only reviewed briefly. Furthermore, our reviews of exact methods and stochastic problems are written fairly concisely, so as to minimise overlap with the reviews of Laporte

(1989) and Berman, Jaillet and Simchi-Levi (1995), who provide very extensive descriptions of these topics.

The remainder of the paper is structured as follows. The next section presents our classification scheme. Sections 3 and 4 look respectively at exact and heuristic approaches for static deterministic problems. Section 5 is devoted to stochastic and dynamic problems. Problems with non-standard hierarchical structures are investigated in section 6. Finally, we list some suggestions for future work in section 7. Each of sections 3 to 6 contains a summary table. These list the most recent major development for each problem version and solution method. Hence, these articles date mainly from the last decade, although some older papers are included for the sake of completeness. These tables also show the problem type and solution method, together with the size of the largest instance solved (in terms of the number of potential facilities and number of customers).

2. Classification and reader's guide

As it is possible in theory to develop a location-routing version of just about all location problems, classifying location-routing problems is at least as difficult a task as that of classifying location problems, with added complexity provided by the variability in the underlying vehicle routing problems. Clearly, there are a number of different ways of classification, involving some element of arbitrary choice. We look at eight aspects of the problem structure, pertaining to the location of facilities, the pattern of vehicle routes or to the entirety of the problem. Furthermore, the type of solution method is also used as a classification criterion. Our aim in choosing these criteria was to identify various strands of research. We found that the resulting groupings of papers correspond well to the – sadly, somewhat disjoint – efforts of the LRP research community. However, we are at pains to point out connections between various areas and methods, to provide a more integrated view of the LRP. We will also take advantage of this section to describe various aspects of the problem.

Our survey of the literature is structured according to this classification scheme. Thus, this section also serves as a reader's guide, explaining the allocation of papers to sections. In particular, readers interested only in certain aspects of, or approaches for, the LRP are recommended to use this section as a guide of quickly finding papers pertaining to their fields of interest.

We begin our classification by looking at four key aspects of location-routing problems. These will form the basis of the structure of the paper.

(a) *Hierarchical structure.* The structure of most location-routing problems consist of facilities servicing a number of customers, these are connected to their depot by means of vehicle tours. No routes connect facilities to each other. However, there is a body of literature that deviates from this structure. Some of these works represent quite complex extensions to the LRP; some others may not even be considered to be part of the LRP. Hence, the above standard hierarchical structure will be assumed through sections 3 to 5, and all deviations from it discussed separately in section 6.

(b) *Type of input data:* this may be *deterministic* or *stochastic*. There is a larger body of literature on the deterministic case. We note that all stochastic papers consider customer demand as the only stochastic variable. Stochastic papers will be the subject of section 5.

(c) *Planning period:* this may be single-period or multi-period. Problems with single or multiple periods are known respectively as *static* or *dynamic*. The vast majority of LRP papers investigate the static case. Dynamic papers will be described in section 5, together with stochastic papers.

(d) *Solution method:* this may be *exact* or *heuristic*. There are more papers using heuristic methods, but exact methods are often very successful for special cases of the LRP. Sections 3 and 4 will look at exact and heuristic methods, respectively, for static deterministic problems. In sections 5

and 6, exact and heuristic approaches are discussed together. In the context of heuristic methods, we wish to point out a peculiarity of this research field: research is so fragmented that only four papers furnish computational comparisons to their peers. Thus, our comparative analysis of heuristics will be more often qualitative than quantitative.

It is clearly not possible to describe together all combinations of papers that are similar in one respect or another. Neither do we wish to follow a complete taxonomy, as this would create a very large number of groupings, each containing only a few papers and this would not allow us to show the logical development of ideas. Thus, the remainder of this section will allow the reader to find papers according to classification criteria further to the ones discussed above. (Numbers in brackets after papers refer to the section or subsection where that paper is described.) We also aim to break down the rigidity of the structure by cross-referring between sections as appropriate.

(e) *Objective function.* The usual objective for location-routing problems is that of overall cost minimisation, where costs can be divided into depot costs and vehicle costs. There are only a few papers where a different objective prevails or consider multiple objectives. These are: Averbakh and Berman (1994, 2002) [3], Averbakh and Berman (1995), Averbakh *et al.* (1994) and Jamil *et al.* (1994) [5.1]. Furthermore, most of the literature on the related problem of transportation-location (see 6.1) is multi-objective.

(f) *Solution space.* This can be *discrete*, *network* or *continuous*. Most of the LRP literature deals with discrete location. However, many works on the round-trip location problem (see section 3) and the travelling salesman location problem (see 5.1 and also Simchi-Levi (1991) in 5.2) are restricted to path or tree networks. Planar location is also often considered for the above problem variants, but apart from these cases only Schwardt and Dethloff (2005) [4.2] and Salhi and Nagy (2007) [4.3] deal with continuous problems.

(g) *Number of depots.* This may be *single* or *multiple*. Most papers on the LRP deal with multiple depots, except Laporte and Nobert (1981), Averbakh and Berman (1994, 2002) [3], Simchi-Levi (1991) [5.2] and Schwardt and Dethloff (2005) [4.2] who restrict themselves to single depots. However, some special cases are solved only for one depot, such as the travelling salesman location problem [5.1] and most round-trip location problems [3]. Furthermore, when multiple depots are considered, it is generally assumed that the number of depots is not given in advance, the exceptions to this being Branco and Coelho (1990) [4.2] and Salhi and Nagy (2007) [4.3].

(h) *Number and types of vehicles.* For most location-routing problems, the number of vehicles is not fixed in advance and a *homogeneous* fleet is assumed. However, a heterogeneous fleet is considered by Bookbinder and Reece (1988), Salhi and Fraser (1996), Wu *et al.* (2002) [4.3], Ambrosino and Scutellà (2005) [6.4] and Gunnarsson *et al.* (2006) [6.3]. Laporte and Nobert (1981) and Averbakh and Berman (2002) [3] investigate problems when the number of vehicles is given in advance. Furthermore, Laporte, Nobert and Pelletier (1983) [3] and Branco and Coelho (1990) [4.2] look at the special case of exactly one vehicle per depot. Of necessity, travelling salesman location problems (see 5.1) also have this structure.

(i) *Route structure.* The usual structure of vehicle routes in an LRP is to start out from a depot, traverse through a number of customer nodes, delivering goods at each customer and finally return to the same depot. For most location-routing problems, this structure holds true, but we note here the following exceptions. Vehicles may traverse given edges rather than nodes (known as *arc routing*), see Levy and Bodin (1989) [4.2] and Ghiani and Laporte (1999) [3]. Vehicles may be allowed *multiple trips*, see Lin, Chow and Chen (2002) [4.4]. Vehicle routes may contain both *deliveries and pickups*, see Mosheiov (1995) [5.1], the round-trip location problem [3] and the many-to-many LRP [6.2].

3. Exact solution methods for deterministic problems

The first deterministic location-routing type problem to be solved to optimality was the *round-trip location problem*, a special case with the following route structure. Vehicles start out from a depot, visit a customer to pick up some load, deliver it to another customer and then return to the depot. This problem occurs frequently in practice (*e.g.* courier service). It was introduced by Chan and Hearn (1977), who assume that customers are located on a plane with rectilinear distances. The special properties of the problem make it possible to find efficiently the minimum value of the objective function. Then, by substituting this value into a linear programme, the optimal solution can be found. Chan and Francis (1976) solve the case when customers are located on a tree graph using a similar procedure. The algorithm of Drezner and Wesolowsky (1982) is based on the numerical solution of differential equations. Further solution algorithms of this type are also presented by Drezner (1982). The last two papers both solve planar problems with a variety of distance norms. While all of the above papers concern the location of a single facility, Kolen (1985) generalises the problem to locating several facilities. An underlying tree network is assumed and a constructive optimal algorithm that iteratively partitions the tree is presented. Round-trip location problems can be solved to optimality for very large problems within reasonable computing times: Drezner (1982) solves problems with up to 5000 pairs of demand points.

Exact methods for more general LRPs are usually based on a mathematical programming formulation. They often involve the relaxation and reintroduction of constraints such as: *(a)* subtour elimination (all vehicle tours must contain a depot), *(b)* chain barring (routes are not allowed to connect one depot to another) and *(c)* integrality (certain variables must be integer – usually binary integer). The following four papers all begin with relaxing these constraints.

The first exact algorithm for the general LRP is by Laporte and Nobert (1981); in this paper a single depot is to be selected and a fixed number of vehicles is to be used. A branch-and-bound algorithm is used. The authors note that the optimal depot location rarely coincides with the node closest to the centre of gravity.

Laporte, Nobert and Pelletier (1983) consider locating several depots, with or without depot fixed costs and with or without an upper limit on the number of depots. For the special case of only one vehicle per depot, it was found to be more efficient to first reintroduce subtour elimination constraints (there would be no chain barring constraints) and then use Gomory cuts to achieve integrality. Otherwise, the authors recommend using Gomory cuts first and then reintroducing subtour and chain barring constraints. On the other hand, the method of Laporte, Nobert and Arpin (1986) applies a branching procedure where subtour elimination and chain barring constraints are reintroduced. (A similar approach is adopted by Laporte, Louveaux and Mercure (1989) for a *stochastic* LRP, see 5.2.)

Laporte, Nobert and Taillefer (1988) use a graph transformation to reformulate the LRP into a travelling salesman type problem. They apply a branch-and-bound algorithm, where in the search tree, each subproblem is a constrained assignment problem and can thus be solved efficiently. This approach is extended to the *dynamic* LRP by Laporte and Dejax (1989), see 5.3.

The *delivery man problem* is a version of the TSP where the objective is to minimise the total waiting time of all customers. The *sales-delivery man problem* combines the above objective with the usual TSP-objective of minimising total tour length. Averbakh and Berman (1994) consider the problem of finding the home base of one or several delivery or sales-delivery men. Polynomial-time algorithms are presented for location on a path.

Ghiani and Laporte (1999) investigate the *Eulerian location problem* introduced by Levy and Bodin (1989) (see 4.2). In this problem the routes, instead of consisting of customer nodes, require the vehicles to traverse given edges (*i.e.* the underlying routing problem is the *arc routing problem*). In this paper, both maximum capacity and route length constraints are absent. The solution method begins by transforming the problem to either the rural postman problem or a relaxation thereof. The solution is then found by constraint relaxation and branch-and-cut.

Averbakh and Berman (2002) introduce the *minmax p-travelling salesmen location problem*, where the objective is to minimise the length of the longest vehicle tour. In their problem, customers are at the vertices of a tree, a single depot must be located on a vertex or an edge of the tree and the number of vehicles is set in advance. An optimal solution is found by reducing the problem to the minimal γ-dividing set problem.

The *plant-cycle location problem* was introduced by Billionnet, Elloumi and Grouz-Djerbi (2005). They consider the problem of simultaneously locating radio-communication stations and designing rings that connect radio antennae to such stations. This problem only differs from the LRP in that instead of vehicle routes, rings of communications links are constructed. There is a maximum limit on the number of antennae per ring, corresponding to "vehicle capacity". Furthermore, the capacity of the stations can be chosen and their costs depend on the chosen size. Optimal solution is found by commercial software. The above problem is further studied by Labbé, Rodríguez-Martin and Salazar-González (2004). Integrality and connectivity constraints are relaxed but a number of valid inequalities are added. An initial solution is found by a simple heuristic and then a branch-and-cut method is used, adding violated inequalities and improving on the routes in each step using the same heuristic. We note that, as no actual vehicle exists in this problem, it could also be viewed as a combined *location and network design problem*. (For networks that are more complex than a set of circuits the location–network design problem is no longer an LRP and thus beyond the scope of this review; the reader is referred to Melkote and Daskin (2001) and to Lee *et al.* (2003).)

In summary, exact methods provide significant insights into problems, but due to the complexity of location-routing they can only tackle relatively small instances. General location-routing instances with up to 40 potential depot locations or 80 customers have been solved to optimality (Laporte *et al.* (1983, 1988)). However, exact methods can be very successful for solving special cases of the LRP, in particular the round-trip location problem. Table 3 summarises, for each problem type and solution method, the most recent paper and the size of the largest problem solved (if available).

Problem type	Solution method	Paper	Facilities	Customers
General deterministic LRP	Cutting planes	Laporte *et al.* (1983)	40	40
	Branch-and-bound	Laporte *et al.* (1988)	3	80
Round-trip location	Numerical optimisation	Drezner (1982)	1	10000
Eulerian location	Branch-and-cut	Ghiani and Laporte (1999)	50	200
Minmax TS location	Graph theoretical	Averbakh and Berman (2002)	1	Not given
Plant cycle location	Branch-and-cut	Labbé *et al.* (2004)	30	120

Table 3. Summary of recent papers on exact methods for deterministic problems.

4. Heuristic solution methods for deterministic problems

Due to the relatively large number of papers devoted to this area, we decided to further classify them according to the solution method employed. We describe our classification scheme in the next subsection. The subsequent three subsections look at clustering-based, iterative and hierarchical heuristics, respectively.

4.1. A classification and an overview of LRP heuristics

We decided to base our classification on how the solution algorithms model the relationship between the locational and the routing subproblems.

Sequential methods first solve the locational problem by minimising the sum of depot-to-customer distances (also known as radial distances) and then solve the routing problem based on the depot locations found. (A more sophisticated approach is to use *route length estimation* formulae instead of radial distances, as advocated by Webb (1968) and Christofides and Eilon (1969) and first used by Watson-Gandy and Dohrn (1973). For more information on this topic, see for example Daganzo (2005).) The sequential solution concept does not allow for feedback from the routing phase to the locational phase. Balakrishnan *et al.* (1987, p.37.) point out that "*the sequential solution of a classical facility location and a vehicle routing model can … lead to a suboptimal design for the distribution system.*" This observation was later supported by the empirical investigations of Salhi and Rand (1989) and Salhi and Nagy (1999) for the static and the dynamic cases respectively. However, Srivastava and Benton (1990) observe that sequential methods are capable in some cases of providing good quality solutions. We also note that solving both locational and routing subproblems to optimality in sequence would still be a heuristic, as it cannot guarantee an optimal solution to the combined problem. We do not classify sequential methods as part of the LRP approach and hence exclude them from our review, except for a few that pertain to interesting special cases. However, we note that sequential methods are useful for benchmarking other heuristics when no other evaluation measure is available.

Clustering-based methods begin by partitioning the customer set into clusters: one cluster per potential depot or one per vehicle route. Then, they may proceed in two different ways:
(a) locating a depot in each cluster and then solving a VRP (or TSP) for each cluster;
(b) solving a TSP for each cluster and then locating the depots.
In some respect, they resemble sequential methods as no feedback takes place. However, clustering is based on some "skeleton" of a routing plan (such as a minimal spanning tree of all customers), so this is a better attempt at integrating locational and routing decisions.

Iterative LRP heuristics decompose the problem into its two constituent subproblems. Then, the methods iteratively solve the subproblems, feeding information from one phase to the other. Clearly, the crux of the problem here is just how information can be compressed from one phase and fed into the other.

Hierarchical heuristics are motivated by the following observations. While iterative methods are clearly an improvement on sequential methods, they "inherit" some of their drawbacks. They let the locational algorithm run until the end and then re-start it taking into account new routing information. Thus, if the routing information is not utilised well in the location phase, the method may go astray. We can also object to iterative methods from a *modelling* point of view. They treat the two constituent components as if they were on the same footing. This is not in line with our view of a *hierarchical* structure, with location as the main problem and routing as a subordinate problem. We conceive heuristic algorithms where the main algorithm is devoted to solving the location problem and refers in each step to a subroutine that solves the routing problem. We believe such hierarchical methods may provide a better model of the real situation and are also likely to give better solutions. These methods sometimes rely on *route length estimation*.

A summary of the most important recent heuristic papers is given in Table 4.

Problem type	Solution method	Paper	Facilities	Customers
General deterministic LRP	Clustering-based	Barreto *et al.* (2007)	15	318
	Iterative	Salhi and Fraser (1996)	199	199
	Hierarchical	Nagy and Salhi (1996b)	400	400
	Hierarchical	Albareda-Sambola *et al.* (2005)	10	30
	Hierarchical	Melechovský *et al.* (2005)	20	240
Plant cycle location	Clustering-based	Billionet *et al.* (2005)	6	70
Planar LRP	Iterative	Salhi and Nagy (2007)	Infinite	199

Table 4. Summary of recent papers on heuristic methods for deterministic problems.

4.2. Clustering-based methods

Bednar and Strohmeier (1979) partition customers into one, two or three clusters, on the basis of Euclidean depot-to-customer distances weighted with customer demands. This is used to see if locations proposed by their client are reasonable. Then, VRPs with the proposed locations are solved using a savings method.

Nambiar, Gelders and Van Wassenhove (1981) consider the case of locating a single depot. First, customers are clustered according to the capacity and the maximum distance constraints of the vehicles. This is based on the idea that every cluster should be reachable from every potential depot location by a feasible vehicle tour. Then, for each potential depot and each cluster a TSP is solved. The depot with the least cost is selected and a VRP is solved to improve the TSP routes. This method is clearly applicable to only a small number of potential depots. (The authors also look at the case of multiple depots, see 4.3.)

Branco and Coelho (1990) look at a special case of the LRP, called the *Hamiltonian p-median problem*, where exactly p facilities must be located and each facility has exactly one route. This is achieved by partitioning the customer set into p Hamiltonian circuits.

The "cluster-routing" heuristic of Srivastava and Benton (1990) and Srivastava (1993) considers in turn $p = m$, m–1, …, 2, 1 depots being open, where m is the number of potential depots. For each value of p, it partitions the customer set into p clusters, based on the minimal spanning tree. For each cluster, the depot is located at the site nearest to the cluster centre. Then, the resulting VRPs are solved using the sweep method. The methods compare favourably against a sequential method.

Min (1996) considers the problem of locating *consolidation terminals*. In this problem, goods from several supply sources are aggregated at terminals before being sent to customers. This problem is somewhat more complicated than the basic LRP in that a number of supply points are present and the allocation of both customers and suppliers to terminals needs to be found. Customers are clustered according to vehicle capacity and the centroid of each cluster is then used in locating the terminals.

Billionnet, Elloumi and Grouz-Djerbi (2005) solve the plant cycle location problem, which is structurally nearly equivalent to the LRP (see section 3). First, a minimal spanning forest problem is solved heuristically. This determines which plants should be open and also finds the allocation of the customers to plants. The resulting vehicle routing problems are then solved using the savings method. The authors also derive an improved lower bound.

The heuristic proposed by Barreto *et al.* (2007) begins by clustering customers according to the capacity of the vehicles. Then, for each cluster, a TSP is solved – optimally for small clusters and heuristically, using the savings method and 3-opt, for large clusters. Finally, depot locations are found by treating each tour as a single customer.

We mention here two papers that are similar to clustering methods in that a one-pass procedure with no feedback is used but information about route structures is nonetheless incorporated in the location algorithm. Levy and Bodin (1989) introduce the *Eulerian location problem*, where the vehicles are required to traverse given edges (rather than nodes). Depots are located based on the partition network (an extension of the underlying road network) and an attractiveness measure relating to the number and weight of the arcs incident to potential depot nodes. Routes are found by a rural postman problem algorithm. Schwardt and Dethloff (2005) solve a *planar* LRP with a single depot and model the relationship of depot and customers by interconnected neuron rings. As the number of routes must be fixed for this approach (each route corresponds to a neuron ring), the method is re-run with varying number of vehicles. The resulting depot location is then improved upon by solving a Weber problem with the end-points of the tours as the fixed points. As the neural network algorithm is a stochastic procedure, the authors repeat their method several times to find the best solution. The results compare favourably to those found by a sequential procedure. Finally, we note that clustering-based methods are also used to solve a dynamic LRP (see 5.3) by Chan, Carter and Burnes (2001) and a road-traing routing problem (see 6.4.) by Semet (1995). Such methods are also used to generate possible starting points for other heuristics.

4.3. Iterative methods

Perl and Daskin (1984, 1985) first introduced the concept of iterating between locational and routing phases. The locational phase is formulated as an ILP and solved to optimality using implicit enumeration. It minimises the sum of distances between depots and the "end-points" of routes found in the routing phase. The routing phase uses a savings-type heuristic generalised for multiple depots. The procedure terminates when in either phase the cost improvement is zero or negligible.

This method was later improved by several authors. Hansen *et al.* (1994) solve both phases heuristically, this allows more time for routing calculations and thus leads to better solution quality. Salhi and Fraser (1996) consider not just the end-points of tours as input to the locational phase but investigate all pairs of customers. They also include the length of tours in calculating the variable costs of the location model. Furthermore, these variable costs are adjusted to take into account a heterogeneous fleet. The locational phase is built on the moves drop and shift and the routing phase is based on a multi-level fleet mix heuristic. Reasonable improvement is found when compared against a sequential method. The method of Wu, Low and Bai (2002) is similar to the above, except that both phases rely on a combined tabu search and simulated annealing framework, but with a simpler neighbourhood structure. On some problem sets, this procedure outperforms Perl and Daskin (1984) and Hansen *et al.* (1994). However, no comparison with Salhi and Fraser (1996) is given.

The "end-point" concept is also used by Salhi and Nagy (2007) who solve a *planar* LRP with a fixed number of depots. In the locational phase, a Weber problem is solved for each depot with the end-points of the tours found in the routing phase as customers. The routing phase consists of a multi-depot savings-based heuristic with several improvement routines. The two phases are repeated until no significant improvement is found. The method is shown to improve on the results of a sequential approach and produces results similar to Schwardt and Dethloff (2005).

A different iterative framework is used by Bookbinder and Reece (1988) who apply Benders decomposition to split the LRP into location-allocation and routing subproblems. (The latter is extended to allow for heterogeneous fleet.) These are solved to optimality in an iterative framework. An advantage of this approach is that it produces upper and lower bounds in each iteration.

We note that an iterative method is also used by Labbé and Laporte (1986) to solve the vehicle routing-allocation problem (see 6.3).

4.4. Hierarchical methods

Nambiar, Gelders and Van Wassenhove (1981) present a method that uses the result of their single-depot clustering heuristic as the starting point. Then, they consider in turn $p = 1, 2, \ldots, m$ depots being open. For each value of p, they reformulate the LRP as a p-median problem with tour lengths as variable costs and solve it using an exact method. Routing is then solved using a savings method. If the cost of the LRP with p depots is more than that with $p-1$, the procedure is stopped. This can be viewed as a hierarchical method, since the routing costs are explicitly included in the locational model. This approach of evaluating every possible move is only feasible if the number of potential depots is fairly small, and hence may not be applicable in practice.

Srivastava and Benton (1990) and Srivastava (1993) present two very similar algorithms based respectively on the moves "drop" and "add" in the locational phase. The routing phase in both is solved using a savings algorithm. In the drop-based version, an "opportunity penalty" for not using the best depot for each customer arc is used to select the depot for closure. (The add-based version works symmetrically.) We note that this is a greedy procedure, as depots dropped/added cannot later be reinstated/closed. Both versions compare favourably against a sequential method.

Chien (1993) uses *route length estimation* for the LRP. A number of heuristics are proposed, consisting of combinations of the following modules. Initial solutions may be generated randomly, with routing costs either calculated fully or estimated using one of two formulae designed and tested

by the author. Initial solutions may also be found using a modified closest-depot rule for the location phase and a savings method for the routing phase. Solutions are then improved upon by a sequence of operations (moving a group of customers from one route to another, inserting a customer from one route to another, swapping two customers, reassigning all customers of a depot to another depot). A comparison of the various combinations is given.

Nagy and Salhi (1996a) propose a hierarchical method called "*nested method*". Nested methods consist of a local search locational algorithm that refers to a routing method when evaluating neighbouring solutions. The locational algorithm is based on tabu search and an add/drop/shift neighbourhood. Thus, decisions made previously can be reversed, unlike in Srivastava (1993). After each move, the routing solution is fully evaluated using a multi-depot VRP algorithm. However, the costs of neighbouring solutions are approximated based on the observation that the impact of a change in location is limited to a "region" and hence the routes are only recalculated for a limited area. Various "region" shapes, based on computational geometrical ideas, are investigated. The method is compared favourably against a sequential method. The above concept is taken further by Nagy and Salhi (1996b), where costs of possible moves are approximated using route length estimation formulae developed by the authors. The basic estimation formula can be enhanced by continued comparison with actual routing costs and its parameters adjusted as necessary. Using such formulae improves the speed of the algorithm considerably, leaving more time to finding better solutions.

Nested methods have subsequently been extended to the *many-to-many location-routing problem* (Nagy and Salhi (1998), see 6.2.) and to the *dynamic* LRP (Salhi and Nagy (1999), see 5.3.). A similar concept is also used by Tuzun and Burke (1999), who employ tabu search in both the location and routing phases but evaluate neighbouring moves according to the sum of the depot-to-customer distances, thus relying on a cruder guidance than Salhi and Nagy (1996a, 1996b). Their method improves on the results of Srivastava (1993). However, no comparison with Salhi and Nagy (1996a, 1996b) is given.

Lin, Chow and Chen (2002) allow vehicles to take multiple trips. First, the minimum number of facilities required is determined. Then, the VRP solution is completely evaluated for all combinations of facilities. Vehicles are allocated to trips by completely evaluating all allocations. If the best routing cost found is more than the setup cost for an additional depot, the algorithm moves on to evaluating all sets of facilities that contain one more depot. The applicability of this method is limited as it relies on evaluating what may well be a large number of depot configurations.

Albareda-Sambola, Díaz and Fernández (2005) apply an ingenious graph transformation to the LRP. An initial solution is found via the linear programming relaxation of their model. The locational neighbourhood is based on the moves add, drop and shift. However, no reference is made to routing when evaluating possible moves. Infeasible solutions are allowed and a penalty term is included in the objective function for depot capacity constraint violation. The overall algorithm is encompassed in a tabu search framework, whereby the locational and routing routines correspond to the diversification and intensification phases.

Melechovský, Prins and Wolfler Calvo (2005) consider an LRP with non-linear depot costs. Initial solutions are found randomly or by a clustering-based procedure. The improvement phase utilises a variable neighbourhood structure, based on moving a chain of customers from one route to another. The widest neighbourhood in this structure, by reallocating all customers of a depot, corresponds to a locational neighbourhood. (In this aspect it is somewhat similar to Chien (1993), who also constructed the locational neighbourhood by reallocating all customers of a depot.) Within each level of neighbourhood, tabu search is used to avoid local optima.

Hierarchical methods are also used for other types of LRPs. All the papers on the many-to-many location-routing problem (see 6.2) rely on a hierarchical solution structure. They are also used in solving stochastic and dynamic LRPs (see section 5) by Salhi and Nagy (1999), Liu and Lee (2003) and Albareda-Sambola *et al.* (2007), while Soiud *et al.* (2007) (see 6.4) implemented a hierarchical heuristic to solve the road-train routing problem.

Finally, we note that in principle, methodologies that either completely evaluate all possible depot combinations or investigate a number of given depot configurations could be classified as hierarchical. These include the pioneering works of Watson-Gandy and Dohrn (1973) and Or and Pierskalla (1979), the dynamic LRP heuristic of Laporte and Dejax (1989) (see 5.3) and the many-to-many problem of Lischak and Triesch (2007) (see 6.2).

5. Stochastic and dynamic problems

The only thing that is certain in life is that it is full of uncertainties! It is often unreasonable to assume that all the parameters of an LRP problem are unchanging and known precisely. Although taking uncertainty into account presents additional difficulties, there are a large number of papers of the *stochastic* LRP. Most of these are devoted to the special case of one depot and one vehicle, known as the *travelling salesman location problem*. We also note that all the papers in the literature consider customer demand as the single input subject to stochastic variation.

Before reviewing these papers, we would like to comment on some striking characteristics of this problem version. Compared to the deterministic LRP, a large proportion of papers uses an exact method of solution. Another striking aspect is that for most heuristics proposed the authors furnish its worst-case bound or optimality gap and its computational complexity. A summary of the most recent major papers for each problem version is presented in Table 5.

Problem type	Solution method	Paper	Facilities	Customers
Travelling salesman location	Exact: graph theoretical	McDiarmid (1992)	Not given	Not given
Probabilistic TS location	Exact: graph theoretical	Averbakh *et al.* (1994)	Not given	Not given
Pickup delivery location	Graph theoretical heuristic	Mosheiov (1995)	1	80
General stochastic LRP	Hierarchical heuristic	Albareda-Sambola *et al.* (2007)	10	100
Stochastic LRP with inventory	Hierarchical heuristic	Liu and Lee (2003)	20	200
Dynamic LRP	Clustering heuristic	Chan *et al.* (2001)	9	52
	Hierarchical heuristic	Salhi and Nagy (1999)	400	400

Table 5. Summary of recent papers on stochastic and dynamic problems.

5.1. Travelling salesman location problems

Travelling salesman location problems consider a set of customers, each time interval a subset of which present a demand for service. This is not known in advance but is described by a probability distribution. The objective is to locate the home base of the salesman such that the expected tour length is minimised. Two models have been considered in the literature:
(1) For each subset of customers a tour is constructed.
(2) An *a priori* tour through all customers is found, and the actual tour will each day skip those customers that do not require service.
We note that solving the first model requires more computational effort than the second one. It is also possible that in practice one does not wish to reoptimise daily, in order to maintain regularity of service. Nevertheless, the results of this model could serve as lower bounds to the second one. From now on, we will use the expression "travelling salesman location problem" (TSLP) for the first model only. The second model will be referred to as the *probabilistic* travelling salesman location problem (PTSLP).

The *travelling salesman location problem* was introduced to the literature by Burness and White (1976). They propose an iterative solution procedure where the facility is relocated in each step as the median of the first and last customers in all possible TSP tours. Planar location with Euclidean or rectilinear distances is assumed. Berman and Simchi-Levi (1986) prove the fundamental theorem of

TSLP namely that at least one of the optimal solutions is located on a vertex of the network. An optimal polynomial-time algorithm for tree networks is also presented. Simchi-Levi and Berman (1988) propose a polynomial-time heuristic for general networks, utilising an approximation formula for the lengths of the TSP tours involved. This work is extended to planar location with Euclidean or rectilinear distances by Simchi-Levi and Berman (1987), while its worst-case bound is improved by Bertsimas (1989). McDiarmid (1992) considers the TSLP with fewer assumptions about probability distributions and presents a linear-time algorithm for tree networks.

Mosheiov (1995) introduces the *pickup delivery location problem*, which combines the TSLP and the TSP with pickups and deliveries. Furthermore, in this problem, apart from the customer set being subject to variation, their demands are stochastic variables. The author extends the TSLP heuristics of Berman and Simchi-Levi (1986) and Simchi-Levi and Berman (1988) to cater for this case and also proposes a heuristic based on ranking customers.

The *probabilistic travelling salesman location problem* was introduced to the literature by Berman and Simchi-Levi (1988). It is based on the underlying probabilistic travelling salesman problem, which entails finding a tour that minimises the expected distance travelled by a salesman who follows the tour but skips customers not requiring service. The authors present a branch-and-bound algorithm for the PTSLP on a network. Bertsimas (1989) reduces the PTSLP to solving n probabilistic travelling salesman problems and proposes two polynomial-time heuristics: a nearest neighbour based one for networks and a spacefilling curve based one for the PTSLP on a Euclidean plane.

Only a few works investigate travelling salesman locations problems with non-standard or multiple objectives. Averbakh and Berman (1995) introduce the multiobjective *probabilistic sales-delivery location problem*, a stochastic extension of their earlier paper (Averbakh and Berman (1994), see section 3). Polynomial-time algorithms for tree location are presented. This is taken further by Averbakh, Berman and Simchi-Levi (1994), who consider a variety of objectives for the probabilistic travelling salesman location problem, such as minimising total or average waiting times or total tour length, and show how their previous heuristics can be adapted to handle all these objectives.

A closely related problem is the *travelling repairperson location problem*, introduced by Jamil, Batta and Malon (1994). This problem combines location with real-time routing. It involves a travelling repairperson who sets out from his/her home base to service randomly arising calls, travelling from customer to customer in the order he/she receives calls for service, only returning to the home base once there are no customers awaiting service. The objective is to find a home base location that minimises the average response time to a call for service. As routes are constructed by emerging customer demand and cannot be planned in advance, this problem is quite rightly modelled as a *queueing-location* rather than a location-routing problem. (See Berman and Krass (2001) and Boffey *et al.* (2007) for reviews of this field.)

5.2. Stochastic location-routing with multiple vehicles

There are relatively few papers that consider more than one vehicle and they treat different problem versions.

In the problem considered by Laporte, Louveaux and Mercure (1989), both depot locations and *a priori* routes must be planned before the exact level of demand is known. This may result in routes exceeding the vehicle capacity, known as a route failure. If this occurs, the vehicle returns to the depot prematurely and then resumes service to the remaining customers. The cost of this additional journey can be viewed as a penalty. The authors' objective function is to minimise depot and *a priori* route costs. This is subject to one of the following two constraints: *(a)* a limit on the probability of route failure or *(b)* a limit on the expected penalty of a route. Solution is by relaxation of the connectivity and integrality constraints, branch-and-bound and reintroduction of violated constraints.

Simchi-Levi (1991) extends the TSLP to several capacitated salesmen and extends the fundamental theorem to this case, that is, at least one of the optimal solutions for the problem is on a

node of the network. A polynomial-time heuristic is presented for the case of networks and then modified to cater for planar location with rectilinear or Euclidean distances.

Liu and Lee (2003) consider a stochastic customer demand and include inventory costs in the LRP. An initial solution is found by clustering the customers, based on an increasing order of their marginal inventory costs. For each cluster, the depot is located nearest to the centre and a TSP is solved. Then, a hierarchical improvement method is used based on the moves drop and shift for the locational phase. Both routing and inventory costs are fully evaluated for possible moves, thus the procedure is much slower than "nested methods" that use route length estimation or other means of reducing the computational burden.

In the problem studied by Albareda-Sambola *et al.* (2007), both depot locations and *a priori* routes are designed before demand is known. The *a posteriori* routes may then omit some customers, if the total demand is such that the vehicle capacity would be exceeded. Unserviced customers result in a penalty. The objective function consists of the sum of depot costs, expected costs of *a posteriori* routes and expected penalty costs. The authors also use approximations for the latter two costs. An initial solution is found by a sequential location-allocation-routing heuristic. Then, this solution is improved upon by local search, consisting of add and swap moves for location and insert, swap and 2-opt for routing.

For an excellent exposition and a more detailed review of stochastic location-routing, including travelling salesman location problems, see Berman, Jaillet and Simchi-Levi (1995).

5.3. Dynamic location-routing

Dynamic problems divide the planning horizon into *multiple periods*. Normally within the planning horizon there is some uncertainty about some of the parameters (typically the customer demands), hence dynamic problems are related to the stochastic problems discussed above. We consider dynamic location-routing a very important area of the LRP. This is because the static (single-period) LRP is very much prone to the criticism that the planning horizons of the location and routing subproblems do not match. By considering a planning horizon for facility location that contain shorter planning intervals for route planning, dynamic LRPs are a much better model of real-life location problems with routing aspects and provide an important means of refuting the above criticism.

We may distinguish between two types of dynamic problems. In one, the depots are located sequentially. In the other, the depots are located at the beginning of the planning horizon and vehicle routes vary with the variations in customer demand. The former case is more applicable if demand is increasing and the latter if demand is fluctuating.

In the problem studied by Nambiar *et al.* (1989) a rapid increase in supply was foreseen. Thus, the authors' solution provided a sequence of depot locations to be opened at different times. An interesting consideration is that they allowed a factory to be closed down when another was opened and to be re-opened later, in line with the real-life situation under investigation. However, the routing considerations were chiefly neglected, thus their method cannot strictly be classified as an LRP.

Laporte and Dejax (1989) also consider multiple planning periods, whereby in each period both the locations and the routes may be changed. They present an ingenuous network representation of the problem, where some of the arcs are "spatial" vehicle routing arcs while some others are "temporal" arcs representing the transformation from one time period to another. The resulting network optimisation problem is solved to optimality following the procedure of Laporte *et al.* (1988). The authors also present a heuristic method. For each time period and for each possible depot configuration, distribution costs are estimated using a route length estimation formula. These costs and the costs of transition from one configuration to another are represented on a network and a least-cost path problem is solved providing the solution as a sequence of depot configurations. The applicability of this heuristic is limited by its exponential computational complexity.

Salhi and Nagy (1999) assume that the depots are fixed throughout the planning horizon but the vehicle routes change following changes in customer demand. It is also assumed that the customer set does not change. A number of solution approaches are investigated:
(a) a locational decision is made on the basis of average (forecasted) demands,
(b) an LRP is solved for each time period (as this violates the assumption of fixed depots, it serves as a lower bound),
(c) a locational decision is made by selecting one of the set of solutions found in *(b)* and
(d) a locational decision is created using some of the depots featuring in the set of solutions found in *(b)*.
The above scenarios were all evaluated using the method of Nagy and Salhi (1996b). It was found that the solution found in *(d)* is very close to the lower bound and thus this selection rule provides a good way of finding a set of depots that behave well under changing conditions.

Chan, Carter and Burnes (2001) use a clustering heuristic to investigate approaches *(a)* and *(b)*, classifying them as *a priori* and *a posteriori* approaches. They conclude that forecasted demands are a reasonable approximation. Ambrosino and Scutellà (2005) consider a multi-level LRP (see 6.4) and apply commercial software to their ILP formulation.

6. Problems with non-standard hierarchical structure

In the location-routing problems discussed thus far, we assumed a structure of facilities servicing a number of customers, who are connected to their depot by means of vehicle tours. No routes connect facilities to each other. (Some LRP papers consider a central pre-located facility to which all facilities to be located must be connected: as this connection is by means of a direct link, its cost can be included in the facility costs and no tour planning is involved. Thus, this is a negligible variation.) This section will look at four problem versions, with very different hierarchical structures: some simpler and some more complex than the LRP models discussed so far. Some of them may even be considered as falling outside the definition of LRP, but are included for the sake of completeness. They differ from each other and from the standard LRP according to whether tour planning is involved in the various layers:
(1) the transportation-location problem does not involve tour planning,
(2) the many-to-many location-routing problem involves tour planning between facilities and customers but also involves inter-facility routes,
(3) the vehicle routing-allocation problem involves tour planning between facilities but not between facilities and customers and
(4) the multi-level location-routing problem involves tour planning at both layers and may even consider more than one level of facility.
We note that all but one of the papers in this section are deterministic and static. Both exact and heuristic approaches are discussed. The latest papers on each problem version are listed in Table 6.

Problem type	Solution method	Paper	Facilities	Customers
Many-to-many LRP	Hierarchical heuristic	Nagy and Salhi (1998)	249	249
		Wasner and Zäpfel (2004)	10	2042
Vehicle routing–allocation	Exact: branch-and-cut	Labbé *et al.* (2005)	150	150
	Sequential heuristic	Murty and Djang (1999)	331	331
Multi-level LRP	Exact: branch-and-bound	Ambrosino and Scutellà (2005)	28	135
Road-train routing	Hierarchical heuristic	Souid *et al.* (2007)	30	100

Table 6. Summary of recent papers on problems with non-standard hierarchical structures.

6.1. The transportation-location problem

The *transportation-location problem* (TLP) combines the problem of locating facilities with that of transporting goods between supply and demand points. Each origin-to-destination route is a path (not necessarily a simple path) through the facilities to be located. This problem is also known as the transshipment-location problem or the path location-routing problem. It occurs especially frequently in *hazardous material transportation*, where it is indeed more sensible not to have stops. In fact, all the combined hazardous LRP models in the literature are, in fact, TLPs. While some authors classify these as falling within the field of LRP, we do not do so, as these papers lack the essential characteristic of tour planning. Thus, we present here merely a brief summary.

Hazardous TLPs have been addressed by Zografos and Samara (1989), ReVelle, Cohon and Shobrys (1991), List and Mirchandani (1991), Stowers and Palekar (1993), Boffey and Karkazis (1993), Kulcar (1996), Giannikos (1998) and Cappanera, Gallo and Maffioli (2004). These papers adopt a multiobjective approach, trying to find an appropriate balance between minimising transportation costs and minimising risk. For a review of hazardous material location and routing, see List *et al.* (1991), Boffey and Karkazis (1995) and Erkut and Verter (1995).

There are relatively few papers on transportation-location problems other than hazardous applications. We refer the readers to the pioneering papers of Maranzana (1964) and Cooper (1972), the multiobjective approach of Ogryczak, Studziński and Zorychta (1989, 1992), the multiproduct problem of Hindi and Basta (1994) and the hub location type problem of Aykin (1995).

6.2. The many-to-many location-routing problem

Nagy and Salhi (1998) introduce the *many-to-many location-routing problem* (MMLRP). In this problem, several customers wish to send goods to others. In the most general case, it is assumed that each customer sends a different commodity to every other customer. This corresponds to the case of postal flow between localities. A network of hubs is to be located, taking into account routing costs. (Inter-hub routes are assumed to be direct while hub-to-customer routes are multi-stop.) Just as the LRP is an approach to locate facilities, the MMLRP is an approach to locate hubs. It is noted that as customers can both send and receive goods, a pickup-and-delivery routing method is required to find routing costs. This is harder to solve than the VRP, as a fluctuating load on the vehicles makes feasibility checks harder to perform. A hierarchical heuristic solution framework, based on the concept of "nested methods" is presented. We note that a number of logistics problems are special cases of the MMLRP, such as the hub location problem (if full-truckload routes are assumed), the freight transport problem (if hubs are fixed) and of course the LRP (if there is no inter-hub flow and either all deliveries or all pickups are zero). This type of problem has application in the design of letter or parcel delivery systems and in fact all the other papers on this problem concern parcel delivery applications. Since they tackle practical applications, no computational comparisons to other papers are given. All authors chose *hierarchical* solution methods.

Bruns, Klose and Stähly (2000) study a problem arising in the parcel delivery operations of a postal service. In this system, parcels travel directly from post offices to parcel processing centres and thence to delivery bases. From there, they are delivered to customers by vehicles making multiple stops. (Processing centres themselves can act as delivery bases.) The problem at hand was to determine the locations of the delivery bases, their allocation to processing centres and the allocation of customer areas to delivery bases. (The processing centre locations are predetermined.) As in this problem the flow from customers (post offices) to delivery bases is via paths and is separate from the flow from bases to customers, the authors are able to reduce their model to an LRP. They investigate the costs incurred and find appropriate approximations for them; in the case of routing costs, this involves designing a route length estimation formula. Thus, the authors are able to further reduce the LRP to a simple plant location problem, where routing costs are subsumed into the assignment costs. This problem is then solved using branch-and-bound.

The problem studied by Wasner and Zäpfel (2004), posed by a parcel delivery service provider, is more closely related to the MMLRP. Vehicles perform both deliveries and pickups and both their home base location and their routes need to be determined. However, all inter-hub flow must travel through a central hub, rather than allowing all hubs to be directly connected to each other. (As this central hub has already been located, this is a minor variation.) A hierarchical solution approach is adopted. Depot locations are found by an add/drop/shift heuristic. Vehicle routes are also found heuristically. The overall method is driven by four feedback mechanisms: *(1)* varies the allocation of customers to depots, *(2)* combines "neighbouring" vehicle tours, *(3)* changes the location of the depots (shift) and *(4)* changes the number of depots (add/drop).

Lischak and Triesch (2007) also investigate a parcel delivery MMLRP. In their problem, parcels between depots may travel directly or via one or two hubs. The majority of the depot locations have been predetermined by the client. Apart from the location of the remaining depots, also the "class" (throughput capacity) of all the depots needs to be determined. The authors choose to completely enumerate all possible depot combinations, using a pickup-and-delivery heuristic to compute routing costs. An attempt to incorporate route length estimation is also made.

Finally, we note that the problems investigated by Min (1996) (see 4.2.) and Gunnarsson *et al.* (2006) (see 6.3.) also involve several sources or destinations, but do not involve tour planning in either the delivery or the collection routes.

6.3. Vehicle routing-allocation problems

In the above papers, inter-hub routes were direct while hub-to-customer routes involved tours. We could turn this around and consider tours involving hubs and customers being allocated to hubs directly. It is debatable whether such problems should be considered part of the LRP, since no tour planning is involved at customer level, but they are both interesting and sufficiently closely related to be included here. Beasley and Nascimento (1996) name this problem the *vehicle routing-allocation problem* (VRAP) and give a review of related works. However, different authors use different names and create slightly different problem versions.

Nambiar *et al.* (1981) consider the allocation of rubber smallholders to rubber collection stations together with designing the routes of collection vehicles, but their solution algorithm ignores the allocation issue. Labbé and Laporte (1986) solve a problem of locating postboxes. They seek to minimise a linear combination of vehicle routing costs (that of the postal collection van) and customers' inconvenience costs (these depend on the sum of distances between customers and their allocated postbox). A sequential and an iterative heuristic are proposed. Murty and Djang (1999) investigate a complex military logistics problem centred on a VRAP. Mobile training simulators traverse training sites, while army units travel directly from their home bases to training sites. A sequential solution method is presented, based on set covering for locating the training sites and a constructive heuristic for routing the simulators. Labbé *et al.* (2005) use a branch-and-cut method to solve a VRAP (called the *median cycle problem*) where the objective is to minimise routing cost subject to an upper bound on the allocation cost. An interesting practical problem, similar to the VRAP, is tackled by Gunnarsson, Rönnqvist and Carlsson (2006). Routes from sources to hubs may involve stops (at other sources or hubs) but routes from hubs to customers are always direct. A heterogeneous fleet consisting of smaller and larger ships, trains and lorries is used. The authors simplify the problem by considering only a small subset of all possible multi-stop routes and associating a variable with each route – thus, their solution algorithm involves no tour planning. The relaxed problem is solved using a commercial ILP solver. Furthermore, three heuristics, based on removing and later reintroducing some of the variables and constraints, are proposed.

The above papers provide a bridge from LRP to the field of *extensive facility location*, since instead of viewing the problem as that of designing a route we could view it as that of locating a cycle. Extensive facility location concerns locating a dimensional structure, for example a path or a cycle on

a network. This falls outside the scope of this paper – we refer to Mesa and Boffey (1996), Labbé *et al.* (1988) and Díaz-Báñez *et al.* (2004) for reviews.

6.4. Multi-level location-routing problems

An imaginative reader may by now have imagined the situation where routing occurs *both* at hub and customer level. Others may well think that this would be a theoretical construct unlikely to occur in practice – however, this is not the case. In the *two-level location-routing problem*, introduced by Jacobsen and Madsen (1980) and Madsen (1983), newspapers are delivered from the factory to transfer points and from these to the customers. The problem consists of:
(a) determining the locations of transfer points,
(b) designing a vehicle route through these points (known as a primary tour),
(c) allocate the customers to transfer points (or directly to the factory),
(d) designing vehicle routes for each of the above customer clusters (known as secondary tours).
We note that the two-level LRP can be viewed as an LRP extension, since it combines the problem of finding the transfer point locations with the customer delivery routing problem. It can also be viewed as an extension to the VRAP where allocation is achieved via tours rather than by direct links. An unusual aspect of the problem is that transfer points can be relocated with little expense. The "tree-tour" heuristic proposed by the authors is based on the observation that if one deletes the last arc from each route, the problem becomes similar to a Steiner tree problem. This tree is constructed by a greedy one-arc-at-a-time procedure. The authors also put forward two sequential heuristics.

The *road-train routing problem*, introduced by Semet and Taillard (1993), can also be viewed as a two-level LRP. This problem concerns designing a route for a vehicle, called a road-train, that is composed of two parts, a truck and a trailer. Some of the customers are not accessible to this vehicle and thus the trailer is detached and left at a customer location (called a "root") while the truck visits a subset of these customers, returning to pick up the trailer. The route of the road-train corresponds to the primary tour and the routes run by the truck alone to secondary tours. The difference from the Jacobsen and Madsen (1980) problem is that here some customers can be served directly by the primary tour. However, similarly to it, facility costs are negligible and the trailers can be parked at different locations each day. Hence, this problem lacks the usual strategic aspect of LRPs. We note that different authors tackle slightly different versions of the problem.

Semet and Taillard (1993) find an initial solution using a sequential procedure and improve this by a tabu search method, where customers (truck or trailer) are reallocated. In terms of the LRP, this method does not distinguish between locational and routing moves. All trailer customers are forced to be located on the primary tour, which may not lead to the best solution. Semet (1995) proposed a clustering-based solution method. First, customers are allocated to roots: this is formulated as an assignment problem and solved using Lagrangean relaxation. Then, the resulting travelling salesman problems are solved. Gerdessen (1996) assumes that all customers have unit demand and each trailer is parked exactly once. Initial solutions are found using a number of sequential heuristics. These are then improved by a selection of VRP improvement heuristics. Chao (2002) uses a cluster-first routing second initial solution and a tabu search improvement phases with customer reallocation moves. Both papers – similarly to Semet and Taillard (1993) – do not distinguish between locational and routing moves. Souid, Hanafi and Semet (2007) allow more than one subtour (truck route) to originate from a root. Similarly to Melechovský *et al.* (2005), a hierarchical variable neighbourhood structure is used, with the smaller neighbourhoods relating to relocating customers within or between secondary tours and the widest one is defined by the moves of adding or dropping a root.

Perhaps the most complex location-routing problem is that of Ambrosino and Scutellà (2005). They consider a *four-level LRP*, where level 1 is the plant, level 2 consists of distribution centres, level 3 contains transfer points and some (mainly large) customers and level 4 consists of customers. Tour planning is present from level 2 downward. The authors also introduce inventory considerations. Both static and dynamic problem cases are treated. Furthermore, a heterogeneous fleet is allowed. The problem is formulated as an ILP and solved to optimality using commercial software.

7. Suggestions for future research

It may be interesting to begin by looking at what the authors of the past three LRP reviews considered promising future research directions and investigating how much of their suggestions have been realised.

The main recommendations of Balakrishnan, Ward and Wong (1987) can be summarised as:

(a) *Investigating multiple planning periods*. This issue, also referred to as the dynamic LRP, has been considered explicitly in Nambiar *et al.* (1989), Laporte and Dejax (1989), Salhi and Nagy (1999) and Ambrosino and Scutellà (2005). The Salhi and Nagy (1999) study also provided an important validation of LRP as a research field, answering criticisms arising from the issue of different planning horizons. However, a more in-depth investigation of dynamic multi-period problems would enhance the standing of location-routing as a research area.

(b) *Explicitly comparing sequential and combined methods*. This has been achieved by Salhi and Rand (1989). It was found that the use of a sequential method leads to worse solutions than a combined method. These results provide an important validation to the field of location-routing.

(c) *Using analytical formulae in a hierarchical framework*. Such "route length estimation" formulae were derived and then used to solve the LRP by Laporte and Dejax (1989), Chien (1993), Nagy and Salhi (1996b), Bruns, Klose and Stähly (2000) and Lischak and Triesch (2007).

Laporte (1989) identifies the following four "promising research areas":

(d) *Development and systematic analysis of LRP heuristics*. A significant number of articles appeared proposing various LRP heuristics. However, a proper analysis and comparison of these is sadly lacking from the literature, with the notable exception of travelling salesman location problems.

(e) *Development of new exact methods based on Lagrangean relaxation or dynamic programming*. No such paper has appeared, although two LRP subproblems were solved by Lagrangean relaxation by Semet (1995) and Bruns, Klose and Stähly (2000). In our view Lagrangean relaxation is likely to be a promising and challenging research approach.

(f) *A study of hierarchical LRPs*. Such studies were investigated in a theoretical framework (called "nested methods") by Nagy and Salhi (1996a, 1996b, 1998). Practical multi-level LRP versions were also tackled by Bruns, Klose and Stähly (2000), Wasner and Zäpfel (2004) and Lischak and Triesch (2007).

(g) *A study of dynamic LRPs*. [Equivalent to suggestion (a) of Balakrishnan *et al.* (1987).]

Min, Jayaraman and Srivastava (1998) list eight "future research directions":

(h) *Stochasticity*. This issue has since been considered by Chan, Carter and Burnes (2001), Liu and Lee (2003) and Albareda-Sambola *et al.* (2007).

(i) *Time windows*. Only Semet and Taillard (2003) considered this issue, for the special case of the road-train routing problem. Perhaps as time windows relate to a much smaller time horizon than facility location, this "horizon mismatch" may have deterred researchers from including this aspect. (Time windows are usually subject to more fluctuation than the routes themselves.)

(j) *Multiple periods*. [Equivalent to suggestion (a) of Balakrishnan *et al.* (1987).]

(k) *Multiple objectives*. To date, the only Hamiltonian LRP with multiple objectives considered in the literature is the sales-delivery location problem of Averbakh and Berman (1994, 1995). We agree that this is an important issue to consider.

(l) *Vertical integration*. This means consideration of both delivery and pickup traffic and has now been considered by Nagy and Salhi (1998), Wasner and Zäpfel (2004) and Lischak and Triesch (2007).

(m) *Horizontal integration*. This essentially means the integration of inventory aspects into the LRP. This has now been considered by Liu and Lee (2003) and Ambrosino and Scutellà (2005).

(n) *Benchmarks for solution efficiency*. Sadly, the LRP literature is very fragmented and no widely accepted benchmarks exist. (There are collections of instances available from Barreto (2003) or Klose (2005).) In the entire subject literature, only four papers (namely Wu *et al.* (2002), Hansen *et al.* (1994), Nagy and Salhi (1996b) and Tuzun and Burke (1999)) give computational comparisons with their peers, making it nearly impossible to properly judge solution quality.

(o) *Application to real-world problems*. A number of such studies have since appeared, confirming the applicability of location-routing (see subsection 1.3). We hope that practitioners realise the importance of using LRP models and methods to solve their location problems! We believe that the applicability of LRP would increase if LRP methods were coupled with an easy-to-use interface (perhaps in a GIS framework).

The above observations paint a mixed picture. Although significant progress has been made, certain challenges have not yet been taken up by the locational analysis community.

In the following, we give some suggestions for future research, which we believe to be worthwhile considering. (Some of these were also suggested by previous reviews, as indicated by the references to the above list given in brackets.) The authors are currently working on some of these problems.

1. *Hierarchical methods and use of route length formulae.* [(c), (f)] The authors' view is that routing plays a subservient role in LRP and *approximation formulae* can often be used instead of vehicle routing algorithms within the search to speed up the process. One possible approach is to refer to the routing stage explicitly whenever necessary and use approximation in other steps. Thus, we feel that continued testing and further development of such formulae would be desirable. In particular, formulae should be extended to cater for vehicle routing extensions such as fleet mix or pickup and delivery. Another interesting topic would be the development of an estimation formula for the arc routing problem. (This would be very different from route length estimation in node routing, since it is the deadheading distance that needs to be estimated.)

2. *Dynamic and stochastic problems.* [(a), (g), (j)] The most important criticism of location-routing is that during the planning horizon of facilities, the customer demands may fluctuate and even the customer set may change. Thus, this type of problems merits further investigation. In particular, little research has been done on the dynamic LRP where depots are located in a sequence. We think that this problem has real-world applicability – we can envisage a growing demand or coverage but the pattern of this growth is not known with certainty. (In fact, one of the papers proposing this problem was a case study.) This problem could best be solved using *robustness analysis*. This methodology is geared towards sequential decision-making and strives to preserve flexibility in an uncertain environment. Thus, it would be very suitable for this type of dynamic LRP. Another interesting problem would be to investigate problems where arc lengths, rather than customer demands, are stochastic. This would model real-life situations better, as travel times tend to depend on travel conditions.

3. *Planar location-routing.* Looking back at our review, we can observe that all papers, with the exception of some TSLP and round-trip location papers and the recent Schwardt and Dethloff (2005) and Salhi and Nagy (2007) papers, deal with discrete location. Yet, in the general location literature, there is a considerable proportion of articles on *continuous location*. It is puzzling why there are hardly any works on planar LRP. Clearly, the LRP is more applicable to discrete problems as the vehicle routing aspect assumes an underlying road network. However, a planar problem could be construed where the customers are on a road network but the facility would be located on a greenfield (brownfield) site.

4. *Integrated methods in logistics.* [(l), (m)] The authors are great believers in integrated methods in logistics. Combining location-routing with other aspects of logistics would be an interesting avenue of research. We are particular interested in solving the *location-routing-inventory* and the *location-routing-packing* problems.

5. *Multiobjective LRP.* [(k)] Very little has been done in this area, yet it can clearly be applied to a variety of problems. On one hand, the problem of *hazardous* material transportation attracted considerable attention in the literature. Some of these works address issues of location and routing in a combined framework, however the underlying routes are not of a multi-stop nature. It would be an interesting problem to solve combined location-routing problems with multi-stop routes where the material transported is either hazardous or obnoxious. On the other hand, we would like

to see development of further models for *delivery-man* or similar problems. For example, the newspaper delivery problem of Jacobsen and Madsen (1980) could be solved using a multiobjective approach, minimising total tour length on one hand and the sum of arrival times at customers on the other.

6. *Competitive LRP.* Competitive location has a considerable literature, giving rise to interesting connections with game theory. Yet, there are no papers in competitive location-routing. One possible explanation for this is that as competitive location focuses on user choice, it is more reasonable to assume that users travel directly to facilities, and hence no routing considerations are required. Perhaps a competitive VRAP, such as the following, is easier to imagine than a competitive LRP. Two transport companies wish to locate circular routes. Customers may just choose the stop nearest to them, or for each possible origin-destination pair they may choose the company that minimises the sum of origin-to-stop and stop-to-destination distances.

7. *Heuristics for Eulerian location.* This has obvious applications to locating postal delivery, road maintenance or waste collection depots. There are only two papers on this problem: one ignores the interdependency of location and routing and the other ignores important constraints. It would be interesting to see iterative and integrated heuristics developed for this problem.

8. *Hybrid methodologies.* Although the location-routing research community is small, research is somewhat fragmented and a number of strands exist. We hope that this review may help in identifying different methodologies and prompt researchers to see if they can be united in some way. For example, the lack of feedback in clustering-based methods could be eliminated by including them in an iterative framework. Or, such methods could provide better starting points for a hierarchical local search algorithm. Another promising avenue would be the combination of exact and heuristic methods, such as heuristic concentration.

9. *Modelling complex situations.* We also believe LRP and similar models should be applied to complex situations. For example, it would be interesting to see an iterative or hierarchical solution to the problem faced by Murty and Djang (1999) rather than the sequential approach adopted by the authors. Similarly, the four-level static and dynamic LRPs introduced by Ambrosino and Scutellà (2005) could be solved heuristically. The MMLRP is also a growing research field, with already some papers published on postal and parcel delivery applications, however it could also be applied to rail-hub location in combined road-rail transport systems. Location-routing type models could be applied to the design of public transport networks, where both metro and connecting bus lines need to be located together.

We may summarise our proposed research agenda as trying to solve *all* locational problems, where appropriate, using the location-routing approach. This review can happily report on a much larger body of work than its precursors, but location-routing is still a developing area. We hope that the challenges posed here will arouse the interest of some of the readers and entice them to work in this challenging and exciting research area!

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