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# Stochastic stability and stabilization of discrete-time singular Markovian jump systems with partially unknown transition probabilities

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## SUMMARY

This paper considers the stochastic stability and stabilization of discrete-time singular Markovian jump systems with partially unknown transition probabilities. Firstly, a set of necessary and sufficient conditions for the stochastic stability is proposed in terms of LMIs, then a set of sufficient conditions is proposed for the design of a state feedback controller to guarantee that the corresponding closed-loop systems are regular, causal, and stochastically stable by employing the LMI technique. Finally, some examples are provided to demonstrate the effectiveness of the proposed approaches. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: singular Markovian jump systems; stability; stabilization; partially unknown transition probabilities; linear matrix inequalities (LMIs)

## 1. INTRODUCTION

Singular systems, which are also referred to as descriptor systems, generalized state-space systems, or differential-algebraic systems, provide convenient and natural representations of practical systems, for example, economic systems, power systems, and circuits systems. Control of singular systems has been an attractive field in control theory and applications [1, 2]. Stability of singular systems is as important as that of normal systems. However, in the singular system, not only asymptotic stability has to be considered but also the regularity and non-impulsiveness/causality are needed to be addressed. Many results have been reported on control for the singular systems [1–6].

In recent years, considerable attention has been paid to Markovian jump systems. It is well-known that the Markovian jump systems are an important class of stochastic systems, which are popular in modeling many practical systems that may experience random abrupt changes in their structures and parameters [7–14]. Singular systems with Markovian jump for the discrete-time case have been studied in [3]. Where the problems of stability, state feedback control and static output feedback control for a class of discrete-time singular Markovian jump systems with completely known transition probabilities are investigated. New necessary and sufficient conditions guaranteeing the systems to be regular, causal, and stochastically stable are proposed in terms of a set of coupled strict LMIs in [3].

In fact, lots of ideal knowledge for the transition probabilities are expected to predigest system analysis. However, to obtain such available knowledge of the transition probabilities is actually problematic, which may be very expensive. For example, in some communication networks, either the

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variation of delays or the packet dropouts can be vague and random in different running periods of networks. It is very hard or costly to obtain all or even part of the elements in the desired transition probabilities matrix. The same problems may appear in other practical jump systems. Therefore, it is meaningful and necessary to further study more general jump systems with partially unknown transition probabilities from control perspectives, rather than having a large complexity to measure or estimate all the transition probabilities. So far, some methods have been developed to deal with the problem of partially unknown transition probabilities [15–20]. The stability and stabilization problems of a class of continuous-time and discrete-time Markovian jump linear systems with partially unknown transition probabilities are investigated in [16]. The partially mode-dependent  $H_\infty$  filtering problem for discrete-time Markovian jump systems with partially unknown transition probabilities via different techniques is concerned in [17], where the unknown elements are estimated. It should be noted that only sufficient conditions are given in [16], while the necessary and sufficient conditions for the stability analysis and stabilization synthesis problems are firstly derived for both continuous-time and discrete-time cases in [15]. However, up to now, necessary and sufficient conditions on the stochastic stability for discrete-time singular Markovian jump systems with partially unknown transition probabilities have not been fully investigated.

In this paper, the problem of stochastic stability and stabilization of discrete-time singular Markovian jumping systems with partially unknown transition probabilities is considered. Firstly, by the convex combination, a set of necessary and sufficient conditions for the stochastic stability is proposed in terms of LMIs, such that the stability criterion developed in Theorem 3.1 is less conservative than the one in Theorem 1 in [18]. Then sufficient conditions are proposed for the design of a state feedback controller, which guarantees that the closed-loop systems with partially unknown transition probabilities are regular, causal, and stochastically stable by employing the LMI technique.

The remainder of this paper is organized as follows. In Section 2, the considered discrete-time singular Markovian jump systems are formulated, some definitions and lemmas are stated. In Section 3, necessary and sufficient conditions on the stochastic stability of the unforced systems with partially unknown transition probabilities are given, and sufficient conditions on the stochastic stability by LMIs are developed. In Section 4, sufficient conditions on the stochastic stabilization are given to design a state feedback controller by LMIs. Some numerical examples are provided to illustrate the validity and the applicability of the developed results in Section 5. Section 6 concludes the paper.

*Notation.* The notation used in this technical note is standard.  $\mathbb{R}^n$  stands for the  $n$ -dimensional Euclidean space, and  $\mathbb{R}^{m \times n}$  represents the set of all  $m \times n$  real matrices. The superscript ‘T’ stands for matrix transposition.  $(\Omega, \mathcal{F}, \mathcal{P})$  is the probability space, where  $\Omega$  represents the sample space,  $\mathcal{F}$  is the  $\sigma$ -algebra of subsets of the sample space, and  $\mathcal{P}$  is the probability measure on  $\mathcal{F}$ .  $\mathbf{N}^+$  represents the set of positive integers.  $\|A\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$  and  $\|A\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|$  represent the induced Matrix 1-Norm and Matrix  $\infty$ -Norm, respectively. The notation  $P > 0$  ( $P \geq 0$ ) implies that  $P$  is a real symmetric and positive definite (semi-positive definite) matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. For simplicity, sometimes  $A_i$ ,  $B_i$ , and  $K_i$  are used to denote  $A(r_t)$ ,  $B(r_t)$ , and  $K(r_t)$ , respectively.

## 2. PROBLEM FORMULATION AND PRELIMINARIES

Fix the probability space  $(\Omega, \mathcal{F}, \mathcal{P})$  and consider the discrete-time singular Markovian jump systems described by

$$Ex(k+1) = A(r_k)x(k) + B(r_k)u(k) \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  is the system state;  $u(k) \in \mathbb{R}^m$  is the control input; The matrix  $E \in \mathbb{R}^{n \times n}$  may be singular, with  $\text{rank}(E) = r \leq n$ ;  $A(r_t)$  and  $B(r_t)$  are known real constant matrices with appropriate dimensions.  $\{r_k, k \geq 0\}$  is the jumping process.  $\{r_k\}$  is a discrete-time homogeneous Markovian process with right discrete trajectories, which takes values in a finite set  $\ell = \{1, 2, \dots, N\}$ , with transition probability matrix  $\pi = [\pi_{ij}]_{N \times N}$ , and  $\pi_{ij} \geq 0$  is defined as

$$\pi_{ij} = Pr\{r_{k+1} = j \mid r_k = i\}$$

where  $\sum_{j=1}^N \pi_{i,j} = 1$ , and the Markovian process transition probability matrix  $\pi$  is

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix}$$

In addition, the transition probabilities of the jumping process are considered to be partially accessed in this paper, that is, some elements in matrix  $\pi$  are assumed to be unknown. For instance, for the systems (1) with four operation modes, the transition probability matrix  $\pi$  may be expressed as

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ ? & ? & ? & \pi_{24} \\ \pi_{31} & ? & \pi_{33} & ? \\ ? & ? & ? & ? \end{bmatrix} \quad (2)$$

where "?" represents the inaccessible elements. For notational clarity,  $\forall i \in \ell$ , we denote  $\ell = \ell_{\kappa}^i \cup \ell_{\nu\kappa}^i$  where

$$\begin{aligned} \ell_{\kappa}^i &\triangleq \{j : \pi_{ij} \text{ is known}\} \\ \ell_{\nu\kappa}^i &\triangleq \{j : \pi_{ij} \text{ is unknown}\} \end{aligned}$$

Moreover, if  $\ell_{\kappa}^i \neq \emptyset$ , it is further described as  $\ell_{\kappa}^i = \{\kappa_1^i, \kappa_2^i, \dots, \kappa_m^i\}$ ,  $\forall 1 \leq m \leq N$ , where  $\kappa_m^i \in \mathbf{N}^+$  represent the  $m$ th known element with the index  $\kappa_m^i$  in the  $i$ th row of matrix  $\pi$ . Because  $\sum_{j=1}^N \pi_{i,j} = 1$  and  $\sum_{j_1 \in \ell_{\kappa}^i} \pi_{i,j_1} + \sum_{j_2 \in \ell_{\nu\kappa}^i} \pi_{i,j_2} = 1$ , we denote

$$h_i = \sum_{j_2 \in \ell_{\nu\kappa}^i} \pi_{i,j_2} = 1 - \sum_{j_1 \in \ell_{\kappa}^i} \pi_{i,j_1} \quad (3)$$

where  $j_1 \in \ell_{\kappa}^i, j_2 \in \ell_{\nu\kappa}^i$ .

**Definition 2.1**

- I. The discrete-time singular Markovian jump systems in (1) with  $u(k) = 0$  are said to be regular if, for each  $i \in \ell$ ,  $\det(zE - A_i)$  is not identically zero.
- II. The discrete-time singular Markovian jump systems in (1) with  $u(k) = 0$  are said to be causal if, for each  $i \in \ell$ ,  $\deg(\det(zE - A_i)) = \text{rank}(E)$ .
- III. The discrete-time singular Markovian jump systems in (1) with  $u(k) = 0$  are said to be stochastically stable if for any  $x_0 \in \mathbf{R}^n$  and  $r_0 \in \ell$ , there exists a scalar  $M(x_0, r_0)$  such that

$$E \left\{ \sum_{k=0}^{\infty} \|x(k)\|^2 \mid x_0, r_0 \right\} < M(x_0, r_0)$$

where  $x(k, x_0, r_0)$  denotes the solution to the systems (1) at time  $k$  under the initial conditions  $x_0$  and  $r_0$ .

- IV. The discrete-time singular Markovian jump systems in (1) with  $u(k) = 0$  are said to be stochastically admissible if they are regular, causal and stochastically stable.

**Definition 2.2**

For  $\Lambda_i \in \mathbf{R}^{n \times n}$ , and  $\sum_{i=1}^n \alpha_i = 1$ , where  $\alpha_i \geq 0$  are scalars, for  $i \in \{1, 2, \dots, n\}$ ,  $\sum_{i=1}^n \alpha_i \Lambda_i$  is said to be convex combination of  $\Lambda_i$ .

01 *Lemma 2.1* ([15])

02 Consider the discrete-time Markovian jump systems  $x(k+1) = A(r_k)x(k)$  with partially unknown  
03 transition probabilities. The corresponding systems are stochastically stable if and only if there  
04 exists a set of matrices  $P_i > 0, i \in \ell$ , such that

$$05 \quad A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \left( 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} \right) P_{j_2} \right) A_i - P_i < 0, \quad j_2 \in \ell_{vk}^i$$

09 *Lemma 2.2* ([3])

10 The discrete-time Markovian jump singular systems (1) with completely known transition  
11 probabilities are stochastically admissible if and only if there exist a set of positive definite matrices  
12  $P_i, i \in \ell$ , and a symmetric and nonsingular matrix  $\Phi$ , satisfying

$$13 \quad A_i^T \left( \sum_{j=1}^N \pi_{i,j} P_j - R^T \Phi R \right) A_i - E^T P_i E < 0$$

14 Define  $R \in \mathbb{R}^{n \times n}$  as the matrix with the properties of  $E^T R^T = 0$  and  $\text{rank}(R) = n - r$ , which  
15 are used in all the subsequent lemmas and theorems.

16 *Lemma 2.3* ([3])

17 Let  $L_i$  be nonsingular matrices with appropriate dimensions, for  $i \in \ell$ . Then, the inequalities  
18  $A_i^T \left( \sum_{j=1}^N \pi_{i,j} P_j - R^T \Phi R \right) A_i - E^T P_i E < 0$  hold if for

$$19 \quad \begin{bmatrix} \Pi_i & A_i^T L_i - L_i^T \\ L_i^T A_i - L_i \sum_{j \in \ell} \pi_{i,j} P_j - L_i - L_i^T \end{bmatrix} < 0$$

20 where  $\Pi_i = A_i^T L_i + L_i^T A_i - A_i^T R^T \Phi R A_i - E^T P_i E$ .

21 *Lemma 2.4*

22 Let  $A$  be a symmetric and positive definite matrix, and  $\lim_{x_{ij} \rightarrow 0} \|X\|_1 = 0$ . Then, we have

$$23 \quad \lim_{\|X\|_1 \rightarrow 0} X^T A X = \mathbf{0}.$$

24 *Proof*

25 From  $A > 0$ , it is easy to see that there exists a nonsingular matrix  $C$  such that  $A = C^T C$ . Denote  
26  $B = X^T A X = X^T C^T C X = (CX)^T C X$ .

27 It is easy to see that the following matrix norm inequality holds

$$28 \quad 0 \leq \|CX\|_1 \leq \|C\|_1 \|X\|_1$$

29 Applying  $\lim_{\|X\|_1 \rightarrow 0} \|X\|_1 = 0$ ,

$$30 \quad 0 \leq \lim_{\|X\|_1 \rightarrow 0} \|CX\|_1 \leq \lim_{\|X\|_1 \rightarrow 0} \|C\|_1 \|X\|_1 = 0,$$

31 which implies  $\lim_{\|X\|_1 \rightarrow 0} \|CX\|_1 = 0$ . Furthermore,

$$32 \quad 0 \leq \lim_{\|X\|_1 \rightarrow 0} \|B\|_1 = \lim_{\|X\|_1 \rightarrow 0} \|X^T A X\|_1 \leq \lim_{\|X\|_1 \rightarrow 0} \|CX\|_\infty \|CX\|_1$$

33 Thus,  $\lim_{\|X\|_1 \rightarrow 0} \|B\|_1 = 0$ , which implies  $\lim_{\|X\|_1 \rightarrow 0} b_{ij} = 0$

34 Hence, the result  $\lim_{\|X\|_1 \rightarrow 0} X^T A X = \mathbf{0}$  follows.  $\square$

3. STABILITY ANALYSIS

In this section, necessary and sufficient conditions on the stochastic stability of the unforced systems (1) with partially unknown transition probabilities are to be presented.

*Theorem 3.1*

The discrete-time singular Markovian jump systems (1) with partially unknown transition probabilities are stochastically admissible if and only if there exist a set of positive definite matrices  $P_i, i \in \ell$ , a symmetric and nonsingular matrix  $\Phi$ , satisfying

$$A_i^T (\bar{P}_i - R^T \Phi R) A_i - E^T P_i E < 0 \tag{4}$$

where  $\bar{P}_i = \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + h_i P_{j_2}, h_i = \sum_{j_2 \in \ell_{\nu\kappa}^i} \pi_{i,j_2} = 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}$ .

*Proof*

Sufficiency. At first, we note that if  $\ell_{\nu\kappa}^i \neq \emptyset$ , then  $\sum_{j_1 \in \ell_k^i} \pi_{i,j_1} < 1$ , it means that the elements in the  $i$ th row are partially known.

If the inequalities in (4) hold, then

$$\sum_{j_2 \in \ell_{\nu\kappa}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} (A_i^T (\bar{P}_i - R^T \Phi R) A_i - E^T P_i E) < 0 \tag{5}$$

$$\sum_{j_2 \in \ell_{\nu\kappa}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} \left( A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + h_i P_{j_2} - R^T \Phi R \right) A_i - E^T P_i E \right) < 0 \tag{6}$$

$$\sum_{j_2 \in \ell_{\nu\kappa}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} \left( A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \left( 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} \right) P_{j_2} - R^T \Phi R \right) A_i - E^T P_i E \right) < 0$$

$$A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \left( 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} \right) \sum_{j_2 \in \ell_{\nu\kappa}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} P_{j_2} - R^T \Phi R \right) A_i - E^T P_i E < 0$$

$$A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \sum_{j_2 \in \ell_{\nu\kappa}^i} \pi_{i,j_2} P_{j_2} - R^T \Phi R \right) A_i - E^T P_i E < 0$$

$$A_i^T \left( \sum_{j \in \ell} \pi_{i,j} P_j - R^T \Phi R \right) A_i - E^T P_i E < 0 \tag{7}$$

where  $0 \leq \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} \leq 1, \forall j_2 \in \ell_{\nu\kappa}^i$  and  $\sum_{j_2 \in \ell_{\nu\kappa}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} = 1$ .

It is clear that the discrete-time singular Markovian jump systems (1) are stochastically admissible by applying Lemma 2.2.

Necessity. Suppose that the discrete-time singular Markovian jump systems (1) are stochastically admissible. Then, we select two nonsingular matrices  $M$  and  $N$  such that

$$E = M \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} N \tag{8}$$

$$A_i = M \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix} N \quad (9)$$

The regularity and causality of the systems (1) imply that  $A_{4i}$  are nonsingular for any  $i \in \ell$ . Then, choose nonsingular matrices

$$\Omega_i = \begin{bmatrix} I & -A_{3i}^T A_{4i}^{-T} \\ 0 & I \end{bmatrix} \quad (10)$$

and let  $\hat{N}_i = N^{-1} \Omega_i^T$ . It can be verified that

$$\hat{E} = M^{-1} E \hat{N}_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \quad (11)$$

$$\hat{A}_i = M^{-1} A_i \hat{N}_i = \begin{bmatrix} \hat{A}_{1i} & A_{2i} \\ 0 & A_{4i} \end{bmatrix} \quad (12)$$

where  $\hat{A}_{1i} = A_{1i} - A_{2i} A_{4i}^{-1} A_{3i}$ .

It can be seen that the stochastic stability of the discrete-time singular Markovian jump systems (1) implies that the discrete Markovian jump systems

$$\eta(k+1) = \hat{A}_{1i} \eta(k) \quad (13)$$

are stochastically stable. Thus, there exist matrices  $\hat{P}_i > 0$ ,  $i \in \ell$ , such that

$$\hat{A}_{1i}^T \hat{P}_i \hat{A}_{1i} - \hat{P}_i < 0$$

where  $\hat{P}_i = \sum_{j \in \ell} \pi_{i,j} \hat{P}_j$ . So, a sufficiently large scalar  $\lambda > 0$  always exists, such that, for  $i \in \ell$ ,

$$\begin{aligned} & \hat{A}_i^T \begin{bmatrix} \hat{P}_i & 0 \\ 0 & \lambda I \end{bmatrix} \hat{A}_i - \hat{E}^T \begin{bmatrix} \hat{P}_i & 0 \\ 0 & \lambda I \end{bmatrix} \hat{E} - \begin{bmatrix} 0 \\ A_{4i}^T \end{bmatrix} 2\lambda \begin{bmatrix} 0 & A_{4i} \end{bmatrix} \\ & = \begin{bmatrix} \hat{A}_{1i}^T \hat{P}_i \hat{A}_{1i} - \hat{P}_i & \hat{A}_{1i}^T \hat{P}_i A_{2i} \\ A_{2i}^T \hat{P}_i \hat{A}_{1i} & A_{2i}^T A_{2i} - \lambda A_{4i}^T A_{4i} \end{bmatrix} < 0 \end{aligned} \quad (14)$$

Let  $M^{-1} = X = [X_1^T, X_2^T]^T$ . Then  $X_2 E = [0, 0]$ ,  $X_2 A_i \hat{N}_i = [0, A_{4i}]$ . Let  $P_i = X^T \begin{bmatrix} \hat{P}_i & 0 \\ 0 & \lambda I \end{bmatrix} X$ ,  $R = Z X_2$ ,  $\Phi = 2\lambda Z^{-T} Z^{-1}$ . Then

$$\hat{N}_i^T \left( A_i^T \left( \sum_{j \in \ell} \pi_{i,j} P_j - R^T \Phi R \right) A_i - E^T P_i E \right) \hat{N}_i < 0 \quad (15)$$

If  $\ell_{\nu\kappa}^i \neq \emptyset$ ,  $\sum_{j_1 \in \ell_{\nu\kappa}^i} \pi_{i,j_1} < 1$ , which means that the elements in the  $i$ th row are partially known. Note that  $\sum_{j_1 \in \ell_{\nu\kappa}^i} \pi_{i,j_1} \leq 1$ . The inequalities (15) can be rewritten as

$$\hat{N}_i^T \left( A_i^T \left( \sum_{j_1 \in \ell_{\nu\kappa}^i} \pi_{i,j_1} P_{j_1} + \sum_{j_2 \in \ell_{\nu\kappa}^i} \pi_{i,j_2} P_{j_2} - R^T \Phi R \right) A_i - E^T P_i E \right) \hat{N}_i < 0$$

where the elements  $\pi_{i,j_1}$ ,  $j_1 \in \ell_{\nu\kappa}^i$  are all known and  $\pi_{i,j_2}$ ,  $j_2 \in \ell_{\nu\kappa}^i$  are all unknown.

$$\begin{aligned}
 & \hat{N}_i^T \left( A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \left( 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} \right) \sum_{j_2 \in \ell_{\nu k}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} P_{j_2} \right. \right. \\
 & \quad \left. \left. - R^T \Phi R \right) A_i - E^T P_i E \right) \hat{N}_i < 0 \\
 & \hat{N}_i^T \left( \sum_{j_2 \in \ell_{\nu k}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} \left( A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + \left( 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} \right) P_{j_2} \right. \right. \right. \\
 & \quad \left. \left. - R^T \Phi R \right) A_i - E^T P_i E \right) \hat{N}_i < 0 \\
 & \hat{N}_i^T \left( \sum_{j_2 \in \ell_{\nu k}^i} \frac{\pi_{i,j_2}}{1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1}} \left( A_i^T \left( \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + h_i P_{j_2} \right. \right. \right. \\
 & \quad \left. \left. - R^T \Phi R \right) A_i - E^T P_i E \right) \hat{N}_i < 0 \tag{16}
 \end{aligned}$$

Therefore, the inequalities (16) are equivalent to the inequalities (4). This completes the proof.  $\square$

**Remark 3.1**

If  $\ell_{\nu k}^i = \emptyset$ , then  $\sum_{j_1 \in \ell_k^i} \pi_{i,j_1} = 1$  for every  $i \in \ell$ . It means that the elements in every  $i$ th row are all known.

$$h_i = \sum_{j_2 \in \ell_{\nu k}^i} \pi_{i,j_2} = 1 - \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} = 0$$

So in this case, Theorem 3.1 is degenerated to Lemma 2.2.

**Remark 3.2**

Necessary and sufficient conditions of the stochastic stability admissible for the systems (1) have been presented earlier. However, Theorem 1 in [18] is only a sufficient condition. So the stability admissible criterion developed in Theorem 3.1 is less conservative than that in Theorem 1 in [18]. It is worth to pointing out that the following Theorem is to be derived by Theorem 3.1, which will be used to derive the main results of next section.

**Theorem 3.2**

Let  $E_1$  and  $E_2$  be the given appropriate matrices, the systems (1) with partially unknown transition probabilities and  $u(k) = 0$  are regular, causal, and stochastically stable if there exist symmetric and positive definite matrices  $Y_i$ ,  $\Psi$  and nonsingular matrices  $G_i$  such that the following set of LMIs holds for each  $i \in \ell$ .

$$\begin{bmatrix} \Xi_i & G_i^T A_i^T - G_i & 0 \\ A_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} < 0 \tag{17}$$

where  $\Xi_i = G_i^T A_i^T + A_i G_i - E_1^T E G_i - G_i^T E^T E_1 - E_2^T R A_i G_i - G_i^T A_i^T R^T E_2 + E_1^T Y_i E_1 + E_2^T \Psi E_2$ ,  $W_i = [\sqrt{\pi_{i1}} G_i^T, \sqrt{\pi_{i2}} G_i^T, \dots, \sqrt{\pi_{ij_1}} G_i^T, \sqrt{h_i} G_i^T]^T$ ,  $\Theta_i = \text{diag}[Y_1, Y_2, \dots, Y_{j_1}, Y_{j_2}]$ .



01 *Proof*

02 By applying Lemma 2.3 to Theorem 3.1 for each  $i \in \ell$ , it follows that inequalities (4) hold if

$$03 \begin{bmatrix} \Pi_i & A_i^T L_i - L_i^T \\ L_i^T A_i - L_i & \bar{P}_i - L_i - L_i^T \end{bmatrix} < 0 \quad (18)$$

04 where  $\Pi_i = A_i^T L_i + L_i^T A_i - A_i^T R^T \Phi R A_i - E^T P_i E$ ,  $\bar{P}_i = \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1} + h_i P_{j_2}$ .

05 Pre-multiplying and post-multiplying (18) by both  $\text{diag}[L_i^{-1} \ L_i^{-1}]^T$  and its transpose, it follows that

$$06 \begin{bmatrix} L_i^{-1} & 0 \\ 0 & L_i^{-1} \end{bmatrix}^T \begin{bmatrix} \Pi_i & A_i^T L_i - L_i^T \\ L_i^T A_i - L_i & \bar{P}_i - L_i - L_i^T \end{bmatrix} \begin{bmatrix} L_i^{-1} & 0 \\ 0 & L_i^{-1} \end{bmatrix} < 0$$

$$07 \begin{bmatrix} \Upsilon_i & G_i^T A_i^T - G_i \\ A_i G_i - G_i^T & G_i^T (\bar{P}_{ki} + h_i P_{j_2}) G_i - G_i - G_i^T \end{bmatrix} < 0 \quad (19)$$

08 where  $L_i^{-1} = G_i$ ,  $\Upsilon_i = G_i^T A_i^T + A_i G_i - G_i^T A_i^T R^T \Psi^{-1} R A_i G_i - G_i^T E^T P_i E G_i$ ,  $\bar{P}_{ki} = \sum_{j_1 \in \ell_k^i} \pi_{i,j_1} P_{j_1}$ .

09 Let  $Y_i^{-1} = P_i$ , by using the Schur complement lemma, it is easy to show that

$$10 \begin{bmatrix} \Sigma_i & G_i^T A_i^T - G_i & 0 \\ A_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} < 0 \quad (20)$$

11 where  $\Sigma_i = G_i^T A_i^T + A_i G_i - G_i^T A_i^T R^T \Psi^{-1} R A_i G_i - G_i^T E^T Y_i^{-1} E G_i$ ,

12  $W_i = [\sqrt{\pi_{i1}} G_i^T, \sqrt{\pi_{i2}} G_i^T, \dots, \sqrt{\pi_{ij_1}} G_i^T, \sqrt{h_i} G_i^T]^T$ ,  $\Theta_i = \text{diag}[Y_1, Y_2, \dots, Y_{j_1}, Y_{j_2}]$ .

13 According to lemma 2.5, choose the appropriate matrices  $E_1$  and  $E_2$  such that

$$14 \begin{aligned} 0 &\leq (G_i^T E^T - E_1^T Y_i) Y_i^{-1} (E G_i - Y_i E_1) \\ &= G_i^T E^T Y_i^{-1} E G_i - E_1^T E G_i - G_i^T E^T E_1 + E_1^T Y_i E_1 \end{aligned} \quad (21)$$

$$15 \begin{aligned} 0 &\leq (G_i^T A_i^T R^T - E_2^T \Psi) \Psi^{-1} (R A_i G_i - \Psi E_2) \\ &= G_i^T A_i^T R^T \Psi^{-1} R A_i G_i - E_2^T R A_i G_i - G_i^T A_i^T R^T E_2 + E_2^T \Psi E_2 \end{aligned} \quad (22)$$

16 It is easy to show that (21) and (22) can be rewritten as

$$17 -G_i^T E^T Y_i^{-1} E G_i \leq -E_1^T E G_i - G_i^T E^T E_1 + E_1^T Y_i E_1 \quad (23)$$

$$18 -G_i^T A_i^T R^T \Psi^{-1} R A_i G_i \leq -E_2^T R A_i G_i - G_i^T A_i^T R^T E_2 + E_2^T \Psi E_2 \quad (24)$$

19 From (20), (23), and (24),

$$20 \begin{bmatrix} \Sigma_i & G_i^T A_i^T - G_i & 0 \\ A_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} \leq \begin{bmatrix} \Xi_i & G_i^T A_i^T - G_i & 0 \\ A_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} \quad (25)$$

21 where  $\Sigma_i = G_i^T A_i^T + A_i G_i - G_i^T A_i^T R^T \Psi^{-1} R A_i G_i - G_i^T E^T Y_i^{-1} E G_i$ ,

22  $\Xi_i = G_i^T A_i^T + A_i G_i - E_1^T E G_i - G_i^T E^T E_1 - E_2^T R A_i G_i - G_i^T A_i^T R^T E_2 + E_1^T Y_i E_1 + E_2^T \Psi E_2$

23 Considering inequalities (25). The inequalities (4) hold if

$$24 \begin{bmatrix} \Xi_i & G_i^T A_i^T - G_i & 0 \\ A_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} < 0$$

25 By Theorem 3.1, the systems (1) with  $u(k) = 0$  are regular, causal, and stochastically stable. This completes the proof.  $\square$

*Remark 3.3*

From the proof of Theorem 3.2, it is easy to see that matrices  $E_1$  and  $E_2$  can be arbitrary. So the matrix inequalities in (17) can be viewed as a standard LMI when the matrices  $E_1$  and  $E_2$  are property chosen. The remaining problem is how to choose the matrices  $E_1$  and  $E_2$ . Define two scalars  $\delta$  and  $\varepsilon$  satisfying

$$\min_{\delta} \|EG_i - Y_i E_1\|_1 \leq \delta, \quad \min_{\varepsilon} \|RA_i G_i - \Psi E_2\|_1 \leq \varepsilon$$

s.t.(17)

We have pointed out that in order to fix the matrices  $E_1$  and  $E_2$ , a matrix equality constraint has to be involved, which forms a minimization problem.

Based on the earlier discussion, the following algorithm is to be presented.

Iterative LMI algorithm:

*Step 1:* For desired decay rate  $\delta \geq 0$  and  $\varepsilon \geq 0$ , give the initial matrices  $E_1$  and  $E_2$ , find a feasible solution for the LMIs (17). Denote the feasible solution as  $(\delta_0, \varepsilon_0, G_{i0}, Y_{i0}, \Psi_0)$ . Take  $G_{i0}, Y_{i0}$ , and  $\Psi_0$  as the iterative initial values.

*Step 2:* Given the initial values  $(\delta_0, \varepsilon_0, G_{i0}, Y_{i0}, \Psi_0)$ , solve the minimization problem:

$$\min_{\delta} \|EG_{i0} - Y_{i0} E_{11}\|_1 \leq \delta_0, \quad \min_{\varepsilon} \|RA_i G_{i0} - \Psi_0 E_{21}\|_1 \leq \varepsilon_0$$

Denote the minimizing solution as  $(E_{11}, E_{21})$ .

*Step 3:* Given the initial matrices  $(E_{11}, E_{21})$ , find a feasible solution for the LMIs (17). Denote the feasible solution as  $(G_{i1}, Y_{i1}, \Psi_1)$  and denote

$$\begin{aligned} \|EG_{i1} - Y_{i1} E_{11}\|_1 &= \delta_1 \\ \|RA_i G_{i1} - \Psi_1 E_{21}\|_1 &= \varepsilon_1 \end{aligned}$$

*Step 4:* If  $\delta_1 \geq \delta_0, \varepsilon_1 \geq \varepsilon_0$ . Then, stop. Otherwise, go to step 2.

*Remark 3.4*

In Theorem 3.2, applying Lemma 2.4, appropriate matrices  $E_1$  and  $E_2$  can guarantee the matrices  $(G_i^T E^T - E_1^T Y_i) Y_i^{-1} (EG_i - Y_i E_1)$  and  $(G_i^T A_i^T R^T - E_2^T \Psi) \Psi^{-1} (RA_i G_i - \Psi E_2)$  in (21) and (22) close to zero, which has reduced the conservatism. It is not only easy to obtain the solutions of (17) and the matrix inequalities of the following theorem but also to reduce the conservatism compared with Theorem 8 in [3], which has used two scalars. Especially, when we choose  $E_1$  and  $E_2$  in terms of  $E_1 = \text{diag}[\alpha, \alpha, \dots, \alpha]$  and  $E_2 = \text{diag}[\beta, \beta, \dots, \beta]$ , it can be seen that matrix parameters play the same role as scalar parameters in handling this problem by applying a set of matrix operations.

## 4. STATE FEEDBACK CONTROL

In this section, the state feedback control for the systems (1) with partially unknown transition probabilities will be studied. Consider the following state feedback controller

$$u(k) = K(r_k)x(k) \quad (26)$$

where  $K(r_k)$  is the feedback gain to be determined. Substituting (26) into systems (1) yields the closed-loop systems

$$Ex(k+1) = (A(r_k) + B(r_k)K(r_k))x(k) \quad (27)$$

Then, by applying Theorem 3.2 to systems (27), the following result can be obtained directly.

*Theorem 4.1*

Consider the systems (1) with partially unknown transition probabilities for each  $i \in \ell$  and  $E_1$  and  $E_2$  be chose matrices. There exists a state feedback controller  $u(k) = K(r_k)x(k)$  for systems (1) such that the corresponding closed-loop systems are regular, causal, and stochastically stable if there exist positive definite matrices  $Y_i$ ,  $\Psi$ , nonsingular matrices  $G_i$  and  $K_i$  satisfying the following matrix inequalities

$$\begin{bmatrix} \bar{\Xi}_i & G_i^T \bar{A}_i^T - G_i & 0 \\ \bar{A}_i G_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} < 0 \quad (28)$$

where  $\bar{\Xi}_i = G_i^T \bar{A}_i^T + \bar{A}_i G_i - E_1^T E G_i - G_i^T E^T E_1 - E_2^T R \bar{A}_i G_i - G_i^T \bar{A}_i^T R^T E_2 + E_1^T Y_i E_1 + E_2^T \Psi E_2$ ,  $\bar{A}_i = A_i + B_i K_i$ .

In order to design a state feedback controller  $u(k) = K(r_k)x(k)$  for systems (1) in the form of LMI, Theorem 4.1 will be replaced by the following theorem.

*Theorem 4.2*

Let  $E_1$  and  $E_2$  be given matrices, the closed-loop systems (27) with partially unknown transition probabilities are regular, causal, and stochastically stable if there exist positive definite matrices  $Y_i$ ,  $\Psi$ , nonsingular matrices  $G_i$  and  $H_i$ , and the gain of the stabilizing state feedback controller  $K_i = H_i G_i^{-1}$ , such that the following coupled of set of LMIs hold for each  $i \in \ell$ ,

$$\begin{bmatrix} \bar{\Xi}_i & G_i^T A_i^T + H_i^T B_i^T - G_i & 0 \\ A_i G_i + B_i H_i - G_i^T & -G_i - G_i^T & W_i^T \\ 0 & W_i & -\Theta_i \end{bmatrix} < 0 \quad (29)$$

where  $\bar{\Xi}_i = G_i^T A_i^T + A_i G_i + B_i H_i + H_i^T B_i^T - E_1^T E G_i - G_i^T E^T E_1 - E_2^T R A_i G_i - G_i^T A_i^T R^T E_2 - E_2^T R B_i H_i - H_i^T B_i^T R^T E_2 + E_1^T Y_i E_1 + E_2^T \Psi E_2$ .

*Remark 4.1*

Analogous to Remark 3.1, when  $h_i = \sum_{j_2 \in \ell_{i,k}} \pi_{i,j_2} = 1 - \sum_{j_1 \in \ell_k} \pi_{i,j_1} = 0$  for every  $i \in \ell$ , the elements of the transition probability matrices are completely known. Then Theorem 3.2, Theorem 4.1, and Theorem 4.2 are degenerated to Theorem 8, Corollary 10, and Theorem 11 of [3], respectively, which means that the results developed in this paper are more general than those for the systems with completely known transition probability matrices.

## 5. NUMERICAL EXAMPLES

In this section, some numerical examples will be given to show the validity of the developed theoretical results.

*Example 1*

Consider the discrete-time singular Markovian jump systems (1) with the following parameters

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -a & -1.25 \\ 2.5 & -b \end{bmatrix}, A_2 = \begin{bmatrix} 0.25 & -0.83 \\ 2.5 & -3.5 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} a & -0.25 \\ b & -3.0 \end{bmatrix}, A_4 = \begin{bmatrix} 1.5 & -0.56 \\ 2.5 & -2.75 \end{bmatrix}, R = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix}$$

The transition probability matrix of form (2) is given by

$$\pi = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.4 \\ ? & 0.2 & 0.3 & ? \\ ? & ? & 0.3 & 0.5 \\ 0.1 & 0.1 & 0.4 & 0.4 \end{bmatrix} \quad (30)$$

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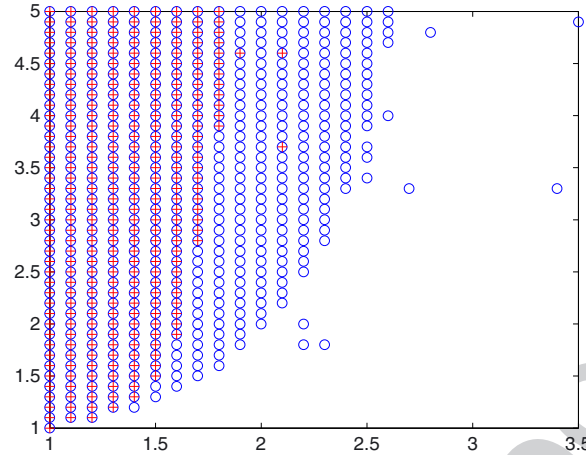


Figure 1. Admissibility analysis with Theorem 3.1 in this paper (O) and Theorem 1 (+) in [18].

The admissibility of unforced systems (1) can be checked using Theorem 3.1 in this paper and Theorem 1 in [18], for several values of pairs (a,b), where  $a \in [1, 4]$  and  $b \in [1, 5]$ . The result is depicted in Figure 1 and reveals that our theorem is less conservative than the previous result.

**F1**

*Example 2*

Consider the discrete-time singular Markovian jump systems (1) with the following parameters

$$E = \begin{bmatrix} 1.45 & 0.5 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} a & b & 6 \\ 5 & 6 & 0 \\ 4 & 5 & 7 \end{bmatrix}, A_2 = \begin{bmatrix} 9 & 5 & 7 \\ -9 & a & -7 \\ 4 & 5 & -b \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 & 0 \\ 0.3 & 0.6 \\ 0.3 & 0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 0.6 \\ 0.7 & 0.1 \end{bmatrix}$$

The transition probability matrix is given by

$$\pi = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{bmatrix} \tag{31}$$

According to Remark 3.3, let

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 2.2 \end{bmatrix}$$

The admissibility of systems (1) can be checked using Theorem 4.2 in this paper and Theorem 11 in [3], for several values of pairs (a,b), where  $a \in [35, 60]$  and  $b \in [59, 63]$ . The result is depicted in Figure 2 and reveals that the chosen  $E_1, E_2$  in terms of matrix in Theorem 4.2 with elements in transition probability matrix  $\pi$  are all known less conservative than the previous result.

**F2**

*Example 3*

Consider the discrete-time singular Markovian jump systems (1) with the parameters

$$E = \begin{bmatrix} 1.45 & 0.5 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 1.5 & 0.4 & 0.6 \\ 0.5 & 0.6 & 0 \\ 0.4 & 0.5 & 0.7 \end{bmatrix}, A_2 = \begin{bmatrix} 0.9 & 0.5 & 0.7 \\ -0.9 & 0.5 & -0.7 \\ 0.4 & 0.5 & -0.2 \end{bmatrix}$$

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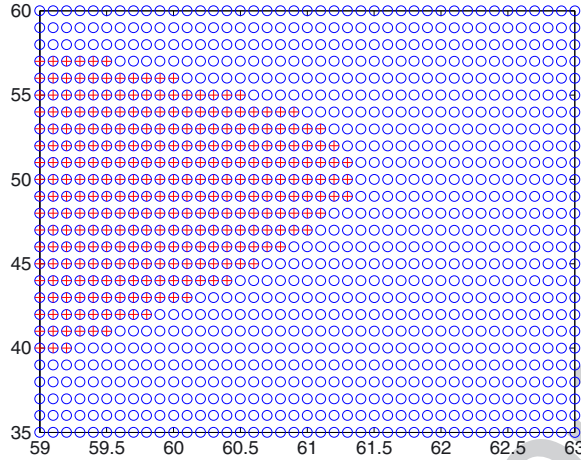


Figure 2. Admissibility analysis with Theorem 4.2 in this paper (O) and Theorem 11 (+) in [3].

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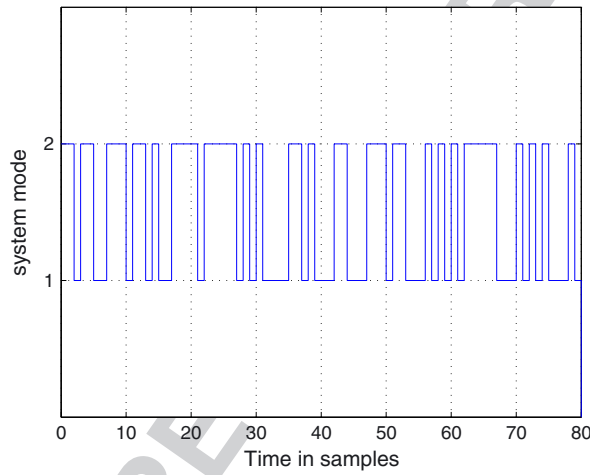


Figure 3. Modes evolution  $r_t$ .

$$R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}, B_1 = \begin{bmatrix} 0.4 & 0 \\ 0.3 & 0.6 \\ 0.3 & 0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 0.6 \\ 0.7 & 0.1 \end{bmatrix}$$

The transition probability matrix of form (2) is given by

$$\pi = \begin{bmatrix} 0.5 & 0.5 \\ ? & ? \end{bmatrix} \quad (32)$$

According to Theorem 4.2, let

$$E_1 = \begin{bmatrix} 1.1 & 0.1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 2.3 \end{bmatrix}$$

Using the LMI toolbox in MATLAB,

$$\Psi = \begin{bmatrix} 4.8341 & 2.6710 & -2.4266 \\ 2.6710 & 3.1340 & -1.4381 \\ -2.4266 & -1.4381 & 26.8138 \end{bmatrix}$$

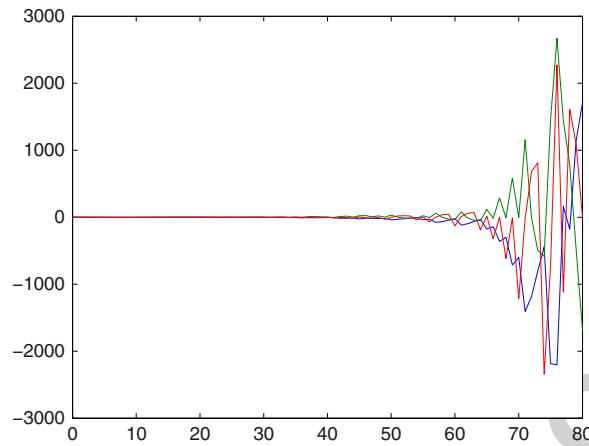


Figure 4. State responses of the open-loop system.

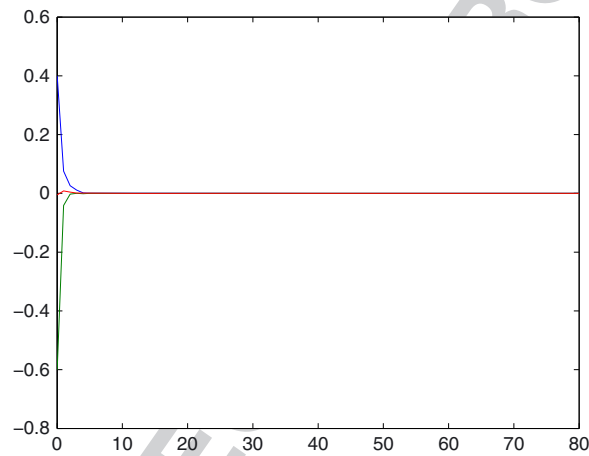


Figure 5. State responses of the closed-loop system.

It is easy to find that  $\Psi > 0$ .

The gains of the stabilizing state feedback controller are chosen as

$$K_1 = \begin{bmatrix} -3.1330 & 0.2068 & -1.6734 \\ 0.6422 & -0.8082 & 1.3129 \end{bmatrix}, K_2 = \begin{bmatrix} -1.0822 & -0.9269 & 2.5771 \\ 2.0982 & -0.2526 & 0.4619 \end{bmatrix}$$

The open-loop systems (27) are diverging from Figure 4. However, after applying Theorem 4.2, trajectory simulation for the closed-loop systems shown in Figure 5 is stochastically admissible with the same Markovian jump process under the given initial condition  $x_0 = [0.4, -0.6, 0.2]^T$ .

F4  
F5

*Example 4*

Consider the discrete-time singular Markovian jump systems (1) with the parameters the same as in Example 2, and the transition probability matrix of form (2) is given by (32)

Compared  $E_1 = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2.1 & 0 \\ 0 & 0 & 2.2 \end{bmatrix}$  with two scalars  $E_1 = 1$  and  $E_2 =$

2 in Theorem 4.2, for several values of pairs (a,b), where  $a \in [31, 43]$  and  $b \in [30, 60]$ . The result which is depicted in Figure 6 reveals that the chosen matrices  $E_1$  and  $E_2$  in Theorem 4.2 are less conservative.

F6

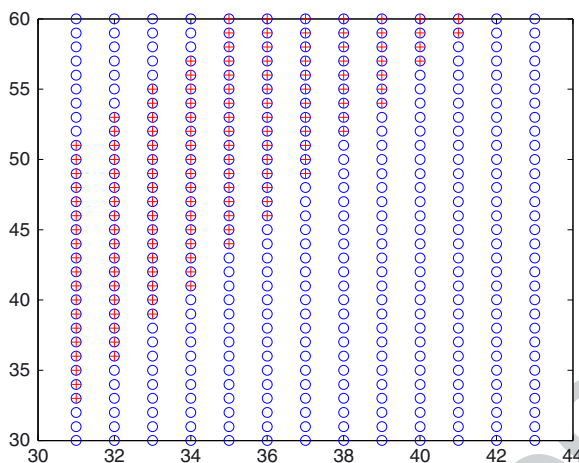


Figure 6. Admissibility analysis matrix parameters (O) and scalar parameters (+) in Theorem 4.2.

## 6. CONCLUSIONS

The stochastic stability and stabilization of discrete-time singular Markovian jump systems with partially unknown transition probabilities have been studied in this paper. The considered systems with partially unknown transition probabilities are more general than the systems with completely known or completely unknown transition probabilities. We give the necessary and sufficient conditions for the stochastic stability analysis by the convex combination in terms of strict LMIs. And sufficient conditions have also been proposed for the design of a state feedback controller, which guarantees that the closed-loop systems are regular, causal, and stochastically stable by employing the LMIs technique. Numerical examples have shown the validity and the applicability of the developed results. It should be noted that in this paper, only the stochastic stability and stabilization problems for discrete-time singular Markovian jump systems without disturbances are considered in this paper. It is well known that disturbances including parameter uncertainties and mismatched disturbances as discussed in [21–23] widely exist in reality, which may destroy system performance. The future work will focus on the study of discrete-time singular Markovian jump systems with mismatched disturbances and partially unknown transition probabilities.

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# Author Query Form

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**Journal: International Journal of Robust and Nonlinear Control**

**Article: rnc\_3146**

Dear Author,

During the copyediting of your paper, the following queries arose. Please respond to these by annotating your proofs with the necessary changes/additions.

- If you intend to annotate your proof electronically, please refer to the E-annotation guidelines.
- If you intend to annotate your proof by means of hard-copy mark-up, please refer to the proof mark-up symbols guidelines. If manually writing corrections on your proof and returning it by fax, do not write too close to the edge of the paper. Please remember that illegible mark-ups may delay publication.

Whether you opt for hard-copy or electronic annotation of your proofs, we recommend that you provide additional clarification of answers to queries by entering your answers on the query sheet, in addition to the text mark-up.

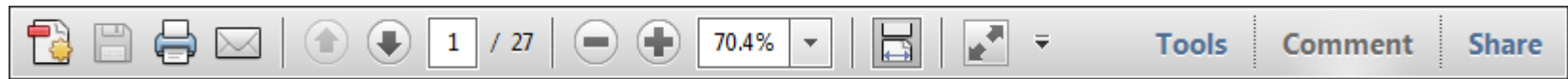
Query No.	Query	Remark
Q1	AUTHOR: Please check if forename/surname order of authors is correct.	
Q2	AUTHOR: Please check the suitability of the suggested short title.	
Q3	AUTHOR: Figure 3 has not been mentioned in the text. Please cite the figure in the relevant place in the text.	
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USING e-ANNOTATION TOOLS FOR ELECTRONIC PROOF CORRECTION

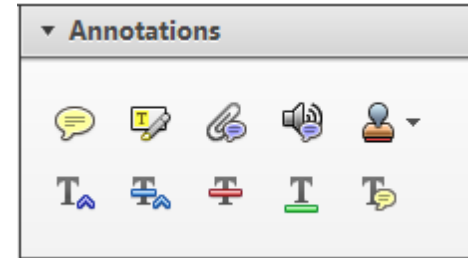
Required software to e-Annotate PDFs: Adobe Acrobat Professional or Adobe Reader (version 7.0 or above). (Note that this document uses screenshots from Adobe Reader X)

The latest version of Acrobat Reader can be downloaded for free at: <http://get.adobe.com/uk/reader/>

Once you have Acrobat Reader open on your computer, click on the [Comment](#) tab at the right of the toolbar:



This will open up a panel down the right side of the document. The majority of tools you will use for annotating your proof will be in the [Annotations](#) section, pictured opposite. We've picked out some of these tools below:



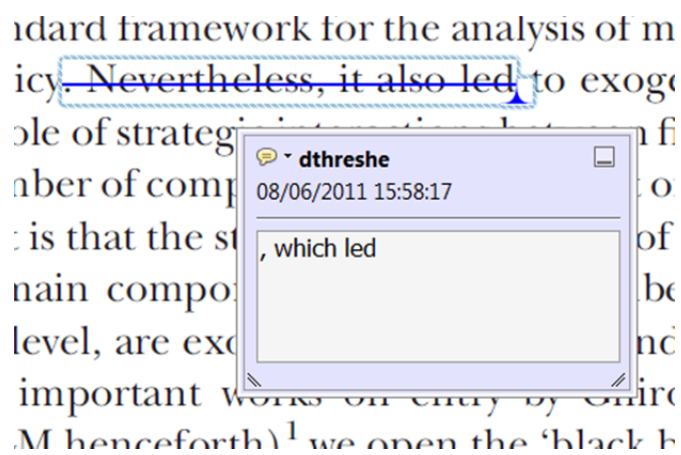
**1. Replace (Ins) Tool – for replacing text.**



Strikes a line through text and opens up a text box where replacement text can be entered.

**How to use it**

- Highlight a word or sentence.
- Click on the [Replace \(Ins\)](#) icon in the Annotations section.
- Type the replacement text into the blue box that appears.



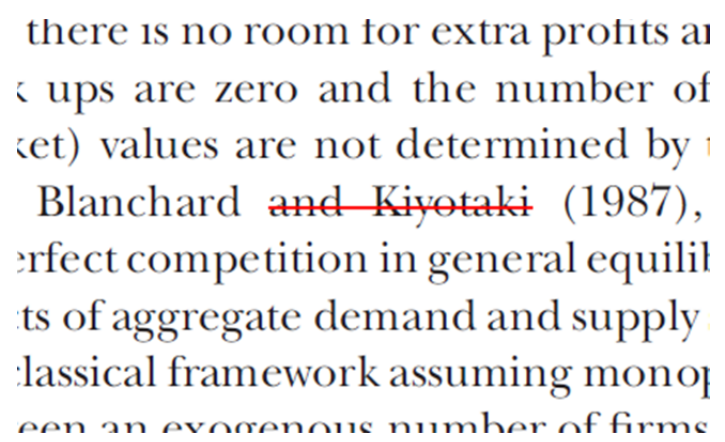
**2. Strikethrough (Del) Tool – for deleting text.**



Strikes a red line through text that is to be deleted.

**How to use it**

- Highlight a word or sentence.
- Click on the [Strikethrough \(Del\)](#) icon in the Annotations section.



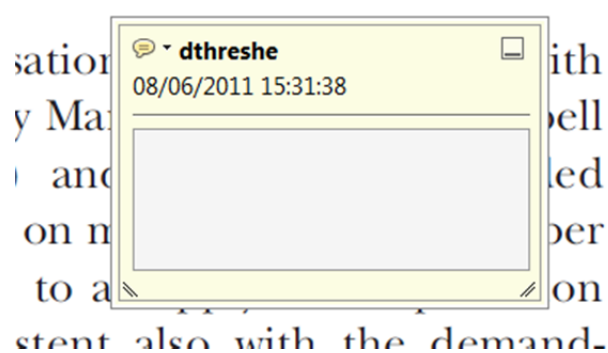
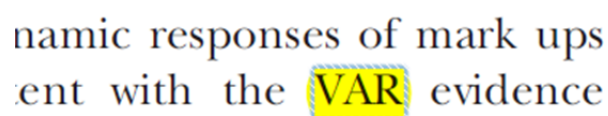
**3. Add note to text Tool – for highlighting a section to be changed to bold or italic.**



Highlights text in yellow and opens up a text box where comments can be entered.

**How to use it**

- Highlight the relevant section of text.
- Click on the [Add note to text](#) icon in the Annotations section.
- Type instruction on what should be changed regarding the text into the yellow box that appears.



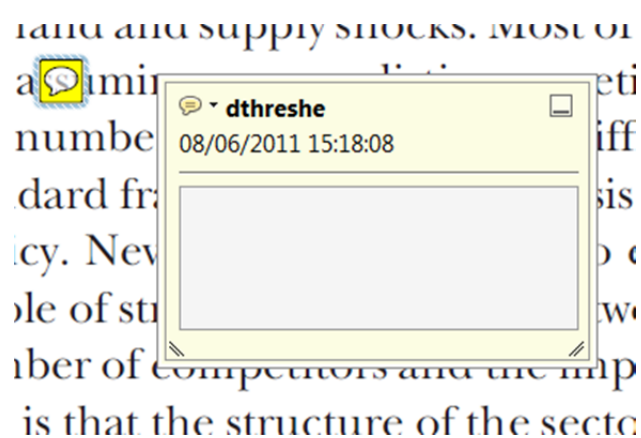
**4. Add sticky note Tool – for making notes at specific points in the text.**



Marks a point in the proof where a comment needs to be highlighted.

**How to use it**

- Click on the [Add sticky note](#) icon in the Annotations section.
- Click at the point in the proof where the comment should be inserted.
- Type the comment into the yellow box that appears.



USING e-ANNOTATION TOOLS FOR ELECTRONIC PROOF CORRECTION

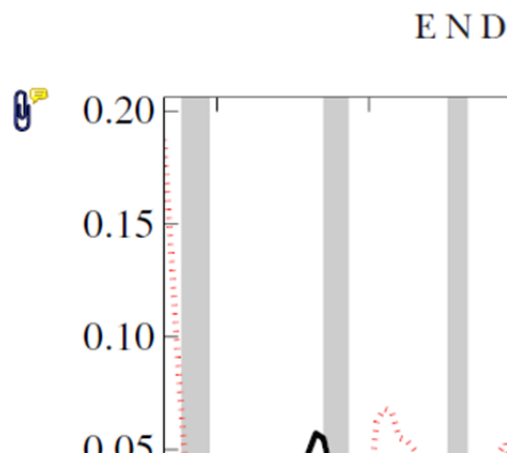
**5. Attach File Tool – for inserting large amounts of text or replacement figures.**



Inserts an icon linking to the attached file in the appropriate place in the text.

**How to use it**

- Click on the [Attach File](#) icon in the Annotations section.
- Click on the proof to where you'd like the attached file to be linked.
- Select the file to be attached from your computer or network.
- Select the colour and type of icon that will appear in the proof. Click OK.



**6. Add stamp Tool – for approving a proof if no corrections are required.**

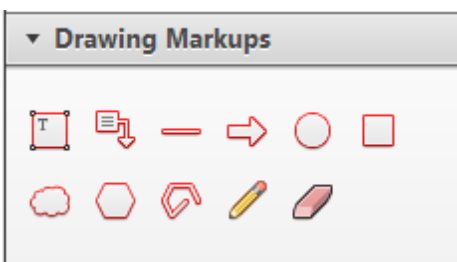


Inserts a selected stamp onto an appropriate place in the proof.

**How to use it**

- Click on the [Add stamp](#) icon in the Annotations section.
- Select the stamp you want to use. (The [Approved](#) stamp is usually available directly in the menu that appears).
- Click on the proof where you'd like the stamp to appear. (Where a proof is to be approved as it is, this would normally be on the first page).

of the business cycle, starting with the  
 on perfect competition, constant return  
 production. In this environment goods  
 extra profits and the number of firms  
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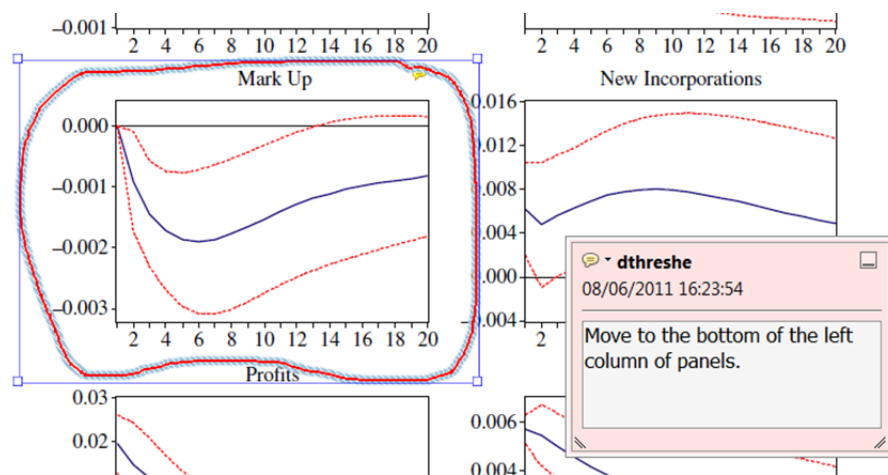


**7. Drawing Markups Tools – for drawing shapes, lines and freeform annotations on proofs and commenting on these marks.**

Allows shapes, lines and freeform annotations to be drawn on proofs and for comment to be made on these marks..

**How to use it**

- Click on one of the shapes in the [Drawing Markups](#) section.
- Click on the proof at the relevant point and draw the selected shape with the cursor.
- To add a comment to the drawn shape, move the cursor over the shape until an arrowhead appears.
- Double click on the shape and type any text in the red box that appears.



For further information on how to annotate proofs, click on the [Help](#) menu to reveal a list of further options:

