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Combinatorial auctions for green supplier selection and bundled order allocation

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ABSTRACT

Outsourcing has become a key strategy for firms seeking efficiency and competitiveness, leading to a growing emphasis on effective supplier selection and order allocation (SSOA). While combinatorial auctions have been recognized as a powerful tool for procurement, their integration into multi-sourcing SSOA decisions remains underexplored. This paper develops a novel Winner Determination Problem model that generalizes prior work by allowing suppliers to submit multiple, potentially even overlapping, bundles. We also propose four original heuristic approaches inspired by Kernel Search, tailored to address both the cost and environmental sustainability dimensions of procurement. Extensive numerical experiments demonstrate the strong performance of the proposed methods, with the weighted frequency-based rarity and value density heuristic achieving the best trade-off between solution quality and computational time. Managerial insights reveal that auction-based SSOA mechanisms can achieve significant cost savings compared to traditional models due to economies of scope, and highlight critical trade-offs between supplier diversification, cost efficiency, and risk management. This study advances the field by offering both methodological and practical contributions, with future research directions suggested in stochastic and dynamic auction settings.

1. Introduction

Outsourcing is constantly evolving, transforming the way in which many businesses operate. It consists of obtaining necessary goods and services through partnerships with external suppliers. This practice helps companies focus on their core business, remain competitive in the modern global market, and procure additional goods and services from partner sources. Outsourcing is becoming increasingly essential for businesses aiming to boost efficiency and cut costs, and it has been gaining momentum in recent years. According to Menhennet (2024), 64% of executive leaders intend to expand their outsourcing capacity within the next two years. This increase is motivated by the demand for specialized skills and the ability to concentrate on core business functions. The same source indicated that 83% of small and medium-sized enterprises in the U.S. planned to outsource part of their business and noted that a similar trend is also being observed in European countries. This growing need creates opportunities for outsourcing providers from emerging economies to access developed-economy markets. A recent survey showed that India, Poland, Mexico, Malaysia, China, and Bulgaria are among the economies currently providing major outsourcing capacity worldwide (Deloitte, 2023).

However, selecting reliable suppliers is crucial, since poor synergy and misalignment in objectives and values can create a high level of risk. A partnership with a supplier is often strategic and may last for an extended period, during which both parties may need to collaborate and support one another in both prosperous and challenging times (Alaei & Setak, 2017). The traditional practice of outsourcing relies on identifying a single supplier to satisfy the entire demand for the product to be procured externally. However, this policy is often characterized by a significant risk and can cause operational disruption. Instead, companies are increasingly moving toward multi-sourcing strategies in which several suppliers are identified to share the responsibility of providing different portions of that product. Thus, not only is the careful selection of the most appropriate suppliers of paramount importance for efficient procurement, but the allocation of each product among suppliers also becomes critical in the multi-sourcing context. On the one hand, choosing a small number of suppliers that cover significant shares of the same product increases the risk of disruption in the company's supply chain. On the other hand, selecting a high

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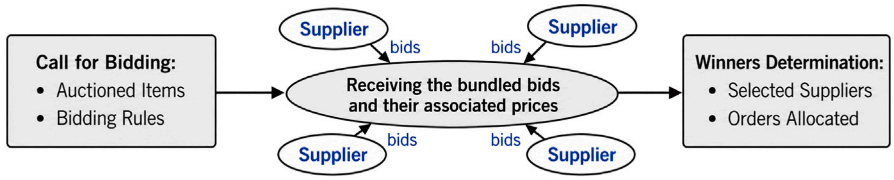

Fig. 1. Phases of the CA in the context of the SSOA.

number of suppliers prolongs the negotiation and coordination times and increases the ordering costs (Hamdan & Cheaitou, 2017c).

The multi-sourcing supplier selection and order allocation (SSOA) problem has often been solved through ranking and negotiation methods. However, auctions have recently been recommended as a fair and efficient mechanism for trading products and allocating resources (Felicetti et al., 2025; Palacios-Huerta et al., 2024). More specifically, since this study concerns a multi-product context, we employ a combinatorial auction (CA) design that allows suppliers to bid on subsets (bundles) of the products and to propose a single price for each bundle (Triki et al., 2023). In this way, suppliers can take advantage not only of economies of scale but also of economies of scope, which account for the synergies and complementarities among the auctioned products and allow them to submit more competitive prices for each bundle. Consequently, CAs are characterized by a higher level of complexity compared to their unbundled counterparts since, to clear the auction and allocate the orders, the auctioneer needs to solve an all-or-nothing combinatorial optimization problem known as the Winner Determination Problem (WDP).

CAs have been applied with remarkable success in several industries. Interested readers are referred to Section 2.3 of this paper and to the recent review by Palacios-Huerta et al. (2024). The latter study highlighted that auction clearing is based on common WDP modeling features, including the following: (i) bids are submitted as packages comprising multiple products, each associated with a single bundle price, (ii) the auctioneer applies an all-or-nothing principle, meaning that each bid is either fully accepted or fully rejected, and (iii) during the clearing phase, each product is assigned to at most one bidder. However, each WDP also exhibits unique features that take into account the specific characteristics of the application under consideration. These features are application-related and represent the way in which the WDP is connected with the proposed SSOA framework. In particular, our model explicitly incorporates variables, parameters, and constraints that reflect the following: (i) the quantities of products offered by each supplier within its bids, which, in this case, define the bid–product–period mapping, (ii) the buyer's demand that must be satisfied, and (iii) inventory management considerations. These elements are embedded in the optimization structure of the SSOA to capture the operational characteristics of the specific application setting.

The dynamics of the bidding and clearing process are shown in Fig. 1. In the first phase, the auctioneer (company) invites suppliers to bid while specifying the set of auctioned products and bidding rules. These include the bid format (bundled in our case), the maximum number of bids to be submitted by each supplier, the format of price submission (sealed in our case), and the auction clearing rule (first-price in our case). In the second phase, each supplier submits one or more sealed-price bids, each specifying a product bundle and its product–period quantities. Finally, in the last phase, the auctioneer clears the auction by solving the WDP in order to identify the selected suppliers and the portions of products allocated to each of them.

This paper focuses on the last phase of the auction process and develops a WDP model to solve the SSOA problem. It also proposes several heuristic approaches for its solution while incorporating green considerations within the SSOA. The framework applies to multi-product procurement settings in which suppliers offer bundled, period-specific quantities; examples across manufacturing, food, healthcare, and infrastructure are reviewed by Naqvi and Amin (2021).

Green considerations have become increasingly relevant in SSOA because supplier-selection and order-allocation decisions affect not only procurement cost, but also sourcing quantities, inventory levels, transportation requirements, and supplier-development trajectories, thereby linking procurement decisions to the United Nations Sustainable Development Goals (SDGs), particularly SDG 12 on responsible consumption and production and SDG 13 on climate action (United Nations, 2015). In practice, however, environmental criteria are usually considered alongside other procurement priorities, such as quality, delivery time, reliability, flexibility, warranty support, and service performance. In such contexts, green objectives can also be translated into practical procurement instruments. For example, buyers may use bonus–malus price signals to reward bids with stronger environmental performance and penalize weaker ones (Scherz et al., 2023), or they may support suppliers through development actions that improve their environmental capabilities over time (Saghiri & Mirzabeiki, 2021).

Although supplier selection in combination with auctions has attracted the attention of several studies, the integrated SSOA problem with CAs has received limited attention in the scientific literature. Indeed, to the best of our knowledge, this integrated SSOA–CA problem has been addressed in only one work, by Gujar and Narahari (2013). However, that study allows each supplier to submit only one bid, which is inconsistent with real-life practice, where suppliers may be interested in bidding on, and possibly winning, several bundles of the products put up for auction. Consequently, this work attempts to fill this gap and makes the following contributions:

- We generalize the single-minded setting of Gujar and Narahari (2013) to allow each supplier to submit and possibly win multiple (even overlapping) bundles of products in the context of SSOA;
- We develop a novel optimization model for the WDP that clears the CA and identifies the winning suppliers and allocated orders while ensuring resilience and diversity;
- We propose four original heuristic variants inspired by the well-known Kernel Search approach, and we define novel scoring methods to order the bids;
- We extend the proposed auction-based SSOA framework to incorporate environmental sustainability through a bi-objective formulation and additional procurement mechanisms, including transport-emission caps, supplier-development qualification, and bonus–malus price signals.

The paper is structured as follows: Section 2 is dedicated to the literature review. Sections 3 and 4 discuss the optimization model and solution approaches, respectively. Section 5 presents the experimental results, and Section 6 offers managerial insights. Finally, Section 7 concludes the paper and suggests future extensions.

2. Literature review

This section reviews SSOA, its integration with environmental requirements and CAs, and the Kernel Search variants underlying Section 4.

2.1. SSOA problem

The SSOA problem and its variants have been extensively studied in the literature, highlighting their importance in efficiently managing supply chains. There are two key challenges in this decision process: first, the accurate identification and selection of suitable suppliers and, second, the allocation of orders to the selected suppliers. The Supplier Selection (SS) problem has attracted the attention of several researchers and practitioners who have mainly relied on well-known multi-criteria ranking approaches such as analytic hierarchy process (Levary, 2008), VIKOR (Chen & Wang, 2009), fuzzy axiomatic design (Kannan, Govindan et al., 2014), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Rouyendegh et al., 2020), or even neural networks (Lee et al., 2009). On the other hand, the Order Allocation (OA) problem has been the subject of several studies that employed mainly simulation techniques (Renna & Perrone, 2015) or optimization approaches such as fix-and-optimize (Thevenin et al., 2022), multi-objective models (Kellner et al., 2019), and stochastic models (Hammami et al., 2014). However, companies can achieve efficiency in their procurement management and ensure competitiveness only if both decisions are made in an integrated manner (Di Pasquale et al., 2020). Nevertheless, the simultaneous solution of the SS and OA makes the problem very challenging from the computational complexity viewpoint. Such a task becomes even more burdensome when (i) we consider the multi-product and multi-period framework (Sahithi et al., 2023), (ii) the procurement process has to comply with strict environmental restrictions (Hamdan et al., 2023) and (iii) the competition among suppliers is ensured through the auction mechanism (Abbaas & Ventura, 2024).

The literature on the integrated SSOA problem is very rich, covering modeling, methodological, and experimental aspects. For the sake of brevity, we refer interested readers to the literature review in the recent work of Abbaas and Ventura (2024) and to the systematic reviews on SS by Saputro et al. (2022), on OA by Di Pasquale et al. (2020), and on the integrated SSOA by Naqvi and Amin (2021) and Nguyen et al. (2024).

2.2. Green aspects in SSOA

The integration of green aspects within the SSOA problem has gained significant attention due to increasing global awareness of the importance of managing resources in an environmentally responsible way. Multi-criteria decision-making and multi-objective optimization are often the most commonly used tools to achieve feasible trade-offs between cost savings and environmental requirements while solving the SSOA problem. For example, Mafakheri et al. (2011) embedded a fuzzy Analytic Hierarchy Process (AHP) within a bi-objective integer linear programming formulation to solve the green SSOA. The AHP approach has also been integrated with a multi-objective genetic algorithm approach in Hamdan and Jarndal (2017) and with a fuzzy TOPSIS in Sadeghi (2018). Real-life case studies have been solved using fuzzy TOPSIS (Kannan, De Sousa Jabbour et al., 2014), the fuzzy axiomatic design approach (Kannan, Govindan et al., 2014), the best–worst method combined with the fuzzy TOPSIS (Lo et al., 2018), the fuzzy goal programming integrated with the intuitionistic fuzzy TOPSIS (Kilic & Yalcin, 2020), the fuzzy AHP (Qazvini et al., 2021), the stochastic AHP-TOPSIS methods (Najafi & Zolfagharinia, 2024), etc.

The multi-objective green SSOA problem with quantity discounts has also been solved by Ali and Zhang (2023), Hamdan and Cheaitou (2017b, 2017c), and Mirzaee et al. (2018). Moreover, its stochastic version was considered by Goodarzi et al. (2022) and Scott et al. (2015). It should be noted that many other works are available in the literature that have focused on integrating green criteria only with the SS problem without considering the OA decisions. Interested readers are referred to recent reviews on this topic produced, for example, by Ograh and Ayarkwa (2022) and Sahoo and Goswami (2024).

2.3. Integrating SSOA with CAs

CAs are a well-known tool for goods and services procurement, trading, exchange, and resource allocation. Although they are characterized by a high level of complexity compared to traditional auctions, CAs have been widely employed in several industries. For example, in the context of trading, CAs have shown remarkable efficiency in treasury bill trading (Menezes, 1995), deregulated energy markets (Musmanno et al., 2010), real estate transactions (Goossens et al., 2014), transportation-service procurement (Triki et al., 2020), etc. Moreover, CAs have generated significant savings or profits when used for allocating resources, such as airport time slots (Rassenti et al., 1982), telecommunications service obligations (Kelly & Steinberg, 2000), spectrum licenses (Günlük et al., 2005), bus and truck routes (Badiee et al., 2023; Cantillon & Pesendorfer, 2005), petrol drilling rights (Cramton, 2010), school food distribution (Olivares et al., 2012), cleaning contracts (Lunander & Lundberg, 2013), tables in busy restaurants (Luzzi et al., 2026), etc.

CAs have also been employed in the context of SS and less frequently in that of SSOA. Bichler et al. (2011) developed a CA for the SS in which suppliers are allowed to express complex cost structures, including volume discounts and bundled offers, through price functions. This approach supports the buyer in optimizing their purchasing decisions by considering both economies of scale and scope through the solution of a binary linear programming (LP) model that determines the optimal procurement decisions. In the same direction, Yu and Wong (2014) introduced a model designed to help buyers purchase multiple products, taking into account the synergy effect among the products. The model is based on the TOPSIS technique to evaluate suppliers using factors like cost, quality, and synergy potential, ensuring that the pre-selection process aligns with the buyer's overall procurement strategy. Similarly, Yu and Wong (2015) presented a model that uses an agent-based system integrating CAs with multi-bilateral negotiation to support the SS process for purchasing multiple products characterized by synergy effects. Subsequently, Alaei and Setak (2017) examined a multi-product SS problem that allows multiple suppliers to bid through a CA mechanism. They developed a binary optimization model to clear the auction and to identify the winning suppliers. However, the major weakness of this study is that their approach allows each product to be assigned to a single supplier.

To the best of our knowledge, the integrated SSOA problem combined with CAs has been considered only by Gujar and Narahari (2013), who developed a multi-product, multiple-sourcing model and allowed the suppliers not only to bid on multiple units of each product but also to offer quantity discounts. The major shortcoming of this work is that each supplier is restricted to submitting only a single bid. For the sake of completeness, it is worth noting that there is a different stream of research in the context of SSOA that focuses on auction design features, and specifically, on developing novel bidding languages and complex function formats that link the demand rate to price changes (Abbaas & Ventura, 2024). In this study, suppliers may submit several all-or-nothing bundle bids, and the model may accept up to a certain number of bids. Our aim here is to make the bidding process as simple as possible, allowing even suppliers who do not have access to advanced analytical tools to define sensible bids and participate in the CA. From this point of view, the closest study to ours is Alaei and Setak (2017). However, we propose meaningful extensions, including solving the SSOA rather than the SS, relaxing the requirement that products be indivisible among suppliers, and incorporating environmental aspects.

2.4. Kernel search method and its variants

Kernel Search (KS) is a general-purpose matheuristic, introduced in Angelelli et al. (2010), that has proved effective in solving many combinatorial optimization problems. It is particularly suitable for problems in which the decision maker has to select a restricted number of items among a very large set of options. The core idea is to identify a small subset of potentially good variables, called the kernel, and to partition all the others into disjoint buckets. At each iteration of the algorithm, a different bucket is selected and a restricted version of the original problem, involving only variables belonging to the kernel and the selected bucket, is solved with a time limit. If some of the variables belonging to the bucket are active in the best solution of the restricted problem, they are added to the kernel. The algorithm terminates when all the buckets have been explored.

The general KS framework is characterized by three main features: (i) the rule used to determine which variables to insert in the kernel, (ii) the number and size of the buckets, and the rules applied to partition the remaining variables into buckets, and (iii) the mechanism employed to update the kernel. In the basic version of KS, proposed by Angelelli et al. (2010), the linear relaxation (LP) of the original problem is exploited to identify the initial kernel composed of the $\mathcal{E}$ variables with the highest values in the optimal solution of the LP. The remaining variables are sorted in a non-decreasing order by their value in the optimal LP solution. The kernel size is non-decreasing, which means that, at each iteration, new promising variables are added to the kernel but no variables are removed from it.

The basic KS has been applied, with very good performance, to several combinatorial optimization problems such as multidimensional knapsack problem (Angelelli et al., 2010), portfolio selection problem (Angelelli et al., 2012), capacitated facility location (Guastaroba & Speranza, 2012b), and multi-plant lot sizing problem with setup carry-over (Carvalho & Nascimento, 2018). In Guastaroba and Speranza (2012a), the authors proposed a variant of KS, named *Improved Kernel Search*, in which KS is exploited to derive information about the desirability of each product to identify the most promising ones. Another improved variant, named *Adaptive Kernel Search*, was proposed in Guastaroba et al. (2017). According to this method, once the size of the kernel becomes too large and the restricted problems become difficult to solve, a kernel update procedure is applied in which variables that were not recently selected in the search process are excluded from the kernel.

3. Winner determination model

In this work, we address the problem of optimizing the procurement process for a retailer managing its demand for multiple products over a multi-period planning horizon. The retailer invites local suppliers to submit bids that specify supply schedules, detailing the quantity of each product to be supplied in each period along with the corresponding bid price. Suppliers have the flexibility to submit multiple bids based on their production capacities, commercial strategies, and other commitments. The retailer's objective is to analyze the submitted bids and select the winners by minimizing the total procurement cost. This total cost includes shortage and inventory-holding costs, the prices paid for winning bids, and the administrative expenses required for management and shipment monitoring. The retailer is committed to fulfilling the total demand for all products by the end of the planning horizon. Additionally, as a recipient of an environmental sustainability award, the retailer integrates a green assessment into the decision-making process. A weight is computed for each supplier–product pair, and the retailer aims to maximize the total environmental sustainability value derived from the selected bids. To ensure robust procurement practices, the retailer imposes constraints to reduce potential disruptions. These include a minimum number of suppliers required for each product and a maximum number of bids that can be accepted from any single supplier. Furthermore, to streamline the bidding process and manage complexity, the retailer informs participants that only a limited number of bids will be accepted. In this section, we first describe the notation and then the mathematical formulation of the WDP used to clear the auction and select the winning suppliers.

3.1. Notation

Table 1 introduces the notation used to define our SSOA model and its environmental components.

We assume that suppliers submit their bids only once at the beginning of the planning horizon. However, they may offer the same product in several periods of the planning horizon. This is captured through parameter Q^b_{tp}, which depends on period index t.

3.2. The economic CA-based SSOA model

Objective function:

$$\min \mathrm{TC} = \sum_{i\in\mathcal{N}}\sum_{p\in\mathcal{P}} F_{ip}Z_{ip} + \sum_{i\in\mathcal{N}}\sum_{b\in\mathcal{B}(i)} P^b X^b + \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}\left(H_{pt}I^+_{pt} + S_{pt}I^-_{pt}\right) \quad (1)$$

Eq. (1) minimizes the total cost (TC), which consists of the administrative cost of managing each selected supplier–product relationship, the cost of accepted bids, and inventory-related costs.

Demand and inventory control constraints:

$$\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}}\sum_{b\in\mathcal{B}(i)} Q^b_{tp}X^b \geq \sum_{t\in\mathcal{T}} D^p_t, \quad p\in\mathcal{P}, \quad (2)$$

$$I^+_{p(t-1)} - I^-_{p(t-1)} + \sum_{i\in\mathcal{N}}\sum_{b\in\mathcal{B}(i)} Q^b_{tp}X^b - I^+_{pt} + I^-_{pt} = D^p_t, \quad p\in\mathcal{P},\quad t\in\mathcal{T}, \quad (3)$$

$$M_1Y^-_{pt} \leq I^-_{pt} \leq M^-_{2,pt}Y^-_{pt}, \quad p\in\mathcal{P},\ t\in\mathcal{T}, \quad (4)$$

$$M_1Y^+_{pt} \leq I^+_{pt} \leq M^+_{2,pt}Y^+_{pt}, \quad p\in\mathcal{P},\ t\in\mathcal{T}, \quad (5)$$

$$Y^+_{pt} + Y^-_{pt} \leq 1, \quad p\in\mathcal{P},\ t\in\mathcal{T}, \quad (6)$$

Constraint (2) ensures that the total quantity selected for each product over all periods is at least equal to the total demand for that product. Constraint (3) updates the inventory levels at the end of each period. It captures the excess units to be held (I^+_{pt}) and the unmet (backlogged) demand (I^-_{pt}). Constraints (4) and (5) link the inventory level decision variables (I^+_{pt} and I^-_{pt}) to their binary variables (Y^+_{pt} and Y^-_{pt}). Constraint (6) ensures that, for each product and at each period, the system can have at most one inventory status: positive inventory, shortage, or neither. These binary variables are introduced to prevent simultaneous positive values of I^+_{pt} and I^-_{pt} for the same product and period. This is particularly relevant in the sustainability-maximization model, where holding and shortage quantities are not penalized in the objective and, without Constraints (4)–(6), unrealistic simultaneous stock and backlog could arise. In the pure cost-minimization setting with strictly positive holding and shortage costs, however, such simultaneous values are already dominated; hence, these variables and constraints may be omitted without changing the optimal cost solution.

Resilience and diversity constraints:

$$\sum_{b\in\mathcal{B}(i)} X^b \leq L_i, \quad i\in\mathcal{N}, \quad (7)$$

$$\sum_{b\in\mathcal{B}(i)}\sum_{t\in\mathcal{T}} Q^b_{tp}X^b \leq M_{3,ip}Z_{ip}, \quad i\in\mathcal{N},\ p\in\mathcal{P}, \quad (8)$$

$$\sum_{b\in\mathcal{B}(i)}\sum_{t\in\mathcal{T}} Q^b_{tp}X^b \geq Z_{ip}, \quad i\in\mathcal{N},\ p\in\mathcal{P}, \quad (9)$$

$$\sum_{i\in\mathcal{N}} Z_{ip} \geq \mathcal{L}_p, \quad p\in\mathcal{P}, \quad (10)$$

$$\sum_{i\in\mathcal{N}}\sum_{b\in\mathcal{B}(i)} X^b \leq B_{\max}. \quad (11)$$

Constraint (7) ensures that each supplier i can be contracted through at most L_i bids. This ensures diversity in the solution and reduces the risk of depending on only a few suppliers. Constraints (8) and (9) link Z_{ip} with X^b such that if bid b offering product p is selected, then Z_{ip} is set to one for the corresponding supplier and product. Constraint (10) ensures that each product is sourced by at least $\mathcal{L}_p$ suppliers. This is to

Table 1
Notation used in the proposed model.

Notation	Description
Sets and indices	
$\mathcal{P}$	Set of products, indexed by p
$\mathcal{T}$	Set of periods in the planning horizon, indexed by t
$\mathcal{N}$	Set of suppliers, indexed by i
$\mathcal{B}(i)$	Set of bids offered by supplier i, indexed by b
A	Set of supplier alternatives for a given product, where $A \subseteq \mathcal{N}$ and $m = \|A\|$
C	Set of criteria, $C = \{C_1, C_2, \dots, C_n\}$, where n is the number of criteria
Parameters and derived quantities	
Q^b_{tp}	Quantity of product p offered by supplier i in period t within bid b ($b \in \mathcal{B}(i)$)
D^p_t	Buyer's demand of product p during period t
P^b	Price of bid b ($b \in \mathcal{B}(i)$) from supplier i
H_{pt}	Unit holding cost for product p in period t
S_{pt}	Unit penalty cost for product p in period t
L_i	Maximum number of bids that can be accepted from supplier i
$\mathcal{L}_p$	Minimum number of suppliers to be used as a source for product p
F_{ip}	Administrative cost for managing supplier i and product p
$B_{\max}$	Maximum number of bids the buyer is willing to accept
M_1	Small positive threshold used to activate nonzero holding or backlog levels; for integer quantities, $M_1 = 1$.
$M^-_{2,pt}$	Backlog upper bound for product p at the end of period t, where $M^-_{2,pt} = \sum_{\tau=1}^{t} D^p_\tau$, $p \in \mathcal{P}, t \in \mathcal{T}$.
$M^+_{2,pt}$	Positive-inventory upper bound for product p at the end of period t, where $M^+_{2,pt} = \max\left\{0, \sum_{\tau=1}^{t}\sum_{i\in\mathcal{N}}\sum_{b\in\mathcal{B}(i)} Q^b_{\tau p} - \sum_{\tau=1}^{t} D^p_\tau\right\}$, $p \in \mathcal{P}, t \in \mathcal{T}$.
$M_{3,ip}$	Supplier–product activation bound, where $M_{3,ip} = \sum_{b\in\mathcal{B}(i)}\sum_{t\in\mathcal{T}} Q^b_{tp}$, $i \in \mathcal{N}, p \in \mathcal{P}$. It represents the maximum quantity of product p that supplier i can provide across its submitted bids.
$\tilde{r}_{ij}$	Triangular fuzzy number, $\tilde{r}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, representing the evaluation of supplier $i \in A$ with respect to criterion C_j
W	Fuzzy weights of criteria, $W = \{\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n\}$
$\tilde{R}$	Fuzzy decision matrix, $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$
$\tilde{V}$	Fuzzy weighted normalized matrix, $\tilde{V} = [\tilde{v}_{ij}]$
A^+, A^-	Fuzzy positive and negative ideal solutions, respectively
d^+_i, d^-_i	Distance of supplier $i \in A$ from A^+ and A^-, respectively
SS^i_p	Relative closeness coefficient of supplier i for product p
SW^b	Environmental sustainability weight of bid b
Decision variables	
X^b	A binary variable equal to 1 if bid $b \in \mathcal{B}(i)$ from supplier i is selected, and 0 otherwise.
I^+_{pt}, I^-_{pt}	Inventory holding and inventory shortage units of product p in period t. Note that the initial inventory level is assumed to be 0 with no holding or shortage ($I^+_{p(t=0)} = 0$, $I^-_{p(t=0)} = 0$).
Y^+_{pt}, Y^-_{pt}	Binary variables used to indicate the inventory status. The variable Y^+_{pt} is equal to 1 if there is inventory holding and Y^-_{pt} is equal to 1 if there is inventory shortage.
Z_{ip}	A binary decision variable indicating which supplier i is contracted to provide product p.

ensure resilience in the solution. Constraint (11) ensures that the total number of selected bids cannot exceed a certain threshold $B_{\max}$.

Decision variable nature constraints:

$$X^b \in \{0,1\}, \quad b \in \mathcal{B}(i),\ i \in \mathcal{N}, \tag{12}$$

$$I^+_{pt}, I^-_{pt} \in \mathbb{Z}_{\geq 0},\ Y^+_{pt} \in \{0,1\},\ Y^-_{pt} \in \{0,1\}, \quad Z_{ip} \in \{0,1\}, \quad p \in \mathcal{P},\ t \in \mathcal{T},\ i \in \mathcal{N}. \tag{13}$$

Finally, Constraints (12)–(13) force the decision variables to be binary or integer. The problem complexity increases rapidly with the number of bids, products, suppliers and time periods.

3.3. The Green SSOA bi-objective model

The retailer evaluates its suppliers and products for sustainability using fuzzy TOPSIS as it accounts for uncertainty in the evaluation and can handle a large number of suppliers and criteria (Hamdan & Cheaitou, 2017a; Nădăban et al., 2016). The process is as follows. The fuzzy TOPSIS method incorporates fuzzy set theory to address uncertainty and vagueness in decision-making. For each product, the retailer creates a set of alternatives corresponding to the suppliers offering this product, denoted by $A \subseteq \mathcal{N}$, with $m = |A|$, and a set of company- and product-related criteria $C = \{C_1, C_2, \dots, C_n\}$. A fuzzy decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ is constructed, where $\tilde{r}_{ij}$ represents the evaluation of supplier $i \in A$ with respect to criterion C_j, expressed as a triangular fuzzy number $\tilde{r}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, where l_{ij}, m_{ij}, and u_{ij} are the lower, middle, and upper bounds, respectively. The criteria weights $W = \{\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n\}$ are also defined as fuzzy numbers.

The fuzzy decision matrix is normalized according to criterion direction. For a benefit-type criterion C_j, the normalized triangular fuzzy rating is $\tilde{r}'_{ij} = (l_{ij}/c^+_j, m_{ij}/c^+_j, u_{ij}/c^+_j)$, where $c^+_j = \max_i u_{ij}$. For a cost-type criterion, it is $\tilde{r}'_{ij} = (a^-_j/u_{ij}, a^-_j/m_{ij}, a^-_j/l_{ij})$, where $a^-_j = \min_i l_{ij}$. The normalized ratings are then weighted to obtain $\tilde{V} = [\tilde{v}_{ij}]$, with $\tilde{v}_{ij} = \tilde{r}'_{ij} \otimes \tilde{w}_j$. The fuzzy positive and negative ideal solutions are derived from $\tilde{V}$. Specifically, if $\tilde{v}_{ij} = (v^l_{ij}, v^m_{ij}, v^u_{ij})$, then $\tilde{v}^+_j = (\max_i v^l_{ij}, \max_i v^m_{ij}, \max_i v^u_{ij})$ and $\tilde{v}^-_j = (\min_i v^l_{ij}, \min_i v^m_{ij}, \min_i v^u_{ij})$. Hence, $A^+ = \{\tilde{v}^+_1, \dots, \tilde{v}^+_n\}$ and $A^- = \{\tilde{v}^-_1, \dots, \tilde{v}^-_n\}$. The closeness coefficient is computed from each supplier's distances to A^+ and A^- and is used as the supplier–product sustainability score SS^i_p. The distances of each supplier $i \in A$ from A^+ and A^- are computed using a distance metric such as the Euclidean distance for fuzzy numbers. The distance of supplier i from A^+ is given by $d^+_i = \sqrt{\sum_{j=1}^{n}\left[d(\tilde{v}_{ij}, \tilde{v}^+_j)\right]^2}$, and the distance from A^- is $d^-_i = \sqrt{\sum_{j=1}^{n}\left[d(\tilde{v}_{ij}, \tilde{v}^-_j)\right]^2}$. Finally, the environmental sustainability score of supplier i for product p, SS^i_p, also known as the relative closeness coefficient, is calculated as $SS^i_p = \frac{d^-_i}{d^+_i + d^-_i}$. Once the environmental sustainability score of each supplier–product pair (SS^i_p) is computed using fuzzy TOPSIS, the retailer computes the environmental sustainability weight of each bid SW^b as shown in expression (14):

$$\mathrm{SW}^b = \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}} SS^i_p Q^b_{tp}, \qquad b \in \mathcal{B}(i), i \in \mathcal{N}. \tag{14}$$

Finally, to account for environmental sustainability, the retailer seeks not only to minimize TC but also to maximize the total environmental sustainability value (SV), which is the total environmental sustainability weight of selected bids (Eq. (15)). In this work, SV refers to environmental sustainability value. If the fuzzy-TOPSIS criteria include the three sustainability pillars, namely economic, environmental, and social criteria, the same quantity is interpreted more generally as the total sustainability value.

$$\max \mathrm{SV} = \sum_{i \in \mathcal{N}} \sum_{b \in \mathcal{B}(i)} \mathrm{SW}^b X^b. \tag{15}$$

Objective functions (1) and (15) along with constraints (2)–(13) form a bi-objective model, which is solved in our study using the ϵ-constraint method. This method (Algorithm 1) is employed to balance two objectives: minimizing TC and maximizing SV. The process begins by establishing two reference environmental sustainability values by solving single-objective models. First, SV^1 is obtained from the solution of the cost-minimization model, i.e., Objective (1) with constraints (2)–(13). This value is the sustainability value associated with the economically optimal solution. Second, SV^2 is obtained by maximizing the sustainability value, i.e., Objective (15), with the same constraints. To sample Ψ points within the sustainability range $[\mathrm{SV}^1, \mathrm{SV}^2]$, we calculate the step size as follows: $\epsilon = \frac{\mathrm{SV}^2 - \mathrm{SV}^1}{\Psi - 1}$. The iterative process begins by setting the imposed sustainability threshold to the upper reference sustainability value, $\theta^1 = \mathrm{SV}^2$, where θ^ψ denotes the imposed sustainability threshold at iteration ψ, ($\psi = 1, \dots, \Psi$). For each iteration, the environmental sustainability objective function is converted into a constraint with a dynamically adjusted lower bound, θ^ψ, ensuring a gradual reduction in sustainability with each step. The constraint for the sustainability score is:

$$\sum_{i \in \mathcal{N}} \sum_{b \in \mathcal{B}(i)} \mathrm{SW}^b X^b \geq \theta^\psi. \tag{16}$$

Each iteration aims to minimize the total cost, using the objective function for cost minimization (expression (1)) while respecting all constraints in Eqs. (2)–(13) and the epsilon constraint in Eq. (16). After solving this constrained problem, the achieved sustainability value is recorded, while the next imposed sustainability threshold is updated by subtracting ϵ from the current threshold, i.e., $\theta^{\psi+1} = \theta^\psi - \epsilon$. The process continues for Ψ iterations, until the imposed threshold reaches the lower reference value, SV^1. This iterative approach provides a series of optimal solutions representing trade-offs between cost and sustainability, allowing the decision-maker to select the most appropriate solution based on their desired balance between the objectives.

Algorithm 1 Bi-objective Optimization using the ϵ-constraint Method.

1: Get SV^1 by solving the model with objective (1) and constraints (2)–(13); SV^1 is the sustainability value of the cost-minimizing solution.
2: Get SV^2 by solving the model with objective (15) and constraints (2)–(13).
3: Specify the desired number of points, Ψ.
4: Calculate ϵ as $\frac{\mathrm{SV}^2 - \mathrm{SV}^1}{\Psi - 1}$.
5: Set iteration counter $\psi = 1$ and $\theta^\psi = \mathrm{SV}^2$.
6: **while** $\psi \leq \Psi$ **do**
7: Convert Objective (15) to a constraint as shown in expression (16), using θ^ψ as the imposed threshold.
8: Optimize using the model defined by objective (1) and constraints (2)–(13) and constraint (16).
9: Record the resulting TC and achieved SV values.
10: $\psi \leftarrow \psi + 1$.
11: Set $\theta^\psi = \theta^{\psi-1} - \epsilon$.
12: **end while**

Beyond the bi-objective formulation, we also examine three environmental sustainability mechanisms that can be embedded in the auction-based SSOA model. These mechanisms represent different managerial approaches: imposing an emission cap, developing selected supplier–product relationships to meet qualification requirements, and using a bonus–malus price signal to favor bids with higher sustainability intensity. Although the three extensions are evaluated separately in this work to isolate their individual effects, they are modular rather than mutually exclusive, and any subset can be integrated within the same WDP by combining the corresponding objective-function modifications and constraints; the unified formulation is provided in Section A of the Supplementary Material.

3.3.1. Transport-emission constrained extension

To examine the environmental implications of the auction-based SSOA model, we extend the WDP with a transport-emission cap. The following additional notation is used:

- e_{ipt}: relative transport-emission-proxy coefficient for one unit of product p supplied by supplier i in period t
- E^b: transport-emission-proxy value of bid b
- $E^{\max}$: maximum admissible transport-emission-proxy value

For a bid $b \in \mathcal{B}(i)$ submitted by supplier i, its transport-emission-proxy value is calculated as

$$E^b = \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} e_{ipt} Q^b_{tp}, \qquad b \in \mathcal{B}(i),\ i \in \mathcal{N}. \tag{17}$$

The transport-emission-proxy-constrained WDP keeps Objective (1) unchanged and adds the following constraint:

$$\sum_{i \in \mathcal{N}} \sum_{b \in \mathcal{B}(i)} E^b X^b \leq E^{\max}. \tag{18}$$

3.3.2. Supplier-development qualification extension

In practice, buyers may support selected suppliers that initially fall below a required sustainability or qualification level, through audits, certification support, training, technical assistance, process improvement, or other supplier-development actions. This view is consistent with the supplier-development literature, which recognizes that buying firms may actively improve supplier capabilities rather than only replace weak suppliers (Glock et al., 2017; Modi & Mabert, 2007). For example, Apple announced a Supplier Employee Development Fund to expand learning opportunities and skills development across its supply chain (Apple, 2022). In sustainable procurement, buyer-led environmental supplier-development programs can also support suppliers in improving their environmental practices (Saghiri & Mirzabeiki, 2021).

To capture this mechanism, we model qualification as an optimized supplier-development decision. A supplier–product relationship can be used if the supplier already satisfies the required qualification level, or if the buyer supports the improvement needed to reach that level. Let $\tau_p \in [0, 1]$ denote the minimum qualification level required for product p. Let U_{ip} be a binary variable equal to 1 if supplier i is selected for development for product p, and 0 otherwise. Let $A_{ip} \geq 0$ denote the qualification improvement achieved for supplier i and product p. Let G^0_{ip} denote the fixed cost of initiating the development action, and G^1_{ip} denote the marginal cost of one unit of qualification improvement. The original cost objective is modified as follows:

$$\min \mathrm{TC}^Q = \mathrm{TC} + \sum_{i \in \mathcal{N}} \sum_{p \in \mathcal{P}} \left(G^0_{ip} U_{ip} + G^1_{ip} A_{ip} \right). \tag{19}$$

The qualification requirement is imposed by

$$SS^i_p + A_{ip} \geq \tau_p Z_{ip}, \qquad i \in \mathcal{N},\ p \in \mathcal{P}. \tag{20}$$

Constraint (20) links qualification to the supplier–product activation decision. If $Z_{ip} = 0$, supplier i is not used for product p, and no improvement is required. If $Z_{ip} = 1$, the supplier–product relationship must either already satisfy $SS^i_p \geq \tau_p$, or the model must choose an improvement level A_{ip} to close the qualification gap. The improvement and development decisions are linked by

$$A_{ip} \leq (1 - SS^i_p) U_{ip}, \qquad i \in \mathcal{N},\ p \in \mathcal{P}, \tag{21}$$

$$U_{ip} \le Z_{ip}, \qquad i \in \mathcal{N},\ p \in \mathcal{P}, \tag{22}$$

$$U_{ip} \in \{0,1\}, \qquad A_{ip} \ge 0, \qquad i \in \mathcal{N},\ p \in \mathcal{P}. \tag{23}$$

Constraint (21) allows qualification improvement only when a development action is undertaken and limits the improvement to the remaining distance from the current score to the upper bound of the qualification scale. Constraint (22) prevents development investment in supplier–product relationships that are not selected. Since $G^1_{ip} > 0$, the optimizer chooses only the minimum improvement needed to satisfy (20). Thus, suppliers that are slightly below the threshold require limited improvement, while suppliers far below the threshold require greater development effort and higher cost.

3.3.3. Bonus–malus extension

Bonus–malus mechanisms are widely used to translate environmental or sustainability performance into economic incentives. In procurement, this logic has been used to adjust tender evaluations or bid prices according to environmental performance (see for example, Scherz et al., 2023; Sustainable Procurement Pledge, 2024). Motivated by this idea, we extend the auction-based SSOA model with a bonus–malus pricing policy that steers bid selection. Let SI^b and $\bar{S}$ be environmental sustainability intensity of bid b, and benchmark environmental sustainability-intensity level, respectively. Let λ^+ and λ^- be bonus and malus rates applied when $SI^b > \bar{S}$ and $SI^b < \bar{S}$, respectively. Let $P^{b,BM}$ be bonus–malus adjusted price of bid b. For a nonempty bid $b \in \mathcal{B}(i)$ submitted by supplier i, the environmental sustainability intensity is defined as the quantity-weighted average environmental sustainability score of the products contained in the bid:

$$SI^b = \frac{\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} SS^i_p Q^b_{tp}}{\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} Q^b_{tp}}, \qquad b \in \mathcal{B}(i),\ i \in \mathcal{N}. \tag{24}$$

The adjusted bid price is then computed as

$$P^{b,BM} = P^b - \lambda^+ P^b \max\{0, SI^b - \bar{S}\} + \lambda^- P^b \max\{0, \bar{S} - SI^b\}, \quad b \in \mathcal{B}(i),\ i \in \mathcal{N}. \tag{25}$$

Eq. (25) gives a favorable price adjustment to bids whose environmental sustainability intensity exceeds the benchmark and an unfavorable adjustment to bids below it. The bonus–malus WDP keeps all constraints (2)–(13) unchanged and replaces the original bid-price term in the objective function with the adjusted bid price:

$$\min \mathrm{TC}^{BM} = \sum_{i\in\mathcal{N}} \sum_{p\in\mathcal{P}} F_{ip} Z_{ip} + \sum_{i\in\mathcal{N}} \sum_{b\in\mathcal{B}(i)} P^{b,BM} X^b + \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} \left(H_{pt} I^+_{pt} + S_{pt} I^-_{pt} \right). \tag{26}$$

Thus, the policy changes the relative attractiveness of bids through the objective function, while leaving the feasible region unchanged. Unlike qualification screening, no bid is excluded solely because its environmental sustainability intensity is below the benchmark.

4. Solution approaches

In this section, we explain our proposed heuristic approaches for solving the SSOA problem. We introduce four original variants of the kernel search method that are based on alternative scoring mechanisms. It is worth noting that classical KS exploits scoring rules based on the optimal solution of the linear relaxation. This is a general and valid scoring method that does not depend on the specific problem addressed. However, problem-specific scoring mechanisms that exploit the unique features of the addressed problem have been shown in the literature to be more effective (see Hanafi et al., 2020; Mancini et al., 2022).

In this work, we propose four modified versions of KS, referred to as *Bundle-based KS*, in which different product and bid features are exploited to identify the kernel and construct the bucket compositions. The main difference between the proposed methods lies in the features considered in each method and in how these features are calculated and combined to determine the method's global score. The four modified methods are described in the following subsections. Each method is tested using three global scoring strategies: the multiplicative method (MM), the weighted method with administrative costs (WM-A), and the weighted method without administrative costs (WM).

The four scoring rules were designed to capture different practical features of submitted bids. M1 protects scarce or difficult-to-source products from being excluded by combining product rarity with price competitiveness. M2 favors bids that cover a larger share of product–period demand while still accounting for value density. M3 prioritizes rare products, but adjusts this priority according to the amount of demand covered by the bid. M4 favors bids whose quantities are closer to the buyer's demand profile, which is useful when excess supply may increase holding costs or poor matching may create shortages. Since the WDP is affected by several interacting factors, including bid composition, product availability, demand levels, costs, planning horizon, and sourcing restrictions, we evaluate the practical scope of these methods empirically across different instance classes in Section 5.

4.1. M1: Frequency-based rarity and value density

This method prioritizes bids that contain rare products, i.e., products appearing in relatively few bids, and combines this preference with the bid's value density to reward competitive pricing. Let $\bar{f}_p$ denote the number of bids in which product p appears at least once over the planning horizon. The raw rarity score of bid b, denoted by f^b, is calculated by summing the inverse product frequency over all product–period pairs included in the bid, i.e., $f^b = \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}: Q^b_{tp}>0} 1/\bar{f}_p$. The corresponding normalized rarity score is $\sigma^{\text{freq}}_b = f^b / \max_b f^b$.

The value density of bid b is calculated as $\rho_b = \sum_{t,p} Q^b_{tp}/P^b$, or as $\rho_b = (\sum_{t,p} Q^b_{tp})/(P^b + \sum_{p:\sum_t Q^b_{tp}>0} F_{ip})$ when administrative costs are included. The normalized value density score is $\sigma^{\text{price}}_b = \rho_b / \max_b \rho_b$. The global score is defined either multiplicatively, i.e., $\sigma_b = \sigma^{\text{freq}}_b \sigma^{\text{price}}_b$ for MM1, or as a weighted sum, i.e., $\sigma_b = \alpha\sigma^{\text{freq}}_b + (1-\alpha)\sigma^{\text{price}}_b$ for WM1-A and WM1, where α is a weight. In WM1-A, F_{ip} is incorporated into ρ_b, whereas in WM1 it is not. Bids are sorted in non-increasing order by their global score σ_b. The top $\mathcal{E}$ bids are assigned to the kernel, and the remaining bids are grouped into buckets according to rank.

4.2. M2: Demand contribution and value density

This method complements the value density with a *demand contribution* score, favoring bids that cover a large portion of the total demand across products and periods. The demand contribution score (δ_b) and consequently its normalized counterpart (σ^{demand}_b) are calculated as:

$$\delta_b = \sum_{p,t} \left(\frac{Q^b_{tp}}{D^p_t} \right), \qquad \sigma^{\text{demand}}_b = \frac{\delta_b}{\max_b \delta_b},$$

where, as above, in WM2-A the administrative cost F_{ip} is incorporated into the value density σ^{price}_b, computed as in M1. The global score is defined either multiplicatively, i.e., $\sigma_b = \sigma^{\text{demand}}_b \sigma^{\text{price}}_b$ for MM2, or as a weighted sum, i.e., $\sigma_b = \alpha\sigma^{\text{demand}}_b + (1-\alpha)\sigma^{\text{price}}_b$ for WM2-A and WM2.

4.3. M3: Rank-based rarity adjusted by demand

This method builds upon the idea of rarity and demand contribution but enhances the scoring granularity by introducing a rank-based rarity index adjusted by demand coverage. The method combines a modified rarity score with the normalized value density σ^{price}_b to determine the global score of each bid. First, the frequency of each product across all bids is computed. The unique values of the product-frequency scores are extracted and ranked in ascending order, assigning higher scores to rarer products. The rarity score width (rw) is defined as the reciprocal of the number of unique frequency values, ensuring equal spacing between score levels. Each product p is then assigned a starting rarity score (s_p^{start}) based on its frequency. For each bid b, the score of a product is adjusted according to how much of the product's total demand the bid covers. Specifically, the increase in rarity score is proportional to the quantity share the bid provides relative to the total demand of that product, scaled and capped at the score width. The adjusted score for product p in bid b is given by: $s_p^b = s_p^{\text{start}} + \min\{rw(\sum_t Q_{tp}^b / \sum_t D_t^p), rw\}$. The total rarity score for bid b is calculated as $f^b = \sum_p s_p^b$ and normalized as $\sigma_b^{\text{rarity}} = f^b / \max_b f^b$. This score reflects both product rarity and product-level demand coverage. The normalized value density score σ_b^{price} is computed as in M1. The global score is defined either multiplicatively, i.e., $\sigma_b = \sigma_b^{\text{rarity}} \sigma_b^{\text{price}}$ for MM3, or as a weighted sum, i.e., $\sigma_b = \alpha\sigma_b^{\text{rarity}} + (1-\alpha)\sigma_b^{\text{price}}$ for WM3-A and WM3.

4.4. M4: Deviation from demand and value density

Rather than using rarity or contribution ratios, this method penalizes deviations between offered quantities and demand. Bids that closely match demand receive higher scores. For each bid b, a deviation score is computed by summing the absolute differences between the quantity offered and the demand across all products and time periods: $\Delta_b = \sum_{p,t} |D_t^p - Q_{tp}^b|$. The total demand $\sum_{p,t} D_t^p$ is used as the normalization reference for the maximum possible deviation. To ensure that bids with smaller deviations get higher scores, the deviation score is inverted and normalized as: $\sigma_b^{\text{deviation}} = \frac{\sum_{p,t} D_t^p}{\Delta_b}$. A lower deviation implies better demand fulfillment. This score is then combined with the normalized value density score σ_b^{price}, computed as in M1. The global score is defined either multiplicatively, i.e., $\sigma_b = \sigma_b^{\text{deviation}} \sigma_b^{\text{price}}$ for MM4, or as a weighted sum, i.e., $\sigma_b = \alpha\sigma_b^{\text{deviation}} + (1-\alpha)\sigma_b^{\text{price}}$ for WM4-A and WM4.

It is worth noting that none of the above methods uses the information provided by the optimal solution of the LP relaxation, as they rely instead on alternative scoring mechanisms to rank the bids. Consequently, our approaches differ substantially from both classical and improved KS methods. In contrast to the variants proposed in the literature, which typically exploit information derived from the relaxed LP solution to guide the search process, the proposed approaches adopt an independent ranking strategy that does not depend on LP-based outcomes. This distinction highlights the methodological novelty of our framework and emphasizes its conceptual departure from traditional KS-based schemes.

5. Numerical experiments

This section presents the computational experiments used to validate the proposed model and evaluate the heuristic approaches. We describe the instance-generation procedure, analyze the effect of instance characteristics on exact-solver complexity, and compare the heuristic methods across both algorithmic configurations and instance classes. All models were implemented in Julia 1.10.0 using JuMP and solved with Gurobi 11.0 on a Windows 11 Enterprise machine with a 12th Gen Intel(R) Core(TM) i5-1235U processor at 1.30 GHz, 16 GB RAM, and a 64-bit x64-based architecture.

5.1. Test-instance generation

In this work, we randomly generated instances to test our model and solution approaches. The process of generating the instances includes creating supplier bids, assigning products to each bid, generating demands and costs, calculating holding and shortage costs, and deriving bid-level environmental sustainability weights. The steps are detailed in Algorithm B.1 in the Supplementary Material and are explained below. Each supplier $i \in \mathcal{N}$ is assigned a random subset of products $\mathcal{P}_i \subseteq \mathcal{P}$, where $|\mathcal{P}_i|$ is uniformly chosen within the range $\{1, \dots, |\mathcal{P}|\}$. The number of bids that each supplier can submit, denoted by $|\mathcal{B}(i)|$, is generated as $|\mathcal{B}(i)| \sim \mathcal{U}\left(\{1, \dots, \min(100, 2^{|\mathcal{P}_i|} - 1)\}\right)$, where $2^{|\mathcal{P}_i|} - 1$ is the total number of nonempty combinations. For each bid $b \in \mathcal{B}(i)$, a subset of products $\mathcal{P}_b \subseteq \mathcal{P}_i$ is selected. The last bid always includes all products offered by supplier i, whereas the remaining bids include random nonempty subsets of $\mathcal{P}_i$ (Alaei & Setak, 2017). To define quantities in the bids, for each supplier i, product $p \in \mathcal{P}_i$, and period $t \in \mathcal{T}$, a base quantity q_{ipt} is randomly generated within the range $[10, 150]$. Then, for each bid $b \in \mathcal{B}(i)$ and each product $p \in \mathcal{P}_b$, a subset of periods $\mathcal{T}_{p,b} \subseteq \mathcal{T}$ is randomly selected. Within each selected period $t \in \mathcal{T}_{p,b}$, the quantity Q_{tp}^b is calculated as $Q_{tp}^b = \left\lceil q_{ipt} r_{ipt}^b \right\rceil$, $r_{ipt}^b \sim \mathcal{U}(0.9, 1.1)$, where r_{ipt}^b is a random multiplier ensuring variability in the assigned quantity across selected periods and bids. If $t \notin \mathcal{T}_{p,b}$, then $Q_{tp}^b = 0$.

The environmental sustainability closeness score (SS_p^i) for each supplier–product pair is generated randomly between 0.25 (low score) and 0.90 (high score). The weight of each bid (SW^b) is then calculated by multiplying the environmental sustainability score of each supplier–product pair by the corresponding quantity offered in the bid and summing over products and periods. Demand, administrative costs, holding costs, and shortage costs were generated by generalizing the rules defined in Hamdan et al. (2023) and Manerba et al. (2018) to suit the multi-period, multi-product characteristics of the SSOA variant studied in this work. For each product $p \in \mathcal{P}$ and period $t \in \mathcal{T}$, the unit product price vc_{pt} is generated randomly from $\mathcal{U}(10, 18)$. The demand D_t^p is calculated using a weighted combination of the maximum and sum of supplier-provided quantities, adjusted by a price-dependent factor (see Eq. (27) below). The adjusted unit price vc_{ipt} for each supplier i, product p, and period t is generated as $vc_{ipt} \sim \mathcal{U}(0.9vc_{pt}, 1.1vc_{pt})$ and is used to calculate the administrative cost (Hamdan et al., 2023; Manerba et al., 2018). The average unit price for each supplier–product pair $\overline{vc}_{ip}$ and overall product $\overline{vc}_p$ are calculated using Eq. (28) and are then used to compute the administrative cost by means of Eq. (29).

Holding costs are drawn as annualized percentages of the average unit price, between 10%–20%, and divided by 12 to obtain period-level costs. Two shortage-cost rules are considered. Under the original rule, annual shortage costs are drawn between 25%–35% of the average unit price and are likewise divided by 12. Under the high-ratio rule, the holding-cost calculation is unchanged, but a shortage-to-holding cost ratio $r_{pt} \in [10, 100]$ is randomly sampled and $S_{pt} = r_{pt} H_{pt}$. The shortage cost acts as a penalty proxy for unmet demand and may represent delayed fulfillment, premium transportation, lost sales, goodwill loss, or future demand loss (Duc et al., 2022; Theodorou et al., 2023).

$$D_t^p = \left\lceil D_t'^p - (D_t'^p - 1) \cdot \frac{vc_{pt}}{\max_{\tau \in \mathcal{T}} vc_{p\tau}} \right\rceil, \tag{27}$$

where

$$D_t'^p = \left\lceil \lambda \max_{i \in \mathcal{N}} q_{ipt} + (1-\lambda) \sum_{i \in \mathcal{N}} q_{ipt} \right\rceil \qquad \text{and} \qquad \lambda \sim \mathcal{U}(0, 1).$$

$$\overline{vc}_{ip} = \frac{\sum_{t \in \mathcal{T}} vc_{ipt}}{|\mathcal{T}|}, \quad \overline{vc}_p = \frac{\sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} vc_{ipt}}{|\mathcal{N}||\mathcal{T}|}. \tag{28}$$

$$F_{ip} = \left(\overline{vc}_p + \frac{\overline{vc}_p}{\overline{vc}_{ip}}\right) \gamma \sum_{t \in \mathcal{T}} q_{ipt}, \text{ where } \gamma = \frac{1}{10|\mathcal{P}|}. \tag{29}$$

The supplier-specific bid limit is generated as $L_i = \lceil 0.3\,|\mathcal{B}(i)| \rceil$, $i \in \mathcal{N}$. Moreover, let $L_p^{\mathbf{max}}$ denote the number of suppliers that can potentially provide product p in the generated instance. Then, the minimum number of suppliers required for product p is set as $\mathcal{L}_p = \min\left\{ L_p^{\mathbf{max}}, \max\{2, \lfloor 0.05\, L_p^{\mathbf{max}} \rfloor\} \right\}$, $p \in \mathcal{P}$. Finally, the bid price (P^b) is calculated using a discounted unit price multiplied by the quantities. For supplier i, let $Q_i^{\max} = \max_{b \in \mathcal{B}(i)} \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} Q_{tp}^b$ denote the maximum quantity offered among all bids submitted by supplier i. A base discount is then defined as

$$\bar{d}^b = 0.005 + \left(\frac{\sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} Q^b_{tp}}{Q^{\max}_i} \right) 0.045.$$

The effective discount applied to each product-period component of bid b is given by

$$df^b_{pt} = \min\left\{0.05,\ \max\left\{0.005,\ \bar{d}^b + \epsilon^b_{pt}\right\}\right\}, \qquad \epsilon^b_{pt} \sim \mathcal{U}(-0.0025, 0.0025). \tag{30}$$

Hence, the discount is capped within $[0.5\%, 5\%]$ and the bid price is expressed as $P^b \leftarrow \sum_{p\in\mathcal{P}} \sum_{t\in\mathcal{T}} Q^b_{tp} vc_{ipt}(1 - df^b_{pt})$.

Table B.1 in the Supplementary Material summarizes the values and distributions used in the instance generation. The details of the generated instances are reported in the Supplementary Material (Table C.1).

5.2. Combinatorial Auction Test Suite (CATS)-based instance generation procedure

To generate benchmark instances with richer combinatorial structure, we adopted a hybrid instance-generation procedure based on the well-known software package CATS (Leyton-Brown et al., 2006). The purpose of using CATS was not to generate the full procurement instance directly, since CATS natively produces bids as bundles of goods with associated bid prices, whereas our model requires supplier identities, product–period quantities, administrative costs, inventory-related parameters, and sourcing constraints. Instead, we used CATS to generate the combinatorial structure of bids and then applied a procurement-specific post-processing layer to recover all parameters required by the proposed WDP model. More precisely, each CATS good, denoted as g, was interpreted as a predefined lot associated with a product–period pair. For a given instance with $|\mathcal{P}|$ products and $|\mathcal{T}|$ periods, we created n^{lot} lots for each (p, t) combination, yielding a total of $|\mathcal{P}||\mathcal{T}|n^{\text{lot}}$ auction goods. Each good g was mapped to a tuple (p_g, t_g, q_g), where p_g is the associated product, t_g is the associated period, and q_g is the quantity represented by that lot. In our implementation, lot quantities were generated randomly to induce heterogeneity across product–period offers while preserving a realistic scale of supply availability. CATS was then used to generate a set of bundle bids over these goods using its arbitrary distribution.

Once the CATS output was generated, each bid was converted into the quantity representation required by our model. Specifically, for each generated bid b, the corresponding quantity parameter Q^b_{tp} was reconstructed by aggregating the quantities of all CATS goods in the bundle that were mapped to product p and period t, i.e., $Q^b_{tp} = \sum_{g\in G_b : p_g = p,\, t_g = t} q_g$, where G_b denotes the set of goods included in bid b. This transformation preserves the combinatorial structure produced by CATS while translating each bundle into a feasible product–period supply schedule. Generated bids were subsequently assigned to suppliers through a product-aware allocation procedure. The assignment was designed to distribute bids across the suppliers in $\mathcal{N}$ while maintaining broad product coverage and avoiding excessive concentration of product availability within a small number of suppliers. This step yields the bid–supplier mapping used in the model and allows the construction of the matrices describing product coverage per bid and product availability per supplier.

The bid prices produced by CATS were retained as the basis for the economic value of each bid, but to align their scale with the procurement cost structure used in our experiments, they were linearly rescaled to the range of supplier-specific proxy costs computed from generated unit cost parameters. This transformation preserves the relative ordering of CATS bid prices while ensuring consistency with the magnitude of the remaining economic parameters in the instance. After generating and assigning the bids, the remaining instance parameters (demand, costs, sourcing limits, and sustainability weights) were constructed separately. The corresponding notation is given in Section 5.1.

5.3. Impact of problem size on complexity

Instances were generated randomly using both Algorithm B.1 and the software package CATS described in Section 5.2. The experimental design starts with a base setting with $|\mathcal{N}| = 5$, $|\mathcal{P}| = 10$, and $|\mathcal{T}| = 6$, and then varies the main size dimensions by increasing the number of suppliers, products, periods, and submitted bids. Specifically, the number of suppliers is increased to 15, 25, and 30; the number of products is increased to 15, 20, and 25; and the number of periods is increased to 12, 18, and 24. The total number of bids and the remaining instance parameters are generated according to the distributions and value ranges reported in Table B.1. The test suite contains 120 instances arranged into four blocks of 30 with the same nominal supplier, product, and period settings. Block 1 uses the bid-generation steps of Algorithm B.1, whereas Blocks 2–4 replace these steps with the CATS-based procedure. For all four blocks, the remaining demand, cost, sourcing, and sustainability parameters follow the corresponding rules in Algorithm B.1 and Table B.1. Blocks 3 and 4 have larger bid pools than Block 2. Blocks 1–3 follow the original shortage-cost rule, whereas Block 4 follows the high-ratio rule.

Table C.1 reports the characteristics of the generated instances together with the solution times, in seconds, obtained using the exact approach. A three-hour time limit was used. Eleven instances reached this limit; for these instances, the best incumbent returned by the MIP solver is used as the reference exact solution. We observe that the number of suppliers and the total number of submitted bids significantly affect the time required to solve the instances with the exact solver, whereas the number of products and periods has a milder and less consistent impact on problem complexity (Table C.1). The variation in solution times among instances with the same nominal dimensions also indicates that exact-solver difficulty is affected not only by the reported aggregate size indicators but also by the generated bid composition, product–period coverage, demand levels, and cost parameters.

5.4. Heuristic performance

Besides the exact solver, we solve the 120 instances using KS, MMs, WM-As, and WMs and report the relative difference (RD) with respect to the reference solution obtained by the exact approach, calculated as $100\frac{\text{Heuristic}-\text{Exact}}{\text{Exact}}$. A negative RD indicates that the heuristic found a better solution than the exact reference due to the time limit. For each instance, we test the following bucket sizes: 8, 10, 15, 20, and 25, and we vary α from 0 to 1 with a step of 0.1. The average RD and time across all instances, calculated for each bucket size based on the minimum RD obtained for the optimal weight combination, are presented in Table 2. This analysis helps in identifying the best bucket size for each method. For instance, eight buckets yield the best results for KS, MM2, MM3, MM4, all WM-A variants, and all WM variants, as shown in Table 2. Only MM1 performs slightly better with 10 buckets.

For the weighted methods, Fig. 2 reports average RD and computation time as functions of α with the best-performing bucket size, $G = 8$. WM1, which combines frequency-based product rarity with value density, gives the lowest mean RD when one common configuration is used across all instances. At $\alpha = 0.3$, it obtains an RD of 5.29% in 64.13 s, compared with 7.03% and 71.11 s for KS. Since this setting assigns greater weight to value density while retaining some weight on rarity, the result suggests that information about scarce products can complement price-based ranking.

Averaged across the five bucket sizes in Table 2, M1 also has the lowest RD under MM, WM-A, and WM, at 7.63%, 4.04%, and 3.42%, respectively. The corresponding averages are 16.96%, 8.49%, and 6.46% for M2; 16.77%, 7.99%, and 6.31% for M3; and 8.39%, 7.05%, and 5.54% for M4. M2 emphasizes the share of demand covered, M3 adjusts rank-based product rarity by demand coverage, and M4 measures how closely a bid's quantities match demand. M2–M4 attain their lowest fixed WM results at $\alpha = 0$, where only value density is

Table 2
Average RD (%) and computation time (s) by bucket size. For each weighted method and instance, the lowest RD over the tested values of α at that bucket size is used, together with the corresponding computation time.

Bucket size	RD (%)					Time (s)				
	KS	M1	M2	M3	M4	KS	M1	M2	M3	M4
Multiplicative methods (MM)										
8	7.03	6.50	12.42	12.21	6.79	71.11	65.07	55.86	49.93	86.22
10	9.56	6.22	13.74	13.97	7.30	56.92	61.36	46.30	31.09	87.13
15	15.03	7.99	17.22	17.47	7.89	24.56	36.53	15.52	11.08	62.62
20	17.50	7.98	19.51	19.07	9.64	23.81	25.67	10.69	8.06	42.87
25	20.47	9.45	21.90	21.16	10.33	19.89	23.08	9.17	8.52	31.62
Average	13.92	7.63	16.96	16.77	8.39	39.26	42.34	27.51	21.74	62.09
Weighted methods with admin cost (WM-A)										
8	7.03	2.88	5.51	5.31	5.21	71.11	30.10	45.04	47.10	83.27
10	9.56	3.34	6.72	5.85	6.13	56.92	28.78	36.11	35.69	80.87
15	15.03	4.26	8.58	8.08	7.46	24.56	23.01	11.82	8.36	80.77
20	17.50	4.52	9.85	9.66	7.81	23.81	19.76	11.11	8.36	49.02
25	20.47	5.21	11.78	11.07	8.62	19.89	14.11	9.14	11.75	45.99
Average	13.92	4.04	8.49	7.99	7.05	39.26	23.15	22.64	22.25	67.98
Weighted methods (WM)										
8	7.03	2.38	4.28	4.37	3.69	71.11	49.60	49.70	48.85	65.83
10	9.56	2.75	5.35	5.09	4.73	56.92	43.33	39.26	20.72	53.83
15	15.03	3.43	6.48	6.31	5.45	24.56	26.12	9.61	12.32	19.47
20	17.50	4.15	7.62	7.53	6.75	23.81	20.21	15.99	16.27	26.32
25	20.47	4.38	8.56	8.23	7.06	19.89	14.22	10.32	10.91	21.27
Average	13.92	3.42	6.46	6.31	5.54	39.26	30.69	24.98	21.81	37.35

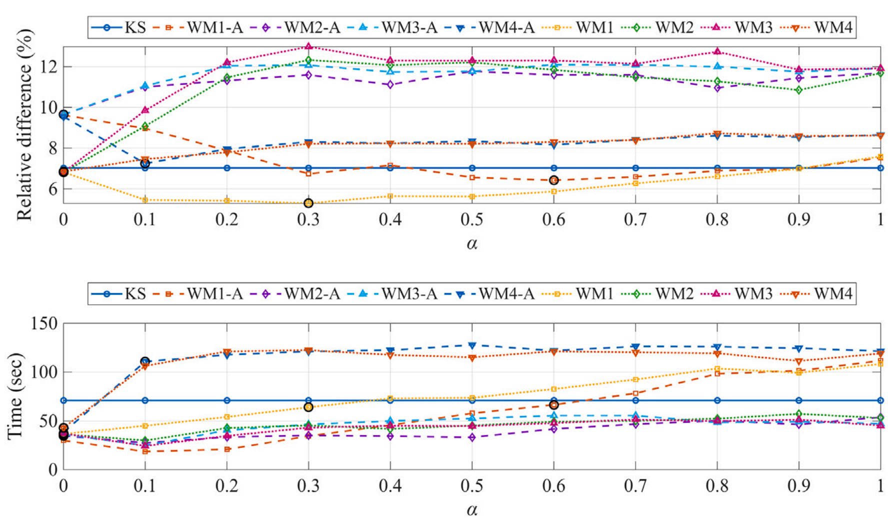

Fig. 2. RD (%) and computation time (sec.) as a function of the weight (α) using the different heuristics for the best bucket size. Circles indicate the minimum RD of the heuristic and the corresponding computation time.

used. Thus, under a common configuration, their additional structural scores do not improve the average result. One possible explanation is that demand coverage or demand matching is evaluated for each bid separately and may not capture how several complementary bids jointly satisfy demand. Moreover, multiplication gives a bid a low overall score whenever either component is low, which may help explain the comparatively high MM RDs of M2 and M3. The consistently lower RDs of WM than WM-A may also reflect that administrative costs are paid once for an activated supplier–product relationship, whereas including the full cost in each bid's ranking score cannot capture its sharing across accepted bids.

The RD–time pattern reflects a trade-off between solution quality and restricted-problem difficulty. For the classical KS, the reported time includes solving the LP relaxation for variable ranking and then running the kernel-search iterations. For the proposed methods, it includes bid scoring/ranking and the same kernel-search structure. Hence, computation time depends not only on the ranking step, but also on the difficulty of the restricted subproblems created by the bucket composition. Under the multiplicative strategy, a bid receives a low score when either component of its score is low. This effect is most evident for MM2 and MM3, which perform worse than KS on average. MM1 and MM4 remain competitive, although MM4 generally requires more computation time because its bucket composition can produce harder restricted subproblems.

Table 2 shows that bucket size 8 gives the lowest RD for every method and strategy except MM1, which performs slightly better with 10 buckets. RD generally worsens as the bucket size increases. M1 and M4 are less sensitive to this increase than M2 and M3, but no method improves consistently with larger buckets. Overall, smaller buckets offer a more precise filtering mechanism, leading to better heuristic performance across most configurations.

Tables D.1 and D.2 in the Supplementary Material provide instance-level results using one fixed configuration for each variant. For reporting purposes, MMC, WMC-A, and WMC denote the lowest RD

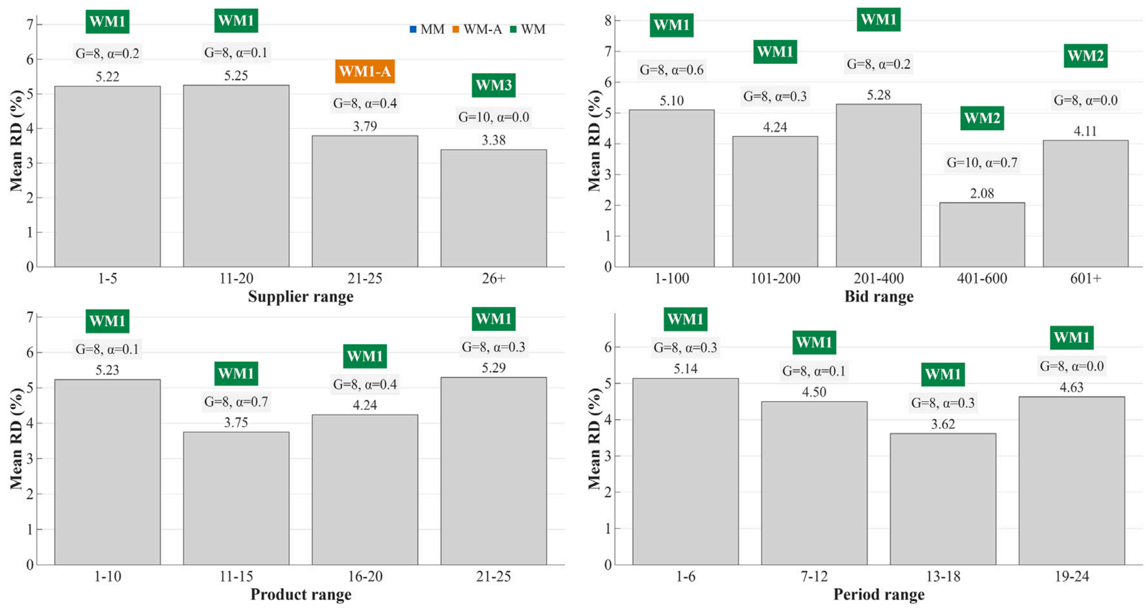

Fig. 3. Best-performing developed heuristic by supplier range, submitted-bid range, product range, and period range.

obtained for each instance among MM1–MM4, WM1-A–WM4-A, and WM1–WM4, respectively. Among the multiplicative variants, MM1 has the lowest mean RD at 6.22%, followed by MM4 at 6.79%. Nevertheless, MM4 is strictly best among MM1–MM4 in 33 instances, including Instances 5, 13, and 23. The MMC summary has a mean RD of 4.32%, showing the potential benefit of selecting a variant according to instance characteristics. Table D.2 shows that WM1 has the lowest mean RD among the fixed weighted configurations, at 5.29%, and obtains zero RD in 18 instances. Each WM variant has a lower mean RD than its corresponding WM-A variant, indicating that including administrative costs in the bid-ranking score is not consistently beneficial. The WMC-A and WMC summaries have mean RDs of 4.52% and 3.64%, respectively. Thus, WM1 is the strongest general-purpose weighted variant, while the retrospective combined summaries show the potential gains from developing a reliable instance-specific selection rule. When evaluated separately on the 30 high-ratio instances in Block 4, WM1–WM3 each attain the lowest mean RD of 5.53% at $G = 8$ and $\alpha = 0$, compared with 5.86% for KS. The overall 120-instance configuration, WM1 with $G = 8$ and $\alpha = 0.3$, gives a mean RD of 6.34% on Block 4. Thus, the weighted methods remain competitive under the higher shortage penalties, although the preferred weight shifts toward pure value density.

Finally, to assess the effect of instance characteristics on heuristic performance, we grouped the 120 instances according to supplier range, product range, period range, and total number of submitted bids. The objective of this analysis is to identify which of the proposed heuristics works best for different classes of instances. For both Fig. 3 and Table D.3 in the Supplementary Material, the selected method is the developed heuristic with the lowest average RD over all tested bucket sizes and, where applicable, all values of α. When two or more methods produce the same average RD within numerical tolerance, the method with the lower average computation time is selected.

Fig. 3 selects WM1 in 13 of the 17 one-dimensional instance classes. The exceptions are the 21–25-supplier class, where WM1-A is selected; the 26-or-more-supplier class, where WM3 is selected; and the two largest bid classes, where WM2 is selected. WM1 is selected in every product-range and period-range class. It therefore remains the safest default, although demand-contribution and rank-based rarity scores perform better in some large supplier or bid pools. The 401–600-bid class contains only two instances and should be interpreted cautiously. Table D.3 provides the corresponding joint-category analysis. Across the 27 joint categories containing at least two instances, a weighted method is selected in 25 categories. WM1 or WM1-A is selected in 21 categories, an M4 variant in five, and WM2 in one. These results reinforce WM1 as the general-purpose choice while indicating potential gains from adapting the heuristic configuration to the instance structure. Because several joint categories contain only two or three instances, the findings should be treated as diagnostic rather than conclusive.

6. Managerial insights

We demonstrate the advantages and disadvantages of the auction-based SSOA model relative to alternative models and configurations. All experiments are conducted on a real-life case adapted from Hosseini et al. (2022), who study a multi-product, multi-period SSOA problem with five suppliers, three products, and two periods under demand and supplier-availability uncertainty. The case provides the base purchase prices, product-specific quantity discount ranges, capacities, fixed ordering costs, holding and shortage costs, supplier-availability probabilities, and supplier environmental sustainability scores. To adapt these data to our auction setting, bid structures were generated using CATS, which is used only to construct feasible bid structures. Bid prices are then calibrated from the case-study price and quantity-discount tables rather than generated by CATS itself (Mansouri et al., 2025; Qian et al., 2023). By contrast, the traditional SSOA benchmark uses the same case inputs but allocates direct supplier–product–period quantities. We used the nine two-period demand states reported in the case study and, for each demand state, we generated three supplier-availability realizations using the reported supplier-availability probabilities, resulting in 27 case-study instances. This preserves the original case dimensions while allowing a broader comparison of the two models under multiple operating conditions. Note that the heuristic performance on the case-study instances (Table D.4 in the Supplementary Material) is consistent with the synthetic-instance results reported in Section 5.4.

Observation 1: Despite the flexibility of the traditional SSOA model, a well-designed auction-based SSOA model achieves greater savings

While the CA-based SSOA allows suppliers to control what they offer, when, and how many units through submitted bids, the traditional SSOA allows the buyer to specify which suppliers to contract,

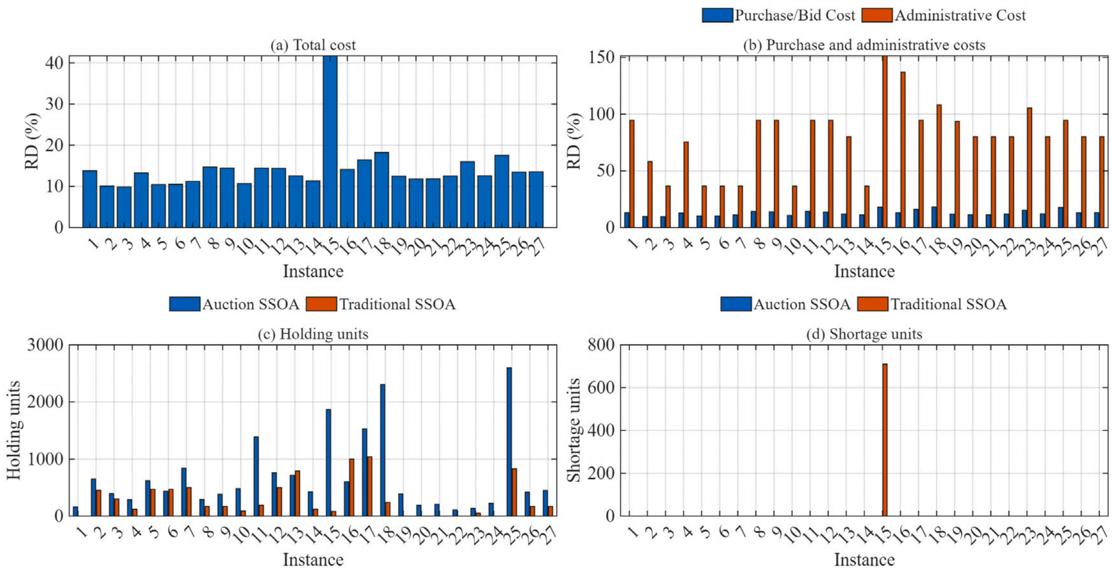

Fig. 4. Average RD between auction-based and traditional SSOA models for total, purchase or bid, and administrative costs, together with the corresponding holding and shortage units.

how many units to order from each, and when to order them through direct supplier–product–period quantity decisions. Thus, the traditional SSOA offers greater flexibility to the buyer in deciding the quantities, provided that the buyer's decisions do not conflict with the supplier's minimum order quantity policy and maximum supply capabilities, thereby helping to reduce procurement costs. However, a major advantage of the CA is that suppliers can express multi-product and multi-period supply schedules through discrete bids, while the resulting bid prices can incorporate the quantity-discount structure reported in the case data. Therefore, if designed properly, the buyer may achieve greater savings than with the traditional SSOA despite the lower purchasing flexibility.

In this section, we compare the auction-based SSOA and the traditional SSOA models. The primary objective of this comparison is to evaluate the impact of the CA approach on key performance metrics, including total cost, shortage levels, and cost distribution. We compare the total cost, purchase or bid, administrative, holding, and shortage costs incurred under both models across all instances. To illustrate the efficiency gains of the auction model, we compute the Relative Difference (RD) in total, purchase or bid, and administrative costs, measured as RD = $\frac{100\times(C_{\text{Traditional}}-C_{\text{Auction}})}{C_{\text{Auction}}}$, where C is the corresponding cost. The results are displayed in Fig. 4. A positive RD value indicates that the traditional model is more expensive than the auction-based model, measured relative to the auction-based cost.

Across all case-study instances, the auction-based SSOA model reduces total cost relative to the traditional model, with RD values ranging from 9.86% to 41.72% and an average of 14.22%, compared with a median of 13.27%. The total-cost RD is positive in all 27 instances, showing that this advantage is observed across the complete case set. The auction-based SSOA model outperforms the traditional benchmark primarily through lower purchase or bid cost, for which the RD ranges from 9.65% to 18.18% with an average of 12.96%. Administrative-cost savings are also consistently observed, with an average RD of 80.40% and values ranging from 36.66% to 151.44%. This indicates that, in the case study, the auction-based formulation not only benefits from discounted bid prices derived from the case-study quantity-discount structure, but also incurs lower administrative costs for the activated supplier–product relationships. Fig. 4 also highlights the observed holding and shortage units under both models. The auction model carries more inventory in 24 of the 27 instances, whereas shortage is eliminated in all auction instances and is observed only once under the traditional model.

The results demonstrate the effectiveness of the auction-based SSOA model in reducing total procurement cost in a case-study-calibrated setting. More specifically, the results show that well-designed bids, constructed from the case-study procurement data and aligned with the buyer's demand profile, can generate persistent savings relative to the traditional SSOA benchmark. At the same time, the comparison reveals a clear operational trade-off: the auction-based model achieves these savings while carrying more holding units, but it avoids the shortage observed in the traditional benchmark. From a managerial perspective, these findings suggest that the auction-based model is most valuable when procurement must be planned under changing conditions, such as demand fluctuations and imperfect supplier availability. In such settings, the mechanism does more than exploit bundle discounts: it also supports a more coordinated sourcing structure that may reduce exposure to shortage, although this interpretation should be treated cautiously because shortage occurred in only one traditional-model solution. Its relative advantage may be smaller in highly stable environments, where direct quantity allocation already provides sufficient flexibility.

Observation 2: Requiring more sources reduces exclusive single sourcing but tends to raise procurement cost

In this section, we analyze the impact of the sourcing decisions on procurement costs, shortage levels, and overall cost efficiency. The minimum number of sources required for each product ($\mathcal{L}_p$) is a critical decision that influences supplier diversification and cost distribution and may also affect supply reliability. We conduct a series of experiments where the minimum number of sources required for each product is varied systematically to assess the impact of sourcing decisions on procurement cost elements. Since we deal with a multiproduct setting, we vary the minimum number of suppliers to be used for each product from one to five as follows.

- **Controlling one product at a time:** The number of sources for each product is varied independently, increasing it from a minimum of one source to a maximum of five sources.
- **Controlling two products simultaneously:** The number of sources for multiple products is varied simultaneously to assess interaction effects.
- **All products:** The number of sources for all products is increased concurrently to evaluate the combined impact on procurement efficiency.

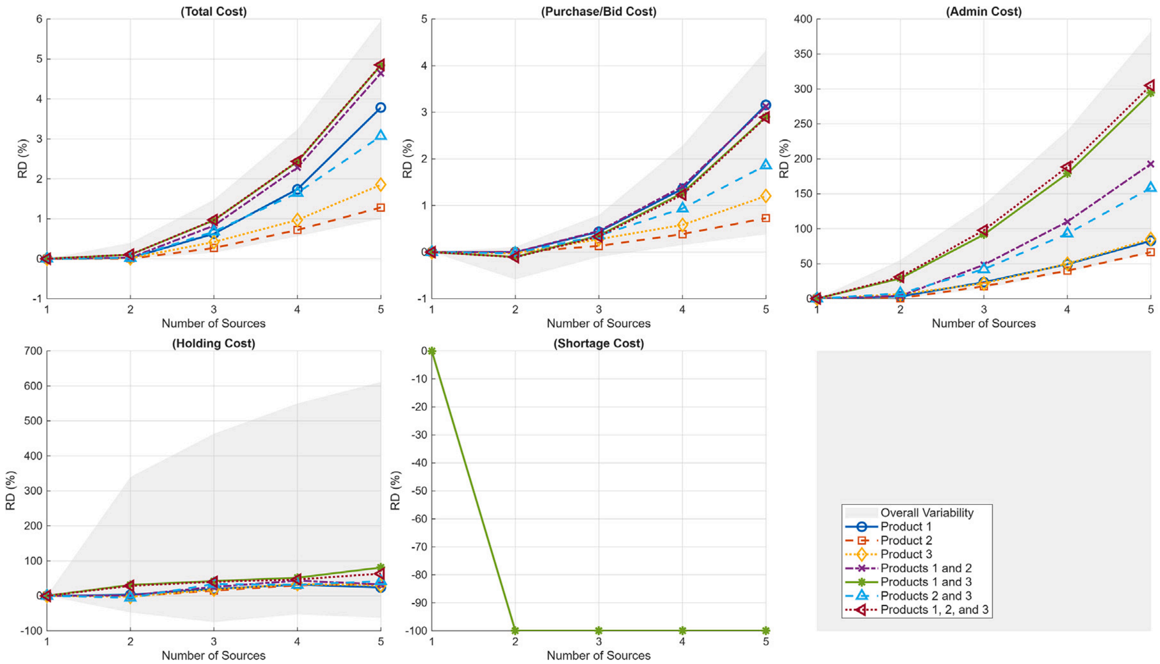

Fig. 5. Average RD and overall variability in cost components under different minimum-source configurations in the case study.

For each sourcing scenario, we measure the RD in costs as follows $\mathrm{RD} = \frac{100\times(C-C_{\text{baseline}})}{C_{\text{baseline}}}$. Here, C_{baseline} is the cost under the corresponding baseline with one required source for the same subset configuration. Thus, when one product is controlled, the baseline fixes that product at one source while the remaining products keep their base case-study source requirement. The same logic applies to the two-product and three-product settings. Fig. 5 shows the average RD across the instances under each scenario and the overall variability for the different cost elements. Across 27 instances, seven product settings, and four nonbaseline source levels, 742 of 756 runs are feasible. Total cost increases in 621 of these runs (83.69%). The increase occurs in all 553 feasible runs requiring three to five sources, but in only 68 of 189 runs requiring two sources. The increase is moderate in magnitude. At the two-source level, the average total-cost RD is only about 0.10% when all three products are controlled simultaneously. At the five-source level, the average total-cost RD reaches 1.28% for Product 2, 1.85% for Product 3, and 3.78% for Product 1 when products are controlled individually. For two-product configurations, the corresponding increases reach 4.64% for Products 1 and 2, 4.84% for Products 1 and 3, and 3.07% for Products 2 and 3. When all three products are constrained simultaneously, the average total-cost RD reaches 4.85%. Overall, the cost increase remains moderate in the present case study, although it becomes more visible as the source requirement increases. Thus, the cost effect is mixed when the controlled products move from one to two required sources, but becomes consistent from three sources onward.

Thus, dual sourcing limits reliance on exclusive single sourcing and may improve supply flexibility, but its total-cost effect is mixed in the present case study. A consistent cost premium emerges only from three required sources onward. Administrative cost increases in 618 of the 742 feasible runs (83.29%), showing that higher coordination overhead is a recurring effect of diversification. Overall, supplier diversification may support supply resilience, but the present experiment does not directly measure disruption recovery or continuity of supply. In the case study, the associated increase in total cost remains limited, even when more products are constrained simultaneously. These results should be interpreted in the context of the case study. In other instances, stronger total-cost increases or more pronounced shortage effects may still occur, depending on the demand profile, supplier availability, and bid composition.

Unlike total cost, purchase or bid cost does not always increase when more suppliers are required. In some configurations, it decreases slightly at the dual-sourcing level. In addition, managing multiple suppliers leads to a substantial rise in administrative expenses, reaching up to 85.63% for Product 3, 192.80% for Products 1 and 2, 294.67% for Products 1 and 3, and 305.26% when all three products are constrained simultaneously at five sources (Fig. 5). Unlike other cost elements, shortage costs do not follow a consistent upward or downward trend. In fact, shortage cost is largely unaffected because shortage is almost absent in these instances. The only clear shortage effect appears when Products 1 and 3 are constrained simultaneously, where the average shortage-cost RD falls by 100% from the two-source level onward. Administrative and holding costs tend to increase as additional supplier–product relationships are required, particularly when Products 1 and 3 or all three products are constrained together. From a managerial perspective, minimum-source requirements reduce dependence on a single supplier but activate more supplier–product relationships and raise coordination cost; this experiment does not measure disruption resilience. They should therefore be set selectively for critical products rather than uniformly.

Observation 3: Substantial cost reductions can be achieved by accepting more bids

Here, we analyze the impact of varying the maximum number of bids that can be accepted on the total cost in an auction-based SSOA problem. For each instance, we conduct 20 experiments with $B_{\max} \in \{1, 2, \ldots, 20\}$. Since the total number of available bids differs across instances, ranging from 177 to 262, the absolute bid cap also corresponds to different percentages of the total bid pool. We compute the RD in total cost relative to the smallest feasible bid-cap setting for each instance, that is, $\mathrm{RD} = 100 \times \frac{\mathrm{TC}_i - \mathrm{TC}_{\text{base}}}{\mathrm{TC}_{\text{base}}}$, where TC_i is the total cost at experiment i, and $\mathrm{TC}_{\text{base}}$ is the total cost at the smallest feasible value of $B_{\max}$. Negative values therefore indicate cost reductions.

Fig. 6 shows the RD in total cost as a function of $B_{\max}$. Relaxing the bid cap produces a cost reduction in all 27 case-study instances. The maximum instance-level reduction ranges from 5.34% to 30.22%, with a median of 14.94%, showing that the magnitude of the benefit is instance-dependent. The largest reduction is observed in Instance 2,

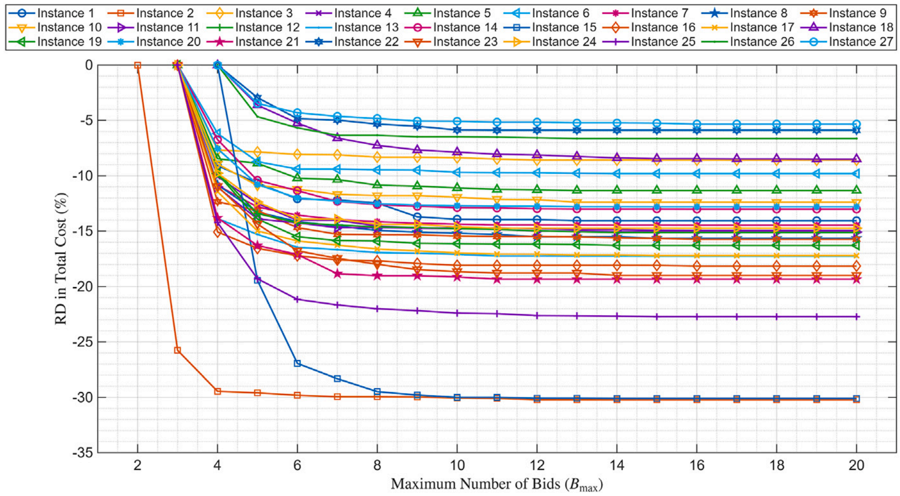

Fig. 6. Effect of the accepted-bid cap on total-cost RD across case-study instances.

where total cost decreases by 30.2% at $B_{\max} = 12$, corresponding to 4.98% of the available bids. A similar reduction of 30.1% is observed in Instance 15 at $B_{\max} = 16$ (8.04% of the available bids), while Instance 25 achieves a reduction of 22.7% at $B_{\max} = 18$ (7.23%).

The results indicate that most of the attainable cost savings are captured at moderate bid caps. We define the near-best threshold as the smallest value of $B_{\max}$ after which the RD at all higher tested caps remains within 0.5 percentage points of the best observed RD. Across the 27 instances, this threshold ranges from 6 to 11 bids, with a median of 8 bids. Beyond the instance-specific threshold, relaxing the cap yields only limited additional savings over the tested range. Some instances are infeasible at very low bid caps. The smallest feasible value of $B_{\max}$ is 2 bids in one instance, 3 bids in 21 instances, and 4 bids in five instances. This demonstrates that the accepted-bid cap must permit a minimum number of bids for a feasible procurement plan to exist. Managers should therefore avoid caps that are so restrictive that they preclude feasible procurement plans or exclude valuable bid combinations. The cap can then be relaxed until further cost improvements become negligible. The appropriate cap remains instance-dependent and should be calibrated empirically rather than imposed as a universal percentage of the available bid pool.

Observation 4: Moderate transport-emission caps can be achieved at limited cost, but tighter caps increase bid fragmentation and may threaten feasibility

This experiment examines the effect of adding a cap on the transport-emission proxy to the auction-based SSOA model. Let $\mathrm{TE}^{\mathrm{NC}} = \sum_{i\in\mathcal{N}} \sum_{b\in\mathcal{B}(i)} E^b X^b$ denote the total proxy value obtained from the corresponding no-cap solution. We impose cap ratios $\kappa \in \{0.95, 0.90, 0.85, 0.80\}$ by setting $E^{\max} = \kappa \mathrm{TE}^{\mathrm{NC}}$. Because direct transport-emission coefficients are unavailable in the case data, we construct a relative proxy from shipment-cost information. Specifically, $e_{ipt} = v^{\min}_{ipt}/\bar{v}^{\mathrm{ref}}$, where $v^{\min}_{ipt}$ is the minimum shipment cost across transport modes for supplier i, product p, and period t, and $\bar{v}^{\mathrm{ref}}$ is the corresponding average over domestic suppliers. Larger values of e_{ipt} are therefore treated as indicating more transport-intensive sourcing within this proxy construction; they are not estimates of physical emissions. We evaluate each cap using the achieved proxy ratio $\mathrm{TE}/\mathrm{TE}^{\mathrm{NC}}$, the RD in total cost, and the selected-bid count.

Fig. 7 shows that the 95% and 90% caps have limited cost effects across the instance set. At the 95% cap, all 27 instances have a total-cost RD of at most 2%; cost increases in 24 instances and remains unchanged in three. At the 90% cap, the mean and median cost RDs are 1.97% and 2.03%, respectively. Although 14 of the 27 instances exceed 2%, none exceeds 5%. The mean achieved proxy ratios at the 95%, 90%, 85%, and 80% cap levels are 0.947, 0.898, 0.848, and 0.798, respectively. As a validity check for the hard constraint, all 24 defined ratios satisfy their targets at each of the 95%, 90%, and 85% levels, as do all 23 defined ratios among the feasible 80% cases. The cost impact becomes more heterogeneous as the cap is tightened. At the 85% cap, the mean and median cost RDs are 4.53% and 3.93%, respectively. The RD exceeds 2% in 24 of the 27 instances, but exceeds 5% in only four and 10% in only one. The largest RD is 29.82%, whereas the second-largest is 5.71%; hence, the pronounced upper-envelope spike is attributable to a single instance rather than representing the typical response. At the 80% cap, the mean and median cost RDs across the 26 feasible instances are 5.39% and 6.05%, respectively. Cost increases in 23 instances and remains unchanged in three; 20 instances exceed 5%, although none exceeds 10%. Thus, the cost effect becomes considerably more widespread under the strictest cap. A possible explanation for the isolated 85% response is that the cap excludes a particularly cost-effective, high-proxy bid combination, but establishing that mechanism would require a separate bid-level comparison for the affected instance.

Figure E.1 in the Supplementary Material reports how the cap affects the size of the selected-bid portfolio. Relative to the corresponding no-cap solution, the selected-bid count increases, remains unchanged, or decreases in 8/9/10 instances at the 95% cap, 8/8/11 instances at the 90% cap, and 12/9/6 instances at the 85% cap. Thus, an increase is not observed in a majority of instances at any of these three levels. At the 80% cap, however, 17 of the 26 feasible instances select more bids than their corresponding no-cap solutions, five select the same number, and four select fewer. The mean and median paired changes are +1.85 and +1.5 bids, respectively. Hence, expansion of the selected-bid portfolio becomes the majority response only under the strictest tested cap.

Managerially, the shipment-cost-based transport-emission proxy can support internal sustainability target setting, but it should not be interpreted as a physical-emissions inventory or as evidence of regulatory carbon compliance. All 27 instances remain feasible at the 95%, 90%, and 85% cap levels, whereas one of the 27 instances is infeasible at the 80% level. Candidate cap ratios should therefore be tested against the available bid pool, demand, and capacity before implementation, with attention to both feasibility and the operational implications of any increase in the selected-bid count.

Observation 5: Supplier-development qualification preserves feasibility by targeting development to selected supplier–product relationships

This experiment evaluates the supplier-development qualification extension of the auction-based SSOA model. The buyer imposes a

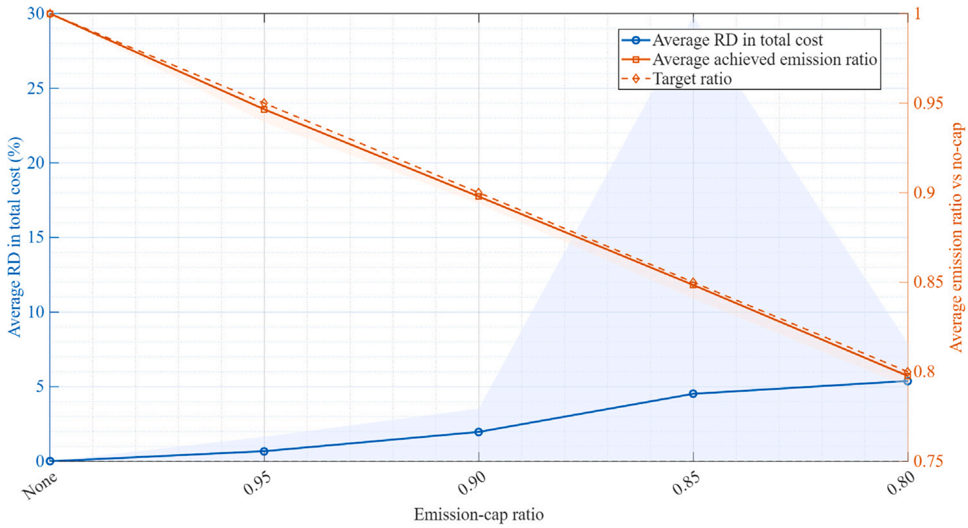

Fig. 7. Mean total-cost RD and mean achieved transport-emission-proxy ratio under the no-cap baseline and each cap. The shaded areas show series-specific min–max ranges across instances for which each measure is defined.

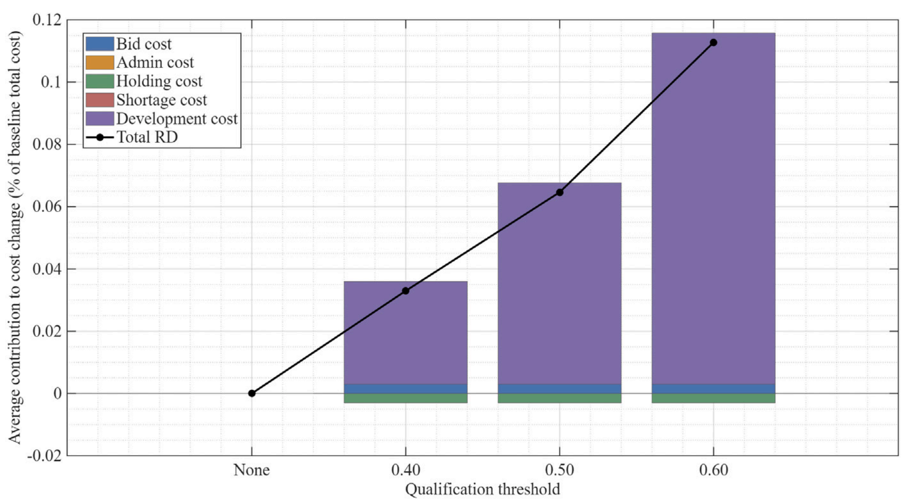

Fig. 8. Average cost-component changes relative to the no-development baseline under supplier-development qualification.

minimum qualification level on each activated supplier–product relationship. A below-threshold relationship can be selected only if the model improves it through U_{ip} and A_{ip}, incurring fixed and effort-based development costs. We set a common threshold for all products, $\tau_p = \tau$, and compare the no-development baseline with $\tau \in \{0.40, 0.50, 0.60\}$. The fixed development cost is set to $G^0_{ip} = 0.05F_{ip}$, and the marginal improvement cost is set to $G^1_{ip} = 0.50F_{ip}$.

Fig. 8 shows that stricter qualification requirements have only mild cost effects. All 27 instances remain feasible at every tested threshold, and total cost increases slightly in every instance. At $\tau = 0.60$, the mean and median cost RDs are 0.113% and 0.112%, respectively, with a range of only 0.099%–0.131%. At every threshold, development cost accounts for at least 90% of the sum of absolute cost-component changes in 26 of the 27 instances, with a median share of 100%. Bid, administrative, holding, and shortage costs remain nearly unchanged on average. This shows that the small total-cost increase is driven mainly by development expenditure rather than by changes in the other cost components. Figure E.2 in the Supplementary Material explains how this targeted response occurs. For each threshold, the left stacked bar reports the supplier–product relationships below the qualification level, while the right stacked bar reports the subset developed in the selected solution. Across the 27 instances, the average numbers of below-threshold and developed relationships are 3 and 3 at $\tau = 0.40$, 7 and 4 at $\tau = 0.50$, and 13 and 6 at $\tau = 0.60$. At $\tau = 0.40$, the two counts are equal in all 27 instances, whereas every instance develops fewer than the number below the threshold at both higher thresholds. The widening gap at the two higher thresholds shows that the model does not develop the entire below-threshold set; development is limited to underqualified relationships used in the selected procurement plan.

The product-level pattern also shows that development pressure is uneven. For Products 1–3, respectively, the average developed counts are 2/0/1 at $\tau = 0.40$, 2/1/1 at $\tau = 0.50$, and 2/2/2 at $\tau = 0.60$. Products 1 and 3 require development in all 27 instances at every threshold, whereas Product 2 requires none at $\tau = 0.40$ but positive development in every instance at $\tau = 0.50$ and 0.60. This suggests that qualification budgets should be allocated selectively across products because development requirements vary across products and thresholds. The selected-bid count also changes only marginally. Relative to the corresponding baseline, it remains unchanged in 25, 24, and 25 of the 27 instances at $\tau = 0.40$, 0.50, and 0.60, respectively. It increases in only two instances at each threshold and decreases once, at $\tau = 0.50$; the median change is zero throughout. Thus, development effort is the principal adjustment, without widespread expansion in the number of selected bids.

Overall, supplier-development qualification allows the buyer to enforce minimum standards without automatically excluding useful underqualified relationships. Managerially, development resources should be directed toward relationships that are both below the required level and used in the selected procurement plan. Stricter qualification targets should therefore be accompanied by an explicit development budget because the required improvement effort rises with the threshold.

Observation 6: A bonus–malus price signal can improve the environmental sustainability profile of the winning bids with a small increase in procurement cost

This experiment evaluates whether a bonus–malus (Section 3.3.3) mechanism can improve the environmental sustainability profile of the selected bid portfolio. We compare the no-policy baseline with three benchmark levels, $\bar{S} \in \{0.40, 0.45, 0.50\}$, and eight bonus–malus rate pairs: four symmetric settings (r, r), where $r \in \{0.05, 0.10, 0.15, 0.20\}$, and four asymmetric settings $(0.05, 0.10)$, $(0.10, 0.05)$, $(0.10, 0.15)$, and $(0.15, 0.10)$.

Figure E.3a in the Supplementary Material reports the average RD in original total cost relative to the no-policy baseline against the average improvement in selected-bid sustainability intensity. The circled points denote globally Pareto-efficient policy settings across all 24 tested benchmark–rate combinations, meaning that no other tested setting has a no-greater average cost increase and a no-smaller average intensity gain, with at least one strict improvement. Among the globally Pareto-efficient settings associated with $\bar{S}$ = 0.40, 0.45, and 0.50, the mean cost premiums are 0.098%, 0.107%, and 0.098%, respectively, and the corresponding mean environmental sustainability-intensity gains are 0.0124, 0.0133, and 0.0123. Across the 20 globally Pareto-efficient settings, the mean setting-specific cost-to-gain ratio is approximately 0.07 percentage points of original total cost per 0.01-unit improvement in selected-bid environmental sustainability intensity. Even the strongest tested rate pair, $(\lambda^+, \lambda^-) = (0.20, 0.20)$, raises mean original total cost by only about 0.23%, while increasing mean selected-bid environmental sustainability intensity by about 0.022. At each benchmark, this rate pair improves intensity in 22 of the 27 instances and leaves it unchanged in five; cost RD exceeds 0.50% in only two instances per benchmark and never exceeds 1%.

Figures E.3b and E.3c in the Supplementary Material show that variation across the tested rate pairs is greater than variation across benchmarks at a fixed rate pair. Weak rates generate limited changes, whereas stronger combined rates generally produce larger environmental sustainability improvements with only small cost increases. The malus contrast examined below is nevertheless benchmark-dependent. For example, at $\bar{S} = 0.45$, increasing the malus rate from 0.05 to 0.15, while keeping the bonus rate fixed at 0.10, raises the environmental sustainability-intensity gain from 0.0065 to 0.0153, while the cost increase rises only from 0.033% to 0.127%. Relative to $\lambda^- = 0.05$, $\lambda^- = 0.15$ improves intensity in 14 of the 27 instances at $\bar{S} = 0.45$, leaves it unchanged in 11, and reduces it in two; the corresponding improved/unchanged/reduced counts at $\bar{S} = 0.50$ are 16/11/0. At $\bar{S} = 0.40$, the average intensity gain instead falls slightly from 0.0113 to 0.0108. Under the strongest symmetric rate pair, $(\lambda^+, \lambda^-) = (0.20, 0.20)$, the average selected-bid count rises modestly from 14.33 in the baseline to 15.85 at every benchmark. At each benchmark, the count increases in 16 of the 27 instances, remains unchanged in six, and decreases in five; the median change is one bid. Thus, the intensity gain is accompanied by only a modest average increase in bid count, although the direction of change varies across instances.

Managerially, the bonus–malus mechanism is a useful intermediate instrument between pure cost minimization and exclusionary qualification screening. It steers awards toward bids with higher environmental sustainability intensity by changing evaluated prices without excluding bids solely because they fall below the benchmark. Because the feasible region remains unchanged, all 27 instances are feasible under all 24 tested policy settings. This makes the mechanism suitable for gradually strengthening sustainability incentives, for example during a transition before stricter environmental sustainability requirements are introduced.

Observation 7: Maximizing the quantity-weighted environmental sustainability value entails a sizeable cost premium

Unlike Observation 6, which examines modest changes in selected-bid sustainability intensity, this experiment traces the frontier between total cost and the quantity-weighted environmental sustainability value SV. Because SV sums supplier–product scores multiplied by selected quantities, it may increase through higher-scoring sourcing, greater selected quantity, or both. An ϵ-constraint procedure minimizes cost while progressively raising the lower bound on SV. Figure E.4 in the Supplementary Material reports the pointwise mean cost–sustainability curve together with the cross-instance min–max envelope. Across the 27 instance-specific frontiers, the mean total cost and environmental sustainability score at the minimum-cost endpoint are 4.39×10^8 and 833.75, respectively; at the maximum-environmental-sustainability endpoint, the corresponding means are 4.41×10^9 and 8595.23. When the percentage changes are calculated within each instance and then summarized across instances, total cost increases by 909.16% on average, with a range of 490.79%–1292.91%, while the environmental sustainability score increases by 934.46% on average, with a range of 507.82%–1361.96%. Thus, moving to the maximum-environmental-sustainability endpoint entails a large cost premium in every tested instance. Within the common-cost region, which covers 44.6% of the plotted grid and is supported by all 27 frontiers, the relative envelope width, calculated at each grid point as $100(\mathrm{SV}_{\max} - \mathrm{SV}_{\min})/|\overline{\mathrm{SV}}|$, has a mean of 4.54%, a median of 4.28%, and a maximum of 11.16%. This indicates limited cross-instance dispersion in sustainability scores when all frontiers are compared at the same absolute cost. Outside this region, the contributor count declines and equals one at each extreme grid point; those extremes therefore should not be interpreted as cross-instance envelopes. For each instance, we calculate the ratio of the percentage sustainability gain to the percentage cost increase over the complete movement from the minimum-cost endpoint to the maximum-environmental-sustainability endpoint. Across the 27 instances, this endpoint ratio has a mean of 1.031, a median of 1.033, and a range of 0.953–1.113. The corresponding instance-specific endpoint cost per additional environmental sustainability-score unit averages 511,024.66, with a median of 512,783.96 and a range of 494,613.07–526,654.12. Thus, over the complete endpoint-to-endpoint movement, the average cost is approximately half a million monetary units per additional score unit; this is not a constant or marginal rate between arbitrary points on a frontier. Managerially, the result suggests that environmental sustainability should be treated as a strategic purchasing decision rather than a cost-free improvement. Each instance-specific Pareto frontier links a proposed environmental sustainability target to the additional procurement cost required to reach it, providing a transparent basis for budget allocation, supplier-engagement planning, and economically defensible target setting.

Observation 8: Sustainability-evaluation uplifts affect optimized procurement outcomes differently across suppliers and decision rules

This experiment changes only the supplier sustainability evaluations; bids, costs, demand, and capacities remain fixed. Because the source case study does not provide a complete criterion-level fuzzy matrix, we construct baseline evaluations for 15 criteria: six economic, five environmental, and four social. These evaluations yield supplier–product scores SS_p^i, which are aggregated into bid weights SW^b using (14). We compare BASE with 15 one-level uplift scenarios, applying an all-criteria, environmental-only, or social-only uplift to each supplier. Targeted ratings move along $VL \rightarrow L \rightarrow G \rightarrow H \rightarrow VH$; VH and

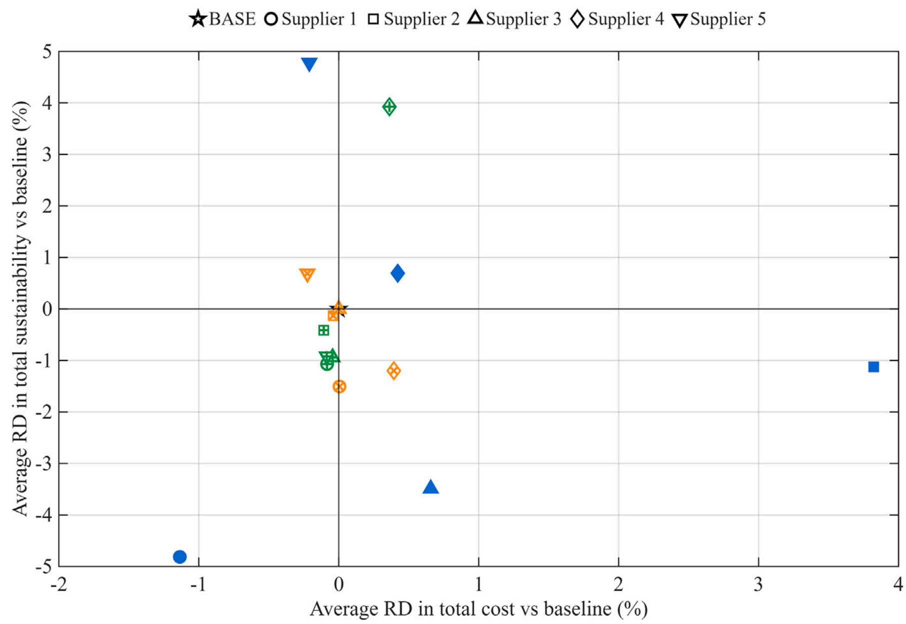

Fig. 9. Mean instance-level RDs in total cost and optimized sustainability value relative to BASE under the sustainability-maximization rule. Sustainability values are evaluated using scenario-specific TOPSIS scores. Marker shape identifies the uplifted supplier; blue filled, green open-plus, and orange open-cross markers denote all-criteria, environmental-only, and social-only uplift, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

all non-targeted ratings remain unchanged. Each scenario is solved under (i) a sustainability-maximization rule that maximizes SV and uses TC only to break ties, and (ii) a balanced rule that minimizes $0.5\frac{TC-TC_{\min}}{TC_{\min}} + 0.5\frac{SV_{\max}-SV}{SV_{\max}}$, with the reference values recalculated for each instance and scenario. All $15 \times 27 \times 2 = 810$ non-baseline runs returned successful solve statuses and finite objective values.

Figs. 9 and E.5 in the Supplementary Material show that changing one supplier's evaluation can alter the optimized procurement outcome even though the bids and the cost, demand, and capacity data remain fixed. Fuzzy TOPSIS recomputes the normalization factors and the positive and negative ideal solutions after each uplift. Consequently, the closeness coefficients of targeted and non-targeted supplier–product pairs may change, after which the optimization model responds to the revised bid weights. Table E.2 illustrates this effect. For Product 1, an all-criteria uplift of Supplier 1 increases its score from 0.400 to 0.643, while changing the non-targeted scores of Supplier 5 from 0.918 to 0.836 and Supplier 4 from 0.529 to 0.492. Under the balanced bi-objective rule, the exact selected-bid vector differs from BASE in 23, 27, 27, 20, and 20 of the 27 instances following the all-criteria uplifts of Suppliers 1–5, respectively. Thus, the average responses are accompanied by frequent, but non-uniform, portfolio changes. Because sustainability RDs compare objective values calculated using different scenario-specific score sets, a negative RD under either rule does not mean that the targeted supplier's assigned ratings deteriorated and does not establish a loss on a common sustainability scale. It means only that the optimized value under the revised score set is lower than the BASE optimized value. These RDs are therefore sensitivity measures and are not directly comparable with the fixed-score frontier in Observation 7. The results below focus on the balanced bi-objective rule, which jointly accounts for normalized cost and revised-score sustainability deviations. In Figure E.5, Supplier 2's all-criteria uplift reduces average cost by 1.21% and increases the optimized sustainability value under the revised scores by 66.88%. This scenario lies in the north-west quadrant in 22 instances and in the north-east quadrant in five, and the winning portfolio changes in all 27 instances. Supplier 3's all-criteria uplift reduces average cost by 1.74% and increases the revised-score sustainability value by 44.09%; it lies in the north-west quadrant in 21 instances, the north-east quadrant in four, and the south-west quadrant in two. Supplier 1 produces a 44.28% average increase in the revised-score sustainability value but an average cost premium of 1.46%; 21 instances lie in the north-east quadrant. The effects are considerably weaker or unfavorable for Suppliers 4 and 5. Supplier 4's all-criteria scenario has a lower revised-score sustainability value in all 27 instances, and the targeted supplier is active after the uplift in only six. Supplier 5 produces a near-neutral average response: its revised-score sustainability value is higher in 14 instances and lower in 13. These findings establish supplier-specific sensitivity under the balanced rule, but they do not identify price, demand coverage, timing, or bundle composition as its cause because those bid-level mechanisms are not analyzed in this experiment. Managerially, this experiment is a screening analysis rather than a direct ranking of supplier-development priorities. Under the balanced rule, the all-criteria uplifts for Suppliers 2 and 3 warrant further analysis, but this ordering differs under the sustainability-maximization rule. Before allocating resources, the buyer should compare the bids and re-evaluate the baseline and uplift-selected portfolios using a common score set to determine whether they provide a genuine sustainability improvement.

7. Conclusions and future developments

This study addresses a gap in the SSOA literature by presenting a comprehensive approach that integrates CAs into the decision process. Unlike prior work that restricts suppliers to a single bid, we generalize the problem to allow suppliers to submit and potentially win multiple, even overlapping, bundles. We contribute a novel optimization model for the WDP that ensures resilience and diversity, and we propose four original heuristic variants inspired by the Kernel Search method. Across 120 synthetic instances, each proposed method performs well in some settings. M1 has the lowest average RD under both multiplicative and weighted strategies, while M4 remains competitive and is strictly best among MM1–MM4 in 33 instances. Smaller bucket sizes generally produce better solutions. The environmental variant of the SSOA problem is also addressed using an ϵ-constraint approach, showing the adaptability of our model to multi-objective settings. Moreover, we consider three sustainability-oriented extensions of the auction-based SSOA model: a transport-emission constraint, supplier-development qualification, and a bonus–malus mechanism. The results highlight that well-designed CA mechanisms can offer significant cost savings compared with traditional SSOA models, particularly when suppliers provide competitive, demand-aligned bids. While auction-based SSOA may result in higher inventory levels, it can reduce or eliminate shortages while taking advantage of bundle discounts. Managers should consider allowing a sufficient number of bids to ensure feasibility and cost-effectiveness. Moreover, while increasing supplier diversity can enhance resilience, it introduces higher administrative and coordination costs, especially when multiple products are constrained. Therefore, strategic sourcing decisions must balance flexibility, cost efficiency, and risk mitigation, tailored to organizational priorities and constraints. Regarding the environmental sustainability-oriented extensions, our managerial analyses show that these mechanisms materially affect procurement decisions, bid attractiveness, and sourcing structures.

For future developments, several avenues could be explored. First, the methods could be extended to stochastic demand settings to capture real-world uncertainty. Second, hybrid metaheuristics that combine Kernel Search with local search or learning-based approaches could further improve performance. Third, incorporating supplier behavior models or learning suppliers' bidding preferences over time would enhance realism. An important limitation of the current study is its static single-shot planning framework, where bids are submitted once at the beginning of the horizon and allocations are determined using the information available at that time. This is suitable when procurement commitments must be fixed in advance, but less suitable when demand, prices, or supplier capacity change over time. A natural extension is a rolling-horizon framework in which the auction and allocation model are periodically re-optimized using updated information. Such an approach could improve robustness under dynamic conditions, but would require repeated bidding, closer supplier coordination, and rules for managing already accepted bids and existing commitments. Future research could compare the cost, stability, and sustainability performance of this dynamic framework against the static single-shot model. Finally, the auction mechanism could be extended to multi-round designs or collaborative procurement contexts.

CRediT authorship contribution statement

Sadeque Hamdan: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Formal analysis, Data curation, Conceptualization. **Chefi Triki:** Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Formal analysis, Conceptualization. **Simona Mancini:** Writing – review & editing, Writing – original draft, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ejor.2026.08.016.

Data availability

The data and results supporting this study are publicly available in Mendeley Data at https://doi.org/10.17632/jg2j65jtg2.

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