



# Kent Academic Repository

Ji, Nan, Xu, De-Zhi, Yan, Xing-Gang, Spurgeon, Sarah and Wang, Lei (2026)  
*A novel dynamic observer based decentralized sliding mode control for nonlinear interconnected systems with uncertainties*. *International Journal of Robust and Nonlinear Control*, 36 (8). pp. 4602-4616. ISSN 1049-8923.

## Downloaded from

<https://kar.kent.ac.uk/113923/> The University of Kent's Academic Repository KAR

## The version of record is available from

<https://doi.org/10.1002/rnc.70438>

## This document version

Author's Accepted Manuscript

## DOI for this version

## Licence for this version

CC BY (Attribution)

## Additional information

## Versions of research works

### Versions of Record

If this version is the version of record, it is the same as the published version available on the publisher's web site. Cite as the published version.

### Author Accepted Manuscripts

If this document is identified as the Author Accepted Manuscript it is the version after peer review but before type setting, copy editing or publisher branding. Cite as Surname, Initial. (Year) 'Title of article'. To be published in **Title of Journal**, Volume and issue numbers [peer-reviewed accepted version]. Available at: DOI or URL (Accessed: date).

## Enquiries

If you have questions about this document contact [ResearchSupport@kent.ac.uk](mailto:ResearchSupport@kent.ac.uk). Please include the URL of the record in KAR. If you believe that your, or a third party's rights have been compromised through this document please see our [Take Down policy](https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies) (available from <https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies>).

## RESEARCH ARTICLE

# A novel dynamic observer based decentralised sliding mode control for nonlinear interconnected systems with uncertainties

Nan Ji<sup>\*1</sup> | De-Zhi Xu<sup>2</sup> | Xing-Gang Yan<sup>3</sup> | Sarah Spurgeon<sup>4</sup> | Lei Wang<sup>1</sup>

<sup>1</sup>School of Automation, Wuxi University, Wuxi, China

<sup>2</sup>School of Electrical Engineering, Southeast University, Nanjing, China

<sup>3</sup>School of Engineering, Mathematics and Physics, University of Kent, Canterbury, United Kingdom

<sup>4</sup>Department of Electronic and Electrical Engineering, University College London, London, United Kingdom

## Correspondence

Nan Ji, School of Automation, Wuxi University, Wuxi, China. Email: jinan1990@foxmail.com

## Abstract

In this paper, a variable structure observer based decentralised sliding mode control is proposed to control a class of nonlinear interconnected systems with matched and mismatched uncertainties. A dynamic observer is initially proposed to estimate the states of the interconnected system. Then a composite sliding surface is designed, and the stability of the sliding motion is analysed based on the regular form of the interconnected system. Using the pre-designed observer, a dynamic decentralised output feedback sliding mode control is presented to drive the interconnected system to the designed sliding surface in finite time, and then the sliding motion is maintained thereafter. The interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties, which are nonlinear and take a more general form, are relaxed. Finally, a numerical practical example is presented to illustrate the advantages of this strategy.

## KEYWORDS:

Interconnected systems, decentralised control, dynamic observer, sliding mode control.

## 1 | INTRODUCTION

From significant advances in science and technology, nonlinear large-scale interconnected systems have been developed such as smart grids, road monitoring systems, aircraft and intelligent manipulators<sup>1</sup>. This class of complex systems consists of many subsystems with different functions and structures. However, it is difficult to analyse and control the interconnected systems because of the existence of nonlinearity, uncertainties and interconnections between subsystems. Moreover, physical systems are affected by uncertainty including modelling errors, parameter variation and external disturbances. The study of nonlinear interconnected systems with uncertainties is thus both challenging and significant.

Centralised control and decentralised control are the two main strategies that may be applied for the control of such interconnected systems. Decentralised control only uses local information while centralised control can employ all the information available across the whole system for control design. Decentralised control does not need data transfer from the other subsystems and can reduce the complexity of calculations in practical applications as well as reduce requirements for data transfer. The structure of decentralised controllers is usually simpler than that of centralised controllers. In general, decentralised control is straightforward to implement in complex and harsh industrial environments. The advantages of decentralised control have resulted in a great deal of research effort that has produced numerous results<sup>2,3,4,5,6</sup>.

In practical systems, the full state of the dynamic system may sometimes not be available for the controller, and in this case, state feedback cannot be directly applied. An observer can be used to estimate the states of the dynamic system. Observers, such

<sup>\*</sup>The original version of this paper was appeared in the Nan Ji's PhD thesis in University of Kent, United Kingdom, September 2023. The current version is its modification and extension in which more detailed information, and new discussion/results have been added.

as unknown input observers, deadbeat observers, and reduced-order observers are useful tools for the control designer when full system state information is not available<sup>7</sup>. An observer has been designed for nonlinear systems with delayed disturbances to facilitate sliding mode control design<sup>8</sup>. A unified  $H_\infty$  dynamic observer has been developed for linear systems with unknown inputs and disturbances<sup>9</sup>. Andrea LA proposed a variable structure observer-based control system to guarantee asymptotic convergence in the presence of both matched and mismatched uncertainties<sup>10</sup>. Xu QM designed a linear observer for a multi-link flexible manipulator to overcome the difficulty of parameter uncertainties, unknown nonlinearities, and exogenous disturbance inputs<sup>11</sup>. Gong LG adopted an extended state observer and adaptive dynamic programming approach to solve the problem of spacecraft output feedback attitude control<sup>12</sup>. Zare K presented four sliding mode observers based on Takagi-Sugeno fuzzy modelling of multi-input multi-output nonlinear systems with non-differentiable operating points<sup>13</sup>. Xu C proposed a strategy based on observer and dynamic event-triggered for the discrete-time multi-agent systems<sup>14</sup>. Tutsoy O presented a robust reduced-order tau observer with the adaptive fault estimator for the unmanned air vehicles<sup>15</sup>. Currently, only limited achievements regarding observers have been applied in decentralised control for interconnected systems.

Sliding mode control (SMC) is known to have numerous advantages, for example, fast response and insensitivity to variation of parameters and disturbances in systems<sup>16,17</sup>. A static output feedback SMC has been designed for time delay systems where the designed controller was independent of the time delay<sup>8</sup>. Kao YG focused on designing an  $H_\infty$  based SMC for neutral-type stochastic systems with Markovian switching parameters and nonlinear uncertainties<sup>18</sup>. Zhu JM used the SMC method for control of uncertain multi-input multi-output linear Markovian jump systems<sup>19</sup>. Incremona GP presented a second-order SMC to improve the closed-loop performance of nonlinear affine systems with quantized uncertainty<sup>20</sup>. Su XJ proposed an SMC for discrete-time switched systems via an event-triggered strategy<sup>21</sup>. Wang XR adopted an incremental nonsingular terminal SMC for multi-input multi-output nonlinear systems with uncertainties, disturbances, and sudden actuator faults<sup>22</sup>. Gao XW developed a chattering-free model free adaptive SMC for a gas collection process with data dropout<sup>23</sup>. Dong H proposed a practical terminal SMC for servo systems<sup>24</sup>. Wang B designed an adaptive fault-tolerant based SMC for a quadrotor unmanned aerial vehicle with actuator faults and model uncertainties<sup>25</sup>. Estrada MA adopted a passive fault-tolerant control based on sliding-mode-based Lyapunov redesign<sup>26</sup>. Gomez MA presented SMC based robust asymptotic stabilization for the time delay systems<sup>27</sup>. It should be mentioned that all of the work above relates to centralised systems.

In recent years, decentralised SMC for interconnected systems has attracted the attention of numerous researchers. An observer based fuzzy neural SMC scheme was designed for interconnected unknown chaotic systems, although the work did not explicitly consider matched uncertainty<sup>28,29</sup>. Global decentralised SMC for interconnected systems using only output information was considered, although again neither matched or mismatched uncertainties were considered<sup>30</sup>. A decentralised control was designed to stabilise a class of fully nonlinear time delay interconnected systems, where strong conditions were imposed on the considered systems and interconnections due to the nature of the static output feedback employed<sup>31</sup>. A new decentralised SMC strategy for complex interconnected systems subject to non-smooth nonlinearities was presented<sup>32</sup>. This strategy requires that the interconnections have a linear structure. A reduced order observer based integral SMC was proposed for interconnected descriptor systems where the mismatched uncertainty was not considered, and the interconnected system needed to have a specific structure<sup>33</sup>. A decentralised SMC for nonlinear interconnected systems was designed using static state feedback, and thus all the states of the interconnected system need to be known for use by the controller<sup>34,35</sup>. Wang T proposed a sliding-mode surface-based decentralised event-triggered control for partially unknown interconnected nonlinear systems via reinforcement learning to achieve great results, though reinforcement learning adjusted the system models via trial and error, it greatly increased computational burden, and the unknown part of this system needed to satisfy the harsh structural characteristics<sup>36</sup>. Xu H presented a finite-time decentralised SMC for interconnected electrical power systems based on a GA (genetic algorithm) design method<sup>37</sup>. The number of subsystems in the interconnected system was limited due to algorithm structure, and the initial values of GA was hard to select during the design process. Although good results in the area of decentralised SMC have been achieved, many previous strategies related to this field have many limitations and defects, the results for nonlinear interconnected systems with unknown states together with matched and mismatched uncertainties are lacking.

In this paper, a dynamic observer based decentralised SMC strategy is specifically proposed to stabilise a class of nonlinear interconnected systems with matched and mismatched uncertainties. A dynamic observer is designed to estimate the states of the interconnected system. A coordinate transformation is used to transform each isolated subsystem into regular form which is convenient for the design of SMC. Using the pre-designed observer, a composite sliding surface is designed, and sufficient conditions are developed to ensure that the sliding motion is uniformly asymptotically stable using the Lyapunov theory. A dynamic decentralised output feedback SMC law is designed to drive the system to the sliding surface in finite time and maintain a sliding motion on it thereafter. The impact of matched and mismatched uncertainties is dealt with during the reachability

analysis under appropriate conditions. The presented results also seek to reduce conservatism. The bounds on the uncertainties are nonlinear and take a more general form to relax the constraints of previous results. Finally, the results of practical simulation are used to demonstrate the effectiveness of the presented approach. To put it simply, the main contributions of this paper are summarized as follows:

- 1) This dynamic observer based decentralised SMC strategy is applied to complex nonlinear interconnected systems in the presence of both matched and mismatched uncertainties, when the system states are unknown.
- 2) The full-order observer is adopted, which can reduce the conservatism of the algorithm compared to some results using the reduced-order observer. Besides that, the uncertainties of nonlinear interconnected systems considered in this paper are nonlinear and have general structures, which indicates that the presented strategy can be effectively used in generalised nonlinear interconnected systems with uncertainties.
- 3) The proposed strategy adopts decentralised control combined with SMC instead of traditional centralised control to eliminate the negative impact of uncertainties, this approach simplifies the complexity of calculations and improve robustness of interconnected systems.

The remainder of this paper is organized as follows. Section ‘System description and preliminaries’ describes the mathematical model of nonlinear interconnected systems with uncertainties and gives some assumptions. Likewise, section ‘Dynamic observer design’ describes the detailed steps to design a novel dynamic observer for nonlinear interconnected systems with uncertainties. Then, section ‘Sliding motion analysis and control design’ gives the design process of decentralised SMC. After that simulation results for lateral flight control system demonstrate the effectiveness of the presented strategy in section ‘Simulation results’. Finally, the section ‘Conclusion’ summarises the whole paper.

## 2 | SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the system

$$\dot{x}_i = A_i x_i + B_i(u_i + G_i \varphi_i(x_i, t)) + E_i f_i(x_i, t) + \sum_{j=1}^n H_{ij}(x_j, t) \quad (1)$$

$$y_i = C_i x_i, \quad i = 1, 2, \dots, n \quad (2)$$

where  $x_i \in \Omega_i \subset R^{n_i}$  ( $\Omega_i$  is a neighbourhood of the origin),  $u_i \in R^{m_i}$  and  $y_i \in R^{q_i}$  with  $m_i \leq q_i < n_i$  are the state, input and output of the  $i$ -th subsystem, respectively.  $A_i$ ,  $B_i$ ,  $C_i$ ,  $E_i$  and  $G_i$  are known constant matrices with appropriate dimensions, where  $B_i$  is of full row rank, and  $C_i$  is of full column rank.  $E_i f_i(x_i, t)$  and  $G_i \varphi_i(x_i, t)$  are mismatched uncertainty and matched uncertainty, respectively, where  $f_i(x_i, t)$  and  $\varphi_i(x_i, t)$  are unknown functions with appropriate dimensions, and the matrices  $G_i$  and  $E_i$  are used to describe the structure of the matched and mismatched uncertainties, respectively.  $\sum_{j=1}^n H_{ij}(x_j, t)$  represents the known nonlinear interconnections. It is assumed that all the nonlinear terms involved in this paper are continuous in the considered domain to guarantee the existence of the solutions of system (1)-(2).

*Assumption 1:* The unknown functions  $f_i(x_i, t)$  and  $\varphi_i(x_i, t)$  satisfy

$$\|f_i(x_i, t)\| \leq \epsilon_i(x_i, t) \quad (3)$$

$$\|\varphi_i(x_i, t)\| \leq \vartheta_i(x_i, t) \quad (4)$$

where known  $\epsilon_i(x_i, t)$  and  $\vartheta_i(x_i, t)$  are Lipschitz with respect to  $x_i$  in the domain  $\Omega_i \subset R^{n_i}$  and uniformly about  $t$ . The known nonlinear interconnection  $H_{ij}(\cdot)$  is Lipschitz with respect to  $x_j$  in the domain  $\Omega_j \subset R^{n_j}$  and uniformly about  $t$  for  $i, j = 1, 2, \dots, n$ .

From Assumption 1, it follows that for any  $x_i, \hat{x}_i, x_j$  and  $\hat{x}_j$  in the considered domain,

$$\|\epsilon_i(x_i, t) - \epsilon_i(\hat{x}_i, t)\| \leq \mathcal{L}_{\epsilon_i}(t) \|x_i - \hat{x}_i\| \quad (5)$$

$$\|\vartheta_i(x_i, t) - \vartheta_i(\hat{x}_i, t)\| \leq \mathcal{L}_{\vartheta_i}(t) \|x_i - \hat{x}_i\| \quad (6)$$

$$\|H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)\| \leq \mathcal{L}_{H_{ij}}(t) \|x_j - \hat{x}_j\| \quad (7)$$

where  $\mathcal{L}_{\epsilon_i}(t)$ ,  $\mathcal{L}_{\vartheta_i}(t)$  and  $\mathcal{L}_{H_{ij}}(t)$  are nonnegative functions in  $R^+ = \{t | t \geq 0\}$ .

*Remark 1:* The matrices  $E_i$  and  $G_i$  are employed to describe the structural characteristics of the nonlinear mismatched and matched uncertainties, respectively. The functions  $f_i(x_i, t)$  and  $\varphi_i(x_i, t)$  are unknown uncertainties with known bounds which will be used later in the control design. The specific values of uncertainties may be difficultly achieved, but based on practical experience and data statistics, the approximate bounds of uncertainties can generally be obtained. This situation is very consistent with the practical application of nonlinear interconnected systems with uncertainties.

*Assumption 2:* The matrix pairs  $(A_i, B_i)$  and  $(A_i, C_i)$  are controllable and observable, respectively, for  $i = 1, 2, \dots, n$ .

Assumption 2 is a basic limitation for the triple  $(A_i, B_i, C_i)$ . A dynamic observer is to be designed in the next section. Then, a decentralised SMC strategy will be proposed.

### 3 | DYNAMIC OBSERVER DESIGN

In view of the observability of the pair  $(A_i, C_i)$  in Assumption 2, there exists a matrix  $L_i$  such that  $(A_i - L_i C_i)$  is stable and thus for any  $Q_i > 0$  the following Lyapunov equation has a unique solution  $P_i > 0$ ,

$$(A_i - L_i C_i)^T P_i + P_i (A_i - L_i C_i) = -Q_i \quad (8)$$

*Assumption 3:* There exist known matrices  $F_i, J_i$  and  $P_i$  such that  $[E_i \ B_i G_i]^T P_i = [F_i \ J_i] C_i$  holds, where  $P_i$  satisfies (8), and the matrices  $B_i, C_i, E_i$  and  $G_i$  are given in equations (1) and (2).

This class of the structure of  $F_i$  and  $J_i$  exists in systems that possess measurable scheduling parameters, decomposable input-output channels or affine parameter dependency. Typical examples include mechanical systems with separable dynamics, power-electronic converters with piecewise-linear operating modes, and aircraft or robotic systems whose dynamics can be expressed in a polytopic form. For such systems, the matrices  $F_i$  and  $J_i$  can be obtained via standard factorization or through solving linear matrix equalities derived from the system representation.

Consider the following dynamic system

$$\dot{\hat{x}}_i = A_i \hat{x}_i + B_i(u_i + \Upsilon_i(\hat{x}_i, y_i, t)) + L_i(y_i - C_i \hat{x}_i) + \Psi_i(\hat{x}_i, y_i, t) + \sum_{j=1}^n H_{ij}(\hat{x}_j, t) \quad (9)$$

where  $\hat{x}_i \in R^{n_i}$ ,  $L_i \in R^{n_i \times q_i}$ , and

$$\Psi_i(\hat{x}_i, y_i, t) = \begin{cases} E_i \frac{F_i(y_i - C_i \hat{x}_i)}{\|F_i(y_i - C_i \hat{x}_i)\|} e_i(\hat{x}_i, t), & F_i(y_i - C_i \hat{x}_i) \neq 0 \\ 0, & F_i(y_i - C_i \hat{x}_i) = 0 \end{cases} \quad (10a)$$

$$F_i(y_i - C_i \hat{x}_i) = 0 \quad (10b)$$

$$\Upsilon_i(\hat{x}_i, y_i, t) = \begin{cases} G_i \frac{J_i(y_i - C_i \hat{x}_i)}{\|J_i(y_i - C_i \hat{x}_i)\|} \vartheta_i(\hat{x}_i, t), & J_i(y_i - C_i \hat{x}_i) \neq 0 \\ 0, & J_i(y_i - C_i \hat{x}_i) = 0 \end{cases} \quad (11a)$$

$$J_i(y_i - C_i \hat{x}_i) = 0 \quad (11b)$$

where  $E_i, F_i, G_i$  and  $J_i$  satisfy Assumption 3 and the known functions  $e_i(\cdot)$  and  $\vartheta_i(\cdot)$  are given in Assumption 1.

Define the state estimation error by  $e_i = x_i - \hat{x}_i$ . It follows from (1) and (9) that the error dynamic is given by

$$\begin{aligned} \dot{e}_i = & (A_i - L_i C_i) e_i + (E_i f_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) + B_i(G_i \varphi_i(x_i, t) - \Upsilon_i(\hat{x}_i, y_i, t)) \\ & + \sum_{j=1}^n (H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \end{aligned} \quad (12)$$

The following results are presented to underpin subsequent analysis.

*Lemma 1:* Suppose that Assumptions 1-3 are satisfied,  $\bar{\lambda}(P_i)$  is the maximum eigenvalue of the matrix  $P_i$ . Then, the following results hold:

- (i)  $e_i^T P_i (E_i f_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) \leq \mathcal{L}_{e_i}(t) \|F_i C_i\| \|e_i\|^2$
- (ii)  $e_i^T P_i B_i (G_i \varphi_i(x_i, t) - \Upsilon_i(\hat{x}_i, y_i, t)) \leq \mathcal{L}_{\vartheta_i}(t) \|J_i C_i\| \|e_i\|^2$
- (iii)  $e_i^T P_i \sum_{j=1}^n (H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \leq \sum_{j=1}^n \bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t) \|e_j\| \|e_i\|$

where  $\mathcal{L}_{e_i}(t)$ ,  $\mathcal{L}_{\vartheta_i}(t)$  and  $\mathcal{L}_{H_{ij}}(t)$  satisfy (5), (6) and (7), respectively.

*Proof.* From Assumptions 1 and 3, combining with equations (10a) and (10b), if  $F_i C_i e_i \neq 0$ ,

$$\begin{aligned} & e_i^T P_i (E_i f_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) \\ &= (F_i C_i e_i)^T f_i(x_i, t) - \frac{(F_i C_i e_i)^T F_i C_i e_i}{\|F_i C_i e_i\|} e_i(\hat{x}_i, t) \\ &\leq \|F_i C_i e_i\| e_i(x_i, t) - \|F_i C_i e_i\| e_i(\hat{x}_i, t) \\ &\leq \mathcal{L}_{e_i}(t) \|F_i C_i\| \|e_i\|^2 \end{aligned} \quad (13)$$

Otherwise if  $F_i C_i e_i = 0$ , then from  $E_i^T P_i = F_i C_i$  in Assumption 3, it follows that

$$e_i^T P_i E_i = (E_i^T P_i e_i)^T = (F_i C_i e_i)^T = 0 \quad (14)$$

Therefore, from the analysis above,

$$e_i^T P_i (E_i f_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) = 0 \leq \mathcal{L}_{e_i}(t) \|F_i C_i\| \|e_i\|^2 \quad (15)$$

Hence conclusion (i) follows.

According to (11a) and (11b), conclusion (ii) can be obtained following a similar analysis as for conclusion (i).

Based on (7) and the fact that the upper-bound for the norm of matrix  $P_i$  is the maximum eigenvalue of  $P_i^{38}$ , it follows that

$$\begin{aligned} & e_i^T P_i \sum_{j=1}^n (H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \\ &\leq \|e_i\| \bar{\lambda}(P_i) \sum_{j=1}^n (\mathcal{L}_{H_{ij}}(t) \|x_j - \hat{x}_j\|) \\ &= \sum_{j=1}^n \bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t) \|e_j\| \|e_i\| \end{aligned} \quad (16)$$

Hence conclusion (iii) follows.  $\square$

*Theorem 1:* Suppose that Assumptions 1-3 are satisfied,  $\underline{\lambda}(Q_i)$  is the minimum eigenvalue of the matrix  $Q_i$ ,  $\bar{\lambda}(P_i)$  is the maximum eigenvalue of the matrix  $P_i$ . Then, there exist positive constants  $\alpha_1$  and  $\alpha_2$  such that

$$\|e_i\| \leq \alpha_2 \exp\{-\alpha_1 t\} \quad (17)$$

if  $(M + M^T)$  is positive definite with  $M = (m_{ij})_{n \times n}$  defined by

$$m_{ij} = \begin{cases} \underline{\lambda}(Q_i) - 2\mathcal{L}_{e_i}(t) \|F_i C_i\| - 2\mathcal{L}_{\theta_i}(t) \|J_i C_i\| - 2\bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t), & i = j \\ -2\bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t), & i \neq j \end{cases} \quad (18a)$$

$$(18b)$$

where  $Q_i$  satisfies equation (8) and  $F_i$  and  $J_i$  are defined in Assumption 3.

*Proof.* For the system (12), consider a Lyapunov function candidate  $V_1 = \sum_{i=1}^n e_i^T P_i e_i$ . Then, the time derivative of  $V_1$  along the trajectories of system (12) is given by

$$\begin{aligned} \dot{V}_1 = & \sum_{i=1}^n \left( -e_i^T Q_i e_i + 2e_i^T P_i (E_i f_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) + 2e_i^T P_i B_i (G_i \varphi_i(x_i, t) \right. \\ & \left. - \Upsilon_i(\hat{x}_i, y_i, t)) + 2e_i^T P_i \sum_{j=1}^n (H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \right) \end{aligned}$$

where equation (8) is used above. From conclusions (i)-(iii) of Lemma 1, it follows that

$$\begin{aligned}
\dot{V}_1 &\leq \sum_{i=1}^n (-\underline{\lambda}(Q_i) \|e_i\|^2 + 2\mathcal{L}_{e_i}(t) \|F_i C_i\| \|e_i\|^2 \\
&\quad + 2\mathcal{L}_{g_i}(t) \|J_i C_i\| \|e_i\|^2 + 2 \sum_{j=1}^n \bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t) \|e_i\| \|e_j\|) \\
&= - \sum_{i=1}^n (\underline{\lambda}(Q_i) - 2\mathcal{L}_{e_i}(t) \|F_i C_i\| - 2\mathcal{L}_{g_i}(t) \|J_i C_i\| - 2\bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t)) \|e_i\|^2 \\
&\quad + \sum_{i=1}^n \sum_{j=1, j \neq i}^n 2\bar{\lambda}(P_i) \mathcal{L}_{H_{ij}}(t) \|e_i\| \|e_j\| \\
&= - \frac{1}{2} [\|e_1\| \|e_2\| \dots \|e_n\|] (M + M^T) [\|e_1\| \|e_2\| \dots \|e_n\|]^T \\
&\leq - \frac{1}{2} \underline{\lambda}(M + M^T) \sum_{i=1}^n \|e_i\|^2
\end{aligned} \tag{19}$$

Given that

$$\sum_{i=1}^n e_i^T P_i e_i \leq \max_i \{\bar{\lambda}(P_i)\} \sum_{i=1}^n \|e_i\|^2 \tag{20}$$

then according to equations (19) and (20), it follows that

$$\begin{aligned}
\dot{V}_1 &\leq - \frac{\underline{\lambda}(M + M^T)}{2\max_i \{\bar{\lambda}(P_i)\}} \sum_{i=1}^n e_i^T P_i e_i \\
&= - \frac{\underline{\lambda}(M + M^T)}{2\max_i \{\bar{\lambda}(P_i)\}} V_1 \\
&= - \alpha_1 V_1
\end{aligned}$$

where

$$\alpha_1 \equiv: \frac{\underline{\lambda}(M + M^T)}{2\max_i \{\bar{\lambda}(P_i)\}} > 0 \tag{21}$$

Based on the analysis above, it follows that

$$V_1(t) \leq \bar{V}_1(0) \exp\{-\alpha_1 t\}$$

Since  $\min_i \{\underline{\lambda}(P_i)\} \|e_i\|^2 \leq e_i^T P_i e_i \leq \sum_{i=1}^n e_i^T P_i e_i = V_1$ , the conclusion  $\|e_i\| \leq \alpha_2 \exp\{-\alpha_1 t\}$  follows by letting

$$\alpha_2 > \sqrt{\bar{V}_1(0) / \min_i \{\underline{\lambda}(P_i)\}} \tag{22}$$

Hence, the result is obtained.  $\square$

*Remark 2:* Theorem 1 shows that the dynamic observer in (9) is an exponential observer for the system (1)-(2). This can be seen from the inequality (17). The proof is also constructive and provides a method to determine the values of  $\alpha_1$  and  $\alpha_2$ . The inequality (17) will be used in the reachability analysis.

## 4 | SLIDING MOTION ANALYSIS AND CONTROL DESIGN

In this section, the dynamic observer (9) is used to form a dynamic feedback control to stabilise the system (1)-(2). The stability of the sliding motion is analysed first. Then a controller is designed to satisfy the reachability condition.

### 4.1 | System transformation

Consider the system (1)-(2). Since  $B_i$  is full row rank, there exist a coordinate transformation  $T_i^{16,39}$  such that

$$T_i B_i = \begin{bmatrix} 0 \\ \bar{B}_i \end{bmatrix} \tag{23}$$

where  $\tilde{B}_i \in R^{m_i \times m_i}$  is a nonsingular matrix for  $i = 1, 2, \dots, n$ .

For systems (1), (2) and (9), consider a transformation

$$\begin{bmatrix} z_i \\ \hat{z}_i \end{bmatrix} := \begin{bmatrix} T_i & 0 \\ 0 & T_i \end{bmatrix} \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix} \quad (24)$$

where  $T_i$  is the nonsingular matrix satisfying (23). Using (24), let  $\tilde{e}_i = z_i - \hat{z}_i$  and  $e_i = x_i - \hat{x}_i$ . It follows that

$$\tilde{e}_i = T_i x_i - T_i \hat{x}_i = T_i e_i \quad (25)$$

In the new coordinates, the systems (1), (2) and (12) can be rewritten as

$$\dot{z}_{i1} = \tilde{A}_{i1} z_{i1} + \tilde{A}_{i2} z_{i2} + \tilde{f}_{i1}(z_{i1}, z_{i2}, t) + \sum_{j=1}^n \tilde{H}_{ij1}(z_{j1}, z_{j2}, t) \quad (26)$$

$$\begin{aligned} \dot{\tilde{e}}_i &= (\tilde{A}_i - \tilde{L}_i \tilde{C}_i) \tilde{e}_i + (\tilde{f}_i(z_{i1}, z_{i2}, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \\ &\quad + \tilde{B}_i(\tilde{\varphi}_i(z_{i1}, z_{i2}, t) - \tilde{\Upsilon}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^n (\tilde{H}_{ij}(z_{j1}, z_{j2}, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \end{aligned} \quad (27)$$

$$\dot{z}_{i2} = \tilde{A}_{i3} z_{i1} + \tilde{A}_{i4} z_{i2} + \tilde{B}_i(u_i + \tilde{\varphi}_i(z_{i1}, z_{i2}, t)) + \tilde{f}_{i2}(z_{i1}, z_{i2}, t) + \sum_{j=1}^n \tilde{H}_{ij2}(z_{j1}, z_{j2}, t) \quad (28)$$

$$y_i = \tilde{C}_i z_i \quad (29)$$

where  $z_{i1} \in \Omega_{z_{i1}} \subset R^{n_i - m_i}$ ,  $z_{i2} \in \Omega_{z_{i2}} \subset R^{m_i}$ ,  $z_i = \text{col}(z_{i1}, z_{i2}) \in \Omega_{T_i} \subset R^{n_i}$ , and

$$\Omega_{T_i} = \Omega_{z_{i1}} \times \Omega_{z_{i2}} = \{(z_{i1}; z_{i2}) | (z_{i1}; z_{i2}) = T_i x_i, x_i \in \Omega_i \subset R^{n_i}\}$$

$\hat{z}_{i1} \in \Omega_{\hat{z}_{i1}} \subset R^{n_i - m_i}$ ,  $\hat{z}_{i2} \in \Omega_{\hat{z}_{i2}} \subset R^{m_i}$ ,  $\hat{z}_i = \text{col}(\hat{z}_{i1}, \hat{z}_{i2}) \in \Omega_{\hat{T}_i} \subset R^{n_i}$ , and

$$\Omega_{\hat{T}_i} = \Omega_{\hat{z}_{i1}} \times \Omega_{\hat{z}_{i2}} = \{(\hat{z}_{i1}; \hat{z}_{i2}) | (\hat{z}_{i1}; \hat{z}_{i2}) = T_i \hat{x}_i, \hat{x}_i \in R^{n_i}\}$$

and

$$\tilde{C}_i = C_i T_i^{-1} \quad (30)$$

$$\tilde{L}_i = T_i L_i \quad (31)$$

$$\tilde{\Psi}_i(\hat{z}_i, y_i, t) = T_i \Psi_i(T_i^{-1} \hat{z}_i, y_i, t) \quad (32)$$

$$\tilde{\varphi}_i(z_i, t) = G_i \varphi_i(T_i^{-1} z_i, t) \quad (33)$$

$$\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) = \Upsilon_i(T_i^{-1} \hat{z}_i, y_i, t) \quad (34)$$

$$\tilde{E}_i = T_i E_i \quad (35)$$

$$\tilde{f}_i(z_i, t) = f_i(T_i^{-1} z_i, t) \quad (36)$$

$$\tilde{A}_i = \begin{bmatrix} \tilde{A}_{i1} & \tilde{A}_{i2} \\ \tilde{A}_{i3} & \tilde{A}_{i4} \end{bmatrix} = T_i A_i T_i^{-1} \quad (37)$$

$$\tilde{f}_i(z_i, t) = \begin{bmatrix} \tilde{f}_{i1}(\cdot) \\ \tilde{f}_{i2}(\cdot) \end{bmatrix} = \tilde{E}_i \tilde{f}_i(z_i, t) \quad (38)$$

$$\tilde{H}_{ij}(z_j, t) = \begin{bmatrix} \tilde{H}_{ij1}(\cdot) \\ \tilde{H}_{ij2}(\cdot) \end{bmatrix} = T_i H_{ij}(T_j^{-1} z_j, t) \quad (39)$$

where  $\tilde{A}_{i1} \in R^{(n_i - m_i) \times (n_i - m_i)}$ ,  $\tilde{A}_{i2} \in R^{(n_i - m_i) \times m_i}$ ,  $\tilde{A}_{i3} \in R^{m_i \times (n_i - m_i)}$ ,  $\tilde{A}_{i4} \in R^{m_i \times m_i}$ ,  $\tilde{f}_{i1}(\cdot) \in R^{n_i - m_i}$ ,  $\tilde{f}_{i2}(\cdot) \in R^{m_i}$ ,  $\tilde{H}_{ij1}(\cdot) \in R^{n_i - m_i}$  and  $\tilde{H}_{ij2}(\cdot) \in R^{m_i}$ .

In the subsequent section, the interconnected system (26)-(29) will be considered from the perspective of sliding mode control.

## 4.2 | Stability of sliding motion

From Assumption 2, the matrix pair  $(A_i, B_i)$  is controllable. Therefore, from the results<sup>40</sup>, the matrix pair  $(\tilde{A}_{i1}, \tilde{A}_{i2})$  is controllable. Then, there exist a matrix  $\tilde{K}_i$  such that  $(\tilde{A}_{i1} - \tilde{A}_{i2} \tilde{K}_i)$  is stable, and thus for any  $\bar{Q}_i > 0$ , the following Lyapunov equation

has a unique solution  $\bar{P}_i > 0$ ,

$$(\tilde{A}_{i1} - \tilde{A}_{i2}\tilde{K}_i)^T \bar{P}_i + \bar{P}_i(\tilde{A}_{i1} - \tilde{A}_{i2}\tilde{K}_i) = -\bar{Q}_i \quad (40)$$

Based on the specific structure of system (26)-(29), the switching function for the system (26)-(29) can be selected as

$$\sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2}, \quad i = 1, 2, \dots, n \quad (41)$$

where  $S_{i1} \in R^{m_i \times (n_i - m_i)}$  and  $S_{i2} \in R^{m_i \times m_i}$  are design parameters with  $S_{i2}$  being nonsingular. The matrix  $S_{i1}$  is selected as

$$S_{i1} = S_{i2}\tilde{K}_i \quad (42)$$

where  $S_{i2}$  is a nonsingular matrix to be designed later, and  $\tilde{K}_i$  satisfies (40). Then, from (40) and (42),

$$(\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1})^T \bar{P}_i + \bar{P}_i(\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1}) = -\bar{Q}_i \quad (43)$$

Then, the composite sliding function for the interconnected system (26)-(29) is given as

$$\sigma(\hat{z}) = \text{col}(\sigma_1(\hat{z}_1), \sigma_2(\hat{z}_2), \dots, \sigma_n(\hat{z}_n)) \quad (44)$$

where  $\sigma_i(\cdot)$  for  $i = 1, 2, 3, \dots, n$  are defined in (41). Thus the composite sliding surface may be written as

$$\{\text{col}(\hat{z}_1, \hat{z}_2, \dots, \hat{z}_n) | \sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2} = 0, \quad i = 1, 2, \dots, n\} \quad (45)$$

Let  $\tilde{e}_{i1} = z_{i1} - \hat{z}_{i1}$  and  $\tilde{e}_{i2} = z_{i2} - \hat{z}_{i2}$ , where  $\tilde{e}_{i1} \in R^{n_i - m_i}$ ,  $\tilde{e}_{i2} \in R^{m_i}$ ,  $\tilde{e}_i = \text{col}(\tilde{e}_{i1}, \tilde{e}_{i2}) \in R^{n_i}$ . The sliding surface can be rewritten as

$$\sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2} = S_{i1}z_{i1} + S_{i2}z_{i2} - S_{i1}\tilde{e}_{i1} - S_{i2}\tilde{e}_{i2}, \quad i = 1, 2, \dots, n \quad (46)$$

When the system is limited to the sliding surface (45) it follows that  $\sigma_i(\hat{z}_i) = 0$ . Thus, from (46),

$$z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}, \quad i = 1, 2, \dots, n \quad (47)$$

Therefore, the sliding mode dynamics of the system (26)-(29) corresponding to the sliding surface (45) can be described by

$$\begin{aligned} \dot{z}_{i1} &= (\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1})z_{i1} + \tilde{A}_{i2}S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{A}_{i2}\tilde{e}_{i2} \\ &\quad + \tilde{f}_{i1s}(z_{i1}, \tilde{e}_i, t) + \sum_{j=1}^n \tilde{H}_{ij1s}(z_{j1}, \tilde{e}_j, t) \end{aligned} \quad (48)$$

$$\begin{aligned} \dot{\tilde{e}}_i &= (\tilde{A}_i - \tilde{L}_i\tilde{C}_i)\tilde{e}_i + (\tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \tilde{B}_i(\tilde{\varphi}_{is}(z_{i1}, \tilde{e}_i, t) \\ &\quad - \tilde{Y}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^n (\tilde{H}_{ijs}(z_{j1}, \tilde{e}_j, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \end{aligned} \quad (49)$$

where

$$\tilde{f}_{i1s}(\cdot) = \tilde{f}_{i1}(z_{i1}, z_{i2}, t)|_{z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}} \quad (50)$$

$$\tilde{H}_{ij1s}(\cdot) = \tilde{H}_{ij1}(z_{j1}, z_{j2}, t)|_{z_{j2} = -S_{j2}^{-1}S_{j1}z_{j1} + S_{j2}^{-1}S_{j1}\tilde{e}_{j1} + \tilde{e}_{j2}} \quad (51)$$

$$\tilde{\varphi}_{is}(\cdot) = \tilde{\varphi}_i(z_{i1}, z_{i2}, t)|_{z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}} \quad (52)$$

$$\tilde{f}_{is}(\cdot) = \tilde{f}_i(z_{i1}, z_{i2}, t)|_{z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}} \quad (53)$$

$$\tilde{H}_{ijs}(\cdot) = \tilde{H}_{ij}(z_{j1}, z_{j2}, t)|_{z_{j2} = -S_{j2}^{-1}S_{j1}z_{j1} + S_{j2}^{-1}S_{j1}\tilde{e}_{j1} + \tilde{e}_{j2}} \quad (54)$$

From Assumption 2 and equations (30), (31), (35) and (37), it is straightforward to obtain

$$\begin{aligned} &(T_i^{-1}\tilde{A}_iT_i - T_i^{-1}\tilde{L}_i\tilde{C}_iT_i)^T P_i + P_i(T_i^{-1}\tilde{A}_iT_i - T_i^{-1}\tilde{L}_i\tilde{C}_iT_i) = -Q_i \\ &\tilde{A}_i^T(T_i^{-1})^T P_i T_i^{-1} - \tilde{C}_i^T \tilde{L}_i^T (T_i^{-1})^T P_i T_i^{-1} + (T_i^{-1})^T P_i T_i^{-1} \tilde{A}_i - (T_i^{-1})^T P_i T_i^{-1} \tilde{L}_i \tilde{C}_i \\ &= -(T_i^T)^{-1} Q_i T_i^{-1} \end{aligned} \quad (55)$$

Let

$$(T_i^{-1})^T P_i T_i^{-1} = \tilde{P}_i \quad (56)$$

$$(T_i^T)^{-1} Q_i T_i^{-1} = \tilde{Q}_i \quad (57)$$

Then, equations (55)-(57) imply that for any  $\tilde{Q}_i > 0$ , the Lyapunov equation

$$(\tilde{A}_i - \tilde{L}_i\tilde{C}_i)^T \tilde{P}_i + \tilde{P}_i(\tilde{A}_i - \tilde{L}_i\tilde{C}_i) = -\tilde{Q}_i \quad (58)$$

has a unique solution  $\tilde{P}_i > 0$ .

*Assumption 4:* It is assumed that

$$\|\tilde{f}_{i1s}(z_{i1}, \tilde{e}_i, t)\| \leq \chi_{i1}(z_{i1}, \tilde{e}_i, t) \|z_{i1}\| + \chi_{i2}(z_{i1}, \tilde{e}_i, t) \|\tilde{e}_i\| \quad (59)$$

$$\sum_{j=1}^n \|\tilde{H}_{ij1s}(z_{j1}, \tilde{e}_j, t)\| \leq \sum_{j=1}^n (\phi_{ij1}(z_{j1}, \tilde{e}_j, t) \|z_{j1}\| + \phi_{ij2}(z_{j1}, \tilde{e}_j, t) \|\tilde{e}_j\|) \quad (60)$$

where  $\chi_{i1}(\cdot)$ ,  $\chi_{i2}(\cdot)$ ,  $\phi_{ij1}(\cdot)$  and  $\phi_{ij2}(\cdot)$  are known functions.

*Theorem 2:* Suppose Assumptions 1-4 are satisfied. Then, the sliding motion dynamics (48)-(49) are asymptotically stable if  $W + W^T$  is positive definite with  $W$  defined by

$$W = \begin{bmatrix} w_{ij}^a & w_{ij}^b \\ w_{ij}^b & w_{ij}^c \end{bmatrix} \quad (61)$$

$$w_{ij}^a = \begin{cases} \underline{\lambda}(\bar{Q}_i) - 2\bar{\lambda}(\bar{P}_i)\chi_{i1}(\cdot) - 2\bar{\lambda}(\bar{P}_i)\phi_{ij1}(\cdot), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\phi_{ij1}(\cdot), & i \neq j \end{cases} \quad (62a)$$

$$w_{ij}^b = \begin{cases} -(2\|\bar{P}_i\bar{D}_i\| + 2\bar{\lambda}(\bar{P}_i)\chi_{i2}(\cdot) + 2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot)), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot), & i \neq j \end{cases} \quad (62b)$$

$$w_{ij}^b = \begin{cases} -(2\|\bar{P}_i\bar{D}_i\| + 2\bar{\lambda}(\bar{P}_i)\chi_{i2}(\cdot) + 2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot)), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot), & i \neq j \end{cases} \quad (63a)$$

$$w_{ij}^b = \begin{cases} -(2\|\bar{P}_i\bar{D}_i\| + 2\bar{\lambda}(\bar{P}_i)\chi_{i2}(\cdot) + 2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot)), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\phi_{ij2}(\cdot), & i \neq j \end{cases} \quad (63b)$$

$$w_{ij}^c = \begin{cases} \underline{\lambda}(\bar{Q}_i) - 2\mathcal{L}_{\tilde{e}_i}(t)\|F_i\tilde{C}_i\| - 2\mathcal{L}_{\tilde{g}_i}(t)\|J_i\tilde{C}_i\| - 2\bar{\lambda}(\bar{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), & i \neq j \end{cases} \quad (64a)$$

$$w_{ij}^c = \begin{cases} \underline{\lambda}(\bar{Q}_i) - 2\mathcal{L}_{\tilde{e}_i}(t)\|F_i\tilde{C}_i\| - 2\mathcal{L}_{\tilde{g}_i}(t)\|J_i\tilde{C}_i\| - 2\bar{\lambda}(\bar{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), & i = j \\ -2\bar{\lambda}(\bar{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), & i \neq j \end{cases} \quad (64b)$$

where  $\tilde{e}_i(z_i, t) = \epsilon_i(T_i^{-1}z_i, t)$ ,  $\tilde{g}_i(z_i, t) = \vartheta_i(T_i^{-1}z_i, t)$  and  $\bar{D}_i = [\tilde{A}_{i2}S_{i2}^{-1}S_{i1}, \tilde{A}_{i2}]$ .  $\bar{P}_i$  and  $\bar{Q}_i$  are defined in (43).  $\tilde{P}_i$  and  $\tilde{Q}_i$  satisfy (58).  $\underline{\lambda}(\bar{Q}_i)$  and  $\underline{\lambda}(\tilde{Q}_i)$  are the minimum eigenvalues of the matrices  $\bar{Q}_i$  and  $\tilde{Q}_i$ , respectively.  $\bar{\lambda}(\bar{P}_i)$  and  $\bar{\lambda}(\tilde{P}_i)$  are the maximum eigenvalues of the matrices  $\bar{P}_i$  and  $\tilde{P}_i$ , respectively.  $\chi_{i1}(\cdot)$ ,  $\chi_{i2}(\cdot)$ ,  $\phi_{ij1}(\cdot)$  and  $\phi_{ij2}(\cdot)$  are given in Assumption 4.  $\mathcal{L}_{\tilde{e}_i}(t) = \mathcal{L}_{\epsilon_i}(t)\|T_i^{-1}\|$ ,  $\mathcal{L}_{\tilde{g}_i}(t) = \mathcal{L}_{\vartheta_i}(t)\|T_i^{-1}\|$  and  $\mathcal{L}_{\tilde{H}_{ij}}(t) = \|T_i\|\mathcal{L}_{H_{ij}}(t)\|T_i^{-1}\|$  are nonnegative continuous Lipschitz functions, where  $\mathcal{L}_{\epsilon_i}(t)$ ,  $\mathcal{L}_{\vartheta_i}(t)$ , and  $\mathcal{L}_{H_{ij}}(t)$  are given in (5)-(7), respectively.

*Proof.* For the system (48)-(49), consider the Lyapunov function candidate  $V_2 = \sum_{i=1}^n (z_{i1}^T \bar{P}_i z_{i1} + \tilde{e}_i^T \tilde{P}_i \tilde{e}_i)$ . Then, the time derivative of  $V_2$  along the trajectories of the dynamic system (48)-(49) is given as

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^n (\dot{z}_{i1}^T \bar{P}_i z_{i1} + z_{i1}^T \bar{P}_i \dot{z}_{i1} + \dot{\tilde{e}}_i^T \tilde{P}_i \tilde{e}_i + \tilde{e}_i^T \tilde{P}_i \dot{\tilde{e}}_i) \\ &= \sum_{i=1}^n \left( -z_{i1}^T \bar{Q}_i z_{i1} + 2z_{i1}^T \bar{P}_i (\tilde{A}_{i2}S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{A}_{i2}\tilde{e}_{i2} + \tilde{f}_{i1s}(\cdot)) + \sum_{j=1}^n \tilde{H}_{ij1s}(\cdot) \right. \\ &\quad \left. - \tilde{e}_i^T \tilde{Q}_i \tilde{e}_i + 2\tilde{e}_i^T \tilde{P}_i (\varpi_1 + \varpi_2 + \varpi_3) \right) \end{aligned} \quad (65)$$

where

$$\varpi_1 = \tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t) \quad (66)$$

$$\varpi_2 = \tilde{B}_i(\tilde{\varphi}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{Y}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \quad (67)$$

$$\varpi_3 = \sum_{j=1}^n (\tilde{H}_{ij1s}(z_{j1}, \tilde{e}_j, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \quad (68)$$

From  $\tilde{e}_i = \text{col}[\tilde{e}_{i1}, \tilde{e}_{i2}]$

$$z_{i1}^T \bar{P}_i (\tilde{A}_{i2}S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{A}_{i2}\tilde{e}_{i2}) = z_{i1}^T \bar{P}_i \bar{D}_i \tilde{e}_i \quad (69)$$

where  $\bar{D}_i = [\tilde{A}_{i2}S_{i2}^{-1}S_{i1}, \tilde{A}_{i2}]$ . Then,

$$z_{i1}^T \bar{P}_i (\tilde{A}_{i2}S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{A}_{i2}\tilde{e}_{i2}) \leq \|\bar{P}_i \bar{D}_i\| \|z_{i1}\| \|\tilde{e}_i\| \quad (70)$$

According to equation (59), it follows that

$$z_{i1}^T \bar{P}_i \tilde{f}_{i1s}(\cdot) \leq \bar{\lambda}(\bar{P}_i)\chi_{i1}(\cdot)\|z_{i1}\|^2 + \bar{\lambda}(\bar{P}_i)\chi_{i2}(\cdot)\|\tilde{e}_i\|\|z_{i1}\| \quad (71)$$

From (60),

$$z_{i1}^T \bar{P}_i \sum_{j=1}^n \tilde{H}_{ij1s}(\cdot) \leq \|z_{i1}\| \bar{\lambda}(\bar{P}_i) \sum_{j=1}^n \varrho_{ij1}(\cdot) \|z_{j1}\| + \|z_{i1}\| \bar{\lambda}(\bar{P}_i) \sum_{j=1}^n \varrho_{ij2}(\cdot) \|\tilde{e}_j\| \quad (72)$$

where  $\tilde{H}_{ij1s}(\cdot)$  is the first  $n_i - m_i$  rows of  $\tilde{H}_{ij}(z_j, t)|_{z_{j2}=-S_{j2}^{-1}S_{j1}z_{j1}+S_{j2}^{-1}S_{j1}\tilde{e}_{j1}+\tilde{e}_{j2}}$ .

Based on equations (10a), (10b) and (32),

$$\tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t) = \tilde{\Psi}_i(\hat{z}_i, y_i, t)$$

$$\tilde{\Psi}_i(\hat{z}_i, y_i, t) = \begin{cases} \tilde{E}_i \frac{F_i \tilde{C}_i \tilde{e}_i}{\|F_i \tilde{C}_i \tilde{e}_i\|} \tilde{e}_i(\hat{z}_i, t), & F_i \tilde{C}_i \tilde{e}_i \neq 0 \\ 0, & F_i \tilde{C}_i \tilde{e}_i = 0 \end{cases} \quad (73a)$$

$$F_i \tilde{C}_i \tilde{e}_i = 0 \quad (73b)$$

where  $\tilde{e}_i(\hat{z}_i, t) = \epsilon_i(T_i^{-1} \hat{z}_i, t)$ .

From equations (11a), (11b) and (34),

$$\tilde{Y}_i(\hat{z}_i, y_i, t) = \begin{cases} G_i \frac{J_i \tilde{C}_i \tilde{e}_i}{\|J_i \tilde{C}_i \tilde{e}_i\|} \tilde{\vartheta}_i(\hat{z}_i, t), & J_i \tilde{C}_i \tilde{e}_i \neq 0 \\ 0, & J_i \tilde{C}_i \tilde{e}_i = 0 \end{cases} \quad (74a)$$

$$J_i \tilde{C}_i \tilde{e}_i = 0 \quad (74b)$$

where  $\tilde{\vartheta}_i(\hat{z}_i, t) = \vartheta_i(T_i^{-1} \hat{z}_i, t)$ .

According to Assumption 3, and equations (30), (35) and (56),

$$\begin{aligned} (T_i^{-1} \tilde{E}_i)^T T_i^T \tilde{P}_i T_i &= F_i \tilde{C}_i T_i \\ \tilde{E}_i^T \tilde{P}_i &= F_i \tilde{C}_i \end{aligned} \quad (75)$$

By the similar reasoning as (75),

$$G_i^T \tilde{B}_i^T \tilde{P}_i = J_i \tilde{C}_i \quad (76)$$

From (73a) and (75), if  $F_i \tilde{C}_i \tilde{e}_i \neq 0$ ,

$$\begin{aligned} \tilde{e}_i^T \tilde{P}_i \varpi_1 &= \tilde{e}_i^T \tilde{P}_i \left( \tilde{E}_i \tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{E}_i \frac{F_i \tilde{C}_i \tilde{e}_i}{\|F_i \tilde{C}_i \tilde{e}_i\|} \tilde{e}_i(\hat{z}_i, t) \right) \\ &= (F_i \tilde{C}_i \tilde{e}_i)^T \tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \frac{(F_i \tilde{C}_i \tilde{e}_i)^T F_i \tilde{C}_i \tilde{e}_i}{\|F_i \tilde{C}_i \tilde{e}_i\|} \tilde{e}_i(\hat{z}_i, t) \\ &\leq \|F_i \tilde{C}_i \tilde{e}_i\| \|\tilde{f}_{is}(z_{i1}, \tilde{e}_i, t)\| - \|F_i \tilde{C}_i \tilde{e}_i\| \|\tilde{e}_i(\hat{z}_i, t)\| \\ &\leq \|F_i \tilde{C}_i \tilde{e}_i\| \|\tilde{f}_{is}(z_{i1}, \tilde{e}_i, t)\| - \|F_i \tilde{C}_i \tilde{e}_i\| \|\tilde{e}_i(\hat{z}_i, t)\| \\ &\leq \|F_i \tilde{C}_i \tilde{e}_i\| \|\tilde{e}_i(z_{i1}, t) - \tilde{e}_i(\hat{z}_i, t)\| \\ &\leq \mathcal{L}_{\tilde{e}_i}(t) \|F_i \tilde{C}_i\| \|\tilde{e}_i\|^2 \end{aligned} \quad (77)$$

where  $\tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) = \tilde{f}_i(z_{i1}, z_{i2}, t)|_{z_{i2}=-S_{i2}^{-1}S_{i1}z_{i1}+S_{i2}^{-1}S_{i1}\tilde{e}_{i1}+\tilde{e}_{i2}}$ . If  $F_i \tilde{C}_i \tilde{e}_i = 0$ ,

$$\tilde{e}_i^T \tilde{P}_i \varpi_1 = (F_i \tilde{C}_i \tilde{e}_i)^T \tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) = 0 \leq \mathcal{L}_{\tilde{e}_i}(t) \|F_i \tilde{C}_i\| \|\tilde{e}_i\|^2 \quad (78)$$

So, from (77) and (78),

$$\tilde{e}_i^T \tilde{P}_i \varpi_1 \leq \mathcal{L}_{\tilde{e}_i}(t) \|F_i \tilde{C}_i\| \|\tilde{e}_i\|^2 \quad (79)$$

By using similar reasoning as used to obtain (79), it follows from (74a), (74b) and (76) that

$$\tilde{e}_i^T \tilde{P}_i \varpi_2 \leq \mathcal{L}_{\tilde{\vartheta}_i}(t) \|J_i \tilde{C}_i\| \|\tilde{e}_i\|^2 \quad (80)$$

According to (7), (39) and (68), in a similar way as used to obtain (80),

$$\tilde{e}_i^T \tilde{P}_i \varpi_3 \leq \sum_{j=1}^n \bar{\lambda}(\tilde{P}_i) \mathcal{L}_{\tilde{H}_{ij}}(t) \|\tilde{e}_i\| \|\tilde{e}_j\| \quad (81)$$

Substituting (70)-(72) and (79)-(81) into (65),  $\dot{V}_2$  can be described as

$$\begin{aligned}
\dot{V}_2 \leq & \sum_{i=1}^n \left( -\underline{\lambda}(\bar{Q}_i) \|z_{i1}\|^2 + 2\|\bar{P}_i \bar{D}_i\| \|z_{i1}\| \|\tilde{e}_i\| + 2\bar{\lambda}(\bar{P}_i) \chi_{i1}(\cdot) \|z_{i1}\|^2 \right. \\
& + 2\bar{\lambda}(\bar{P}_i) \chi_{i2}(\cdot) \|\tilde{e}_i\| \|z_{i1}\| + 2\|z_{i1}\| \bar{\lambda}(\bar{P}_i) \sum_{j=1}^n \phi_{ij1}(\cdot) \|z_{j1}\| \\
& + 2\|z_{i1}\| \bar{\lambda}(\bar{P}_i) \sum_{j=1}^n \phi_{ij2}(\cdot) \|\tilde{e}_j\| - \underline{\lambda}(\tilde{Q}_i) \|e_i\|^2 + 2\mathcal{L}_{\tilde{e}_i}(t) \|F_i \tilde{C}_i\| \|\tilde{e}_i\|^2 \\
& \left. + 2\mathcal{L}_{\tilde{\theta}_i}(t) \|J_i \tilde{C}_i\| \|\tilde{e}_i\|^2 + 2 \sum_{j=1}^n \bar{\lambda}(\bar{P}_i) \mathcal{L}_{\tilde{H}_{ij}}(t) \|\tilde{e}_i\| \|\tilde{e}_j\| \right) \\
= & - \sum_{i=1}^n \left( \underline{\lambda}(\bar{Q}_i) - 2\bar{\lambda}(\bar{P}_i) \chi_{i1}(\cdot) - 2\bar{\lambda}(\bar{P}_i) \phi_{ij1}(\cdot) \right) \|z_{i1}\|^2 \\
& - \sum_{i=1}^n \left( \underline{\lambda}(\tilde{Q}_i) - 2\mathcal{L}_{\tilde{e}_i}(t) \|F_i \tilde{C}_i\| - 2\mathcal{L}_{\tilde{\theta}_i}(t) \|J_i \tilde{C}_i\| - 2\bar{\lambda}(\tilde{P}_i) \mathcal{L}_{\tilde{H}_{ij}}(t) \right) \|\tilde{e}_i\|^2 \\
& + \sum_{i=1}^n \left( 2\|\bar{P}_i \bar{D}_i\| + 2\bar{\lambda}(\bar{P}_i) \chi_{i2}(\cdot) + 2\bar{\lambda}(\bar{P}_i) \phi_{ij2}(\cdot) \right) \|\tilde{e}_i\| \|z_{i1}\| \\
& + \sum_{i=1}^n \sum_{j=1, j \neq i}^n 2\bar{\lambda}(\bar{P}_i) \phi_{ij1}(\cdot) \|z_{i1}\| \|z_{j1}\| + \sum_{i=1}^n \sum_{j=1, j \neq i}^n 2\bar{\lambda}(\bar{P}_i) \phi_{ij2}(\cdot) \|z_{i1}\| \|\tilde{e}_j\| \\
& + \sum_{i=1}^n \sum_{j=1, j \neq i}^n 2\bar{\lambda}(\tilde{P}_i) \mathcal{L}_{\tilde{H}_{ij}}(t) \|\tilde{e}_i\| \|\tilde{e}_j\| \\
= & - \frac{1}{2} [\|z_{11}\| \|z_{21}\| \cdots \|z_{n1}\| \|\tilde{e}_1\| \|\tilde{e}_2\| \cdots \|\tilde{e}_n\|] (W + W^T) \\
& \cdot [\|z_{11}\| \|z_{21}\| \cdots \|z_{n1}\| \|\tilde{e}_1\| \|\tilde{e}_2\| \cdots \|\tilde{e}_n\|]^T
\end{aligned} \tag{82}$$

Hence, the conclusion follows from  $W + W^T > 0$ .  $\square$

Theorem 2 shows that the sliding motion of system (26)-(29) relating to the sliding surface (45) is asymptotically stable. In the next section, a sliding mode controller will be designed to drive the system to the sliding surface (45).

*Remark 3:* Theorem 2 provides a set of sufficient conditions under which the sliding mode is asymptotically stable. The function matrix  $W$  in Theorem 2 only depends on the known coefficient matrices and related parameters of the bounds of uncertainties and interconnections. The coefficient matrices are determined by the given system. The condition that  $W + W^T > 0$  with  $W$  defined in (61)-(64b) implies the limitation to the uncertainties and interconnections. Theorem 1 also follows the similar situation.

### 4.3 | Reachability analysis

The objective now is to design a control law such that the reachability condition<sup>1</sup>

$$\sum_{i=1}^n \frac{\sigma_i^T(\hat{z}_i) \dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} < 0 \tag{83}$$

is satisfied where  $\sigma_i(\hat{z}_i)$  is the composite sliding function given in (45). If condition (83) is satisfied by some controllers, the system (26)-(29) will be driven to the sliding surface and maintain a sliding motion on it thereafter.

Based on the estimated state  $\hat{z}_i = \text{col}(\hat{z}_{i1}, \hat{z}_{i2})$  and  $\tilde{e}_i = \text{col}(\tilde{e}_{i1}, \tilde{e}_{i2})$  given by (26)-(29), the following decentralised sliding mode control is proposed

$$\begin{aligned} u_i = & -(S_{i2}\tilde{B}_i)^{-1} \left( (S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})\hat{z}_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})\hat{z}_{i2} \right. \\ & + \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} (\|S_{i1}\|\|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\| + \|S_{i2}\|\|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\| + \sum_{j=1}^n \|S_{j1}\|\|\tilde{H}_{ji1}(\hat{z}_i, t)\| \\ & \left. + \|S_{i2}\tilde{B}_i\|\|\tilde{Y}_i(\hat{z}_i, y_i, t)\| + \sum_{j=1}^n \|S_{j2}\|\|\tilde{H}_{ji2}(\hat{z}_i, t)\| + k_i(\hat{z}_i, t) \right) \end{aligned} \quad (84)$$

where  $k_i(\hat{z}_i, t)$  is the control gain to be determined later.  $\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)$  denotes the first  $n_i - m_i$  rows of  $\tilde{\Psi}_i(\hat{z}_i, y_i, t)$ , and  $\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)$  represents the last  $m_i$  rows of  $\tilde{\Psi}_i(\hat{z}_i, y_i, t)$ .

*Theorem 3:* Suppose that Assumptions 1-4 are satisfied. The control law in (84) drives the system (26)-(29) to the sliding surface (45) and maintains a sliding motion on it thereafter if

$$\begin{aligned} k_i(\hat{z}_i, t) > & \|T_i\|\alpha_2 \exp\{-\alpha_1 t\} \left( \|S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3}\| + \|S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4}\| \right. \\ & \left. + (\|S_{i1}\| + \|S_{i2}\|)\|\tilde{A}_i - \tilde{L}_i\tilde{C}_i\| \right) \end{aligned} \quad (85)$$

where  $\alpha_1$  and  $\alpha_2$  are given in (21) and (22), respectively.

*Proof.* From the definition of  $\sigma_i(\hat{z}_i)$  in (41) and system (26)-(28), it follows that

$$\begin{aligned} & \sum_{i=1}^n \frac{\sigma_i^T(\hat{z}_i)\sigma_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\ = & \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i1}(\tilde{A}_{i1}z_{i1} + \tilde{A}_{i2}z_{i2} + \tilde{f}_{i1}(\cdot) + \sum_{j=1}^n \tilde{H}_{ij1}(\cdot)) \right. \\ & + S_{i2}(\tilde{A}_{i3}z_{i1} + \tilde{A}_{i4}z_{i2} + \tilde{B}_i(u_i + \tilde{\varphi}_i(\cdot)) + \tilde{f}_{i2}(\cdot) + \sum_{j=1}^n \tilde{H}_{ij2}(\cdot)) \\ & - S_{i1}(\tilde{D}_{i1}\tilde{e}_i + (\tilde{f}_{i1}(\cdot) - \tilde{\Psi}_{i1}(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t))) \\ & + \sum_{j=1}^n (\tilde{H}_{ij1}(\cdot) - \tilde{H}_{ij1}(\hat{z}_{j1}, \hat{z}_{j2}, t))) \\ & - S_{i2}(\tilde{D}_{i2}\tilde{e}_i + (\tilde{f}_{i2}(\cdot) - \tilde{\Psi}_{i2}(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t))) \\ & \left. + \tilde{B}_i(\tilde{\varphi}_i(\cdot) - \tilde{Y}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^n (\tilde{H}_{ij2}(\cdot) - \tilde{H}_{ij2}(\hat{z}_{j1}, \hat{z}_{j2}, t))) \right) \\ = & \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( (S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})z_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})z_{i2} \right. \\ & + S_{i1}\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) + S_{i1}\sum_{j=1}^n \tilde{H}_{ij1}(\hat{z}_j, t) + S_{i2}\tilde{B}_i u_i + S_{i2}\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t) \\ & \left. + S_{i2}\tilde{B}_i\tilde{Y}_i(\hat{z}_i, y_i, t) + S_{i2}\sum_{j=1}^n \tilde{H}_{ij2}(\hat{z}_j, t) - (S_{i2}\tilde{D}_{i2} + S_{i1}\tilde{D}_{i1})\tilde{e}_i \right) \end{aligned} \quad (86)$$

where  $D_{i1}$  represents the first  $n_i - m_i$  rows of  $(\tilde{A}_i - \tilde{L}_i \tilde{C}_i)$ .  $D_{i2}$  denotes the last  $m_i$  rows of  $(\tilde{A}_i - \tilde{L}_i \tilde{C}_i)$ . Substituting the control law (84) into (86),

$$\begin{aligned}
& \sum_{i=1}^n \frac{\sigma_i^T(\hat{z}_i) \dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\
&= \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( (S_{i1} \tilde{A}_{i1} + S_{i2} \tilde{A}_{i3}) \tilde{e}_{i1} + (S_{i1} \tilde{A}_{i2} + S_{i2} \tilde{A}_{i4}) \tilde{e}_{i2} \right. \\
&\quad + (S_{i1} \tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i1}\| \|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\|) \\
&\quad + (S_{i1} \sum_{j=1}^n \tilde{H}_{ij1}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \sum_{j=1}^n \|S_{j1}\| \|\tilde{H}_{ji1}(\hat{z}_i, t)\|) \\
&\quad + (S_{i2} \tilde{\Psi}_{i2}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i2}\| \|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\|) \\
&\quad + (S_{i2} \tilde{B}_i \tilde{Y}_i(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i2} \tilde{B}_i\| \|\tilde{Y}_i(\hat{z}_i, y_i, t)\|) \\
&\quad + (S_{i2} \sum_{j=1}^n \tilde{H}_{ij2}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \sum_{j=1}^n \|S_{j2}\| \|\tilde{H}_{ji2}(\hat{z}_i, t)\|) \\
&\quad \left. - (S_{i1} D_{i1} + S_{i2} D_{i2}) \tilde{e}_i - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} k_i(\hat{z}_i, t) \right) \tag{87}
\end{aligned}$$

where  $\tilde{e}_{i1} = z_{i1} - \hat{z}_{i1}$  and  $\tilde{e}_{i2} = z_{i2} - \hat{z}_{i2}$ .

From equation (24)

$$\|T_i^{-1}(z_i - \hat{z}_i)\| = \|T_i^{-1} \tilde{e}_i\| = \|e_i\| \tag{88}$$

Then, from equations (17) and (88),

$$\|\tilde{e}_i\| \leq \|T_i\| \|T_i^{-1} \tilde{e}_i\| \leq \|T_i\| \alpha_2 \exp\{-\alpha_1 t\} \tag{89}$$

In the equation (87),

$$\begin{aligned}
& \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} ((S_{i1} \tilde{A}_{i1} + S_{i2} \tilde{A}_{i3}) \tilde{e}_{i1} + (S_{i1} \tilde{A}_{i2} + S_{i2} \tilde{A}_{i4}) \tilde{e}_{i2}) \\
& \leq \sum_{i=1}^n (\|S_{i1} \tilde{A}_{i1} + S_{i2} \tilde{A}_{i3}\| + \|S_{i1} \tilde{A}_{i2} + S_{i2} \tilde{A}_{i4}\|) \|\tilde{e}_i\| \tag{90}
\end{aligned}$$

From equations (73a) and (73b),

$$\begin{aligned}
& \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i1} \tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i1}\| \|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\| \right) \\
&= \sum_{i=1}^n \left( \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} S_{i1} \tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \|S_{i1}\| \|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\| \right) \leq 0 \tag{91}
\end{aligned}$$

Based on equation (39) and from the fact that

$$\sum_{i=1}^n \sum_{j=1}^n S_{i1} \tilde{H}_{ij1}(\hat{z}_j, t) = \sum_{i=1}^n \sum_{j=1}^n S_{j1} \tilde{H}_{ji1}(\hat{z}_i, t), \tag{92}$$

it follows that

$$\begin{aligned}
& \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i1} \sum_{j=1}^n \tilde{H}_{ij1}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \sum_{j=1}^n \|S_{j1}\| \|\tilde{H}_{ji1}(\hat{z}_i, t)\| \right) \\
&= \sum_{i=1}^n \left( \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} S_{i1} \sum_{j=1}^n \tilde{H}_{ij1}(\hat{z}_j, t) - \sum_{j=1}^n \|S_{j1}\| \|\tilde{H}_{ji1}(\hat{z}_i, t)\| \right) \leq 0 \tag{93}
\end{aligned}$$

Taking into consideration (73a) and (73b),

$$\begin{aligned} & \sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i2} \tilde{\Psi}_{i2}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i2}\| \|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\| \right) \\ &= \sum_{i=1}^n \left( \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} S_{i2} \tilde{\Psi}_{i2}(\hat{z}_i, y_i, t) - \|S_{i2}\| \|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\| \right) \leq 0 \end{aligned} \quad (94)$$

Following similar reasoning as used to obtain (94), it follows that

$$\sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i2} \tilde{B}_i \tilde{Y}_i(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \|S_{i2} \tilde{B}_i\| \|\tilde{Y}_i(\hat{z}_i, y_i, t)\| \right) \leq 0 \quad (95)$$

$$\sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} \left( S_{i2} \sum_{j=1}^n \tilde{H}_{ij2}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|} \sum_{j=1}^n \|S_{j2} \tilde{H}_{ji2}(\hat{z}_j, t)\| \right) \leq 0 \quad (96)$$

$$\sum_{i=1}^n \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|} (S_{i1} D_{i1} + S_{i2} D_{i2}) \tilde{e}_i \leq (\|S_{i1}\| + \|S_{i2}\|) \|\tilde{A}_i - \tilde{L}_i \tilde{C}_i\| \|\tilde{e}_i\| \quad (97)$$

Based on the equations (90)-(91) and (93)-(97),

$$\begin{aligned} & \sum_{i=1}^n \frac{\sigma_i^T(\hat{z}_i) \dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\ & \leq \sum_{i=1}^n \left( (\|S_{i1} \tilde{A}_{i1} + S_{i2} \tilde{A}_{i3}\| + \|S_{i1} \tilde{A}_{i2} + S_{i2} \tilde{A}_{i4}\|) \|\tilde{e}_i\| \right. \\ & \quad \left. + (\|S_{i1}\| + \|S_{i2}\|) \|\tilde{A}_i - \tilde{L}_i \tilde{C}_i\| \|\tilde{e}_i\| - k_i(\hat{z}_i, t) \right) \\ & = \sum_{i=1}^n \left( (\|S_{i1} \tilde{A}_{i1} + S_{i2} \tilde{A}_{i3}\| + \|S_{i1} \tilde{A}_{i2} + S_{i2} \tilde{A}_{i4}\| \right. \\ & \quad \left. + (\|S_{i1}\| + \|S_{i2}\|) \|\tilde{A}_i - \tilde{L}_i \tilde{C}_i\|) \|\tilde{e}_i\| - k_i(\hat{z}_i, t) \right) \end{aligned} \quad (98)$$

Hence, from equations (85), (89) and (98),

$$\sum_{i=1}^n \frac{\sigma_i^T(\hat{z}_i) \dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} < 0 \quad (99)$$

Thus, the composite reachability condition is satisfied.  $\square$

*Remark 4:* In this section, a dynamic decentralised output feedback SMC is presented to drive an interconnected system to the designed sliding surface in finite time using a pre-designed observer. A sliding motion is then maintained thereafter. The interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds on the uncertainties, which are nonlinear and take more general forms, are relaxed. This strategy improves the robustness of the whole system and manages the impact of matched and mismatched uncertainties by appropriate reachability analysis. It should be noted that the effect of the mismatched uncertainty can not be rejected by designing controller. Actually the limitation to the mismatched uncertainty is necessary for sliding phase.

## 5 | SIMULATION RESULTS

Consider the following lateral flight control system composed of two subsystems, the generalised lateral mode at the cruising flight condition for civil airliners can be described as<sup>41</sup>:

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} -7 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (u_1 + \varphi_1(t, x_1)) + \begin{bmatrix} 0.9126 & 1.1776 \\ 1 & 1.2 \\ 0.5 & 1 \end{bmatrix} f_1(x_1, t) + \begin{bmatrix} 0.14(\|x_{21}\| + \sin(x_{13})) \\ 0 \\ 0.07(\sin(x_{21}) + \|x_{11}\|) \end{bmatrix} \end{aligned} \quad (100)$$

$$y_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \quad (101)$$

$$\begin{aligned} \dot{x}_2 &= \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 + \begin{bmatrix} 4.5 & 2.25 \\ 1 & 0.5 \end{bmatrix} f_2(x_2, t) + \begin{bmatrix} 0.05(\|x_{11}\| + \|x_{22}\|) \\ 0.04\sin(x_{12}) \end{bmatrix} \end{aligned} \quad (102)$$

$$y_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \quad (103)$$

where  $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$  and  $x_2 = \text{col}(x_{21}, x_{22})$  represent bank angle, sideslip angle, washout filter output, roll rate, yaw rate, respectively. It is assumed that flight attitude angle, washout filter output and flight attitude rate are available as the outputs of the system (100-103). Input signals  $u_1$  and  $u_2$  are the perturbed aileron deflection command and the perturbed rudder deflection command, respectively. The dynamic coefficients denote the Boeing 707 aircraft cruising at the specific speed<sup>41</sup>. The uncertainties are assumed to satisfy

$$\|f_1(\cdot)\| \leq \epsilon_1(\cdot) = 0.2(\sin(x_{12}) + \|x_{13}\|) \quad (104)$$

$$\|f_2(\cdot)\| \leq \epsilon_2(\cdot) = 0.02\|x_{22}\| \quad (105)$$

$$\|\varphi_1(\cdot)\| \leq \vartheta_1(\cdot) = 0.13\sin(x_{11}) \quad (106)$$

$$\mathcal{L}_{\epsilon_1}(\cdot) = 0.4 \quad (107)$$

$$\mathcal{L}_{\epsilon_2}(\cdot) = 0.02 \quad (108)$$

$$\mathcal{L}_{\vartheta_1}(\cdot) = 0.13 \quad (109)$$

and

$$\mathcal{L}_{H_{1j}}(\cdot) = 0.313 \quad (110)$$

$$\mathcal{L}_{H_{2j}}(\cdot) = 0.108 \quad (111)$$

where

$$H_{1j} = \begin{bmatrix} 0.14(\|x_{21}\| + \sin(x_{13})) \\ 0 \\ 0.07(\sin(x_{21}) + \|x_{11}\|) \end{bmatrix} \quad (112)$$

$$H_{2j} = \begin{bmatrix} 0.05(\|x_{11}\| + \|x_{22}\|) \\ 0.04\sin(x_{12}) \end{bmatrix} \quad (113)$$

By direct verification and using the matrices below,  $(A_1, B_1)$  and  $(A_2, B_2)$  are controllable,  $(A_1, C_1)$  and  $(A_2, C_2)$  are observable. Thus Assumption 2 is satisfied.

$$A_1 = \begin{bmatrix} -7 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (114)$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \quad (115)$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (116)$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (117)$$

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (118)$$

$$C_2 = [1 \ 1] \quad (119)$$

Choose

$$L_1 = \begin{bmatrix} -3.5979 & 5.3943 \\ 0.2988 & -1.7078 \\ 1.3056 & 2.2992 \end{bmatrix} \quad (120)$$

$$L_2 = \begin{bmatrix} -0.5 \\ 1.0000 \end{bmatrix} \quad (121)$$

By calculation,  $(A_1 - L_1 C_1)$  and  $(A_2 - L_2 C_2)$  are stable. For  $Q_1 = I_3$  and  $Q_2 = I_2$ , the solutions of Lyapunov equation (8) are

$$P_1 = \begin{bmatrix} 0.1497 & 0.0668 & -0.0960 \\ 0.0668 & 0.1339 & -0.0789 \\ -0.0960 & -0.0789 & 0.3081 \end{bmatrix} \quad (122)$$

$$P_2 = \begin{bmatrix} 0.3333 & 0.2000 \\ 0.2000 & 0.8000 \end{bmatrix} \quad (123)$$

Let

$$E_1 = \begin{bmatrix} 0.9126 & 1.1776 \\ 1 & 1.2 \\ 0.5 & 1 \end{bmatrix} \quad (124)$$

$$F_1 = \begin{bmatrix} 0.1554 & -0.0125 \\ 0.1604 & 0.1004 \end{bmatrix} \quad (125)$$

$$E_2 = \begin{bmatrix} 4.5 & 2.25 \\ 1 & 0.5 \end{bmatrix} \quad (126)$$

$$F_2 = \begin{bmatrix} 1.7 \\ 0.85 \end{bmatrix} \quad (127)$$

$$J_1 = [-0.0875 \ 0.3081] \quad (128)$$

$$G_1 = I_1 \quad (129)$$

According to (18a)-(18b),

$$(M + M^T) = \begin{bmatrix} 1.0658 & -0.3111 \\ -0.3111 & 1.4084 \end{bmatrix} \quad (130)$$

where  $(M + M^T)$  is positive definite, thus Theorem 1 is satisfied. Based on the equations (21) and (22),  $\alpha_1 = 1.1$  and  $\alpha_2 = 0.15$ .

The system (100)-(103) satisfies equation (23). Consider the transformation matrices  $T_1$  and  $T_2$  given by

$$T_1 = I_3 \quad (131)$$

$$T_2 = I_2 \quad (132)$$

Then,

$$\begin{bmatrix} z_1 \\ \hat{z}_1 \end{bmatrix} := \begin{bmatrix} T_1 & 0 \\ 0 & T_1 \end{bmatrix} \begin{bmatrix} x_1 \\ \hat{x}_1 \end{bmatrix} \quad (133)$$

$$\begin{bmatrix} z_2 \\ \hat{z}_2 \end{bmatrix} := \begin{bmatrix} T_2 & 0 \\ 0 & T_2 \end{bmatrix} \begin{bmatrix} x_2 \\ \hat{x}_2 \end{bmatrix} \quad (134)$$

where  $z_1 = \text{col}(z_{11}^a, z_{11}^b, z_{12}) = \text{col}(x_{11}, x_{12}, x_{13})$ ,  $z_2 = \text{col}(z_{21}, z_{22}) = \text{col}(x_{21}, x_{22})$ ,  $\hat{z}_1 = \text{col}(\hat{z}_{11}^a, \hat{z}_{11}^b, \hat{z}_{12}) = \text{col}(\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13})$  and  $\hat{z}_2 = \text{col}(\hat{z}_{21}, \hat{z}_{22}) = \text{col}(\hat{x}_{21}, \hat{x}_{22})$ . From (131)-(134), it is straightforward to determine that, for  $i = 1, 2$ ,

$$\tilde{C}_i = C_i \quad (135)$$

$$\tilde{L}_i = L_i \quad (136)$$

$$\tilde{E}_i = E_i \quad (137)$$

$$\tilde{f}_i(\cdot) = E_i f_i(\cdot) \quad (138)$$

$$\tilde{A}_i = A_i \quad (139)$$

$$\tilde{B}_i = B_i \quad (140)$$

$$\tilde{H}_{ij}(\cdot) = H_{ij}(\cdot) \quad (141)$$

$$\mathcal{L}_{\tilde{e}_i}(\cdot) = \mathcal{L}_{e_i}(\cdot) \quad (142)$$

$$\mathcal{L}_{\tilde{g}_i}(\cdot) = \mathcal{L}_{g_i}(\cdot) \quad (143)$$

$$\mathcal{L}_{\tilde{H}_{ij}}(\cdot) = \mathcal{L}_{H_{ij}}(\cdot) \quad (144)$$

Based on the equations (73a)-(74b),

$$\tilde{\Psi}_1(\cdot) = \begin{cases} \tilde{E}_1 \frac{F_1 \tilde{C}_1 \tilde{e}_1}{\|F_1 \tilde{C}_1 \tilde{e}_1\|} \tilde{e}_1(\cdot), & F_1 \tilde{C}_1 \tilde{e}_1 \neq 0 \\ 0, & F_1 \tilde{C}_1 \tilde{e}_1 = 0 \end{cases} \quad (145a)$$

$$\quad (145b)$$

$$\tilde{\Psi}_2(\cdot) = \begin{cases} \tilde{E}_2 \frac{F_2 \tilde{C}_2 \tilde{e}_2}{\|F_2 \tilde{C}_2 \tilde{e}_2\|} \tilde{e}_2(\cdot), & F_2 \tilde{C}_2 \tilde{e}_2 \neq 0 \\ 0, & F_2 \tilde{C}_2 \tilde{e}_2 = 0 \end{cases} \quad (146a)$$

$$\quad (146b)$$

$$\tilde{\Upsilon}_1(\cdot) = \begin{cases} G_1 \frac{J_1 \tilde{C}_1 \tilde{e}_1}{\|J_1 \tilde{C}_1 \tilde{e}_1\|} \tilde{g}_1(\cdot), & J_1 \tilde{C}_1 \tilde{e}_1 \neq 0 \\ 0, & J_1 \tilde{C}_1 \tilde{e}_1 = 0 \end{cases} \quad (147a)$$

$$\quad (147b)$$

Consider the equations (59)-(60),

$$\chi_{11}(\cdot) = \chi_{12}(\cdot) = 0.4444 \quad (148)$$

$$\chi_{21}(\cdot) = 0.1006 \quad (149)$$

$$\chi_{22}(\cdot) = 0.2012 \quad (150)$$

$$\varrho_{1j1}(\cdot) = \varrho_{1j2}(\cdot) = 0.14 \quad (151)$$

$$\varrho_{2j1}(\cdot) = \varrho_{2j2}(\cdot) = 0.1 \quad (152)$$

Choose  $\bar{Q}_1 = I_2$  and  $\bar{Q}_2 = I_1$ ,  $S_{11} = 0$ ,  $S_{12} = 1$ ,  $S_{21} = 2$  and  $S_{22} = 2$ . According to equation (43)

$$\bar{P}_1 = \begin{bmatrix} 0.0714 & 0 \\ 0 & 0.0833 \end{bmatrix} \quad (153)$$

$$\bar{P}_2 = 0.1667 \quad (154)$$

Based on equations (56) and (57), it can be seen that  $\tilde{P}_1 = P_1$ ,  $\tilde{P}_2 = P_2$ ,  $\tilde{Q}_1 = Q_1$  and  $\tilde{Q}_2 = Q_2$ .

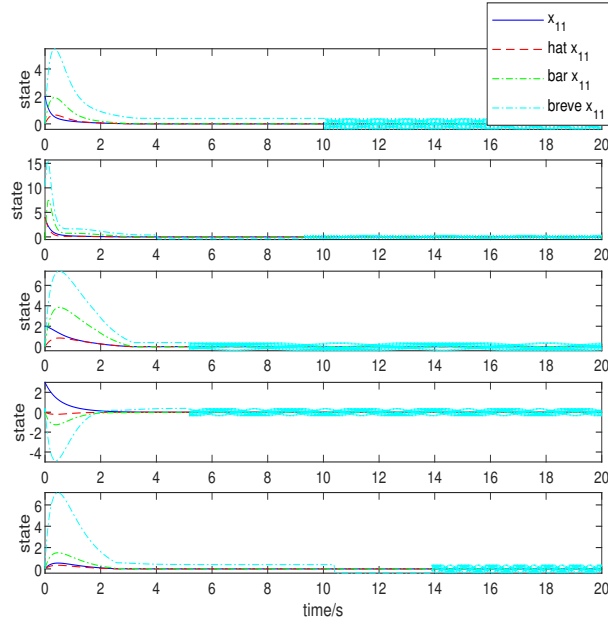


**Figure 1** Time responses of states.

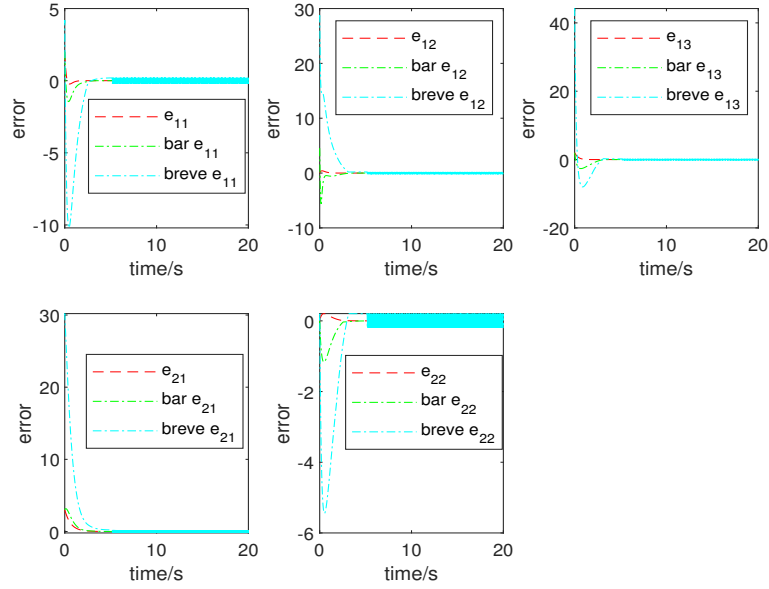


**Figure 2** Time responses of observation errors.

Based on the equations (61)-(64b), it follows that

$$W + W^T = \begin{bmatrix} 1.8052 & -0.0567 & -0.6659 & -0.0567 \\ -0.0567 & 1.8662 & -0.0567 & -1.1436 \\ -0.6659 & -0.0567 & 0.8201 & -0.4339 \\ -0.0567 & -1.1436 & -0.4339 & 1.4084 \end{bmatrix} \quad (155)$$

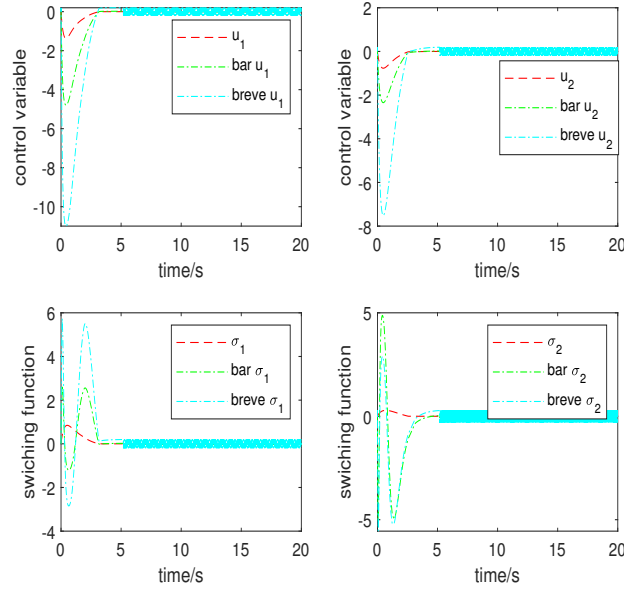


**Figure 3** The control laws (Upper); the switching functions (Bottom).

where  $W + W^T$  is positive definite. Then, Theorem 2 is satisfied. From (85),  $k_1(\cdot)$  and  $k_2(\cdot)$  are selected as

$$k_1(\cdot) = 1.97952e^{-1.1t} + b_1 \quad (156)$$

$$k_2(\cdot) = 2.59864e^{-1.1t} + b_2 \quad (157)$$

where  $b_1$  and  $b_2$  are selected as 0.1 and 0.1, respectively. Then, the control law (84) can be described as

$$u_1 = - \left( \hat{z}_{11}^a + \frac{\sigma_1(\cdot)}{\|\sigma_1(\cdot)\|} (\|\tilde{\Psi}_{12}(\cdot)\| + \|\tilde{Y}_1(\cdot)\| + \|0.07(\sin(\hat{z}_{21})) + \|\hat{z}_{11}^a\|\| + k_1(\cdot)) \right) \quad (158)$$

$$u_2 = -0.5 \left( -2\hat{z}_{21} + 2\hat{z}_{22} + \frac{\sigma_2(\cdot)}{\|\sigma_2(\cdot)\|} (2\|\tilde{\Psi}_{21}(\cdot)\| + 2\|\tilde{\Psi}_{22}(\cdot)\| + 0.1(\|\hat{z}_{11}^a\| + \|\hat{z}_{22}\|) + 0.08\|\sin(\hat{z}_{11}^b)\| + k_2(\cdot)) \right) \quad (159)$$

where  $\sigma_1(\cdot) = \hat{z}_{12}$ ,  $\sigma_2(\cdot) = 2\hat{z}_{21} + 2\hat{z}_{22}$ ,  $\tilde{\Psi}_{12}(\cdot)$  is the last row of  $\tilde{\Psi}_1(\cdot)$ ,  $\tilde{\Psi}_{21}(\cdot)$  is the first row of  $\tilde{\Psi}_2(\cdot)$ ,  $\tilde{\Psi}_{22}(\cdot)$  is the last row of  $\tilde{\Psi}_2(\cdot)$ .

For simulation, the initial conditions are selected as  $z_1 = \text{col}(2, 4, 2)$ ,  $z_2 = \text{col}(3, 0)$ ,  $\hat{z}_1 = \text{col}(0, 4, 0)$  and  $\hat{z}_2 = \text{col}(1, 0)$ . Figure 1 shows the time responses of the states of the lateral flight control system, it should be mentioned that  $\bar{x} = \text{col}(\bar{x}_{11}, \bar{x}_{12}, \bar{x}_{13}, \bar{x}_{21}, \bar{x}_{22})$  and  $\check{x} = \text{col}(\check{x}_{11}, \check{x}_{12}, \check{x}_{13}, \check{x}_{21}, \check{x}_{22})$  denote the estimated states of the lateral flight control system using the strategies<sup>33,42</sup>, respectively. Compared to the methods<sup>33,42</sup>, it can be found that the estimated states in the presented method have the greater tracking ability for the states of this kind of interconnected systems with uncertainties. Figure 2 shows the time responses of observation errors. Especially,  $\bar{e} = \text{col}(\bar{e}_{11}, \bar{e}_{12}, \bar{e}_{13}, \bar{e}_{21}, \bar{e}_{22})$  and  $\check{e} = \text{col}(\check{e}_{11}, \check{e}_{12}, \check{e}_{13}, \check{e}_{21}, \check{e}_{22})$  denote the observation errors related to states of the lateral flight control system using the approaches<sup>33,42</sup>, respectively. From figure 2, compared to the existing methods<sup>33,42</sup>, the observation errors in this paper are smaller and adjusted to 0 faster. Figure 3 presents the control laws and sliding functions,  $\bar{u} = \text{col}(\bar{u}_1, \bar{u}_2)$  and  $\bar{\sigma} = \text{col}(\bar{\sigma}_1, \bar{\sigma}_2)$  present the control laws and sliding functions via the method<sup>33</sup>, respectively.  $\check{u} = \text{col}(\check{u}_1, \check{u}_2)$  and  $\check{\sigma} = \text{col}(\check{\sigma}_1, \check{\sigma}_2)$  are the control laws and sliding functions in the approach<sup>42</sup>, respectively. According to figure 3, it is easily found that this lateral flight control system can be driven to the designed sliding surface in finite time, and the stable sliding motion is maintained thereafter. According to figures 1-3, another noteworthy issue is that, compared with the approach proposed in this paper, the existing strategy<sup>42</sup> exhibits the pronounced chattering in this lateral flight control system. The results of this practical simulation show the effectiveness of the presented strategy.

## 6 | CONCLUSIONS

A dynamic observer based decentralised SMC strategy is proposed to stabilise a class of nonlinear interconnected systems in the presence of matched and mismatched uncertainties. The dynamic observer is designed to estimate the states of the interconnected system. A coordinate transformation is used to transform each isolated subsystem into regular form which is convenient for SMC design. Then, a composite sliding surface is designed incorporating the states from the dynamic observer. A decentralised feedback SMC law based on the estimated states and outputs of the system is designed to drive the system to the sliding surface in finite time and maintain a sliding motion on it thereafter. The interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties, which are nonlinear and take more general forms, are relaxed. The presented method improves the robustness of nonlinear interconnected systems in the presence of uncertainties. However, this paper ignores the problems of chattering and time-delay, and the uncertainties of interconnected systems must be bounded, while the interconnections are known. In the future, this method will be modified to deal with the problems of chattering and time-delay for practical interconnected systems. Besides that, future work will also focus on studying cases that the bounds of the uncertainties and the interconnections are themselves unknown, the proposed method will be improved to enhance its generality.

## ACKNOWLEDGMENTS

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by Wuxi University Research Start-up Fund for Introduced Talents (2024r026).

## CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

## DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

## References

1. Yan XG, Spurgeon SK, Edwards C. *Variable Structure Control of Complex Systems*. Springer . 2017.
2. Yu TH, Cao DG, Huang WH. Robust decentralized stabilization for large-scale time-delay system via impulsive control. *IMA Journal of Mathematical Control and Information* 2019; 36(4): 1181–1198.
3. Chen ZH, Cheng J, Tan J, Cao Z. Decentralized finite-time control for linear interconnected fractional-order systems with input saturation. *Journal of the Franklin Institute* 2020; 357(10): 6137–6153.
4. Shao YM, Jia XL, Ju XX, Shi XC. Global decentralized control for uncertain large-scale feedforward nonlinear time-delay systems via output feedback. *IMA Journal of Mathematical Control and Information* 2022; 39(1): 155–170.
5. Jiang H, He X, Song Q. Decentralized learning control for large-scale systems with gain-adaptation mechanisms. *Information Sciences* 2023; 623: 539–558.
6. Hayano A, Takai S. A General Architecture for Intersection-Based Decentralized Supervisory Control of Discrete Event Systems. *IEEE Transactions on Automatic Control* 2024; 69(1): 674–680.
7. Xiao TF, Li XD. Eigenspectrum-based extended Luenberger observers for a class of distributed parameter systems. *Journal of Process Control* 2020; 96(12): 15–22.
8. Yan XG, Spurgeon SK, Edwards C. Memoryless Static Output Feedback Sliding Mode Control for Nonlinear Systems With Delayed Disturbances. *IEEE Transactions on Automatic Control* 2014; 59(7): 1906–1912.

9. Nan G, Mohamed D, Holger V, Marouane A. New unified  $H_{\infty}$  dynamic observer design for linear systems with unknown inputs. *Automatica* 2016; 65(3): 43-52.
10. Andrea LA, Keyvan M. Robust observer-based control of nonlinear dynamical systems with state constraints. *Journal of the Franklin Institute* 2017; 354(16): 7385-7409.
11. Xu QM, Jing ZL, Hu SQ. Stability analysis of nonlinear dynamic system with linear observer for a multilink flexible manipulator. *International Journal of Non-Linear Mechanics* 2018; 103(7): 27-36.
12. Gong LG, Wang Q, Dong CY. Spacecraft output feedback attitude control based on extended state observer and adaptive dynamic programming. *Journal of the Franklin Institute* 2019; 356(10): 4971-5000.
13. Zare K, Shasadeghi M, Izadian A, Niknam T, Asemani MH. Switching TS fuzzy model-based dynamic sliding mode observer design for non-differentiable nonlinear systems. *Engineering Applications of Artificial Intelligence* 2020; 96(11): 103990.
14. Xu C, Qin Y, Su H. Observer-based dynamic event-triggered bipartite consensus of discrete-time multi-agent systems. *IEEE Transactions on Circuits and Systems II: Express Briefs* 2022; 70(3): 1054-1058.
15. Tutsoy O, Asadi D, Ahmadi K. Robust reduced order thau observer with the adaptive fault estimator for the unmanned air vehicles. *IEEE Transactions on Vehicular Technology* 2023; 72(2): 1601-1610.
16. Utkin V, Guldner J, Shi J. *Sliding mode control in electro-mechanical systems*. CRC press . 2009.
17. Fei ZY, Wu Z, Zhao XD, Zong GD, Lin-Shi XF. Reachability-guaranteed sliding mode control for asynchronously switched uncertain systems. *IEEE Transactions on Automatic Control* 2025; 70(2): 921-932.
18. Kao YG, Xie J, Wang CH, Hamid RK. A sliding mode approach to  $H_{\infty}$  non-fragile observer-based control design for uncertain Markovian neutral-type stochastic systems. *Automatica* 2015; 52(2): 218-226.
19. Zhu JM, Yu XH, Zhang TP, Cao ZQ, Yang YQ, Yi Y. Sliding mode control of MIMO Markovian jump systems. *Automatica* 2016; 68(6): 286-293.
20. Incremona GP, Cucuzzella M, Ferrara A. Second order sliding mode control for nonlinear affine systems with quantized uncertainty. *Automatica* 2017; 86(12): 46-52.
21. Su XJ, Liu XX, Shi P, Song YD. Sliding mode control of hybrid switched systems via an event-triggered mechanism. *Automatica* 2018; 90(4): 294-303.
22. Wang XR, Van Kampen EJ, Chu QP. Quadrotor fault-tolerant incremental nonsingular terminal sliding mode control. *Aerospace Science and Technology* 2019; 95(12): 105514.
23. Gao XW, Weng YP. Chattering-free model free adaptive sliding mode control for gas collection process with data dropout. *Journal of Process Control* 2020; 93(9): 1-13.
24. Dong H, Yang X, Gao H. Practical terminal sliding-mode control and its applications in servo systems. *IEEE Transactions on Industrial Electronics* 2022; 70(1): 752-761.
25. Wang B, Shen Y, Li N. An adaptive sliding mode fault-tolerant control of a quadrotor unmanned aerial vehicle with actuator faults and model uncertainties. *International Journal of Robust and Nonlinear Control* 2023; 33(17): 10182-10198.
26. Estrada MA, Fridman L, Moreno JA. Passive fault-tolerant control via sliding-mode-based Lyapunov redesign. *IEEE Transactions on Automatic Control* 2024.
27. Gomez MA, Cruz-Ancona CD. Closing the gap from nominal to robust asymptotic stabilization of time delay systems. *International Journal of Robust and Nonlinear Control* 2024; 34(10): 6416-6430.
28. Lin D, Wang XY. Observer-based decentralized fuzzy neural sliding mode control for interconnected unknown chaotic systems via network structure adaptation. *Fuzzy Sets and Systems* 2010; 161(15): 2066-2080.

29. Feng JH, Zhao DY, Yan XG, Spurgeon SK, Han Y. Decentralized static output feedback sliding mode control for a class of nonlinear interconnected systems in the generalized regular form. *Transactions of the Institute of Measurement and Control* 2025.
30. Mahmoud MS, Aminuddin Q. Decentralized sliding mode output feedback control of interconnected discrete delay systems. *Automatica* 2012; 48(5): 808-814.
31. Yan XG, Spurgeon SK, Edwards C. Decentralised stabilisation for nonlinear time delay interconnected systems using static output feedback. *Automatica* 2013; 49(2): 633-641.
32. Liu WJ. Decentralized Sliding Mode Control for Multi-Input Complex Interconnected Systems Subject to non-smooth Nonlinearities. *Asian Journal of Control* 2018; 20(3): 1171-1181.
33. Gao R, Zhang QL, Li JH, Zhai D. Integral sliding mode control for interconnected descriptor systems based on a reduced-order observer. *International Journal of Systems Science* 2019; 50(10): 19471–1960.
34. Feng JH, Zhao DY, Yan XG, Spurgeon SK. Decentralized sliding mode control for a class of nonlinear interconnected systems by static state feedback. *International Journal of Robust and Nonlinear Control* 2020; 30(6): 2152–2170.
35. Ji N, Yan XG, Mao ZH, Zhao DY, Jiang B. Decentralised state feedback stabilisation for nonlinear interconnected systems using sliding mode control. *International Journal of Systems Science* 2022; 53(5): 1017–1030.
36. Wang T, Wang H, Xu N. Sliding-mode surface-based decentralized event-triggered control of partially unknown interconnected nonlinear systems via reinforcement learning. *Information Sciences* 2023; 64(1): 119070-119099.
37. Xu H, Yu T, Ren C. Finite-time decentralized sliding mode control for interconnected systems and its application to electrical power systems: a GA-assisted design method. *IEEE Transactions on Circuits and Systems I: Regular Papers* 2024; 71(6): 2835-2847.
38. Horn RA, Johnson CR. *Matrix analysis*. Cambridge university press . 2012.
39. Yan XG, Edwards C. Adaptive sliding-mode-observer-based fault reconstruction for nonlinear systems with parametric uncertainties. *IEEE Transactions on Industrial Electronics* 2008; 55(11): 4029–4036.
40. Edwards C, Spurgeon S. *Sliding mode control: theory and applications*. Crc Press . 1998.
41. Unal G. Integrated design of fault-tolerant control for flight control systems using observer and fuzzy logic. *Aircraft Engineering and Aerospace Technology* 2021; 93(4): 723–732.
42. Sun K, Feng JH, Zhao DY. Decentralised sliding mode control for a class of nonlinear interconnected systems with unstable internal dynamic. *International Journal of Industrial and Systems Engineering* 2024; 48(1): 83-99.

