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Model-Free Adaptive State Feedback Control for a Class of Nonlinear Systems

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Abstract—This paper investigates state feedback control for a class of discrete-time multiple input and multiple output nonlinear systems from the perspective of model-free adaptive control and state observation. The design of a dynamic state feedback control can be efficiently carried out using dynamic linearization and state observation. The stability of the proposed method is guaranteed by theoretical analysis. Numerical simulation tests and experimentation on a continuous stirred tank reactor are carried out to validate the effectiveness of the proposed approach.

Note to Practitioners—The growth in the scale of factories and the complexity of associated production processes increases the complexity and time involved in associated mathematical modelling. Data driven approaches to control remove the need to model processes. To the best of the authors' knowledge, existing approaches to model-free adaptive control (MFAC) of general systems are all based on an input-output control paradigm. These methods thus cannot guarantee the stability of the system state. The purpose of this study is to develop a novel Model-Free Adaptive Control (MFAC) approach to achieve control of the system state. In this paper, the assumptions required to achieve model-free adaptive control by state feedback are presented mathematically. A controller design and the associated stability proof are then presented. Numerical simulation and experimentation is conducted to validate the effectiveness of the proposed approach. In future research, state feedback data control in the presence of random disturbances will be investigated.

Index Terms—State feedback control, model-free adaptive control, state observer, nonlinear systems.

I. INTRODUCTION

INDUSTRIALISATION has produced factories of increasing scale which also incorporate production processes of growing complexity. This increase in scale and complexity results in greater difficulty in modelling processes based on

traditional physical principles. Computer technology is now widely used in industrial processes, which provides ready access to considerable amounts of data. This has created a focus on data-driven control (DDC) theory, which reduces dependence on the availability of mathematical models for development of control strategies [1], [2], [3].

Compared with traditional model based control, DDC is a fully data based control that only uses available process data to formulate a control strategy. Model information is unnecessary which is attractive in practice as the time to controller implementation may be reduced as a consequence. DDC approaches can be broadly classified according to the type of data used. Offline DDC uses offline data to establish a dynamic model beforehand and then designs a corresponding control. This approach results in high computational efficiency [4], [5], [6]. However, offline DDC exhibits poor adaptability due to the use of a model which has been developed offline and is thus fixed. Online DDC uses real-time data to identify the system parameters and update the controller parameters. In this way, the closed-loop system can adapt to changes in the process conditions. This enhances the robustness of the closed loop system at the expense of computational efficiency [7], [8], [9].

With increasing developments in computer technology, the constraints of computational efficiency with online DDC reduce and implementation becomes a more straightforward proposition. Online data-driven control methods are growing in popularity and efficiency. The iterative feedback tuning method is a data driven control method that uses measured data from the closed-loop control system and searches for optimal controller parameters based on gradient iteration [10]. A database of pre-computed motion is used for path planning in high dimensional problems to alleviate computational load [11]. Adaptive power control is introduced into energy management to reduce energy consumption of nodes and extend the life of nodes in [12].

While iterative learning control exhibits high precision in processing under repetitive conditions [13], [14], [15], it requires strict consistency between the initial condition and the desired trajectory for each cycle. So-called unfalsified switching control selects the controller to meet the current specific performance requirements from a candidate control set [16], [17]. However the method exhibits strong constraints on the admissible control set and control reversibility. Model-free adaptive control (MFAC) generates a dynamic linearization, which can avoid the restriction of the above online control

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methods [18], [19], [20]. The basic idea of MFAC is to obtain an equivalent dynamic linearization model at each working point of a general discrete-time nonlinear system [21]. Only the I/O data of the system are used in the control design of the MFAC [20], [22]. Due to the above advantages, MFAC has been successfully applied in various systems, including an unmanned surface vehicle [23], autonomous car [24], urban road traffic network [25], and chemical process [26].

The main feature of MFAC is to introduce pseudo partial derivatives to simplify the linearized model structure and avoid the design of a high-order controller. Pseudo partial derivatives are robust to the time variability of system parameters, structure and time delay [27]. The stability and convergence of MFAC has been proved [28], [29]. This design principle has been used extensively and combined with many control approaches, such as, MFAC sliding mode control [30], [31], [32], MFAC model predictive control [33] and MFAC iterative learning control [34]. However, in industrial processes it may not be possible to measure all the states directly. This lack of knowledge may impact on product quality and even safety. Indeed, existing MFAC may only guarantee the input-output stability without considering the system state, which may limit the application of MFAC. For example, in a motor control problem, only the speed of the motor can be controlled by using the input output control approach. The internal state, current, is not controlled, which may lead to instability and may even damage the motor in the case of an overload. Note that stability analysis of a MFAC based on full form dynamic linearization was proposed in [20], but this does not consider the unmeasurable state of the system. An extended state observer has been used in some MFAC approaches, such as [35] and [36], but these methods were used to observe the modeling error and disturbance rather than the system internal states. A state feedback model-free control was proposed in [37], but this requires that the system state is measured, that is, the system output equals the system state, which cannot be achieved in many real cases. A Kalman filter is used in [38] to filter the measurement noise using an identified model but the system states were not observed. Further, the system class is relatively constrained. The problem of state feedback control is still an open problem in MFAC studies.

In light of the above analysis, the objective of this research is to design a state feedback model-free adaptive control (SFMFAC) for a class of multiple input and multiple output (MIMO) nonlinear systems. As with traditional MFAC, this method only uses input and output information to ensure state stability and expands the scope of applications which may be controlled by MFAC. The contributions of this paper can be summarized as: (i) Compared with traditional MFAC in [33], a new model free state observer is derived and the systematic design method is given, which paves the way for model free adaptive state feedback control. Further, in light of the proposed state observer, a new model free adaptive state feedback control is developed which is more practical than the existing ones in the sense of ensuring internal state stability. (ii) Compared with the existing data driven state feedback control in [38] and [39], the stability analysis has been given in

details to lay a theoretical foundation for practical application. Hence, this method is an effective expansion of existing MFAC approaches which may be more suitable for systems requiring the internal states to be stable, such as the previously mentioned motor control system.

The outline of this paper is as follows: Section II describes the MFAC with state estimation problem, and gives the basic assumptions. Section III designs a novel data-driven observer. Section IV derives a model-free adaptive state feedback control. Section V gives simulation and experimental results and is followed by conclusions in Section VI.

II. PROBLEM FORMULATION AND DYNAMICAL LINEARIZATION

Consider a MIMO nonlinear discrete-time system described by

$$\begin{aligned} y(k+1) = F(y(k), \dots, y(k-n_y), u(k), \dots, \\ u(k-n_u), d(k), \dots, d(k-n_d)) \end{aligned} \quad (1)$$

where $F(y(k), \dots, y(k-n_y), u(k), \dots, u(k-n_u), d(k), \dots, d(k-n_d)) \in \mathbb{R}^p$ is an unknown nonlinear function, which is abbreviated as $F(\cdot)$, $u(k) = [u_1(k), u_2(k), \dots, u_m(k)]^T \in \mathbb{R}^m$ is the control input, $y(k) = [y_1(k), y_2(k), \dots, y_p(k)]^T \in \mathbb{R}^p$ is the system output, $d(k) = [d_1(k), d_2(k), \dots, d_m(k)]^T \in \mathbb{R}^m$ is the external disturbance, $p \geq m$ and $n_y, n_u, n_d > 0$ are the orders of the output, input and disturbance of the system, respectively.

Lemma 1 [33]: For system (1), if the following conditions are satisfied:

(I) The partial derivatives of $F(\cdot)$ with respect to the inputs $u(k)$ are continuous and non-zero.

(II) The generalized Lipschitz condition needs to be satisfied for system (1). For any k , $\|\Delta y(k+1)\| < b\|\Delta u(k)\|$, where $b > 0$, $\Delta y(k+1) = y(k+1) - y(k)$. For any k , $\|\Delta u(k)\| \neq 0$, where $\Delta u(k) = u(k) - u(k-1)$.

(III) Function $F(\cdot)$ is locally continuous and bounded.

Then there exists a bounded matrix $\Phi(k)$, whereby system (1) is converted to

$$\Delta y(k+1) = \Phi^T(k) \Delta u(k) \quad (2)$$

where $\Phi(k) \in \mathbb{R}^{m \times p}$ is called the pseudo-partial-derivative (PPD) matrix.

Remark 1: It can be seen from (2) that the disturbance is not considered when the system is dynamically linearized, which will reduce the robustness of the system. Further, system (1) and (2) describe a typical output feedback system, which cannot guarantee the stability of the internal state of the controlled plant.

In terms of the system state, (1) can be equivalently rewritten as the nonlinear system

$$\begin{aligned} z(k+1) = F'(z(k), \dots, z(k-n_z), u(k), \dots, \\ u(k-n_u), d(k), \dots, d(k-n_d)) \end{aligned} \quad (3)$$

where $F'(z(k), \dots, z(k-n_z), u(k), \dots, u(k-n_u), d(k), \dots, d(k-n_d)) \in \mathbb{R}^n$ is an unknown nonlinear function, which is referred to as $F'(\cdot)$ for ease of exposition,

$z(k) = [z_1(k), z_2(k), \dots, z_n(k)]^T \in \mathbb{R}^n$ is the system state, $n_z > 0$ is the order of the system state and $y(k) = C^*z(k)$ with $C^* \in \mathbb{R}^{p \times n}$ is a known constant matrix.

According to Lemma 1 [33], system (3) satisfies the following conditions:

Assumption 1: The partial derivatives of $F'(\cdot)$ with respect to the control inputs $z(k)$, $u(k)$ and $d(k)$ are continuous and non-zero.

Assumption 2: The generalized Lipschitz condition needs to be satisfied for system (3), that is, $\|\Delta z(k+1)\| < b'\|\Delta X(k)\|$ for any k and $\|\Delta X(k)\| \neq 0$, where $X(k) = [z^T(k), u^T(k), d^T(k)]^T$, $\Delta X(k) = X(k) - X(k-1)$, $\Delta z(k+1) = z(k+1) - z(k)$, $b' > 0$, $\Delta d(k) = d(k) - d(k-1)$, and the external disturbance satisfies $\|\Delta d(k)\| \leq d_{\max}\|\Delta y(k)\|$.

Assumption 3: Function $F'(\cdot)$ is locally continuous and bounded.

Remark 2: Assumptions 1-3 are general conditions imposed in the DDC method. Assumption 1 is a common condition in control system design. Assumption 2 limits the speed at which the function changes. These inequalities are commonly used in MFAC [18], [20]. The assumption on the disturbance is commonly used in control design [40].

Lemma 2: For system (3) satisfying Assumptions 1-3, there exists $A^*(k)$, $B^*(k)$ and $d^*(k)$, so that system (3) can be written as

$$\begin{cases} \Delta z(k+1) = A^*(k)\Delta z(k) + B^*(k)\Delta u(k) + d^*(k) \\ \Delta y(k) = C^*\Delta z(k) \end{cases} \quad (4)$$

where $\|A^*(k)\| \leq b_1^*$, $\|B^*(k)\| \leq b_2^*$, $b_1^*, b_2^* > 0$.

Proof: From the definition of $\Delta z(k+1)$,

$$\begin{aligned} \Delta z(k+1) &= F'(z(k), \dots, z(k-n_z), u(k), \\ &\quad \dots, u(k-n_u), d(k), \dots, d(k-n_d)) \\ &\quad - F'(z(k-1), \dots, z(k-n_z-1), u(k-1), \\ &\quad \dots, u(k-n_u-1), d(k-1), \dots, d(k-n_d-1)) \end{aligned}$$

Based on the differential mean value theorem [41], it follows that

$$\Delta z(k+1) = \frac{\partial F'^*}{\partial z(k)} \Delta z(k) + \frac{\partial F'^*}{\partial u(k)} \Delta u(k) + \frac{\partial F'^*}{\partial d(k)} \Delta d(k) + \psi(k) \quad (5)$$

where $\frac{\partial F'^*}{\partial z(k)}$, $\frac{\partial F'^*}{\partial u(k)}$ and $\frac{\partial F'^*}{\partial d(k)}$ are given in the Appendix, and

$$\psi(k) = F_d - F_{k-1}$$

where F_d and F_{k-1} are given in the Appendix. $\psi(k)$ can be written as

$$\psi(k) = h^T(k)\Delta X(k) \quad (6)$$

where $h^T(k) \in \mathbb{R}^{n \times (n+2m)}$ is a common auxiliary variable used in MFAC [33] and $\Delta X(k) = [\Delta z^T(k), \Delta u^T(k), \Delta d^T(k)]^T$.

Since $\|\Delta X(k)\| \neq 0$, there exists a solution $h^{*T}(k)$ to (6).

Let $\Phi^*(k) = h^{*T}(k) + [\frac{\partial F'^*}{\partial z(k)}, \frac{\partial F'^*}{\partial u(k)}, \frac{\partial F'^*}{\partial d(k)}]$, then (3) can be written as

$$\Delta z(k+1) = A^*(k)\Delta z(k) + B^*(k)\Delta u(k) + D^*(k)\Delta d(k) \quad (7)$$

where $A^*(k) = \frac{\partial F'^*}{\partial z(k)}$, $B^*(k) = \frac{\partial F'^*}{\partial u(k)}$, and $D^*(k) = \frac{\partial F'^*}{\partial d(k)}$. Full details can be found in the Appendix.

From Assumption 2, $\|[A^*(k), B^*(k), D^*(k)]\| < b'$ can be obtained. In addition $\|A^*(k)\| \leq b_1^*$, $\|B^*(k)\| \leq b_2^*$, $\|D^*(k)\| \leq b_3^*$, $b_1^*, b_2^*, b_3^* > 0$. Defining $d^*(k) = D^*(k)\Delta d(k)$ and $y(k) = C^*x(k)$, (4) can be obtained. Q.E.D.

It can be seen from the proof of Lemma 2 [18] and Lemma 2 that the dynamic linearized model does not lose any information of the nonlinear system. Therefore, the dynamic linearized model is the equivalent of the original nonlinear model.

From Lemma 2, the parameter matrix of the system has been obtained. The following Assumption is then given.

Assumption 4: $\text{rank}(C^*) = p$.

In light of [42], there exists a coordinate transformation $x(k) \mapsto T_c z(k)$, and the system (4) becomes:

$$\begin{cases} \Delta x(k+1) = A(k)\Delta x(k) + B(k)\Delta u(k) + d'(k) \\ \Delta y(k) = C\Delta x(k) \end{cases} \quad (8)$$

where $T_c \in \mathbb{R}^{n \times n}$, $A(k) = T_c A^*(k) T_c^{-1}$, $B(k) = T_c B^*(k)$, $d'(k) = T_c d^*(k)$, $C = C^* T_c^{-1} = [0 \ T]$, $T \in \mathbb{R}^{p \times p}$ is an orthogonal matrix.

Let $x(k) = [x_1^T(k), x_2^T(k)]^T$, $x_2(k) = T^T y(k)$. According to Assumption 2 and Lemma 2, there exist positive constants b_1 and b_2 making $\|A(k)\| \leq b_1$, $\|B(k)\| \leq b_2$, $\|d'(k)\| \leq b_3\|\Delta y(k)\|$.

The following may be stated as the control objective of this paper: To develop a new state-feedback model-free adaptive control (SFMFAC) for a class of MIMO non-linear systems, whereby the internal state will be observed by a model-free state observer by appealing to the dynamic linearization method. Accordingly, the stability of the system output and unknown state is proved using the proposed approach.

Remark 3: The existing data driven state feedback control can only satisfy a kind of special system, whose dynamic model is [39]:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i), & 1 \leq i \leq n-1 \\ \dot{x}_n = u + f_n(\bar{x}_n) \\ y = x_1 \end{cases}$$

The system parameter matrix A in the model has a lower triangular form, which is very special. In this paper, the proposed approach has been derived for a class of general nonlinear discrete system. The control design don't rely on model at all.

Remark 4: As the disturbance is considered within the dynamic linearization, the robustness of the system is enhanced. This also facilitates the design of a disturbance observer. The linear equation (8) represents a dynamically linearized model from Assumptions 1-4, and does not correspond directly to the actual model. This is a common technique in MFAC [20], [33]. The system output can track a desired trajectory while the stability of the unknown states is also achieved by use of an appropriate controller. Further, the corresponding stability of the control system is analyzed, which provides a theoretical underpinning for the proposed approach.

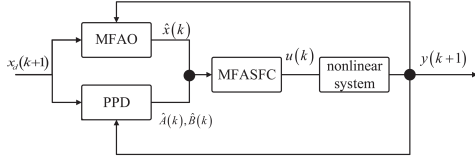


Fig. 1. Block diagram of model-free adaptive state feedback control.

III. MODEL-FREE ADAPTIVE STATE OBSERVER DESIGN

Because only system I/O data can be used, the design of a state feedback controller cannot be achieved using the method of MFAC. It is necessary to first develop a model-free adaptive observer (MFAO). A block diagram of the proposed approach is given in Fig. 1.

The MFAO is designed as follows

$$\begin{aligned} \Delta \hat{x}(k+1) &= \hat{A}(k) \Delta \hat{x}(k) + \hat{B}(k) \Delta u(k) \\ &\quad + L(\Delta y(k) - C \Delta \hat{x}(k)) + \hat{d}'(k) \end{aligned} \quad (9)$$

where $\Delta \hat{x}(k) = \hat{x}(k+1) - \hat{x}(k)$, $\hat{x}(k) = [\hat{x}_1^T(k), \hat{x}_2^T(k)]^T$ is the estimate of $x(k)$, $\hat{x}_1(k) \in \mathbb{R}^{n-p}$, $\hat{x}_2(k) \in \mathbb{R}^p$, $\hat{A}(k)$ is the estimate of $A(k)$, $\hat{B}(k)$ is the estimate of $B(k)$ and $L \in \mathbb{R}^{n \times p}$ is the observer gain, $\hat{d}'(k)$ is the estimate of $d'(k)$.

The disturbance observer is given by

$$\hat{d}'(k) = \hat{d}'(k-1) + G \Delta y(k) \quad (10)$$

where $G \in \mathbb{R}^{n \times p}$ is the gain matrix of the observer.

The unknown PPD matrix $\Phi'(k) = [A(k), B(k)]$ can be estimated by deriving a new projection algorithm which is a modification of the one presented in [18]. The PPD matrix objective function is given by

$$J(\Phi'(k)) = \|\Delta x'(k) - \Delta x'(k-1)\| + \mu \|\Delta \Phi'(k)\| \quad (11)$$

where $\Delta \Phi'(k) = \Phi'(k) - \Phi'(k-1)$, $\mu > 0$ is a weighting factor, $x'(k) = [\hat{x}_1^T(k), T^T y(k)]^T$, $\Delta x'(k) = x'(k) - x'(k-1)$ and $\hat{\Phi}'(k) = [\hat{A}(k), \hat{B}(k)]$ is an estimate of $\Phi'(k)$.

Consider the criterion $\frac{\partial J(\Phi'(k))}{\partial \Phi'(k)} = 0$, which yields

$$\begin{aligned} \hat{A}(k) &= \hat{A}(k-1) + \frac{\eta \Delta x'(k) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \hat{A}(k-1) \Delta x'(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \hat{B}(k-1) \Delta u(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \end{aligned} \quad (12)$$

$$\begin{aligned} \hat{B}(k) &= \hat{B}(k-1) + \frac{\eta \Delta x'(k) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \hat{A}(k-1) \Delta x'(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \hat{B}(k-1) \Delta u(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \end{aligned} \quad (13)$$

where $\eta \in (0, 1]$ is the step-size of the PPD matrix. $\hat{A}_{ij}(k) = \hat{A}_{ij}(0)$ if $\text{sign}(\hat{A}_{ij}(k)) \neq \text{sign}(\hat{A}_{ij}(0))$ where $\hat{A}_{ij}(k)$ is the element of row i and column j of $\hat{A}(k)$. $\hat{B}(k) = \hat{B}(0)$ if

$\text{sign}(\hat{B}(k)) \neq \text{sign}(\hat{B}(0))$ where $\hat{B}_{ij}(k)$ is the element of row i and column j of $\hat{B}(k)$.

Theorem 1: Consider system (8) which is assumed to satisfy Assumptions 1-4. If $L' \geq \|\hat{A}(k)\| + d_{\max} \|C\| + \gamma'$ where L' is the minimum eigenvalue of LC , $\gamma' = \|\tilde{A}(k)\| + \gamma$, and $\gamma = \max\{\|\tilde{B}(k)\| + (b_3 - \|G\|)\|C\|, \|\tilde{A}(k)\|\}$, $2b_3 < \|G\| < 6b_3$, then $x(k)$, $A(k)$ and $B(k)$ can be estimated by $\hat{x}(k)$, $\hat{A}(k)$ and $\hat{B}(k)$, respectively. Further, the estimation errors, $\tilde{A}(k) = \hat{A}(k) - A(k)$, $\tilde{B}(k) = \hat{B}(k) - B(k)$, $\tilde{d}'(k) = \hat{d}'(k) - d'(k)$ and $\tilde{x}(k) = \hat{x}(k) - x(k)$ are bounded.

Proof: From (8) and (11), it can be seen that

$$\begin{aligned} \tilde{A}(k) &= \hat{A}(k-1) - A(k) \\ &\quad + \frac{\eta \Delta x'(k) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta (\hat{A}(k-1) \Delta x'(k-1) + \hat{B}(k-1) \Delta u(k-1))}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad \times \Delta x'^T(k-1) \end{aligned}$$

It follows that

$$\begin{aligned} \tilde{A}(k) &= \tilde{A}(k-1) - \frac{\eta \tilde{A}(k-1) \Delta x'(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \tilde{B}(k-1) \Delta u(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} - \Delta A(k) \end{aligned}$$

where $\Delta A(k) = A(k) - A(k-1)$.

In a similar way

$$\begin{aligned} \tilde{B}(k) &= \tilde{B}(k-1) - \frac{\eta \tilde{B}(k-1) \Delta u(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \\ &\quad - \frac{\eta \tilde{A}(k-1) \Delta x'(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} - \Delta B(k) \end{aligned}$$

where $\Delta B(k) = B(k) - B(k-1)$.

Taking the norm of both sides of the above equations yields

$$\begin{aligned} \|\tilde{A}(k)\| &\leq \left\| \tilde{A}(k-1) \left(I - \frac{\eta \Delta x'(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right) \right\| \\ &\quad + \left\| \frac{\eta \tilde{B}(k-1) \Delta u(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right\| + \|\Delta A(k)\| \\ \|\tilde{B}(k)\| &\leq \left\| \tilde{B}(k-1) \left(I - \frac{\eta \Delta u(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right) \right\| \\ &\quad + \left\| \frac{\eta \tilde{A}(k-1) \Delta x'(k-1) \Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right\| + \|\Delta B(k)\| \end{aligned}$$

According to Lemma 2 and (8), it follows that

$$\begin{cases} \Delta A(k) \leq 2b_1 \\ \Delta B(k) \leq 2b_2 \end{cases} \quad (14)$$

Consider $\eta \in (0, 1]$ and $\mu > 0$, such that

$$\left\| I - \frac{\eta \Delta x'(k-1) \Delta x'^T(k-1)}{\mu + \|\Delta x'(k)\|^2 + \|\Delta u(k)\|^2} \right\| \in (0, 1)$$

It is further obtained that

$$\left\| \frac{\Delta x'(k-1)\Delta u^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right\| \in \left(0, \frac{1}{2}\right)$$

$$\left\| \frac{\Delta u(k-1)\Delta x'^T(k-1)}{\mu + \|\Delta x'(k-1)\|^2 + \|\Delta u(k-1)\|^2} \right\| \in \left(0, \frac{1}{2}\right)$$

Thus, there exists a positive constant $0 < d_1 < 1$ and $0 < d_2 < 1$ for any $k > 1$ such that the following inequality holds

$$\begin{aligned} \|\tilde{A}(k)\| &\leq d_1 \|\tilde{A}(k-1)\| + d_1 \|\tilde{B}(k-1)\| + \|\Delta A(k)\| \\ &\leq d_1^2 \|\tilde{A}(k-2)\| + d_1^2 \|\tilde{B}(k-2)\| \\ &\quad + d_1 \|\Delta A(k-1)\| + d_1 \|\Delta B(k-1)\| + \|\Delta A(k)\| \\ &\leq d_1^k \|\tilde{A}(0)\| + d_1^k \|\tilde{B}(0)\| + \frac{2b_1(1-d_1^k)}{1-d_1} \\ &\quad + \frac{2b_2(1-d_1^{k-1})}{1-d_1} \end{aligned} \quad (15)$$

In a similar way,

$$\begin{aligned} \|\tilde{B}(k)\| &\leq d_2^k \|\tilde{A}(0)\| + d_2^k \|\tilde{B}(0)\| \\ &\quad + \frac{2b_2(1-d_2^k)}{1-d_2} + \frac{2b_1(1-d_2^{k-1})}{1-d_2} \end{aligned} \quad (16)$$

where b_1 and b_2 are given by (14).

It can be obtained that the time-varying estimation errors relating to the parameter matrices $\tilde{A}(k)$, $\tilde{B}(k)$ are bounded. According to Assumption 2, Lemma 2 and (8), $A(k)$, $B(k)$ are bounded. Hence, $\hat{A}(k)$, $\hat{B}(k)$ are also bounded.

Define a Lyapunov function as

$$\begin{aligned} V(k) &= \tilde{x}^T(k)\tilde{x}(k) + \tilde{d}'^T(k)\tilde{d}'(k) \\ &\quad + \frac{1}{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j)} + \frac{1}{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j)} \end{aligned} \quad (17)$$

where $\tilde{A}_i(k)$ and $\tilde{B}_i(k)$ denote the i_{th} column vector of $\tilde{A}(k)$ and $\tilde{B}(k)$, respectively.

From (17), it can be seen that

$$\begin{aligned} \Delta V(k+1) &= V(k+1) - V(k) \\ &= \frac{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j) - \sum_{i=1}^n \sum_{j=0}^{k+1} \tilde{A}_i^T(j)\tilde{A}_i(j)}{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j) \sum_{i=1}^n \sum_{j=0}^{k+1} \tilde{A}_i^T(j)\tilde{A}_i(j)} \\ &\quad + \frac{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j) - \sum_{i=1}^m \sum_{j=0}^{k+1} \tilde{B}_i^T(j)\tilde{B}_i(j)}{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j) \sum_{i=1}^m \sum_{j=0}^{k+1} \tilde{B}_i^T(j)\tilde{B}_i(j)} \\ &\quad + \Delta \tilde{x}^T(k+1)\Delta \tilde{x}(k+1) + \Delta \tilde{d}'^T(k+1)\Delta \tilde{d}'(k+1) \end{aligned}$$

Let $\Delta V(k+1) = \Delta V_1(k+1) + \Delta V_2(k+1) + \Delta V_3(k+1)$ with

$$\begin{aligned} \Delta V_1(k+1) &= \frac{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j) - \sum_{i=1}^n \sum_{j=0}^{k+1} \tilde{A}_i^T(j)\tilde{A}_i(j)}{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j) \sum_{i=1}^n \sum_{j=0}^{k+1} \tilde{A}_i^T(j)\tilde{A}_i(j)} \\ &\quad + \frac{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j) - \sum_{i=1}^m \sum_{j=0}^{k+1} \tilde{B}_i^T(j)\tilde{B}_i(j)}{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j) \sum_{i=1}^m \sum_{j=0}^{k+1} \tilde{B}_i^T(j)\tilde{B}_i(j)} \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta V_2(k+1) &= (\tilde{x}(k) + (\hat{A}(k) - LC)\Delta \tilde{x}(k) \\ &\quad + \tilde{A}(k)\Delta x(k) + \tilde{B}(k)\Delta u(k) + \tilde{d}'(k))^T \\ &\quad \times (\tilde{x}(k) + (\hat{A}(k) - LC)\Delta \tilde{x}(k) + \tilde{A}(k)\Delta x(k) \\ &\quad + \tilde{B}(k)\Delta u(k) + \tilde{d}'(k)) - \tilde{x}^T(k)\tilde{x}(k) \end{aligned} \quad (19)$$

$$\begin{aligned} \Delta V_3(k+1) &= \tilde{d}'^T(k+1)\tilde{d}'(k+1) - \tilde{d}'^T(k)\tilde{d}'(k) \\ &= (\tilde{d}'(k+1) - \tilde{d}'(k))^T(\tilde{d}'(k+1) + \tilde{d}'(k)) \\ &= (G\Delta y(k+1) - \Delta d'(k+1))^T \\ &\quad (2\tilde{d}'(k+1) - G\Delta y(k+1) + \Delta d'(k+1)) \\ &\leq ((\|G\| - 2b_3)\|\Delta y(k+1)\|) \\ &\quad ((6b_3 - \|G\|)\|\Delta y(k+1)\|) \\ &\leq 0 \end{aligned} \quad (20)$$

It can be seen that

$$\begin{aligned} \Delta V_1(k+1) &= \frac{-\sum_{i=1}^n \tilde{A}_i^T(k+1)\tilde{A}_i(k+1)}{\sum_{i=1}^n \sum_{j=0}^k \tilde{A}_i^T(j)\tilde{A}_i(j) \sum_{i=1}^n \sum_{j=0}^{k+1} \tilde{A}_i^T(j)\tilde{A}_i(j)} \\ &\quad + \frac{-\sum_{i=1}^m \tilde{B}_i^T(k+1)\tilde{B}_i(k+1)}{\sum_{i=1}^m \sum_{j=0}^k \tilde{B}_i^T(j)\tilde{B}_i(j) \sum_{i=1}^m \sum_{j=0}^{k+1} \tilde{B}_i^T(j)\tilde{B}_i(j)} < 0 \end{aligned}$$

Since system (8) is obtained by coordinate transformation of system (4), system (8) satisfies Assumption 2.

$$\begin{aligned} &\|\tilde{A}(k)\Delta x(k) + \tilde{B}(k)\Delta u(k) + \tilde{d}'(k)\| \\ &= \|\tilde{A}(k)\Delta \tilde{x}(k) + \tilde{B}(k)\Delta u(k) + \tilde{d}'(k) - \tilde{A}(k)\Delta \hat{x}(k)\| \\ &\leq \|\tilde{A}(k)\| \|\Delta \tilde{x}(k)\| \\ &\quad + \|\tilde{B}(k)\Delta x(k) + (b_3 - \|G\|)\|C\| \|\Delta x(k)\| - \tilde{A}(k)\Delta \hat{x}(k)\| \end{aligned} \quad (21)$$

Let $\gamma = \max\{\|\tilde{B}(k)\| + (b_3 - \|G\|)\|C\|, \|\tilde{A}(k)\|\}$. Since $\tilde{B}(k)$ is bounded, then (21) can be written as

$$\begin{aligned} &\|\tilde{A}(k)\Delta x(k) + \tilde{B}(k)\Delta u(k)\| \\ &\leq \|\tilde{A}(k)\| \|\Delta \tilde{x}(k)\| + \gamma \|\Delta \tilde{x}(k)\| \\ &\leq \gamma' \|\Delta \tilde{x}(k)\| \end{aligned}$$

Applying (17) to (20), it follows that

$$\begin{aligned} \Delta V_2(k+1) &\leq (\tilde{x}(k) + (\|\hat{A}(k)\| + \gamma' - L')\|\Delta\tilde{x}(k)\|\mathbf{1}_n)^T \\ &\quad \times (\tilde{x}(k) + (\|\hat{A}(k)\| + \gamma' - L')\|\Delta\tilde{x}(k)\|\mathbf{1}_n) - \tilde{x}^T(k)\tilde{x}(k) \end{aligned}$$

where $\mathbf{1}_n \in \mathbb{R}^n$ is an n -dimensional column vector with all elements 1, $L' \geq \|\hat{A}(k)\| + \gamma'$, then $\Delta V_2(k+1) < 0$. $\Delta V(k+1) = \Delta V_1(k+1) + \Delta V_2(k+1) + \Delta V_3(k+1) < 0$, then $\tilde{A}(k)$, $\tilde{B}(k)$, $\tilde{d}'(k)$ and $\tilde{x}(k)$ are bounded. Q.E.D.

Remark 5: From the proof of Theorem 1, it can be seen that the tracking error is guaranteed to be bounded. The problem of the coupling between the state and system parameters is solved through coordinate transformation and the adaptive algorithm (10-12). The coupling of $\hat{A}(k)$ and $\hat{B}(k)$ is solved by the recursive inequalities (15) and (16). The system parameter estimation errors are affected by the initial values including $\hat{A}(0)$, $\hat{B}(0)$ and $\hat{x}(0)$. From (15) and (16), it can be seen that the bounds on the estimation errors are also a function of b_1 and b_2 . However, the parameters b_1 and b_2 are not explicitly used in the design and it will be seen in the simulation and experimental testing that it is possible to render the estimation errors small using the proposed design framework.

IV. MODEL-FREE ADAPTIVE STATE FEEDBACK CONTROL DESIGN

Let $x_d(k)$ be the desired state trajectory and define the tracking error as

$$e(k) = x_d(k) - x(k) \quad (22)$$

The optimal controller is obtained from the criterion function:

$$J(u(k)) = \|x_d(k+1) - \hat{x}(k+1)\| + \mu' \|\Delta u(k)\| \quad (23)$$

where $\Delta u(k) = u(k) - u(k-1)$, $\mu' > 0$ is a weighting factor.

From $\frac{\partial J(u(k))}{\partial u(k)} = 0$, it follows that

$$\begin{aligned} u(k) &= u(k-1) \\ &\quad + \frac{\eta' \hat{B}^T(k)(x_d(k+1) - \hat{x}(k) - \hat{A}(k)\Delta\hat{x}(k) - \hat{d}'(k))}{\mu' + \|\hat{B}(k)\|^2} \end{aligned} \quad (24)$$

where $\eta' \in (0, 1]$ is the step-size of the control.

The resulting algorithm is presented step by step as follows:
Algorithm 1:

Step 1 (initialization): Let $k = 0$ and set the initial value of the system parameters.

Step 2 (state estimation): Solve for $\hat{x}(k)$ by using the MFAO in (9).

Step 3 (PPD matrix estimation): Solve for $\hat{d}'(k)$, $\hat{A}(k)$ and $\hat{B}(k)$ using (11) (11) and (12). If $(\hat{A}(k); C)$ is unobservable, let $\hat{A}(k) = \hat{A}(0)$. If $(\hat{A}(k); \hat{B}(k))$ is uncontrollable, let $\hat{A}(k) = \hat{A}(0)$ and $\hat{B}(k) = \hat{B}(0)$.

Step 4 (control design): Solve for $u(k)$ from (24).

Step 5 (implementation): Apply the control to the system. Move to the control interval $k = k+1$, return to step 2 and repeat the procedure. End of Algorithm 1.

Remark 6: Algorithm 1 defines a model-free state feedback control for a general nonlinear system. The input and output of the system are strictly needed to be known during Steps 2-4. From Algorithm 1, it can be seen that the proposed control design procedure is straightforward. For practical application, the control can be directly designed according to Algorithm 1 and the controller can be implemented straightforwardly in any computer language. As long as the system satisfies the assumptions in this paper, the control can be directly used in the system. The proposed method can be extended to the continuous time system. In real systems, the plants are time continuous, but most of the controllers are digital computers and are essentially discrete. With the help of sampling technology, the state information of the plants can be transformed into discrete data, which can be used to design the proposed control. With the help of the holder, the discrete time control can be converted into the continuous time variable, which makes the discrete controller applicable to the continuous systems. Definitely, the proposed method can be extended to the continuous time system.

Remark 7: A method by which $\hat{A}(0)$ and $\hat{B}(0)$ are chosen has been given in [20] and [33]. They can be chosen by using inverse calculation PID parameters and diagonal dominance condition methods as detailed in [20] and [33]. The selection of $\hat{x}(0)$ is trial and error, which is an interesting issue and will be further studied in the authors' future work. According to Lemma 2 and (8), it can be obtained that $A(k)$ and $B(k)$ are bounded. When $(\hat{A}(k); C)$ is unobservable or $(\hat{A}(k); \hat{B}(k))$ is uncontrollable, Algorithm 1 Step 3 will select $\hat{A}(k) = \hat{A}(0)$ and $\hat{B}(k) = \hat{B}(0)$ where $\hat{A}(0)$ and $\hat{B}(0)$ are bounded. Accordingly, $\|\tilde{A}(k)\|$ and $\|\tilde{B}(k)\|$ are still bounded. It can be seen from Theorem 1 and Theorem 2 that the system can be guaranteed to be stable as long as $\|\tilde{A}(k)\|$ and $\|\tilde{B}(k)\|$ are bounded.

Remark 8: How to deal with the observation error is very important especially in the presence of the external disturbance. In this paper, the authors used a disturbance observer to cope with the external disturbance. In light of the disturbance observer, the observation error will be bounded and can be stabilized under the proposed model free adaptive state feedback control. Robust control is a good approach to deal with the observation error however it is model based method. Note that the proposed approach is a model free type control.

Theorem 2: For system (8), under Assumptions 1-4 and considering Theorem 1, if the state feedback control is designed as (24), $x(k)$ is estimated by (9), and $d'(k)$, $A(k)$, $B(k)$ are estimated by (10)-(13), then $e(k)$ is bounded and converges to $\Omega = \{e(k) | \|e(k)\| \leq \frac{S}{1 - \|\Gamma_n - \Sigma(k)\|} + \delta\}$, where S is the upper bound of $\| -A(k)\Delta x(k) - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta\hat{x}(k)) - \tilde{d}'(k) + \Delta x_d(k+1) \|$, δ is a small positive constant and $\Sigma(k) = \frac{\eta' B(k) \hat{B}^T(k)}{\mu' + \|\hat{B}(k)\|^2}$.

Proof: Define a Lyapunov function as

$$V'(k) = e^T(k)e(k) \quad (25)$$

Substituting from (8) into equation (25)

$$\Delta V'(k+1) = e^T(k+1)e(k+1) - e^T(k)e(k)$$

$$\begin{aligned}
&= (e(k) - A(k)\Delta x(k) - B(k)\Delta u(k) \\
&\quad - \tilde{d}'(k) - \Delta x_d(k+1))^T \\
&\quad \times (e(k) - A(k)\Delta x(k) - B(k)\Delta u(k) \\
&\quad - \tilde{d}'(k) - \Delta x_d(k+1)) \\
&\quad - e^T(k)e(k)
\end{aligned} \tag{26}$$

Substituting $\Sigma(k) = \frac{\eta' B(k)\tilde{B}^T(k)}{\mu' + \|\tilde{B}(k)\|^2}$ and taking $\Delta u(k)$ into (26), it follows that

$$\begin{aligned}
\Delta V'(k+1) &= ((I_n - \Sigma(k))e(k) - A(k)\Delta x(k) - \tilde{d}'(k) + \Delta x_d(k+1) \\
&\quad - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta \hat{x}(k)))^T \\
&\quad \times ((I_n - \Sigma(k))e(k) - A(k)\Delta x(k) - \tilde{d}'(k) + \Delta x_d(k+1) \\
&\quad - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta \hat{x}(k))) - e^T(k)e(k)
\end{aligned} \tag{27}$$

Let

$$\begin{aligned}
Z(k) &= -A(k)\Delta x(k) - \tilde{d}'(k) + \Delta x_d(k+1) \\
&\quad - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta \hat{x}(k))
\end{aligned}$$

It follows that

$$\begin{aligned}
Z(k) &= -A(k)\Delta \tilde{x}(k) - A(k)\Delta \hat{x}(k) - \tilde{d}'(k) + \Delta x_d(k+1) \\
&\quad - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta \hat{x}(k)) \\
&\leq (-A(k))\Delta \tilde{x}(k) - (-A(k))\Delta \hat{x}(k) - \tilde{d}'(k) \\
&\quad - \Sigma(k)(\Delta x_d(k+1) + \tilde{x}(k) - \hat{A}(k)\Delta \hat{x}(k)) + \Delta x_d(k+1)
\end{aligned}$$

Since the elements in $Z(k)$ are bounded according to Theorem 1, Assumption 2 and (8), $\|Z(k)\|$ has an upper bound S . Then

$$\begin{aligned}
\Delta V'(k+1) &\leq (\|(I_n - \Sigma(k))e(k)\| + S) \\
&\quad \times (\|(I_n - \Sigma(k))e(k)\| + S) - e^T(k)e(k) \\
&\leq (\|(I_n - \Sigma(k))\| \|e(k)\| + S) \\
&\quad \times (\|(I_n - \Sigma(k))\| \|e(k)\| + S) - e^T(k)e(k) \\
&= (\|(I_n - \Sigma(k))\| \|e(k)\| + S)^2 - \|e(k)\|^2
\end{aligned} \tag{28}$$

Since $\Sigma(k) > 0$ and $\|\Sigma(k)\| < 1$, so $\|I_n - \Sigma(k)\| < 1$. If $(\|(I_n - \Sigma(k))\| \|e(k)\| + S) < \|e(k)\|$, that is $\|e(k)\| > \frac{S}{1 - \|I_n - \Sigma(k)\|}$, $\Delta V'(k+1) < 0$ and $\|e(k+1)\|$ is decreasing. As $\|e(k+1)\|$ decreases, $\Delta V'(k+1) > 0$, and then $\|e(k+1)\|$ will increase until $\|e(k)\| > \frac{S}{1 - \|I_n - \Sigma(k)\|}$. This implies that $\|e(k)\|$ will always remain near $\frac{S}{1 - \|I_n - \Sigma(k)\|}$. In conclusion, $e(k)$ is bounded and lies in the set $\Omega = \{e(k) \mid \|e(k)\| \leq \frac{S}{1 - \|I_n - \Sigma(k)\|} + \delta\}$.

Remark 9: From the proof of Theorem 2, it can be seen that the tracking error is guaranteed to be bounded. All elements in $Z(k)$ are bounded, so S must exist. The value of S is affected by $\hat{A}(k)$, $\tilde{B}(k)$ and $\tilde{x}(k)$, but the size of S does not affect the stability of the system. The bounded results is enough for real applications. In many practical applications, it will only require Lyapunov stability in order to avoid energy waste and actuator wear. For example, in the chemical production process, it often requires that the controlled variable is kept within a certain range rather than a fixed value. The experimental

TABLE I VALUES OF THE SYSTEM PARAMETERS			
$x_{11}(0) = 1$	$x_{12}(0) = 1$	$x_{21}(0) = 1$	$x_{22}(0) = 1$
$\hat{x}_{11}(0) = 0$	$\hat{x}_{12}(0) = 0$	$\hat{x}_{21}(0) = 0$	$\hat{x}_{22}(0) = 0$
$x_{d11}(k) = 2$	$x_{d12}(k) = 4$	$x_{d21}(k) = 4$	$x_{d22}(k) = 15.294$

results also verify the effectiveness of the proposed method in practical applications.

V. SIMULATION AND EXPERIMENTAL VERIFICATION

To validate the proposed approach, comparisons are made between the proposed control and a classical MFAC scheme [20].

A. Simulation

Case 1: Analysis of disturbance rejection capability.

A MIMO nonlinear system is given by [33]

$$\begin{cases}
x_{11}(k+1) = \frac{x_{11}^2(k)}{1 + x_{11}^2(k)} + 0.3x_{12}(k) + d^*(k) \\
x_{12}(k+1) = \frac{x_{11}^2(k)}{1 + x_{12}^2(k) + x_{21}^2(k) + x_{22}^2(k)} + u_1(k) \\
x_{21}(k+1) = \frac{x_{21}^2(k)}{1 + x_{21}^2(k)} + 0.2x_{22}(k) + d^*(k) \\
x_{22}(k+1) = \frac{x_{21}^2(k)}{1 + x_{11}^2(k) + x_{12}^2(k) + x_{22}^2(k)} + u_2(k) \\
y_1(k) = x_{11}(k) + w_1 \\
y_2(k) = x_{21}(k) + w_2
\end{cases}$$

where $x_{11}(k)$ and $x_{21}(k)$ are measured values from the sensors, $x_{12}(k)$ and $x_{22}(k)$ are the internal states, $u_1(k)$ and $u_2(k)$ are the inputs, $w_1(k) = 0.01x_{11}(k)\frac{1}{\sqrt{2\pi}}e^{(-\frac{k}{2})}$ and $w_2(k) = 0.01x_{21}(k)\frac{1}{\sqrt{2\pi}}e^{(-\frac{k}{2})}$ are the random noises, $y_1(k)$ and $y_2(k)$ are the outputs.

The disturbance is given by

$$\begin{cases}
d^*(k) = 0.6 & \text{if } 100 < k < 200 \text{ and } 1000 < k < 1400 \\
d^*(k) = 0 & \text{else}
\end{cases}$$

The initial matrices are

$$\begin{aligned}
\hat{A}(0) &= \begin{bmatrix} 0 & 0 & 0.02 & 0 \\ 0 & 0 & 0 & 0.02 \\ 1 & 0 & 0.02 & 0 \\ 0 & 1 & 0 & 0.02 \end{bmatrix} \\
\hat{B}(0) &= \begin{bmatrix} 0.2 & 0.04 \\ 0.04 & 0.2 \\ 0.2 & 0.04 \\ 0.04 & 0.2 \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{aligned}$$

The remaining system parameters are given in Table I.

The parameters of the SFMFAC and MFAC are listed in Table II.

TABLE II
VALUES OF THE CONTROLLER PARAMETERS

SFMFAC	$\eta' = 0.2$	$\mu' = 0.1$	$\eta = 0.001$	$\mu = 0.1$
	$L = [0.3 \ 0; 0 \ 1; 0.8 \ 0.1; 0 \ 1]$		$G = [0 \ 0; 0.1 \ 0; 0 \ 0; 0 \ 0.1]$	
MFAC	$\eta' = 0.2$	$\mu' = 0.1$	$\eta = 0.001$	$\mu = 0.1$

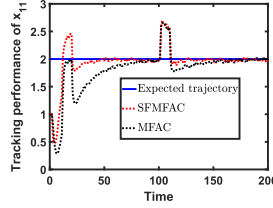


Fig. 2. Tracking performance of x_{11} .

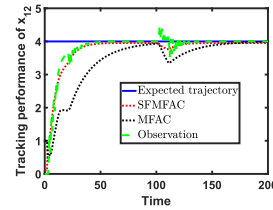


Fig. 3. Tracking performance of x_{12} .

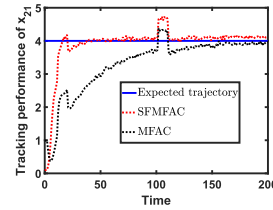


Fig. 4. Tracking performance of x_{21} .

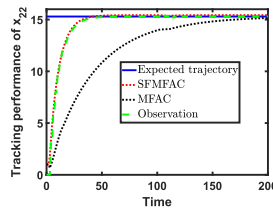


Fig. 5. Tracking performance of x_{22} .

The simulation results show that the SFMFAC algorithm can effectively deal with the disturbance and measurement noise. Fig. 2 - Fig. 5 show the tracking performance. Each state of the SFMFAC is stabilized into a neighbourhood of the expected value. From Fig. 2-5, it can be seen that the performance of SFMFAC is better and has a more rapid convergence rate than MFAC. The robustness of SFMFAC is also better than that of MFAC in the presence of the external disturbance. Under the influence of measurement noise, SFMFAC and MFAC both have good tracking performance. The advantages of the proposed approach are particularly noticeable for the unmeasured system states x_{12} and x_{22} . As MFAC is an output feedback control, it has reduced ability to guarantee the stability of

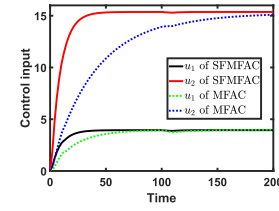


Fig. 6. Time response of control inputs.

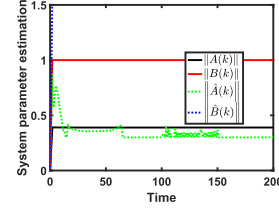


Fig. 7. System parameter estimation.

the unmeasured states. Fig. 6 shows the corresponding control signals. Fig. 7 shows that the system parameter estimates track the corresponding actual parameter values using the proposed method, which verifies that the estimation errors $\hat{A}(k)$ and $\hat{B}(k)$ can converge to a small region.

Case 2: Comparison of the control performance of the internal states.

A wheeled mobile robot (WMR) system is used in this case, as shown in Fig. 8. The dynamical model can be written as

$$\begin{cases} w(k) = \frac{v_l(k) - v_r(k)}{D'} \\ \theta(k) = \theta(k-1) + w(k) * T_s \\ v(k) = \frac{v_l(k) + v_r(k)}{2} \end{cases}$$

where $v(k)$ and $w(k)$ are respectively the linear and angular velocities of the mobile robot, $\theta(k)$ is the angular position, $v_l(k)$ and $v_r(k)$ are respectively the linear velocities of the left and right wheels of the mobile robot, D' is the distance between the two wheels, T_s is the sampling time, $w(k)$ and $\theta(k)$ are the outputs, $v(k)$ is the internal state, and $v_l(k)$ and $v_r(k)$ are the control inputs.

The initial matrices are

$$\begin{aligned} \hat{A}(0) &= \begin{bmatrix} 1 & 0.1 & 0.1 \\ -0.1 & 1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \\ \hat{B}(0) &= \begin{bmatrix} 1 & 0.1 \\ -0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

The remaining system parameters are given in Table III.

The parameters of the corresponding SFMFAC and MFAC are listed in Table IV.

The simulation results show that the SFMFAC algorithm effectively deals with the problem where the speed does not return to zero after the WMR system adjusts the angular

TABLE III
VALUES OF THE SYSTEM PARAMETERS

$v(0) = 1$	$w(0) = 1$	$\theta(0) = 1$
$\hat{v}(0) = 0$	$\hat{w}(0) = 0$	$\hat{\theta}(0) = 0$
$v_d(k) = 0$	$w_d(k) = 0$	$\theta_d(k) = 2$

TABLE IV
CONTROLLER PARAMETERS

SFMFAC	$\eta' = 0.2$	$\mu' = 1$	$\eta = 0.4$	$\mu = 1$
	$L = [1 \ 0; 1 \ 0; 0 \ 0.5] \ G = [0 \ 0; 0.1 \ 0; 0 \ 0.1]$			
MFAC	$\eta' = 0.2$	$\mu' = 1$	$\eta = 0.4$	$\mu = 1$

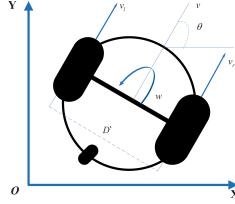


Fig. 8. WMR system platform.

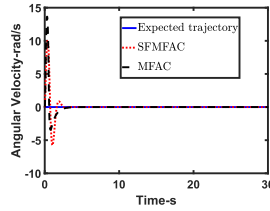


Fig. 9. Tracking performance of angular velocity.

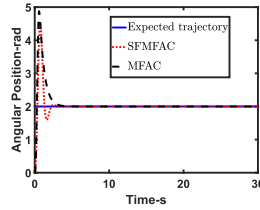


Fig. 10. Tracking performance of angular position.

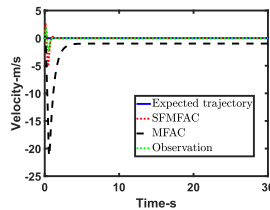


Fig. 11. Tracking performance of velocity.

position. Fig. 9 - Fig. 11 show the tracking performance. The observed states and system states in the SFMFAC converge to the desired trajectory from the initial values with a rapid convergence rate. MFAC cannot guarantee the velocity converges to 0, which means that the WMR will keep moving forward. Fig. 12 shows the control signals of the proposed controller. Fig. 13 shows that the estimation errors can converge to a small

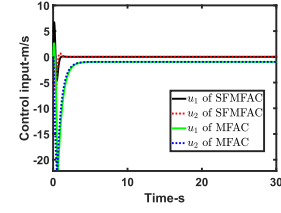


Fig. 12. Time response of control inputs.

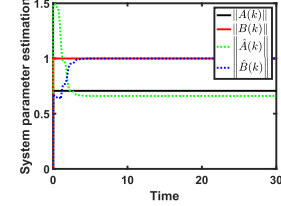


Fig. 13. System parameter estimation.

TABLE V
VALUES OF THE SYSTEM PARAMETERS

$x_1(0) = 0.5$	$x_2(0) = 1$	$x_3(0) = -0.05$	$x_4(0) = 0.05$
$\hat{x}_1(0) = 0.6$	$\hat{x}_2(0) = 0.6$	$\hat{x}_3(0) = -0.1$	$\hat{x}_4(0) = 0.6$

region by the proposed method. This comparison demonstrates that the proposed approach can control the internal states whilst traditional MFAC cannot.

Case 3: A stabilization problem for a continuous system.

A translating oscillator with a rotating actuator system is used in this case [43]. The dynamical model can be written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{k^*}{m_r + m_c} \left(\frac{m_r l_r \sin x_3}{m_r + m_c} - x_1 \right) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{1}{\delta(x_3)} \left((m_r + m_c)u - m_r l_r \cos x_3 \right. \\ \quad \left. \times \left(m_r l_r x_4^2 - k^* \left(x_1 - \frac{m_r l_r \sin x_3}{m_r + m_c} \right) \right) \right) \\ y = x_3 \end{cases}$$

where $m_c = 1.3608 \text{ kg}$, $m_r = 0.096 \text{ kg}$, $l_r = 0.0592 \text{ m}$, $k^* = 186.3 \text{ N/m}$, $\delta(x_3) = (1 + m_r l_r^2)(m_c + m_r) - m_r^2 l_r^2 \cos^2(x_3)$. The sampling time of the zero order hold is 0.01s. The remaining system parameters are given in Table V.

The initial matrices are

$$\begin{aligned} \hat{A}(0) &= \begin{bmatrix} 1 & 0.01 & 0.0001 & 0.0001 \\ 0.0001 & 800 & 0.0001 & 10 \\ 0.0001 & 0.0001 & 1 & 0.01 \\ 0.0001 & 10 & 0 & 800 \end{bmatrix} \\ \hat{B}(0) &= \begin{bmatrix} 0.1 & 0.0001 \\ 0.0001 & 10 \end{bmatrix} \\ C &= [0 \quad 1 \quad 0 \quad 0] \end{aligned}$$

The parameters of the SFMFAC and MFAC are listed in Table VI.

TABLE VI
VALUES OF THE CONTROLLER PARAMETERS

SFMFAC	$\eta' = 1$ $\mu' = 0.01$	$\eta = 0.001$ $\mu = 2$
	$L = [0.16; 0.01; 0.16; 0.01]$	$G = [0.1; 0; 0; 0.1]$
MFAC	$\eta' = 1$ $\mu' = 0.01$	$\eta = 0.001$ $\mu = 2$

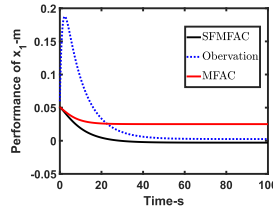


Fig. 14. Stabilizing performance of x_1 .

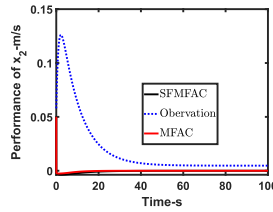


Fig. 15. Stabilizing performance of x_2 .

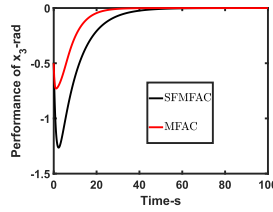


Fig. 16. Stabilizing performance of x_3 .

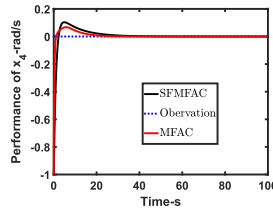


Fig. 17. Stabilizing performance of x_4 .

The simulation results show that the SFMFAC can deal with the stabilization problem of a continuous system. Fig. 14-17 show the performance of the SFMFAC and MFAC, respectively. It can be seen that the estimation errors in the system parameters are bounded. Fig. 14 shows the observed states and system states with the SFMFAC converge to a smaller region than with MFAC. Fig. 18 shows the control signals of the SFMAC and MFAC, respectively.

B. Experimental Trial

In this section, the applicability of the proposed method will be further validated using the ‘‘Saponification reaction continuous stirred tank (CSTR) experimental platform’’ shown

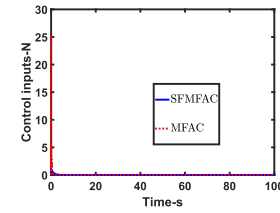


Fig. 18. Time response of control inputs.



Fig. 19. Experimental equipment.

TABLE VII
CONTROLLER PARAMETERS

SFMFAC	$\eta' = 0.4$ $\mu' = 1$	$\eta = 0.1$ $\mu = 1$	$L = [1 \ 0; 1 \ 0; 0 \ 1; 0 \ 1]$
MFAC	$\eta' = 0.4$ $\mu' = 1$	$\eta = 0.1$ $\mu = 1$	

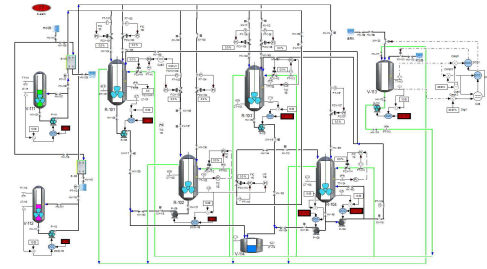


Fig. 20. Experimental operation interface.

in Fig. 19. The operational interface is shown in Fig. 20. There are four tanks. They can be connected in series, parallel, or series and parallel by changing a switch, which is convenient for the verification and testing of control algorithms. Only one tank is used in this experiment. The two tanks containing raw materials are marked V111 and V112 and contain sodium hydroxide and ethyl acetate as required for saponification. V113 is a jacketed tank of water for heat exchange. This experiment seeks to control the temperature and concentration of a reactor. The system dynamic equations of the CSTR have been given in [44]. Set the sampling time as $T_s = 0.45s$. The temperature is used as the output of the system where the initial value is 26 degrees Celsius and the desired value is 29 degrees Celsius. The concentration is the internal state of the system where the initial value is 1.0 kmol/m^3 , and the set value is 2.0 kmol/m^3 . The flow of heat from V113 is the input to the system. The classical MFAC [33] scheme runs under the same experimental conditions for comparison. The parameters of the SFMFAC and MFAC are given in Table VII. Fig. 21 shows the tracking performance of the solution temperature. It can be seen that the proposed method ensures the temperature converges to

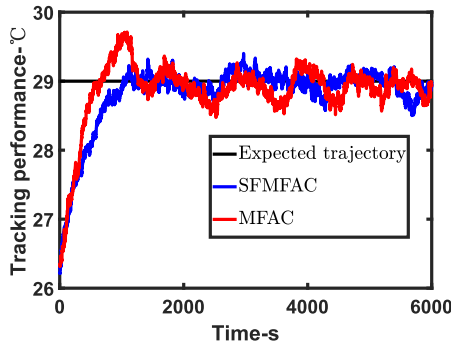


Fig. 21. The output temperature of the CSTR.

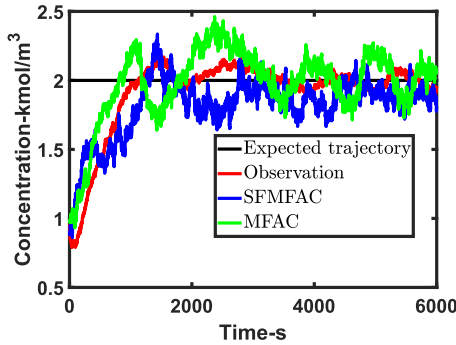


Fig. 22. Concentration tracking performance.

TABLE VIII
ITSE FOR THE TWO METHODS

	Tracking performance	Concentration
SFMFAC	664.98	650.98
MFAC	716.79	701.64

the desired trajectory. The steady-state error with SFMFAC is almost identical to that of MFAC, but the overshoot with SFMFAC is much smaller. The tracking performance of the concentration of the solution is shown in Fig. 22. From the results, the observed state and the actual state converge to the desired trajectory. The steady-state error and overshoot with SFMFAC is much smaller than with MFAC. The Integral of Time Squared Error (ITSE) for the SFMFAC and MFAC are given in Table VIII. It can be seen that the ITSE for the SFMFAC is much smaller than for the case of MFAC. The experimental results further validate the effectiveness of the proposed method.

VI. CONCLUSION

This paper has proposed a state feedback model-free adaptive control for a class of discrete-time MIMO nonlinear systems. Unlike with the usual model-free adaptive control technique, the proposed controller deliberately includes state feedback. The disturbance, output and internal state of the system are considered in the control design. The design of the controller only needs the relationship between the state and output and I/O data to stabilize the tracking error of the system state. Simulations and an experimental trial demonstrate the

advantages of the proposed approach. In the future, more work will be done on robustness and a model free robust control will be studied.

APPENDIX

Detailed explanation of (5) as follows [18]:

$$F_k = F'(z(k), z(k-1), \dots, z(k-n_z), u(k), u(k-1), \dots, u(k-n_u), d(k), d(k-1), \dots, d(k-n_d))$$

$$F_z = F'(z(k-1), z(k-1), z(k-2), \dots, z(k-n_z), u(k), u(k-1), \dots, u(k-n_u), d(k), d(k-1), \dots, d(k-n_d))$$

$$F_u = F'(z(k-1), z(k-1), z(k-2), \dots, z(k-n_z), u(k-1), u(k-1), u(k-2), \dots, u(k-n_u), d(k), d(k-1), \dots, d(k-n_d))$$

$$F_d = F'(z(k-1), z(k-1), z(k-2), \dots, z(k-n_z), u(k-1), u(k-1), u(k-2), \dots, u(k-n_u), d(k-1), d(k-1), d(k-2), \dots, d(k-n_d))$$

$$F_{k-1} = F'(z(k-1), z(k-2), \dots, z(k-n_z-1), \dots, u(k-1), u(k-2), u(k-n_u-1), d(k-1), d(k-2), \dots, d(k-n_d-1))$$

From the definition of $\Delta z(k+1)$,

$$\Delta z(k+1) = F_k - F_z + F_z - F_u + F_u - F_d + F_d - F_{k-1}$$

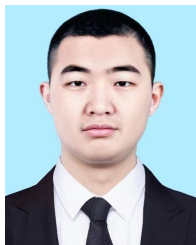
For (5), $\frac{\partial F'^*}{\partial z(k)}$, $\frac{\partial F'^*}{\partial u(k)}$, and $\frac{\partial F'^*}{\partial d(k)}$ can be written as follows

$$\begin{aligned} \frac{\partial F'^*}{\partial z(k)} &= \begin{bmatrix} \frac{\partial F'_1}{\partial z_1(k)} & \frac{\partial F'_1}{\partial z_2(k)} & \cdots & \frac{\partial F'_1}{\partial z_n(k)} \\ \frac{\partial F'_2}{\partial z_1(k)} & \frac{\partial F'_2}{\partial z_2(k)} & \cdots & \frac{\partial F'_2}{\partial z_n(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F'_n}{\partial z_1(k)} & \frac{\partial F'_n}{\partial z_2(k)} & \cdots & \frac{\partial F'_n}{\partial z_n(k)} \end{bmatrix}, \\ \frac{\partial F'^*}{\partial u(k)} &= \begin{bmatrix} \frac{\partial F'_1}{\partial u_1(k)} & \frac{\partial F'_1}{\partial u_2(k)} & \cdots & \frac{\partial F'_1}{\partial u_m(k)} \\ \frac{\partial F'_2}{\partial u_1(k)} & \frac{\partial F'_2}{\partial u_2(k)} & \cdots & \frac{\partial F'_2}{\partial u_m(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F'_n}{\partial u_1(k)} & \frac{\partial F'_n}{\partial u_2(k)} & \cdots & \frac{\partial F'_n}{\partial u_m(k)} \end{bmatrix}, \\ \frac{\partial F'^*}{\partial d(k)} &= \begin{bmatrix} \frac{\partial F'_1}{\partial d_1(k)} & \frac{\partial F'_1}{\partial d_2(k)} & \cdots & \frac{\partial F'_1}{\partial d_m(k)} \\ \frac{\partial F'_2}{\partial d_1(k)} & \frac{\partial F'_2}{\partial d_2(k)} & \cdots & \frac{\partial F'_2}{\partial d_m(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F'_n}{\partial d_1(k)} & \frac{\partial F'_n}{\partial d_2(k)} & \cdots & \frac{\partial F'_n}{\partial d_m(k)} \end{bmatrix}, \end{aligned}$$

where $\frac{\partial F'^*_{i1}}{\partial z_{j1}(k)}$ is the partial derivative of F' to state z'_{j1} , $z'_{j1} \in [z_{j1}(k), z_{j1}(k-1)]$, $\frac{\partial F'^*_{i2}}{\partial u_{j2}(k)}$ is the partial derivative of F' to input u'_{j2} , $u'_{j2} \in [u_{j2}(k), u_{j2}(k-1)]$, $\frac{\partial F'^*_{i3}}{\partial d_{j3}(k)}$ is the partial derivative of F' to disturbance d_{j3} at a point in the interval $[d_{j3}(k), d_{j3}(k-1)]$, $i1 = i2 = i3 = 1, \dots, n$, $j1 = 1, \dots, n$, $j2 = j3 = 1, \dots, m$.

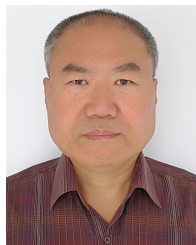
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