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Central Bank Reserves and Currency Volatility

Alex Ferreira¹ · Arie Gozluklu² · João Mainente³ 

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Abstract

We investigate the effects of sterilised Central Bank interventions designed to smooth exchange rate volatility but not aimed at a particular trend in the spot rate. We present a model in which the intervention flow is a nonlinear mapping of the market order flow. Simulations from our model show that small daily drifts that can arise from various reasons beyond the central bank's control lead to major changes in both the stock of foreign reserves and the distribution of currency returns. The likelihood of a major impact increases with the Central Bank's inability or reluctance to adopt a neutral policy regarding the exchange rate drifts. Using detailed intervention data from the Central Bank of Brazil, we provide empirical support for the link between exchange rate drifts and Central Bank interventions.

Keywords Central Bank · Currency interventions · Foreign reserves · Order flow

JEL Classification F31 · F33 · F37

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1 Introduction

Many Central Banks (CBs) report to the International Monetary Fund (IMF) that they follow a flexible exchange rate regime. However, some of the major CBs in emerging markets as well as developed economies also declare that they are active in the foreign exchange market. The most commonly stated reason for intervention is to smooth exchange rate *volatility*, with no aim to affect the (trend) level of the exchange rate (Chutasripanich and Yetman 2015). Deliberate actions to affect the trend would result in beggar-thy-neighbour policies which can ultimately lead to currency wars and accusations of currency manipulation (see Dominguez 2020 for the challenges in identifying manipulative interventions). Concerns about exchange rate volatility, on the other hand, are closely linked to a country's currency and sovereign risk as well as economic performance (e.g., Della Corte et al. 2018; Alla et al. 2019), especially for those countries that rely heavily on foreign credit to maintain growth, e.g., Brazil.¹ In fact, CB intervention in the form of reserve accumulation has become commonplace in the post-crisis period.

We formally investigate the *de facto* flow and stock implications of a policy that closely follows this *de jure* rhetoric. Our main research question is whether a policy function aimed at (smoothing) the second moment of the exchange rate change is neutral with regards to the first moment of the distribution. We also ask whether there are any significant collateral implications for the balance sheets of CBs of such a reaction (intervention) function in line with Chen and Lin (2019) work. The latter has been at the center of policy discussions in response to the Global Financial Crisis.

Standard open macroeconomic models suggest that an interventionist policy that does not aim at impacting the level of the exchange rate would result in zero net accumulation of foreign assets. However, there are numerous examples of trends in the stocks of foreign exchange reserves held by CBs across the world. There were many known speculative attacks on emerging economies in the 1990s and 2000s leading to sharp decreases in reserves. They mostly culminated with currency pegs being abandoned in favor of more flexible exchange rates. In the 2000s, these trends were mostly positive in emerging markets (Obstfeld et al. 2010; Menkhoff 2013), including countries that report to the IMF as floating regimes.

As we will discuss in detail, our hybrid macro-microstructure model can capture many of these episodes. However, our empirical analysis starts by focusing on one specific case, namely the Central Bank of Brazil (CBB)². We have two main reasons to start with the CBB case. First, although self-reporting to the IMF as a “floating system”, CBB accumulated around 150 billion USD (United States dollars) of foreign

¹ Exchange rate volatility can also impact on the balance sheet of private business, economic growth, international trade, inflation, inflation expectations, etc. A crucial paper that explains why volatility is particularly distressful for emerging economies is Calvo and Reinhart (2002).

² We also propose a generalization that would apply to any CB, focusing on the Exchange Equalisation Account (EEA) run by the Bank of England (BoE) as a further example for developed economies in the Appendix A. The EEA case is interesting in this regard given that the BoE self-reports and is classified as a free floater by the IMF but it had accumulated an increasing amount of foreign reserves. The government's endowment to the EEA-BoE for the purchase of foreign reserves has increased to six billion GBP (Great British Pound), *per year* since 2010.

reserves through spot market operations between 2009 and 2011. This is a typical case where we observe a difference between *de jure* and *de facto* classifications. Second, the CBB recently published a new dataset containing the full information on its exchange rate market intervention, which allows us to test if foreign reserves changes are good proxies for the direction of the interventions.

The increase in the stock level of the CBB's reserves could seem excessive if analyzed solely from the objective of smoothing volatility. For example, estimates show that the net impact of one billion buyer-initiated over seller-initiated trades of USD by the CBB (or the CBB order flow) leads to an approximate 0.5% increase³ in the exchange rate. Because the CBB's foreign reserve stock at the end of 2020 was around 360 billion USD, a 1% offsetting order flow *per day* could, in principle, be generated over a total of 180 business days. Hence, the accumulation of foreign assets may imply that there is either a much higher weight placed on other objectives or there are serious - intended or unintended - collateral effects of the smoothing volatility policy on the stocks of foreign reserves. Our paper investigates the latter hypothesis.

We understand that the CB's goal of smoothing volatility is secondary to the main monetary policy objective: namely inflation control⁴. The (conventional) instrument used to achieve this primary goal is the interest rate which is frequently under the discretion of a monetary policy committee. However, the smoothing problem requires a different instrument or variable, frequently under the responsibility of a different sector of the CB. In our model, we assume that this variable is the CB order flow⁵. This particular modeling strategy has the advantage of incorporating both the traditional portfolio and signaling channels of the sterilized intervention policy literature (see, for example, Sarno and Taylor (2001)) in an underlying microstructure framework based on Evans and Lyons (2002). Our paper thus complements not only those numerous papers on policy reaction functions for the nominal interest rate but also those that investigate the effects of foreign exchange interventions.

The CB reaction function in our model primarily depends on the impact of the market order flow and an exogenous target for the exchange rate volatility. The target represents a constraint to a benevolent monetary authority, which minimizes the present value of future losses caused by excess exchange rate volatility. Both our analytical and simulation results reveal that if the underlying data generation process of the exchange rate does not contain a drift, net foreign exchange reserve accumulation will, on average, be zero. This will also be the case if there is indeed a known drift and the CB adopts a neutral drift policy concerning its impact on the exchange rate. However, the cost of a neutral policy will be higher than the target exchange rate volatility. These impacts on the level of foreign reserves contrast with periods in which the CB applies non-neutral drift policies. Even small daily drifts, arising from speculative attacks, persistent swings in commodity prices, changes in terms of trade,

³This impact is around half of the magnitude that is estimated for the corresponding market order flow - see Barroso (2018) and Kohlscheen and Andrade (2014).

⁴Alla et al. (2019) show that FX intervention not only reduces the volatility of the exchange rate but also of inflation and output gap in a New Keynesian model.

⁵The net of buyers over sellers initiated trades, based on the high-frequency evidence (Chari 2007)

global financial cycles, or Peso problems, for instance, can imply large appreciating or depreciating *ex post* trends in exchange rates⁶ and/or substantial trends in the average stock level of foreign reserves. The paper delivers a clear message. There is a high probability of a change in the stock of foreign reserves when the CB controls volatility and has a limited ability to learn the drift process under uncertainty. Simulations and counterfactual exercises show that the likelihood that the CBB and the EEA-BoE affect the level (up) and the volatility (down) of the exchange rate is high.

The rest of this paper is structured as follows: the next section briefly reviews the related theoretical and empirical literature. It provides the justification for the model, econometric estimations, and calibrations. The third section introduces our smoothing volatility model. Forth section presents the simulation results and their corresponding analysis. The fifth section covers a brief analysis of drift in exchange rates and reserve dynamics. The last section includes concluding remarks.

2 Optimal Reserves and FX Intervention Policies

The literature on exchange rate intervention can be divided into two main categories. One focuses on devising flow policies that set out to achieve optimal reserve (stock) levels. The other aims at macro-management according to different policy objectives (Blanchard et al. 2015): foreign exchange flows are deployed to affect the exchange rate, which can be understood as the intermediary instrument of macro-management. Stock levels are thus the result of optimal flow policies. We propose that CB order flows are the (endogenous) instruments by which smoothing interventions are carried out. Our paper can thus be placed in the macro-management literature. Our literature review, however, starts with a discussion about a few papers in the first category.

Optimal Stocks Models that follow from this tradition are generally based on the theory of optimal inventory (Arrow et al. 1958). The benchmark model assumes non-linear fixed costs of portfolio adjustment which leads to (*S-s*) type policies. In summary, optimal stock levels are replenished up to an upper level *S* only if they fall below a lower threshold *s*. The middle regime corresponds to a zone of inaction. Intervention flows are thus purely determined by stock level considerations (see, for example, Hamada and Ueda (1977) and Miller and Orr (1966)). Social welfare considerations for the optimal stock of reserves are a key feature of the most recent models. A common reason relates to precautionary savings for consumption smoothing during currency crises or sudden stop episodes (Jeanne and Rancière 2011; Calvo et al. 2013; Alfaro and Kanczuk 2019). But there are several other related motives: maintaining financial stability (Obstfeld et al. 2010; Domanski et al. 2016), decreasing the impact of debt roll-over risk during sudden stops (Hur and Kondo 2016) and/or reaping dynamic gains from real depreciations, Levy-Yeyati et al. (2013) and Korinek and Servén (2016), among others. Sudden stops were common in emerging

⁶A separate finance literature documents the profitability of *momentum* strategies in the foreign exchange market (e.g., Menkhoff et al. 2012) and its link to political risk (Filippou et al. 2018).

economies in the 1980s, while the 1990s were characterized by various episodes of speculative attacks (Amador et al. 2016).

Sterilised Interventions Although largely used for macro-management objectives, their impact is still contentious. The literature that empirically investigates the impact of sterilized interventions on exchange rates is also very large, so we will highlight a few key observations. First, CB interventions have been more relevant for emerging economies than for developed ones Menkhoff (2013), Daude et al. (2016) though there are recent examples from developed economies (e.g., Switzerland, Japan). Second, the literature broadly suggests the existence of two simultaneous channels by which sterilized interventions affect the exchange rate: the portfolio balancing and the signaling channels (see Sarno and Taylor 2001). Third, it also recognizes the importance of the microstructure of currency markets in understanding the mechanisms through which these channels are linked to exchange rates. Last but not least, there is some evidence that CB behavior is asymmetric for exchange rate shocks, with central banks *leaning against the wind* more strongly on depreciation shocks (Chen and Lin 2019).

Microstructure of FX Markets Exchange rate models that draw on Evans and Lyons (2002)'s approach provide an alternative answer to the enigma in international macroeconomics about the apparent disconnect between macroeconomic fundamentals and the short-term behavior of exchange rates (Obstfeld and Rogoff 2000). In these models, there is no Walrasian auctioneer calling a vector of prices that leads to zero excess demand. Instead, there is a price discovery process through order flow, which reflects portfolio adjustments. The basic idea is that, at the beginning of a typical trading period, customers observe their particular state of nature, form their expectations, and place orders in the currency market. Customer orders result in negotiations within the interbank tier of the market and consequentially, form order flow. The latter aggregates all of the idiosyncratic, scattered *private* pieces of information regarding underlying price pressures. The aggregated information helps dealers to learn about the unobserved state of nature. The portfolio balancing effect occurs because the equilibrium price largely reflects demands given by optimal risk-sharing considerations. As order flow aggregates exogenous unexpected changes in private agents' incomes, it conveys signals about fundamental changes in the economy. The most simple specification in which order flow contemporaneously affects exchange rate changes has been shown to be empirically robust. Feedback between order flow and exchange rate changes could lead to endogeneity by simultaneity or other dynamic (lagged) specifications. Estimates of price-impact parameters in the simpler specification, however, frequently have the expected signs, and the model retrieves R^2 coefficients that can be greater than 50%.

Our assumption that the Central Bank order flow is an *endogenous* instrument for conducting *smoothing intervention* policy is a novelty. Although this instrument has been recognized by the literature (see Lyons 2006, chapter 8), few papers develop a comprehensive microstructure model with a CB that intervenes through initiated trades (for example, Evans and Lyons (2001) and Pasquariello (2010)). Interven-

tions in Evans and Lyons (2001) are secretly and randomly routed to a dealer at the beginning of a particular trading day. CB orders are exogenous and follow a distribution with zero mean and constant variance. In Pasquariello (2010), interventions are announced, non-secret, and occur with an exogenous probability. They are aimed at optimizing a function that weighs the CB trade-off between wealth maximization and quadratic deviations of the exchange rate from a fundamental value. Dealers are assumed to be risk-neutral liquidity providers to customers. CB interventions can increase the unconditional volatility of the exchange rate in Pasquariello (2010)'s model. The higher volatility is an optimal response of profit-maximizing market makers to extract rent from order flows. Conversely, in our model, CB intervention is an optimal response to keep the second moment of the exchange rate distribution within a certain level. This way, the CB “trims” excess volatility.

As will be shown in the next sections, our model is based on a loss function that places a positive weight on the amount of currency risk faced by economic agents. Deviations from what would have been the observed “free” or “market” exchange rate volatility concerning an “optimal” level thus decrease social welfare. We understand that a comprehensive model of interventions, aimed at understanding the smoothing volatility problem, should include the two channels described above in a microstructure framework.

CBB Case While we propose a general framework, we apply our model to the Central Bank of Brazil case, which is an example of an emerging economy that is sensitive to commodity shocks⁷.

As mentioned above, there are two main reasons to start with the case of the CBB. First, it *explicitly* states that one of its policy objectives, within a framework of inflation targets and floating exchange rates, is to avoid excessive currency volatility. Notice that this rhetoric is not at odds with the classification provided by the IMF for floating regimes, according to which “...intervention may be either direct or indirect, and such intervention serves to moderate the rate of change and prevent undue fluctuations in the exchange rate, but policies targeting a specific level of the exchange rate are incompatible with floating.” (IMF 2022, p. 64). Also, there is evidence from a large sample of countries that interventions designed to smooth volatility are effective for economies with this sort of monetary arrangement (see, for instance, Berganza and Broto 2012, Fratzscher et al. 2019 and Arango-Lozano et al. 2020). There is a growing literature on this topic particularly for the CBB, with several estimates showing that interventions are effective (Nedeljkovic and Saborowski 2019). Some of these estimates will later be used for the calibration of our model. For example, it has been documented that the estimated effect of CB interventions has a similar magnitude, albeit slightly smaller, to that of the market's order flow (see Barroso 2018 and Kohlscheen and Andrade 2014). Second, the CBB recently published a dataset

⁷The EEA-BoE case is the one of a developed economy that has been suffering from exchange rate volatility due to political uncertainty – for instance the Brexit referendum and Liz Truss mini-budget. The EEA-BoE case contrasts with the CBB's type of active management of reserves, but we argue that they share the same smoothing principle (see Appendix B).

containing full information on its daily intervention since the adoption of a flexible rate (from January 1999). As far as we know, little has been done to explore using this dataset. The closest paper is by Nedeljkovic and Saborowski (2019) which uses this dataset to evaluate the effectiveness of spot and derivative FX interventions but does not focus on the implications for CBs reserve accumulation within a microstructure framework of exchange rates.

The simple model put forward in the next section thus aims at helping CBs understand the foreign reserve (stock) implications of optimal (flow) policies to smooth exchange rate volatility. Our model is designed to understand interventions in countries that follow a *de facto* floating exchange rate regime, thus our results can be applied to a wider range of classifications.

3 Order Flow Rules

In this section, we build a simple theoretical model with an underlying FX microstructure framework. We consider that the effect of the CB intervention is analogous to that of a private market order flow, i.e. it occurs through the net result of buyer-initiated purchases over seller-initiated trades of foreign reserves. The model explicitly assumes that the signaling/portfolio channel, as characterized in the previous section, is given using a variable that is conceptually identical to the market order flow.

Social Loss Function Assume that society's welfare (cost) objective function for each period is given by the piecewise loss function

$$L_t^{ps} =: \begin{cases} L_t^s(\cdot), & \text{if } E_t \text{ is true,} \\ 0, & \text{otherwise.} \end{cases}$$

where

$$L_t^s(\cdot) =: \left[(\Delta s_t)^2 - \sigma^2 \right]^2,$$

$t = 1, 2, \dots, T$ is a time subscript which represents the end of period (day), E_t is the event realisation that will be defined later, Δ stands for the first difference, s_t is the natural logarithm of the spot exchange rate, $(\Delta s_t)^2$ represents realised variance (Andersen et al. 2003) and σ^2 is the volatility targeted by the CB. This level is constant and exogenously determined.

The FX Market Microstructure models of the exchange rate (Evans and Lyons 2002; Evans 2011) suggest that the daily reduced-form exchange rate change is a linear function of public news and the order flow. Hence, our next hypothesis is that

$$\Delta s_t =: i_t^n + \epsilon_t, \quad (1)$$

and

$$\epsilon_t =: \beta x_t + \theta I_t, \quad (2)$$

where i_t^n represents public news on the interest rate differential, x_t corresponds to the market's (or interdealer's) order flow and I_t is the CB intervention; β and θ , which are strictly positive, stand for the market's and CB's price impact on the exchange rate, respectively. We assume that i_t^n is a white-noise exogenous process, in line with the hypothesis that interventions are sterilized. Therefore, i_t^n is a zero-average and stationary process⁸. We also consider that the market order flow, x_t , is divided into two different parts

$$x_t =: \gamma_t + \eta_t, \quad (3)$$

where η_t is the order flow in the absence of the drift term, in turn represented by γ_t . We start by not making assumptions about the data-generating process of the exogenous drift. However, we understand that drifts could exist for several reasons: swings in commodity prices that affect terms of trade could generate small but persistent daily order flow trends or, for example, Peso problems that impact on the level of hedge or speculative demands for foreign exchange. These characteristics can make the drift behave in a non-stationary way. As we develop ahead, the drift in our simulations is a constant and time-invariant parameter. This way, we can focus on how the size of interventions is different for specific values of drift and not on their dynamics.

We incorporate dynamics in the conditional volatility⁹ of the exchange rate by considering that the market order flow behaves according to

$$\eta_t =: v_t \sqrt{h_t}, \quad (4)$$

where $v_t \sim \mathcal{N}(0, 1)$ and h_t follows a GARCH (p, q) process by assumption, where p and q are the orders of the conditional volatility terms and h_0 is given. Given that i_t^n represents public news, the GARCH process of the order flow is the main stochastic element of the model and is stationary. This way, all the variables in our model are stationary.

Now, consider the set of assumptions below.

Assumption 1 The Benevolent Representative. The problem of the monetary authority is to choose the sequence of CB order flows, $\{I_t\}_{t=1}^T$ in order to minimize the expected present value of the society's welfare costs, subject to the linear constraints given by Eqs. 1, 2, 3 and 4:

⁸We also test the stationarity of this process by calculating the interest rate differential of Brazil's SELIC rate and the United States' Federal Fund Rates. The *Augmented Dicky-Fuller's* test indicates that there's not enough evidence to say that the series is trend stationary nor that it has drift. Hence the series is stationary.

⁹We presented in the next section some evidence for this assumption.

$$\min_{\{I_t\}_{t=1}^T} \mathbb{E} \left\{ \sum_{t=1}^T \left(\frac{1}{1+\delta} \right)^{t-1} \left[(\Delta s_t)^2 - \sigma^2 \right]^2 \times \mathbb{I}(E_t), \quad \Omega_1^{CB} \right\} \quad (5)$$

where $\mathbb{E}$ is the expectation operator; the parameter δ is the society's subjective intertemporal discount factor, $\delta \in \mathbb{R}^+$; Ω_1^{CB} is the initial information set of the CB with $\{\beta, \theta, \sigma, i_t^n, x_t\} \in \Omega_t^{CB}, \forall t$; $\mathbb{I}$ is the indicator function, i.e. for any event realization (E_t) , $\mathbb{I}(E_t) = 1$ if E_t is true and $\mathbb{I}(E_t) = 0$ otherwise, where $E_t =: |i_t^n + \beta x_t| > \sigma$.

Assumption 2 $(\sigma \wedge \theta) \in \mathbb{R}^+$.

Assumption 3 Let the CB **actual intervention function** be given by

$$I_t^* =: \begin{cases} I_t^{(+)}, & \text{if } E_t \text{ is true and } \min\{|I_t^{(+)}|, |I_t^{(-)}|\} = |I_t^{(+)}|, \\ I_t^{(-)}, & \text{if } E_t \text{ is true and } \min\{|I_t^{(+)}|, |I_t^{(-)}|\} = |I_t^{(-)}|. \end{cases}$$

where

$$I_t^{(+)} =: \frac{-(i_t^n + \beta x_t) + \sigma}{\theta}, \quad (6)$$

and

$$I_t^{(-)} =: \frac{-(i_t^n + \beta x_t) - \sigma}{\theta}. \quad (7)$$

Proposition 1 Under Assumptions Eqs. 1, 2 and 3, I_t^* minimises the society's loss function and has the lowest impact on the level of CB reserves.

The proof can be found in Appendix C.

Discussion Note that, by Assumption 1, the CB knows the values of all parameters and that it learns both i_t^n and x at the end of each period t . It is an implicit assumption that private information transmitted from customers to dealers is only publicly revealed through the realization of order flow. Hence, dealers only know for sure that the CB rule is triggered at the same time that they observe the market order flow. Strategic interactions between dealers and the CB can certainly have interesting implications for exchange rate determination [see, for instance, Vitale (1999), for a model in which the CB targets the level of the exchange rate]. However, they are not considered in our simple model. One could think that the CB observes the interdealer order flow simultaneously with the market but before dealers can react to its pre-committed rule. Finally, note that future values of i_t^n , x_t , and $L_t^s(\cdot)$ are all independent of I_t , therefore the end-of-period objective function can be written as a single-period problem. This implies that there is no binding recursive structure for the interventions, although such dynamics could be incorporated. The assumption

that the CB is fully aware of the market order flow and the interest differential means that the problem is deterministic. The CB problem at each end of period t can thus be simply restated as $\min_{I_t} L_t^{ps}(\cdot)$, or

$$\min_{I_t} \left[(\Delta s_t)^2 - \sigma^2 \right]^2 \times \mathbb{K}(E_t).$$

First order conditions imply that when the government chooses $\sigma = 0$, the CB optimal solution is to choose a fixed exchange rate regime. This is not considered in our analysis due to Assumption 2. Since L_t^s is a quartic function, there are two possible solutions when volatility is strictly positive. They both represent a reaction from the CB in the opposite direction of the market order flow which does not eliminate volatility but rather controls or “smooths” it.

As can be inferred from Eq. 5, the monetary authority has a symmetrical loss every time the realized variance is greater than the target. Assumption 3, however, imposes restrictions on the choices that are available to the policymaker, implying that some variables will follow a non-linear process. This assumption draws insights from Kearns and Rigobon (2005). Note that the positive root of the first order condition is represented by $I_t^{(+)}$ and the negative one by $I_t^{(-)}$. Because of the quadratic nature of the problem, the CB could, in principle, react to an increase in the order flow above σ by choosing $I_t^{(-)}$, producing an exchange rate change of $-\sigma$. It could also adopt a policy rule like $I_t^{(+)}$ as a response to a negative order flow generating a $+\sigma$ variation. The above possibilities can be seen in Fig. 1 which graphs two $L_t^s(\cdot)$ functions, one for $\sigma = 0\%$, which has a global minima at zero and another for $\sigma = 2\%$, with two local minima: one at -2% and another at $+2\%$. Assumption 3 neither considers that the CB chooses between the two critical points illustrated by Fig. 1 randomly, nor

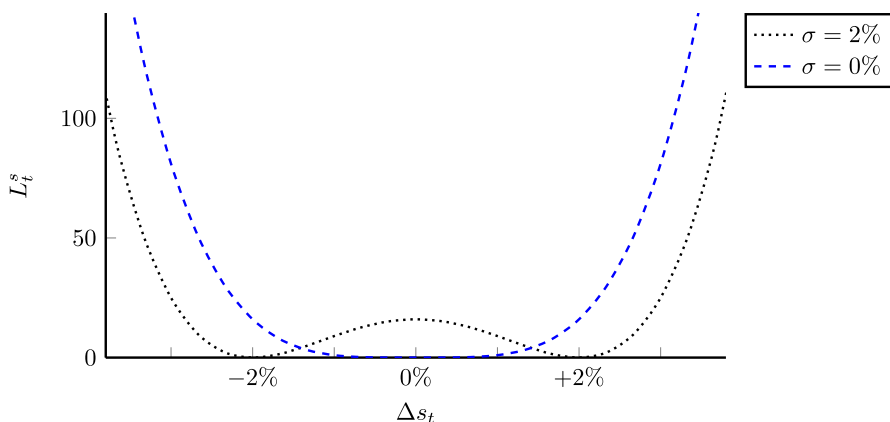


Fig. 1 Graphical Analysis of two Loss Functions. *Notes:* For $\sigma = 0\%$, there is one global minima at 0% . For $\sigma = 2\%$, there are two local minima, one at -2% and another at $+2\%$

that one of the roots will be ignored. It restricts the CB choice to be the one with the minimal impact on the level of foreign reserves¹⁰.

Proposition 1 thus establishes that the CB's actual intervention function is also an optimal solution to its problem, given the society's loss function. The rule is analogous to choosing the optimal intervention according to the minimal impact on the level of foreign reserves. So, for example, if $i_t^n + \beta x_t > 2\%$ when $\sigma = 2\%$, the CB will choose $I_t^{(+)}$ rather than $I_t^{(-)}$. By doing so the CB also preserves the direction of the exchange rate change as determined by the market when applying its policy rule. It follows that the optimization problem can be thought of as being given in two steps. The first is to identify the order flows that offset the higher-than-target volatility. The second is to choose the one that preserves the direction of the change. The piecewise loss function together with the assumption of a benevolent CB restricts intervention flows to occur only when the market or "free" volatility is above the desired level so that we can finally summarize

$$I_t = \begin{cases} I^{(+)} & \text{if } i_t^n + \beta x_t > +\sigma, \\ I^{(-)} & \text{if } i_t^n + \beta x_t < -\sigma, \\ 0 & \text{if } +\sigma \geq i_t^n + \beta x_t \geq -\sigma. \end{cases}$$

This hypothetical behavior imprints non-linear dynamics into the reduced form exchange rate, which is consistent with the findings of the literature (see, for instance, Reitz and Taylor 2008; Kohlscheen 2012) and with the results that are shown in Appendix D. In summary, our hypotheses imply that $\pm\sigma$ represents reflexive barriers for the exchange rate. As will become clearer later on with our numerical simulations, the addition of a zero mean and independent random variable to σ , to randomize the optimal volatility level, will not change our results, on average.

In the next section we explore, through model simulations, the decision-making process of the CBB. We calibrate the model parameters using some of our own estimates as well as values found in the related literature. We provide simulations as an alternative to the algebraic analytical evaluation of the problem. We start with the case of the CBB assuming rules that are not neutral with respect to the drift. Appendix E discusses the case when the CB is uncertain about the drift term or when it is certain but applies a neutral rule. Simulations will allow us to assess whether the observed accumulation of reserves in the period between 2009 to 2011 is in line with the objective of controlling excess exchange rate volatility.

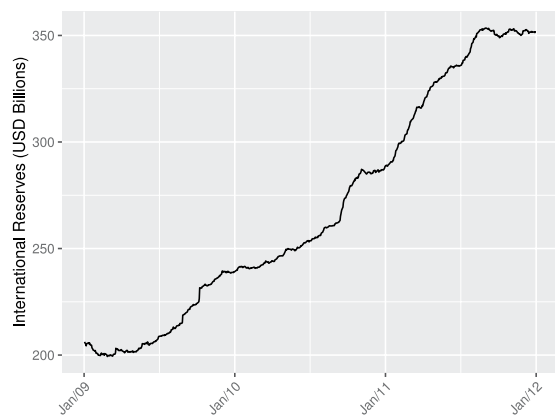
¹⁰The terminal level is given by $R_T =: R_0 + \sum_{t=1}^T I_t$, where $R_0 \in \mathbb{R}^+$ is the initial amount of reserves. If R_T turns out to be negative by borrowing, one can think of it as net reserves. We do not impose restrictions on the level of the reserves nor investigate the optimal inventory choice. We aim to study the stock consequences of a flow policy that is obtained when the level of reserves does not represent a binding constraint. Hence, we allow R_T to take any possible value in the set of Real numbers. However, we are implicitly assuming that the government intertemporal budget constraint is satisfied, such that there are no extreme realizations of x_t which could threaten the (fiscal) capability of the CB buffering order flow shocks through I_t .

4 The Case of the CBB

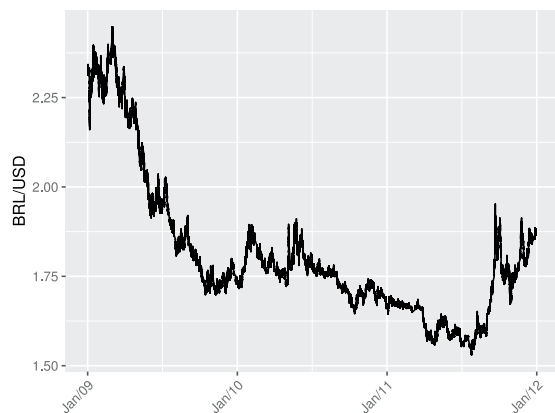
Simulations are performed for the CBB to “match” some key statistics for the sample period that goes from January 2009 to December 2011. Data is daily (from Bloomberg and CBB time series system) and spans from 01/01/2009 to 30/12/2011. We collect the nominal BRL/USD exchange rate for every 5-minute interval starting at 10:00 am on a total of 729 days, with 84 observations on each day. Days in which there were no transactions, due to holidays, for example, are excluded. We chose this period because of the significant increase in the amount of foreign reserves held by the CBB, as shown in Fig. 2a.

Visual inspection of Fig. 2b suggests a non-stationary behavior for the exchange rate. In Table 1 below we report the summary statistics for both the level and return of BRL/USD exchange rate as well as the realized volatility calculated over the sample period, 01/01/2009 - 30/12/2011. The last row shows the autoregressive coefficients

Fig. 2 CBB Reserves and the Exchange Rate. *Notes:* Both figures present daily data between 01/01/2009 and 29/12/2011. Figure 2a shows reserves calculated by the CBB using the liquidity concept. Figure 2b presents the spot exchange rate collected from Bloomberg. *Source:* CBB and Bloomberg



(a) Foreign Reserves



(b) BRL/USD Exchange Rate

Table 1 Summary Statistics for the BRL/USD

	Level	Return	Realized Vol.
Minimum	1.7134	-4.4347%	0.2418%
P(25)	1.8244	-0.5382%	0.5784%
Median	1.8929	-0.0296%	0.7901%
Average	1.9361	-0.0038%	0.8856%
P(75)	1.9880	0.5660%	1.0879%
Maximum	2.4418	4.1006%	3.3080%
AR(1)	1.0000	-0.0525***	0.7480***

Notes: This table shows the summary statistics for the BRL/USD exchange rate. In the last row, we report the autoregressive coefficient as well as Kapetanios and Shin (2008) unit root test for SETAR specification (Ha: Stationary), ****p*-value < 0.01; realized volatility is computed as the square root of the daily sum of squared returns

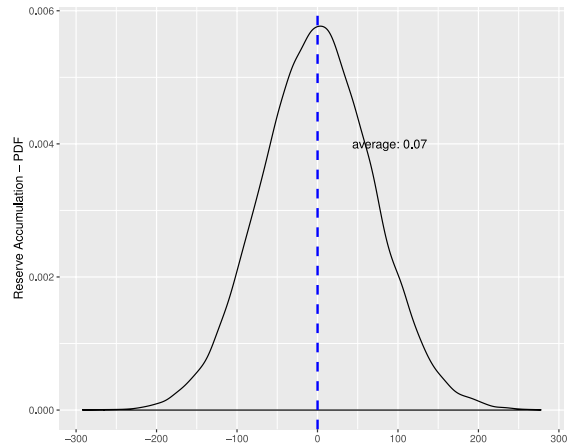
for each time series and the asterisks indicate the results of the Augmented Dickey-Fuller (ADF) test, considering a constant and a time trend. One cannot reject the null hypothesis of non-stationarity for the level of the spot rate, while the return and the realized volatility seem to be (trend) stationary. The assumption that the exchange rate and, by deduction, the order flow have a conditional volatility structure is not only based on the findings of the literature reviewed in the last section. We also find evidence to reject the null hypothesis of no conditional volatility (heteroskedasticity) using the Lagrange Multiplier test. Furthermore, the null of linearity is rejected at the 10% significance level for a Self Exciting Threshold Autoregressive (SETAR) with three lags, SETAR(3). As mentioned, Appendix D provides evidence that the BRL/USD exchange rate can be represented by a non-linear model, as predicted by our theory.

This interval is characterized by a marked increase in commodity prices, favorable terms of trade, and large surpluses in Brazil's trade balance. IPEA (the Institute for Applied Economic Research of the Brazilian government) index of commodity prices, which excludes oil, rose from 166.61 in Jan 2009 to 226.6 in December 2011. During this same time interval, the Brazilian terms of trade, measured as the ratio of export to import prices, increased from 97.05 to 124.61, according to FUNCEX - a Brazilian foundation that performs international trade studies. These facts help to explain the accumulated surplus in the trade balance during the aforementioned period, which reached 71.1 billion USD.

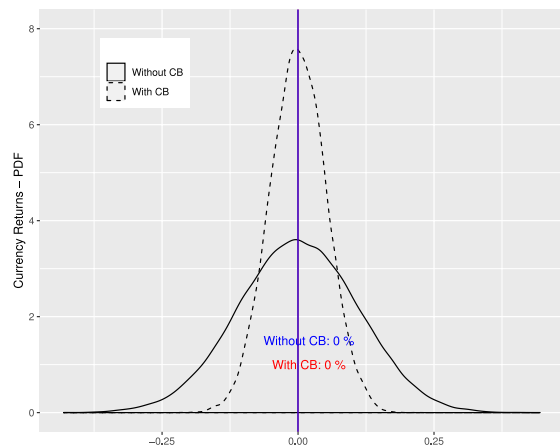
Between January 2009 and December 2011, the CBB reserves increased by approximately 150 billion USD, while the currency appreciated by about 19.5% (29.8% between January 2009 and July 2011).

Both Kohlscheen (2014) and Aizenman et al. (2012)'s findings strongly point to the view that the CBB accumulation period could be driven by drifts in commodity exports. Visual inspection of Fig. 3 in Kohlscheen (2014) reveals a strong trend in a real commodity price index. This stochastic trend is highly correlated with the real appreciation of the BRL against the USD. Aizenman et al. (2012) show that the accumulation of international reserves substantially affects the dynamics of the equilibrium real exchange rate (REER) after commodity terms of trade shocks (CTOT). Their results for Latin America over the 1970-2009 period reveal that the accumula-

Fig. 3 Simulation results for $\theta = \beta = 100(\text{bp})$ and $\gamma = 0$. *Notes:* The figure shows the distributions of the averages of the 20,000 simulations for 1,000 days each. Figure 3a presents the distribution of the accumulated reserves by the CB. Figure 3b presents a comparison between the daily average currency returns, in percentage changes, with and without CB intervention



(a) Accumulation of foreign reserves



(b) Currency returns

tion of foreign reserves “not only helps buffer the direct effect of CTOT transitory shocks on the REER, but also decreases the half-life of the shock, substantially lowering the volatility of the REER after the shock (p.1999).”

Ex post and in-sample, it is almost certainly possible to fit optimal non-zero drifts in the exchange rates which will ameliorate forecasts for the driftless random walk. One can also find evidence that out-of-sample forecasts with drifts will perform better than a naive driftless process¹¹. The results are often sensitive to the starting point and sample split, however, the conclusions put forward above are generally valid. Note that the observed exchange rate is already a variable that has been subject to the

¹¹ We have performed various forecasting exercises for the BRL/USD rate using mean absolute errors and mean squared errors which corroborate these conclusions. They are not reported in the paper but are available with the authors upon request.

intervention of the CBB. Hence, if the CBB applied a non-neutral policy function to some extent, at least part of the drift in the exchange rate will “disappear” during the period, having been “transferred” to the level of foreign reserves.

4.1 Drift vs. No Drift: Simulations

To simplify, our simulation exercises consider that $i_t^n = 0$ and that γ_t is constant, $\forall t$ ¹². We assume that $\gamma_t = \frac{\gamma}{\beta}$ so that γ also stands for the exchange rate drift¹³. We calibrate the GARCH parameters in a way to reflect the frequency of CBB interventions observed in the period.

The no Drift Case We start by explaining the calibration of a “benchmark case”. This is summarized by $\gamma = 0$, $\alpha_0 = 6$, $\alpha_1 = 0.5$, $\sigma = 2\%$ and $\beta = \theta = 100(\text{bp})$. The assumption of $\theta = \beta$ implies that the information content of the CB order flow is the same as the market’s. The CB’s order flow will be of the same magnitude but with an opposite sign, neutralizing the market order flow and leaving an exchange rate change, in absolute terms, equal to $|\sigma|$. We measure β and θ in basis points (bp). The value 100 means that the impact of one billion USD order flow on the exchange rate is 1%. The choice of $\gamma = 0$ is analogous to the hypothesis of no long-run trend in the level of the nominal exchange rate, in other words, a random walk without a drift. The exchange rate change, however, presents conditional time-dependent variance and unconditional constant variance. We choose ranges for β and θ using estimates from the literature (Ventura and Garcia 2012; Kohlscheen 2012; Barroso 2018). The parameter σ was calibrated to 2%, which corresponds to the concept of narrow bands, as defined in Reinhart and Rogoff (2004). Note that this is very close to the average of the interval given by the minimum and maximum values of the realized volatility, 3.308% and 0.2418%, respectively, as reported in Table 1.

For the calibration of the GARCH parameters, we use values for α_0 and α_1 that generate a proportion of around 50% intervention days out of the 1,000 in the sample¹⁴. We report results only for the ARCH(1) model which assumes $\alpha_0 = 6$ and $\alpha_1 = 0.5$

¹²The simulations done here reflect our approach to daily interventions. We acknowledge and thank the referee’s comment regarding the relevance of understanding whether, at a lower frequency, the results are robust. We believe that intervention dynamics and identification occurs at this high frequency level, however we do simulations with month equivalent parameters and obtain similar results.

¹³The results for the case in which the CB adopts a neutral policy when γ_t is imperfectly known will not be shown. However, inference on the likely consequences of a policy rule like Eqs. 24 and 25 in Appendix E is straightforward. One part of γ_t would be absorbed in the exchange rate, and the remaining in the level of foreign assets.

¹⁴This proportion is chosen arbitrarily as we are not aware of any exact number for the period under investigation. Kohlscheen (2012) analyses the impact of CBB daily intervention flows for the period that goes from the 2nd of January 2002 until the 30th of November 2011. Out of a total of 2,399 trading days, there were 56% with CBB interventions (1345 days). Kohlscheen (2012) reported that CBB net purchases amounted to 254.5 billion USD during this period. Fratzscher et al. (2019) reports a proportion of 19.1% days with CB intervention. Interestingly, net purchases of currencies by the CBs were observed in 76.1% of the 113,842 trading days of their sample.

¹⁵ These parameter values imply that the market order flow has a zero mean and an unconditional standard deviation of 3.46 billion USD $\left(\sqrt{\frac{\alpha_0}{1-\alpha_1}} = \sqrt{\frac{6}{1-0.5}} \approx 3.46\right)$. The implied unconditional standard deviation of the exchange rate change is 3.46% for $\beta = 100(\text{bp})$. For a normal distribution with a zero mean, which is a reasonable approximation for the analysis of the unconditional or the average case, this implies that 68% of the exchange rate changes will be inside the interval $[-3.46\%, +3.46\%]$. Approximately 56.4% and 57.3% of the observations will be outside of the aforementioned interval, for both a zero and a -0.05% daily drift, respectively, if σ is set at 2%. This is a reasonable choice since we are aiming at a conditional distribution that replicates around 50% of the days with intervention.

The main steps employed in the simulations can be summarized as follows: we initially create 1.000 order flow observations (η_t) , calibrating the ARCH(1) parameters as explained above; using the order flow impact parameter (β) , we simulate both the exchange rate variation and the realized volatility that would have prevailed at the end of each period in the absence of the CB intervention; we then evaluate the policy response function according to the cases given by the actual intervention function, i.e. following the conditional choice of j given by Proposition 1; the CBB intervention order flow is aggregated over the 1.000 days to calculate the impact on the stock of foreign reserves. We also compare currency returns with and without intervention; we repeat these steps 20.000 times to obtain a measure of the distributions of the foreign reserve impact and the currency returns effect.

The results for the benchmark case are summarized in Fig. 3. When there are only stochastic trends, i.e. there is no long-run deterministic trend in the nominal exchange rate, the accumulation of foreign reserves is, on average, close to zero. The reason is that there is no significant predominance in either direction of the interventions, on average, when the exchange rate follows a driftless random walk. Finally, it can be seen that the CB intervention lightens the weight of the tails of the currency returns distribution, relative to the outcome that would have prevailed in the absence of a policy reaction function.

The magnitude of the signaling channel is given by the relative size of the θ parameter for that of the market. If interventions are secret, there is no clear reason why $\theta \neq \beta$. However, non-secret interventions could drive a wedge between θ and β . We analyse alternative scenarios for θ , $\theta < \beta$ and $\theta > \beta$, keeping other parameters constant. The next simulation, however, assumes $\theta = \beta$ and a deterministic drift $\gamma \neq 0$. In what follows, we only present results for negative values of γ , since the results obtained with the opposite sign and same magnitude, *ceteris paribus*, are symmetrical.

The Drift Case For illustration, we maintain the assumption $\theta = \beta$, and when $\gamma < 0$, the currency presents a long run appreciation trend. It is thus reasonable to expect that the CB would intervene predominantly in the positive (buying) direction since the

¹⁵ Results with different specifications for the GARCH(p,q) model are available upon request. We do not report these results as they mainly differ only in terms of magnitude - richer dynamics tend to magnify the impact of interventions, by increasing volatility.

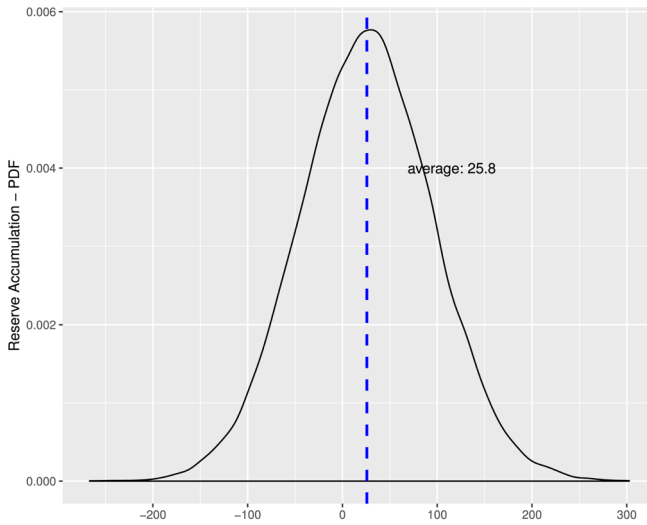
probability that the exchange rate will exceed the optimal volatility through the lower limit will be larger than through the upper threshold. If the frequency of intervention is higher in one particular direction, we would observe an accumulation or depletion of foreign reserves over time, depending on the sign and magnitude of the long-run deterministic trend. In the case of an appreciation trend, the CB would initiate buy trades more often than sell ones, accumulating stocks. This case is illustrated in Fig. 4 where $\gamma = -0.05\%$. In other words, even if η_t is zero on a particular day, the currency would still appreciate by 0.05% on that day. Over the 1.000 days, one would expect a cumulative (not compounded) appreciation of 50%.

As can be observed, the average number of buys is greater than in the benchmark scenario, as is the average accumulation, which is around 25.8 billion USD during the period. Only in a few out of the 20.000 simulations do the CB accumulate reserves that are over and above 150 billion USD. With regards to the shape of the currency returns distribution, one can see that the impact is qualitatively similar to the previous one. It is important to stress that, if the CB had not intervened, the average simulated appreciation in this period would have been 50.2%, a value close to the theoretical expected value ($\gamma \times 1.000$), while the simulated appreciation with the CB intervention is only 24.7%.

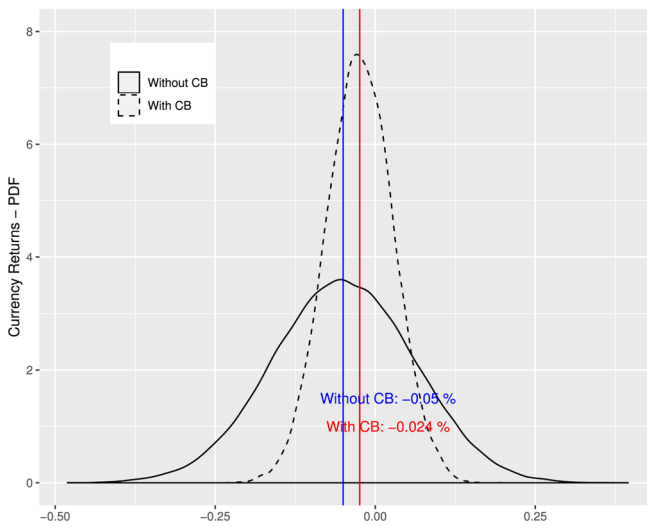
Other results are qualitatively similar to the previous cases so we do not discuss them individually. Each row of Table 2 presents selected descriptive statistics for each of the (20.000×1.000) simulations using different values for γ , β , and θ . This table shows that the larger the long-run trend, the higher the stock of foreign reserves is. It is interesting to note that the probability of the CBB accumulating 150 billion USD or more of foreign reserves is high (being larger than 40% in two cases) when the signaling channel is weak (in other words, when θ is small). One can see that our simulation results in Table 2 successfully explain the average direction of the changes in the observed data, both for the exchange rate and the level of foreign reserves.

There are many possible explanations for these findings that are complementary to the ones put forward in the introduction. One is that, during the period of analysis, the market already expected some daily intervention from the CBB, which significantly weakened the price impact channel. This could be due to their knowledge of the small negative deterministic daily time trend γ . This was an outstanding period for commodity exporters like Brazil, who experienced improved terms of trade and fast growth of the *quantity* exported. One hypothesis is that it would have been harder for the CBB to provide an effective signal through buyer-initiated trades; in other words, θ could be a negative function of the absolute value of γ .

Another plausible explanation is that the CBB had another objective, different from the one stated in Proposition 1. This could be, for example, the goal of accumulating foreign reserves for precautionary reasons, as previously stated. In that case, the CBB could deliberately offset the aforementioned appreciating deterministic trend. In that sense, our paper complements the optimal stock literature by showing how CBs could decide optimally on their intervention flows to build an optimal stock. In other words, we substitute the inaction zone (i.e., between the S-s region) for an active smoothing policy: leaning against the volatility of the wind. Another final possibility is that the CB acts less aggressively, i.e. by placing passive orders



(a) accumulation of foreign reserves



(b) currency returns

Fig. 4 Simulation results for $\theta = \beta = 100(\text{bp})$ and $\gamma = -0.05\%$. *Notes:* The figure shows the distributions of the averages of the 20.000 simulations for 1.000 days each. Figure 4a presents the distribution of the accumulated reserves by the CB. Figure 4b presents a comparison between the daily average currency returns, in percentage changes, with and without CB intervention

Table 2 Summary of the Simulation Results: the CBB case

γ (%)	β (bp)	θ (bp)	Reserve Accumulation - billions USD (b)	$P(b) > 150$ billions USD (%)	Proportion of Days with Intervention (%)	Cumulative Return ($-\Delta s_t$ %) "with"	Cumulative Return ($-\Delta s_t$ %) "without"	Standard deviation Δs_t (%) "with"	Standard deviation Δs_t (%) "without"	Min-Max daily return interval Δs_t (%) "without"
-0.05	100	20	131.09	47.83	51.37	27.45	64.68	1.63	3.46	[-17.23, 17.38]
-0.05	80	20	103.78	42.69	41.69	34.63	65.85	1.54	2.77	[-13.83, 13.97]
0.00	100	20	4.08	34.04	51.36	0.48	1.65	1.63	3.46	[-17.34, 17.34]
-0.05	60	20	72.50	31.01	28.54	42.82	65.20	1.39	2.07	[-10.32, 10.45]
-0.05	100	40	61.76	30.80	51.36	27.63	63.78	1.63	3.45	[-17.26, 17.32]
0.00	80	20	-0.63	27.35	41.72	-0.20	-0.14	1.54	2.77	[-13.85, 13.83]
-0.05	80	40	51.62	21.72	41.68	33.48	64.33	1.54	2.76	[-13.78, 13.86]
0.00	100	40	-0.91	19.62	51.35	0.28	1.34	1.63	3.46	[-17.27, 17.25]
-0.05	100	60	43.51	18.01	51.37	28.01	65.64	1.63	3.46	[-17.30, 17.46]
0.00	60	20	-0.39	16.65	28.54	0.05	-0.16	1.39	2.07	[-10.42, 10.41]
0.00	80	40	0.46	11.49	41.73	-0.15	-0.17	1.54	2.77	[-13.85, 13.87]
0.00	100	60	0.71	10.20	51.36	0.68	1.37	1.63	3.46	[-17.40, 17.37]
-0.05	80	60	34.36	8.32	41.70	33.27	63.77	1.54	2.77	[-13.85, 13.93]
-0.05	60	40	36.26	7.31	28.54	43.44	66.09	1.39	2.07	[-10.34, 10.46]
0.00	80	60	0.35	3.53	41.68	0.01	-0.14	1.54	2.76	[-13.85, 13.84]

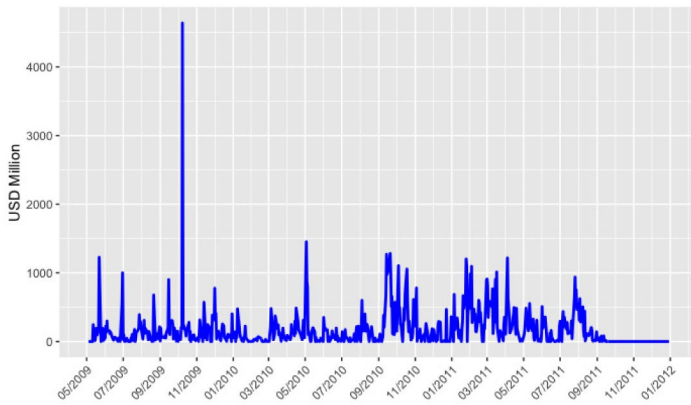
Notes: Each line contains average results of 20,000 simulations for a number of 1,000 days. γ is the daily constant percentage change - the drift; β and θ refer to the impact of a one billion USD order flow on the exchange rate ($\beta = \theta = 100$ (bp), for example, implies that a 1 billion USD order flow changes the exchange rate by 1%); cumulative returns are the averages of the compounded daily results over a total of 1,000 days in percentage terms (hence, different from the average returns that are actually presented in Figs. 2 to 5); "with" and "without" refer to the cases of CB intervention (the proportion of days shown in the sixth column of the table) and no CB intervention (zero interventions), respectively; Column 5 presents the average probability of an accumulation of 150 billion USD in foreign reserves or more and it is expressed in percentages (column 4 is the average reserve accumulation). Maximum and minimum values of the exchange rate "with" the CBB correspond to the [-2%, +2%] interval, respectively. It must be stressed that adding a random number with a zero mean and small standard deviation to this chosen level of intervention ($\pm 2\%$) to randomize the latter interval will not change our quantitative results, in average; the ARCH model was parameterized according to the reference case. We set the seed equal to one in the R script, for reproducible tests

and providing liquidity to the market rather than by initiating trades. This would explain larger changes in reserves without corresponding changes in prices.

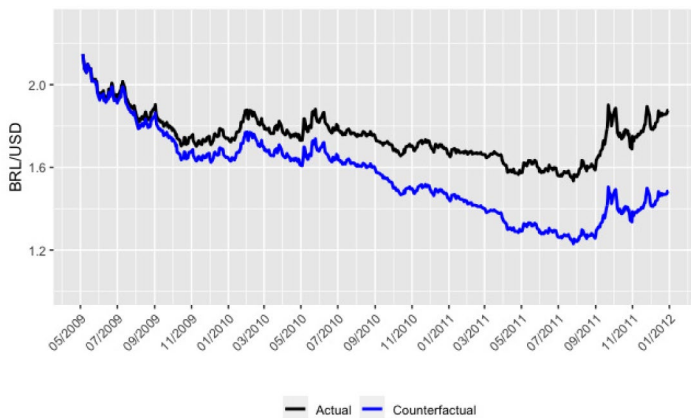
A Counterfactual Analysis We build counterfactual scenarios for the BRL/USD using our model's central idea. By reversing or compensating the impact of I_t on ϵ_t , we can calculate a path for the exchange rate that is free from the intervention. CBB publicly reports daily data on net interventions in the spot market in their "Time Series Management System, v2.1", code 17843. Data spans from the 11th of May 2009, onwards. There are 545 out of 666, or 81.8% days with interventions until December 2011. Figure 5a shows the volume of contracts during this period. A closer inspection of the figure reveals that daily interventions are frequently below 1 billion USD. A notable exception is October 8th 2009, when the CBB bought a net amount of 4.6 billion USD. Net buys reached 36.5 billion USD in that year, showing that this was an important event. We have not found any mention of this intervention flow in the news, so we cannot explain this outlier based on public information. Most likely, this large event was due to private information that was available to market participants only.

Another noticeable characteristic of Fig. 5a is that the totality of the interventions corresponds to episodes of net buying of dollars. The total amount of net buys during this period is 95 billion USD. This could be explained due to the context in which Brazil was inserted at the moment. After the 2008-09 financial crisis, the commodities market experienced a boom driven by imports from China. The latter had direct impacts – by expanding the imports of commodities from Latin America – and indirect impacts – by consuming more commodities as a whole, which drove their prices up as well (Gallagher and Porzecanski 2009). This generated positive terms of trade for Latin American countries and allowed the CBB to constantly assume buying positions in the spot market.

The counterfactual case of the CBB is straightforward. We used the time series explained above as a proxy for the CBB order flow (Nedeljkovic and Saborowski 2019). We then added the daily change due to the CBB order flow, given its market impact θ , to the actual variation of the exchange rate. We thus compensated the impact of the CB intervention by using the opposite sign for these trade flows. For example, if the CB bought one billion USD, given our calibrated θ of 20 bps, we added 0.2% to the actual change in the exchange rate. We decided to be more conservative concerning the market impact of the CBB order flow. Larger price impacts would increase the magnitude of our findings. Figure 5b presents our counterfactual path for the BRL/USD exchange rate. During the period of analysis, the BRL appreciated 12.7%, while the counterfactual appreciation reached 30.9%. This result can be explained by the combination of two main ingredients. First, the fact that the totality of the CBB interventions was of net buying, generating positive order flows. Second, the assumption that the impact of the intervention occurs like the order flow implies that shocks have a long-lasting effect on the exchange rate. Note that this result is somewhat similar to the case presented in the first line of the Table 2, in which $\gamma = -0,05\%$, $\beta = 100$ and $\theta = 20$ - we would need to run the simulations for 666 days to retrieve a level



(a) CBB Daily intervention in the Spot Market



(b) BRL/USD Counterfactual

Fig. 5 Actual Intervention and the Counterfactual Exchange Rate. *Notes:* Fig. 5a shows the daily FX spot market intervention by the CBB. Contracts in USD million. Figure 5b presents the daily exchange rate with a counterfactual scenario constructed using actual CBB (daily) intervention data. We compensated the impact of the intervention by adding the CBB order flow back to the actual exchange rate change, assuming $\theta=20\text{bps}$. *Source:* Central Bank of Brazil and own calculations

of appreciation that is closer to the observed. Finally, the actual volatility during the period was 4.8% and the counterfactual 5.20%¹⁶

5 Reserve Accumulation and Exchange Rate Drifts

In the previous section, we present that a key element that distinguishes the simulation results is the presence of a drift element in the exchange rate. In this section, we first explore whether an exchange rate drift can be related to the direction of the exchange rate intervention using the CBB intervention dataset. We then compare the actual intervention data with the accumulation of reserves in Brazil. Note that although Central Banks (under free-floating) do not intervene to impact the trend component of the foreign exchange rate, as our theoretical model illustrates, the reserve accumulation might still depend on short-term exchange rate drifts.

We obtain similar results of the impact of the drifts in both exchange rate interventions and reserve accumulation, where both are negatively related with FX drifts¹⁷. Most of the interventions in the FX market in other countries resemble spot purchases/sales. We then assume that most of the reserve changes are due to exchange market actions. This allows us to further expand our analysis to cases outside the CBB to test the existence of this correlation in an expanded sample.

5.1 Data

The dataset used in this section comprises the daily intervention operations by the CBB; the exchange rate broad index panel data collected by the Bank of International Settlements (BIS); the level of foreign reserves retrieved from the International Financial Statistics by the IMF (IFS); the Chicago Board Options Exchange Volatility Index (VIX) which measures the market's expectations of volatility based on option prices in the S&P 500; the Economic Policy Uncertainty Index (EPU) by Baker et al. (2016), Davis (2016) and others, which is a newspaper's text-based measure of economic policy uncertainty, and the Geopolitical Risk Index (GPR) by Caldara and Iacoviello (2022), another text-based measure of risk, but associated with geopolitical events and risks.

The CBB data corresponds to daily information on all types of intervention carried out during the time frame considered. We have information on the time of day, volume offered and accepted and the exchange (or interest if it was a swap contract) rate offered by the CBB. This data is presented in Table 3 and indicates that across

¹⁶ It is worth noting that Chamon et al. (2017) also created a counterfactual exchange rate using a different methodology. Their synthetic control approach used data of covariates (currencies) that were not subject to the massive interventionist currency swap program implemented by the CBB between August 2013 and April 2015. They found that interventions over-appreciated the currency at the beginning of this program by 10% relative to their counterfactual level. The average effect on volatility was found to be zero when this was measured as the squared change in the log of the exchange rate. However, they present results showing a decrease of around 10% in the option-implied volatility measure. This impact was not observed when the program was extended.

¹⁷ In our notation, a negative change in the exchange rate is equivalent to an appreciation of it.

Table 3 Total volume by mode of intervention

	Accepted Volume (USD Billion)	Share (%)
Traditional Swap	1175	57.0
Spot Market	367	17.7
Reversed Swap	231	11.2
Pre Fixed	163	7.9
Post Fixed	70	3.4
PTAX	45	2.2
Others	22	0.6

Notes: Accepted volume corresponds to the actual USD volume in billions in an operation. Traditional Swap corresponds to currency swaps where the CBB is in the long position, whilst Reversed Swap the CBB is in the short position. Pre-fixed and Post Fixed correspond to the type of remuneration in Sale with Repurchase operations. Spot Market and PTAX are spot inter-banking market operations where the rates used were market rate and PTAX rate, respectively. Data corresponds to the period between January 1999 and December 2021

the sample, the preferred intervention methods were traditional currency swaps (57% of the total period volume) followed by spot market operations and reversed Swaps (17.7% and 11.2%). Given that, we focus our analysis on these three types of interventions. Furthermore, traditional swaps (TS) resemble spot sales in terms of their intended impact – protect against currency depreciation from an increase in foreign currency demand. Reverse swaps (RS), on the other hand, resemble spot purchases for the same reason. We then combine these two series into one total spot data as being RS – TS.

The BIS data corresponds to the daily effective exchange rate¹⁸ of countries with the IMF's de facto classification of *Independently floating*¹⁹. The panel covers 36 advanced countries and emerging markets²⁰ between January 1999 and December 2021 exchange rate measured as the price of one unit of foreign currency, hence a positive change corresponds to a currency depreciation. The reserves data correspond to monthly total reserves excluding gold, measured in US dollars. Furthermore, we use the VIX as a measure of common market volatility that might cause changes in reserve accumulation dynamics, and the EPU and GPR indexes measure country-specific uncertainty associated with economic policies and geopolitical situations.

We create two important variables using this dataset:

The 1-month Exchange Rate Drift Component We use the month average of daily FX returns. If we consider that the $\Delta s_t = \gamma_t + \varepsilon_t$ $\varepsilon_t \sim \mathcal{N}(0, \sigma_t)$, then $\gamma_t = \mathbb{E}(s_t)$. This would be the most parsimonious form of estimating the drift component.

The 1-month Exchange Rate Volatility The intra-month realized volatility generated using the squared daily returns of the exchange rate.

¹⁸For the methodological note on how the effective exchange rate index is constructed, see the latest update by Klau and Fung (2006).

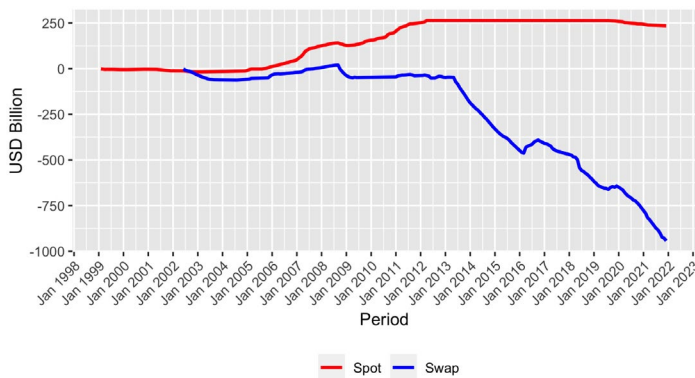
¹⁹Available in the <https://www.imf.org/external/np/mfd/er/2006/eng/0706.htm>.

²⁰The countries are Austria, Australia, Belgium, Brazil, Canada, Switzerland, Chile, Cyprus, Czech Republic, Germany, Spain, Finland, France, United Kingdom, Greece, Hungary, Ireland, Israel, Iceland, Italy, Japan, Korea, Malta, Mexico, Netherlands, Norway, New Zealand, Philippines, Poland, Portugal, Sweden, Slovenia, Turkey, United States, the Euro area and South Africa.

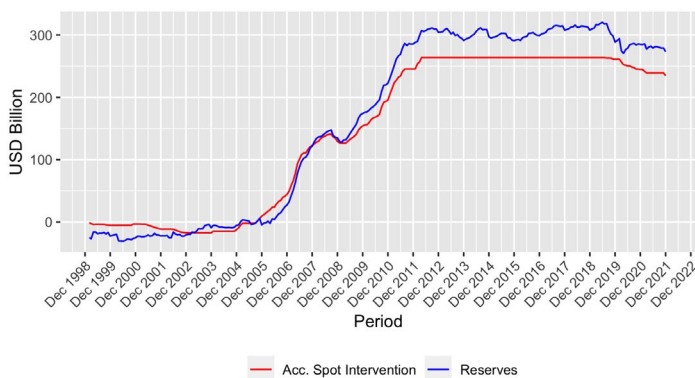
Figure 6a plots the evolution of the accumulated intervention in the currency market since the beginning of the series. We notice that spot interventions accumulated a net purchase of around US\$250 billion (a similar number to the amount of reserve accumulation), whereas Swap interventions are more significant when the CBB is leaning against a depreciation of the currency. Notice also that both the evolution of reserves and accumulated spot intervention present a close relation as illustrated in Fig. 6b.

5.2 Empirical Analysis

We first test whether the exchange rate drift variable created in the previous section impacts the sign of the CBB intervention, comparing the results for Spot and Swap interventions separately. We analyse both form of intervention this way, given the fact that swap contracts do not generate actual cash-flows. We also include controls,



(a) Accumulated Intervention by the CBB



(b) Accumulated Intervention and Reserves

Fig. 6 Intervention and Reserves. *Notes:* Fig. 6a shows the total accumulated intervention since January 1999. Figure 6b presents the comparison between accumulated spot interventions and accumulated reserves. Source: World Bank

such as changes in VIX, EPU, and GPR as well as country and time-fixed effects in the regressions. Using the fact that spot interventions and reserve changes seem to be strongly correlated, we replace spot interventions with the change in reserve and expand the analysis to consider emerging markets and advanced economies separately, following the IMF's most recent classification for grouping the countries.

We estimated the model using ordinary least squares at the country-month level with a set of fixed effects, and splitting the sample into developed and developing countries in order to mitigate as many confounding factors as possible.

Table 4 presents the results for the Brazilian case using different specifications. It's possible to see that both change in reserves and spot intervention are negative correlated with the drift variable, and the result is robust with the inclusion of controls and a linear trend. The result points toward to 100 basis points of drift (1 percentage point) being correlated with a decrease of around 400 million in reserves. The same drift is associated with an US\$ 300 million intervention.

Table 5 shows that the drift coefficient is significant across all specifications of the emerging markets. It is also significant for advanced economies, but only for the pooled specification with no time fixed effects.

Coefficients associated with economic uncertainty – VIX, Economic Policy, and Geopolitical Risk – are insignificant for developed economies. We find different results in some of the emerging market specifications. The first difference between the VIX and the 1-month exchange rate volatility is significant, for example, when all controls are included in the regression. These variables negatively impact the level of foreign reserves after controlling for drifts. These results could reflect the fact that reserves are being used to smooth out the effect of sudden capital movements when VIX and volatility are increasing. Overall, our findings show that exchange rate drifts and reserve accumulation are conditionally correlated. This result is stronger for the set of emerging countries and reflects our theoretical framework with respect to the

Table 4 Drift Impact on Spot intervention and Reserve Change

	$\Delta \text{Reserve}_t$			Spot Intervention		
FX Drift $_{t-1}$	-451.9*** (130.8)	-454.8*** (131.6)	-384.9*** (134.2)	-391.1*** (72.0)	-396.8*** (72.4)	-380.5*** (74.7)
ΔVIX_{t-1}			-3.053** (1.473)			-1.04 (0.819)
ΔEPU_{t-1}			-1.072** (0.530)			-0.028 (0.295)
GPR_{t-1}			4.692 (6.634)			3.661 (3.692)
Linear Trend		-0.000686 (0.003)	-0.00156 (0.003)		-0.00134 (0.002)	-0.00224 (0.002)
Constant	1.168*** (0.255)	1.57 (1.746)	1.882 (1.834)	1.005*** (0.141)	1.789* (0.960)	2.177** (1.020)
Observations	294	294	286	294	294	286
R-squared	0.039	0.039	0.074	0.092	0.094	0.106

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. We present absolute change in reserve in order to have comparison with spot market interventions, that can assume negative values and doesn't have an initial stock value. Source: Central Bank of Brasil

Table 5 Drift Impact on Reserve Accumulation – Developed vs. Emerging Markets

	Advanced Economies – $\Delta Reserve_t$		Emerging Market Economies – $\Delta Reserve_t$		
$FX\ Drift_{t-1}$	-2.872 (2.624)	-2.918 (2.634)	-4.661* (2.480)	-3.438 (2.703)	-3.41 (2.703)
ΔVIX_{t-1}			-0.006 (0.009)	-5.093** (2.395)	-3.454*** (0.756)
$FX\ Vol_{t-1}$				-0.00708 (0.009)	-3.481*** (0.767)
ΔEPU_{t-1}				25.49 (20.830)	-2.662*** (0.857)
GPR_{t-1}				-0.008 (0.007)	-0.018*** (0.005)
Constant	0.002 (0.002)	0.002 (0.002)	0.002 (0.002)	0.001 (0.002)	0.001 (0.004)
Fixed Effects					
Time	Yes	Yes	No	No	Yes
Country	No	Yes	Yes	No	Yes
Observations	7,685	7,685	7,685	7,685	2,200
R-squared	0.052	0.054	0.002	0.054	0.222

Note: Standard errors in parentheses. ADF Unit Root test suggests stationarity for GPR. *** p<0.01, ** p<0.05, * p<0.1

relationship between drifts and CB interventions in the form of order flows. The following section presents a model that explains changes in the level of foreign reserves when volatility control is a policy objective and exchange rates potentially exhibit drifts.

6 Concluding remarks

We present a simple yet innovative model that can help CBs across the world understand the stock and flow implications of a “flexible” exchange rate regime that adopts a smoothing volatility policy. Our results imply that the exchange rate follows non-linear dynamics and that small daily trends in the exchange rate can have a large impact on the stock of foreign reserves in a short period. Simulations show that either a non-neutral or an unaware (with respect to the drift) interventionist smoothing policy affects both moments of the exchange rate returns. These effects can be large according to our findings.

We also show that, on average, the effect of CB interventionist policies can be restricted to volatility only, without changing the level of the exchange rate and the amount of foreign reserves, if there are no deterministic trends in the data-generating process of the exchange rate. During periods of large swings in the stocks of foreign assets our findings point to 1) small but persistent drifts in the order flow; 2) the CB effectively offsetting the impact of these trends for other reasons than only being unaware of the drifts, such as the intentional build-up of precautionary savings; 3) a combination of a high price impact of the market and a low signaling channel of the CB. These explanations are non-exhaustive and non-mutually exclusive.

We used our model to create counterfactual scenarios for CBs under scrutiny. They show that the change in the exchange rate would have been 17.8% and 8% lower in the CBB and BoE-EEA cases, respectively. Volatility would have been 0.4% and 1.9% higher, respectively. Accumulation of foreign reserves prepares the CBs for future contingencies in the event of infrequent negative outcomes in the FX market. Our paper also shows that these contingencies could be better addressed by a thorough understanding of the underlying microstructure parameters during normal and stress periods.

A policy to smooth volatility can also be seen as an attempt by policymakers to “round the corners” of the triangle representing the Mundell trilemma (Klein and Shambaugh 2015). Ultimately, the geometrical shape that represents the available policy choices can be linked to the drift parameter in our model. A country can choose to have an open capital account, independent monetary policy, and relatively stable exchange rates without increasing the likelihood of a major balance sheet problem. Within the structure of our model, this is attainable only when there are no drifts, outliers, or rare large events that raise exchange rate volatility and trigger persistent or large-scale central bank interventions. If the central bank absorbs such drifts through its intervention rule, causing systematic or substantial changes in reserves, a quadrilemma (Aizenman 2013) may emerge—where maintaining financial stability requires significant accumulation or depletion of international reserves. In this context, an additional vertex representing financial stability might require the (de)accu-

mulation of substantial international reserves. Our model also rationalizes the idea that a global financial cycle, as suggested by Rey (2015), could reduce the trilemma into a dilemma. The reason is that persistently tight or loose monetary policies in the center economies could export exchange rate drifts to the smaller open economies. In turn, these drifts will significantly increase the probability of massive changes in the level of foreign reserves of these smaller markets, thus raising concerns about macroeconomic instability.

Appendix A The case of the Bank of England's EEA

In this section, we focus on the modern era of EEA account. There are several recorded cases of intervention by the BoE during the Bretton Woods period and after.

Perhaps the most famous one occurred on September 16th during the 1992 crisis (also known as black Wednesday) when the UK crashed out of the ERM. After selling foreign currency, mostly Deutsche Marks, worth the equivalent of 2.26 billion USD on Monday and 302 million USD on Tuesday, the Bank of England sold an additional 24.56 billion USD on Wednesday. Interest rates increased to 15% on that day and Britain suspended its membership of the European Exchange-Rate Mechanism, which was created in 1979. As it is officially recognised, the only recorded intervention in the UK since February 1993 is the coordinated G7 intervention to support the Japanese Yen on 22 September 2000.

There are a few points with respect to the contemporary role of the EEA that are worth noting. The first is that it is more open in historical terms with respect to its management, as can be inferred from their annual reports. However, its operations are still carried out in an opaque manner and there is a degree of subjectivism in the definition of intervention. According to the BoE, market intervention occurs when trades are “intended to influence the exchange rate”. In their view, as far as trades are carried out without the intention of influencing the exchange rate, active trading does not qualify as intervention. In Appendix B, we argue that even in the case of active trading, the EEA-dealers still have an incentive to moderate volatility.

Second, it is important to emphasize that HM treasury has increased holdings of foreign reserves from 2000 until 2020. The endowment to the EEA amounts to 6 billion GBP annually from 2010 until 2020. Figure 7a shows that the UK Government reserves have been increasing, especially in the post-crisis period. Recent data show that monthly amounts outstanding of Central Government foreign currency total reserves have been quite volatile. Although volumes seem small by comparison to the daily recorded turnover in the London foreign exchange rate market²¹, the EEA-BoE can aggressively engage in trades and trigger large order flows.

The decision to accumulate reserves is likely to be a precautionary response to the uncertainty generated by the 2007 recession and the European Union Sovereign Debt crisis that started in 2010. The rate of increase is higher after the first shaded area in the graph which highlights the aforementioned period. The second shaded

²¹The figures for the most recent BIS Triennial survey show that the daily average turnover in 2019 in the UK was 3.576 trillion USD, the highest in the world. It amounts to 43.1% of the global FX activity.

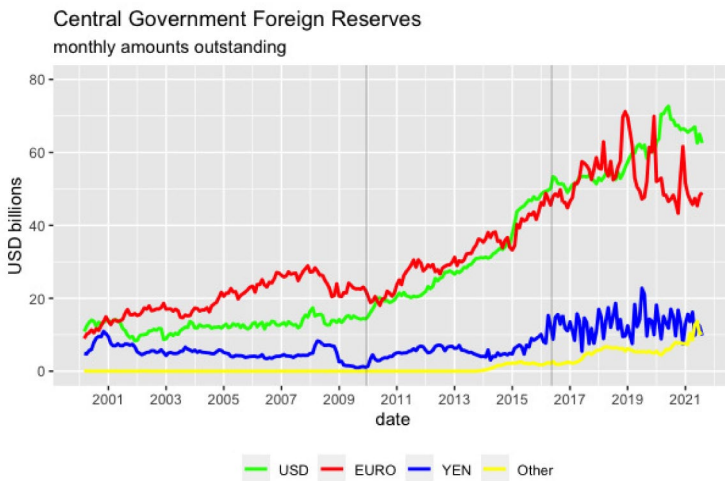
area, which corresponds to the period of the 2016 Brexit referendum vote, shows a subsequent fall in both USD and EUR reserves followed by a volatile period for Euro reserves. The Brexit process presumably helped to extend this perceived need for a large cushion of reserves at least until 2020. There are other observations from Fig. 7a. First, a big spike in January 2015, which coincides with the removal of the CHF peg. Second, a larger increase in the volatility of Yen denominated reserves and an increase in gold reserves (other), presumably linked to the uncertainty around Brexit. Figure 7b shows that the reserves held by the BoE are smaller and mostly concentrated on Euros. The USD spikes around 2008 and 2009 are likely to be due to the CB Swap lines with the FED during the crisis period.

We also created a counterfactual exercise for the GBP/USD rate, similar to the BRL/USD case. Figure 8 presents the observed (black) and the counterfactual path (blue) for the GBP/USD exchange rate. The blue line was generated under the assumption that the EEA-dealers traded by “trimming” the excess volatility subject to its endowment constraint (six billion GBP *per* year). Appendix B presents a model that formalises this idea. For the EEA-BoE case, we considered that $\theta = 50$ bps and assumed that the annual endowment was used to moderate the 50% largest daily exchange rate changes during each year. The size of the CB order flow was then determined by the relative weight of each of these changes (in absolute value). Hence, larger changes received proportionally more of the endowment. The actual depreciation of the GBP between 2010 and 2020 was 15%. Our counterfactual exchange rate increased by 12% during the same period. The counterfactual volatility was 2.3% smaller than the actual, 5.9% against 8.2%, respectively. The volatility is estimated by adding the monthly squared return for the period: “realized volatility.”

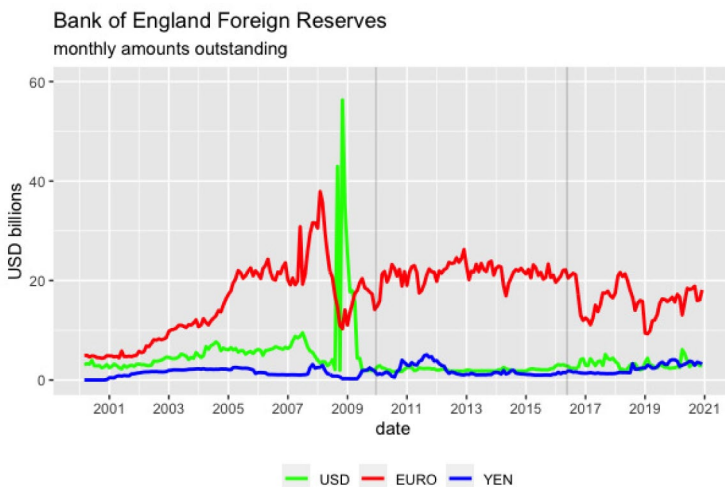
We present another useful exercise with our model in Appendix B. We consider that EEA-dealers intervene in the market with the objective of moderating sterling’s volatility during a crash scenario. We analyze the impact on the stock of EEA foreign reserves for different scenarios using our smoothing volatility model. The choice of scenarios consider exceptional historical moments of financial distress and political uncertainty and is also explained in the appendix. The simulations reveal that a combination of a positive (*ex post*) drift with a high level of volatility of the order flow can lead to a rapid depletion of the UK foreign reserves. This is most likely when the impact of the EEA-BoE order flow is low with respect to that of the market’s.

B The EEA-BoE Problem

There are three main ingredients to consider in a model that aims at explaining the BoE-EEA portfolio decision. The first is that the BoE acts on behalf of the EEA-HM in the management of the foreign reserves. We also understand that the execution of this policy is performed through some brokers/dealers. We will henceforth refer to them as EEA-BoE dealers or simply as EEA-dealers. Second, the EEA sets historical benchmark weights for the currency composition of their portfolio, allowing for some deviations (see EEA 2018, page 10, for instance). The third main ingredient is the scope for active management of the reserves: “The Treasury sets a target



(a) Uk Government FX Reserves



(b) Bank of England FX Reserves

Fig. 7 UK Foreign Exchange Reserves Pre- and Post-Crisis. *Notes:* The first shaded area represents the beginning of the accumulation period by the EEA. It amounts to an increase of six billion GBP per year. The second shaded area represents the date of the 2016 Brexit referendum. Source: Bank of England

for active management return as part of the SLA, above the cost of managing the reserves” (EEA 2018, p.7).

In order to illustrate the role of the EEA in the UK’s currency market, suppose that there are n risky assets available to the EEA-dealer and single period returns $R = [R_1, R_2, \dots, R_n]^\top$. Consider that returns are random variables determined by $R_i = \beta_i x_i$, where $i = 1, 2, \dots, n$ and $\beta_i > 0$ stands for the order flow impact on the

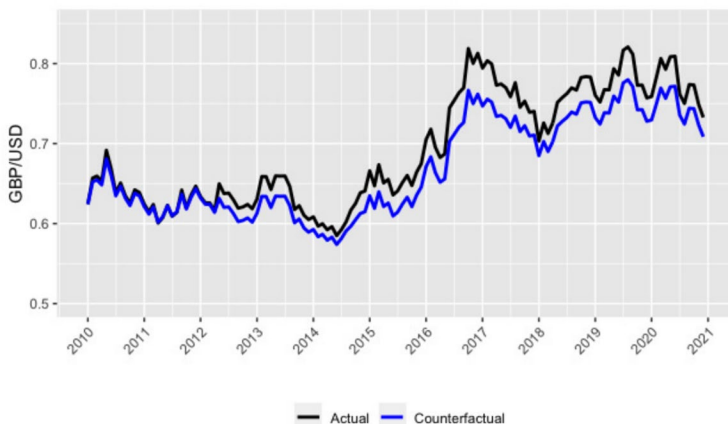


Fig. 8 GBP/USD Counterfactual. *Note:* Daily exchange rate with a counterfactual scenario constructed using information of the yearly BoE-EEA endowment. We compensated the impact of the intervention by adding the BoE-EEA order flow back to the actual exchange rate change, assuming $\theta=50$ bps. Detailed explanation is provided in the text. Source: Bank of England and own calculations

domestic price of currency i (i.e., the nominal exchange rate) and x_i is the “market” order flow in currency i ; $x_i \sim \mathcal{N}(0, \sigma_{x_i}^2)$. We assume that the “market” order flow excludes the flows that are created by the EEA-dealer when it is exclusively trading on behalf of the EEA.

The net amount of foreign currency i bought by the EEA-dealer is denominated as x_i^d . If all trades occur in the most aggressive way, in other words, by means of dealer initiated trades, x_i^d would represent the EEA-dealer’s contribution to overall order flow. The variance-covariance matrix of these risky assets is given by $\text{Cov}[R] = \Sigma$. Portfolio variance is $\text{Var}[R_p] = \omega^\top \Sigma \omega = \sigma_p^2$ where $\omega^\top = (\omega_1, \omega_2, \dots, \omega_n)$ is the matrix of the weights, i.e. the dealer’s control variables, with $\sum_{i=1}^n \omega_i = 1$. Note that portfolio returns are expressed by $R_p = \omega^\top R = \sum_{i=1}^n \omega_i R_i$ and $\mu = \mathbb{E}R$. Pre-determined benchmark shares by the EEA are given by $a^\top = (a_1, a_2, \dots, a_n)$.

Assume that there is a single representative dealer who acts on behalf of the EEA-BoE. Consider that the BoE-EEA imposes constraints on their dealers with respect to a target mean return and that the objective is a loss function which weights the portfolio variance with penalties from deviations from benchmark shares. The EEA-dealer’s problem can thus be stated as

$$\{\omega_1, \dots, \omega_n\} \underset{\text{minimize}}{\frac{1}{2}} [\omega^\top \Sigma \omega + (\omega^\top - a^\top) \Lambda (\omega^\top - a^\top)^\top], \quad (\text{A1})$$

$$\text{subject to } \omega^\top \mu = \mu_p, \quad (\text{A2})$$

$$\omega^\top 1_n = 1, \quad (\text{A3})$$

where

$$\mathbf{\Lambda} =: \begin{bmatrix} p_1 & 0 & \cdots & 0 & 0 \\ 0 & p_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & p_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & p_n \end{bmatrix},$$

μ_p is the target return, $p_1, p_2, \dots, p_n$ are the penalties given by the BoE-EEA to deviations of the chosen weights from their benchmark portfolio, $\mathbf{x}^d{}^\top =: (x_1^d, x_2^d, \dots, x_n^d)$ and $\mathbf{s}^q{}^\top =: (s_{1t}^q, s_{2t}^q, \dots, s_{nt}^q)$ are vectors that represent the net dollar amount per currency and price quotes in Eq. 9, respectively; if s_i^q is the best price (quote) at which buy and sell orders are traded, when $x_i^d > 0$, s_i^q refers to the best ask (minimum) price and when $x_i^d < 0$, s_i^q is the best bid (maximum) price. The exogenous value B is the endowment from the EEA (HM) to the EEA-dealer, thus $\mathbf{x}^d{}^\top \mathbf{s}^q = B$ represents the resource constraint. Note also that we define

$$\omega_i =: \frac{x_i^d s_i^q}{B}, \quad (\text{A4})$$

for $i = 1, 2, \dots, n$.

Construct the Lagrangian

$$\mathcal{L}(\omega, \lambda_1, \lambda_2) = \frac{1}{2} [\omega^\top \Sigma \omega + (\omega^\top - \mathbf{a}^\top) \mathbf{\Lambda} (\omega^\top - \mathbf{a}^\top)^\top] + \lambda_1 (\omega^\top \boldsymbol{\mu} - \mu_p) + \lambda_2 (\omega^\top \mathbf{1}_n - 1), \quad (\text{A5})$$

and take the first order conditions

$$\nabla_{\omega} \mathcal{L}(\bullet) = \Sigma \omega^* + \mathbf{\Lambda} (\omega^* - \mathbf{a}) - \lambda_1 \boldsymbol{\mu} - \lambda_2 \mathbf{1}_n = \mathbf{0}_n, \quad (\text{A6})$$

$$\frac{\partial \mathcal{L}(\bullet)}{\partial \lambda_1} = \omega^{*\top} \boldsymbol{\mu} - \mu_p = 0, \quad (\text{A7})$$

$$\frac{\partial \mathcal{L}(\bullet)}{\partial \lambda_2} = \omega^{*\top} \mathbf{1}_n - 1 = 0. \quad (\text{A8})$$

From Eq. 13 we have

$$\omega^* = (\Sigma + \mathbf{\Lambda})^{-1} \mathbf{\Lambda} \mathbf{a} + \lambda_1 (\Sigma + \mathbf{\Lambda})^{-1} \boldsymbol{\mu} + \lambda_2 (\Sigma + \mathbf{\Lambda})^{-1} \mathbf{1}_n. \quad (\text{A9})$$

Observe that if $\mathbf{\Lambda}$ was a null matrix, the problem would be reduced to the typical optimal portfolio selection theory (Markowitz 1952). From Eqs. 14 and 15 we can write:

$$\begin{bmatrix} \mu^\top V^{-1} \mu & 1_n^\top V^{-1} \mu \\ 1_n^\top V^{-1} \mu & 1_n^\top V^{-1} 1_n \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \mu_p - a^\top \Lambda^\top V^{-1} \mu \\ 1 - a^\top \Lambda^\top V^{-1} 1_n \end{bmatrix}, \quad (A10)$$

where $V =: \Sigma + \Lambda$. Note that V is positive semi-definite since both Σ and Λ are positive semi-definite, the latter by construction (both are square symmetric matrices). Given that $1_n^\top V^{-1} \mu = \mu^\top V^{-1} 1_n$, by using Cramer's rule we have

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{(1_n^\top V^{-1} 1_n)(\mu_p - a^\top \Lambda^\top V^{-1} \mu) - (1_n^\top V^{-1} \mu)(1 - a^\top \Lambda^\top V^{-1} 1_n)}{(\mu^\top V^{-1} \mu)(1_n^\top V^{-1} 1_n) - (1_n^\top V^{-1} \mu)^2} \\ \frac{(\mu^\top V^{-1} \mu)(1 - a^\top \Lambda^\top V^{-1} 1_n) - (1_n^\top V^{-1} \mu)(\mu_p - a^\top \Lambda^\top V^{-1} \mu)}{(\mu^\top V^{-1} \mu)(1_n^\top V^{-1} 1_n) - (1_n^\top V^{-1} \mu)^2} \end{bmatrix}. \quad (A11)$$

From the solution of the Lagrange multipliers in Eq. 18 one can see that optimal weights are a complicated function of the model parameters. Because comparative statics are also not straightforward, we analyze the model using numerical simulations.

Table 6 shows the optimal weights that have been calculated using a combination of different target mean returns and two different penalty levels. The upper part of the table shows the weights that would have been chosen were all the penalties set to zero. In the Table 6 notes we explain how the model is calibrated using descriptive statistics of the observed series of the relevant currency pairs. The lower part show the results with quadratic penalties, which is equivalent to setting $p_1 = p_2 = p_3 = 2$.

The analysis of Table 6 shows that deviations from optimal weights can be very small even when penalties for deviations from the benchmark weights are very large.

Table 6 Numerical Analysis

No penalties: $p_1 = p_2 = p_3 = 0$							
μ_p	4.0	5.0	6.0	7.0	8.0	9.0	10.0
USD share (%)	78.6	66.1	53.7	41.2	28.8	16.3	3.9
EUR share (%)	49.9	49.2	48.6	47.9	47.3	46.6	45.9
YEN share (%)	-28.5	-15.4	-2.3	10.9	24.0	37.1	50.2
σ_p	2.4	2.4	2.5	2.6	2.9	3.1	3.5
μ_p/σ_p	1.68	2.10	2.43	2.66	2.80	2.87	2.89
Quadratic Penalties: $p_1 = p_2 = p_3 = 2$							
μ_p	4.0	5.0	6.0	7.0	8.0	9.0	10.0
USD share (%)	64.7	57.9	51.2	44.5	37.8	31.0	24.3
EUR share (%)	63.8	57.4	51.1	44.7	38.3	31.9	25.5
YEN share (%)	-28.5	-15.4	-2.3	10.9	24.0	37.1	50.2
σ_p	13.7	12.0	10.5	9.4	8.8	8.8	9.4
μ_p/σ_p	0.29	0.42	0.57	0.74	0.91	1.02	1.07

Notes: We calculate the compounded yearly returns using end of month spot exchange rates from the BoE: GBP/USD (XUMLUSS), GBP/EUR (XUMLERS) and GBP/YEN (XUMLJYS) from January 1999 until December . The averages of these yearly returns during the aforementioned twenty years period were: 6.170% for the GBP/USD, 6.173% for the GBP/EUR and 13.8% for the GBP/YEN. Return correlations are 36% between the GBP/USD and GBP/EUR rates, 74% between GBP/USD and GBP/YEN and, finally, 40% between the GBP/EUR and GBP/YEN. The averages of the yearly standard deviations are 0.299, 0.305 and 0.510 for the GBP/USD, GBP/EUR and GBP/YEN, respectively. Benchmark weights are set at 40% for both the GBP/USD and GBP/EUR rates and 20% for the GBP/YEN

The reason for this low sensitivity is that target returns are binding restrictions to the problem of minimizing the variance of the portfolio. This is more obvious with regards to the YEN share in the optimal portfolio, which is quite stable across penalties for given target returns (μ_p). However, the YEN has a relatively large amount of variation across different target mean returns. The share spans from -28.5% (a short position) when the target mean return is 4% to a maximum of 50% of the portfolio when the target mean return is set to 10%. The reason is that, although much more volatile, the return on the YEN is almost twice as large as the return on the EUR and the USD. After the YEN, the USD has the most volatile share across these monotonically increasing target mean returns. This is because, although the USD represents a slightly smaller risk than the EUR (see the notes in Table 6), it also has a lower expected mean return. It follows that its share is substantially reduced when the target mean return increases.

Finally, the comparison between the last lines of both the upper and lower parts of Table 6 reveals a deterioration of the $\frac{\mu_p}{\sigma_p}$ ratio. For a given risk free rate, this can also be interpreted as a worsening of the Sharpe ratio. This is due to the imposition of penalties for deviations from historical or “rule of thumb” benchmarks when evaluating the performance of EEA-BoE dealer’s active management. As can be inferred, this considerably decreases taxpayers welfare if it is evaluated by optimal portfolio allocation considerations.

Two Assets In order to provide an analytical solution to the previous problem, we will also analyze the simpler two assets case. In order to be a meaningful analysis, we also drop the assumption of a target mean return. This could be a more realistic hypothesis since EEA-dealers should not have a permanent incentive to directly affect returns in their favour by acting in the most aggressive way, i.e. by initiating trades. By narrowing the analysis down to the $n = 2$ case, we can also write

$$\Sigma = \begin{bmatrix} R_1^2 & \rho|R_1||R_2| \\ \rho|R_1||R_2| & R_2^2 \end{bmatrix},$$

where the vertical bars represent the absolute value function indicating non-negative values; ρ is the correlation parameter, $-1 < \rho < 1$. The variable R_i^2 for $i = 1, 2$ can also be understood as the realized volatility for assets 1 and 2. One can rewrite Eq. 8 as an unconstrained problem by, for example, substituting both $\omega_2 = 1 - \omega_1$ and the resource constraint solved for x_1^d , i.e $\omega_1 = (x_1^d s_1^q)/B$, in the objective function. The optimal net amount of currency 1 demanded by the EEA-dealer can thus be written as:

$$x_1^{d*} = \frac{B}{s_1^q} \frac{(p_1 a_1 + R_2^2 - \rho|R_1||R_2|)}{(p_1 + R_1^2 + R_2^2 - 2\rho|R_1||R_2|)}. \quad (\text{A12})$$

As can be seen in Eq. 19, the optimal net amount of currency 1 for the EEA-dealer is a negative function of the variance of the currency 1 returns. Consequentially it is also a negative function of the corresponding variance of the market order flow of that

specific currency. The fraction $\frac{B}{s_1^q}$ can be interpreted as the BoE-EEA currency drift, provided that the optimal amount of currency is obtained through initiated trades. For instance, if $s_1^q = 0.78$ - which is close to the observed GBP/USD exchange rate at the end of December, 2020 - $B = 6$ billion GBP and $\beta = 100$, the daily drift would be 0.77% per year or 0.003% per working day (250).

In order to better understand the model results, consider the following comparative static analysis. Taking the partial derivative of the optimal weight with respect to R_1 gives

$$\frac{\partial \omega^*}{\partial R_1} = \frac{-\{[2R_1 - 2\rho|R_2| \operatorname{sign}(R_1)](a_1 p_1 + R_2^2 - \rho|R_1||R_2|)\}(p_1 + R_1^2 + R_2^2 - 2\rho|R_1||R_2|)}{(p_1 + R_1^2 + R_2^2 - 2\rho|R_1||R_2|)^2 - [\rho|R_2| \operatorname{sign}(R_1)]}, \quad (\text{A13})$$

where $\operatorname{sign}(R_1) = \frac{R_1}{|R_1|}$ for $R_1 \neq 0$. This derivative, evaluated at $R_1 = R_2 = 1$ and $a_1 = 0.5$, for instance, gives

$$\left. \frac{\partial \omega^*}{\partial R_1} \right|_{R_1=R_2=1, a_1=0.5} = -\frac{1}{p_1 - 2\rho + 2}. \quad (\text{A14})$$

It is possible to see from Eq. 21 that an increase in currency's 1 risk (standard deviation) decreases the optimal weight for $\rho < 1$ since $p_1 \geq 0$. Also, note that the higher the penalty for deviations from the benchmark portfolio, the lower is the partial effect of a change in risk on the optimal weight. The above equation shows that optimal weights and, thus, optimal demands for currency 1 will negatively depend on its own return volatility. Suppose that currency 1 represents the returns on the GBP/USD and currency 2 the returns on the GBP/EUR rate. It is reasonable to suppose that the correlation for the GBP/EUR and GBP/USD currency returns is positive because of systematic risk, common preference shocks or any other non-idiosyncratic reason.

C Proof of Proposition 1

Proof First order conditions of the CB problem under Assumption 1 give $I_t^{(+)}$ and $I_t^{(-)}$ as critical points. The second derivatives evaluated at these points are $\frac{d^2 L_t^s(I_t^{(+)})}{dI_t^2} = \frac{d^2 L_t^s(I_t^{(-)})}{dI_t^2} = 8\sigma^2\theta^2$. They are strictly positive under Assumption 2, satisfying the second order conditions. The value attained by the loss function at these critical points is zero, thus $L_t^s(\cdot)$ is convex in the neighborhood of the critical points, which correspond to local minima. Thus under Assumptions 1 and 2, the CB minimizes $L_t^{ps}(\cdot) = L_t^s(\cdot)$ when $|i_t^n + \beta x_t| > \sigma$ by choosing either $I_t^{(+)}$ or $I_t^{(-)}$. Now assume that $i_t^n + \beta x_t < -\sigma$. Let us show that the CB will choose $I_t^{(-)}$. We have that $|I_t^{(+)}| = \frac{-(i_t^n + \beta x_t) + \sigma}{\theta} > -\frac{(i_t^n + \beta x_t)}{\theta}$ and $\frac{-(i_t^n + \beta x_t)}{\theta} > \frac{-(i_t^n + \beta x_t) - \sigma}{\theta} = |I_t^{(-)}|$

Table 7 SETAR Results

	Low		Mid		High		AIC
	Constant	Auto regressive	Constant	Auto regressive	Constant	Auto regressive	
Symmetric Threshold	0.0000	-0.0150**	0.0000	0.0693	0.0001**	-0.0702***	-837,187
Asymmetric Threshold	0.0000	-0.0238*	0.0000	-0.0491	0.0001**	-0.0659***	-837,189

Notes: This table shows the estimated coefficients for the SETAR models with and without the restriction of symmetric thresholds. Considering asymmetrical threshold yields an AIC selection criterion marginally better. p-value: '***' < 0.01, '**' < 0.05, '*' < 0.1

therefore, by Assumption 3, $I_t^* = I_t^{(-)}$. The proof for the case $i_t^n + \beta x_t > \sigma$ is analogous. $\square$

D The Empirical Non-linear Model

The theoretical model suggests estimating a general unrestricted non-linear model for the reduced form of the exchange rate that includes several lags and constant terms. We can thus posit the following SETAR model:

$$\Delta s_t =: \begin{cases} \phi_{10} + \phi_{11}\Delta s_{t-1} + \cdots + \phi_{1L}\Delta s_{t-L} + \xi_t & \text{if } \Delta s_t \leq -\sigma, \\ \phi_{20} + \phi_{21}\Delta s_{t-1} + \cdots + \phi_{2L}\Delta s_{t-M} + \xi_t & \text{if } -\sigma < \Delta s_t \leq +\sigma, \\ \phi_{30} + \phi_{31}\Delta s_{t-1} + \cdots + \phi_{3L}\Delta s_{t-H} + \xi_t & \text{if } \Delta s_t > +\sigma, \end{cases} \quad (\text{A15})$$

where L, M, H are the maximal lags in the low, intermediary and high regimes, respectively and ξ_t is an *i.i.d* $\sim (0, \sigma_\xi)$ shock (see Hansen 1999 for a review of non-linear approaches such as the SETAR model above). We estimate the reduced-form model Eq. 22 using the data described in Section 4. We choose thresholds that trigger a regime change with one lag. This choice is based on a best fit grid search using all possible values in the sample (i.e. the values of the lags of the dependent variable itself), with at least 15% observations in each regime. Table 7 shows the values of the estimated coefficients. The auto regressive coefficients of both outer regimes are statistically significant and lower than one, while the same coefficient for the middle regime is not significant. This suggests that in the outer regimes the exchange rate return process has a mean reversion characteristic that it is not observed in the middle regime (random walk). Table 7 also shows that a SETAR model that considers a asymmetric restriction for the threshold yields a AIC selection criterion only marginally different than the specification with symmetric threshold. Results presented in Table 7 suggest that our model is close to capturing the observed behaviour of the exchange rate.

E Drifts, Uncertainty and Neutral Policies

In the simulations, we will assume for simplicity that the drift term is constant. This subsection discusses the case when the drift is variable. It analytically investigates the impact of the CB uncertainty with regards to the drift on the model results²². One straightforward way to characterize the learning process of the CB is to suppose a state space representation. A simple way to proceed with the analysis is to consider that γ_t and η_t are both independent zero mean processes and to ignore the conditional volatility in η_t . The rational expectation forecast of the CB can thus be given by a typical signal extraction problem

$$\mathbb{E}(\gamma_t | x_t) = \hat{\gamma}_t = \kappa x_t, \quad (\text{A16})$$

where σ_γ^2 and σ_η^2 are the unconditional variances of γ_t and η_t , respectively and $\kappa =: \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\eta^2}$.

Now, suppose that the CB does not wish to react to this drift term. In fact, this is the “de jure” policy of many floating countries. The CB would choose a neutral policy, for example, to avoid suspicion that it is interfering with free market forces. Notice that this comes, according to our model’s set up, at the social cost of accepting excess volatility. In this case, the loss function and the resulting rules can be modified to be “neutral” with regards to the effect of the drift term as follows

$$I_t^{(N+)} =: \frac{-\beta(x_t - \hat{\gamma}_t) + \sigma}{\theta} = \frac{-\beta(1 - \kappa)x_t + \sigma}{\theta}, \quad (\text{A17})$$

and

$$I_t^{(N-)} =: \frac{-\beta(x_t - \hat{\gamma}_t) - \sigma}{\theta} = \frac{-\beta(1 - \kappa)x_t - \sigma}{\theta}, \quad (\text{A18})$$

when $i_t^n + \beta x_t > +\sigma$ and $i_t^n + \beta x_t < -\sigma$, respectively. Note in Eqs. 24 and 25 that we are also implicitly assuming a neutral exchange rate policy with respect to news or drifts in the interest rate differential.

In order to analyze these neutral rules, consider first the case in which $\sigma_\eta^2 \rightarrow 0$. It follows that all variation in the order flow will be due to changes in the drift term and hence the signal is perfectly informative ($\kappa \rightarrow 1$). The drift will almost certainly be known by the CB. The modified rule then prescribes that the CB order flow must be $I_t^{(N+)} = \frac{+\sigma}{\theta}$ and $I_t^{(N-)} = \frac{-\sigma}{\theta}$ for the two threshold cases shown above, respectively. The corresponding reduced forms will be, $\Delta s_t^{(N+)} = i_t^n + \beta x_t + \sigma$ and

²²For an investigation about the effectiveness of Central Bank interventions under uncertainty see Vargas-Herrera and Villamizar-Villegas (2020). Their theoretical and empirical results lend support to the conclusion that uncertainty with regards to exchange rate fundamentals increases the effect of interventions on both the level and the volatility of the exchange rate.

$\Delta s_t^{(N-)} = i_t^n + \beta x_t - \sigma$. The neutral rule and the hypothesis of a perfectly informed CB both imply that the exchange rate will entirely absorb the impact of the drift term. In the contrasting case when $\sigma_\eta^2 \rightarrow \infty$, however, there is too much noise in the signal and the CB is not able to learn anything about the drift from the observation of the market order flow. The signal will thus be completely uninformative ($\kappa \rightarrow 0$). The neutral rules that are associated with a perfectly uninformed CB are thus analogous to the ones in which the CB is perfectly informed and chooses not to react to the drift term.

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