New Journal of Physics
The open access journal at the forefront of physics

Deutsche Physikalische Gesellschaft DPG   IOP Institute of Physics

Quantum-assisted domination games on cycle graphs

To cite this article before publication: C Weeks *et al* 2026 *New J. Phys.* in press https://doi.org/10.1088/1367-2630/ae73a9

Quantum-assisted domination games on cycle graphs

C Weeks[1,2], P Strange[1,2], P Drmota[3], and J Quintanilla*[1,2]

[1]University of Kent, Physics of Quantum & Materials Research Group, School of Engineering, Mathematics & Physics, Ingram Building, Canterbury, Kent, CT2 7NH, United Kingdom
[1]University of Kent, Quantum Applications Research Centre (QuARC), Ingram Building, Canterbury, Kent, CT2 7NH, United Kingdom
[3]Department of Physics, University of Oxford, Clarendon Laboratory, Parks Road, Oxford, OX1 3PU, United Kingdom

April 12, 2026

Abstract

Quantum entanglement allows for correlations between distant objects that go beyond any classical theory. These additional correlations can be exploited to gain practical advantage in certain non-local games. In recent years there has been interest in games defined on graph structures and involving mobile agents. One of these is the graph domination game, where quantum advantage has been discovered recently on some finite graphs by numerical optimization [1]. Here we study quantum advantage in the 1-step, 2-player version of this game, focusing on cycle graphs. We study it numerically, analytically and through the use of noisy, intermediate scale quantum (NISQ) processors. We find explicit strategies and show that they realise the numerical bounds that were found recently for the case of small graphs [1]. We then generalise our strategies to cycles of arbitrary size. Finally, we run our strategies for 5-, 6-, and 7-site cycles on NISQ hardware and find measurable advantage (compared to the optimal classical strategies) in all cases.

1 Introduction

Non-locality is a central concept in quantum mechanics. Its origin traces back to the Einstein-Podolsky-Rosen (EPR) "paradox" [2]. EPR argued that in order to make quantum mechanics compatible with locality it would have to be completed with the addition of local hidden variables (LHV). Bell subsequently proved that some predictions of quantum mechanics are incompatible with LHV theories [3]. The experimental verification of Bell's predictions established the existence of intrinsically non-local correlations in quantum systems. The experiments were recognised with the award of the Nobel prize for physics to Aspect, Clauser and Zeilinger in 2022 [4]. We now understand non-local Bell correlations as a distinctive manifestation of the more general phenomenon of quantum entanglement: the fact that the state of some quantum systems cannot be described by stating the states of its constituent parts individually.

Quantum game theory considers game-theoretic scenarios where two or more players take individual actions and their success depends on the actions of all players. Using entanglement enables outcomes not attainable in classical settings. Examples include the quantum prisoner's dilemma [5] and the quantum volunteer's dilemma [6]. A subset of quantum games are non-local, incomplete-information games where spatially separated players try to cooperate without communicating with the other players. In this case they can exploit entanglement to generate non-local correlations that they can use to improve their coordination. Examples include the odd-cycle game [7] and the magic square game [8], both of which have been demonstrated experimentally [9,10]. A recent development in non-local games are those concerning mobile agents, specifically rendezvous [1,11] and graph domination [1]. Here, players attempt to achieve a joint task by coordinating their moves on a graph.

*Email: j.quintanillakent.ac.uk

The original domination problem was formulated in the context of pure graph theory [12]. in this context, a graph $G = (V, E)$ consists of vertices V and edges joining them, $E \subseteq \{\{u, v\} \mid u, v \in V\}$ [13]. Graphs provide a powerful framework for modelling pairwise interactions and networks. Graph theory is the study of such graphs including their properties and structure. The domination problem described in Ref. [12] is purely combinatorial and static. A dominating set in a graph is a set of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set. The classical graph domination problem asks for the smallest such set, or more generally, for structural and complexity properties of dominating sets. This original version of graph domination has many applications, for instance to facility location and coverage problems [14, 15].

Since the early 2000s, there has been interest in dynamic variants of graph domination involving mobile agents constrained to move along the edges of the graph [16]. For example, the eternal domination problem, introduced in 2005 [17], models guards that must continually relocate in response to attacks while ensuring that the occupied vertices remain a dominating set. More recently, in distributed computing and mobile robotics, researchers have considered algorithms in which robots collectively move so as to form a dominating set, often under additional algorithmic or memory constraints [18]. Unlike classical domination problems [12], the challenge in these agent-based domination problems is not only combinatorial (whether a dominating set exists) but also algorithmic and probabilistic (whether a team of moving agents can find one efficiently).[1]

A recent development is the introduction by Viola and Mironowicz of a graph-domination game where players have access to entangled quantum resources [1].[2] In the game they considered, two players are placed randomly on the nodes of a graph. They are tasked with coordinating their movements in order to dominate as much of the graph as possible. A node is dominated when there is a player on that node or on a node connected to it by an edge, as illustrated in Fig. 1a. The degree of success of a given strategy is the number of dominated nodes after the players have moved. Strategies can be characterised by their "domination number": the average number of dominated nodes after many runs of the game. It was shown that this increases when quantum resources are available. Importantly, the qubits can be entangled before the players learn which vertices have been assigned to them. Afterwards, no further interaction or communication is required. The obtained quantum advantage is therefore not purely formal: it represents a real advantage that could be gained in real-life situations where such games need to be played, without the need to change the nature of the game; only the resources available to each player locally need to be modified. These games thus offer a form of 'quantum operational advantage', in which quantum resources allow access to an improved result that is unobtainable using classical means.

In Ref. [1] semi-definite programming was used to find bounds on the maximum domination obtainable in a single step by two players on various graphs, including cycle graphs with up to 13 vertices (C_{13}). However, explicit protocols were not reported - though in the case of C_5 the probabilities of all possible moves were provided. Here we build upon this work by finding explicit strategies that realise these bounds. Moreover, we generalise our single-step domination strategies to cycles of arbitrary size. We use these strategies to run experiments using NISQ hardware, finding that present-day qubits are of a high enough quality to realise most of the predicted advantage for small cycle graphs.

2 Theory

We start by defining the rules of the game. The graphs the players will move on are unweighted and undirected. Each player will be allowed one move from their random starting positions (1-step game). These initial positions are taken from a uniform probability distribution (in particular, it is possible that the players start on the same location, though even then they do not know the location of the other player). Waiting is not an allowable action. After all players have moved, their collective domination of the graph is checked. The nodes of the graph are labelled and the graph topology is known to all players. The players have access to the label of the site they are on and can use that label

[1] Mobile-agent graph domination games can be regarded as a sub-class of turn games based on graph domination - for another recent example see [19].

[2] A quantum-assisted graph domination is not to be confused with domination of so-called "quantum graphs" [20].

to determine their actions. Finally, the players do not have to follow the same rules when making their decisions (player-asymmetric game).[3]

Quantum advantage emerges when the players share an entangled quantum system [1]. Specifically, each player holds an n-level system, where n is the number of edges for each node of the graph. After a player measures their qudit they use the result to decide which of the n edges to travel along. Before measurement, each player is allowed to perform an arbitrary rotation on their qudit which may depend on their identity and the site they are on. If the two qudits are entangled, this crucial step allows players to encode information about their location in the shared quantum state – the underlying source of the quantum advantage.

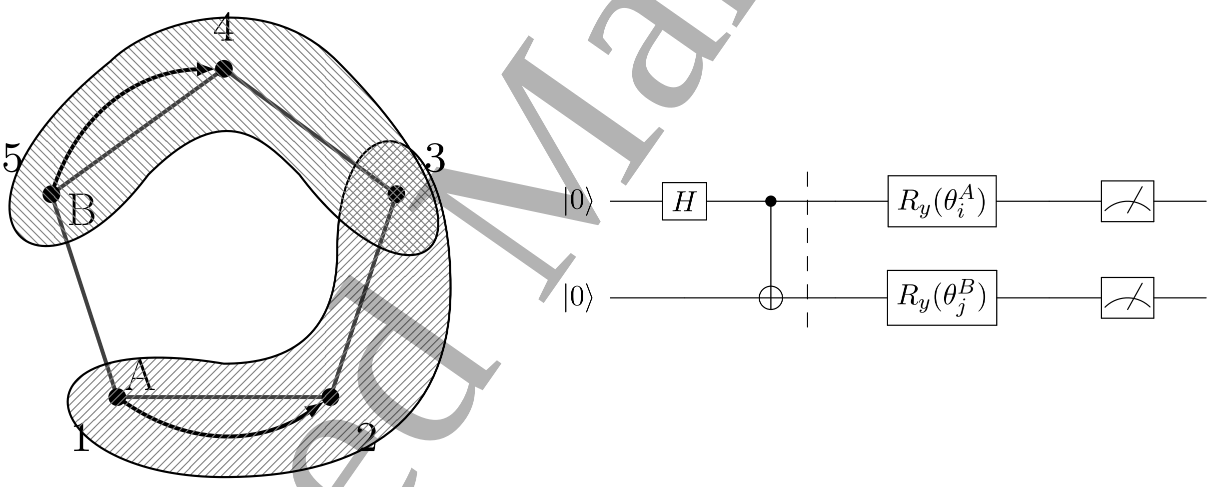
In this paper, we will consider cycle graphs only, so the players will share a pair of qubits ($n = 2$). The players always start by initialising their qubits in the Bell state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right). \tag{1}$$

They then get separated and each player carries out a local unitary operation on their qubit. We take these to consist of rotations around the y measurement axis.

Let us first consider the 5-vertex cycle graph C_5. This is the simplest non-trivial graph for the domination game.

The classical version of the game is illustrated in Fig. 1a.The optimal classical strategy (shown in Fig. 4 of Ref. [1]) yields an average domination of 4.6 nodes. The quantum approach for all cycles is illustrated in Fig. 1b. For C_5, it achieves a domination number $D \approx 4.67$ [1].



(a) (b)

Figure 1: Classical and quantum-assisted approaches to the graph domination game. (1a) shows a play-through of the classical domination game. The players Alice (A) and Bob (B) are randomly placed (on sites 1 and 5, in this example). They then use a pre-agreed strategy to decide their moves. In our example, A moves from 1 to 2 and B moves from 5 to 4, which corresponds to the optimal classical strategy shown in Fig. 4 of Ref. [1]. They then check how much of the graph is dominated between them (shaded area). In this case, they dominate all 5 nodes of the graph. (1b) shows the quantum circuit A and B can use to gain quantum advantage. Before A and B get separated (left of the dashed line) they perform a joint operation on their two qubits that places them in the entangled state of Eq. (1). Afterwards the players get assigned their starting nodes and A(B) rotate their qubit around the y axis by an angle θ_i^A (θ_j^B) that depends on the index i (j) of the site they are in. They then measure their qubit in the computational basis and use the result to decide whether to move clockwise (1) or anti-clockwise (0).

[3] This implies no loss of generality as player-symmetric strategies are a subset of player-asymmetric ones.

		1		2		3		4		5	
		0	1	0	1	0	1	0	1	0	1
1	0	3	5	4	4	3	5	5	4	4	5
	1	5	3	4	5	5	4	3	5	4	4
2	0	4	4	3	5	4	5	4	5	3	5
	1	4	5	5	3	4	4	5	3	5	4
3	0	3	5	4	4	3	5	5	4	4	5
	1	5	4	5	4	5	3	4	4	5	3
4	0	5	3	4	5	5	4	3	5	4	4
	1	4	5	5	3	4	4	5	3	5	4
5	0	4	4	3	5	4	5	4	5	3	5
	1	5	4	5	4	5	3	4	4	5	3

Table 1: The domination table for the C_5 graph where players independently decide whether to move clockwise or anti-clockwise by flipping a coin (or examining a qubit). The number at the top of each column represents Alice's site, and the number at the front of each row represents Bob's site. 0 and 1 correspond to clockwise and anti-clockwise moves, respectively.

The average domination for any two-player, single-step strategy on a cycle graph is given by

$$D = \frac{1}{N^2} \sum_{a,b=1}^{N} \sum_{n,m=0}^{1} C_{a,b,n,m} P_{a,b}(n,m) \tag{2}$$

where N is the number of sites on the graph, $C_{a,b,n,m}$ is the domination achieved when players on sites a and b make moves n and m respectively, and $P_{a,b}(n,m)$ is the probability that the collective move indexed by n, m is made when the players are initially at a, b.

In order to find the explicit strategy that realises the optimal bounds found by Viola and Mirionowicz, we can employ a similar heuristics to that used for the rendezvous problem in Ref. [11]. We start by creating the "domination table", Tab. 1, which shows the domination number depending on the sites Alice and Bob land on and on whether they move clockwise (1) or counter-clockwise (0).[4] A random coin-flip leads to an unweighted average of all the numbers on the table ($D = 4.2$). The optimal classical strategy shown in Fig. 4 of Ref. [1] corresponds to picking a specific row (column) for each of Alice's (Bob's) possible starting sites. For instance, for the case where Alice starts on Site 1 we pick 0 (counter-clockwise motion) and for the case where Bob starts on Site 5 we pick 1 (clockwise motion). This is the situation illustrated in Fig. 1a and leads to a higher average domination number $D = 4.6$. Note, however, that this means Alice's and Bob's moves are uncorrelated: Bob will move clockwise whenever he starts the game at Site 5, independently of Alice's location. In the quantum game all combinations of starting sites and measurement outcomes are possible but they have different probabilities given by

$$\begin{pmatrix} P_{i,j}(00) & P_{i,j}(01) \\ P_{i,j}(10) & P_{i,j}(11) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cos^2 \frac{\theta_j^B - \theta_i^A}{2} & \sin^2 \frac{\theta_j^B - \theta_i^A}{2} \\ \sin^2 \frac{\theta_j^B - {}'\theta_i^A}{2} & \cos^2 \frac{\theta_j^B - \theta_i^A}{2} \end{pmatrix}. \tag{3}$$

Here, $P_{i,j}(\sigma_A \sigma_B)$ denotes the probability of Alice's reading being σ_A and Bob's being σ_B ($\sigma_A, \sigma_B = 0, 1$). It depends on the angles θ_i^A and θ_j^B which are functions of the indices i, j of their respective landing sites.

Any given quantum-assisted graph-domination strategy is completely specified by the functions

$$\theta^A : i \mapsto \theta_i^A \tag{4}$$

$$\theta^B : j \mapsto \theta_j^B \tag{5}$$

[4] A similar table for a larger graph (C_{10}) has been provided in the Appendix for illustration purposes.

These in turn determine the probabilities in Eq. (3). With this, we can straight-forwardly obtain the average domination number by substituting the probabilities into Eq. (2) alongside each corresponding square from Tab. 1. Simplifying this equation yields:

$$\begin{aligned} D_5 = \frac{1}{25}\Bigg\{ & 3\sum_{i=1}^{5}\cos^2\left(\frac{\theta_i^B-\theta_i^A}{2}\right) + 4\sum_{i=1}^{5}\left[\cos^2\left(\frac{\theta_i^B-\theta_{(i+1)}^A}{2}\right)+\cos^2\left(\frac{\theta_i^B-\theta_{(i-1)}^A}{2}\right)\right] \\ & +5\sum_{i=1}^{5}\left[\cos^2\left(\frac{\theta_i^B-\theta_{(i+2)}^A}{2}\right)+\cos^2\left(\frac{\theta_i^B-\theta_{(i-2)}^A}{2}\right)+\sin^2\left(\frac{\theta_i^A-\theta_i^B}{2}\right)\right] \\ & +\frac{9}{2}\sum_{i=1}^{5}\left[\cos^2\left(\frac{\theta_i^A-\theta_{(i+1)}^B}{2}\right)+\cos^2\left(\frac{\theta_i^A-\theta_{(i-1)}^B}{2}\right)\right] \\ & +\frac{7}{2}\sum_{i=1}^{5}\left[\cos^2\left(\frac{\theta_i^A-\theta_{(i+2)}^B}{2}\right)+\cos^2\left(\frac{\theta_i^A-\theta_{(i-2)}^B}{2}\right)\right]\Bigg\}. \end{aligned} \tag{6}$$

A cycle graph is periodic with a finite number of equivalent sites. To take account of this all site labels are written modulo n_v. For example for $n_v = 5$ if $i = 4$, then $i - 2 = 2$ and $i + 2 = 1$.

Note that, as a result of the symmetry of the graph, the graph domination number depends only on the differences between the measurement angles. However, Alice and Bob do not get to pick that difference because they do not know the site at which the other player is located. They can only pick their own angle using a previously-agreed strategy that is optimal on average. Therefore, an absolute angle has to be associated with each site, not just angle differences. From that point of view, the C_5 symmetry means there are many degenerate strategies, all of them equally successful: a constant angle could be added to the angle associated with every site and the resulting strategy would be equivalent. However, the strategies (choices of angles) used by Alice and Bob cannot be altered independently: the same constant angle must be added by both players.

We have optimised numerically Eq. (6) with respect to the 10 angles $\theta_1^A, \dots \theta_5^A, \theta_1^B, \dots, \theta_5^B$ using the Broyden–Fletcher–Goldfarb–Shanno algorithm [21]. The results suggest that the optimal angles are given by

$$\theta_i^A = (i-1)\,\theta_5 \tag{7}$$

$$\theta_i^B = \pi + \theta_i^A \tag{8}$$

where

$$\theta_5 = \frac{2\pi}{5} \tag{9}$$

is a fixed angle increment. We have confirmed analytically that this is indeed a maximum, and it coincides numerically with the previously discovered [1] bound, $D \approx 4.67361$. The difference of π between the angles used by Bob and Alice ensures that the players spread out when they start together, which is desirable for graph domination. Moreover, the probabilities obtained by substituting Eqs. (7,8) into (3) are those given in Ref. [1] for this game.

We now generalise (7,8) to

$$\theta_i^A = (i-1)\,\theta_n \tag{10}$$

$$\theta_i^B = \pi + \theta_i^A. \tag{11}$$

Recognizing that the structure of the domination table is invariant as a function of number of vertices when the players are close to each other enables us to derive a generalization of equation (6) for any cycle graph ($n_v > 3$). The average domination number can be rewritten as a Fourier series by application of basic trigonometric identities:

$$D \;=\; 6 + \frac{1}{n}\left(-8 + \cos(\theta) - \frac{1}{2}\cos(2\theta) - \cos(3\theta) - \frac{1}{2}\cos(4\theta)\right)$$

$$+ \quad \frac{1}{n^2}\left(-\cos(\theta)+\cos(2\theta)+3\cos(3\theta)+2\cos(4\theta)\right.$$
$$\left. - \quad 2\cos((n-4)\theta)-3\cos((n-3)\theta)-\cos((n-2)\theta)+\cos((n-1)\theta)\right). \tag{12}$$

Here, $\theta \equiv \theta_{i+1}^{A/B} - \theta_i^{A/B}$ is the constant difference between the angle set by Alice/Bob when they land on site $i+1$ and at site i. The above expression can be written more compactly as

$$D_n(\theta) = \lambda_n + \sum_{l=1}^{n-1} \mu_l \cos(l\theta). \tag{13}$$

The constant λ_n and the coefficients μ_n can be deduced from the longer form. For convenience, they are given in Table 2 for the first 7 values of n. We have verified numerically that the above equation gives an optimal strategy for every value of n for which the bounds are known [1] i.e. $5 \leq n \leq 13$. Interestingly, the angle increment is not always given by the straight-forward generalisation of (9),

$$\theta_n = \frac{2\pi}{n}. \tag{14}$$

That expression is valid for $n \leq 10$ only. For $n = 11, 12, 13$ we find instead

$$\theta_n = \frac{4\pi}{n}. \tag{15}$$

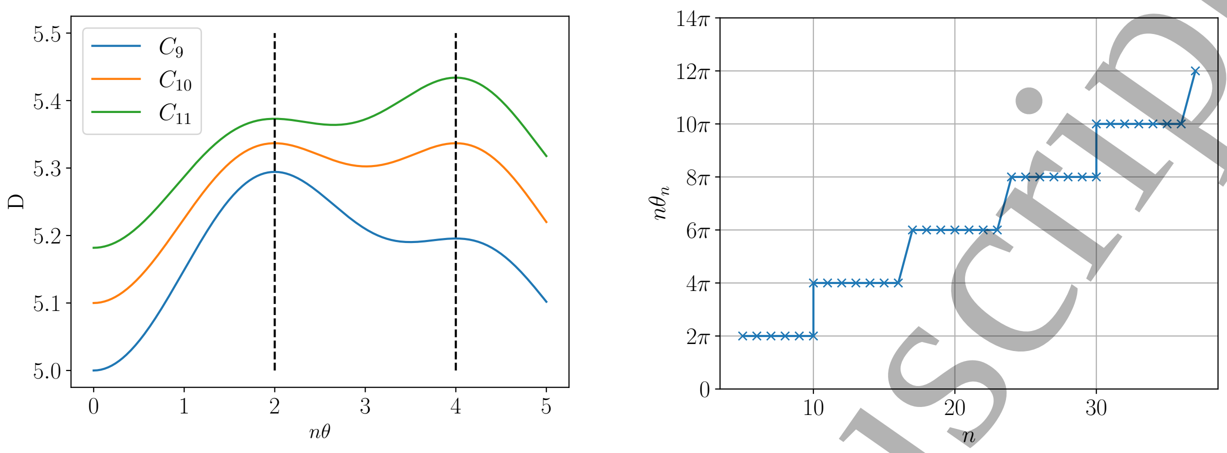
The transition is illustrated in Fig. 2a which shows the optimization for $n = 9, 10, 11$ (for $n = 10$, both choices of the angle optimize the domination number). Beyond $n = 13$ the optimal domination numbers are not known [1]. We can, however, hypothesize that the optimal strategy is still consistent with Eqs. (10,11) and determine numerically the value of θ that optimizes (12). This value, multiplied by the number of sites on the graph, is displayed in Fig. 2b for cycles with $n \leq 37$, displaying an interesting stepped behaviour. Analysis of equation (12) shows that the value of n at which the steps in $n\theta$ occur is determined by the relative phase of the terms in $1/n$ and those in $1/n^2$. n does not occur in the argument of the cosines in the terms in $1/n$. These terms interfere, but the resulting wave, to a good approximation, does not change as a function of n. n does occur in some of the terms of order $1/n^2$. The wave that results from the interference of these terms does change with n. In fact the peaks in these terms increase in number and the lower peaks move to lower angles as a function of n. Each time a minimum in the $1/n^2$ terms passes the peak in the $1/n$ terms a step in the value of $n\theta$ at which dominance is a maximum occurs. We see that the steps will occur when n changes by about 6.67 and that if a step occurs at a particular value of n one will also occur at $n + 20$. This can be thought of as the players more strongly correlating their decisions by shortening the distance the correlations span. This is beneficial as past a certain distance, there is no advantage in correlating their actions, as any combination of moves would result in maximum domination.

The predicted performance for these optimized values of θ is shown in Fig. 3. We cannot discard that better strategies might be available for $n > 13$. The steps observed in Fig. 2b have a visible effect on domination. This is further illustrated by comparing the optimal quantum strategy to the dashed line, which shows the performance of the quantum strategy that is optimal before the first jump ($\theta = \frac{2\pi}{n}$) when applied to larger graph sizes. Clearly, the successive shifts in player strategies as the number of sites increases plays a crucial role in maximising the amount of quantum advantage.

3 Simulations

In this section we will describe simulations of our graph-domination strategies using both classical and quantum processors. The former allow us to confirm convergence under ideal conditions. The latter allow us to verify that the predicted quantum advantages can be realised using current qubit technology. The *modus operandi* for these simulations is analogous to that used for quantum-assisted rendezvous in Ref. [11].

Our simulations of graph domination strategies on the simplest non-trivial cycle graph, C_5, are shown in Fig. 4. We observe clear convergence of the classical simulations towards the predicted



(a) (b)

Figure 2: Optimization of the angle increment θ for maximising the domination number $D(n,\theta)$. (a) Domination number as a function of the angle for $n = 9, 10, 11$, showing a shift in the optimal value of θ from the one given in Eq. (9) ($n = 9$) to the one given in Eq. (15) ($n = 11$). (b) Optimal value of θ as a function of n.

n	5	6	7	8	9	10	11
λ_n	85	138	203	280	369	470	583
μ_1	2	5	6	7	8	9	10
μ_2	$-\frac{9}{2}$	-4	$-\frac{5}{2}$	-3	$-\frac{7}{2}$	-4	$-\frac{9}{2}$
μ_3	-3	-6	-6	-5	-6	-7	-8
μ_4	$\frac{1}{2}$	-2	$-\frac{9}{2}$	-4	$-\frac{5}{2}$	-3	$-\frac{7}{2}$
μ_5		1	-1	-3	-2	0	0
μ_6			1	-1	-3	-2	0
μ_7				1	-1	-3	-2
μ_8					1	-1	-3
μ_9						1	-1
μ_{10}							1

Table 2: Coefficients in Eq. (13) for cycle graphs with $5 \leq n \leq 11$.

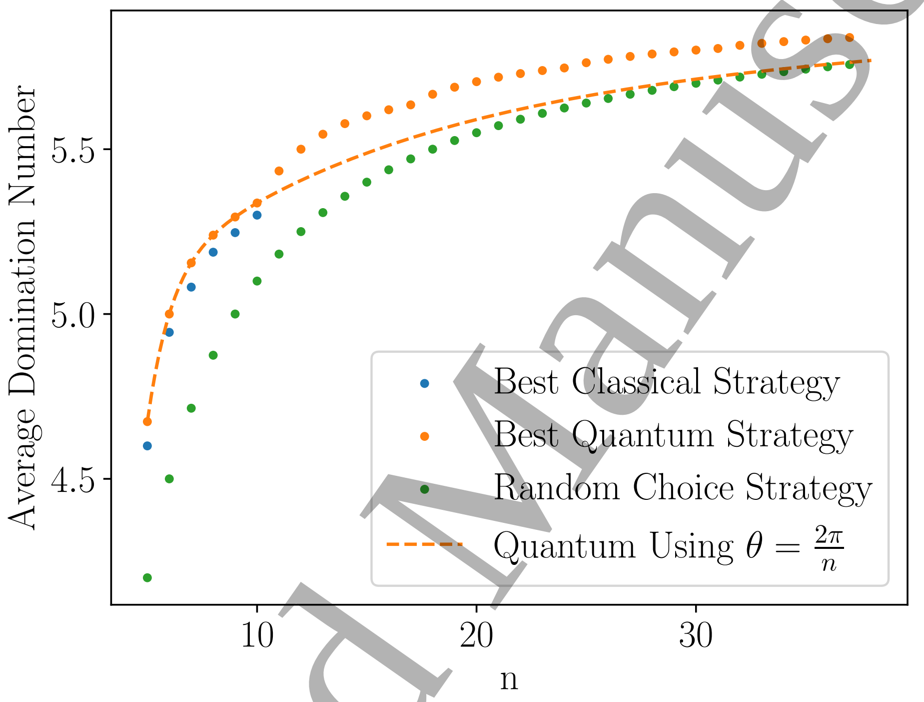


Figure 3: Theoretical prediction of the average domination number achieved by our optimised quantum strategy for cycle graphs compared to a coin-tossing strategy (random choice) and to the optimal classical strategy, when known. In each case, the domination number is given as a function of the number n of vertices in the graph. The dashed line has been obtained by substituting (14) into (12) and it illustrates the performance of the quantum strategy that is optimal for small graphs ($n \leq 10$) when deployed on larger graphs.

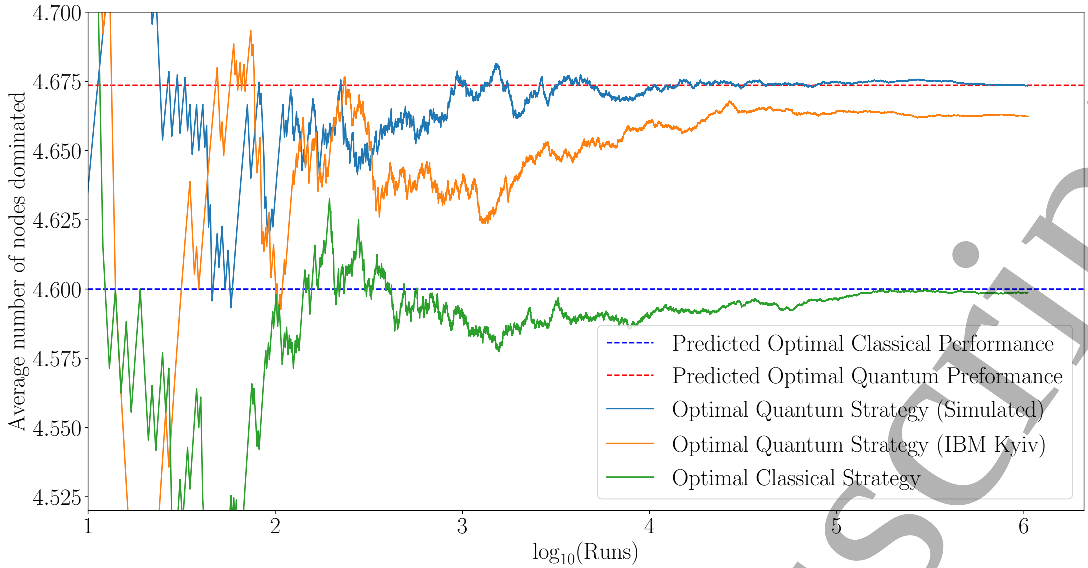


Figure 4: Performance of optimal strategies for the graph domination game on a 5-site cycle. The dashed lines indicate the predicted averages over many runs for the optimal classical and quantum strategies, as indicated. The solid lines show: simulations of the optimal classical and quantum strategies using a classical computer; and a simulation of the optimal quantum strategy using the best found superconducting quantum processor, IBM Kyiv. The difference between the simulated and hardware results is chiefly accounted for by gate and measurement errors, with decoherence playing a minimal role, as discussed in Section 4.

average domination numbers, with the performance of the classical and quantum strategies clearly differentiated after averaging over > 1000 runs. Similar convergence behaviour was obtained for C_6 and C_7, albeit somewhat slower for larger graphs (see Figs. 6,7 in Appendix B). For the quantum simulations we used four different superconducting quantum processors, namely `ibm_kyiv` [22], `ibm_marrakesh` [23], `ibm_brisbane` [24] and `ibm_fez` [25], as well as, in the case of C_5, one trapped-ions quantum processor, `IONQ Aria1` [26]. The converged results for the graphs C_5, C_6, and C_7 after averaging over $10^{20} \approx 10^6$ runs are shown in Fig. 5. `ibm_marrakesh`, `ibm_brisbane`, and `ibm_fez` show similar performance, with `ibm_kyiv` outperforming the other three. `IONQ Aria1`, used for C_5, shows performance below all three superconducting quantum processors. In all cases, the experimentally obtained domination numbers are closer to the optimal quantum strategy than the classical one.

To quantify the observed quantum advantage A we use the following formula [1]:

$$A = \frac{Q - C}{C - R} \tag{16}$$

Here Q, C, and R are the average domination numbers with the optimal quantum strategy, the optimal classical strategy, and random choice, respectively. This equation can be rewritten as $A = (Q - R)/(C - R) - 1$. Thus, both strategies (classical and quantum) are compared to a random choice. If $A = 0$, both fare equally and there is no quantum advantage. More generally, if the optimal classical strategy represents an M-fold improvement over random choice the optimal quantum strategy represents and $[M + A(M - 1)]$-fold improvement. The achieved quantum advantages are summarised in Table 3. Our explorations on cloud-based quantum hardware show that most of the available quantum advantage can be realised. This is auspicious both for the possibility of realising the advantage in real-life scenarios and using quantum processors to simulate complex graph-domination strategies beyond the reach of classical computers (i.e., those involving large numbers of entangled qubits). Although the limited gate fidelities and finite qubit coherence times of the NISQ devices used here are sufficient to demonstrate the quantum advantage conceptually on small graphs, experimental errors are

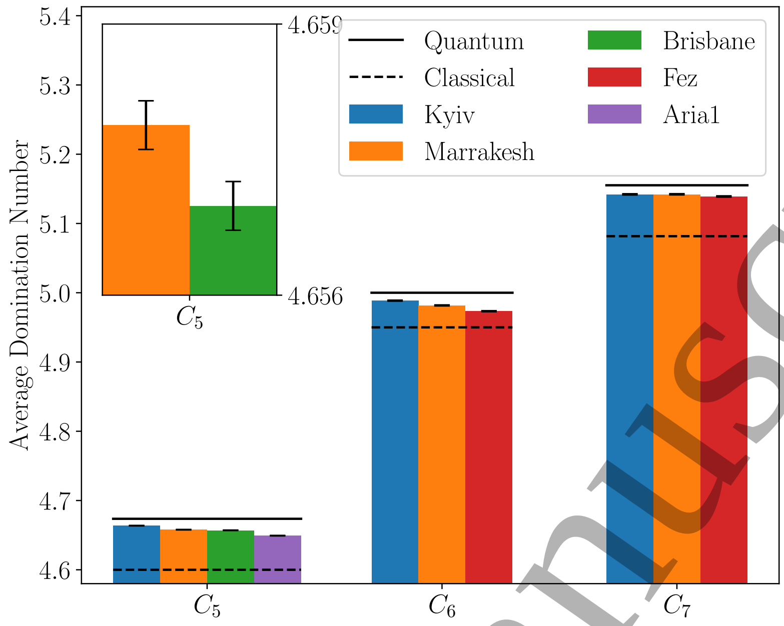


Figure 5: Performance of optimal strategies for the graph domination game on 5, 6 and 7 cycle graphs simulated on different quantum processors. The dashed line indicates the predicted average for the optimal classical strategy and the solid line indicates the optimal quantum strategy. Coloured bars show the average domination number obtained from $10^{20} \approx 10^6$ runs. Error bars represent one standard deviation (see inset for scale).

	C_5	C_6	C_7
Predicted	**18%**	**11%**	**20%**
IBM Marrakesh	14%	7%	16%
IBM Brisbane	14%	-	-
IBM Fez	-	5%	15%
IBM Kyiv	15%	9%	16%
IONQ Aria1	12%	-	-

Table 3: Predicted values of the quantum advantage A for graph domination on 5-, 6-, and 7-site cycles compared to the values achieved after averaging over $10^{20} \approx 10^6$ runs of the simulated game using different quantum processors.

the limiting factor for practical applications. Moreover, current hardware is not yet field-deployable; quantum networking, i.e., the ability to entangle qubits in different processors, will become an essential capability. To this end, significant progress is underway to create room-temperature optical interfaces between remote trapped ions [27, 28], neutral atoms [29], and color centres in diamond [30], as well as cryogenic links between superconducting qubits [31]. As the number of available high-quality qubits grows and quantum networks become more powerful, the simulation of more complex strategies will come within reach for experimentation.

4 Discussion

It is worth trying to develop an intuitive understanding of the origin of quantum advantage in our 2-player, mobile-agent games. Since the players only learn their starting locations after they get separated, and at that point they can no longer communicate, it is counter-intuitive that, by manipulating entangled qubits, they are able to improve their level of coordination, compared to a pre-shared deterministic strategy. One way to understand this is to regard the local rotated qubit measurement as a way to encode local information in the shared system. This introduces correlations between the moves made by the each player that are informed by the local information provided by both players, even though the players themselves cannot retrieve that information. Ultimately, what makes this possible is the fact that although the maximally entangled 2-qubit state has zero global entropy, the entropy of each qubit, considered separately, is maximal – a property of quantum systems with no classical analogue.

The gap between the predicted quantum advantage and the realised quantum advantage can be explained in a depolarising model, where an error would result in the execution of the random strategy, whereas no error would execute the optimal quantum strategy. In the case of C_5, the random strategy and the optimal quantum strategy realise $D = 4.2$ and $D = 4.67361$, respectively. Using IBM Marrakesh as the middling performer from Figs. 4,5 for example, the observed domination number of $D \approx 4.658$ is reproduced by a depolarising probability $p_{\text{depol}} \approx 3.3\,\%$. The dominant mechanisms responsible for the suboptimal performance of the hardware are

- decoherence due to unwanted interaction of the qubits with the environment;
- gate errors due to imperfect implementation of quantum operations;
- and state preparation and measurement (SPAM) errors.

The extent of decoherence can be estimated by comparing the execution time of the quantum circuit to the decoherence timescales (i.e. the relaxation time T_1 and dephasing time T_2). Focusing again on IBM Marrakesh, the average qubit has $T_1 \sim 179\,\mu\text{s}$ and $T_2 \sim 89\,\mu\text{s}$ [23] setting a much longer decoherence timescale than the average $0.18\,\mu\text{s}$ each circuit took to run. This strongly suggests that the effect of decoherence is negligible. To measure gate error, we model the circuit as a sequence of independent noisy gates with single qubit fidelities of 0.9997 [23] and two-qubit fidelity of 0.997 [23]. This results in approximately a $0.65\,\%$ chance for there to be a gate-induced error, thus contributing significantly to the overall decrease in performance on NISQ hardware. The largest contributing factor however, are SPAM errors. Assuming $1.337\,\%$ failure probability per qubit, [32] this leads to $\approx 2.7\,\%$ error chance, which represents the largest contribution to the suboptimal performance. We therefore estimate the total error at probability at $p_{\text{err}} \approx 3.4\,\%$, which is in excellent agreement with $p_{\text{depol}} \approx 3.3\,\%$ obtained above.

In the context of quantum computing [33], the term "quantum advantage" usually refers to the time complexity of a problem, when compared to the best known classical algorithms. Examples of this are Shor's algorithm [34] for factorisation and Grover's algorithm [35] for searching unstructured databases. The quantum advantage we have found in this work is not of that type – in fact, since playing our game requires only two qubits, it is perfectly possible to simulate the whole game on a classical machine (the blue line in Fig. 4). Instead, it is an example of a different type of advantage enabled by entanglement; its non-local nature provides advantage in situations where communication is limited and correlated outcomes are targeted. The paradigmatic example is quantum cryptography [36].

We could call such advantages "quantum operational advantages" to distinguish them for those of the purely computational kind. The use of entanglement to improve the probability of success of coordinated moves by distant parties without exchange of signals [1,11,37,38] falls under that category.

Non-local games provide a natural framework for modelling performance improvements in distributed computing tasks, where players receive a local input and must produce a global output [39]. More generally, graph theoretical frameworks are uniquely suited to analyse interconnected systems in a wide range of fields due to their ability to capture complex relationships. Graph domination problems often arise in the context of resource allocation tasks [14, 15], where external factors can be dynamically changing. For instance, collision avoidance problems [40, 41], can be re-cast as a graph domination game (the problem we have tackled here is indeed closely related to collision avoidance on a cycle). In general, the optimal outcome cannot be achieved if real-time communication between the resources is impossible, restricted, unreliable, or unsafe. This issue can arise in critical infrastructure networks, such as ambulances, military units, and electrical power distribution grids. Using quantum resources, the communication requirements can be reduced [42, 43] and, on graphs, nonlocal correlations appear to create the 'telepathic' ability that coordinates movements, improving the average payoff. Recently, the theory of quantum non-local games has been generalised to the case where communication is not impossible but suffers from high latency which is relevant to real-world applications including high frequency trading, distributed computing, computer architecture, and distributed control systems [44].

Our work connects to a broader line of results on coordinating mobile agents [11, 37, 38], including rendezvous games where a quantum advantage was first identified by Mironowicz [38] and demonstrated on real NISQ hardware in Ref. [11]. These belong to the family of cooperative *non-local games* (players receive private inputs and cannot communicate) in which entanglement improves performance [7]. By contrast, *quantum strategic-form games* consider complete-information, simultaneous-move settings [45], with the classic quantum Prisoner's Dilemma [5] and, more recently, the quantum Volunteer's Dilemma [6] providing canonical examples featuring a tension between cooperative and selfish incentives. Some games are cooperative in objective yet strategic-form in structure—such as the Kolkata Restaurant Problem [46]—and thus bridge these two perspectives.

5 Conclusions and outlook

In conclusion, we have identified explicit strategies that realise a quantum operational advantage for mobile agents attempting to dominate circular graphs of arbitrary size, and reproduced them successfully on NISQ hardware.

There is a rich outlook for the further development of this topic from the point of view of possible applications. In networked settings, quantum strategies can often be *embedded* into existing classical protocols to yield measurable advantages in situations where communication is constrained. Entanglement distribution is a key network resource [47–49]; the non-local, cooperative games studied here provide primitives for leveraging that resource under minimal signalling. More broadly, the operational research literature [50] offers real-world problems amenable to game-theoretic treatments, including collision avoidance [40, 41, 51], facility location [14], and related coverage tasks [15], all of which map naturally to domination on graphs.

The techniques presented here invite extensions to non-circular graphs (including mixed degree) and to games involving more than two players.

The extension to higher-degree (including mixed-degree) graphs is a fertile area for further development. For a graph consisting of only degree D vertices, each player will need $\log_2(D)$ qubits. For mixed degree graphs, in order to split options evenly regardless of the degree of the vertex the player is on, $\log_2[\mathrm{LCM}(D_1, D_2, \cdots, D_n)]$ qubits will be needed per player, where n is the total number of vertices, D_j is the degree of the jth vertex, and LCM stands for the least common multiple (LCM).

Another way to generalise our approach, in the case of more than two players, is to use a multipartite state, such as a Greenberger-Horne-Zeilinger (GHZ) state [52] or W state [53]. In this arrangement, each player still controls only one qubit. Alternatively, players could hold more than one qubit, each providing pairwise entanglement with one of the qubits held by another player. This approach would require more qubits and necessitate novel strategies to be developed, but would maximise the correlations between any pair of players due to monogamy of entanglement [54].

Acknowledgements

We would like to thank Joshua Tucker, Elizabeth Chipperfield, and Carlos Perez Delgado for useful discussions. CW acknowledges a studentship awarded by the Engineering and Physical Sciences Research Council (EPSRC) (Grant Number: EP/W52461X/1). This work was supported by the National Quantum Computing Centre through its Quantum Computing Access Program (project number ACA0014).

References

[1] P Mironowicz. Entangled rendezvous: a possible application of bell non-locality for mobile agents on networks. *New Journal of Physics*, 25(1):013023, jan 2023.

[2] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777–780, May 1935.

[3] J. S. Bell. On the Einstein Podolsky Rosen paradox. *Physics Physique Fizika*, 1:195–200, Nov 1964.

[4] Nobel Prize Outreach AB. The Nobel Prize in Physics 2022. NobelPrize.org, 2022. Accessed: 2026-04-02.

[5] Jens Eisert, Martin Wilkens, and Maciej Lewenstein. Quantum Games and Quantum Strategies. *Phys. Rev. Lett.*, 83(15):3077–3080, October 1999.

[6] Dax Enshan Koh, Kaavya Kumar, and Siong Thye Goh. Quantum volunteer's dilemma. *Phys. Rev. Res.*, 7:013104, Jan 2025.

[7] P. Drmota, D. Main, E. M. Ainley, A. Agrawal, G. Araneda, D. P. Nadlinger, B. C. Nichol, R. Srinivas, A. Cabello, and D. M. Lucas. Experimental Quantum Advantage in the Odd-Cycle Game. *Phys. Rev. Lett.*, 134(7):070201, February 2025.

[8] N. David Mermin. Simple unified form for the major no-hidden-variables theorems. *Phys. Rev. Lett.*, 65:3373–3376, Dec 1990.

[9] Jia-Min Xu, Yi-Zheng Zhen, Yu-Xiang Yang, Zi-Mo Cheng, Zhi-Cheng Ren, Kai Chen, Xi-Lin Wang, and Hui-Tian Wang. Experimental demonstration of quantum pseudotelepathy. *Phys. Rev. Lett.*, 129:050402, Jul 2022.

[10] Fatih Ozaydin. Quantum pseudo-telepathy in spin systems: Magic square game under magnetic fields and dzyaloshinskii-moriya interaction, 2016.

[11] Joshua T. Tucker, Paul Strange, Piotr Mironowicz, and Jorge Quintanilla. Quantum-assisted Rendezvous on Graphs: Explicit Algorithms and Quantum Computer Simulations. *New Journal of Physics*, 2024.

[12] Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater. *Fundamentals of Domination in Graphs*. Pure and Applied Mathematics - A Series of Monographs and Textbooks. Marcel Dekker, Inc., New York - Basel, 1998.

[13] R.J. Wilson. *Introduction to Graph Theory*. Longman, 1996.

[14] B. A. Sabarish, B Kailassh, Kirthana Baktha, and Y Janaki. Recommendations of location for facilities using domination set theory. In *2017 International Conference on Communication and Signal Processing (ICCSP)*, pages 1540–1544, 2017.

[15] Teresa W Haynes, Stephen T Hedetniemi, and Michael A Henning. *Domination in graphs: Core concepts*. Springer monographs in mathematics. Springer International Publishing, Cham, 2023.

[16] William F. Klostermeyer and C. M. Mynhardt. Eternal and Secure Domination in Graphs. In *Topics in Domination in Graphs*, pages 445–478. Springer, Cham, Switzerland, October 2020.

[17] Wayne Goddard, Sandra M. Hedetniemi, and Stephen T. Hedetniemi. Eternal security in graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 52:169–180, 2005.

[18] Prabhat Kumar Chand, Anisur Rahaman Molla, and Sumathi Sivasubramaniam. Run for cover: dominating set via mobile agents. In *International Symposium on Algorithmics of Wireless Networks*, pages 133–150. Springer, 2023.

[19] João Marcos Brito, Thiago Marcilon, Nicolas A. Martins, and Rudini Sampaio. The Normal Domination Game in graphs. *J. Comput. System Sci.*, 157:103751, May 2026.

[20] Ananta Maity, Sukumar Mondal, Tofigh Allahviranloo, and Sovan Samanta. Domination in quantum graphs. In *Quantum Theory and Fuzzy Systems: Traversing Uncertainty in Group Decision-Making and Social Networks: Quantum and Fuzzy Approaches to Social Network Analysis and Group Decisions*, pages 367–377. Springer, 2025.

[21] Scipy minimize documentation. https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html. Accessed: 22/09/2025.

[22] IBM Quantum. IBM quantum processor: ibm_kyiv (Eagle r3 architecture). https://quantum-computing.ibm.com/, 2025. Accessed via the IBM Quantum Platform. Processor type: Eagle r3.

[23] IBM Quantum. IBM quantum processor: ibm_marrakesh (Heron r2 architecture). https://quantum-computing.ibm.com/, 2025. Accessed via the IBM Quantum Platform. Processor type: Heron r1.

[24] IBM Quantum. IBM quantum processor: ibm_brisbane (Eagle r3 architecture). https://quantum-computing.ibm.com/, 2025. Accessed via the IBM Quantum Platform. Processor type: Eagle r3.

[25] IBM Quantum. IBM quantum processor: ibm_fez (Heron r2 architecture). https://quantum-computing.ibm.com/, 2025. Accessed via the IBM Quantum Platform. Processor type: Heron r1.

[26] IONQ. IONQ aria1 quantum processor. https://azure.microsoft.com/en-us/products/quantum-computing/, 2025. Accessed via Microsoft Azure Quantum platform. Processor type: IonQ Aria1.

[27] D. Main, P. Drmota, D. P. Nadlinger, E. M. Ainley, A. Agrawal, B. C. Nichol, R. Srinivas, G. Araneda, and D. M. Lucas. Distributed quantum computing across an optical network link. *Nature*, 638(8050):383–388, Feb 2025.

[28] V. Krutyanskiy, M. Galli, V. Krcmarsky, S. Baier, D. A. Fioretto, Y. Pu, A. Mazloom, P. Sekatski, M. Canteri, M. Teller, J. Schupp, J. Bate, M. Meraner, N. Sangouard, B. P. Lanyon, and T. E. Northup. Entanglement of trapped-ion qubits separated by 230 meters. *Phys. Rev. Lett.*, 130:050803, Feb 2023.

[29] Tim van Leent, Matthias Bock, Florian Fertig, Robert Garthoff, Sebastian Eppelt, Yiru Zhou, Pooja Malik, Matthias Seubert, Tobias Bauer, Wenjamin Rosenfeld, Wei Zhang, Christoph Becher, and Harald Weinfurter. Entangling single atoms over 33 km telecom fibre. *Nature*, 607(7917):69–73, Jul 2022.

[30] Arian J. Stolk, Kian L. van der Enden, Marie-Christine Slater, Ingmar te Raa-Derckx, Pieter Botma, Joris van Rantwijk, J. J. Benjamin Biemond, Ronald A. J. Hagen, Rodolf W. Herfst, Wouter D. Koek, Adrianus J. H. Meskers, René Vollmer, Erwin J. van Zwet, Matthew Markham, Andrew M. Edmonds, J. Fabian Geus, Florian Elsen, Bernd Jungbluth, Constantin Haefner, Christoph Tresp, Jürgen Stuhler, Stephan Ritter, and Ronald Hanson. Metropolitan-scale heralded entanglement of solid-state qubits. *Science Advances*, 10(44):eadp6442, 2024.

[31] P. Magnard, S. Storz, P. Kurpiers, J. Schär, F. Marxer, J. Lütolf, T. Walter, J.-C. Besse, M. Gabureac, K. Reuer, A. Akin, B. Royer, A. Blais, and A. Wallraff. Microwave quantum link between superconducting circuits housed in spatially separated cryogenic systems. *Phys. Rev. Lett.*, 125:260502, Dec 2020.

[32] IBM Quantum. IBM quantum processor: ibm_brussels (Eagle r3 architecture). `https://quantum-computing.ibm.com/`, 2025. Accessed via the IBM Quantum Platform. Processor type: Eagle r3.

[33] E.G. Rieffel and W.H. Polak. *Quantum Computing: A Gentle Introduction*. Scientific and Engineering Computation. MIT Press, 2014.

[34] P.W. Shor. Algorithms for quantum computation: discrete logarithms and factoring. In *Proceedings 35th Annual Symposium on Foundations of Computer Science*, pages 124–134, 1994.

[35] Lov K. Grover. A fast quantum mechanical algorithm for database search. In *Proceedings of the Twenty-Eighth Annual ACM Symposium on Theory of Computing*, STOC '96, page 212–219, New York, NY, USA, 1996. Association for Computing Machinery.

[36] S. Pirandola, S. Pirandola, U. L. Andersen, L. Banchi, M. Berta, D. Bunandar, R. Colbeck, D. Englund, T. Gehring, C. Lupo, C. Ottaviani, J. L. Pereira, M. Razavi, J. Shamsul Shaari, J. Shamsul Shaari, M. Tomamichel, M. Tomamichel, V. C. Usenko, G. Vallone, P. Villoresi, and P. Wallden. Advances in quantum cryptography. *Adv. Opt. Photonics*, 12(4):1012–1236, December 2020.

[37] Časlav Brukner, NIKOLA PAUNKOVIĆ, Terry Rudolph, and Vlatko Vedral. Entanglement-assisted orientation in space. *International Journal of Quantum Information*, 4(02):365–370, 2006.

[38] P Mironowicz. Entangled rendezvous: a possible application of bell non-locality for mobile agents on networks. *New Journal of Physics*, 25(1):013023, jan 2023.

[39] Francisco Ferreira da Silva and Stephanie Wehner. Entanglement improves coordination in distributed systems. In *ACM Conferences*, pages 14–20. Association for Computing Machinery, New York, NY, USA, September 2025.

[40] Tianlei Miao. Graph search algorithms in pathfinding and collision avoidance. Accessed: 30/09/2025.

[41] Akif Durdu, Mesut Tuncer, and Berat Yıldız. Graph-based collision avoidance algorithm among swarm agents. In *2023 8th International Symposium on Electrical and Electronics Engineering (ISEEE)*, pages 19–24, 2023.

[42] Richard Cleve and Harry Buhrman. Substituting quantum entanglement for communication. *Phys. Rev. A*, 56(2):1201–1204, August 1997.

[43] Peng Xue, Yun-Feng Huang, Yong-Sheng Zhang, Chuan-Feng Li, and Guang-Can Guo. Reducing the communication complexity with quantum entanglement. *Phys. Rev. A*, 64(3):032304, August 2001.

[44] Dawei Ding, Zhengfeng Ji, Pierre Pocreau, Mingze Xu, and Xinyu Xu. Quantum nonlocality under latency constraints. *arXiv preprint arXiv:2510.26349*, 2025.

[45] David A Meyer. Quantum games and quantum algorithms. *arXiv preprint quant-ph/0004092*, 2000.

[46] Puya Sharif and Hoshang Heydari. *An Introduction to Multi-player, Multi-choice Quantum Games: Quantum Minority Games & Kolkata Restaurant Problems*, pages 217–236. Springer Milan, Milano, 2013.

[47] Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, and Anton Zeilinger. Experimental quantum teleportation. *Nature*, 390(6660):575–579, Dec 1997.

[48] Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters. Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels. *Phys. Rev. Lett.*, 70:1895–1899, Mar 1993.

[49] Indrakshi Dey, Nicola Marchetti, Marcello Caleffi, and Angela Sara Cacciapuoti. Quantum game theory meets quantum networks, 2024.

[50] Fotios Petropoulos, Gilbert Laporte, Emel Aktas, Sibel A. Alumur, Claudia Archetti, Hayriye Ayhan, and Maria Battarra et al. Operational research: methods and applications. *Journal of the Operational Research Society*, 75(3):423–617, 2024.

[51] Mohammad Reza Rezaee, Nor Asilah Wati Abdul Hamid, Masnida Hussin, and Zuriati Ahmad Zukarnain. Comprehensive review of drones collision avoidance schemes: Challenges and open issues. *IEEE Transactions on Intelligent Transportation Systems*, 25(7):6397–6426, 2024.

[52] Daniel M Greenberger, Michael A Horne, and Anton Zeilinger. Going beyond bell's theorem. In *Bell's theorem, quantum theory and conceptions of the universe*, pages 69–72. Springer, 1989.

[53] Wolfgang Dür, Guifre Vidal, and J Ignacio Cirac. Three qubits can be entangled in two inequivalent ways. *Physical Review A*, 62(6):062314, 2000.

[54] V. Vedral. *Introduction to Quantum Information Science*. Introduction to Quantum Information Science. OUP Oxford, 2006.

469

A C_{10} Domination Table

		1		2		3		4		5		6		7		8		9		10	
		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
1	0	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4
	1	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4
2	0	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3
	1	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5
3	0	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4
	1	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6
4	0	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5
	1	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6
5	0	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6	6	6
	1	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6
6	0	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6	6	6
	1	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6
7	0	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6	6	6
	1	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6
8	0	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6	5	6
	1	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5
9	0	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5	4	6
	1	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4
10	0	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3	4	4	3	5
	1	4	4	3	5	4	6	5	6	6	6	6	6	6	6	6	5	6	4	5	3

Table 4: The domination table for the cycle 10 graph where players independently decide whether to move clockwise or anti-clockwise by flipping a coin (or examining a qubit). The number at the top of each column represents Alice's site, and the number at the front of each row represents Bob's site

470

B C_6 and C_7 convergence plots

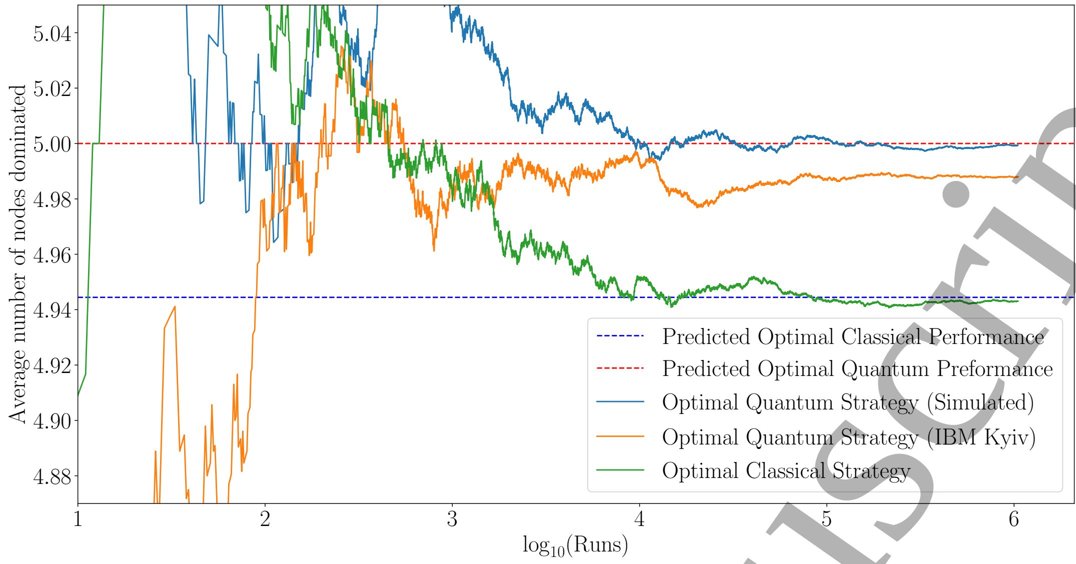


Figure 6: Performance of optimal strategies for the graph domination game on a 6-site cycle. The dashed lines indicate the predicted averages over many runs for the optimal classical and quantum strategies, as indicated. The solid lines show: simulations of the optimal classical and quantum strategies using a classical computer; and a simulation of the optimal quantum strategy using the best found superconducting quantum processor, IBM Kyiv.

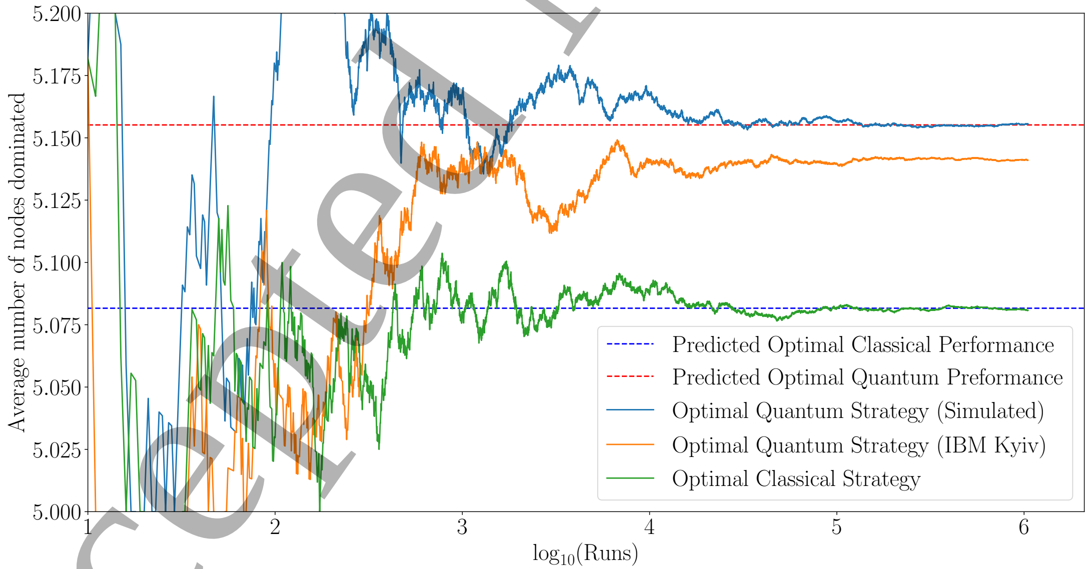


Figure 7: Performance of optimal strategies for the graph domination game on a 7-site cycle. The dashed lines indicate the predicted averages over many runs for the optimal classical and quantum strategies, as indicated. The solid lines show: simulations of the optimal classical and quantum strategies using a classical computer; and a simulation of the optimal quantum strategy using the best found superconducting quantum processor, IBM Kyiv.