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SYMMETRY AND INTERDISCIPLIANRITY

A MATHEMATICAL EXPOSITION OF BALANCE IN THE PERCEPTION OF AVIAN EGG SHAPE AND ITS RELATIONSHIP TO HUMAN NOTIONS OF "HARMONIC" STRUCTURES: FROM SYMMETRY TO ASYMMETRY

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Abstract: *Eggs are regarded to be harmonious and aesthetic structures, with their shape used in art, architecture, design and engineering applications. Considering that they combine many different shapes, from symmetrical (ellipsoid) to asymmetrical (though bilaterally symmetrical), we herein aimed to develop mathematical principles that allow us to formalize a definition of the degree of harmony in egg-shaped figures. Proposing a "principle of balance", in which the basis of the most aesthetic shape demonstrated equality of volumes/projection areas in pointed and blunt ends, we developed computational dependences, with which one can select the parameters of eggs so that their shape meets these principles. We demonstrate that two famous artworks do not adhere to such principles but describe how it could, theoretically, be*

possible to alter the egg shapes in them so that their aesthetic perception may be reconsidered. We generated, by artificial intelligence, an egg shape defined on the condition that it most represents the most aesthetic shape to the human eye, finding that it fully complied with our balance principles. Since "harmony" is a subjective term and individual perceptions of it may vary, we consider the appropriateness of formalization in determining the degree of aesthetics of egg shape, while adhering to the general perception of a bird's egg due to its asymmetric geometric structure. Aspects of our findings may have practical applications in agriculture, architecture, design and/or engineering.

Keywords: avian eggs, egg geometry, symmetry vs. asymmetry, egg images, egg profiles, egg shape balance, egg volume, egg normal projection, egg-inspired engineering.

1 INTRODUCTION

The unique "ovoid" shape of a bird's egg extends beyond the niche of consumers of this food product to an object involving the perception of "harmony", with positive aesthetic effects on the feelings of the individual observing it. Why this should be the case is matter of conjecture, but perhaps it is due to the mythologization of the egg in several ancient cultures. Life emerges from it, and it is even considered, by some, to be a kind of "container of the Universe" (Leeming, 2010). While the question of the purported oviform shape of the Universe may never be resolved, the egg shape of other objects observed through empirical science, e.g., supermassive black holes (Songsheng and Wang, 2023; Tlemissov and Kovář, 2024), the carbon cage of isomer 2 of $Tb_3N@C_{84}$ (Beavers et al., 2006) and a number of others (Merrick, 2010), have been confirmed.

With the above in mind, it is perhaps not surprising that the egg itself and egg-shaped entities are not uncommon in art works (Gilbert, 1974; Herz-Fischler, 1990; Eberhart, 2020; Hodos, 2020; Grady, 2023), religious objects (Kitat, 2014; Omran, 2015), architectural structures (Freiberger, 2007; Petrović et al., 2011; Feleki and Nagy, 2016; Obradović and Martinenko, 2023) and designer objects (Grady, 2023; Archiproducts, 2025). Furthermore, the ovoid shape is considered in the creation of egg-shaped resonant chambers meant for medical healing of the body. According to Lynn (2023), the principle of the design of these *'combines bio resonance with the ancient wisdom of sacred geometry.'*

The explanation for the popularity of egg shapes among our surroundings (beyond actual eggs) cannot logically be the widespread occurrence of this shape in nature: there are, after all, many other shapes of surrounding objects in our world, both natural and man-made.

An alternative reason could be perhaps, our subject views of its aesthetics and elegance (Gobithaasan et al., 2009; Ali et al., 2010). A review of studies pertaining to the level of human aesthetic perception of shapes suggested the main issue to be studied was the analysis of the preference for a particular shape among a range of other disparate objects. It appears, for instance, that the preference for different geometric figures among, say, a circle, triangle, and square (Manippa and Tommasi, 2023), as well as a wider range of objects and geometric shapes (Trautmann, 2021) lay with rounded objects. That is, respondents attributed the greatest emotional satisfaction to geometric shapes such as circle and/or ellipse.

We are not aware of studies suggesting the aesthetic preference of ovoids, or indeed one ovoid over another. Again, ovoids are characterized by smooth changes in the curvature of their contours, which is usually preferred when assessing the aesthetics of objects (Song and Kim, 2025). This therefore inspired us to investigate this issue more deeply, given our background in egg-shape-related research. We thus considered existing methods and approaches, and how they might be applied to concept of aesthetics or "harmony."

Perhaps the simplest, yet quite frequently used, principle of harmonic perception in the disciplines of art and architecture is the correspondence of certain geometric parameters to the principle of the "golden ratio". Since the golden ratio, also called the divine ratio, roughly 1.618 (or its reciprocal 0.618), is reflected in nature, such artistic and architectural objects that conform to it are seen as "harmonious." The human eye and mind are believed, inherently, to find this proportion appealing, particularly in a face or building that is perceived as beautiful (e.g., Livio, 2002a, 2002b; Freiberger, 2007; Stakhov, 2014; Hwang and Park, 2021; Subedi, 2024).

Although we can find no direct evidence to this effect in the relevant literature, we are given to understand (from informal conversations) that more established architects tend to adhere to, and apply, the golden ratio routinely, whereas younger ones tend to do so less.

If true, this may reflect a certain "rebelliousness of youth" or perhaps a changing of attitudes. In our own studies (Narushin et al., 2022a), we described the attempts of various researchers to link together the concept of the golden ratio with egg shape.

We concluded, however, that the ratio of the length (*L*) and maximum breadth (*B*) varied among birds. We nonetheless selected an egg whose *L*/*B* value was 1.618 and it turned out that of a bean goose (*Anser fabalis*) (Figure 1a). We did not try to establish whether this egg was considered to be aesthetically superior to the shape of the domestic goose (*Anser anser*) egg from our previous study (Narushin et al., 2023a), for which the *L*/*B* value is 1.393 (Figure 1b). Nonetheless, we could perhaps imagine experiments whereby this could be achieved.

Our further attempts to establish whether the neutral axis of the shell of avian eggs conformed to the golden ratio led to the conclusion that this was true only for chickens, domestic geese and some species of falcon, but no others (Narushin et al., 2022a). So, some birds (principally geese) lay "golden eggs", but some do not.

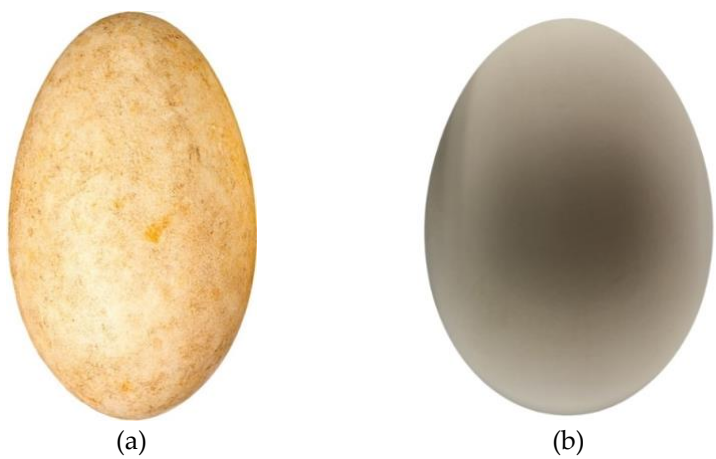


Figure 1: Comparison of subjective harmonic perception of an egg: (**a**) a bean goose (*Anser fabalis*) egg that has a "golden ratio" (1.618) of its length to maximum breadth. Image credit: Wikimedia Commons (2018a); and (**b**) a domestic goose (*Anser anser*) egg, from research by Narushin et al. (2023a).

Peculiar and indicative in this respect are the psychological experiments by Kimber (1995) who experimented with a group of students. They were asked to draw an egg profile in the hope of proving that the golden ratio is such an elegant form that the ratio of egg contours would be manifested in the imagination of an ordinary person when attempting to draw one from memory.

This experiment, however, led to an result in which, by and large, this was not the case. The first person of whom we are aware to use a clear mathematical relationship to assess the level of aesthetics of both visual and audio objects was Birkhoff (1933).

In his interpretation, the level of aesthetics of an object (M) can be computed by the relationship of its order (O) and complexity (C), i.e., $M = O/C$.

The respective components included in the assessed object structure of O and C are selected based on certain conditions and the nature of the object. For example, Birkhoff (1933) interpreted the level of C as *'the number of units in the object which require a conscious act of attention.'*

The O values of the assessed object is a complex of certain positive (or negative) feelings that the object evokes. According to Douchová (2015), who provided an extensive analysis of Birkhoff's (1933) book in terms of assessing the aesthetic component of polygons, the concept of O should be implied as *'basic properties of objects, such as symmetry, repetition, similarity, contrast, identity, balance, repetitive parts.'* Many subsequent studies were aimed at refining and mathematically interpreting the values of O and C for calculating the *aesthetic measure* (M). For example, for assessing polygons, Douchová (2015) proposed that the C value be taken equal to the number of lines on which the edges of a given polygon lie, and O be conditionally assessed by the magnitude of vertical and rotational symmetry, balance and some other parameters that can be expressed as numbers.

Gobithaasan et al. (2008) paid more attention to the calculation of the C value, which, when assessing smooth lines, should, according to the authors, include the number of inflection points and extremes, and the presence or absence of loops and cusps. The authors (Gobithaasan et al., 2008) provided a specific example of computing the M level of several types of spiral curves in their work, as a result of which the cornu spiral was recognized as the most aesthetic.

An even more mathematically complex computation was proposed by Harada and Yoshimoto (2002). The authors suggested to estimate the aesthetic value of a curve by determining the relationship between the length frequencies of a segmented curve with regards to its radius of curvature. The method gained popularity in assessing the beauty of monotonic planar curves (e.g., Gobithaasan et al., 2013); however, it is more applicable when comparing different types of lines rather than egg-shaped profiles.

In our work (Narushin et al., 2021a), we demonstrated that the functional dependence reflecting variations in the radii of curvature of an ovoid surface is practically identical for any chicken egg regardless of the ratio of the main geometric parameters.

Who then determines the criteria for assessing which shape is more aesthetic or harmonious? The experts who write the papers? Key opinion leaders or most of the population? What if this changes over time and how do we then use mathematics to define this concept of harmony so that we can apply it to other scenarios?

A purely mathematical method of evaluation can perhaps be considered acceptable for artificially created objects, where the optimal shape can be whatever the author(s) wishes. It has applications when evaluating empirically completely different geometric objects that have obvious differences in form. With natural objects that have a similar shape, such as eggs, however, this cannot be so straightforward.

After all, one of the main principles of aesthetics is to achieve a certain similarity of a certain object or work with a natural object. In this regard, such natural objects as avian eggs, by default, are already aesthetic. But how to choose the *most* aesthetic among fundamentally similar looking objects? This may sound frivolous but could have profound implications e.g. in the profitability of industries that produce table eggs, or in the perception of the beauty of a building.

If we accept symmetry as one of the main criteria of aesthetic perception, then the image of an egg meets this requirement only with respect to its longitudinal axis, thus exhibiting bilateral symmetry (Shi et al., 2023). Despite this egg's bilateral symmetry, the transverse axis only emphasizes the *asymmetry* of the egg shape. The only parameter that we can think of that can equilibrate such a geometric discrepancy is that of "*balance*". This category of aesthetics has received special attention from all authors who used the Birkhoff's (1933) formula in its classical or modified form.

The question of balance has also been considered in psychological aspects of perception, for example, between order and complexity on aesthetic appreciation (e.g., Van Geert et al, 2023), as well as psychophysical ones, for example, between technical qualities and emotional meaning (e.g., Koenderink, 2026).

Balance may be reasonably considered the basis for the harmonious perception of any geometric form for several reasons (e.g., Hübner and Fillinger, 2019; Recraft, 2025; DSM, 2025):

- *Visual stability*. That is, when an object or composition is balanced, it appears more stable and whole. This creates a sense of order and completeness, which is intuitively perceived as pleasant.
- *Direction of attention*. Balance helps properly distribute the viewer's attention, allowing the eye to move smoothly throughout the form.
- *Emotional perception*. Balance is most often associated with calmness and stability.
- *Natural associations*. People subconsciously seek balance because it is often found in nature.
- *Functionality and form*. In design and architecture, balance not only pleases the eye, but also suggests that the object "works" properly.

The bird's egg, however, at least on the face of it, is a striking example of a figure that does not appear balanced – as mentioned above, it is fundamentally asymmetrical. How then do we rationalize the fact that most people find it aesthetically pleasing? In other words, how do we find the balance, if balance is to be found?

The principles of balance of an asymmetrical figure are widely used in art, design and engineering (e.g., Lidwell et al., 2010; White, 2011; Sharma, 2017; Garbin et al., 2020). One of the main solutions by which one can balance an asymmetrical figure is to counterbalance it. Every figure has a point where its weight is visually balanced.

Therefore, if such a center of gravity is located at a conditional central point, the figure appears to be in equilibrium. With the above in mind the purpose of this study was to evaluate the geometric parameters of the oviform shape both in two and three dimensions in order to establish where (and if) the balance exists volumetrically (3D) and/or on a plane (2D) relative to its transverse axis. A more detailed and visual representation of the design of the planned studies is presented in Figure 2.

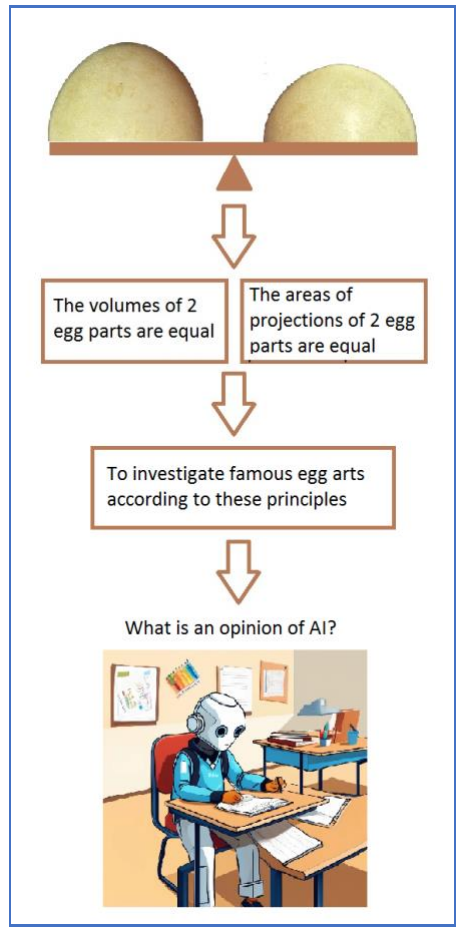


Figure 2: A research framework diagram and a description of the research ideas in the article. Bottom image generation: Pixlr (2024, October 21; Pixlr Pte Ltd., Singapore; https://pixlr.com/ (accessed on 4 June 2025)).

2 THEORETICAL INVESTIGATIONS

In considering the theoretical aspects of the bird egg balance, we made several assumptions outlined below. The first one concerns its gravimetric constancy, which implies that the mass of matter in each particle of an object remains unchanged. That is, we assumed that the egg density is uniformly distributed throughout its volume. The second was that the transverse axis, perceived as the center of mass/volume distribution of the main parts of the egg, passes through its maximum breadth, *B* (Figure 3).

Exactly such a division of the egg into, respectively, blunt and pointed parts is distinguishing both for the mathematical interpretation of its shape (e.g., Narushin et al., 2020, 2021b, 2023b) and for its perception as a holistic structural object (e.g., Romanoff and Romanoff, 1949). At the same time, the geometric *y*-axis passes through the middle of the egg length, *L*, mathematically dividing the *x* values into negative (to the left of the *y*-axis) and positive (to the right of the *y*-axis) values.

Herewith, the *B* axis is shifted from the *y*-axis by a certain value *w*. In addition to the specified geometric parameters *B*, *L* and *w*, we need another one, i.e., the diameter of the egg at a point located from its pointed end by a value of *L*/4 (Figure 3).

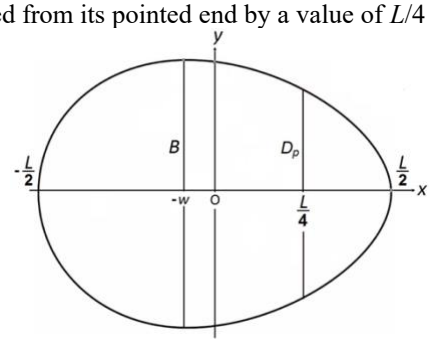


Figure 3: Schematic representation of an egg with the main measured parameters.

Let us consider separately the compliance of a bird's egg with the balance condition of both a volumetric (3D) and a plane (2D) geometric object.

2.1 Balance harmony for 3D egg shape

If an egg shape is to be considered “harmonious” or aesthetically pleasing then, here, we are assuming that there is balance between the volumes of the two fundamental parts, i.e., its blunt and pointed ends. In other words, that the volumes to the left and right of the axis *B* are equal.

We have already considered how to define the volumes of such parts in previous studies (Narushin et al., 2024a), based on the equations with which these parts can be described mathematically (Narushin et al., 2023b).

For the blunt part (y_b):

$$y_b = \pm \frac{B}{2} \cdot \sqrt{\frac{L^2 - 4x^2}{L^2 + 8wx + 4w^2}}, \tag{1}$$

where B is the maximum breadth of the egg, L is its length, and w is the parameter that displays the separation between two vertical lines that represent the egg's maximum breadth and half length. Formula (1) corresponds to the x values in the interval $[-L/2 \ldots -w]$ (Figure 3). At the same time, the pointed end (y_p), characterized by the interval $x = [-w \ldots L/2]$ (Figure 3), conforms to the following function:

$$y_p = \pm \frac{B_p}{2} \cdot \sqrt{\frac{L^2 - 4x^2}{L^2 + 8w_p x + 4w_p^2}}. \tag{2}$$

For equation (2), the values of B_p and w_p do not correspond to the analogous B and w in Eqn1. While the parameters of equation (1) can be measured relatively easily, directly on the studied egg or on its image, the values of B_p and w_p can be estimated as follows (Narushin et al., 2023b):

$$B_p = B \cdot \sqrt{\frac{1 - 8\frac{w_p}{L} \cdot \frac{w}{L} + 4\left(\frac{w_p}{L}\right)^2}{1 - 4\left(\frac{w}{L}\right)^2}}, \tag{3}$$

$$w_p = L\left[\frac{\left(\frac{D_p}{B}\right)^2\left(1 - 4\left(\frac{w}{L}\right)^2\right) + 3\frac{w}{L}}{3 - 4\left(\frac{D_p}{B}\right)^2\left(1 - 4\left(\frac{w}{L}\right)^2\right)} - \sqrt{\left(\frac{\left(\frac{D_p}{B}\right)^2\left(1 - 4\left(\frac{w}{L}\right)^2\right) + 3\frac{w}{L}}{3 - 4\left(\frac{D_p}{B}\right)^2\left(1 - 4\left(\frac{w}{L}\right)^2\right)}\right)^2 - \frac{1}{4}}\right], \tag{4}$$

where D_p is the diameter corresponding to the point $x = L/4$ (Figure 3).

The expediency of including the parameter D_p in the mathematical model of the egg contour was considered in detail in Narushin et al. (2021b, 2023b), since it is precisely owing to it that one can indirectly judge the degree of conicity of the bird's egg.

Using integral geometry formulae (e.g., Piessens et al., 1983), we calculated the corresponding volumes of the egg parts, i.e., for blunt (V_b) and pointed (V_p) ones (Narushin et al., 2024a), which, after some mathematical transformations (see their details in Supplementary Data A) can be written as follows:

$$V_b = \frac{\pi L B^2 \left(1-2\frac{w}{L}\right)}{128\left(\frac{w}{L}\right)^2}\left(1+2\frac{w}{L}+8\left(\frac{w}{L}\right)^2-\frac{\left(1-2\frac{w}{L}\right)\left(1+2\frac{w}{L}\right)^2}{4\frac{w}{L}}\cdot\ln\left|\frac{1+2\frac{w}{L}}{1-2\frac{w}{L}}\right|\right), \quad (5)$$

$$V_p = \frac{\pi L B^2}{128}\cdot\frac{1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2}{\left(\frac{w_p}{L}\right)^2\left[1-4\left(\frac{w}{L}\right)^2\right]}\left(\left(1+2\frac{w}{L}\right)\left[1+4\left(\frac{w_p}{L}\right)^2-2\frac{w_p}{L}\left(1-2\frac{w}{L}\right)\right]-\frac{\left[1-4\left(\frac{w_p}{L}\right)^2\right]^2}{4\frac{w_p}{L}}\ln\frac{\left(1+2\frac{w_p}{L}\right)^2}{\left|1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2\right|}\right) \quad (6)$$

Let us consider the premise that the ratio of the volumes of the two parts must be equal to one another. In other words, the condition for observing the principle of balance is presented as follows:

$$\frac{V_p}{V_b} = 1\,. \quad (7)$$

Then,

$$\frac{V_p}{V_b} = \frac{\left(\frac{w}{L}\right)^2\left[1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2\right]\left(\left(1+2\frac{w}{L}\right)\left[1+4\left(\frac{w_p}{L}\right)^2-2\frac{w_p}{L}\left(1-2\frac{w}{L}\right)\right]-\frac{\left[1-4\left(\frac{w_p}{L}\right)^2\right]^2}{4\frac{w_p}{L}}\ln\frac{\left(1+2\frac{w_p}{L}\right)^2}{\left|1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2\right|}\right)}{\left(\frac{w_p}{L}\right)^2\left(1-2\frac{w}{L}\right)\left[1-4\left(\frac{w}{L}\right)^2\right]\left(1+2\frac{w}{L}+8\left(\frac{w}{L}\right)^2-\frac{\left(1-2\frac{w}{L}\right)\left(1+2\frac{w}{L}\right)^2}{4\frac{w}{L}}\cdot\ln\left|\frac{1+2\frac{w}{L}}{1-2\frac{w}{L}}\right|\right)} \quad (8)$$

That is, based on Eqn8, to meet the condition of Eqn7, the shape index (B/L) is not required. Therefore, eggs of any shape can meet the condition of balance of their parts. For a better analysis of Eqn8, we expressed it graphically to try and simplify it.

For this purpose, we used the simulated database of virtual eggs compiled from our previous study (Narushin et al., 2026a).

Substituting into Eqn8 and, accordingly, into Eqn4 the simulated w/L values from the permissible range [0, 0.16] and those of D_p/B from the corresponding range [0.60, 0.86] (Narushin et al., 2026a), we obtained a row of variants for the V_p/V_b ratio as seen in

Figure 4. Hereby, to avoid division by 0, the minimum w/L value was taken equal to 0.001, and the maximum possible value was limited to 0.13, since we were interested in the condition expressed by Eqn7, i.e., when $V_p/V_b = 1$.

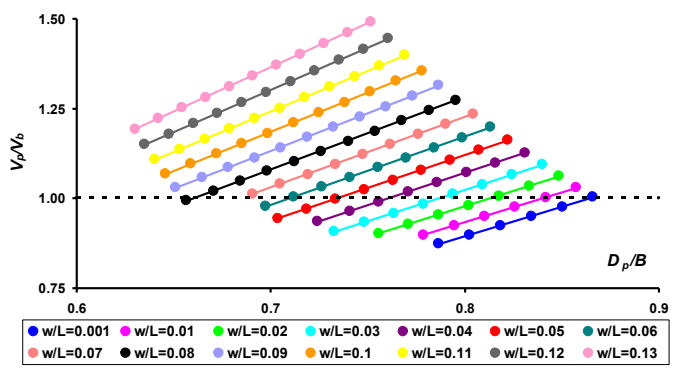


Figure 4: Graphical visualization of the functional relationships for possible simulated values of w/L, D_p/B and V_p/V_b in Eqn8.

The functional dependences $D_p/B = f(V_p/V_b)$ shown in Figure 4 can be described by a linear equation of the following form:

$$\frac{V_p}{V_b} = a\frac{D_p}{B} - b\,, \tag{9}$$

where a and b are some coefficients, the values of which, along with the coefficients of determination (R^2) for the conforming simulated w/L values, are provided in Table 1.

w/L	*a*	*b*	R^{2*}
0.001	1.6472	0.4229	0.99980
0.010	1.6793	0.4100	0.99980
0.020	1.7064	0.3872	0.99980
0.030	1.7401	0.3682	0.99980
0.040	1.7900	0.3607	0.99980
0.050	1.8358	0.3480	0.99980
0.060	1.8960	0.3446	0.99990
0.070	1.9609	0.3426	0.99990
0.080	2.0152	0.3302	0.99990
0.090	2.0913	0.3319	0.99990
0.100	2.1728	0.3349	0.99990
0.110	2.2602	0.3394	0.99990
0.120	2.3538	0.3453	0.99990
0.130	2.4541	0.3528	0.99999

Table 1: Values of coefficients *a* and *b* from Eqn9 depending on the simulated *w*/*L* values.

If we express the functional dependences of the coefficients *a* and *b* on the simulated *w*/*L* values, they can be approximated by the second-order equations presented in Figure 5.

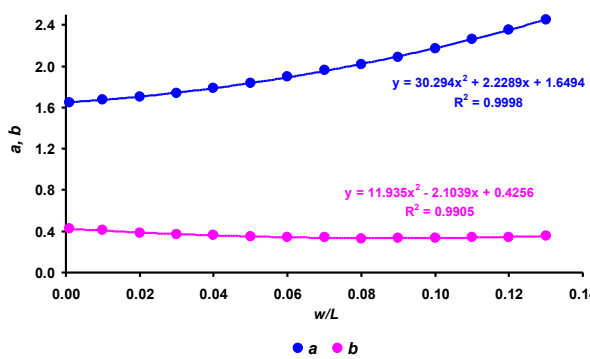


Figure 5: Graphical dependences of the coefficients *a* and *b* from Eqn9 on the simulated *w*/*L* values.

As a result of substituting the obtained dependences for *a* and *b* into Eqn9, we obtained a simpler form for the calculation equation (8) as follows:

$$\frac{V_p}{V_b} \approx \left[30.294\left(\frac{w}{L}\right)^2 + 2.229\frac{w}{L} + 1.649\right]\frac{D_p}{B} - 11.935\left(\frac{w}{L}\right)^2 + 2.104\frac{w}{L} - 0.426 \cdot \qquad (10)$$

The correlation coefficient between the V_p/V_b values computed using Eqn8 and, accordingly, Eqn10 was 0.9996. The resultant equation (10) is much more convenient for subsequent analysis. Based on the condition $V_p/V_b = 1$, Eqn10 can be transformed into the following:

$$\frac{D_p}{B} \approx \frac{\left(\frac{w}{L}\right)^2 - 0.176\frac{w}{L} + 0.119}{2.538\left(\frac{w}{L}\right)^2 + 0.187\frac{w}{L} + 0.138}. \tag{11}$$

Eqn11 demonstrates the relationship between D_p/B and w/L values under the condition of achieving the egg shape balance, i.e., when $V_p/V_b = 1$. Since the boundaries of possible variability of the D_p/B value are limited by the following specific interval (Narushin et al., 2026a):

$$\frac{D_p}{B} = \left[0.593\frac{1-0.97\frac{B}{L}}{1-\frac{B}{L}}, \frac{\sqrt{3}}{2\sqrt{1+2\frac{w}{L}+4\left(\frac{w}{L}\right)^2}}\right], \tag{12}$$

it is advisable to check for compliance with the above condition (Eqn12) when selecting a balanced egg contour.

Let us also deduce the inverse dependence of the determination of the w/L values on the D_p/B value. For this, we use Eqn11, from which it follows:

$$\frac{w}{L} \approx \frac{\sqrt{1.337\frac{D_p}{B} - \left(\frac{D_p}{B}\right)^2 - 0.326} - 0.16\frac{D_p}{B} - 0.151}{4.343\left(\frac{D_p}{B} - 0.394\right)}. \tag{13}$$

A detailed derivation of Eqn13 is presented in Supplementary Data B.

Thus, Eqn13, like Eqn11, demonstrates the relationship between the D_p/B and w/L values provided that the egg shape balance is achieved, i.e., when $V_p/V_b = 1$.

Since the boundaries of possible variability of the w/L value are limited by the following interval (Narushin et al., 2026a):

$$\frac{w}{L} = \left[0{,}0.09\left(\frac{B}{L}\right)^{-1} \right], \tag{14}$$

the respective check should be made to ensure that it meets this condition (Eqn14) when selecting a balanced egg contour.

In this subsection we covered the balance of a 3D image of an egg. What if we are talking about a picture – a flat image? Let us consider the approach with a 2D image of an egg as outlined below.

2.2 Balance harmony for 2D egg shape

At the beginning of section 2.1, we postulated that the volume either side of axis B should be equal to satisfy the criteria of "most harmonious" (Eqn7). Similar criteria are therefore met in a plane figure, i.e. the areas either side of axis B (in Figure 3) are equal (Eqn18).

To compute the balance of plane figures, instead of the volumes of the blunt and pointed parts of the egg, we should operate with the area of their projections onto the plane that we designated as A_b and A_p. Thus, the primary task here is reduced to determining the computation formulae with which these parameters could be determined. Using the formulae of integral geometry for symmetrical figures, we derived the following calculation dependences:

$$A_b \approx BL\left[0.3927 - 0.6663\frac{w}{L} - 0.2011\left(\frac{w}{L}\right)^2 \right], \tag{15}$$

$$A_p \approx BL\sqrt{\frac{1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2}{1-4\left(\frac{w}{L}\right)^2}}\cdot\left[0.591\frac{w}{L}+0.445+\frac{0.03\frac{w}{L}-0.012}{\frac{D_p}{B}-0.819\left(\frac{w}{L}\right)^2+0.591\frac{w}{L}-0.65} \right]. \tag{16}$$

The detailed derivation of Eqns 15 and 16 is demonstrated in Supplementary Data C.
Using computation formulae (15) and (16), we can write the result of their relationship

as follows:

$$\frac{A_p}{A_b} \approx \frac{\sqrt{\dfrac{1-8\dfrac{w_p}{L}\cdot\dfrac{w}{L}+4\left(\dfrac{w_p}{L}\right)^2}{1-4\left(\dfrac{w}{L}\right)^2}}\cdot\left[0.591\frac{w}{L}+0.445+\dfrac{0.03\dfrac{w}{L}-0.012}{\dfrac{D_p}{B}-0.819\left(\dfrac{w}{L}\right)^2+0.591\dfrac{w}{L}-0.65}\right]}{0.3927-0.6663\dfrac{w}{L}-0.2011\left(\dfrac{w}{L}\right)^2}. \quad (17)$$

Similar to the balance condition for volumetric (3D) eggs (Eqn7), we can expect that the balance principle for plane (2D) images is observed when

$$\frac{A_p}{A_b}=1. \quad (18)$$

Let us simplify the obtained Eqn17 for further, more convenient mathematical processing. To do this, we initially presented it in the form of graphical dependences, using data from the simulated virtual egg database compiled from our previous study (Narushin et al., 2026a). Substituting into Eqn17 and, accordingly, into Eqn4 the simulated w/L values from the permissible range [0, 0.16] and those of D_p/B from the respective range [0.60, 0.86] (Narushin et al., 2026a), we produced the variants of the A_p/A_b ratios as seen in Figure 6.

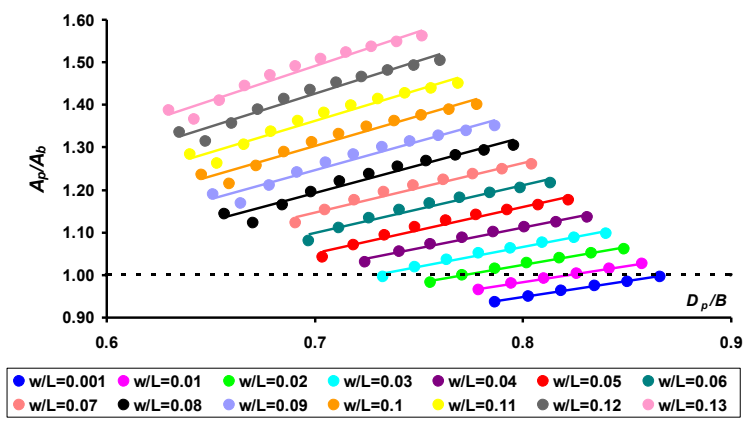


Figure 6: Graphical visualization of the functional relationships of possible simulated values of w/L, D_p/B and A_p/A_b in Eqn17.

In this case, to avoid division by 0, the minimum w/L value was taken equal to 0.001, and the maximum possible value was limited to 0.13, since we were interested in the condition expressed by Eqn18, i.e., when $A_p/A_b = 1$.

The functional dependences $D_p/B = f(A_p/A_b)$ shown in Figure 6 can be described by a linear equation of the following form:

$$\frac{A_p}{A_b} = a' \frac{D_p}{B} + b', \tag{19}$$

where a' and b' are some coefficients, the values of which, along with the coefficients of determination (R^2) for the appropriate w/L values, are presented in Table 2.

w/L	*a*	*b*	R^{2*}
0.001	0.7484	0.3488	0.997
0.010	0.7669	0.3698	0.997
0.020	0.8348	0.3556	0.992
0.030	0.9272	0.3242	0.984
0.040	0.9617	0.3422	0.983
0.050	1.0871	0.2907	0.970
0.060	1.1220	0.3132	0.971
0.070	1.1603	0.3354	0.971
0.080	1.3142	0.2721	0.956
0.090	1.3633	0.2920	0.957
0.100	1.4187	0.3100	0.957
0.110	1.4784	0.3279	0.958
0.120	1.5478	0.3421	0.958
0.130	1.6268	0.3529	0.958

Table 2: Values of coefficients *a* and *b* from Eqn19 depending on the simulated *w/L* values.
* Coefficient of determination.

If we express the functional dependences of the coefficients a' and b' on the w/L value, they can be approximated by the equations plotted in Figure 7.

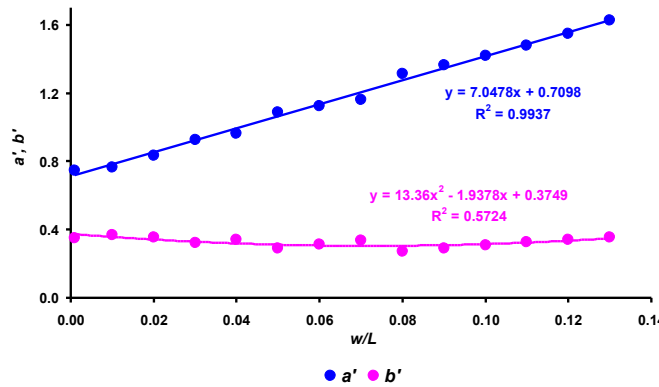


Figure 7: Graphical dependences of the coefficients *a'* and *b'* from Eqn19 on the simulated *w*/*L* values.

As a result of substituting the obtained dependences for *a'* and *b'* into Eqn19, we obtained a simpler form of Eqn17 as follows:

$$\frac{A_p}{A_b} \approx \left(7.048\frac{w}{L} + 0.71\right)\frac{D_p}{B} + 13.36\left(\frac{w}{L}\right)^2 - 1.938\frac{w}{L} + 0.375\,. \qquad (20)$$

Considering the condition of Eqn18, the obtained Eqn20 can be written as follows:

$$\left(7.048\frac{w}{L} + 0.71\right)\frac{D_p}{B} + 13.36\left(\frac{w}{L}\right)^2 - 1.938\frac{w}{L} + 0.375 \approx 1\,. \qquad (21)$$

From this, we obtained a relationship between the values D_p/B and w/L, for which the 2D balance condition is satisfied. In particular,

$$\frac{D_p}{B} \approx \frac{0.047 + 0.145\frac{w}{L} - \left(\frac{w}{L}\right)^2}{0.528\frac{w}{L} + 0.053}\,. \qquad (22)$$

We have demonstrated above that the values of the *Dp*/*B* and *w*/*L* ratios can have a number of limitations (Eqns 12 and 14) and, therefore, it is sometimes necessary to be more flexible in choosing the dependent parameter. For this purpose, it is advisable to have the possibility of calculating *w*/*L* depending on the value of *Dp*/*B*.

The inverse dependence of the determination of the w/L values on D_p/B value has the following form:

$$\frac{w}{L} \approx 0.264\left[\sqrt{\left(\frac{D_p}{B}\right)^2 - 1.308\frac{D_p}{B} + 0.749} - \frac{D_p}{B} + 0.275\right]. \quad (23)$$

A more detailed deduction of equation (23) can be seen in Supplementary Data D.
The computed w/L values (Eqn23) for the selected egg contour should be checked for compliance with condition (14).

3 EXPERIMENTAL INVESTIGATIONS

3.1 Methodology

The experimental studies described below are based on the principles of measuring the main geometric parameters of bird egg images. Using Microsoft Picture Manager software, the values of L and B, the distance (w) of the B axis shift from the egg center, and the egg diameter (D_p) at the $L/4$ point located from the pointed end were measured with an accuracy of 1 pixel, according to the diagram shown in Figure 3. Since all geometric parameters of the equations for balance computations are presented in an index, i.e. dimensionless, form as their ratios, recalculation of pixel measurements into metric ones was not performed.

3.2 Volumetric balance of eggs with an example of the Dalí egg

Suppose we want to depict an egg that resembles a chicken egg and meets the requirements of visual harmony, i.e., the condition $V_p/V_b = 1$ (Eqn7) or $A_p/A_b = 1$ (Eqn18). Romanoff and Romanoff (1949), based on many measurements of geometric parameters, presented the so-called *standard chicken egg* that possesses the following values of the egg indices, i.e., the ratios of the main geometric parameters: $B/L = 0.737$, $w/L = 0.044$, and $D_p/B = 0.810$ (Narushin et al., 2023b). Using the standard chicken egg parameters, the respective computation according to Eqn10 resulted in $V_p/V_b = 1.11$, demonstrating non-compliance with the balance condition (Eqn7). That is, in this standard chicken egg case, the pointed part of such an egg is slightly more massive than the blunt one (Figure 8a).

Leaving the w/L value unchanged, we computed the D_p/B value (Eqn11), which would meet the fulfillment of the Eqn7 condition. This value was 0.749, which fits into the permissible interval (Eqn12). The graphical interpretation of this profile is shown in Figure 8b. Now, let us assume that we are more satisfied with the option that the value of the D_p/B index remains unchanged, i.e., equal to 0.810. Correspondingly, we calculated the w/L value following Eqn13, which resulted in 0.021 and the appropriate profile shown in Figure 8c. Checking the obtained value of w/L = 0.021, according to the condition determined by Eqn14, demonstrated the required compliance of this value.

There may be many other possible values of the w/L and D_p/B indices that warrant the balance (Eqn7). For example, an intermediate variant, when w/L = 0.030, led to D_p/B = 0.786, with the balanced egg profile shown in Figure 8d.

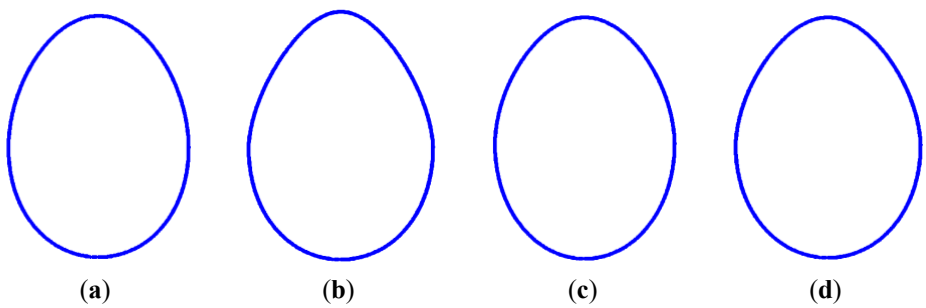


Figure 8: Graphic interpretation of profiles for (**a**) the standard chicken egg (when B/L = 0.737, w/L = 0.044, and D_p/B = 0.810) and its variants plotted in compliance with the principles of balance based on the following index values: (**b**) B/L = 0.737, w/L = 0.044, D_p/B = 0.749; (**c**) B/L = 0.737, w/L = 0.021, D_p/B = 0.810; and (**d**) B/L = 0.737, w/L = 0.030, D_p/B = 0.786.

What could be more adequate in assessing the degree of harmony than a work of art? After all, a generally recognized creation product of a famous author is not just a beautiful combination of elements, but a complex system of relationships that evokes a feeling of satisfaction and integrity in a viewer.

There are many three-dimensional compositions depicting eggs in the world. Salvador Dalí (1904–1989) is rightfully considered one of the masters who repeatedly used eggs in his artworks. One of them is the sculpture of an egg on the roof of his House Museum in Portlligat, Cadaqués, Spain (Figure 9).

Figure 9: Egg from the Salvador Dalí House Museum in Portlligat, Cadaqués, Spain. Image credit: Wikimedia Commons (2013).

If we measure to what extent an egg created by Dalí (Figures 9 and 10a) conforms to the conditions of harmony outlined (Eqn7, Eqn18). The measurements made allowed us to establish the following values of the three coefficients: B/L = 0.66, w/L = 0.13, and D_p/B = 0.76. The ratio of the volumes of the two halves of such an egg (Eqn8) is 1.51, which clearly is somewhat removed from our own purported notions of "harmony" outlined in sections 2.1 and 2.2 (Eqn7, Eqn18).

To achieve the condition V_p/V_b = 1, the D_p/B value computed according to Eqn11 must be equal to 0.551. However, this value is not within the permissible limits of variability (Eqn12), according to which the minimum D_p/B value is limited to 0.63. This value can be achieved by decreasing the w/L ratio. Based on Figure 4, the highest possible value of this index is 0.08. The appropriate recalculation of the required D_p/B value to meet the requirements of the balance condition (Eqn7) was 0.658. In this case, the image of such a corrected contour meets the balance conditions of the volumes of its two halves (Figure 10b).

If, however, we want to preserve the original ratio D_p/B = 0.76 adopted by the author, we can recalculate w/L using Eqn13. In this case, the balance will be maintained at w/L = 0.04. The corresponding image of the egg contour corresponding to the values of indices B/L = 0.66, w/L = 0.04, and D_p/B = 0.76 is shown in Figure 10c. If we adopt some intermediate option, e.g., at w/L = 0.06, the D_p/B value computed using Eqn11 will be equal to 0.708 (Figure 10d).

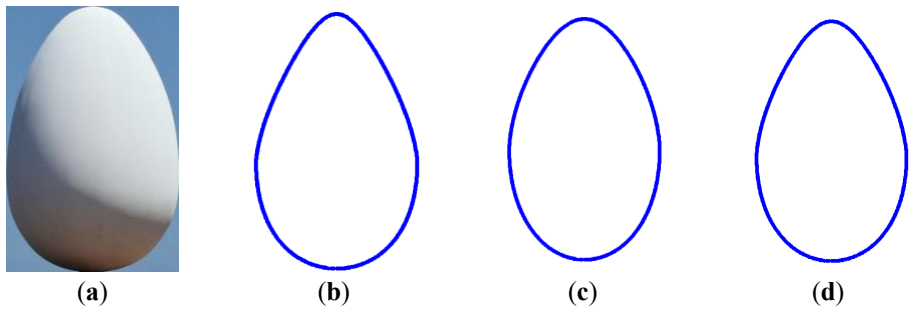


Figure 10: Egg from the Salvador Dalí House Museum in Portlligat, Cadaqués, Spain (**a**; image credit: Wikimedia Commons, 2013) and egg contours made taking into account the harmony balance conditions based on the following index values: (**b**) $B/L = 0.66$, $w/L = 0.08$, $D_p/B = 0.658$; (C) $B/L = 0.66$, $w/L = 0.04$, $D_p/B = 0.76$; and (D) $B/L = 0.66$, $w/L = 0.06$, $D_p/B = 0.708$.

As mere scientists, we would not be so presumptuous as to challenge Salvador Dalí and his own artistic vision of what any given artwork should be. Nevertheless, it does beg the question of whether, if surveyed, how many people would find more aesthetically pleasing a "mathematically corrected" version of the masterpiece. It seems likely that those already familiar with the work of out would not wish to change their point of view, but a "blind" test might yield interesting results.

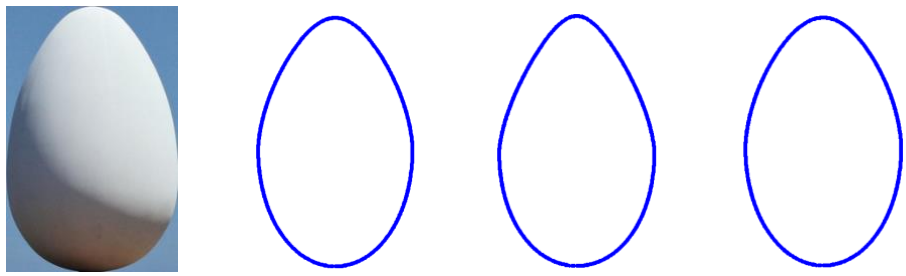
3.3 Balance of plane egg images with examples from Dalí and Francesca

For plane images of the egg contour, the obtained values should also be checked for compliance with the Eqn18 condition.

Having an arsenal of computation formulae (Eqns 22 and 23) that secure the egg contour balance for a plane image, let us establish how the image of the 3D Dalí egg (Figure 10) would change if we had chosen to depict it according to our own "harmony" principles.

If the imbalance of the volumetric copy of the egg, i.e., the ratio of the volumes of its two halves, was 1.51 (Eqn8), the computed ratio of the areas of similar parts of the plane image (Eqn17) was already equal to 1.59. That is, there is a mathematical discrepancy in the balance of the two halves of the egg when they are correspondingly depicted in 3D and 2D formats.

Naturally, this also affected the possibilities of "rectifying" this situation, and our appropriate attempts are presented in Figure 11.



(a) **(b)** **(c)** **(d)**

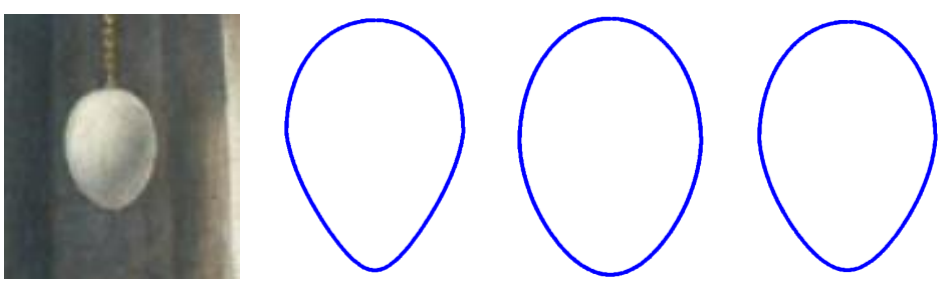
Figure 11: Egg from the Salvador Dalí House Museum in Portlligat, Cadaqués, Spain (**a**; image credit: Wikimedia Commons, 2013) and egg contours made taking into account the harmony balance conditions based on the following index values: (**b**) $B/L = 0.66$, $w/L = 0.024$, $D_p/B = 0.76$; (**c**) $B/L = 0.66$, $w/L = 0.038$, $D_p/B = 0.70$; and (D) $B/L = 0.66$, $w/L = 0.02$, $D_p/B = 0.78$.

Now let us consider another example, namely, a plane egg image depicted in the famous painting *Brera Madonna* by Piero della Francesca (1415–1492) (Figure 12).

The computation according to Eqn20 demonstrated an imbalance expressed by a discrepancy between the areas of the plane projections of the pointed and blunt halves of the egg at a level of 1.16. We used the following indices when depicting the egg: $B/L = 0.75$, $w/L = 0.05$, and $D_p/B = 0.80$.

Our attempts to "correct" this image, leaving one of the original values w/L or D_p/B unchanged, or changing both at once, in order to maintain the balance condition according to Eqn18, are presented in Figure 13.

Figure 12: *Brera Madonna* (*La Vergine con il Bambino e santi*) by Piero della Francesca (1415–1492). Image credit: Wikimedia Commons (2018b). The egg is directly above the Madonna's head.



(**a**) (**b**) (**c**) (**d**)

Figure 13: Egg from *Brera Madonna* by Piero della Francesca (**a**; image credit: Wikimedia Commons, 2018b) and egg contours made taking into account the harmony balance conditions based on the following index values: (**b**) $B/L = 0.75$, $w/L = 0.05$, $D_p/B = 0.68$; (**c**) $B/L = 0.75$, $w/L = 0.016$, $D_p/B = 0.80$; and (D) $B/L = 0.75$, $w/L = 0.026$, $D_p/B = 0.75$.

3.4 Mathematical imbalance between balances

The discrepancy between the two approaches (volumetric and plane) to computing the balance of egg contours is apparent if we compare the data of the w/L and D_p/B indices obtained by recalculating egg variants in 3D and 2D formats (Figures 10 and 11). We attempted to evaluate the degree of this imbalance mathematically, for which we compared two equations (10) and (20), presenting them in the form of a ratio that we decided to name the *transformation coefficient* (τ):

$$\tau = \frac{\frac{V_p}{V_b}}{\frac{A_p}{A_b}} \approx \frac{\left[30.294\left(\frac{w}{L}\right)^2 + 2.229\frac{w}{L} + 1.649\right]\frac{D_p}{B} - 11.935\left(\frac{w}{L}\right)^2 + 2.104\frac{w}{L} - 0.426}{\left(7.048\frac{w}{L} + 0.71\right)\frac{D_p}{B} + 13.36\left(\frac{w}{L}\right)^2 - 1.938\frac{w}{L} + 0.375}. \quad (24)$$

Computation of possible simulated values of τ for the entire sets of w/L and D_p/B values demonstrated an average transformation coefficient value of 0.925 ± 0.034. This value can be used as a constant value when recalculating the volumetric egg image balance into the plane one. Such recalculation can be useful when creating sketches of egg images with their subsequent transfer to a sculptural product. It is possible that in some cases a more accurate computation of the τ value may be required, and Eqn24 can be used for these purposes.

In order to simulate and assess the degree of influence of each index, i.e., w/L and D_p/B, on the τ value, we presented the obtained results of computing this coefficient according to Eqn24 in the form of graphical dependences shown in Figure 14.

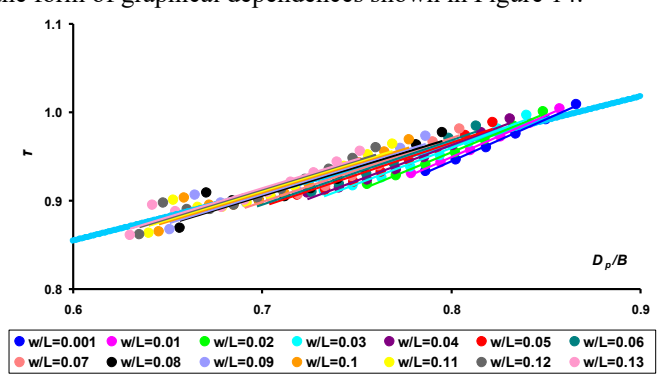


Figure 14: Graphical visualization of the values of the transformation coefficient τ depending on the simulated values of w/L and D_p/B of bird eggs.

Despite some discrepancy between the data conforming to various w/L values, they all fit, with a sufficient degree of accuracy, into a single trend line (the blue line in Figure 14) that can be approximated by the following relationship:

$$\tau \approx 0.5445\frac{D_p}{B} + 0.5277\,. \tag{25}$$

The agreement between the τ calculation results according to Eqn24 and Eqn25 was at the level of $R = 0.949$. Thus, the τ value tends to increase somewhat in the case of an egg image of a more rounded shape.

3.5 Which bird's egg is the most harmonious?

The realm of bird eggs is so vast that it is likely to contain an array of representatives of various levels of balance of the two parts, i.e., pointed and blunt. We thus decided to find two extreme variants when an egg that best meets the balance condition and, alternatively, the least. Since a bird egg is a volumetric structure, it was decided to use the equality of the volumes of its halves, rather than the areas of its projected shape, and, accordingly, adhere to the requirements of Eqn7 as a balance condition.

For these purposes and in line with our earlier studies (Narushin et al., 2024b, 2026b), we utilized a database of digital images of bird eggs from the Natural History Museum of Wiesbaden, Wiesbaden, Germany (Wikimedia Commons, 2014) and the Muséum de Toulouse, Toulouse, France (Wikimedia Commons, 2019). The geometric parameters of 444 eggs from 444 bird species, 89 families, and 30 orders are thus available to us; these represent approximately 4%, 36%, and 73% of the world's total biodiversity of species, families, and orders, respectively.

Eggs chosen as the most harmonious (balanced) eggs were the northern shoveler (*Anas clypeata*; Figure 15a) and the Tataupa tinamou (*Crypturellus tataupa*; Figure 15b). In both species, the ratio of the volumes of the pointed and blunt egg parts was exactly 1.

(**a**) (**b**)

Figure 15: Images of ideal harmonious (balanced) eggs: (**a**) northern shoveler (*Anas clypeata*), and (**b**) Tataupa tinamou (*Crypturellus tataupa*). Image credit: Wikimedia Commons (2012a,b).

On the other hand, the least harmonious, from the viewpoint of the equality of the balances of the two halves, was the egg of the pygmy cormorant (*Microcarbo pygmaeus*) (Figure 16), for which the V_p/V value was equal to 1.414.

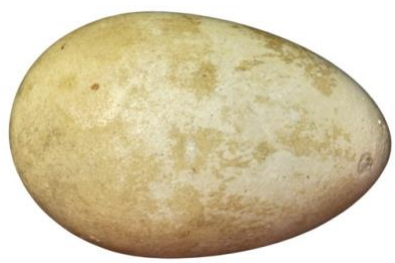


Figure 16: An image of a "least harmonious" egg, that of the pygmy cormorant (*Microcarbo pygmaeus*). Image credit: Wikimedia Commons (2012c).

Of the 444 bird egg images examined, only 50 demonstrated a value of $V_p/V_b \leq 1$. For the remaining 394 species, this was $V_p/V_b > 1$, from which we concluded that a "weighted" appearance of the pointed part is more typical for actual bird eggs.

We also tested how much the egg shape index (B/L) affects the value of the V_p/V_b ratio. In doing so, we presented the variants when V_p/V_b is less and, accordingly, greater than 1 separately (Figure 17). An analysis of the obtained graphical dependences suggested the absence of any regularity in the egg shape, for which the volume of the pointed part was less than the volume of the blunt one. That is, both eggs with a pointed end and almost round ones can correspond to the condition $V_p/V_b \leq 1$. As for most eggs, for which the volume of the pointed end prevailed over the volume of the blunt one, a clear tendency for the value of V_p/V_b to increase as their shape becomes more elongated was observed.

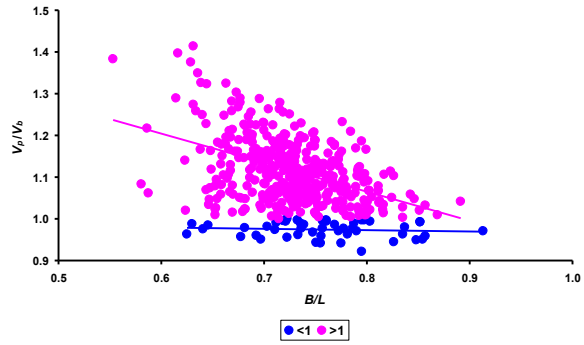


Figure 17: Dependence of the V_p/V_b ratio on the B/L index value for egg categories in which $V_p/V_b < 1$ (blue dots and trend line) and $V_p/V_b > 1$ (purple dots and trend line).

Since during our theoretical research we demonstrated that the shape index (B/L) did not affect the value of the V_p/V_b ratio (Eqn7), the produced relationships between these parameters in actual eggs may be useful in creating their artistic representations.

We were also interested in the estimation of a possible evolutionary momentum in developing a more (or less) balanced egg shape over millions of years of speciation. What if the egg shape was originally intended to be balanced, but under the influence of natural factors it evolved into something else? Or, conversely, the egg gradually evolved into the most balanced form of its shape. For these reasons, the indication of a specific bird species' emergence time was considered; data from this indicator was evaluated in line with the TimeTree 5 web resource for species divergence times (Kumar et al., 2022).

The results of this analysis are presented in Figure 18. Judging by the nature of the trend line (the purple line in Figure 18), no evolutionary patterns in the change in the V_p/V_b value occurred. Throughout the entire time stage of evolution, the ratios of the volumes of the two parts of the egg were found in various ranges (from 0.9 to 1.4). Fluctuations around the average value (trend line) did not have the appearance of some kind of pattern, but were, most likely, random. That is, in relation to the bird egg, the hypothesis about the desire to achieve some kind of perfection, in other words, harmony of form in the process of evolutionary development, was not confirmed.

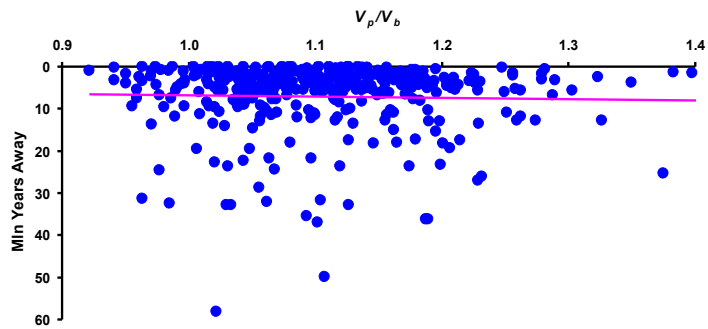


Figure 18: Plots of the relationship between species origin time (million years) and V_p/V_b value.

3.6 What does AI think about the egg shape balance?

Both ordinary users and scientists have recently turned to artificial intelligence (AI) on many issues of broad interest. AI-assisted knowledge is based on an integral assessment of a vast amount of information available on the Internet. We thus also decided to address our request to a GenAI tool (ChatGPT, 2025; OpenAI, San Francisco, CA, USA) with a request to provide a volumetric and plane image of a bird's egg that would be "ideal" in shape. We also asked AI to justify its decision to recognize the ideality of the generated images (Figure 19).

(**a**) (**b**)

Figure 19: Images of the "ideal" eggs generated by ChatGPT (2025; OpenAI) for the volumetric (**a**) and plane (**b**) versions.

In this case, we decided to deliberately not carry out the process of preliminary or subsequent deep learning of the AI, to engage in scientific polemics with it about methods of aesthetic perception of objects, or to ask it to duplicate the image of an "ideal" egg.

We were interested in an artificial model of an inexperienced connoisseur of bird eggs, whose knowledge is based on publicly available sources of information and is not "spoiled" or biased by the opinions of representatives of various theories. That is, our idea was to reproduce how an ordinary mind imagines an "ideal" egg without special skills and preliminary training.

The AI substantiated its vision of the ideality of the generated egg images. From the biological approach viewpoint, it stated: *'The egg is an elongated ellipsoid with a slight asymmetry, featuring a thicker, more reinforced shell on the blunt end for protection and a thinner, tapered pointed end for easier hatching.'* In terms of artistic perception, AI argued: *'A slightly elongated shape (like a chicken or duck egg) is perceived as harmonious – it is asymmetrical, but balanced.'*

Unlike the masterpieces depicted in Figures 9 and 12, the geometric dimensions of the AI-generated images conformed to identical results of their ratios, both for the volumetric version (Figure 19a) and for the plane one (Figure 19b): $B/L = 0.660$, $w/L = 0.041$, and $D_p/B = 0.760$. The performed computation of the balance level yielded a surprisingly ideal correspondence of the artificial 3D egg to the volumetric harmony, with the V_p/V_b value being equal to 1. The comparative result for the plane egg image conformed to $A_p/A_b = 0.95$ that was understandable. Apparently, AI perceived the request for a plane image as a need to change its external texture, and not its geometric shape. Nevertheless, the AI-produced output may be considered as a form of indirect confirmation of the fact that it is the balance of the volumetric parts of the egg that can serve as the principle of its shape harmony, at least when we ask a computer, rather than a famous artist.

4 DISCUSSION

The effect of aesthetic perception of a certain object is quite difficult to fit into the framework of a mathematical or even just logical conclusion. Indeed, it is practically impossible to do this using some universal indicator. Nevertheless, in our particular case, there was some elimination of "objective factors that affect aesthetic evaluation."

We tried to decide on the choice of criterion for evaluating the most harmonious form of a bird's egg. That is, a number of factors, which, according to many researchers, might be considered objective, can be excluded, e.g., the naturalness of the object and the smoothness and roundness of its forms or the similarity to a circle and an ellipse, the most harmonious geometric figures. Nevertheless, bird eggs demonstrate a huge variety of forms, for the comparison of which it was necessary to select some kind of calculation approach.

A fairly universal criterion of "golden section" does not withstand a competition when evaluating the harmony of an egg, as evidenced by both studies reported in the literature and a purely visual comparison of similar eggs. Thus, by a process of gradual elimination, the only remaining criterion was *balance*, which we decided to examine more closely.

The results presented here shed a new light on how we perceive the shape of an egg. Our basic premise that the most aesthetically pleasing or "harmonious looking" egg, where the volumes or areas either side of the axis *B* seemed to be borne out when considering an "ideal" egg generated by AI, but less so when considering works of art.

The results presented above also challenge the reader to judge whether the eggs in Figure 15 are more aesthetically pleasing to the eye than that in Figure 16. Similarly, if we surveyed several hundred people from different demographics, what would their response be to this question? This then brings us to the question of what is so special, extraordinary and inspiring about the egg? Why is it so popular in artworks, in architecture and even functional everyday objects?

Eggs certainly have deep symbolism, being associated (cf. *ab ovo*) with birth, life, fertility, beginnings, perfection and mystery (Jackson, 2017). In different cultures, eggs are associated with creation myths and the idea of rebirth, e.g., in the Christian tradition, the Easter egg symbolizes the resurrection (Scala, 2022; Shkolna et al., 2023). The contours of the egg represent an ideal natural shape with smooth lines that evoke a sense of harmony, tranquility and serenity (Breitenbach, 2019).

Moreover, the simplicity and smoothness of the lines make it easy to combine egg contours with other objects, to fit them into certain plots and motifs, making it an object and tool for various creative experiments (Elephant, 2019; Hage, 2021; mmERCH, 2024).

The question of whether a mathematical approach does in fact, fundamentally, help to improve our perception of a shape is open to question. Would Dali's or Francesca's eggs have been considered even more aesthetically pleasing if they conformed to the principle of balance outlined here? It might be argued that many works of art were all the better for considering (either consciously or sub-consciously) the principles of the golden ratio.

However, pre-determined ideals of beauty or "harmony" are not always relevant and can be very subjective and cultural. A further consideration therefore is how may a generally accepted notion of beauty or harmony be arrived at? As scientists we have, above, suggested sets of experiments of opinion-based surveys with levels of experimental control. Any artist or designer considering the issue of egg shape would presumably have their own vision of how exactly a given shape evoking an egg should look.

Herewith, the harmonic component may well be of primary importance to them, particularly as harmony is important for creating aesthetic pleasure, expressiveness and meaning of the created and contemplated work. In this study, we aimed, and were able, to develop an algorithm that allows generating egg contours that correspond to the principles of the greatest aesthetic effect based solely on the concept of shape balance. Whether we have shed any light on the notion of harmony remains to be seen.

Without a doubt, the level of perception of an artistic object will be influenced by many subjective factors. Here, perhaps, the number of judgments will be proportional to the number of individuals. Someone is more comfortable with the appearance of a standard chicken egg, which a person encounters almost every day. Someone prefers ellipsoid (symmetrical) eggs, like, for example, emu eggs, and they seem more harmonious to them, due to the absolute symmetry along both the horizontal and vertical axes. Our authors' group is delighted with the so-called pear-shaped (or conical) eggs, like, for example, guillemot eggs.

We have devoted several studies to both the mathematical description and the search for possible reasons for this far from ordinary shape of guillemot eggs (Narushin et al., 2023c, 2024c). In this respect, we decided not to try to find some universal criterion for individual perception of the shape of an egg, but to focus on the results of *a priori* information corresponding to the opinion of most respondents.

By incorporating the additional and impartial help of AI-assisted confirmation of the methodological hypothesis of a balanced egg, we consider this study as another example of successful AI application in research (e.g., Wang et al., 2023). We accept that it is possible that AI-generated images may be more in alignment with the ideals of scientists and mathematicians than of artists, architects and the general population.

We suggest nonetheless that using the balance principle and the proposed index, i.e., the ratio of the volumes of the pointed and blunt parts of a bird's egg (Vp/Vb), may be appropriate in assessing qualitative or other characteristics in biological and agricultural research. Indeed, several studies (e.g., Narushin et al., 2023a), have demonstrated that morphometric parameters of eggs, especially when combined in the form of indices impact on the ability of birds to fly (Stoddard et al., 2017, 2019), in nesting characteristics of wild species (Montgomerie et al., 2021) and in many aspects of agriculture. In other words, the Vp/Vb index can be considered as one of many tools in applied and fundamental oological research, regardless of its aesthetic nature.

The perception of a bird's egg due to its asymmetric geometric structure, which has significant differences depending on the variation of the minimum set of egg parameters (Narushin et al., 2023c) is one that will undoubtedly be continued by many authors, we included. In this regard, the solution we express here to represent balance is one possible approach for combining the foundations of mathematics to comprehend the harmony of the egg, as has been repeatedly applied by scientists of various fields over many years, decades and centuries (reviewed in Birkhoff, 1933; Harada and Yoshimoto, 2002, 2003; Gobithaasan et al., 2008, 2013; Brinkmann, 2009; Douchová, 2015; Hübner and Ufken, 2023).

5 CONCLUSIONS

Considering the popularity of the bird's egg in art, architecture, design and engineering, we proposed a concept of its harmonious and aesthetic perception using the balance of its structural halves, pointed and blunt, the border between which runs along the maximum breadth axis. The developed mathematical dependences allowed us to compute variants of a balanced egg shape, both in its volumetric (3D) and plane (2D) execution. The presented transformation coefficient, τ, with the average value of 0.925, demonstrated the imbalance between the perception of the harmonious execution of the egg in 3D and 2D formats.

The method of mathematical computation of the balanced egg shape can serve as a convenient tool not only for creating artificial models and images of the egg shape, but also for researching the possible relationship between the degree of balance of a particular egg and its incubation and/or other properties in the relevant fields of design, architecture, agriculture and egg-inspired engineering (Narushin et al., 2022b). The role of mathematics in our perception of the egg as an aesthetically pleasing, "harmonious" object (or in some cases, quite the opposite) is another matter entirely. The described mathematical approaches and derived equations for determining the balance of an egg image will also find application in school curricula, allowing for the use of non-trivial mathematical approaches, as well as artificial intelligence, to assess the degree of perception of various geometric objects.

Supplementary **Materials:** Supplementary Data A–D.

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Abbreviations: Gen AI Generative Artificial Intelligence; Eqn(s) Equation(s).

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