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Decentralised static output feedback tracking control for nonlinear interconnected systems using sliding mode techniques

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ABSTRACT

In this paper, a sliding mode technique based decentralised static output feedback control scheme is proposed for system output tracking for a class of nonlinear interconnected systems where both nonlinear nominal subsystems and unknown interconnections are considered. A composite sliding surface is designed in terms of tracking error at first and then the corresponding sliding motion stability is analysed based on the Lyapunov approach. In order to guarantee the reachability, a decentralised output tracking control is designed based on system output and the pre-given desired output signals such that the effect of the interconnections can be tolerated and the tracking error dynamical systems are driven to the sliding surface maintaining a sliding motion thereafter. The proposed method depends on available information only and can reduce conservatism by fully exploring and adopting known information. An appropriate coordinate transformation is employed to facilitate the sliding mode control design. The effectiveness of the proposed approach is demonstrated through simulation applied to a river pollution system.

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1. Introduction

The prevailing rapid growth and technological advancements in the 21st century have created a need for increased research in controlling large-scale nonlinear interconnected systems. Such complex systems arise widely in practice, including automated highway systems, quadrupedal locomotion, river pollution systems, and power systems (Yan et al., 2017). This has motivated extensive research on nonlinear large-scale systems that can be modelled as collections of interconnected nonlinear subsystems. It should be noted that many practical systems are significantly influenced by external disturbances and parameter variations, which can substantially degrade system performance. Therefore, the design of control schemes that ensure strong robustness against various uncertainties is of considerable practical importance for interconnected systems.

Attempts have been made to address these aforementioned challenges. One of them is the development of robust control method using sliding mode techniques. The sliding mode control is deployed in

control design of a nonlinear interconnected system due to its unique features such as high robustness and reduced-order sliding mode dynamics with relatively simple design process (see, e.g. Edwards & Spurgeon, 1998; Yan et al., 2017). The sliding mode control is categorised by a suit of discontinuous feedback control laws which alters the systems dynamics on purposes, thereby forcing the system to slide along a predefined surface (Edwards & Spurgeon, 1998; Song et al., 2022). The sliding mode control design methodology consists of two phases: the reaching phase where the system transitions from an initial state to the pre-designed sliding surface, and the sliding phase when the system moves on the designed sliding surface (Utkin et al., 2017). The sliding mode control is robust and insensitive to matched uncertainties, which are a class of uncertainties in the input channels within the system. The sliding mode dynamics are a reduced-order system which facilitates the stability analysis of sliding motion (Spurgeon, 2014; Yan et al., 2017). Sliding mode control can also be employed to deal with unmatched uncertainties which are those outside

the input channels (see, e.g. Azar & Zhu, 2015; Yan et al., 2017).

When analysing complex nonlinear interconnected systems, it is often highly challenging and/or impossible to obtain analytical expressions for their solutions. This is primarily due to the intricate nonlinear nature of these systems and complex interconnections, which usually involve high-dimensional state spaces and intricate interactions between their components. Over the past few decades, there has been considerable progress in the field of feedback control for nonlinear interconnected systems, specifically in relation to achieving specific design objectives. Notably, this progress has been observed across various control tasks and formulations, with primary emphasis on state feedback control and, to a lesser extent, output feedback control. In Mirkin et al. (2011), decentralised state feedback adaptive sliding mode control for large-scale interconnected systems with nonlinear interconnections and time-delay was developed. Other studies utilised decentralised sliding mode control to tackle dead-zone input nonlinearities, interconnected nonlinearities, and matched/unmatched uncertainties simultaneously, to guarantee asymptotic stabilities of sliding motion, thus reducing the conservativeness of interconnected terms using structure characteristics (Feng et al., 2020). All the works mentioned above assume that system states are accessible for decentralised stabilisation control design.

In spite of the considerable applications, the system state variables are limited in their availability for practical systems. Beyond this, some of the variables are challenging, expensive, and difficult to measure. Although an observer could be used to estimate unknown states, additional resources would be required which would also significantly increase the system dimension (Yan et al., 2014). Lee (1995) adapted the sliding mode techniques for a class of interconnected systems, where it was required that the uncertainties and interconnections were matched. However, the major setback of this method is the assumption that the interconnections and isolated subsystems were both considered to be linear. In Yan et al. (1998), Yan and Dai (1998) and Yan et al. (2013), static output feedback controllers were adopted to improve the robustness and stability of a fully nonlinear interconnected systems, where only stabilisation is considered. In the decentralised output feedback

control, each subsystem is controlled independently based on its local output information without relying on information from the other subsystems to achieve its control objectives (Yan et al., 2017). However, for a complex interconnected system, the performance of each subsystem is influenced by the other subsystems through their interconnections, making the dynamics control more challenging and possibly resulting in instability (Yan et al., 1998). In Yan et al. (2004), the decentralised sliding mode control has been developed and the system is fully nonlinear with a more general structure, but the tracking control problem is not considered in Yan et al. (2004).

It is well known that the tracking problem is much more challenging when compared with stabilisation. Super twisting control was designed for trajectory tracking of a nonlinear quadrotor under assumption that all the system states are available for design in Xu et al. (2025), and an observer based fuzzy PID tracking control scheme is proposed for a class of linear system in Wang et al. (2024). Previous research also has investigated trajectory tracking and output tracking for linear large-scale systems with model reference control in Pagilla et al. (2007) and adaptive fuzzy techniques in Ren et al. (2019), using decentralised adaptive output feedback controllers. The limitations in the approaches above were that they needed the considered systems to have a special structure and the corresponding isolated subsystems to be linear. Moreover, the results for output tracking for interconnected systems using sliding mode techniques are very few. In Ding et al. (2023), a decentralised output-tracking problem for nonlinear large-scale systems with unknown interconnections was studied using sliding mode control, under the assumption that all system states are available for controller design. In practice, however, full-state measurements are rarely accessible; moreover, some state variables may lack physical meaning and thus cannot be directly measured. To address these limitations, this paper proposes a decentralised static output feedback control scheme that relies solely on locally measured outputs.

The main contributions of this paper are summarised as follows:

- A decentralised output-feedback control framework is developed for output tracking of a class of nonlinear large-scale interconnected systems with

both matched and unmatched uncertainties, eliminating the need for state observers or full-state measurements.

- A composite sliding surface based on the output-tracking error is constructed, and sufficient conditions are derived to guarantee uniformly ultimately bounded sliding motion.
- Nonlinear nominal subsystems and unknown nonlinear interconnections bounded by the system outputs are jointly addressed, allowing for time-varying reference signals while preserving a fully decentralised controller structure suitable for practical implementation.
- The proposed controllers exhibit strong robustness against nonlinearities and uncertainties by exploiting partition of the known nonlinear terms in the isolated subsystems and the inherent robustness properties of sliding-mode control.

The problems considered in this work involve several technical challenges, including nonlinear subsystem dynamics, unmatched and nonlinear interconnections, decentralised design constraints, time-varying reference signals, and the absence of full-state measurements. These challenges significantly limit conventional decentralised sliding-mode control methods, which typically rely on restrictive assumptions or observer-based designs. The novelty of this paper lies in overcoming these challenges within a unified output-feedback framework. Finally, a river pollution system is used to demonstrate the practicality and effectiveness of the proposed approach through simulation.

Notation: The notation used in this paper is standard. For a square matrix A , $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimum and maximum eigenvalues of the matrix A , respectively. The expression $A > 0$ means that A is symmetric positive definite and I_n denotes the $n \times n$ identity matrix. The symbols \mathcal{R}^n and $\mathcal{R}^{n \times m}$ denote the sets of n -dimensional real vectors and $n \times m$ dimensional real matrices, respectively. $\text{col}(\cdot)$ represents a column vector. Lastly, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

2. Preliminaries

In this section, some preliminary results are to be summarised at first which will be used for further analysis

and design. Consider a simple linear control system given by

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx \quad (2)$$

where variables $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$, and $y \in \mathcal{R}^p$ ($p \geq m$) are system states, control inputs, and outputs, respectively. The matrices $A \in \mathcal{R}^{n \times n}$, $B \in \mathcal{R}^{n \times m}$, and $C \in \mathcal{R}^{p \times n}$ are known real constants with appropriate dimensions, where both B and C are of full rank.

Now, consider a special case where system (1)–(2) is a square plant, which means that in system (1)–(2), the dimension of the input equals the dimension of the output denoted as $m = p$. It is clear to see that, if $\text{rank}(CB) = m$, it can be shown that there exists a nonsingular linear coordinate transformation $\tilde{x} = \tilde{T}x$, such that the system (1)–(2) can be transformed to a new system with respect to the new coordinates \tilde{x} as follows

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$$

$$y = \tilde{C}\tilde{x}$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} = \tilde{T}A\tilde{T}^{-1}, & \tilde{B} &= \begin{bmatrix} 0 \\ B_2 \end{bmatrix} = \tilde{T}B, \\ \tilde{C} &= [C_1 \quad C_2] = C\tilde{T}^{-1} \end{aligned} \quad (3)$$

Here the matrix $\tilde{A}_{11} \in \mathcal{R}^{(n-m) \times (n-m)}$, while the matrices $B_2 \in \mathcal{R}^{m \times m}$ and $C_2 \in \mathcal{R}^{m \times m}$ are nonsingular, and the coordinate transformation matrix \tilde{T} can be obtained through basic matrix theory. From $\text{rank}(CB) = m$, it follows that CB is an invertible matrices.

Thus, it follows from Edwards and Spurgeon (1998) that in the new coordinates $\tilde{x} \rightarrow z = \hat{T}\tilde{x} = \hat{T}\tilde{T}x$, where \hat{T} is nonsingular defined as

$$\hat{T} = \begin{bmatrix} I_{(n-m)} & 0 \\ C_1 & C_2 \end{bmatrix} \quad (4)$$

where C_1 and C_2 are given in (3), the system (1)–(2) can be described by

$$\dot{z} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} z + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \quad (5)$$

$$y = [0 \quad I_m] z \quad (6)$$

It is clear to see that system (5)–(6) is now in the well known regular form, and specifically, the matrix $A_{11} \in \mathcal{R}^{(n-m) \times (n-m)}$ is Hurwitz stable, and $B_2 \in \mathcal{R}^{m \times m}$ is nonsingular. More importantly, it is easily to see from output Equation (6) that the system output can be separated from system state which can facilitate to fully employ the output information in output feedback analysis and control design.

3. System description and basic assumptions

Consider the nonlinear large-scale system composed of N -interconnected subsystems modelled by

$$\dot{x}_i = A_i x_i + F_i(x_i) + B_i(u_i + \Delta G_i(x_i)) + \sum_{\substack{j=1 \\ j \neq i}}^N H_{ij}(x_j) \quad (7)$$

$$y_i = C_i x_i, \quad i = 1, 2, \dots, N \quad (8)$$

where $x = \text{col}(x_1, x_2, \dots, x_N)$, $x_i \in \mathcal{R}^{n_i}$, $u_i \in \mathcal{R}^{m_i}$ and $y_i \in \mathcal{R}^{m_i}$ are the states, control inputs, and outputs of the i th subsystem, respectively. The system triple (A_i, B_i, C_i) has appropriate dimensions with both B_i and C_i being of full rank. In the i th subsystem, the function $F_i(x_i)$ denotes a known nonlinear term. $\Delta G_i(x_i)$ represents the matched uncertainty of the i th isolated subsystem which is acting in the input channel. The term $\sum_{\substack{j=1 \\ j \neq i}}^N H_{ij}(x_j)$ describes the unknown nonlinear interconnection between the i th subsystem and other subsystems. It is assumed that all the nonlinear functions are sufficiently smooth such that the system (7)–(8) has a unique continuous solution.

Remark 1: To simplify the statements, similar to the work in Ding et al. (2023), it is assumed that the dimension of each subsystem input is equal to the output dimension of the subsystem in the considered system (7)–(8) which is called square system. It should be pointed out that this limitation is not inherent as the approaches developed in this paper can be extended to the general case by applying the methodology in Yan et al. (2004).

In general case, the sliding mode dynamics will involve part of system output y_i in (8). Thus, the study will become more complicated, and simultaneously, it will provide more flexibility in sliding surface design.

A detailed information regarding this is available in the work in Edwards and Spurgeon (1998) and Yan et al. (2004).

It is worth noting that square systems arise naturally in many practical applications, such as bioreactor systems (Gauthier et al., 1992) and river pollution systems (Ding et al., 2023) etc. In addition, all single-input single-output (SISO) systems studied in classical control theory are inherently square. In particular, square plant structures have been widely adopted in sliding-mode control design, as they greatly facilitate the transformation of the system into a regular form and enable the direct construction of appropriate sliding surfaces (Edwards & Spurgeon, 1998; Yan et al., 2004).

The following assumptions are imposed on the nonlinear interconnected subsystems (7)–(8).

Assumption 1: All the invariant zeros of the systems triple (A_i, B_i, C_i) lie on the left half-plane for $i = 1, 2, \dots, N$.

Since $B_i \in \mathcal{R}^{m_i \times m_i}$ and $C_i \in \mathcal{R}^{m_i \times m_i}$ are full-rank, it follows that $\text{rank}(C_i B_i) = m_i$, and thus $C_i B_i$ is nonsingular.

Under Assumption 1 and from the preliminaries explained in Section 2, there exists a nonsingular coordinate transformation

$$z_i = T_i x_i \quad (9)$$

such that the triple (A_i, B_i, C_i) with respect to the new coordinates z_i is presented as

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \quad C_i = [0 \quad I_{i2}] \quad (10)$$

where $A_{i11} \in \mathcal{R}^{(n_i-m_i) \times (n_i-m_i)}$ is Hurwitz stable, the square matrices $B_{i2} \in \mathcal{R}^{m_i \times m_i}$ is nonsingular and $I_{i2} \in \mathcal{R}^{m_i \times m_i}$ is an identity matrix for $i = 1, 2, \dots, N$. The nonsingular matrix T_i in (9) can be obtained using the method given in Edwards and Spurgeon (1998).

Remark 2: It should be pointed out that the structure of the transformed triples in (10) have three important properties:

- the triples in (10) have the regular form;
- all matrices $A_{i11} \in \mathcal{R}^{(n_i-m_i) \times (n_i-m_i)}$ for $i = 1, 2, \dots, N$ are Hurwitz stable;

- the output matrices make the output separated from system states.

These are very important and will help to fully employ the output information in the system analysis and sliding mode control design later.

Assumption 2: Suppose that $F_i(x_i)$ in (7) has the decomposition $F_i(x_i) = \Gamma_i(y_i)x_i$, where $\Gamma_i(\cdot) \in \mathcal{R}^{n_i \times n_i}$ is a continuous function matrix.

Remark 3: The limitation imposed on the nonlinear term F_i in Assumption 2 enables that the functions $\Gamma_i(\cdot)$ can be potentially employed in output feedback control design to reject the effects of the nonlinear terms in the isolated subsystem. Note that Assumption 2 will hold if $f_i(0) = 0$, and the function $f_i(\cdot)$ is sufficiently smooth in its definition domain for $i = 1, 2, \dots, N$.

Assumption 3: There exist known non-negative continuous functions $\rho_i(\cdot)$ and $\eta_{ij}(\cdot)$ such that

- (i) $\|\Delta G_i(x_i)\| \leq \rho_i(y_i)$
- (ii) $\|H_{ij}(x_j)\| \leq \eta_{ij}(x_j) \quad (j \neq i)$

where $\eta_{ij}(x_j)$ satisfies $\eta_{ij}(x_j) \leq \gamma_{ij}(x_j)\|x_j\|$ ($j \neq i$) for some known continuous function $\gamma_{ij}(\cdot)$ for $i, j = 1, 2, \dots, N$ and $j \neq i$.

Remark 4: Assumption 3 ensures that the uncertainties $\Delta G_i(x_i)$ in (7)–(8) are bounded by known functions of system outputs (here are $\rho_i(y_i)$ for $i = 1, 2, \dots, N$), which ensures that these bounds can be fully employed in the control design to reject the effects of the corresponding uncertainties even if static output feedback is employed.

Assumption 4: The desired output signals $y_{id}(t)$ are differentiable and bounded in $t \in \mathcal{R}$.

The Assumption 4 is a limitation to the desired tracking signals, which is satisfied in most cases in reality and thus it is reasonable.

4. Tracking system formation analysis

In this section, considering the structure characteristics of the triples in (10) presented above, a nonlinear

interconnected system relating to tracking error is to be formed and then the problem considered in this paper will be proposed.

Under Assumptions 1–2, consider the nonlinear interconnected system in (7)–(8). From (Spurgeon, 2014), in the new coordinate $z = \text{col}(z_1, z_2, \dots, z_n)$, the system (7)–(8) is described by

$$\begin{aligned} \dot{z}_i = & \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} z_i + \begin{bmatrix} \Gamma_{i1}(y_i) \\ \Gamma_{i2}(y_i) \end{bmatrix} z_i \\ & + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} (u_i + \Delta G_i(T_i^{-1}z_i)) + \sum_{\substack{j=1 \\ j \neq i}}^N T_i H_{ij}(T_j^{-1}z_j) \end{aligned} \quad (11)$$

$$y_i = \begin{bmatrix} 0 & I_{i2} \end{bmatrix} z_i \quad (12)$$

where $A_{i11} \in \mathcal{R}^{(n_i-m_i) \times (n_i-m_i)}$ is stable, the submatrices $B_{i2} \in \mathcal{R}^{m_i \times m_i}$ is an invertible matrix, and $I_{i2} \in \mathcal{R}^{m_i \times m_i}$ is an identity matrix, the number 0 represents zero matrices with appropriate dimensions.

$$\begin{bmatrix} \Gamma_{i1}(y_i) \\ \Gamma_{i2}(y_i) \end{bmatrix} = T_i \Gamma_i(y_i) T_i^{-1}$$

where $\Gamma_{i1}(y_i) \in \mathcal{R}^{(n_i-m_i)}$, $\Gamma_{i2}(y_i) \in \mathcal{R}^{m_i}$, and the matrices T_i are the transformation matrices defined in (9) for $i = 1, 2, \dots, N$.

Since A_{i11} is stable for $i = 1, 2, \dots, N$, for any $Q_i > 0$, the following Lyapunov equation has a unique solution $P_i > 0$ such that

$$A_{i11}^T P_i + P_i A_{i11} = -Q_i. \quad i = 1, 2, \dots, N \quad (13)$$

For further analysis, system (11)–(12) is rewritten in the following partitioned form:

$$\dot{z}_{i1} = A_{i11}z_{i1} + A_{i12}z_{i2} + \Gamma_{i1}(y_i)z_i + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij1}(z_j1, z_j2) \quad (14)$$

$$\begin{aligned} \dot{z}_{i2} = & A_{i21}z_{i1} + A_{i22}z_{i2} + \Gamma_{i2}(y_i)z_i \\ & + B_{i2}(u_i + \Delta G_i(T_i^{-1}z_i)) + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_j1, z_j2) \end{aligned} \quad (15)$$

$$y_i = z_{i2} \quad (16)$$

where z_i is partitioned as $z_i = \text{col}(z_{i1}, z_{i2})$, $z_{i1} \in \mathcal{R}^{n_i - m_i}$ and $z_{i2} \in \mathcal{R}^{m_i}$, and

$$\delta_{ij}(z_j) = \begin{bmatrix} \delta_{ij1}(z_j) \\ \delta_{ij2}(z_j) \end{bmatrix} = T_i H_{ij}(x_j)|_{x_j = T_j^{-1} z_j} \quad (17)$$

with $\delta_{ij1}(z_j) \in \mathcal{R}^{n_i - m_i}$ and $\delta_{ij2}(z_j) \in \mathcal{R}^{m_i}$ for $i \neq j$ and $i, j = 1, 2, \dots, N$. For system (14)–(16), define the output tracking error state e_i as the difference between the system output and the desired output signals given by

$$e_i(t) = y_i(t) - y_{id}(t), \quad i = 1, 2, \dots, N. \quad (18)$$

It then follows that the first derivative of (18) with respect to time yields

$$\dot{e}_i(t) = \dot{y}_i(t) - \dot{y}_{id}(t), \quad i = 1, 2, \dots, N. \quad (19)$$

From (14)–(16) and (19), it follows that

$$\dot{z}_{i1} = A_{i11}z_{i1} + A_{i12}y_i + \Gamma_{i1}(y_i)z_i + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij1}(z_{j1}, y_j) \quad (20)$$

$$\begin{aligned} \dot{e}_i &= A_{i21}z_{i1} + A_{i22}y_i + \Gamma_{i2}(y_i)z_i \\ &+ B_{i2}(u_i + \Delta G_i(T_i^{-1} \text{col}(z_{i1}, y_i))) \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_{j1}, y_j) - \dot{y}_{id} \end{aligned} \quad (21)$$

for $i = 1, 2, \dots, N$, where $A_{i11} \in \mathcal{R}^{(n_i - m_i) \times (n_i - m_i)}$ is stable and $A_{i21} \in \mathcal{R}^{(n_i - m_i) \times m_i}$, and $\delta_{ij1}(\cdot)$ and $\delta_{ij2}(\cdot)$ are defined in (17).

It should be noted that system (20)–(21) has a well-known regular form that will facilitate the design of the sliding mode control. In the following, the focus will be on system (20)–(21). The objective is to design a decentralised static output feedback control

$$u_i = u_i(t, y_i) \quad i = 1, 2, \dots, N$$

using sliding mode techniques such that the tracking error $e_i(t)$ defined in (18) converges to zero when $t \mapsto \infty$, and the system (20)–(21) is uniformly-ultimately bounded even in the presence of uncertainties and interconnections.

5. Sliding mode tracking dynamics analysis

In this section, a composite sliding surface will be proposed for the interconnected system (20)–(21) based

on the output tracking errors. Then, the corresponding sliding motion will be analysed. Consider a composite sliding surface proposed by,

$$\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} = 0 \quad (22)$$

where $e_i(t)$ is the tracking error defined in (18) for $i = 1, 2, \dots, N$. It is clear to see that when system (20)–(21) is constrained to the sliding surface (22), the system output y_i in (12) can track the desired signal y_{id} .

From the sliding mode control theory, when sliding motion occurs, from (22), $y_i = y_{id}$. Further from the specific structure of system (20)–(21), the sliding motion of the system (20)–(21) regarding to the sliding surface (22), is governed by the sliding mode dynamics

$$\begin{aligned} \dot{z}_{i1} &= A_{i11}z_{i1} + A_{i12}y_{id} + \Gamma_{i1}(y_{id}) \begin{bmatrix} z_{i1} \\ y_{id} \end{bmatrix} \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij1}(z_{j1}, y_{jd}) \end{aligned} \quad (23)$$

Following Assumption 3(ii), there exist continuous functions $\alpha_{ij}(\cdot)$ and $\beta_{ij}(\cdot)$ ($j \neq i$) such that

$$\|\delta_{ij1}(z_{j1}, y_{jd})\| \leq \alpha_{ij}(z_{j1}, y_{jd})\|z_{j1}\| + \beta_{ij}(z_{j1}, y_{jd})\|y_{jd}\| \quad (24)$$

where $\delta_{ij1}(\cdot)$ is determined in (17) for $j \neq i$, $i, j = 1, 2, \dots, N$.

Remark 5: It should be noted that the bound on the uncertain interconnections $\delta_{ij1}(\cdot)$ in (24) is related to the desired signal y_{jd} for $j = 1, 2, \dots, N$. This is consistent with the reality as different tracking signals will have a different environment which may produce different disturbances.

Obviously, the sliding mode dynamics in (23) are a nonlinear interconnected system composed of N subsystems in which each subsystem has an order $(n_i - m_i)$ and thus is a reduced-order system when compared with the corresponding subsystem of (1)–(2) which has order n_i for $i = 1, 2, \dots, N$. In order to

reduce the conservatism, partition $\Gamma_{i1}(y_{id})$ in (23) as

$$\Gamma_{i1}(y_{id}) = [\Gamma_{i11}(y_{id}) \quad \Gamma_{i12}(y_{id})] \quad (25)$$

where $\Gamma_{i11}(\cdot) \in \mathcal{R}^{(n_i-m_i) \times m_i}$ and $\Gamma_{i12}(\cdot) \in \mathcal{R}^{(n_i-m_i) \times (n_i-m_i)}$. Then

$$\Gamma_{i1}(y_{id}) \begin{bmatrix} z_{i1} \\ y_{id} \end{bmatrix} = \Gamma_{i11}(y_{id})z_{i1} + \Gamma_{i12}(y_{id})y_{id} \quad (26)$$

The result for stability of the sliding mode dynamics is now ready to be presented.

Theorem 5.1: Consider the sliding mode dynamics given in (23). Under Assumptions 1–3, the sliding motion governing by (23) is uniformly-ultimately bounded if there is a domain of the origin $\Omega_S \subset \mathcal{R}^{\sum_{i=1}^N (n_i-m_i) \times N}$ such that the function matrix $W^T(\cdot) + W(\cdot) > 0$ for $(z_{11}, z_{12}, \dots, z_{1N}) \in \Omega_S \setminus \{0\}$, where $W := (w_{ij})_{N \times N}$ with its elements w_{ij} defined by

$$w_{ij} = \begin{cases} \lambda_m(Q_i - R_i(\cdot)) - 2\|P_i\|\alpha_{ij}(\cdot), & i = j \\ -2\|P_i\|\alpha_{ij}(\cdot), & i \neq j \end{cases} \quad (27)$$

where the matrices P_i and Q_i are positive definite satisfying the Lyapunov Equation (13), the function matrix $R_i(\cdot) \in \mathcal{R}^{n_i \times n_i}$ is defined by

$$R_i(\cdot) := P_i\Gamma_{i11}(\cdot) + \Gamma_{i11}^T(\cdot)P_i$$

with $\Gamma_{i1}(\cdot)$ defined in (25), and $\alpha_{ij}(\cdot)$ satisfies (24) for $i \neq j$ and $i, j = 1, 2, \dots, N$.

Proof: For system (20)–(21) and the desired output signals given in Assumption 4, the analysis above has shown that system (23) is the reduced-order sliding mode dynamics associated with the sliding surface given in (22). It is now only needed to prove that system (23) is uniformly-ultimately bounded.

Consider that the Lyapunov function candidate as

$$V_{(z_{11}, z_{21}, \dots, z_{N1})} = \sum_{i=1}^N z_{i1}^T P_i z_{i1} \quad (28)$$

where $P_i > 0$ satisfies (13) for $i = 1, 2, \dots, N$. Taking the time derivative of $V_{(z_{11}, z_{21}, \dots, z_{N1})}$ along the trajectories of system (23) gives

$$\dot{V}_{(28)} = \sum_{i=1}^N \left(\dot{z}_{i1}^T P_i z_{i1} + z_{i1}^T P_i \dot{z}_{i1} \right)$$

$$\begin{aligned} &= \sum_{i=1}^N \left\{ \left(A_{i11}z_{i1} + A_{i12}y_{id} + \Gamma_{i1}(y_{id}) \begin{bmatrix} z_{i1} \\ y_{id} \end{bmatrix} \right. \right. \\ &\quad \left. \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij1}(z_{j1}, y_{jd}) \right)^T P_i z_{i1} \right. \\ &\quad \left. + z_{i1}^T P_i \left(A_{i11}z_{i1} + A_{i12}y_{id} + \Gamma_{i1}(y_{id}) \begin{bmatrix} z_{i1} \\ y_{id} \end{bmatrix} \right. \right. \\ &\quad \left. \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij1}(z_{j1}, y_{jd}) \right) \right\} \quad (29) \end{aligned}$$

Then from (26), Equation (29) can be rewritten by

$$\begin{aligned} \dot{V}_{(28)} &= \sum_{i=1}^N \left\{ z_{i1}^T \left(A_{i11}^T P_i + P_i A_{i11} \right) z_{i1} \right. \\ &\quad \left. + z_{i1}^T \left(P_i \Gamma_{i11}(\cdot) + \Gamma_{i11}^T(\cdot) P_i \right) z_{i1} \right. \\ &\quad \left. + 2z_{i1}^T P_i \left(A_{i12} + \Gamma_{i12}(\cdot) \right) y_{id} \right. \\ &\quad \left. + 2 \sum_{\substack{j=1 \\ j \neq i}}^N z_{i1}^T P_i \delta_{ij1}(z_{j1}, y_{jd}) \right\} \\ &= \sum_{i=1}^N \left\{ -z_{i1}^T \left(Q_i - R_i(\cdot) \right) z_{i1} \right. \\ &\quad \left. + 2z_{i1}^T P_i \left(A_{i12} + \Gamma_{i12}(\cdot) \right) y_{id} \right. \\ &\quad \left. + 2 \sum_{\substack{j=1 \\ j \neq i}}^N z_{i1}^T P_i \delta_{ij1}(z_{j1}, y_{jd}) \right\} \end{aligned}$$

where $\Gamma_{i11}(\cdot)$ and $\Gamma_{i12}(\cdot)$ are defined in (25) and the Equation (13) is used above. By (24), it follows that

$$\begin{aligned} \dot{V}_{(28)} &\leq \sum_{i=1}^N \left\{ -\lambda_m(Q_i - R_i(\cdot)) \|z_{i1}\|^2 \right. \\ &\quad \left. + 2\|z_{i1}\| \|P_i(A_{i12} + \Gamma_{i12}(\cdot))\| \|y_{id}\| \right. \\ &\quad \left. + 2 \sum_{\substack{j=1 \\ j \neq i}}^N \|z_{i1}\| \|P_i\| \|\delta_{ij1}(z_{j1}, y_{jd})\| \right\} \\ &\leq \sum_{i=1}^N \left\{ -\lambda_m(Q_i - R_i(\cdot)) \|z_{i1}\|^2 \right. \end{aligned}$$

$$\begin{aligned}
& + 2 \left\| P_i (A_{i12} + \Gamma_{i12}(y_{id})) \right\| \|z_{i1}\| \|y_{id}\| \\
& + 2 \sum_{\substack{j=1 \\ j \neq i}}^N (\alpha_{ij}(z_{j1}, y_{jd}) \|z_{j1}\| \\
& + \beta_{ij}(z_{j1}, y_{jd}) \|y_{jd}\|) \|P_i\| \|z_{i1}\| \left. \right\} \\
\leq & \sum_{i=1}^N \left\{ -\lambda_m(Q_i - R_i(\cdot)) \|z_{i1}\|^2 \right. \\
& + 2 \left\| P_i (A_{i12} + \Gamma_{i12}(y_{id})) \right\| \|z_{i1}\| \|y_{id}\| \\
& + 2 \sum_{\substack{j=1 \\ j \neq i}}^N \alpha_{ij}(z_{j1}, y_{jd}) \|P_i\| \|z_{i1}\| \|z_{j1}\| \\
& + 2 \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(z_{j1}, y_{jd}) \|P_i\| \|z_{i1}\| \|y_{jd}\| \left. \right\} \\
\leq & - \sum_{i=1}^N \left\{ \lambda_m(Q_i - R_i(\cdot)) - 2 \|P_i\| \alpha_{ij}(\cdot) \right\} \|z_{i1}\|^2 \\
& + 2 \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \alpha_{ij}(z_{j1}, y_{jd}) \|P_i\| \|z_{j1}\| \|z_{i1}\| \\
& + 2 \sum_{i=1}^N \|P_i (A_{i12} + \Gamma_{i12}(\cdot))\| \|y_{id}\| \|z_{i1}\| \\
& + 2 \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(z_{j1}, y_{jd}) \|P_i\| \|y_{jd}\| \|z_{i1}\| \\
= & \frac{1}{2} (\|z_{11}\|, \|z_{21}\|, \dots, \|z_{N1}\|) \\
& \times (M^T + M) \begin{pmatrix} \|z_{11}\| \\ \|z_{21}\| \\ \vdots \\ \|z_{N1}\| \end{pmatrix} \\
& + 2 \sum_{i=1}^N \left(\|P_i (A_{i12} + \Gamma_{i12})\| \|y_{id}\| \right. \\
& \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(\cdot) \|P_i\| \|y_{jd}\| \right) \|z_{i1}\| \tag{30}
\end{aligned}$$

where the matrix W is defined in (27). From Young's inequality:

$$\begin{aligned}
& 2 \sum_{i=1}^N \left(\|P_i (A_{i12} + \Gamma_{i12})\| \|y_{id}\| \right. \\
& \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(\cdot) \|P_i\| \|y_{jd}\| \right) \|z_{i1}\| \\
\leq & 2 \left(\sum_{i=1}^N \left(\|P_i (A_{i12} + \Gamma_{i12})\| + \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(\cdot) \|P_i\| \right)^2 \right. \\
& \left. \times \|y_{jd}\|^2 \right)^{\frac{1}{2}} \left(\sum_{i=1}^N \|z_{i1}\|^2 \right)^{\frac{1}{2}} \tag{31}
\end{aligned}$$

Then substitute (31) into (30), it yields

$$\begin{aligned}
\dot{V} |_{(28)} \leq & -\frac{1}{2} \lambda_m(W^T + W) \sum_{i=1}^N \|z_{i1}\|^2 \\
& + \sigma \left(\sum_{i=1}^N \|z_{i1}\|^2 \right)^{\frac{1}{2}} \tag{32}
\end{aligned}$$

$$= -\left(\frac{1}{2} \lambda_m(W^T + W) - \sigma \right) \left(\sum_{i=1}^N \|z_{i1}\|^2 \right)^{\frac{1}{2}} \tag{33}$$

where

$$\begin{aligned}
\sigma := & 2 \left\{ \sum_{i=1}^N \left(\|P_i (A_{i12} + \Gamma_{i12})\| \right. \right. \\
& \left. \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij}(z_{j1}, y_{jd}) \|P_i\| \right)^2 \|y_{jd}\|^2 \right\}^{\frac{1}{2}} \tag{34}
\end{aligned}$$

for $i, j = 1, 2, \dots, N$

It can be observed that \dot{V} is negative definite if $\sigma < \frac{1}{2}(\lambda_m(W^T + W))$ in the considered domain. Accordingly, it is straightforward to conclude that the sliding mode of the system (20)–(21) associated with the sliding surface (22) that is, system (23) is uniformly-ultimately bounded. ■

Remark 6: It is straightforward to see that the sliding mode dynamics (23) are a nonlinear interconnected system, which is independent of system control. The result developed in Theorem 5.1 is local. The stability domain of the sliding motion are determined by the matrix $W(\cdot)$ which confirms that the unmatched interconnection $\delta_{ij1}(\cdot)$ and unmatched nonlinear term $\Gamma_{i11}(\cdot)$ affect the sliding motion stability.

6. Decentralised sliding mode control design

The objective in this section now is to design a decentralised static output feedback sliding mode control such that the system state is driven to the sliding surface (22) and maintains a sliding motion on it thereafter.

Consider the interconnected systems in (20)–(21) with N subsystems in the domain Ω where Ω is a domain of the origin in $R^{\sum_{i=1}^N n_i}$, containing the sliding patch Ω_s , given by

$$\Omega = \{(z_1, z_2, \dots, z_N) \mid \|z_{i1}\| \leq \chi_i, z_{i2} \in \mathcal{R}^{m_i} \text{ for } i = 1, 2, \dots, N\} \quad (35)$$

for some positive constant χ_i and $z_i = \text{col}(z_{i1}, z_{i2})$ for $i = 1, 2, \dots, N$. The reachability condition (see, e.g. Konwar, 2017; Yan et al., 2017) is described as;

$$\sum_{i=1}^N \frac{e_i^T(t) \dot{e}_i(t)}{\|e_i(t)\|} < 0 \quad (36)$$

In order to fully use system output information to reduce conservatism, consider the output matrix C_i given in (12). From the transformation matrix (9) $x_i = T_i^{-1} z_i = T_i^{-1} T_i x_i$ and $z_i = \text{col}(z_{i1}, z_{i2})$ and $y_i = z_{i2}$, it follows that

$$x_i = T_i^{-1} z_i = T_i^{-1} T_i x_i = T_i^{-1} \begin{bmatrix} z_{i1} \\ y_i \end{bmatrix} \quad (37)$$

where T_i is nonsingular. Further partition $\Gamma_{i2}(\cdot)$ as

$$\Gamma_{i2}(y_i) = [\Gamma_{i21}(y_i) \quad \Gamma_{i22}(y_i)] \quad (38)$$

where $\Gamma_{i21}(y_i) \in \mathcal{R}^{m_i \times m_i}$ and $\Gamma_{i22}(y_i) \in \mathcal{R}^{m_i \times (n_i - m_i)}$. Then

$$\Gamma_{i2}(y_i) \begin{bmatrix} z_{i1} \\ y_i \end{bmatrix} = \Gamma_{i21}(y_i) z_{i1} + \Gamma_{i22}(y_i) y_i \quad (39)$$

The proposed decentralised static output feedback sliding mode control law for the interconnected system in (21) is given by

$$u_i = -B_{i2}^{-1} \{ \|A_{i21} + \Gamma_{i21}(y_i)\| \chi_i + (A_{i22} + \Gamma_{i22}(y_i)) y_i + K_i(y_i) + \|B_{i2}\| \rho_i(y_i) - \dot{y}_{id} \} \text{sgn}(y_i - y_{id}) \quad (40)$$

for $i = 1, 2, \dots, N$, where $\rho_i(y_i)$ is defined in Assumption 3(i), $K_i(y_i)$ is the controller gain to be designed later and χ_i is defined in (35). The symbol $\text{sgn}(\cdot)$ denotes the usual signum function. The function matrices $\Gamma_{i21}(\cdot)$ and $\Gamma_{i22}(\cdot)$ are defined in (38).

It should be mentioned that the i th control gain K_i in (40) is only dependent on the i th subsystem output y_i . It is clear to see that the designed controller u_i in (40) depends only on system local information: outputs y_i and desired signal y_{id} and thus is decentralised.

Then the reachability result can be obtained as follows.

Theorem 6.1: Consider the nonlinear interconnected system (20)–(21). Under Assumptions 1–3, the decentralised output feedback controller (40) can drive the system (20)–(21) to the composite sliding surface (22) and maintains sliding motion on it afterward, in the domain Ω (35), if the controller gain $K_i(y_i)$ satisfies

$$\sum_{i=1}^N K_i(y_i) - \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \|T_i\| \|\eta_{ij}(x_j)\| > 0 \quad (41)$$

where $\eta_{ij}(\cdot)$, $i \neq j$ is determined in Assumption 3(ii) and T_i is given in (9) for $i, j = 1, 2, \dots, N$.

Proof: From the analysis above, it only needs to prove that the reachability condition (36) is satisfied. From (21), the tracking error dynamics can be described by

$$\dot{e}_i = A_{i21} z_{i1} + A_{i22} y_i + \Gamma_{i2}(y_i) \begin{bmatrix} z_{i1} \\ y_i \end{bmatrix} + B_{i2}(u_i + \Delta G_i(T_i^{-1} \text{col}(z_{i1}, y_i)))$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_{j1}, y_j) - \dot{y}_{id} \quad i = 1, 2, \dots, N. \quad (42)$$

Substituting the control (40) into (42), the follows that

$$\begin{aligned} \dot{e}_i = & \left\{ (A_{i21} + \Gamma_{i21}(\cdot)) z_{i1} + (A_{i22} + \Gamma_{i22}(\cdot)) y_i \right. \\ & - (\|A_{i21} + \Gamma_{i21}(\cdot)\| \chi_i + (A_{i22} + \Gamma_{i22}(\cdot)) y_i \\ & + K_i(y_i) + \|B_{i2}\| \rho_i(y_i) - \dot{y}_{id}) \operatorname{sgn}(y_i - y_{id}) + B_{i2} \\ & \left. \times \Delta G_i(T_i^{-1} \operatorname{col}(z_{i1}, y_i)) + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_{j1}, y_j) - \dot{y}_{id} \right\} \end{aligned} \quad (43)$$

From (43),

$$\begin{aligned} & \sum_{i=1}^N \frac{(e_i)^T \dot{e}_i}{\|e_i\|} \\ &= \sum_{i=1}^N \frac{1}{\|y_i - y_{id}\|} (y_i - y_{id})^T \\ & \quad \times \left\{ (A_{i21} + \Gamma_{i21}(\cdot)) z_{i1} + (A_{i22} \right. \\ & \quad + \Gamma_{i22}(\cdot)) y_i - (\|A_{i21} + \Gamma_{i21}(\cdot)\| \chi_i \\ & \quad + (A_{i22} + \Gamma_{i22}(\cdot)) y_i + K_i(y_i) \\ & \quad + \|B_{i2}\| \rho_i(y_i) - \dot{y}_{id}) \operatorname{sgn}(y_i - y_{id}) \\ & \quad + B_{i2} \Delta G_i(T_i^{-1} \operatorname{col}(z_{i1}, y_i)) \\ & \quad \left. + \sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_{j1}, y_j) - \dot{y}_{id} \right\} \\ &= \sum_{i=1}^N \frac{1}{\|y_i - y_{id}\|} \\ & \quad \times \left\{ (y_i - y_{id})^T (A_{i21} + \Gamma_{i21}(\cdot)) z_{i1} - \|A_{i21}\| \right. \\ & \quad + \Gamma_{i21}(\cdot) \chi_i (y_i - y_{id})^T \operatorname{sgn}(y_i - y_{id}) \\ & \quad + (y_i - y_{id})^T (A_{i22} + \Gamma_{i22}(\cdot)) y_i - (A_{i22} \\ & \quad + \Gamma_{i22}(\cdot)) y_i (y_i - y_{id})^T \operatorname{sgn}(y_i - y_{id}) + (y_i - y_{id})^T \end{aligned}$$

$$\begin{aligned} & \left. \times \left(\sum_{\substack{j=1 \\ j \neq i}}^N \delta_{ij2}(z_{j1}, y_j) + B_{i2} \Delta G_i(T_i^{-1} \operatorname{col}(z_{i1}, y_i)) \right) \right. \\ & - (K_i(y_i) + \|B_{i2}\| \rho_i(y_i)) (y_i - y_{id})^T \\ & \quad \times \operatorname{sgn}(y_i - y_{id}) - (y_i - y_{id})^T \dot{y}_{id} \\ & \quad \left. + \dot{y}_{id} (y_i - y_{id})^T \operatorname{sgn}(y_i - y_{id}) \right\} \\ & \leq \sum_{i=1}^N \frac{1}{\|y_i - y_{id}\|} \\ & \quad \times \left\{ \|(y_i - y_{id})^T (A_{i21} + \Gamma_{i21}(\cdot)) z_{i1}\| - \|A_{i21}\| \right. \\ & \quad + \Gamma_{i21}(\cdot) \chi_i \|y_i - y_{id}\| \\ & \quad + \|(y_i - y_{id})^T (A_{i22} + \Gamma_{i22}(\cdot)) y_i\| \\ & \quad - \|(A_{i22} + \Gamma_{i22}(\cdot)) y_i\| \|y_i - y_{id}\| \\ & \quad - \|(y_i - y_{id})^T \dot{y}_{id} + \dot{y}_{id}\| \|y_i - y_{id}\| \\ & \quad + \left(\sum_{\substack{j=1 \\ j \neq i}}^N \|(y_i - y_{id})^T \|\delta_{ij2}(z_{j1}, y_j)\| \right. \\ & \quad \left. + \|(y_i - y_{id})^T \|B_{i2} \Delta G_i(\cdot)\| \right) \\ & \quad \left. - (K_i(y_i) + \|B_{i2}\| \rho_i(y_i)) \|y_i - y_{id}\| \right\} \\ & \leq \sum_{i=1}^N (-\|B_{i2}\| \rho_i(y_i) \\ & \quad + \|B_{i2}\| \|\Delta G_i(T_i^{-1} \operatorname{col}(z_{i1}, y_i))\| \\ & \quad + \sum_{i=1}^N \left(-K_i(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^N \|\delta_{ij2}(z_{j1}, y_j)\| \right) \end{aligned} \quad (44)$$

where the fact that (see Lemma 1 in Yan and Edwards (2008))

$$\begin{aligned} & (y_i - y_{id})^T \operatorname{sgn}(y_i - y_{id}) \\ & \leq \|y_i - y_{id}\|, \quad i = 1, 2, \dots, N \end{aligned}$$

is employed above.

It is clear to see from (35) that in the considered domain Ω

$$\begin{aligned} & (A_{i21} + \Gamma_{i21}(\cdot)) z_{i1} \\ & \leq \|A_{i21} + \Gamma_{i21}(\cdot)\| \|z_{i1}\| \leq \|A_{i21} + \Gamma_{i21}\| \chi_i \end{aligned} \quad (45)$$

for $i = 1, 2, \dots, N$. From (37), (17) and Assumption 3, it follows that

$$\begin{aligned} & \|\delta_{ij2}(z_{j1}, y_j)\| \\ & \leq \|H_{ij}(x_j)\| \leq \|T_i\| \|H_{ij}(x_j)\| \leq \|T_i\| \|\eta_{ij}(x_j)\| \end{aligned} \quad (46)$$

$$\begin{aligned} & \|B_{i2} \Delta G_i (T_i^{-1} \text{col}(z_{i1}, y_i))\| \\ & \leq \|B_{i2}\| \|\Delta G_i (T_i^{-1} \text{col}(z_{i1}, y_i))\| \leq \|B_{i2}\| \rho(y_i) \end{aligned} \quad (47)$$

Substituting the inequalities (46)–(47) into (44) yields

$$\begin{aligned} & \sum_{i=1}^N \frac{(e_i)^T \dot{e}_i}{\|e_i\|} \\ & \leq \sum_{i=1}^N \left(\left(\|B_{i2}\| \rho_i(y_i) - \|B_{i2}\| \rho_i(y_i) \right. \right. \\ & \quad \left. \left. - K_i(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^N \|\delta_{ij2}(z_{j1}, y_j)\| \right) \right) \\ & \leq - \left(\sum_{i=1}^N K_i(y_i) + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \|T_i\| \|\eta_{ij}(x_j)\| \right) \end{aligned} \quad (48)$$

As the control gains $K_i(y_i)$ for $i = 1, 2, \dots, N$ satisfy (41), it follows that, in the domain Ω , the reachability condition (36) is satisfied. Hence, the result follows. ■

From the proof above, it is easy to observe the motivation of the control design. Actually, the term $\|A_{i21} + \Gamma_{i21}(y_i)\| \chi_i$ is employed to reject the terms involving unknown variable z_{i1} , the term $(A_{i22} + \Gamma_{i22}(y_i))y_i$ is used to cancel the known terms in the isolated subsystems, the term $\|B_{i2}\| \rho_i(y_i)$ is used to deal with the effect of the matched uncertainties, and the control gain $K_i(y_i)$ is mainly used to deal with the interconnections.

Remark 7: It is clear to see that the control law u_i designed in (40) is only related to the output of the i th subsystem, y_i , the desired output signal y_{id} and its derivative \dot{y}_{id} for $i, j = 1, 2, \dots, N$. Thus, it is a decentralised output feedback control. Also, the control u_i has employed the bounds of uncertainties to

cancel/reduce the effects of uncertainties to enhance the robustness.

Remark 8: Theorem 5.1 shows that the sliding mode dynamics is uniformly-ultimately bounded. Theorem 6.1 shows that the designed control (40) can drive the system to the designed sliding surface (22). Theorems 5.1 and 6.1 together guarantee the uniformly-ultimately bounded stability of the closed loop system formed by applying the control (40) to the system (20)–(21), irrespective of the uncertainties and the interconnections within the subsystems.

Remark 9: The partition (25) results in (26) in which the known term $\Gamma_{i12}(y_{id})y_{id}$ is separated to reduce the conservatism in the sliding motion stability analysis. The partition (38) results in (39) in which the known term $\Gamma_{i22}(y_i)y_i$ is separated which can be cancelled by the designed controller to enhance the robustness in reachability analysis.

7. Simulation example

In order to illustrate the developed methodology above and demonstrate the performance of the system control law designed in Section 6 the proposed algorithm is applied to a river pollution consisting of regions as shown in Figure 1 (see, Jamshidi (1996)). It should be pointed out that the water quality of a river is mainly dependent upon the concentrations of oxygen and pollutants. In a simplified manner, this problem can be stated as the task of controlling pollutants discharged at different places along the river in such a way that the river pollution remains within a given tolerance. Then, the river pollution system can be modelled by (see, e.g. Ding et al., 2023; Jamshidi, 1996).

$$\dot{x}_1 = \underbrace{\begin{bmatrix} -1.32 & 0 \\ -0.32 & -1.2 \end{bmatrix}}_{A_1} x_1 + \underbrace{\begin{bmatrix} 0 \\ 0.1 \end{bmatrix}}_{B_1} (u_1 + \Delta G_1(x_1)) \quad (49)$$

$$y_1 = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_1} x_1 \quad (50)$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} -1.32 & 0 \\ -0.32 & -1.2 \end{bmatrix}}_{A_2} x_2 + \underbrace{\begin{bmatrix} 0.1 \\ 0 \end{bmatrix}}_{B_2} (u_2 + \Delta G_2(x_2))$$

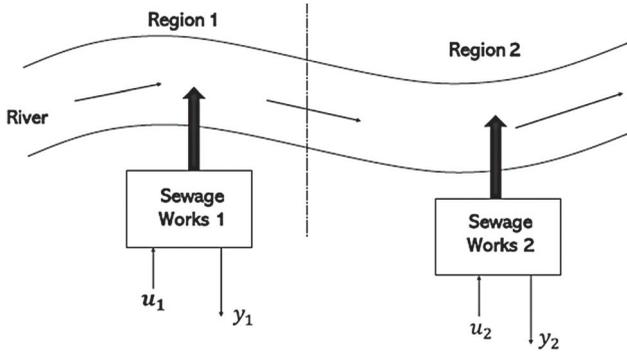


Figure 1. Diagram of the River Pollution.

$$+ \underbrace{\begin{bmatrix} 0 & 0.9 \\ 0.9 & 0 \end{bmatrix}}_{H_{21}} x_1 \quad (51)$$

$$y_2 = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C_2} x_2 \quad (52)$$

where $x_1 = \text{col}(x_{11}, x_{12})$, $x_2 = \text{col}(x_{21}, x_{22})$ and $x = \text{col}(x_1, x_2)$. The variables x_{i1} and x_{i2} represent the concentration of biochemical oxygen demand and the concentration of dissolved oxygen of the region, respectively, and the control u_i are the biochemical oxygen demand of the effluent discharge into the river, ΔG_i represent any matched uncertainties and $H_{ij}(x_j)$ represent interconnections respectively for $j \neq i$, $i, j = 1, 2$. It is assumed that the concentration of biochemical oxygen demand for the two regions are measurable.

It is clear to see that there is not any nonlinear term $F_i(x_1)$ appearing in (49) and (51) when compared with system (7)–(8). Thus, Assumption 2 is not required. It is straightforward to verify that Assumption 1 is satisfied. The coordinate transformation matrices T_i are chosen as ($z_i = T_i x_i$) for $i = 1, 2$

$$T_1 = T_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (53)$$

Then, in the new coordinates $z = \text{col}(z_1, z_2)$ where $z_1 = \text{col}(z_{11}, z_{12})$ and $z_2 = \text{col}(z_{21}, z_{22})$, system (49)–(52) can be described in the regular form in (14)–(16) with

$$\begin{bmatrix} A_{111} & A_{112} \\ A_{121} & A_{122} \end{bmatrix} = \begin{bmatrix} -1.2 & -0.32 \\ 0 & -1.32 \end{bmatrix}, \quad \begin{bmatrix} \Gamma_{11}(\cdot) \\ \Gamma_{12}(\cdot) \end{bmatrix} = 0, \\ B_{12} = 0.1, \quad \begin{bmatrix} \delta_{11}(\cdot) \\ \delta_{12}(\cdot) \end{bmatrix} = 0 \quad (54)$$

$$\begin{bmatrix} A_{211} & A_{212} \\ A_{221} & A_{222} \end{bmatrix} = \begin{bmatrix} -1.2 & -0.32 \\ 0 & -1.32 \end{bmatrix}, \quad \begin{bmatrix} \Gamma_{21}(\cdot) \\ \Gamma_{22}(\cdot) \end{bmatrix} = 0, \\ B_{22} = 0.1, \quad \begin{bmatrix} \delta_{21}(\cdot) \\ \delta_{22}(\cdot) \end{bmatrix} = \begin{bmatrix} 0.9z_{12} \\ 0.9z_{11} \end{bmatrix} \quad (55)$$

Now, consider the system in the domain

$$\Omega = \{(z_{11}, z_{21}) \mid \|z_{11}\| < 0.04, \text{ and } \|z_{21}\| < 0.04\} \quad (56)$$

and the matched uncertainties $\Delta G_1(\cdot)$ and $\Delta G_2(\cdot)$ are added to the system in order to illustrate the results obtained which is assumed to satisfy

$$\|\Delta G_1(\cdot)\| \leq |5.2y_1|, \quad \|\Delta G_2(\cdot)\| \leq |\cos^2(y_2)|$$

By direct calculation,

$$a_{11} = a_{12} = a_{21} = a_{22} = \beta_{11} = \beta_{22} = \beta_{12} = 0, \\ \beta_{21} = 0.9$$

It is easy to verify that Assumption 3 is satisfied. The sliding mode dynamics are derived as

$$\dot{z}_{11} = -1.2z_{11} - 0.32y_{1d} \quad (57)$$

$$\dot{z}_{21} = -1.2z_{21} - 0.32y_{2d} + 0.9\|z_1\| \quad (58)$$

The initial states are chosen as $z_1(0) = \text{col}(0, 1)$ and $z_2(0) = \text{col}(0, 0)$, and the desired output signals y_{id} are set as

$$y_{1d} = 3e^{-0.5t}, \quad y_{2d} = \frac{7}{6}e^{-0.5t} + 1$$

From $A_{111} = A_{211} = -1.2$ and by solving the Lyapunov equation in (13), it is obtained that $P_1 = P_2 = 0.416$. The matrix W defined in (27) can be calculated directly. It is straightforward to verify that $W^T + W > 0$. Thus the conditions in Theorem 5.1 is satisfied. By further calculation, the proposed decentralised static output sliding mode control law can be given by

$$u_1 = -10 \{1.32y_1 + (0.1(|5.2y_1|) + 1) - 1.5e^{-0.5t}\} \\ \times \text{sgn}(y_i - y_{id}) \quad (59)$$

$$u_2 = -10 \left\{ 1.32y_2 + (0.1(|\cos^2(y_2)|) + 1) \right. \\ \left. - 2 \left(\frac{7}{6}e^{-0.5t} + 1 \right) \right\} \text{sgn}(y_i - y_{id}) \quad (60)$$

the control gains are chosen as $K_1(y_1) = K_2(y_2) = 1$. It is clear to see that the control (59)–(60) above are in decentralised format.

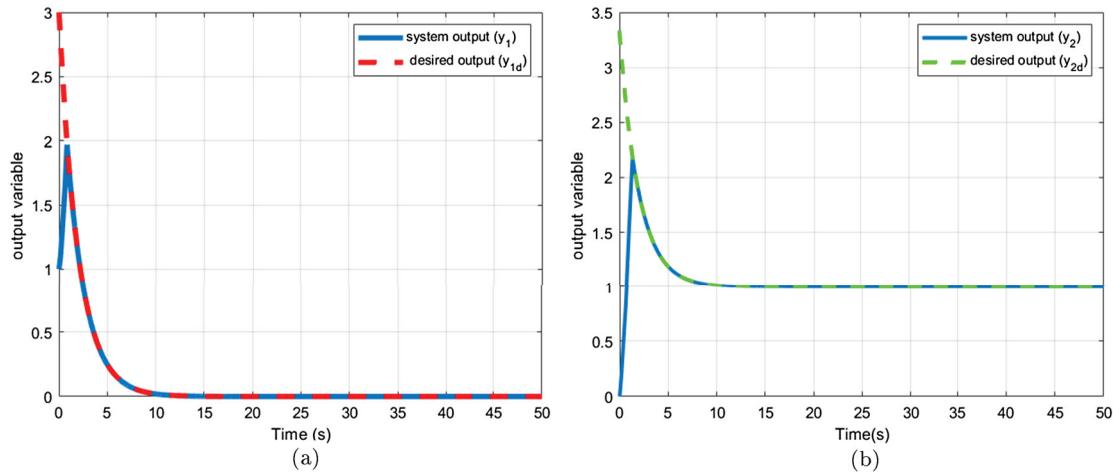


Figure 2. Time responses of the controlled system outputs and the desired outputs. (a) Subsystem 1 output and the desired output response. (b) Subsystem 2 output and the desired output response.

For simulation purposes, the function

$$\frac{(y_i - y_{id})}{\|y_i - y_{id}\| + \epsilon_i}$$

is used to approximate the discontinuous function $\text{sgn}(y_i - y_{id})$ for $i = 1, 2$ to avoid chattering where the smoothing constant ϵ_i is taken as 0.005 is a small positive constant. Figure 2(a,b) show that the system outputs can track the corresponding desired output signals very well. The simulation results show the effectiveness of the proposed methodology.

8. Conclusion

The paper has proposed a sliding mode control strategy to tackle the output feedback tracking control of a class of large-scale nonlinear interconnected systems. The sliding surface is first synthesised. Then, a decentralised output feedback control strategy was developed, accompanied by the formulation to satisfy the reachability condition tailored for large-scale systems. This newly proposed controller demonstrates enhanced robust performance capabilities, particularly in handling uncertainties and system interconnections using static output feedback for tracking. The decentralised output feedback tracking control technique presented in this work was applied to water quality control. to demonstrate the effectiveness of the approach through simulation.

Disclosure statement

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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