CAUSES AND CONSEQUENCES OF INEQUALITY IN MACROECONOMICS

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Abstract

Interests in inequality in macroeconomics has been rapidly growing, due to its deepening in the reality. Bewley-Hugget-Aiyagari model is a classical model in this study. Despite being innovative with realistic assumptions such as idiosyncratic shock and borrowing constraint, it also has drawback such that it does not generate high enough wealth inequality under reasonable calibrations. Also, since Krusell and Smith (1998) argued, there has been a a consensus that the aggregates are the most important rather than heterogeneity. However, as lots of research have followed to overcome the model's limitations there are now a large number of heterogeneous agent papers, investigating the causes of inequality as well as the effect of fiscal and monetary policy.

This thesis stands in line with these studies, exploring topics regarding the causes and consequences of inequality. First chapter suggests loss aversion preference as a sources of high wealth inequality. Using Aiyagari type heterogeneous agents dynamic stochastic general equilibrium (DSGE) model, it shows that with loss aversion, agents have an extra motive for precautionary savings especially among the wealthier agents and hence, it affects the upper tail of the wealth distribution. Quantitative analysis shows that, under standard parameterisation of the utility function, the model achieves a significant increase in the wealth inequality and the savings rate along with the degree of loss aversion. Higher kurtosis and positive skewness suggest a thicker upper right tail, implying more wealth concentration among wealthier agents. The shape of distribution in some Cumulative Density Function (CDF) and Probability Density Function (PDF) figures also imply that loss aversion can generate a bimodal distribution, with rather less inequality at the very top between 0.1 percent and 1 percent, as implied by Inverted Pareto coefficients.

The second chapter assumes that there is ex-ante inequality in the income and wealth. Differentiating two household types as in existing Two-Agent New Keynesian (TANK) models, it analytically shows that the supply side inflation from the essential sector can shrink the overall demand and aggravate the inequality, as we recently observed. By differentiating essential and normal consumption goods under quasi-linear utility function, it leads to deriving the threshold income as a function of essential goods price. By this threshold income, households are split into two types, i.e., the rich (unconstrained) and the poor (constrained). Hence, household fraction is determined by the threshold income, rather than being an exogenous probability parameter, it is now changeable by the essential price. And it suggests a new methodology for inequality analysis. Having firms in both essential and normal sectors to follow the standard New Keynesian (NK) model with price rigidity, this chapter discovers that the essential mark-up shock, among other shocks, contracts the overall consumption, raises up the fraction of the poor households, and can increase consumption and income gap.

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Chapter 1

Introduction

As inequality has deepened, the importance of developing an analytical framework that both theoretically studies this phenomenon and reflects it in macroeconomic policy has emerged. As a result, there has been a rising literature in this area for the past 30 years, and now it has become a part of the mainstream. With heterogeneity, the channel through which the policy affects the macroeconomic variables differs, then the outcomes, which affect the inequality again. This implies that there is the need for alternative policy rules taking into account the heterogeneity, or inequality in this context.

Along with this background, HANK models have been developed from the classical Bewley-Huggett-Aiyagari model, which assumes idiosyncratic shock and borrowing constraint with continuous time framework, to show how the transmission channels of monetary policy can differ from the Rational Agent New Keynesian (RANK) model. However, their computational heaviness, methodological complexity, and lack of analytical exposition have made them rather difficult to apply or utilise. As an alternative, TANK models which exogenously assumes a fraction of constrained households from unconstrained, i.e., hit by borrowing constraint to remain as hand-to-mouth agents, have gained attention. There are active research on alternative policy rules within these frameworks as well.

This thesis has been written with intention to contribute to this flow of knowledge. In particular, it aims to extend existing models by filling in certain gaps that have been overlooked in previous research with analytical perspective. It is well known that when idiosyncratic shock exists and consumption is not fully insured due to market friction such as borrowing constraint, wealth distribution is generated. However, as households further away from the borrowing constraint are supposed to have less precautionary savings, there is a limitation in the degree of wealth inequality generated, weaker than observed in reality. To attribute this to such as the asset return shock seems insufficient in terms of persistence and magnitude. Instead, I find the reason in a strong motive for smooth consumption over infinite time (over generations) which would serve as a significant source of precautionsry savings motive. In other words, I explicitly consider the loss aversion in the model. Secondly, in classifying households into two types (the poor and the rich) depending on whether they are binding constraint or not, there have not been yet explanation in existing TANK papers about why and to what extent hand-to-mouth poor households are vulnerable to economic shocks. I pay attention to the essential goods as a means to address those questions; it has low price elasticity, so for the poor the impact of its price change on the consumption and welfare is quite direct, whereas the rich are able to mitigate it by adjusting other normal consumption or savings decision. Based on this observation, I show that by exploiting the properties of quasi-linear utility function, the fraction of the households can be expressed as a function of essential price.

In addition, this thesis seeks to overcome the methodological challenges imposed by such extended models, and to provide policy implications. By including loss aversion preferences, I derive the conditions under which the general equilibrium exists, and incorporated these conditions and an additional state variables to complete the computation. Furthermore, I show how the fraction changes by the change of income distribution, which is assumed to be exogenously given, so that the transition occurs between the rich and the poor. They are incorporated into aggregation and log-linearisation, so that the individual policy function and income are allowed to differ between the agents in the model.

At the same time, with the view that the economic research must contribute not only to the theoretical progress, methodological advancement, and policy implications but also to the explanation of the real-world problems, the specific topics are chosen to be topical: Why does wealth inequality arise so largely? How do different types of households and goods sectors respond differently when supply-side shocks affect the prices?

In fact, heterogeneity is the concept used not only for inequality but also for other topics such as firm heterogeneity, but in this thesis it is understood only in the context of inequality. Then why is it titled ”Causes and Consequences of Inequality”? With regard to causes, research has focused on how to elaborate idiosyncratic shock, by heterogeneous asset return, discount rate, wider earnings series, or occupational heterogeneity. However, in the pursuit of a more fundamental cause, I paid attention to the need to establish a link with bahavioural economics, i.e., loss aversion. With respect to the consequences, while the literature from HANK to TANK mainly focus on the transmission channel of the policy, I sought to show that once the heterogeneity is assumed and inequality is generated, even well-known NK shock can exacerbate the aggregate and distributional variables.

Chapter 2 demonstrates that when loss aversion preferences is included in the utility function of households in the traditional Aiyagari model, wealth inequality close to the observed data can be generated at the stationary general equilibrium, through inter-temporal decision even without large earnings inequality. It highlights several meaningful distributional features as well as aggregate features. In particular, it shows that even agents farther away from the borrowing constraint have a strong precautionary savings motive due to loss aversion. This is the main contribution of this chapter, as it identifies features that could not be explained by the Aiyagari model. It also corresponds with the empirical findings that small differences in savings rates lead to very large wealth inequality in the long term. Its polarised wealth distribution also provides a foundation for TANK model which follows in the next chapter. In terms of methodology, this chapter derives more structural and endogenous conclusions than simply expanding the shock process. To do it, it extensively examines loss aversion functions and their modelling, and derives theoretical conditions under which general equilibrium exists when loss aversion preference is incorporated into the utility function.

Chapter 3 analytically demonstrates that in the TANK model with two types of households and two types of goods, supply-side shock to the essential price can aggravate both aggregate demand and inequality. The decline in demand comes from the reduced essential consumption. Noting that the normal goods sector are the same as RANK model in terms of modelling, the responses from the essential sector are the main contribution of this chapter; different characteristics of different sectors, when combined with the heterogeneity in of households, generate the difference in macroeconomic implications. For example, the poor can consume only the essential goods whereas the rich do so only until they reach a certain threshold. This chapter then analytically derives conditions under which the consumption, income and wealth inequality worsen. These results also entail methodological innovation, such as the active use of properties of quasi-linear utility function. In particular, I derived the threshold income by which households, who prioritise essential goods over normal goods in general, no longer consume essential goods. In addition, based on that this is a function of essential price and exogenously given income distribution, the fraction of households types are endogenised. This is a modelling contribution in the sense that it endogenises a parameter previously assumed as exogenous. It suggests that the policy implication should take into account the fraction of each households type in each sector and behavioural differences.

In addition, apart from both chapters answer the specific questions by modifying existing models from the perspectives of causes and consequences of inequality, they share following common features. First, both chapters expand microfoundations of heterogeneous agent models by introducing behavioural elements into the utility function. Chapter 2 emphasises loss aversion, and the Chapter 3 uses quasi-linear utility function, in order to analyse how behavioural features that are not captured by standard CRRA preference structure, can alter the outcomes in macroeconomic variables as well as inequality. Second, related to above, both chapters focus on inter-temporal savings decision rather than labour earnings or labour market dynamics. It is not only to simplify the channels in the model, but mainly because I have more interest in the different outcomes when the households' policy functions are only available in consumption-savings decision. Finally, both chapters offer new perspectives on policy implications. The results emphasise the importance of initial savings rates, and suggest that the attention should be paid not only to aggregate inflation but also to managing the sectoral price levels, particularly that of essential goods. Although the thesis does not compare fiscal and monetary policy effects in detail, it should point to a new direction to consider.

Chapter 2

Loss Aversion and Wealth Inequality

2.1 Introduction

It is well known that wealth inequality is high and, by some measures, has increased in recent decades. The recent World Inequality Report 2022 (Chancel et al. 2022) shows that, in 2021, the global top 10 percent of the wealthiest owned more than 75.6 percent of the total wealth, and the top 1 percent of the wealthiest owned 37.8 percent of the total wealth. The bottom 50 percent only owned less than 2 percent during the same period. The wealth concentration has been exceeding income since the 1980's, during which the per-adult average income increased by 1.3 percent a year between 1987 and 2017, while the per-adult average wealth increased by 1.9 percent. (Alvaredo et al. 2017) This is because the growth rate of wealth increased by the size of wealth. For example, its growth rate for the top 1 percent was 2.6 percent while for the top 0.1 it was 3.5 percent, and for the top 0.01 percent it was 4.7 percent for the same period. (Alvaredo et al. 2017) Chancel et al. 2022 emphasises the extreme growth of the very top of the wealth distribution, especially during the Covid-19 pandemic. They have also shown that inequality is a global phenomenon at all levels of economic development in many countries.

It will ultimately be impossible for any economy to sustain with increasing inequality, although the speed and the level would vary. Furthermore, with increasing political concerns, governments would seek an appropriate analytical framework to tackle inequality and implement policies to address it. However, there is not yet a widely agreed model to explain the cause of inequality, although a lot of papers have emerged to address it. To explore the literature in this field, the canonical starting point is a Bewley-type heterogeneous agent DSGE model.(Bewley 1983) This is the first macroeconomic paper to induce the distribution out of the aggregate variables. Huggett (1993) and Aiyagari (1994) both used innovative strategy to apply a stochastic component to the households problem, i.e. the idiosyncratic risk. In addition, they introduced the borrowing constraint as the lower bound of the asset. However, the limitation was that with only these ingredients, the model could not generate wealth distribution and precautionary savings that matched the real data. In particular, under the reasonable earnings process calibration, it failed to achieve the thick upper right tail observed in the empirical data. Many following studies have attempted to solve these problems, as is discussed in detail in Section 2.2.

In line with this research using heterodeneous agent DSGE models, this chapter focuses on the behavior of households, more specifically loss aversion. This originated in the prospect theory, by which the agents' preference is determined not only by the absolute level of consumption, but also by the relative gain or loss from the reference point (Kahneman and Tversky 1979). Loss aversion postulates an asymmetric structure in its preference so that agents dislike the loss more than they like the gain. Its intuition for the wealth inequality in the heterogeneous

DSGE model is as follows: Agents, in the face of idiosyncratic shocks and being subject to borrowing constraint, do not like to make a loss, either in their consumption or the utility of it. As a result, there is an additional precautionary saving motive to the original Aiyagari model. Moreover, if this motive increased disproportionately, thus more among the wealthier, there would be more precautionary savings among them than the wealth poor, causing even thicker right tail in the wealth distribution. This is in contrast to the existing interpretation of the relatively thin right tail found in the Aiyagari model, as it is regarded that the precautionary saving motive disappears for wealthy people positioned far away from the borrowing constraint. Controlling other factors that might change the wealth inequality such as earnings inequality and the discount rate, this chapter found that such behavioural factor of loss aversion effectively gives rise to more precautionary savings especially among the wealthier, thus an even larger accumulation of wealth. The poor, on the other hand, are drawn to the left, as a result of loss aversion in consumption, losing more assets. And this result was obtained under the necessary theoretical conditions for the existence of general equilibrium, which still hold under the utility function with loss aversion and the Markov chain idiosyncratic shock process. However, at the equilibrium, the interest rate turned out not to be lower and the savings rate also did not meaningfully increase, suggesting there were not much changes in the aggregate features. Furthermore, a possible correlation between risk aversion and loss aversion in generating larger inequalities was found in the computational results. The structure is as follows: Section 2.2 presents the Literature Review, Section 2.3 introduces the Data, Section 2.4 describes the model and analyses the equilibrium, Section 2.5 covers the computational strategy, Section 2.6 presents and discusses the results, and Section 2.7 concludes the work.

2.2 Literature Review

In macroeconomics, inequality has been addressed within the class of heterogeneous DSGE models. The heterogeneous model was first developed by Bewley (1983), and then used by Huggett (1993) and Aiyagari (1994) to generate the distribution. They show that when the financial market is incomplete and agents face idiosyncratic shock and borrowing constraint, agents accumulate wealth for precautionary motives and then generate wealth distribution in the steady state equilibrium as a result. In addition, the aggregate savings rate, as well as wealth accumulation, becomes greater than in the complete market. However, as Aiyagari himself was aware, under reasonable calibration on the earnings process, wealth distribution does not show high enough inequality in the data, especially in the upper right tail. Huggett (1993) induces the wealth distribution in solving the households' problem with upper and lower bounds. The existence of upper limit is required for the stationary distribution. On the other hand, Aiyagari does not postulate anything about the upper limit of the asset; [1] however, the wealth distribution does not feature the wealth accumulation among the wealthiest. Krusell and Smith (1998) emphasise why the heterogeneous agent model is important compared to the representative agent model, analysing how the aggregate variable is related to the distributive results. However, in their approximate aggregation, they argue at the end that only the mean of the wealth distribution and the aggregate productivity shock matter. A mismatch between the Aiyagari model and the data is also noted in their paper, and their solution is to exogenously input the heterogeneous discount rates alongside the stochastic process with transition probability.

Some empirical studies have carefully matched the distribution of wealth to

[1] He rather emphasises the condition under which the cross-sectional average asset goes to infinity. The way he ensures the steady state is to use a bisection method when updating the return rate.

the skewed earning out of the stochastic process, as suggested by Aiyagari (1994). However, the top right tail of earnings was required to be much thicker to match the chosen moments of wealth distribution. For example, Castaneda, Diaz-Gimenez, and J. Rios-Rull (2003) estimated the properties of the top earnings in their OLG model and found that the top 0.039 percent of earners have around 1,000 times the average labour endowment of the bottom 61 percent, while this ratio is about 400-450 in the World Wealth and Income Database (WWID) by Alvaredo et al. (2017). Diaz, Pijoan-Mas, and Rios-Rull (2002) estimated that the top 6 percent earn more than 40 times the labour earnings of the bottom 50 percent, while 5 percent of households in the WWID earn about 20 times the average earnings of the bottom 50 percent. With respect to this mismatch, Stachurski and A. A. Toda (2019), in their theoretical paper, has proven that under canonical assumptions and with CRRA preferences in the original Aiyagari model it is impossible to generate thicker right tail in the wealth distribution than the income because the wealth distribution inherits the income.

A large body of literature has attempted to solve this problem, for example by modifying the earnings/income process, or the financial market, implementing a wider range of returns to assets, or modifying the utility function, etc. As mentioned above, Krusell and Smith (1998) have suggested choosing the random values of discount factors so that they meet certain conditions in the empirics. Castaneda, Diaz-Gimenez, and J. Rios-Rull (2003) have investigated the earnings process, and Cagetti and De Nardi (2006) and Quadrini and J. Rios-Rull (2015) have studied the impact of heterogeneous occupation. On the other hand, Benhabib, Bisin, and Zhu (2011), Benhabib, Bisin, and Zhu (2015), Benhabib and Bisin (2018), Jones (2015), and Acemoglu and Robinson (2015) have scrutinised the Pareto distribution, which is regarded to be largely related to fat-tailed income and wealth distribution. Among those Gabaix et al. (2016) suggested analytical measurement of the transition speed to the fat tail, taking type dependence and scale dependence as reasons for why random growth theory did not incarnate the rapid concentration on the top. Type dependence refers to the presence of high income growth types, and scale dependence means that heterogeneous amplifiers determinie high incomes among highly talented agents(e.g. CEO), as the firms' value added is amplified by the CEO effect.

With respect to financial/capital market modification, Benhabib, Bisin, and Zhu (2015) implemented the idiosyncratic capital income risk process and argued it complements their previous papers, which focused on overlapping generation economies with bequest (Benhabib, Bisin, and Zhu (2011), Benhabib, Bisin, and Zhu (2013)). Hubmer, P. Krusell, and Smith (2016) suggested transitory shock to earnings as well as an exogenous Pareto-shaped tail in earnings. Mengus and Pancrazi (2015) suggested the "partial-insurance equilibrium" to explain the relationship between income and wealth inequality in theory. Setting up the Aiyagari (1994) economy with idiosyncratic shock, they assumed that there is an insurance market in which agents first decide whether to participate or not, depending on the expected gains as a function of their wealth and the fixed participation cost. Based on some properties that characterise this economy, they formed recursive problems leading to partial-insurance equilibrium. They concluded that poor households are not able to obtain any insurance, middle-class households actively participate in the contingent asset market to be fully insured, and most interestingly, rich households prefer to self-insure by accumulating a large stock of risk-free asset.

Benhabib and Bisin (2018) found in their literature review that skewed earnings with stochastic returns to wealth is the most important for the wealth accumulation. They also explored empirical evidences in the literature, and found that earnings has a thinner right tail than wealth, there is a positive relationship between the rate of return to wealth and the size of wealth, and the idiosyncratic component of capital return is composed of principal residence, unincorporated private business equity, and private equity.

What we can see from the literature review is that nearly all research tried to exogenously add the extra heterogeneity to the existing Aiyagari model as a means of modification. In more interest in finding the cause of high wealth inequality that can endogenously generate, I focused on the households' behaviour, specifically loss aversion. Loss aversion was firstly developed by Kahneman and Tversky (1979) as a part of prospect theory, who asserted that agents take utility compared to the reference point, and the disutility of the loss is larger than the utility of the gain. Hence, the utility function is derived to have an s-shape, piecewise centre at the reference point, with slopes in the local region of loss being much steeper than those in the region of gain. There has been application of the loss aversion to macroeconomics, mainly in financial economics. Berberis, Huang, and Santos (2001) explained high mean and volatility of stock market by the loss aversion in the investment based on the past investment as the reference point. Pagel (2016) later derived the similar result in which higher equity premium was generated by the loss aversion in the consumption based on the expectation of future consumption, following Koszegi and M. Rabin (2006, 2007, 2009).

However, to the best of my knowledge, no papers have worked on the heterogeneous DSGE model with loss aversion, although there are some papers that have modified the utility function in other ways. For example, Diaz, Pijoan-Mas, and Rios-Rull (2002) investigated how internal habit formation under idiosyncratic income shock affects precautionary savings and wealth distribution, which unfortunately turned out to reduce inequality. Caballe and Moro-Egido (2014) implemented aspiration and habit in their model with bequest, to see the wealth distribution under idiosyncratic income shock. Angelopoulos, Lazarakis, and Malley (2019) studied the effect of peer pressure among classified occupations on inequalities in the UK. Both Caballe and Moro-Egido (2014) and Angelopoulos, Lazarakis, and Malley (2019) have shown that the positive impact of aspiration on the wealth ineuqlaity, but it was not as exponential as needed.

This chapter is positioned, along with the vast literature in this class of studies, to suggest that loss aversion plays a role in generating the thick-tailed wealth distribution.

2.3 Data

As briefly introduced in Section 2.1, the wealth inequality among households has worsened in many aspects across the world. First, according to Chancel et al. (2022), there have been significant increase in the share of wealth by the wealthy people since 1995. As shown in Table 1, in 2021, the top 1 percent owned 37.8 percent of the wealth in the world, top 10 percent owned 75.6 percent, the middle 40 percent owned 22.4 percent, and the bottom 50 percent owned only 2 percent. While half of global population owned nearly nothing, the top 10 percent own nearly three quarters of it. It is because the wealth at the very top of the distribution has grown faster than the average. Between 1995-2021, the average wealth of top 0.01 percent grew by 5 percent and top 0.1 percent by 4 percent, whereas top 1 percent grew by 3.2 percent, and top 10 percent by 3 percent. On the other hand, the average wealth of the middle 40 percent increased by 3.8 percent, and the bottom 50 percent by 3.7 percent. The increase in the wealth inequality has been driven by the rapid increase of the top 1 percent share.

Table 1: Global Distribution of Wealth

Quantile	Share in total Wealth (%)	Average Wealth (per adult, euro)	Threshold
Full population	100	72,913	
Bottom 50%	2.0	2,908	
Middle 40%	22.4	40,919	11,954
Top 10%	75.6	550,920	124,876
Top 1%	37.8	2.8M	893,338

(**Source** : Extract from Chancel et al. (2022))

The wealth inequality also has been exceeding the income inequality. According to Chancel et al. (2022), the global top 1 percent share of income has increased since 1980's and was 21 percent in 2020, while the bottom 50 percent income share as well as top 0.1 percent were about 8 percent. It implies that the wealth concentration has been processed faster than the income. In addition, one of their diagram shows an elephant curve of the average wealth growth rate from 1995-2021 against each wealth quantile, where we can see its decline between the top 1 and 30 percent(Chancel et al. 2022, Figure 4.2). It shows that despite the regional difference, the middle wealth group, especially in rich countries, has squeezed for the period. And such regional difference was notable between the U.S and the Europe, as the wealth share of middle 40 percent in the U.S dropped from 34 percent in 1980 to 28 percent in 2020 while in Europe they remained around 40 percent.

Although worsening wealth inequality is a global phenomenon today, its trajectory and extent should be different between countries. The U.S has been the most prominent in terms of recent aggravation, with the reduction in the middle wealth group discussed above, as well as rising top wealth share (35 percent in 2020, compared to about 22 percent in Europe), while the bottom 50 percent share has remained low(Chancel et al. 2022). Kuhn and J. Rios-Rull (2020) produced values

Table 2: Inequality in the U.S (2019)

Measurement	Earnings	Income	Wealth
Coefficient of Variation	2.57	4.83	7.52
Variance of logs	1.51	1.02	5.04
Gini Coefficient	0.65	0.57	0.85

(**Source** : Extract from Kuhn and J. Rios-Rull (2020))

of various measures of inequality in the U.S for 2019, using Survey of Consumer Finances data, including coefficient of variation, variance of logs and Gini coefficient. The coefficient of variation is sensitive to the right tail of the distribution, whereas the variance of logs is sensitive to the left tail of the distribution. Gini coefficient is sensitive to the change around the median. Table 2 summarises their values, which all show how wealth is largely concentrated compared to income or earnings. And Gini coefficients in Table 2 are the moments this chapter will aim to replicate, to show how loss aversion is effective in generating the wealth inequality with reasonable income inequality (Section 2.6.5).

Both WID reports (Chancel et al. (2022) and Alvaredo et al. (2017)) find the reasons for such high wealth inequality first in the income inequality, differentials in the savings rate and the rate of return to the investment that have size effect (so called as snowball effect), under the change of policy environment such as financial de-regulation, change of tax policy and privatisation since 1980's in the U.S and the Europe. Among them, the difference in the savings rate is regarded as the major factor that has a long-term effect. Assuming the current wealth growth rate of each group, the top 0.1 percent turns out to own more than the global middle 40 percent by the end of the century.(Chancel et al. 2022)(Figure 4.6) It cannot be indefinite because under this scenario the very top group will end up taking all the private wealth. Such high savings rate among the wealthiest [2] reveals

[2] In France, the top 10 percent saved 20-30 percent of their annual income since 1970's, while

their precautionary savings motive, which is not sufficiently derived in the existing Aiyagari model. I find its cause in the loss aversion preference among households; although it is equally present in all agents, in the face of idiosyncratic shock in a frictional market, it makes the result in which only some agents accumulate the wealth by large.

2.4 Model

As introduced earlier, the research presented in this chapter modified the Aiyagari model so that it could have the utility function with loss aversion in it.

2.4.1 Households

Ex-ante, identical households want to solve the dynamic problem while having loss aversion preference compared to the reference point. That is, households determine whether the consumption they decide on today is a gain or loss using a rule they had set up to compare it to the reference point. In addition, they are assumed to dislike loss more than they would like the gain, i.e. loss aversion. To reflect this preference, the loss aversion utility function v was added to the classical constant relative risk aversion (CRRA) utility function with a weight θ so that the total utility U is composed as follows:

$$U(c_t, x_t) = (1-\theta)u(c_t) + \theta v(c_t, x_t) \tag{1}$$

$$\text{where } \; u(c_t) = \frac{c_t^{1-\mu} - 1}{1-\mu}$$

the middle 40 percent savings rate dropped from 15 percent in 1970 to 5 percent in 2012. In the U.S, while the top 1 percent savings rate has been stable, the bottom 90 percent savings rate was negative (-5 percent) in the mid-2000's before bouncing back to about 0 percent in the late 2000's.(Chancel et al. 2022)

$$x_t : \text{reference point}, \quad \mu : \text{CRRA coefficient}$$

There is the CRRA utility function re-scaled by $(1-\theta)$ and another term, $v(c_t, x_t)$ weighted by θ. $v(c_t, x_t)$ is a function of c and x for households to compare c to x and determine whether to perceive it as a gain or loss. In addition, placing $z_t = z(c_t, x_t)$ makes $v(c_t, x_t) = v(z_t)$, from which we can explain that households perceive z_t as a gain (or loss) when the result of the realised consumption is larger (or smaller) than the reference point. Then the loss aversion is performed through the utility function v, which is the loss aversion function.

Having $v(z_t)$ as the loss aversion utility function, I explored its necessary functional properties in a more formal way, as Bowman, Minehart, and Rabin (1999) proposed. [3]

A1. Monotonicity : $v(z)$ is continuous and strictly increasing for all $z \in R$, where $v(0) = 0$
A2. Diminishing Marginal Sensitivity : $v(z)$ is twice differentiable for all $z \neq 0, v''(z) \leq 0$ for all $z > 0$, and $v''(z) \geq 0$ for all $z < 0$
A3. Loss Aversion :

$$v(z_1) + v(-z_1) < v(z_2) + v(-z_2) \text{ for all } z_1 > z_2 > 0, \text{and} \lim_{z \to 0} \frac{v'(-z)}{v'(z)} = \lambda > 1$$

A2 implies that the function is concave in the positive domain (gain) and convex in the negative domain (loss). And it implies that agents are risk-averse in situations involving a sure gain, but risk-*loving* in situations involving a sure

[3] The time subscription t was eliminated in this part for clarity and convenience, without any loss of applicability in the exposition of the model in this chapter.

loss. (Bowman, Minehart, and Rabin 1999)[4] Note that it does not require twice differentiability at the origin, $z = 0$, the reference point.[5] The essential property is A3, describing its asymmetric property of the model. As it goes farther from the origin, the loss is more disliked than the gain, thus the disutility in the loss part becomes even larger. Therefore, the loss aversion coefficient λ is always greater than 1 since the disutility from the loss is larger than the utility from the gain over the same size of z. It can be presented graphically as in Figure 1. We can see that the curve as a whole is continuous and strictly increasing for all z. However, it has the concave curve on the positive side for gain ($z > 0$), and the convex curve on the negative side for loss ($z < 0$), showing its utility from the gain and disutility from the loss, both with diminishing marginal sensitivity to changes in consumption (A2). It has larger curvature on the loss because the disutility from loss is larger than the same amount of gain (A3). In addition, there is a kink at $z = 0$ due to the different slopes on two sides, derived from the loss aversion index $\lambda > 1$. [6]

Having discussed the definition and properties of loss aversion, the next step is to find the functional form that meets such conditions with respect to this. Kobberling and Wakker (2005) suggested the constant absolute risk aversion (CARA) which is an exponential utility function, specified as follows:

$$v(z) = \begin{cases} \frac{1-e^{-\eta z}}{\eta} \text{ for all } z \geq 0 \\ \lambda(\frac{e^{\phi z}-1}{\phi}) \text{ for all } z < 0 \end{cases} \tag{2}$$

[4] Note that this is risk loving behaviour associated with the loss. That is, agents dislike risky loss less than the loss for certainty.

[5] Related to this, I emphasise that A1.-A.3 are conceptually different from the assumptions needed for its solvability. (B1.-B.5 in 1.4.3.)

[6] Especially when the parameter values are the same for both functions (more discussed in Section 2.5.2.).

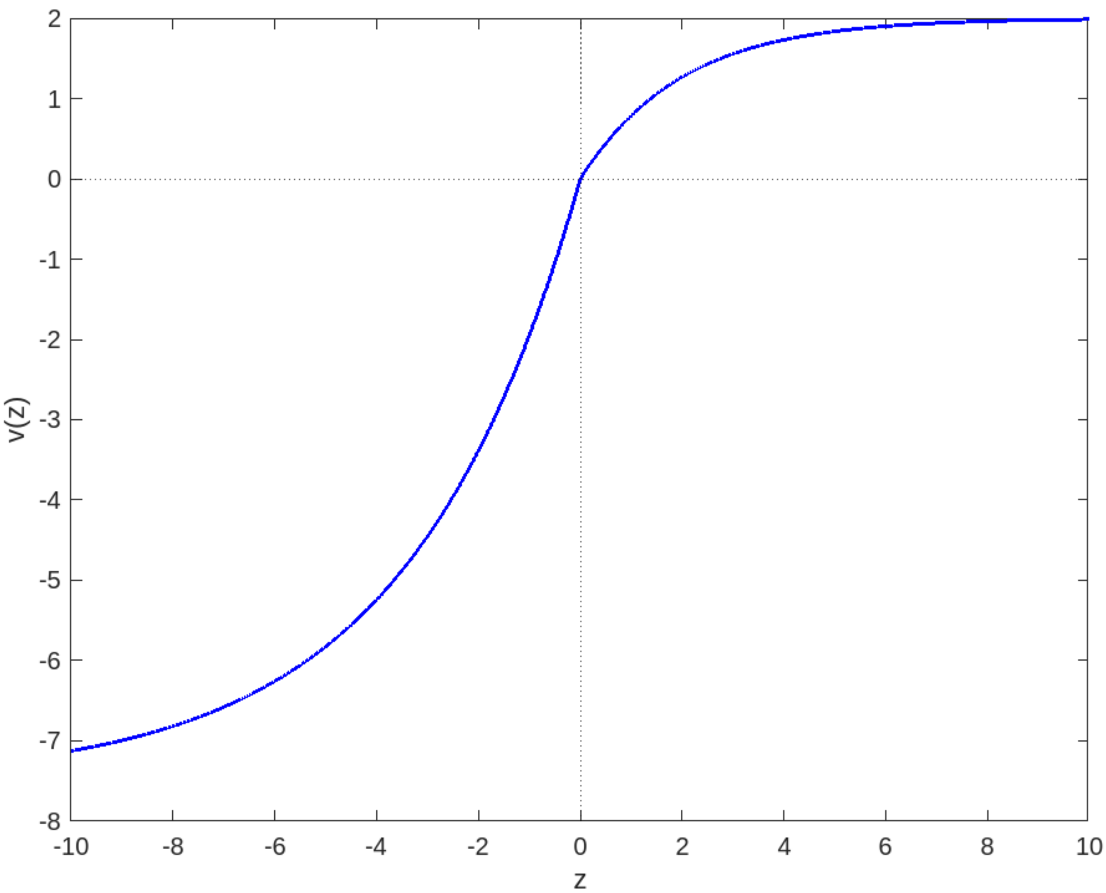


Figure 1: Loss Aversion Utility Function

where η is the controlling parameter of concavity in gain and ϕ is the controlling parameter of convexity in loss. Kobberling and Wakker (2005) argued that while the CRRA function is commonly used in macroeconomics, unless two CRRA coefficients in the gain and loss domains are the same, the utility in the gain is greater than the loss in it, which is contrast to the property needed for loss aversion (A3). [7]

To define $z_t = z(c_t, x_t)$, a function of the present consumption decision c_t and the reference point x_t, we need to first define x_t, the reference point. I let the

[7] They also referred to Rabin (2000, p. 1287) who said that CRRA utility should not be used in domains with large stakes and small stakes near 0.

past consumption as the reference point throughout this chapter, i.e. $x_t = c_{t-1}$. As recognised in Koszegi and M. Rabin (2009), this status quo is one of the most commonly used forms of reference point as well as a fixed reference point (Chen 2015). Compared to the aggregate past consumption as another status quo reference point as in the habit formation literature, it is known for its excessive sensitivity around the reference point and smoother consumption as a result, while keeping its asymmetry (Chen 2015). And this is a useful feature to generate precautionary savings. Pagel (2017), when she applied expectations-based reference point following Koszegi and M. Rabin (2006, 2007, 2009) to a dynamic macroeconomic model for the study of life-cycle consumption, also emphasised excess smoothness and sensitivity when the gain/loss is assessed compared to the expectation made in the previous period.[8] However, it is computationally more challenging to solve in heterogeneous agent general equilibrium models like the Aiyagari model; for example, while Pagel (2017) chooses i.i.d. shock for agent's expectation, the idiosyncratic shock in the Aiyagari model is the Markov chain process. Park (2023) modified Pagel (2017)'s partial equilibrium model into the general equilibrium model, but still under the state-independent shock for two periods. Although it will be contributing for future research if one can model the expectation as the reference point to follow the Markov chain process in the Aiyagari model because it may show how precautionary savings evolves alongside the reference point, taking past consumption as its proxy (expected present consumption made in the past) still achieves the smoothness of consumption needed here, while keeping the model simpler and computationally solvable.

[8]The utility function consists of the summation of three component; 1) the utility from the realised current consumption (same as the model without loss aversion), 2) the prospective gain-loss utility between the realised current consumption and the utility of expected current consumption made in the previous period, 3) the gian-loss utility over the entire future consumptions updated and discounted at the present period, compared to the expectation made in the previous period. The distribution of the reference point equals the agent's rational beliefs about consumption.

Now, we can think of several functional forms for z_t. One simple way is to take the difference of two arguments so $z_t = c_t - c_{t-1}$. In this world, households think they gain (or lose) if they consume more (or less) than in the previous period, while disliking loss more than gain. The commonly used specification is $z_t = u(c_t) - u(c_{t-1})$, that is, z_t is the difference between the utility of two consumptions. Households still perceive they gain (or lose) if they consume more (or less) than in the previous period since the CRRA function is monotonous, but by the extent of the difference in their utility/welfare instead of the difference in the consumption. Taking this into account for the whole utility function U_t, households decide on their present consumption of the period. To conclude, we can rewrite the households problem in Equation (1) as

$$U(c_t \mid c_{t-1}) = (1-\theta)u(c_t) + \theta v(c_t - c_{t-1}) \tag{3}$$

or,

$$U(c_t \mid c_{t-1}) = (1-\theta)u(c_t) + \theta v(u(c_t) - u(c_{t-1})) \tag{4}$$

Having past consumption as reference point, households consider gain/loss represented by z_t, through loss aversion function v in their optimisation, then make consumption or savings decision by their total utility function U subject to their budget and borrowing constraints, as will be discussed later.

An implication to note about each specification of z_t is the following: As the CRRA function is concave, whether z_t in either specification represents a stronger (weaker) experience of gain or loss with a greater (smaller) difference depends on where c_t and c_{t-1} are located on its horizon (assuming they are smooth enough not to jump between or far from each other). For example, if c_t and c_{t-1} are near the origin, $(u(c_t)-u(c_{t-1}))$ will be larger than (c_t-c_{t-1}). Therefore, $v(u(c_t)-u(c_{t-1}))$ will result in a greater impact of loss aversion on the total utility function U_t than its counterpart $v(c_t-c_{t-1})$. More discussion of this will be made in Section 2.6.4, in relation to the results of wealth inequality.

Equation (5) shows what the budget constraint looks like. Households first earn from labour supply, l_t, which is assumed to be exogenous, thus completely inelastic in this model. The wage rate is assumed to be fixed, but it is combined with idiosyncratic earnings or productivity shock, $s_t \in S = [s_0, s_1, s_2, ...s_n]$, out of the Markov chain, $\Omega \equiv \Omega(s_t, s_{t+1})$, to generate heterogeneity. The second term shows the capital income from past savings $((1+r_t-\delta)k_t)$, as well as the amount of borrowing that each agent can obtain, as in Φ.

Although Aiyagari (1994) explicitly distinguished household financial assets from physical capital in the production function, they were consolidated here as if households are the owners of capital to lend to the company, which is reasonable in terms of general equilibrium, without loss of generality. In addition, although Aiyagari (1994) postulated the borrowing constraint as being the negative value of financial assets [9], we could take the borrowing constraint as $\Phi > 0$ due to its nature of physical capital.

[9] Aiyagari (1994) originally suggested $\Phi \equiv min(b, \frac{wl_{min}}{r})$, i.e. borrowing constraint is the smaller value of either fixed (ad-hoc) borrowing constraint or the present value budget balance $(\frac{wl_{min}}{r})$.

They all constitute the total resource, y_t, that households can utilise to make decisions about consumption and the savings. The reason for excluding the labour supply decision is to simplify the model and abstract the impact of saving decisions on the income and wealth inequality from the earnings ineuqality, in the face of idiosyncratic shocks and the borrowing constraint.

$$s.t. \;\; c_t + k_{t+1} = \underbrace{w s_t l_t + (1 + r_t - \delta) k_t + \Phi}_{\equiv y_t} \tag{5}$$

$s_t \sim$ idiosyncratic productivity shock , $\Phi \geq 0$: borrowing constraint

w: (fixed) wage rate, l_t: labour supply, k_t: capital, δ: depreciation rate

Now we can build up the recursive form of the household problem as:

$$V(k_t, s_t, x_t) = \max_{c_t, k_{t+1}} \Big\{ U(c_t(k_t, s_t, x_t)) + \beta \sum_{s \in S} \Omega(s_t, s_{t+1}) V(k_{t+1}, s_{t+1}, x_{t+1}) \Big\} \tag{6}$$

$$\begin{aligned} \text{where } \; U(c_t(k_t, s_t, x_t)) &\equiv U(c_t(k_t, s_t, c_{t-1})) \\ &= (1-\theta) u(c_t(k_t, s_t)) + \theta v(c_t(k_t, s_t) - c_{t-1}) \end{aligned} \tag{7}$$

or

$$\begin{aligned} \text{where } \; U(c_t(k_t, s_t, x_t)) &\equiv U(c_t(k_t, s_t, c_{t-1})) \\ &= (1-\theta) u(c_t(k_t, s_t)) + \theta v(u(c_t(k_t, s_t)) - u(c_{t-1})) \end{aligned} \tag{8}$$

subject to

$$c_t = w s_t l_t + (1 + r_t - \delta) k_t - k_{t+1} + \Phi \tag{9}$$

Equation (6) shows the Bellman equation for the households' problem. This depends on three state variables: current capital/asset, exogenous idiosyncratic shock, and reference point as another endogenous state variable. For time t, the (total) utility function U is the function of consumption and the reference point, and consumption is the function of the state variables of capital and shock, that is, $c_t = c_t(k_t, s_t)$. The future sum of the utility function was replaced by the next period value function $V'(k_{t+1}, s_{t+1}, x_{t+1})$ and discounted by β. Importantly, this term is also subject to the Markov chain transition probability denoted by Ω.

Equation (7) shows how the total utility function is specified, as already discussed. This is the sum of the weighted normal utility function and the reference dependent loss aversion function. Note that the reference point is the past consumption; thus, it has state variables from the previous period, i.e. $c_{t-1} = c_{t-1}(k_{t-1}, s_{t-1})$. The specification of the loss aversion function in Equation (7) corresponds to Equation (3) and Equation (8) corresponds to Equation (4). Equation (9) is from the recalling Equation (5).

Once the households' problem was solved by the Bellman equation of Equation (6)- Equation (8), the policy function for consumption is derived as $c_t = h(k_t, s_t, c_{t-1})$. In addition, the law of motion of the capital (asset) can be obtained as:

$$k_{t+1} = y_t - h(k_t, s_t, c_{t-1}) = [(1 + r_t - \delta)k_t + ws_t l_t + \Phi] - h(k_t, s_t, c_{t-1}) \tag{10}$$

Whether this can be aggregated to the capital supply curve depends on whether a stationary distribution of capital exists. Aiyagari (1993) showed that for his model, if the relative risk aversion coefficient is bounded, the distribution is bounded, so there is a unique stationary distribution. Whether it still applies to the model with loss aversion is discussed in Section 2.4.3. For now, we assume its existence to continue the discussion about the capital supply function.

Assuming there exists a stationary distribution of capital, we can obtain the average capital given the interest rate and wage. Aiyagari (1994) already demonstrated how the capital supply curve shifts to the right with a precautionary saving motive and the capital goes to infinity as r converges to the rate of time preference ($\frac{1-\beta}{\beta}$). Therefore, a straightforward priori assumption with loss aversion is that if households have even more precautionary motive, the capital supply curve will shift even further to the right, yielding a higher saving rate with a lower interest rate in the general equilibrium, and this will be examined in Section 2.6.1.

One thing to note about the labour market is, due to its idiosyncratic nature, the expected value of stochastic earnings (productivity) shock (s_t) for an individual equals its cross-sectional average per capita and is therefore constant[10]. Furthermore, the labour supply for each household, l_t, is completely inelastic, therefore constant regardless of the earnings rate $w \cdot s_t$. From these we can derive that the individual's expected value of labour supply with idiosyncratic (productivity) shock is constant, thus can normalise it to unity, i.e. $E(s_t l_t) = 1$. In other words, the per capita labour supply with idiosyncratic shock is constant to be unity. This was inserted into the firms' problem in the next section.

2.4.2 Firms

The standard firms' problem is that firms are assumed to be identical, which can be solved using the static maximisation problem as in Equation (11) to yield the

[10]Aiyagari (1994), p. 664

capital demand function, having labour supply as 1:

$$\max_{K_t} \Pi_t = Y_t - w \cdot 1 - (r_t + \delta)K_t \tag{11}$$

$$s.t. \quad Y_t = F(K_t, 1) = K_t^{\alpha}$$

$$\alpha : \text{the capital share}, \ \delta : \text{the depreciation rate}$$

$$r_t = F_{K_t} - \delta \tag{12}$$

Note that the solution for the firms' problem is summarised by aggregate capital, K^*, in the stationary general equilibrium.

2.4.3 Equilibrium

As mentioned above, if the relative risk aversion coefficient is bounded in the Aiyagari model there exists a unique stationary distribution thus the general equilibrium. Having modified the objective function to contain loss aversion, however, it is not guaranteed that the same exposition still applies for the general equilibirum to exist. To answer this, I used Acikgoz (2018)'s findings in search for the conditions to ensure the existence and the uniqueness of the stationary distribution while allowing modifications as in Equations (4), (5), (8) and (9). In his theoretical paper, Acikgoz (2018) derived a number of conditions by which the equilibrium is guaranteed to exist in Aiyagari type models :

B1. Utility function $U : \mathbb{R}_{++} \to \mathbb{R}$ is continuously differentiable, strictly increasing, and strictly concave, with $\lim_{c \downarrow 0} U'(0) = \infty$ and $\lim_{c \to \infty} U'(c) = 0$.

B2. The interest rate and the wage level satisfy $w > 0, r > -1$ and $\beta(1 + r) < 1$.

B3. The utility function is twice continuously differentiable and satisfies $\liminf_{c \to \infty} -\frac{U''(c)}{U'(c)} = 0$.

B4. The Markov chain process Ω is irreducible and $\Omega(\underline{s}, \underline{s}) > 0$ where $\underline{s}$ is the lower bound of s.

B5. The production function F is constant-returns-to scale (CRS), strictly increasing, strictly concave, continuously differentiable, and satisfies $\lim_{K\to 0} F(K,1) = 0$, $\lim_{K\to 0} F_K = \infty$, and $\lim_{K\to\infty} F_K < \delta$.

Using the policy iteration method operator based on the Euler equation (Coleman 1990), Acikgoz (2018) showed that the boundedness of state space, which Schechtman and Escudero (1977) found to hold under certain assumptions on utility function and i.i.d. shocks, still holds under the weaker assumptions on preferences, including those unbounded both from above and below. Acikgoz (2018) also found that it holds under the arbitrary Markov chain processes not only under i.i.d. shocks. Such compactness of the state space, in conjunction with the strictly positive probability of hitting the borrowing constraint and the positive probability of remaining in the worst scenario inducing its persistence, is sufficient to prove the existence of the stationary distribution, regardless of the level of utility being bounded or not. The last condition on the persistence of the worst state also ensures the uniqueness of stationary distribution.

How the model in this chapter satisfies **B1-B5** is as following:

Lemma 1. A piecewise CARA gain loss function in Equation (2) is asymptotically continuously twice differentiable if it uses the following smooth approximation:

$$v_k(z) = \gamma(z)(\frac{1-e^{-\eta z}}{\eta}) + (1-\gamma(z))\lambda(\frac{e^{\phi z}-1}{\phi})$$

where the switching function $\gamma(z) = \frac{1}{1+e^{-kz}}$, as $k \to \infty$.

The proof is provided in Appendix A.1, applying the similar method to Rosenblatt-Wisch (2008). It shows that despite the kink at the reference point, $v(c_t, x_t)$ can be approximately smoothed as the exponent parameter k in the switching function goes to infinity, to become differentiable. This is called a smooth approximation method and enables the non-differentiable problems to be asymptotically solved. And given this lemma, it is obvious that $U(c_t, x_t)$ in Equation (1) is continuously twice differentiable.

Proposition 1. Given Lemma 1, the utility function specified as Equation (1) and (2) asymptotically satisfies B1 iff $c_t < \frac{\mu(1-\theta)}{\theta\phi z\lambda e^{\phi z}} + \frac{\mu}{\phi}$.

The proof is provided in Appendix A.2. It implies that the utility function incorporating the piecewise CARA loss aversion function does have equilibrium as long as the consumption choice is bounded by a certain combined value of the parameters. To my knowledge, this derivation is one of the novelties that this chapter contributes, especially when using CARA for the loss aversion function, while being versatile with respect to the definition of z. Personal equilibrium in Koszegi and M. Rabin (2006, 2007, 2009) and Pagel (2017) use piecewise linear function of expectations-based reference point. This condition on consumption is inserted inside the iteration (Section 2.5.1) to satisfy Proposition 1 in the computation.

Proposition 2. Given Lemma 1, the utility function specified in Equation (1) and (2) asymptotically satisfies B3.

The proof is provided in Appendix A.3. Note that B3 is a sufficient condition for boundedness of state space when the shock follows the Markov process, but not a necessary condition. In fact, even without Proposition 2, the compactness of state space is already satisfied in the computation process (Section 2.5.1), which ensures that the influence of stochastic earnings on consumption/savings gets smaller as the wealth gets large.[11]

B2 is satisfied by parameter values in the computation process as explained in 1.5.(Table 3) Note that unlike the condition on consumption in Proposition 1, I did not insert the condition on interest rate inside the iteration to see the resulting interest rates as reported in Table 5. B4 is already present in the Aiyagari model as well as the modified model in this chapter. B5 is ensured by the properties of Cobb-Douglas production function.

In developing his proofs, Acikgoz (2018) derived some important properties regarding the capital supply function, which had been informally described by Aiyagari (1994). First, due to the lower bound of the consumption (policy) function, the asset decreases nearer the borrowing constraint, independent of the level of impatience. It also becomes one factor enabling the borrowing constraint to be hit with strictly positive probability (Acikgoz (2018), Proposition 2.1). Another property to note is that the upper bound of the aggregate capital supply goes to infinity as the interest rate converges to the time preference rate (discount rate in his paper), $\overline{r} = \frac{1-\beta}{\beta}$). In other words, $\lim_{r \uparrow \overline{r}} K(r) = \infty$ where $K(r)$ is the aggregate capital as a function of the interest rate $r \in (-1, \frac{1-\beta}{\beta})$(Acikgoz (2018), Proposition 6). This is the state where households with accumulated assets are

[11] Acikgoz (2018) has mentioned that CARA violates B3 as it sometimes has the asset blow to the infinity even before the interest rate converges to the discount rate. However, provided that B3 is only the sufficient condition and not the necessary condition to prove the compactness of state space which is already in the computation, we can ignore his comment on CARA violating it. In fact, it turned out that the savings rate did not appear to be blowing up. (Section 2.6.1) Another reason might be that the specification of this chapter combines CRRA and CARA instead of CARA alone; that is, the total utility function does not cause the asset to blow to the infinity because CRRA in it provides some boundedness. However, this chapter does not pursue further theoretical clarification about it.

unlikely to be affected by idiosyncratic shocks for their consumption decisions. It also explicitly explains why the precautionary saving motive occurs, as suggested by Aiyagari (1994). Households, in anticipation of the probability of hitting borrowing constraint, will save more to reach this state of largely accumulated assets.

Acikgoz (2018) argued that these limits are independent of the level of impatience imposed by Assumption 2, and Assumption 2 is not necessary for the existence of equilibrium but only for solving the households problem. He has proven that these limits can be derived from the weak inequality of Assumption 2 ($\beta(1+r) \leq 1$) when $r > 0$. Based on this, he proved Lemma 1 for the upper bound of capital supply. [12] According to him, the utility function with loss aversion in CARA[13] should have properties at the upper bound, because CARA sometimes goes to infinity even before the interest rate converges to the time preference rate. However, it remains unclear whether this model should or does meet Assumption 2 to solve the households problem that produces properties at the lower bound of consumption; it is beyond the scope of this chapter. However, an intuitive explanation for this model is that if the loss aversion specified as in Equations (3) and (4) imposes additional motivation to smooth the consumption, the decrease in the asset around the lower bound of consumption will be even more expedited, so impatience in Assumption (2) may not even be needed. Nevertheless, it is only an a priori assumption and will be revisited when analysing the results in Section 2.6.1.

In consequence, it can be argued that as long as satisfying the assumptions he suggested, the Aiyagari type model with loss aversion as in this chapter still must generate the unique stationary capital distribution with the properties explained

[12] In contrast, Stachurski and A. A. Toda (2019) has argued that Assumption 2 is essential to have those two limits as well as the equilibrium.

[13] This is the functional form of loss aversion utility, v, borrowed in this chapter. In proving Assumption 3, Acikgoz (2018) explained that CARA violates Assumption 3 due to the absence of a wealth effect.

above. Furthermore, as his capital supply function features similar properties to the Aiyagari model which was based on risk aversion (Assumption 3), and the loss aversion is essentially to reinforce precautionary saving motive in addition to the risk aversion, we can infer a priori that capital supply function will have the same properties of upward sloping.

Having theoretically proven the existence and uniqueness of stationary capital distribution, and having the capital demand function from the firms' problem, we can now clear the capital market. This is sufficient to yield the general equilibrium. [14] While the stationary distribution is generated computationally in the next section, the formal description is as follows: the joint distribution reaches stationarity when the next period distribution Λ' equals the present distribution Λ in recursive form, taking into account the transition probability Ω, the law of motion, and the policy function h.

$$\begin{aligned} \Lambda'(k_{t+1}, s_{t+1}, x_{t+1}) &\equiv \Lambda'(k_{t+1}, s_{t+1}, c_t) \\ &= \sum_{k_t} \sum_{s_t} \sum_{c_{t-1}} \Lambda(k_t, s_t, c_{t-1}) \times \Omega(s_t, s_{t+1}) \\ &\times 1\Big\{k_{t+1} = y_t - h(k_t, s_t, c_{t-1})\Big\} \times 1\Big\{c_t = h(k_t, s_t, c_{t-1})\Big\} \end{aligned} \tag{13}$$

The left hand side shows the joint distribution of three state variables in the next period; i.e. capital, the idiosyncratic shock, and the reference point. To note, the reference point in the next period is the *present* consumption. It is defined as the joint distribution of the current period with state variables of capital, shocks and past consumption, taking account of all possible values of them that will be

[14] Acikgoz (2018) gave an additional discussion about the uniqueness of the equilibrium, using the CRRA specification in the utility function, but this was not imposed or further investigated in this chapter. According to Acikgoz - also see Light (2018), the multiplicity of equilibrium is caused when the income effect dominates the substitution effect, i.e., depends on the level of prudence (given as $\mu > 1$) in conjunction with the earning process. He suggested that the calibration strategy should use capital/output ratio as a target, to adjust the discount rate β.

realised in the next period, as represented by the summations for each state variable. Such transitions are made by the transition probability Ω for shock, and the policy function h for both capital and the reference point (consumption). This then induces the unique stationary distribution, denoted as Λ^* where $\Lambda' = \Lambda$. Note that, although the distribution is stationary and unique, individual household agents can move within it. This is why the policy function does not a have stationary notation $*$. In aggregation, however, the average capital supply at the equilibrium is stationary; thus, it has a fixed value of K^*.

Concluding all the discussion above, the recursive competitive stationary general equilibrium is defined as 1) given (w, r), the policy function $c_t = h(k_t, s_t, c_{t-1})$ solves the household's dynamic problem; 2) given (w, r), K^* solves the firm's static problem; 3) the unique stationary distribution Λ^* is induced by the Markov chain Ω and the policy function $c_t = h(k_t, s_t, c_{t-1})$; and 4) K^* clears the capital market.

2.5 Computation

2.5.1 Coding Strategy

The VFI Toolkit for MATLAB by Kirkby (2020)[15] provides a highly versatile set of codes for the Aiyagari model and VFI with loss aversion, and is thus useful to modify for the model of this chapter. After setting up the parameters, the toolkit creates the exogenous state variable s_t, i.e. idiosyncratic shock, with a grid size of 21, as well as its Markov chain process through Tauchen method, to approximate the AR(1) process as specified below:

$$s_t = \rho s_{t-1} + e_t, e_t \sim \mathcal{N}(0, \sigma^2) \tag{14}$$

[15] https://www.vfitoolkit.com

where ρ is the autocorrelation coefficient and σ is the standard deviation of innovation e_t.

It also generates the endogenous state variable grids, i.e. capital with 256 discrete points. Then, starting from the initial guess, it iterates over the equilibrium price, which is the interest rate r,[16] until the stationary distribution is obtained. Once it reaches the equilibrium price, it can compute the inequality measures such as the Gini coefficient. Aiyagari (1994) also used a similar iteration.

Incorporating loss aversion into Aiyagari model, following points were taken into account for modification. First, an additional endogenous state variable was introduced, namely the past consumption as reference variable in a grid size of 51. Then, the endogenous state variable became a matrix of capital (asset) and past consumption instead of a single vector of capital (asset). Second, resulting quantities such as the stationary distribution, inequality measures, and figures, were adjusted to be compatible with the new dimension. Third, the utility function was expanded to include the loss aversion term in two different specifications. It was also ensured that the model complies with the condition set out in Proposition 1 (Equation A.2.5), as well as as the condition such that the value of past consumption as the state variable (c_{t-1}) is not too far from the value of consumption as the choice variable of the last iteration (c'_{t-2}).

Then it was experimented with different values of parameters of interest, θ and μ, the weight of the loss aversion utility function compared to the CRRA function, and the CRRA coefficient, respectively. μ was chosen from 2 to 4 as it is mostly around 3 in the literature, and θ was simply chosen between $[0, 1]$. This was run on two different specifications of the reference point, z, as in Equations (3) and (4). Other calibrations followed the conventions and were discussed in

[16]It is set up such that the initial guess $r_0 = 0.04$.

more detail in the next section (2.5.2).

2.5.2 Parameterisation

The whole parameterization is summarized in Table 3, with left columns for the parameters of interest iterations were run over, and the right columns for the fixed parameters calibrated from the literature. As this chapter basically replicates and modifies Aiyagari (1994), which Kirkby (2020) computed in his code, most of the values of the fixed parameters are based on it. Probably following conventions in macroeconomics, Aiyagari (1994) did not specifically comment on how he calibrated α, β and δ, but discussed σ and ρ for Equation (10) in more detail. (Aiyagari 1994, p.675-676, 681) In particular, he concluded that $\sigma = [0.2, 0.4]$ is a reasonable range at an annual rate, according to Heaton and Lucas (1992)'s estimates using PSID data, as well as others. He ran the iterations over combinations of $\sigma = [0.2, 0.4]$ and $\rho = [0, 0.3, 0.6, 0.9]$, and explicitly pointed out that the model cannot generate the observed Gini coefficients, taking an example when $\sigma = 0.2, \rho = 0.6$. And these are the parameter values that VFI Toolkit chose for this chapter to follow. Next, according to VFI Toolkit, q is the number of standard deviations of the AR(1) variable covered by the range of discrete points, as Aiyagari (1993, footnote 33, p.25) implicitly says he uses $q = 3$. Lastly, the model period is taken to be one year.

I discuss the calibration of loss aversion parameters. The CARA coefficients (η and ϕ), being allowed to be between 0 and 10, (Babcock, Choi, and Feinerman 1993, Table 1) were set as 1 to simplify without loss of generality, because their calibration depends on an explicit choice of consumption units ($z_t = c_t - c_{t-1}$) or utility ($z_t = u(c_t) - u(c_{t-1})$), neither of which is specified in the analysis. What is more important is the fact that the same parameter is used in the gain and loss

Table 3: Parametrisation

Variable parameters	Value	Fixed parameters	Value	Source
θ	0, 0.3, 0.5, 0.7	β	0.96	VFI toolkit based on Aiyagari(1993, 1994)
μ	2, 3, 4	α	0.36	
		δ	0.08	
		σ	0.2	
		ρ	0.6	
		q	3	
		η	1	Kobberling and Wakker (2005)
		ϕ	1	
		λ	2.25	Chen (2015)

domains, so the role of the loss aversion index (λ) is distinguished from the role of (different) curvatures of the gain and loss function in featuring the loss aversion, while still being along with A1-A3 from Bowman, Minehart, and Rabin (1999). In fact, $\frac{v(-z_t)}{v(z_t)} = \lambda$ for all z_t, not just in the limit of $z \to 0$, as well as Arrow-Pratt measure $-\frac{v''(z_t)}{v'(z_t)} = \eta = \phi = 1$ for all z_t. The loss aversion index (λ) was calibrated to be 2.25 following Chen (2015) and Tversky and Kahneman (1992). [17]

2.6 Results

2.6.1 Aggregate vs distributional effect

In stationary equilibrium, the effect of loss aversion weighted by θ turns out to be ambivalent between aggregate and distributional variables. Although the results in aggregate variables such as the savings rate and interest rate are not the same as a priori assumption, inequality measures such as Gini coefficient and Pareto

[17] Recently, Brown et al. (2024) conducted a meta-analysis of 150 articles and estimated the loss aversion coefficient to be 1.955 with the 95 percent interval of [1.820, 2.102] to which λ in this chapter is close.

Table 4: Savings Rate

		$z_t = c_t - c_{t-1}$				$z_t = u(c_t) - u(c_{t-1})$			
μ \ θ	Bench -mark 1)	0 2)	0.3	0.5	0.7	0	0.3	0.5	0.7
2	23.98	24.09	21.17	22.37	32.31	23.44	21.13	38.67	20.38
3	24.26	24.26	24.29	22.64	27.0	23.71	24.54	21.90	19.6
4	24.47	24.27	23.81	23.14	26.26	24.29	24.35	25.44	12.7

1) Benchmark is the original Aiyagari (1994) model throughout the tables in this section.
2) Both benchmark and the model where $\theta = 0$ are conceptually identical. The difference in the results purely come from the computational issues because the modified code has a larger state space with additional state variables of past consumption and numerical variability as a result. It also has two conditions constraining the value of present and past consumptions in the code, as discussed in 2.5.1 (last sentence of the third paragraph).

coefficient show an increase along with θ. Specifically, the savings rate in Table 4 does not necessarily increase in all values of θ compared to the original Aiyagari model (under the "benchmark"). In Table 5, some interest rates even exceed the rate of time preference, $\frac{1-\beta}{\beta} \approx 0.0416$, especially when the savings rate is lower. This is clearly different from the a priori assumption, where the aggregate capital supply curve was anticipated to shift more to the right than the Aiyagari model with upward slope. In contrast, Tables 4-5 clearly show the effect of loss aversion on income and wealth distribution, especially when $\mu = 2, 3$ and $\theta = 0.5, 0.7$. One possible reason for this result is that, while loss aversion does not increase the average capital supply out of precautionary savings as a whole, it changes the distribution of it. That is, it may be that agents in the upper quantiles have become keener to save for precautionary purpose than in the Aiyagari model, whereas agents in the low quantiles struggled more to save enough not to be affected by the persistence of worst shock and hitting the borrowing constraint, because they have become less flexible in reducing the consumption due to the loss aversion in it. And this is what this chapter discovers in the following sections.

Table 5: Interest Rate

		$z_t = c_t - c_{t-1}$				$z_t = u(c_t) - u(c_{t-1})$			
μ \ θ	Bench -mark	0	0.3	0.5	0.7	0	0.3	0.5	0.7
2	4.0	4.05	4.18	4.32	4.56	4.04	4.11	4.18	4.37
3	3.89	3.97	4.18	4.33	4.70	3.97	4.04	4.08	4.21
4	3.77	3.88	4.18	4.40	4.79	3.89	3.96	4.02	4.13

2.6.2 Income and Wealth inequality

Before looking into the inequality results in detail, I first introduce the measures of income and wealth inequality used in this chapter, i.e. Gini coefficients and Pareto coefficients.

Gini coefficient. Gini coefficient measures the inequality of a variable (e.g. income or wealth) by a single index number from 0 to 1, where 0 signifies perfect equality and 1 signifies perfect inequality, thus the higher the Gini coefficient, the greater the inequality of a variable. Its mathematical expression is as follows:

$$G = \frac{1}{2\bar{x}} \frac{1}{N^2} \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} |x_i - x_j| \tag{15}$$

where x_i is a variable of interest (e.g. income or wealth, $\bar{x}$ is the mean of it, and N is the number of population in the sample.[18] It is the average absolute difference between all pairs of observations. It is known to be sensitive to the change around the median of the distribution, because it weighs all observations equally.

In practice, it is commonly explained and computed using the Lorenz curve, as the ratio of the area between the line of equality (45 degree diagonal) and the

[18]The notation in this section is irrelevant to the ones used in other parts of this chapter. It is to help understanding the concept of inequality indices explained here.

Lorenz curve, and the whole triangular area under the line of equality. If the former is called A and the area under the Lorenz curve is B, then $G = \frac{A}{A+B} = \frac{A}{0.5} = 2A$. And this is how the Gini coefficient was computed in this chapter, by summing the height of it at each quantile.

(Inverted) Pareto coefficient. I start from the Pareto coefficient, mostly used to analyse the upper tail of distribution. It is the exponent, α, of the Pareto distribution function,

$$F(x) = 1 - (\frac{\bar{x}}{x})^{\alpha} \tag{16}$$

where F is the Pareto CDF of x, again a variable of interest such as income or wealth, and $\bar{x}$ is a lower bound of it. It has a inverse relationship with inequality, i.e. the higher α is, the thinner the tail is (Atkinson and Jenkins 2020).

More straightforward and commonly used inequality measure is Inverted Pareto Coefficient, b, defined as

$$b = \frac{\mathbb{E}[x|x > \bar{x}]}{\bar{x}} \tag{17}$$

where x is again a variable of interest (income or wealth) and $\bar{x}$ is a lower bound of it. It is the ratio of the average income or wealth above a certain threshold $\bar{x}$ and the threshold itself. It is in a positive relationship with inequality, i.e. the higher b is, the thicker the upper tail, thus the income/wealth is more concentrated on the top. In addition, it is known that $b = \frac{\alpha}{\alpha-1}$.(van der Wijik(1939)'s Law, as cited in Blanchet, Piketty, and Fournier (2022))

In practice, α is estimated by using $\alpha = 1/[1 - log(S0.01/S0.001)/log(10)]$, where $S0.01$ is the share of top 1 percent and $S0.001$ is the share of top 0.1 percent. (Atkinson and Jenkins 2020; Vries and A. Toda 2021). It is estimated well when analysing the very top of the distribution (Atkinson and Jenkins 2020), so α in this estimation is (only) suitable for measuring the inequality within the rich. (top 0.1 percent within the top 1 percent) Using this estimation method and the above equation ($b = \frac{\alpha}{\alpha-1}$), I produced Inverted Pareto coefficients from VFI toolkit.

As shown in Tables 6-7, loss aversion clearly generates higher wealth and income Gini coefficients compared to the original Aiyagari model, along with θ, the weight of the gain-loss utility function. [19] However, while the Inverted Pareto coefficient greater than 1 implies a thick upper tail, it does not show a significant change compared to the original Aiyagari model, in some cases modest declines. It means that the within-the-top inequality of the top 1 percent and 0.1 percent remains relatively the same or rather improved. More details follow on each variable of interest.

First to note, the earnings inequality does not show any difference from the original Aiyagari model, with a Gini coefficient of 0.11 and the Inverted Pareto coefficient of 1.39. This is because the labour supply was assumed to be inelastic in both models, so the dispersion of earnings is attributed to the idiosyncratic shock process only, which was identically inserted in all the values of θ. The implication therefore remains the same as discussed earlier: Despite the identical response of the full inelastic labour supply, which is independent of the idiosyncratic shock iteslf, households end up with inequality in income and wealth solely through heterogeneous saving decisions in the face of shock. In addition, the degree of heterogeneity in that saving decision should be related to loss aversion, as will be

[19] It ranges from 0.3-0.7. The results when loss aversion was weighted too low, close to zero, or too full, close to one, are not reported because they are extreme or invalid.

Table 6: Gini Coefficients

μ \ θ	Bench -mark	$z_t = c_t - c_{t-1}$: 0	0.3	0.5	0.7	$z_t = u(c_t) - u(c_{t-1})$: 0	0.3	0.5	0.7
Income									
2	0.32	0.35	0.43	0.43	0.64	0.34	0.41	0.56	0.56
3	0.31	0.34	0.42	0.38	0.56	0.34	0.40	0.53	0.58
4	0.30	0.33	0.37	0.34	0.56	0.32	0.37	0.50	0.35
Wealth									
2	0.38	0.42	0.54	0.53	0.72	0.41	0.51	0.62	0.70
3	0.37	0.41	0.51	0.46	0.65	0.40	0.48	0.65	0.73
4	0.36	0.39	0.44	0.41	0.66	0.39	0.44	0.60	0.52

discussed later.

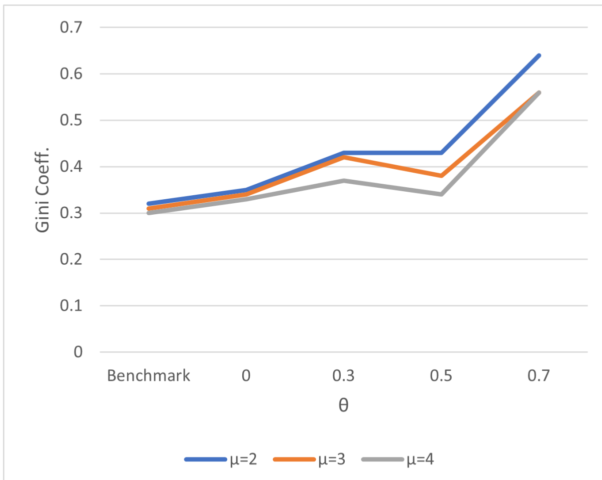
Income contains labour earnings and capital income as they are available resources in the income constraint.[20] Its inequality measured by the Gini coefficient increases along with the degree of loss aversion, highest when $\theta = 0.7$ in the majority cases, in both specifications of the gain-loss term z (Table 4). For example, when $z_t = c_t - c_{t-1}$, the Gini coefficient increases from 0.32 in the benchmark to 0.64 by 0.32 (when $\mu = 2$). When $z_t = u(c_t) - u(c_{t-1})$, again it increases from 0.31 to 0.58 by 0.27 (when $\mu = 3$). The results of the Inverted Pareto coefficient of income are mostly the same or less than in the benchmark case, except when it slightly increases from 1.38 to 1.39 ($z_t = c_t - c_{t-1}$ and $\mu = 3, \theta = 0.5$). The income dispersion within the top group, i.e. between the top 0.1 percent and top 1 percent, appears to have reduced.

A similar pattern is found for wealth inequality. When $z_t = c_t - c_{t-1}$, the Gini wealth coefficient increases the most from 0.38 to 0.72 by 0.36 (when $\mu = 2, \theta = 0.7$), and from 0.37 to 0.73 (by 0.36) when $z_t = u(c_t) - u(c_{t-1})$ and $\mu = 3, \theta = 0.7$ (Table 6). In addition, increases in wealth Gini coefficients tend to be more

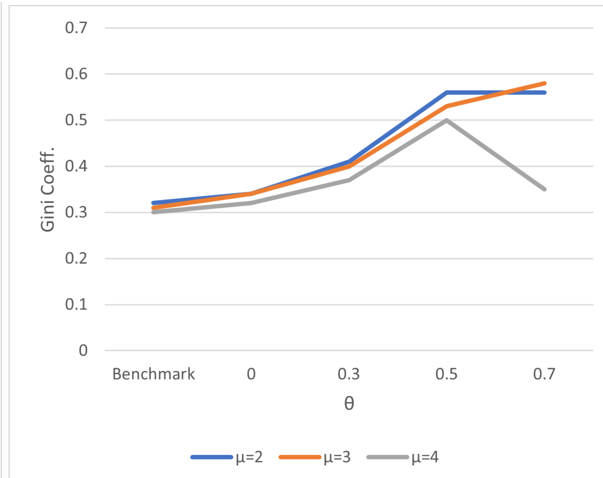
[20] We can exclude the borrowing because it is assumed to be zero by the borrowing constraint.

prominent than income coefficients with a greater jump. On the other hand, the Inverted Pareto wealth coefficient in the same sections decreases from 1.39 to 1.33 and from 1.38 to 1.29 respectively (Table 7). Like in the income, the wealth of very top group moved toward more equal.

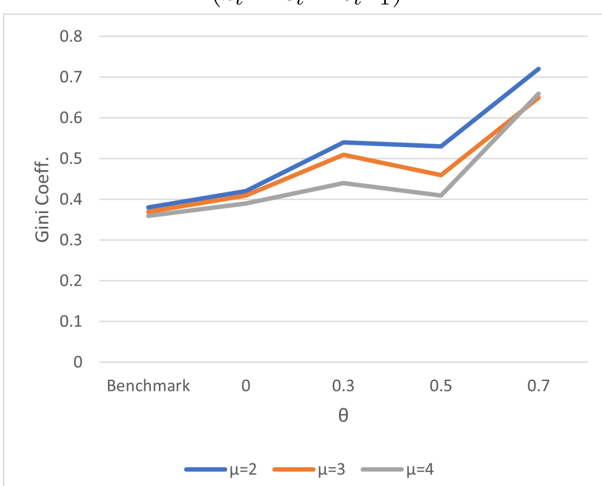
The results of the Gini coefficients clearly show the role of loss aversion in generating more wealth inequalities compared to the original Aiyagari model. In addition to the *existing* precautionary saving motives in the face of idiosyncratic shocks, agents now have loss aversion either on their consumption or utility from it. They dislike feeling lost compared to the past consumption or the utility from past consumption (more than they like gain from them), so their consumption needs to be even smoother. And to make it, they have *additional* precautionary savings motive. However, although the degree of such additional motive must be identical to all agents along with θ because the same θ value is applied to all, their final choice of consumption / savings turn out to differ depending on where they are positioned in the distribution transition. The farther away from the borrowing constraint, the more savings may occur than the Aiyagari model without loss aversion. On the other hand, at the very bottom near the borrowing constraint, dissaving may occur more. Then, as the economy approaches stationary equilibrium, it will move towards the state where the most savings are made by the wealthier, making the distribution more concentrated to the right, even though the increase in the aggregate savings is minimal. In addition, the results of the Inverted Pareto coefficients tell us that in that process, the top above 1 percent form a relatively similar level of income and wealth than in the Aiyagari model. To better understand this, section 2.5.3 investigates the shape of distribution.



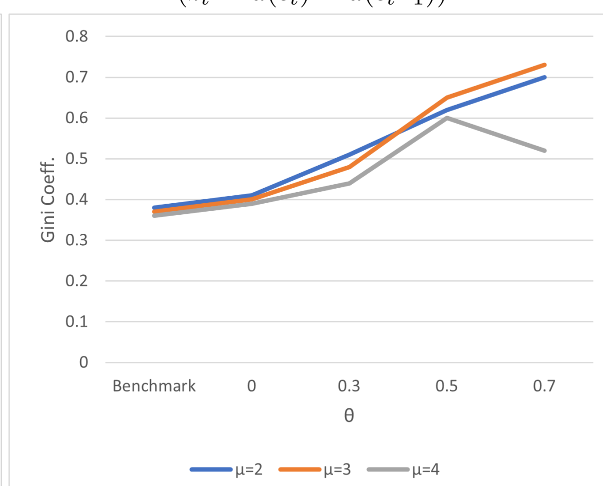
(a) Income Gini Coefficient
($z_t = c_t - c_{t-1}$)



(b) Income Gini Coefficient
($z_t = u(c_t) - u(c_{t-1})$)



(c) Wealth Gini Coefficient
($z_t = c_t - c_{t-1}$)



(d) Wealth Gini Coefficient
($z_t = u(c_t) - u(c_{t-1})$)

Figure 2: Change of Income and Wealth Gini Coefficient

Table 7: Inverted Pareto Coefficients

		$z_t = c_t - c_{t-1}$				$z_t = u(c_t) - u(c_{t-1})$			
μ \ θ	Bench -mark	0	0.3	0.5	0.7	0	0.3	0.5	0.7
Income									
2	1.39	1.38	1.33	1.36	1.33	1.36	1.35	1.39	1.30
3	1.38	1.36	1.34	1.39	1.32	1.37	1.36	1.35	1.29
4	1.38	1.37	1.36	1.38	1.33	1.37	1.36	1.28	1.35
Wealth									
2	1.39	1.38	1.33	1.36	1.33	1.36	1.35	1.35	1.30
3	1.38	1.36	1.34	1.39	1.32	1.37	1.36	1.29	1.29
4	1.38	1.37	1.36	1.38	1.33	1.37	1.36	1.28	1.35

2.6.3 Shape of Distribution

High positive skewness and kurtosis provide important details about the shape of the distribution. Kurtosis is a measure of the tailedness of a distribution relative to the normal distribution; if it is greater than 3, it is fat-tailed, and vice versa. As shown in Table 8, when $z_t = c_t - c_{t-1}$, the kurtosis increases from its corresponding benchmark value and tends to increase more for wealth than for income. Furthermore, it tends to have a higher increase from the benchmark when $z_t = u(c_t) - u(c_{t-1})$ than $z_t = c_t - c_{t-1}$, both in income and wealth. For example, when $\mu = 3$ and $\theta = 0.7$, it increases from 2.22 to 5.28 for income and from 2.37 to even 8.87 for wealth.

On the other hand, skewness measures how biased the distribution is to the left (negative value) or to the right (positive value). All results in Table 9 are positive, and the benchmark results show that they are already more skewed to the right for wealth than for income. With loss aversion, the skewness is aggravated to the right tail, more for wealth than income, and by the largest at $\mu = 2, \theta = 0.7$ when $z_t = c_t - c_{t-1}$ and at $\mu = 3, \theta = 0.7$ when $z_t = u(c_t) - u(c_{t-1})$. Overall, changes in kurtosis and skewness reinforce the priori assumption that loss aversion causes

Table 8: Kurtosis

		$z_t = c_t - c_{t-1}$				$z_t = u(c_t) - u(c_{t-1})$			
μ \ θ	Bench -mark	0	0.3	0.5	0.7	0	0.3	0.5	0.7
Income									
2	2.25	2.36	2.92	2.86	5.31	2.34	2.78	3.82	4.53
3	2.22	2.34	2.86	2.52	4.47	2.31	2.67	3.94	5.28
4	2.19	2.3	2.50	2.35	4.86	2.28	2.48	3.43	2.36
Wealth									
2	2.41	2.58	3.54	3.30	5.95	2.56	3.29	4.09	6.45
3	2.37	2.55	3.29	2.80	5.55	2.52	3.02	5.70	8.87
4	2.34	2.50	2.77	2.55	6.16	2.46	2.74	4.25	3.09

Table 9: Skewness

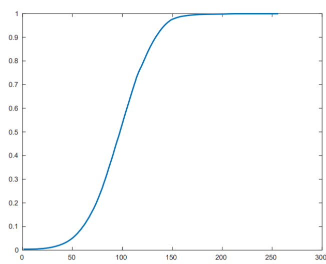
		$z_t = c_t - c_{t-1}$				$z_t = u(c_t) - u(c_{t-1})$			
μ \ θ	Bench -mark	0	0.3	0.5	0.7	0	0.3	0.5	0.7
Income									
2	0.63	0.69	0.91	0.97	1.82	0.67	0.86	1.42	1.41
3	0.62	0.68	0.92	0.84	1.39	0.66	0.83	1.10	1.47
4	0.60	0.65	0.79	0.73	1.50	0.63	0.74	1.04	0.65
Wealth									
2	0.76	0.83	1.18	1.18	2.01	0.81	1.10	1.52	1.94
3	0.73	0.81	1.12	1.01	1.71	0.80	1.01	1.58	2.29
4	0.71	0.78	0.95	0.87	1.87	0.77	0.90	1.33	1.04

more concentration of the wealth towards the right tail with greater dispersion. It clearly shows the effectiveness of the model in explaining why the super-rich stack their assets.

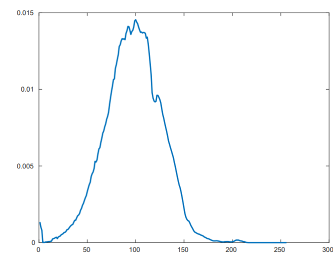
This shape of wealth distribution is more clearly found in wealth CDFs and PDFs (Figure 4-7), compared to the ones of Aiyagari model as benchmark (Figure 3). CDF figures in Figures 4 and 6 show that the steeper the curve, the more concentrated the wealth around that point. As the inequality increases along μ or θ, the CDFs show a flatter and longer curve toward the right, implying its thicker right tail but with less dispersion at the top, after a sharp increase around the middle group. The PDF figures of the wealth in Figures 5 and 7 show the details of the distribution shape. One distinguishing feature is that the distribution becomes more bi-modal or multi-modal, i.e. there are points out of population where the the level of wealth more coincides. In particular, when it is the case of high wealth inequality, it shows a high density on the left side as well as smaller spike(s) on the right side. In consequence, we can conclude that wealth inequality increases by loss aversion, in the way the large size of poor group and smaller size of rich group have formed themselves with squeezed middle group, i.e. wealth is polarised.

To explain why let me first recapitulate the baseline. Household agents already know that the borrowing constraint can be hit with strictly positive probability, as the worst shock that can happen to them will also probably be persistent. In addition, savings will decrease at this point due to the minimum consumption, which is greater than zero. Hence, households inevitably have precautionary savings motives so that they are not affected by idiosyncratic shocks in making their consumption-saving decision. This is independent of patience, prudence, convexity of utility function, etc. (Acikgoz 2018) Note that β is fixed throughout the iterations in my analysis.

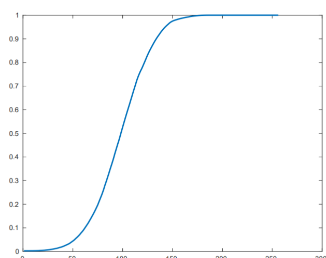
Given that, household agents now have loss aversion in their preference. In addition to the situation where they have to dissave or hit the borrowing constraint, they are also concerned about a situation whereby they have to decrease consumption compared to the past. Hence, it is natural to think that this will boost the precautionary saving motive even further, especially for those who are not binding the borrowing constraint. That is, households will save even more (consume less) to avoid decrease in consumption except those who are already too close to borrowing constraint so hard to make savings while keeping the past consumption.



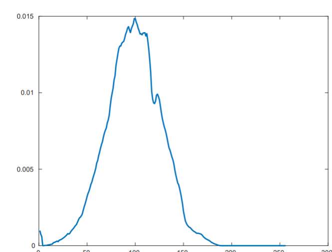
(a) CDF when $\mu = 2$



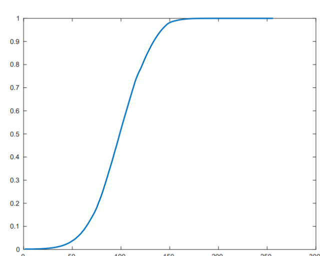
(b) PDF when $\mu = 2$



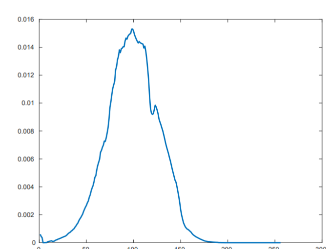
(c) CDF when $\mu = 3$



(d) PDF when $\mu = 3$

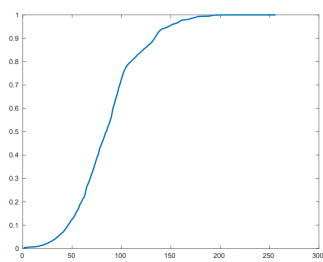


(e) CDF when $\mu = 4$

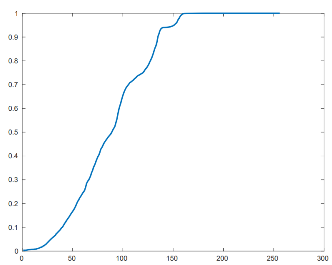


(f) PDF when $\mu = 4$

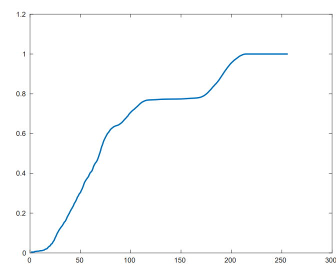
Figure 3: Wealth CDF and PDF (Benchmark)



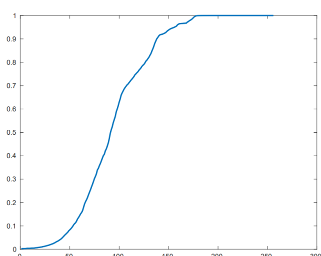
(a) $\mu = 2, \theta = 0.3$



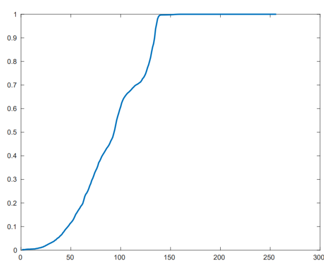
(b) $\mu = 2, \theta = 0.5$



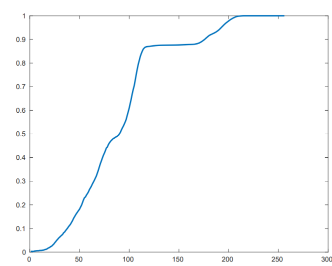
(c) $\mu = 2, \theta = 0.7$



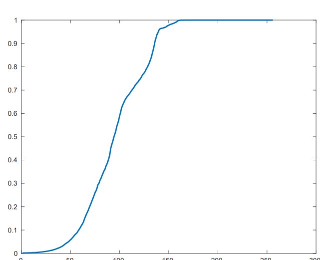
(d) $\mu = 3, \theta = 0.3$



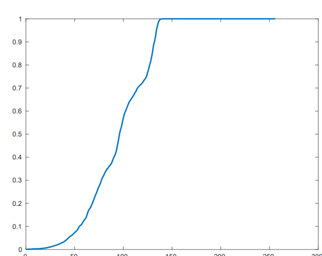
(e) $\mu = 3, \theta = 0.5$



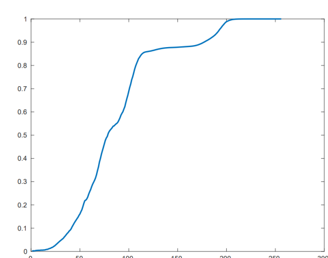
(f) $\mu = 3, \theta = 0.7$



(g) $\mu = 4, \theta = 0.3$

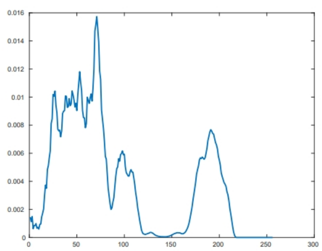


(h) $\mu = 4, \theta = 0.5$

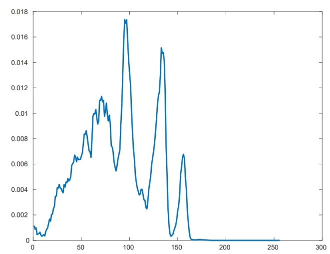


(i) $\mu = 4, \theta = 0.7$

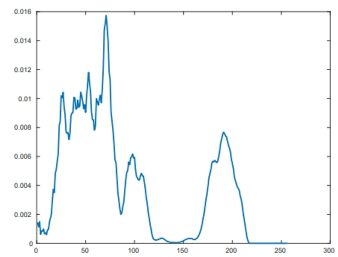
Figure 4: CDF of wealth ($z_t = c_t - c_{t-1}$)



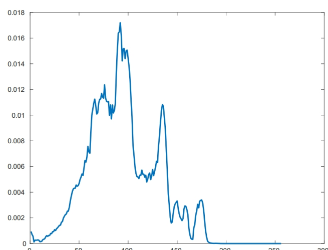
(a) $\mu = 2, \theta = 0.3$



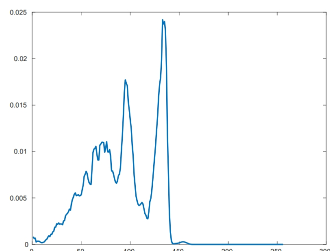
(b) $\mu = 2, \theta = 0.5$



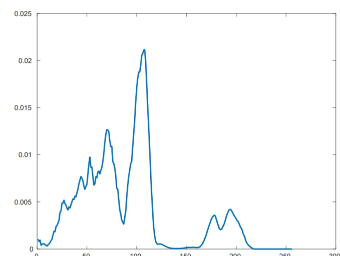
(c) $\mu = 2, \theta = 0.7$



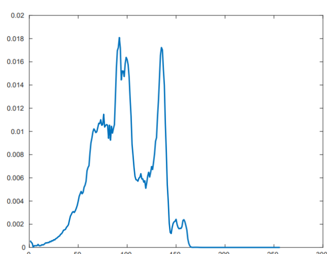
(d) $\mu = 3, \theta = 0.3$



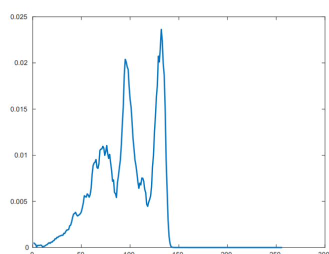
(e) $\mu = 3, \theta = 0.5$



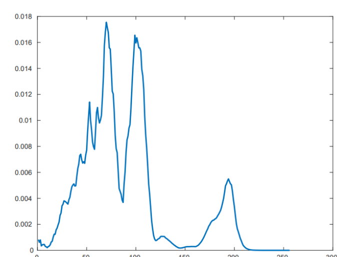
(f) $\mu = 3, \theta = 0.7$



(g) $\mu = 4, \theta = 0.3$

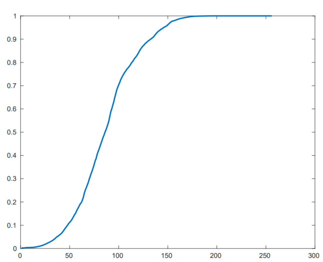


(h) $\mu = 4, \theta = 0.5$

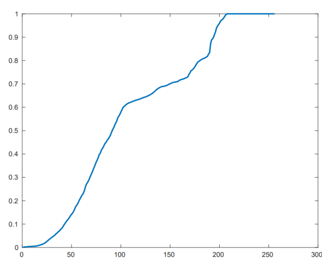


(i) $\mu = 4, \theta = 0.7$

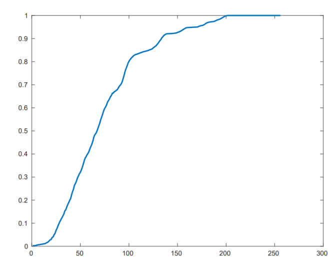
Figure 5: PDF of wealth ($z_t = c_t - c_{t-1}$)



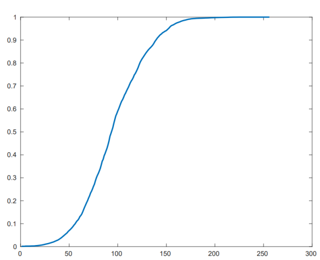
(a) $\mu = 2, \theta = 0.3$



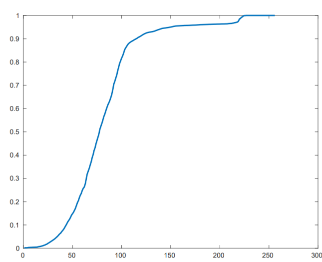
(b) $\mu = 2, \theta = 0.5$



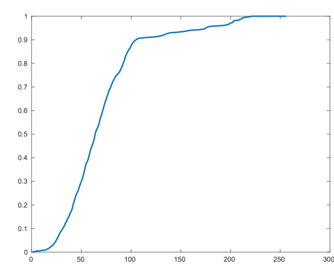
(c) $\mu = 2, \theta = 0.7$



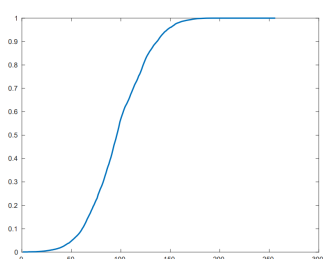
(d) $\mu = 3, \theta = 0.3$



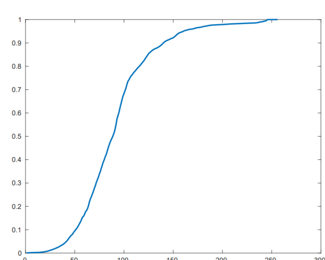
(e) $\mu = 3, \theta = 0.5$



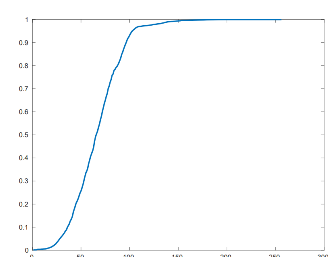
(f) $\mu = 3, \theta = 0.7$



(g) $\mu = 4, \theta = 0.3$

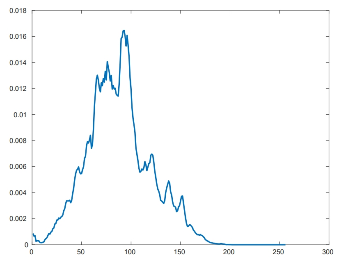


(h) $\mu = 4, \theta = 0.5$

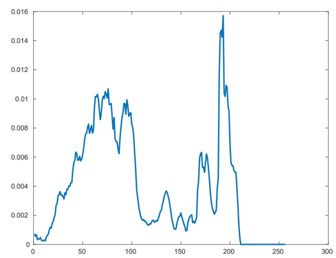


(i) $\mu = 4, \theta = 0.7$

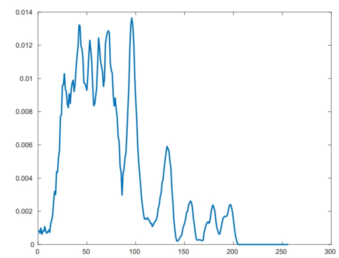
Figure 6: CDF of wealth ($z_t = u(c_t) - u(c_{t-1})$)



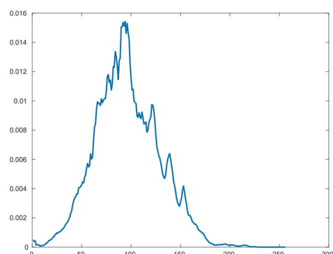
(a) $\mu = 2, \theta = 0.3$



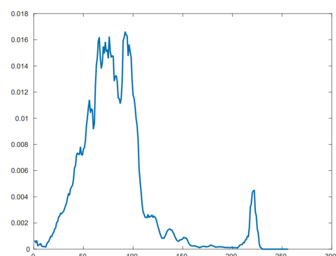
(b) $\mu = 2, \theta = 0.5$



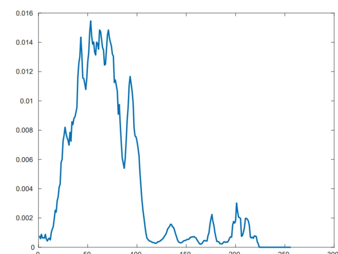
(c) $\mu = 2, \theta = 0.7$



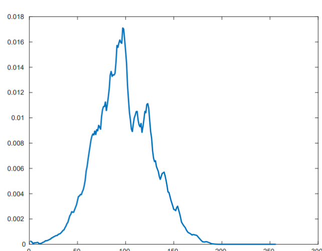
(d) $\mu = 3, \theta = 0.3$



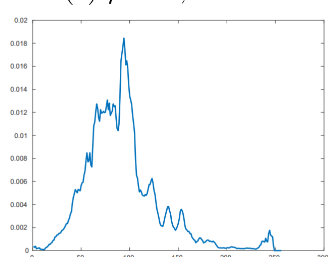
(e) $\mu = 3, \theta = 0.5$



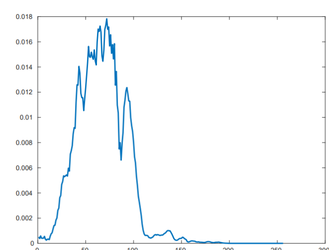
(f) $\mu = 3, \theta = 0.7$



(g) $\mu = 4, \theta = 0.3$



(h) $\mu = 4, \theta = 0.5$



(i) $\mu = 4, \theta = 0.7$

Figure 7: PDF of wealth ($z_t = u(c_t) - u(c_{t-1})$)

As a result, the wealth is more concentrated on the right tail and skewed to the right. But in doing so, they experience the polarisation with squeezed middle group and spikes at the top. And in doing so, as Inverted Pareto coeffficients imply, the level of accumulated wealth within the top group is more equal or as unequal as the benchmark Aiyagari model while the whole distribution being skewed to the right with thicker tail.

One might question whether the higher inequality comes not from artificial jaggedness. As the increase in the state space (due to the presence of additional endogenous state variable of past consumption) causes the numerical solution to become more approximative, that is, it contains more approximation error, the PDFs of modified model must have more jaggedness, as shown in Figures 5 and 7, compared to the benchmark in Figure 3.[21] However, as we can see in Figures 5 and 7, there is a clear formation of the bimodal distribution along with θ, consistently across the values of μ, showing its economic significance attributed to model dynamics, not random jaggedness.

2.6.4 Combined effect with risk aversion

On a separate note, this tendency toward higher income/wealth inequality along the degree of loss aversion is aggravated when combined with risk aversion, but in a hump-shaped rather than linear way. The CRRA coefficient μ in the baseline utility increases the Gini income / wealth coefficients, especially when $\mu = 2$, across the majority of the values of θ (Tables 6, Figure 2). However, this feature does not appear when $\theta = 0$ or in the original Aiyagari model as shown in Figure 3, where CDF and PDF in various values of μ appear identical. These suggest

[21] This is the same reason for why the results where $\theta = 0$ are different from the benchmark, as explained in Note 2) under Table 4.

that loss aversion makes the risk aversion function work in another dimension to generate income/wealth inequality. Risk aversion takes a role *only through* loss aversion, rather differently by different specifications of z_t.

One reason for such a difference by specifications could be found in the functional form of the CRRA function. For example, when it is additively separated over two variables c_1 and c_2, the marginal rate of substitution between two variables become the ratio of themselves exponential to μ, the CRRA coefficient.

$$U = u(c_1) + u(c_2) = \frac{c_1^{1-\mu}}{1-\mu} + \frac{c_2^{1-\mu}}{1-\mu}$$

$$MRS_{12} = \frac{MU_2}{MU_1} = \frac{u'(c_2)}{u'(c_1)} = \Big(\frac{c_1}{c_2}\Big)^{\mu} \tag{18}$$

Hence, μ is like the elasticity of MRS to the ratio of consumption. The larger μ, the more MRS moves for the same amount of change in the consumption ratio. Having said this, let z_t take a similar ratio form instead of the differential form, so $z_t = \frac{u(c_t)}{u(c_{t-1})} = \Big(\frac{c_t}{c_{t-1}}\Big)^{1-\mu}$ compared to $z_t = \frac{c_t}{c_{t-1}}$. Then, if μ is large, the change of $z_t = \frac{u(c_t)}{u(c_{t-1})}$ is greater than $z_t = \frac{c_t}{c_{t-1}}$ for the same change in consumption from the reference point (past consumption). That is, the agents perceive the gain/loss more when $z_t = \frac{u(c_t)}{u(c_{t-1})}$ than when $z_t = \frac{c_t}{c_{t-1}}$. Furthermore, as the loss aversion utility function $v(z_t)$ increases in z_t in both specifications, we can say that the role of loss aversion in $z_t = \frac{u(c_t)}{u(c_{t-1})}$ in generating inequality is greater than in $z_t = \frac{c_t}{c_{t-1}}$.

Taking z in the differential specification used in this chapter, first normalise it by its reference point:

$$z_t = u(c_t) - u(c_{t-1})$$

$$z'_t = \frac{z_t}{u(c_{t-1})} = \frac{u(c_t)}{u(c_{t-1})} - 1 \tag{19}$$

$$z_t = c_t - c_{t-1}$$

$$z'_t = \frac{z_t}{c_{t-1}} = \frac{c_t}{c_{t-1}} - 1 \tag{20}$$

Equations (17) and (18) show that z_t in the differential forms should have the characteristics as z in the ratio forms. And it explains why $z_t = u(c_t) - u(c_{t-1})$ has a higher income and wealth inequality than $z_t = c_t - c_{t-1}$ at $\mu = 3$ rather than at $\mu = 2$.

Lastly, fixing all other variables including the CARA coefficients and θ, hump-shapes over μ arise because when loss averse agents become extremely risk averse, they are eager to save up to their maximum, which then insures them earlier than in other scenarios, so they are relatively equal in income/wealth at the end of the steady state. The important implication here is that although loss aversion is by nature different from risk aversion, it appears to have some correlation with risk aversion when it functions as a motive for precautionary savings. [22] Furthermore, if the agents know that more utility occurs when the consumption is not reduced compared to when it increases, then their β might become meaningless (independent) or at least lose its power to allow a large amount of asset accumulation. However, providing more details on this issue is beyond the research of this chapter.

[22]This point might be related to some of the wealthy making risky decisions in their investments when there is a stochastic process in the asset return. As they are insured not to lose their consumption level by such investment, they can feature a risk-loving behaviour in their investment decisions, which could bring about even higher income/wealth inequality depending on the realised return.

2.6.5 Empirical and policy implication

To summarsie, this chapter introduces the loss aversion into the standard Aiyagari model to examine its effect on the precautionary savings and income/wealth inequality through simulation. The degree of loss aversion is determined by the index of loss aversion which is calibrated by the literature. Its functional form, CARA is chosen following the literature as it meets the conditions to be loss aversion function. The weight of CARA loss aversion function in the whole utility function, denoted as θ, measures how much it takes part in the whole preference, thus the size of the cause of loss aversion for the wealth inequality. The equilibrium is achieved in the way it satisfies the conditions for the existence of general equilibrium according to Acikgoz (2018). The simulation at the stationary equilibrium shows the desired results such that when θ is experimented over different values, it generates higher wealth inequality than Aiyagari (1994). In addition, the rich turn out to do more precautionary savings than the poor, and as a result of it, the capital distribution features a bimodal shape. Separately, simulation results over μ imply that loss aversion affects wealth inequality in correlation with risk aversion in this model framework.

Higher wealth Gini coefficients in the results fit existing empirical results. According to Kuhn and J. Rios-Rull (2020) as discussed in Section 1.3, the Gini coefficient in the US is 0.65 for earnings, 0.57 for income, and 0.85 for wealth. As already explained, the earnings Gini coefficient mainly comes from the exogenous shock process in this chapter, due to the model setting with inelastic labour supply. The income Gini coefficient is mostly about 0.4-0.6, matching well their results. In particular, it is 0.56 when $(\mu, \theta) = (3, 0.7), (4, 0.7)$ for $z_t = c_t - c_{t-1}$, and 0.56, 0.56, 0.58 when $(\mu, \theta) = (2, 0.5), (2, 0.7), (3, 0.7)$ respectively for $z_t = u(c_t) - u(c_{t-1})$.

Considering that the earnings inequality in this chapter is smaller than Kuhn and J. Rios-Rull (2020), and there are other income sources in their paper than private interest income as in this chapter, such as private and public transfer taking 14 percent and business income taking 13.8 percent, it generates quite a good match. Finally, the Gini wealth coefficient in Kuhn and J. Rios-Rull (2020) is 0.85 and the closest ones in this chapter are 0.72 when $(\mu, \theta) = (2, 0.7)$ for $z_t = c_t - c_{t-1}$, and 0.7 and 0.73 when $(\mu, \theta) = (2, 0.7)$ and $(\mu, \theta) = (3, 0.7)$, respectively, for $z_t = u(c_t) - u(c_{t-1})$.

Considering that the income and wealth Gini coefficients are generated only by private saving decisions with risk-free interest income in this model, we can argue that there will be even larger wealth inequality with risky assets (for different rate of returns) in portfolio, showing even clearer bimodal shape in the long term distribution, where we should have not reached yet. In other words, the model shows the long term stationary equilibrium of polarisation, which will be even more extreme with additional elements in reality, driven by heterogeneous saving behaviour due to loss aversion. And this is what Section 1.3 casts a question about, where the difference in the savings rate, as one of main reasons for wealth inequality, derives from. In conclusion, the model has realistic explanatory power: Loss aversion is a valid additional cause for the higher wealth inequality in the Aiyagari model especially among the wealthier as we observe in the data, and it is well reflected in my specification weighted by θ in terms of modelling.

This result reinforces the importance of the policy implications of the Aiyagari model, although it is beyond the research question of this chapter. That is, it implies that if the financial market is deepened to insure agents from idiosyncratic shock, the need for the precautionary savings of agents with loss aversion will decrease more significantly than those without it (like in the original Aiyagari model), especially among the wealthier. Hence, the effect of the policy will be larger than the original Aiyagari model, more persuasively suggesting the direction to which the financial market needs to be developed. Redistribution policy by progressive tax and/or transfer could be understood in this regard, too. On the other hand, the policy effort to raise the savings rate, financial market participation, and the rate of return to investment among the middle and the bottom will also alleviate the wealth inequality although it will not change the financial friction present in the market or the idiosyncratic shock itself. Examples on this side would be an improvement in the public/private pension schemes (in terms of accrual rate or pension rate of return) and asset portfolio development, etc.

2.7 Conclusion

To sum up, in attempt to address the gap in wealth inequality between the theoretical Aiyagari (1994) model and the empirical data, this chapter modifies the households' preference by incorporating the loss aversion in additive form. Exploring the properties of loss aversion, it suggests CARA utility function for gain/loss function and chooses past consumption as the reference point, based on relevant literature. The gain/loss is specified as either the difference between the present and past consumption or the difference between the utility of those consumptions. The labour supply is assumed to be inelastic. Having developed the model as a heterogeneous agent general equilibrium model, it shows how this model can have a stationary general equilibrium following Acikgoz (2018), proposing a novel condition to satisfy (Proposition 1). Then it computationally simulates the model to analyse the state of equilibrium, both aggregate and distributional variables, compared to the original Aiyagari model. In doing so, it follows Aiyagari (1994) for basic calibration, decides the CARA coefficients the way the loss aversion index solely takes the role of asymmetry in the loss aversion function, ensures that the (derived) necessary equilibrium conditions are satisfied, and experiments over the range of values of the loss aversion weight and the CRRA coefficient.

As a result, it shows wealth inequality is generated by large with loss aversion in both specifications, even with small earnings inequality. Although the aggregate savings rate and the interest rate do not move as anticipated, the underlying distributional characteristics suggest some implications why loss aversion causes higher wealth inequality than the original Aiagari model; it generates additional precautionary savings motive especially among the wealthier. PDFs present additional spike(s) in the upper and lower parts shaping bimodal distribution, which are distinctive enough to argue that loss aversion polarise agents in generating higher wealth inequality in the long term, with squeezed middle group. Inverted Pareto coefficients imply that within the very top of this distribution, the wealth dispersion can even decrease. Furthermore, we could even argue that it suggests that the idea of two types of agents (like in the TANK models) might not be just an arbitrary assumption. In relation to this, the next chapter starts with households agents being ex-ante heterogeneous with different level of income and wealth from the initial state. Then it develops a TANK model in which two types of agents as well as their fraction are endogenously determined, to discuss what consequences it makes in the aggregate as well as distributional variables in the face of economic shock.

As an extra note, it is also found that the risk aversion coefficient affects income/wealth inequality but only in combination with loss aversion, leaving an interesting implication for future research. Another extension of this research could be the labour market, as long as the model still satisfies the equilibrium conditions. Overall, the results support previous empirical studies (Kuhn and J.

Rios-Rull 2020) on inequality quite well, drawn by additional precautionary savings motive especially among the wealthier, demonstrating its explanatory power in generating inequality commonly observed today. Lastly, it leaves the policy implication such that the policy effort to address the market friction imposed by the borrowing constraint in the face of idiosyncratic shock will be larger than the original Aiyagari model when taking account of the impact of loss aversion on the wealth inequality.

Chapter 3

Distributional Impact of Inflation

3.1 Introduction

This chapter was motivated by the recent increase in the prices of energy, raw materials, and food since the war by Russia against Ukraine. It was called a cost of living crisis that has affected many households, especially those with lower incomes. It is also featured by a supply-side shock caused by firms' raising mark-ups. As these are essential goods for living that are hard to reduce, some households especially farther in the low end have been even worse off from the marginal situation, in terms of both income and consumption. Inequality has worsened, while the aggregate economy has contracted.

It shows that the properties of essential goods are the factors that affect the recent economic phenomenon following its increase in price. In particular, it implies that there is little change in demand due to its small price elasticity, so when the price is solely determined by producers, the welfare cost to the households happens differently by their income. It also implies that there is a channel between the essential sector and the aggregate economy.

To address these, this chapter first analyses the households problem by splitting the types of goods into essential and normal goods. By introducing a quasi-linear utility function, we can not only make the preference structurally different for each type of goods but also differentiate the types of households by deriving threshold income; the poor (constrained) households and the rich (unconstrained). In consequence, there are three different policy functions, i.e. the poor's essential consumption, the rich's essential consumption, and the rich's normal consumption. Although the idea of two household types originates from the existing TANK model, it is novel that the household types are endogenously determined by the threshold income instead of exogenous probability parameter. In other words, the fraction is endogenised.

The production side is also divided into two sectors of essential goods and normal goods, and it is assumed that both sectors are identical except for the parameter values and demand functions. It is also assumed that they follow the standard NK model with price rigidity as in other TANK papers. Two sectors are aggregated by the size of the market.

Having the modeling strategy as above, this chapter investigates the impact of inflation through the mark-up shock of each sector and the total productivity shock on the consumption, employment and demand of each sector as well as the whole economy. It also looks at the change in consumption, income and wealth inequality as a result of it. In doing so, it analytically shows that the assumptions made for the essential goods, such as its low price elasticity and priority over the normal goods by which the poor can only consume the essentials, play an important role in shrinking consumption and increasing inequality. It also discovers how price stickiness and risk aversion affect such consequences through this channel, among other parameters. It then briefly suggests the need for an alternative policy approach that considers sectoral characteristics.

Its structure is as follows. Section 3.2 reviews the literature, Section 3.3 describes the households' problem, and Section 3.4 explains the firms' problem. In Section 3.5 the general symmetric equilibrium is defined and the variables are log linearised. Then impacts of each shock and the factors affecting them are analysed in Section 3.6. Section 3.7 concludes.

3.2 Literature Review

First, this chapter lies in the line of recent innovation in heterogeneous models. In an effort to develop the traditional Bewley-Hugget-Aiyagari model for policy implementation, Heterogeneous agent New Keynesian Model (HANK) model was suggested by Kaplan, Moll, and Violante (2018), where they asserted that the monetary expansion brings about the increase in the aggregate demand through income effect from the increased employment by firms rather than households' inter-temporal decision. They introduced continuous time and solved numerically to analyse the distribution at the equilibrium. Since then, a variety of topics have been conducted using a variety of HANK models: monetary policy transmission (Auclert 2018; Kaplan, Moll, and Violante 2018), the effects of transfer payment (Oh and Reis 2012), deleveraging and liquidity traps (Lorenzoni and Guerrieri 2017), dynamics of inequality (Auclert and Rognile 2018), heterogeneous portfolios (Luetticke 2021), fiscal multipliers (Hagedorn, Manovskii, and Mitman 2019), automatic stabilizers (McCay and R. Reis 2016, 2021).

Among them, Auclert (2018) showed how HANK model can be analytically represented in his analysis of the transmission channel of monetary expansions. On the other hand, Bilbiie (2008) suggested the TANK model, using discrete time and preserving tractability for analysis. It was shed a new light on as an alternative to resolve the drawbacks of HANK models, i.e. complexity and heavy computation. And it has been understood that the resulting features have the same implications as HANK (Cantore and Freund 2021; Debortoli and Gali 2017). Debortoli and Gali (2017), for example, elucidated that TANK captures well the aggregate implications of HANK since the supply blocks are the same in both models as NK models with divine coincidence.[1] It is particularly so under the policy rule of price stability. Bilbiie and Ragot (2021) investigated with the TANK model to conclude that the cost of price stability is beyond inflation vs. real activity when constrained households depend on liquidity because it hinders the value of consumption insurance. Most recently, Bilbiie (2025) studied the cyclical features of inequality and risk and showed that if the risk is countercyclical, the need for precautionary savings decreases, thus the effect of monetary policy amplifies aggregate demand. Given that, to ensure the determinancy of monetary policy and rule out the forward-guidance puzzle, he suggested price-level targeting and debt-quantity targeting.

Whilst in line with it, the most novel and distinctive feature of this chapter is that the fraction for two types of households is not an exogenous parameter but is endogenously derived from the price of essential goods. That is, the derivation of fraction of households is related to the sectoral differentiation of the economy, i.e., essential and normal goods market. By doing so, this chapter attempts to build up an explicit mechanism in which the poor (constrained) households become constrained by structure instead of assumption. Such sectoral analysis was also attempted by Chan, Diz, and Kanngiesser (2024) but they differentiated production inputs (energy from labour) not consumption goods, also unrelated to endogenising the fraction. The contribution of this chapter lies in that it deepens

[1] Policy that stabilises the inflation also reduces the output gap

existing models with respect to how such a fraction is determined for households to behave differently. In addition, properties of quasi-linear utility function are utilised to establish such a channel. The quasilinear utility function is used largely in the incentive literature and public economics, but not much in macroeconomics. Just as one relevant research, L. Maliar and S. Maliar (2003) have shown that the general equilibrium exists if the linear variable of quasi-linear utility function has heterogeneity. My approach is different from them in the sense that I set up a two-step problem where all ex-ante heterogeneous households first solve the problem with quasi-linear preference to determine essential consumption as well as their types. The linear variable is taken as the budget constraint for the next problem of normal goods. To my best knowledge, this is the first research applying quasi-linear utility function to the model with heterogeneity, as well as to endogenising the fraction of the households type.

Other distinct features include that while the distributional impact of inflation was mostly analysed from the demand side (Alan and Ragot 2010; Doepke and Schneider 2005; Gotllieb 2012), this chapter looks into the supply shocks such as mark-ups and total productivity. Chan, Diz, and Kanngiesser (2024) were the most similar in this sense, as they took mark-up shocks in the energy sector and total productivity shock. That is, the (traditional) distortion in the NK models, not the idiosyncratic shock, can generate the inequality when there is heterogeneity. Considering the reduction in the essential consumption as a result of positive mark-up shock, which makes aggregate demand shrink to that extent, we can say that the result of this chapter reinforces the complementarity of high mark-up and lack of consumption insurance. (Bilbiie and Ragot 2021) Lastly, on top of the alternative policy rules explored and suggested (Bilbiie 2025; Bilbiie and Ragot 2021), this chapter indicates that there is a need for a different approach by sectors.

3.3 Households

3.3.1 Model assumptions

The model assumes that there are two types of agents, the poor and the rich. However, unlike existing TANK models (Chan, Diz, and Kanngiesser 2024), the fraction of each type is not an exogenous assumption but is determined endogenously by a threshold income that is also endogenously derived within the model. This endogenisation is enabled by exploiting the property of quasi-linear utility function, as discussed in Section 3.3.2 and Section 3.3.3.

It is also assumed that there are two types of goods, i.e. essential goods, denoted as $C_{e,t}$, and normal goods, denoted as $C_{n,t}$. Essential goods are what agents must consume for living, for example, food, clothes, bills, fuel, and transport.[2] Normal goods refer to all the other goods agents can choose to consume, depending on their (pure) preference. Again, quasi-linear utility function enables them to be analysed in one preference, because it has two arguments in its additive form, e.g. a and b in $U = a + u(b)$, by which one can start to consume a if and only if b is fully consumed first. They stop consuming b when it no longer increases the utility.(Varian 1992, p.18) The originality of this chapter largely attributes to the property of quasi-linear utility function that behaves differently on each argument.

Regarding the initial state, it is assumed that there is a mass of *ex-ante heterogeneous* households $m \in [0, 1]$ in the economy, in income and wealth. Instead of a random fraction of the constrained and unconstrained households as in the TANK literature(Bilbiie 2008, 2025; Bilbiie and Ragot 2021; Chan, Diz, and Kanngiesser 2024), it is assumed that there is a random distribution given exogenously, denoted as $f_t(x)$, where x is income or wealth. $f_t(x)$ could be thought of as the resulting

[2] Rent and mortgage interest/repayment was excluded for simplification.

stationary distribution from the previous chapter, from which this chapter starts the discussion of the consequence of inequality. However, although it is considered to help understand the endogenous fraction by which the rich and the poor are segmented out of it, it is not the main interest in this chapter whether it evolves or not because agents do not need to know it, nor are there idiosyncratic shocks or borrowing constraints as in the previous chapter or other HANK literature. There is no shock entering the decision problem.

3.3.2 Quasi-linear utility function

I assume that all households have the same quasi-liner utility function as specified in Equation (21). By its structure, they first consume the essential goods $C_{e,t}$, then start to consume *the rest* of income, denoted by X_t. It may be an unfamiliar concept to have a variable such as X_t as an argument for the utility function. However, it is quite useful to analyse two types of agents in relation. The poor is defined as the constrained agents who can only consume the essential goods out of all their income. The rich is defined as the unconstrained agents who can consume more than essential goods. Hence, from the amount of their essential goods consumption under quasi-linear utility function, we can derive the condition for the threshold income under which the poor is placed. [3] In addition, by having X_t instead of normal consumption $C_{n,t}$ for the linear term of the quasi-linear utility function, we can practically have a choice variable that is compatible with linearity and can even solve the dynamic problem described below.

Once households solve this problem every period and determine $C_{e,t}$ as well as the type of agent they belong to, only the rich, unconstrained agents with $X_t > 0$

[3] In this model, the poor has no other option but to be 'hand-to-mouth', it is not their choice but they are forced to do so by structure.

move onto the next problem, the classical dynamic consumption-savings problem but subject to *the rest*, X_t, not Y_t. At this stage, they solve for the normal goods consumption, $C_{n,t}$, risk-free bonds, B_{t+1}. To note, it is not simply that they change the preference as they become the poor or the rich. And it is not about the different preferences on the same consumption goods. By the structure of quasi-linear utility function over two arguments, only the rich become able to be in a position to consume the normal goods against savings having satiated their needs for essential goods. Although there can be a transition between two types of agents, they first solve the same problem every period.

Formally, all households $m \in [0, 1]$ first solve the problem at the beginning of period t.

$$\max_{C_{e,t}, X_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_{e,t}, X_t) \tag{21}$$

$$\text{where} \quad U(C_{e,t}, X_t) = X_t + u(C_{e,t}), \quad u(C_{e,t}) = \frac{{C_{e,t}}^{1-\sigma} - 1}{1-\sigma} \quad (\sigma > 1)$$

$$s.t. \;\; P_{e,t} C_{e,t} + X_t = Y_t \equiv W_t N_t + (1 + R_t) B_t + D_t \tag{22}$$

$P_{e,t} > 0$: nominal price for essential goods $W_t > 0$: wage $N_t > 0$: labour supply

$R_t > 0$: nominal interest rate $B_t \geq 0$: risk-free bond $D_t \geq 0$: dividend

$U(C_{e,t}, X_t)$ is the quasi-linear utility function. Note that inside the quasi-linear utility function, $C_{e,t}$ follows its own Constant Relative Risk Aversion (CRRA) utility function with risk aversion coefficient σ, while X_t is linear. Households first earn their income from the labour earnings $W_t N_t$, but can also earn income from the purchase of bonds in the last period $(1 + R_t)B_t$, or the dividend distributed from the monopolistic firms (D_t), although those last two terms can be zero for some households.[4] β is a discount rate. N_t is the number of hours worked that is assumed to be identical across the agents in terms of productivity. I also assume that the wage W_t is determined by monopolistic firms, and households labour supply always meets the demand taking W_t as given, thus N_t is completely elastic.[5] Also, as this chapter looks into the distributional effect of inflation, all variables are nominal.

The rich's problem for normal consumption and savings is:

$$\max_{C_{n,t}, B_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(C_{n,t}) \tag{23}$$

$$v(C_{n,t}) = \frac{{C_{n,t}}^{1-\mu} - 1}{1 - \mu} \quad (\mu > 1, C_{n,t} > 0)$$

$$s.t. \quad P_{n,t} C_{n,t} + B_{t+1} = X_t = Y_t - {P_{e,t}}^{\frac{\sigma-1}{\sigma}} \tag{24}$$

As discussed earlier, only the rich, unconstrained households proceed to solve the next dynamic problem for normal goods, $C_{n,t}$, against savings through bond purchase B_{t+1}. They can also borrow, as well as earn dividend from the firms.

[4]Even some households initially with positive bond savings and dividend can also turn out to be the poor, i.e. forced to be hand-to-mouth, depending on the solution.

[5]Disutility from labour is not considered because if there was, labour supply will respond to the change of essential goods price, which will affect the income and consumption of both goods, as well as their inequalities. It will also affect ω_t, the fraction of the poor. Hence, such simplification is needed to look at the impact of preference structure with two goods through consumption/savings decision only, keeping tractability.

[6] They take another CRRA function $v(C_{n,t})$ with its own CRRA coefficient, μ, which may be different from σ. As shown in the budget constraint Equation (24), X_t becomes the new budget for the rich after consuming essential goods.

3.3.3 Two Households types

Before discussing the solution, let us analyse two important derivations from the properties of quasi-linear utility function and our assumptions in relation to its structure.

Threshold income. Due to the quasi-linear functional form of U_t, household agents cannot choose X_t, i.e. $X_t = 0$, until $C_{e,t}$ is consumed enough. And all income will be used to consume $C_{e,t}$ until then. Hence, from the budget constraint in Equation (22),

$$X_t = 0, \quad C_{e,t} = \frac{Y_t}{P_{e,t}} \tag{25}$$

Substituting this into the quasi-linear utility function $U(C_{e,t}, X_t)$ in Equation (21), we have

$$U(C_{e,t}, X_t) = u(C_{e,t}) = u\Big(\frac{Y_t}{P_{e,t}}\Big) = \frac{\left(\frac{Y_t}{P_{e,t}}\right)^{1-\sigma} - 1}{1-\sigma} \tag{26}$$

Taking the marginal utility with respect to the income Y_t from Equation (25),

[6] To note, the poor, in contrast, whether they previously had income from earnings only or interest payment/dividend, consume all of their income on the essential goods (Footnote 3). Therefore, they have no more access to the financial market, having earnings as the only source of income.

$$\frac{\partial U(C_{e,t}, X_t)}{\partial Y_t} = \frac{\partial u(C_{e,t})}{\partial Y_t} = (\frac{Y_t}{P_{e,t}})^{-\sigma} \cdot \frac{1}{P_{e,t}} = \frac{Y_t^{-\sigma}}{P_{e,t}^{1-\sigma}} \tag{27}$$

Equation (27) shows the effect of an increase in income on the utility when all income is consumed on $C_{e,t}$, which is a function of Y_t and $P_{e,t}$. As long as it is greater than 1, the agents will continue to spend all their income on $C_{e,t}$. As it converges to 1, they will stop consuming $C_{e,t}$ and move on to consuming X_t.[7]

From this exposition, we can derive a threshold income, $\bar{Y}_t$, the level of income from which agents have consumed enough $C_{e,t}$ so start to spend the remaining income, $Y_t - \bar{Y}_t$, on *the rest*, X_t.

$$\frac{\partial u(C_{e,t})}{\partial Y_t} = \frac{Y_t^{-\sigma}}{{P_{e,t}}^{1-\sigma}} = 1 \tag{28}$$

$$\therefore \bar{Y}_t = {P_{e,t}}^{\frac{\sigma-1}{\sigma}}$$

In other words, provided $P_{e,t}$ and the CRRA functional form for $u(\cdot)$ as given, we can determine the value of the threshold income, $\bar{Y}_t$.

Endogenous fraction. We can now define the poor, constrained households as a fraction of the households whose (initial) income is $Y_t \leq \bar{Y}_t$, and the rich, unconstrained households as those with $Y_t > \bar{Y}_t$. Recall the initial income distribution $f_t(Y_t)$, if its CDF is $F_t(Y_t)$, then the size of the fraction for the poor, denoted as ω_t, is the CDF value of the threshold income, i.e. $\omega_t = F_t(\bar{Y}_t)$. Hence,

[7] For related discussion, see Varian (1992)

ω_t is not an exogenous probability as in the existing TANK literature but is dependent on $\bar{Y}_t$, or $P_{e,t}$ (Equation (28)), so $\omega_t(P_{e,t}) = F_t(\bar{Y}_t(P_{e,t}))$ $(\omega_t'(P_{e,t}) > 0)$. Its time subscript t also implies that it is subject to change over time depending on $P_{e,t}$. Recall that $f_t(\cdot)$ is a random distribution given as an initial condition; while the initial heterogeneity in terms of the whole distribution of income/wealth is exogenously given, the fraction of the poor (constrained) versus the rich (unconstrained) is endogenously derived, dependent on the distribution and the essential price $P_{e,t}$. Also, it is important to note that while $f_t(\cdot)$ can change depending on the households' solving the problem, its stationarity is not required for the equilibrium as long as $\omega_t(P_{e,t})$ is stationary. The heterogeneity we are interested in is the fraction between two types of households, not the entire distribution. This is one of the originalities for which this chapter sits between the HANK and TANK literature. It will be revisited in Section 3.5.1.

If we rearrange the households set $m \in [0,1]$ to $m' \in [0,1]$ where they are realigned by the level of income from the lowest to the highest as in the income Lorenz curve, then $\omega_t(P_{e,t})$ becomes a quantile in the Lorenz curve below which the poor are defined and vice versa.[8] The poor are the set of agents m' such that $m' \in [0, \omega_t(P_{e,t})]$ and the rich are the set of agents m' such that $m' \in (\omega_t(P_{e,t}), 1]$.

3.3.4 Solutions

Agents first solve the problem in Equation (21-22). By the definition of threshold income and two types of households, the poor consume only the essential goods $C_{e,t}$. The rich can consume X_t in addition to fixed $\bar{C}_{e,t}$, because for the rich $C_{e,t}$ remains constant (Equation (29)), denoted as $\bar{C}_{e,t}$. X_t is not zero anymore but

[8] Let the Lorenz curve at time t be $L_t(p)$ for quantile $p \in [0,1]$. Then, $L_t(p) \equiv \frac{\int_0^{F^{-1}(p)} Y_f(Y_t)dY_t}{\mu}$ where μ is the average income. At threshold income $\bar{Y}_t$, Lorenz curve implies that $F^{-1}(p) = \bar{Y}_t$. Since $\omega_t = F(\bar{Y}_t)$, $p = \omega_t$.

solved by substituting $Y_{e,t}$ by $\bar{Y}_{e,t}$ as in Equation (28).

$$\bar{C}_{e,t} = \frac{\bar{Y}_t}{P_{e,t}} = P_{e,t}^{-\frac{1}{\sigma}} \tag{29}$$

$$X_t = Y_t - \bar{Y}_t = Y_t - P_{e,t}{}^{\frac{\sigma-1}{\sigma}} \tag{30}$$

The essential consumption for an individual agent in each type of households is

$$C^P_{e,t}(m') = \frac{Y_t(m')}{P_{e,t}} \qquad (\forall m' \in [0, \omega_t(P_{e,t})] \Leftrightarrow Y_t(m') \leq \bar{Y}_t) \tag{31}$$

$$C^R_{e,t}(m') = \bar{C}_{e,t} \qquad (\forall m' \in (\omega_t(P_{e,t}), 1] \Leftrightarrow Y_t(m') > \bar{Y}_t) \tag{32}$$

$Y_t(m')$ is the income of an individual household. Note that $C^R_{e,t}(m')$ is one fixed value of $\bar{C}_{e,t}$ for all $m' \in (\omega_t(P_{e,t}), 1]$ while $C^P_{e,t}$ varies between 0 and $\bar{C}_{e,t}$, depending on the poor's income given the price $P_{e,t}$.

Normal consumption is made only by the rich, unconstrained households. Denoting the individual rich's normal consumption as $C^R_{n,t}(m') \quad (m' \in (\omega_t(P_{e,t}), 1])$, it is determined by solving the classical dynamic model in Equation (24), with the below Euler Equation:

$$\beta\, \mathbb{E}_t \left[\left(\frac{C^R_{n,t}(m')}{C^R_{n,t+1}(m')} \right)^{\mu} \frac{1 + R_t}{\Pi_{n,t+1|t}} \right] = 1 \tag{33}$$

$$\text{where} \qquad \Pi_{n,t+1|t} \equiv \frac{P_{n,t+1}}{P_{n,t}}$$

Aggregating the consumption of each goods for each type of households, as well as the relevant discussions, are provided in Section 3.5.1.

3.4 Firms

Both essential goods and normal goods are produced by monopolistically competitive firms within their own market. However, two markets are completely separate so there is no competition between two goods markets. Similarly, they are sold by competitive final goods packers but again their markets are separate between two goods. Then it follows the standard NK model.

Final goods packers produce the aggregate final essential or normal goods, i.e. $Z_{e,t}$ or $Z_{n,t}$, by combining a continuum of varieties $Z_{e,t}(i)$ or $Z_{n,t}(j)$, respectively $(i, j \in [0, 1])$. The final product packers take the CES production function with the substitution elasticity, denoted by ϵ_e or ϵ_n.

$$Z_{e,t} = \left[\int_0^1 (Z_{e,t}(i))^{1-\frac{1}{\epsilon_e}} di \right]^{\frac{\epsilon_e}{\epsilon_e - 1}} \tag{34}$$

$$Z_{n,t} = \left[\int_0^1 (Z_{n,t}(j))^{1-\frac{1}{\epsilon_n}} dj \right]^{\frac{\epsilon_n}{\epsilon_n - 1}} \tag{35}$$

$$\text{where} \quad \epsilon_e, \epsilon_n > 1$$

Minimising costs implies a demand function for each variety.

$$Z_{e,t}(i) = \left(\frac{P_{e,t}(i)}{P_{e,t}}\right)^{-\epsilon_e} Z_{e,t} \tag{36}$$

$$Z_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\epsilon_n} Z_{n,t} \tag{37}$$

where $P_{e,t} \equiv \left(\int_0^1 (P_{e,t}(i))^{1-\epsilon_e} di\right)^{\frac{1}{1-\epsilon_e}}$, $P_{n,t} \equiv \left(\int_0^1 (P_{n,t}(j))^{1-\epsilon_n} dj\right)^{\frac{1}{1-\epsilon_n}}$

Final goods producers are monopolistically competitive and each of them produces each variety of final goods, $Z_{e,t}(i)$ and $Z_{n,t}(j)$, using the labour, deciding on the wage by the marginal product of labour. They are subject to the nominal rigidities a la Calvo, with the probability that they reset their price as $(1 - \rho_e)$ and $(1 - \rho_n)$ respectively. [9] They are also subject to the demand schedule from Equation (34) and (35), i.e. they produce to meet the demand instantly. Having those above as ingredients, each of them solves inter-temporal price setting problem. As the solution process is identical for each type of goods whose market is independent of each other, I show the essential goods case first, then only replicate the results for the normal goods.

Essential goods firms produce $Z_{e,t}(i)$ following the function below:

$$Z_{e,t}(i) = exp(\varepsilon_t^{TFP}) N_{e,t}(i)^{1-\alpha} \tag{38}$$

[9] Wage setting process is not assumed in this chapter because 1) it conflicts with the labour market assumptions made in this chapter to focus on the impact of preference structure, 2) market friction is assumed by Calvo pricing as the (basic) NK model, 3) for tractability as the labour market dynamics is not the focus of this chapter. In relation to the second point, Blanchard and Gali (2005) has shown that the divine coincidence of the basic NK model disappears when it is extended to have the wage rigidity. Recent Bilbiie (2008) used Rotemberg pricing that does not take account of wage setting process, either.

where ε_t^{TFP} is the productivity shock, $N_{e,t}(i)$ is the labour firm i used for the production of essential goods with coefficient $1-\alpha$.

And they maximise the profit:

$$\max_{P^*_{e,t}(i)} E_t\Sigma_{s=0}^{\infty}\frac{\rho_e^s}{1+R_{t,t+s}}(P^*_{e,t}(i)Z_{e,t+s|t}(i)-\Omega_{e,t+s}(Z_{e,t+s|t}(i))) \tag{39}$$

$$\text{where} \quad Z_{e,t+s|t}(i)=\left(\frac{P^*_{e,t}(i)}{P_{e,t+s}}\right)^{-\epsilon_e}Z_{e,t+s}$$

The first order condition yields

$$E_t\Sigma_{s=0}^{\infty}\frac{\rho_e^s}{1+R_{t,t+s}}\left(Z_{e,t+s|t}(i)-\epsilon_e Z_{e,t+s|t}(i)+\epsilon_e\Omega'_{e,t+s}(Z_{e,t+s|t}(i))\frac{Z_{e,t+s|t}(i)}{P^*_{e,t}(i)}\right)=0 \tag{40}$$

$$\Rightarrow \quad E_t\Sigma_{s=0}^{\infty}\frac{\rho_e^s}{1+R_{t,t+s}}Z_{e,t+s|t}(i)\left(P^*_{e,t}(i)-\frac{\epsilon_e}{\epsilon_e-1}\Omega'_{e,t+s}(Z_{e,t+s|t}(i))\right)=0$$

Rewriting

$$E_t\Sigma_{s=0}^{\infty}\frac{\rho_e^s}{1+R_{t,t+s}}Z_{e,t+s|t}(i)\left(\frac{P^*_{e,t}(i)}{P_{e,t-1}}-\frac{\epsilon_e}{\epsilon_e-1}\Pi_{e,t+s|t-1}\mathcal{MC}_{e,t+s|t}\right)=0 \tag{41}$$

$$\text{where} \quad \Pi_{e,t+s|t-1}=P_{e,t+s}/P_{e,t-1}$$

$$\Omega_{e,t+s}(Z_{e,t+s|t}(i))=\tau_{e,t+s}\left[W_{t+s}\left(\frac{Z_{e,t+s|t}(i)}{exp(\varepsilon_t^{TFP})}\right)^{\frac{1}{1-\alpha}}\right]$$

$$\left(\tau_{e,t+s}\equiv\tau_e\varepsilon_{e,t+s}^{\mathcal{M}} \ , \quad \mathcal{M}_e\equiv\frac{\epsilon_e}{1-\epsilon_e}\right)$$

$$\mathcal{MC}_{e,t+s|t} = \Omega'(Z_{e,t+s|t}(i))/P_{t+s}$$
$$= \tau_{e,t+s}\left[\frac{W_{t+s}}{P_{t+s}} \cdot \frac{1}{1-\alpha} \cdot \frac{1}{exp(\varepsilon_t^{TFP})}\left(\frac{Z_{e,t|t+s}(i)}{\exp(\varepsilon_t^{TFP})}\right)^{\frac{\alpha}{1-\alpha}}\right]$$

$\Omega_{e,t+s}(Z_{e,t+s|t}(i))$ is the total cost function for a unit of final essential goods which is determined by the demand function from the final essential goods packers at the time of $t+s$, since the producing firms are forward-looking for the price setting. It has $\tau_{e,t+s}$, a shock to the total cost that is isomorphic to a shock to the price markup, $\varepsilon^{\mathcal{M}}_{e,t+s}$(Chan, Diz, and Kanngiesser 2024). $\mathcal{MC}_{e,t+s|t}$ is the corresponding marginal cost in real terms.

3.5 Equilibrium

3.5.1 Aggregation and market clearing

Essential consumption

I first aggregate essential consumption for the poor and the rich

$$C_{e,t}^P = \int_0^{\omega_t(P_{e,t})} \frac{Y_t(m')}{P_{e,t}} dm' \qquad (Y_t^P \le \bar{Y}_t) \tag{42}$$

$$C_{e,t}^R = \int_{\omega_t(P_{e,t})}^1 \bar{C}_{e,t} dm' = \int_{\omega_t(P_{e,t})}^1 \frac{\bar{Y}_t}{P_{e,t}} dm' = \int_{\omega_t(P_{e,t})}^1 P_{e,t}^{-\frac{1}{\sigma}} dm' = (1-\omega_t(P_{e,t}))P_{e,t}^{-\frac{1}{\sigma}} \tag{43}$$

The whole essential consumption is the sum of $C^P_{e,t}$ and $C^R_{e,t}$,

$$
\begin{aligned}
C_{e,t} &= C^P_{e,t} + C^R_{e,t} \\
&= \int_0^{\omega_t(P_{e,t})} \frac{Y^P_t(m')}{P_{e,t}} dm' + (1-\omega_t(P_{e,t}))P_{e,t}^{-\frac{1}{\sigma}}
\end{aligned} \tag{44}
$$

There are a few points to note at this stage. First, $\omega_t(P_{e,t})$ is not used as the probability or weight to aggregate essential consumption for the poor, unlike the TANK literature. Being endogenously derived from the threshold income that is by itself a function of essential goods price, and out of exogenous income distribution, essential consumption of the individual poor can vary depending on the income distribution. This feature breaks the link between the fraction of poor households and the fraction of essential consumption made by them; that is, the fact that the fraction of poor households is ω_t does not necessarily mean that the share of essential consumption of the poor is ω_t. Second, while ω_t increases and $\bar{C}_{e,t}$ decreases by the change of $P_{e,t}$, whether the total $C_{e,t}$ increases or decreases depends on the size of each term in Equation (44). It will be analysed in the log-linearised form in the next section.

Normal consumption

Normal consumption is only made by the rich.

$$
C_{n,t} = C^R_{n,t} = \int_{\omega_t(P_{e,t})}^1 C^R_{n,t}(m') \qquad \forall m' \in (\omega_{e,t}(P_{e,t})), 1] \tag{45}
$$

More points to discuss are as follows: first, total normal consumption is expressed as the sum of individual normal consumption, again allowing the possibility that it can vary depending on the distribution. Second, the individual agent's position within and/or between the poor and the rich is also changeable depending on their labour earnings and savings decision each period, in addition to $P_{e,t}$ that determines threshold income. [10] However, as long as $\omega_t(P_{e,t})$ remains the same, such transition or change of distribution is not taken account into the equilibrium. Third, in spite of the possibility of such a transition, there is no precautionary savings among households in this model, with market friction occurring only in the price stickiness (no borrowing constraint). [11] That is, although agents are perfectly forward-looking, due to the sticky price, some of the rich can fall into the poor. Moreover, by structure, the poor consume all of their income on the essential goods unless they become the rich by the increase in their earnings above the threshold income.

Given above, market clearing conditions for the essential goods are as following.

Individual essential goods i

$$Z_{e,t}(i) = \left(\frac{P_{e,t}(i)}{P_{e,t}}\right)^{-\epsilon_e} Z_{e,t} \tag{46}$$

Final essential composite goods

$$C_{e,t} = Z_{e,t} = \left[\int_0^1 (Z_{e,t}(i))^{1-\frac{1}{\epsilon_e}} di\right]^{\frac{\epsilon_e}{\epsilon_e - 1}} \tag{47}$$

[10] Once they belong to the poor the only source of income becomes to be earnings because the poor do not have access to the financial market.

[11] Note that the fact that the poor is constrained without access to the financial market is the result of solving their problem (a result of modelling decision), rather than explicit assumption they take into account for decision.

Labour market for the essential goods sector

$$\begin{aligned} N_{e,t} &= \int_0^1 N_{e,t}(i)di \\ &= \int_0^1 \left(\frac{Z_{e,t}(i)}{exp(\varepsilon_t^{TFP})}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Z_{e,t}}{exp(\varepsilon_t^{TFP})}\right)^{\frac{1}{1-\alpha}} \left(\int_0^1 \left(\frac{P_{e,t}(i)}{P_{e,t}}\right)^{-\frac{\epsilon_e}{1-\alpha}} di\right) \end{aligned} \tag{48}$$

To note, firms are assumed to equally distribute the labour demand between the poor and the rich [12] $N_{e,t} = N_{e,t}^P = N_{e,t}^R$.

Price dispersion of the essential goods sector

$$v_{e,t} = \left(\int_0^1 \left(\frac{P_{e,t}(i)}{P_{e,t}}\right)^{-\frac{\epsilon_e}{1-\alpha}} di\right) \leq 1 \tag{49}$$

[13]

Aggregate price index of the essential goods sector

$$P_{e,t} = \left(\rho_e P_{e,t-1}^{1-\epsilon_e} + (1-\rho_e)(P_{e,t}^*)^{1-\epsilon_e}\right)^{\frac{1}{1-\epsilon_e}} \tag{50}$$

[12]In relation to footnote 9, given this assumption and the completely elastic labour supply, income of the poor from the next period is likely to be the same. However, we do not continue to investigate this as long as $\omega_t(P_{e,t})$ remains the same.

[13]From the last expression in Equation (48) we can derive

$$Z_{e,t} = exp(\varepsilon_t^{TFP}) N_{e,t}^{(1-\alpha)} \left(\int_0^1 \left(\frac{P_{e,t}(i)}{P_{e,t}}\right)^{-\frac{\epsilon_e}{1-\alpha}} di\right)$$

which has the price dispersion in the last term, the distortion from sticky price.

It is the weighted average of lagged prices and the prices set by the price adjusters.

Assuming the normal goods packers and producers have the same technology, pricing strategy and market properties as above, it is summarised as follows:

Individual normal goods j

$$Z_{n,t}(i) = \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\epsilon_n} Z_{n,t} \tag{51}$$

Final normal composite goods

$$C_{n,t} = Z_{n,t} = \left[\int_0^1 (Z_{n,t}(j))^{1-\frac{1}{\epsilon_n}} dj\right]^{\frac{\epsilon_n}{\epsilon_n - 1}} \tag{52}$$

Labour market for the normal goods sector

$$N_{n,t} = \left(\frac{Z_{n,t}}{exp(\varepsilon_t^{TFP})}\right)^{\frac{1}{1-\alpha}} \left(\int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\frac{\epsilon_n}{1-\alpha}} dj\right) \tag{53}$$

$$(N_{n.t} = N_{n,t}^P = N_{n,t}^R)$$

Price dispersion of the normal goods sector

$$v_{n,t} = \left(\int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}}\right)^{-\frac{\epsilon_n}{1-\alpha}} dj\right) \leq 1 \tag{54}$$

Aggregate price index of the normal goods sector

$$P_{n,t} = \left(\rho_n P_{n,t-1}^{1-\epsilon_n} + (1-\rho_n)(P_{n,t}^*)^{1-\epsilon_n}\right)^{\frac{1}{1-\epsilon_n}} \tag{55}$$

Other market clearing conditions

$$B_t = 0 \tag{56}$$

$$D_{e,t}(i) = P_{e,t}(i)Z_{e,t}(i) - \Omega_{e,t}(Z_{e,t}(i)) \tag{57}$$

$$D_{n,t}(i) = P_{n,t}(i)Z_{n,t}(i) - \Omega_{n,t}(Z_{n,t}(i)) \tag{58}$$

3.5.2 Equilibrium

Symmetric equilibrium

General equilibrium is defined by an allocation of

$$\{C_{e,t}, C_{n,t}, N_t, Z_{e,t}, Z_{n,t}, Z_{e,t}(i), Z_{n,t}(j), B_{t+1}\}_{t=0}^{\infty}$$

given non-negative prices $\{P_{e,t}, P_{n,t}, P_{e,t}(i), P_{n,t}(j), R_t, W_t\}_{t=0}^{\infty}$, such that (i) both the poor and the rich households maximise utilities subject to their budget constraint, (ii) final goods packers and the producers maximise their profits, (iv) all firms that are given the opportunity to change prices choose the same price, (v) the fraction $\omega_t(P_{e,t})$ is constant, and (vi) all markets clear.

Condition (iv) is called as the symmetric equilibrium and enables firms to collapse to one representative. One thing to note for this model is that it does not apply to the households income distribution because it is assumed to be exogenously given, so cannot be identical across the agents. We cannot collapse them into a single variable, and they are taken into account in the log-linearisation in the next section.

Consumption gap

Borrowing fron Chan, Diz, and Kanngiesser (2024), consumption gap is defined as the ratio of consumption between the poor and the rich in equilibrium.

$$\begin{aligned}\Gamma_t^{CON} &\equiv C_t^R / C_t^P \\ &= (C_{e,t}^R + C_{n,t}^R)/C_{e,t}^P\end{aligned} \tag{59}$$

Furthermore, consumption gap for the essential good is the ratio of essential consumption between the poor and the rich.

$$\begin{aligned}\Gamma_{e,t}^{CON} &\equiv C_{e,t}^R / C_{e,t}^P \\ &= P_{e,t}^{-\frac{1}{\sigma}} / \frac{Y_t^P}{P_{e,t}}\end{aligned} \tag{60}$$

Income gap

Income gap is the ratio between the income of the rich and the poor, and is related to the consumption gap Γ_t^{CON}. [14]

[14] With respect to $Y_t^P = W_t N_t$, it would not be completely correct specification of Y_t^P because every period there can be some households who fall from the rich to the poor, whose income come from other than labour earnings. However, since they will not save until they earn more than the threshold income $\bar{Y}_t$ again, we can consider that their income comes only from labour earnings for majority.

$$
\begin{aligned}
\Gamma_t^{INC} \equiv \frac{Y_t^R}{Y_t^P} &= \frac{W_t N_t + (1+R_{t-1})B_t + D_t}{W_t N_t} = \frac{C_{e,t}^R + C_{n,t}^R + B_{t+1}}{C_{e,t}^P} \\
&= \frac{C_{e,t}^R + C_{n,t}^R}{C_{e,t}^P} + \frac{B_{t+1}}{C_{e,t}^P} \\
&= \Gamma_t^{CON} + \frac{B_{t+1}}{C_{e,t}^P}
\end{aligned} \tag{61}
$$

3.5.3 Log-linearised model summary

The following are log-linearised key equations of the model, where lower case variables with a hat subscript refer to the log-deviation from the steady state with a bar subscript. The derivations are provided in Appendix B.

Essential consumption

Assuming that $F(\cdot)$ is differentiable,

$$
\begin{aligned}
\hat{c}_{e,t} &= \int_0^{\omega_t} \hat{y}_t(m') \frac{Y_t(m')}{Y_t^P} dm' + \left(\frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}}{\bar{Y}^P} \cdot \eta \cdot \frac{\sigma - 1}{\sigma} - 1 \right) \hat{p}_{e,t} \\
&- \frac{1}{\sigma} \left(1 + \frac{\eta(\sigma-1)}{1-\bar{\omega}} \right) \hat{p}_{e,t} \\
&= \int_0^{\omega_t} \hat{y}_t(m') \frac{Y_t(m')}{Y_t^P} dm' + \frac{1}{\sigma} \left[\eta(\sigma-1) \left(\frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}}{\bar{Y}^P} - \frac{1}{1-\bar{\omega}} \right) - 1 \right] \hat{p}_{e,t}
\end{aligned} \tag{62}
$$

where $\eta = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1$

Normal consumption (linearised Euler equation)

$$
\hat{c}_{n,t} = \hat{c}_{n,t}^R = \Sigma_{s \geq 0} \mathbb{E}_t \left[\left(\frac{C_{n,t+s+1}^R(\bar{\omega})}{\bar{C}_n^R} \cdot \bar{\omega} \eta \frac{\sigma-1}{\sigma} + \frac{1}{\mu} \right) \hat{\pi}_{e,t+s|t+s+1} - \frac{1}{\mu} (\hat{R}_{t+s} - \hat{\pi}_{n,t+s|t+s+1}) \right] \tag{63}
$$

Total consumption

$$\begin{aligned}\hat{c}_t &= \hat{c}_{e,t} + \hat{c}_{n,t} \\ &= \int_0^{\omega_t} \hat{y}_t(m') \frac{Y_t(m')}{Y_t^P} dm' - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \left(\frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}}{\bar{Y}^P} \cdot \eta - 1 \right) \cdot \frac{\sigma - 1}{\sigma} \cdot \hat{p}_{e,t} \\ &+ \Sigma_{s \geq 0} \mathbb{E}_t \left[\left(\frac{C^R_{n,t+s+1}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega} \eta \frac{\sigma - 1}{\sigma} + \frac{1}{\mu} \right) \hat{\pi}_{e,t+s|t+s+1} - \frac{1}{\mu} (\hat{R}_t - \hat{\pi}_{n,t+s|t+s+1}) \right]\end{aligned} \tag{64}$$

Phillips curve

$$\begin{aligned}\hat{\pi}_{t-1|t} &= \gamma_t \hat{\pi}_{e,t-1|t} + (1-\gamma_t) \hat{\pi}_{n,t-1|t} \\ &= \frac{\gamma_t (1 - \beta \rho_e)}{\rho_e} \cdot \frac{1-\alpha}{1-\alpha+\alpha\epsilon_e} \cdot \hat{\mathcal{MC}}_{e,t} + \gamma_t \beta E_t \hat{\pi}_{e,t|t+1} \\ &+ \frac{(1-\gamma_t)(1-\beta\rho_n)(1-\rho_n)}{\rho_n} \cdot \frac{1-\alpha}{1-\alpha+\alpha\epsilon_n} \cdot \mathcal{M}\hat{\mathcal{C}}_{n,t} + (1-\gamma_t)\beta E_t \hat{\pi}_{n,t|t+1}\end{aligned} \tag{65}$$

where γ_t is the market share of essential goods, i.e. $\gamma_t = \frac{C_{e,t}}{C_{e,t}+C_{n,t}}$

Employment

$$\hat{n}_t = -\frac{1}{\alpha} \hat{w}_t + \frac{1}{\alpha} \cdot \frac{\sigma - 1}{\sigma} \left(\frac{\eta + 1 - \bar{\omega}}{1 - \bar{\omega}} \right) \hat{p}_{e,t} + \frac{1-\alpha}{\alpha} \hat{\gamma}_t + \frac{1}{\alpha} \hat{\varepsilon}_t^{TFP} \tag{66}$$

where $\eta = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1$

Consumption gap

Consumption gap in Equation (59) is log linearised to

$$
\begin{aligned}
\hat{\Gamma}_t^{CON} &= \hat{c}_t^R - \hat{c}_t^P = \hat{c}_{n,t} + \hat{c}_{e,t}^R - \hat{c}_{e,t}^P \\
&= \Sigma_{s\geq 0}\mathbb{E}_t\left[\left(\frac{C^R_{n,t+s+1}(\bar{\omega})}{\bar{C}^R_n}\cdot\bar{\omega}\eta\frac{\sigma-1}{\sigma}+\frac{1}{\mu}\right)\hat{\pi}_{e,t+s|t+s+1}-\frac{1}{\mu}(\hat{R}_{t+s}-\hat{\pi}_{n,t+s|t+s+1})\right] \\
&-\frac{1}{\sigma}\left(1+\frac{\eta(\sigma-1)}{1-\bar{\omega}}\right)\hat{p}_{e,t}-\int_0^{\omega_t}\hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm'-\left(\frac{\bar{Y}(\bar{\omega})\cdot\bar{\omega}}{\bar{Y}^P}\cdot\eta\cdot\frac{\sigma-1}{\sigma}-1\right)\hat{p}_{e,t} \\
&= \Sigma_{s\geq 0}\mathbb{E}_t\left[\left(\frac{C^R_{n,t+s+1}(\bar{\omega})}{\bar{C}^R_n}\cdot\bar{\omega}\eta\frac{\sigma-1}{\sigma}+\frac{1}{\mu}\right)\hat{\pi}_{e,t+s|t+s+1}-\frac{1}{\mu}(\hat{R}_{t+s}-\hat{\pi}_{n,t+s|t+s+1})\right] \\
&-\int_0^{\omega_t}\hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm'-\left[\frac{1}{\sigma}\left(1+\frac{\eta(\sigma-1)}{1-\bar{\omega}}\right)+\left(\frac{\bar{Y}(\bar{\omega})\cdot\bar{\omega}}{\bar{Y}^P}\cdot\eta\cdot\frac{\sigma-1}{\sigma}-1\right)\right]\hat{p}_{e,t}
\end{aligned}
\tag{67}
$$

The term $\hat{c}_t^R - \hat{c}_t^P$ is the consumption gap for the essential good.

$$
\begin{aligned}
\hat{\Gamma}_{e,t}^{CON} &= \hat{c}_{e,t}^R - \hat{c}_{e,t}^P \\
&= -\int_0^{\omega_t}\hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm'-\left[\frac{1}{\sigma}\left(1+\frac{\eta(\sigma-1)}{1-\bar{\omega}}\right)+\left(\frac{\bar{Y}(\bar{\omega})\cdot\bar{\omega}}{\bar{Y}^P}\cdot\eta\cdot\frac{\sigma-1}{\sigma}-1\right)\right]\hat{p}_{e,t}
\end{aligned}
\tag{68}
$$

Income gap

Log linearising Equation (61),

$$
\begin{aligned}
\hat{\Gamma}_t^{INC} &= \hat{\Gamma}_t^{CON} + \hat{b}_{t+1} - \hat{c}_{e,t}^P \\
&= \hat{c}_{n,t} + \hat{\Gamma}_{e,t}^{CON} + \hat{b}_{t+1} - \hat{c}_{e,t}^P
\end{aligned}
\tag{69}
$$

3.6 Impacts of supply side price increase

The main interest of this chapter is the effect of inflation from the supply side, particularly from the essential goods sector. I consider shocks to the mark-ups of each sector and the total productivity in order, to look into the effect of each shock on the inflation, demand, and income/consumption inequality.

3.6.1 Aggregate variables

Shocks to mark-ups and TFP are

$$\hat{\varepsilon}^{\mathcal{M}}_{e,t} = \eta^{\mathcal{M}}_{e} \hat{\varepsilon}^{\mathcal{M}}_{e,t-1} - \zeta^{\mathcal{M}}_{e} e^{\mathcal{M}}_{e,t} \quad , \quad e^{\mathcal{M}}_{e,t} \sim \mathcal{N}(0,1) \tag{70}$$

$$\hat{\varepsilon}^{\mathcal{M}}_{n,t} = \eta^{\mathcal{M}}_{n} \hat{\varepsilon}^{\mathcal{M}}_{n,t-1} - \zeta^{\mathcal{M}}_{n} e^{\mathcal{M}}_{n,t} \quad , \quad e^{\mathcal{M}}_{n,t} \sim \mathcal{N}(0,1) \tag{71}$$

$$\hat{\varepsilon}^{TFP}_{t} = \eta^{TFP} \hat{\varepsilon}^{TFP}_{t-1} - \zeta^{TFP} e^{TFP}_{t} \quad , \quad e^{TFP}_{e,t} \sim \mathcal{N}(0,1) \tag{72}$$

Essential goods sector

To analyse the effect of mark-up shocks in the essential goods sector, first log-linearise the expression for $\mathcal{MC}_{e,t+s|t}$ in Equation (41) for time $t-1|t$, then simplify to

$$\hat{\mathcal{MC}}_{e,t} = \tau_e \hat{\varepsilon}^{\mathcal{M}}_{e,t} + \hat{w}_t - \hat{\pi}_{e,t-1|t} + \alpha \hat{n}_{e,t} - \frac{\alpha}{1-\alpha} \hat{\varepsilon}^{TFP}_{t} \tag{73}$$

Substituting into Equation (B.3.3) we obtain PC for the essential sector

$$\begin{aligned}\hat{\pi}_{e,t-1|t} =& \frac{\kappa_e \alpha(1-\alpha)}{(1-\alpha)(1+\kappa_e)+\alpha\epsilon_e}\hat{n}_{e,t} + \frac{\beta(1-\alpha+\alpha\epsilon_e)}{(1-\alpha)(1+\kappa_e)+\alpha\epsilon_e}E_t\hat{\pi}_{e,t|t+1} \\ &+ \frac{\kappa_e(1-\alpha)}{(1-\alpha)(1+\kappa_e)+\alpha\epsilon_e}\hat{w}_t + \frac{\kappa_e\tau_e(1-\alpha)}{(1-\alpha)(1+\kappa_e)+\alpha\epsilon_e}\hat{\varepsilon}^{\mathcal{M}}_{e,t} \\ &- \frac{\kappa_e\alpha}{(1-\alpha)(1+\kappa_e)+\alpha\epsilon_e}\varepsilon_t^{\hat{TFP}}\end{aligned} \tag{74}$$

$$\text{where} \quad \kappa_e = \frac{(1-\beta\rho_e)(1-\rho_e)}{\rho_e}$$

Mark-up shock in the essential sector first increases the sectoral price, depending on the production technology, the elasticity of substitution, the discount rate, and the price adjustment rate. Note that the effect on the price is greater as the price is stickier to change (lower ρ_e). And its impact on the (total) inflation $\pi_{t-1|t}$ will be adjusted by γ_t, as in Equation (B.3.5).

Recalling Equation (62) the essential consumption,

$$\hat{c}_{e,t} = \int_0^{\omega_t} \hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm' + \frac{1}{\sigma}\left[\eta(\sigma-1)\left(\frac{\bar{Y}(\bar{\omega})\cdot\bar{\omega}}{\bar{Y}^P} - \frac{1}{1-\bar{\omega}}\right) - 1\right]\hat{p}_{e,t}$$

The effect of essential price in demand appears in the last term; it depends on the relative size of threshold income as well as the fraction of the poor in the steady state, differenced by the inverse of the fraction of the rich in the steady state and multiplied by the coeffient $\eta(\sigma-1)$, which is negative as a whole. One in the second term comes from the price effect for the poor's essential consumption. As the first term inside the bracket is negative, the overall impact on the essential sector demand is negative. Note that the poor's consumption is reduced by $\left(\frac{\bar{Y}(\bar{\omega})\cdot\bar{\omega}}{\bar{Y}^P}\cdot\eta\cdot\frac{\sigma-1}{\sigma}-1\right)$ (Equation B.1.6) becuase the effect from the transition at the upper bound (ω_t) turns out to be smaller than the price effect, and the rich's essential consumption is also reduced by $\frac{1}{\sigma}\left(1+\frac{\eta(\sigma-1)}{1-\bar{\omega}}\right)$ (Equation B.1.7) due to the price effect as well as the fall in the fraction of the rich. Note that the transition between the types of households is endogenous, different from Debortoli and Gali (2017)'s exogenous swifter.

On the other hand, productivity shock causes the price to drop, again the size depending on the parameters as shown in the last term of Equation (74). And by the same analysis the essential consumption increases.

Normal goods sector

The Phillips curve of the normal goods sector (Equation (B.3.4)) leads to an equation similar to that of essential goods, as follows.

$$\begin{aligned}\hat{\pi}_{n,t-1|t} =& \frac{\kappa_n \alpha(1-\alpha)}{(1-\alpha)(1+\kappa_n)+\alpha\epsilon_n}\hat{n}_{n,t} + \frac{\beta(1-\alpha+\alpha\epsilon_n)}{(1-\alpha)(1+\kappa_n)} \\ &+ \alpha\epsilon_n E_t \hat{\pi}_{n,t|t+1} + \frac{\kappa_n(1-\alpha)}{(1-\alpha)(1+\kappa_n)+\alpha\epsilon_n}\hat{w}_t + \frac{\kappa_n \tau_n (1-\alpha)}{(1-\alpha)(1+\kappa_n)+\alpha\epsilon_n}\hat{\varepsilon}^{\mathcal{M}}_{n,t} \\ &- \frac{\kappa_n \alpha}{(1-\alpha)(1+\kappa_n)+\alpha\epsilon_n}\hat{\varepsilon}^{TFP}_t\end{aligned} \tag{75}$$

$$\text{where} \quad \kappa_n = \frac{(1-\beta\rho_n)(1-\rho_n)}{\rho_n}$$

The normal consumption given by Equation (63).

$$\hat{c}_{n,t} = \hat{c}^R_{n,t} = \Sigma_{s\geq 0}\mathbb{E}_t\left[\left(\frac{C^R_{n,t+s+1}(\bar{\omega})}{\bar{C}^R_n}\cdot\bar{\omega}\eta\frac{\sigma-1}{\sigma}+\frac{1}{\mu}\right)\hat{\pi}_{e,t+s|t+s+1}-\frac{1}{\mu}(\hat{R}_{t+s}-\hat{\pi}_{n,t+s|t+s+1})\right]$$

The effect of the mark-up shock on the normal goods price is shown in the second term, in the the same manner as the standard NK model: Mark-up shock causes inflation, just with its impact on total inflation being adjusted by $(1-\gamma_t)$. Then the change in the sectoral demand is determined by the agents' inter-temporal decision based on the expectation about the future; if agents expect the real interest rate will fall as a result of inflation, they will substitute the future consumption by the current consumption of normal goods. Even so, it has an additional term to the standard NK model that depends on the expected change in the essential price. While its impact on the total inflation is adjusted by γ_t, if it lowers the expected real interest rate, the present normal consumption will increase. Note that its coefficient differs from the normal sector inflation, which implies following: Since the essential consumption has the priority over X_t and the fraction ω_t (or threshold income $\bar{Y}_t$ in η) also changes by $p_{e,t}$, both the consumption decision and the fraction of the rich change as a result of positive mark-up shock in the essential price. Combined with the inter-temporal effect as described above, it determines how largely the current normal consumption should increase. In conclusion, we can see that the essential price plays a role in normal consumption in this model, while normal prices does not impact the essential consumption.

The total productivity shock has the opposite effect on the price and the demand through the channel that is already explained in the analysis of the essential mark-up shock case.

Total employment

Equation (66) implies that the markup shock as well as the productivity shock have a positive impact on employment because their increased profit allows more capacity to employ.

3.6.2 Consumption inequality

Equations (67) and (68) show the effect of inflation on the consumption gap. Note that Equation (68) is the last two terms of Equation (67). When it was triggered by the mark-up shock in the essential goods sector, both $\hat{\Gamma}_t^{CON}$ and $\hat{\Gamma}_{e,t}^{CON}$ change as summarised by the coefficient in the last term. It depends on whose essential consumption shrinks more than the other type. In addition, if the mark-up shock in the essential sector increases the normal consumption as explained above, it will enlarge the consumption gap further (first term), compared to the essential consumption gap.

On the other hand, the mark-up shock in the normal goods sector does not change the poor's consumption, and the change in the consumption gap solely depends on the rich's normal consumption from their inter-temporal decision. In other words, it will become larger relative to the future consumption gap, depending on their expectation.

The total productivity shock reduces inflation in both sectors, thus the effect on the consumption gaps is opposed to the mark-up shock. The total consumption gap will also drop if the current consumption of normal goods by the rich falls against the future, as shown in the first term of Equation (67).

3.6.3 Income inequality

For income inequality, I look into two features, first the fraction of the poor $\omega_t(P_{e,t})$, and the income gap Γ_t^{INC}, derived in Equation (69). $\omega_t(P_{e,t})$ is a reasonable indicator because it changes with the price of essential goods that is related to the income threshold $\bar{Y}_t$. Inflation driven by the essential sector mark-up shock will increase $\omega_t(P_{e,t})$, thus a larger number of households will become the poor, with income by which they can only manage the essential goods. Note that $P_{n,t}$

does not play any role in the change of $\omega_t(P_{e,t})$ nor $\varepsilon^{\mathcal{M}}_{n,t}$

The income gap in Equation (69) shows that the impact of essential mark-up shock depends on four factors: the change of normal consumption ($\hat{c}_{n,t}$), essential consumption gap ($\hat{\Gamma}^{CON}_{e,t}$), savings ($\hat{b}_{t+1}$) and the poor's essential consumption ($\hat{c}^{P}_{e,t}$). $\hat{c}_{n,t}$ increases by the essential mark-up shock while $\hat{b}_{t+1}$ and $\hat{c}^{P}_{e,t}$ decrease. Hence, if $\hat{\Gamma}^{CON}_{e,t}$ increases and the overall size effect is large enough to cancel out the decrease in $\hat{b}_{t+1}$, the income gap will be positive

In sum, we investigated how the price increase of essential goods, when caused by the mark-up shock, can have a negative impact on inequalities. If the reduction in the poor's essential consumption is bigger than the rich's, essential consumption gap will increase. The increase in normal consumption will increase the overall consumption gap in addition to the essential consumption gap. Then consumption and income gaps and the fraction of the poor increase as a result of it, although income gap still depends on how much the rich's savings decreases. Also, the last point has another implication for the wealth inequality in the opposite direction. The impact of mark-up shocks in the normal sector causes the inter-temporal substitution among the rich, then its result in the consumption gap is positive and the income gap is neutral because the decrease in the savings cancels out the increase in the normal consumption. The total productivity shock decreases the inflation so the results in the consumption gap is the opposite to the mark-up shocks, but the income gap can be aggravated depending on the rich's saving behaviour, and the wealth inequality as well.

3.6.4 Policy implication

Although this chapter focuses on the impact of inflation caused by supply shocks, it is still worth looking into the effect of policy. Assuming Taylor rule for the monetary policy, the only channel in this model is the intertemporal substitution effect among the rich agents in the normal sector. There is no direct impact on the essential sector, which means that the policy effect on the aggregate economy can be more restrictive than the models without sectoral differentiation.[15] The impact on inequality is that while consumption inequality responds in the negative direction to the policy rate, the income inequality depends on both the change of normal consumption and the savings which move to the opposite direction each other.

If there was a capital investment in firms' production, the lower cost of investment could increase the employment as in Kaplan, Moll, and Violante (2018) and the essential consumption as a result. In this case, it will reduce the consumption inequality and the income inequality even further, reinforcing such a policy implication in terms of distribution. We could also consider the policy mix with the fiscal policy to manage the price stability while increasing essential consumption. In general, we can conclude that this chapter shows the need for an analysis framework based on sectoral characteristics, its relation to heterogeneous types of agents, and the importance of its results in terms of both aggregates and distribution, in pursuit of a more effective policy reaction.

[15] Bilbiie (2025)'s modified Taylor rule (price-level targeting) might be more suitable for the essential sector that is directly affected by its current price level, rather than the chance of it or the future expectation.

3.7 Conclusion

In an attempt to investigate the effect of supply side inflation on aggregate economy as well as distribution, this chapter models a two-step problem for two types of goods and two types of households. First, all households who are ex-ante heterogeneous about income and wealth solve maximises quasi-linear utility function, which plays an important role for consumption decision on each type of goods and differentiating two types of households. When agents solve for essential consumption, their household types are determined to be the rich (unconstrained) or the poor (constrained) by the threshold income derived from it. Then only the rich move on to the next dynamic decision problem between the normal goods and the savings. The novelty is the endogenously derived threshold income, so the fraction of each household is not an exogenous parameter as in the existing TANK literature so far. In addition, it links the different behaviour of households with the different characteristics of two goods, which gives rise to the different impacts on two sectors in equilibrium.

The price of each sector is assumed to be determined by the monopolistic firms' pricing a la Calvo. Each sector solves identical NK problems, but with different parameters and demand function. The price for the whole economy is determined as the weighted average by the market size (consumption).

After log-linearisation for general equilibrium analysis, I could discover the changes of the aggregate and distributional variables by the supply side shocks such as mark-ups and total productivity. The most important element was the price of essential goods caused by mark-up shock. Not only did it aggravate the essential consumption of the poor and the rich, but also appeared to determine the consumption gap and the income gap. Due to the fact that the savings are only made by the rich as a result of their inter-temporal decision over normal goods and risk-free bonds, wealth inequality between the rich and the poor responds alongside the income inequality. Although there are opposing forces and the final result will depend on the parameter sizes and the dynamics of each variable over time, this chapter still would have a contribution such that it analytically shows the channels through which the inequality is affected by the supply side inflation that we have recently experienced, by suggesting a novel model framework.

Chapter 4

Conclusion

Under the title ”Causes and consequences of inequality in macroeconomics”, the thesis has examined a wide body of related literature and explored the specific subtopics in each chapter using both classical and the most recent models.

On the side of causes, I extended the calssical Aiyagari model to incorporate loss aversion preference and investigated how this modification changes the results. The aim was to be conceptually fundamental and methodologically endogenous. To incorporate the loss aversion into the utility function of classical CRRA, I followed the status-quo formulation and employed the CARA utility function, which has properties that the loss aversion needs that CRRA does not have. The reference point and the gain/loss specification were defined relative to the past consumption or utility. To ensure the existence of stationary general equilibrium of this modified model, I showed how the model satisfies Acikgoz (2018)'s criteria partly by proving Lemma 1 and Proposition 1-3, which constitutes a theretical contribution of the thesis.

For computational analysis of equilibrium, I modified the original Aiyagari VFI toolkit by adding the past consumption as an additional variable, also by implementing the derived conditions for the existence of stationary equilibrium. Then, the various inequality measures, such as the Gini coefficient, Pareto coefficient (between the top 1 percent and 0.1 percent), kurtosis, and skewness, were analysed over the earnings, income and wealth. Earnings inequality remained unchanged due to the model assumption, but income and wealth inequality have worsened significantly in Gini coefficient, which is sensitive to the middle group. However, the Pareto coefficient showed little change or even improvement, suggesting that the inequality increased primarily through middle-group decline while the top-group remained relatively unaffected. Kurtosis revealed fatter right tails, and skewness confirmed its bias to the right in distributions, consistent with rising inequality. PDF figures showed bimodal patterns in the long run, providing evidence of polarisation and offering a theoretical ground for the TANK model framework analysed in the subsequent chapter.

These results demonstrate the loss aversion generates a precautionary saving motive even for the agents far from the borrowing constraint, thereby amplifying inequality well beyond the Aiyagari model. These are also line with empirical research that the differential in the savings rate causes the inequality by large in the long term, matching the recent U.S income and wealth Gini coefficients. It suggests that if the financial market is deepened in the way consumption is more insured, the policy effect on inequality will be larger than the original Aiyagari model without loss aversion.

On the side of consequences of inequality, I analysed the effect of supply side price shock on aggregate variables and inequalities, when the income distribution is exogenously given (including polarisation from the previous chapter). For this purpose, I started from the TANK model with constrained and unconstrained households agents, while also distinguishing between essential and normal goods sectors. Supply side shocks include both mark-up shocks and total productivity shock. The core idea is that, given the low price elasticity of demand for essential goods, its price increase will exacerbate both the welfare and inequalities, more significantly than the models without such sectoral distinction. To formalise this, I used quasi-linear utility function, which allowed to derive threshold income - the level of income beyond which households cease to consume the essential goods- as a function of essential price. Then, the poor households were defined as those below the threshold income, who cannot help but consume all the income on the essential goods. Only the rich have the ability to consume normal goods or save. From this threshold income and the income distribution that was assumed to be exogenously given, I derived an endogenous fraction of the poor households.

This approach is one of the most novel parts of this thesis because there has been no research attempting to link HANK and TANK models analytically in this way, beyond the borrowing constraint framework. Consequently, unlike other TANK models in which all households are assumed to be identical, individual consumption and income may vary between households in this model, as in HANK models. This requires aggregation by household types and allows for transitions between two types. However, the equilibrium is still defined in the TANK format, as the state where the fraction is stationary rather than the entire income distribution. Based on this, I derived optimal solutions for individual households and firms, aggregated them, and log-linearised to analyse the equilibrium.

The most notable finding concerns mark-up shocks in the essential sector. Its price increase reduces consumption for both the poor and the rich. This affects not only essential consumption by both types but also normal consumption by the rich. The change of essential consumption gap depends on which type of households reduced consumption more, and the change of total consumption gap that includes the change of normal consumption depends on the expectation about future essential price. Income gap increases if the increase in the essential consumption gap exceeds the reduction in the savings by the rich. The fraction of the poor clearly increased.

In conclusion, it demonstrates that when inequality is ex-ante present, introducing the sectoral distinction allows us to show that even without idiosyncratic shock, the standard NK shocks alone can reduce the welfare and worsen the inequality. As for the policy implication, the results highlight the need for the sector specific policy approach, and suggest that for essential goods, the price level may be more important than the price stability.

Future research could enrich these insights by incorporating empirical study, extending the framework to include the labour market dynamics, or examining the fiscal and monetary policy rules.

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Appendix A

Proof of Lemma 1, Proposition 1 and Proposition 2

A.1 Proof of Lemma 1: Continuous twice differentiability of piece-wise CARA loss aversion function through smooth approximation

Recall Equation (2),

$$v(z) = \begin{cases} v^+(z) = \frac{1-e^{-\eta z}}{\eta} & \text{for all } z \geq 0 \\ v^-(z) = \lambda(\frac{e^{\phi z}-1}{\phi}) & \text{for all } z < 0 \end{cases}$$

Let a switching function $\gamma(z) = \frac{1}{1+e^{-kz}} \in C^{\infty}(\mathbb{R})$, which is indefinitely continuously differentiable, then we have $\lim_{k\to\infty} \gamma(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$

Use $\gamma(z)$ to define $v_k(z)$, to asymptotically express $v(z)$ in one equation,

$$\begin{aligned} v_k(z) &\equiv \gamma(z)v^+(z) + (1-\gamma(z))v^-(z) \\ &= \gamma(z)\Big[\frac{1-e^{\eta z}}{\eta}\Big] + (1-\gamma(z))\Big[\lambda\Big(\frac{e^{\phi z}-1}{\phi}\Big)\Big] \end{aligned} \tag{A.1.1}$$

$$\text{then} \quad \lim_{k\to\infty} v_k(z) = v(z) \tag{A.1.2}$$

From Equation (A.1.1.) we can derive the first and second derivatives,

$$v_k'(z) = \gamma'(z)(v^+(z) - v^-(z)) + \gamma(z)v^{+\prime}(z) + (1-\gamma(z))v^{-\prime}(z) \tag{A.1.3}$$

where

$$\gamma'(z) = \frac{ke^{-kz}}{(1+e^{-kz)^2}}$$

$$v^{+\prime}(z) = e^{-\eta z}$$

$$v^{-\prime}(z) = \lambda e^{\phi z}$$

And

$$v_k''(z) = -\gamma''(z)(v^+(z)-v^-(z))+2\gamma'(z)(v^{+\prime}(z)-v^{-\prime}(z))+\gamma(z)v^{+\prime\prime}(z)+(1-\gamma(z))v^{-\prime\prime}(z) \tag{A.1.4}$$

where

$$v^{+\prime\prime}(z) = -\eta e^{-\eta z}$$

$$v^{-\prime\prime}(z) = \lambda\phi e^{\phi z}$$

$$\gamma''(z) = \frac{k^2e^{-kz}(e^{-kz}-1)}{(1+e^{-kz})^3} \in \mathbb{R}$$

Since both Equation (A.1.3) and (A.1.4) consist of continuous functions for all z in additive and multiplicative form, $v_k(z)$ is continuously twice differentiable for all z, including 0.

Since Equation (A.1.2) holds, $v(z)$ is *asymptotically* continuously twice differentiable for all z as $k \to \infty$ □

A.2 Proof of Proposition 1

Based on Acikgoz's (2018) proof, we can derive the conditions for the existence of a stationary distribution in the Aiyagari model with gain-loss utility. The key element is that the convexity assumption in B1 still holds under loss aversion as it brings about the condition on consumption, as in Proposition 1.

The proof requires the following: $U(c_t) \in C^2(\mathbb{R}), U'(c_t) > 0, U''(c_t) < 0, \lim_{c_t \to 0} U'(c_t) = \infty, \lim_{c_t \to \infty} U'(c_t) = 0$

Since Lemma 1 holds for $v(z)$ and $u(c_t)$ is CRRA function which is continuous and twice differentiable, $U(c_t)$, a weighted sum of two, is also asymptotically continuous and twice differentiable as $v_k(z) \to v(z)$.

Then the first and second derivatives of $U(c_t)$ are following:

$$\begin{aligned} \frac{\partial U(c_t \mid c_{t-1})}{\partial c_t} \equiv U'(c_t) &= (1-\theta)u'(c_t) + \theta v'(z)u'(c_t) \\ &= [(1-\theta) + \theta v'(z)]u'(c_t) \end{aligned} \tag{A.2.1}$$

$$\frac{\partial^2 U(c_t \mid c_{t-1})}{\partial c_t^2} \equiv U''(c_t) = [(1-\theta) + \theta v'(z)]u''(c_t) + \theta v''(z)u'(c_t) \tag{A.2.2}$$

It is easy to show $U'(c_t) > 0$ because $v(z)$ is strictly increasing as well as u.

$\lim_{c_t \to 0} U'(c_t) = \infty$, and $\lim_{c_t \to \infty} U'(c_t) = 0$ also hold because of $u'(c_t)$ in Equation (A.2.1).

For the convexity condition, first denote the first term in Equation (A.2.2) as a and the second as b for convenience. Then it depends on the sign and size of terms a and b. $a < 0$ because $v'(z) > 0$ and $u''(c_t) < 0$. $b > 0(b < 0)$ if $v'' > 0(v'' < 0)$, i.e. when there is loss (or gain), as $u'(c_t) > 0$ always. Hence, when there is gain, the convexity condition holds as standard; otherwise, it depends on whether $a > b$ or not. Now, we apply the functional form of $v(z)$ for the loss to analyse:

$$\begin{aligned} v(z) &= \lambda(\frac{e^{\phi z} - 1}{\phi}) \\ v'(z) &= \lambda e^{\phi z} > 0 \\ v''(z) &= \phi \lambda e^{\phi z} > 0 \end{aligned} \tag{A.2.3}$$

$$\Rightarrow U''(c_t) = -[(1-\theta) + \theta \cdot \lambda e^{\phi z}]\mu c_t^{-\mu-1} + \theta \cdot \phi \lambda e^{\phi z} \cdot c_t^{-\mu} < 0 \tag{A.2.4}$$

$$\therefore c_t < \frac{\mu(1-\theta)}{\theta \phi z \lambda e^{\phi z}} + \frac{\mu}{\phi} \quad \square \tag{A.2.5}$$

When there is no loss aversion, $\theta = 0$, then $c_t < \infty$, so there is no restriction to hold the convexity. When there is full loss aversion, as the strictest case that can be imposed, then $\theta = 1$, resulting in $c_t < \frac{\mu}{\phi}$. Hence, based on the functional form chosen, we can conclude that the model is sufficient as long as the consumption is below the relative parameter value consisting of the degree of risk aversion, the degree of loss aversion, CARA coefficients and the weight of loss aversion. This condition was inserted in the computational process.

A.3 Proof of Proposition 2

If Lemma 1 holds $U(c_t)$ is asymptotically twice continuously differentiable as discussed in A.2.

From (A.2.1) and (A.2.2),

$$\frac{U''(c_t)}{U'(c)} = \frac{[(1-\theta) + \theta v'(z)]u''(c_t) + \theta v''(z)u'(c_t)}{[(1-\theta) + \theta v'(z)]u'(c_t)} \tag{A.3.1}$$

Since $u'(c_t) = c_t^{-\mu}$ and $u''(c_t) = -\mu \cdot c_t^{-\mu-1}$, both $\lim_{c_t \to \infty} u'(c_t) = \lim_{c_t \to \infty} u''(c_t) = 0$.

Therefore, as c_t goes to infinity (A.3.1) converges to 0.

$$\therefore \lim_{c \to \infty} -\frac{U''(c_t)}{U'(c_t)} = 0 \qquad \square$$

Appendix B

Log-linearisation

B.1 Essential consumption

B.1.1 The poor's essential consumption

Log-linearised poors' essential consumption is derived as following, where a bar without time subscript refers to the steady state:

$$\begin{aligned} C^P_{e,t} &= \int_0^{\omega_t(P_{e,t})} C^P_{e,t}(m')dm' \\ &= \int_0^{\omega_t(P_{e,t})} \frac{Y_t(m')}{P_{e,t}}dm' \\ &= \frac{Y^P_t}{P_{e,t}} \end{aligned} \tag{B.1.1}$$

$$\begin{aligned} logC^P_{e,t} &= logY^P_t - logP_{e,t} \\ \hat{c}^P_{e,t} &= \hat{y}^P_t - \hat{p}_{e,t} \end{aligned} \tag{B.1.2}$$

$$\begin{aligned} dY_t^P &= Y_t(\omega_t) \cdot d\omega_t + \int_0^{\omega_t} dY_t(m')dm' \\ \frac{dY_t^P}{Y_t^P} &= \frac{Y_t(\omega_t)d\omega_t}{Y_t^P} + \frac{\int_0^{\omega_t} dY_t(m')dm'}{Y_t^P} \end{aligned} \tag{B.1.3}$$

Since $\frac{dY_t^P}{Y_t^P} = dlogY_t^P$ and $\frac{dY_t(m')}{Y_t(m')} = dlogY_t(m')$,

$$\begin{aligned} dlogY_t^P &= \frac{Y_t(\omega_t)d\omega_t}{Y_t^P} + \int_0^{\omega_t} dlogY_t(m')\frac{Y_t(m')}{Y_t^P}dm' \\ \hat{y}_t^P &= \frac{\bar{Y}(\bar{\omega})}{\bar{Y}^P} + \int_0^{\omega_t} \hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm' \\ &= \frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}\hat{\omega}_t}{\bar{Y}^P} + + \int_0^{\omega_t} \hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm' \qquad (\because d\omega_t = \bar{\omega}\hat{\omega}_t) \end{aligned} \tag{B.1.4}$$

Recall that

$$\begin{aligned} \omega_t &= F(\bar{Y}_t) \\ \frac{d\omega_t}{\omega_t} &= \frac{F'(\bar{Y}_t)d\bar{Y}_t}{\omega_t} \\ \therefore \hat{\omega}_t &= \frac{F'(\bar{Y}_t)}{\bar{\omega}} \cdot \hat{\bar{y}}_t = \eta \cdot \hat{\bar{y}}_t = \eta \cdot \frac{\sigma - 1}{\sigma}\hat{p}_{e,t} \qquad \left(\eta \equiv \frac{F'(\bar{Y}_t)}{\bar{\omega}} = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1\right) \end{aligned} \tag{B.1.5}$$

$$\begin{aligned} \therefore \hat{c}_{e,t}^P &= \int_0^{\omega_t} \hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm' + \frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}}{\bar{Y}^P} \cdot \eta \cdot \frac{\sigma - 1}{\sigma}\hat{p}_{e,t} - \hat{p}_{e,t} \\ &= \int_0^{\omega_t} \hat{y}_t(m')\frac{Y_t(m')}{Y_t^P}dm' + \left(\frac{\bar{Y}(\bar{\omega}) \cdot \bar{\omega}}{\bar{Y}^P} \cdot \eta \cdot \frac{\sigma - 1}{\sigma} - 1\right)\hat{p}_{e,t} \end{aligned} \tag{B.1.6}$$

B.1.2 The rich's essential consumption

$$\begin{aligned} C_{e,t}^R &= (1-\omega_t)P_{e,t}^{-\frac{1}{\sigma}} \\ logC_{e,t}^R &= log(1-\omega_t) - \frac{1}{\sigma}logP_{e,t} \\ dlogC_{e,t}^R &= dlog(1-\omega_t) - \frac{1}{\sigma}dlogP_{e,t} \\ \hat{c}_{e,t}^R &= -\frac{\hat{\omega}_t}{1-\bar{\omega}} - \frac{1}{\sigma}\hat{p}_{e,t} \\ &= -\frac{1}{\sigma}\Big(1+\frac{\eta(\sigma-1)}{1-\bar{\omega}}\Big)\hat{p}_{e,t} \end{aligned} \tag{B.1.7}$$

B.2 Normal consumption

Recall the Euler equation for the rich individual $m' \in (\omega_t, 1]$, as well as the aggregation of it.

$$\mathbb{E}_t\left[\beta\left(\frac{C_{n,t}^R(m')}{C_{n,t+1}^R(m')}\right)^{\mu}\frac{1+R_t}{\Pi_{n,t|t+1}}\right] = 1 \tag{B.2.1}$$

$$\text{where} \qquad \Pi_{n,t|t+1} \equiv \frac{P_{n,t+1}}{P_{n,t}}$$

$$C_{n,t} = C_{n,t}^R = \int_{\omega_t(P_{e,t})}^{1} C_{n,t}^R(m') \tag{B.2.2}$$

Linearise Equation (B.2.1),

$$\begin{aligned} &\mathbb{E}_t[\mu(\hat{c}_{n,t}^R(m') - \hat{c}_{n,t+1}^R(m')) + \hat{R}_t - \hat{\pi}_{n,t|t+1}] = 0 \\ &\Rightarrow \hat{c}_{n,t}^R(m') = \mathbb{E}_t\Big[\hat{c}_{n,t+1}^R(m') - \frac{1}{\mu}(\hat{R}_t + \hat{\pi}_{n,t|t+1})\Big] \end{aligned} \tag{B.2.3}$$

Linearise Equation (B.2.2),

$$
\begin{aligned}
\frac{dC^R_{n,t}}{C^R_{n,t}} &= -\frac{C^R_{n,t}(\omega)d\omega}{C^R_{n.t}} + \int_{\omega}^{1} \frac{dC^R_{n,t}(m')dm'}{C^R_{n,t}} \\
dlogC^R_{n,t} &= -\frac{C^R_{n,t}(\omega)d\omega}{C^R_{n,t}} + \int_{\omega}^{1} dlogC^R_{n,t}(m')dm' \qquad \text{(B.2.4)} \\
\hat{c}^R_{n,t} &= -\frac{C^R_{n,t}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\hat{\omega}_t + \int_{\bar{\omega}}^{1} \hat{c}^R_{n,t}(m')\frac{\bar{C}^R_n(m')}{\bar{C}^R_n}dm'
\end{aligned}
$$

Substitute Equation (B.2.3) for Equation (B.2.4),

$$
\begin{aligned}
\hat{c}^R_{n,t} &= -\frac{C^R_{n,t}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\hat{\omega}_t + \int_{\bar{\omega}}^{1} \mathbb{E}_t[\hat{c}^R_{n,t+1}(m') - \frac{1}{\mu}(\hat{R}_t - \hat{\pi}_{n,t|t+1})] \times \frac{\bar{C}^R_n(m')}{\bar{C}^R_n}dm' \\
&= -\frac{C^R_{n,t}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\hat{\omega}_t + \mathbb{E}_t[\hat{c}^R_{n,t+1} + \frac{C^R_{n,t+1}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\hat{\omega_{t+1}}] - \frac{1}{\mu}\mathbb{E}_t[\hat{R}_t - \hat{\pi}_{n,t|t+1}] \\
&= \mathbb{E}_t[\hat{c}^R_{n,t+1} + \frac{C^R_{n,t+1}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\eta\frac{\sigma-1}{\sigma} \cdot \hat{p}_{e,t+1}] - \frac{1}{\mu}\mathbb{E}_t[\hat{R}_t - \hat{\pi}_{n,t|t+1}] \\
&- \frac{C^R_{n,t}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\eta\frac{\sigma-1}{\sigma} \cdot \hat{p}_{e,t} \\
&= \mathbb{E}_t\left[\hat{c}^R_{n,t+1} + \left(\frac{C^R_{n,t+1}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\eta\frac{\sigma-1}{\sigma} + \frac{1}{\mu}\right)\hat{\pi}_{e,t|t+1} - \frac{1}{\mu}(\hat{R}_t - \hat{\pi}_{n,t|t+1})\right] \\
&= \Sigma_{s\geq 0}\mathbb{E}_t\left[\left(\frac{C^R_{n,t+s+1}(\bar{\omega})}{\bar{C}^R_n} \cdot \bar{\omega}\eta\frac{\sigma-1}{\sigma} + \frac{1}{\mu}\right)\hat{\pi}_{e,t+s|t+s+1} - \frac{1}{\mu}(\hat{R}_{t+s} - \hat{\pi}_{n,t+s|t+s+1})\right]
\end{aligned}
$$

(B.2.5)

where $\eta = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1$

B.3 Phillips Curve

To obtain the Phillips curve of this economy for two sectors, first find the Phillips curve for each essential and normal good, then aggregate those two by market size.

Phillips curve for the essential good sector is derived by log-linearising the first-order condition as in Equation (41). Also, since it assumes that the economy is in the steady state $\beta = \frac{1}{1+R_{t,t+1}}$.

$$\hat{P}^*_{e,t}(i) - \hat{P}_{e,t-1} = (1-\beta\rho_e)E_t\sum_{s=0}^{\infty}(\beta\rho_e)^s(\hat{\mathcal{MC}}_{e,t|t+s} + (\hat{P}_{e,t+s} - \hat{P}_{e,t-1})) \quad \text{(B.3.1)}$$

$$\hat{\mathcal{MC}}_{e,t|t+s} = \hat{\mathcal{MC}}_{e,t+s} - \frac{\alpha\epsilon_e}{1-\alpha}(\hat{P}^*_{e,t} - \hat{P}_{e,t+s})$$

Log-linearising the aggregate price index in Equation (50),

$$\hat{\pi}_{e,t-1|t} = (1-\rho_e)(\hat{P}^*_{e,t} - \hat{P}_{e,t-1}) \quad \text{(B.3.2)}$$

Combining Equations (B.3.1) and (B.3.2), we can derive the Phillips curve for the essential good sector,

$$\hat{\pi}_{e,t-1|t} = \frac{(1-\beta\rho_e)(1-\rho_e)}{\rho_e}\cdot\frac{1-\alpha}{1-\alpha+\alpha\epsilon_e}\hat{\mathcal{MC}}_{e,t} + \beta E_t\hat{\pi}_{e,t|t+1} \quad \text{(B.3.3)}$$

Then the Phillips curve of the normal good sector is analogous.

$$\hat{\pi}_{n,t-1|t} = \frac{(1-\beta\rho_n)(1-\rho_n)}{\rho_n}\cdot\frac{1-\alpha}{1-\alpha+\alpha\epsilon_n}\hat{\mathcal{MC}}_{n,t} + \beta E_t\hat{\pi}_{n,t|t+1} \quad \text{(B.3.4)}$$

Let γ_t be the share of the essential good market, that is, $\gamma_t = \frac{C_{e,t}}{C_{e,t}+C_{n,t}}$, then the Phillips curve for the whole economy is

$$\begin{aligned}\hat{\pi}_{t-1|t} &= \gamma_t \hat{\pi}_{e,t-1|t} + (1-\gamma_t)\hat{\pi}_{n,t-1|t} \\ &= \frac{\gamma_t(1-\beta\rho_e)}{\rho_e} \cdot \frac{1-\alpha}{1-\alpha+\alpha\epsilon_e} \cdot \hat{\mathcal{MC}}_{e,t} + \gamma_t \beta E_t \hat{\pi}_{e,t|t+1} \\ &+ \frac{(1-\gamma_t)(1-\beta\rho_n)(1-\rho_n)}{\rho_n} \cdot \frac{1-\alpha}{1-\alpha+\alpha\epsilon_n} \cdot \hat{\mathcal{MC}}_{n,t} + (1-\gamma_t)\beta E_t \hat{\pi}_{n,t|t+1}\end{aligned} \tag{B.3.5}$$

B.4 Employment

Using Equation (62) and manipulating the fact that $\hat{c}^P_{e,t} = \hat{y}^P_t - \hat{p_{e,t}} = \hat{w}_t + \hat{n}^P_t - \hat{p_{e,t}} = \hat{w}_t + \hat{n}_t - \hat{p_{e,t}}$ [1] and $v_{e,t} = 1$ at the steady state, we can derive the log-linearised employment.

$$\begin{aligned}\hat{c}_{e,t} &= \hat{c}^P_{e,t} + \hat{c}^R_{e,t} \\ &= \hat{y}^P_t - \hat{p}_{e,t} - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \frac{1}{\sigma}\hat{p}_{e,t} \\ &= \hat{w}_t + \hat{n}_t - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \frac{1-\sigma}{\sigma}\hat{p}_{e,t}\end{aligned} \tag{B.4.1}$$

[1] $Y^P_t = W_t N^P_t = W_t N_t$ (Firms are assumed to distribute the labour demand equally betwen the poor and the rich (Section 3.5.1)). It would not be completely correct specification of $\hat{y}^P_t$ because every period there can be some households who fall from the rich to the poor, whose income come from other than labour earnings. However, since they will not save until they earn more than the threshold income $\bar{Y}_t$ again, we can consider that their income comes only from labour earnings for majority.

where $\eta = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1$

$$\begin{aligned}
Z_{e,t} &= \left[\int_0^1 (exp(\varepsilon_t^{TFP})N_{e,t}(i)^{1-\alpha})^{1-\frac{1}{\epsilon_e}} di\right]^{\frac{\epsilon_e}{\epsilon_e-1}} \\
\Rightarrow logZ_{e,t} &= log\left[\int_0^1 (exp(\varepsilon_t^{TFP})N_{e,t}(i)^{1-\alpha})^{1-\frac{1}{\epsilon_e}} di\right]^{\frac{\epsilon_e}{\epsilon_e-1}} \\
&= \frac{\epsilon_e}{\epsilon_e - 1} log\left[\int_0^1 (exp(\varepsilon_t^{TFP})N_{e,t}(i)^{1-\alpha})^{1-\frac{1}{\epsilon_e}} di\right] \\
&= \frac{\epsilon_e}{\epsilon_e - 1}\left[\int_0^1 log(\exp(\varepsilon_t^{TFP})N_{e,t}(i)^{1-\alpha})^{1-\frac{1}{\epsilon_e}} di\right] \qquad \text{(B.4.2)} \\
&= \int_0^1 log(\exp(\varepsilon_t^{TFP})N_{e,t}(i)^{1-\alpha}) di \\
&= \int_0^1 (1-\alpha) logN_{e,t}(i) di + \varepsilon_t^{TFP} \\
\therefore \hat{z}_{e,t} &= (1-\alpha)\int_0^1 \hat{n}_{e,t}(i) di + \hat{\varepsilon}_t^{TFP} \\
&= (1-\alpha)\hat{n}_{e,t} + \hat{\varepsilon}_t^{TFP}
\end{aligned}$$

Combining Equation (B.4.1) and (B.4.2) and use $\gamma_t = \frac{C_{e,t}}{C_{e,t}+C_{n,t}} = \frac{N_{e,t}}{N_{e,t}+N_{n,t}}$ we obtain

$$
\begin{aligned}
(1-\alpha)\hat{n}_{e,t} + \hat{\varepsilon}_t^{TFP} &= \hat{w}_t + \hat{n}_t - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \frac{1-\sigma}{\sigma}\hat{p}_{e,t} \\
(1-\alpha)(\hat{\gamma}_t + \hat{n}_t) + \hat{\varepsilon}_t^{TFP} &= \hat{w}_t + \hat{n}_t - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \frac{1-\sigma}{\sigma}\hat{p}_{e,t} \\
(\because \hat{\gamma}_t &= \hat{n}_{e,t} - \hat{n}_t) \\
(1-\alpha)\hat{n}_t &= \hat{w}_t + \hat{n}_t - \frac{\hat{\omega}_t}{1-\bar{\omega}} + \frac{1-\sigma}{\sigma}\hat{p}_{e,t} - (1-\alpha)\hat{\gamma}_t - \hat{\varepsilon}_t^{TFP} \\
\hat{n}_t &= -\frac{1}{\alpha}\hat{w}_t + \frac{1}{\alpha}\cdot\frac{\hat{\omega}_t}{1-\bar{\omega}} - \frac{1}{\alpha}\cdot\frac{1-\sigma}{\sigma}\hat{p}_{e,t} + \frac{1-\alpha}{\alpha}\hat{\gamma}_t + \frac{1}{\alpha}\hat{\varepsilon}_t^{TFP} \\
&= -\frac{1}{\alpha}\hat{w}_t + \frac{1}{\alpha}\cdot\frac{\sigma-1}{\sigma}\left(\frac{\eta+1-\bar{\omega}}{1-\bar{\omega}}\right)\hat{p}_{e,t} + \frac{1-\alpha}{\alpha}\hat{\gamma}_t + \frac{1}{\alpha}\hat{\varepsilon}_t^{TFP}
\end{aligned}
\tag{B.4.3}
$$

where $\eta = \frac{f(\bar{Y}_t)}{\bar{\omega}} < 1$