The pros and cons of the Preston–Biggins egg shape model: a reconsideration case based on mathematical modeling and simulation

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Abstract

The Preston egg profile model developed from an interdisciplinary research over 70 years ago remains a popular mathematical tool among formulae to define geometric bird egg contours. Biggins and colleagues transformed this into a mathematical formula with broader applicability (Preston–Biggins model), now considered to be the most accurate in describing egg shapes. However, an important principle of universality of mathematics in egg shape computations is compliance with the properties of a geometric solid of revolution; we previously postulated this model as the *Main Axiom of the Mathematical Formula of the Bird's Egg*. Here, we tested the conformity of the Preston–Biggins model with this Main Axiom. The theoretical verification failed, however, as it was impossible to solve theoretically the fourth-order polynomial. Using numerical and simulation methods, we examined geometric egg profiles with several variants of their linear dimensions. When applying the Preston–Biggins model, high accuracy of actual egg profiles was demonstrated. For example, the root mean square error (RMSE) calculated for a standard chicken egg was equal to 0.007. This value increased slightly when evaluating conical eggs (0.01), but the accuracy of reproducing such a contour was still at a fairly high level. The calculated values of the relative root mean square error (RRMSE) ranged between 0.003 and 0.007. However, the Main Axiom principle was violated, leading to discrepancy between the maximum breadth and the value of its distance from the egg center. The pros of this model thus lie in the fact that it is applicable to real-world egg profiles, the cons in that it does not conform to the universality principle encapsulated in the Main Axiom.

Keywords: Avian eggs; egg geometry; Preston model; standard egg shape; Main Axiom of the Mathematical Formula of the Bird's Egg; solid of revolution

1. Introduction

Deriving the most ideal mathematical relationship for describing the contours of birds' eggs is a discipline now entering its second century. This is a subset of the broader field that can be conditionally characterized as the use of the principles of mathematical modeling and algorithms in the interdisciplinary research of bird eggs and their quality (e.g., [1–3]), for which we have chosen a conditional term "Eggology" [4]. In an ideal world, a mathematical model should take into account both the physiological characteristics and the principles of egg formation [5–8]. It should accurately display the curves of egg profiles in nature and should also be a standard figure that obeys all universal geometric laws.

1.1. Preston model

The name of the Preston model for egg shape [9] should not be confused with Preston models of the same name in other spheres of science and practice, such as glass making and economy [10–12]. The founder of this egg research-related model [9] is Frank W. Preston (14 May 1896 – 1 March 1989), an English-American engineer and a prominent figure in glass technology, ornithology, ecology, and nature conservation [10,13–15]. He presented three main mathematical functions based on the product of the circle formula and polynomials of different orders, from linear to cubic. The accuracy of this mathematical egg shape description depends on the degree of the polynomial, of which the most accurate is cubic. Preston presented these formulae in parametric form that might have somewhat limited their practical application. Todd and Smart [7] transformed the parametric notation of the Preston model into a Cartesian one that is generally agreed to be more convenient for further mathematical transformations and graphical representation.

The Preston model owes a new round of popularity to Biggins et al. [16,17]. These authors first demonstrated that a number of other models that appeared in publications on this topic (e.g., [18–20]) are often variations of the Preston model. The difference between them lies in a slightly different mathematical interpretation of the function multiplied by the equation of a circle [16]. Secondly, they [17] demonstrated the accuracy of the Preston model by using it, in practice, to describe a large number of real bird egg contours. What the authors did not do, however, was to provide specific approximation formulae and their graphical interpretations that could contribute to their more detailed analysis. Furthermore, this research group did not evaluate the geometric universality of this model, which would enable to consider it a mathematical representation of a certain standard oviform geometrical figure.

1.2. Principles of mathematical standardization of an egg profile

The growing number of mathematical models developed to describe the contours of bird eggs has required their unification and standardization. Biggins et al. [16,17] proposed, in their works, to interpret the principle of universality of the egg model as one that gives a minimal error in describing specific profiles. At the same time, the authors, having set themselves the task of achieving maximum accuracy in reproducing the form, did not pay attention to the basic principles inherent in geometric figures. Mytiai and Matsyura [21] relied on a combination of different geometric shapes whose outer contours corresponded to the profiles of eggs of different shapes. Despite the resultant similarity, the authors limited themselves to a geometric model, without its mathematical description.

In our previous studies [22], we proposed a basic concept of standardization of the egg model that was a step towards developing a mathematical formula that describes a standard geometric figure resembling the profile of any particular egg. In doing so, the main focus was on observing the basic geometric principles inherent in all standard geometric solids of revolution (i.e., a solid figure formed by rotating a plane image around a straight line (axis) of revolution). In particular, a basic modeling postulate was proposed that we [23] called the *Main Axiom of the Mathematical Formula of the Bird's Egg* (hereafter the *Main Axiom*). This Main Axiom, in essence, lies in the fact that the extremum of the function that describes the egg contour geometry should conform to the value of $B/2$, i.e., half the maximum diameter (breadth) of the egg. Failure to comply with the principles of the Main Axiom leads to a violation of the basic dimensions of the egg in the model, most often, the maximum breadth, B. Since the latter parameter is used to determine the volume, V, and surface area of the egg, S (e.g., [24]), the result is overestimated (or underestimated) compared to what it actually is in nature. This problem can only be prevented by complying with the principles of the Main Axiom.

In this regard, this Main Axiom-based modeling approach can become a fundamental mathematical tool in assessing the adequacy and compliance with the principles of universality and standardization relevant to any mathematical model of the geometric profile of a bird's egg. The goal of the present interdisciplinary study was thus to test the conformity of the Preston (Preston–Biggins) model with the Main Axiom.

The recent advent of simulation modeling methods, mathematical apparatus for performing applied tasks, and computer technology with appropriate software that allows processing large volumes of data has given impetus to the transfer of a large number of studies from the real world to the virtual one. At the same time, the modeling/simulation approach has affected all possible areas of scientific disciplines. These include social [25] and biological [26–28] systems, forecasting issues [29,30], and software development [31].

Previously, we succeeded in implementing the simulation approach to determine the interrelationships between density and other physical parameters of the egg (e.g., [32]). Substituting possible variations of various parameters of a bird's egg into the theoretical density formula, namely, the ratio of maximum breadth to length, the ratio of length to shell thickness, shell density and contents density, the variation limits of which are well known, we obtained an extensive set of virtual data conforming to actual eggs. This approach simplified the derivation of empirical dependences and allowed us to expand the sample dataset. Therefore, we have also applied here the simulation method to the goal of revisiting the Preston–Biggins egg shape model.

2. Theory

As follows from Preston [9], his basic equation for mathematical interpretation of any egg shape that can be named as the Preston model looks as follows:

$$\begin{aligned} y &= a\sin\theta \\ x &= b\cos\theta(1 + c_1\sin\theta + c_2\sin^2\theta + c_3\sin^3\theta) \end{aligned} \qquad (1)$$

where a is the semi-major axis, b is the semi-minor axis, θ is the so-called eccentric angle, and c_1, c_2 and c_3 are coefficients.

Preston [9] implemented mathematical transformations of a circle into an ellipse and then into an ovoid (egg shape) locating the corresponding geometrical figure vertically. Thus, the egg length, L, half of which was denoted by Preston [9] as a, was located along the y-axis. However, it is more common and suitable if an egg profile is located horizontally, i.e., along the x-axis. Therefore, we decided to rewrite Eqn1 as follows:

$$\begin{aligned} x &= a\sin\theta \\ y &= b\cos\theta(1 + c_1\sin\theta + c_2\sin^2\theta + c_3\sin^3\theta) \end{aligned} \qquad (2)$$

Our intention, and incentive, was to convert Eqn2 from parametric into Cartesian coordinates. Considering that

$\sin\theta = \dfrac{x}{a}$ and, then, $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\dfrac{x^2}{a^2}}$,

the corresponding substitution gives the following formula:

$$y = \frac{b}{a}\sqrt{a^2 - x^2}\left(1 + \frac{c_1}{a}x + \frac{c_2}{a^2}x^2 + \frac{c_3}{a^3}x^3\right). \qquad (3)$$

The obtained equation (Eqn3) is practically identical to the results of other authors [7,33] who carried out similar mathematical transformations of the Preston model (Eqn2).

Before coming to more regular symbols of *B* and *L*, we needed to assure ourselves what egg dimensions were symbolized by Preston as *b* and *a*. According to the given explanations in Preston [9], *b* is the radius at the egg equatorial part. As the egg equatorial diameter lays on the *y*-axis, i.e., $x = 0$, we decided to mark its dimension as $B_0 = 2b$. Also, considering $L=2a$, Eqn3 was rewritten as follows:

$$y = \frac{B_0}{2L}\sqrt{L^2 - 4x^2}\left(1 + \frac{2c_1}{L}x + \frac{4c_2}{L^2}x^2 + \frac{8c_3}{L^3}x^3\right). \quad (4)$$

To determine the coefficients c_1, c_2 and c_3 in Eqn4, we need a system of three equations. In contrast, Biggins et al. [17], investigating the Preston model (Eqn1), assumed that the optimal solution for such a determination would be a modified computation with four equations, when *b* is not measured but calculated as an additional unknown coefficient. These equations were grounded on four measurements of egg diameters, D_1 to D_4. According to Biggins et al. [17], *L* is divided into eight equal parts. Therefore, considering that the *x*-axis is split with the *y*-axis at the midpoint of *L*, the respective values for *x* for the diameters D_1 to D_4 will be equal to $-3L/8$, $-L/8$, $L/8$, and $3L/8$. Biggins et al. [17] assumed that the egg orientation is directed with its pointed end to the positive meanings of *x*. As follows from Preston [9], the egg profile was oriented with its pointed end to the negative values of *y*. Thus, while changing the axes (Eqn2), we decided to consider the same orientation of egg profiles as was initially used by Preston [9], i.e., towards the negative *x* values. The scheme of such measurements is shown in Fig. 1, in which we also included the location of *B* that is shifted from the egg equator at some distance $x = w$.

As above mentioned, Biggins et al. [17] included an additional coefficient c_0 to the cubic polynomial of Eqn3 and, correspondingly, Eqn4 as well. In accordance with our notation, such a coefficient would be equal to $c_0 = B_0/2$, which is logically coming after the conforming substitution of $x = 0$ into Eqn4. Considering that Biggins et al. [17], while evaluated the coefficients c_0 to c_3, used the variations of *L* from −1 to +1, we decided to implement the similar approach but considered the variations of *x* from $-L/2$ to $L/2$ and Eqn4 as an initial egg model. Then, Eqn4 can be presented as follows:

$$y = \frac{\sqrt{L^2 - 4x^2}}{L}\left(\frac{B_0}{2} + \frac{2k_1}{L}x + \frac{4k_2}{L^2}x^2 + \frac{8k_3}{L^3}x^3\right). \quad (5)$$

The new coefficients k_1 to k_3 imply a corresponding recalculation of the coefficients c_1 to c_3 multiplying them by $B_0/2$.

To distinguish the Preston model (Eqn3) from Eqn5 (and its further mathematical transformations), we decided to name the latter one as the Preston–Biggins model, of course, without diminishing the contribution of other authors of the paper by Biggins et al. [17].

Mathematical transformations of Eqn5 to find the unknown coefficients were performed by Biggins et al. [17] for their version of the egg model using a conventional value of L = 2. Since we decided to explore a more generalized egg model version that includes an actual L value, we set out to follow a similar path in determining the Eqn5 coefficients as presented in detail in Supplementary Material S1. As a result of determining the coefficients in equation (5) through the values of the measured egg diameters D_1 to D_4, the Preston–Biggins model took the following form:

$$\begin{aligned} y = \frac{\sqrt{L^2-4x^2}}{L}\Bigg(& -\frac{1}{8\sqrt{7}}(D_1+D_4)+\frac{9}{8\sqrt{15}}(D_2+D_3) \\ & +\frac{1}{L}\left[\frac{9}{\sqrt{15}}(D_3-D_2)-\frac{1}{3\sqrt{7}}(D_4-D_1)\right]x \\ & +\frac{8}{L^2}\left[\frac{1}{\sqrt{7}}(D_1+D_4)-\frac{1}{\sqrt{15}}(D_2+D_3)\right]x^2 \\ & +\frac{64}{L^3}\left[\frac{1}{3\sqrt{7}}(D_4-D_1)-\frac{1}{\sqrt{15}}(D_3-D_2)\right]x^3\Bigg). \end{aligned} \tag{6}$$

Our next, and crucial, task was to test if the Preston–Biggins model conforms to the Main Axiom. To determine the extremum point, one should find the derivative of the function and equate it to zero [23]. The only option that makes the derivative of the egg model equal to zero implies that $x = w$ (Fig. 1).

Considering that the obtained Eqn6 does not include the parameter w, additional transformations were undertaken. As follows from Fig. 1, the extremum of Eqn6 should conform to $B/2$ when $x = w$. By making this substitution, the polynomial in the parentheses in Eqn6 can be reconsidered in the following way:

$$\begin{aligned}&-\frac{1}{8\sqrt{7}}(D_1+D_4)+\frac{9}{8\sqrt{15}}(D_2+D_3)\\&+\frac{1}{L}\left[\frac{9}{\sqrt{15}}(D_3-D_2)-\frac{1}{3\sqrt{7}}(D_4-D_1)\right]w\\&+\frac{8}{L^2}\left[\frac{1}{\sqrt{7}}(D_1+D_4)-\frac{1}{\sqrt{15}}(D_2+D_3)\right]w^2\\&+\frac{64}{L^3}\left[\frac{1}{3\sqrt{7}}(D_4-D_1)-\frac{1}{\sqrt{15}}(D_3-D_2)\right]w^3\\&=\frac{LB}{2\sqrt{L^2-4w^2}}\end{aligned} \quad . \tag{7}$$

Let us determine a derivative of Eqn6. To make it easier, we considered this in the form of Eqn5. Then,

$$\frac{\partial y}{\partial x}=\frac{\sqrt{L^2-4x^2}}{L}\left(\frac{2k_1}{L}+\frac{8k_2}{L^2}x+\frac{24k_3}{L^3}x^2\right)-\frac{4x}{L\sqrt{L^2-4x^2}}\left(\frac{B_0}{2}+\frac{2k_1}{L}x+\frac{4k_2}{L^2}x^2+\frac{8k_3}{L^3}x^3\right). \tag{8}$$

As follows from the Main Axiom, Eqn8 is equal to zero when $x = w$. Considering Eqn7, Eqn8 can be rewritten as follows:

$$\sqrt{1-4\left(\frac{w}{L}\right)^2}\left[k_1+4k_2\frac{w}{L}+12k_3\left(\frac{w}{L}\right)^2\right]-B\frac{\frac{w}{L}}{1-4\left(\frac{w}{L}\right)^2}=0\,. \tag{9}$$

Thus, to show if the Preston model and, correspondingly, the Preston–Biggins model, as its specified version, both conform to the Main Axiom [23], we will need to prove that Eqn9 is true. For that, the experimental procedure using the simulation approach was considered as outlined below.

3. Methods

In our previous studies [34,35], we encountered the difficulty of accurately measuring *w* using both the egg itself and its digital image. Since the adequacy of estimating the dependence (9) relies on the accuracy of measuring *w*, we decided here to use a different approach. It consisted of analyzing images of eggs whose dimensions are known in advance. One of these is the so-called "standard egg", the dimensions and image of which are presented by Romanoff and Romanoff [36] as derived from a number of measurements of hens' eggs. The basic dimensions of the standard eggs in accordance with Romanoff and Romanoff [36] are: L = 5.7 cm, B = 4.2 cm, and w = 0.25 cm. In our previous work by Narushin et al. [23], we recreated a digital image of the standard egg based on both the interdependences between the parameters presented by Romanoff and Romanoff [36] and the image they provided. The next step was to measure the diameters D_1

to D_4 similar to what was proposed by Biggins et al. ([17]; Fig. 1). This measurement was fairly easy to perform given the numerical series of *x*'s and *y*'s in the digital image of the egg.

In addition to the standard egg, we also decided to check several modifications of egg profiles by corresponding transformation of the standard egg implementing thus such mathematical tool as the simulation method described elsewhere [32]. Values of the basic egg dimensions were still the same, i.e., L = 5.7 cm and B = 4.2 cm, but the value of the distance w was increased. Considering that the variability of the ratio of w/L is limited within a range of 0 to 0.2 [22,37–39], we chose the following three value options for w: 0.50; 0.65 and 0.80 cm. The egg profiles with the specified parameters were generated according to the method presented in Narushin et al. [23]. Moreover, to give an egg diversity in the degree of its conicity, we arbitrarily chose the following three options for the diameter measured at a point located at a distance of $L/4$ from the sharp end of the egg: 3.25, 3.15, and 2.90 cm. For each obtained image, the corresponding diameters D_1 to D_4 were measured, and the required coefficients included in Eqn9 were calculated.

To measure the goodness of fit of egg contours constructed according to the Preston–Biggins model (Eqn6) in comparison with the generated profiles, the root mean square error (RMSE) was used (e.g., [40]):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - y'_i)^2}{N}} \tag{10}$$

where y_i and y'_i represent the observed (generated) and predicted (Eqn6) values, and N represents the number of data points.

Since future studies may require comparing error values across different datasets, the relative root mean square error (RRMSE) was also calculated for this purpose (e.g. [41,42]):

$$RRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - y'_i)^2}}{\frac{1}{N}\sum_{i=1}^{N} y_i} \tag{11}$$

4. Results and discussion

The profile of the standard egg [36] was visualized graphically in Fig. 2 (blue line). The values of the measured diameters of the standard eggs were as follows: D_1 = 2.462 cm, D_2 = 3.913 cm, D_3 = 4.143 cm, and D_4 = 2.964 cm.

After substituting the values of the egg geometrical parameters into Eqn6, a theoretical profile of the egg was obtained (Fig. 2, yellow line). As follows from Fig. 2, the theoretical contour is copying the true one rather accurately. The RMSE value was equal to 0.007 and that of RRMSE to 0.004, respectively.

Despite such accuracy, our goal was to estimate the concordance of the Preston–Biggins model (Eqn6) to the conditions of the Main Axiom. For this, the provisions of Eqn9 should be fulfilled. The recalculation showed the result being equal to −0.104 instead of the supposed zero. Thus, despite the high accuracy of the conformation of the theoretical profile to the actual one, its provision to the basic geometrical law was questionable.

Other possible modifications of egg profiles are presented in Fig. 3 (blue lines). The measurements of the diameters D_1 to D_4 in each of the egg contours and their corresponding substitution into Eqn6 allowed to plot theoretical profiles that are also depicted in Fig. 3 (yellow lines).

Reproduction of egg profiles with the Preston–Biggins theoretical model (Eqn6) also showed high accuracy results. The RMSE and RRMSE values were respectively 0.005 and 0.003 for the egg #1, 0.009 and 0.006 for the egg #2, and 0.010 and 0.007 for the egg #3.

After substitution of the measured parameters into Eqn9, the respective recalculations led to the results that also differed from zero. Such a discrepancy constitutes −0.244 for egg #1 (Fig. 3A), −0.333 for egg #2 (Fig. 3B), and −0.403 for egg #3 (Fig. 3C). The obtained results and the analysis of Eqn9 demonstrated that the only chance of being equal to zero was when w = 0. This means that the egg should be exactly in the form of an ellipsoid. The respective increase of the w value led to noncompliance with the Main Axiom. Herewith, the more the dimension w, the higher noncompliance was observed.

To evaluate the "losses" of such noncompliance, we decided to determine the meanings of x values for each of the studied egg profiles. To do this, Eqn8 was equalized to zero and transformed into the following one:

$$x^4 + \frac{24k_2}{L^2} \cdot \frac{L^3}{64k_3} x^3 + \frac{8k_1 - 12k_3}{L} \cdot \frac{L^3}{64k_3} x^2 + (B_0 - 4k_2) \cdot \frac{L^3}{64k_3} x - k_1 L \cdot \frac{L^3}{64k_3} = 0. \qquad (12)$$

Unfortunately, it became very difficult to find a theoretical solution for Eqn12. After all, to solve quaternary equations, one has to perform many intermediate steps, including the transition to equations of lower degree. This makes the formulas too complex and cumbersome for practical use. Therefore, a numerical procedure was considered. Substituting the corresponding data of each of the considered eggs (Figures 2 and 3) into Eqn12, the following polynomials were obtained:

– for the standard chicken egg, with *w* = 0.25 cm (Fig. 2),

$$x^4 - 48270.17x^3 + 1055917.03x^2 + 19199063.48x - 4288367.79 = 0, \quad (13)$$

wherefrom *x* = 0.22071;

– for the egg #1, with *w* = 0.50 cm (Fig. 3A),

$$x^4 + 50260.60x^3 + 2023693.18x^2 + 21515441.66x - 8218748.67 = 0, \quad (14)$$

wherefrom *x* = 0.36906;

– for the egg #2, with *w* = 0.65 cm (Fig. 3B),

$$x^4 + 60918.83x^3 + 1362289.40x^2 + 11291442.43x - 5532622.57 = 0, \quad (15)$$

wherefrom *x* = 0.46352;

– for the egg #3, with *w* = 0.80 cm (Fig. 3C),

$$x^4 + 81315.35x^3 + 2273335.58x^2 + 13082221.29x - 9232608.86 = 0, \quad (16)$$

wherefrom *x* = 0.63425.

Thus, the noncompliance with the Main Axiom resulted in the shift of the *B*-axis from its right position. Within our examples, the errors between the actual and calculated values of *w* varied from 12 to 29%. Such discrepancies in the calculated values of *w* also affected the error in calculating *B*. If we substitute the calculated values of *x* (Eqns 13 to 16) into Eqn6, taking into account that the *y* value in this case will correspond to *B*/2, we can estimate the deviation of the calculated *B* value from the true one. In the context of our examples with the considered egg images, the deviation of the calculated *B* value from its true value (4.2 cm) was 0.2 to 0.5%.

We also assumed that if the diameter D_3 were to coincide with the maximum breadth of the egg, *B*, this would lead to compliance with the Main Axiom. However, these diameters in Fig. 3B and 3C are almost identical, while the degree of discrepancy does not become smaller. The deviation of the *w* value from the required value was 29% in Fig. 3B and 21% in Fig. 3C.

In principle, the magnitude of this error can be taken as acceptable. However, if we are considering the universality criterion, even small discrepancies in the key provisions of the geometric standard, i.e., the Main Axiom, do not allow us to classify this model as ideal. Also, it should be taken into account that the main geometric dimensions, *L* and *B*, and, often, *w*, are used to calculate other parameters of the egg, in particular, its *V* and *S* (e.g., [34,35,35,43,44]). In this regard, discrepancies in the true and calculated dimensions of the egg can result in a very significant error, unacceptable if it is necessary to perform very accurate mathematical calculations. In addition, the error may increase depending on the size of the egg under study and the ratio of its parameters.

When carrying out these studies, we realized how difficult it is to find a compromise between the applied task, say, to ensure the accuracy of reproduction of the egg profile, and the theoretical completeness of the obtained result, that is, when it is necessary, perhaps in some cases and at the expense of accuracy, to ensure compliance with physical laws. In this regard, a researcher independently chooses what is more important in accordance with the designated goals. However, if we try to achieve mathematical accuracy and not deviate from the rules of geometry of bodies of revolution, the optimal universal model describing the contours of a bird's egg with a minimum set of parameters is the universal dependence proposed by us in ref. [23].

Considering the popularity of the topic of mathematical description of geometric contours of bird eggs, as well as the developed approaches to the application of this research direction, the present study findings can become the basis for the implementation of the following scientific investigation:

1. A study of other popular models used for the mathematical description of egg contours from the view point of their accuracy and compliance with the principles of constructing geometric shapes.
2. In view of the convenience and popularity of the mathematical approach to developing the egg model used by Preston that consists of a combination of the circle equation with a polynomial of the 1st, 2nd or 3rd degree, it will be very relevant to perform a research on replacing the polynomial with another function so that the result of such a 'symbiosis' satisfies the requirements of the Main Axiom.

5. Concluding Remarks

If the universality of the mathematical egg model is understood to be its correspondence to a certain standard geometric figure of rotation, the Preston and/or Preston–Biggins model, despite an accurate reproduction of actual egg contours, cannot be considered to meet this criterion. This is due to the discrepancy between this model (Eqn6) and the Main Axiom, the provisions of which have been expressed in the work by Narushin et al [23]. According to the Main Axiom, the extremum of the function that describes the geometry of the egg contour should conform to the value of $B/2$, i.e., half the maximum diameter of the egg. Finding the extremum point involves determining the derivative of the function and equating it to zero. In this case, the solution should consist in the equality of the argument to the w value that, along with the values of B and L, is one of the fundamental parameters of the universal egg formula. The w value conforms to the shift of the vertical axis until it coincides with the maximum diameter. That is, the main disadvantage of the Preston model is the impossibility of its usage as a standard figure of a solid of revolution that has an oviform profile.

On the other hand, in the case when the practical need is limited to visual representation of the egg contour or does not require exceptional accuracy of further recalculations, the use of the Preston model (Eqn4), especially its modification in the form of the Preston–Biggins formula (Eqn6), is quite acceptable and even advisable. The only limiting factor of its practical application is the needed measurement of four egg diameters. Nevertheless, this procedure is quite feasible when using digital images of the eggs being examined.

In conclusion, therefore, the pros of the Preston (Preston–Biggins) model lie in the fact that it is applicable to real-world egg profiles and widely useable. The cons, however, lie in the fact that, according to these studies, it does not conform to the universality principle encapsulated in the Main Axiom of the Mathematical Formula of the Bird's Egg and applicable to a geometric solid of revolution. Implementation of the numerical and simulation methods was also instrumental in determining the pros and cons of the popular mathematical egg shape model.

Acknowledgements

Not applicable.

Funding

No funds, grants, or other support was received.

Data availability

All data supporting the findings of this study are available within the paper and its Supplementary Material S1.

Declarations

Conflict of interest

The authors have no relevant financial or non-financial interests to disclose.

Author contributions

Conceptualization, investigation, methodology, software, validation and writing – original draft: V.G.N. and S.T.O. Data curation: V.G.N. and S.T.O. Formal analysis: V.G.N. Project administration: M.N.R. Supervision: D.K.G. Visualization, V.G.N., S.T.O. and M.N.R. Writing – review & editing: V.G.N., S.T.O., M.N.R. and D.K.G.

Supporting information

Additional supporting information may be found in the online version of this article.

Supplementary Material S1. Determining the Eqn5 coefficients.

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