



# Kent Academic Repository

**Roitzheim, Constanze (2005) *Arbeitsgemeinschaft mit aktuellem Thema: Modern Foundations for Stable Homotopy Theory: Mathematisches Forschungsinstitut Oberwolfach Report No. 46/2005, organised/edited by John Rognes (Oslo) and Stefan Schwede (Bonn)*. In: Schwede, Stefan, ed. *Oberwolfach Reports*. *Oberwolfach Reports*, 2 (4). European Mathematical Society Publishing House, pp. 2603-2604.**

## Downloaded from

<https://kar.kent.ac.uk/49545/> The University of Kent's Academic Repository KAR

## The version of record is available from

<https://doi.org/10.4171/OWR/2005/46>

## This document version

Publisher pdf

## DOI for this version

## Licence for this version

UNSPECIFIED

## Additional information

## Versions of research works

### Versions of Record

If this version is the version of record, it is the same as the published version available on the publisher's web site. Cite as the published version.

### Author Accepted Manuscripts

If this document is identified as the Author Accepted Manuscript it is the version after peer review but before type setting, copy editing or publisher branding. Cite as Surname, Initial. (Year) 'Title of article'. To be published in *Title of Journal*, Volume and issue numbers [peer-reviewed accepted version]. Available at: DOI or URL (Accessed: date).

## Enquiries

If you have questions about this document contact [ResearchSupport@kent.ac.uk](mailto:ResearchSupport@kent.ac.uk). Please include the URL of the record in KAR. If you believe that your, or a third party's rights have been compromised through this document please see our [Take Down policy](https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies) (available from <https://www.kent.ac.uk/guides/kar-the-kent-academic-repository#policies>).

## Rigidity for the stable homotopy category

CONSTANZE ROITZHEIM

If two model categories  $\mathcal{C}$  and  $\mathcal{D}$  are Quillen equivalent, then their homotopy categories  $Ho(\mathcal{C})$  and  $Ho(\mathcal{D})$  are equivalent. But if  $Ho(\mathcal{C})$  and  $Ho(\mathcal{D})$  are equivalent categories, can anything be said about the underlying model structures? For the stable homotopy category  $Ho(\mathcal{S})$  (i.e. the homotopy category of spectra) there is the following result:

### Rigidity Theorem [1]

Let  $\mathcal{C}$  be a stable model category and

$$\Phi : Ho(\mathcal{S}) \longrightarrow Ho(\mathcal{C})$$

be an equivalence of triangulated categories. Then  $\mathcal{S}$  and  $\mathcal{C}$  are Quillen equivalent.

This means that all higher homotopy information in  $\mathcal{S}$  such as Toda brackets,  $K$ -theory or mapping spaces is already encoded in the triangulated structure of the stable homotopy category.

In this talk we will construct the most important category-theoretic tool used in the proof of the Rigidity Theorem:

### Universal Property of Spectra [2]

Let  $\mathcal{C}$  be a stable model category,  $X \in \mathcal{C}$  a fibrant and cofibrant object. Then there is a Quillen adjoint functor pair

$$X \wedge - : \mathcal{S} \rightleftarrows \mathcal{C} : \text{Hom}(X, -)$$

with  $X \wedge S^0 \simeq X$ .

In the set-up of the Rigidity Theorem this Quillen pair will provide the desired Quillen equivalence. More precisely, for  $X = \Phi(S^0)$ , the composition

$$Ho(\mathcal{S}) \xrightarrow{L(X \wedge -)} Ho(\mathcal{C}) \xrightarrow{\Phi^{-1}} Ho(\mathcal{S})$$

of the left derived Quillen functor and the given equivalence  $\Phi$  is an endofunctor of the stable homotopy category sending the sphere to itself. Hence, as the following talks will show, it must be a self-equivalence of  $Ho(\mathcal{S})$ . Consequently, the derived functor of the Quillen functor  $X \wedge -$  is an equivalence of categories which means that  $\mathcal{S}$  and  $\mathcal{C}$  are Quillen equivalent.

## REFERENCES

- [1]
- [2]