# Advanced Synergy-Driven Algorithms for Bid Generation in Transportation Auctions

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**Abstract**

Securing transportation services is crucial for supply chain management (SCM), as it constitutes the most significant expense within this domain. Implementing an effective bid generation strategy in combinatorial auctions (CAs) as a procurement mechanism to secure these services is vital for enhancing SCM efficiency. This study presents an advanced approach for quantifying synergy within the transportation network, aiming to identify the optimal bundles of auctioned contracts (AC). An efficient synergy-based bid generation algorithm has been developed to determine the optimal bids to be submitted within a transportation CA framework. This approach addresses two scenarios: one that considers service time limitations and another that allows for flexibility in early time constraints for the AC. The study investigates the potential of implementing a discount system by achieving savings through a relaxed time approach. It also examines the system's effectiveness in promoting more efficient transportation by minimizing travel distances on roadways in relaxed early-time cases. This research provides an efficient solution to the bid generation problem (BGP) with high dimensionality. Results indicate that the BGP having 550 booked contracts (BC), 800 AC, and 220 cities is solved within a reasonable timeframe. Furthermore, the study found that the computational complexity of the synergy-based BGP relies more on the ratio of the number of AC to BC rather than solely on the number of AC.

**Keywords:** Bid generation; Combinatorial auction; Transportation procurement; Synergy; Algorithms; Discounts.

## 1. Introduction

The combinatorial auction (CA) mechanism is gaining popularity in the transportation industry due to its ability to handle complex and interdependent bidding scenarios. This mechanism facilitates the procurement of bundles that exploit synergies within the transportation network. Buyers can express their preferences for these bundles, optimizing the procurement process and enabling sellers to enhance their resource allocation and operations independently. Consequently, this results in more accurate pricing and efficient procurement. However, the intricacies of the transportation network, characterized by multiple services and constraints, must not be overlooked. CA is a flexible and nuanced tool that captures such complexities while ensuring fairness and transparency in the bidding process, thereby establishing trust among all parties involved. It is noteworthy that CA is particularly effective in the spot market (Badiee et al., 2023; Wang & Wang, 2015), whereas most companies tend to prefer negotiation for long-term agreements.

Bid generation is one of the main challenges that bidders face in CAs. It involves determining the optimal combination of services to bid on, considering the complexity of the transportation network (Triki et al., 2023). However, this problem is computationally complex, necessitating the use of sophisticated optimization algorithms to generate efficient and effective bids. Various factors influence the optimization of the CA bidding process, including time windows, cost constraints, and service availability; these factors contribute to increased challenges and time consumption. This burden can be particularly significant for fleet providers, especially smaller companies. According to Trucking Industry Trends & Statistics (2021), most carrier providers are not considerably large, with only 2.6% owning more than 20 trucks. The complexity of the problem escalates exponentially as the number of potential service bundles increases, making it challenging to compute the optimal solution within a reasonable timeframe, even for a limited number of contracts. Researchers and practitioners have developed various optimization techniques, such as integer programming (IP), branch-and-bound algorithms, column generation, and heuristic algorithms, to efficiently generate and evaluate bids for service bundles to overcome this complexity (Chang, 2009; Hammami et al., 2021; Mesa-Arango & Ukkusuri, 2013; Song & Regan, 2005). Nevertheless, effective bid generation in CAs still requires further investment in technology and methodologies to achieve optimal outcomes while procuring transportation services (Hasan et al., 2023).

One of the most effective approaches for generating bids is synergy approximation (Triki, 2016; Yan et al., 2021). This method enhances bid effectiveness by considering the interdependencies among contracts within the transportation network. Transport providers typically have pre-existing agreements for transportation contracts and maintain their own networks to fulfill these obligations. However, the spot market can create opportunities to accommodate additional requests through the CA process for transportation procurement. In this context, the synergy approximation method is useful for finding the optimal alignment of auctioned contracts (AC) with existing booked contracts (BC), allowing transport providers to optimize their bids and increase the likelihood of winning an auction. There are several methods to quantify synergies, such as minimizing empty travel (Wang & Xia, 2005), pairwise synergy quantification (An et al., 2005; Triki et al., 2014), and synergy approximation (Chang, 2009). However, the development of an analytical formula for quantifying synergy has received scant attention in the literature. Recently, Keskin et al. (2023) introduced an analytical method to determine synergy values by calculating synergy based on the attainable revenues of AC and selecting the bundle of AC that offers the best synergy, specifically the one that yields the highest revenue. They claimed that their approach relies on a linear model for calculating synergy, which is easier for carriers to implement compared to complex combinatorial optimization models. However, quantifying the value of synergy solely in terms of revenue is not entirely realistic, as revenues depend on future decisions regarding bundle pricing. Thus, there remains a need to establish synergy values through a more appropriate analytical method.

The transportation industry is a major contributor to global carbon emissions. Research indicates that, among all sectors, transportation consistently ranks high in its carbon footprint (IEA, 2020). The primary concern is that these emissions increase greenhouse gases (GHGs) associated with freight transportation in the supply chain (Demir et al., 2011), which are notorious for their role in intensifying global warming. Recognizing these ramifications, global leaders have rallied behind initiatives such as the Paris Agreement, which aims to combat escalating global temperatures. In response, stakeholders have adopted a variety of emission control strategies. For instance, a report from the World Bank highlights the existence of over 60 diverse policies geared toward emission reduction (World Bank, 2020). Widely adopted strategies include carbon taxes and cap-and-offset mechanisms. While necessary, these regulations introduce additional complexity to operations within the transportation domain. This underscores the urgent need for more efficient bidding strategies in CA, ensuring that the sector remains both compliant and efficient in this evolving landscape.

In this study, we address a significant research gap by introducing a model that represents the ratio between the total effective distance and the total minimum distance required to cover the effective distance of any given bundle. The term "effective distance" refers to the sum of all contract distances from their pickup points to their delivery locations, excluding any empty travel. It is important to note that the total distance required to serve the contracts must include the empty travel. This ratio ensures optimal synergy calculations, maximizing effective travel distance while minimizing empty travel. By reducing the amount of empty travel within the transportation network, the system can lower operational costs. This approach minimizes total travel distance, optimizes transportation operations, and significantly reduces carbon emissions. Addressing travel distances impacts our ability to manage challenges posed by carbon regulations, thereby enhancing the overall efficiency of transport solutions. The synergy formula introduced in this study provides a strategic framework for reducing travel distances. Consequently, this method improves both economic and environmental sustainability within the transportation industry. We propose a synergy-based bid generation problem (BGP) algorithm that effectively enhances computational performance and identifies efficient solutions.

Additionally, the discount mechanism for optimizing transportation procurement is an effective technique, as highlighted by researchers such as Yang et al. (2019), and it has the potential to stimulate market demand (Garrido, 2007). Accordingly, this mechanism can help optimize transportation networks by reducing empty movements. As a result, emissions associated with these empty movements will be reduced.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature, highlighting prior research in this area. Section 3 presents the problem statement, while Section 4 proposes an optimal ordering problem algorithm. Subsequently, Sections 5 and 6 discuss the synergy quantification model and algorithm, as well as the bid generation method, respectively. Section 7 provides a detailed discussion of the computational experiments conducted for the proposed algorithms, along with numerical illustrations. Finally, Section 8 concludes the research findings and outlines future scopes.

## 2. Literature Review

The literature review aims to revisit issues associated with the proposed approach in the current study. This includes examining methods for BGP solutions, quantifying synergy in BGP, and exploring opportunities for discounts in the procurement of transportation services. These services are essential for identifying the rationale behind this study.

### 2.1 BGP Methods

The CA is a widespread mechanism in the transportation service procurement industry that allows for the simultaneous allocation of multiple services or goods. The BGP is a crucial aspect of CA in transportation procurement, where the main challenges involve allocating contracts to carriers and optimally assigning transport tasks. Previously, researchers have proposed different models to tackle the challenges associated with the BGP.

Initially, Song and Regan (2003) introduced a simple IP model for lane selection, which is a fundamental aspect of BGP. Later, Song and Regan (2005) extended the model to include both pre-assigned and non-pre-assigned lanes. They employed a modified branch-and-bound solution approach, enabling them to explore the solution space and determine optimal results. The routing aspect of BGP was further developed by Wang and Xia (2005), who formulated a nonlinear programming model. They used the nearest insertion technique and proposed optimal fleet assignment algorithms to solve the problem. Lee et al. (2007) and Kwon et al. (2005) explored routing in BGP, considering its multi-dimensional phenomenon, and developed a column-generation technique for solving the problem, yielding more efficient solutions. Conversely, the pricing of bundle bids and minimum cost flow formulations were the main focuses of Chen et al. (2009) and Chang (2009), respectively. These models enriched the literature by examining various BGP components.

Mesa-Arango and Ukkusuri (2013) proposed a mixed-integer linear programming approach for lane selection, along with a scenario-based LP model (Mesa-Arango & Ukkusuri, 2015), to address lane volume and price uncertainties. Their solution algorithm includes the branch-and-price technique and introduces a new approach of dependent sampling using the Latin hypercube technique. Stochastic modeling in the context of BGP concerning auction clearing prices was first introduced by Triki et al. (2014). They proposed both exact and heuristic solution methods according to the size of the input instances. Subsequently, Wang and Wang (2015) suggested an encoding strategy for bundling a BGP model by proposing a quantum evolutionary algorithm and a genetic algorithm for optimization. They successfully presented an alternative solution technique within the BGP framework. To determine the price interval that carriers should target for each bid, Ben Othmane et al. (2019) suggested a three-stage heuristic. By considering the pricing component of the issue, this technique provided a fresh perspective in the literature.

A new BGP that considers a heterogeneous fleet model was introduced by Hammami et al. (2019). Subsequently, Hammami et al. (2021, 2022) extended the previous study by addressing the stochastic nature of clearing prices and proposed a two-stage modeling approach. Consistent with their earlier work (Hammami et al., 2019), the first stage was deterministic and focused on solving a contract selection problem, whereas the second stage dealt with clearing prices through chance constraint modeling to capture the stochasticity. The BGP problem for multi-period scenarios was introduced by Mamaghani et al. (2019), who proposed an advanced tabu search approach to obtain improved and faster solutions. Further, Lyu et al. (2022) introduced another stochastic model that considers load uncertainty and developed a Benders decomposition that incorporates Pareto-optimal cuts to reduce solution complexity.

Since the BGP has been investigated over the years using numerous modeling and solution methodologies, its complexity in the transportation process remains unavoidable. Therefore, the development of modeling techniques and BGP solutions is still needed. This urgent issue prompts the current study to propose advanced algorithms that are easy to implement. We will first establish the feasibility of bundles before determining their optimality. Next, we will employ algorithms to assess the synergy of any bundle with the contracts of a pre-assigned transportation network. Finally, we will use an algorithm to establish the best bids for potential submission in an auction.

### 2.2 Synergy Approximation in BGP

The synergy approximation method has garnered attention as a viable solution to the bid generation challenge in the procurement of transportation services despite its limited coverage in the existing literature. The concept of synergy was first introduced to transportation procurement under CA by Song and Regan (2003, 2005), who concentrated on lowering repositioning expenses for empty trucks. This idea was further developed by Wang and Xia (2005) and An et al. (2005) to minimize the total empty trip distance and maximize profitability. They established complementarity metrics between lanes to identify first- and second-order synergy values. Subsequently, Chang (2009) proposed a decision support system that used an approximation method for calculating synergy values within a formulation for minimum cost flow. Triki et al. (2014) evaluated pairwise synergies in two distinct ways: one based on Euclidean distance and the other on the number of hops. They estimated bundle synergy by calculating the average pairwise synergies among contracts within a bundle.

Afterward, Mesa-Arango and Ukkusuri (2015) sought to reduce empty carrier movements by detecting and clustering synergistic contracts that, when used together, minimize empty trips. They created an interconnectivity metric for each contract pair to represent the value of grouping them. Subsequently, Triki (2016) focused on quantifying synergy through location-based techniques, such as contract classification based on geographical location and determining cluster center points. Yan et al. (2021) enhanced Mesa-Arango and Ukkusuri's (2015) concept of in-vehicle consolidation by defining synergy within bundles and computing synergy expressions while considering vehicle capacity. To establish a novel bundle expression, Hammami et al. (2021) merged and altered formulations from An et al. (2005), Triki (2016), and Triki et al. (2014). Their novel definition of bundle synergy enabled a more precise assessment of synergy values, thereby enhancing decision-making for carriers participating in transportation CA. Researchers have also quantified synergy for simultaneous independent single auctions (Kuyzu et al., 2015; Olcaytu & Kuyzu, 2018, 2021). They used synergy values to choose the most beneficial bundles and determine bids for concurrent independent single auctions.

In their recent work, Keskin et al. (2023) integrated new issues, such as time window restrictions and movement direction, to quantify the synergy of auctioned bundles with BC. They introduced an analytical formula instead of relying on approximation expressions to calculate the synergy value according to bundle revenues and proposed algorithms. The authors claimed that their approach is easy to implement without requiring extensive mathematical modeling. The literature shows that the synergy quantification method can effectively resolve the BGP in CA. These techniques consider the interdependence among various bid components, enabling the formulation of more effective bids that can raise winning possibilities while lowering expenses. However, more studies are required to examine whether these techniques can be applied in various procurement circumstances within the transportation sector and to assess their potential for broader practical implementation.

### 2.3 Discounts in Transportation Procurement Auctions

Discount mechanisms play a critical role in the transportation industry. They serve as strategic tools used by service providers to make their bids more competitive and appealing to clients. This approach facilitates the formation of long-term relationships and the securing of recurring contracts, ultimately benefiting both carriers and shippers.

Yang et al. (2019) studied the total volume discount in the context of WDP (Winner Determination Problem) in transportation service procurement. Later, Yang and Huang (2020) considered the volume discount in an incremental quantity discount to the centralized planning problem as a variant of the WDP in transportation service procurement. Carriers offering the most considerable quantity discounts have the best chance of winning serving lanes, and the discount serves as a mechanism to enhance bid-winning chances in CA. Additionally, bids that incorporate volume discounts permit the specification of supply curves, meaning they establish varying unit prices for different quantities of an item sold (Bichler et al., 2009). In a later study, Yang and Huang (2021) examined the WDP by integrating shipping distance and volume-based discounts to optimize transportation costs. More recently, Triki et al. (2023) explored a WDP for discounted bids aimed at reducing empty travel within the transportation network. Furthermore, Qian et al. (2021) analyzed discount factors related to sustainability and responsiveness scores to determine winners amid accidental uncertainty. Additionally, Yin et al. (2021) addressed quantity discounts for fourth-party logistics providers facing disruptions to select third-party logistics for transportation services. They reported that disruption probabilities increased as discount parameters decreased, emphasizing the importance of fortification strategies when third-party logistics providers offer quantity discounts.

The literature reveals that discounts have primarily been discussed in relation to the WDP to create more opportunities for winning bids and to reduce transportation costs. Qian et al. (2021) presented an exception by examining sustainability and responsiveness phenomena through a discount factor. In another work, Garrido (2007) introduced a discount on transportation prices to stimulate demand for early shipments, claiming that demand can be triggered when the average discounted amount is one-third of the regular market price. However, the integration of discount policies with BGPs has not been addressed in the literature, even though bidders often offer discounts. Therefore, the integration of discount mechanisms with BGP in transportation auctions to optimize the network by eliminating unnecessary empty movements represents a promising area for further study.

### 2.4 Research Gaps and Contributions

Synergy quantification in Keskin et al. (2023) is implemented by calculating the minimum travel time required to serve the AC between two sequentially BC. However, minimizing travel distances is more appropriate for reducing costs and emissions. The authors calculated synergy based on the contracts' revenues, which conflicts with the pricing of the selected bundles for bids, typically determined after the auction has concluded. Keskin et al. (2023) examined the issue of time windows for the BC rather than for the AC. Conversely, this study investigates time restrictions for AC and explores potential early service times along with possible discounts. The current research presents several novel aspects, as outlined below.

- A new formula has been introduced to calculate synergy, defined as the ratio between effective distance (the sum of the distances of all contracts from the origin to the delivery point of any tour, excluding empty trips) and actual distance (the total route distance, including empty trips) for serving any bundle. The optimal synergy is achieved by minimizing empty trips while maximizing effective travel distance.
- This study introduces algorithms that quantify the synergy of AC within the existing transportation network. These algorithms produce optimal results for any bundle of AC situated between two consecutive BCs. A synergy-based BGP algorithm has been introduced to select the final bid. It operates with high efficiency, demonstrating the capability to solve large-scale problems (e.g., 550 BC, 800 AC, and 220 cities) in a reduced timeframe.

- This study examines cases involving time windows and relaxed time windows for AC to explore early shipment opportunities that may offer potential discounts. This approach aims to reduce additional empty travel and emissions from the transportation system, thereby enhancing environmental sustainability. To the best of the author's knowledge, this is the first study in transportation auction research that focuses on discount opportunities in conjunction with the BGP.

The summary of related articles addressing BGP for transportation procurement under CA using synergy methods is reported in Table 1. We utilized the SCOPUS database with the keywords << Bid Generation Problem >> OR << Bid Construction Problem >> AND << Combinatorial Auction >> AND << Transporation >> AND << Synergy >> for the selection of the related articles and identified 10 articles. After reviewing the abstracts, we selected six out of the 10 articles. Additionally, by examining the references of these selected articles, we identified three more relevant articles.

Table 1 shows the uniqueness of this study compared to previous research. It focuses on economic and operational efficiencies by enhancing the new synergy estimation formula for the transportation network. The proposed synergy-based BGP algorithm can identify exact solutions for large instances (BC = 550, AC = 800, and number of cities [NC] = 220) within a few seconds, which is an impressive achievement, as previously proposed methods have never successfully solved instances of this size. The pioneering approach of this work lies in the synergy formula, which quantifies the exact value and integrates discounts in BGP. This study demonstrates superior computational performance compared to existing approaches in the literature, which have addressed up to 400 AC, taking 179,131 seconds. It is important to note that the problem setup in this study is not entirely comparable to those in previous works; however, our proposed problem setup favors us in determining better performance.

**Table 1: Comparison table**

| Articles | Focus | Modeling type | | Solution | | Mechanism | | | Computation | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Maths Prog. | Algorithm | Exact | Approx. | Synergy | | Discount | Size (BC-AC-NC) | Time (sec.) |
| | | | | | | Approx. | Technique | | | |
| Wang and Xia (2005) | Nearest insertion method | ✓ | ✓ | - | ✓ | ✓ | Complementarity | - | 10-30-NA | - |
| Chang (2009) | Fleet dimensionality (1000) | ✓ | - | ✓ | ✓ | ✓ | Partial pairwise | - | 38-10-15 | 33360 |
| Triki et al. (2014) | Computational performance | ✓ | ✓ | - | ✓ | ✓ | Pairwise | - | NA-400-NA | 179131.0 |
| Triki (2016) | Clustering locations | Analytical | | - | ✓ | ✓ | Location-based | - | 8-6-16 | - |
| Olcaytu and Kuyzu (2018) | Efficient synergy-based bidding method | ✓ | ✓ | - | ✓ | ✓ | Costs per auctioned lanes | - | 90-10-100 | 6035 |
| Yan et al. (2020) | In-vehicle consolidations | ✓ | ✓ | - | ✓ | ✓ | Profit-distance-based synergy | - | 10-9-NA | 12024 |
| Olcaytu and Kuyzu (2021) | Efficient heuristic for generating bids | - | ✓ | - | ✓ | ✓ | No formula | - | 90-10-100 | 177 |
| Hammami et al. (2021) | Exact non-enumerative solution | ✓ | ✓ | ✓ | ✓ | ✓ | Pairwise | - | 25-25-NA | 10150.95 |
| Keskin et al. (2023) | Quantify the synergy among the contracts | ✓ | ✓ | ✓ | - | ✓ | Revenue per distance | - | 8-6-16 | < 1 |
| Liu et al. (2024) | DFS-genetic algorithm for BGP | ✓ | ✓ | - | ✓ | ✓ | No formula | - | 67-187-80 | 77.23 |
| **This paper** | **Computational performance, empty movements, and emissions** | - | ✓ | ✓ | - | **Exact** | **Effective distance per total travel distance** | ✓ | **550-800-220** | **< 1** |

*AC=Auctioned contracts, *BC=Booked contracts, *NC=Number of cities, DFS=Depth-first search

## 3. Problem Statement and Notations

The CA mechanism for procuring transportation services has already been established within the realm of supply and logistics management. This study explores a BGP under a CA framework for a single-round auction concerning full truckload (TL) shipments. Carriers maintain transportation networks for pre-BC $i \in I = \{1, 2 \dots m\}$, and routes are selected to serve them. Additionally, the spot market creates opportunities through the CA process for some new contracts $j \in J = \{1, 2 \dots n\}$ to the carrier's provider. Accordingly, interested companies can participate in the CA to obtain these new contracts; however, generating bundle bids is not linear because they must comply with auction rules and optimize their resources. One of the most convenient ways to optimize resources is to identify feasible bundles within their existing network of pre-BC.

We assume that the BC are pre-ordered based on their designated time windows. Since these contracts have already been selected for their tour and service times, the time limitations are strict; i.e., the vehicle can arrive at any time before the specified time limit, but the service will start as scheduled. In practice, a transportation network always consists of some empty travel among the contracts. This presents an opportunity to convert this empty travel into new contracts from the spot market, thereby creating an additional business scope and mitigating the negative impact of carbon emissions by reducing instances of empty travel. Therefore, if feasible, the possible bundle of auctions can be chosen to utilize any available space within the existing network. The synergy between the BC and the auctioned bundle of contracts is vital in selecting the bundle of contracts for potential bids.

Figure 1 shows the transportation network associated with the contracts. Each contract is represented as a directed edge connecting two locations/cities. However, for simplicity, we present the edges (*i*, *i+1*, etc.) as BC, while each dot signifies AC (in reality, each dot includes two cities/locations). The dotted circles indicate possible bundles of AC, whereas the solid/shaded circles represent the feasible spaces between any two consecutive BC. Synergy measures how well the bundles align with the BC; the essential criteria for assessing synergy include time windows, travel distance, and the direction of flows. This study proposes algorithms to calculate synergy concerning empty movements, aiming to find the bundles that best fit the BC.

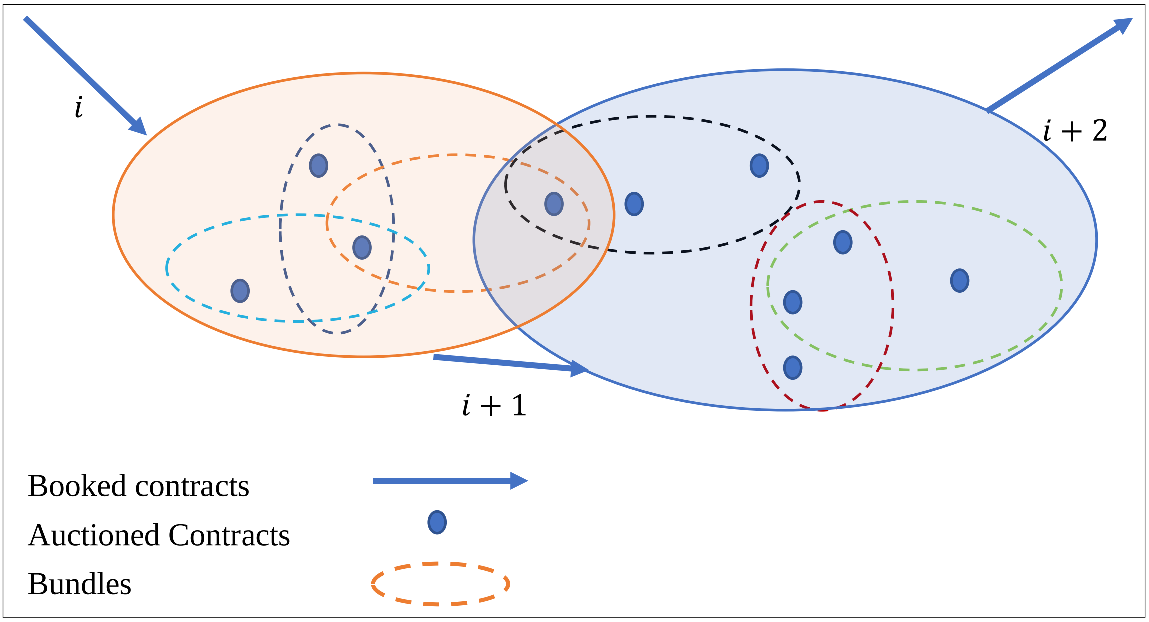


**Figure 1: Transportation network**

It is essential to emphasize that our problem is not standardized due to its sensitivity to real-world factors, such as the geographical area or industrial context in which it arises. Additionally, in our problem formulation, we assume that each contract, whether booked or auctioned, has a designated time window. BC has fixed time limitations for services, while auction contracts have specific earliest and latest times for reaching the entry point. The notations and their interpretations for the current study are presented in Table 2.

**Table 2: Indexes, parameters, and variables**

| Indices | Descriptions |
|---|---|
| $i$ | Booked contract |
| $j$ | Auctioned contract |
| $k$ | Any contracts either $i$ or $j$ |
| $b$ | Any bundle of auctioned contracts |
| $\alpha$ | Space between consecutive booked contracts (e.g., between $i$ and $i+1$) |
| $k'$ | Size of the bundle |
| **Sets** | **Description** |

| | |
|---|---|
| $I$ | Set of booked contracts |
| $J$ | Set of auctioned contracts |
| $B$ | Set of bundles |
| **Parameters** | **Description** |
| $m$ | Number of booked contracts |
| $n$ | Number of auctioned contracts |
| $sp_k$ | Start point of any contract $k \in I \cup J$ |
| $ep_k$ | End point of any contract $k \in I \cup J$ |
| $t_k^{e/l}$ | Early "$e$" / Latest "$l$" time limitation for any contract $k \in I \cup J$ |
| $w_k$ | The weight of any contract $k \in I \cup J$ is the distance of $k$ |
| $t_d$ | Time to travel a unit distance "$d$" (determined by speed) |
| **Variable** | **Description** |
| $\beta_b^\alpha$ | Optimized permutation of bundle $b$ in space $\alpha$ |
| $\tau_\beta^\alpha$ | Traveling time for $\beta$ in space $\alpha$ |
| $S_\beta^\alpha$ | Synergy of bundle $\beta$ in space $\alpha$ |
| $x_{ib}$ | Binary, equal to 1 if bundle $b$ is selected; otherwise, 0 |

### 3.1 Problem Formulation and Solution Methodology

Our goal is to find the optimal bid by maximizing the synergy of a transportation network, where the AC must be assigned within the existing network of pre-BC. The formulation of the problem is structured below.

**Decision variable:** $x_{\alpha b}$, is a binary variable, where $x_{\alpha b} = 1$ if the bundle b is selected in space $\alpha$; otherwise, $x_{\alpha b} = 0$.

**Objective:** Maximize the total synergy ( $Max,\ Z = \sum_{\alpha,b} S_b^\alpha\, x_{\alpha b}$ ) of the selected bundles across several spaces of the transportation network.

**Subject to the constraints**

- No common contract will be included in the selected bundles.
- At most, one bundle may be selected in the space between consecutive BC.
- Time window constraints of contracts and their corresponding trip directions must be followed when optimizing network synergy. This adherence will be ensured outside the optimization model through the use of a synergy calculation algorithm and an optimal ordering algorithm.

Based on the problem setup considered, we structure the solution methodology by following the steps below to achieve a computationally efficient BGP solution.

- First, we determine the order of feasible bundles between two consecutive BC in each space by solving the optimal ordering problem. The objective is to minimize the travel distance of any feasible AC bundle. Constraints include the feasibility of the contracts to be served together and the direction of travel. Section 4 describes the details of the optimal ordering problem algorithm.
- Second, the synergy quantification formula is proposed as the ratio of effective travel distance to total travel distance, thereby minimizing empty travel while maximizing the synergy value, as presented in Section 5. The method to calculate the synergy of the auctioned bundles is presented in Sub-section 5.1, where a novel algorithm is proposed. This algorithm filters feasible bundles within a specific space of consecutive BC by imposing time window constraints, and it determines the optimal order of the bundles using the algorithm for the optimal ordering problem (Sub-section 4.1). Subsequently, the proposed formula calculates the corresponding bundle's synergy. Finally, it stores the optimal order and the associated synergy for all feasible bundles within each space.
- Third, a bid-generation algorithm (described in Section 6) optimizes total synergy by selecting the most effective bundles across different spaces. The synergy calculation algorithm determines and stores the input parameters for the BGP algorithm. However, a common contract issue may arise among the bundles. To address this, an ILP model maximizes total synergy by ensuring that no

common contracts exist within the selected bundles and that only one bundle is selected per space. Finally, our approach processes the optimal bid and the order of their visit.

The overview of the solution methodology is presented in Figure 2.

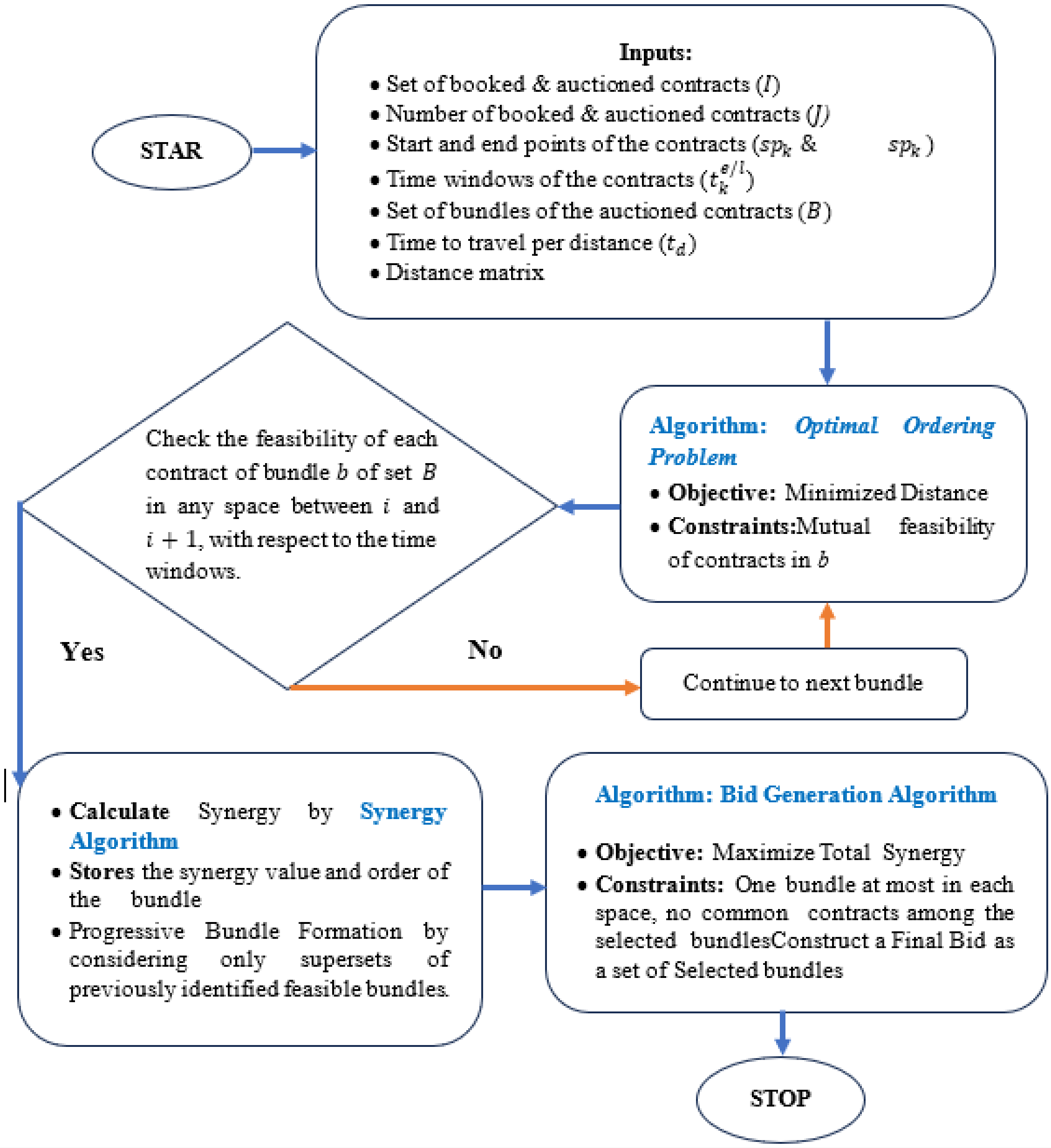


**Figure 2: Solution methodology**

The optimal ordering, synergy calculation, and bid generation algorithms execute the solution methodology to obtain an exact solution. The optimal ordering algorithm determines the exact solution by employing exhaustive enumeration, iterating through all possible orders to find the one with the minimum travel distance. Additionally, the bid generation algorithm determines the optimal solution by solving an ILP. In Sub-section 5.2, we demonstrate that our synergy algorithm is an exact method through inductive iteration for all feasible solutions. While exhaustive enumeration of all possible bundles could serve as an alternative approach to prove exactness, it is computationally less efficient than our inductive iteration method.

## 4. Optimal Ordering Problem

The relative order in which the AC should be addressed must be determined before calculating feasibility, travel time, empty distance, etc. For this reason, we need to identify the optimal ordering of the contracts before proceeding with synergy quantification. Let's consider a bundle, denoted as $b$, as any unordered subset of $J$. To calculate the minimum traveling distance required to "cover" all $j$ in $b$, we must establish a specific order for the contracts. This order depends on the bundle's designated covering space, denoted as [α]. For each space, we commence at the endpoint of the current BC, $ep_i$, and conclude at the starting point of the next BC, $sp_{i+1}$.

### 4.1. Optimal Ordering Algorithm

The following steps are essential for constructing the algorithm to find the optimal order of auction contracts within a bundle.

- We consider each AC $j$ in bundle $b$ as an individual point rather than as an edge or arc connecting two points. In this representation, the distances between points become asymmetric.
- The distance between any contracts $k$ to $v$, where $k, v \in I \cup J$, corresponds to the distance from the endpoint of the first contract, $ep_k$, to the starting point of the next contract, $sp_v$. Conversely, the distance in the opposite direction from contract $v$ to $k$ is the effective distance from $ep_v$ to

$sp_k$. Additionally, when considering the distance from the starting point $sp_k$ to the endpoint $ep_k$ of any contract, $k$ will be counted as the distance weight of that contract, denoted as $w_k$.

**Function** Arrange($b$, $\alpha$):<br>
  *Create contract distance matrix $\mathcal{D}$ with dimensions $NC \cdot NC$.*<br>
  **for** *each pair of auctioned contracts* $j, j' \in b$ **do**<br>
    $D[j][j'] \leftarrow$ *the distance from* $ep_j$ *to* $sp_{j'}$.<br>
    $D[j'][j] \leftarrow$ *the distance from* $ep_{j'}$ *to* $sp_j$.<br>
  **for** *each pair of booked and auctioned contracts* $i, j$ **do**<br>
    $D[i][j] \leftarrow$ *the distance from* $ep_i$ *to* $sp_j$.<br>
    $D[j][i] \leftarrow$ *the distance from* $ep_j$ *to* $sp_i$.<br>
  ***Create contracts weights*** $w(k)$***:***<br>
  **for** *all* $k$ **do** $w(k) \leftarrow$ *the distance from* $sp_k$ *to* $ep_k$*;*<br>
  *Redefine the bundle in space* $\alpha$ *in between any two consecutive book contracts* $i$ *and* $i+1$<br>
  $b(\alpha) \leftarrow b' = [i, list(b), i+1]$*; where position of* $i$ *and* $i+1$ *is fixed*<br>
  ***Calculate Travel distance to any contract*** $j \in b'$***:***<br>
  **for** *each pair of contracts* $j, j' \in b'$ **do**<br>
    *Travel distance* $TD[j][j'][\alpha] = D[j][j'] + w(j')$<br>
  ***Calculate Arrival Time to any contract*** $j \in b'$***:***<br>
  **for** *Any contracts* $j \in b'$ **do**<br>
    *Arrival Time at* $j$, $AT[j = 1][\alpha] = t_i^e + [TD[i][j][\alpha]] * t_d$<br>
    *Start Time at* $j$, $ST[j] = max[AT[j], t_j^l]$<br>
    $AT[j(>1)][\alpha] = ST[j-1] + [TD[j-1][j][\alpha]] * t_d$<br>
  ***Determine Optimal order of*** $b'$***:***<br>
  **for** *Any contracts* $j \in b'$ **do**<br>
    *If* $t_j^e \leq ST[j][\alpha] \leq t_j^l$<br>
    *Optimal Order of* $b'$*, which is also optimal order of* $b$<br>
    $\beta_b^\alpha \leftarrow Argmin(TD[b'] : b' \leftarrow [i, list(b), i+1])$<br>
  *Calculate* $\Delta_\beta^\alpha \leftarrow D[\beta_b^i]$ *; Travel distances of the optimal visiting order.*<br>
  *Calculate* $\tau_\beta^\alpha \leftarrow \delta[ep_i, sp_{\beta[1]}, ep_{\beta[1]}, ..., sp_{\beta[k]}, ep_{\beta[k]}, sp_{i+1}]$ *; Travel time of* $\beta_b^\alpha$<br>
  **return** $(\beta_b^\alpha, \tau_\beta^\alpha, \Delta_{\beta \in b}^\alpha, AT[j][\alpha])$

**Figure 3: Algorithm 1 for optimal ordering problem**

To determine the optimal order of the bundle, the algorithm proposed in Figure 3 formulates the optimal ordering problem with the following elements:

- **Input:** The algorithm accepts bundles of AC as input.
- **Output:** The optimal order to minimize the travel distance for each feasible bundle.

- **Computing the contract distance matrix**: In the first step, Algorithm 1 focuses on creating a contract distance matrix $D$ of order $NC \times NC$, where $NC$ is the total number of contracts.
- **Computing contract weights and bundle sets in space:** In the second step, we calculate the weight of each contract $k$ as the distance between the start and end cities of that contract, i.e., $w_k$. Then, we construct an ordered set of bundles, where each bundle $b$ is placed in any space $\alpha$ between two consecutive BC. Specifically, for every space, the AC of bundle $b$ is placed between two BC $i$ and $i+1$, with their positions fixed. All possible combinations of contracts in b create various orders of the new bundles $b' = \text{Book}[i] + \text{list}(b) + \text{Book}[i+1]$; the $\text{list (b)}$ denotes a certain order of the contracts in $b$.
- **Computing travel distance:** In the third step, we calculate the travel distance between the contracts by adding the distance between them, as determined from the contract distance matrix $D,$ to the weight of the first city. This method allows us to determine the travel distance for both bundle $b'$ and bundle $b$.
- **Computing arrival time:** In the fourth step, we compute the arrival time for each contract within the bundle. The departure time of the first contract of the ordered set $b'$ in any space is the latest time at which that BC can be served, and this time is considered the arrival time for that contract. For all subsequent contracts, their arrival time is determined by adding the arrival time of their previous contract to its travel time from the preceding contract.
- **Computing the optimal order of the bundle:** According to the order and arrival time of any contracts in the bundle $b'$, the feasibility of the bundle is checked. This ensures that the arrival time for any contract must be between the earliest and latest times of the contract after starting from the first BC in space. We determine the optimal order that minimizes travel distance and record information regarding the bundle order ($\beta_b^\alpha$), travel distance, travel time, and arrival time for each contract. The travel time for the optimal order $\tau_\beta^\alpha$ is $\delta[q_1, q_2, \dots, q_r]$, i.e., time starting from the point $q_1$ to $q_r$ passing through all other points in the given order. Here, *r* is the order of the final point, and the corresponding order of the points is $[q_1, q_2, \dots, q_r]$.

## 5. Synergy Quantification

The synergies between BC and bundles of AC need to be determined. Synergy quantification is assessed by comparing the effective travel distance with the minimum possible travel distance for any given bundle $b \in B$. Specifically, for each bundle of AC, the synergy attributable to each BC is determined by calculating the ratio of the bundle's effective travel distance to the shortest total distance required to service the bundle across the spaces from $i$ to $i+1$, i.e., $\frac{\sum_{j \in b} w_j}{\Delta_\beta^\alpha}$. The largest value of this ratio indicates the best synergy, while the lowest value signifies a lesser synergy of bundle $b \in B$ with any given space. The following numerical example illustrates the process of quantifying synergy across possible routes.

**Example:** Consider a case involving three contracts, $C_1$, $C_2$, and $C_3$. In the original route, the effective distances for these contracts are 10 km, 8 km, and 6 km, respectively, totaling 24 km of loaded travel. However, this route also incurs an additional 9 km of empty travel between the locations of these contracts, as the distance from $C_1$ to $C_2$ is 5 km and from $C_2$ to $C_3$ is 4, leading to a cumulative distance of 33 km. The synergy ratio for the possible route $C_1 \rightarrow C_2 \rightarrow C_3$, calculated as the effective distance divided by the total distance, is approximately 0.73. This indicates that only 73% of the total travel is effective, while 27% constitutes empty travel, thereby highlighting an opportunity for optimization.

To improve efficiency, we consider an optimized route that minimizes the empty distances between contracts. The empty travel distance from $C_1$ to $C_3$ is 3 km, and from $C_3$ to $C_2$ is 2 km, resulting in a total empty distance of 5 km. Consequently, the overall travel distance drops to 29 km for the route $C_1 \rightarrow C_3 \rightarrow C_2$. This reduction in empty travel contributes to a higher synergy ratio of approximately 0.83, meaning that 83% of the total travel is now effective.

Thus, the optimized route clearly demonstrates how minimizing empty travel enhances operational synergy and improves overall efficiency. Our algorithm is designed to find the optimal synergy value, ensuring minimal empty travel while maximizing effective travel concerning AC within any bundle.

Our approach to quantifying synergy in contract bidding is more advanced than existing studies, as most synergy approaches are not precise but rather approximate (Hammami et al., 2021; Triki et al., 2014). Unlike Chang (2009), our method avoids arbitrary values and comprehensively incorporates key metrics, including empty distance and effective distance, which are available to bidders. Additionally, unlike the method outlined by Keskin et al. (2023), our model does not rely on "revenue," which is often unavailable to bidders during synergy assessments. Contrary to Triki (2016), who employs geometrical clustering of contracts that may fail in cases of uniformly located contracts or a lack of clusters, the proposed synergy quantification in this study considers both the directionality and the feasible time windows of contracts, offering a more nuanced analysis. An indecisiveness arises between distance-based and hop-based methods of pairwise synergy in Triki et al. (2014), and they did not propose any formula to calculate pairwise synergy. Our framework presents a singular, deterministic formula for synergy calculation, confidently articulating its mathematical and practical implications.

### 5.1. Synergy Calculation Algorithm

The study proposes Algorithm 2, illustrated in Figure 4, to efficiently calculate the synergy of feasible AC bundles within the space of pre-ordered BC. The index of spaces is redefined as $i$ (ranging from 1 to $m-1$) and serves as an iteration counter (the same index of BC). This dual purpose allows the progression of $\alpha$ and $i$ to serve iterations in both spaces and BC. However, we use $\alpha$ in the notation to identify the spaces where needed. The algorithm is described below.

- **Input:** The algorithm accepts two inputs, $I$ and $J$, which represent sets of BC and AC, respectively.
- **Output:** The algorithm produces the output $T$, which stores the synergy values for each bundle along with their optimal order.
- **Computing synergy**: Algorithm 2 focuses on computing synergy values among the BC and AC. It iterates over the spaces between BC and performs the following steps:

- **Filtering contract sets:** For each space $(i)$, the algorithm filters the set $(J)$ of AC into $(J_i)$, applying constraints to ensure that only feasible contract combinations are considered. These constraints involve timing and distance parameters, which help to eliminate infeasible bundles early in the process.
- **Iterative bundle evaluation:** The function starts with the smallest possible bundle size and iteratively evaluates all subsets of $(J_i)$, progressively increasing the subset size. This step-by-step increase allows for a thorough examination of all possible combinations, ensuring that no potentially synergistic bundles are overlooked.
- **Arrangement and feasibility check**: For each subset $(b)$ of AC, the algorithm arranges them in a specific order $(\beta_b^\alpha)$, travel time $(\tau_\beta^\alpha)$, and travel distance $(\Delta_\beta^\alpha)$. It also assesses their feasibility against logistical constraints such as serving times and distances. Infeasible bundles are promptly discarded, streamlining the process to focus only on viable options.
- **Synergy calculation**: Upon identifying a feasible bundle, the function calculates its synergy value $(S_\beta^\alpha)$, which reflects the time efficiency of serving the bundle compared to the latest permissible time for any end-BC in space $(i)$. Finally, it stores the calculated synergy and the optimal order of all feasible bundles in $T$.
- **Progressive bundle formation:** The function enhances its selection criteria by considering only new bundles that are supersets of previously identified feasible bundles. This approach ensures a logical and efficient buildup in the search for optimal bundles. The expression $F_{i,k'+1} \leftarrow \{b' : |b'| = k' + 1 \wedge \forall b'' \subset b', b'' \in \bigcup_{p \le k'} F_{i,p}\}$ is used to update the set of feasible bundles for a given space $(i)$ as the algorithm progresses. It adds to $F_{i,k'+1}$ all new bundles $b'$ of size $k' + 1$ that are built upon smaller, previously identified feasible bundles $(b'')$ from earlier steps $(p \le k')$. For example, given starting points $F_{\{i,1\}}$, $F_{\{i,2\}}$, and $(F_{\{i,3\}})$, which represent collections of validated subsets of size up to 3, we aim to construct $(F_{\{i,4\}})$ for space $(i)$ using subsets of size 4. Suppose $F_{\{i,1\}} = \{\{1\},\{2\},\{3\}\}$, $F_{\{i,2\}} =$

$\{\{1, 2\}, \{2, 3\}\}$, and $F_{\{i,3\}} = \{\{1, 2, 3\}\}$. To build $F_{\{i,4\})}$, we must ensure that all size 3 subsets ($b''$) of a candidate size 4 subset ($b'$) are in the union of $F_{\{i,1\}}, F_{\{i,2\}}$, and $F_{\{i,3\}}$. Now, the construction of $F_{\{i,4\}}$ is outlined below.

i. Considering elements 1, 2, and 3, we introduce a new element, 4, to create a subset ($b' = \{1, 2, 3, 4\}$).

ii. We check all size-3 subsets of $b'$: $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}$, and $\{2, 3, 4\}$.

iii. $\{1, 2, 3\}$ is validated in $F_{\{i,3\}}$, but subsets including 4 are not present in earlier sets, meaning ($b'$) fails the inclusion criteria for $F_{\{i,4\}}$.

Thus, $b' = \{1, 2, 3, 4\}$ cannot be added to $F_{\{i,4\}}$ since not all of its size-3 subsets have been validated in previous sets, leaving $F_{\{i,4\})}$ empty under the given criteria. This step ensures that as we construct larger bundles, they are built on the foundation of smaller bundles that have already been vetted for feasibility, thereby maintaining coherent and manageable growth in the complexity of the solution space.

In summary, the algorithm illustrated in Figure 4 aims to determine the feasibility of combining BC and AC. It subsequently computes synergy values based on various combinations of AC for each BC. The output table *T* presents the synergies of each feasible bundle across all spaces.

**Input :** Set of Booked Contracts $I$, Set of Auctioned Contracts $J$
**Output:** Synergy for all feasible contract bundles

**Function** `ComputeSynergy()`**:**
  **for** $i \leftarrow 1$ **to** $m-1$ **do**
    $J_i \leftarrow$ filtered set $J$ with the following mutually independent constraints for each $j$ in $J$;
      $t_i^e + (w(i) + D[i][j]) \cdot t_d \leq t_j^l$;
      $t_j^e + (w(j) + D[j][i+1]) \cdot t_d \leq t_{i+1}^l$;
      $t_i^e + (w(i) + D[i][j] + w(j) + D[j][i+1]) \cdot t_d \leq t_{i+1}^l$;
    Initialize $F_{i,1}$ to potential feasible bundles in space $i$ with size 1;
    **for** $k' \leftarrow 1$ **to** $|J_i|$ **do**
      **for** *all bundle* $b \in F_{i,k'}$ **do**
        $(\beta_b^{\alpha=i}, \tau_\beta^{\alpha=i}) \leftarrow \text{arrange}(b, \alpha = i)$;
        **if** $[t_{i+1}^l - (t_i^e + w(i) * t_d + \tau_\beta^\alpha)] < 0$ **then**
          $\mathrm{F}_{i,k'} \leftarrow F_{i,k'} \setminus \{b\}$
        **else**
          $\mathrm{S}_\beta^\alpha \leftarrow \frac{\sum_{j \in b} w(j)}{\Delta^i_{\beta \in \mathrm{b}}}$;
          Store $(\alpha, \beta_b^\alpha) \mapsto \mathcal{S}_\beta^\alpha$ in table $T$;
      $F_{i,k'+1} \leftarrow \{b' : |b'| = k' + 1 \wedge \forall b'' \subset b', b'' \in \bigcup_{p \leq k'} F_{i,p}\}$;
  **return** $T$;

**Figure 4: Algorithm 2 for synergy calculation**

## 5.2. Proof of Correctness

To formalize the propositions regarding the proof of correctness for the "ComputeSynergy" function, we focus on two main aspects: feasibility and synergy calculation. These propositions are designed to formally establish the integrity and intended output of the algorithm based on its inputs and operational logic.

### ***5.2.1.*** *Proposition 1: Feasibility of Bundles*

For every bundle $b$ in $F_{i,k'}$ at each iteration $k'$ (the size of the bundle), b is deemed feasible under the constraints established by the algorithm, specifically the time and distance constraints.

**Proof:**

- **Base case**: At $k' = 1$, $F_{\{i,1\}}$ consists of individual contracts that independently satisfy the feasibility constraints. This serves as our base case, establishing that each element in $F_{\{i,1\}}$ is feasible by definition.

- **Inductive step**: Assume that $F_{i,k'}$ contains only feasible bundles of size $k'$. When constructing $F_{i,k'+1}$, a bundle $b'$ of size $k' + 1$ can only be formed if every subset $b'' \subset b'$ of size $k'$ (or smaller) is already included in $\bigcup_{p \leq k'} F_{i,p}$. This ensures that $b''$ has been previously validated as feasible. Hence, the construction rules preserve the feasibility of $b'$ as it is based on the feasibility of its subsets, thereby maintaining the proposition through inductive reasoning.

*5.2.2. Proposition 2: Accurate Synergy Calculation and Storage*

The synergy $S_\beta^\alpha$ for each feasible bundle b in $F_{i,k'}$ is accurately calculated and stored in a table $(T)$, reflecting the correct computation of synergies according to the algorithm's specifications.

**Proof:**

- **Verification:** Given that a bundle is deemed feasible, its synergy $S_\beta^\alpha$ is calculated using the provided formula: $S_\beta^\alpha = \frac{\sum_{j \in b} w_j}{\Delta_\beta^\alpha}$. This formula involves the sum of weights $w_j$ for contracts $(j)$ in bundle $b$, adjusted by the optimal travel distance $b$, $\Delta_\beta^\alpha$. This calculation directly follows the algorithm's specifications, ensuring accuracy.

- **Storage integrity:** Once $S_\beta^\alpha$ is computed for $b$, it is stored in $(T)$ alongside its identifying parameters $(\alpha, \beta_b^\alpha)$. The algorithm's design ensures storage integrity by uniquely associating each synergy value with its corresponding bundle and parameters, thereby preventing misalignment or overwriting.

### *5.2.3. Corollary 1: Comprehensive Coverage and Optimization*

By iterating through all possible bundle sizes and ensuring each step's correctness (as outlined in Propositions 1 and 2), the "ComputeSynergy" function not only covers the entire feasible solution space but also accurately computes and records the optimal synergy values for contract bundles within the specified constraints.

- **Justification:** The algorithm's iterative and inductive logic ensures that no feasible bundle is overlooked and each bundle's synergy is accurately computed and stored. This comprehensive approach, underpinned by the correctness of its feasibility verification and synergy computation, guarantees that the algorithm's output is both complete and optimized within the operational constraints.

These propositions and the accompanying corollary outline the theoretical foundation that affirms the exactness and correctness of "ComputeSynergy" by methodically verifying its feasibility and accurately computing the synergies. Additionally, higher efficiency can be achieved by systematically reducing the solution space and focusing computational resources on promising paths, thus outperforming exhaustive enumeration methods in terms of computational efficiency and resource utilization.

## 6. Bid Generation

Once we have computed and saved the synergy of all feasible ordered bundles of AC across various spaces, our next step is to build the optimal bid. This bid comprises the best selection of feasible bundles. Although it may seem straightforward to include the bundles with the highest synergy in each space, we encounter challenges known as the Common Contract Problem. Common AC may exist in multiple bundles, each exhibiting maximum synergy within their respective spaces. In this case, it is not possible to include all of these bundles in the optimal bid, as covering an AC multiple times is not allowed. To resolve this problem, we propose Algorithm 3, as shown in Figure 5.

**Input** : Synergy Table $T$ mapping space $i$ and bundle $b$ to its synergy $S^i_{\beta \in b}$ and indicating if contract $j$ is included in bundle $b$ as $C_{jb}$

**Output:** Final Bid Configuration

**Function** `ExtractData(`$T$`)`:

  **for** *each space* $i$ *and bundle* $b$ *in* $T$ **do**

    Extract $S^i_\beta$ from $T$ and populate $S$

    For each contract $j$ in $b$, extract $C_{jb}$ from $T$ and populate $C$

  **return** $S, C$

**Function** `GenerateFinalBid(`$S, C$`)`:

  Define binary variables $x_{ib}$ for each space $i$ and bundle $b$

  `// Objective Function`

  Maximize $Z = \sum_{i \in I} \sum_b S^i_\beta \cdot x_{ib}$

  `// Constraints`

  **for** *each space* $i$ **do**

    $\sum_b x_{ib} \leq 1 \quad \forall i$ `// One bundle per space`

  **for** *each contract* $j$ **do**

    $\sum_i \sum_b C_{jb} \cdot x_{ib} \leq 1 \quad \forall j$ `// No common contracts`

  $x_{ib} \in \{0, 1\} \quad \forall i, b$ `// Binary variables`

  Solve the ILP problem

  **for** *each space* $i$ **do**

    **if** $x_{ib} = 1$ *for some* $b$ **then**

      Mark bundle $b$ as selected for space $i$

  **return** Selected bundles for each space

$S, C \leftarrow$ `ExtractData(`$T$`)`

$FinalBid \leftarrow$ `GenerateFinalBid(`$S, C$`)`

**return** $FinalBid$

**Figure 5: Algorithm 3 for bid generation**

The algorithm formulated employs an ILP approach to select a set of bundles that maximizes total synergy while ensuring that no common contracts exist across the various spaces. Below is a step-by-step explanation that utilizes both the provided pseudocode and a descriptive explanation:

- **Input:** A synergy table $T$ that maps each space $i$ and bundle $b$ to its synergy $S_\beta^{i(=\alpha)}$. A binary indicator $C_{jb}$ that is equal to 1 if contract $j$ is included in bundle $b$ and 0 otherwise.
- **Output:** The final bid configuration comprises a selection of bundles that maximize total synergy while satisfying all constraints.
- **Data collection:** The following steps outline the process for collecting the relevant input data:
    - Go through the synergy table $T$, which was determined using Algorithm 2.

- Extract the synergy value $S_{\beta}^{i(=\alpha)}$ for each space $i$ and bundle $b$.
- Determine whether each contract j ∈ $b$, $(C_{jb})$ is common across the bundles with a (1,0) parameter.

- **Decisions:** Create binary decision variables $x_{ib}$ for each space $i$ and bundle $b$. These variables will be equal to 1 if bundle $b$ is selected for space $i$; otherwise, they will be 0.
- **Objective function:** Formulate the ILP model to maximize the total synergy $Z$, defined as the sum of the synergy values $S_{\beta}^{i(=\alpha)}$ for the selected bundles across all spaces.
- **Constraints:** The first constraint ensures that, at most, one bundle is selected for each space (constraint for each space $i$). The second constraint confirms that each contract $j$ appears in at most one selected bundle across all spaces (constraint for each contract $i$). The third constraint enforces binary constraints on $x_{ib}$ to ensure that the variables can only take values of 0 or 1.
- **Run the process:** After solving the ILP, iterate through each space $i$ and mark bundle $b$ as selected if $x_{ib} = 1$. Return the selected bundles for each space as the final bid configuration.

Thus, the algorithmic process for selecting bundles is reported as the final bid configuration, which represents the optimal choice of bundles aimed at maximizing synergy.

### 6.1. Discount Opportunities

The flexibility of time windows for fulfilling AC can lead to cost savings for bidders, providing them with greater opportunities to improve their network synergy. However, deviations from these time windows may not always be acceptable to the auctioneer, as they could jeopardize their ability to meet commitments, leading to increased costs. In such cases, the bidder may propose a new service time, which could be either earlier or later than the original time window, potentially accompanied by discounts. However, we only consider early-time relaxation to ensure that there is no violation of on-time demand mitigation due to late deliveries. The auctioneer must then decide whether to accept the offer, as they may incur extra costs associated with early delivery. The algorithm used to calculate the savings that justify a potential discount is shown in Figure 6.

**Input** : Set of Booked Contracts $I$, Set of Auctioned Contracts $J$, Constructed Final Bid $B$, Flexible Time, $FT > 0$;
**Output:** Savings for Potential Early Shipment Bid

Calculate the weights of all contracts $k \in (B \cup I)$, $W_B = \sum w(k)$;
Calculate total travel distance of $B$, $D_B$;
Calculate the Overall synergy of bid $B$, $S_B = \frac{W_B}{D_B}$;
Store $\leftarrow S_B$;
Calculate New Early time ,$NE[j] = max(0, t_j^e - FT)$ for all $j \in J$;
Insert $t_j^e = NE[j]$ for all $j \in J$;
Run Synergy Calculation Algorithm & store results in Table $T'$;
Run Bid Construction Algorithm & store results in ordered set $B'$;
Calculate the Overall synergy of revised bid $B'$, $S_{B'} = \frac{W_{B'}}{D_{B'}}$;
Calculate the percentage of savings in total trip per loaded trip;
$Savings = \left(\frac{S_{B'} - S_B}{S_B}\right) \times 100$;
**return** (savings);

**Figure 6: Algorithm 4 for relaxed time windows for auctioned contracts**

In the first step, we calculate the overall synergy of the already selected bid in conjunction with the proposed time windows. Next, we relax the time window constraints as defined in Algorithm 4, illustrated in Figure 6. We then check the revised bid along with its relevant information and the overall synergy of the revised bid. The new synergy value $S_{B'}$ must be greater than or equal to the previous $S_B$, i.e., $S_{B'} \geq S_B$. Finally, we calculate the possible savings in the final step of the algorithm. This algorithm describes a process for calculating potential savings by adjusting shipment times within a flexible time (FT) window. Detailed explanations are provided below.

- **Calculate contract weights**: Sum the weights (the distance from the start city to the end city of each contract) of contracts $w(k)$ for all $k \in (B \cup I)$. This is denoted by $W_B = \sum w(k)$.
- **Total travel distance**: Calculate the total travel distance for bid ( $B$ ), referred to as $D_B$. The distance of the route consists of the BC and AC orders included in the final bid.

- **Overall synergy:** Compute the overall synergy of the bid ( $B$ ), denoted as $S_B = \frac{W_B}{D_B}$, which represents the ratio of the total weight of contracts ( $W_B$) to the total travel distance ( $D_B$). Additionally, store the initially calculated synergy $S_B$ for future comparison.
- **Adjust early time**: For each contract $j \in J$, calculate the new earliest start time $NE[j]$ by reducing the current earliest time ( $t_j^e$) by introducing the parameter $FT(> 0\ )$, ensuring that it does not fall below zero. Then, update the earliest time $NE[j] = \max\{0, t_j^e - FT\}$ for all contracts $j \in J$.
- **Synergy re-calculation**: Execute the synergy calculation algorithm using the updated earliest times and store the synergy values of $j \in J$ in a new table $T'$.
- **Construct a revised bid**: Use the bid construction algorithm to create an ordered set ($B'$) that reflects the revised bid with updated timelines. Calculate the overall synergy of the revised bid as $S_{B'} = \frac{W_{B'}}{D_{B'}}$.
- **Computed savings:** Compute the savings using the formula: *savings* = $\frac{100(S_{B'} - S_B)}{S_B}$. This formula yields a percentage improvement in synergy resulting from adjusted times. Specifically, the effective distance or loaded travel increases the total travel rate (effective + empty travel). The calculated savings are returned, signifying the benefit of adjusting the earliest start times within the given FT window.

Thus, this algorithm maximizes efficiencies by potentially shifting delivery times within a permissible range, thereby increasing the overall synergy of the route. These savings can be used to offer potential discounts for shippers/auctioneers, contingent upon a new service time that may violate the specified earliest time limitation.

To illustrate the savings that can be achieved by using the above method, we present an example based on the following contracts:

- Contract 1 (Auctioned): Current earliest time $t_1^e = 9:00\text{AM}$, Distance: $w_1 = 40$ km
- Contract 2 (Auctioned): Current earliest time $t_2^e = 10:00\text{AM}$, Distance: $w_2 = 20$ km
- Contract 3 (Booked): Distance $w_3 = 50$ km
- Contract 4 (Booked): Distance $w_4 = 30$ km

The constructed "final bid" B includes all four contracts. Thus, the distances (weights) of all the contracts are $W_B = 140$ $(\text{i.e.}, w_1 + w_2 + w_3 + w_4 = 40 + 20 + 50 + 30)$. We also assume that the total travel distance for bid B is $D_B = 300$. Consequently, the overall synergy of bid B is:

$$S_B = \frac{W_B}{D_B} = \frac{140}{300} = 0.46$$

If the parameter *FT* is set to 2 hours, this indicates that the company can propose to serve the AC either 2 hours earlier or later, depending on their operational strategy. Then, the new earliest times for Contracts 1 and 2 are calculated as follows:

- $NE[1] = \max(0, t_1^e - FT) = \max(0, 9:00\text{AM} - 2 \text{ hours }) = 7:00\text{AM}$
- $NE[2] = \max(0, t_2^e - FT) = \max(0, 10:00\text{AM} - 2 \text{ hours }) = 8:00\text{AM}$

We can use this to evaluate the overall synergy of the revised bid $B'$ considering the new earliest time. Assuming that the total travel distance after adjustments decreases to 270 units while the total weight remains the same, we have $S_{B'} = \frac{W_{B'}}{D_{B'}} = \frac{140}{270} \approx 0.51$. Additionally, we can calculate the savings percentage for the total trip per loaded trip:

$$\text{Savings } = \left(\frac{S_{B'} - S_B}{S_B}\right) \times 100 \approx 11\%\text{B}$$

ased on the travel savings, the company may decide to offer a maximum discount that is proportional to an 11% reduction in distance, provided that customers accept the newly proposed time windows (which are earlier in this case). Therefore, the logistics company can reduce costs and improve scheduling efficiency by offering flexibility in service times. This arrangement also benefits customers who are willing to adjust their delivery windows in exchange for a lower delivery cost.

## 7. Results and Discussion

For the numerical illustration, we used test data where the BC were pre-ordered. We assumed that the AC has both early and late service times, allowing them to be scheduled between any BC according to feasibility, while the real geographic nature remains fixed. Conversely, BC has fixed service times to serve them as scheduled before. In this case, the vehicle may arrive before the designated service time; however, service will start only at the given time, necessitating that the vehicle wait until then. There were 12 contracts among eight cities, including seven BC and five AC. The details of these contracts are presented in Tables 3 and 4.

**Table 3: Pre-ordered booked contracts**

| Contracts | Start city | End city | Service time |
|---|---|---|---|
| $C_1'$ | City 6 | City 7 | 0 |
| $C_2'$ | City 7 | City 3 | 50 |
| $C_3'$ | City 8 | City 1 | 150 |
| $C_4'$ | City 4 | City 3 | 220 |
| $C_5'$ | City 2 | City 4 | 400 |
| $C_6'$ | City 3 | City 5 | 650 |
| $C_7'$ | City 4 | City 6 | 750 |

**Table 4: Auctioned contracts to fit in the existing network**

| Contracts | Start city | End city | Earliest time | Latest time |
|---|---|---|---|---|
| $C_1$ | City 1 | City 2 | 10 | 150 |
| $C_2$ | City 2 | City 3 | 50 | 150 |
| $C_3$ | City 4 | City 2 | 20 | 300 |
| $C_4$ | City 3 | City 8 | 200 | 350 |
| $C_5$ | City 6 | City 7 | 400 | 500 |

These tables provide a comprehensive overview of AC and BC, detailing the relevant times and cities. The distances between each pair of cities are presented in Table 5, while Table 6 contains the contract distance matrix, which includes the distance weights of the contracts.

**Table 5: Distances between the cities**

| **Cities** | City 1 | City 2 | City 3 | City 4 | City 5 | City 6 | City 7 | City 8 |
|---|---|---|---|---|---|---|---|---|
| City 1 | 0 | 141 | 139 | 83 | 97 | 151 | 85 | 181 |
| City 2 | 141 | 0 | 116 | 46 | 116 | 121 | 56 | 87 |
| City 3 | 139 | 116 | 0 | 83 | 105 | 142 | 78 | 89 |
| City 4 | 83 | 46 | 83 | 0 | 109 | 46 | 44 | 120 |
| City 5 | 97 | 116 | 105 | 109 | 0 | 104 | 139 | 62 |
| City 6 | 151 | 121 | 142 | 46 | 104 | 0 | 150 | 144 |
| City 7 | 85 | 56 | 78 | 44 | 139 | 150 | 0 | 90 |
| City 8 | 181 | 87 | 89 | 120 | 62 | 144 | 90 | 0 |

All of our experiments were conducted using the default Python 3 runtime on Google Colab, which is equipped with two virtual CPUs and 12 GB of RAM. For connectivity, we utilized a personal computer featuring an Intel(R) Core(TM) i3-7100U CPU @ 2.40GHz and 8 GB of RAM. The bid combines the feasible bundle of AC that yields the highest total synergy across all spaces. In our current arrangement, there were six spaces (calculated as the number of BC minus one), and the selected bundles from a total of $2^5$, after performing Algorithms 1 and 2, were presented along with their corresponding synergy values.

**Table 6: Contract distance matrix and the weights of the contracts**

| Contracts | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1'$ | $C_2'$ | $C_3'$ | $C_4'$ | $C_5'$ | $C_6'$ | $C_7'$ | Weights |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_1$ | 0 | 0 | 46 | 116 | 121 | 121 | 56 | 87 | 46 | 0 | 116 | 46 | 141 |
| $C_2$ | 139 | 0 | 83 | 0 | 142 | 142 | 78 | 89 | 83 | 116 | 0 | 83 | 116 |
| $C_3$ | 141 | 0 | 0 | 116 | 121 | 121 | 56 | 87 | 46 | 0 | 116 | 46 | 46 |
| $C_4$ | 181 | 87 | 120 | 0 | 144 | 144 | 90 | 0 | 120 | 87 | 89 | 120 | 89 |
| $C_5$ | 85 | 56 | 44 | 78 | 0 | 150 | 0 | 90 | 44 | 56 | 78 | 44 | 150 |
| $C_1'$ | 85 | 56 | 44 | 78 | 150 | 0 | 0 | 90 | 44 | 56 | 78 | 44 | 150 |
| $C_2'$ | 139 | 116 | 83 | 0 | 142 | 142 | 0 | 89 | 83 | 116 | 0 | 83 | 78 |
| $C_3'$ | 0 | 141 | 83 | 139 | 151 | 151 | 85 | 0 | 83 | 141 | 139 | 83 | 181 |
| $C_4'$ | 139 | 116 | 83 | 0 | 142 | 142 | 78 | 89 | 0 | 116 | 0 | 83 | 83 |
| $C_5'$ | 83 | 46 | 0 | 83 | 46 | 46 | 44 | 120 | 0 | 0 | 83 | 0 | 46 |
| $C_6'$ | 97 | 116 | 109 | 105 | 104 | 104 | 139 | 62 | 109 | 116 | 0 | 109 | 105 |
| $C_7'$ | 151 | 121 | 46 | 142 | 0 | 0 | 150 | 144 | 46 | 121 | 142 | 0 | 46 |

$(Bundle, Synergy, Space) \equiv$

$[(\{C_1\}, \{0.5\}, \{1\}); (\{C_1, C_2\}, \{0.52\}, \{2\}); (\{C_4, C_3\}, \{0.30\}, \{3\}); (C_4, C_3, \{0.52\}, \{4\}); (\{C_5\}\{0.54\}, \{5\})]$

Therefore, we found the bundles with the highest synergy in spaces 1, 2, 3, 4, and 5; however, no bundles were feasible for space six. Additionally, these sets of bundles share common contract issues. After performing Algorithm 3, we obtained the final bid:

$(Bundle, Synergy, Space) \equiv$
$[(\{C_1\}, \{0.5\}, \{1\}); (\{C_2\}, \{0.36\}, \{2\}); (\{C_3\}, \{0.26\}, \{3\}); (\{C_4\}, \{0.50\}, \{4\}); (\{C_5\}, \{0.54\}, \{5\})]$.

Table 7 shows the bids and visiting orders of the contracts. There are two cases to determine the different routes for the optimal bid: Case 1 involves strict time windows, while Case 2 allows for relaxed early time windows for AC, where the early time can be relaxed by an $FT (> 0)$ in hours. The time windows for the BC remain unchanged; therefore, the time windows for the AC will be redefined as $NE[j] = \max\{0, t_j^e - FT\}$ for all contracts $j \in J$. For this specific example, we set $FT = 400$ hours to provide maximum space for early service.

**Table 7: Optimal bid, routes, and travel distances**

| Cases | Bid (ordered set) | Route | Overall synergy |
|---|---|---|---|
| **Case 1** | $B = \{C_1, C_2, C_3, C_4, C_5\}$ | $\boldsymbol{C'_1} \to C_1 \to \boldsymbol{C'_2} \to C_2 \to \boldsymbol{C'_3} \to C_3 \to \boldsymbol{C'_4} \to C_4 \to \boldsymbol{C'_5} \to C_5 \to \boldsymbol{C'_6} \to \boldsymbol{C'_7}$ | $S_B = 0.607$ |
| **Case 2** | $B' = \{C_1, C_2, C_4, C_3, C_5\}$ | $\boldsymbol{C'_1} \to C_1 \to C_2 \to \boldsymbol{C'_2} \to C_4 \to \boldsymbol{C'_3} \to \boldsymbol{C'_4} \to C_3 \to \boldsymbol{C'_5} \to C_5 \to \boldsymbol{C'_6} \to \boldsymbol{C'_7}$ | $S_{B'} = 0.686$ |
| $Savings = \dfrac{100(S_{B'} - S_B)}{S_B} \approx 13\%$ | | | |

Table 7 confirms that the order of AC $\{C_1, C_2, C_3, C_4, C_5\}$ represents the optimal bidding strategy for the bidder in Case 1. However, for Case 2, the selected contract orders were $\{C_1, C_2, C_4, C_3, C_5\}$. It is necessary to determine the optimal route for fulfilling the bid, which includes AC within the existing transportation network of pre-booked agreements that have already been ordered. Each case presents a unique route, as shown in Table 7, and the overall synergy of the route is important for estimating service costs. The synergy for the first case, given the specified time windows, was **0.607**, whereas it was **0.686** for the second case. This indicates an improvement, as the travel distance of the vehicle per loaded trip was reduced by (0.686 - 0.607) = **0.079**. Therefore, the flexibility in time windows creates an opportunity to save costs and reduce emissions by enhancing overall synergy. In this instance, the percentage reduction in costs was around 13%, representing savings per distance cost for the bidder. However, since the AC serves in various spaces of the existing network, the time window for any space was limited to two consecutive BC if $NE[j] = 0$.

In some cases, early shipment is usually acceptable according to the situation and cost considerations. However, this can create extra costs because of additional time in inventory management due to early shipment, and late shipments can also incur costs because of lost sales opportunities. The flexibility in time windows can help reduce costs for the carrier service provider, creating a trade-off between bidders and auctioneers. This trade-off allows for the potential use of savings—proportional to distance reduction—as discounts offered by carriers to auctioneers. Additionally, minimizing travel, specifically for empty trips, can decrease emission costs for both parties.

### 7.1 Results Validation

To validate the results, this study examined a real case analyzed by Triki (2016) in his proposal for location-based synergy techniques. The data included 14 contracts across 16 cities, including eight BC and six AC, as shown in Table 8 below.

**Table 8: Contracts by connecting a pair of cities**

| Booked contracts | Start city | End city |
|---|---|---|
| $L'_1$ | Al Amarat | Rusayl |
| $L'_2$ | Al Amarat | Barka |
| $L'_3$ | Salalah | Muscat |
| $L'_4$ | Sohar | Musandam |
| $L'_5$ | Muscat | Dubai |
| $L'_6$ | Muscat | Abu Dhabi |

| | | |
|---|---|---|
| $L'_7$ | Muscat | Fahud |
| $L'_8$ | Muscat | Sohar |
| **Auctioned contracts** | **Start city** | **End city** |
| $L_1$ | Muscat | Nizwa |
| $L_2$ | Dubai | Ras Al-Khaimah |
| $L_3$ | Sohar | Al Burimi |
| $L_4$ | Salalah | Hima |
| $L_5$ | Salalah | Duqum |
| $L_6$ | Al Amarat | Al Rustaq |

As the data lacks information regarding time windows, we performed our experiment using a modified approach where the contract time limitations were relaxed. We limited the bundle size to no more than two to maintain comparability with Triki (2016). To determine the coordinates of the cities listed in Table A.1, we used the Opencage.geocoder library. This process led to a distance matrix for the cities presented in Table A.2 and allowed us to create the contract distance matrix, which incorporates the weights of the contracts shown in Table A.3 (see Appendix). Table 9 displays the bundles in various spaces that exhibit the highest synergy.

**Table 9: Bundles with the highest synergy**

| Spaces | Bundles | Synergy |
|---|---|---|
| 1 | $\{L_1, L_6\}$ | 0.48 |
| 2 | $\{L_4, L_5\}$ | 0.33 |
| 3 | $\{L_1, L_6\}$ | **0.49** |
| 4 | $\{L_4, L_5\}$ | 0.31 |
| 5 | $\{L_4, L_5\}$ | 0.33 |

| | | |
|---|---|---|
| 6 | $\{L_4, L_5\}$ | 0.34 |
| 7 | $\{L_4, L_5\}$ | 0.37 |

The table shows that the highest synergy with the bundle $\{L_1, L_6\}$ is **0.49** among all selected bundles, where $L_1$: $\{Muscat - Nizwa\}$ and $L_6$: $\{ Al\ Amarat - Al\ Rustaq\}$. This finding aligns with the results reported by Triki (2016). Therefore, it is verified that our proposed synergy algorithm performs correctly. After the execution of the bid generation algorithm, the study obtained the final bid, as shown in Table 10.

**Table 10: Optimal bid and route**

| Spaces | Bundles | Synergy |
|---|---|---|
| 1 | $\{L_6\}$ | 0.48 |
| 2 | $\{L_4\}$ | 0.33 |
| 3 | $\{L_1\}$ | **0.49** |
| 4 | $\{L_3\}$ | 0.31 |
| 5 | $\{\ \}$ | 0.33 |
| 6 | $\{L_2\}$ | 0.34 |
| 7 | $\{L_5\}$ | 0.37 |
| **Route** | | **Overall synergy** |
| $L'_1 \to L_6 \to L'_2 \to L_4 \to L'_3 \to L_1 \to L'_4 \to L_3 \to L'_5 \to L'_6 \to L_2 \to L'_7 \to L_5 \to L'_8$ | | 0.487 |

The final route follows the cities "*Al Amarat➔Rusayl➔Al Amarat➔Al Rustaq➔Al Amarat➔Barka➔Salalah➔ Hima➔Salalah➔Muscat➔Muscat➔Nizwa➔Sohar➔ Musandam➔Sohar➔ AlBurimi➔Muscat➔Dubai➔Muscat➔AbuDhabi➔Dubai➔Ras Alkhaima➔Muscat➔ Fahud➔Salalah➔Duqum➔ Muscat➔Sohar*."

In addition, we examined other test data reported in the same works by Triki (2016) and Keskin et al. (2023), as shown in Figure 7 and Table A.4 in the Appendix.

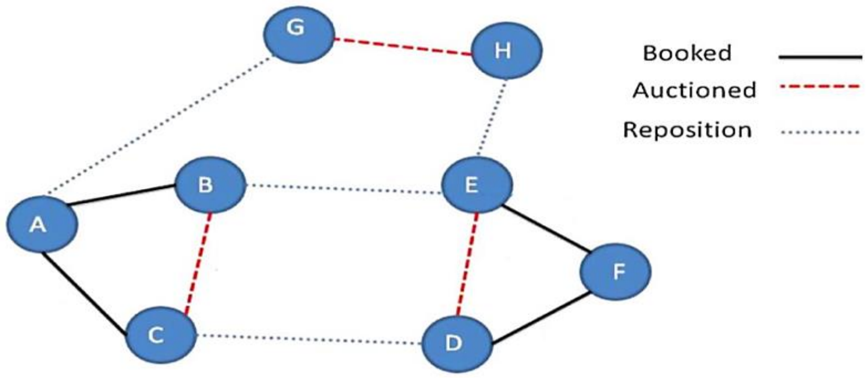


**Figure 7: Network of the illustrative example (from Triki, 2016)**

We applied our proposed algorithm to determine the synergy of bundles ($2^3$-1) of sizes up to two and obtained the optimal bundles in spaces as $(Bundle, Synergy, Space) \equiv$ $[(\{BC\},\{0.37\},\{1\}); (\{BC,DE\},\{0.57\},\{2\}); (\{DE\},\{0.46\},\{3\})]$.

This indicates the maximum synergy associated with the bundle $\{BC, DE\}$, which is also referenced in Triki (2016) and Keskin et al. (2023). However, this study produced a final set of bundles by executing Algorithm 3.

$(Bundle, Synergy, Space) \equiv [(\{BC\},\{0.37\},\{1\}); (\{GH\},\{0.21\},\{2\}); (DE\},\{0.46\},\{3\})]$.

Therefore, the proposed algorithm for calculating synergy can be successfully implemented to identify the optimal bundle for the bidder in a specific space of their existing transportation network. The final set of bundles, after removing common contract issues, represents a potential bid for the overall route.

This has been achieved with remarkable efficiency, as the execution time of our approach is only 0.012 seconds. Conversely, the computational times of the three strategies developed by Keskin et al. (2023) to solve the same instance amount to 0.022, 0.041, and 0.032 seconds, respectively. The most noteworthy outcome is that our algorithm enhances route synergy by minimizing total travel distances and accordingly reducing empty travels.

### 7.2 High Dimensionality Experiments

In the previous section, we validated our algorithms for a small size of instances. However, it is essential to also assess the performance of the proposed algorithm on larger dimensions of the instances regarding solution quality and execution time compatibility. Due to the unavailability of benchmarking data, we generated input data to test the algorithms. The data generation process considered the following actions:

- Contract points are uniformly distributed within a circle of radius R(= 1000), applicable to both BC and AC. These points are set by first determining a random angle $\theta$ and a radius $\rho$, which are adjusted to ensure uniform distribution across the circle's area. The angle is chosen from between $0$ and $2\pi$ radians, where $\theta = random.uniform\ (0, 2\pi)$. The radius is calculated using the formula $\rho = \sqrt{random.\ uniform\ (0,1)} * R$ to maintain uniformity. These polar coordinates $(\rho, \theta)$ are then converted to Cartesian coordinates $(x, y)$. Finally, the Euclidean distance between each pair of points/cities is calculated to create a distance matrix.
- Contract distances are determined by the shortest path from a starting point to any random location within the designated circle. The term "empty distance" refers to the second shortest path: one that connects AC and another that links BC to AC. For example, if a BC connects point A to point B, the next job is to pick up goods from point C (an AC). If point C is the second closest auctioned point to B, then the path from B to C is classified as the empty distance. Further, if there is another AC at point D and it is the second closest to point C after the delivery at C has been completed, then the distance from C to D is considered the following empty distance.

- Transitioning from spatial to temporal dynamics, we assume that the interval between consecutive BC follows a normal distribution. This assumption regarding the time gap reflects the stochastic nature of contract scheduling and completion, thereby adding a temporal dimension to our spatial framework. Temporal spareness (TS) is defined as any parameter greater than 1, which increases the time space between two consecutive BC ($i$). Initially, we set $TS = 10$ to perform a high-dimensionality experiment that generates time windows for BC.

i.e., $t_i^{service} = t_{i-1}^{service} + (empty\ travel\ time\ between\ i\ and\ i - 1) * TS$

- Set the time horizon equal to the latest time of the last BC in the sequence. Additionally, establish a time limit based on the ratio of the time horizon to the total number of contracts. Next, define the time windows for the AC ($j$) as follows:

$t_j^e = random.uniform(0, time\ horizon)$

and $t_j^l = t_j^e + random.uniform(0, time\ limit)$

This study quantifies the probability density for the distance "$d$" between contract points, as identified in García-Pelayo (2005), by integrating our spatial assumptions into a coherent mathematical framework. The setup of the test instances is described in Table 11.

**Table 11: Test instance profile**

| **Instances ID** | **Number of booked contracts** | **Number of auctioned contracts** | **Number of cities** |
|---|---|---|---|
| 1 | 100 | 150 | 40 |
| 2 | 150 | 250 | 60 |
| 3 | 200 | 350 | 80 |
| 4 | 250 | 450 | 100 |
| 5 | 300 | 500 | 120 |
| 6 | 350 | 550 | 140 |

| | | | |
|---|---|---|---|
| 7 | 400 | 600 | 150 |
| 8 | 450 | 650 | 170 |
| 9 | 500 | 700 | 200 |
| 10 | 550 | 800 | 220 |

The computational results are presented in Table 12. The objective of the problem, including route decisions based on effective distances, empty distances, and overall synergy, was analyzed for both cases: strict time windows and relaxed early time windows for AC. These factors were determined across several columns, with the final column displaying the calculated savings.

**Table 12: High dimensionality experiment results**

| **ID** | **Case 1: With strict time windows** | | | | | **Case 2: Relaxed earliest time of auctioned contracts (FT = 100 hours)** | | | | | **Savings** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Obje-ctive | Effective distance | Empty distances | Overall synergy | CPU Time (sec.) | Obje-ctive | Effective distance | Empty distances | Overall synergy | CPU Time (sec.) | |
| **1** | 23.60 | 153407.13 | 121344.87 | 0.56 | 0.16 | 34.16 | 190380.64 | 137697.27 | 0.58 | 3.25 | 3.93 |
| **2** | 40.05 | 255203.74 | 211604.00 | 0.55 | 0.33 | 62.22 | 326675.53 | 231213.84 | 0.59 | 7.40 | 7.11 |
| **3** | 54.42 | 396344.84 | 325213.63 | 0.55 | 0.47 | 76.42 | 491514.35 | 364793.63 | 0.57 | 3.01 | 4.50 |
| **4** | 75.45 | 455241.92 | 364821.60 | 0.56 | 0.33 | 104.40 | 586841.27 | 413726.75 | 0.59 | 5.55 | 5.65 |
| **5** | 84.13 | 553935.45 | 454749.43 | 0.55 | 0.53 | 118.29 | 671586.12 | 484350.23 | 0.58 | 10.65 | 5.79 |
| **6** | 91.73 | 600344.46 | 485784.38 | 0.55 | 1.05 | 135.74 | 771445.23 | 554039.18 | 0.58 | 15.58 | 5.30 |
| **7** | 108.69 | 677987.50 | 564465.27 | 0.55 | 0.50 | 154.09 | 822873.60 | 625450.35 | 0.57 | 6.15 | 4.12 |
| **8** | 115.11 | 763090.63 | 635676.42 | 0.55 | 0.49 | 170.83 | 935570.86 | 707821.59 | 0.57 | 4.22 | 4.35 |
| **9** | 123.51 | 831645.76 | 703781.57 | 0.54 | 0.68 | 179.72 | 1012931.79 | 784167.83 | 0.56 | 8.65 | 4.06 |
| **10** | 139.83 | 985539.01 | 825498.34 | 0.54 | 0.92 | 197.29 | 1179760.76 | 899494.65 | 0.57 | 20.39 | 4.27 |

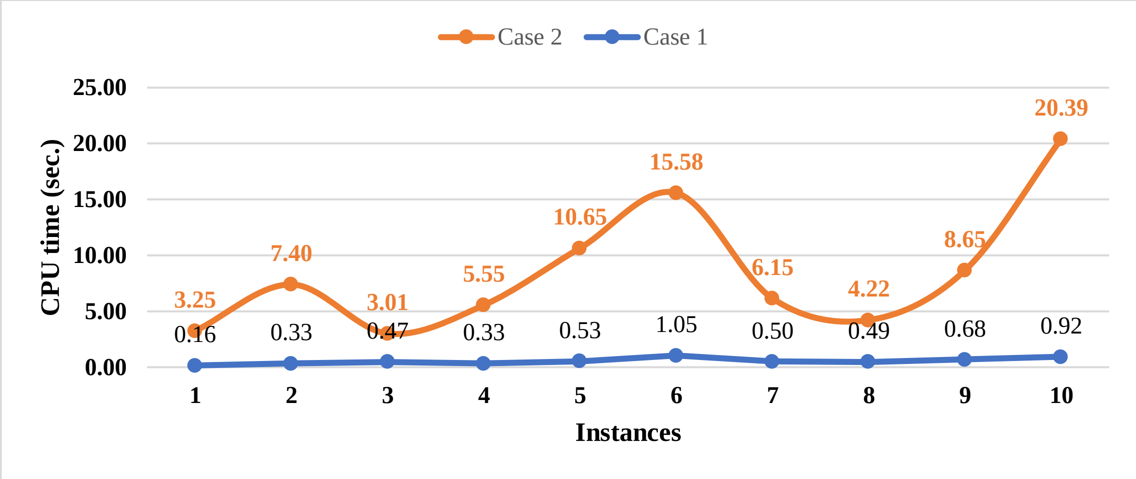


**Figure 8: CPU time Case 1 vs. Case 2**

Table 12 shows that CPU times remain reasonable, even at higher dimensions for both cases (see Figure 7). However, on average, Case 2 requires more CPU time than Case 1. Despite this, Case 2 exhibited better overall network synergy (see Figure 8); Specifically, in Case 2, the effective distance (travel) increases per the total travel distances. Therefore, relaxing the early time limitations of AC creates an opportunity for cost savings by reducing empty travel and enhancing synergy. The savings achieved can be offered as discounts to customers, subject to their acceptance of early shipments compared to the defined early time limitations. Figure 9 shows the savings resulting from the relaxed time windows.

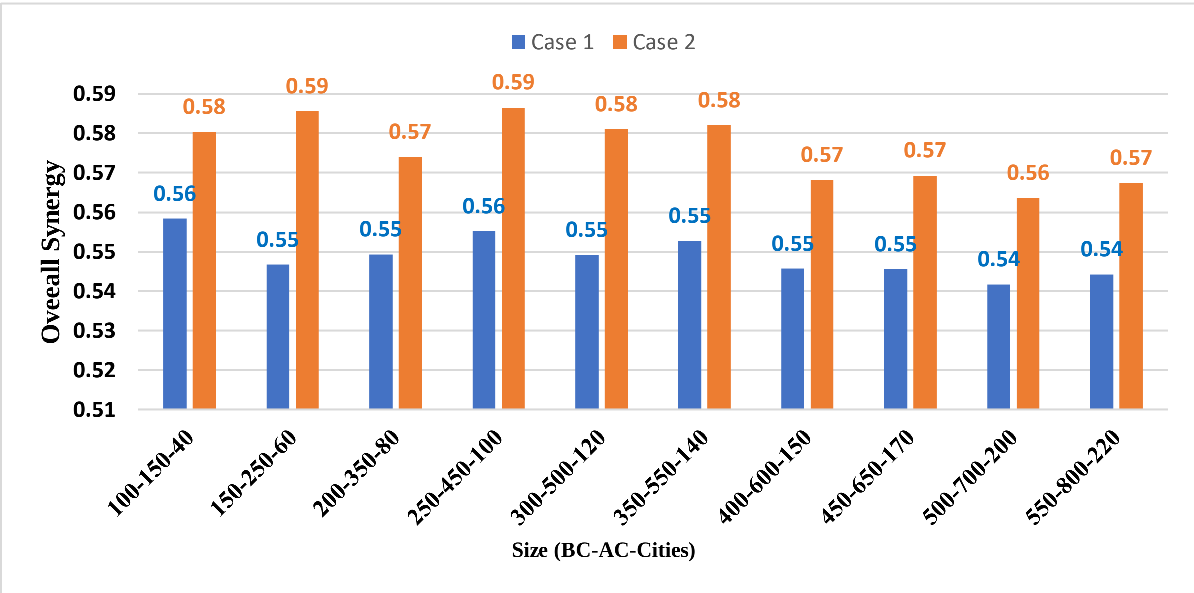


**Figure 9: Synergy comparison across the cases**

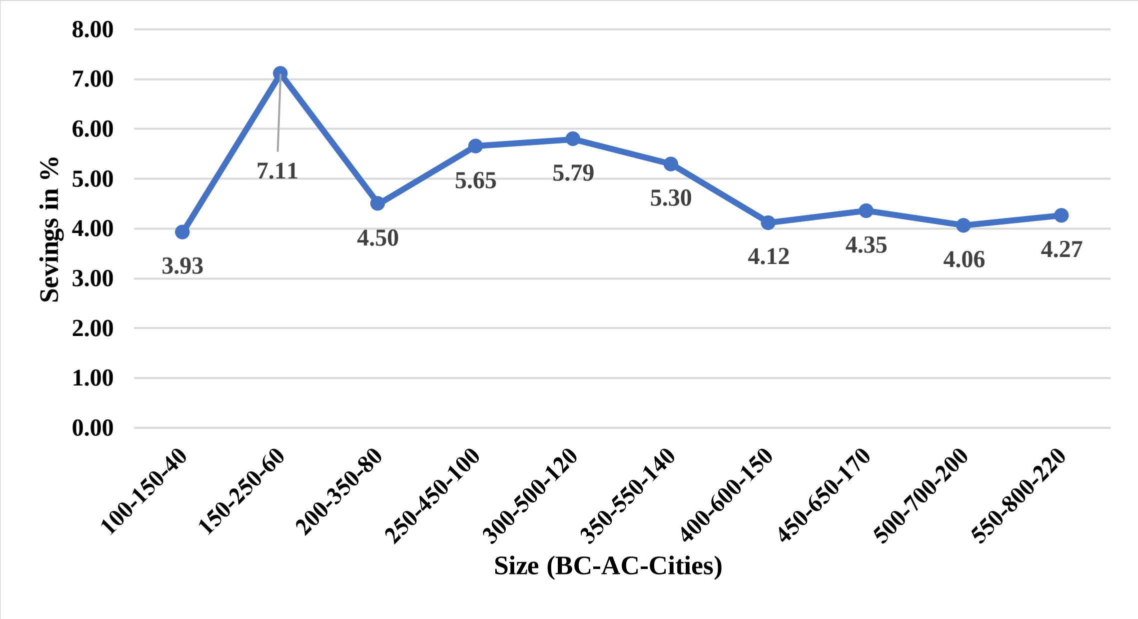


**Figure 10: Increased effective distance (%) per total distance traveled in Case 2 against Case 1 across the problem instances**

Although the BGP traditionally has high computational complexity, our problem setup and proposed algorithms can efficiently execute solutions, even in seconds, for large-dimensional instances. Figure 7 shows that the computational CPU time increases in a convincing manner. Thus, our approach is highly efficient for finding the optimal bid for the auction of new contracts within the existing network pre-BC. However, computational performance depends on several factors, such as data nature and size, modeling/algorithm, and problem design. Having confirmed that our algorithms and problem design perform well, we conducted a sensitivity analysis on various parameters of the generated data to assess the impact of these factors on CPU time. Table 13 presents CPU times for different instance sizes, the ratio of AC to BC, and the NC involved. We determined the CPU time by executing each instance at least five times on five distinct random datasets, and the average time is reported for analysis.

**Table 13: Reporting CPU times with the changes in dimensional parameters**

| Fixed ratio (FR) of auctioned-to-booked contracts | | | Variable ratio (VR) of auctioned-to-booked contracts | | | Variable number of cities (VC) | | |
|---|---|---|---|---|---|---|---|---|
| **ID (FR)** | **Instances (BC, AC, NC)** | **CPU time** | **ID (VR)** | **Instances (BC, AC, NC)** | **CPU time** | **ID (VC)** | **Instances (BC, AC, NC)** | **CPU time** |
| 1 | **(20, 40, 100)** | 0.04 | 1 | **(20, 20, 100)** | 0.02 | 1 | **(50, 150, 50)** | 0.23 |
| 2 | (30, 60, 100) | 0.04 | 2 | (20, 40, 100) | 0.03 | 2 | (50, 150, 60) | 0.29 |
| 3 | (40, 80, 100) | 0.04 | 3 | (20, 60, 100) | 0.08 | 3 | (50, 150, 70) | 0.15 |
| 4 | (50, 100, 100) | 0.10 | 4 | (20, 80, 100) | 1.04 | 4 | (50, 150, 80) | 0.32 |
| 5 | (60, 120, 100) | 0.08 | 5 | (20, 100, 100) | 0.18 | 5 | (50, 150, 90) | 0.35 |
| 6 | (70, 140, 100) | 0.13 | 6 | (20, 120, 100) | 1.00 | 6 | (50, 150, 100) | 0.28 |
| 7 | (80, 160, 100) | 0.15 | 7 | (20, 140, 100) | 3.65 | 7 | (50, 150, 110) | 0.19 |
| 8 | (90, 180, 100) | 0.20 | 8 | (20, 160, 100) | 2.80 | 8 | (50, 150, 120) | 0.18 |
| 9 | (100, 200, 100) | 0.18 | 9 | (20, 180, 100) | 5.46 | 9 | (50, 150, 130) | 0.24 |
| 10 | (110, 220, 100) | 0.22 | 10 | (20, 200, 100) | 22.39 | 10 | (50, 150, 140) | 0.19 |
| 11 | (120, 240, 100) | 0.21 | 11 | (20, 220, 100) | 44.65 | 11 | (50, 150, 150) | 0.33 |
| 12 | (130, 260, 100) | 0.24 | 12 | (20, 240, 100) | 99.39 | 12 | (50, 150, 160) | 0.31 |
| 13 | (140, 280, 100) | 0.22 | 13 | (20, 260, 100) | 304.36 | 13 | (50, 150, 170) | 0.21 |
| 14 | (150, 300, 100) | 0.34 | 14 | (20, 280, 100) | 697.59 | 14 | (50, 150, 180) | 0.28 |
| 15 | **(160, 320, 100)** | 0.29 | 15 | **(20, 300, 100)** | 1475.37 | 15 | **(50, 150, 190)** | 0.18 |

*AC = Auctioned Contracts number, BC = Booked Contracts number, *NC = Number of Cities

Figure 10 shows CPU time changes with a larger number of problem instances, while sizes span with a fixed ratio between the number of AC and BC. The initial setup was (BC = 20, AC = 40, and NC = 100), and the final configuration was (BC = 160, AC = 320, and NC = 100), meaning the ratio (= AC/BC) remains constant at two. Simultaneously, the problem size has increased eightfold regarding the contract number; however, an exact solution for the problem can still be found within seconds for all instances. Thus, while the problem size exhibits a gradual overall increase, it has a minimal impact on computational time (CPU time).

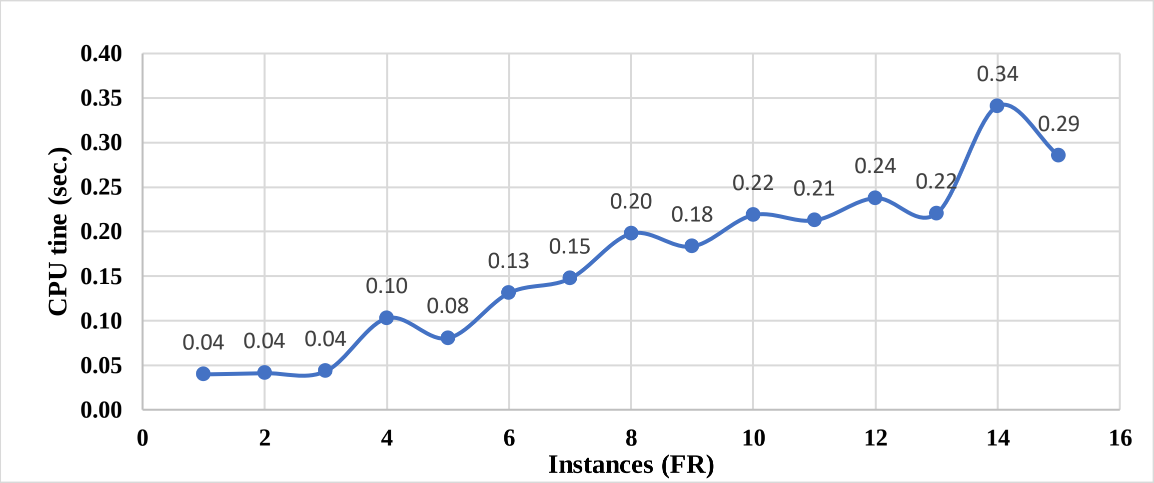


**Figure 11: Changes in CPU time with problem sizes when the ratio (= AC/BC) is fixed**

On the other hand, Figure 11 shows the CPU time changes with problem dimensions while the ratio of contract numbers increased by one across the instances. The initial test setup was (BC = 20, AC = 20, and NC = 100), while the final was (BC = 20, AC = 300, and NC = 100). CPU time remains relatively stable up to a ratio of nine, after which it increases exponentially. Consequently, the difference between the number of AC and BC emerges as the main deciding factor for the computational performance. Therefore, a high difference in the contract number (AC-BC) or the ratio (AC/BC) leads to an increase in computation CPU time. However, computation remains efficient up to a certain ratio value (nine).

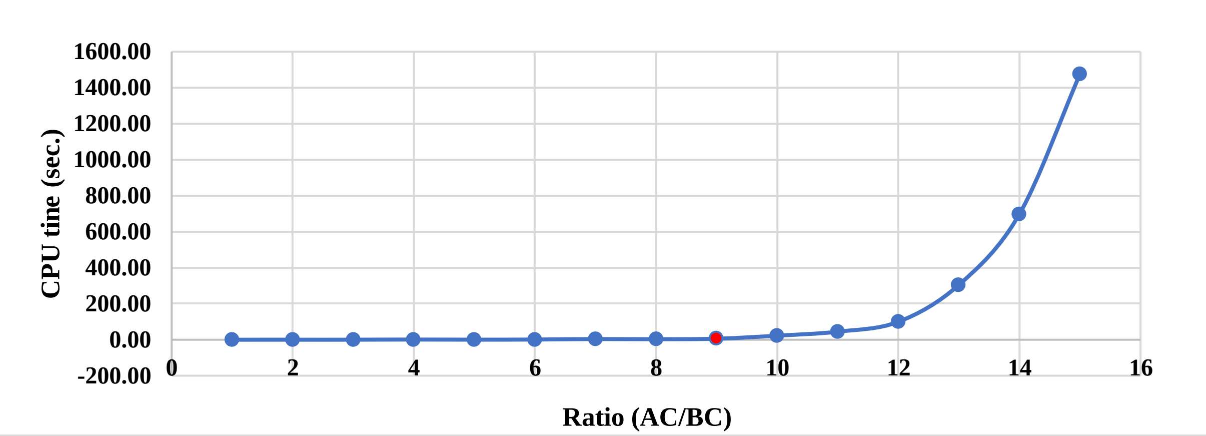


**Figure 12: Changes in CPU time with an increment of ratio (= AC/BC) by one**

In addition, we tested the problem instances using incremental values for the NC to assess the impact of that dimension on the computational performance. Figure 12 shows the CPU time changes as the problem size increases, with the NC increasing by ten for each instance. It is also noteworthy that the instances achieved exact solutions within one second. Thus, the NC appears to have a minimal impact on computational performance.

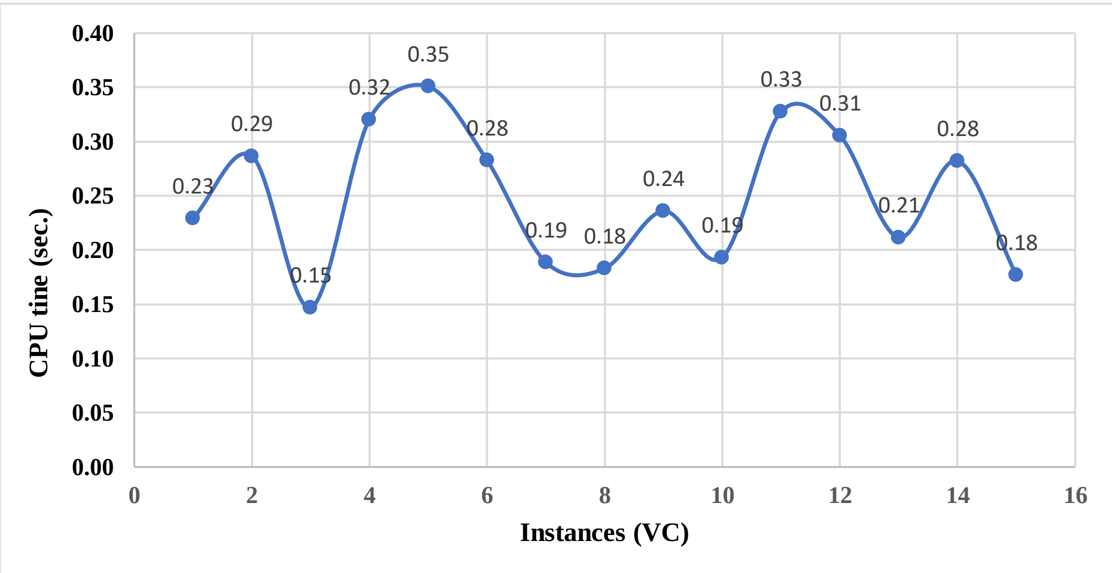


**Figure 13: Changes in CPU time with an increment of the number of cities by 10**

**Table 14: CPU times for variable TS**

| TS | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| **CPU (sec.)** | 0.17 | 0.39 | 0.63 | 0.80 | 3.20 | 1.92 | 3.26 | 2.44 | 1.54 | 2.14 |

Moreover, we performed a computational study on several key parameters related to data design, including $TS$ and $FT$. The time-space between consecutive BC depends on the values of the parameter $TS$ (> 1), i.e., the bigger the values of TS, the larger the time-space. We examined the initial setup with parameters set to BC = 200, AC = 350, and NC = 80, using $TS = 5$ as a starting point. We then incremented the values of $TS$ as follows $((TS = 5, 10, 15, 20, 25, 30, 35, 40, 50)$ and reported the average CPU time from five random executions in Table 14, which are visualized in Figure 13.

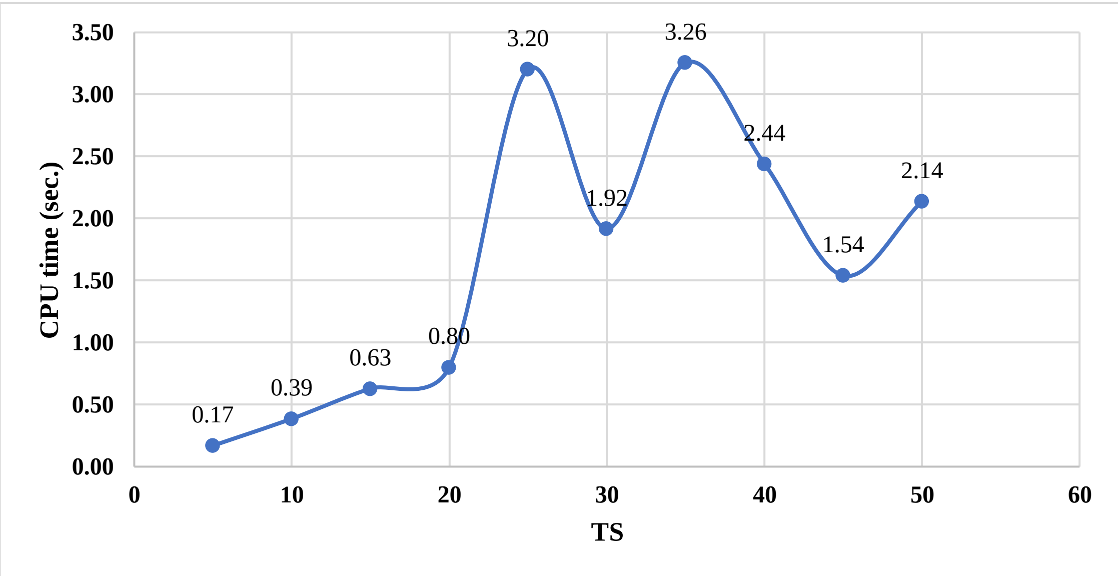


**Figure 14: Changes in CPU time with varying TS**

Figure 13 shows the impact of TS on computational performance. It is evident that while the overall performance increases, the CPU time remains relatively stable, ranging from 0.17 seconds to 3.26 seconds in our experiment. Thus, it can be concluded that $TS$ reasonably impacts computational performance.

The parameter $FT$ (> 0) is a critical deciding factor for bidders in the approach of FT windows (Case 2). We performed computational execution on the initial setup (BC = 200, AC = 350, and NC = 80) with $FT = 20$, while varying $FT = (20, 25, 30, 35, 40, 45, 50)$. Table 15 reports the average CPU time and overall synergy values for the different FT settings. Figures 14 and 15 illustrate the changes in CPU times and overall synergy as FT values increase.

**Table 15: CPU times and overall synergy for variable FT**

| FT | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|
| **CPU Time (sec.)** | 1.87 | 3.08 | 11.71 | 61.47 | 250.25 | 434.85 | 1986.96 |
| **Overall Synergy** | 0.575 | 0.577 | 0.582 | 0.584 | 0.599 | 0.617 | 0.617 |

Figure 14 illustrates that CPU time increases exponentially with varying values of FT, indicating significant computational costs associated with higher FT values. Conversely, overall synergy improved as FT values increased (Figure 15). Therefore, changes in FT yielded a dual outcome: while higher synergy results were cost-effective, computational performance was compromised. Nonetheless, this analysis can assist decision-makers in selecting the optimal FT value for their operations, balancing the associated benefits and drawbacks.

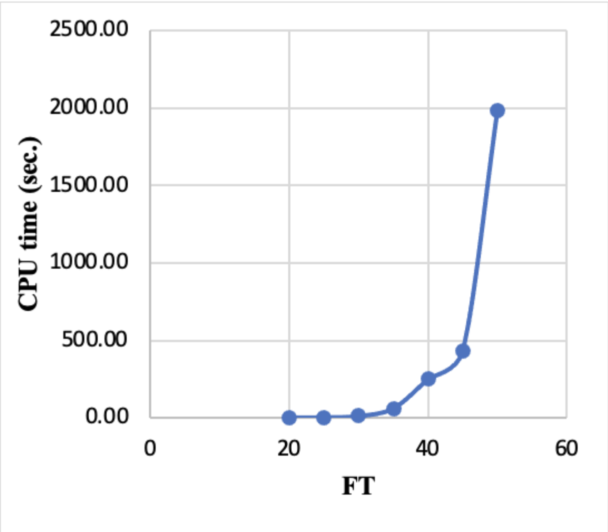


**Figure 15: CPU Time vs. FT**

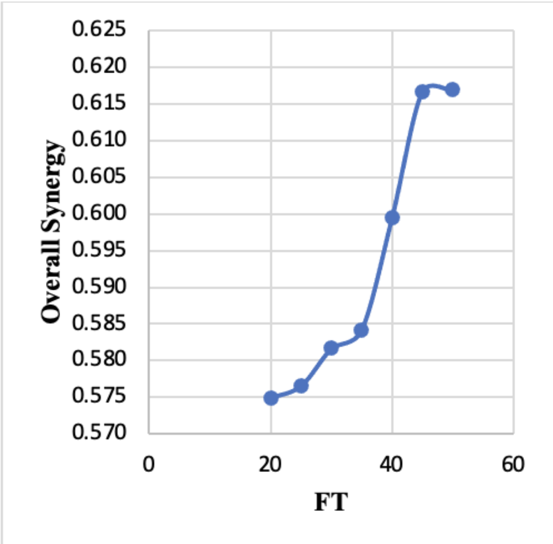


**Figure 16: Overall Synergy vs. FT**

### 7.3 Managerial Insights

This study identified a crucial pathway for addressing the challenges of BGP in procuring transportation services through a CA process. The numerical analysis provides the following managerial insights.

- **Computational performance**: The proposed approaches yield exact solutions for the BGP at significantly high dimensions, achieving rapid execution times for problem sizes of 550 BC, 800 AC, and 220 NC without compromising solution quality. It has been established that the number of AC is not a significant issue in the solution process; rather, the ratio of AC to BC (AC/BC) is a critical factor influencing computational performance in this specific problem setup. Throughout the experiments, a ratio of nine is identified as a crucial threshold, beyond which computational time increases exponentially while it remains relatively stable and less time-consuming up to this point. Thus, decision-makers should consider this ratio to optimize similar problems.
- **Solution quality**: Solution quality is a critical factor in the computational performance of large instances. Enhancing computational performance for NP-hard problems like BGP is essential. In contrast, this arrangement finds exact solutions for all instance sizes reported in this work. Comparatively, performance is improved relative to existing approaches in the literature for this specific problem, although the problem setup may not be entirely comparable to other works. Therefore, operational managers can adjust the problem as proposed in this work to achieve better solutions.
- **Discount opportunity**: This work discusses a specific case of the proposed approach, where time windows can be relaxed to address the early time limitations of AC. In practice, this relaxation occurs for various reasons, including cost efficiency. This study examines the mentioned case to identify improved network synergy, which can reduce operational costs for bidders by minimizing total travel distances and, consequently, emissions. The resulting savings in both costs and emissions present an opportunity for auctioneers and shippers to adjust their time windows to benefit from early shipments. Naturally, they will consider the feasibility of such adjustments.

Therefore, successfully implementing the proposed time window adjustments creates a trade-off between the parties involved.

- **Key decisions:** This study identified crucial decision factors, such as optimal bundles in each space, optimal bids, and route design concerning network synergy. Accordingly, decision-makers can choose options regarding whether to serve specific spaces or complete routes. Additionally, high-dimensional experiments assess the impact of several factors on computational costs, including the ratio (AC/BC), TS, and FT parameters.
- **Stakeholders' implications:** The expected positive effects of our approach on stakeholders' operations and auction outcomes are evident. The complexity associated with the bidding process often poses a barrier for many carriers, leading them to either avoid participating in the CAs or limit their participation to trivial or unsuccessful bids. Our approach serves as a simple yet powerful tool to help define efficient bundles that maximize the synergies within their transportation networks, significantly enhancing success during the auction clearing phase. Consequently, carriers can utilize their fleet capacity more effectively, thereby increasing their profits. Likewise, when shippers and auctioneers receive efficient and competitive bids, they can expect improved auction outcomes, resulting in significant savings in transportation procurement costs.

Thus, the above discussion confirms that the current approach benefits decision-makers in designing a cost-efficient system that delivers high computational performance and quality solutions.

## 8. Conclusions

This study presents an efficient method for bid generation within the CA mechanism for procuring transportation services, which is crucial for effective transportation management. It addresses new aspects of the synergy quantification method applicable to transportation procurement systems. A novel formula quantifies the synergy between existing and potential new contracts within a predefined transportation network of already BC. The proposed synergy calculation and bid generation algorithms demonstrate significant potential for cost efficiency by minimizing total travel distances. This approach is comparatively easy to implement due to its simplicity, avoiding complex mathematical formulations. More importantly, the method exhibits highly efficient computational performance, yielding exact solutions for 550 BC, 800 AC, and 220 cities with remarkably fast CPU times. Our approach provides exact solutions even for large-scale problems, while traditional methods have often failed to do so. This distinguishes our method from existing techniques by ensuring superior computational performance. Additionally, it identifies a ratio factor that drives the computational performance in relation to the problem's high dimensionality. Thus, the suggested problem setup, along with the proposed algorithms, creates a new dimension in solving the BGP more efficiently and in a quality manner.

On the other hand, identifying potential discount opportunities through the time relaxation approach enhances the whole system by creating opportunities for cost reduction. This improvement necessitates a trade-off between bidders and auctioneers, as potential deals are initiated with the flexibility of the service time window. The study shows that average costs per effective distance are reduced by four to 13 percent in relaxed time scenarios. Therefore, logistics companies may adopt this approach to reduce costs and improve scheduling efficiency by offering flexibility in service times along with potential discounts.

Future research can extend this study by modifying the synergy quantification formula to include various aspects, such as emissions considerations for individual carrier providers and their respective routes. Additionally, explicitly imposing carbon regulations to determine winning bids will introduce a new phenomenon in transportation procurement research. Integrating the pricing aspects of bids and considering a multi-auction context (Musmanno et al., 2010) also represent promising avenues for future investigation.

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## APPENDIX

**Table A.1: Location of the cities**

| SN | Cities | Coordinates |
|---|---|---|
| 1 | Al Amarat | (23.5005, 58.4002) |
| 2 | Rusayl | (23.5523, 58.2000) |
| 3 | Barka | (23.6865, 57.8814) |
| 4 | Fahud | (22.3512, 56.4842) |
| 5 | Musandam | (26.1917, 56.2446) |
| 6 | Al Burimi | (24.2500, 55.7939) |
| 7 | Sohar | (24.3643, 56.7468) |
| 8 | Abu Dhabi | (24.4667, 54.3667) |
| 9 | Dubai | (25.2048, 55.2708) |
| 10 | Ras Alkhaima | (25.6741, 55.9804) |
| 11 | Nizwa | (22.9336, 57.5353) |
| 12 | Muscat | (23.6105, 58.5874) |
| 13 | Al Rustaq | (23.3908, 57.4244) |
| 14 | Salalah | (17.0151, 54.0924) |
| 15 | Hima | (19.9496, 56.3919) |
| 16 | Duqum | (19.6012, 57.7778) |

**Table A.2: Distance matrix of real case cities (values are rounding the fractions)**

| Cities | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 21 | 57 | 234 | 369 | 278 | 194 | 424 | 369 | 344 | 109 | 23 | 100 | 847 | 91 | 437 |
| **2** | 21 | 0 | 36 | 221 | 353 | 257 | 173 | 403 | 349 | 325 | 97 | 40 | 81 | 841 | 101 | 440 |
| **3** | 57 | 36 | 0 | 206 | 323 | 221 | 138 | 368 | 314 | 292 | 91 | 73 | 57 | 838 | 129 | 452 |
| **4** | 234 | 221 | 206 | 0 | 426 | 222 | 225 | 319 | 339 | 372 | 126 | 257 | 150 | 642 | 214 | 333 |
| **5** | 369 | 353 | 323 | 426 | 0 | 220 | 209 | 269 | 147 | 63 | 384 | 371 | 332 | 1040 | 452 | 747 |
| **6** | 278 | 257 | 221 | 222 | 220 | 0 | 98 | 147 | 118 | 159 | 230 | 293 | 191 | 820 | 329 | 554 |
| **7** | 194 | 173 | 138 | 225 | 209 | 98 | 0 | 242 | 176 | 164 | 178 | 205 | 128 | 859 | 260 | 538 |
| **8** | 424 | 403 | 368 | 319 | 269 | 147 | 242 | 0 | 123 | 211 | 365 | 440 | 333 | 826 | 469 | 644 |
| **9** | 369 | 349 | 314 | 339 | 147 | 118 | 176 | 123 | 0 | 88 | 341 | 380 | 297 | 915 | 433 | 672 |
| **10** | 344 | 325 | 292 | 372 | 63 | 159 | 164 | 211 | 88 | 0 | 342 | 349 | 292 | 978 | 420 | 697 |
| **11** | 109 | 97 | 91 | 126 | 384 | 230 | 178 | 365 | 341 | 342 | 0 | 131 | 52 | 748 | 106 | 370 |
| **12** | 23 | 40 | 73 | 257 | 371 | 293 | 205 | 440 | 380 | 349 | 131 | 0 | 121 | 868 | 102 | 452 |
| **13** | 100 | 81 | 57 | 150 | 332 | 191 | 128 | 333 | 297 | 292 | 52 | 121 | 0 | 787 | 138 | 421 |
| **14** | 847 | 841 | 838 | 642 | 1040 | 820 | 859 | 826 | 915 | 978 | 748 | 868 | 787 | 0 | 782 | 483 |
| **15** | 91 | 101 | 129 | 214 | 452 | 329 | 260 | 469 | 433 | 420 | 106 | 102 | 138 | 782 | 0 | 35 |
| **16** | 437 | 440 | 452 | 333 | 747 | 554 | 538 | 644 | 672 | 697 | 370 | 452 | 421 | 483 | 351 | 0 |

**Table A.3: Contrcat distance matrix of the case study (values are rounding the fractions)**

| **Contracts** | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ | $L'_1$ | $L'_2$ | $L'_3$ | $L'_4$ | $L'_5$ | $L'_6$ | $L'_7$ | $L'_8$ | **Weight** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_1$ | 0 | 341 | 178 | 748 | 748 | 109 | 109 | 109 | 748 | 178 | 131 | 131 | 131 | 131 | 131 |
| $L_2$ | 349 | 0 | 164 | 978 | 978 | 344 | 344 | 344 | 978 | 164 | 349 | 349 | 349 | 349 | 88 |
| $L_3$ | 293 | 118 | 0 | 820 | 820 | 278 | 278 | 278 | 820 | 98 | 293 | 293 | 293 | 293 | 98 |
| $C_4$ | 465 | 593 | 490 | 0 | 406 | 445 | 445 | 445 | 406 | 490 | 465 | 465 | 465 | 465 | 406 |
| $L_5$ | 452 | 672 | 538 | 483 | 0 | 437 | 437 | 437 | 483 | 538 | 452 | 452 | 452 | 452 | 483 |
| $L_6$ | 121 | 297 | 128 | 787 | 787 | 0 | 100 | 100 | 787 | 128 | 121 | 121 | 121 | 121 | 100 |
| $L'_1$ | 40 | 349 | 173 | 841 | 841 | 21 | 0 | 21 | 841 | 173 | 40 | 40 | 40 | 40 | 21 |
| $L'_2$ | 73 | 314 | 138 | 838 | 838 | 57 | 57 | 0 | 838 | 138 | 73 | 73 | 73 | 73 | 57 |
| $L'_3$ | 0 | 380 | 205 | 868 | 868 | 23 | 23 | 23 | 0 | 205 | 0 | 0 | 0 | 0 | 868 |
| $L'_4$ | 371 | 147 | 209 | 1040 | 1040 | 369 | 369 | 369 | 1040 | 0 | 371 | 371 | 371 | 371 | 209 |
| $L'_5$ | 380 | 0 | 176 | 915 | 915 | 369 | 369 | 369 | 915 | 176 | 0 | 380 | 380 | 380 | 380 |
| $L'_6$ | 440 | 123 | 242 | 826 | 826 | 424 | 424 | 424 | 826 | 242 | 440 | 0 | 440 | 440 | 440 |
| $L'_7$ | 257 | 339 | 225 | 642 | 642 | 234 | 234 | 234 | 642 | 225 | 257 | 257 | 0 | 257 | 257 |
| $L'_8$ | 205 | 176 | 0 | 859 | 859 | 194 | 194 | 194 | 859 | 0 | 205 | 205 | 205 | 0 | 205 |

**Table A.4: Distance matrix for cities A, B, C, D, E, F, G, H**

| **Cities** | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** |
|---|---|---|---|---|---|---|---|---|
| **A** | 0 | 32 | 36 | 89 | 95 | 102 | 74 | 104 |
| **B** | 32 | 0 | 41 | 71 | 65 | 76 | 47 | 72 |
| **C** | 36 | 41 | 0 | 61 | 83 | 81 | 87 | 103 |
| **D** | 89 | 71 | 61 | 0 | 47 | 28 | 93 | 82 |
| **E** | 95 | 65 | 83 | 47 | 0 | 26 | 60 | 35 |
| **F** | 102 | 76 | 81 | 28 | 26 | 0 | 83 | 60 |
| **G** | 74 | 47 | 87 | 93 | 60 | 83 | 0 | 45 |
| **H** | 104 | 72 | 103 | 82 | 35 | 60 | 45 | 0 |