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Output feedback integral sliding mode predictive control for linear discrete time systems

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***Abstract*—In this paper, a new output feedback integral sliding model predictive control (OF-ISMPC) strategy is proposed for linear discrete-time systems when the states are unmeasured, which avoids the utilization of a state observer. The proposed OF-ISMPC can ensure strong robustness while at the same time achieving optimal performance of the controlled system. The role of the output feedback integral sliding mode control (OF-ISMC) is to compensate for any matched disturbances. The role of the output feedback robust model predictive control (OF-RMPC) is to solve the optimization problem with constraints. Moreover, the impact of unmatched disturbances on the system is reduced by defining the integral sliding surface as a function of an optimal projection matrix. The input-to-state practical stability for the closed-loop system is proved. Finally, numerical simulations and experimental results are provided to validate the effectiveness of the proposed approach.**

***Index Terms*—integral sliding mode control, robust model predictive control, output feedback control**

I. Introduction

In industrial process control applications, many control systems must simultaneously achieve robust stability and optimal performance under constraints. On the one hand, the system may be affected by external disturbances, which may cause problems such as introducing errors between the desired behaviour and the actual behaviour, product quality degradation and equipment failure. On the other hand, in any industrial process, variables such as temperature, flow and composition may have a limited range of regulation. The above two potentially competing properties are challenging in the theory and application of process control. It is necessary to design an optimal control method with strong robustness that can accommodate constraints.

To achieve robustness, many control methods have been developed, such as robust control [1, 2], adaptive control [3, 4] and sliding mode control (SMC) [5, 6]. The SMC approach is known to deliver strong robustness, indeed total invariance, to matched disturbances. Integral SMC (ISMC) [7, 8] can force the system state to lie on the sliding surface from the initial time instant. This eliminates the reaching phase inherent in classical SMC and thus enhances robustness. However, the ISMC approach cannot easily handle constraints and cannot guarantee optimal system performance. Robust model predictive control (RMPC) [9, 10] is an optimal control method with the capability to handle constraints. The integral sliding mode predictive control (ISMPC) method has been proposed which combines ISMC with RMPC. This method not only produces the properties of strong robustness and straightforward implementation inherent in ISMC, but also has the advantages of RMPC in being able to solve constrained optimization problems. In recent years, ISMPC research has obtained rich results [11–19] and has been successfully applied to practical systems such as vehicle autonomous driving and obstacle avoidance [20, 21], three-dimensional trajectory tracking control for unmanned underwater vehicles [22, 23], and mobile robotic systems [24]. However, the methods are based on an assumption that the states can be measured. Note that not all system states are measurable in practice. In this case, it is necessary to design an output feedback ISMPC (OF-ISMPC). The majority of OF-ISMPC methods employ a state observer [25–27]. The implementation of the state observer may increase the system order as well as increasing the computational cost and design difficulty of the controller. In order not to increase the system order, in this paper a new OF-ISMPC method is proposed, which avoids the use of an observer.

The proposed OF-ISMPC produces a simpler control algorithm in comparison with one that includes observer dynamics in the formulation of the control law, and may be more effective and useful for practical systems. However, this new OF-ISMPC may be difficult to design. Firstly, an integral sliding surface must be designed for the ISMPC and when the states cannot be measured, this integral sliding surface is usually deployed by using an observed state [28–30]. Reachability of an integral sliding mode based upon observed states may be difficult to guarantee. Secondly, under the output feedback scheme, a sliding surface is designed by using only the output information [31–33]. For the case of ISMPC, the system trajectories will start on the sliding surface from the initial instant, which means the methods in [31–33] cannot be applied for the design of the integral sliding surface. Thirdly, it is not easy to design a RMPC controller using only output information, because the existence of both prediction errors and system disturbances complicate the robust stability conditions and physical constraints in OF-RMPC optimization. Therefore, it is challenging to design an OF-ISMPC for linear

discrete-time systems subject to disturbances without violating the given constraints. The integral term in the integral sliding surface is difficult to handle as it is a function of the state and it cannot be replaced by an integral term which is a function of the output directly, since the output matrix is generally not invertible. In addition, the disturbances have been divided into matched and unmatched disturbances, but only the matched disturbances can be directly compensated for by the ISMC. The unmatched disturbances are difficult to compensate because they do not enter the same channel as the control input. However, unmatched disturbances must be considered in the design of the integral sliding surface to reduce their influence on the system performance, which further increases the difficulty in the design of an integral sliding surface. Further, the reachability of the integral sliding mode and the stability of the closed-loop system are complex to prove. In summary, there are the following challenges in designing an OF-ISMPC: How to design the integral sliding surface and reduce the impact of unmatched disturbances when the states are unavailable, how to determine an OF-RMPC controller that can accommodate constraints while ensuring optimal performance of the system under the influence of external disturbances, how to prove the reachability of the integral sliding mode and the stability of the closed-loop system with constraints.

In light of the above analysis, the contributions of this paper are summarized as follows: (i) When the state is not measurable, an OF integral sliding surface is designed based on a predicted nominal output, which avoids the use of an observer. (ii) The norm of the equivalent disturbances are considered within a projection theorem and an optimal projection matrix is obtained, which can minimize the impact of unmatched disturbances on the system. (iii) The OF-RMPC controller is obtained by a Lyapunov-like sufficient condition of input-to-state practical stability (ISpS) which can effectively handle the constraints and guarantee optimality. (iv) The reachability of an OF based integral sliding mode is proved according to the Lyapunov stability theorem, while the stability of the closed-loop system is proved based on the principle of ISpS. In addition, the overall optimal global control performance is further guaranteed by designing an OF-ISMPC. Compared with the case of state feedback ISMPC [15, 17], the proposed OF-ISMPC solves the problem of system state availability. Compared with ISMPC based on output tracking [26], this paper obtains an OF-RMPC controller in a real sense by using output information only. Compared with the case of state observer based OF-ISMPC [25, 27], the proposed OF-ISMPC may be expected to have better performance. Furthermore, the design of a complex state observer is avoided as compared to the former, and the stability of the closed-loop system is demonstrated in further detail as compared to the latter. Compared with the output feedback RMPC [34–36], the proposed OF-ISMPC has strong robustness to disturbances. Because the matched disturbance is completely compensated for by the OF-ISMC, the OF-RMPC can be designed on a system subject to reduced disturbances.

The remainder of this paper is organized as follows. In Section II, the problem formulation is given. An OF-ISMPC method is proposed in Section III, which includes the design process and stability analysis of the OF-ISMC and OF-RMPC. In Section IV, simulation and experimental results are given and finally in Section V, conclusions are drawn.

II. Problem formulation

Consider the linear discrete-time system described as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + d(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbf{X} \subseteq \mathbf{R}^n$ is the state vector, $u(k) \in \mathbf{U} \subseteq \mathbf{R}^m$ is the control input vector and $y(k) \in \mathbf{Y} \subseteq \mathbf{R}^p$ is the output vector with $m < p < n$, $d(k) \in \mathbf{D} \subseteq \mathbf{R}^n$ denotes a vector of unknown disturbances. $\mathbf{U}$ and $\mathbf{Y}$ are constraint sets, where $\mathbf{U}$ is a compact set containing the origin and $\mathbf{D}$ is a compact set. $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$ and $C \in \mathbf{R}^{p\times n}$ are the state transition, input distribution and output distribution matrices, respectively, where (A, B) is controllable and $rank(C) = p$.

For system (1), the following assumption is made:

Assumption 1: $rank(CB) = rank(B) = m$ and all the invariant zeros of the triple (A, B, C) are inside the unit disk.

Remark 1: The pair (A, B) in system (1) is controllable. As $rank(CB) = rank(B) = m$ and all the invariant zeros of the triple (A, B, C) are inside the unit disk, the nominal system is relative degree one and minimum phase. These are common conditions required to attain a stable output feedback controller [26, 33, 37].

The disturbances $d(k)$ are projected into matched and unmatched spaces [38, 39]

$$\begin{aligned} d(k) &= d_m(k) + d_u(k) \\ d_m(k) &= BB^+ d(k) \\ d_u(k) &= B^\perp B^{\perp+} d(k) \end{aligned} \tag{2}$$

where B^+ is understood as the left inverse of B, that is $B^+ = \left(B^T B\right)^{-1} B^T$ and the columns of $B^\perp \in \mathbf{R}^{n\times(n-m)}$ span the null space of B^T, and $I_n = BB^+ + B^\perp B^{\perp+}$. $d_m(k) \in \mathbf{D}_m$ denotes the matched disturbances while $d_u(k) \in \mathbf{D}_u$ indicates the unmatched disturbances. $d(k)$ is unknown but it is bounded as follows:

$$\mathbf{D}(\rho) = \{d(k) \in R^n : \|d(k)\| \le \rho, \rho \ge 0, k = 0, 1, \cdots\} \tag{3}$$

where ρ defines the upper bound on the disturbances. $\mathbf{D}(\rho)$ describes disturbances with bounded norm which is challenging for the design of the integral sliding surface.

Note that it is important to consider the above framework, which is motivated from largely theoretical considerations, in the light of practical application. The formulation involves a discrete time system representation (1) and thus the sample period must be chosen appropriately for any specific plant under consideration as a poor choice will undoubtedly affect the level of performance that can be achieved, as would be the case in any sampled control implementation. It is also the case that rank conditions and conditions on the stability of invariant zeros are assumed. Again these can be viewed as a structural property of a given system and analysis of the system structure and careful positioning of sensors can mitigate any potential issues in satisfying these assumptions.

Based on the above description, the constraint control law can be defined as:

$$u(k) = u_{ISM}(k) + u_{MPC}(k) \tag{4}$$

where the component $u_{ISM}(k) \in \mathbf{U}^I$ is generated by the ISMC controller to deal with matched disturbances, whereas $u_{MPC}(k) \in \mathbf{U}^R$ is generated by the RMPC controller to achieve optimal control satisfying constraints. This defines an OF-ISMPC, $u(k)$ and $\mathbf{U}^R = \mathbf{U} - \mathbf{U}^I$. The block diagram on the implementation of the OF-ISMPC is shown in Fig. 1.

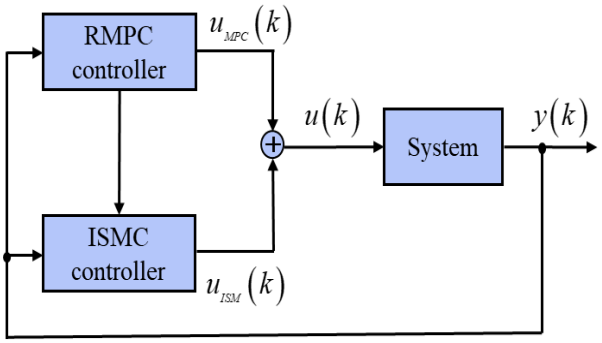


Fig. 1: Block diagram of OF-ISMPC method.

To ensure the reachability of the integral sliding mode, $u_{ISM}(k)$ is usually selected to be of the following form:

$$u_{ISM}(k) = -\mathrm{T}\mathrm{sgn}(s(k)) \tag{5}$$

where $\mathrm{T} \in \mathbf{R}^{m\times m}$ is a matrix to enforce the sliding motion and $s(k)$ is the integral sliding surface.

The second element of the control (4) is designed as follows:

$$u_{MPC}(k) = Ky(k) \tag{6}$$

where $K \in \mathbf{R}^{m\times p}$ is a output feedback gain matrix. It is derived from the solution of the following optimization problem $W_N(x)$:

$$J_N^*(k) = \min_{U^R(t)} \max_{d_{ue}(t)\in D_{ue}} J_N(k) \tag{7}$$

$$\begin{aligned} &s.t\ u_{MPC}(k) \in \mathbf{U}^R, y(k) \in \mathbf{Y}, , x(N) \in \mathbf{X}_f, \\ &d_{ue}(k) \in \mathbf{D}_{ue}, k = 0, \cdots, N-1, \end{aligned} \tag{8}$$

where

$$\begin{aligned} J_N(k) = &\sum_{i=0}^{N-1} \Big[x(k+i|k)^T Q x(k+i|k) \\ &+u_{MPC}^T(k+i|k) R u_{MPC}(k+i|k)\Big] + F(x(k+N|k)) \\ = &\sum_{i=0}^{N-1} L(x(k+i|k), u_{MPC}(k+i|k)) + F(x(k+N|k)) \end{aligned},$$

$Q \in \mathbf{R}^{n\times n}$ and $R \in \mathbf{R}^{m\times m}$ are symmetric positive definite matrices, N is the prediction horizon and $\mathbf{X}_f \subseteq \mathbf{X}$ is the terminal set. The terminal cost function $F(x(k+N|k))$ is selected as follows:

$$F(x(k+N|k)) = x(k+N|k)^T P x(k+N|k) \tag{9}$$

The control objective is to design a control law (4) for the linear discrete-time system (1) under the constraint sets (8) and in the presence of the disturbances (3), to ensure that the closed-loop system is ISpS.

Remark 2: When the state information is not measurable, the design of the control law (4) must depend on the output information $y(k)$ only. From (5), it can be seen that $u_{ISM}(k)$ is a function about the sliding mode surface $s(k)$. This leads to the fact that the design of the integral sliding surface using output information. Furthermore, the unmatched disturbances $d_u(k)$ must be considered as they impact on the performance of system (1). These present challenges to both the design of the integral sliding surface and the reachability proof for an integral sliding mode.

III. Design of discrete-time OF-ISMPC

In this section, the design of an OF-ISMC will be presented with guaranteed reachability of the integral sliding mode. Subsequently, the OF-RMPC will be obtained by solving an optimization problem and ultimately, the stability analysis of the closed-loop system will be provided.

A. OF-ISMC component

For system (1), the integral sliding surface using output information is defined as follows:

$$\begin{cases} s(k) = Gy(k) + \sigma(k) \\ \sigma(k+1) = \sigma(k) + G(y(k) - y_p(k+1)) \\ \sigma(0) = -Gy(0) \end{cases} \tag{10}$$

where $s(k) = [s_1(k), s_2(k), \cdots s_m(k)]^T$, $G \in \mathbf{R}^{m\times p}$ is a projection matrix and $y_p(k+1)$ is a predicted nominal output.

Assumption 2: The designed matrix G in (10) must satisfy the condition $\det(GCB) \neq 0$.

The expression for $y_p(k+1)$, which is determined solely from the state and the input of the nominal system, can be derived as follows:

$$y_p(k+1) = y_p(k+1|k) = Cx_p(k+1|k) \tag{11}$$

where $x_p(k+1|k)$ is the predicted value of the state. This can be obtained from the following equation:

$$\begin{aligned} x_p(k+1|k) &\approx x(k+1|k) \\ &= A^{k+1}x(0) + \sum_{i=1}^{k+1} A^{i-1} B u_{MPC}(k-i) \end{aligned} \tag{12}$$

Here, the state provides the initial value for the iterative calculation of the RMPC, and does not require high accuracy. A detailed description follows for the OF-RMPC component.

The prediction error of the state is defined as follows:

$$e(k) = x(k|k) - x_p(k|k-1) \tag{13}$$

and the error of the predicted nominal output is:

$$e_o(k) = y(k|k) - y_p(k|k-1) \tag{14}$$

The boundedness of $e(k)$ can be guaranteed by the following Lemma 1, inspired from [36]. From (13) and (14), it is easy to see that $e_o(k)$ is also bounded. Therefore, the integral sliding surface can be obtained from this information.

Lemma 1: For system (1), if $x_p(k+1|k)$ can be obtained by (12), then $e(k)$ is bounded.

The detailed proof is given in Appendix A.

According to (10), the OF-ISMC $u_{ISM}(k)$ is designed as follows:

$$u_{ISM}(k) = -(GCB)^{-1}(\rho\|GC\| + \varepsilon)\,\mathrm{sgn}(s(k)) \tag{15}$$

where ε is a positive constant.

Theorem 1: Suppose that both the system (1) and the integral sliding surface (10) satisfy Assumption 1 and Assumption 2, and that the parameter ε in (15) satisfies $\varepsilon > \|GCA\|\|e(k)\|$. Then, reachability of the discrete-time integral sliding mode is guaranteed by control law (15). Moreover, it ensures that system trajectories remain within the quasi-sliding mode domain:

$$\mathbf{\Gamma} = \{|s_i(k)| \le \delta, \delta = 2\varepsilon + 2\rho\|GC\|, i = 1, 2, \cdots, m\} \tag{16}$$

Proof

According to (1) and (10):

$$\begin{aligned} &s(k+1) - s(k) \\ =&Gy(k+1) + \sigma(k+1) - Gy(k) - \sigma(k) \\ =&Gy(k+1) - Gy_p(k+1) \\ =&GCAe(k) + GCBu_{ISM}(k) + GCd(k) \end{aligned} \tag{17}$$

Substituting (15) into (17) yields

$$\begin{aligned} s(k+1) - s(k) = &GCAe(k) - (\rho\|GC\| + \varepsilon)\,\mathrm{sgn}(s(k)) \\ &+ GCd(k) \end{aligned} \tag{18}$$

The Lyapunov function is chosen as follows

$$V(k) = s^T(k)\,s(k) \tag{19}$$

then,

$$\Delta V(k) = s^T(k+1)\,s(k+1) - s^T(k)\,s(k) \tag{20}$$

$\Delta V(k) < 0$ is equivalent to $s^T(k)(s(k+1) - s(k)) < 0$ [40].

Since

$$\begin{aligned} &s^T(k)(s(k+1) - s(k)) \\ =&s^T(k)\,GCAe(k) - s^T(k)(\rho\|GC\| + \varepsilon)\,\mathrm{sgn}(s(k)) \\ &+s^T(k)\,GCd(k) \\ \le&\|GCA\|\|s(k)\|\|e(k)\| - \varepsilon\|s(k)\| - \rho\|GC\|\|s(k)\| \\ &+\|GC\|\|s(k)\|\|d(k)\| \\ \le&(\|GCA\|\|e(k)\| - \varepsilon)\|s(k)\| \end{aligned} \tag{21}$$

From $\varepsilon > \|GCA\|\|e(k)\|$, it can be obtained that $s^T(k)(s(k+1) - s(k)) < 0$, that is $\Delta V(k) < 0$. Thus, the reachability of the integral sliding mode is guaranteed.

From (18), it follows that

$$\begin{aligned} s_i(k+1) = &s_i(k) - (\rho\|GC\| + \varepsilon)\,\mathrm{sgn}(s_i(k)) \\ &+ \|GC\|\|d(k)\| + \|GCA\|\|e(k)\| \end{aligned} \tag{22}$$

where $s_i(k)$ is denoted as the $i-th$ element of the vector $s(k)$.

The monotonicity of the system trajectory will be analyzed below for the two cases of $s_i(k) > 0$ and $s_i(k) < 0$.

When $s_i(k) > 0$ for any $k \ge 0$, it follows that

$$\begin{aligned} s_i(k+1) &= s_i(k) - \rho\|GC\| - \varepsilon + \|GC\|\,d_i(k) \\ &\quad + \|GCA\|\|e(k)\| \\ &\ge s_i(k) - \rho\|GC\| - \varepsilon - \|GC\|\,d_i(k) \\ &\quad + \|GCA\|\|e(k)\| \\ &\ge s_i(k) - \varepsilon - 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &\ge s_i(k) - 2\varepsilon - 2\rho\|GC\| \end{aligned} \tag{23}$$

Let $\delta = 2\varepsilon + 2\rho\|GC\|$. From (23), if $s_i(k) > \delta$, it can be obtained that $s_i(k+1) > 0$. Together with (21), this implies that $0 < s_i(k+1) < s_i(k)$ is satisfied. At this point the system trajectory will move monotonically decreasing towards the quasi-sliding mode domain.

If $0 < s_i(k) < \delta$, the following two inequalities are established:

$$\begin{aligned} s_i(k+1) &\ge s_i(k) - \varepsilon - 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &\ge -\varepsilon - 2\rho\|GC\| \\ &> -2\varepsilon - 2\rho\|GC\| = -\delta \end{aligned} \tag{24}$$

$$\begin{aligned} s_i(k+1) &= s_i(k) - \rho\|GC\| - \varepsilon + GC\|d(k)\| \\ &\quad + \|GCA\|\|e(k)\| \\ &\le s_i(k) - \varepsilon + \|GCA\|\|e(k)\| \\ &\le \varepsilon + 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &< 2\varepsilon + 2\rho\|GC\| = \delta \end{aligned} \tag{25}$$

From (24) and (25), it can be obtained that $-\delta < s_i(k+1) < \delta$. This implies that the system does not escape from the quasi-sliding mode domain once it enters.

Besides, when $s_i(k) < 0$ for any $k \ge 0$, it follows that

$$\begin{aligned} s_i(k+1) &= s_i(k) + \rho\|GC\| + \varepsilon + \|GC\|\|d(k)\| \\ &\quad + \|GCA\|\|e(k)\| \\ &\le s_i(k) + 2\rho\|GC\| + \varepsilon + \|GCA\|\|e(k)\| \\ &\le s_i(k) + 2\varepsilon + 2\rho\|GC\| \end{aligned} \tag{26}$$

For (26), if $s_i(k) < -\delta$, it can be obtained that $s_i(k+1) < 0$. Together with (21) this implies that $s_i(k) < s_i(k+1) < 0$ is satisfied. At this point the system trajectory will move monotonically increasing towards the quasi-sliding mode domain.

If $-\delta < s_i(k) < 0$, the following two inequalities are established:

$$\begin{aligned} s_i(k+1) &\le s_i(k) + \varepsilon + 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &\le \varepsilon + 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &< 2\varepsilon + 2\rho\|GC\| = \delta \end{aligned} \tag{27}$$

$$\begin{aligned} s_i(k+1) &= s_i(k) + \rho\|GC\| + \varepsilon + \|GC\|\|d(k)\| \\ &\quad + \|GCA\|\|e(k)\| \\ &\geq s_i(k) + \rho\|GC\| + \varepsilon - \|GC\|\|d(k)\| \\ &\quad + \|GCA\|\|e(k)\| \\ &\geq s_i(k) + \varepsilon + \|GCA\|\|e(k)\| \\ &\geq -\varepsilon - 2\rho\|GC\| + \|GCA\|\|e(k)\| \\ &> -\varepsilon - 2\rho\|GC\| \\ &> -2\varepsilon - 2\rho\|GC\| = -\delta \end{aligned} \tag{28}$$

From (27) and (28), $-\delta < s_i(k+1) < \delta$ is satisfied. It also means that once the trajectory enter the quasi-sliding mode domain, it will stay within the domain.

From the above analysis, it follows that when $s_i(k) > \delta$ or $s_i(k) < -\delta$, $s_i(k+1)$ has the same sign as $s_i(k)$. That is, the system trajectory will move monotonically towards the domain, and enter it in finite time. Meanwhile, from (24), (25), (27) and (28), it can be obtained that when $|s_i(k)| < \delta$ for any $k \geq 0$, $|s_i(k+1)| < \delta$ is also satisfied. Therefore, the control law (15) can guarantee the system reaches the quasi-sliding mode domain $\mathbf{\Gamma}$ and cannot escape from the domain [40][41]. Q.E.D.

Remark 3: Theorem 1 ensures the existence and reachability conditions are satisfied for the ISMC. It is important to note that $s(0) = 0$ in (10), which means the system trajectories always start on the sliding surface. Therefore, the designed integral sliding surface can eliminate the reaching phase and ensure robustness throughout the entire response of the system, simultaneously.

By solving the equation $s(k+1) - s(k) = 0$ in (17) for $u_{ISM}(k)$, the following equivalent control is obtained

$$\begin{aligned} u_{eq}(k) &= -B^{+}d(k) - (GCB)^{-1}GCd_u(k) \\ &\quad - (GCB)^{-1}GCAe(k) \end{aligned} \tag{29}$$

Substituting (29) into (1):

$$\begin{aligned} &x(k+1) = Ax(k) + Bu_{MPC}(k) - BB^{+}d(k) \\ &- B(GCB)^{-1}GCd_u(k) - B(GCB)^{-1}GCAe(k) \\ &+ BB^{+}d(k) + d_u(k) \\ &= Ax(k) + Bu_{MPC}(k) - B(GCB)^{-1}GCAe(k) \\ &+ \left(I - B(GCB)^{-1}GC\right)d_u(k) \end{aligned} \tag{30}$$

The following sliding dynamics can then be obtained:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu_{MPC}(k) + d_{eq}(k) \\ &\quad - B(GCB)^{-1}GCAe(k) \end{aligned} \tag{31}$$

where $d_{eq}(k) = \left(I - B(GCB)^{-1}GC\right)d_u(k)$ is the vector of equivalent disturbances.

It can be seen from (31) that the ISMC component of the controller compensates for the matched disturbances. The unmatched ones are replaced by equivalent disturbances. Since the equivalent disturbances $d_{eq}(k)$ are closely related to the projection matrix G, their reduction depends on the choice of G.

Inspired by [38], the following lemma is given.

Lemma 2: Apply the control (15) based on the integral sliding surface (10) to system (1). The choice $G = (CB)^{T}$ or $G = (CB)^{+}$ minimizes the norm of $d_{eq}(k)$, i.e.

$$d_{eq}^{*}(k) = \underset{G \in R^{m\times p}}{\arg\min} \left\| \left(I - B(GCB)^{-1}GC\right)d_u(k) \right\|_2 \tag{32}$$

Then, the following dynamical system is used to develop the second component of the control in (4)

$$\begin{cases} x(k+1) = Ax(k) + Bu_{MPC}(k) + d_{ue}(k) \\ y(k) = Cx(k) \end{cases} \tag{33}$$

where $d_{ue}(k) = \left(I - B(CB)^{+}C\right)d_u(k) - B(CB)^{+}CA\,e(k)$. The boundedness of $d_{ue}(k)$ depends on $d_u(k)$ and $e(k)$, and the boundedness of $e(k)$ has been proved in Appendix A. So if $d_u(k)$ is bounded, then $d_{ue}(k)$ is also bounded. This will be discussed in the next subsection.

The dynamical system (33) is an equivalent system of the system (1) following application of the ISMC component of the control (4). Note that the matched disturbances have been compensated by the action of $u_{ISM}(k)$, while the impact of the unmatched disturbances have been minimized. The second element of the controller $u_{MPC}(k)$ in (33) will be developed as a solution to a constrained optimization problem for systems with disturbances reduction. Therefore, the benefits of OF-ISMC and OF-RMPC can be maximized by using input allocation [42].

B. OF-RMPC component

Following local application of the OF-ISMC control, the original system has been transformed into (33) and the second OF-RMPC element of the control will be designed for a dynamical system with reduced disturbances.

For $e(k)$ in (13), the set $\mathbf{E}$ has the following form

$$\mathbf{E}(\rho_e) = \{e(k) \in R^n : \|e(k)\| \leq \rho_e, \rho_e \geq 0\} \tag{34}$$

where $\rho_e = \left\|S^{\frac{1}{2}}\right\| + \left\|AS^{\frac{1}{2}} + BYS^{-\frac{1}{2}}\right\|$ is the upper bound on the predicted error (see Appendix A).

From (2) and (3), $\mathbf{D}_u$ has the following form:

$$\mathbf{D}_u(\rho) = \{d_u(k) \in R^n : \|d_u(k)\| \leq b\rho = \rho_u, \rho_u \geq 0\} \tag{35}$$

where $b = \left\|B^{\perp}B^{\perp+}\right\|$ and ρ_u is the upper bound of the unmatched disturbances.

Then, the boundedness of $d_{ue}(k)$ is proved by the following Proposition 1.

Proposition 1: For system (33), if $d_{ue}(k) = \left(I - B(CB)^{+}C\right)d_u(k) - B(CB)^{+}CAe(k)$, then $d_{ue}(k)$ is bounded and $d_{ue}(k) \in \mathbf{D}_{ue}$ where the set $\mathbf{D}_{ue}$ is represented as follows

$$\begin{aligned} \mathbf{D}_{ue}(\rho_{ue}) &= \left\{d_{ue}(k) \in R^n : \|d_{ue}(k)\| \leq \|L\|\rho_u + \left\|\bar{L}\right\|\rho_e \right. \\ &\quad \left. = \rho_{ue}, \rho_{ue} \geq 0\right\} \end{aligned} \tag{36}$$

with $L = \left(I - B(CB)^{+}C\right)$ and $\bar{L} = B(CB)^{+}CA$.

The detailed proof follows in Appendix A.

Following from the control in (15)

$$\begin{aligned} \mathbf{U}^{I} = \Big\{ & u_{ISM}(k) \in \mathbf{U}^{I} : \mathbf{U}^{I} = \mathrm{T} \times (a_1, \cdots, a_m)^{T}, \\ & a_1, \cdots, a_m \in \{-1, 0, 1\}\Big\} \end{aligned}$$

and it follows that $\mathbf{U}^R$ can be obtained as follows:

$$\mathbf{U}^R = \left\{ u_{MPC}(k) \in \mathbf{U}^R : \mathbf{U}^R = \mathbf{U} - \mathbf{U}^I \right\} \tag{37}$$

where $\mathbf{U}^I$ and $\mathbf{U}^R$ are polytopic sets. The details can be seen in [43].

According to (37) and $y(k) \in \mathbf{Y}$, the constraints can be represented as follows:

$$\begin{aligned} &\|u_{MPC}(k)\|_2 \le u_{MPC\max} \\ &\|y(k)\|_2 \le y_{\max} \end{aligned} \tag{38}$$

where $u_{MPC\max}$ and $y_{\max}$ are the maximum input of the RMPC and maximum output, respectively.

To design the OF-RMPC, the following Lemma is given.

Lemma 3: [44] For system (33) controlled by (6), $\mathbf{X}_f$ is assumed to be an admissible robust invariant set and the origin is contained within it. Consider that $F(x(k+N|k))$ is an ISpS-Lyapunov function for all $x(k) \in \mathbf{X}_f$ and all $d_{ue}(k) \in \mathbf{D}_{ue}$. At each time step k, there exists a constant α such that $F(x(k+N|k))$ satisfies the following inequality:

$$\begin{aligned} F(x(k+i+1|k)) - F(x(k+i|k)) \le \\ - L(x(k+i|k), u_{MPC}(k+i|k)) \\ + \alpha d_{ue}^T(k+i|k)\, d_{ue}(k+i|k) \end{aligned} \tag{39}$$

where $L(x(k), u_{MPC}(k)) \ge \alpha_3(\|(x(k), u_{MPC}(k))\|)$, being $\alpha_3(\cdot)$ a $\mathbf{K}_\infty$ -function.

Taking summation on both sides of inequality (39) from $i=0$ to $i=N-1$, and according to (7) the following is obtained:

$$\begin{aligned} &\max_{d_{ue}\in D_{ue}} J_N(k) \le F(x(k|k)) \\ &+ \alpha \sum_{i=0}^{N-1} \left[d_{ue}^T(k+i|k)\, d_{ue}(k+i|k) \right] \\ &\le x^T(k|k) P x(k|k) + \alpha N \rho_{ue}^2 \end{aligned} \tag{40}$$

Let $\Phi(k) = x^T(k|k) P x(k|k) + \alpha N \rho_{ue}^2$, then

$$\max_{d_{ue}\in D_{ue}} J_N(k) \le \Phi(k) \tag{41}$$

It can be seen from (41) that $\Phi(k)$ is essentially an upper bound of $J_N(k)$. Then, the following inequality can be obtained:

$$J_N^*(k) \le \min_{U^R(t)} \Phi(k) \tag{42}$$

Therefore, the solution of the optimization problem $W_N(x)$ can be transformed into the minimization of $\Phi(k)$

$$\min_{U^R(t)} \Phi(k), s.t. (25), (27) \tag{43}$$

If there exists a nonnegative variable γ satisfying $\Phi(k) \le \gamma$, then, the following Theorem can be presented to solve the optimization problem.

Theorem 2: For system (33), if there exist scalars γ, $u_{MPC\max}$, $y_{\max}$ and positive definite matrices $Y>0$, $M>0$, $S>0$, Y_1, M_{11}, then a feedback gain K to solve the OF-RMPC can be obtained from the solution of the following LMIs:

$$\min_{\gamma, Y, M, S, Y_1, M_{11}} \gamma \tag{44}$$

$$\begin{bmatrix} \gamma^{-1}\alpha N \rho_{ue}^2 - 1 & x^T(k|k) \\ x(k|k) & -S \end{bmatrix} \le 0 \tag{45}$$

$$\begin{bmatrix} -S & 0 & SA^T + Y^T B^T & & \\ 0 & -\alpha\gamma^{-1} I & I & & \\ AS + BY & I & -S & & \\ R^{\frac{1}{2}} Y & 0 & 0 & & \\ Q^{\frac{1}{2}} S & 0 & 0 & & \\ & & Y^T R^{\frac{1}{2}} & SQ^{\frac{1}{2}} & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & -\gamma I & 0 & \\ & & 0 & -\gamma I & \end{bmatrix} \le 0 \tag{46}$$

$$\begin{bmatrix} -u_{MPC\max}^2 I & Y \\ Y^T & -S \end{bmatrix} \le 0 \tag{47}$$

$$\begin{bmatrix} -y_{\max}^2 I & CAS + CBY \\ SA^T C^T + Y^T B^T C^T & -S \end{bmatrix} \le 0 \tag{48}$$

where $K = Y_1 M_{11}^{-1}$, $M = \begin{bmatrix} M_{11} & 0 \\ M_{21} & M_{22} \end{bmatrix}$, $Y = \begin{bmatrix} Y_1 & 0 \end{bmatrix}$.

Proof

Equation (43) is equivalent to the following optimization proposition:

$$x^T(k|k) P x(k|k) + \alpha N \rho_{ue}^2 \le \gamma \tag{49}$$

Let $S = \gamma P^{-1}$, then

$$x^T(k|k) S^{-1} x(k|k) + \gamma^{-1}\alpha N \rho_{ue}^2 \le 1 \tag{50}$$

According to the Schur Complement Lemma, (50) can be transformed into the following:

$$\begin{bmatrix} \gamma^{-1}\alpha N \rho_{ue}^2 - 1 & x^T(k|k) \\ x(k|k) & -S \end{bmatrix} \le 0 \tag{51}$$

which is equivalent to (45). The Hamilton function $H(x(k), u(k), d_{ue}(k))$ is defined as follows:

$$\begin{aligned} &H(x(k+i|k), u(k+i|k), d_{ue}(k+i|k)) \\ =&F(x(k+i+1|k)) - F(x(k+i|k)) \\ +&x^T(k+i|k) Q x(k+i|k) \\ +&u_{MPC}^T(k+i|k) R u_{MPC}(k+i|k) \\ -&\alpha d_{ue}^T(k+i|k)\, d_{ue}(k+i|k) \end{aligned} \tag{52}$$

Assume $i=0$, and substitute (6), (9), (33) into (52). Then, (52) can be rewritten as follows:

$$\begin{aligned} &H(x(k|k), u(k|k), d_{ue}(k|k)) = \\ &\begin{bmatrix} x^T(k|k) & d_{ue}^T(k|k) \end{bmatrix} \Xi \begin{bmatrix} x(k|k) \\ d_{ue}(k|k) \end{bmatrix} \end{aligned} \tag{53}$$

where

$$\Xi = \begin{bmatrix} (A+BKC)^T P (A+BKC) - P + Q + C^T K^T R K C & (A+BKC)^T P \\ P(A+BKC) & P - \alpha I \end{bmatrix}$$

The Hamilton function $H(x(k), u(k), d_{ue}(k)) \le 0$ can be obtained from (39), and then the following inequality is

established:

$$\left[\begin{array}{c}(A+BKC)^T P(A+BKC)-P+Q+C^T K^T RKC \\ P(A+BKC) \\ (A+BKC)^T P \\ P-\alpha I\end{array}\right]\leq 0 \tag{54}$$

Substitute $P=\gamma S^{-1}$ into (54) to yield:

$$\left[\begin{array}{c}(A+BKC)^T S^{-1}(A+BKC)-S^{-1}+\Upsilon \\ S^{-1}(A+BKC) \\ (A+BKC)^T S^{-1} \\ S^{-1}-\alpha\gamma^{-1} I\end{array}\right]\leq 0 \tag{55}$$

where $\Upsilon=\gamma^{-1}Q+C^T K^T\gamma^{-1}RKC$. Using the Schur Complement Lemma

$$\left[\begin{array}{ccccc}-S^{-1} & 0 & (A+BKC)^T & & \\ 0 & -\alpha\gamma^{-1}I & I & & \\ (A+BKC) & I & -S & & \\ R^{\frac{1}{2}}KC & 0 & 0 & & \\ Q^{\frac{1}{2}} & 0 & 0 & & \\ & & C^T K^T R^{\frac{1}{2}} & Q^{\frac{1}{2}} & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & -\gamma I & 0 & \\ & & 0 & -\gamma I & \end{array}\right]\leq 0 \tag{56}$$

Multiplying both sides of (56) by the diagonal matrix $diag\left\{\begin{array}{ccccc} S & I & I & I & I \end{array}\right\}$, it follows that

$$\left[\begin{array}{ccccc}-S & 0 & S(A+BKC)^T & & \\ 0 & -\alpha\gamma^{-1}I & I & & \\ (A+BKC)S & I & -S & & \\ R^{\frac{1}{2}}KCS & 0 & 0 & & \\ Q^{\frac{1}{2}}S & 0 & 0 & & \\ & & SC^T K^T R^{\frac{1}{2}} & SQ^{\frac{1}{2}} & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & -\gamma I & 0 & \\ & & 0 & -\gamma I & \end{array}\right]\leq 0 \tag{57}$$

Since $rank(C)=p$, there exists an invertible matrix $Z\in\mathbf{R}^{n\times n}$ such that $CZ=\left[\begin{array}{cc} I & 0 \end{array}\right]$, where $I\in\mathbf{R}^{p\times p}$ is a unit matrix.

Then, letting $S=ZM$, it can be obtained that:

$$\begin{aligned} KCS=KCZM&=K\left[\begin{array}{cc} I & 0 \end{array}\right]M \\ &=K\left[\begin{array}{cc} M_{11} & 0 \end{array}\right] \\ &=\left[\begin{array}{cc} Y_1 & 0 \end{array}\right]=Y \end{aligned} \tag{58}$$

Hence, $KCS=Y$ and (57) is equivalent to (46). It follows that the output feedback gain matrix $K=Y_1M_{11}^{-1}$ and

$$u_{MPC}(k)=Y_1M_{11}^{-1}y(k) \tag{59}$$

Since α, N and ρ_u are constants, $\Phi(k)\leq\gamma$ can be rewritten as:

$$x^T(k|k)S^{-1}x(k|k)\leq 1 \tag{60}$$

In addition, in order to solve the optimization problem (44), the constraints (38) need to be considered. For $\|u_{MPC}(k)\|_2\leq u_{MPC\max}$, the following holds

$$\begin{aligned} \max_{i\geq 0}\|u_{MPC}(k+i|k)\|_2^2&=\max_{i\geq 0}\|Ky(k+i|k)\|_2^2 \\ &=\max_{i\geq 0}\|KCx(k+i|k)\|_2^2 \\ &\leq\max_{x\in\mathbf{X}}\left\|YS^{-1}x(k+i|k)\right\|_2^2 \end{aligned} \tag{61}$$

Let $h=S^{-\frac{1}{2}}x$. The constraint on the input variable can be converted into:

$$\begin{aligned} \max_{i\geq 0}\|u_{MPC}(k+i|k)\|_2^2&\leq\max_{x\in\mathbf{X}}\left\|YS^{-1}x\right\|_2^2 \\ &=\max_{h^Th\leq 1}\left\|YS^{-\frac{1}{2}}h\right\|_2^2 \\ &\leq\lambda_{\max}\left(S^{-\frac{1}{2}}Y^TYS^{-\frac{1}{2}}\right) \\ &\leq u_{MPC\max}^2 \end{aligned} \tag{62}$$

According to the Schur Complement Lemma, it follows that

$$\left[\begin{array}{cc}-u_{MPC\max}^2 I & YS^{-\frac{1}{2}} \\ S^{-\frac{1}{2}}Y^T & -I\end{array}\right]\leq 0 \tag{63}$$

Multiplying both sides of (63) by the diagonal matrix $diag\left(I,\ S^{\frac{1}{2}}\right)$ yields:

$$\left[\begin{array}{cc}-u_{MPC\max}^2 I & Y \\ Y^T & -S\end{array}\right]\leq 0 \tag{64}$$

and the input constraint $\|u_{MPC}(k)\|_2\leq u_{MPC\max}$ is guaranteed.

For $\|y(k)\|_2=\|y(k+i|k)\|_2\leq y_{\max}$, it follows that

$$\begin{aligned} &\max_{i\geq 1}\ \|y(k+i|k)\|_2^2=\max_{i\geq 1}\ \|Cx(k+i|k)\|_2^2 \\ =&\max_{i\geq 0}\ \|C(Ax(k+i|k)+Bu_{MPC}(k+i|k))\|_2^2 \\ =&\max_{i\geq 0}\ \|C(A+BKC)x(k+i|k)\|_2^2 \\ =&\max_{i\geq 0}\ \left\|C\left(A+BYS^{-1}\right)x(k+i|k)\right\|_2^2 \\ =&\max_{h^Th\leq 1}\ \left\|C\left(A+BYS^{-1}\right)S^{\frac{1}{2}}h(k+i|k)\right\|_2^2 \\ \leq&\lambda_{\max}\left(S^{\frac{1}{2}}\left(A+BYS^{-1}\right)^TC^TC\left(A+BYS^{-1}\right)S^{\frac{1}{2}}\right) \\ \leq&y_{\max}^2 \end{aligned} \tag{65}$$

Note that $x(k+1,k)=Ax(k+0,k)+Bu(k+0,k)$. It follows that in the second equation in (65) the range of i is changed form $i\geq 1$ to $i\geq 0$.

Then, according to the Schur Complement Lemma yields:

$$\left[\begin{array}{cc}-y_{\max}^2 I & S^{\frac{1}{2}}\left(A+BYS^{-1}\right)^TC^T \\ C\left(A+BYS^{-1}\right)S^{\frac{1}{2}} & -I\end{array}\right]\leq 0 \tag{66}$$

Multiply both sides of (66) by the diagonal matrix $diag\left(I,\ S^{\frac{1}{2}}\right)$. It follows that

$$\left[\begin{array}{cc}-y_{\max}^2 I & CAS+CBY \\ SA^TC^T+Y^TB^TC^T & -S\end{array}\right]\leq 0 \tag{67}$$

Therefore, the output constraint $\|y(k)\|_2 \leq y_{\max}$ is guaranteed. Q.E.D.

By considering Theorem 2, the optimization problem $W_N(x)$ can be solved:

$$\min_{\gamma,Y,M,S,Y_1,M_{11},a_u,a_y} \gamma \quad s.t.(32),(33),(34),(35) \tag{68}$$

Remark 4: The disturbances $d_{ue}(k)$ cannot be compensated by the ISMC and the corresponding effects on the constraint system need to be considered. The explicit OF-RMPC law is given by Theorem 2. From the proof of Theorem 2, it can be seen that the system states and the disturbances are expressed by the Hamiltonian function (52). In Theorem 2, LMIs (45), (47) and (48) ensure that the system does not violate constraints in the presence of the disturbances. The $d_{ue}(k)$ is treated by LMI (46). The input constraints and output constraints are handled by LMIs (47) and (48), respectively. In solving these LMIs, efficient solving tools in MATLAB can be utilized, such as YALMIP [45] and SDPT3 [46].

Remark 5: In Theorem 2, inequality (45) must be iteratively calculated with state information $x(k|k)$ as the initial value. It is worth noting that the initial value does not need to be very precise in the iterative calculation, so the value of the current state $x(k|k)$ can be replaced by the predicted state $x_p(k|k-1)$, that is, $x(k|k) \approx x_p(k|k-1)$. $x_p(k|k-1)$ is predicted from previous information. At time k, $u(k-i), i = 1,\dots k$, and $x(0)$ are available. Then, $x_p(k|k-1)$ can be obtained from this information based on the nominal prediction model: $x_p(k|k-1) = A^k x(0) + \sum_{i=1}^{k} A^{i-1} B u_{MPC}(k-i)$. Further, $y_p(k+1|k)$ can be obtained by $x_p(k+1|k)$ as $y_p(k+1|k) = Cx_p(k+1|k) = C(Ax_p(k|k-1) + Bu_{MPC}(k|k))$, which is defined in (10). This is a common approach from [36]. Although the predicted value of the state is used in inequality (45), it is only used as an initial value in the design process of the RMPC. The order of the system (33) is not increased by the method so that the complexity of the controller can be effectively reduced.

Choose an arbitrarily small positive constant ϵ as the iteration error. Then, the algorithm can be given in Algorithm 1.

Algorithm 1 OF-ISMPC for linear discrete-time systems

Step 1 Set $k = 0$ and $K = 0$;
Step 2 **For** each $k \in [\ 0 \quad k_\infty\ )$ **do**
(i) Exchange output information and initial estimates K, set iteration number $t = 0$ and set $K = K^t$;
(ii) Predict the initial values $x_p(k|k-1)^{(t)}$, obtain $A(k)^t$ and $B(k)^t$;
(iii) Solve LMI problems (68), obtain γ^t, Y^t, M^t, S^t, Y_1^t, M_{11}^t and K^t;
(iv) **If** $\left\|K^t - K^{t-1}\right\| \leq \epsilon$, then, go to next step;
else
Exchange K, set $t = t+1$, return to (ii) and iterate;
end
end
Obatin $u_{MPC}(k) = Ky(k)$;
Step 3 In the current feedback output $y(k)$, an integral sliding surface $s(k)$ is obtained based on the OF-RMPC $u_{MPC}(k)$;
Step 4 According to (32), obtain G, then the OF-ISMC $u_{ISM}(k)$ can be obtained;
Step 5 Implement the OF-ISMPC $u(k) = u_{ISM}(k) + u_{MPC}(k)$;
Step 6 Go to $k = k+1$, return to step 2 and repeat.

C. Closed loop system stability analysis

In order to analyze the stability of the equivalent system (33), the following theorem is presented.

Theorem 3: For system (33) subject to constraints (38), controlled by the OF-RMPC law $u_{MPC}(k) = Y_1 M_{11}^{-1} y(k)$ which is given by Theorem 2, considering Lemma 3, if the terminal cost $F(x(k+N|k))$ is an ISpS-Lyapunov function, then, the optimal cost $J_N^*(k)$ is an ISpS-Lyapunov function, and then the system (33) is ISpS for all initial states $x(0) \in \mathbf{X}_{MPC}$ and for $d_{ue}(k) \in \mathbf{D}_{ue}(\rho)$.

Proof.

Due to the invariance of the terminal set $\mathbf{X}_f$, $\mathbf{X}_{MPC}$ is a robust positively invariant set for system (33), in which $\mathbf{X}_{MPC}$ is including the origin as an interior point [47].

The optimization problem $W_N(x)$ is rewritten as follows:

$$\begin{aligned} J_N^*(k) = \min_{U^R(t)} \max_{d_{ue}(t)\in D_{ue}} & \left\{ \sum_{i=0}^{N-1} L(x(i|k), u_{MPC}(i|k)) \right. \\ & \left. + F(x(N|k)) \right\} \\ & s.t u_{MPC}(k) \in \mathbf{U}^R, y(k) \in \mathbf{Y}, x(N) \in \mathbf{X}_f, \\ & d_{ue}(k) \in \mathbf{D}_{ue}, k = 0, \cdots, N-1, \end{aligned} \tag{69}$$

where $x(i|k)$ denotes the predictive state at time period $k+i$ i.e. $x(i|k) = x(k+i|k)$.

$u_{MPC}^*(i|k) = K_i^*(y)$ is the optimal solution to $W_i(x)$.

Then,

$$\begin{aligned} J_N^*(k) &\geq \min_{U^R(t)} J_N(k) \\ &\geq L(x(i|k), K_i^*(y)) \\ &\geq \lambda_{\min}(Q) x^T(i|k) x(i|k) \end{aligned} \tag{70}$$

From (42), the following inequality can be obtained:

$$J_N^*(k) \leq \lambda_{\min}(P) x^T(k|k) x(k|k) \tag{71}$$

It follows that:

$$\begin{aligned} &\lambda_{\min}(Q) x^T(i|k) x(i|k) \leq L(x(i|k), K_i^*(y)) \\ &\leq J_N^*(k) \leq \lambda_{\min}(P) x^T(i|k) x(i|k) \end{aligned} \tag{72}$$

which satisfies the first requirement for an ISpS-Lyapunov function (88) as described in Appendix B.

The optimal cost of i-steps is defined as follows:

$$J_i^*(k) = \min_{U^R(t)} L_{\max -i}(k) \tag{73}$$

where $L_{\max -i}(k) =$

$$\left\{\max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i|k\right),u_{MPC}\left(i|k\right)\right)+J_{i-1}^{*}\left(k\right)\right]\right\}.$$

$\Delta J_i^*(k) = J_i^*(k) - J_{i-1}^*(k)$ for all $x(i|k) \in \mathbf{X}_{MPC}$.

When $i=1$, the following inequality can be obtained:

$$\begin{aligned}&\Delta J_1^*(k) = J_1^*(k) - J_0^*(k)\\&\leq \max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(1|k\right),u_{MPC}\left(1|k\right)\right)+F\left(x\left(1|k\right)\right)\right]\\&-F\left(x\left(0|k\right)\right)\leq 0\end{aligned} \tag{74}$$

Assuming that $\Delta J_i^*(k) \leq 0$ for all $x(i|k) \in \mathbf{X}_{MPC}$, then

$$\begin{aligned}&\Delta J_{i+1}^*(k) = J_{i+1}^*(k) - J_i^*(k)\\&=\max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i+1|k\right),K_{i+1}^{*}\left(y\right)\right)+J_i^*(k)\right]\\&-\max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i|k\right),K_{i}^{*}\left(y\right)\right)+J_{i-1}^*(k)\right]\\&\leq\max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i|k\right),K_{i}^{*}\left(y\right)\right)+J_i^*(k)\right]\\&-\max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i|k\right),K_{i}^{*}\left(y\right)\right)+J_{i-1}^*(k)\right]\\&\leq\max_{d_{ue}(t)\in D_{ue}}\left[J_i^*(k)-J_{i-1}^*(k)\right]\\&=\max_{d_{ue}(t)\in D_{ue}}\left[\Delta J_i^*(k)\right]\leq 0\end{aligned} \tag{75}$$

Hence, $J_{i+1}^*(k) \leq J_i^*(k)$. It follows that $J_i^*(k)$ is a monotonically decreasing function.

Because $x(N|k) \in \mathbf{X}_{MPC}$ and $d_{ue}(k) \in \mathbf{D}_{ue}(\rho)$, it is clear that $x(N|k+1) \in \mathbf{X}_{MPC}$. According to the monotonicity result, it can be obtained that:

$$\begin{aligned}&J_N^*(k+1) - J_N^*(k)\\&=J_N^*(k+1) - \max_{d_{ue}(t)\in D_{ue}}\left[L\left(x\left(i|k\right),K_{N}^{*}\left(y\right)\right)+J_{N-1}^*(k)\right]\\&\leq J_N^*(k+1) - L\left(x\left(i|k\right),K_{N}^{*}\left(y\right)\right) - J_{N-1}^*(k)\\&\leq \Delta J_N^*(k+1) - L\left(x\left(i|k\right),K_{N}^{*}\left(y\right)\right)\\&\leq -L\left(x\left(i|k\right),K_{N}^{*}\left(y\right)\right)\end{aligned} \tag{76}$$

which satisfies the second requirement for an ISpS-Lyapunov function (88) as described in Appendix B.

From (72) and (76), it can be seen that the optimal cost $J_i^*(k)$ is an ISpS-Lyapunov function. According to Theorem 1 in [44], a sufficient condition for the system to be ISpS is that admits an ISpS-Lyapunov function. Therefore, it can be deduced that the system (33) is ISpS. Q.E.D.

In this section, the role of the OF-ISMC is to compensate for matched disturbances, while an OF-RMPC is allocated to solve the optimization problem for systems with unmatched disturbances. To design the OF-ISMPC, an integral sliding surface is designed which uses output information directly and an optimal projection matrix is selected on the integral sliding surface. The solution of an optimization problem is effectively transformed into a set of LMI solutions. The equivalent system is proved to be ISpS. This is the main result of this paper.

Remark 6: Applying the OF-ISMC law (15) to system (1), then the equivalent system (33) is obtained based on Lemma 2. By Theorem 1, it can be seen that the trajectory of system (1) is stabilized in the quasi-sliding mode domain $\mathbf{\Gamma}$. From Theorem 3, it can be obtained that the system (33) is stabilized in $\mathbf{X}_{MPC}$. Therefore, it can be ensured that the closed-loop system is stabilized in the set $\mathbf{X}_C = \mathbf{\Gamma} \cup \mathbf{X}_{MPC}$.

IV. Simulation and Experimental Results

In this section, simulation tests and an experimental trial are presented to demonstrate the validity of the proposed method. In order to verify the feasibility of the proposed OF-ISMPC, the simulation results are compared with the state observer based OF-ISMPC [27] and output feedback RMPC [35]. In addition, a continuous stirred tank reactor (CSTR) experiment is used to further verify the effectiveness of the proposed method.

A. Discrete-time system numerical simulation

The proposed method, observer-based OF-ISMPC and output feedback RMPC are applied to a heat exchanger system with shell and tube configuration as considered in [48]. With a sampling period of $T_s = 0.1$, the following required discrete-time system is obtained:

$$\begin{aligned}x(k+1) &= \begin{bmatrix} 0.9909 & 0.0861 \\ -0.1722 & 0.7326 \end{bmatrix} x(k)\\&+\begin{bmatrix} 0.0091 \\ 0.1722 \end{bmatrix} u(k) + d(k)\\ y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)\end{aligned} \tag{77}$$

where the disturbance $d(k) = \begin{bmatrix} -0.4 \\ 0.8 \end{bmatrix} \sin(1.4k)$ is decomposed into matched and unmatched components as follows:

$$\begin{aligned}d(k) =& \underbrace{\begin{bmatrix} 0.0028 & 0.0527 \\ 0.0527 & 0.9972 \end{bmatrix}}_{BB^+} d(k)\\&+\underbrace{\begin{bmatrix} 0.9972 & -0.0527 \\ -0.0527 & 0.0028 \end{bmatrix}}_{B^\perp B^{\perp +}} d(k)\end{aligned} \tag{78}$$

The first component is matched and will be compensated by $u_{ISM}(k)$ while the second is unmatched.

The system initial states are selected as $x(0) = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$. For OF-ISMC (15), the gain $\rho = 1$ is selected so that the condition $\|d(k)\| \leq \rho$ can be guaranteed. For OF-RMPC (59), the output feedback gain K is obtained through Theorem 2. Then, the following OF-ISMPC control law is implemented:

$$\begin{aligned}u(k) =& -(GCB)^{-1}\left(\rho\|GC\| + 0.1\right)\mathrm{sgn}\left(s(k)\right)\\&-0.9802y(k)\end{aligned} \tag{79}$$

where $G = (CB)^T = 0.0091$.

The cost function $J_N(k)$ is shown in (7) and the positive definite weighting matrices are chosen as $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = 1$. For observer-based OF-ISMPC, the initial estimated states are selected as $\hat{x}(0) = \begin{bmatrix} 2.2 & -1.4 \end{bmatrix}^T$, $u(k) = -0.2\mathrm{sgn}(s(k)) + \begin{bmatrix} -0.4194 & -0.4199 \end{bmatrix}\hat{x}(k)$ and a Luenberger observer is used where the observer gain $L = \begin{bmatrix} 2.6235 & 1.6438 \end{bmatrix}^T$. For output feedback RMPC, $u(k) = \begin{bmatrix} -0.4194 & -0.4199 \end{bmatrix}\hat{x}(k)$ can be obtained by solving a set of LMIs. The control horizon is 1 and the prediction horizon is 10. The simulation results for the proposed OF-ISMPC, the

state observer based OF-ISMPC and output feedback RMPC are shown in Figs.2-8. The Integral of Time and Absolute Error (ITAE) for all three methods are listed in TABLE I, and the expression for ITAE can be found in Appendix B.

TABLE I: ITAE for proposed method, observer-based OF-ISMPC and output feedback RMPC.

Method	$x_1(k)$	$x_2(k)$
Proposed method	41.6086	35.1959
Observer-based OF-ISMPC	46.7781	44.4059
Output feedback RMPC	60.6288	56.0750

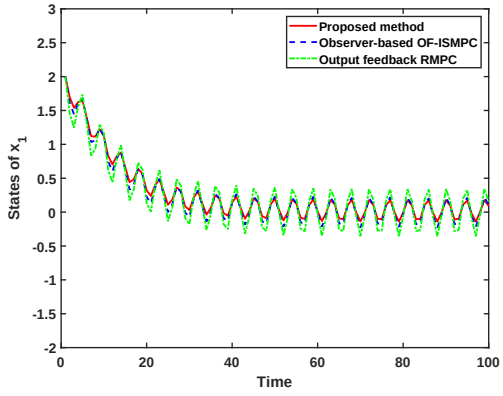


Fig. 2: Time response of the state $x_1(k)$.

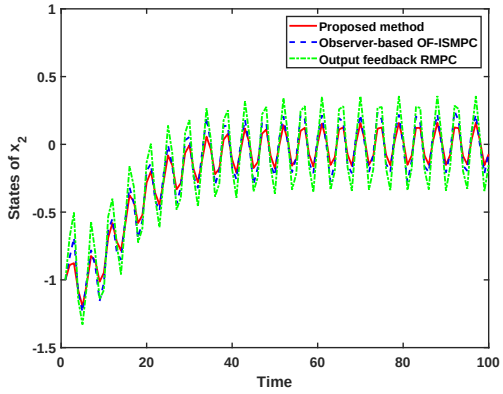


Fig. 3: Time response of the state $x_2(k)$.

In the following simulation results, the solid line represents the response using the proposed method, the dotted line represents the results using the observer-based OF-ISMPC, and the dash-dotted line represents the response with the output feedback RMPC. From Figs. 2 and 3, it can be seen that the proposed method provides better disturbance rejection than the other two control methods. As can be seen from Figs. 4 and 5, the proposed method has better control accuracy than the observer-based OF-ISMPC and output feedback RMPC, because the proposed method does not increase the system order. Figs.2, 3, and 4 show that the proposed method has stronger robustness than output feedback RMPC

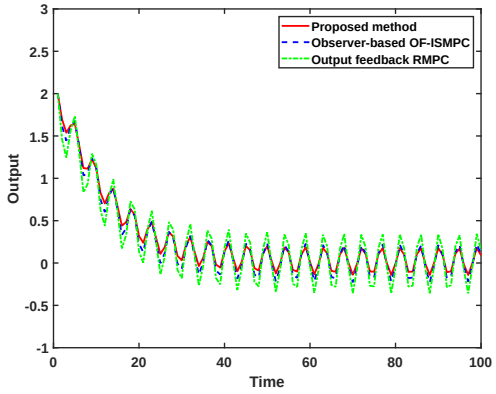


Fig. 4: Time response of the output $y(k)$.

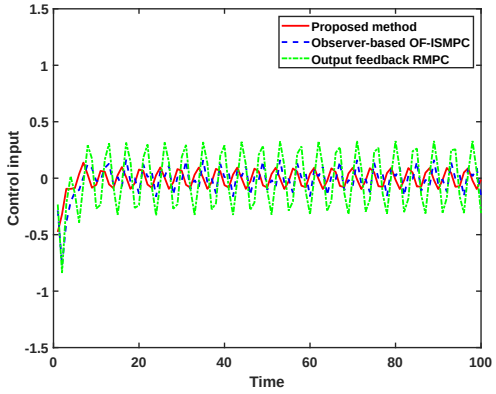


Fig. 5: Time response of the control input $u(k)$.

for disturbances, because the ISMC compensates the matched disturbances. The proposed method has prediction errors (see Remark 5), so that the overshoot of its state and output responses are slightly higher than that of the observer-based OF-ISMPC before 15s. Lemma 1 shows that the prediction error is bounded, so the control accuracy will improve as time increase. Fig. 6 shows the convergence of the integral sliding variable $s(k)$ when the system uses output information. Fig. 7 shows the stabilization domains of the three methods for the given system, and it can be seen that the proposed method is superior to the other two methods. Due to the better control accuracy demonstrated by the proposed method (in Fig. 5), it achieves a faster initial response speed compared to observer-based ISMPC. Additionally, this method is able to completely compensate for the matched disturbances, making it more robust than OF-RMPC and the controller therefore ensures a smaller stabilization domain. When $k = 100$, the cost function values of the three methods are $J_{Proposed\ method}(100) = 45.4843$, $J_{Observer-based\ OF-ISMPC}(100) = 65.9277$ and $J_{Output\ feedback\ RMPC}(100) = 103.5219$, respectively. It can be seen that the proposed method can maintain better performance. Meanwhile, from TABLE I, the tracking performance of the proposed method is better. To sum up, the simulation results and ITAE show that the proposed method is superior to the other two methods.

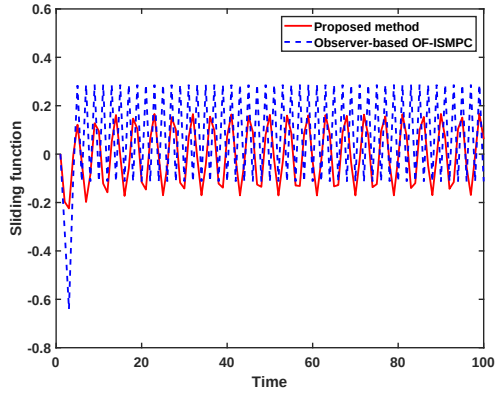


Fig. 6: Time response of the sliding function $s(k)$.

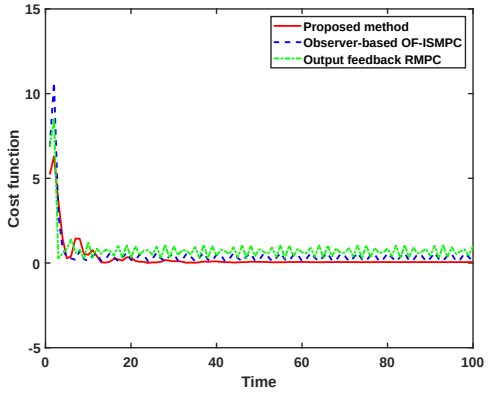


Fig. 8: Time response of the cost function $J(k)$.

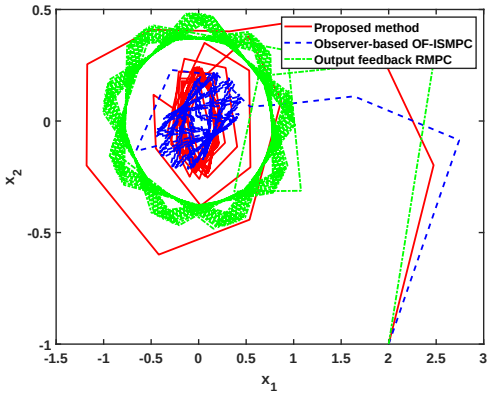


Fig. 7: The stability domain of the system.

B. CSTR system experiment

The applicability of the proposed method will be further validated by the following experiment. The research team at the China University of Petroleum (East China) has developed an experiment platform, which is shown in Fig. 9. The experimental platform has four reactors (R101, R102, R103, R104), which can be connected in series, parallel or series and parallel to verification and testing the control algorithms. The operational interface is shown in Fig. 10. Only one reactor (R101) is used in the experiment. V111 and V112 are header tanks containing ethyl acetate and sodium hydroxide, respectively. The dynamic equations of the CSTR system have been given in [49]. V113 is a water tank jacket used for heat transfer, and the heat flow from V113 is used as the input to the system. Set the sampling time as $T_s{}' = 0.45s$. The initial temperature conditions for deployment of the proposed method, Observer-based OF-ISMPC and output feedback RMPC are 23.4 degrees Celsius, 24.1 degrees Celsius and 25.6 degrees Celsius, respectively. The corresponding desired set-points are all 30 degrees Celsius. Since the temperature is affected by many environmental factors, the initial temperature conditions for the three experiments are different. The tracking performance of the temperature as shown in Fig. 11. It can be seen that the tracking performance of the proposed method is superior. From TABLE II, it can be seen that the ITAE of the proposed method is the smallest. Therefore, experimental results further validate that the proposed method has better control performance.



Fig. 9: Experimental equipment.

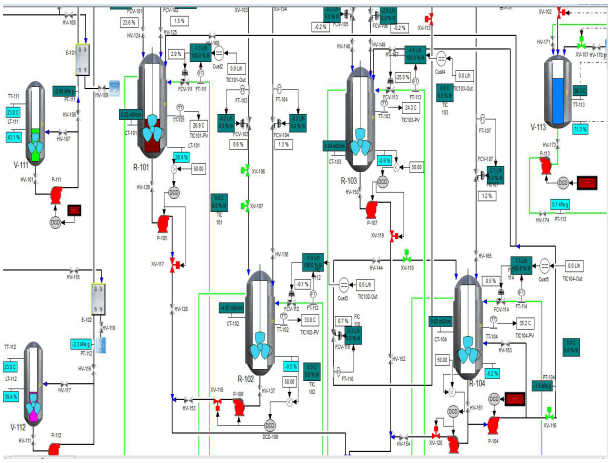


Fig. 10: Experimental operation interface.

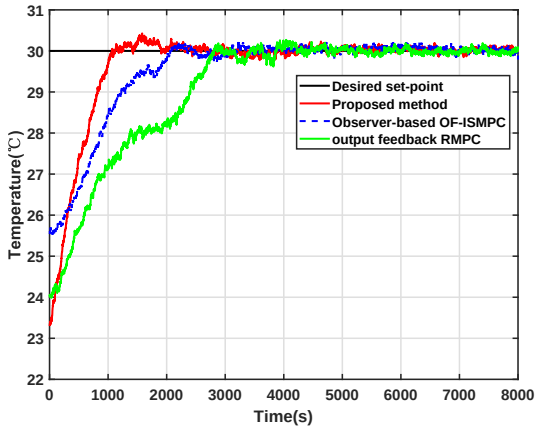


Fig. 11: The output temperature of the CSTR.

TABLE II: ITAE results of three methods for experiment.

Method	Traking performance
Proposed method	23.3018
Observer-based OF-ISMPC	24.0167
Output feedback RMPC	25.5635

V. Conclusion

In this paper, a new OF-ISMPC strategy has been proposed for cases where the system states are not measurable. The main objective is to develop a new control with strong robustness which also guarantees optimal performance without violating constraints. The key elements of the proposed control algorithm are the design of an OF-ISMC and a corresponding OF-RMPC. The OF-ISMC compensates for the matched disturbances that affect the system and enforces a sliding mode from the initial time instant. The OF-RMPC ensures global optimization of the equivalent system. The ISpS of the closed-loop system is proved. The simulation results show that the proposed method can deal with an unmeasurable system state exhibiting strong robustness to disturbances. The experimental results further illustrate the effectiveness of the proposed method for linear discrete-time systems when the state is unmeasurable.

Future work will consider the development of a decentralised approach to achieve the robustness and performance benefits whilst reducing computational complexity.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A
Other Proofs

Proof of Lemma 1: The predicted value of the state $x_p(k|k-1)$ can be obtained by the following equation:

$$x_p(k|k-1) = Ax(k-1|k-1) + Bu_{MPC}(k-1|k-1) \tag{80}$$

Substitute (80) into (13) to yield:

$$\begin{aligned} e(k) &= x(k|k) - x_p(k|k-1) \\ &= x(k|k) - [Ax(k-1|k-1) \\ &\quad + Bu_{MPC}(k-1|k-1)] \\ &= x(k|k) - (A+BKC)x(k-1|k-1) \end{aligned} \tag{81}$$

Then,

$$\begin{aligned} \|e(k)\| &= \|x(k|k) - [(A+BKC)x(k-1|k-1)]\| \\ &\le \|x(k|k)\| + \|(A+BKC)x(k-1|k-1)\| \end{aligned} \tag{82}$$

According to $h = S^{-\frac{1}{2}}x$ and $KC = YS^{-1}$, it can be obtained that:

$$\begin{aligned} \|e(k)\| &\le \left\|S^{\frac{1}{2}}h(k|k)\right\| \\ &\quad + \left\|\left(AS^{\frac{1}{2}} + BYS^{-\frac{1}{2}}\right)h(k-1|k-1)\right\| \\ &\le \left\|S^{\frac{1}{2}}\right\| \|h(k|k)\| \\ &\quad + \left\|AS^{\frac{1}{2}} + BYS^{-\frac{1}{2}}\right\| \|h(k-1|k-1)\| \\ &\le \left\|S^{\frac{1}{2}}\right\| + \left\|AS^{\frac{1}{2}} + BYS^{-\frac{1}{2}}\right\| \end{aligned} \tag{83}$$

$\|h(k|k)\| \le 1$ and $\|h(k-1|k-1)\| \le 1$ can be derived from (60). If $\|Y\|$ and $\|S\|$ are bounded, then $\left\|S^{\frac{1}{2}}\right\|$ and $\left\|AS^{\frac{1}{2}} + BYS^{-\frac{1}{2}}\right\|$ are also bounded. Then, the predicted error $e(k)$ is bounded.

For $e_o(k)$, it follows that

$$\begin{aligned} e_o(k) &= y(k|k) - y_p(k|k-1) \\ &= C(x(k|k) - x_p(k|k-1)) \\ &= Ce(k) \end{aligned} \tag{84}$$

According to (13), it is straight forward to prove that $e_o(k)$ is bounded. Q.E.D.

Proof of Proposition 1: For $d_{ue}(k)$ in (33), it follows that

$$\begin{aligned} \|d_{ue}(k)\| &= \left\|Ld_u(k) - \bar{L}e(k)\right\| \\ &\le \|Ld_u(k)\| + \left\|\bar{L}e(k)\right\| \\ &\le \|L\| \|d_u(k)\| + \left\|\bar{L}\right\| \|e(k)\| \end{aligned} \tag{85}$$

According to (34) and (35), it can be obtained that

$$\|d_{ue}(k)\| \le \|L\| \rho_u + \left\|\bar{L}\right\| \rho_e = \rho_{ue} \tag{86}$$

where $L = \left(I - B(CB)^+C\right)$, $\bar{L} = B(CB)^+CA$. Q.E.D.

Appendix B
Definitions

Definition 1: [50] A continuous function α: $\mathbf{R}_+ \to \mathbf{R}_+$ is a K-function, if $\alpha(0) = 0$, $\alpha(s) > 0$ for all $s > 0$ and it is strictly increasing. A function $\alpha(\cdot)$ is a K_∞-function if it is

a K-function and $\alpha(s) \to \infty$ when $s \to \infty$. A continuous function β: $\mathbf{R}_+ \times \mathbf{Z}_+ \to \mathbf{R}_+$ is a KL-function if $\beta(s,t)$ is a K-function in s for every $t \geq 0$, it is strictly decreasing in t for every $s > 0$ and $\beta(s,t) \to 0$ when $t \to \infty$.

Definition 2: [44] The system (1) is input-to-state practically stable (ISpS), if there exist a KL -function $\beta(\cdot,\cdot)$, a K -function $\tilde{\gamma}(\cdot)$ and a constant $c \geq 0$ such that

$$\|x(k)\| \leq \beta(\|x_0\|, k) + \tilde{\gamma}(\|\mathbf{w}\|_\infty) + c \tag{87}$$

where $\|x(k)\|$ denotes a norm of a given vector $x(k)$, disturbances signals $\mathbf{w} = \{d(0), d(1), \ldots, d(N-1)\}$ and $\|\mathbf{w}\|_\infty = \sup\{\|d(k)\|, k = 0, 1, \ldots, N-1\}$.

Definition 3: [44] If there exist a function $V(\cdot)$: $\mathbf{R}^n \to \mathbf{R}_+$, K_∞-functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\alpha_3(\cdot)$ and K-functions $\sigma(\cdot)$, for all $x(0) \in \mathbf{R}^n$, $d(k) \in \mathbf{D}$ satisfy the following conditions:

$$\begin{aligned} &\alpha_1(\|x(k)\|) \leq V(x(k)) \leq \alpha_2(\|x(k)\|) \\ &V(x(k+1)) - V(x(k)) \leq -\alpha_3(\|x(k)\|) + \sigma(\|d(k)\|) \end{aligned} \tag{88}$$

then, the function $V(\cdot)$: $\mathbf{R}^n \to \mathbf{R}_+$ is an ISpS-Lyapunov function.

Definition 4: [51] The Integral of Time and Absolute Error (ITAE) is mathematically given by:

$$ITAE = \int_0^\infty t \cdot |e(t)| dt \tag{89}$$

where t is the time and $e(t)$ is the error which is calculated as the difference between the set point and the output.

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