Reimagining Archimedes: an innovative and accurate calculation of volumes and asserting another standard method for defining the surface area of quail and any avian eggs

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Abstract

Egg-related research promises unique opportunities for food science and technology. There is an urgent need to develop non-destructive methodologies for defining key egg parameters, e.g., egg volume (*V*) and surface area (*S*), based only on egg images. Herewith, *V* can be measured using the Archimedes' principle (i.e., dipping in water), while *S* can be inferred using formulae that include *V* as one of its variables. Although the Archimedes' principle is the best approach for determining *V*, dipping an egg into water cannot be practicable. In this study, we derived the appropriate mathematical approaches to calculate *V* and *S* based on measurements of quail eggs' linear parameters. The proposed calculation formulae are suitable for eggs of any shape and species. This innovative procedure can be employed as the basis of the most accurate of all existing methods for computing *S* and is suitable for both analytical and industrial measurements of *V*.

Keywords: Japanese quail eggs; egg quality; egg volume; egg surface area; topology; non-destructive testing technology

1. Introduction

Non-destructive quality evaluation of agro-food products is critical for further progress in agriculture and food industry (Mollazade et al., 2012). For this purpose, a variety of non-invasive methods have been developed by employing novel technologies, e.g., thermal imaging (Vadivambal and Jayas, 2011), low-field nuclear magnetic resonance and magnetic resonance imaging (Ezeanaka et al., 2019), light backscattering imaging (Mollazade et al., 2012) and many others. A number of non-destructive techniques for egg quality assessment have also been proposed (e.g., Abdel-Nour et al., 2011), although their straightforward and effective implementation in wide practice is often impeded by sophistication and high cost, which suggests further search for relevant methods including mathematics-assisted assays (Narushin, 1997a; Narushin et al., 2020a, 2021a,c, 2024).

Increasing the mathematical accuracy of calculation in fields such as engineering and physics can sometimes make it more accessible for wide use and facilitate further progress. For example, a sufficient value of π for the vast majority of practical calculations is 3.14; however, the current accuracy of this constant exceeds 100 trillion decimal places (Iwao, 2022) and a greater level of accuracy beyond these two decimal places has widespread use. The colloquial term 'eggology' (e.g., Williams, 1971; Jurriëns, 2016; Richmond, 2020) embraces a range of disciplines of poultry science and ornithology (and its oological subdiscipline). It also implies egg-inspired engineering solutions for thin-walled vessels (e.g., Narushin et al., 2022), buildings and other architectural and technological structures (Freiberger, 2007; Petrović et al., 2011; Levine et al., 2022; Juračka et al., 2023) and even objects of art (Gilbert, 1974; Herz-Fischler, 1990). In all the above fields, mathematical formulae for computing the egg volume (V) and surface area (S) are key. Publications continue to emerge incorporating these calculations. The earliest equations for V and S were established by Romanoff & Romanoff (1949), with further formulae developed over the next 50 years being reviewed by Narushin (1997a). Later, a few more dependencies, with theoretical background, appeared (Narushin, 1997b, 2001, 2005), while their further variations increased in their scope after 2010. These encompassed the use of topology principles (Narushin et al., 2020a), digital imaging analysis (Soltani et al., 2015; Zhang et al., 2016; Dangphonthong & Pinate, 2016; Zlatev, 2018; Chan et al., 2018; Narushin et al., 2020a), mathematical equations inferred for other objects but modified for egg shape (Maulana et al., 2015; Narushin et al., 2020b; Petrović & Malešević, 2023, Shi et al., 2022), and specialized equations meant directly for the geometric description of the egg profile (Troscianko, 2014; Narushin et al., 2021b,c, 2022a,b; Shi et al., 2023).

Judging from the logic of classical geometry, the calculation formulae for V and S based on the mathematical description of the egg's contours clearly take precedence. The formulae of integral geometry were derived as early as the 17th century (Chisholm, 1911), and have remained largely unchallenged since. In this context is the famous Archimedes' principle to measure V by dipping an object in water (Kireš, 2007; Archimedes, 2009; Fig. 1A). Although Archimedes' principle is the best approach for determining V (and our prior formulae for S of eggs also include V as one of its variables; e.g., Narushin et al., 2020a, 2021c), dipping an egg into water can be undesirable in experimental and commercial settings due to washing off the cuticle and the complexity in automating the measurement process. We thus recently developed and previously tested a number of approaches for calculating *V* and *S* based on non-invasive measurements of chicken eggs (e.g., Narushin et al., 2020a, 2021c).

In our most current work (Narushin et al., 2023a), we developed, in our opinion, an accurate and practical tool to describe the geometric shape of any egg using applied mathematics approaches. Nonetheless, despite a decent number of options for calculating these two parameters, the present study focusses on the integral transformation of egg shape formulae into computed dependences for determining *V* and *S*. It should also be noted that the vast majority of such calculations were previously performed using chicken eggs as an experimental model. As a consequence, the eggs of other poultry species have been undeservedly neglected. In order to address this imbalance, at least in part, we chose, in this study, domestic Japanese quail eggs to test our theoretical hypotheses and developments. Their shape is quite unique as it varies from conical to classic oval (Narushin et al., 2024). In this regard, among quail eggs one can find examples of almost the entire range of shapes inherent in bird eggs that exist in nature. Thus, the objective of our present research was to develop mathematical dependencies for calculating *V* and *S* for eggs of any geometric shape, along with testing their adequacy and precision on Japanese quail eggs. Novel technologies for assessing quail egg quality and other traits are critical for further improvements in quail breeding and production (e.g., Baumgartner and Bondarenko, 1989; Ryabokon et al., 2005; Podstreshnyi et al., 2010; Podstreshnyi and Tereshchenko 2012a,b; German et al., 2022).

2. Theory

In this investigation, we adhered to a fundamental principle laid down in our previous studies (Narushin et al., 2022a, 2023a). It lies in the fact that mathematical transformations are much easier to carry out not on a whole egg, but on its two halves, blunt and pointed. In this regard, we conferred the respective subscript '*b*' (for ‘blunt’) or '*p*' (for ‘pointed’) to all subsequent measured or calculated parameters characteristic of one or another half.

According to Narushin et al. (2022a), the contours of the corresponding half of any egg can be described mathematically by the following expressions:

- for the blunt end

$$y_b = \pm\frac{B}{2}\cdot\sqrt{\frac{L^2-4x^2}{L^2+8wx+4w^2}}\,, \tag{1}$$

where B is the egg maximum breadth, L is the egg length, and w is the parameter that displays the distance between two vertical lines that represent the egg's maximal breadth and half its length. Formula (1) conformed to x values in the interval $[-L/2\ldots-w]$.

- for the pointed end

$$y_p = \pm\frac{B_p}{2}\cdot\sqrt{\frac{L^2-4x^2}{L^2+8w_p x+4w_p^2}}\,. \tag{2}$$

The pointed end was characterized by the interval $x = [-w\ldots L/2]$.

For Eqn2, the values of B_p and w_p did not correspond to those of B and w in Eqn1. If the parameters of Eqn1 could be relatively easily measured either directly on the investigated egg or on its image, then the values of B_p and w_p were calculated. Since in subsequent calculations it would be more convenient to operate with specific variables (or their index representations), we slightly modified the format of the equations for B_p and w_p in comparison with that presented in Narushin et al. (2023a) as follows:

$$B_p = B\cdot\sqrt{\frac{1-8\frac{w_p}{L}\cdot\frac{w}{L}+4\left(\frac{w_p}{L}\right)^2}{1-4\left(\frac{w}{L}\right)^2}}\,, \tag{3}$$

$$w_p = L\left(\frac{\left(\frac{D_p}{B}\right)^2\left(1-4\left(\frac{w}{L}\right)^2\right)+3\frac{w}{L}}{3-4\left(\frac{D_p}{B}\right)^2\left(1-4\left(\frac{w}{L}\right)^2\right)}-\sqrt{\left(\frac{\left(\frac{D_p}{B}\right)^2\left(1-4\left(\frac{w}{L}\right)^2\right)+3\frac{w}{L}}{3-4\left(\frac{D_p}{B}\right)^2\left(1-4\left(\frac{w}{L}\right)^2\right)}\right)^2-\frac{1}{4}}\right), \tag{4}$$

where D_p is the diameter corresponding to the point $x = L/4$.

Further, we implemented the formulae of integral geometry (e.g., Turkawi, 2020) as follows:

$$V = \pi\int_a^b y^2\,\mathrm{d}x \tag{5}$$

and

$$S = 2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,\mathrm{d}x\,. \tag{6}$$

The above equations were inferred relying on the thoroughness and accuracy of a series of intermediate mathematical calculations that, in order not to burden this article, can be seen in detail in Supplementary Data A. The resulting final mathematical dependencies had the following form:

$$V = \frac{\pi}{128}\left[\left(8.917 - 29.998\frac{w}{L}\right)\left(\frac{D_p}{B}\right)^2 + \left(2.459 + 88.647\frac{w}{L}\right)\frac{D_p}{B} - 36.26\frac{w}{L} + 12.453\right]LB^2 \qquad (7)$$

$$S = \pi BL\left(0.389 + 0.188\frac{B}{L} - 0.063\frac{w}{L} + 0.365\frac{D_p}{B} + 0.114\frac{D_p}{L} - 0.168\frac{w}{L}\cdot\frac{B}{L} + 0.46\frac{w}{L}\cdot\frac{D_p}{B} + 0.484\frac{w}{L}\cdot\frac{D_p}{L}\right) \qquad (8)$$

To assess the adequacy of the theoretical study carried out, a series of experimental studies were performed as outlined below.

3. Materials and Methods

Experimental testing of the above theoretical investigation was executed using 54 quail eggs from F_2 progenies of a Japanese and Texas breed cross aged two to three months and produced as a reference population for identification of molecular genetic mechanisms underlying economically important traits in quails, including traits of egg quality and reproduction (German et al., 2022, 2023; Prituzhalova et al., 2023; Volkova et al., 2023). Using a digital caliper gauge (Fig. 1A), *L* values were determined with an accuracy of 0.1 mm, and the eggs were photographed (Fig. 1B) as implemented elsewhere (e.g., Narushin et al. 2020a). After that, we employed these images to measure values of *L*, *B*, *w* and D_p with an accuracy of 1 pixel using the Microsoft Picture Manager software as described elsewhere (e.g., Narushin et al. 2020b; Narushin et al. 2021d). Knowing *L*, respectively in mm and pixels, the measurements and other geometric parameters, i.e., *B*, *w* and D_p, were converted into the metric system. Values of *V* were measured by the Archimedes' principle also known as the hydrostatic weighing method (e.g., Handrich, 1989; Kireš, 2007; Archimedes, 2009; Sukhon et al., 2011), i.e., by weighing eggs in water (Fig. 1A).

A **B**

Fig. 1. Experimental setting for quail egg examination. (A) Hydrostatic weighing (a digital caliper gauge is also shown next to the digital scale). (B) Digital photographing.

A mean percentage error, ε (e.g., Makridakis et al., 1982), was used to evaluate how closely the computed and true parameter values corresponded:

$$\varepsilon = \frac{1}{n} \cdot \sum_{1}^{n} \left| \frac{v_1 - v_2}{v_1} \right| \cdot 100\% \quad (9)$$

,

where n is number of measurements (according to the number of eggs examined, i.e., 54), and v_1 and v_2 are values of the respective parameters, which are represented by the direct measurement (v_1) and the computed one (v_2).

The datasets under study were processed using the STATISTICA 5.5 program (StatSoft, Inc./TIBCO, Palo Alto, CA, USA) and computational Microsoft Excel tools.

4. Results and Discussion

As a result of the measurements, the following means and variations of parameters were obtained: L = 3.31 ± 0.105 cm; B = 2.56 ± 0.075 cm; w = 0.19 ± 0.045 cm; D_p = 1.99 ± 0.075 cm; and V = 11.04 ± 0.877 cm^3. Having at our disposal a set of initial parameters required to calculate V, we undertook a comparative analysis of direct (Archimedes' principle; Fig. 1B) and indirect (calculation by Eqn7) results. Based on the calculated mean percentage error (Eqn9), it was concluded that the two methods coincided fairly accurately. The value of ε was 1.1 ± 0.75% that was more than acceptable for both practical and analytical purposes.

4.1. Egg Volume

Previously, Narushin et al. (2022a) already derived a universal equation for calculating V that, taking into account the designation of parameters adopted in this article, looks like this:

$$V = 0.992\left(\left(\frac{D_p}{B} - 0.426\right)\frac{w}{L} + 0.396\frac{D_p}{B} + 0.182\right)LB^2 . \qquad (10)$$

We compared how these calculations, i.e., Eqn7 vs Eqn10, differ from each other. The calculation of the error value when comparing the Archimedes' method with the calculation by Eqn10 showed almost identical results with the new calculation formula (Eqn7), ε = 1.1 ± 0.72%. It was possible that this similarity was due to the small size of quail eggs. In this regard, we carried out an additional check using an experimental sample from our previous study (Narushin et al., 2020a) conducted on chicken eggs. Again, the results were practically the same: the error was ε = 1.8 ± 1.55% for Eqn7, and ε = 1.8 ± 1.59% for Eqn10.

Then, we investigated theoretically the magnitude of the differences between the two variants for calculating V, i.e., using Eqns 7 and 10. To do this, we introduced the following coefficients, K_{V1} and K_{V2} for LB^2 in Eqn7 and Eqn10, respectively:

$$K_{V1} = \frac{\pi}{128}\left[\left(8.917 - 29.998\frac{w}{L}\right)\left(\frac{D_p}{B}\right)^2 + \left(2.459 + 88.647\frac{w}{L}\right)\frac{D_p}{B} - 36.26\frac{w}{L} + 12.453\right], \qquad (11)$$

$$K_{V2} = 0.992\left(\left(\frac{D_p}{B} - 0.426\right)\frac{w}{L} + 0.396\frac{D_p}{B} + 0.182\right). \qquad (12)$$

Subsequently, we substituted all possible values of w/L and D_p/B combinations into both formulae. In Supplementary Data A, we demonstrated the possible magnitudes of variability for both parameters, w/L and D_p/B, as well as the relationship between them. Visualization of the comparative analysis of the data obtained as a result of the calculation for Eqn11 and Eqn12 is presented in Fig. 2.

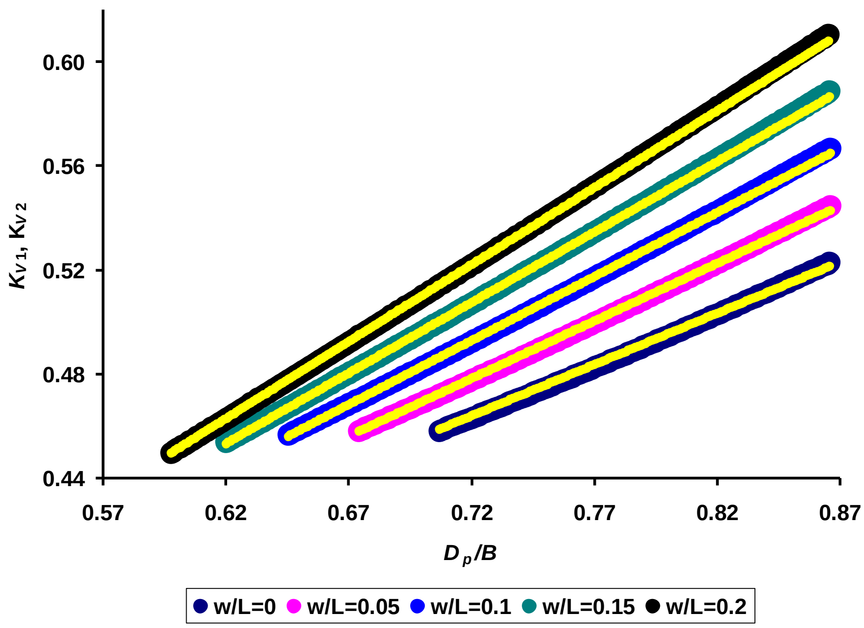

Fig. 2. Visualization of the comparative analysis of the data obtained as a result of the calculation using Eqn11 (colored contours according to the legend below the graph) and Eqn12 (basic yellow lines).

Based on the graphical dependencies (Fig. 2), the calculated data obtained using both formulae (Eqn11 and Eqn12) were almost indistinguishable. Obviously, within this range of D_p/B values, the polynomials of the 2nd (Eqn11) and 1st order (Eqn12) practically coincided. Minimal differences were observed in the area of maximum D_p/B values. However, testing this discrepancy on quail eggs did not confirm any difference in ε when calculating V according to Eqn7 and Eqn10 for eggs with higher D_p/B ratios. Some, though not significant, increase in ε values for chicken V was observed using Eqn10 as compared to Eqn7. Thus, we could safely recommend both formulae (Eqn7 and Eqn10) for use with some minimal advantage of Eqn7.

4.2. Egg Surface Area

It was not practicable to carry out a fully-fledged analysis demonstrating the adequacy of the inferred theoretical dependence (8) for calculating the S value in the same way as it was done for calculating V. This was due to the lack of accurate methods for measuring S directly. At one time, this parameter was determined by pasting the egg surface with adhesive tape, after which its area was measured (Murray, 1925). However, this method, in addition to its complexity, is also extremely inaccurate. A big hope was to implement the S measurement by using digital imaging technology. However, our previous digital imaging-assisted studies (Narushin et al., 2020a, 2021b,c) showed that the results produced were far from highly accurate. This conclusion was made after comparing the results obtained on ellipsoid eggs. For the geometric shape of an ellipsoid, there is a standard formula for calculating S (e.g., Tee, 2004), the correctness of which was beyond doubt. Its version, adapted by us (Narushin et al., 2020a) for the egg parameters, had the following form:

$$S_{el1} = \frac{\pi LB}{2}\left(\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}} + \frac{B}{L}\right), \qquad (13)$$

where S_{el1} is the surface area of an ellipsoid (in cm^2), L is its length (in cm), and B is its breadth (in cm).

Although the formula (8) was also derived by us based on the methods of classical integral geometry, it would be extremely premature to accept it unconditionally as a reference determination of S for any egg. In this regard, we attempted different approaches to find an alternative.

4.3. Comparison of *S* Calculation Results for Ellipsoids

Since there is a reference calculation for ellipsoids (Eqn13), we assumed that there should be a full concordance of the results obtained when using Eqn8 for strictly ellipsoidal eggs. This was easy enough to check, since the w/L value for the ellipse is zero, and the D_p/B ratio was quite simple to calculate after substituting $x = L/4$ into the classical ellipse formula, which led to $D_p/B = 0.866$ (Narushin et al., 2022a). Substituting these values into Eqn8, we were able to calculate S of ellipsoidal eggs that we denoted as S_{el2}:

$$S_{el2} = \pi BL\left(0.705 + 0.287\frac{B}{L}\right). \qquad (14)$$

Comparing Eqns 13 and 14, we were able to identify coefficients K_{S1} and K_{S2} for a product of B and L that can be written as follows:

$$K_{S1} = \frac{\pi}{2}\left(\frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}} + \frac{B}{L}\right), \qquad (15)$$

$$K_{S2} = \pi\left(0.705 + 0.287\frac{B}{L}\right). \qquad (16)$$

Knowing that the variability of the B/L ratio for eggs of the avian realm ranges between 0.5 and 1, as we repeatedly cited for similar calculations (e.g., Narushin et al., 2021d), we substituted these values into both Eqns 15 and 16, resulting in the respective graphic dependencies (Fig. 3).

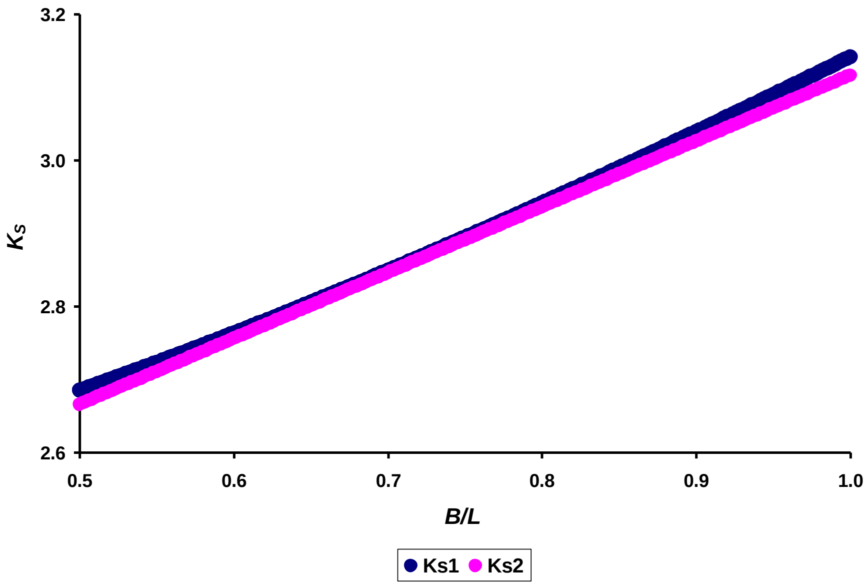

Fig. 3. Visualization of the comparative analysis of the data obtained as a result of calculating the coefficients K_{S1} and K_{S2} respectively for Eqn15 (blue line) and Eqn16 (pink line).

The values of coefficients K_{S1} and K_{S2} practically coincided (Fig. 3), at least in the area of B/L = [0.6…0.86] that conformed to the largest group of bird eggs. The error in calculating the coefficient K_{S2} relative to K_{S1} was 0.33 ± 0.20%, suggesting the adequacy of formula (8) for this stage of its testing.

4.4. Using Topology Methods When Calculating *S*

The use of topological principles (e.g., Gamelin and Greene, 1999) was actively employed by us in previous egg studies for computing *S* (Narushin, 1993, 1997b, 2001, Narushin et al., 2020a). In accordance with this technique, the egg is conditionally transformed into a certain standard geometric figure of the same volume, for which the standard formulae for calculating *S* are known. As such figures, in relation to chicken eggs of a classical oval shape, Narushin et al. (2020a) explored several models, the most accurate of which turned out to be an ellipse. In this case, the *S* value was calculated based on the formula for ellipsoids (Narushin et al., 2020a) as follows:

$$S = 2.418\left(\frac{B}{L}\right)^{\frac{2}{3}} \cdot \left(\frac{L}{B} \cdot \frac{\arcsin\sqrt{1-\frac{B^2}{L^2}}}{\sqrt{1-\frac{B^2}{L^2}}} + 1 \right) \cdot V^{\frac{2}{3}}, \tag{17}$$

where *S* is the egg surface area (in cm^2), *V* is its volume (in cm^3), *L* is its length (in cm), and *B* is its maximum breadth (in cm).

For eggs with a more conical shape, it was proposed to use the previous Narushin's model (Narushin, 1997b), according to which we presented the derived formulae for calculating V and S (Narushin et al., 2022c) as follows:

$$V = \frac{4.44}{9.37 - \left(\frac{B}{L}\right)^2} LB^2 , \qquad (18)$$

$$S = \frac{0.44\left(\left(\frac{L}{B}\right)^2 + 8.49\right)}{\left(\frac{L}{B}\right)^2 + 0.35} L^2 . \qquad (19)$$

A series of mathematical transformations was executed for equations (18) and (19), the detailed course of which is presented in Supplementary Data B. This enabled us to reveal the following relationship between these two indicators:

$$S = \frac{0.163\left(1 + 8.49\left(\frac{B}{L}\right)^2\right)\left(9.37 - \left(\frac{B}{L}\right)^2\right)^{\frac{2}{3}}}{\left(\frac{B}{L}\right)^{\frac{4}{3}}\left(1 + 0.35\left(\frac{B}{L}\right)^2\right)} V^{\frac{2}{3}} \qquad (20)$$

If, as we noted above, the D_p/B ratio for ellipsoid eggs was around 0.866, then, it can also be obtained for the Narushin's model (Narushin, 2001) after appropriate substitution of $x = L/4$ (Fig. 4) into the basic equation:

$$y = \pm\sqrt{L^{\frac{2}{n+1}} x^{\frac{2n}{n+1}} - x^2} \ , \qquad (21)$$

where the n value is calculated depending on the B/L ratio (see more details in Supplementary Data C).

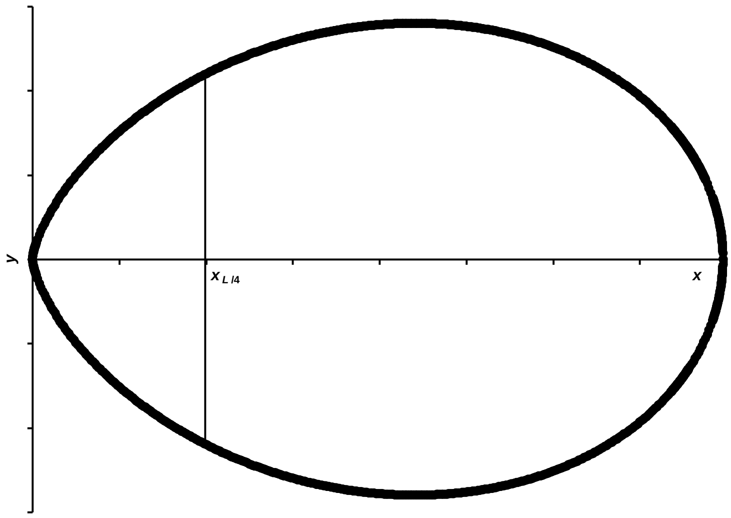

Fig. 4. Geometrical interpretation of the Narushin's model (Eqn21).

As a result of the appropriate mathematical transformations, the detailed course of which is presented in Supplementary Data C, the following expression was produced for computing D_p/B:

$$\frac{D_p}{B} = 0.136\left(\frac{B}{L}\right)^2 + 0.071\frac{B}{L} + 0.667\,. \tag{22}$$

Recalculation according to formula (22), by substituting the B/L data obtained in the experimental sample of quail eggs in the range [0.74…0.83], demonstrated that the D_p/B values were in the interval [0.79…0.82] and clearly did not reach ellipsoid shape, for which D_p/B should approach 0.866. Thus, the best option for comparing the adequacy of the resulting Eqn8 will be the topological recalculation of the S value using the Narushin's model (Eqn20). Comparison of the results of computing the S value according to Eqn8 with the data obtained from Eqn20 showed an average error of 1.0% that was an excellent result suitable both for analytical and, moreover, for industrial applications.

Since we failed to find classic oval shapes among quail eggs that were more akin to an ellipsoid shape, we used the experimental data obtained on chicken eggs again (Narushin et al, 2020a). The variability of D_p/B of eggs of this species was 0.82 to 0.86, which made it possible to use the topological method of recalculating S according to Eqn17. Comparison of the results of calculating the S value using Eqn8 with the data obtained according to Eqn17 led to an average error of 1.2% that also fully satisfied the accuracy requirements.

The resulting errors might be completely due to the calculation formulae (17) and (20) that were used as references. In any case, Eqn8 can be recommended as a baseline S calculation for eggs of all shapes and sizes. An analysis of Eqn8 demonstrated the feasibility of estimating another index, D_p/L, which, in addition to a set of B/L, w/L and D_p/B ratios, also had a direct effect on S.

When analyzing Eqn8, the topological approach to recalculate the S value using the V value measured by the Archimedes' principle was the most effective method that can be conditionally attributed to a category of direct (or "almost direct") measurements. The known geometric figures used in this case (ellipsoid and Narushin's model) were not universal for all egg shape that exist in nature, while the derived calculation formulae for V (Eqn7) and S (Eqn8) met the universality criterion. Hence, we decided to use Eqns 7 and 8 to generate a reference computation algorithm for S of any egg. Detailed mathematical transformations of equations Eqns 7 and 8 are presented in Supplementary Data D, which resulted in an equation that reflected the relationship of the two main characteristics of the egg, i.e., S and V, as follows:

$$S = \left[6.438 - 2.666\frac{B}{L} + 1.867\frac{w}{L} - 0.44\frac{D_p}{B} - 0.134\frac{D_p}{L} - 0.683\frac{B}{L}\cdot\frac{w}{L} - 2.578\frac{w}{L}\cdot\frac{D_p}{B} + 1.29\frac{w}{L}\cdot\frac{D_p}{L} + 1.369\left(\frac{B}{L}\right)^2 + 0.336\left(\frac{w}{L}\right)^2 + 0.233\left(\frac{D_p}{B}\right)^2\right]V^{\frac{2}{3}} \qquad (23)$$

The resulting function can be considered a kind of standard that allows one to determine the true S values based on the results of measuring the V values for the eggs under study.

Within the framework of the data produced by measuring the quail eggs, we discovered that there was the error of $0.53 \pm 0.15\%$ between the calculation results according to Eqn23 and Eqn17, i.e., when an egg topologically converted into an ellipsoid. While comparing the obtained reference S calculation with the figure of rotation built on the basis of Narushin's model (Eqn20), we found the value $\varepsilon = 0.29 \pm 0.12\%$. The appropriate visualization of these differences is provided in Fig. 5.

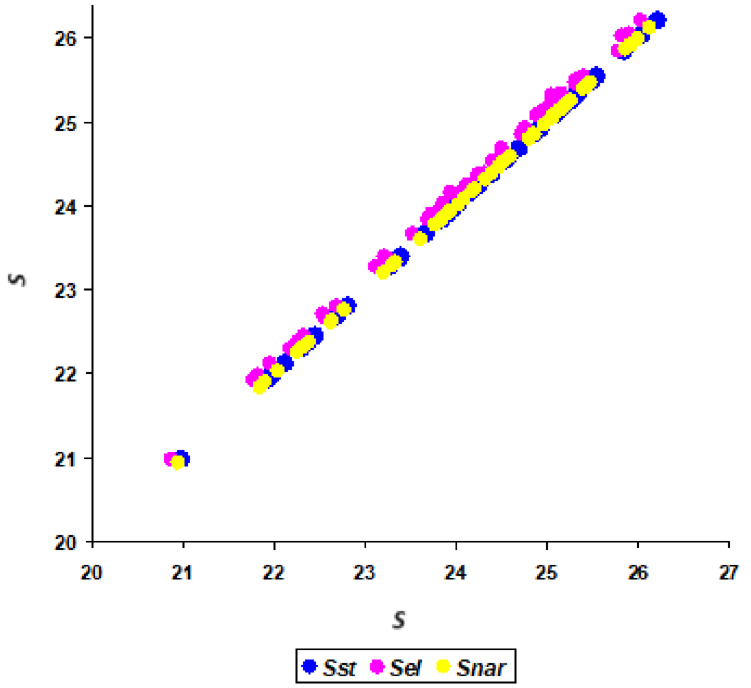

Fig. 5. Comparative visualization of the data resulted from the S calculation for quail eggs using Eqn23 (S_{st}, blue line), Eqn17 (S_{el}, pink line), and Eqn20 (S_{nar}, yellow line).

5. Conclusions

Our findings provide an innovative approach for non-destructive egg quality assessment, whereas the subsection of practical egg geometry was enriched with two more novel formulae for defining V and S of the egg ovoid. At the same time, the proposed computation algorithm was adequate for any nuances of the egg shape in quail and other birds, from spherical to conical (pyriform). Moreover, taking into account the characteristic features of the egg profile for birds of different species, the derived dependencies can be significantly simplified. To do this, it suffices to determine the average values of two characteristic indices, i.e., the ratios w/L and D_p/B, the inheritance of which is one of the unique features that distinguish one avian species from another. In this case, it remains only to measure the two main parameters of the egg, L and B.

Also, we believe that we finally developed a computation method (Eqn8) that, along with the Archimedes' principle for determining V accurately, can be used as a reference for an adequate S assessment of eggs. The need to measure four parameters to determine S might cause some inconvenience in practical applications. However, the results of its use can serve as a standard when compared with other calculation models.

CRediT authorship contribution statement

Valeriy G. Narushin: conceptualization, data curation, formal analysis, investigation, methodology, resources, software, visualization, writing – original draft, writing – review & editing. **Natalia A. Volkova**: data curation, funding acquisition, investigation, methodology, project administration, supervision, validation, writing – review & editing. **Anastasia N. Vetokh**: investigation, validation. **Alan Yu. Dzhagaev**:

investigation, validation, visualization. **Danila A. Sotnikov**: investigation. **Ludmila A. Volkova**: investigation, validation. **Stefan T. Orszulik**: formal analysis, methodology, software. **Darren K. Griffin**: supervision, writing – review & editing. **Michael N. Romanov**: project administration, validation, visualization, writing – original draft, writing – review & editing. **Natalia A. Zinovieva**: funding acquisition, project administration, supervision, writing – review & editing.

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Data availability

Data will be made available upon request.

Declarations

Human and Animal Rights

These experiments comply with the ARRIVE guidelines and were carried out in accordance with the U.K. Animals (Scientific Procedures) Act, 1986 and associated guidelines, EU Directive 2010/63/EU for animal experiments, and the authors verify that such guidelines have been followed.

Competing Interests

The authors declare no conflicts of interest.

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Not applicable.

Appendix A. Supplementary information

Supplementary data associated with this article can be found in the online version at

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