TOWARDS FACILITATED OPTIMISATION

BY

Sheema Noorain

Kent Business School
Department Of Analytics, Operations and Systems

Supervisors:
Professor Maria Paola Scaparra
and
Professor Kathy Kotiadis

ABSTRACT

Optimisation modelling in healthcare has addressed a diverse range of challenges inherent to decision-making and supports decision-makers in determining the best solution under a variety of constraints. In contrast, optimisation models addressing planning and service delivery issues in mental healthcare have received limited attention. Mental healthcare services in England are routinely facing issues relative to scarcity of available resources, inequities in their distribution, and inefficiencies in their use. Optimisation modelling has the potential to support decision making and inform the efficient utilisation of scare resources. Mental healthcare services are a combination of several subsystems and partnerships comprising of numerous stakeholders with a diversity of interests. However, in optimisation literature, the lack of stakeholder involvement in the development process of optimisation models is increasingly identified as a missed opportunity impacting the practical applicability of the models and their results. This thesis argues that simulation modelling literature offers alternative modelling approaches that can be adapted to optimisation modelling to address the shortcoming highlighted. In this study, we adapt PartiSim, a multi-methodology framework to support facilitated simulation modelling in healthcare, towards facilitated optimisation modelling and test it using a real case study in mental healthcare. The case study is concerned with a Primary Care Mental Healthcare (PCMH) service that deploys clinicians with different skills to several General Practice (GP) clinics. The service wanted support to help satisfy increasing demand for appointments and explore the possibility of expanding their workforce.

This research puts forward a novel multimethodology framework for participatory optimisation, called PartiOpt. It explores the adaptation and customisation of the and PartiSim framework at each stage of the optimisation modelling lifecycle. The research demonstrates the applicability and relevance of a ‘conceptual model’ to optimisation modelling, highlighting the potential of facilitated optimisation as a methodology. This thesis argues for the inclusion of conceptual modelling in optimisation when dealing with real world practice-based problems. The thesis proposes an analytics-driven optimisation approach that integrates descriptive, predictive, and prescriptive analytics stages. This approach is utilised to construct a novel multi-skill multi-location optimisation model. By applying the analytics-driven optimisation approach to the case study, previously untapped resource potential is uncovered, leading to the identification of various strategies to improving service efficiency. The successful conceptualisation of an optimisation model and the quantitative decision support requirements that emerged in the initial stages of the study drive the

analytics-driven optimisation. Additionally, this research also presents a facilitative approach for stakeholder participation in the validation, experimentation, and implementation of a mathematical optimisation model. Reflecting on the adaptation and subsequent amendments to the modelling stages, the final PartiOpt framework is proposed. It is argued that this framework could reduce the gap between theory and practice for optimisation modelling and offers guidance to optimisation modellers on involving stakeholders in addressing real world problems.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

To Salma.

For ceaselessly guiding me home.

ACKNOWLEDGEMENTS

I must start with an emphatic and immense thank you to my PhD supervisors: Maria Paola Scaparra and Kathy Kotiadis. The support, guidance, and patience you have extended over the past five years have been immeasurable. It has been an absolute honour to have you play a crucial role in shaping my journey. As strong and accomplished women in your respective fields, you serve as a tremendous source of inspiration. While the thesis undoubtedly reflects the outcome of our collaboration, I have also gained invaluable life lessons from both of you that cannot be framed in tangible terms. I would like to extend a heartfelt thank you to Paola for making the collaboration with the NHS a reality. I would also like to express my deep appreciation to Kathy, whose work serves as the foundation for this thesis. I sincerely hope that I have honoured her contributions to the best of my abilities.

I would like to express my gratitude to the University of Kent for awarding me the Vice-Chancellor’s Research Scholarship and a Postgraduate Research Bursary, which provided invaluable support. I extend my thanks to the Kent and Medway NHS and Social Care Partnership Trust for their keen interest and involvement, and for supporting this endeavour. A special appreciation to the PGR team at the Kent Business School for their unwavering support and goodwill. I am also thankful to the Department of Analytics, Operations and Systems for nurturing my academic aspirations over the past seven years.

I am also grateful to the members of my cohort who embarked on their own individual journeys alongside me. A special thank you goes to Betul, Sim, Idorenyin, and Charles for their friendship, selfless assistance, feedback, and ever forthcoming moral support. It has been an absolute delight witnessing each of your accomplishments. Thanks also to my dearest friend Matthew, whose positive and enriching presence has a profound impact on my life.

Lastly, I express my sincerest gratitude and appreciation to my beloved Ammi and Abba. I stand on the foundations laid by you and I face each day in this world, knowing that you have my back. You hold a special place in my heart and thank you for always doing your best. I love you to Jannah and back. Thank you also to my cherished brother. Your profound wisdom and gentle nature have been a constant source of protection for me. I love you dearly and take great pride in you.

DECLARATION

Chapters 2 to 6 are designed as independent papers and along with the rest of the chapters, they collectively form the entirety of the thesis. Consequently, there may be occasional overlaps in the content within these chapters. Chapters 2 and 3 have already been published, while Chapters 4, 5 and 6 delve into a focused subject matter for a single case study.

Journal Publications:

Noorain, S., Paola Scaparra, M. and Kotiadis, K., 2022. Mind the gap: a review of optimisation in mental healthcare service delivery. *Health Systems*, pp.1-34.

Conference Proceedings:

Noorain S, Kotiadis K, and Scaparra M.P. 2019. "Application of Discrete-Event Simulation for Planning and Operations Issues in Mental Healthcare". In Proceedings of *the 2019 Winter Simulation Conference*.

Chapter 1: Thesis Overview

1.1. Introduction

Mental illness significantly impacts individuals, society, and the economy. Before COVID-19, mental health services in England were already under considerable strain and facing issues of inadequate resourcing, access to care and overall patient outcomes (British Medical Association, 2020a). Many of these issues have worsened due to the pandemic (British Medical Association, 2020b). The pandemic has greatly affected the mental health and overall well-being of individuals, posing a significant and enduring public health concern (McCartan et al., 2021; Pierce et al., 2021). Recent data shows that the number of people contacting the National Health Service (NHS) seeking help for mental health problems is now at a record high (NHS Digital, 2022). These needs arise within the context of underfunded mental health services facing a care backlog, waiting lists, and a stretched, exhausted, and understaffed workforce. Workforce capacity has been a long-term concern, and shortages represent the biggest threat to national ambitions to improve mental healthcare (HM Government, 2021; NHS Confederation, 2022). Primary care, now at the forefront of the predicted increase in mental health presentations, plays a vital role in early intervention, reducing subsequent mental health problems, and proving to be a cost-effective approach (Park et al., 2020; Van't Veer-Tazelaar et al., 2010).

Mental healthcare services primarily rely on their human workforce, which is a mix of collaboration between psychosocial and biomedical providers with varying levels of skills, spread across numerous locations. The system is a combination of several subsystems and partnerships comprising of numerous stakeholders with a diversity of interests. Scarcity of available resources, inequities in their distribution, and inefficiencies in their use, are the three major obstacles to better mental healthcare (British Medical Association, 2017; Carbonell et al., 2020; Gask, 2005). Effective deployment of existing services has been recognised as having the potential to close this gap and reduce unmet need (British Medical Association, 2020a; Kakuma et al., 2011).

Optimisation modelling in healthcare has been used to address a diverse range of challenges inherent to decision-making in healthcare (Capan et al., 2017). Optimisation models can support decision-makers in determining the best solution under a variety of constraints, often by simultaneously considering multiple factors (Earnshaw & Dennett, 2003; Tüzün & Topcu, 2018). Optimisation models have been employed in emergency room planning, primary, outpatient and home health to address issues such as determine resource quantity to meet demand, determining size and composition of staff, creating shift rosters, allocating appointments, assigning care workers to patients, and scheduling patient visits (Cissé et al., 2017; Grieco et al., 2021; Leeftink et al., 2020; Marynissen & Demeulemeester, 2019; Samudra et al., 2016; Zhu et al., 2019). In comparison, the application of optimisation modelling to address modelling service delivery issues in mental healthcare is an area of research that has received limited attention (Bradley et al., 2017; Long & Meadows, 2018; Noorain et al., 2022). In mental healthcare, optimisation modelling has the potential to support decision making and inform the efficient utilisation of scare resources, but there is a lack of research that sufficiently captures the complexities of the system.

In traditional optimisation modelling, the lack of stakeholder involvement leads to missed opportunities throughout the modelling life cycle, starting with inadequate primary data collection, leading to the development of a realistic model, as opposed to a real case study, culminating in a lack of real implementation of the models (Amideo et al., 2019; Çoban et al., 2021). In healthcare, building the model in isolation along with the use of opaque model design approaches is found to be associated with stakeholder resistance to trust the outcomes and implement the findings (Carter & Busby, 2022). Recently, case studies of optimisation modelling with stakeholder participation (Abuabara et al., 2022) along with the use of Problem Structuring Methods (PSMs) (Cardoso-Grilo et al., 2019) for conceptual modelling and scenario generation (Amorim-Lopes et al., 2021) utilising the multimethodology approach have emerged. In comparison, simulation modelling has developed several approaches that involve clients in the modelling process through facilitated simulation (Lane et al., 2019; Proudlove et al., 2017; Robinson et al., 2012; Robinson et al., 2014; Tako et al., 2019; Tako et al., 2021; Willis et al., 2018). Particularly relevant to this study is the Participative Simulation (PartiSim) multimethodology framework that uses Soft Systems Methodology (SSM) to engage stakeholders in developing a Discrete-event Simulation (DES) model through facilitated workshops (Kotiadis et al., 2014; Kotiadis & Tako, 2018; Tako & Kotiadis, 2015). In facilitated simulation modelling, involving stakeholder groups is associated with improved information flow, better model quality, and acceptance and implementation of improvements identified in the study (Pessôa et al., 2015; Robinson et al., 2014). To the best of our knowledge, a facilitated modelling approach to support optimisation modelling does not exist. However, a facilitative, and participative framework such as PartiSim can be adapted to support the development of optimisation models in contexts where involving stakeholders is necessary.

The work presented here introduces a novel multi-methodology framework by adapting PartiSim towards facilitated optimisation modelling. The proposed multi-methodology framework is termed PartiOpt and provides a structured process for developing optimisation models by involving stakeholders through facilitated workshops. The study also required complementing optimisation with other analytics tools. The framework was developed in collaboration with a real-world Primary Mental Healthcare (PCMH) service provided by the Kent and Medway Mental Healthcare Trust (KMPT) based in Kent, UK. The service works alongside General Practice (GP) clinics and primary care partners and interfaces with KMPT services to provide care to people experiencing mild/moderate mental health conditions who do not require secondary care mental health services. At the time of the study, several change imperatives had highlighted the need for integrating mental health services into primary healthcare to foster closer integration of primary, secondary, and tertiary mental healthcare service, and improve patient access to services (NHS England, 2020). The stakeholders involved in the study included individuals from various levels in the organisation, such as local commissioners, trust executives, service managers, and clinicians. Given that the service had begun as an ad-hoc experiment, and evolved on a need basis, it did not have an overall operational design, or in other words, a narrative with consensus. Stakeholders were keen to evaluate the performance of the service and consider opportunities for improvement. The evaluation was prompted by a county level commissioning decision to increase funding for the service to enable KMPT (the providers) to hire more clinicians, based on population level demand forecasts. In light of this impending expansion, stakeholders sough decision support to determine 'how' the service has been functioning thus far, 'what' if any can be improved, and the direction of improvement, if any.

The proposed multi-methodology contributes a structured framework by drawing on existing research, thereby addressing a key gap in optimisation literature. Specifically, the participative and facilitative multi-methodology framework developed in this research, will adapt, and extend the well-established PartiSim framework for the optimisation modelling lifecycle. The adaptation will introduce new tools and alter existing tools to fit the requirements of an optimisation model. From a methodological perspective to the best of the researcher's knowledge, a facilitated optimisation multi-methodology framework covering the entire modelling lifecycle does not exist. Additionally, a comprehensive case study chronicling the application of multiple OR techniques in mental healthcare is a significant contribution of this study.

1.2. Aims and Objectives

The overarching intention of this research is to introduce a structured framework that could aid optimisation modellers to consider all the key steps in the development of models. Additionally, the approach will allow stakeholders to actively engage in the model development process. This intention is further elaborated into aims and objectives. The aims are:

1) To develop a comprehensive understanding of literature on the application of Operations Research (OR) methodologies in mental healthcare, with the objective of adapting and extending the PartiSim multi-methodology framework for optimisation modelling.
2) To develop a facilitative multi-methodology framework that provides a pathway for developing optimisation models with stakeholder participation, particularly in the context of mental healthcare.

Towards these aims, the following five objectives will be met. It is expected that by meeting these five objectives, the aims of this thesis will be realised.

Objective 1: To investigate existing knowledge on the mental healthcare services and identify characteristic of the system that are relevant to the study. (In relation to aim 1).

The mental healthcare system will be thoroughly investigated to identify features that are unique to the system as well as those that are comparable to other healthcare systems. This task will help contextualise the mental healthcare landscape, predominantly in relation to the UK. The findings will also be influential in engaging with stakeholders in the case study; identifying the social, clinical, political, and policy implications within the system; and classifying existing literature.

Objective 2: To investigate the application of simulation and optimisation modelling techniques in mental healthcare to identify gaps and opportunities. (In relation to aim 1).

An extensive review on the application of simulation and optimisation modelling in mental healthcare is conducted. Since the study adapts the PartiSim framework, a review of Discrete-Event simulation modelling is conducted to identify the extent of its application and gauge the potential for including simulation modelling techniques as an adjunct to the optimisation model. Additionally, a detailed review of optimisation techniques in healthcare is conducted to identify the scope and depth of existing research. This is followed by a review on the application of optimisation modelling specifically to mental healthcare. A comparative analysis is carried out to locate opportunities for transferability while highlighting differences between the systems warranting special attention. Literature review on the two modelling techniques also helps recognise similarities and differences between the two modelling lifecycles, aiding the adaptation of PartiSim to optimisation.

Objective 3: To iteratively adapt PartiSim for optimisation modelling through a case study in mental healthcare services. (In relation to aim 2).

The PartiSim framework involves six stages, and each stage is considered individually for adaptation. The study begins by considering conceptual modelling for formulating an optimisation model. New participative tools are developed to enable problem definition in stakeholder workshops. The mapping of these new developments along with the utilisation of existing PartiSim tools are demonstrated and discussed. Following model development, the experimentation and implementation stages are also adapted for optimisation. New modes of conducting workshops are considered while also exploring new implementation tools.

Objective 4: To investigate and identify OR/analytics techniques to support the development of an optimisation model for mental healthcare service provision, to be embedded within the overarching participative and facilitative multi-methodology framework. (In relation to aim 2).

This objective covers the model coding stages in the framework. OR/analytics techniques are considered to support the development of an optimisation model. Using the conceptual model of the system, an optimisation model is developed and potentially mapped to an existing model in literature. Optimisation model being a prescriptive analytics technique, the study investigates the feasibility of employing descriptive and predictive analytics techniques in a multi-methodology framework, thereby creating a multi-methodology within the overarching multi-methodology. The utility, composition and choice of the OR/analytics techniques is determined by the outputs of the problem structuring/conceptual modelling stages of the framework.

Objective 5: To propose a Participative Optimisation (PartiOpt) multi-methodology framework for developing optimisation models (In relation to aim 2).

To realise this objective, the adapted framework is proposed based on the knowledge gained from its application to the mental healthcare system. The final framework will be refined and requisite stages along with respective tools will be described. Reflections on the proposed framework will highlight limitations and areas of further work. The impact of the case study will also be examined.

1.3. Research Methodology

This thesis adopts a multi-paradigm, multi-methodology process, which can bridge 'soft' and 'hard' OR practices. Mingers and Rosenhead (2004) define a multimethodology as *"In use, multimethodology is a creative process of design, based on competence in a range of methods. Each project or intervention is seen as a unique situation…for which a particular combination of methods, or parts of methods, needs to be constructed. This is an on-going process throughout the project, as events occur, and the situation evolves".* Multimethodologies are seen as a possible means to facilitate rapid problem structuring, the analysis of alternative process design and then the specification through to implementation of systems solutions (Small & Wainwright, 2014). In OR, mixing and employing multiple methodologies have been deployed consistently in interventions (Gomes Júnior & Schramm, 2021; Henao & Franco, 2016; Yearworth & White, 2013).

Multimethodology is supported by critical realism, a theoretical framework that facilitates the use of multiple research methods from different methodical approaches (Mingers, 2001a; Mingers, 2004). Critical Realism introduces a more nuanced version of a realist ontology by offering a pathway between positivism on the one hand and interpretivism on the other (Archer et al., 2013; Bhaskar, 2010). From a critical realist perspective, most important is how quantitative and qualitative methods are used (Pratschke, 2003). In this view, the strength of quantitative techniques is that they can be used to develop reliable descriptions and provide accurate comparisons. Specifically, in the exploratory phase of an intervention, quantitative techniques can identify patterns and associations that may otherwise be masked. This may help to tease out new and unexpected causal mechanisms and can also be used to test out theories about how causal mechanisms operate under particular sets of conditions (Mingers, 2004). The key strength of qualitative techniques, from a critical realist perspective, is that they are open ended. This may allow themes to emerge during an inquiry that could not have been anticipated in advance. Qualitative techniques can help to illuminate complex concepts and relationships that are unlikely to be captured by predetermined response categories or standardised quantitative measures.

The critical realist approach is highly compatible with a case study research strategy. In literature, a critical realist approach to a case study involves developing research question or questions that identifying a research phenomenon of interest, in terms of discernible events, and asking what causes them to happen (Easton, 2010). The case study then involves provisionally identifying entities involves, their powers, liabilities, and relationships. Research then proceeds by capturing data with respect to ongoing or past events, always asking why they happened or are happening and considering the problems and issues associated with interpreting the empirical data back to the real entities and their actions (Yin, 2009). Hence, the research process is one of continuous cycles of research and reflection, or of moving back and forth between the diverse stages of the research project. (Verschuren, 2003). The result is the identification of one or more mechanisms that can be regarded as having caused the events.

This study adopts the facilitated mode of conducting the OR intervention. This is a process by which OR interventions are conducted jointly with clients in a facilitated mode (Franco & Montibeller, 2010). This choice has been made based on the nature of the problem situation and as facilitated mode is particularly suited for evaluating strategic decision options such as the impact of enhancing the primary care mental health service. Furthermore, the PartiSim framework that is being adapted in this study is a multimethodology framework for building Facilitated DES models. Therefore, the proposed research methodology matches that of PartiSim.

The methodology described above is best suited to this study, particularly as a key aim of this research is to explore theory (combining Soft and Hard OR techniques to develop a multi-methodology) in relation to practice. For the case study, this involves evaluating the impact of the primary care mental health team on bridging the primary and secondary care, analysing the operational efficiency of the service, and evaluating alternative futures for the service. The mental health setting of this research provides rich data about the generalisability and transferability of the proposed framework for planning mental health service delivery.

The PartiOpt multimethodology will be proposed by following an iterative adaptation process, starting with a reflective consideration of the transferability of stages, activities, and tools from PartiSim for optimisation. After identifying appropriate components from PartiSim, necessary modifications and additions for PartiOpt will be determined. Subsequently, an initial PartiOpt multimethodology will be developed for each stage of the modelling lifecycle. This initial multimethodology will then be applied to the case study. Feedback and reflections from the case study will guide further adjustments to the multimethodology, thereby leading to the incorporation of knowledge gained from the application. Finally, based on the insights that will be acquired, modifications to refine the multimethodology will be proposed.

1.4. Thesis Structure

This section describes the contents of each chapter of the thesis. Chapter's 2 to 6 are research papers. Specifically, Chapters 2 and 3 are published, Chapter 4 is set to be submitted and Chapter 5 and 6 are in view for submission. Essentially, Chapters 2-6 are structured as a stand-alone paper and together with the remaining chapters, they make up the thesis. The stand-alone chapters have their own introduction and literature review section that is tailored to a focused subject matter. The following is a description of each chapter.

Chapter Two: Literature Review - Discrete-Event Simulation in Mental Healthcare

This chapter presents a critical review of simulation modelling, specifically Discrete-Event Simulation, in mental healthcare. It begins by providing background on mental healthcare services, followed by a literature review on the application of simulation techniques to healthcare. The chapter describes the search strategy and selection criteria for the review. The selected articles are then categorised based on the publication characteristics, simulation study's objectives, modelling scope, model type, stakeholder engagement and implementation. The discussion focuses on gaps in literature relative to operational efficiency, stakeholder engagement, and methodological pluralism, followed by a conclusion. This chapter contributes to the first two objectives.

Chapter Three: Literature Review - Optimisation Modelling in Mental Healthcare

This chapter present a meta-analysis of optimisation literature in healthcare, followed by a critical review of optimisation modelling in mental healthcare. The chapter begins with a discussion on the history of optimisation modelling in healthcare in terms of problems addressed as categorised by planning levels, model types, problem types such as models that address planning, scheduling, routing, and supply chain management issues. The chapter then goes on to discuss mental healthcare services in terms of features such as care setting, uncertainty, risks and considers the difference between physical and mental health. The review of optimisation in mental healthcare is initiated with a discussion of the method adopted for review, followed by a description of results. The chosen articles are categorised based on planning level and planning decisions addressed, the type of care setting, model objectives and constraints, model formulation, and solution algorithms employed. The chapter ends with a discussion on gaps and opportunities identified through the review. In particular, the discussion covers the importance of incorporating uncertainty and risk in mental health optimisation models, highlights the need for models to address timely access and continuity of care in services. The discussion also considers the challenge of developing models for multi-layered mental healthcare systems and identifies opportunities for developing new modelling and solution methodologies to address the challenges of mental healthcare delivery. Managerial insights are then highlighted followed by the conclusion.

Chapter Four: Conceptual Modelling for Optimisation

This chapter illustrates the conceptual modelling of the optimisation model for the mental healthcare case study. The chapter begins by introducing the notion of a 'conceptual model' and highlights its relevance to the case study and to optimisation modelling. The literature review section covers several intersecting topics in OR literature that are relevant to the proposed framework. In particular, the section discusses relevant literature on facilitated modelling in OR, outlines the conceptual modelling stages on the PartiSim framework, examines facilitation, participation and multimethodology in optimisation modelling, and finally investigates the application of the above in mental healthcare modelling. The literature review provides essential background information and rationale for the proposed framework, specifically for the initial stages of problem structuring and conceptual modelling for optimisation. The chapter then continues by describing specific aspects of the case study that are under consideration and provides details of the first three stages of the framework corresponding to problem structuring and conceptual modelling. The next section describes how the framework was applied in the case study, starting from the initial exploration of the problem situation to the mapping of workshop outputs to the objectives of the study, and to conceptualise an optimisation model. In the discussion section, a case for conceptual modelling in optimisation is made by drawing from historical OR practices. The section includes an examination of how facilitated optimisation can be a viable methodology, and reflections on the proposed framework are followed by the conclusion.

Chapter Five: Analytics-Driven Optimisation Modelling

This chapter proposes an analytics-driven optimisation approach that integrates the three stages of descriptive, predictive, and prescriptive analytics. The approach was developed following a successfully conceptualisation optimisation model and driven by the quantitative decision support requirements that emerged in the initial stages of the study. The chapter begins by making a case for the development of an analytics-driven optimisation modelling approach to address the full range of quantitative analysis needs of the mental healthcare service under consideration. The literature review section briefly reflects on optimisation in mental healthcare, followed by an examination of literature on personnel scheduling in healthcare, with an emphasis on multi-skill multi-location personnel scheduling which is the type of model built for the case study. The section ends with an examination of analytics driven approaches to optimisation modelling in healthcare. The next section provides background on the case study and a problem statement for the integrated optimisation approach. An overview of the proposed approach is presented next followed by a discussion on the outputs of each analytics stage within the approach: descriptive, predictive, and prescriptive. The next section presents the mathematical formulation of the optimisation problem along with the notation used. Experiments with the model in terms of scenario analysis are highlighted in the next section, which also includes details on how and which data was used in each scenario. Computational results of the model with respect to scenarios are provided in the next section. The discussion focuses on the contributions of the integrated approach and future research directions. The chapter ends with some conclusive remarks.

Chapter Six: How do stakeholders interact with optimisation models?

This chapter explores stakeholder engagement and interaction with the solved model. Specifically, the chapter presents the adaptation of the post-model coding stages of the PartiSim framework for optimisation modelling in healthcare. A facilitative approach for stakeholder participation, focusing on the validation, experimentation, and implementation of a mathematical optimisation model is then derived and its effectiveness demonstrated using a real case study in mental healthcare. The chapter begins by introducing limitations, gaps and opportunities in optimisation model validation and implementation, while also highlighting how simulation modelling has successfully addressed these challenges. Proposal to address these gaps by drawing from existing research is outlined and specific contributions are outlined. The next section provides background literature by first presenting the Post-Model Coding stages in PartiSim including experimentation and implementation of the model. Next, literature on optimisation model validation, experimentation, and implementation is explored by examining the present state of each theme relative to the use of Facilitation and Soft OR methodologies to these corresponding themes. The aim is to identify developments, gaps and recognise opportunities, while foregrounding the adaptation of the PartiSim framework. An overview on the adaptation to optimisation is provided in the next section followed by a description of how the framework was applied to the case study. The framework is then proposed in the next section, along with a reflection on the adaptations, and examination of the footprint of conducting facilitated workshop virtually. The chapter ends with some conclusive remarks.

Chapter Seven: Summary and Conclusion

This chapter provides a summary of the research and discusses its contribution. This chapter highlights how the research aim and objectives have been achieved. It concludes with a reflection on the overall adaptation of PartiSim to optimisation and the researcher's evolution over the course of the study. Limitations and key future research areas are also discussed.

Chapter 2: Application of Discrete-Event simulation for planning and operations issues in mental healthcare

Abstract

Mental health disorders are on the rise around the world. Inadequate service provision and lack of access have led to wide gaps between the need for treatment and service delivery. Despite the popularity of Discrete-event Simulation (DES) in healthcare planning and operations, there is evidence of limited application of DES in planning for mental healthcare services. This paper identifies and reviews all the papers that utilize DES modelling to address planning and operations issues in mental healthcare services. The aim is to contribute a roadmap for the future application of DES in mental healthcare services, with an emphasis on planning and operations.

2.1. Introduction

Mental disorders are an enormous burden to society. They account for 30% of non-fatal disease burden worldwide and 10% of overall disease burden, including death and disability (Mnookin et al. 2016). In addition to the health impact, mental disorders cause a significant amount of economic burden through health spending, social spending, and through the loss of labour (World Health Organization and Calouste Gulbenkian Foundation 2017). From a service planning and delivery point of view, the era of advanced deinstitutionalization brings with it significant challenges to provide high-quality coordinate care (OECD/EU 2016). Individuals who have a varying range of health and social needs must be organized by providers of care across three settings: care provided in the community, inpatient care and secure care, in a locked setting. For healthcare professionals in mental healthcare, improving efficiency of operations by optimally allocating scarce resources and improving access to treatment while minimizing delivery costs becomes imperative to delivering high quality care.

Discrete-event simulation has long been a popular and widely accepted tool of decision support for decision-makers in healthcare operations planning, even before the widespread availability of computers and development of advanced simulation software (Papageorgiou 1978; Tunnicliffe 1980; Günal and Pidd 2010). Despite its popularity, there is evidence of limited application of DES (six papers have been found) in operations planning for Mental Healthcare Services (MHSs) (Long and Meadows 2018). This knowledge gap warrants attention as DES has the potential to analyse and improve health services (Jacobson et al. 2013). We conducted a systematic review to determine the extent to which studies have used DES within MHSs. This paper builds on the review by Long and Meadows (2018) by contributing additional insights and a tailored roadmap for the future application of DES for planning and operations issues in Mental Healthcare (MH).

This paper is organized into a further five sections. Section 2.2 provides an overview of background literature on MH and simulation modelling in MHSs. Section 2.3 describes the search methodology employed for the literature review. Section 2.4 offers an analysis and description of findings from the articles chosen to be reviewed. Section 2.5 discusses the future research directions for the application of DES in MH. Section 2.6 concludes this paper.

2.2. Background Literature

2.2.1. Mental Healthcare Services

Mental disorders often follow a chronic course, albeit with periods of relapse and remission which can mimic acute disorders. Management of mental disorders- more particularly than other medical conditions- is said to require a balanced combination of three fundamental ingredients of care: pharmacological; psychological; and psychosocial interventions (World Health Organization 2001). Therefore, the needs of people with mental illness are multiple and varied and differ at different stages of the illness. These needs are met mainly through community-based services within a local setting. Community mental health can comprise of a variety of services such as outpatient services, acute inpatient services, long-term care, nursing services, mental health teams, therapy services, and community hospitals in co-ordination with a number of external partners including primary care, specialist care, social care, voluntary services, emergency services, education, housing, and the justice system (Thornicroft et al. 2016; Carter 2018).

From an operational aspect, there is little uniformity in the delivery of services (Carter 2018). It has been reported that in a single geographical location no two mental health service providers deliver the same set of services (Carter 2018). This discord between how services are structured is both a global and national phenomenon. Patterns of services and provision of treatment for mental health not only differ between high- vs. low- and middle income countries, but also high- vs. low-resource areas within countries (Patel et al. 2018). A single global model of mental health care provision simply does not exist (Thornicroft et al. 2016).

Additionally, a range of barriers limit the provision of care specifically for the MH sector, which include inadequate funding, high workload pressure on mental health workers, and understaffing among others (BMA 2017). For patients with mental health conditions, there remain a number of system-wide challenges. These include, long waiting times, poor integration across services, bed shortages and inadequate service provision, to name a few (BMA 2017). With rising healthcare costs and continued prevalence of mental health disorders worldwide, the need to make comprehensive decisions in service delivery and for robust resource allocation add to the ever-increasing pressure to deliver quality care. The mental healthcare system consists of multiple stakeholders, inter-related and interconnected components, with complex interactions. Hence, OR techniques such as DES, can and should play a significant role in helping MH service managers to evaluate efficiency of existing systems, examine staffing levels, and investigate complex relationships in the system.

2.2.2. Simulation in Mental Healthcare

A number of reviews published in the timeframe 2009-2019, have explored the application of DES in a wide array of healthcare settings (Brailsford et al. 2009; Cardoen et al. 2010; Günal and Pidd. 2010; Mustafee et al. 2010; Katsaliaki and Mustafee 2011; Fakhimi and Mustafee 2012; Mielczarek and Uziałko-Mydlikowska 2012; Mielczarek 2016; Long and Meadows 2018). In striking contrast, analysis of these reviews reveals that prior to the review authored by Long and Meadows (2018), the paper by authors Mielczarek and Uziałko-Mydlikowska (2012) was the only one that cited a study related to mental health.

MHS planning has been largely neglected by the discipline of Operations Research (OR), which by extension also holds true for DES (Bradley et al. 2017). A similar conclusion was arrived at by authors Long and Meadows (2018), having reviewed 160 papers that employed simulation modeling methods such as Markov modelling; Monte Carlo Simulation; Microsimulation; DES; Agent Based Modelling (ABM); and System Dynamics (SD) in mental healthcare. The authors found widespread applications in areas of medical decision making and epidemiology. However, application of simulation in healthcare system design, planning and operations were found to be relatively underrepresented (Long and Meadows 2018). Furthermore, the authors identified 19 articles that applied DES, of which four journal articles, one conference proceedings paper and one PhD thesis applied DES to address planning and operations issues in MHSs. The application of ABM and SD to inform mental health policy has also been reviewed by authors Langellier et al. (2019). They provide a narrative synthesis of eight articles included in their review and highlight opportunities for expanded use of complex systems approaches in mental healthcare (Langellier et al. 2019). Along similar lines, this paper aims to further contribute to the budding literature in MHS planning by reviewing and analyzing literature specific to the application of discrete-event simulation.

2.3. Search strategy and methodology

We conducted a systematic review of literature to identify studies that utilized DES within MHSs. We retrieved relevant studies from a number of databases. The search strategy was designed to capture publications not only from OR journals but also to include articles from medical journals. The search term utilized was “discrete-event simulation” AND “mental health*”. Articles published between 2000 and 2018 were included. Figure 1 summarizes the search strategy employed for selecting articles (Liberati et al. 2009). The selection procedure included two screenings to determine the eligibility of the articles. In the first screening, articles were included if the answer to the questions: (i) has DES been applied; and (ii) has DES been applied to MHS was affirmative. Those excluded from the analysis were articles that were reviews, opinion pieces, debates and methodology focused papers. Furthermore, articles’ whose primary focus was to model epidemiology, disease progressing, screening, health promotions and hospital overcrowding where mental health clinics were not a key focus were also excluded. In the second round of screening, articles were excluded if they primarily dealt with health economics. Following screening, ten papers were selected for review. Data extracted for each paper is presented in Table 1.

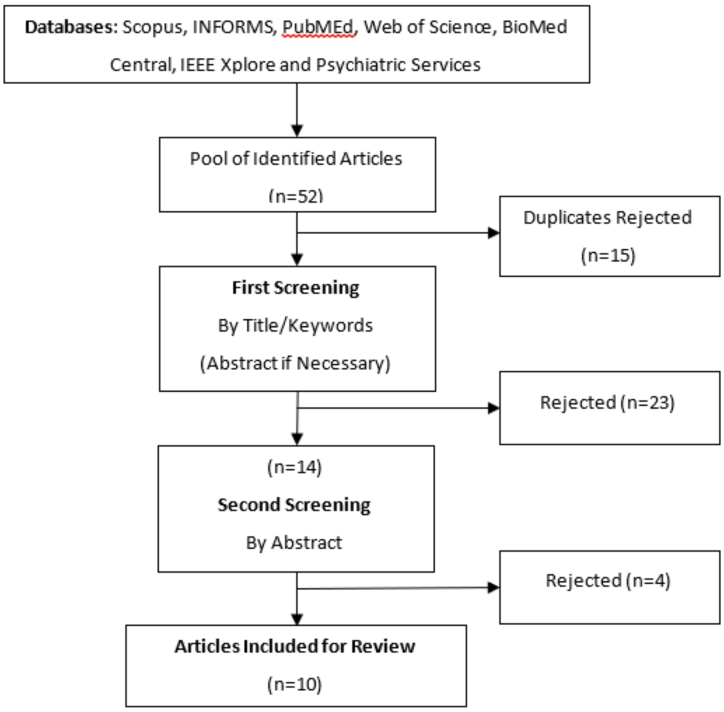

Figure 1: Flow diagram of review paper selection.

2.4. Results

2.4.1. Publication Characteristics

A total of ten publications were retrieved dating from 2000 to 2018 of which seven were published in journals and three were conference publications. Interestingly, of the seven journal publications, only one was from an Operations Research journal and six were from non-OR journals. Additionally, majority of publications were from the USA (seven papers), with Australia, Canada and UK constituting for one paper each.

2.4.2. Study Objectives

We categorized six papers as being predominantly concerned with capacity planning whilst four papers featured resource allocation issues. Studies focused on capacity planning largely involved increasing bed capacity to understand potential impacts on patient flow through the system (Kuno et al. 2005; La et al. 2016; Paton and Tiffin 2018; Roh et al. 2018). Furthermore, two studies examined capacity in terms of prospective requirement of practitioners to satisfy patient demand for a service (Patten and Meadows 2009) and investigated the optimum panel size (list of assigned patients) for a psychiatrist providing treatment to Post Traumatic Stress Disorder (PTSD) patients (Dursun et al. 2013). The lack of beds in mental healthcare services is a contentious issue where service providers have to find tradeoffs between increasing health outcomes for patients by decreasing waiting times and costs associated with increasing bed capacity. Especially when delay in treatment poses considerable health risks to patients with mental health conditions.

Resource allocation is the second most investigated issue in MHS planning, wherein authors Konrad et al. (2017) have explored the impact of projected increase in patient volumes on resources, whilst authors Chepenik and Pinker (2017) developed a model to predict potential benefits of additional clinical staff to patient flow. Resource allocation has also been conducted along with rationalizing budgets for MHSs (Troy et al. 2017) and to improve service (Kim et al. 2013).

Most studies reviewed in this paper have marked the beginning of DES in various aspects of MHSs, for instance: authors Konrad et al. (2017) have modelled an integrated clinic, thereby addressing a gap in simulation as well as in mental health; Troy et al. (2017) have applied simulation on a granular level for a large mental healthcare network for resource allocation; Dursun et al. (2013) used DES to design a panel (list of assigned patients) for a psychiatrist, a phenomenon commonly only associated with physicians in primary care; Roh et al. (2018) have addressed a gap in literature by considering the transition process for patients from an emergency department into external community and inpatient settings; and Patten and Meadows (2009) have demonstrated how service planning can be conducted by utilizing epidemiologic data.

Table 1: Summary of classification of review articles

Title and Authors	Purpose	Modelling Scope	Stakeholder Engagement	Implementation	Model's Input Parameters	Study Findings
Kuno et al. (2005)	Capacity Planning	Multi-Unit (Hospital and Residential Units)	✘	Suggested	▪ Length of Stay (LoS). ▪ Bed capacity ▪ Transition rate (between facilities).	▪ Comparison of various bed capacity options. ▪ Increased bed capacity improved system performance.
Patten and Meadows (2009)	Capacity Planning and Patient Demand	Service Network	✘	Suggested	▪ Population size. ▪ Treatment acceptability rate. ▪ Recurrence rate.	▪ Linked epidemiology data to service planning. ▪ Estimated number of therapists required.
Dursun et al. (2013)	Capacity Planning	Single Unit (Clinic)	✓	Conceptualized	▪ Panel size ▪ Treatment engagement (%)	▪ The number of patients a psychiatrist should provide care to was identified.
Kim et al. (2013)	Service Redesign and Resource Allocation	Single Unit (Clinic)	✓	Conceptualized	▪ Clinical hours. ▪ Staff composition.	▪ Analysis of trade-offs between long service time and increasing staffing costs. ▪ Extending clinic hours by two and an additional psychiatrist were recommended.
La et al. (2016)	Capacity Planning	Single Unit (Hospital)	✓	Conceptualized	▪ Bed capacity.	▪ A 165% increase in bed capacity required to reduce patient wait time. ▪ Emphasized DES's potential to solve complex operational problems in MH.

Troy et al. (2017)	Resource Allocation and Budgetary Evaluation	Service Network	✓	Conceptualized	▪ Staff composition ▪ Clinic location	▪ Experimentation revealed underutilized staff that were reallocated. ▪ Rationalized staffing levels and improved service levels.
Konrad et al. (2017)	Resource Allocation	Multi-Unit (Integrated Clinic)	✓	Conceptualized	▪ Patient volumes	▪ Expanding patient coverage required four additional providers. ▪ Inform the MH community to the benefits of DES.
Chepenik and Pinker (2017)	Resource Allocation	Single Unit (Psychiatric Emergency Service)	✘	Conceptualized	▪ Number of practitioners	▪ Modest addition of one half-time clinician produced biggest increase in patient flow metrics. ▪ Explained service bottlenecks
Paton and Tiffin (2018)	Capacity Planning	Single Unit (Clinic)	✘	Suggested	▪ Referral rate. ▪ LoS.	▪ Substantial increase in in-patient capacity needed to reduce wait times. ▪ Call for a more complex approach within DES framework.
Roh et al. (2018)	Capacity Planning	Multi-Unit (Hospital ED and Inpatient Wards)	✓	Conceptualized	▪ Patient arrival rate ▪ ED inpatient admissions (%) ▪ Inpatient LoS (%)	▪ Boarding time increase with high arrival rates and LoS. ▪ Over-utilized inpatient units push urgent care for MH into the emergency department.

Clearly, all of the papers are primarily motivated by improving the quality of services being studied and demonstrating the utility of DES in mental health as opposed to enhancing the DES method and models.

2.4.3. Modelling Scope and Model Type

Scope represents the extent to which the MH system has been captured in models. Five articles under review were modelled on a single unit (such as MH clinics, Hospitals, Psychiatric Emergency Services) and four articles modelled multiple units in the MHS network (e.g. hospitals, residential units and inpatient wards).

Additionally, DES models have broadly been classified into four types based on the purpose they serve. Based on this classification, models developed in eight of the ten articles were grouped as 'Throwaway Models', that is, models that are developed for the duration of a study to investigate one or more issues that are being address (Robinson 2014). In contrast, models from the two remaining studies were classified as "Generic Models", that is models developed in a particular context that can be used across a number of organizations (Robinson 2014). Thus, the service planning model linking epidemiology data to service planning developed by authors Patten and Meadows (2009) and the model built by Troy et al. (2017), to rationalize staffing levels were generic models that could potentially be applied across organization in the context of MH.

2.4.4. Stakeholder Engagement and Implementation

Stakeholder engagement is said to play a key role in the success of a simulation project (Robinson and Pidd 1998). Six out of ten papers from this review describe varying degrees of stakeholder engagement. The paper that described a relatively high stakeholder engagement was authored by La et al. (2016). They describe the number of stakeholders that participated and enumerate who the stakeholders were while stating reasons for their involvement. A total of nine meetings were held at various points in the study. These allowed for goal communication and data collection as well as conceptualizing scenarios for analysis. Likewise, authors Konrad et al. (2017) have described adequate levels of stakeholder engagement with staff for a number of purposes including, data collection, conceptual model validation, base scenario modelling and incorporating feedback via a number of model iterations. On the other hand modest levels of engagement have been described by authors Dursun et al. (2013), Kim et al. (2013), Troy et al. (2017) and Roh et al. (2018), typically through model validation, reviewing model's results, and interviews to quantify service parameters, validation of model's assumptions and for conceptualizing service changes.

Moreover, the nature of stakeholder engagement varies across studies. That is, we deduced from the description of the engagement that La et al. (2016) engaged with stakeholders in a group, while other authors engaged on a one-on-one basis (Dursun et al. 2013; Konrad et al. 2017; Roh et al. 2018). However, for authors Kim et al. (2013) and Troy et al. (2017), we were unable to deduce the nature of stakeholder engagement owing to the lack of a detailed description.

None of the papers being reviewed reported the use of their models in practice. This is in line with previous findings (Wilson 1981; Taylor et al. 2009). The papers were classified based on the three-level scale of implementation described by Brailsford et al. (2009). Accordingly, seven studies have 'conceptualized' (discussed with a client organization) their model's results by describing the likelihood for improvement in services, if utilized. On the other hand, three studies have 'suggested' (theoretically proposed by authors) their model's usefulness, specifically in the context of MHS.

2.4.4.1. Sponsor and Funding

The primary initiator (sponsor) of seven of these studies was the health services, although sources of funding for these studies were not reported. Furthermore, one study was judged to be solely of academic origin, although the authors utilize data that was consolidated by the government, the study itself was an academic venture (Patten and Meadows 2009).

Moreover, we found evidence of two studies that were sponsored and funded by government initiation/support via grants and/or by health services. Specifically, the study conducted by Kuno et al. (2005) was government funded and the study conducted by La et al. (2016) had elements of funding and support from government as well as health services. While the number of articles being analysed here is modest to come to a conclusion, it is however, indicative of a possible recognition from the mental health community and to some extent, the government of DES modelling' s offerings. In support of this argument, Konrad et al. (2019) have highlighted the coming together of academics and clinicians in their study as having been successful in applying DES, which is not typically used in mental health workforce planning and have advocated for more such partnerships across mental health settings. Perhaps, future research can look to this study for academic-clinical partnerships in the context of mental health.

2.5. Discussion

The previous section illustrates the underrepresentation of DES in operations and planning of MHSs. The papers reviewed so far have made a case for robust application of DES to the mental health community as well as to researchers and practitioners alike. Having said that, the application of simulation modelling to MHSs is anything but straightforward. The structural ambiguity of mental health service provision, pose significant challenges to model transferability and adaptability. However, certain contextual and structural similarities can be drawn from application of simulation to social care (Onggo 2012); stroke care systems (Churilov and Donnan 2012); and long-term care (Patrick et al. 2015). Each of these care systems consists of a diverse range of disparate services, which constitute interrelated parts of a whole system. Notwithstanding these similarities, it is important to recognize that mental healthcare services encompass elements of acute care, chronic care, social care and long-term care, which makes direct reapplication of previous research a matter of further inquiry.

This section will draw on existing literature of DES and its application in healthcare, while examining the potential for reapplication or adaptation to aspects of MHSs. The subsequent roadmap has been conceived by carefully considering the complex dynamics within the system, while also acknowledging the characteristics of the MHSs discussed in the review. Besides, the roadmap is also consistent with emerging trends in modelling healthcare systems (Arisha and Rashwan 2016).

2.5.1. Operational Efficiency

Variations across mental health services have had a negative impact on workforce productivity, operational efficiency while adding to the escalating mental health related costs (Lagomasino 2010). According to the analysis, most studies have primarily focused on capacity planning and resource allocation. In contrast, only one study focused on service design. Whereas, DES has been utilized for these purposes in other areas of healthcare (Mustafee et al. 2010), such applications in mental health are negligible. For instance, DES has been used to evaluate service design options for stroke care pathways to determine the most effective alternative that reduces in-hospital delays (Monks et al. 2012); and DES was used to design a more efficient hospital pharmacy by comparing changes in staffing levels and skill-mix depending on workload (Reynolds 2011). Such evidence-informed analysis of service design and delivery alternatives, have the potential to improve outcomes and cut costs (Pitt 2016). Future research could focus on this aspect of MHSs as care pathways of mental health patients are highly variable. This is especially important as patients with mental health disorders present with considerable risks and poor quality of treatment can lead to poor outcomes (Gilburt 2015).

Length of Stay (LoS) has been a key performance indicator that most studies have tried to reduce owing to the financial constraints of increasing bed capacities. Lack of care in the community and decreasing provision of social care are said to prolong LoS (Paton and Tiffin 2018). However, such influences have not been modelled or studied and can be a promising area of future research.

2.5.1.1. Quality Improvement

In response to huge pressures due to severe financial constraint and workforce shortage facing MHSs, a growing number are turning to 'quality improvement' (QI) approaches to achieve service improvements (Green et al. 2012; Ross and Naylor 2017). QI tools include, 'Plan-Do-Study-Act Cycle'; Six-Sigma; Lean methodology etc. (Varkey et al. 2007). In essence, these efforts proceed on the basis of anecdotal accounts of successful strategies and require multiple iterations to attain reliable improvements, which are likely to incur additional costs. Although such efforts in mental healthcare services are in their early days, there is limited evidence of impact (Ross and Naylor 2017).

Alternatively, evidence in simulation literature demonstrates the potential for DES and QI as complementary methodologies that can be used together as they have similar motivations: to improve process and service delivery (Robinson et al. 2012). Indeed, the integration of DES and QI has also been advocated for by the medical community as well (Rutberg et al. 2015) and there exist a number of instances in literature where such efforts have been successfully employed in healthcare (Robinson et al. 2012; Baril et al. 2016). This integration can help an already financially constrained mental health service in selecting the best option of service improvement by using DES, without having to dissipate precious resources.

2.5.2. Stakeholder Engagement

In MH, delays in decision making on improvements to patient pathways owing to stakeholder concerns and feedback, have been known to have substantial impacts on costs and patients' health (Carter 2018). From the analysis, it appears that most papers have given limited attention to stakeholder engagement in terms of identifying relevant stakeholders, describing their level of decision-making or involving them explicitly from the outset of the study. The fragmented nature of MHSs across different local areas and the presence of a range of partners and stakeholders warrants cooperation and integration, to achieve long-term efficiency and greater operational productivity (Carter 2018). Therefore, future research offers ample opportunities to improve limitations of stakeholder engagement so far and enhance stakeholder engagement in the application of DES to MHSs. This could not only be beneficial to improving a DES models' quality and with it, the chance of a successful outcome, it could also help MHS providers and decision makers tackle some of their productivity issues.

Stakeholder engagement is considered a key factor in simulation studies, and is critical to successful model implementation (Young et al. 2009). There is evidence of a direct causal link between weak or low stakeholder engagement and lack of implementation. Early involvement of stakeholders is often recommended for a simulation study. This is truer so in health care than in other areas of application as it increases the risk of loss of interest in the final results and recommendations (Roberts 2011). Furthermore, it is also suggested to involve a diverse group of stakeholders whose interests add an additional dimension to a simulation study (Roberts 2011).

In literature, there are instances of simulation studies that utilize Problem Structuring Methods (PSMs) for stakeholder engagement through facilitated modelling (Kotiadis et al. 2014; Robinson et al. 2014; Tako and Kotiadis 2015). Interestingly, PSMs are already being applied within mental health for systems improvement and policy (Powell and Mustafee, 2017). Future research could use PSMs in combination with DES through facilitated modelling in MH.

2.5.3. Methodological Pluralism

Several aspects of mental health services that need further investigation have been identified by the studies that have been reviewed here. Most authors recognize the preliminary nature of their application and call for a more comprehensive approach.

The dynamic structure of MHSs, often generates a number of inefficiencies at boundaries between different services and service providers in the system (Carter 2018). Therefore focusing on the wider mental healthcare continuum by modelling service integration and examining the interdependencies in the system could be a promising future research direction. For instance, service use by patients with mental illness is associated with habitual no-shows, which has a negative effects on both the patient and the service (Gondolf 2009). Such analyses have not been incorporated into simulation models of MHSs so far. Although, statistical analysis of such factors is usually conducted on a standalone basis (Crabb and Hunsley 2006). Coupling statistical analysis of demand factors such as age, gender, ethnicity with DES modelling could provide invaluable insight into the operational dynamics associated with them.

Additionally, service improvements in MHSs are currently being carried out without thoroughly analyzing the impacts of implemented changes (Ross and Naylor 2017). It is also reported that these improvements are being carried out in isolation or in single units (Gilburt 2015). The combination of such practices can be detrimental to MHSs that are under immense pressure. Increasingly in healthcare, similar issues are being tackled by acknowledging that it is rarely possible to capture multiple aspects of a problem, and by employing hybrid simulation by combining two or more simulation methods such as DES, system dynamics (SD) and agent-based modelling (ABM) for one intervention (Brailsford et al. 2018). Indeed such advantages of hybrid simulation are progressively being discussed in literature while also being used to explore links between health and social care systems (Brailsford et al. 2013). Moreover, similar inquiries can also be found in mental healthcare, wherein hybrid simulation has been used for cost-effectiveness analysis of integrating mental health into primary care (Aringhieri et al. 2018). By further adapting approaches that address multiple aspects of service delivery in MH, current limitation could be overcome.

Furthermore, under the current system wide financial constraints facing MHSs, resource planning is essential to deliver quality care (Dunn et al. 2016). Increasingly, simulation-optimization approaches are being used for identifying effective improvement factors in planning healthcare service resources (Fu et al. 2015). Simulation methods such as DES can be employed to model critical activities and scarce resources and optimization methods such as linear programming can be used to provide optimal resource configurations that best improve performance. For instance, authors Ozcan et al. (2016) have used the simulation-optimization approach to evaluate and improve the performance of a surgery-based pathway. Simulation allowed for system variability to be tracked and for the evaluation of resource utilization. Whereas optimization allowed for the identification of optimal capacity decisions in delivering performance. This integration of simulation and optimization could be another interesting area of future research.

2.6. Conclusion

Mental illness is the next major global health challenge. Worldwide, there is widespread commitment to fill the gaps between the need for treatment and service delivery. Operations and service planning issues in mental healthcare present plenty of opportunities for researchers as well as practitioners, not only for the application of DES, but also for combining DES with other suitable methods that capture multiple aspects of the service delivery system. Our review analyzes the application of DES modelling for planning and operations issues in mental healthcare services so far. The analysis highlights several limitations and contributes a roadmap for the application of DES to tackle issues of operational efficiency and productivity in MHSs. We encourage simulation researchers to direct their efforts towards tackling operations and planning of MHSs. This could be a step in the right direction towards addressing important problems faced by mental healthcare.

Chapter 3: Mind the gap: a review of optimisation in mental healthcare service delivery.

Abstract

Well-planned care arrangements with effective distribution of available resources have the potential to address inefficiencies in mental health services. We begin by exploring the complexities associated with mental health and describe how these influence service delivery. We then conduct a scoping literature review of studies employing optimisation techniques that address service delivery issues in mental healthcare. Studies are classified based on criteria such as the type of planning decision addressed, the purpose of the study and care setting. We analyse the modelling methodologies used, objectives, constraints and model solutions. We find that the application of optimisation to mental healthcare is in its early stages compared to the rest of healthcare. Commonalities between mental healthcare service provision and other services are discussed, and the future research agenda is outlined. We find that the existing application of optimisation in specific healthcare settings can be transferred to mental healthcare. Also highlighted are opportunities for addressing specific issues faced by mental healthcare services.

3.1. Introduction

Mental health is a significant global concern, not only for public health but also for economic development and societal welfare. Mental health disorders are on the rise around the world. Failure to respond to this growing crisis could cause lasting harm to individuals, societies, and economies worldwide. This crisis has been exaggerated by the COVID-19 pandemic (Pfefferbaum & North, 2020). The gap between the need for treatment and its provision is a global issue. The World Health Organisation (WHO) estimates that between 35% and 50% of people with severe mental health problems in developed countries and 76-85% in developing countries receive no treatment (World Health Organisation, 2019). Untreated mental health problems account for 13% of the total global burden of disease (Ibid). Concern for mental health as a pressing public health issue is also building as the magnitude of the problem is put in economic terms. The World Economic Forum estimates that the costs associated with mental illness at $2.5 trillion in 2010 can grow to $6 trillion in 2030 (Bloom et al., 2011). Mental illness costs exceed the cost of any other non-communicable disease, including cardiovascular disease, chronic respiratory disease, cancer, and diabetes (McDaid, Park, & Wahlbeck, 2019).

At present, as the world confronts the COVID-19 pandemic, experts predict a looming mental health crisis on the horizon (Ahluwalia C. Sangeeta, Farmer M. Carrie, & Abir Mahshid, 2020). Before COVID-19 emerged, statistics on mental health conditions were already stark. As the situation unfolds, there is emerging evidence that healthcare workers are at significant risk of adverse mental health outcomes (Ho, Chee, & Ho, 2020; Kang et al., 2020; Lai et al., 2020). For patients living with existing mental health challenges, the pandemic carries a high risk of symptoms worsening, mental or emotional deterioration or full-blown relapse (Yao, Chen, & Xu, 2020). This constantly changing landscape has increased levels of loneliness, depression, harmful alcohol and drug use, and self-harm or suicidal behaviour (World Health Organization, 2020). Globally, the pandemic has exposed glaring health disparities and highlighted the weaknesses in seemingly robust healthcare systems (Tandon, 2020). Simultaneously, the pandemic has highlighted the significance of mental health and the pressing need for parity with other health services (Moreno et al., 2020). While several initiatives to strengthen mental health services have sprung up, the response has been hampered by the historical underinvestment (United Nations, 2020). The COVID-19 pandemic is markedly a turning point, moving mental health up the list of global health priorities. As countries struggle to rebuild their damaged economies, they are being urged to accept the reality of the financial toll of mental ill-health and invest in efficient and good quality services (The Lancet Global Health, 2020).

Operational Research (OR) encompasses a wide range of problem-solving techniques and algorithms that are applied in the pursuit of improved decision-making and efficiency. Over the last two decades, OR methodologies have been applied extensively to various health care systems. In contrast, the mental/psychological care services have been noted as an area of neglect in OR (Bradley et al., 2017). For instance, existing reviews explore the application of specific OR methodologies, such as simulation (Langellier et al., 2019; Long & Meadows, 2018; Noorain, Kotiadis, & Scaparra, 2019), and Data Envelopment Analysis (García-Alonso, Almeda, Salinas-Pérez, Gutierrez-Colosia, & Salvador-Carulla, 2019), on mental healthcare services. In contrast, a comprehensive review of the application of optimisation methodologies to mental healthcare in the OR literature is lacking. We aim to provide a comprehensive and up-to-date account of the application of optimisation for planning and delivery in mental/psychological healthcare services.

The contributions of this review are threefold. First, we provide a comprehensive overview of the application of optimisation in healthcare so far. Through this, we highlight gaps in existing optimisation literature and examine future research directions. Second, we analyse the context of mental healthcare services to identify unique features and investigate if similar features have been considered in the healthcare literature. Our primary contribution though results from a scoping review on the application of optimisation techniques in mental healthcare services to identify issues for researchers to analyse, study and model.

The remainder of this paper is organised as follows. Section 3.2 provides background information on the topic by examining existing optimisation literature in healthcare and analysing the context of mental healthcare services. Section 3.3 describes the search methodology employed in this review, followed by Section 3.4, which provides a thematic overview and presents an analysis of optimisation model components such as the objective function, model constraints, model formulation and solutions methodologies employed by the articles under review. Section 3.5 draws on the similarities between mental healthcare services and other healthcare settings and sets the agenda for future research. Section 3.6 presents some conclusive remarks.

3.2. Background

This section is intended to serve four purposes: to provide a brief overview of planning levels, to describe the components of an optimisation model, to illustrate the use of optimisation in healthcare, to demonstrate the unique characteristics of mental illness, and to explore opportunities of synergy between the application of optimisation and mental healthcare services.

3.2.1. Planning Levels

The optimisation literature is often organised based on four hierarchical planning levels, including various planning decisions (Cissé et al., 2017; Hans, Van Houdenhoven, & Hulshof, 2012). The four hierarchical levels are strategic, tactical, operational offline, and operational online (Hulshof, Kortbeek, Boucherie, Hans, & Bakker, 2012). Planning on a strategic level addresses structural decisions with a long planning horizon, whereas planning on a tactical level involves the translation of strategic planning decisions into guidelines that facilitate operational planning (Hans et al., 2012). Operational planning involves short-term decision-making, reflecting the execution of tactical blueprints. Offline operational is about *advance* planning or operations, whereas online operational planning deals with reactive decision making in response to events that cannot be planned in advance (Cardoen, Demeulemeester, & Beliën, 2010; Hulshof et al., 2012).

3.2.2. Optimisation Model

There are three main components in an optimisation model: objective function, decision variables, and constraints. An optimization model seeks to find the values of decision variables that optimize (maximize or minimize) an objective function among a set of values of the decision variables that satisfy given constraints (Winston & Goldberg, 2004). To illustrate, consider a simplified example. A hospital emergency room would like to minimize costs associated with scheduling nurses. The optimisation model would include the 'objective function' (goal) to minimize costs related to nurses. The 'decision variable' would be the number of nurses to be deployed and, the 'constraints' would be the limits on the number of nurses required for a shift.

The optimisation model is formulated using a wide range of prominent techniques, including linear programming (Dantzig, George B., 1951), integer programming (Wolsey & Nemhauser, 1999), dynamic programming (Bellman, 1966), stochastic programming (Kall, Wallace, & Kall, 1994), network programming (Bertsekas, 1991), combinatorial optimization (Wolsey & Nemhauser, 1999), and nonlinear programming (Bazaraa, Sherali, & Shetty, 2013). The type and complexity of a model will dictate the solution method of choice. Exact algorithms such as simplex (Dantzig, George B., Orden, & Wolfe, 1955), branch and cut (Padberg & Rinaldi, 1991), and branch and bound (Little, John DC, Murty, Sweeney, & Karel, 1963) are employed to solve optimisation problems to optimality. If a model is too complex to be solved by exact algorithms, the search for an optimal solution is abandoned to seek a reasonable solution using heuristics or metaheuristics. Heuristics and metaheuristics use a collection of intelligent rules of thumb to find a suitable solution quickly (Horst & Pardalos, 2013). Examples include column generation heuristic (Taillard, 1999), Tabu Search (Glover, Fred, 1986), and simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983).

3.2.3. Optimisation in Healthcare

A survey of recent literature reviews on the application of optimisation techniques in healthcare is presented in this section. Articles were identified on Scopus, and then a backward search was performed using the initial pool of papers to find additional reviews. We selected review articles for analysis if they were published in the period (2011-2019) and analysed the application of OR methodologies to planning issues in healthcare. In the past decade, 19 literature reviews on the application of Operations Research/Management (OR/OM) in healthcare have been published. These include reviews that are generic and specific in their scope. Generic reviews examine the nature of the application of OR/OM techniques to healthcare (Brailsford & Vissers, 2011; Hulshof et al., 2012; Rais & Vianaa, 2011). In contrast, specific reviews are spread across application areas such as planning and scheduling, routing and scheduling, and supply chain management.

This analysis is primarily concerned with surveying articles that review research that has applied optimisation techniques in healthcare. Therefore, reviews with a specific scope are given preference. These include review articles from various application areas—specifically, nine reviews on planning, scheduling and routing and six on supply chain management. We build a comprehensive picture of the optimisation landscape in healthcare to identify the progress so far, highlight gaps and analyse the direction of future research. Table 2 depicts critical gaps and limitations specified in each research area that were identified in each review article.

3.2.3.1. Planning, Scheduling & Routing

In healthcare, planning, scheduling & routing decisions have been explored extensively. We classified reviews into several themes based on the area of research. We find that the planning of operating rooms has received the most attention (Cardoen et al., 2010; Samudra et al., 2016; Zhu, Fan, Yang, Pei, & Pardalos, 2019), followed by routing and scheduling in home healthcare (Cissé et al., 2017; Fikar & Hirsch, 2017; Gutiérrez & Vidal, 2013). We also identified reviews on physician scheduling (Erhard, Schoenfelder, Fügener, & Brunner, 2018) and appointment scheduling (Ahmadi-Javid, Jalali, & Klassen, 2017). Furthermore, we also include reviews on two budding research areas, namely the multi-appointment scheduling problems in hospitals (Marynissen & Demeulemeester, 2019) and multi-disciplinary planning and scheduling (Leeftink, Bikker, Vliegen, & Boucherie, 2020). Based on this classification, an analysis of literature reviews is presented in this section.

Operating Room Planning & Scheduling

Operating theatres are a hospital’s most significant cost and revenue centre, with substantial impacts on a hospital's performance as a whole (Macario, Vitez, Dunn, & McDonald, 1995). Several reviews have examined the literature on operating room planning and surgical care scheduling (Cardoen et al., 2010; Samudra et al., 2016; Zhu et al., 2019). This literature primarily deals with two categories of patients, namely elective or non-elective and inpatient or outpatient. Furthermore, operating room planning and surgical scheduling address a variety of issues, including the determination of resource quantity (surgeons, nurses, rooms, equipment, operations time) needed to meet demand; allocation of operating room capacity to various medical disciplines; assigning definite dates for operations; determining the start time of the operations and the allocation of resources. Zhu et al. (2019) observe that most research has been directed towards scheduling problems at the operational level (Kroer, Foverskov, Vilhelmsen, Hansen, & Larsen, 2018; Roshanaei, Luong, Aleman, & Urbach, 2017). Moreover, Samudra et al. (2016) notice that a large part of the literature is aimed at decision-making on a patient level (Agnetis et al., 2012). Particularly to the assignment of dates and room (Banditori, Cappanera, & Visintin, 2013).

Performance measures are primarily in the interest of the three stakeholders in the system: hospital administrators, medical staff, and patients (Wachtel & Dexter, 2009). Consequently, performance measures that are considered as model objectives were financial, utilisation, levelling (resource occupancy), throughput, idle time, makespan (completion time), preferences, waiting time, and patient deferrals. Likewise, uncertainty related issues are a significant component of operating room planning and scheduling. Therefore, models account for uncertainties relative to surgery duration (deviation between actual and planned) (Denton, Viapiano, & Vogl, 2007), patient arrival (unpredictable arrival of outpatients) (Beliën, Demeulemeester, & Cardoen, 2009), resources (availability, applicability and usability of human and material resources) (Cardoen et al., 2010), and more recently, uncertainty relative to care requirement (Holte & Mannino, 2013). The most considered type of uncertainty in models is duration uncertainty, followed by arrival uncertainty. Although the arrival of non-elective patients generates significant operational deficiencies, few studies have modelled this (Arenas et al., 2002; Pham & Klinkert, 2008). Generally, planning and scheduling of elective patients has received more attention when compared to non-elective and outpatients (Lamiri, Grimaud, & Xie, 2009). This trend in research is despite the ongoing shift from inpatient to outpatient care (Koenig & Gu, 2013).

Physician Scheduling

Shortages in medical personnel are ubiquitous in most industrialized countries. The scarcity of physicians adds increasing pressure on managers to find efficient and effective ways to schedule their workforce. Therefore, physician scheduling has received a fair amount of attention over the last decade. Erhard et al. (2018) surveyed physician scheduling in hospitals by classifying them as problems of staffing (determining size and composition), rostering (creating shift rosters), and re-planning (short-term adjustments to schedules). Research in this area mainly concentrates on building mid-and long-term rosters (Bruni & Detti, 2014; Brunner & Edenharter, 2011), thereby foregoing the incorporation of realism in models. Moreover, model objectives/goals are either financial (minimizing wage costs, overtime, outside resource usage) or non-financial (minimizing demand under coverage, roster changes and maximising employee preference). As for constraints, models consider two types, hard (non-negotiable) and soft (negotiable). Hard constraints are classified into two types: compulsory, including meeting demand, single shift per period, restricted backwards rotation, and minimum rest periods. At the same time, soft constraints are relative to ergonomics (preference, weekends off, days off, forward rotation, shift duration limits) and fairness (distribution of unpopular shifts, free weekends etc.). The most frequently used modelling methodologies are Integer Programming (Dexter, Wachtel, Epstein, Ledolter, & Todd, 2010), Mixed-Integer Programming (Bard, Shu, Morrice, & Leykum, 2017) and Linear Programming (Topaloglu, 2009). As for solution algorithms employed to solve models, exact algorithms (Shamia, Aboushaqrah, & Bayoumy, 2015) are preferred over heuristic algorithms (Samah, Zainudin, Majid, & Yusoff, 2012). However, since demand cannot be fully controlled, using deterministic demands to generate schedules is noted as a drawback. Moreover, the review highlights the increasing willingness of hospitals to provide data and conduct experimental studies. Specifically, of the 68 studies, 64 used real life data to test the performance of the proposed theoretical model and 24 (more than a third) reported on the implementation results in hospitals (Erhard et al., 2018).

Appointment Scheduling

Outpatient Appointment System (OAS) problems have been studied since 1952 (Bailey, 1952). An early review classified appointment systems into three categories based on their environment: primary care, speciality care and elective surgical care (Gupta, D. & Denton, 2008). While surgeries can be scheduled as either inpatient or outpatient, the other two types are predominantly outpatient. Surgical/operating theatre scheduling is addressed in the above section, here we discuss appointment scheduling in primary care and specialist care (outpatient). In the latest and most up-to-date review of literature by Ahmadi-Javid et al. (2017), it is observed that most OAS studies deal with operational decisions that are related to the execution of plans on an individual patient level. These include allocating of patients to servers/resources (Riise, Mannino, & Lamorgese, 2016), determining appointment day and time (Chen & Robinson, 2014; Kuiper, Kemper, & Mandjes, 2015), patient acceptance/rejection (Qu, Peng, Shi, & LaGanga, 2015), and patient selection from the waiting list (Saure, Patrick, Tyldesley, & Puterman, 2012). Furthermore, several studies also address problems at a tactical level, resulting in the determination of characteristics of the OAS that maximises resource utilization and accessibility (Wiesche, Schacht, & Werners, 2017). Additionally, performance measures often pertinent to the three main stakeholders: patients, system owners and staff are used in OAS models. We also found that studies have used patient waiting time as a measure of patient satisfaction (Kemper, Klaassen, & Mandjes, 2014), revenue is calculated as a measure of the number of patients seen (Balasubramanian, Muriel, & Wang, 2012), and costs are a measure of physician idle time (Vink, Kuiper, Kemper, & Bhulai, 2015). The most common performance measures used in OAS studies are the patient waiting time, staff idle time, overtime (Anderson et al., 2015), number of patients seen, number of patients rejected (Gocgun & Puterman, 2014). Although exact methods are used extensively, they are most often used to compare some given policy and develop efficient algorithms (Huh, Liu, & Truong, 2013; Truong, 2015). Ergo, due to the complexity of OAS problems, most studies employ heuristic/metaheuristic/approximate methods (Anderson, Zheng, Yoon, & Khasawneh, 2015; Azadeh, Farahani, Torabzadeh, & Baghersad, 2014; Castro & Petrovic, 2012).

When compared to early review papers (Cayirli & Veral, 2003; Gupta, D. & Denton, 2008), several milestones concerning future research have been achieved in this last decade. In particular, models now incorporate environmental factors such as patient preferences, cancellations, no-shows, and indirect patient waiting (time between appointment request and allocation) (Anderson et al., 2015; Erdogan, Gose, & Denton, 2015). Although this area is growing and expanding, OAS has many open and complex research questions. For instance, Ahmadi-Javid et al. (2017) advocate adopting more realistic assumptions relative to environmental factors. They also highlight the need to include interruptions (writing up notes, talking with support staff, or emergency patient arrivals) into existing optimisation models (Klassen & Yoogalingam, 2013; Luo, Kulkarni, & Ziya, 2012). Along similar lines, the review also highlights the need to study the effects of disruptions to OASs. Specifically, disruptions relative to natural disasters (earthquakes, floods and terrorist attacks) likely result in very high-level demands of urgent walk-ins; disruptions caused by economic or financial crises; and social events that could result in complete stoppage or severely reduce the availability of resources.

Home Health Care (HHC) Routing & Scheduling

Owing to a shifting trend in many countries where healthcare services are transitioning from a hospital setting to homes, HHC is a promising and growing research area (Genet et al., 2011). Providers of HHC dispense a range of services, including healthcare provider care, nursing, therapy (physical or occupational), medical social services, health aides, attendant care, volunteer care, nutrition and meal support, medical equipment and supplies, laboratory and pharmaceutical services, and transportation (John Hopkins Medicine, 2020). Based on three planning levels (strategic, tactical and operational), several issues are addressed in literature: 1) partitioning of HHC service territory into patient clusters and assigning resources to each cluster; 2) identifying resource (people or materials) levels and assigning resources to districts, and 3) assigning care workers to patients and scheduling patient visits assigned to each care worker. Issues relative to HHC overlap considerably with the problems addressed in logistics (Gutiérrez & Vidal, 2013). Therefore, reviews focusing primarily on the operational level of decision-making are discussed (Cissé et al., 2017; Fikar & Hirsch, 2017). A broader logistics oriented review is analysed under supply chain management.

In the past decade, an increasing number of studies have addressed routing and scheduling issues in HHC (Cissé et al., 2017). The presence of certain salient features such as "full continuity of care", where a unique care worker visits a patient over a planning horizon, generate challenges when modelling the system. Despite the challenges associated with incorporating such features into a model, Cissé et al. (2017) find that most researchers use several of the above features in their model's objectives and constraints (Mankowska, Meisel, & Bierwirth, 2014). Additionally, Fikar & Hirsch (2017) identify that most models are tested on data originating from real-world operations. However, the models have not considered uncertainty relative to travel time, care service duration, emergencies, and unavailability of workers or patients. Nevertheless, some studies consider uncertainties concerning when and where, in the future, patients will request care (Hewitt, Nowak, & Nataraj, 2016).

Multi-Appointment Scheduling in Hospitals (MASPH)

Unlike HHC, MASPH is gaining momentum in the academic literature, as observed in the review by Marynissen & Demeulemeester (2019). MASPH problems address a patient's need to sequentially visit multiple resource types in a hospital setting to receive treatment or diagnosis, for example, cancer treatments. Because MASPH is only just gaining momentum, it is currently only found in a limited number of hospital departments that have systems that directly address this. Several hospital resources are considered in MASPH including, doctors, specialists, beds, medical devices, diagnostic resources, chemotherapy chairs, and linear accelerators (used for radiotherapy). By extension, hospital departments included are rehabilitation (Braaksma, Kortbeek, Post, & Nollet, 2014; Kortbeek, van der Velde, M F, & Litvak, 2017), diagnostic facilities (Azadeh et al., 2014; Azadeh, Baghersad, Farahani, & Zarrin, 2015), oncology (Leeftink, Vliegen, & Hans, 2019; Suss, Bhuiyan, Demirli, & Batist, 2018), and operating rooms (Burdett & Kozan, 2016; Kazemian et al., 2017). From a patient's perspective, services that are considered for scheduling are either diagnostic or treatment. Furthermore, three types of patients are identified: outpatient, inpatient and emergency patients. In outpatient procedure planning, the main challenges are uncertain service times and patient no-shows (Tsai & Teng, 2014). For inpatient planning, most work has focused on minimising the length of stay (Conforti, Guerriero, Guido, Cerinic, & Conforti, 2011). For emergency patients, although their arrival is unforeseen, studies have focused on scheduling diagnostic laboratories tied to the emergency department (Azadeh et al., 2014). Studies also address the scheduling of different treatment steps in a treatment path of a triaged emergency patient following the assignment of a treatment path (Luscombe & Kozan, 2016). In contrast to other application areas where exact methodologies are popular for solving models, because of the complexity, most models are solved using metaheuristics (Azadeh et al., 2015) and multi-agent models (Kanaga & Valarmathi, 2012).

3.2.3.2. Supply Chain Management

Supply chain management in healthcare refers to the information, supplies and finances involved with the acquisition and movement of goods, and services from the point of supply to the end-user, to enhance clinical outcomes while controlling costs (De Vries & Huijsman, 2011; Dobrzykowski, Deilami, Hong, & Kim, 2014). These processes might relate to physical goods like drugs, pharmaceuticals, medical devices, health aids, and patients' flow (Beier, 1995). In this section, we examine reviews relative to a specific component of SCM, logistics. Furthermore, activities associated with logistics, such as facility location and inventory management, are inspected.

Logistics

It is defined as the process of planning, implementing, and controlling procedures for the efficient and effective transportation and storage of goods including services, and related information from the point of origin to the point of consumption based on customer requirements (Cordeau, Pasin, & Solomon, 2006; CSCMP, 2013). This definition includes inbound, outbound, internal, and external movements. This section reviews literature articles on Home Health Care logistics (Gutiérrez & Vidal, 2013) and material logistics in hospitals (Volland, Fügener, Schoenfelder, & Brunner, 2017).

Home health care logistics literature includes decision support across three contexts. These include 1. 'design and planning decisions': dealing with issues of facility location and districting (Blais, Lapierre, & Laporte, 2003); 2. 'resource planning and allocation': relative to issues of staff and inventory management (Chahed, Marcon, Sahin, Feillet, & Dallery, 2009; Kommer, 2002), and 3. 'service scheduling': concerned with staff routing and scheduling (Bredström & Rönnqvist, 2008). Gutiérrez & Vidal (2013) note that although most models support staff routing and scheduling decisions, a significant impact on system performance has not been observed. Therefore, a call for diversification of future research in strategic and tactical levels has been issued.

Volland et al. (2017) review literature on activities associated with handling physical goods in hospitals. These physical goods are related to the care of patients, including items such as pharmaceuticals, medical consumables, food, laundry, sterile items, laboratory samples, waste etc. The review categorized publications into four research topics, of which three employ optimization models. (1) 'Supply & procurement': relative to purchasing (Rego, Claro, & de Sousa, 2014), and aspects of the interface between drug manufacturers and wholesalers (Li, X., Zhao, Zhu, & Wyatt, 2011). (2) 'Inventory Management': includes literature on inventory policy (Rosales, Magazine, & Rao, 2014). (3) 'Distribution and Scheduling': distribution within (Lapierre & Ruiz, 2007) and outside a hospital (Medaglia, Villegas, & Rodríguez-Coca, 2009), and handling of sterile devices (Ozturk, Begen, & Zaric, 2014). A significant rise in the application of optimisation techniques has been observed. Wherein most Optimization techniques are applied in streams (2) and (3). Optimization in inventory management has primarily sought to minimize costs. Specifically, heuristics are applied to minimize the total, ordering, and inventory costs (Baboli, Fondrevelle, Tavakkoli-Moghaddam, & Mehrabi, 2011; Kelle. While in 'Distribution and Scheduling', some models have sought to minimise costs associated with transportation, the number of routes, and travel mileage (Augusto & Xie, 2009; Medaglia et al., 2009; Shih & Chang, 2001).

Facility Location

In its own right, this is an established topic of research within Operations Research. In healthcare, facility location problems concentrate on three main areas. These include healthcare facility location (involving community health clinics, primary care or specialist clinics, public and private hospitals), ambulance location and, hospital layout planning (Güneş, Melo, & Nickel, 2019). In essence, facility location problems locate a set of facilities (resources) to minimize/maximize specific objectives while fulfilling a set of demands concerning some constraints (Laporte, Nickel, & Saldanha-da-Gama, 2019). Objectives most commonly applied in healthcare facility location are: 1) minimize access cost for healthcare consumers, 2) maximise population with access to a healthcare facility, and 3) maximize the equity in access (Güneş et al., 2019). Increasingly, facility location has been proposed within the context of logistics as a sub-activity in several healthcare settings (Melo, Nickel, & Gama, 2007). These settings, along with their respective review papers, are supply chain (De Vries & Huijsman, 2011; Dobrzykowski et al., 2014), pharmaceutical supply chain (Lemmens, Decouttere, Vandaele, & Bernuzzi, 2016; Narayana, Pati, & Vrat, 2012; Shah, 2004), healthcare waste management (Thakur & Ramesh, 2015) and emergency response (Daskin & Dean, 2005; Li, X. et al., 2011).

We surveyed two reviews on the emergency and non-emergency facilities location (Ahmadi-Javid, Seyedi, & Syam, 2017; Li, X. et al., 2011). Li et al. (2011) conclude that heuristics (Jia, Ordóñez, & Dessouky, 2007), simulation, and exact algorithms (Alsalloum & Rand, 2006) have been used to solve models that emphasized providing coverage for emergency calls. They also found that simulation has been used to either evaluate the performance of policies derived from the solutions of optimisation models (Maxwell, Henderson, & Topaloglu, 2009) or in conjunction with heuristics to provide better quality solutions (Slocum et al., 2021). Through their analysis, Ahmadi-Javid, et al. (2017) observe that cost minimization is a major objective used in location problems (Ghaderi & Jabalameli, 2013; Mestre, Oliveira, & Barbosa-Póvoa, 2015), and the minimization of distance (or time) is considered a key factor in enhancing efficiency and effectiveness of locations (Beheshtifar & Alimoahmmadi, 2015; Smith, Harper, & Potts, 2013). Furthermore, a large number of models are built using Integer Linear Programming (ILP) and Mixed-Integer Linear Programming (MILP) (Ares, De Vries, & Huisman, 2016; Beheshtifar & Alimoahmmadi, 2015; Mestre et al., 2015), as opposed to stochastic programming (Mitropoulos, Mitropoulos, & Giannikos, 2013) or dynamic programming (Elalouf, Hovav, Tsadikovich, & Yedidsion, 2015).

Inventory Management

This is another sub-activity of logistics management in supply chain management, with a focus on end-customer demand. Here, the aim is to improve customer service while lowering relevant costs (Cordeau et al., 2006). In the context of Healthcare, inventory management refers to the management and control of a large number and variety of stocked items. When needed, not having the supplies in-stock can seriously impact the quality of care (Moons, Waeyenbergh, & Pintelon, 2019), with consequences such as loss of life (Guerrero, Yeung, & Guéret, 2013).

Table 2: Key Future Research Directions from Optimisation Related Literature Reviews

	Publication	Key Future Research Directions
Planning & Scheduling	**Cardoen, B., Demeulemeester, E. and Beliën, J., 2010**. **Operating room planning and scheduling**: A literature review. ***European Journal of Operational Research***, *201*(3), pp.921-932.	- Account for stochastic activity duration. - Research non-elective patient scheduling - Model integrated facilities & resources
	Samudra, M., Van Riet, C., Demeulemeester, E., Cardoen, B., Vansteenkiste, N. and Rademakers, F.E., 2016. **Scheduling operating rooms**: achievements, challenges and pitfalls. ***Journal of Scheduling***, 19(5), pp.493-525	- Consideration of stochastic arrivals & patient bulking (leaving waiting list) - Research on outpatient and non-elective. - Inclusion of behavioural factors as performance measures. - Model integrated system (outpatient & inpatient) - Apply stochastic programming for real-life problems.
	Ahmadi-Javid, A., Jalali, Z. and Klassen, K.J., 2017. **Outpatient appointment systems** in healthcare: A review of optimization studies. ***European Journal of Operational Research***, *258*(1), pp.3-34.	- Models to incorporate continuity of care, patient preferences, patient walk-ins. - Models to include environmental variables (no-shows, patient & physician unpunctuality). - Consider environmental factors such as disruption (natural disasters, economic or financial crises, social events) - Develop novel multi-decision models to address real-life situations.
	Erhard, M., Schoenfelder, J., Fügener, A. and Brunner, J.O., 2018. State of the art in **physician scheduling**. ***European Journal of Operational Research***, *265*(1), pp.1-18.	- Estimation of realistic demand and demand fluctuation. - Models to incorporate physician absenteeism and break assignment - Consider simulation-optimization as an alternative solution approach. - Models to develop flexible shifts.
	Leeftink, A.G., Bikker, I.A., Vliegen, I.M.H. and Boucherie, R.J., 2018. **Multi-disciplinary planning** in health care: a review. ***Health Systems***, pp.1-24.	- Account for variability in the care pathway and resource capacity with stochastic or robust programming - Model multi-disciplinary care outside hospitals. - Explore applicability of methods across health areas
	Marynissen, J. and Demeulemeester, E., 2019. Literature review on **multi-appointment scheduling** problems in **hospitals**. ***European Journal of Operational Research***, 272(2), pp.407-419.	- Account for emergency patients in inpatient and outpatient scheduling by reserving capacity. - Monitor and report system performance before and after implementation - Report on implementation.
	Zhu, S., Fan, W., Yang, S., Pei, J. and Pardalos, P.M., 2019. **Operating room planning** and **surgical case scheduling**: a review of literature. ***Journal of Combinatorial Optimization***, *37*(3), pp.757-805.	- Models to incorporate stochastic surgical duration. - Research non-elective patient scheduling - Focus on resource (human and material resource) uncertainty - Focus on uncertain medical requirements by patients.

Routing & Scheduling	**Cissé, M., Yalçındağ, S., Kergosien, Y., Şahin, E., Lenté, C. and Matta, A., 2017**. OR problems related to **Home Health Care**: A review of relevant **routing and scheduling** problems. ***Operations Research for Health Care***, *13*, pp.1-22.	- Capture uncertainty aspects (travel time between locations, care service duration, emergencies, workers' or patients' unavailability) with stochastic models. - Account for cancellation of appointments or last-minute absence of care workers.
	Fikar, C. and Hirsch, P., 2017. **Home health care routing and scheduling**: A review. ***Computers & Operations Research***, 77, pp.86-95.	- Models to consider emergencies, cancellation, unavailability of nurses & traffic delays. - Include ecological & social criteria.
Supply Chain Management	**Li, X., Zhao, Z., Zhu, X. and Wyatt, T**., **2011**. Covering models and optimization techniques for **emergency response facility location and planning**: a review. ***Mathematical Methods of Operations Research***, *74*(3), pp.281-310.	- Models to incorporate different priorities requiring different types of services. - Models to consider survival rate as an objective function. - Incorporate equity in facility distribution.
	Gutiérrez, E.V. and Vidal, C.J., 2013. **Home health care logistics management**: Framework and research perspectives. ***International Journal of Industrial Engineering and Management***, 4(3), pp.173-182.	- Model long-term resource location and allocation issues. - Integrated analysis of logistic decision across planning levels - Models to include realistic features (patient pathologies, service references & legal work regulations)
	Ahmadi-Javid, A., Seyedi, P. and Syam, S.S., 2017. A survey of **healthcare facility location**. ***Computers & Operations Research***, 79, pp.223-263.	- Dynamic location models accounting population migration, changes in management objectives, transportation & facility capacities, patient population. - Statistical methods to estimate input parameters. - Models to include multiple services and service quality. - Capture realistic assumptions such as uncertain & multi-type demand, & multiple servers.
	Volland, J., Fügener, A., Schoenfelder, J. and Brunner, J.O., 2017. **Material logistics in hospitals**: a literature review. ***Omega***, 69, pp.82-101.	- Heuristics to address large-scale, real-life complex logistics problems. - Improve and incorporate forecasting mechanism to capture demand. - Employ optimisation to determine product characteristics or to define an optimal degree of outsourcing. - Models to incorporate lead times.
	Ahmadi, E., Masel, D.T., Metcalf, A.Y. and Schuller, K., 2019. **Inventory management** of **surgical supplies and sterile instruments** in **hospitals**: a literature review. ***Health Systems***, *8*(2), pp.134-151.	- Stochastic models to incorporate operational and/or disruption risk factors. - Models to incorporate stochastic demand for instruments. - Models to determine location and quantity of supplies to stock. - Consider inventory cost and service levels simultaneously.
	Saha, E. and Ray, P.K., 2019. Modelling and analysis of **inventory management systems in healthcare**: A review and reflections. ***Computers & Industrial Engineering***, p.106051.	- Develop integrated model considering all types of medical products should be considered (e.g., pharmaceuticals, medical equipment, surgical instruments). - Heuristics to consider randomness and complexities (patient arrivals, illness, treatment stages, treatment responses). - Model uncertainty (demand for medicines, patient conditions, & physician prescribing behaviour) using robust optimization and probabilistic programming.

We survey two reviews. One, on inventory systems across various inventory items such as pharmaceuticals, medical equipment, surgical instruments, and other medical and surgical supplies (Saha & Ray, 2019). Two, inventory management of surgical supplies and sterile instruments (Ahmadi, Masel, Metcalf, & Schuller, 2019). Saha & Ray (2019) find that heuristics solve inventory problems under uncertainties (Rosales, Magazine, & Rao, 2015) and solve inventory allocation problems for surgical supplies stored in multiple locations. Through their analysis, Ahmadi et al. (2019) observe that early studies examined classical inventory models that relied on simplified assumptions, leading to far from practical solutions (Burns, Cote, & Tucker, 2001; Machline, 2008). On the other hand, research incorporating stochastic models did not specify which sources of uncertainty they considered (Little, James & Coughlan, 2008; Rappold, Van Roo, Di Martinelly, & Riane, 2011). The review also demonstrated several strategies towards cost reduction and standardizing practices utilized by practitioners (Eiferman, Bhakta, & Khan, 2015; Park & Dickerson, 2009).

3.2.4. Mental Healthcare

In this section, the distinctive features of mental healthcare are elaborated. In particular, we examine the nature of service models in mental healthcare, the causes and diagnosis of mental illnesses and their impacts on services, risks associated with mental illness and their consequences on service delivery, and finally, the integrated nature of psychological and physical health.

3.2.4.1. Care Setting

The care of patients with mental illness has been subject to significant changes in the West over the last two centuries. In particular, from the 1960s onwards, many countries implemented the policy of deinstitutionalisation, which led to the movement of patients from large inpatient institutions into the community by establishing community services (Fakhoury & Priebe, 2007). Presently, it is widely recognised that effective mental healthcare services cannot be delivered exclusively within a hospital setting or exclusively within the community (Abdulmalik & Thornicroft, 2016). An optimal mix of hospital and community services is recommended (Thornicroft & Tansella, 2013). Yet, such a mix has only been achieved in a few high-income countries, where the relatively high availability of workforce and financial resources have been matched by political willingness to increase community care (Saxena, Thornicroft, Knapp, & Whiteford, 2007). A diverse collection of service delivery models are currently in use in both low-middle-income countries and high-income countries (Carter, 2019; Cohen et al., 2011).

Mental health care relies on its human resources rather than advanced technology or equipment. The mental health workforce is a mix of collaboration between psychosocial providers and biomedical providers wherein the workforce is generally composed of three groups of individuals (Gask, 2005; Kakuma et al., 2011). The first includes specialist workers, such as psychiatrists, neurologists, psychiatric nurses, psychologists, mental health social workers, and occupational therapists (Kakuma et al., 2011). The second group is composed of non-specialist health workers, such as general practitioners/doctors, nurses, lay health workers, and caregivers (Gupta, N., Bhalla, & Rosenheck, 2019). The final group is formed of other professionals such as community-level resources that include formally structured bodies such as international and indigenous non-governmental organisations (NGOs) (Patel & Thara, 2003). The heterogeneity of service models across the world is no doubt challenging to the modelling of such services.

3.2.4.2. Uncertainty

Unlike the rest of medicine, a psychiatric diagnosis does not have any specific identifiable biological or psychological markers (Timimi, 2014). This is reflected through the diagnoses listed on major psychiatric diagnosis manuals such as Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-5) (American Psychiatric Association, 2013) and International Classification of Diseases (ICD-10) (World Health Organization, 1994). Unlike the rest of medicine, where the cause of a symptom is clarified by diagnosis, the cause of various mental disorders does not share the same scientific security (Clark, Cuthbert, Lewis-Fernández, Narrow, & Reed, 2017). The heterogeneous use of diagnostic manuals further complicates this, wherein DSM-5 is primarily used in the United States, and ICD-10 is used internationally. The widely used diagnostic manuals have been subject to various criticisms, particularly for being fundamentally descriptive systems, based primarily on self-reported symptoms and observed signs (Clark et al., 2017).

The two widely used systems of diagnosis are increasingly bringing into question issues of clinical validity, reliability, impact on treatment and outcomes, and the uniformity of prognoses. Many psychiatrists have called for a shift from the current paradigm of a mental health diagnosis that focuses on the biomedical cause of mental disorders because of evidence-based research (Bracken et al., 2012). Research on the causes of mental illness has shown that it arises from several factors, including biological, behavioural, psychosocial, and cultural factors that interact in complex ways (Canino & Alegría, 2008). Research has also highlighted that, unlike the rest of medicine, outcomes of mental illnesses are not definable but are complex and variable combinations of psychological problems (Clark et al., 2017).

With diagnoses of psychological disorders often overlapping and criteria frequently changing, the uncertainty created by these factors is a particular concern in mental healthcare (American Psychiatric Association, 2013).

3.2.4.3. Risks

Research has linked service availability and quality of care to patient safety (Brickell & McLean, 2011). Although a lack of awareness on the issue of patient safety has been highlighted, researchers have identified many risk factors for patient safety in mental healthcare (Callaly, Arya, & Minas, 2005). Several patient risk factors from acute medical care settings apply to mental healthcare and are frequently adopted. However, safety issues exist that are unique to mental healthcare. Studies have identified medical errors to be the foremost risk to patients in hospitals for physical disorders. While in mental health, the main concern is self-destructive behaviour (suicide and attempted suicide), violence and self-harm (Brickell & McLean, 2011; Flewett, 2010). Furthermore, critical differences in risks between physical and mental healthcare are the prevalence of patients who do not believe they are ill and refuse treatment; staff safety is directly related to the specific manifestations of mental illnesses (Briner & Manser, 2013).

3.2.4.4. Physical and Mental Health

Although mental and physical illnesses have fundamental differences, as described above, they have been found to influence each other in several ways. Lifestyle changes in the modern population are said to contribute to poor physical health, affecting the incidence rates for mental illness (Hidaka, 2012). Research into the cross-effects between physical and psychological health has a strong link (Ohrnberger, Fichera, & Sutton, 2017). Studies have also found in-direct pathways through which mental health affects physical health and vice versa (Ohrnberger et al., 2017). Several reviews and studies have highlighted that for people with severe mental illness, including schizophrenia and bipolar disorder, there are higher morbidity and mortality rates of cardiovascular diseases than the general population (De Hert et al., 2011). They also have high rates of infectious diseases, diabetes, respiratory disease, some forms of cancers and HIV (Cournos, McKinnon, & Sullivan, 2005). On the other end of the spectrum, a similar trend can be observed. Here, for patients with physical disorders, particularly those with severe disorders such as stroke, cancer, and acute coronary syndrome, depression is prevalent and harms the course of these diseases. This information is integral to understanding the differences between physical and mental health and highlighting the connections and influence of one on the other as it shapes the service provision to tackle these complex and debilitating associations. There is much 'physical' in 'mental' disorders and much 'mental' in 'physical' disorders (Kendell, 2001).

An acknowledgement of links described above has resulted in the re-conceptualization of care delivery into models of integrated care that involve co-location and interdisciplinary working of various health professionals, from mental health, physical health and social care (Hetrick et al., 2017). Although this integration improves outcomes, there are problems of sharing responsibility, uncertainties regarding the boundary between services and roles (Pomare, Ellis, Churruca, Long, & Braithwaite, 2018).

3.2.5. Summary

This section demonstrates how optimisation methodologies have a diverse history of application in healthcare. The application of optimisation methodologies has evolved to accommodate and address the ever-changing and often shifting contextual priorities of healthcare services. We have also examined the distinctive characteristics of mental healthcare and associated services.

The optimisation literature appears to have examined characteristics similar to mental health services compared to other healthcare settings. However, a comprehensive account reviewing the optimisation literature in the context of mental healthcare services does not exist. With mental healthcare being one of the immediate healthcare priorities, the application of optimisation methodologies can address major obstacles of imbalances and inefficiencies often associated with mental healthcare services. Therefore, intending to identify the application of optimisation to mental healthcare services thus far, we conduct a literature review to define future research opportunities for the application of optimisation methodologies.

3.3. Method of Review

A literature search was conducted on Scopus and Web of Science for papers published any time before December 2020, with a particular focus on articles that applied optimisation methodologies to mental healthcare service delivery. Table 3 contains a sample search strategy used across search engines and depicts the search results for each query. Additionally, a backwards referencing search and manual search of reference lists were conducted from the relevant articles, which yielded results.

Table 3: Sample Search Queries

Web of Science	(ALL=("heuristic" OR "metaheuristic") AND ALL=("mental health*" OR "community mental health*" OR "psychi*") AND ALL=("service*" OR "planning" OR "allocation" OR "scheduling" OR "design"))	22
	(ALL=("optimization" OR "optimisation") AND ALL=("mental health*" OR "community mental health*" OR "psychi*") AND ALL=("service*" OR "planning" OR "allocation" OR "scheduling" OR "design"))	40
	(ALL=("programming" OR "non-linear programming" OR "nonlinear programming" OR "linear programming") AND ALL=("mental health*" OR "community mental health*" OR "psychi*") AND ALL=("service*" OR "planning" OR "allocation" OR "scheduling" OR "design"))	153
	(ALL=("mathematical model*" OR "mathematical program*") AND ALL=("mental health*" OR "community mental health*" OR "psychi*") AND ALL=("service*" OR "planning" OR "allocation" OR "scheduling" OR "design"))	11

The identified articles underwent a set of rigorous screenings, based on two key inclusion criteria's: 1. an optimisation methodology is used; 2. the problem addressed has a mental healthcare service delivery focus. A similar inclusion criterion has been previously employed by Bradley et al. (2017). Moreover, only papers published in peer-reviewed journals and full papers in conference proceedings were included. Articles with a primary focus on epidemiology, prevention, screening, alcohol and substance abuse, and smoking cessation were not included. Following two screenings, a total number of 13 articles are included in the review, as depicted in Figure 2. Of the 13 articles published between 1976 and 2020, 7 were published before the 2000s and six after. Geographically, the majority of all articles were based on research conducted in the USA, followed by the UK.

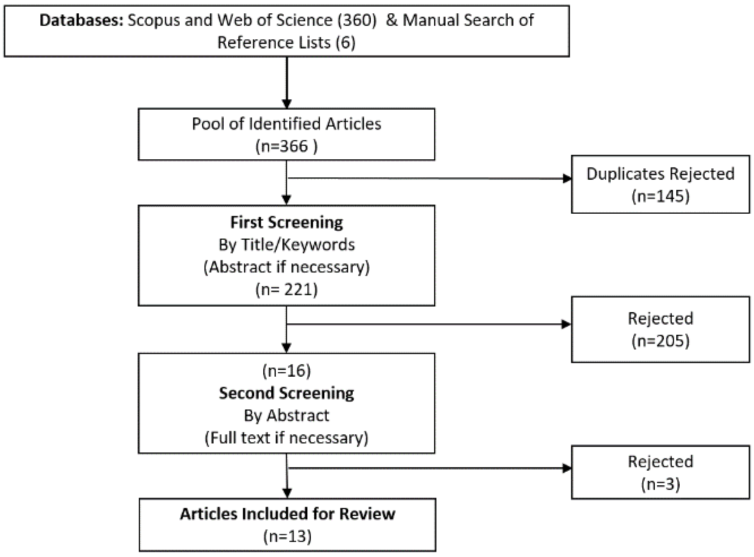

Figure 2: Search Strategy

3.4. Analysis

To analyse and classify the literature under review, taxonomies employed by existing literature reviews were referenced. Specifically, reviews on the application of OR methodologies to a specific healthcare context such as home care and those addressing a particular problem such as scheduling were drawn upon. We first describe a general overview of the literature, followed by an in-depth analysis of the optimisation models. Themes such as the level of planning, the type of planning decision, and the care setting where the study was conducted are described in this section and summarised in Table 4.

3.4.1. Planning Level & Planning Decisions

Seven studies were conducted before the 2000s. Specifically, the deinstitutionalisation of mental healthcare services- a dramatic movement of patients from state mental hospitals to the community- that began in the 1960s steered the development of optimisation models to provide transitional support. Four studies address planning decisions on a strategic level, and three address tactical level planning decisions (See Table 4). As for the classification of articles post-2000s, six are identified, three of which address decisions on a tactical planning level, two on an offline operational level, and one on both tactical and offline operational level. Notably, studies addressing strategic level decisions are absent in recent mental healthcare literature. A similar trend is observed in healthcare, where operational level planning has received the most attention. Researchers have called for more diversification in strategic and tactical planning. In contrast, online operational level planning has not been investigated in the existing mental healthcare literature. Overall, the sporadic distribution of articles and the restricted number of publications on research in mental healthcare service planning and delivery as opposed to physical healthcare are telltale signs of the limited attention given to this aspect of healthcare.

The classification of planning decisions is based on the taxonomic classification described in a review by Hulshof et al. (2012). We identify a variety of decisions across three planning levels. First, studies have tackled decisions on placement policy, regional coverage, and capacity dimensioning on a strategic level. Placement policy decisions aim to establish types of patients to the right treatment at an appropriate time through cost-effective means. Herein patients are classified based on their diagnosis, care required, and the location where care can be dispensed. These early optimisation models enabled mental health planners to simultaneously evaluate several uncertain parameters resulting from changing government fiscal policies and the availability of funds. In particular, optimisation models were used to analyse several policies to fulfil what was termed the "goal of deinstitutionalisation" of reassigning noncritical patients to non-residential services while meeting demand using available resources (Franz, Rakes, & Wynne, 1984; Specht, 1993). Regional coverage involves decisions on the number, type, and location of care facilities. In our review, under conditions of centralized geographic demand, client accessibility and convenience strategies are assessed. In particular, Muraco et al. (1977) demonstrated that the deconcentrating of mental health services under conditions of centralised demand resulted in pseudo concentration that masked the actual concentration in the service delivery system. Capacity dimensioning involves the testing of alternative scenarios for staff size or availability to fulfil projected demand. Lyon & Young (1976) described a model for allocating staff within a large psychiatric hospital. The model formulation incorporated a patient needs survey for various therapeutic activities and activity analysis of staff functions.

Several planning decisions have not been addressed on a strategic planning level in mental healthcare compared to healthcare. For instance, we found no evidence of studies addressing the ‘Facility Layout’ and ‘Care Unit Partitioning’ decision. The facility layout concerns the positioning and organisation of various physical areas in a facility. The decision related to dividing an inpatient facility into care units is called care unit partitioning decisions. These decisions are critical elements of in-care mental health safety and harm reduction. In particular, designing facilities with increased visibility to allow staff to monitor and observe at-risk patients closely has the potential to help minimise the risk of suicides (Reiling, Hughes, & Murphy, 2008).

Additionally, ‘case mix’ which is the volume and composition of patient groups that the facility serves; ‘panel size’, the number of potential patients; and ‘service mix’, the particular services a facility provides, are all decisions that are yet to be addressed in mental healthcare literature. These decisions on a strategic level aim to help maintain a minimum standard of service while efficiently using scarce resources. Mental healthcare services could greatly benefit from deploying optimisation models to address these decisions, especially given the issues of accessibility and reduced resource availability.

Second, on a tactical level, admission control decisions, appointment scheduling, and staff-shift schedule have been addressed. Admission control relates to determining rules on which a patient can be admitted from a waiting list into a service. Hertz & Lahrichi (2009) proposed a model that balances nurses' workload who provided long-term and short-term care to five categories of patients, including patients with serious mental health problems. At the same time, several studies developed models that allocated resources and various treatment modalities to patients categorised based on their needs and diagnosis. In particular, Heiner et al. (1981) developed a resource allocation and evaluation model for several clusters of intellectually disabled patients in a multi-service delivery system based on efficiency, effectiveness, and equity measures. Leff et al. (1986) developed a planning model that allocated services to chronically mentally ill patients to improve care outcomes. Appointment scheduling has involved the development of a blueprint used to specify a time and date for patient consultation/treatment. Samorani & LaGanga (2015) set out to overbook appointments optimally given no-show predictions of patients in a large mental health centre with a high no-show rate. Pagel et al. (2012) allocate appointments subject to waiting times to maximise desired clinical outcomes in a primary mental healthcare system. Scheduling of shifts to staff determines which shifts are to be worked and by how many employees. A shift-staff schedule is developed for medical residents specialising in psychiatry at a medical university, spanning 365 days by Cohn et al. (2009).

We could not find studies that addressed several tactical level decisions such as capacity allocation, patient routing, and unused capacity (re)allocation that are available in healthcare literature. Particularly relevant planning mental healthcare services are capacity allocation decisions where resource capacities settled on the strategic level are subdivided over patient groups. For many countries, increasing mental healthcare provision and ensuring that resources are distributed equitably are priorities (Anselmi et al., 2020). Increasingly, the geographical distribution of resources is encouraged to reflect need. As such, an optimisation model for capacity allocation can be a means to achieve equitable distribution of access.

Lastly, operational decisions on staff-to-shift assignment, assessment and intake, visit scheduling, and patient-to-appointment location scheduling are addressed. Based on the staff-shift schedule, specific dates and times associated with shifts are assigned to staff in staff-to-shift assignments. For instance, Bester et al. (2007) use their model to build duty rosters for nurses at a psychiatric facility that includes a fairness component. Assessment and intake decisions include a process wherein a patient referred to a service is assessed for eligibility (based on the placement policy), care requirements are determined, and a care provider is assigned. Such a model was built by Hertz & Lahrichi (2009) to assign a care provider with a workload-balancing component. Similar to staff-shift scheduling, visit scheduling involves determining which staff member will perform a visit on which day and time. Visit scheduling was modelled for travelling physicians (Li, Y., Kong, Chen, & Zheng, 2016) and home care workers (Hertz & Lahrichi, 2009). Based on the appointment schedule blueprint developed on a tactical level, the scheduling of a particular patient to a specific location has been addressed in the literature. Specifically, to improve access to care, Li et al.

(2016) investigated the problem of scheduling patients with chronic mental disorders to an outreach clinic location.

Some operational planning decisions that have not been considered in mental healthcare are decisions associated with short-term care planning, as observed in the context of home care services (Hertz & Lahrichi, 2009). Another set of planning decisions that have not been modelled in mental healthcare is scheduling a combination of appointments as observed in cancer care (Petrovic, Morshed, & Petrovic, 2011) and a series of appointments modelled in rehabilitation care (Chien, Tseng, & Chen, 2008). In particular, individuals with severe complex mental health needs often require support from several different agencies in the community. Internationally, "case management" policies have been devised to promote patient-centred care coordination and care planning for individuals with complex health needs combining multiple chronic conditions with psychosocial or mental health comorbidities (Hudon et al., 2017). Broadly speaking, case management is an umbrella term for various care models that ensure that service users are provided with coordinated, effective and efficient care based on an assessment of their needs. In mental healthcare, such a care model is concerned with comprehensively coordinating services to meet a patient's mental health needs. Variants are found in the USA (Rapp & Wintersteen, 1989), the UK (Department of Health, 1990), Australia (Rickwood, 2006), New Zealand (Mental Health Commission, 2012), and the Netherlands (Van Veldhuizen, 2007). Although policy aspirations have created an expanded mental health system that encompasses large-scale care provision to people living in the community, a significant gap exists between policy aspirations and operational practices (Jones, Hannigan, Coffey, & Simpson, 2018). Services face challenges in designing operations that support staff and service users in realising personalised care. As discussed in the previous sections, MASPH is only just gaining momentum in the hospital settings to address a patient's need to access multiple resources. In contrast, models for settings outside the hospital are still missing. In particular, the gap does extend to care provided in the community and to mental healthcare.

In this section, we have established that optimisation models in mental healthcare have focused mainly on the application area of planning, scheduling and routing. We have demonstrated that a good proportion of healthcare optimisation literature has also focused on this area. We have found several parallel and gaps in the levels of planning and decision types between the two contexts in this area. However, we found no evidence of models for the supply chain management of mental healthcare. Notably, inventory management does not apply to mental healthcare, as it does not involve the use of medical instruments.

However, facility location as an area of research is highly relevant to mental healthcare as it is to extant healthcare. Among the challenges associated with reduced access or discontinuity of care in mental healthcare services is the geographical distance to services (Carbonell, Navarro-Pérez, & Mestre, 2020). Existing optimisation models for healthcare facility location have been deployed to minimize access costs for healthcare consumers, maximise population with access to a healthcare facility, and maximize the equity in access (Güneş et al., 2019). This extensively researched area of application is unexplored in mental healthcare and is a promising avenue for future research.

3.4.2. Care Setting

Articles are classified into three care settings based on the number and distribution of care units that were the focus of modelling: single care unit, multi-care unit, and multi-site care network. A single care unit refers to one health centre, for instance, a single outpatient clinic, as observed in the study by Samorani & LaGanga, (2015). On the other hand, multi-care units refer to a single care organisation with multiple subunits, for instance, a psychiatric hospital with several wards, as seen in studies conducted by Lyons & Young (1976) and Bester et al. (2007). Moreover, a multi-site care network comprises multiple care units distributed over a geographic area. Most articles under review have built models to address planning issues in such networks, such as a regional hierarchical care system (Muraco et al., 1977); a conceptual network of community mental healthcare system spanning across local, state and federal bodies (Wolpert & Wolpert, 1976); a system composed of numerous distinct mental healthcare providers (Heiner et al., 1981; Leff et al., 1986); and a care system consisting of a hospital and community mental health care centre (Franz et al., 1984; Specht, 1993). More recently, models have been used to address planning issues in a network of multiple sites such as psychiatric hospitals (Cohn et al., 2009; Pagel et al., 2012); home care services (Hertz & Lahrichi, 2009); and outpatient speciality clinics (Li, Y. et al., 2016). Remarkably, models in mental healthcare literature are spread over a range of care settings, reflective of the diversity inherent in the services. In contrast, modelling multiple care settings is a relatively recent development in other healthcare services.

3.4.3. Model Objectives

This section describes objective functions used in optimisation models in planning mental healthcare services so far. Optimisation models can have single or multiple objective functions. In a single objective function model, the optimal decision is taken based on one objective. In a multi-objective function model, more than one objective must be satisfied (Hwang & Masud, 2012). Our analysis has found that 5 of the 13 papers have used multi-objective function models, as seen in Table 5. Furthermore, the objective functions are divided into five observed categories: maximization of patient outcomes, maximization of constraint/goal satisfaction, minimization of costs, maximization of resource allocation and utilisation, and minimization of patient dissatisfaction. Table 5 depicts the objective functions for each article.

Table 4: Optimisation in Mental Healthcare Literature Thematic Overview

Author	Planning Level	Planning Decision	Care Setting
Lyons & Young (1976)	Strategic	Capacity Dimensioning - Staff	Multi-care units
Wolpert and Wolpert (1976)	Tactical	Admission Control	Multi-site care network
Muraco et al. (1977)	Strategic	Regional Coverage - Care Centre Location	Multi-site care network
Heiner et al. (1981)	Tactical	Admission Control	Multi-site care network
Franz et al. (1984)	Strategic	Placement Policy	Multi-site care network
Leff et al. (1986)	Tactical	Admission Control	Multi-site care network
Specht (1993)	Strategic	Placement Policy	Multi-site care network
Bester et al. (2007)	Operational (Offline)	Staff-to-Shift Assignment	Multi-care units
Cohn et al. (2009)	Tactical	Staff-Shift Schedule	Multi-site care network
Hertz & Lahrichi (2009)	Tactical	Admission Control	Multi-site care network
	Operational (Offline)	Assessment and Intake Visit Scheduling - Short-Term Care Plan - Staff-to-Visit Assignment	
Pagel et al. (2012)	Tactical	Appointment Scheduling	Multi-site care network
Samorani & LaGanga (2015)	Tactical	Appointment Scheduling	Single-care unit
Li et al. (2016)	Operational (Offline)	Patient-to-Appointment Location Scheduling Visit Scheduling - Staff-to-Visit Assignment	Multi-site care network

3.4.3.1. Maximising Constraint/Goal Satisfaction

As described by the Donabedian framework, quality of care includes the organisation of care (or structure), the influence of structure on care delivery processes, and patient-level health care outcomes (Kilbourne et al., 2018; McDonald et al., 2007). Therefore, to provide safe, effective, patient-centred, timely, efficient, and equitable care, services are faced with diverse priorities and competing goals. 4 of the 13 papers under review define multiple goals in their objective. Specifically, Franz et al. (1984) and Specht (1993) explore multi-objective optimisation using goal programming for resource allocation. Both models maximize and prioritise diverse, conflicting goals, including budget, patient load, patient admission/reassignment, community education, demand satisfaction, staff and service capacity. More recently, Cohn et al. (2009) found the most feasible schedule that satisfies constraints of staff availability, staff capacity, staff preference, and demand satisfaction while also emphasising schedule fairness. Similarly, Hertz & Lahrichi (2009) model fairness as a function of workload balancing, measured by minimising travel load, caseload and visit load of staff.

3.4.3.2. Maximising Patient Outcomes

In mental healthcare, patient outcomes measure whether the care received by a patient has led to improvements in their symptoms – e.g., improvement or remission – or functioning or treatment completion (Kilbourne et al., 2018). These measures assist service providers in planning, monitoring and adjusting treatment options. Similar outcomes have been modelled as objective functions of 4 articles. Leff et al. (1986) define an objective function in which patient outcome is the maximisation of the net forward movement of a patient in a care system in terms of functional improvement or regression. Along similar lines, Heiner et al. (1981) define an objective function that maximises the aggregate improvement in the functioning of each patient cluster (also called the deinstitutionalisation objective). Wolpert et al. (1976) define an objective function that maximises outcomes by reducing patient dependency on mental healthcare, social welfare and law enforcement. More recently, maximising the number of patients who complete treatment was considered as the objective in the model built by Pagel et al. (2012).

3.4.3.3. Minimizing Costs

Whilst mental illness accounts for 13% of health care costs globally, it receives on average 3% of healthcare funding in mid, high-income countries and 0.5% in low-income countries (World Health Assembly, 2012). When mental health issues are recognised and responded to, they have sizeable impacts on budgets associated with treatments delivered in inpatient, outpatient, community and primary care settings (Knapp & Lemmi, 2019). Consequently, economic costs associated with mental disorders and disease are generally distinguished between direct and indirect costs (Trautmann, Rehm, & Wittchen, 2016). Direct costs—also referred to as 'visible costs'—are associated with diagnosis and treatment in the healthcare system, including the use of hospital services, medication, staff time, ambulances, psychotherapy, and primary and community care (Ride, Kasteridis, Gutacker, Aragon, & Jacobs, 2019). Indirect costs—also called 'invisible costs'—include reduced labour supply, premature mortality, reduced health-related quality of life, lost output, lost tax revenue, transfer payments, and unpaid care by family or friends (Emily & Valerie, 2014). Costs associated with treatment in the mental healthcare system have been used in 3 of 13 papers we review. Specifically, Muraco et al. (1977) define a single objective function that minimises costs incurred by a client when travelling to treatment centre locations. Bester et al. (2007) describe a multi-objective function, which is a combination of remuneration costs and accumulated nurse dissatisfaction—a measure of mismatch between their schedule preference—corresponding to current and previous assignments. More recently, Samorani & LaGanga (2015) maximised the profits of a mental healthcare centre by overbooking appointments on a schedule. Herein, profits are maximised to minimise costs associated with patient waiting time and clinic overtime, besides also maximising the number of patients seen.

Table 5: Classification of Literature Based on Objective Functions

	Max. Objectives			Min. Objectives		
Authors	Resource Allocation & Utilisation	Constraint Satisfaction	Patient Outcomes	Costs	Patient Rejection/dissatisfaction	Patient Travel Time
Lyons & Young (1976)	✓					
Wolpert & Wolpert (1976)			✓			
Muraco et al. (1977)				✓		
Heiner et al. (1981)	✓		✓			
Franz et al. (1984)		✓				
Leff et al. (1986)			✓			
Specht (1993)		✓				
Bester et al. (2007)				✓		
Cohn et al. (2009)		✓				
Hertz & Lahrichi (2009)		✓				
Pagel et al. (2012)			✓			
Samorani & LaGanga (2015)				✓		
Li et al. (2016)					✓	✓
	2	4	4	3	1	1

3.4.3.4. Maximise Resource Allocation & Utilization

Many countries face the challenge of providing adequate human, material, technological, and financial resources for delivering essential mental health services. Lack of funding—

described in minimizing costs—is compounded with a worldwide shortage of human resources for mental health (World Health Organization, 2018b). Therefore, mental healthcare services often consider allocating and utilising their human resources with *outcome* vs *output and productivity* (Daniels, 2016; Davies, 2006). Specifically, a fundamental trade-off is between meeting patient needs (medical outcome) and contractual agreements (outputs) in allocating staff. Of the papers under review, Heiner et al. (1981) define an objective function that minimises deviation from improvement in patient outcome by ensuring equitable distribution of resources (staff and services). Besides, efficient utilization of resources is about maximising productivity by matching staff to appropriate roles by considering skill mix, role design, staff shortages and inequities, and service requirements. Lyons & Young (1976) modelled an objective function that maximizes staff utilisation by maximising an aggregate appropriateness score for all personnel performing activities. Herein, appropriateness levels for 12 types of personnel for the performance of 13 planned therapeutic activities in the service were fed to the model.

3.4.3.5. Minimize Patient Rejection/Dissatisfaction

A recent study allocates optimal appointment locations for patients and includes the minimization of the penalty associated with unsatisfied appointment requests, in addition to minimizing the travel time for patients (Li, Y. et al., 2016).

3.4.4. Model Constraints

Constraints are generally interpreted as limits or boundaries governing the system being modelled. The nature of these limits is diverse and includes limits on the availability of resources, funding, time-based limits (temporal) and capacity. In this review, constraints have been grouped based on their primary focus and the nature of their application. Specifically, constraints have focused on the service provider, staff and patient, and the nature of the constraints are relative to service delivery, geography and temporality. Table 6 provides a detailed overview of the constraints with respect to their publication.

3.4.4.1. Service Delivery Constraints

Service delivery constraints have been considered in relation to the service provider and service staff. Constraints used in models are relative to budget, resource availability, service capacity, assignment of tasks to staff, assignment of service packages and mandatory services.

A recent study produced country-level estimates in the Americas for the proportion of total disease burden to the health expenditure and found a striking imbalance in the ratio between disease burden of mental health and allocated spending (Vigo, Kestel, Pendakur, Thornicroft, & Atun, 2019). The spending ranged from 3:1 in Canada and the USA to 435:1 in Haiti, with a median of 32:1 across 30 countries (Vigo et al., 2019). Because of such historical imbalances, models in literature have given considerable attention to budgeting. Articles under review have examined the allocation of a fixed budget (Franz et al., 1984; Heiner et al., 1981; Leff et al., 1986; Lyons & Young, 1976; Specht, 1993; Wolpert & Wolpert, 1976). These articles have included a diversity of budgetary constraints in their models, such as the maximum allowable monetary expenditure (Franz et al., 1984; Specht, 1993; Wolpert & Wolpert, 1976), per capita budget (Leff et al., 1986), total available personnel budget (Lyons & Young, 1976); and government-mandated budget (Heiner et al., 1981).

Evidence indicates that mental health workers account for only 1% of the global health workforce. Approximately 45% of the global population resides in a country with less than one psychiatrist per 100,000 people (World Health Organization, 2018b). Two articles have defined limits on the availability of resources. Leff et al. (1986) set an upper limit on the amount of personnel available in the service, whereas (Lyons & Young (1976) have a fixed number of beds. Furthermore, mental health services continually experience rising demands and, in many cases, exceed available capacity. For instance, bed occupancy rates for inpatient services regularly exceeds recommended levels to maintain safety standards, highlighting the significant pressure the system is under (World Health Organization, 2018b). Therefore, a variety of service capacity constraints, such as the limitation on service provider capacity (Heiner et al., 1981), number of available service hours (Li, Y. et al., 2016; Lyons & Young, 1976) and the maximum number of appointments (Pagel et al., 2012), have been formulated.

From a staffing perspective, constraints focusing on task assignment, staff preferences, the sequence of shifts and skill requirements have been considered. Assignment of tasks to the staff mainly specifies permissible values of the maximum and the minimum number of staff per task or tasks per staff. Examples include constraints that specify the minimum and the maximum number of shift assignments for a nurse (Bester et al., 2007) and constraints limiting any physician's assignment to an outpatient appointment (Li, Y. et al., 2016). Additionally, staff preference constraints have been used to model vacation and weekly shift assignment requirements in the appointment-scheduling problem (Cohn et al., 2009). Instances of constraints corresponding to skill requirements include defining the minimum number of nurses of a particular rank to be assigned to a shift (Bester et al., 2007) and assigning a patient to a type of nurse (Hertz & Lahrichi, 2009).

Several countries rely on government policies that specify values, principles and objectives of a population’s mental health (Zhou, Yu, Yang, Chen, & Xiao, 2018). These policies are implanted in several domains such as service organizing, service provision, service quality, human resources etc. (Zhou et al., 2018). Countries face several challenges in the implementation of these policies. Articles under review have modelled such features as constraints related to mandatory (government or service organisation) service hours for a service or groups of services. Heiner et al. (1981) formulate the mandated number of service hours for each individual in a patient cluster—based on functional skills, social skills and motor disabilities.

In contrast, Cohn et al. (2009) model the mandatory coverage for a network of hospitals as a constraint by specifying the compulsory presence of one primary and backup member of staff on any given day. A minimum limit on the number of three different types of appointments to be allocated is included by Pagel et al. (2012), and mandatory patient follow-up constraints are outlined by Specht (1993). Lastly, constraints ensuring appropriate service assignment to patients are defined in the resource allocation model built by Leff et al. (1986). A constraint ensures that patients at a functional level are assigned suitable service packages. Whereas, for booking outpatient appointments, a constraint allocates at most a single appointment slot to a patient (Samorani & LaGanga, 2015).

3.4.4.2. Temporal Constraints

In this section, temporal constraints relative to service providers, staff and patients are examined. These constraints are based on time relationships between entities. Specifically, these are used to orient an event on a timeline, specify the duration of an event, and determine the order of an event to other events. There are two main types of temporal constraints, sequencing and real-time (Kuhn et al., 2015). Sequencing constraints specify the order in which a sequence of actions or events is allowed to take place. For instance, a sequential constraint would specify that two night shifts should not be scheduled in sequence. On the other hand, a real-time constraint may specify the explicit references to time. For instance, an event must take place 10 minutes before another event. From a service provider perspective, Samorani & LaGanga (2015) have included a lead-time (time between initiation and completion of process) constraint for booking appointment requests, which ensures that any request is assigned to at most one of the days that follow its arrival.

From a staff perspective, because of limited resource availability, staff are said to experience ‘brain drain’ resulting in low morale and high turnover. This leads to a significant obstacle in retaining staff required to deliver services (Thornicroft, Deb, & Henderson, 2016). For instance, the National Health Service in the UK has recorded a drop of 11% in the mental healthcare nursing workforce between 2009 and 2019 (Buchan, Gershlick, Charlesworth, & Seccombe, 2019). Therefore, the prevention of overburdening workloads is a critical challenge in managing the workforce. In addition to addressing capacity issues described in the previous section, the distribution of tasks/work to staff is defined within a model through temporal constraints. Cohn et al. (2009) have included restrictions on the number of daily and weekly on-calls for staff. While studies published before the 2000s have formulated constraints that limit the number of hours staff spend supervising or receiving supervision (Lyons & Young, 1976) and constraints on total time available for psychiatrists to dispense services (Franz et al., 1984; Specht, 1993).

Finally, for a patient, shorter waiting times are said to affect patient outcomes positively. This is particularly so for conditions such as psychosis and in services for children and adolescents (Reichert & Jacobs, 2018; Schraeder & Reid, 2015). Waiting times have been observed to be a contributing factor to high rates of 'no shows', greater likelihood of disengaging from services and worsening of conditions (Schraeder & Reid, 2015). By reducing waiting time, services have the potential for efficiency gains and cost savings. Furthermore, studies have found that rapid access reduces the 'no show' rates falling by more than half and reduces crisis hospitalisations (Williams, Latta, & Conversano, 2008).

Additionally, from an economic point of view, poor outcomes related to an extended waiting period, which prevents patients from working, has associated costs (OECD, 2020; Reichert & Jacobs, 2018). While waiting time is often incorporated into the objective function, waiting times as temporal constraints have been included by Pagel et al. (2012) to facilitate introducing a new care systems model. These constraints specify the maximum allowable increase in waiting time for patients and define waiting-time periods for different service types.

3.4.4.3. Geographic constraints

Although mental health services do not adhere to a distinguished model of providing care, most services are in inpatient or community settings. While accessibility to services is impacted negatively by waiting lists, equally important is the uneven geographical distribution of service locations and staff (Samartzis & Talias, 2020). Geographic constraints in mental healthcare optimisation literature have primarily been associated with planning models built to aid deinstitutionalisation.

Table 6: Classification of Literature Based on Model Constraints

	Service Provider								Staff						Patient
	Delivery					Geographic		Temporal	Delivery				Geographic	Temporal	Temporal
Authors	Budget	Resource Availability	Service Capacity	Mandatory Service Requirements	Service Package Assignment	Demand Coverage	Patient Load Balancing	Lead Time	Staff Assignment Thresholds	Staff Preference	Shift Sequence	Skill Requirement	Location Preference	Worktime Thresholds	Patient Waiting Time
Lyons & Young (1976)	×	×	×											×	
Wolpert & Wolpert (1976)	×														
Muraco et al. (1977)						×	×								
Heiner et al. (1981)	×		×	×											
Franz et al. (1984)	×					×	×						×	×	
Leff et al. (1986)	×	×			×										
Specht (1993)	×			×		×	×							×	
Bester et al. (2007)									×		×	×			
Hertz & Lahrichi (2009)												×			
Pagel et al. (2012)			×	×											×
Cohn et al. (2009)				×						×			×	×	
Samorani & LaGanga (2015)					×			×						×	
Li et al. (2016)			×						×				×		

Therefore, they have been applied to a large region consisting of a network of care services. Franz et al. (1984) and Specht (1993) have considered two types of constraints. The first type satisfies patient demand in a region and increases the number of patients reached by community-based educational programmes. Second increases the flow/transition of patients from institutional care to community care. A single article addresses facility location of community mental health services in a geographical area by incorporating demand coverage constraints to equally assign demand amongst community centres (Muraco et al., 1977). In contrast, staff-related geographical constraints have taken the form of location preferences. For instance, preferences are taken into consideration for determining appointment locations for medical residents (Cohn et al., 2009), community staff (Franz et al., 1984) and physicians (Li, Y. et al., 2016).

3.4.5. Model Formulation

Five types of optimization techniques have been employed by the studies included in our review: linear, integer, mixed-integer, goal and stochastic programming. Linear programming is an optimization technique to determine the value of decision variables that maximize or minimize a linear objective function where decision variables are subject to linear constraints (Chong & Zak, 2004; Vanderbei, 2020). Linear programming is employed in various application areas, including production planning, inventory control, and workforce planning (Mula, Poler, García-Sabater, & Lario, 2006; Taha, 2017). Of the articles under review, linear programming has been used for locating care centres (Muraco et al., 1977), assigning patients to services (Heiner et al., 1981; Leff et al., 1986; Wolpert & Wolpert, 1976), and scheduling appointments (Pagel et al., 2012). Furthermore, Integer programming is the same as linear programming except that all decision variables are constrained to be integers. When some but not all decisions are restricted to be integers, the optimisation technique is referred to as mixed-integer programming (Taha, 2017). Integer programming is often used to formulate scheduling problems (Vanderbei, 2020). In this review, scheduling of patient appointments (Li, Y. et al., 2016) and staff (Bester et al., 2007; Cohn et al., 2009) have been modelled using integer programming. While staff dimensioning (Lyons & Young, 1976) and assigning patients to services are addressed using mixed-integer programming (Hertz & Lahrichi, 2009).

Goal programming can be thought of as an extension of linear programming to handle multiple, conflicting objectives. A target value to be achieved is specified for each goal, and unwanted deviations are then minimized (Winston & Goldberg, 2004). Often, goal programming is used to provide the best satisfying solution under conditions of multiple goal priorities. Among the 13 articles under review, two have used goal programming to analyse alternative placement policies (Franz et al., 1984; Specht, 1993). Stochastic programming constitutes a framework for modelling optimization models in the presence of uncertainty (Ruszczynski & Shapiro, 2003). Decision problems addressed by stochastic programming are canonically expressed as “some decisions must be made today, but important information will not be available until after the decision is made” (King & Wallace, 2012). Samorani & LaGanga (2015) incorporate uncertainty regarding appointment cancellation and no-show probability by formulating a model using stochastic programming.

The optimisation techniques used to formulate problems in a mental healthcare setting are similar to techniques used in extant healthcare. However, as can be observed, optimisation in mental healthcare is limited and sporadically dispersed. Therefore, it appears that the choice of formulation technique is essentially a reflection of ‘when’ the study was conducted and corresponds to the progressive development of optimisation techniques. Even so, the more recent study by Samorani & LaGanga (2015) is an exemplar in healthcare research for having been the first to integrate predictive analytics, optimisation and overbooking for scheduling.

3.4.6. Solution Algorithm

Once the model is defined, it can be solved by a solution algorithm. Formalised by Turing (1937) and Church (1936), an algorithm is a finite set of well-defined instructions for accomplishing a task. In optimisation, an algorithm's goal is to find a solution with minimal or maximal evaluation time (Rothlauf, 2011). Solution algorithms for optimization problems can be roughly distinguished into two types: exact algorithms and heuristics. Articles under review have been categorised based on the type of solution algorithm deployed, as seen in Table 7. Most often, the solution algorithm of choice speaks to the complexity and size of a problem. This section will explore each model solution based on the type.

3.4.6.1. Exact Solution Algorithms

Exact solution algorithms are designed in such a way that they guarantee finding an optimal solution in a finite amount of time. To do this, exact algorithms conduct an exhaustive search of every single solution in the solution space. Exact solutions algorithms were employed by 10 (of 13) articles under review. The most used algorithm was simplex (n=6), whereas branch-and-bound, branch-and-cut, column generation and nested decomposition were used once by four different articles.

Wolpert and Wolpert (1976), Heiner et al. (1981) and Pagel et al. (2012) have solved their linear programming problem by directly applying the simplex algorithm. The simplex algorithm effectively solves Linear Programming (LP) problems with continuous decision variables (Dantzig, G. B., 1998). In particular, the algorithm finds an optimal solution to a problem, where the objective function depends linearly on the continuous decision variables. Specifically, the algorithm sequentially tests multiple values in a set of feasible values to improve the objective function until the optimal solution is found. Franz et al. (1984) and Specht (1993) used the goal programming variant of the simplex algorithm, which operates on multiple objective functions, where each objective is ranked. The algorithm prioritises goals with a higher priority, unlike in LP, where an objective function is optimized.

Lyons and Young (1976) employ the Branch and Bound (B&B) algorithm to solve a mixed-integer programming problem. B&B is a common enumerative approach to solving LP problems with discrete decision variables. Solving a problem using B&B involves recursively decomposing a problem into sub-problems, which are then solved using LP methods like the simplex algorithm (Land & Doig, 2010). Hertz and Lahirichi (2009) used Branch and Cut (B&C) to solve a mixed-integer programming problem. B&C algorithms combine B&B with cutting planes methods. Specifically, cutting plane methods add additional constraints (cutting planes) to a problem. The original constraints are replaced by alternative constraints closer to producing a feasible integral solution and exclude fractional solutions (Mitchell, 2002). Leff et al. (1986) deployed a nested decomposition algorithm (Glassey, 1973) to solve the resource allocation model. Decomposition algorithms split a problem into a master problem and one or more slave problems. The solution of the master problem is then fed to the slave problem to determine feasibility (Dantzig, G. B. & Wolfe, 1961).

Samorani & LaGanga (2015) used column generation to solve an integer programming problem. This approach is selected for scheduling outpatient appointments to keep the number of constraints low. A column generation algorithm is typically applied to problems where it is not possible to consider all variables explicitly (Desaulniers, Desrosiers, & Solomon, 2006). Therefore, a problem is split into two problems: the restricted master problem and the sub-problem. The master problem works only with a sufficient subset of variables. In contrast, the sub-problem is created to identify new promising variables with reduced negative cost, which are then added to the master problem and resolved. This process is repeated until no negative reduced cost variables are identified.

Table 7: Classification of Literature Based on Solution Algorithms

Author	Model Formulation	Solution Type	Solution Algorithms	Solver
Lyons & Young (1976)	Mixed-Integer Programming	Exact	Branch and Bound	
Wolpert and Wolpert (1976)	Linear Programming		Simplex	
Heiner et al. (1981)	Linear Programming		Simplex	
Franz et al. (1984)	Goal programming		Simplex	IBM's MPSX
Leff et al. (1986)	Linear Programming		Nested Decomposition	
Specht (1993)	Goal programming		Simplex	
Cohn et al. (2009)	Integer Programming		Simplex	CPLEX
Pagel et al. (2012)	Linear Programming		Simplex	Microsoft Excel
Muraco et al. (1977)	Linear Programming	Heuristics	Alternating Heuristic	
Li et al. (2016)	Integer Programming		Primal and Local Search Heuristics	CPLEX
Bester et al. (2007)	Integer Programming	Metaheuristics	Tabu Search	Microsoft Visual Basics
Hertz and Lahirichi (2009)	Mixed-Integer Programming	Exact & Metaheuristic	Branch and Cut & Tabu Search	CPLEX
Samorani and LaGanga (2015)	Stochastic programming	Exact and Heuristic	Column Generation & Heuristic	

3.4.6.2. Heuristics

For large problems, which cannot be solved using exact algorithms, heuristics are employed. Heuristics do not guarantee an optimal solution and generally return suboptimal solutions. Furthermore, heuristics are often problem-specific. In literature, two types of heuristics are distinguished: construction heuristics and improvement heuristics (Rothlauf, 2011). Construction heuristics build a complete solution from scratch by following a step-wise creation process. On the other hand, improvement heuristics start with a complete solution and then try to improve the solution iteratively. Three studies have utilised heuristics to solve their optimisation problems.

Samorani & LaGanga (2015) develop a new heuristics policy to schedule outpatient appointments. Since the 'column generation' approach took a long time to solve—if the rejection of patients is not allowed—a new heuristic policy was developed and solved to near optimality. The heuristic schedule predicted shows in the near future and predicted no-shows into the future. This new procedure was found to outperform the exact solution. Further, Muraco et al. (1977) deployed an 'alternating heuristic' represented by alternate steps of location assignment and demand allocation, which continues until an optimal minimal configuration is achieved within the given constraints. This heuristic was used to find a location with minimum transport and then assign a service centre to each location, followed by the allocation of demand to these centres.

Li et al. (2016) employ both construction and improvement heuristics to construct physician assignments in an outpatient care network. Specifically, several column generation based primal heuristic algorithms were used to construct assignments, followed by several local search algorithms to improve the assignments further. In particular, heuristics that are based on exact methods are called primal heuristics (Joncour, Michel, Sadykov, Sverdlov, & Vanderbeck, 2010). In contrast, local search heuristics are applied to problems that are formulated to find a solution that maximises a criterion among several candidate solutions. Notably, the algorithm moves from solution to solution in the space of candidate solutions by applying local changes, until a time-bound elapses or an optimal solution is found.

3.4.6.3. Metaheuristic

Improvement heuristics that use a search strategy that is general, widely applicable and problem-invariant are denoted as metaheuristics (Glover, Fred W. & Kochenberger, 2006). Two of the articles under review have employed metaheuristics. Bester, Nieuwoudt et al. (2007) developed a decision support tool for nurse rostering that is underpinned by the tabu search method. While Hertz & Lahirichi (2009) use tabu search for a patient assignment. Specifically, tabu search is a metaheuristics search method that builds on local search by relaxing its basic rule (Glover, Fred W. & Kochenberger, 2006). Not unlike local search, tabu search takes a potential solution and checks its immediate neighbours in the hope of finding a solution. However, unlike local search, tabu search will accept moves that worsen the solution if no other improving move is available. Besides, the method uses a list of prohibitions (termed tabu list) to discourage the solution from returning to previously visited solutions.

The choice of solution methods is dependent on how complex, large, and computationally cumbersome the problem is. The increase in computer power has also increased the scope of solvable applications. As can be observed in Table 7, early applications mainly deployed Simplex to solve their optimisation problem. More recently, the complexity of solutions is reflected in the type, algorithm of choice and the use of specialised software packages such as CPLEX. As noted earlier, the application of optimisation to mental healthcare is trailing compared to other healthcare settings.

3.5. Discussion

The application of optimisation to mental healthcare is in its nascent stages. We have assembled a purposefully broad-ranging future research agenda, drawing on several significant trends and characteristics from healthcare literature. For the future development of optimisation models in healthcare, we outline actionable themes such as incorporating uncertainty and risk, timely access to care, continuity of care, multiple care settings, integrated care settings and, new modelling and solution methodologies.

3.5.1. Incorporating Uncertainty and Risk in Mental Health Optimisation Models

Models are beginning to incorporate dynamic aspects of the healthcare system by integrating sources of uncertainty and risk in application areas such as inventory management, facility location, and planning and scheduling of operating rooms. Uncertainties have been included in optimisation models in several care settings such as cancer care (Mahmoudzadeh, Purdie, & Chan, 2016), surgical care (Koppka, Wiesche, Schacht, & Werners, 2018), in the management of operation theatres (Kroer et al., 2018) and home healthcare (Yuan, Liu, & Jiang, 2015). In the context of mental healthcare, it appears that some studies have incorporated uncertainty either explicitly or implicitly. A recent study has explicitly modelled uncertainty regarding appointment cancellation and no-shows by using patient progress indicators to make no-show predictions (Samorani & LaGanga, 2015). Models built in the context of deinstitutionalisation have incorporated uncertainty implicitly corresponding to funding and budgets. Effectively, uncertainty in mental healthcare optimisation models is lagging in both scope and depth compared to broader healthcare. Particularly challenging to model is the uncertainty associated with diagnosing psychological disorders, which influences treatment pathways and subsequent treatment outcomes. Furthermore, mental health services' co-location and interdisciplinary nature pose uncertainties regarding the boundaries between services and roles. Indeed, healthcare literature has deemed it necessary to integrate uncertainty to expand the scope of application. Essentially, this assertion extends to mental healthcare.

Risk factors in mental healthcare are predominantly related to the self-destructive behaviour of patients and staff safety relative to specific manifestations of mental illness. These risk factors are often associated with risk categories, including (but not limited to) individual risk factors, demographic variables, treatment history, and social variables (Franklin et al., 2017). Risk assessment tools are a central practice in mental health services. Often, they are used as a helpful adjunct to inform management plans (Appleby, Kapur, & Shaw, 2018). In mental health optimisation literature, Leff et al. (1986) use a similar approach to categorise patients based on a spectrum of functional levels, starting from ‘dangerous’ to ‘Recovering’. Patients from each category are then assigned to specific service packages. Even so, in recent studies, no such consideration of risk has been considered. In healthcare literature, risks associated with various care settings have been included in optimisation models in multiple contexts.

For instance, the risk of surgery cancellation (Wang, Y., Tang, & Fung, 2014), operational risk (Ahmadi et al., 2019), and longer procedure times have been modelled. In the context of mental healthcare, future research could look to existing models that incorporate risk. Besides, the inclusion of risk relative to both patients and staff is an essential strand of consideration for future research.

3.5.2. Models to Address Timely Access for Mental Health Services

Several parallels can be drawn between the service provision of cancer care and specialist mental health care. Recent initiatives to improve specialist mental health services align with some principles that underpin good practice in cancer care (Larkin, Boden, & Newton, 2017). While acknowledging clinical differences between the two care systems, it has been argued that comparisons between cancer care and mental illnesses such as psychosis provide a valuable lens to examine service provisions (Larkin et al., 2017). Not unlike mental healthcare, cancer care combines hospital care, outpatient care, and home care (Gospodarowicz et al., 2015). Although cancer treatment is mainly hospital-based, and mental healthcare is mainly community-based. In both cancer care and specialist mental healthcare, the ethos of providing timely access to care is yet another parallel (Mulville, Widick, & Makani, 2019; National Academies of Sciences, Engineering, and Medicine, 2018). A substantial amount of research utilizing OR methods for cancer treatment planning and scheduling can be found in the literature (Saville, Smith, & Bijak, 2019).

Despite the rhetoric of providing timely access to care, patients are often unable to access care on time, and long waiting times are a challenging barrier to improving mental health outcomes (British Medical Association, 2017). Instances from cancer care that have also explored improving access to treatment present a possibility for adaptation. Future research could consider optimising the location of treatment centres using performance measures like total demand-weighted distance, and total distance travelled (Cotteels, Peeters, Coucke, & Thomas, 2012).

3.5.3. Modelling Continuity of Care for Mental Health Patients

Continuity of care is considered a prerequisite for providing high-quality care and is regarded as a guiding principle in planning and delivering services in mental healthcare (Biringer, Hartveit, Sundfør, Ruud, & Borg, 2017; Freeman, Weaver, & Low, 2002). This aspect of mental health services warrants further inclusion in model development. Specifically, in home healthcare and outpatient care literature, continuity of care constraints are often used to assign patients to care workers (Ahmadi-Javid et al., 2017; Cissé et al., 2017). Furthermore, continuity of care for mental health patients can be extended to a patient's care pathway across multiple services in the mental healthcare network, which could include social services, community services, outpatient and inpatient mental health services (Slade, Leese, Cahill, Thornicroft, & Kuipers, 2005). When organizing treatment pathways, multidisciplinary teams are faced with a similar challenge of ensuring continuity of care. Examples of multi-disciplinary planning include modelling capacity fluctuations and planning care pathways (Leeftink et al., 2020). Future research could explore the applicability of such instances to planning mental healthcare service delivery.

3.5.4. Models to Consider Multi-Layered Mental Healthcare Systems

One of the main characteristics of care settings in mental healthcare is the interconnectedness of services. Also present are multiple types and levels of workers who work in tandem. It is known that mental healthcare is primarily focused on providing care in the community through several channels such as in a patient's home, on the telephone, and at local clinics. In this context, to tackle common mental health issues or complex mental health issues in the community, patients increasingly receive care at their home, by telephone, and at local clinics by 'wrapping services' around primary care (Edwards, 2014). Herein teams of multidisciplinary skill-mix mental health staff are developed in collaboration with secondary care, around groups of primary care practices that serve a specified population in a geographic location (World Health Organization, 2018a; World Health Organization, 2018c).

From a modelling perspective, incorporating features that are characteristic of complex systems in models is challenging. However, similar structural and workforce issues exist in healthcare literature, which are transferrable to mental healthcare services. For instance, parallels can be drawn from existing applications of optimisation in community services (Palmer, Fulop, & Utley, 2018), home healthcare (Cissé et al., 2017), outpatient Care (Ahmadi-Javid et al., 2017) and owing to the multidisciplinary nature of the teams, from multi-disciplinary planning (Leeftink et al., 2020). Besides, in situations where multiple workers with a mixed skill set are required to provide services to patients in multiple locations, future research could investigate the possibility of applying multi-skilled multi-location models. Such models have been developed to address the food safety inspector scheduling problem (Cheng & Kuo, 2016) and for scheduling airline customer service agents to locations in a large international airport terminal (Kuo, Leung, & Yano, 2014).

The integrated nature of mental healthcare services poses another modelling challenge: developing models that aid decision-making across different systems and planning levels. In essence, integrated care delivery involves coordinating services across multiple healthcare professionals, organizations, and sectors and prioritizing patient needs and preferences (Tsasis, Evans, & Owen, 2012). There is a wide-ranging consensus in extant healthcare optimisation literature to develop models that aid decision-making across integrated systems. An example of optimization applied to an integrated care system can be found in a study by Braaksma et al. (2014), who present a methodology to plan treatment for a multidisciplinary rehabilitation centre and present an integer linear programming approach to implement combination appointments. Additionally, Marynissen & Demeulemeester (2019) have positioned the MASPH literature as an additional dimension to the spectrum of integrated healthcare. Several authors have encouraged future researchers to build models that capture realistic assumptions (multiple servers, multi-type demand, and uncertainty). This limitation also extends to capturing variability in care pathways. Although modelling the integration of services is an emerging application area, existing models can be adapted to model integrated care in mental healthcare. In addition, future work would need to consider the boundaries between healthcare professionals and organisations and incorporate multi-level modelling and mixed methods, which involve some recognition and appreciation for the dynamic complexity of the mental healthcare system. Future research can be guided by a recent review article on clinical pathway modelling. Aspland, Gartner, & Harper (2021) propose a taxonomy of clinical pathway problems to improve the handling of multiple scopes within one model while encouraging interaction between the disjoint care levels.

The findings suggest that future work should consider industrial engineering integrated with OR techniques. So far, this review has identified opportunities from several healthcare settings where optimisation models can be transferred to mental healthcare. Through this analysis, we have established that research gaps that were identified can also be extended to mental healthcare service planning. In particular, models are far from comprehensively tackling complex real-world problems in healthcare planning. Several reviews have highlighted the absence of models that include environmental factors such as patient no-shows, emergencies, resource absenteeism, unpunctuality, unavailability and traffic delays. Moreover, given the disruptions caused by the current public health crisis, researchers call attention to the absence of models that consider factors such as disruption relative to natural disasters, economic or financial crises, and social events.

3.5.5. Developing New Modelling and Solution Methodologies to Address Challenges of Mental Healthcare Delivery

This review has identified the need to formulate complex models that capture mental healthcare systems. Increasingly, optimisation methods capable of solving complex real-world problems in healthcare are being developed and deployed. By examining the latest advances in healthcare modelling, this section will attempt to carve out methodological avenues for future research in mental healthcare planning.

In mental health care, lack of standardised information technology data sources and limited scientific evidence for mental health quality measures are critical barriers to measuring and improving mental health care quality (Kilbourne et al., 2018). Worldwide, quality of care in mental healthcare is suboptimal with persistent gaps in access to and receipt of mental health services (Demyttenaere et al., 2004; Wang, P. S. et al., 2007; Whiteford et al., 2013). Therefore, to close existing gaps, mental healthcare systems worldwide are also rolling out service standards similar to those in physical health services. Services are looking to increase capacity and set up access and waiting time standards (NHS England, 2014). Although significant advances are currently underway to identify mental health care quality measures, several obstacles are yet to be overcome. These systemic factors are challenging to model since quality measures are inextricably linked to measures of performance, which inform model building.

As evidenced earlier, optimisation models for mental healthcare planning are predominantly deterministic; they do not capture the uncertainties inherent in the system. In other strands of healthcare optimisation literature, uncertainty related to service duration, patient preferences, patient arrivals, interruptions etc., have been modelled using methodologies such as stochastic programming and robust optimization. Specifically, an optimization problem is stochastic if some or all parameters are uncertain, but they follow a probability distribution (Birge & Louveaux, 2011). For instance, stochastic programming has been used for staffing and scheduling homecare employees by considering uncertain demand (Restrepo, Rousseau, & Vallée, 2020) and for operating room scheduling in the presence of cancellations and resource unavailability (Xiao, van Jaarsveld, Dong, & van de Klundert, 2016). On the other hand, in the presence of unreliable data in a system with uncertainty, a robust optimization model can be used. Such a model aims to make a feasible decision no matter the constraints and is optimal for the worst-case objective function (Gabrel, Murat, & Thiele, 2014). For instance, physician capacity planning at a tactical level, in the presence of unreliable data and uncertain patient demand, is modelled using robust optimization

(Aslani, Kuzgunkaya, Vidyarthi, & Terekhov, 2020). Formulating planning problems by utilising such methods could be considered for future research.

More than half the articles under review have used exact solution algorithms, while the rest have employed heuristics, metaheuristics, or hybrid algorithms. Notably, when the time or cost of determining the optimal solution is too large in decision problems, an acceptable and feasible solution is preferred (Capan et al., 2017). In this context, optimization models in healthcare are increasingly being solved by more than one solution method. Specifically, hybrid optimization approaches that combine exact and heuristic methods to deal with the complexity are used (Feldman, Liu, Topaloglu, & Ziya, 2014). Additionally, to solve large-scale problems, heuristics and metaheuristics are the methods of choice for their ability to provide satisficing solutions (Saha & Ray, 2019; Volland et al., 2017). While the use of such instances has been found in mental healthcare literature, in comparison to other strands of healthcare literature, it is limited in both size and scope.

Researchers have recently identified the need to take a holistic approach that integrates planning decisions and have developed hybrid models that combine several OR techniques. In particular, forecasting, simulation and optimization are used in combination for capacity planning in a hospital (Ordu, Demir, Tofallis, & Gunal, 2020). Metaheuristics are used alongside simulation to schedule walk-in patients in clinics (Amaran, Sahinidis, Sharda, & Bury, 2016; Peng, Qu, & Shi, 2014). Such approaches often build on gaps identified in particular strands of research (Uriarte, Zúñiga, Moris, & Ng, 2017). Notably, such approaches are lacking in mental healthcare research.

Globally, the increased awareness of the unmet need for mental health services is leading to the growth of several strategies that focus on coordination and communication between health services. Such care models are often collectively termed “mental health integration”, “behavioural health integration”, or “integrated care” (Unützer, Carlo, & Collins, 2020). In such a care setting, multiple stakeholders with diverse perspectives and views are likely to influence decision-making. Therefore, researchers have developed multi-methodology frameworks that combine hard and soft OR methods to gather information and knowledge about the system and help reflect multiple stakeholders' diversity of concerns (Pessôa, Lins, da Silva, Angela Cristina Moreira, & Fiszman, 2015). Simulation is often combined with Problem Structuring Methods (PSM) (Sachdeva, Williams, & Quigley, 2007; Tako & Kotiadis, 2015). More recently, Soft Systems Methodology (SSM) tools were used to structure the medical training problem's objectives and specifications. The information was then fed to formulate a mixed integer-programming problem (Cardoso-Grilo, Monteiro, Oliveira, Amorim-Lopes, & Barbosa-Póvoa, 2019). It is noteworthy that the combination of optimisation with PSMs is only just beginning in healthcare literature, with only one such application so far.

3.5.6. Managerial insights

Increasingly, to ensure safe, sustainable, and productive staffing of mental healthcare services, the planning priority is to make sure 'the right people, with the right skills, are in the right place at the right time' (Baker & Pryjmachuk, 2016). Central to achieving this goal is appropriate workforce planning and deployment. The planning and deployment of a skilled workforce are some of the most challenging problems a manager faces that have real-life implications. Our review has revealed that most planning models in mental healthcare were used in real world practical contexts. However, we have decidedly established that these models have a narrow scope and use simplified assumptions. Workforce planning in healthcare is a well-researched application area. Models have considered multiple skills, shifts, and criteria to build realistic models incorporating stochasticity and uncertainty (De Bruecker, Van den Bergh, Beliën, & Demeulemeester, 2015). Such considerations are missing in mental healthcare models and is a prominent area of future research.

Additionally, when it comes to model building in practice, managers and researchers should be aware of a range of factors that differentiate healthcare modelling from other industries. Factors include the importance of using problem structuring, problems associated with data collection, and interpreting the model and its results (Virtue et al., 2013). The importance of using problem-structuring methods to facilitate stakeholder participation is the focus of many future research directions, also echoed by this article (Júnior & Schramm, 2021). In addition, by drawing on our own experience of building an optimisation model for a mental healthcare service, we acknowledge and confirm the importance of these practical factors described above. Notably, in our experience, problem-structuring methods such as SSM proved invaluable in eliciting stakeholder participation throughout the entire modelling cycle (Ranyard, Fildes, & Hu, 2015). Equally, we wish to emphasise the effort and difficulty associated with collecting data. We recognise that most services routinely collect crucial data. However, significant resources are required to understand and clean said data before being used in the model (Onggo & Hill, 2014). Likewise, researchers and managers ought to be aware of the intricacies of communicating technical information to stakeholders (Herrera, McCardle-Keurentjes, & Videira, 2016). Problem-structuring and multi-methodology methods have endeavoured to bridge this gap, and optimisation modellers can draw from these studies (Howick & Ackermann, 2011). However, it is worth noting that such applications have developed visually interactive simulation models. In comparison, optimisation models do not have visualisation capabilities and are therefore challenging to translate (Waisel, Wallace, & Willemain, 2008).

3.6. Conclusion

This paper provides a scoping review of the application of optimisation methodologies in mental healthcare services. Half of the reviewed studies were conducted in the immediate period following deinstitutionalisation (the 1960s onwards). The research appears to have resumed in the past decade. We also survey the landscape on the application of optimisation to healthcare and provide an overview in Table 2. Through this survey, we identify gaps in current literature and highlight opportunities for transferability of existing applications to the context of mental healthcare services. Features associated with mental healthcare are also presented and contrasted with healthcare to identify similar characteristics or problems that have been addressed in other healthcare settings, which have the potential of being transferred.

After establishing the background for the mental healthcare setting, we then conduct a scoping review and classify the identified studies. Articles are organised through a generic analysis of various characteristics. The number of publications associated with mental healthcare planning and delivery is restricted and sporadically distributed compared to physical healthcare, which indicates the limited attention habituated to this aspect of healthcare. We then conduct an in-depth analysis of the optimisation models built for mental health services and find that the models are predominantly deterministic; they do not capture the complexities inherent in the system. We draw parallels between psychological and physical health to identify opportunities for transferability and propose a broad research agenda. Based on the analysis of existing literature, features of mental healthcare services, and the results of our review, we find that although opportunities for transferability exist, gaps in healthcare optimisation literature also extend to mental healthcare. Although COVID-19 is a physical health crisis, it has seeds of a major mental health crisis as well. Mental health services have had to switch to providing care remotely. While such approaches can be effective and scalable, they are not the answer for all mental health needs. Other tried and tested modalities of care continue to be of importance. Good mental health is a critical aspect of recovery from COVID-19. The pandemic could turn into an opportunity to catalyse change and comprehensively address the barriers that have prevented the widespread delivery of efficient services. Indeed, now is the time to expand access to provide cost-effective delivery of effective mental health services. OR techniques have a proven track record for their ability in aiding decision-making at strategic, tactical and operational levels. Healthcare managers can use optimisation models to plan patient pathways, efficiently manage and deploy their workforce, and evaluate the introduction of new treatment modalities such as telemedicine. Through this review, we have outlined a host of future research questions that optimisation modelling can answer. However, we do not assume to have identified all of them. This review is an open call to optimisation modellers and to the OR community to help support future planning of mental health services.

Chapter 4: Facilitative conceptual model development for mathematical optimisation: A case study in mental healthcare

Abstract

Facilitative modelling is an established mode of conducting an intervention in Operational Research (OR), especially in health care. Only some Hard OR approaches, such as simulation, have developed frameworks and guidance to support the facilitators in the modelling process. In this study, we introduce a facilitative framework for collaboratively conceptualising a mathematical optimisation model with stakeholders. Our novel approach to optimisation is demonstrated through a real case study in mental healthcare delivery. We showcase the practical implementation of our framework, which incorporates Soft Systems Methodology tools adapted from PartiSim. Additionally, we describe the development of new tools specifically tailored to conceptualise an optimisation model. These tools are utilised in workshops with a diverse group of stakeholders, including mental health clinicians and managers, to collectively frame the problem and arrive at a conceptual model. We analyse how the combined use of adapted and newly developed tools aligns with the overall framework and reflect on the generalisability of our approach towards facilitated conceptual modelling for optimisation.

4.1. Introduction

In the last four decades, optimisation modelling in healthcare has been used to address a diverse range of challenges inherent to decision-making in healthcare (Capan et al., 2017). Optimisation is an efficient tool for healthcare planning by assisting the decision maker in determining the best solution under a variety of constraints, often by simultaneously considering multiple factors (Earnshaw & Dennett, 2003; Tüzün & Topcu, 2018). Several healthcare settings employ optimisation models to address various service delivery issues. In emergency room planning, optimisation models are used to determine resource quantity (surgeons, nurses, rooms, equipment, operations time) needed to meet demand, allocate operating room capacity to various medical disciplines, assign definite dates for operations, determine the start time of the operations and the allocation of resources (Samudra et al., 2016; Zhu et al., 2018). In the primary, outpatient and home health care settings, problems of staffing (determining size and composition), rostering (creating shift rosters), allocating appointments by maximising resource utilisation and accessibility, assigning care workers to patients, and scheduling patient visits are addressed (Cissé et al., 2017; Grieco et al., 2021; Leeftink et al., 2020; Marynissen & Demeulemeester, 2019).

Notwithstanding the range, depth, and significant benefits of employing optimisation modelling in healthcare, translating the models into action is an active challenge. Managing healthcare involves planning and coordinating several scarce resources (often expensive and highly specialised) and considering multiple stakeholders with often conflicting goals (Brailsford & Vissers, 2011; Clarkson et al., 2018; Eldabi, 2009). It is suggested that the lack of stakeholder involvement in traditional optimisation modelling leads to missed opportunities throughout the modelling life cycle, starting with inadequate primary data collection, leading to the development of a realistic model, as opposed to a real case study, culminating in a lack of real implementation of the model (Amideo et al., 2019). Furthermore, research has highlighted model inaccuracies, model simplifications, and challenges in the input data as negatively impacting the uptake of model recommendations by decision-makers (Käki et al., 2019). In the context of healthcare, stakeholder resistance to viewing model outcomes as evidence is exacerbated if the model is constructed in isolation and the rationale for model design is not transparent (Carter & Busby, 2022).

In a simulation study, conceptual modelling (CM) is a key first stage, followed by model coding, experimentation, and implementation (Robinson, 2014). A conceptual model is described as "a non-software specific description of the computer simulation model (that will be, is or has been developed), describing the objectives, inputs, outputs, content, assumptions and simplifications of the model" (Robinson, 2013). The aim of CM is to abstract a simulation model from the real world system that is being modelled (Robinson, 2020). CM as a process and tool assists in communication and trust building with stakeholders, aids model validation, supports credibility and transparency of the modelling process (Harper et al., 2021; Kotiadis et al., 2014; Pessôa et al., 2015).

Simulation studies involve stakeholders for CM using participative approaches involving Soft Systems Methodology (SSM) and through facilitated modelling (den Hengst et al., 2007; Holm & Dahl, 2011; Kotiadis, 2007; Lehaney & Paul, 1994; Lehaney & Paul, 1996; Oliveira et al., 2023; Proudlove et al., 2017; Robinson et al., 2012; Robinson et al., 2014; Tako et al., 2010; Tako & Kotiadis, 2012; van der Zee, 2010). Healthcare has been the key application area for facilitated simulation studies with several case studies (Lane et al., 2019; Proudlove et al., 2017; Robinson et al., 2012; Robinson et al., 2014; Tako et al., 2019; Tako et al., 2021; Willis et al., 2018). Particularly relevant to this study is the multi-methodology PartiSim framework combines Discrete-Event Simulation (DES) and SSM across six stages and contains a strong element of model conceptualisation (Kotiadis et al., 2014; Kotiadis & Tako, 2016; Kotiadis & Tako, 2018; Tako et al., 2010; Tako & Kotiadis, 2015).

In optimisation modelling, a handful of case studies have employed multimethodology approaches to enable stakeholder participation. Abuabara et al., (2022) develop a participatory approach that combines linear optimisation with Parsimonious Analytic Hierarchy Process to address a standard "Diet Problem". The proposed methodological framework engages with stakeholders in three workshops. In a different study, Cardoso-Grilo et al., (2019) develop a multi-methodological framework that combines the structuring of objectives and specificities of the medical training problem with a Soft Systems Methodology. The study essentially presents a participative approach to conceptualise a specific optimisation problem. Both studies are significant as reference points for participatory and multimethodology approaches that integrate "Soft OR" with optimisation. We argue that a facilitated conceptual modelling approach for optimisation modelling is not currently available. Additionally, optimisation modelling can address issues of stakeholder involvement and opaque modelling processes by borrowing from simulation to realise the benefits intrinsic to participative and facilitative approaches.

This study considers how a group of stakeholders can be involved in the conceptualisation of an optimisation model. We evaluate how and if the conceptual modelling elements of the PartiSim framework can be adapted to optimisation. The adaptation will be tested on a real case study in mental healthcare, an application area overlooked in OR healthcare modelling (Bradley et al., 2017; Long & Meadows, 2018; Noorain et al., 2019; Noorain et al., 2022). To the best of our knowledge, facilitated modelling approaches for optimisation have not been documented or formalised in the scientific literature. This study complements existing research efforts in different ways. First, it investigates the adaptation of CM element of PartiSim to conceptualise an optimisation model. Second, it contributes to the understanding of facilitated modelling for optimisation. Third, it expands multimethodology literature by combining SSM with optimisation modelling, and last, it contributes a case study in mental healthcare.

The rest of the study is structured as follows. Section 4.2 presents background literature on facilitated modelling in OR; PartiSim; Soft OR, Participatory and Facilitated Approaches in Optimisation; and mental healthcare modelling. In Section 4.3, an overview of adapting PartiSim's CM stages for optimisation is presented. In Section 4.4 we describe the case study, and presents the application. In Section 4.5, we reflect on the adaptation and propose a CM framework for optimisation. The section also highlights the contribution and present directions for future research. Section 4.6, presents some conclusive remarks.

4.2. Literature Review

4.2.1. Facilitated Modelling in OR

Facilitated modelling is an established intervention tool involving group dialogue, facilitation, and participatory modelling, that enables the OR researcher-facilitator to carry out the whole intervention jointly with the client (Franco & Montibeller, 2010; Franco & Rouwette, 2011; Tavella & Franco, 2015). The facilitated modelling process involves developing models jointly with a client group, often face-to-face, with or without the assistance of computer support (Eden, 1982; Eden & Radford, 1990; Franco & Montibeller, 2010). Facilitated modelling is positioned as being antithetical to the traditional way of conducting OR interventions which adopts the expert mode. In the expert mode, the OR researcher/consultant uses Hard OR methods and models to conduct an 'objective' analysis of the client's problem situation, together with the recommendation of optimal (or quasi-optimal) solutions to alleviate that problem situation (Ackoff, 1979; Franco & Montibeller, 2010; Rosenhead & Mingers, 2001). On the other hand, in facilitated modelling a subjective analysis of the problem (incorporating many views) is carried out with a view to identifying desirable and feasible outcomes (Eden & Sims, 1979; Eden, 1992; Franco & Montibeller, 2010).

There are several types of facilitated modelling approaches, including facilitated problem structuring, facilitated system dynamics, facilitated discrete-event simulation, and facilitated decision analysis. In the facilitated mode, often a problem structuring intervention is framed by several interacting features such as a group of people involved in the problem structuring and, who outline the multiple perspectives. The intervention practice is mainly expressed by the interaction between the facilitator (or facilitators), and client group and the tools and techniques allow the construction of visually interactive models (Abuabara & Paucar-Caceres, 2021; Eden & Ackermann, 2004; Franco & Montibeller, 2010; Hindle & Franco, 2009; Smith & Shaw, 2019). PSM approaches include SSM (Checkland, Peter & Poulter, 2020; Checkland, Peter B., 1989), Strategic Options Development and Analysis (SODA) (Abuabara & Paucar-Caceres, 2021; Ackermann & Eden, 2020), and Strategic Choice Approach (SCA) (Friend & Hickling, 2012).

Facilitated modelling in the System Dynamics (SD) tradition is termed group model building. A term that is used to refer to the construction of an SD model whilst working directly with a group of clients (Vennix, 1995; Vennix, 1999). Two reviews cover over 130 empirical studies and identify how GMB improves communication, learning and agreement between participants in group decision processes (Rouwette et al., 2002; Scott et al., 2016). In healthcare, there are several examples of facilitated modelling with stakeholders, including the evaluation of cataract treatment in the Netherlands (van Nistelrooij et al., 2013), state health policy making (Minyard et al., 2014), improving acute patient flows (Lane & Husemann, 2018), strategic workforce planning (Willis et al., 2018), and understanding foodborne transmission mechanisms for Norovirus (Lane et al., 2019).

Drawing on the extensive work of facilitation in system dynamics, the DES community have developed facilitated approaches to DES modelling, particularly in the context of healthcare. In general, the lifecycle of creating a simulation model is structured into four key stages: conceptual modelling, model coding, experimentation, and implementation (Robinson, 2014). Existing research in facilitated DES has developed several approaches for one or more key stages of the modelling lifecycle. SimLean is an approach that integrates DES to support facilitated-group with lean-based improvement (Robinson et al., 2012; Robinson et al., 2014). Kotiadis et al. (2014) developed processes and tools for the conceptual modelling stage. Tako and Kotiadis (2015) present the PartiSim framework for facilitated simulation using SSM to involve stakeholders over the entire simulation project life cycle. A PartiSim influenced approach that expands the scope of facilitation to the model coding stage is introduced by Proudlove et al. (2017). Kotiadis and Tako (2018) further elaborate on the processes followed as part of the experimentation and implementation stages of the PartiSim framework (and the simulation modelling lifecycle). More recently, Tako et al. (2019) have introduced a new facilitated simulation approach called SIMTEGR8, which stands for "Simulation for Great Care", which is a combination of SimLean and PartiSim.

Optimisation models are traditionally built in the expert mode (Currie et al., 2020; Smith & Shaw, 2019). However, issues with model outcome uptake and implementation have been linked to a lack of stakeholder involvement, while calls for integrating optimisation with techniques that support stakeholder involvement are gaining momentum (Amideo et al., 2019; Carter & Busby, 2022; Çoban et al., 2021; Noorain et al., 2019). More generally, to expand the scope of OR practice, researchers are being encouraged to utilise Problem Structuring Methods (PSMs) to facilitate stakeholder participation (Gomes Júnior & Schramm, 2021; Ranyard et al., 2015). To the best of our knowledge, facilitated modelling approaches for optimisation have not been documented or formalised in the scientific literature.

4.2.2. Participative Simulation (PartiSim)

PartiSim is a multimethodology facilitative and participative framework for conducting facilitated DES projects (Tako & Kotiadis, 2015). The framework is structured to follow the simulation modelling lifecycle of four key stages: conceptual modelling, model coding, experimentation, and implementation (Robinson, 2014). The framework consists of a set of consecutive, and iterative modelling stages, that blend modelling with project management as seen in Figure 3. It is supported by toolsets (tools, manuals, and scripts) that provide guidance to modellers. Tools have been adapted from Soft System Methodology (SSM), and new bespoke tools have been developed to enable undertaking a simulation study in a facilitated environment. Appendix A, Appendix B, and Appendix C include screenshots of tools that are prescribed for Stages 1, 2 and 3 respectively. Furthermore, several workshops enable stakeholders to engage in the study starting with problem structuring through to implementation.

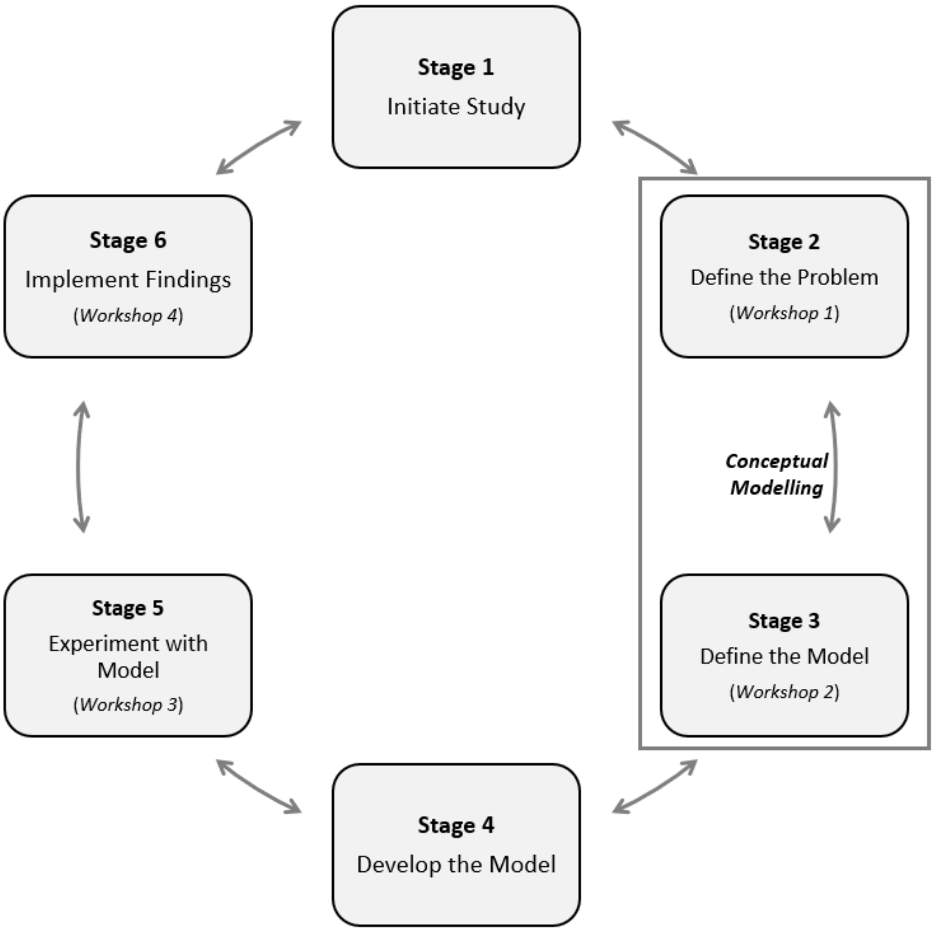

Figure 3: Stages of the PartiSim Modelling Framework Tako et al., (2010)

Each stage is accompanied by guidance in the form of scripts to support the modelling team achieve the prescribed outputs. As seen in Table 8, the framework has several tools to obtain the outcomes, such as CATWOE and root definition (Figure 32, Appendix B). The CATWOE is a mnemonic, the first letters of which consist of the elements of the system under study, namely Customers, Actors, Transformation process, Worldview, Owners, and Environmental constraints. These identified elements are then assembled into a root definition that defines the key transformation process that is the key activity that takes place in the system. The root definition acts as a mission statement for the system and follows the format "do X by using Y to achieve Z" (Checkland, Peter & Scholes, 1999).

Additionally, the framework also includes the Performance Measurement Model (PMM) tool (Figure 34, Appendix C). The PMM is derived from SSM's Purposeful Activity Model (PAM), which is a 'device' to stimulate, feed and structure debate' (Checkland, Peter & Scholes, 1999) assisted by the tools CATWOE, root definition and measures of performance. Kotiadis (2007) extend SSM's performance measures to create a PMM (Figure 33, Appendix C). The performance measures are, typically known as 3Es are Efficacy, Efficiency, and Effectiveness are criteria used to define and monitor performance of the system represented in the PAM. In the PMM, the 3Es are broken down into monitoring activities (to examine performance measures identified), determine if activities (to assess the need for action), and suggest action to be taken (Kotiadis, 2007; Kotiadis et al., 2013). Using the PMM, objectives for a simulation study are obtained.

An essential draw of PartiSim framework is the proposed guidance to adopt different roles of the project team, including roles for stakeholders. Essentially, the project team consists of two teams: the modelling and stakeholder team and descriptions for roles within the project team are elucidated extensively. Additionally, the framework provides guidance on organising facilitated workshops, including duration, location, and material required for the workshop. Although such guided frameworks are now gaining traction in simulation modelling, no such framework exists in optimisation modelling.

Table 8: A participative and facilitative conceptual modelling framework for discrete event simulation studies in healthcare (Kotiadis et al., 2014)

Stage and purpose	***CM support activities***	***Facilitation support activities***	***Tools***	***CM outputs/deliverables***
1. Initiate Study	The modelling team undertake:	Determine a list of key stakeholders to be involved in study and timescales.	Information Collection Tool	Preliminary understanding of the problem situation.
Purpose:	• informal meetings and/or	Modelling team and stakeholder team roles are decided.		
• Identify stakeholder team	• on-site observations and/or			
• Identify key problem situation(s)	• one-to-one interviews			
	• with project champion and key stakeholder(s), to address preliminary information needs			
1a. Pre-workshop stage *Purpose*: Preparations for Workshop 1	Modelling team prepare preliminary materials for tools to be used in workshop 1	Workshops 1 and 2 venues and time slots are determined.		
		Stakeholders are invited to workshops		
		Facilitator prepares for the workshop 1.		
2. Define system (Workshop 1) *Purpose*:	Participating stakeholders take part in a facilitated workshop process to:	During the workshop the facilitator guides the group of stakeholders through the process by proposing activities and providing tools, so they design/determine the deliverables.	Problem statement form	General study objective(s)
Agree on the problem situation and the wider system, within which it exists.	• Brainstorming problem area (s) to be addressed and identify study objectives		CATWOE and root definition	A bounded system within which the problem to be addressed exists
	• Define system boundaries		Care system model	
2a. Post-workshop1/Pre-workshop 2 stage *Purpose*: Disseminate workshop 1 outputs and prepare workshop 2	Modelling team re-draw tools and disseminate workshop outputs to stakeholders	The modelling team liaises with the stakeholder team over correctness of workshop 1 outputs.		
	Prepare preliminary materials for tools used in workshop 2	Workshop 2 venue and timeslot is confirmed. The facilitator prepares for workshop 2		
3. Specify conceptual model (Workshop 2) *Purpose*: Define specific	Participating stakeholders take part in a facilitated workshop process to:	During the workshop the facilitator guides the group of stakeholders through the process	Performance measurement model (PMM)	Model inputs and outputs and model content

elements of the conceptual model	• Put forward and agree on performance measures to address the problem identified in workshop 1	by proposing activities and providing tools, so they design/determine the deliverables.	Study objectives form	Model objectives
	• Identify inputs, outputs and model content		Patient Flow diagram	A preliminary list of assumptions and simplifications
	• Define the model objectives			A communicative model
	• Produce communicative model (discuss model contents, model scope and level detail)			A list of data requirements
	• Discuss responsibility for data collection.			
3a. Post-workshop 2 stage *Purpose*: Disseminate workshop 2 outputs and refine conceptual model	Modelling team prepare report detailing: • Refined CM outputs from stage 2.a and stage 3 • Data requirements	The modelling team liaises with the stakeholder team over correctness of workshop 2 outputs.		An agreeable to all (study participants) and feasible conceptual model describing DES study

4.2.3. Optimisation Modelling

Optimisation modelling consists of four phases: problem definition, problem formulation, problem solution, and model validation, as seen in Figure 4 (Horst & Tuy, 2013; Nocedal & Wright, 2006; Williams, 2013; Zamanifar & Hartmann, 2020). An optimisation model is made up of three basic ingredients: decision variables, objective function, and constraints. An objective function is the quantity that is to be maximised. The controllable inputs are the set of decision variables which affects the value of the objective function. Additionally, the uncontrollable inputs are parameters, which may be fixed input values. Constraints are relations between decision variables and the parameters. The basic goal of the optimisation process is to find values of the variables (the output) that minimise or maximise the objective function while satisfying the constraints. This result is called an optimal solution.

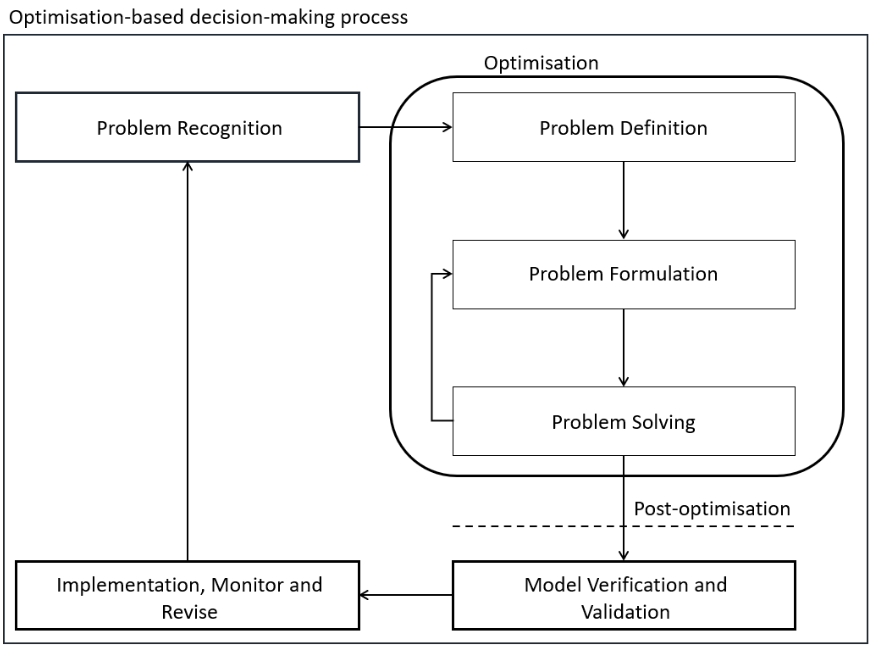

Figure 4: Optimisation Modelling Phases, adapted from Zamanifar & Hartmann, (2020)

In optimisation-based decision-making process, after recognising a problem, the task of problem definition includes identifying model components. Next, a mathematical translation of the defined problem and established set of relationships are formulated. In this step, an analyst will set up relationships among various decision variables and translate them into mathematical equations. In the problem solution phase, a matching and robust solution approach is selected and implemented with the aim of the quality of the model and the feasibility of outcomes, simultaneously. Following this, the model's outputs are verified and validated, and the model is implemented.

4.2.4. Soft OR, Participatory and Facilitated Approaches in Optimisation

Optimisation models have been combined with other quantitative methods, such as simulation and forecasting (Amaran et al., 2016; Ordu et al., 2020; Peng et al., 2014). Although such studies are not commonplace, several novel applications have been developed in the last decade. Similarly, the combination of optimisation with Soft OR methods is limited. We found three studies that have used tools adapted Soft OR and applied it to define an optimisation model with and without a participative modelling approach. As seen in Table 9, we classify the articles by considering the optimisation model building lifecycle and further segregating them based on factors including the methods employed, PSMs in the problem definition phase of optimisation modelling, and if the study included participatory or facilitated approaches.

Table 9: Soft OR, Participatory & Facilitated Approaches for Defining an Optimisation Model

Authors	**Methods**	**PSM/Soft OR used at what stage**	**Participatory**	**Facilitated**
Mabin (2009)	Evaporating Clouds (EC) & MILP	Problem Definition	×	×
Cardoso-Grilo (2019)	SSM (CATWOE) & MILP	Problem Definition	✓	×
Abuabara (2022)	SSM (Rich Picture), AHP, Linear Programming	Problem Definition	✓	✓

Three studies have used Soft OR approached in the ‘Problem Definition’ phase. Mabin (2009) have explored the role of a process known as Evaporating Cloud (EC) to enhance OR modelling in a theoretical study. EC is a schematic approach to conflict resolution, also known as the Conflict Resolution Diagram, using which assumptions underlying the problem situation are surfaced, to then “evaporate” apparent conflicts, dilemmas, or trade-offs. In the study, EC is used to structure and provide a check on the assumptions underlying a facilities location problem formulated as a mixed integer programming model. The study makes a case for using soft approaches to better understand the system for both modeller and owner. Furthermore, the process is purported to lead to better outcomes as it goes beyond just ‘optimising’ a notional system based on unquestioned assumptions and implicit constraints.

Cardoso-Grilo et al. (2019) developed a multimethodology framework to assist in the planning of medical training for a National Health Service Structure. The framework combines the structuring of the objectives and specificities of the medical training problem with Soft Systems Methodology using the CATWOE tool, leading to the formulation of a multi-objective MILP model. Considering the specificities of countries based on a National Health Service structure, a multi-objective planning model informed how many vacancies should be opened/closed per year in medical schools and in each specialty. The use of SSM tools to build optimisation models is a relatively new development. The proposed methodology was participatory as it incorporated local knowledge into research and planning, and included collaborative activities in an iterative, flexible design (Cornwall & Jewkes, 1995; Harper et al., 2021) In the study, data for the CATWOE tool was obtained using policy statements and official documents and authors mention conducting a workshop to develop policy questions for the model, but no other details were provided. However, the study does not explicitly state the adaptation of the facilitated mode of intervention involving a group of stakeholders for model building.

More recently, Abuabara (2022) develop a novel participatory framework, which combines linear optimisation with the Parsimonious Analytic Hierarchy Process (PAHP). The framework includes Rich Pictures used for initial problem structuring, then 4 main sequential activities using PAHP as a participatory tool for formulating a linear programming model. The study examines a food planning problem for several families in the context of the COVID-19 pandemic. The participatory linear programming model maximises diet preferences subject to constraints reflecting the lockdown-specific context. The framework employed Rich Picture to survey the problem at hand, followed by a set of three workshops, each with a defined task. Several examples of workshop questions are presented, to develop: 1) a tree of values and attributes, and 2) workable daily meal alternatives. The workshop outputs are provided as data inputs for the MCDA tool, AHP, to rank the various meal alternatives, culminating in the development of an optimisation model.

4.2.5. Mental Healthcare Modelling

Several distinguishing features of the mental healthcare system add to the complexity associated with modelling it. One of the main characteristics of care settings in mental healthcare is the interconnectedness of services with multiple types and levels of workers who work in tandem. Patients are primarily provided care in the community through several channels, such as in a patient’s home, on the telephone, and at local clinics. Common or complex mental health issues are tackled in the community. Teams of multidisciplinary skill-mix mental health staff are developed in collaboration with secondary care around groups of primary care practices that serve a specified population in a geographic location (World Health Organization, 2018a; World Health Organization, 2018c). The integrated nature of mental healthcare services is challenging as it involves coordinating services across multiple healthcare professionals, organisations, and sectors and prioritising patient needs and preferences (Tsasis et al., 2012). Naturally, mental healthcare services are increasingly developing strategies focusing on coordination and communication between health services. These integrated care settings involve multiple stakeholders with diverse perspectives and concerns likely to influence decision-making (Unützer et al., 2020).

In OR, modelling service delivery of mental/psychological care services is an area of neglect (Bradley et al., 2017). A small pool of existing reviews explores the application of specific OR methodologies, such as simulation (Langellier et al., 2019; Long & Meadows, 2018; Noorain et al., 2019) and optimisation modelling (Noorain et al., 2022) on mental healthcare services. Noorain et al. (2022) identify the need to formulate complex models that capture the mental healthcare systems. Furthermore, gaps identified in optimisation literature, extend to mental healthcare and several opportunities for transferability have been identified.

Future research in planning service delivery for mental healthcare services must account for context specific idiosyncrasies. For instance, the co-location and interdisciplinary nature of MH services pose uncertainties regarding the boundaries between services and roles. Furthermore, future research could consider the integrated nature of MH services and build models to optimise the location of treatment centres or improve access to care or to aid decision-making across different systems and planning levels. It is suggested that future research could develop holistic approaches that integrates planning decisions and developed hybrid models by combining several OR techniques (Ordu et al., 2020). Additionally, optimisation modelling can turn to multimethodology frameworks to better reflect the diversity of concerns of multiple stakeholders by combining hard and soft OR methods to gather information and knowledge about the system, and structure complex problem situations (Pessôa et al., 2015).

4.3. Overview of PartiSim Adaptation for Optimisation

In this study, we adapted the conceptual modelling stages of the PartiSim framework for optimisation (Kotiadis et al., 2014). To tailor the three CM stages to optimisation, we began by comparing the components of a conceptual model for DES with optimisation, as seen in Table 3. Any updates and additions to the activities and tools in the framework were driven by the need to tailor these to accommodate the identified differences.

In simulation modelling, a conceptual model is defined as "*a non-software specific description of the simulation model that is to be developed, describing the objectives, inputs, outputs, content, assumptions and simplifications of the model*" (Robinson, 2014). The key components of a conceptual model are objectives, inputs, outputs, and model content (Robinson et al., 2010). The objectives can be divided into two, one which describes the purpose of the simulation model and the modelling project and second which include timescales of the project and the nature of the model and its use. The inputs are elements of the model that can be altered to effect an improvement in, or provide a better understanding of, the problem situation; also known as experimental factors. These are often determined by the objective. Outputs report the results from a run of the simulation model and are used to determine whether the model objective is being achieved and to point out the reasons why the objective is not being achieved if they are not (Robinson, 2014). In other words, identifying responses leads to the determining of responses required from the model. The model content is made up of the components that are represented in the model and their interconnectedness. These are split into two dimensions such as the scope of the model, where a boundary of the real system that is to be included in the model (e.g., entities and events) is determined, and the level of detail, where details of each component within the model's scope is defined (Robinson, 1994; Robinson, 2014). Table 10 summarises the main components of conceptual modelling for DES and provides a comparison of each corresponding component to an optimisation model.

As depicted in Table 10, an optimisation model is made up of three key components: objective function, decision variables, and constraints (Hillier, 1967). An objective is performance measure that is maximised or minimised, decision variables represent controllable choices that the model seeks to optimise, and constraints define the limitations, requirements., or restrictions the model must satisfy. Besides these core components, an optimisation model also includes inputs that are parameters with a fixed value and be associated with the objective function, constraints, or decision variables. Additionally, outputs are results that are obtained after the model is solved and take the form of an optimal solution which is a set of values assigned to decision variables and the objective value. When conceptualising an optimisation model, this could include the identification of expected model outcomes.

Table 10: Differences in Conceptualising DES against Optimisation

Conceptual Modelling for Discrete-Event Simulation	Conceptual Modelling for Optimisation
Determining the **objectives** of the simulation model and modelling study.	Determining **model objectives** that represent performance measures to be maximised or minimised
Determining the **model content** (scope and level of detail), identifying any assumptions and simplifications	Determining **decision variables** that represent controllable choices that the model seeks to optimise. Identifying **constraints** that define the limitations, requirements., or restrictions the model must satisfy.
Identifying the **outputs** or responses required from the model.	Identifying expected model **outcomes**, including variables that quantify the objective function to be calculated by the model.
Identifying the model **Inputs**; also known as experimental factors	Identifying **input** parameters that have a fixed value or constants. - Can be associated with the objective function, constraints, or decision variables. - Derived from historical data, expert knowledge, or input assumptions.

Generally, the components can be mapped and there are similarities in the overarching structure of requirements within the two approaches. However, the contents of an optimisation model must be defined specifically across the three aforementioned component and the scope and level are encompassed within the determination of the three key components. Both optimisation and simulation provide recommendations for specific action. Optimisation modelling utilises mathematical techniques to represent real-world situations. It considers essential variables, constraints, and trade-offs to determine a feasible optimal solution that accomplishes specific objectives (Pinedo, 2012; Winston, 2022). In simulation modelling, users explore and observe how a system responds to various inputs by evaluating multiple scenarios, with an aim to gain insights into system behaviour and performance.

Therefore, by acknowledging these structural differences, we recognised that stage 3 of the PartiSim framework would need to be customised in order to extract a conceptual model for optimisation. Stage 1 and 2 were generic in their exploration of the problem situation, and so we did not consider any modifications. In the next sections, we provide an overview of the adaptation grouped by stages that we determined to be transferrable (1 & 2), followed by an elaboration on stage 3 with a description of tools that were omitted and why. We then explore the development of new activities and that we considered to be more appropriate for conceptualising an optimisation model. Additional modifications to the framework that were based on lessons learnt through the case study are explored in the discussion.

4.3.1. Initiate Study (Stage 1) and Define the Problem (Stage 2)

An optimisation modelling study begins with problem exploration. Given the exploratory nature of this stage in the optimisation modelling life cycle, we deduced that the prescribed activities and tools in the 'initiate study' stage tools could be appropriately unrestricted and complementary to allow for an exploration of the problem context. Therefore, we did not consider making any modifications to Stage 1. Specifically, we recognised that the "information collection tools" (see Figure 27, Figure 28, Figure 29, Figure 30, and Figure 31, in Appendix A) could be used to explore and understand the problem situation and the crucial task of identifying relevant stakeholders would be aided by following the prescribed process.

Assuming a generic exploration of the problem context takes place in the first stage, in the second stage, we believed that we could use the activities and tools to define system boundaries and identify the problem to be addressed. In the workshop, we hoped that we would be able to facilitate an understanding of the system by synthesising stakeholders' perceptions. We determined after inspection that by using the Problem statement form, CATWOE, and root definition tools, we could gain a macro level understanding of what would make up the conceptual model (Appendix B). After the workshop, in sharing the outputs for reviewing/updating the outputs, stakeholders would be able to confirm the emergent view of the system, therefore, validating the data collected. From a modelling perspective, we recognised that the activities and tools prescribed in Stage 2 stage could be suitable to draw boundaries around the system under study and outputs from the workshop, including CATWOE and the care systems model could determine the scope and serve as springboards to conceptualise an optimisation model stage 3 (Appendix B). .

4.3.2. Define Conceptual Model (Stage 3)

In this stage, we recognised that to obtain a conceptual model of an optimisation model, tools would need to be tailored to accommodate the differences identified earlier. Therefore, we began by considering which of the activities and tools could be retained, which might not be suitable and what, if any, new tools needed to be developed. The stage prescribes 3 tools: PMM, study objectives form, and patient flow diagram (Appendix C).

We began by considering the PMM tools, using which, the 'efficacy', 'efficiency' and 'effectiveness' of the system is determined (Kotiadis, 2007; Kotiadis et al., 2013). These are further broken down into performance criteria of monitoring, determine if, and suggested change activities. We recognised that by using the PMM we could explore the performance of the system in terms of an optimisation model. Specifically, this activity could identify potential inputs and model performance measures. In particular, the monitoring and determine if activities can be indicators of elements in the system that need to be included in the model either as decision variables (controllable inputs) or parameters (fixed inputs). Furthermore, we also recognised that the 'suggested changes' could be seen as early indicators of 'scenarios' that could be explored by experimenting with the model in subsequent stages.

In PartiSim, the next set of activities are: Identify inputs, outputs, and model content; Define the model objectives; and produce communicative model (discuss model contents, model scope and level detail). We realised that this is tailored to the components of a DES as described earlier. After inspecting the tools 'Study objectives form' (Figure 33, Appendix C) and Patient Flow diagram, we determined they were not suitable for optimisation. At the time, we believed that we needed an activity that gathered information on model inputs that could identify controllable inputs using which we could derive a set of decision variables. To obtain this information, we would need to design a form or identify discussion points that support the emergence of relevant inputs and decisions in the workshop. Additionally, we realised that outputs of the CATWOE could be invoked to guide stakeholders in this activity. Specifically, stakeholders could be asked to consider Customers (C), Actors (A), and Owners (O) of the system, and depending on which of these are going to be included in the model, more information on the chosen elements could be obtained. Ideally, this activity along with the PMM, would highlight the data requirements leading to a discussion on how these identified data needs can be met.

We also needed an activity that focuses on obtaining constraints. Again, outputs of CATWOE could be invoked to obtain this information from stakeholders. Specifically, constraints could be captured by further expanding on Environmental (E) constraints of the Transformation Process (T). As for identifying the objective, we reasoned that since it is typical in optimisation modelling for a specific constraint to become the primary objective function and vice versa, we could derive the model's objective through the constraint's activity. Alternatively, it could be that the objective identified in workshop 1 would not require further elaboration. For these reasons, we did not consider developing a separate activity or tool to obtain the objective.

The last activity for this stage in the PartiSim framework involves the development of a communicative model in the form of a Patient Flow Diagram. Communicative models are a representation of the conceptual model. These take the form of stock and flow or causal loop diagrams for System Dynamics (SD) (Coyle, 1997), process flow diagrams for Discrete-event Simulation (DES) (Oscarsson & Moris, 2002), and state diagrams for Agent Based Modelling (ABM) (Triebig & Klügl, 2009). We recognised that it would be necessary to develop a visual representation of the conceptualised optimisation model after the relevant information was gathered. We also identified that this would essentially be deconstruction of the mathematical structure and would integrate the information obtained through the new activities. It was decided that further exploration of this activity would be considered during the application of the case study. And lastly, no updates were considered for the data collection activity.

4.4. Primary Mental Healthcare Service Case Study

The framework was developed in collaboration with a real-world PCMH service provided by the Kent and Medway Mental Healthcare Trust (KMPT) based in Kent, UK. The service works alongside GP clinics and primary care partners and interfaces with other KMPT services to provide care to people experiencing mild/moderate mental health conditions who do not require secondary care mental health services. During initial meetings with the project champion, it was highlighted that the trust was keen to evaluate the performance of the service and consider opportunities for improvement because of several change imperatives that highlighted the need for integrating mental health services into primary healthcare to foster closer integration of primary, secondary, and tertiary mental healthcare service, and improve patient access to services (NHS England, 2020). This evaluation was prompted by a county level commissioning decision to increase funding for the service to enable KMPT (the providers) to hire more clinicians and improve efficiency to prepare for an increase in demand based on population level forecasts.

Through these exploratory meetings, we gathered that the service had begun as an ad-hoc experiment and evolved on a need basis. This trajectory of development meant that appropriate considerations to service design and operational specifications were not given, as such it did not have an overall operational design, or a narrative with consensus. At the time of the study, the service comprised of 12 multi-skilled clinicians deployed who were deployed to 65 General Practice (GP) clinic locations to provide patient consultations across four types of appointments. The project champion wanted support to determine 'how' the service has been functioning thus far, 'what', if anything, can be improved, and the direction of improvement, if any. However, there was significant uncertainty and a lack of clarity around current and future capacity and demand. Before officially starting the study, we discerned that there were pointers to a kind of workforce planning model, given the emphasis on expansion and/or efficiency of the pool of clinicians. However, from a modelling perspective, there was no immediate pathway to an optimisation model. Equally, it was evident that involving stakeholder's and gathering their perspectives was essential. Moreover, because the service is a bridge between primary, secondary, and tertiary care services, the shaping of this service was of concern to many stakeholders. We adapt the PartiSim framework which has a strong CM element, to help explore the system under consideration and develop a conceptual model that captures key operational details and characteristics in order to extract an optimisation model.

The next sections provides an account of implementing the participative and facilitative approach to develop a conceptual model for an optimisation model. The study began in April 2019 both the workshops were conducted in September 2019. We describe how new activities and tools were shaped by outputs from previous stages, and discuss the development of new activities, tools and a communicative model.

4.4.1. Initiate Study Stage

In this stage, initiation activities included several on-site visits to gather a preliminary understanding of issues in the service. With the project champion, we identified several key shareholders using the stakeholder information forms (Table 41 and Table 42, Appendix D). Stakeholders from policy, strategic, tactical, and operational levels from the organisation were chosen for participation. Specifically, participants included public health consultants, representatives from the West Kent Care Commissioning Group (WKCCG), who fund the system under consideration, executives from KMPT, PCMH service manager, clinicians from the service, and IT personnel from KMPT. Additionally, reading material associated with the service and the wider system were supplied by the project champion (Table 40, Appendix D)

One-on-one interviews were conducted with these key stakeholders at the behest of the project champion to identify issues in the service using the 'Situation of Interest' form (Table 39, Appendix D). The range of participant was crucial is gaining an exhaustive and all-encompassing view of the service. A number of problems were identified through this process including the lack of a standard PCMH service model that could be extended across the whole county; limited information on past and present performance of the service and the clinical workforce; underutilisation of available data and its potential to provide benchmarks. The presence of underutilised data was repeatedly highlighted by the stakeholder group as significant to the study. Therefore, we explored availability of data and accessibility options using a form (Table 42, Appendix D). Information collected in this stage was wide in its scope and explored the PCMH service and its relationships to other services. Giving us a big picture view of the service and allowing the capture of complexities and interconnectedness within the system.

At this point in the study, it became clear that data was going to play an important role in providing stakeholders a picture of the past, present and future. As stated before, the emphasis on the distribution and efficient utilisation of workforce capacity appeared to be critical concerns. This type of problem lends itself to mathematical programming, as opposed to DES which is usually used to model systems that involve queues. Moreover, we determined that the optimisation model would likely require integration with other analytics tools and techniques to fully utilise the available data.

4.4.2. Pre - Define the PMCH Service Workshop

Following the initial problem exploration, stakeholders were invited to the first facilitated workshop to define the system. This workshop took place in-person and was organised to take place on the premises of the University of Kent. We determined that it would be of duration 2 hours and eight key stakeholders would be gathered in a large room with a u-shaped seating arrangement. Breaks and refreshments would also be scheduled. The modelling team would be composed of the first author (novice facilitator and modeller), supported by the second author (expert facilitator & PhD supervisor) and third author (expert optimisation modeller). In preparing for the workshop the PhD candidate (myself) prepared a script under the guidance of the expert facilitator. As described earlier, tools for this stage were not modified as we determined that existing tools were transferrable to exploring the problem situation. The prescribed tools included preliminary examples and prompts to facilitate discussion between stakeholders. Appendix G depicts images from the workshops.

4.4.3. Defining the PMCH Service - Workshop 1

The aim of this workshop was to develop a common understanding of the PCMH service among project members and to agree on the overall study objectives. The workshop was structured around three prescribed activities: agree problem statement, define the system and draw a system model. Stakeholders were informed that this would be required to participate, brainstorm, and contribute throughout the workshop. The group was provided with handouts to help them gather their thoughts.

In the first activity, stakeholders were asked to consider the questions "What major problems is the mental health local care service in West Kent facing, where quantitative information is not available to support your planning and decision making?". Each stakeholder was asked to brainstorm and put forward three problems, and to support their thinking, several example issues were provided. After brainstorming, the group was invited to discuss the issues they had put forward and identify top three issues that could be considered in this study. Several issues were put forward and captured (Table 43, Appendix E). However, stakeholders were asked to choose three and the choice was informed by what was possible to deliver and whether data was available to take it forward. The top three issues that the group settled on were:

1. Discharge from the service being too long for a service that is meant to be short-term.
2. To streamline messy timetable in primary care by exploring issues with allocated primary care clinic locations.
3. This relates to the previous problem, necessary to opportunities to centralise primary care mental health practitioners perhaps by pooling patients or identifying bases (locations) to reduce travel load and improve patient accessibility.

In the next activity, stakeholders were taken through the process of defining the key elements in the PCMH service using the CATWOE tool. We began this activity by asking participants to decide on which/what is the Transformation that was to be the focus of this study. The group was given examples of what the T is and how to define one. Stakeholders were also informed that only one Transformation would be explored further. Stakeholders provided several transformation that they considered to be relevant to the PCMH service (Figure 38, Appendix E). To help the group narrow it down to one, they were asked to consider what captured the essence of their service and should be considered in more detail. Figure 5 represents the Transformation that was chosen following a discussion.

Figure 5: Transformation of the PCMH service

In the next activity, the group is asked to define the elements in the system that support the Transformation and are supported by the CATWOE tool. To prompt discussion on the Weltanschauung element, the group is asked to the following questions: What is your belief for this T to hold true?, Why do you think this T should exist?, What would happen if this service was not in place?, How would it affect the individuals that you serve in this service, if it did not exist?. The C, A, O, and E elements were defined next, resulting in Table 11. Specifically, the first two columns were defined in the workshop and the information in the third column chosen to be taken forward during the post-workshop stage. In the next part of the session, the group was asked to define a root definition for the service. They were prompted to assemble this using the CATWOE elements that had already been defined. The following statement is the root definition of the PCMH service as conceived by stakeholders: The Primary Care Mental Health Service (PCMHS) ***owned*** jointly by Kent and Medway NHS and Social Care Partnership (KMPT) and West Kent Clinical Commissioning Group (WKCCG) ***operated by*** mental health specialist, ***assisted by*** professionals from the mental health & social care network, ***to support*** people with mental health needs ***by providing*** the right intervention by the right professional at the right time and location ***in order to*** dispense an effective clinical pathway ***whilst*** recognizing financial, workforce, structural and organizational constraints.

The next activity is to draw a systems model (SM) which is a mapping of activities that is equivalent to a Purposeful Activity Model (PAM) in SSM. To produce the SM model, stakeholders are directed to build on the CATWOE and root definition that were previously put forward. The group was provided with a list of verbs to help formulate the definition, and examples of a sample SM were provided.

Table 11: CATWOE outputs

CATWOE Elements		Relevant Element Chosen
Customer	- General Practitioners in Primary Care (PC) - People with mental health needs and their carers/relatives	- General Practitioners in Primary Care
Actors	- Advanced Care Practitioners (ACPs) - Mental health Nurses - Allied Health Professionals - Social prescribers - Voluntary sector - Health & social care coordinators - Secondary care mental health - Public Health	- Advanced Care Practitioners - Mental health Nurses Collectively termed as 'Clinicians'
Transformation Process	To provide the right intervention by the right professional at the right time and location.	Need met by deploying mental healthcare clinicians in primary care
Weltanschauung	- To provide an efficient patient pathway resulting in quality interventions that decreases patient & system risk. - To achieve parity between mental and physical health.	To provide an efficient patient pathway resulting in quality interventions that decreases patient and system risk.
Owners	- KMPT - West Kent Clinical Commissioning Group (WKCCG) - Primary Care Networks (PCN) - Long Term: - Integrated Care Partnerships (ICPs) & Integrated Care Systems (ICSs)	KMPT
Environmental Constraints	- Clinical Workforce - KPIs - Funding - Data - Contracts - Not being a strategic priority for ICPs	- Clinical Workforce - Contracts - KPIs

Next, the group were invited to brainstorm and produce a mapping of activities that take place in the system, and to identify activities that do not currently taking place that either need to be considered in the future or need improvement. Clinical, managerial and data collection activities were identified and are represented in Figure 6. Through this session, stakeholders admitted that the figure represented an ideal service, while the current service lacked several key activities and dependencies. Establishing roles and developing standard operating procedures were two activities that stakeholders wanted to improve most.

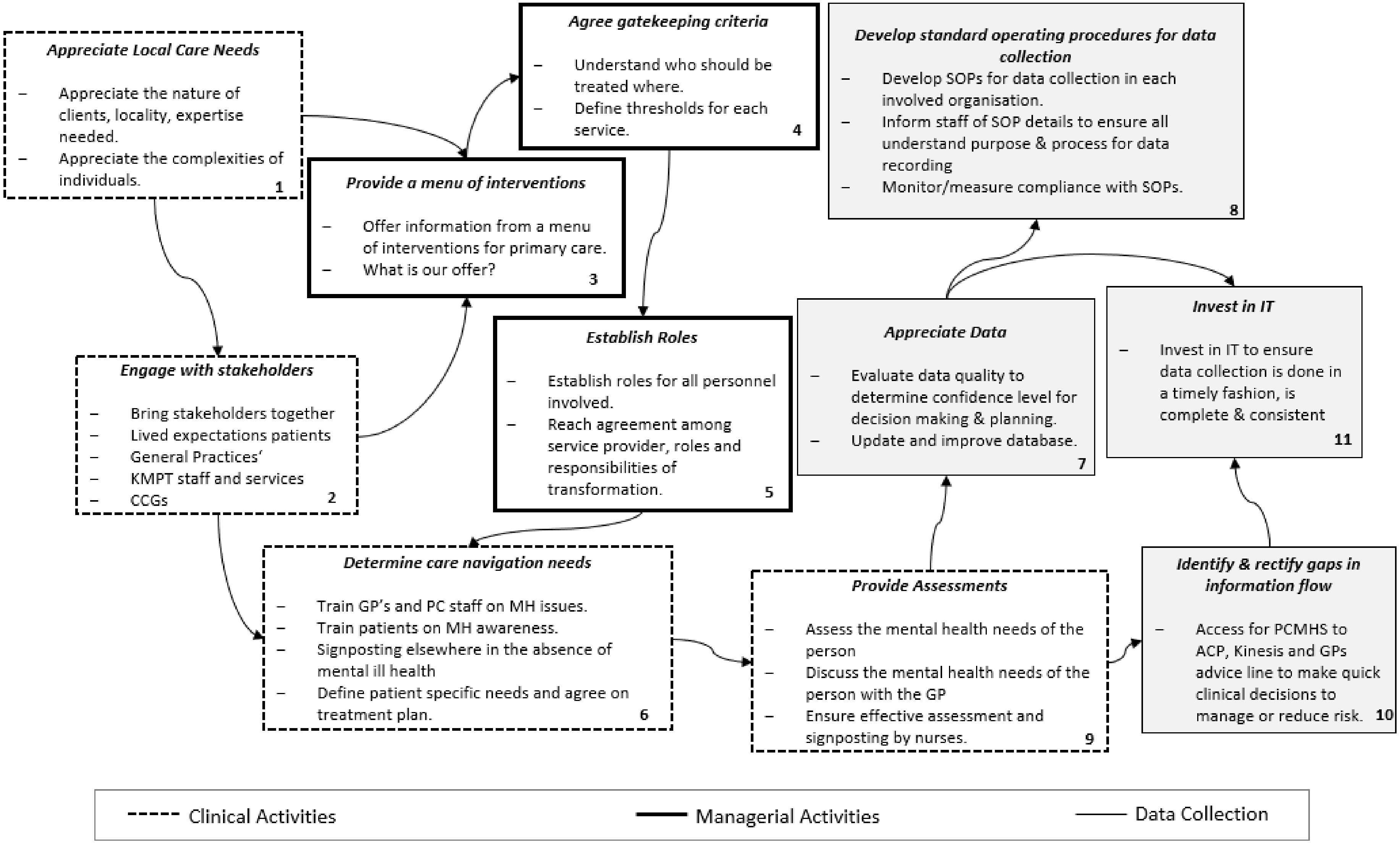

Figure 6: PCMH Service Systems Model

4.4.4. Post - Define the PCMH Service Workshop

A report containing the outputs of the workshop is shared with stakeholders via email. When sharing the report, the group is directed to confirm the outputs, thereby agreeing to the focus of the study, and to inform us of any changes that should be made to the outputs. In essence, the first workshop helped draw a boundary and helped consolidated the scope and generate a joint focus within the modelling and stakeholders' groups. In preparing for the next workshop, we recognised that it was essential to capture the operational details and characteristics of system elements that would make up the conceptual model. Specifically, we needed to discuss and agree on the inputs, objectives, decisions, constraints, and outputs of the CM. Based on the outputs of the workshop, we determined that for the modelling activity, two directions could be considered: 1) Utilising available data and apply analytics to derive insights that were keenly sought by stakeholders and 2) to examine MH clinician capacities (Actors) in relation to demand from clinic locations (Customers). We suspected that the PMM model could give us an idea of specific aspects that required analysis. Additionally, to conceptualise a workforce planning optimisation model that could investigates capacities in relation to demand, we needed to capture operational details and characteristics of the Actors (A) and Customers(C) and explore the Environmental Constraints (E) that limit or regulate A and C.

In order to successfully capture the relevant information, we identified that the PMM, along with CATWOE and root definition would support the extraction of conceptual model components, and so these tools was not modified (Figure 32, Appendix B and Figure 34, Appendix C). We then identified the need for three new activities that were tailored to capture details specific to an optimisation model. Following the development of the PMM, we would direct stakeholders to consider the CATWOE output and expand on the Actors (A) and Customers (C) that were chosen for this study. Stakeholders would specifically be asked to consider characteristics and operational details relative to the Actors and Customers that are directly impacted or could impact the performance of the system. We hoped that this would lead to the extraction of model input data, using which, stakeholders would be asked to identify aspects of the Actors and Customers were controllable, leading to the identification of decision variables. In the workshop, we would capture the information emerging from this activity on a flip-board and a suitable representation that was sufficiently generalisable would be produced after the workshop.

For the next activity, we would extract constraints as boundaries on the system, Actors, and Customers. To support this activity, we developed a 'constraint form' that was pre-populated with prompts and examples to stimulate a discussion. For constraints, the group would be asked to expand on the Environmental Constraint (E) from the CATWOE output. On the back of all preceding activities, stakeholders would be asked to define a potential objective for the model. The group would be directed to consider the Transformation (T) or goal of the service and the PMM model. Stakeholders would be asked to consider how best they could achieve this goal, given what they can control and within the boundaries that have been defined. They would also be directed to consider which of the performance measures defined in the PMM has the most significance towards achieving the goal of the service. The objective and decision variables could be captured on a preliminary communicative model which we term the "Optimisation Component Map".

4.4.5. Defining the Conceptual Model - Workshop 2

The aim of the second workshop is to identify key elements of conceptual model prior to developing a formal optimisation model. The optimisation model components include inputs, objective, constraints, decision variables, and outputs, mapped onto the communicative model we term Optimisation Component Map. To achieve this output, we considered the employing the following activities: draw the PMM model; identify inputs, and decision variables; define the model constraints and objectives, produce a communicative model in the form of an Optimisation Component Map; and discuss data collection.

We began by introducing the 3Es to the stakeholder group using an examples and handouts were provided for further clarification. Stakeholders were asked to consider how they would define the Efficacy, Efficiency, and Effectiveness of the service. And the agreed definitions for the 3Es are depicted in column 1 of Table 12. Next, stakeholders were asked to examine which aspects of the system need to be monitored, controlled or changes to achieve these performance criteria. Each stakeholder was provided with a form with the same structure as Table 12. Following a discussion where stakeholders debated the aspects they put forward, an agreed PMM model was produced as by populating columns 2, 3, and 5 in Table 12. Stakeholders identified potential changes that the service could make to achieve intended service performance.

Table 12: 3E's of the PCMH System

	Monitoring Activities	Determine if activities	Suggested Changes
Efficacy (E1) – What? Will ensure a smooth and timely flow of patients through mental health service?	What do you need to monitor (measure) to know that the system is providing the intended care?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think are needed to ensure that the system provides the intended care to patient?
	I would like to monitor: – Length of Stay – Referral to service – **Clinician utilisation** – Serious Incidence Reports	I would be able to determine if: – Patients are seamlessly moving through the systems. – **Clinicians are conducting appropriate interventions and referrals to tackle mental health issues in primary care.**	I would suggest: **To analyse and streamline nurse-patient allocation based on patient needs and geographic concentration of referrals**.
Efficiency (E2) – How? Ensuring high quality clinical outcomes by establishing efficiency in the process. (Making Every Contact Count)?	What do you need to monitor (measure) to know that the system works efficiently?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think is needed to ensure that the system works efficiently?
	I would like to monitor: – **Waiting time (backlog)** – Length of Stay – **Average number of patients seen in a week** – **Clinician case load (patients and locations)**	I would be able to determine if: – Meeting clinical targets of providing first appointment within 4 weeks of referral – Patients are being discharged from the service within six months. – **Clinician capacity is utilised based on their band**	I would suggest: - **Examine and fix bottlenecks in the patient pathway on both the demand and supply side.** - **Hire more clinicians if existing capacity is insufficient.** - Consider having a single assessment procedure that devolves the need to have an assessment in secondary care
Effectiveness (E3) – Why? Ensure improvement of health and well-being of the target population?	What do you need to monitor (measure) to improve population well-being?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think is needed to ensure that the system ensures population well-being?
	I would like to monitor: – Signposting timelines – Number of people being discharged from SC after assessment and referral. – Discharge post screening and assessment – Number of referrals from GP to PCMH.	I would be able to determine if: – If expectation set based on mental health needs data in population are being satisfied. – If pressures on CMHT have been alleviated owing to PCMHS. – If GPs are aware and actively referring to PCMHS	I would suggest: – To find out which GP's have low referral rates and make them aware of the service, so they do not refer to CMHT. – To explore the possibility of having a permanent mental health presence in GP services. Particularly for those with high referral rates.

The PMM model paves the way for abstracting a system that could be modelled. To support this goal, we decided to deploy two activities: 1) define the inputs and decisions, 2) define the constraints and objective. In the first activity, stakeholders were asked to recall the Customers (C) and Actors (A) element from the CATWOE output, they had previously defined in the first workshop. To make the discussion more accessible, the group was asked to first consider the Actors, collectively called ‘clinicians’, who make up the workforce in the service. We then asked the group to expand on clinician characteristics and their operational details. To prompt a discussion, for characteristics, the group was directed to provide details of clinician duties & responsibilities, who they report to, and what their role entails. For operational details relative to clinicians, the group was asked for specifics on clinician employment category, how many clinicians were currently employed in each category, is there a specific pattern to the geographical distribution of clinicians, and what type of appointments the group could conduct.

Similarly for the Customer, in this case the GP clinics, clinicians were asked to expand on the location of these GP clinics and the describe the operational links between the clinicians (A) and the GP locations. For instance, the group was asked to consider the following questions: what defines the relationship between you the provider of the service and customers, the clinics? Where are the clinics located? How many clinicians are assigned to each location? How is this assignment made? How is the caseload/demand from these clinics distributed? Do clinicians have preferences for locations?

We had prepared some initial questions to prompt a discussion on the Actors and Customers. However, several of the follow-up questions that were eventually asked were generated based on the information provided by stakeholders. During the workshop, we noted down the information that was discussed on a flipchart and after the workshop, the information was collated into a form, seen in Table 13. Given that this was a first attempt at conceptualising an optimisation model by involving stakeholders, besides having an idea of what sort of information was required and how the discussion could be prompted, the input form had not been pre-designed for use in the workshop.

Returning back to the workshop, using the information that been provided by the stakeholders, we then asked them to consider for which operational aspect could the model provide decision support to meet their service goal of “Providing the right intervention by the right professional at the right time and location”. Stakeholders were asked if, for instance, they would like the model to help them decide how many clinics should be assigned to each clinician? Or if they needed support determining clinician caseload, or when the clinician should go to specific clinics?

Table 13: Inputs Form Output

Actors	Characteristics	Operational Details
Advanced Clinical Practitioner (ACP)	- Senior Specialist Clinician. Can diagnose, prescribe, mental health assessment, signpost, & risk assessment - Team lead & supervisor for Band 7. - Report to service manager	- Band 7/8a - 3 ACPs on the team - Can conduct Assessments & Follow-ups & Telephone.
Mental Health Specialist Nurse	- Mental health assessment, signpost & risk assessment. - Reports to ACP & service manager - Supervisor for Band 6	- Band 7 - 4 on the team - Can conduct Assessments, Follow-ups, Community & Telephone.
Mental Health Specialist Nurse	- Signposting, advice, physical health checks, risk assessment - Reports to band 7 & service manager	- Band 6 - 5 on the team - Can conduct Follow-ups, Community & Telephone.
MH Team		- 12 in total - Number of clinics assigned to each clinician [not known] - Number of patients on each clinician's caseload [not known] - Clinicians prefer or go to specific clinics [not known] - Each clinician decides when to hold appointments and clinics
Customers		
General Practice (GP) Clinics	- Scattered across West Kent. - Patients referred to PCMH service by GP's	- 65 clinics currently served by clinicians. - Locations divided across two geographical patches [not known] - Distance between clinics [not known] - Number of clinicians assigned to each clinic [not known] - Demand frequency from each clinic [not known] - Four types of appointments. Durations vary [not known]

During the discussion, the group agreed that they did not have control over how many referrals (demand) they received from clinics and that they required quantitative support to help tackle the expected increase in demand from clinics. Equally, they did not want to disturb the current allocation of clinics and caseloads to clinicians as it would disrupt patient care. We then prompted them to consider what decision they did control, and many clinicians in the group reported making daily decisions about which appointments they would conduct at various locations. This was currently being done individually and on an ad-hoc basis. They did not have information at the time to support the efficiency of this process and its impact on the service. We noted this as a potential set of decisions for the conceptual model and suggested that this is a feasible direction for exploring model support in deciding an efficient strategy of deploying clinicians to clinics.

In the next activity, we aimed to further explore and capture Environmental (E) constraints acting on the Transformation Process (T). We began this activity by asking stakeholders to consider the question "What are the regulations and limitations governing the service that must be respected in order to achieve the service goals?". To direct the conversation, stakeholders are asked to expand on the environmental constraints identified in the CATWOE exercise and particularly rules relative to the Actors and Customers covered in the last activity. Prior to the workshop, the facilitation team developed a few potential categories of constraints to direct the discussion. To support this activity, we provided stakeholders with the "Environmental Constraints" form that had been pre-populated with some categories of optimisation modelling constraints. These categories are based on an initial understanding of the system that was gathered during the first stage of the framework, coupled with the knowledge of optimisation modelling. We found that the prepared groups/categories provided stakeholders with a reference point to begin identifying similar constraints. For instance, we proposed that the PCMH service could have constraints relative to "location", "clinician skills", "timeframe", and "preferences" (clinician and clinic locations). To prompt a discussion for each pre-defined group, we asked questions such as "*Does the service have rules for visiting clinic locations? For example, is there a maximum and minimum number of locations a clinician should visit in a planning period?*". Table 14 is the final output of this exercise. The discussion led to the identification of two additional groups of constraints, "caseload" and "appointments". For each group example prompt questions are provided.

In the last activity, stakeholders were asked to consider the performance measures identified in the PMM activity along with the Transformation goal. Stakeholders were then asked to consider several questions about attaining this goal such as "how can this goal be satisfied?"; "Can we consider maximising clinician capacity utilisation to meet the goal?"; "Is there a specific performance measure('s) that needs special attention to help us achieve this goal?". Stakeholders debated the suggestions and examined the significance towards meeting the defined goal. Several stakeholders agreed that it was more important for appointment requests to be met in time and for there to be a minimum amount of carryover of appointment demand from one month to the next. Although clinician utilisation was an important aspect of the service, a more pressing concern was to see patients within a short period of receiving an appointment request. The mapping of these activities to the "optimisation component map" was not undertaken in the workshop because of time constraints. Instead, we agreed to use the information that's been gather so far and do the mapping in the post-workshop stage.

Table 14: Constraints Form

Environmental Constraints (E)	
Location E.g., specify a min & max number of locations to visit per clinician	–Each nurse covers locations in a cluster. –Clinician travel need to be in geographic proximity of no more than 7 miles. –Clinicians can visit a min of 1 and max of 2 clinics in a working day
Clinician Skills E.g., can a junior clinician conduct all types of appointments?	– Lower band clinicians do not diagnose or prescribe (Assessments). –ACP's provide support to complex patients. (assessments) –Clinicians in higher bands do not conduct community
Timeframe E.g., specify the min or max number of appointments per day	–2 clinics in a day (morning & afternoon) –On average, clinicians to have 4 face-to-face appointments in a day
Preference E.g., specify clinician and location preferences	–Clinicians must be assigned appointments in clinics within their designated geographical patch.
Appointments E.g., are the same type of appointments provided at all clinic locations?	–4 appointment types apply to all 65 clinic locations. –Appointments should be allocated based on skill. –Clinicians should have 4 appointments in a day. –Telephone screenings conducted by band 6 clinicians –Patients to be given appointment within 4 weeks
Caseload E.g., specify max and min number of patients and clinics on a clinician's caseload	–Each nurse may support 11 clinic locations. –More nurses to be deployed to locations with higher volumes.

At the end of the workshop, we had a discussion on where data or if data is available for the key components that were identified. It was agreed that the service would provide operational data for the last 4 years as they were keen to evaluate service performance. The same data could also be used for model building. Stakeholders were also informed that the outputs of the workshop would be collated in a report that would also outline the direction of the model building activity.

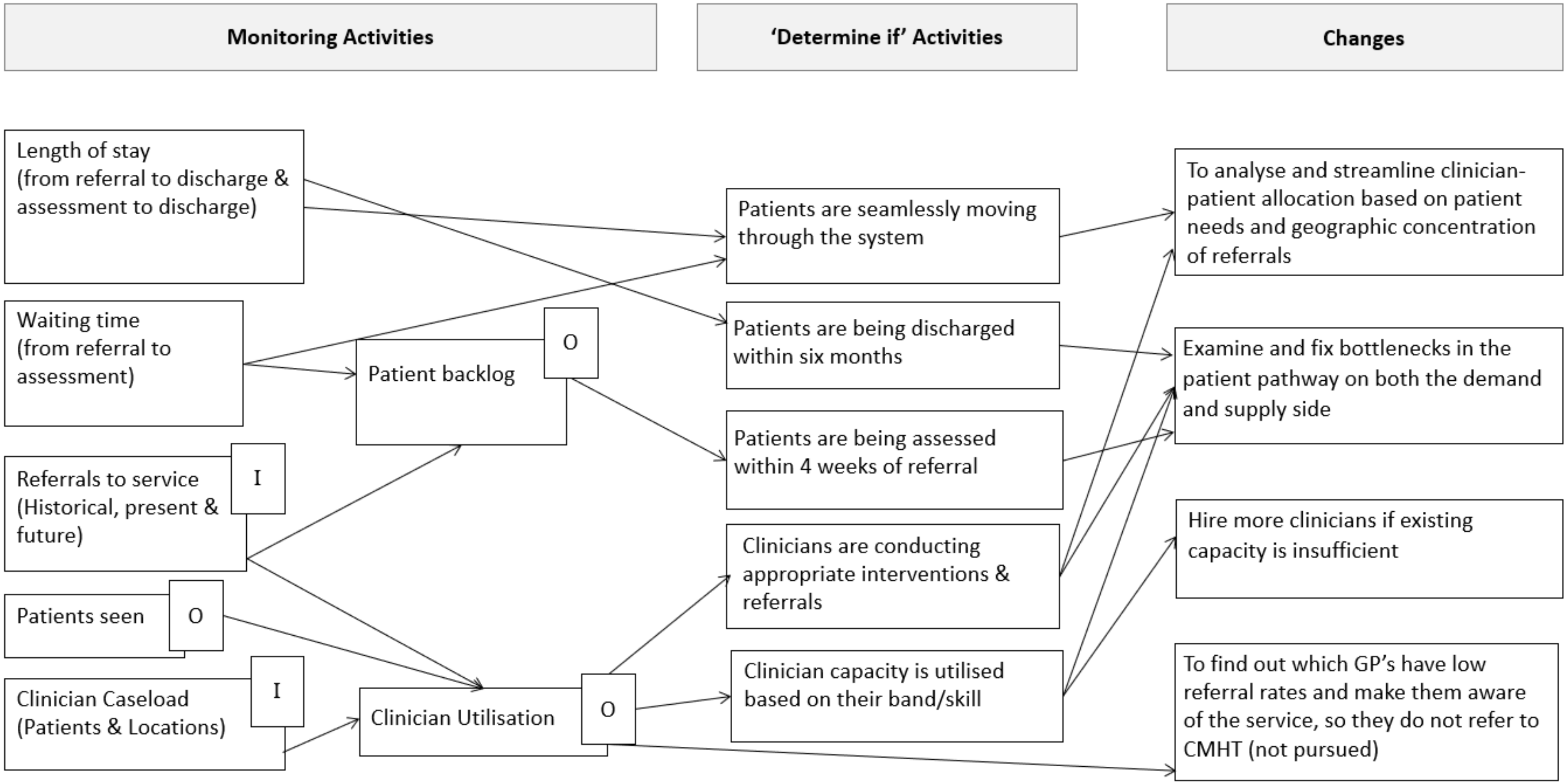

Figure 7: Performance Measurement Model of the PCMH System

4.4.6. Post - Defining the Conceptual Model Workshop

In this stage, outputs from workshop 2 were gathered, and several key activities were undertaken: identify activities from the PMM that can be included in the study; draw the PMM model with the chosen activities and identify potential inputs and outputs that could be mapped to the conceptual model; collate information collected for inputs and represent it in a format that can be validated by stakeholders; map information collected on model objectives, constraints, and decision onto a preliminary 'optimisation component map'; liaise with project champion for data collection based on identified components.

After inspecting the outputs obtained in Table 7, we determined that the model could address the highlighted activities. These were then included in the visually (see Figure 5) and we also identified potential inputs and outputs to the model, based on the overall information that was gathered in Workshop 2. We realised that monitoring activities "Referrals to service (also demand from clinicians) could be provided to the model as inputs to determine patient backlog levels and provide insights on how many patients can be seen given various demand values and under current service specifications. Additionally, the model could also be provided with clinician caseload data to evaluate utilisation of resources. Since the service had large amount of historical data, we determined that length of stay, waiting times, and service demand figures could be analysed by applying descriptive tools to the data. We resolved to also use predictive tools to forecast future service demand, given the emphasis on preparedness for an impending rise in demand.

Table 15: Optimisation Component Map (Preliminary)

Objective	Constraints	Decisions
To meet demand for primary care from clinics	- Limits on clinician availability to satisfy referral demand. - Clinicians must be assigned appointments based on their skill. - Clinicians must be assigned appointments in clinics within their designated geographical patch. - Clinicians cannot travel to clinics that are too far from each other. - Limits to how many appointments clinicians can have in a day & how many clinics they can visit. - Limits on the number of clinics on a clinician's caseload. - Limits on number of clinicians to be assigned to any given clinic	- We can decide what clinic a clinician goes to. - Also, for which appointment. - We can also decide when the clinician can hold these appointments.

Information collected during the inputs and decisions activity was gathered and presented in the form of a table as seen in Table 13. We believe that this table is sufficiently generic to be used as a tool in workshops. It can be noticed in the table, that some input descriptions are suffixed with [not known]. This is because explicitly values were not provided during the workshop, and the information was identified to be gathered from other sources. These points were highlighted in the report as requiring further clarification from stakeholders.

The components of the model explored that were individually explored in the workshop were consolidated mapped onto the preliminary "Optimisation Component Map", depicted in Table 15. Through the mapping activity, the modelling team also identified a preliminary list of assumptions for the model, and these were shared with stakeholders for approval. The list of assumptions and simplification are depicted in Table 16.

Table 16: Model Assumptions and Simplifications

Assumptions
Schedule generation/planning is centralised
Four-week planning period with each week consisting of 5 working days
A given working day is divided into two slots (AM & PM)
Clinician availability is standardised
Demand for appointments from clinics is known at the start of each planning period
Appointment durations are standardised
Physical space to run clinics at GP location assumed to be available

Figure 8 represents the flow of information leading to the development of a communicative model. This preliminary map contains the objective in column one that is derived from the Transformation (T); the second column contains constraints, that include a list of a high-level summaries derived from the constraints form; and the third column contains decisions, that would be identified as controllable inputs in the second activity.

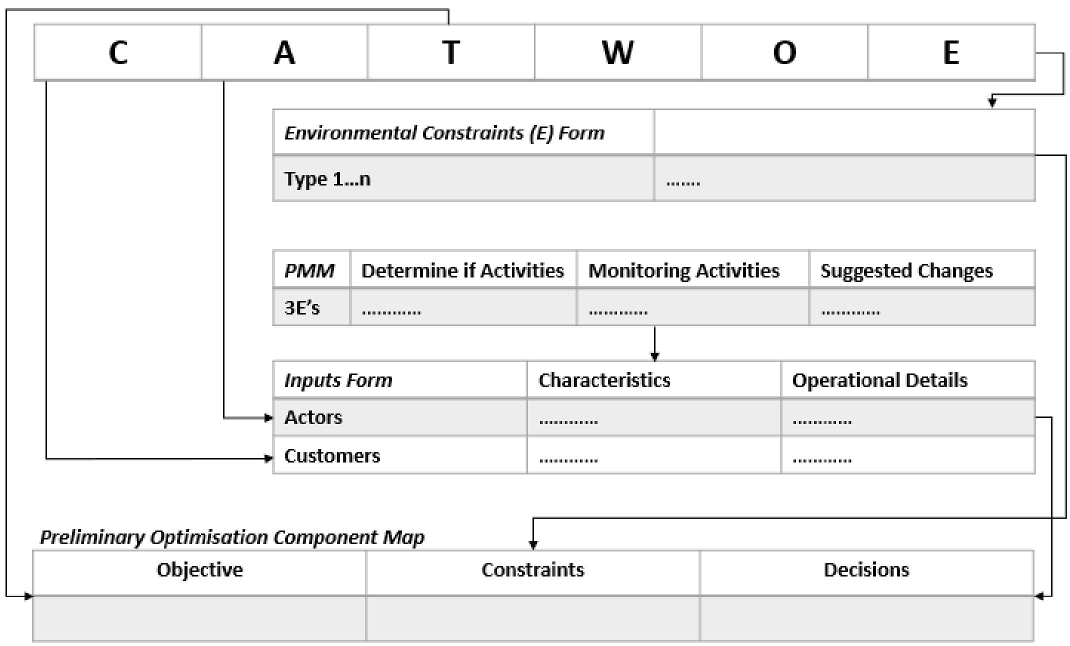

Figure 8: Workshop 2 Information Flow to Specify a Conceptual Model

Chapter 5 describes the transformation of the conceptual model into a mathematical formulation and provides additional details on inputs that were fed to the model, describes the coding of the model on a solves, and illustrates the outcomes of the solved model. Table 17 presents the mathematical formulation of the model that was derived from the conceptual model. In particular, the table described the objective, decision variables and constraints that were formulated.

Table 17: Mapping of the final Conceptual Model leading to Mathematical Formulation

Objective	
- Minimising unmet demand	- The objective of the model (1) is to minimise the number of unassigned appointments. In other words, to minimise unmet demand. $Min\ \sum_{a\in A}\sum_{l\in L} F_l^a - \sum_{a\in A}\sum_{c\in C}\sum_{l\in L}\sum_{s\in S} X_{cls}^a$
Decision Variables	
- "Who" (clinician) goes "where" (clinic) for "what" (appointment type) and "when" (week, day, & shift)	- $\boldsymbol{Y_{cls}} = \begin{cases} 1, if \text{ clinician } c \in C\ is \text{ assigned to clinic location } l \in L\ in \text{ shift } s \in S \\ 0, \text{otherwise} \end{cases}$
- Integer value showing how many appointments to each clinician, "where" (clinic) for "what" (appointment type) and "when" (week, day, & shift)	- $\boldsymbol{X_{cls}^a}$ = number of appointments of type $a \in A$ assigned to clinician $c \in C$ at clinic location $l \in L$ in shift $s \in S$
Constraints	
- Assign demand from clinic locations based on available clinician capacities.	- Constraints (2) ensure that the sum of durations of all appointments assigned to a shift does not exceed the length of each shift. $\sum_{a\in A} R_a X_{cls}^a \le L_s\ \forall c \in C, \forall s \in S, \forall l \in L$ (2) - Constraints (3) make sure that appointments assigned to any shift in a clinic do not exceed the demand of appointments in that clinic. $\sum_{c\in C}\sum_{s\in S} X_{cls}^a \le F_l^a\ \forall l \in L, \forall a \in A$ (3)

	- Constraints (4) assigns demand based on available clinician hours and captures any unassigned hours in the slack variable Z_c^-. $\sum_{l\in L}\sum_{s\in A}\sum_{a\in A} R_a X_{cls}^a + Z_c^- = H_c \;\; \forall c \in C$ (4) - Constraints (5) prevent the allocation of appointments to clinicians in each location and shift unless the clinician has been assigned to the location ($Y_{cls} = 1$). $M\, Y_{cls} \geq X_{cls}^a \;\; \forall l \in L, \forall c \in C, \forall a \in A, \forall s \in S$ (5)
- Max shifts per day	- Constraints (6) set the maximum number of shifts that can be assigned to a clinician per day. $\sum_{l\in L} Y_{cls} \leq St_{cd} \;\; \forall c \in C, \forall s \in S_d$ (6) - Constraints (7) ensure that a clinician can only be assigned to 1 or 0 shifts in a clinic location. $\sum_{c\in C} Y_{cls} \leq 1 \;\; \forall s \in S, \forall l \in L$ (7)
- Clinician travel constraints between clinic locations - Max travel distance between clinics	- Constraints (8)-(9) prevent the assignment of a clinician to locations that are too far away on the same day. $(Y_{cl_1s_1} + Y_{cl_2s_2}) - 1 \leq M_1 V_{cdl_1l_2} \;\; \forall c \in C, \forall s_1, s_2 \in S, \forall l_1, l_2 \in L: l_1 \neq l_2, \forall d \in D$ (8) $T_{l_1l_2} - T_{max} \leq M_2(1 - V_{cdl_1l_2}) \;\; \forall c \in C, \forall l_1, l_2 \in L: l_1 \neq l_2, \forall d \in D$ (9)
- Max clinicians per clinic	- Constraints (11) limit the number of clinicians that can be assigned to a clinic location. $\sum_{c\in C} W_{cl} \leq N_l \;\; \forall l \in L$ (11)
- Max clinics per clinicians	- constraints (12) limit the number of clinic locations that can be assigned to a clinician. $\sum_{l\in L} W_{cl} \leq N_c \;\; \forall c \in C$ (12)
- Assign appointments based on clinic location preference	- Constraints (13) ensure that clinicians are only assigned to clinic locations that they cover. $W_{cl} \leq P_{cl} \;\; \forall c \in C, \forall l \in L$ (13)
- Assign shifts based on clinicians' availability	- Constraints (14) and (15) ensure that clinicians are only assigned to shifts based on availability. $Y_{cls} \leq Q_{cs} \;\; \forall l \in L, \forall c \in C, \forall s \in S$ (14) $Q_{cs} \leq H_{cs} \;\; \forall c \in C, \forall s \in S$ (15)

- Assign appointments based on clinician skills.	- Constraints (16) and (17) assign appointments to clinicians based on their skill level. $M\, U_c^a \geq X_{cls}^a \qquad \forall l \in L, \forall c \in C, \forall s \in S, \forall a \in A$ (16) $U_c^a \leq B_c^a \qquad \forall c \in C, \forall a \in A$ (17)

4.5. Discussion

In section 4.5.1, we begin by reflecting on the application and development of the facilitated conceptual modelling approach for an optimisation model. Based on learning through the application, we then propose the framework along with the requisite activities, tools, and prescribed outputs. In section 4.5.2, we consider conceptual modelling for optimisation and lastly in section 4.5.3, a discussion for facilitated optimisation is presented.

4.5.1. Reflections on the Proposed Framework

Through the case study, we have provided proof of transferability for the first two stages of the PartiSim framework, which can be adapted to optimisation without any modification in order to initiate the study and define the system. In the first stage, by following the prescribed process, we were to gain a good understanding of the PCMH service and the investigate the wider context within which the service operates. Identification of the stakeholder team was a key activity that led to the recognition of several. In the second stage, we were able to use the information gathered in the initial stages of the study to direct the conversation, supported by the tools. In preparing material for the workshop, it was determined that some of the activities, tools, and prescribed outputs within stage 3 would need to be tailored to illicit information relevant to an optimisation model.

These new additions were developed by acknowledging the differences in the structures of a conceptual model between DES and optimisation. These new additions are not stand alone or disconnected from the outputs of the first workshop. In fact, they activity draw from preceding stages. Specifically, the CATWOE tool, and PMM drove the thought process in how to guide stakeholders to efficiently utilise these new tools. Table 18 is the proposed framework for conceptualising an optimisation model. The highlighted text within Table 18 represents modifications that were made to the original framework. Specifically, we introduce three new activity and tools. 1) Activity to 'Identify inputs and decision variables of the model' and propose a corresponding tool 'Inputs Form'; 2) Activity to 'Define the model constraints and objectives', supported by the 'Constraint Form'; Activity to 'Produce optimisation model component mapping' by using the 'Optimisation Component Map'. In the remaining section, we will discuss the practical implementation of these new activities and share the valuable insights we gained from them. We will briefly revisit the development process of the 'Input Form' that emerged after the workshop and suggest modifications based on the lessons we learnt. Additionally, we will introduce a more generalisable 'constraint Form'. Similarly, building upon the knowledge gained after workshop 2, we have enhanced the design of the communicative model and propose these improvements to be included in the framework.

4.5.1.1. Reflections on the Input Form

The input form was designed based on the information gathered in the workshop. The activity was structured to capture characteristics and operational details relative to the Actors(A) and Customers (C), in this case study were clinicians and clinics. Table 13 was the first iteration of this form. We proposed improvements to the form based on the insights derived from the data collection activity. Following the workshop, we liaised with the project champion to identify data sources for the components that had been identified in this activity and to conduct an analysis on the elements in the PMM that were not being captured in the optimisation model. Through this dialogue with the stakeholder team, it became apparent that data would need to be derived from multiple sources. Specifically, some of the data was known with certainty (such as the number of clinicians in the service, the number of appointments etc). The rest would need to be derived from the following sources: historical data using descriptive tools, expert knowledge, predictive tools, new data collection etc.

Considering this knowledge, we realised that it would have been useful to have this conversation within the workshop as it has implications for the next stage: model coding. Specifically, we recognised that to build a realistic optimisation model with the potential for implementation, the modelling process would require the combined use of multiple analytics tools. Therefore, the new proposed ‘Input Form’ contains an extra column with a list of potential data options, using which additional data sources and the need to compliment the optimisation model with analytics techniques can be explored. Table 45 (Appendix F) is the proposed improved ‘Input Form’. The first iteration of this design emerged after workshop 2, and the latest iteration is deemed most suitable for capturing the whole breath of data requirements and inform the modelling approach.

Table 18: Conceptual Modelling for Optimisation, adapted from PartiSim

Stage and purpose	***CM support activities***	***Tools***	***CM outputs/deliverables***
1. Initiate Study Purpose: • Identify stakeholder team • Identify key problem situation(s)	The modelling team undertake: • informal meetings and/or • on-site observations and/or • with project champion and key stakeholder(s), to address preliminary information needs	Information Collection Tool	Preliminary understanding of the problem situation.
1a. Pre-workshop stage *Purpose*: Preparations for Workshop 1	Modelling team prepare preliminary materials for tools to be used in workshop 1		
2. Define system (Workshop 1) *Purpose*: Agree on the problem situation and the wider system, within which it exists.	Participating stakeholders take part in a facilitated workshop process to: • Brainstorming problem area (s) to be addressed and identify study objectives. • Define system boundaries	- Problem statement form - CATWOE and root definition - Care system model	- General study objective(s) - A bounded system within which the problem to be addressed exists
2a. Post-workshop1/Pre-workshop 2 stage *Purpose*: Disseminate workshop 1 outputs and prepare workshop 2	Modelling team re-draw tools and disseminate workshop outputs to stakeholders Prepare preliminary materials for tools used in workshop 2		
3. Specify conceptual model (Workshop 2) *Purpose*: Define specific elements of the conceptual model	Participating stakeholders take part in a facilitated workshop process to: • Put forward and agree on performance measures to address the problem identified in workshop 1 • **Identify inputs and decision variables of the model** • **Define the model constraints and objectives** • **Produce optimisation model component mapping** • Discuss responsibility for data collection.	- Performance measurement model (PMM) - **Inputs Form** - **Constraints Form** - **Optimisation Component Map**	- **Model inputs, decision variables and outputs** - **Model objectives and constraints** - A preliminary list of assumptions and simplifications - **A communicative model** - A list of data requirements
3a. Post-workshop 2 stage *Purpose*: Disseminate workshop 2 outputs and refine conceptual model	Modelling team prepare report detailing: • Refined CM outputs from stage 2.a and stage 3 • Data requirements		An agreeable to all (study participants) and feasible conceptual model describing an optimisation model

4.5.1.2. Reflections on the Constraint Form

The “constraint form” uses the CATWOE output to draw a connection between the real system and the definition of constraints for the model. In determining the prompts to prepopulate the model for use in the workshop, only constraints that seemed relevant to the specific problem situation were considered. Using the knowledge about the PCMH service, that was gained in the prior stages, the form considered the following constraint prompts “location”, “clinician skills”, “timeframe”, “preference”, “appointments”, and “caseload”. A more generalised form that can be adapted for any optimisation model would include a combination of typical optimisation model constraints such as: Budget, Human resources, Physical resources, Time, Location / geographical, Preference / utility, Demand, Capacity, and Structural as seen in the Table 46 (Appendix F). Depending on the application context, each category can include a specific prompt to support stakeholders with identifying relevant constraints in the system of concern.

4.5.1.3. Reflections on the Communicative Model

This stage required the development of an ad-hoc, novel communicative model depicting the conceptualisation, which was named “Optimisation Component Map”. The first iteration of the map consisted of three key components of the optimisation model: objectives, constraints, and decision. After the second workshop, we gathered more information on the model inputs and realised they came from different sources. Additionally, during initial model formulation activity of Stage 4, we added more detail to the conceptual model as we recognised that we would build mixed-integer optimisation model. The output of this model would be a clinician to appointment and clinic location schedule. Therefore, we updated the optimisation model map and added the remaining components of an optimisation model: Inputs and Outputs, as seen in Table 19.

Robinson (2020) have identified that the representation of the conceptual model in a way that is meaningful, comprehensive, and communicative is one of the grand challenges of conceptual modelling. We believe this is equally true for CM in optimisation modelling. The proposed a communicative model is a transparent representation of all components in the optimisation model. This iteration of the map was used in subsequent stages of the framework. We believe that the map was a useful tool in communicating the optimisation model and improved stakeholder understanding of the mathematical formulation presented in Chapter 5. However, we recognise that the communicative model could be further improved by conducting more case studies. Given how the modifications and subsequent improvement have been informed by the case study. We believe that more studies are needed to develop greater agreement and uniformity in representing a conceptual model for optimisation.

Table 19: Optimisation Component Map (Refined)

Inputs	Objective	Constraints	Decisions	Outputs
From Data: - Set of Clinicians, locations, appointment types, days, shifts - Demand per Clinic Location - Clinic locations assigned to clinicians - Appointment durations - Clinician skills and appointment matrix - Clinician to clinic preference **Stakeholder Supplied:** - Max shifts per day - Max clinics per clinicians - Max clinicians per clinic - Clinicians' availability - Max travel distance between clinics - Shift Duration **Generated:** - Distance between clinics	Minimising Unmet Demand	- Assign demand from clinic locations based on available clinician capacities. - Assign appointments based on clinician skills - Assign appointments based on clinic location preference - Clinician travel constraints between clinic locations - Max shifts per day - Max clinics per clinicians - Max clinicians per clinic - Assign shifts based on clinicians' availability - Max travel distance between clinics	"Who" (clinician) goes "where" (clinic) for "what" (appointment type) and "when" (week, day, & shift)	Planning Schedule

4.5.1.4. Reflections on the overall application

The development of this framework doubly benefitted from having the co-creator of the PartiSim framework, also an expert facilitator, on the modelling team. The first author, who was a novice facilitator at the start of the project had access to expert guidance and procedural insights throughout the project. Equally, the presence of an expert in the field of optimisation significantly contributed to bridging the gap between optimisation, soft OR and facilitation. Although we did not explore this aspect of the development process to the extent that it is possible, it is an interesting area of research that could yield important guidance to both new and seasoned researchers. It has been established that success in group facilitation is still dependent on the modellers leading each intervention (Tako & Kotiadis, 2012). Therefore, future research could investigate the impact of roles and modelling team composition by drawing in existing literature on the facilitated process (Tavella & Papadopoulos, 2015a; Tavella & Papadopoulos, 2015b).

As stated before, the PartiSim framework provides a list of *stages* which include specific *activities*, suggested *tools* to be used with corresponding manuals to support the process of using the tools to reach to the prescribed dedicated *outputs* for each stage. The framework also provides some scripts, that are aimed mainly at the facilitator. Scripts are different from the tools or manuals in that they include advice to support the facilitation process for activities that do not require any specific tools to be used (Tako & Kotiadis, 2015; Tako & Kotiadis, 2018). In facilitated modelling practice, scripts capture the expertise of facilitators in concise statements, enabling the transfer of knowledge and experience to beginners for effective workshop management (Hovmand et al., 2012; Smith & Shaw, 2019). They provide instructions on connecting activities to design the workshop and scheduling them throughout the event. The proposed framework is primarily focused on adapting the stages, activities, and tools to optimisation modelling. It goes some way in providing guidance to support the use of the tools to reach prescribed outputs, but is limited in the exploration of scripts, which is an opportune area for future work as facilitated optimisation evolves. Future research could explore opportunities for cross-fertilisation between Group Model Building and Facilitated DES, an avenue identified as having potential as PartiSim evolves (Kotiadis & Tako, 2018). Additionally, future work could also draw from studies that examine scripts the use of scripts to manage facilitated workshops for novice facilitators (Tavella & Papadopoulos, 2015b)

4.5.2. Conceptual Modelling for Optimisation

We contribute a framework for the conceptualising an optimisation model in a healthcare context, by adapting the PartiSim multi-methodology. However, as discussed in Section 2, for this category of OR methods, limited attention is given to problem exploration and conceptual modelling, where these are not regarded as key activities. In the systems view, the OR process consists of activities spanning four phases, namely, the problem situation, the conceptual model, the formal model, and the solutions and recommendations (Landry et al., 1983; Sagasti & Mitroff, 1973). In contrast to optimisation, simulation modelling processes prioritise extensive exploration of the problem situation and the development of a conceptual model before delving into creation of a formal model (Jones et al., 2022; Kotiadis, 2007; Kotiadis & Robinson, 2008; Kotiadis et al., 2014; Robinson, 2008; Robinson et al., 2010; Robinson, 2013; Robinson, 2014; Sterman, 2002; Tako & Robinson, 2009; Tako & Kotiadis, 2012).

Although the development of a conceptual model receives limited attention in optimisation modelling, the notion of a "conceptual model" is applicable to mathematical programming, as it is to other modelling techniques in OR (Landry et al., 1983). In conceptual modelling for simulation literature, no single definition of CM exists (Robinson et al., 2015). However, Robinson (2013) broadly defines CM as a non-software specific description of the computer model that will be or has been developed, that describes the objectives, inputs, content, assumptions, and simplifications of the model. In our case study, the conceptualisation of the optimisation involved defining the non-software specific description of nearly all the aforementioned aspects of CM for simulation modelling. In terms of model content, the CM developed for an optimisation model involves further describing the components decision variables and constraints, which are key structural elements alongside model objective.

In literature, optimisation models are mainly developed for 'prototype problems', that address a specific managerial situation. In this modelling process, the 'formal model' takes precedence, while the conceptual model is a secondary or a non-issue (Oral & Kettani, 1993). A formal model is a translation of the conceptual model into mathematical symbols, computer languages, or both. The purpose of a formal model is to study the problem and obtain solutions to formulate recommendations. Many well-known problems fall into this category, such as vehicle scheduling problem (Kliewer et al., 2006), travelling salesman problem (Mosheiov, 1994), quadratic assignment problem (Loiola et al., 2007), transportation problem (Masson et al., 2016), location problem (Bélanger et al., 2019), and some production planning and scheduling problems (Wu et al., 2022). These problems are well-conceptualised and 'formal models' can be easily formulated, with the focus mainly on solution techniques and implementation procedures (Oral & Kettani, 1993).

Looking at the multi-methodology proposed by Cardoso-Grilo et al. (2019) from this perspective, we argue that their study develops a conceptual model for a 'medical training problem'. In other words, their study presents a sophisticated reformulation of a specific problem that supports health care workforce planning. Similarly, the participatory approach developed by Abuabara et al., (2022) addresses the 'diet problem', which is a classical application of linear programming in OR. Our approach is shares some similarities with these aforementioned studies, as it involves the conceptualisation and eventual development of an optimisation model for a specific context. However, our study did not aim to address a specific type of 'problem'. Instead, the tools were employed to develop a conceptual model that could be applied to any type of optimisation model, tailored to fit a specific problem type.

Future research can use our framework for applications where stakeholder participation is deemed crucial in the conceptualisation of an optimisation model. Moreover, our approach is suitable for application for situations where a pragmatic and expedient conceptualisation of an optimisation model is required, regardless of problem type. Alternatively, as demonstrated in our case study, researchers can use our facilitative and participative approach to develop a conceptual model and identify an existing problem in literature, that aligns with the conceptualisation.

Our work presents an important case study in linking conceptual modelling research with conceptual modelling practice. In CM literature, this aspect has been deemed a mini challenge (Robinson, 2020). The collaboration between academia and practice as implemented in our case study is a prime example of work where equal focus was granted to the method and tool development as well as the practical application of CM.

4.5.3. Towards Facilitated Optimisation

Our work contributes the first facilitative and participative multi-methodology framework for building an optimisation model, in the context of mental healthcare. In doing so, we demonstrate that the facilitated mode of intervention can be applied to optimisation modelling, thereby contributing to the facilitated modelling literature (Franco & Montibeller, 2010; Lane & Husemann, 2018; Robinson et al., 2012). Furthermore, the combination of optimisation with SSM, adds to the limited pool of studies that mix soft OR approaches with optimisation (Abuabara et al., 2022; Amorim-Lopes et al., 2021; Cardoso-Grilo et al., 2019; Ferreira, 2013). Our framework employs tools from SSM to enable the development of a conceptual model for mathematical programming. By drawing from the PartiSim framework, we build on existing knowledge without re-inventing the wheel (Kotiadis & Tako, 2018; Tako & Kotiadis, 2015). Granted that the PartiSim framework was developed particularly for simulation modelling. We argue that the framework is compatible for optimisation as both modelling methods follow mostly similar modelling cycles (Pidd, 1997). Furthermore, Smith and Shaw (2019) have also demonstrated that that DES and Linear Programming (LP), are modelling approaches that are more similar than not when compared across several characteristics. For instance, although DES and LP build distinct models of a system, both approaches build objective models that represent a situation using interconnected quantitative variables. Additionally, we can link optimisation modelling to the stages of SSM based on existing research that compares the four stages involved in a DES modelling lifecycle with the 4 stages of SSM and established that both approaches cover the same stages at varying levels of detail (Tako & Kotiadis, 2015). We propose that our framework can be applied for building optimisation models in other contexts characterised by multiple stakeholders with diverse perspectives and concerns likely to influence decision-making; where the problem definition is not well defined or even absent, as was the case in the mental healthcare service that was the focus of our case study. For example, the framework could be advantageous to other application in healthcare services (Unützer et al., 2020) and disaster management (Amideo et al., 2019; Çoban et al., 2021). Additionally, given that our framework is an adaptation of the PartiSim, future researchers considering both DES and optimisation for their intervention can draw from the work presented here and the original framework. Researchers looking to integrate optimisation with hybrid simulation can draw from the representation method that can aid the modeller in defining the modelling frame (i.e., the combination of methods forming the hybrid model) (Jones et al., 2022).

4.6. Conclusion

This paper presents a facilitative and participative multi-methodology framework, adapted from PartiSim, for conceptualising an optimization model. The framework comprises tools and processes specifically designed for the initial stages of optimisation modelling. We provide proof of the practicality of the framework through a case study in mental healthcare, showcasing its effectiveness in conceptualising an optimization model. Additionally, the case study illustrates how the framework can be applied to structure problem situations in collaboration with stakeholders. We discuss the modifications made to the PartiSim framework to suit optimisation modelling needs. We argue that conceptual modelling plays a crucial role in developing a formal model. The proposed framework emphasises the usefulness of having more rigour for this stage of optimisation. The participative and facilitative elements of the framework are particularly valuable in contexts where stakeholder engagement is essential for optimization model development, such as healthcare and disaster management, where a sense of progress is prioritised over achieving optimality. We identify areas for potential improvement that can streamline the path to facilitated optimisation modelling based on the specific application context. Furthermore, we encourage researchers to conduct further investigations utilising real case studies to explore the potential of conceptual modelling for optimisation.

Chapter 5: An integrated optimisation and analytics approach for planning mental healthcare services.

ABSTRACT

Mental health services worldwide, including in the UK, face significant constraints that necessitate effective resource planning for delivering high-quality care. Applying analytics to healthcare has the potential of improving efficiency while enhancing the quality of care. However, achieving this vision is particularly challenging in the context of mental healthcare. This paper focuses the evaluation and redesign of a Primary Care Mental Health (PCMH) service located in Kent, UK. To address this problem, we propose an analytics driven approach that integrates the three stages of descriptive, predictive, and prescriptive analytics with an optimisation model. Through a case study, we illustrate how the integrated approach served as a valuable tool for experimentation within the PCMH service. The findings of our novel multi-skill multi-location model demonstrate the benefits of utilising optimised workforce planning to reduce unmet demand. We discuss the adaptability analytics approach and the potential applicability of the optimisation model in mental health and other care settings.

Keywords: Healthcare Analytics; Optimisation; Multi-Skill Multi-Location; Integrated analytics approach; Mental Healthcare Service Planning

5.1. Introduction

Mental illness has a significant impact on individuals, society, and the economy. Primary care is now at the forefront of the predicted increase in mental health presentations (Park et al., 2020). Early intervention in primary care reduces subsequent mental health problems and is cost-effective (Van't Veer-Tazelaar et al., 2010). In the UK, one mental health care model of care involves distributing mental health professionals from secondary care to primary care locations (Naylor et al., 2020). These heterogeneous mental health professionals work across multiple General Practitioner (GP)/primary care clinic locations in a geographic patch. The planning and scheduling of such healthcare workers have received limited attention in Operations Research (OR) (Al-Yakoob & Sherali, 2008; Cheng & Kuo, 2016). Furthermore, the application of OR techniques to planning mental healthcare services does not receive the same attention when compared to other healthcare services (Bradley et al., 2017; Howells et al., 2022; Long, K. M. & Meadows, 2018; Noorain et al., 2019; Noorain et al., 2022).

Before COVID-19, mental health services in England were already under considerable strain. Issues included inadequate resourcing, patients' ability to access care and overall patient outcomes (British Medical Association, 2020a). Many of these issues have worsened due to the pandemic (British Medical Association, 2020b). The impacts of the COVID-19 pandemic on people's mental health and wellbeing are a significant public health concern; while some are transient, others are likely to be long-term (McCartan et al., 2021; Pierce et al., 2021). Recent data shows that the number of people contacting the NHS seeking help for mental health problems is now at a record high (NHS Digital, 2022). These needs arise within the context of underfunded mental health services facing a care backlog, waiting lists, and a stretched, exhausted, and understaffed workforce. Workforce capacity has been a long-term concern, and shortages represent the biggest threat to national ambitions to improve mental healthcare (HM Government, 2021; NHS Confederation, 2022). Potential solutions could be found in telemedicine, which remains controversial, despite its efficiency in reducing the impact of the pandemic (Omboni et al., 2022). In OR literature, studies addressing telemedicine with respect to operational efficiency are limited (Zhou et al., 2023).

In OR literature, optimisation modelling has a long history of supporting healthcare decision makers seeking to develop more efficient healthcare systems (Cissé et al., 2017; Grieco et al., 2021; Leeftink et al., 2020; Marynissen & Demeulemeester, 2019). Despite the widespread use of optimisation techniques in healthcare contexts, application in mental healthcare is still sparse (Noorain et al., 2022).

In this paper, we consider the problem of evaluating and redesigning a mental health service for a primary care network in Kent, UK. Our approach demonstrates how OR can be used to support improvements to mental healthcare services. Specifically, our contribution is threefold. First, we contribute a novel multi-skill multi-location optimisation model that schedules itinerant mental health clinicians to multiple geographical locations across a planning horizon. Second, we contribute a real case study using real data, adding to the limited pool of optimisation literature applied to mental healthcare. Third, we develop an integrated three-stage optimisation-based analytics approach that combines descriptive, predictive, and prescriptive analytics. The integrated approach draws upon the principles and practices of OR and takes a holistic view of the problem situation (Hindle, Giles, Kunc, Mortensen, Oztekin, & Vidgen, 2020; Hindle, Giles A. & Vidgen, 2018). Furthermore, the case study enabled the development and demonstrated the utility of the analytics approach, which has the scope to be extended to other healthcare contexts with similar features.

The remainder of this paper is organised as follows. In Section 5.2, we provide a literature review on the various components of this study, such as the application of OR and analytics approaches in mental healthcare. We also discuss a specific body of scheduling literature upon which our optimisation model draws. Section 5.3 provides contextual background on the collaboration with a PCMH service. Section 5.4 describes the various elements of the integrated approach. Since the novel optimisation model is a central contribution, Section 5.5 provides a comprehensive account of the prescriptive element, including model formulation, and Section 5.6 examines the computational results from the scenario analysis. Section 5.7 discusses the study's contributions and Section 5.8 provides some conclusive remarks.

5.2. Literature Review

We drew on several relevant literature themes to positioning our study. We begin by focusing on optimisation in mental healthcare, followed by a broader examination of personnel scheduling in healthcare, particularly on multi-skill multi-location scheduling. We then examine the literature for studies in healthcare that have applied analytics approaches spanning all three types of analytics.

5.2.1. Optimisation in Mental Health

Operational Research (OR) has contributed significantly to designing and organising processes, optimising operations, and managing healthcare systems (Hulshof et al., 2012; Rais & Viana, 2011). Optimisation for healthcare planning enables simultaneous consideration of multiple constraints and sensitivity analysis to find the best solution (Kahraman et al., 2018) and have been used to determine resource quantity, allocate capacity, scheduling, and allocating appointments to support planning of emergency rooms, primary, outpatient and home health (Cissé et al., 2017; Grieco et al., 2021; Leeftink et al., 2020; Marynissen & Demeulemeester, 2019).

Despite the widespread use of optimisation in healthcare, application in mental healthcare (MH) is only beginning (Noorain et al., 2022). MH systems are generally composed of a diverse range of services that comprise interrelated parts of a whole system and primarily rely on human resources, including a heterogeneous mix of specialists, non-specialists, and community workers (Gask, 2005; Gupta et al., 2019; Kakuma et al., 2011). A handful of studies have developed optimisation models in MH, such as to schedule appointments with patient no-show predictions (Samorani & LaGanga, 2015), allocate appointments subject to waiting times (Pagel et al., 2012), assign staff to shifts by considering preferences (Cohn et al., 2009) and workload balancing (Hertz & Lahrichi, 2009), schedule visits to outreach clinic locations (Li et al., 2016) and patients' homes (Hertz & Lahrichi, 2009), and build duty rosters for psychiatric nurses (Bester et al., 2007). It has been established that the strength of optimisation planning models in MH so far is that they were developed in real world practical contexts but had a narrow scope and used simplified assumptions (Noorain et al., 2022).

5.2.2. Personnel Scheduling in Healthcare

In OR, scheduling problems have been studied extensively (Van den Bergh et al., 2013). In healthcare, personnel scheduling problems have been considered for nurses (Burke et al., 2004; De Causmaecker & Berghe, 2011; Kellogg & Walczak, 2007), and physicians (Brunner et al., 2009; Brunner & Edenharter, 2011; Erhard et al., 2018; Thielen, 2018). Studies have also examined the scheduling of patients in outpatient clinics (Ahmadi-Javid et al., 2017; Cayirli & Veral, 2003), and operating rooms (Cardoen et al., 2010; Samudra et al., 2016; Zhu et al., 2018). Most nurse scheduling problems are addressed within the context of hospitals. Specifically, nurses are allocated to periods of work over a planning period. Such problems consider skill categories, shift types, coverage constraints, work regulations, nurse preferences etc. Although physician scheduling is part of the larger field of personnel scheduling, specific such as demand volatility, cost-intense resource, strict adherence to preferences differentiates it from other types (Erhard et al., 2018). The commonality between nurse and physician scheduling is that the schedules are developed in the context of a single location - a hospital.

5.2.2.1. Multi-skill Multi-location Personnel Scheduling

In literature, skills are classified into two categories: hierarchical and categorical. In hierarchical skills, workers with higher skills can do more complex tasks than workers with lower skills. With categorical skills, there is no difference in skill level, and a worker's skills define which tasks they can perform (De Bruecker et al., 2015). Workforce staffing and scheduling incorporating skills have been studied extensively in healthcare (De Bruecker et al., 2015; Respicio et al., 2018; Vermuyten et al., 2018). However, many recently published studies on personnel scheduling consider a single location, often a department (Dahmen et al., 2018; Restrepo et al., 2017) and few consider multi-department or multi-location (Nearchou et al., 2020).

To the best of our knowledge, Franz et al. (1989) have studied the first multi-skill multi-location situation in healthcare. A case study presents an application to schedule a hierarchically skilled workforce in a rural healthcare area comprising several clinic locations by minimising travel costs and maximising staff preferences. More recently, Maenhout & Vanhoucke (2013) studied the case of scheduling different category nurses to several wards in a hospital. The transfer of nurses depends on costs associated with staff shortages per ward. Similarly, Wright & Mahar (2013) centralise the scheduling of nurses across multiple departments at a hospital, with an objective that considers schedule desirability and costs.

Studies that consider personnel scheduling across multiple departments or locations enable the transfer of employees under specific rules (Van den Bergh et al., 2013). The skills feature is often considered along with the location feature, but not always. In recent studies, skills and the movement of workers are incorporated into models via transfer and labour costs, predominantly addressing problems in the service industry (Attia et al., 2019; Bard & Wan, 2008; Dahmen et al., 2020; Nearchou et al., 2020). Models have also included objectives such as fairness, worker preference and satisfaction when considering the movement of a multi-skilled workforce (Al-Yakoob & Sherali, 2007; Al-Yakoob & Sherali, 2008; Cheng & Kuo, 2016; Kuo et al., 2014). Al-Yakoob and Sherali (2007; 2008) assign a hierarchical workforce across several gas stations, where the model satisfies demand while minimising employee dissatisfaction. Kuo et al. (2014) schedule multi-skilled employees to various stations at an international airport by minimising staffing shortages and skills mismatches. Cheng and Kuo (2016) develop a model for scheduling food safety inspectors at an airport with travel restrictions relative to fairness, preference, and skill matches.

In general, the heterogeneity in skills is relative to the heterogeneity of tasks, activities, departments, or locations in each application. Of the non-healthcare studies discussed above, many focus on formulating and solving complex problems by developing novel or heuristic solutions. Although the healthcare application considering multi-skill multi-location planning are limited, the focus has been on examining policies that would be necessary for today's scheduling environments. Of the three healthcare applications, two consider nurses with categorical skills and a single unit location consisting of multiple departments or wards. Present-day features of mental health services have not been considered. We address this gap in workforce planning in mental healthcare and present research demonstrating practical scheduling policies motivated by real data.

5.2.3. Analytics Driven Approaches to Optimisation Modelling in Healthcare

In OR literature, the interest in mixing methods is evidenced by the pool of publications, particularly in healthcare (Brailsford et al., 2019; Morgan et al., 2017; Yearworth & White, 2013). Lately, we have seen the combination of traditional OR and analytics methods for interventions. Analytics, as a discipline, is composed of three distinct stages: descriptive analytics, or the study of systems, organisations and phenomena according to historical data; predictive analytics, or the informed estimation of future values of variables or configurations of systems to aid in anticipation of as yet unknown events; and prescriptive analytics, or the design of policies, guidelines or practices based on optimal or best possible values of decision variables (Liberatore & Luo, 2010).

When considering the application of each of the three stages of analytics in healthcare, researchers have found that predictive analytics is most prevalent, followed by prescriptive and then descriptive (Galetsi & Katsaliaki, 2020; Lepenioti et al., 2020). For this study, we investigate the intersection of descriptive, predictive, with optimisation in healthcare. Specifically, we examine literature from 2015 to present, by performing a literature search on Scopus, resulting in the identification of 13 articles as seen in Table 20. Studies that do not include a predictive element but did contain descriptive analysis along with optimisation were excluded (DuBois et al., 2021; Mazloumian et al., 2022; Restrepo et al., 2020; Zimmerman et al., 2021). On examination, it became evident that the extent of analysis conducted using historical data was spread across three categories where historical data was used as inputs for the predictive analytics techniques and for estimating optimisation model parameters, using summary statistics. In contrast, we found that only three studies conducted data analysis and produced visualisations to uncover patterns and trends in the data (Lee et al., 2015; Sir et el., 2017; Uriarte et al., 2017). Data was also used to validate either the predictive model (Andersen et al., 2019; Mizan and Taghipour 2022; Lee et al., 2015; Ordu et al., 2020) or both the predictive and prescriptive optimisation model (Elleuch et al., 2021; Lee et al., 2015; Ordu et al., 2020). All three types of analysis using historical data have been classified under descriptive (or data processing) to demonstrate what's been done and to highlight what could be done. The 'predictive' column indicates which type of method has been applied, and the 'optimisation model' column provides information on the type of model developed.

The integrated application of the three types of analytics, particularly with optimisation modelling is gaining momentum in the last decade as depicted in Table 20. These studies are based on real data from case studies, often conducted with stakeholder involvement (Lee et al., 2015; Ordu et al., 2015). In comparison to other application areas such as supply chains and manufacturing, combining predictive analytics with optimisation in healthcare is lagging (Galetsi & Katsaliaki, 2020; Lepenioti et al., 2020).

Table 20: Integrated Descriptive ∩ Predictive ∩ Prescriptive Approaches in Healthcare

Author(s)	Descriptive (or Date Processing)			Predictive	Optimisation Model
	Parameter Estimation + Input for Predictive	Data Analysis + Visualisation	Model Validation		
Ahmed & Frohn, (2021)	✓			ML	MOIP
Andersen et al., (2019)	✓		✓ (P)	MCS	ILP
Elleuch et al., (2021)	✓		✓ (P + O)	ANN	FIM
Jang (2019)	✓			ML	RO
Lee et al., (2015)	✓	✓	✓ (P + O)	ML	MINLP
Mizan and Taghipour (2022)	✓		✓ (P)	ML	MOMILP
Moradi et al., (2022)	✓		✓ (P)	ML	MILP
Olya et al., (2022)	✓		✓ (P)	ML	MIP
Ordu et al. (2020)	✓		✓ (P + S)	F	IP
Samorani & LaGanga, (2015)	✓		✓ (O)	DM	SO
Sir et al., (2017)	✓	✓		CART	MIP
Uriarte et al., (2017)	✓	✓		DM	DES + SMO
Wang et al., (2021)	✓			R	MIP

ANN: Artificial Neural Network; ML: Machine Learning; F: Forecasting; DM: Data Mining; R: Regression; CART: Classification and Regression Tree; MCS: Markov Chain Simulation; MIP: Mixed-Integer Programming; DES: Discrete-event Simulation; SMO: Simulation-based Multi-Objective Optimisation; SO: Stochastic Optimisation; IP: Integer Programming; MOMILP: Multi-Objective Mixed Integer Linear Programming; MINLP Mixed-Integer Nonlinear Programming; RO: Robust Optimisation; FIM: Fuzzy Interval Mathematical Model; ILP: Integer Linear Programming; MOIP: Multi-Objective Integer Programming.

Of the 13 articles identified in the review, two have used data from mental healthcare services (Samorani and LaGanga 2015; Wang et al., 2021). Samorani and LaGanga (2015) examine the scheduling of appointments in outpatient clinics given no-show predictions, while Wang et al., (2021) consider short-term physician scheduling by regressing daily demand. Within both articles, the primary focus is on developing and testing the optimisation model with uncertain inputs. We argue that use of descriptive analytics to gain insights on the systems under consideration is minimal. The analytics-driven optimisation modelling approach proposed in this paper is novel in its emphasis on multiple elements of a problem situation. In particular, the proposed approach explores the system to gain insights, uses historical data to estimate parameters (optimisation model) and as input for predictive analytics, and uses the model to validate the prescriptive and predictive models.

5.3. Background and Problem Statement

Primary care providers continue to encounter barriers when referring patients to secondary mental healthcare settings (Pomerantz et al., 2008). At the time of the study, several change imperatives had highlighted the need for integrating mental health services into primary healthcare (NHS England, 2020). Several key benefits of the primary care mental health model were highlighted, including the closer integration of primary, secondary, and tertiary mental healthcare and improved patient access to services.

Several new models of PCMH services are in various stages of development across the UK. There is significant variability in the service models of newly established PCMH services. These services exist on a spectrum between a simple attached specialist working within the primary care setting and a fully integrated multidisciplinary team drawn from primary and specialist services. The deployment of these models represents a significant opportunity to consider how services can be redesigned to ensure the most effective and appropriate care provision.

One such PCMH service was the focus of our study. We developed this approach in collaboration with a real-world PCMH service provided by the Kent and Medway Mental Healthcare Trust (KMPT) based in Kent, UK. The service works alongside GP clinics and primary care partners and interfaces with KMPT services to provide care to people experiencing mild/moderate mental health conditions who do not require secondary care mental health services. The workforce comprises 12 multi-skilled clinicians deployed to 65 GP clinic locations to provide patient consultations across four types of appointments.

During problem exploration, stakeholders reported that the service was on the verge of expansion on two fronts: expanding service capacity by hiring more clinicians and adding more clinic locations to their service provision. These advances were being made in response to county-level population health forecasts' predictions. However, stakeholders conveyed high levels of uncertainty and a lack of clarity around current and future capacity and demand. Furthermore, stakeholders were aware of differences in clinician experience and variations in operating procedure. As such, stakeholders were keen to understand the service quantitatively, uncover operational patterns and explore opportunities to improve service efficiency. The service was driven by the goal of providing the right intervention, by the right professional, at the right time and location. Stakeholders sought our help to investigate current efficiency and consider several options for service transformation.

At the time, the PCMH service was reporting high-level summaries of Key Performance Indicators (KPIs) generated from heterogeneous data sources. The lack of integration and visualisation presented the stakeholders a disjointed view of the service. There was tacit consensus amongst stakeholders that service data was inadequately leveraged for better decision-making. As such, we identified a course of actions that interlinked three areas of investigation: the presence and underutilisation of historical data with a potential for generating insights, elements of uncertainty regarding future service transformation, and the planning of a multi-skilled workforce across multiple locations. We further describe the components of the analytical framework in the next section.

5.4. Overview of Analytics-Driven Optimisation Approach

We developed an analytics driven optimisation modelling approach interlinking the descriptive, predictive, and prescriptive analytics, as seen in Figure 9. In this section, we describe the techniques, tools and processes that are embedded within the approach.

The descriptive analytics phase involved collecting and integrating heterogenous data from several electronic patient record systems used by the PCMHS to manage administrative and clinical processes. The process begins with identifying sources, followed by the transformation of raw data, through data linkage, into a format that enables historical data analysis. Critical components within the service are identified for the data analysis, and performance measures relative to these components are explored using techniques such as visualisation, statistical summaries, and drill-down tables. Furthermore, data gathered in this stage is used as inputs to the predictive and prescriptive stages.

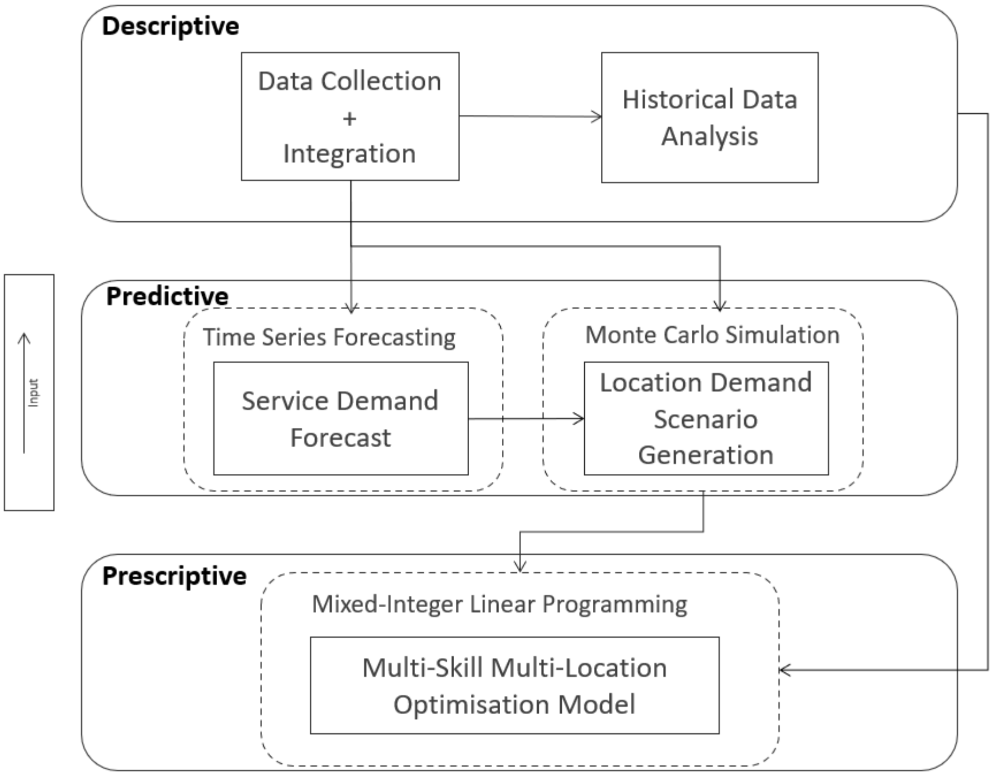

Figure 9: Analytics-Driven Optimisation Modelling Approach

The predictive analytics stage is predominantly concerned with the demand for appointments. Here, a monthly service level demand forecast is generated using historical data and applying time series forecasting. We use Monte Carlo simulation to fill gaps in historical data to determine daily demand for each clinic location. Guided by the upper and lower bounds of the predictive demand forecast and stakeholder recommendation, location-level demand scenarios for four different appointments are generated. These scenarios are used to examine the impact of changes in operational specification on the service using the optimisation model.

Finally, in the prescriptive analytics stage, we build a multi-skill multi-location optimisation model using Mixed-Integer Linear Programming (MILP). Based on skills, the model allocates clinicians to clinic locations on a given day and shift and assigns appointments to clinicians. Inputs to the model are obtained from both preceding stages of the multi-methodology. The main aim of this stage is to use the model to compare service performance for several alternative operational strategies. The next sections will examine specific findings of each stage and its impacts on the overall analytical strategy.

5.4.1. Descriptive Analytics

Historical data spanning four years was extracted from the service's electronic patient record system used by the PCMH service to manage administrative and clinical processes. This data included information about referrals made to the service, clinician utilisation, and patient appointment logs. The data was anonymised and altered only to depict the operational perspective of the service. We began by performing data profiling to clarify the structure, content, relationships, and derivation rules. We then conducted data linkage to join records and create a multi-dimensional integrated dataset. For this purpose, data was migrated to MySQL and transformed. The analysis was then conducted on this enhanced dataset. During the preliminary data analysis, we identified several gaps in information related to clinician working patterns. To redress these gaps, individual clinicians were required to provide a "Job Plan" detailing the division of working hours to various activities, including patient appointments, over a four-week planning horizon.

Two defining operational guidelines of the PCMHs service were that a patient is assessed within four weeks of a referral and that it offers short-term interventions lasting at most six months. As such, we examined the waiting times for all active patients in the service, as seen in Figure 10. A significant proportion of patients wait two months before their first appointment (an assessment), and the second-highest duration is three months, followed by one month and three weeks. Additionally, some patients wait six months to a year, as highlighted by the red bars in Figure 10. On average, 90-150 patients are waiting to be assessed at any given month in the services' timeline.

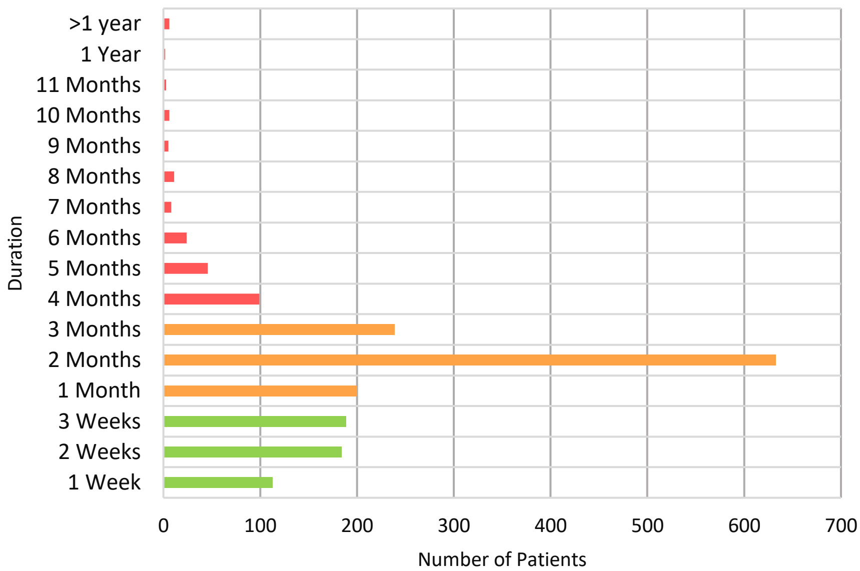

Figure 10: Patient Waiting Time

The length of stay of patients in the service is depicted in Figure 11. Most patients stay in the service for two months following their assessment, closely followed by three, four, and five months. However, the second highest length of stay is two years. Upon further investigation, we found that in each month, alongside the arrival of new referrals, clinicians were having to catch up with the backlog from the previous month, thereby creating delays in the service and leading to longer waiting times.

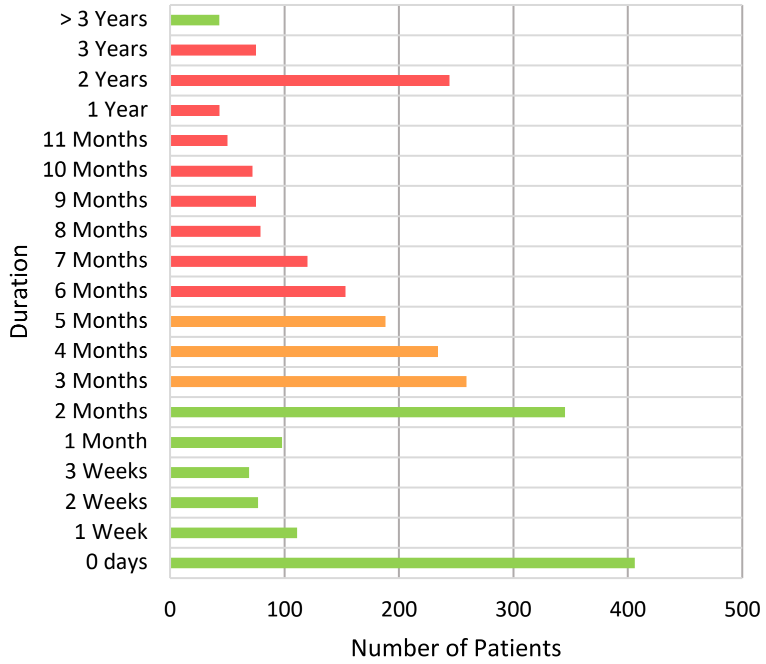

Figure 11: Patient Length of Stay

Next, we looked at clinician related analytics. At the time of this analysis, the services employed 12 multi-skilled clinicians, who were grouped into hierarchical categories called bands. Table 21 depicts the 12 clinicians with a code (C1, C2,.., C12) and their corresponding band. In addition to representing skill class, bands also provide information on clinician employment type. We have five bands: 8a, 7, 6a, 6b and 6c. Clinicians in bands 6b and 6c worked part-time, while all others worked full-time.

Table 21: Clinician Code and Band

Clinician	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
Band	B-8a	B-8a	B-8a	B-7	B-7	B-7	B-7	B-6a	B-6a	B-6b	B-6c	B-6c

Individual clinician working patterns were grouped and quantified based on the time clinicians allocated to each task. Figure 12 represents the clinician's availability for appointments. Although clinicians also conduct other activities, in this study, we only consider the planning of appointments, as these make up the bulk of their duties and are directly related to service efficiency. As seen in Figure 12, the expected availability for appointments is highly variable across all clinicians and clinicians of the same band. For instance, although clinicians C4, C5, C6 and C7 belong to band 7, the hours allocated for patient appointments are highly variable. A small degree of variability in clinician activities was taken as a given by stakeholders. However, this analysis revealed the extent of individual variability perceived by each clinician. Stakeholders believed clinician availability for appointments in Figure 12 were either over or underestimated. Therefore, standardisation of clinician availabilities was proposed based on emergent patterns observed in job plans for clinicians at a single skill level and historical data. For instance, considering a 3-hour clinic shift, Band 8a clinicians often have five available weekly shifts, and Band 7 clinicians will have seven weekly shifts.

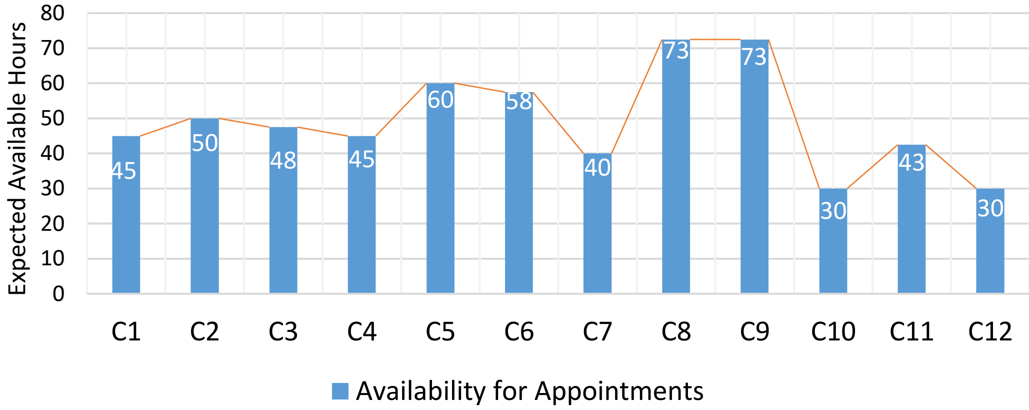

Figure 12: Clinician Availability for Appointments

We then analysed the clinician caseload for active patients in the service, as seen in Figure 13. 12 clinicians conduct appointments at 65 different clinic locations (denoted by L1, L2,.., L65). As such, a clinician's caseload was examined based on two overlapping variables: the number of distinct clinic locations assigned to the clinician and the number of patients managed by a clinician within those locations.

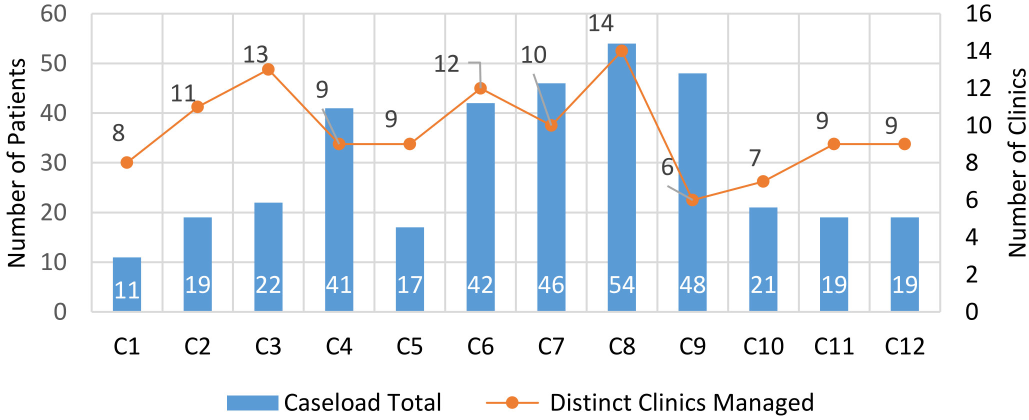

Figure 13: Clinician Caseload and Location Distribution

At the time of this analysis, only active patients were considered. As can be seen in Figure 13, the distribution of clinic locations and patients among clinicians within a band varies. For instance, clinicians C1, C2, and C3, belonging to band 8a, have 11, 19 and 22 patients from 8, 11, and 13 different clinic locations, respectively. In other words, clinician C1 manages 11 patients across 8 clinic locations, whereas clinician C3 manages 22 patients across 13 clinic locations. An additional insight derived from Figure 13 is the distribution ratio of clinics to clinicians. For instance, clinician C1 has 11 patients from 8 different clinics, meaning the clinician most of the times, travels to a location to see one patient only. The same pattern can be observed for clinicians C2 and C5. This insight makes a case for not only a redistribution of caseload but also a reduction in the dispersal of different clinics managed by each clinician. A further drill-down on the distribution of clinicians to clinic locations based on the demand (active patients) was conducted, as seen in Figure 14.

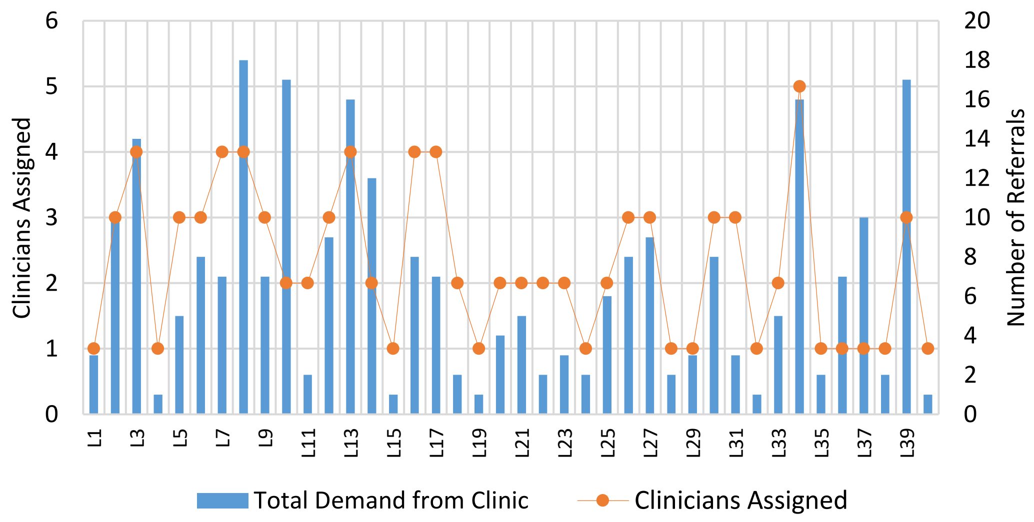

Figure 14: Clinic Demand against Assigned Clinicians

We found that the number of clinicians managing referrals from a clinic did not necessarily correspond to the demand. In other words, as seen in Figure 14, two clinics with different referral frequencies were allocated the same number of clinicians. For instance, locations L34 and L10 have 18 active referrals. However, two clinicians are deployed in location L10, while five have been allocated to location L34. Although this analysis revealed undesirable variation in the distribution of clinics covered by each clinician and clinician caseload, an operational decision was made to preserve these allocations, seen in Figure 13 and Figure 14, in favour of continuity of care. Besides, a higher clinician count in a clinic with low referrals could be due to historically high referral frequencies, but at the time, these did not match the active referrals.

One last feature stakeholders were keen to explore was appointment durations for each appointment type offered by the clinic. The four appointment types are: Assessment (A), Follow-up (F), Community (C), and Telephone (T). An assessment appointment is the first appointment where a clinician sees a newly referred patient and assesses the patient's needs and risks. Here the clinician will determine if the patient can immediately be signposted to other services or if a follow-up appointment is required. A follow-up appointment is either conducted in person or over the telephone, which the service dubs a telephone appointment. A patient can have several follow-up appointments following an assessment based on the outcomes of the assessment. A community appointment is given to patients who require ongoing medication to be administered by a clinician to support the transition of a patient from secondary care to the community. During workshops, clinicians stated varying preferences for appointment durations for each type. Clinicians perceived that most appointments were over 60 minutes. Therefore, a graphical summary of duration is represented in a box-and-whiskers diagram using historical service data, as seen in Figure 15. The approximate upper range values for appointments of type A, F, T and C are 60, 60, 35, and 45 minutes respectively. We were particularly concerned with each appointment type's 90th and 75th percentile values.

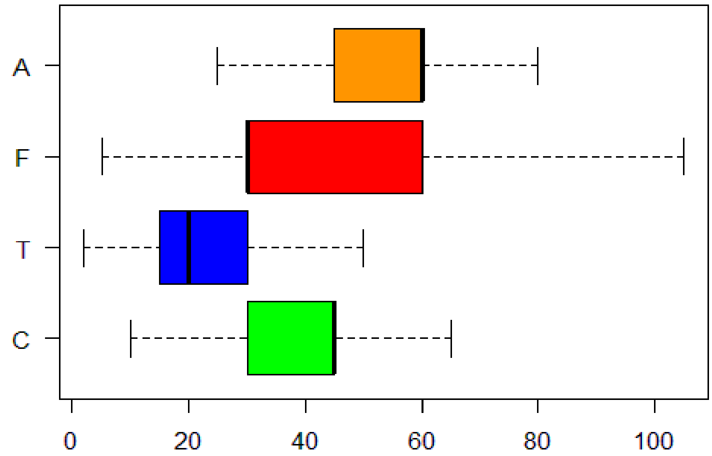

Figure 15: Appointment Type Duration Box Plot

Table 22 shows that 75% of all assessments were less than or equal to 60 minutes. Similarly, follow-up, telephone, and community appointments were 45, 30, and 45 minutes, respectively. The analysis provided empirical evidence to challenge clinicians' perceptions and to incentivise the standardisation of appointment durations.

Table 22: Appointment Types Duration Distribution

	75th Percentile (Pa)	90th Percentile (Pb)
Assessment (A)	60	60
Follow-up (F)	45	60
Telephone (T)	30	45
Community (C)	45	60

5.4.1.1. Summary

The descriptive analytics provided some critical insights about clinician capacity and utilisation drawn from a snapshot in the PCMH service timeline. As such, the insights are specific to the chosen period. Some of the imbalances can be attributed to time-specific circumstances, such as the hiring and training of new clinicians, new clinics signing up for the service etc. Nevertheless, the investigation did show a lack of consistency in clinician profiles relative to their bands, the number of clinicians allocated to each clinic, and perceptions of appointment time durations.

5.4.2. Predictive Analytics

The electronic patient record system accurately captured patient demand for the service. We could use patient referral information to capture demand. We extracted monthly aggregate referrals from January 2018 to May 2020. We found that referrals peak at the start of the year, continue through May, and then taper off during summer, only to rise again at the beginning of winter. These properties were applied to all future predictions to assess risks or opportunities to guide decision-making. Using the" R" software, a monthly forecast for future referral values is obtained using ARIMA modelling. Specifically, an ARIMA (1,1,0)(0,0,2) [12] model provided the best fit and included a differenced non-seasonal AR(1) term, a seasonal MA(2) term ,and the seasonal period is S = 12. The model outputs predict—with some certainty—future referral values to be within a range, as seen in Figure 16. Naturally, all projections are attached with a degree of error. These error values further enhance the forecast by providing upper and lower bounds. We also note that the forecast does not capture the impact of COVID-19.

Having forecasted monthly demand, we then set out to disaggregate the demand values to generate daily demand values for 65 clinic locations across the four appointment types. Using Monte-Carlo simulation, we established the probability distributions for the four appointment types at each location using historical data. Several demand scenarios were developed, guided by the probability estimates and the upper and lower bounds of monthly demand forecasts.

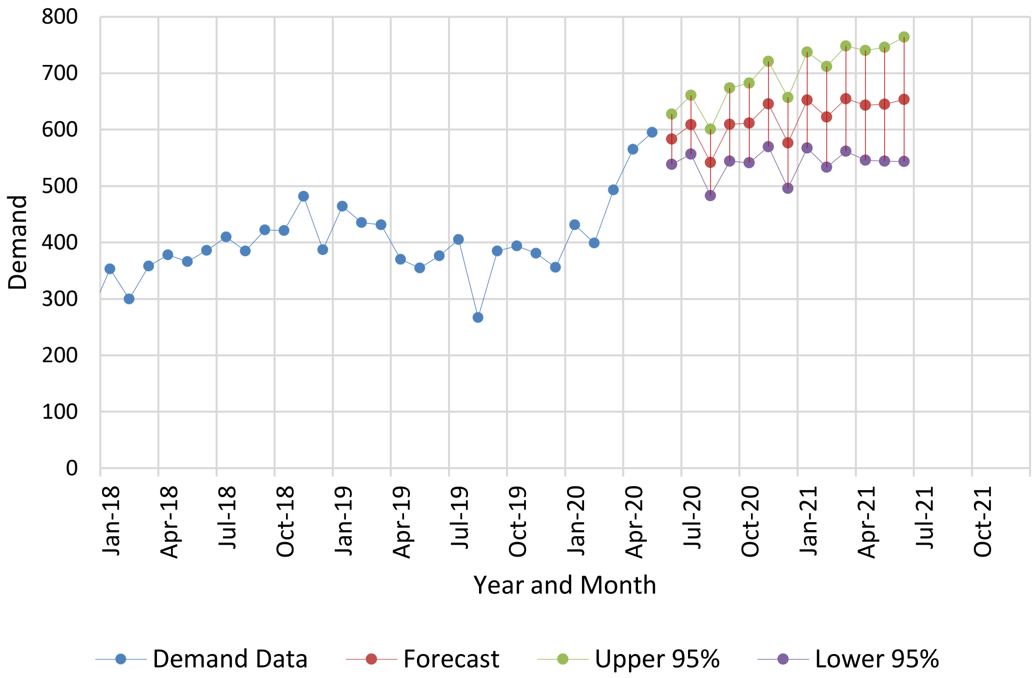

Figure 16: Service Level Demand Forecast

5.5. Prescriptive Analytics: Optimisation Model

We built a multi-skill multi-location optimisation model using Mixed-Integer Linear Programming. Based on skills, the model allocates clinicians to locations at a given day and shift and assigns appointments to clinicians. Using this model, we examine the utilisation of resources under current operational specifications. We then determine if workforce deployment can be improved to match service needs and explore several service design alternatives to eliminate identified variations.

As stated before, the model considers a service with 12 multi-skilled clinicians deployed to 65 primary care clinic locations to conduct four types of appointments. Clinicians are assigned appointments based on their skill set. For instance, clinicians in bands 8a, and 7 have the skills to conduct appointment type 'assessment', while clinicians 6a, 6b and 6c cannot. Additionally, 'community' appointments are conducted only by clinicians in bands 6a, 6b and 6c. And clinicians 6b and 6c, only conduct 'Telephone' and 'Community' appointments. Furthermore, the service operates on weekdays (Monday to Friday) from 9 AM until 6 PM. A working week consists of 5 days, and a clinician's working day is split into two shifts: AM and PM. In addition to consulting with patients at a clinic, clinicians also carry out other activities. Our application does not include these activities as they are pre-scheduled and fixed. However, these activities are considered when determining clinician availability in each shift. Clinicians travel to locations split over two geographic patches to hold clinics across the two shifts (AM and PM).

Furthermore, each clinician manages patients from a predetermined set of clinic locations. They consult with patients for four kinds of appointments in each shift. It is assumed that the demand for each type of appointment at each clinic location throughout the planning horizon is known based on the predictive analytics stage results. On a given working day, a clinician can be assigned two shifts, either in one clinic or in two separate clinics. A clinician cannot be assigned to two locations if the travel distance is greater than a threshold value. The model determined the optimal allocation of clinicians to clinics, appointments and shifts over a planning horizon to minimise the number of unassigned appointments. Although the model is inspired by this real application, it is general in scope and can handle different numbers of shifts, appointment types, skills etc.

5.5.1. Model Formulation

The model formulation used the following notation.

Notation:

- $\boldsymbol{A}$: Set of appointment types, each with a skill requirement, indexed by a
- $\boldsymbol{C}$: Set of clinicians , indexed by c
- $\boldsymbol{L}$: Set of clinic locations, indexed by l
- $\boldsymbol{S}$: Set of shifts, indexed by s
- $\boldsymbol{D}$: Set of days in the planning horizon, indexed by d
- $\mathbf{L}_{\boldsymbol{s}}$: Length of shift s
- $\mathbf{S}_{\boldsymbol{d}}$: Set of shifts $s \in S$ for each day $d \in D$
- $\boldsymbol{F}_{\boldsymbol{l}}^{\boldsymbol{a}}$: Demand for appointment type $a \in A$ in clinic location $l \in L$
- $\boldsymbol{B}_{\boldsymbol{c}}^{\boldsymbol{a}}$: 1, if clinician $c \in C$ is skilled for appointment type $a \in A$, 0 otherwise
- $\mathbf{T}_{\mathbf{l_1 l_2}}$: Distance between clinic locations $l_{1,} l_2 \in L$: $l_1 \neq l_2$
- $\mathbf{R}_{\boldsymbol{a}}$: Duration of appointment type $a \in A$
- $\mathbf{P}_{\boldsymbol{cl}}$: 1, if clinician $c \in C$ can be assigned to clinic location $l \in L$, 0 otherwise
- $\mathbf{T}_{\mathbf{max}}$: Maximum travel distance between clinics
- $\mathbf{St}_{\mathbf{cd}}$: Maximum number of shifts per day $d \in D$ per clinician $c \in C$
- $\mathbf{N}_{\mathbf{c}}$: Maximum number of clinic locations that can be assigned to a clinician
- $\mathbf{N}_{\boldsymbol{l}}$: Maximum number of clinicians that can be assigned to a clinic location
- $\mathbf{H}_{\boldsymbol{c}}$: Total available hours per clinician $c \in C$
- $\mathbf{H}_{\boldsymbol{cs}}$: 1, if clinician $c \in C$ is available in shift $s \in S$, 0 otherwise

Decision variables:

- $\boldsymbol{Y_{cls}} = \begin{cases} 1, \text{if clinician } c \in C \textit{ is } \text{assigned to clinic location } l \in L \textit{ in} \text{ shift } s \in S \\ 0, \text{otherwise} \end{cases}$
- $\boldsymbol{X^a_{cls}} = $ number of appointments of type $a \in A$ assigned to clinician $c \in C$ at clinic location $l \in L$ in shift $s \in S$
- $\boldsymbol{W_{cl}} = \begin{cases} 1, \text{if clinic location } l \in L \textit{ is } \text{assigned to clinician } c \in C \\ 0, \text{otherwise} \end{cases}$
- $\boldsymbol{Q_{cs}} = \begin{cases} 1, \text{if clinician } c \in C \textit{ is assigned to } \text{shift } s \in S \\ 0, \text{otherwise} \end{cases}$
- $\boldsymbol{U^a_c} = \begin{cases} 1, \text{if clinician } c \in C \textit{ is assigned to } \text{appointment type } a \in A \\ 0, \text{otherwise} \end{cases}$
- $\boldsymbol{Z^-_c} = Unassigned$ hours for each clinician $c \in C$
- $\boldsymbol{V_{cdl_1l_2}} = \begin{cases} 1, \text{if clinician } c \in C \textit{ can be } \text{assigned to both clinic locations } l_1 \text{ and } l_2 \in \text{L on the same day } d \in D \\ 0, \text{otherwise} \end{cases}$

Mathematical Model:

$$Min \sum_{a\in A}\sum_{l\in L} F^a_l - \sum_{a\in A}\sum_{c\in C}\sum_{l\in L}\sum_{s\in S} X^a_{cls} \tag{1}$$

Subject to:

$$\sum_{a\in A} R_a X^a_{cls} \le L_s \qquad \forall c \in C, \forall s \in S, \forall l \in L \tag{2}$$

$$\sum_{c\in C}\sum_{s\in S} X^a_{cls} \le F^a_l \qquad \forall l \in L, \forall a \in A \tag{3}$$

$$\sum_{l\in L}\sum_{s\in A}\sum_{a\in A} R_a X^a_{cls} + Z^-_c = H_c \qquad \forall c \in C \tag{4}$$

$$M\, Y_{cls} \ge X^a_{cls} \qquad \forall l \in L, \forall c \in C, \forall a \in A, \forall s \in S \tag{5}$$

$$\sum_{l\in L} Y_{cls} \le St_{cd} \qquad \forall c \in C, \forall s \in S_d \tag{6}$$

$$\sum_{c\in C} Y_{cls} \le 1 \qquad \forall s \in S, \forall l \in L \tag{7}$$

$$\left(Y_{cl_1s_1} + Y_{cl_2s_2}\right) - 1 \le M_1 V_{cdl_1l_2} \qquad \forall c \in C, \forall s_1, s_2 \in S, \forall l_1, l_2 \in L: l_1 \ne l_2, \forall d \in D \tag{8}$$

$$T_{l_1l_2} - T_{max} \le M_2\left(1 - V_{cdl_1l_2}\right) \qquad \forall c \in C, \forall l_1, l_2 \in L: l_1 \ne l_2, \forall d \in D \tag{9}$$

$$Y_{cls} \le W_{cl} \qquad \forall l \in L, \forall c \in C, \forall s \in S \tag{10}$$

$$\sum_{c \in C} W_{cl} \leq N_l \qquad \forall l \in L \quad (11)$$

$$\sum_{l \in L} W_{cl} \leq N_c \qquad \forall c \in C \quad (12)$$

$$W_{cl} \leq P_{cl} \qquad \forall c \in C, \forall l \in L \quad (13)$$

$$Y_{cls} \leq Q_{cs} \qquad \forall l \in L, \forall c \in C, \forall s \in S \quad (14)$$

$$Q_{cs} \leq H_{cs} \qquad \forall c \in C, \forall s \in S \quad (15)$$

$$M\, U_c^a \geq X_{cls}^a \qquad \forall l \in L, \forall c \in C, \forall s \in S, \forall a \in A \quad (16)$$

$$U_c^a \leq B_c^a \qquad \forall c \in C, \forall a \in A \quad (17)$$

$$Y_{cls} \in \{0,1\} \qquad \forall c \in C, \forall l \in L, \forall s \in S \quad (18)$$

$$X_{cls}^a \in \mathbb{Z} \qquad \forall l \in L, \forall c \in C, \forall a \in A, \forall s \in S \quad (19)$$

$$W_{cl} \in \{0,1\} \qquad \forall c \in C, \forall l \in L \quad (20)$$

$$Q_{cs} \in \{0,1\} \qquad \forall c \in C, \forall s \in S \quad (21)$$

$$U_c^a \in \{0,1\} \qquad \forall c \in C, \forall a \in A \quad (22)$$

$$Z_c^- \in \mathbb{Z} \qquad \forall c \in C \quad (23)$$

$$V_{cdl_1l_2} \in \{0,1\} \qquad \forall c \in C, \forall d \in D, \forall l_{1,} l_2 \in L \quad (24)$$

The objective of the model (1) is to minimise the number of unassigned appointments. In other words, to minimise unmet demand. Constraints (2) ensure that the sum of durations of all appointments assigned to a shift does not exceed the length of each shift. Constraints (3) make sure that appointments assigned to any shift in a clinic do not exceed the demand of appointments in that clinic. Constraints (4) assigns demand based on available clinician hours and captures any unassigned hours in the slack variable Z_c^-. Constraints (5) prevent the allocation of appointments to clinicians in each location and shift unless the clinician has been assigned to the location ($Y_{cls} = 1$). In these constraints M_1 denotes a large constant that can be set, for example, to the value F_l^a. Constraints (6) set the maximum number of shifts that can be assigned to a clinician per day. Constraints (7) ensure that a clinician can only be assigned to 1 or 0 shifts in a clinic location. Constraints (8)-(9) prevent the assignment of a clinician to locations that are too far away on the same day. Note that for the constraints to work, the constant M_2 in (9) can be set equal to $T_{l_1l_2}$. Constraints (10) - (13) are location specific. Constraints (11) limit the number of clinicians that can be assigned to a clinic location. In contrast, constraints (12) limit the number of clinic locations that can be assigned to a clinician. Constraints (13) ensure that clinicians are only assigned to clinic locations that they cover. Constraints (14) and (15) ensure that clinicians are only assigned to shifts based on availability. Constraints (16) and (17) assign appointments to clinicians based on their skill level. Finally, constraints (18)-(24) define the domain of the variables.

The verification and validation of the model with the stakeholders took place during the COVID-19 pandemic. At the time, wide-ranging changes were being reported in the organisation of mental health services, including the pausing of services that were deemed "non-essential", staff deployment to new and unfamiliar roles, and move to remote working (Liberati et al., 2021). Given the unprecedented nature of the pandemic and a complete lack of face-to-face appointments, the service decided to offer longer appointments to patients to counteract the lack of in-person contact. During the pandemic, the operational policy changed to accommodate a potential increase in demand. The organic but unexpected change in service operations called for some indication of resource utilisation and service capacity, therefore, leading to the development of a COVID model variant to support planning operations during a pandemic.

The initial model also dubbed the "Non-COVID" model, assumes that clinicians deliver the service as usual. On the other hand, the COVID variant of the model assumes that all demand/appointments are of one type (telephone-based) and does away with constraints on the clinic location and clinician travel. Constraints (8) and (9) are removed, and there is only one kind of appointment (telephone), which further simplifies constraints (2)-(4) and (16)-(17). The model was coded and solved on CPLEX (see Appendix H for code).

5.5.2. Inputs and Scenario Generation

Based on discussion with stakeholders, several alternative service design options were derived for scenario analysis. In the descriptive stage of the methodology, we identified problems relating to clinic shift duration, appointment duration, and clinician hours. We were guided by stakeholders to explore these issues as well as an increase in demand. Scenarios were generated using experimental model input values derived from the prescriptive and descriptive analytics stages. For the analysis, data from the service for a specific planning period of 4 weeks (1st to 30th of May 2021) was extracted to compare service improvements and model performance. This period was considered for its high demand values. For appointment durations, two profiles (Pa and Pb) discussed in Section 5.4.1, Table 22 were used. For clinician availability over a 4-week planning period, stakeholders supplied shift durations of 2.5 and 3 hours for the Non-COVID and COVID scenarios, and weekly shifts for clinicians grouped by band are based on the discussion in Section 5.4.1, Figure 12. Table 23 depicts the standardisation strategy for clinician availability. In practice, the service did not have standardised specifications for appointment durations, clinician availability, and appointments were being scheduled on an ad-hoc basis by each clinician.

Table 23: Standardised Clinician Availability

Clinician Band	Standardised Availability			
	Available Shifts (4 Weeks)	Weekly Shifts	Available Hours (2.5hr shift)	Available Hours (3hr shift)
8a	20	5	50	60
7	28	7	70	84
6a	32	8	80	96
6b	12	3	30	36
6c	16	4	40	48

5.5.2.1. The Scenarios

Table 24 summarises the scenario options. Scenarios 1 to 4 use the perceived clinician availability, while scenarios 5 to 8 use standardised availability in Table 23. Then in scenarios 1, 2, 5 and 6 we use current appointment demand, whereas scenarios 3, 4, 7, and 8 uses an increase of about 15% which was the result of the forecasting and stakeholders' intuition. For each of the previous combinations we consider two duration profiles (Pa and Pb). Scenarios 9-16 in the table are COVID-variant counterparts of scenarios 1-8. Note that in this case, the appointment duration profiles Pa and Pb only include telephone appointments.

Table 24: Summary of Scenario Options

Scenarios		Clinician Available Hours	Appointment Demand	Appointment Duration
Non-COVID	COVID	$(\mathbf{H}_c)$	$(\boldsymbol{F}_l^a)$	$(\mathbf{R}_a)$
1	9	Perceived	Current	Pa
2	10	Perceived	Current	Pb
3	11	Perceived	Increased	Pa
4	12	Perceived	Increased	Pb
5	13	Standardised	Current	Pa
6	14	Standardised	Current	Pb
7	15	Standardised	Increased	Pa
8	16	Standardised	Increased	Pb

Table 25 depicts scenario specification for the non-COVID model variant. The second column has clinician available hours, the third has the number of appointments for each type across all locations and the appointment duration with total duration of appointments is given in the last column.

Table 25: Non-COVID Scenario Specifications

Scenario	Clinician Available Hours													Appointment Demand					Appointment Duration				
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	Total	A	F	T	C	Total	A	F	T	C	Total Hrs
1	45	50	48	45	60	58	40	73	73	30	43	30	593	53	264	269	6	592	60	45	30	45	390
2	45	50	48	45	60	58	40	73	73	30	43	30	593	53	264	269	6	592	60	60	45	60	525
3	45	50	48	45	60	58	40	73	73	30	43	30	593	137	425	96	41	699	60	45	30	45	535
4	45	50	48	45	60	58	40	73	73	30	43	30	593	137	425	96	41	699	60	60	45	60	675
5	47	47	47	67	67	67	67	75	75	40	30	30	663	53	264	269	6	592	60	45	30	45	390
6	47	47	47	67	67	67	67	75	75	40	30	30	663	53	264	269	6	592	60	60	45	60	525
7	47	47	47	67	67	67	67	75	75	40	30	30	663	137	425	96	41	699	60	45	30	45	535
8	47	47	47	67	67	67	67	75	75	40	30	30	663	137	425	96	41	699	60	60	45	60	675

Likewise,

Table 25 displays scenario specifications for the COVID model variant.

Table 26: COVID Scenario Specifications

Scenario	Clinician Available Hours													Total Appointment Demand	Appointment Duration	Total Appointment Hours
	C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	*C8*	*C9*	C10	C11	*C12*	Total			
9	45	50	48	45	60	58	40	73	73	30	43	30	711	592	45	444
10	45	50	48	45	60	58	40	73	73	30	43	30	711	592	60	592
11	45	50	48	45	60	58	40	73	73	30	43	30	711	699	45	524
12	45	50	48	45	60	58	40	73	73	30	43	30	711	699	60	699
13	47	47	47	67	67	67	67	75	75	40	30	30	795	592	45	444
14	47	47	47	67	67	67	67	75	75	40	30	30	795	592	60	592
15	47	47	47	67	67	67	67	75	75	40	30	30	795	699	45	524
16	47	47	47	67	67	67	67	75	75	40	30	30	795	699	60	699

5.6. Computational Results

The ultimate output of the model, as seen in Figure 17, is a planning schedule that decides "who" (clinician) goes "where" (clinic), for "what" (appointment type) and "when" (shift, day, and week). A specific colour represents each clinician, while a pattern indicates the appointment type. To compute the distance between clinics, a distance matrix was developed based on the location of each GP location. Distances (in miles) were generated from Google Maps using JavaScript.

As highlighted in the results of the descriptive analytics, the service has between 90-150 patients on the waiting list at any given time, and patients wait between 2-8 weeks for their first appointment. By building a 'baseline' using historical operational data from the service, we aim to investigate the possibility of reducing patient waiting times and retrospectively examine clinician utilisation against availability to prevent carry-over of the waiting list to the next planning period. The following sections discuss the scenario outputs for the two model variants.

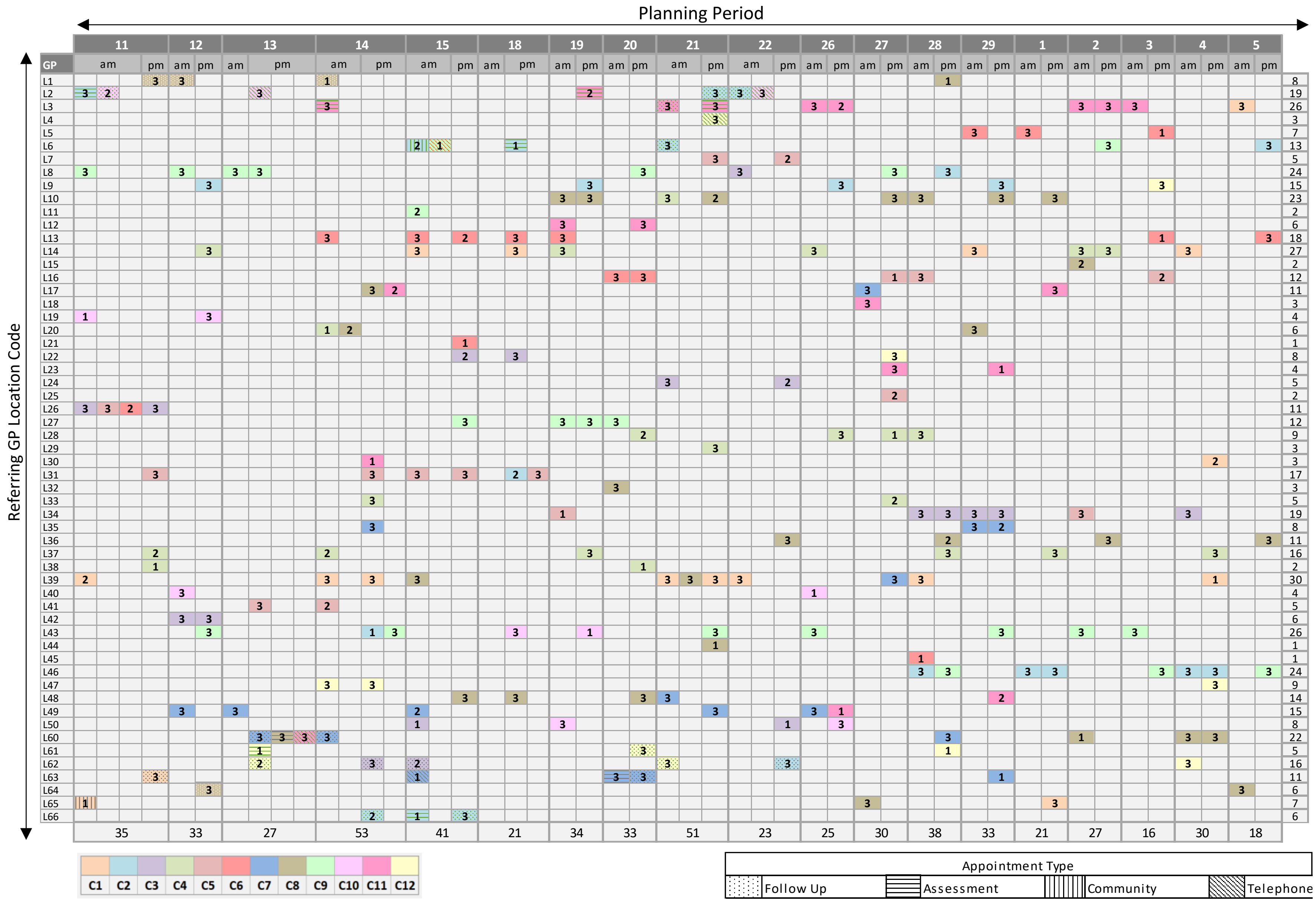

Figure 17: Sample Model Output

5.6.1. Non-COVID Model Scenarios Results

We begin by establishing the utility of using the optimisation model to schedule clinicians by comparing the model's outputs with the historical appointment assignment data from the service. Since scenario's 1, 2, 5 and 6 use historical demand, we compare the number of appointments conducted by each clinician and compare these to the number of appointments assigned by the model, as seen in . We then examine scenario results primarily using clinician unassigned hours and unmet demand measures summarised in Table 27. For all scenarios, we also compare the impact of 'perceived' against standardised availabilities on clinician utilisation, as seen in Figure 19.

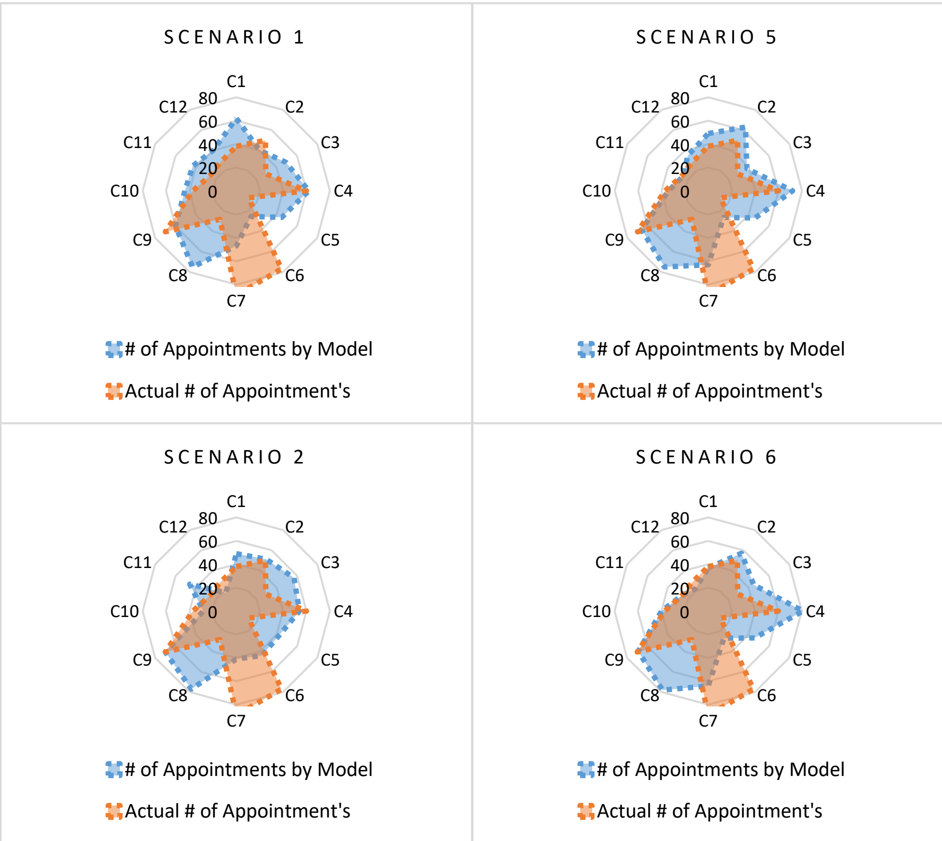

Figure 18: Number of historical Appointment Allocated vs Model Allocation

For each scenario in Figure 18, the orange area depicts the variation in number of appointments conducted by each clinician during the chosen planning period of 4 weeks (1st to 30th of May 2021). The blue area is the assignment of appointments produced by the model. The variation between each clinician is clearly discernible in each scenario. Consistently low values can be seen for clinicians C1, C3, C5 and C8. Meanwhile clinicians C5 and C7 conducted a higher number of appointments compared to other clinicians.

The model's allocation of appointments irons out most under and over assignment of appointments. We can also observe a scalable increase in the number of appointments for clinicians C2, C3, C4, C5, C8 and C11 as seen in each scenario. However, disparities persist in the distribution of appointment between clinicians of the same band. However, when clinician's availability is standardises, peaks observed in scenario 1 (C1) and scenario 2 (C11) are removed as seen in scenarios 5 and 6.

Model results in Table 27 indicates that in scenarios 1 and 2, which are baseline, we observe that the unmet demand is zero or minimal (15). These highlight that if appointments are allocated optimally, with current demand, the waiting list should not grow by 90-150 appointments as noted in the descriptive analytics. Standardisation of clinician availability in most cases results in the elimination of unmet demand as seen in scenarios 6 and 7 as opposed to 2 and 3. In scenario 8 with increased demand and long appointment duration, even if standardisation doesn't eliminate unmet demand, it reduces it significantly (from 93 to 50). Given these findings, the model shows that in each planning period, clinicians have the available capacity to offer appointments to patients on the waiting list, as seen in scenarios 1, 5, 6 and 7.

Table 27: Non-Covid Model Output Summary

Scenario	Unassigned Hours per Clinician													Unmet Demand Per Appointment Type				Total Unmet Demand
	C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	*C8*	*C9*	*C10*	*C11*	*C12*	*Total*	*A*	*F*	*T*	*C*	
1	12	12	30	2	18	38	13	8	32	18	10	11	203	0	0	0	0	0
2	0	1	11	0	22	17	1	0	16	7	0	10	83	6	9	0	0	15
3	0	1	5	0	7	28	0	0	26	8	1	3	78	20	0	0	0	20
4	0	0	0	1	2	3	0	0	4	0	0	1	11	9	79	0	5	93
5	18	11	18	21	39	44	10	22	45	10	20	15	273	0	0	0	0	0
6	3	17	6	1	20	44	11	9	9	9	1	11	138	0	0	0	0	0
7	8	5	8	9	32	22	0	5	29	9	0	1	128	0	0	0	0	0
8	0	0	1	0	23	10	0	0	4	0	0	1	38	10	37	0	3	50

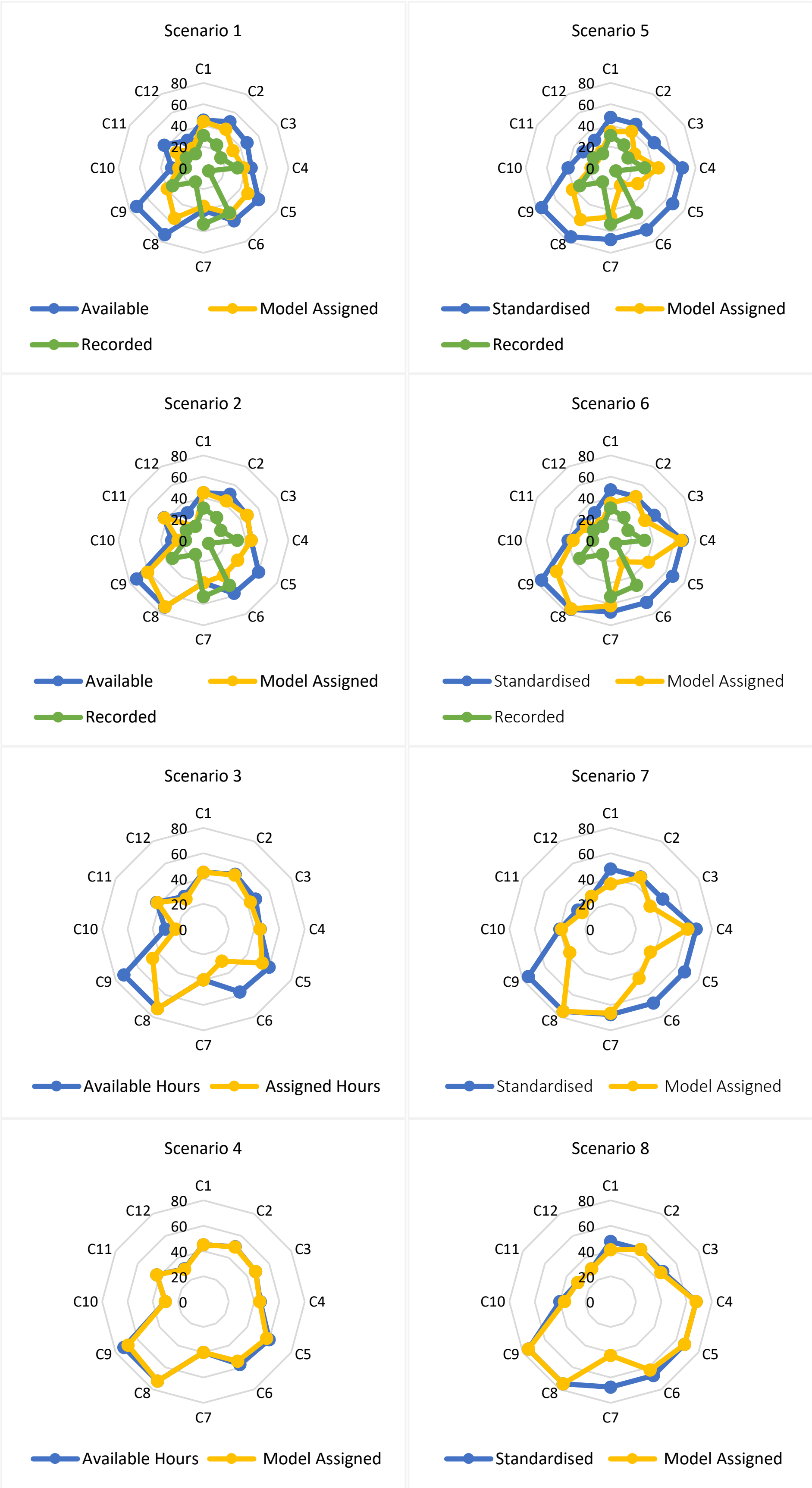

Figure 19: Appointment Hours vs Available Clinician Hours

The improvements to clinician utilisation because of standardisation are depicted in Figure 19. In the figure, the blue line indicates hours available, the yellow line indicates hours assigned by the model and the green line is the recorded hours (service databases) for each clinician. The model is realistic and comparable to the real situation because it does not allocate appointments fairly among clinicians of the same band. With standardisation, the distribution of appointments to clinicians is improved, but variations persist. Specifically, the distribution of assigned hours among clinicians of the same band is not uniform. For example, in scenario 6, band 7 clinician C4 and C7 are fully utilised while C6 has significant spare capacity. Therefore, our approach uncovers that the uneven distribution plays a consistent role in the unfair allocation of appointments.

5.6.2. COVID-19 Model Scenarios Results

Table 28 summarises the outputs of the eight COVID scenarios. Compared to the non-COVID model results, where demand is unmet in four scenarios, in the COVID model unmet demand only occurs in two scenarios. Although, the number of unassigned appointments in the COVID model solutions is lower and the service efficiency seems improved, a model that only includes telephone appointments was not a viable long-term solution for the service as they felt that it would not benefit patients or prove meaningful to clinicians of varied skillset. Nonetheless, the analysis did provide stakeholders value as it provided insights into the service's performance under COVID versus Non-COVID situations.

Table 28: Covid Model Output Summary

Scenario	Unassigned Hours per Clinician													Total Unmet Demand
	C1	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*	*C8*	*C9*	*C10*	*C11*	*C12*	*Total*	
9	7	13	13	18	45	41	6	14	56	20	29	6	267	0
10	9	0	10	0	32	34	0	0	21	5	3	5	119	0
11	1	20	9	21	39	38	2	0	35	11	3	9	187	0
12	0	4	0	0	15	11	0	0	34	3	0	0	67	55
13	14	13	18	41	59	53	28	32	49	18	11	17	351	0
14	3	11	9	13	37	38	16	23	30	12	8	3	203	0
15	15	41	5	32	37	44	11	2	50	6	23	5	271	0
16	0	1	10	0	22	34	0	0	32	1	0	0	100	4

5.7. Discussion and Future Research Directions

The growing repositories of data have fuelled interest in analytics, which has proven valuable for businesses, governments, and communities (Davenport, 2013). Numerous studies have emphasised the influence of analytics and its potential for Operations Research (OR) (Liberatore & Luo, 2010; Mortenson et al., 2015; Ranyard et al., 2015; Vidgen et al., 2017).

Within the field of OR, there has been a surge in interest in methods such as business analytics and artificial intelligence since 2015, alongside traditional methodologies like optimisation, simulation, and decision analysis (Romero-Silva & De Leeuw, 2021). Analytics is widely regarded as highly influential and relevant to OR(Burger et al., 2019; Hindle et al., 2020). However, a gap exists in effectively combining these two disciplines (Hindle et al., 2020; Vidgen et al., 2017). Meanwhile, industries such as airlines, retail, finance, and marketing have increasingly leveraged analytics methodologies to drive system-level innovation, while the healthcare sector has been slower to adopt these advancements, though it is making significant progress (Copenhaver et al., 2019).

This article contributes to healthcare analytics by demonstrating the integration of descriptive, predictive, and prescriptive analytics, an area with limited prior cases (Galetsi & Katsaliaki, 2020; Lepenioti et al., 2020). The proposed approach employs these analytics stages in a logical sequence to guide the development of a prescriptive optimisation model. This integrated approach incorporates existing and supplementary data while incorporating stakeholder assumptions and experiences into formulating alternative scenarios through optimisation modelling. The proposed approach initially employs descriptive analytics to identify system problems and highlight data gaps. Stakeholders gain an understanding of performance measures and challenge perceptions of clinician utilisation. In the predictive stage, we extrapolate current demand trends through forecasting, providing stakeholders with a view of future service demand. Additionally, the Monte-Carlo simulation generates missing location-specific demand data. The prescriptive optimisation model benefits from the contextual and parametric information provided by the descriptive and predictive analytics tools, influencing the model and its results (Abbasi et al., 2016; Grover et al., 2018). The strength of this integrated approach lies in the collaboration between different OR methods across all three analytics stages (see Figure 1). It offers a structured framework for developing analytics-driven optimisation models in various contexts. Furthermore, the approach can be enhanced by exploring other analytics techniques, including Machine Learning.

The findings of our multi-skill multi-location model demonstrate the advantages of using science-based workforce planning instead of ad-hoc procedures employed by the PCMH service. Moreover, our approach generates reliability in assessing the impact of service changes using the optimisation model. The model and corresponding scenario analysis identify and quantify the trade-offs that stakeholders should consider, providing several recommendations to improve service efficiency. By optimally allocating clinicians, the service can better manage patients on the waiting list and effectively respond to increasing demand. These benefits can be realised by standardising service specifications, such as appointment durations, clinician shift duration, and clinician availability. The optimisation model and scenario analysis demonstrate how incremental changes to service specifications can eliminate operational inefficiencies, leading to improvements in mental healthcare delivery. The results also indicate that expanding service capacity may not be strictly necessary under current demand levels.

Our model emphasises the incorporation of first-order hard constraints derived from clinical and cultural factors. The outputs are adapted to match the decision and workflow processes inherent to clinicians. However, there are aspects of the optimisation model that can be further adapted and improved in future modelling efforts. For example, variations in clinic locations and caseload distribution emerge as the main factors hindering a fairer distribution of appointments. In our application, redistribution of clinics and caseload was not possible to maintain continuity of care for patients and foster the clinician-patient relationship. This feature was set aside for future consideration. Future research could allow the model to decide locations for clinicians and incorporate workload distribution fairness, as seen in existing studies (Cheng & Kuo, 2016; Ladier et al., 2014).

We also explored increased workforce flexibility by considering the relaxation of skill-based constraints, although it was not relevant to the PCMH service. However, this analysis can provide insights for other contexts. By relaxing the skill requirements for each appointment type, we investigated whether unmet demand in critical scenarios could be resolved. Our findings indicate that by upskilling clinicians, unmet demand can be eliminated or significantly reduced. Additionally, allowing higher-skilled clinicians to conduct low-skilled appointments did not affect overall unmet demand. If considered feasible, future research could incorporate strategies such as substitution and cross-training, commonly employed to increase workforce flexibility (Bard & Purnomo, 2005; Bard & Wan, 2008; Burke et al., 2010; De Bruecker et al., 2015; Golalikhani & Karwan, 2013; Krishnamoorthy et al., 2012). For services considering expanding their workforce in response to rising demand, the optimisation model can highlight bottlenecks, guide decisions on the number and type of clinicians required and identify necessary training.

Future research could explore the development of ad-hoc solution methodologies to address larger problems. While our application could be solved using commercial software, larger problem instances or additional complexities, such as fairness and workforce flexibility, may require the deployment of novel heuristics (Attia et al., 2019; Dahmen et al., 2018; Nearchou et al., 2020). Furthermore, incorporating demand as a stochastic element within the model itself could be a promising area of future research. In our application, demand is generated in the predictive analytics stage and supplied as input to the optimisation model. Robust optimisation, which considers uncertain demand, is an approach that could be explored (Cappanera et al., 2018).

Enabling the ongoing use of the analytics-driven optimisation approach within the service presents a challenge. For the PCMH service to utilise our approach continuously, they must possess continuous analytics capabilities, which was not the case. Our involvement was an 'Aspirational' venture for the PCMH service, focusing on understanding the situation at a particular point in time without considering their ongoing needs for using the model. To further the insights gained from the study, the PCMH service can embed analytics and better utilise information to identify challenges and justify actions (Shanks & Bekmamedova, 2012). With the exponential growth in health data volumes, analytics-driven insights must be integrated into organisational processes and closely linked to operational management for them to trigger new actions (LaValle et al., 2011; Shanks & Bekmamedova, 2012). A top-down approach to analysing, structuring, mapping, and innovating an organisation's analytics capability is integral to successful analytics implementation (Hindle et al., 2020). Methodologies such as the Business Analytics Methodology (BAM) can help healthcare organisations develop a sustained analytics strategy that aligns with their value system, business goals, and data utilisation capabilities (Hindle et al., 2020). However, it's essential to recognise that analytics is primarily a means to an end, with the ultimate goal being the improvement of decision-making and workflow processes (Copenhaver et al., 2019). In our case study, considering the organisation's analytics maturity, the findings of our approach were translated into practical operational policies for clinicians to implement (Long, E. F. et al., 2022). In organisations with appropriate analytics capabilities, prototypes of decision support tools can be considered to enable sustained implementation of new process designs, as seen in large medical centres (Copenhaver et al., 2019).

Our case study contributes to the limited application of optimisation modelling in mental healthcare service planning (Bradley et al., 2017; Howells et al., 2022; Long, K. M. & Meadows, 2018; Noorain et al., 2019; Noorain et al., 2022). Additionally, we present a novel multi-skill multi-location optimisation model that considers practical scheduling policies using real data, a first in the context of mental healthcare (Al-Yakoob & Sherali, 2008; Cheng & Kuo, 2016). The approach is relevant not only to mental healthcare but also to other care contexts with varied workforce compositions, inter-organisational teams, and integrated care delivery such as primary mental healthcare, community care, and multidisciplinary care teams (Leeftink et al., 2020; Palmer et al., 2018). Moreover, the approach is relevant in care contexts where multiple organisations collaborate to design and deliver integrated care across localities and neighbourhoods(Charles, 2020; Wright & Turner, 2021). By addressing practical challenges and considering future research directions, this study contributes to the advancement of analytics-driven optimisation models in diverse contexts.

The analytics-driven optimisation modelling approach can also serve as a tool to investigate the impact of introducing telemedicine in mental healthcare. The COVID-19 pandemic has led to increased demand for mental healthcare, and telemedicine is reported to enhance access and reduce waiting times (Aknin et al., 2022; Hohman et al., 2022). In our study, we modelled the PCMH service operating with telephone consultations, which is conceptually similar to exploring telemedicine in practice. As telemedicine gains interest, future research could adapt our model to explore hybrid systems that combine in-person and telephone consultations.

5.8. Conclusion

This paper introduces an analytics-driven optimisation modelling approach to assess and redesign a mental healthcare service. The effectiveness of this approach is demonstrated through a case study, where we find that the analysed mental healthcare service had untapped capacity to address an increase in demand without the need for additional resources. Further, the case study highlights the potential for efficiency improvements by reallocating workload. By employing the analytics-driven optimisation modelling approach, other similar care contexts such as primary mental health, community, multidisciplinary care teams and integrated care models can be evaluated. Future research should aim to incorporate complexities into the model and integrate various descriptive and predictive techniques within the framework. We have presented a practical optimisation model with recognised benefits that can be extended to other contexts. We encourage researchers to undertake further research utilising real case studies to explore the potential of analytics-driven optimisation modelling.

Chapter 6: How do stakeholders interact with optimisation models? A case study in mental healthcare

ABSTRACT

In healthcare optimisation literature, the implementation of models poses a significant challenge, with limited or no information provided on stakeholder engagement and interaction with the solved model. Additionally, there is a lack of insight into real-world challenges that hinder implementation, and the process of implementation itself is inadequately described. In this study, we have adapted the post-model coding stages of the PartiSim framework specifically for optimisation modelling in healthcare. By leveraging this framework, we have derived a facilitative approach for stakeholder participation, focusing on the validation, experimentation, and implementation of a mathematical optimisation model. To demonstrate the effectiveness of our approach, we have conducted a real case study involving mental health care delivery. In our case study, we illustrate how stakeholders engaged with the optimisation model following its development using tools based on Soft Systems Methodology (SSM) adapted from the PartiSim framework. This chapter builds upon the adaptation of PartiSim stages, aligning them with the model building lifecycle. In Chapter 4, we present the conceptualisation of the optimisation model, while Chapter 5 delves into the model coding and the analytics process employed to construct and solve a multi-skill multi-location model. The focus of this chapter is to provide insights into the post-model coding stages and present a framework comprising custom-made tools to support the validation, experimentation, and implementation of an optimisation model. By highlighting stakeholder interactions and incorporating the PartiSim framework, we contribute to addressing the existing gaps in healthcare optimisation literature.

6.1. Introduction

Optimisation modelling has found extensive application in various domains of Operational Research (OR), including healthcare, manufacturing, logistics, transportation, supply chain management, and others (Abdalkareem et al., 2021; Archetti et al., 2022; Bortolini et al., 2018; Carter & Busby, 2022; Govindan et al., 2017; Noorain et al., 2022). In the optimisation literature, the primary focus of studies is to address a range of problems by formulating mathematical models, and developing sophisticated solution techniques and algorithms that can identify optimal or near-optimal solutions (Amin & Zhang, 2012; Liu et al., 2021; Marynissen & Demeulemeester, 2019; Saha & Ray, 2019; Schwerdfeger & Boysen, 2020; Soleimani et al., 2022).

The modelling lifecycle in hard OR disciplines is divided into four phases: conceptual modelling, model coding, experimentation, and implementation (Robinson, 2014). Amideo et al., (2019) have aligned the steps involved in optimisation modelling with these four phases. Specifically, in the conceptual modelling phase, the focus is on problem recognition, problem definition, and data collection. Model coding pertains to the formulation and solution of the model. During the experimentation phase, the model is validated and verified using data or case studies, and sensitivity analysis is conducted. Finally, the implementation phase involves disseminating the model to stakeholders.

After model coding, an optimisation model undergoes rigorous validation and verification processes to ensure accuracy and reliability. Validation involves assessing the model's ability to predict system behaviour and confirming proper functionality (Gass, 1983; Pala et al., 1999; Robinson, 1997; Sargent, 2020; Taha, 2017). In optimisation modelling, there are several different interpretations of model validation ranging from model validation by 'computational testing', or comparing solution algorithm performance, sensitivity analysis, validation by simulation, testing using benchmark datasets or using real-data (Harris & Claudio, 2022; Humagain et al., 2020; Kim & Mehrotra, 2015; Zamanifar & Hartmann, 2020). A recent analysis of model validation literature has found that data plays an important role in current validation practices (Eker et al., 2019). However, it is highlighted that current model validation procedures lack methodological reliability and future studies are encouraged to provide a deeper analysis on how a model fits its purpose conceptually and technically. Several studies recommend the utilisation of soft and participatory approaches to involve stakeholders throughout the optimisation modelling lifecycle (Amideo et al., 2019; Carter & Busby, 2022), given that such approaches have positively impacted the uptake of models as seen in simulation modelling (Harper et al., 2021; Pessôa et al., 2015; Robinson et al., 2014).

The implementation of an optimisation model involves translating the model's recommendations into actionable plans, integrating them into existing operational systems, and ensuring smooth execution (Taha, 2017). In domains such as manufacturing, transportation, production, and logistics, implementation of optimisation models is widespread due to increased computational power alongside established data systems, centralised decision-making structures, and cultures more amenable to change (Archetti et al., 2022; Kuo et al., 2023). In OR healthcare literature, several reviews have pointed out that only a few studies discuss implementation of model outcomes and recommendation. This is because the practical use of models in healthcare is unique, due to factors such as ethical considerations, patient-centric focus, and engagement of diverse stakeholders (Bradley et al., 2017; Brailsford & Vissers, 2011; Lamé et al., 2016; Long & Meadows, 2018; Mahdavi et al., 2013; Mohiuddin et al., 2017; Palmer et al., 2018). In healthcare optimisation literature, model implementation is a significant challenge for reasons such as complexity of the models, ill-fitted performance measures in models, failure to understandably report on method-related assumptions, lack of performance data prior to model development, and lack of practical relevance to stakeholders (Ahmadi-Javid et al., 2017; Marynissen & Demeulemeester, 2019; Samudra et al., 2016). Studies that do report on implementation, provide little detail about the process of implementation and so researchers are encouraged to provides information on the behavioural factors that intersect with actual implementation, and to discuss real-world challenges that impede implementation (Samudra et al., 2016; Zhu et al., 2019). Some studies found that implementation success was associated with data-driven modelling strategies, engaging, and receiving buy-in from leadership, and through receiving feedback on potential changes from services within the organisation (Zenteno, Ana C., Carnes, Levi, Daily, Price, Moss, & Dunn, 2015). In general, stakeholder involvement and ethical considerations play a critical role in healthcare optimisation, while transportation, logistics, and manufacturing domains emphasise collaboration, real-time data integration, and stakeholder engagement (Aringhieri et al., 2022).

In OR, Soft approaches or problem structuring methods have been utilised to enable stakeholders to provide inputs, challenge assumptions, and collectively build a shared understanding of the problem and the model's representation of the real-world system (Ackermann, 2012; Dyson et al., 2021; Powell & Mustafee, 2017). Participatory approaches to modelling involve enhanced stakeholder engagement throughout the model's lifecycle. By involving those affected by the model's outcomes, participatory approaches ensure that the model aligns with the stakeholders' needs, captures their knowledge and perspectives, and incorporates their feedback. Driven by an interest in supporting decision-makers facing complex problems, the simulation modelling community have developed several participatory research practices. In facilitated simulation participative methodologies such as PartiSim (Kotiadis et al., 2014; Kotiadis & Tako, 2018; Tako et al., 2010; Tako & Kotiadis, 2015), SimLean (Robinson et al., 2012) and Simtegr8 (Tako et al., 2019) have successfully demonstrated how stakeholders can be involved in the modelling lifecycle while also focusing on implementation and change. Similarly, facilitated modelling in System Dynamics

(SD), termed group model building (GMB) refers to the construction of an SD model whilst working directly with a group of clients (Rouwette et al., 2002; Scott et al., 2016; Vennix, 1995; Vennix, 1999). There are several examples of GMB in healthcare (Lane & Husemann, 2018; Lane et al., 2019; Minyard et al., 2014; van Nistelrooij et al., 2013; Willis et al., 2018).

Compared to other Hard OR methods such as simulation modelling, the utilisation of Soft OR and participatory approaches in optimisation modelling is limited (Amideo et al., 2019; Çoban et al., 2021; Jones et al., 2022; Noorain et al., 2022; Robinson, 2008; Robinson, 2014; Sterman, 2002; Tako & Kotiadis, 2012; Tako & Kotiadis, 2015; Vennix, 1999). The opportunities offered by facilitation for involving stakeholders in optimisation modelling have not been fully explored, especially by considering existing work that have developed participative approaches (Franco & Montibeller, 2010; Tako & Kotiadis, 2015). In this study, our contributions focus on the post-model coding stages of the PartiSim framework. We present evidence demonstrating the feasibility of model validation, building scenarios, and considering model implementation in collaboration with stakeholders by following a structured facilitated approach. Through the case study, we offer insights into real-world factors that impact the actual implementation of optimisation models. Furthermore, we emphasise how stakeholder engagement through workshops fosters acceptance and support for the model's recommendations. Notably, we contribute a case study where facilitated workshops were conducted in a virtual setting.

The rest of the article is structured as follows. Section 6.2 presents a background, including a literature review on the use facilitation, soft OR, and PSM’s for model validation and implementation of an optimisation model. We then examine the post-model coding stages of the PartiSim framework. Section 6.3 provides an overview on the adaptation. In Section 6.4 we describe the development of the framework with the use of a case study in mental healthcare. In Section 6.5, we discuss the proposed framework, reflect on the adaptations, and examine the footprint of conducting facilitated workshop virtually. In Section 6.6 we present some conclusive remarks.

6.2. Background

In this section, the post-model coding stages of the PartiSim framework are explored. Stages 5 and 6 comprise the post-model coding stages. Appendix I and Appendix J contains screenshots of tools that are prescribed for Stages 5 and 6. Additionally, these stages include validation, experimentation and/or implementation of the model. Following an exploration of PartiSim, we examine literature on optimisation model validation, experimentation, and implementation and analyse the present state of each theme. We also explore the application of Soft OR, participatory and facilitative methodologies to these corresponding themes. The aim is to identify developments, gaps and recognise opportunities, while foregrounding the adaptation of the PartiSim framework.

6.2.1. Optimisation in Mental Health

In stage 4 of the PartiSim framework, the conceptual model is converted into a computer simulation model using specialist software (Tako & Kotiadis, 2015). This activity is primarily driven by the modeller and does not involve a workshop. However, lines of communication with the project champion and other requisite members of the stakeholder team are maintained for data collection purposes. During this stage, the conceptual model could undergo modifications where elements of the model could be refined, updated, simplified, or removed based on their relevance. Furthermore, in preparation of the 3rd workshop, the project champion is presented with the model for validation and preliminary scenarios for the experimentation stages for clarification, as part of the pre-workshop 4.a stage.

Following the model coding stage, the PartiSim framework consists of two stages that include two workshops for experimentation (stage 5) and implementation (stage 6) (Kotiadis & Tako, 2018). Collectively, these stages are described as post-model coding to reflect their position on the framework's timeline. Table 29 provides a description of each stage's purpose, with associated activities, suggested tools, and prescribed outputs. Manuals with instruction on using the suggested tools and scripts containing advice for facilitators to support the facilitation process are also provided (Tako & Kotiadis, 2018).

Stage 5 involves a workshop and is primarily associated with model validation and verification and for choosing scenarios for experimenting with the model. In the model validation activity, the structure and contents of the model are demonstrated to stakeholders to gather confidence in the model and its results. Using the Model Validation tool (Figure 46 and Figure 47, Appendix I), stakeholders are invited to reflect on the model and suggest changes that improve the model validity. Although the full acceptance of the model is not to expected, in some instances, it could become evident that additional data collection or coding changes are necessary based on stakeholder suggestions. In such cases, the focus of the workshop shifts towards identifying ways to improve the model and/or the conceptual model. The workshop and remaining activities will need to be rescheduled to continue model validation after requisite changes have been made.

The next activity is to rate performance measures, supported by the Rating the Performance Measures tool (Figure 48, Appendix I). Here, stakeholders revisit performance measures identified during conceptual modelling and rate them according to their importance. This activity is said to contribute to identifying scenarios that achieve improvement of performance measures and also help in reducing the solution space. The next activity is where stakeholders debate desirable and feasible scenarios, using the Debating the Alternative Scenarios tool (Figure 47 and Figure 50, Appendix I), after being shown the preliminary future scenarios. In encouraging debate, this activity is intended to help stakeholders determine the feasible and desirable solution space. The solution space is the total range of conditions under which the model might be run (Robinson, 2014). It is a region that represents all possible combinations of values of the experimental factors. In the post workshop 3 sub-stage 5.a, following the successful completion of these activities a report outlining the model results and findings is prepared and sent to stakeholders for reflection.

The final stage is undertaken in a workshop setting and is concerned with the implementation of findings. Stakeholders are invited to reflect on the learning achieved so far during the simulation study and debate their plans for the future. The aim is to move the stakeholder away from the model and its finding towards gaining an understanding on the present and future implications of each scenario. To this end, three activities are conducted in the workshop: review learning and changes implemented, risk analysis and feasibility of change, and agree action trail.

In the first activity, the facilitator creates awareness of the learning generated throughout the study. A prescribed script is available in the framework to support facilitators. In the next activity, the risks and feasibility of change in potential scenarios are discussed to agree on a preferred scenario/scenarios to be pursued. This activity is supported by Feasibility and Risks Scale tool (Figure 51, Appendix J). The aim of this activity is to allow tacit knowledge to surface so that an action to tackle this change can be assigned in the next activity. Once an agreement on a promising scenario (or scenarios) is reached, additional analysis to explore any additional changes for the implementation of scenario is undertaken. This activity utilises the Barriers to Change tool (Figure 50, Appendix J) to explore the expected benefits of any additional changes. If the stakeholder group is sufficiently confident in the chosen scenario, an action and communication plan tool is utilised to record the action and the responsibility for it.

Table 29: Post-model coding stages in PartiSim (Kotiadis & Tako, 2018)

Stage & Activities	Activities	Tools	Outputs
4. Model Coding *Purpose:* Conceptual model is converted into a computer model	-Data collection (modeller and stakeholders) -Build simulation model on the computer (modeller)		Model Results
4.a Pre-workshop 3 sub-stage *Purpose:* Preparations for workshop 3	Prepare preliminary materials for use in workshop 3 (stage 5): -Liaise with the project champion over correctness of model and its results (modeller and project champion) -Review preliminary scenarios with project champion scenarios -Prepare preliminary materials for use in the next workshop		Model validation and verification Preliminary future scenarios
5. Experimentation stage (workshop 3) *Purpose:* Define alternative scenario to experiment with model	Stakeholders are invited to: - Validate the simulation model and its results - Rate performance measures (linked to model results)	Model validation tool Rating the Performance Measures tool with manual Debating the Alternative Scenarios tool with manual	Model validation and verification Alternative future scenarios
5.a Post-workshop 3/Pre-workshop 4 sub-stage *Purpose:* Refine alternative scenarios and prepare for workshop 4	Modelling team: - Tweak or correct simulation model - Implement additional scenarios suggested (based on stakeholder feedback from workshop 3.) - Liaise with the stakeholder team over correctness of model results - Prepare preliminary materials for use in workshop 4		New alternative future scenarios Revised simulation model

6. Implementation stage (workshop 4) Purpose: Define an implementation plan	Stakeholders are invited to: - Review learning and changes implemented - Risk analysis and feasibility of change - Agree action trail	Script for identifying changes in the system Feasibility and Risks Scale tool with manual	Agreeable and feasible scenario(s) to be taken forward
		Barriers to Change tools with manual Action and Communication Plan tool with manual	Action plan with deliverables (including due date and person responsible)

6.2.2. Model Validation and Experimentation

In OR, validation is an important activity undertaken to ensure a model is sufficiently accurate for the intended application of the model (Balci, 1994; Gass, 1977; Gass, 1983; Gass, 1993; Landry et al., 1983; Landry et al., 1996; Oral & Kettani, 1993; Sargent, 1984; Sargent, 2013; Taha, 2017; Tsioptsias et al., 2016; Whitner & Balci, 1989). This activity is integral to the model development process in OR, including for optimisation and simulation modelling. However, the utilisation and application of validation differs between the two approaches. Table 30 presents a comparison of validation between optimisation and simulation by drawing on descriptions put forward by Robinson (2014) and Oral and Kettani (1993). It should be noted that forms of validation are not necessarily named explicitly in literature (Oral & Kettani, 1993; Tsioptsias et al., 2016). Therefore, the type of validation is often implicit and embedded within the modelling process.

Data validity is an activity that spans all stages of model development and mainly concerns the availability, reliability, appropriateness, sufficiency, maintainability, correctness, completeness, and cost of data (Balci, 1994; Landry et al., 1983; Oral & Kettani, 1993; Pala et al., 1999; Robinson, 1997; Sargent, 2013; Tsioptsias et al., 2016). In simulation modelling, it involves ensuring sufficient accuracy of contextual data and the data necessary for model implementation and validation for the purpose at hand (Robinson, 2014). In optimisation, data validation is linked to model and solution validation (Dominguez-Ballesteros et al., 2002). Data plays a key role in assessing the robustness and logical acceptability of model results. Data instances are utilised to exercise the models and ensure their reliability. Issues with model formulation and solution are diagnosed through model and data debugging (Dominguez-Ballesteros et al., 2002).

Conceptual model validation is an integral activity that is conducted in the simulation modelling process (Balci, 1994; Gass, 1983; Landry et al., 1983; Pace, 2004; Pala et al., 1999; Robinson, 1997; Sargent, 2013; Tsioptsias et al., 2016). This activity involves determining that the content, assumptions, and simplifications of the proposed model are sufficiently accurate for the purpose at hand (Robinson, 2014). In optimisation, conceptual modelling is a secondary or non-issues, as formulation and developing solutions are key activities. Therefore, the emphasis is on formulation validation, which is conducted when prototype OR models, that are well conceptualised managerial problem, are reformulated (Arrigo et al., 2022). The main concern is to determine the degree to which the new formulation correctly and accurately describes the well-conceived problem (Oral & Kettani, 1993). In reformulating a prototype problem, the logical and conceptual validity is implicitly maintained as the concept and definition of a well-defined problem is not changed.

Table 30: Comparison of Validation between Simulation and Optimisation

Simulation	Optimisation
Data Validation Determining data required is sufficiently accurate for purpose at hand.	**Data Validation** Diagnosing issues between model formulation and solution. Debugging mismatches between model and data.
Conceptual Validation Determining the content, assumptions and simplifications of the proposed model are sufficiently accurate.	
	Formulation Validation Determination of degree to which new formulation's accuracy to a well-conceptualised model
Verification Process of testing the fidelity with which the conceptual model is converted into a computer model.	**Verification** Related to the verification of claims made by modellers about the merits of the model and/or solution.
White-box Validation Determining that the parts of the computer model represent real-world elements with sufficient accuracy.	**Operational Validation** Involves the examination of model results (often termed "computational results") to confirm they do not contradict the model builder's user's and/or associated "experts" perceptions of reality.
Black-box Validation Determining that the overall model represents the real world with sufficient accuracy.	
Experimentation Validation Determining that the experimental procedures are providing results that are sufficiently accurate.	**Experimentation Validation** Comparing the performance of several solution algorithms, output comparison with similar models, analysing the sensitivity of the solutions to input variations, and by using simulation models, benchmark data or real-world test-data.
Solution Validation Determining that the results from the model of the proposed solution are sufficiently accurate.	

In simulation, after a model is conceptualised and transformed into a computer model, it undergoes verification, which is seen as a subset of the wider issue of validation (Robinson, 2014). This process tests the level of fidelity in converting the conceptual model into a computer model (Gass, 1983; Landry et al., 1983; Oral & Kettani, 1993; Pala et al., 1999). In contrast, for optimisation, verification is associated with the development of abstract concepts and generalisations rather than for a current and immediate 'problem situation' (Oral & Kettani, 1993). Often the contribution is theoretical with research having to justify that the proposed 'model' and solutions are useful and useable (Kimbrough et al., 2008). A more technical view is related to the verification of claims made by modellers about the merits of the model and/or solution. This could involve comparing the suggested contribution with other existing works, highlighting the superiority and accessibility of a solution, and outlining the contribution to the extension of pertinent knowledge (Mishra et al., 2015; Samadi-Dana et al., 2017).

Operational validity in simulation is concerned with model usability, usefulness, timeliness, synergism, speed, effort, and costs of the model (Gass, 1983; Landry et al., 1983; Sargent, 2020). It is conducted across two activities: White-box validation, which is intrinsic to model coding; and black-box validation which is performed once the model code is complete (Robinson, 2014). White-box validation determines if the constituent parts of the computer model represent the corresponding real world elements, while black-Box Validation determines that the overall model represents the real world with sufficient accuracy for the purpose at hand (Kleijnen, 1995; Robinson, 1997; Sargent, 2013). For optimisation, operational validation involves the examination of model results (often termed "computational results") to confirm they do not contradict the model builder's user's and/or associated "experts" perceptions of reality (Alkaabneh & Diabat, 2023; Kim & Mehrotra, 2015). Additionally, it also includes systematically comparing model results against corresponding real-world observations. Operational validity is strongly linked to experimental validity and often there is considerable overlap between the two for simulation as well as optimisation model (Landry et al., 1983; Sargent, 2020).

Experimental validity for simulation is an indication of the quality, efficiency, sufficient accuracy and robustness of solutions, mechanisms and techniques used. Quality is determined by the level of insight gained, sensitivity to changes in values of model parameters, acceptability, applicability, and usefulness in leading to a "decision" (Robinson, 2014). Experimental validity is associated with running sensitivity analysis, or designing experiments etc. In optimisation, experimentation can take the form of comparing the performance of several solution algorithms, output comparison with similar models, analysing the sensitivity of variables, and by using simulation models, benchmark data or real-world test-data (Humagain et al., 2020; Zamanifar & Hartmann, 2020). Several non-mutually exclusive categories of validation experiments are conducted to yield information on a model's ability to replicate real world outcomes. For instance, validation experiments where the model is used to generate several solutions for a series of parameter sets, to study the magnitude of adjustments between alternative scenarios (McCarl & Apland, 1986).

Presently, within healthcare optimisation literature, operational and experimentation validation corresponds to testing and is concerned with whether researchers used data, either theoretical or real, to examine their model's effects (Harris & Claudio, 2022). Testing is said to serve two purposes. First, it can provide insights into the computational efficiency of the model. Second, it can show whether or not the model improves the desired performance measures (Samudra et al., 2016). As such, studies examine the impact of specific changes to the problem setting by selecting a set of parameters of interest and assuming changes to these parameters. This process is termed scenario analysis and includes multiple scenarios, settings or options which are compared to each other with respect to performance measures (Banditori et al., 2014; Cardoso et al., 2012; Duma & Aringhieri, 2019; Laesanklang & Landa-Silva, 2017).

Solution validation, in simulation, involves comparing the implemented solution to the final model of the proposed solution (Robinson, 2014). This type of validation occurs after implementation and is not inherent to the simulation study itself. However, it provides valuable feedback to the modeller. In optimisation model, a direct mapping of solution validation is not possible. However, comparisons can be drawn to the experimentation validation, with its emphasis on the performance of solution algorithms.

In facilitated simulation, model validation and experimentation are critical activities undertaken during facilitated workshops with stakeholders (Franco & Montibeller, 2010; Happach et al., 2012; Kotiadis & Tako, 2018; Robinson et al., 2014; Tako & Kotiadis, 2015). In comparison, there exists a gap in optimisation literature addressing the process of involving stakeholders for model validation and experimentation in a facilitated setting. To the best of our knowledge, two studies use participatory approaches during the problem definition stage of the optimisation modelling cycle. Cardoso-Grilo (2019) develop a conceptual model for a 'medical training problem' and present a sophisticated reformulation of a specific problem that supports health care workforce planning. Similarly, the participatory approach developed by Abuabara et al., (2022) addresses the 'diet problem', which is a classical application of linear programming in OR. Through deploying participatory approaches, these studies conduct implicit conceptual and formulational model validation for a specific type of problem.

Similarly, to the best of our knowledge, we found one study that conducts operational model validation and experimentation through scenario analysis using SSM and participatory methods of online surveys, interviews, and a facilitated workshop (Amorim-Lopes et al., 2021). The study builds on the multimethodology of enhanced optimisation proposed by Cardoso-Grilo (2019), in the context of Human Health Resource (HHR) planning by embedding scenario building within the mathematical planning model. In particular, the study provides concepts and a path to generate scenarios by capturing the views and stimulating the involvement of relevant stakeholders and experts, to produce coherent combinations of parameters for scenarios.

As optimisation modelling advances towards developing approaches to involve stakeholders in model building, the development and validation of conceptual models in a structured facilitated setting will become increasingly important. Optimisation modelling can draw from facilitated simulation to address identified limitations.

6.2.3. Model Implementation

Table 31 presents a comparison of model implementation between optimisation and simulation modelling, in healthcare. In simulation modelling, implementation has been extensively examined (Brailsford, 2005; Brailsford et al., 2013; Brailsford et al., 2009; Eldabi, 2009; Jahangirian et al., 2012; Long & Meadows, 2018; Long et al., 2020; Monks et al., 2015; Moretto et al., 2019; Soorapanth et al., 2023; Tako & Robinson, 2015; Thompson et al., 2016; van Lent et al., 2012). The word implementation takes on different meanings including the practical use of model results to inform a real-world decision, stakeholders learning about the problematic situation and/or an agreed action plan, and it is also used to describe the process of coding a model in computer software (Brailsford et al., 2019; Kotiadis & Tako, 2010; Long et al., 2020; Tako & Kotiadis, 2015).

Researchers have found several factors affect the implementation of simulation models, including stakeholder involvement, organisational and problem characteristics, relevance to stakeholders, availability of quality data, perceived usefulness of the model, model validation (with data, expert opinion and sensitivity analysis), well defined model scope and modelling process (Brailsford, 2005; Brailsford et al., 2013; Long et al., 2020; van Lent et al., 2012). These challenges are being addressed in several ways, as depicted in Table 3. For instance, studies have utilised Soft OR tools to involve stakeholders in the modelling process through facilitated modelling (Jones et al., 2022; Kotiadis, 2007; Kotiadis & Robinson, 2008; Kotiadis et al., 2014; Kotiadis & Tako, 2016; Kotiadis & Tako, 2021; Robinson, 2008; Robinson et al., 2010; Robinson, 2013; Robinson, 2014; Sterman, 2002; Tako & Robinson, 2009; Tako & Kotiadis, 2012; Tako & Kotiadis, 2015; Tako & Kotiadis, 2021; Vennix, 1999); pursued flexible definitions of implementation success; and realising unexpected ways in which a model adds value to a situation (Kotiadis & Tako, 2010; Long et al., 2020; Tako & Kotiadis, 2015).

Table 31: Comparison of Model Implementation between Optimisation & Simulation

Simulation	Optimisation
What constitutes implementation	
- Model results inform real-world decisions. - Stakeholder learning about the problematic situation and/or an agreed action plan. - Process of coding a model in computer software	- Developing good solution procedures - Implementing well-known algorithms, testing prototypes (software products) in real-world environments, applying the model in practice. - Developing nearly realistic models.
Factors identified as affecting implementation	
- Organisational & problem characteristics. - Relevance to stakeholders - Availability of quality data - Stakeholder involvement - Perceived usefulness of model - Model validation with data, expert opinion & sensitivity analysis - Well defined model scope & modelling process - Tension for change and leadership engagement	- Lack of information on behavioural factors intersecting implementation. - Lack of data-driven modelling strategies - Lack of engagement with stakeholders - Lack of reporting in literature on problems encountered during implementation. - Limited understanding of whether models work in practice. - Lack of performance data prior to implementation for comparing results - Gap between theory & practical implementation
How are implementation challenges being addressed?	
- Uncovering factors and barriers affecting implementation - Evaluating dynamic interplay of implementation - Rigorous post-implementation studies. - Methodological development formalising methods of analysis - Collecting data on intangible benefits. - Reframing success through collaborative model-building leading to critical learning incidents for the client. - Development of participative and facilitated modelling. - Growing consensus on flexibility in the definition of implementation success - Realising and pursuing other unexpected possible ways in which a model adds value in a situation. - Cost evaluation of implementation strategy - Utilising implementation science approaches to examine how and why key decision makers adopt modelling.	- Encouragement for testing models with real data - Some emerging instances of model validation involving stakeholders using participatory approaches (surveys, interviews & facilitated workshops) - Some advancements in involving stakeholders in multi-methodology (SSM tools with optimisation) approaches for testing models using fit-for-purpose scenarios. - Reviews highlighting the need for involving stakeholders in model building

In healthcare optimisation, model implementation has been identified as a significant challenge (Ahmadi-Javid et al., 2017; Marynissen & Demeulemeester, 2019; Samudra et al., 2016). As seen in Table 31, implementation can have a range of meanings. For instance, it is viewed as a function of developing good solution procedures in terms of quality, speed, and implementing nearly realistic models (Ahmadi-Javid et al., 2017). It can also include the implementation of well-known mathematical algorithms, developing of a prototype (a software product), and testing prototypes in real world environments (Humagain et al., 2020).

Several factors have been recognised to contribute to issues in the implementation of optimisation models in healthcare. These include a lack of information on how behavioural factors intersect with implementation, the need for data-driven modelling strategies, stakeholder engagement, reporting of implementation problems, insufficient performance data for comparing results prior to implementation, limited understanding of the practical effectiveness of models, and a general gap between theory and practical implementation (Harris & Claudio, 2022; Kortbeek et al., 2017; Marynissen & Demeulemeester, 2019; Samudra et al., 2016; Visintin et al., 2017; Zenteno, Ana C. et al., 2015; Zenteno, Ana Cecilia et al., 2016; Zhu et al., 2019).

Optimisation modelling is just getting started in addressing these challenges with implementation. Recent advances have focused on using increasing the utilisation of data-driven approaches, using Soft OR tools with stakeholder participation in conceptual modelling and scenario building, and applying comprehensive validation and testing procedures (Abuabara et al., 2022; Amideo et al., 2019; Amorim-Lopes et al., 2021; Cardoso-Grilo et al., 2019; Çoban et al., 2021; Humagain et al., 2020). There is growing consensus in the community to develop procedures that involves stakeholders in the process. However, these remain open challenges and areas of research that have the potential to fill the gap between theory and practice for optimisation models.

6.3. Overview of PartiSim Adaptation for Optimisation

PartiSim was developed using an action research approach (Tako & Kotiadis, 2015). Action research is based on action, evaluation, and critical analysis of practices based on data collected to introduce improvements in relevant areas. It enables the creation of knowledge. As such, the creators of PartiSim first developed the framework, then specific modelling activities, followed by a phase of testing and reflection. The proposed framework was a result of the amendments made during the development, while also paving the way for future work to further develop and improve the framework.

The developers of PartiSim advise that in adopting the framework for an intervention, the individuals conducting the intervention could consider an ongoing loop of reflecting on their knowledge and experience by asking questions such as “What did I do well?” and “What should I have done differently to engage clients? (Kotiadis & Tako, 2021) The loop that leads to better practice in case studies, starts and ends at the same point, with reflection. As such, the rest of this chapter follows a similar loop of reflection on the adaptation of the post-model coding stages to optimisation modelling, followed by the description of application, and ending with a reflection of the adaptation.

When considering the adaptation of model validation tools for optimisation, we determined that the tools would need to be more specific to each optimisation model component. In workshop 3 of Stage 5, stakeholders are given a demonstration of the simulation model and then invited to reflect on contents. Using the model validation tools, stakeholders are then asked to comment on a particular aspect they want to update and/or change. The validation of an optimisation model is dissimilar to a simulation model, particularly since there is no visual component that could be utilised to enable stakeholder understanding. To tailor the validation to the optimisation framework, we determined that validation of the model and its output would need to be conducted by examining each component of the model and then viewing it as a whole. For instance, stakeholders would first be given an overview of the model and its solution, followed by a component-by-component analysis to determine if it accurately represents the system under consideration and if the behaviour is being captured appropriately. Therefore, each component is examined with a dedicated validation form. Therefore, the validation tools from PartiSim, would need to be extended to each optimisation model component.

Furthermore, it is agreed that of the five optimisation model components: inputs, constraints, objectives, decisions, and outputs; constraints and inputs would be most critical. Specifically, constraints are critical, as too many restrictions would not yield a feasible solution, while relaxing constraints may generate solutions that are not feasible in practice. Often constraints can be moved into the objective and vice versa. Therefore, model validation would need to begin by debating the constraints where stakeholders are asked to verify the accuracy of the formulation, followed by the validation of input parameters, as constraints essentially represent relationships between parameters and decision variables.

Material for workshop 3 was readied with the anticipation that it would be a fairly smooth process because of activities conducted in the conceptual modelling and model coding stages. Therefore, it was anticipated that in the workshop, the model and its outputs would be validated, additional data needs would be identified, and scenarios would be defined. Equally, it was recognised that the process of validating objectives, decisions, constraints, and inputs may be lengthy, and it would not be possible to easily modify the model during the workshop for immediate re-validation.

For scenario building, PartiSim links scenarios to the Performance Measurement Model (PMM), where stakeholders identify performance measures and establish monitoring and control activities to support those measures (Kotiadis, 2007; Kotiadis et al., 2013). These activities are grouped as monitoring activities (to examine performance measures identified), determine if activities (to assess the need for action), and suggest action to be taken. In PartiSim, stakeholders rank each performance measure to determine which simulation scenario is most feasible and desirable. This is done using the Rate Performance Measures tool (Figure 48, Appendix I). In adapting this approach to optimisation, we acknowledged that each scenario will be associated with a combination of model inputs. Each combination of input parameters would play a critical role in scenario building and impact decision variables and subsequent model solutions. Therefore, by recognising that scenario analysis for optimisation grants more emphasis on inputs, rather than outputs (performance measures), it was determined that the Rate Performance Measures tool would not be used. Moreover, in PartiSim, alternative scenarios are generated using the Debate Alternative Scenario tools (Figure 47 and Figure 50, Appendix I). This tool links each scenario with performance indicators. Based on understanding that the scenario generation for optimisation would be more focused on identifying input combinations, it was determined that the scenario tools from PartiSim would need to be updated for optimisation.

Furthermore, it was determined that while performance measures may not directly influence the scenario building activity for optimisation in the same manner as PartiSim, each scenario would still incorporate the performance activities identified in the PMM. Initially, a baseline scenario would be constructed to represent the existing system, allowing stakeholders to assess examine the identified performance measures (monitoring activity) to assess the need for action (determine if activities). Based on the 'suggested actions' specific in the PMM, the modelling team would develop additional scenarios that improve the baseline. In workshop 3, stakeholders would be walked through this process as depicted in Figure 20. Specifically, using the model outputs from the baseline, the group would be asked if the service is performing to the degree specified in the PMM. With the suggested changes in view, stakeholders would be asked if the suggestions are still relevant, and if yes, which of the model inputs that are within their control, could be changed in the model, to then examine the expected impact of these changes.

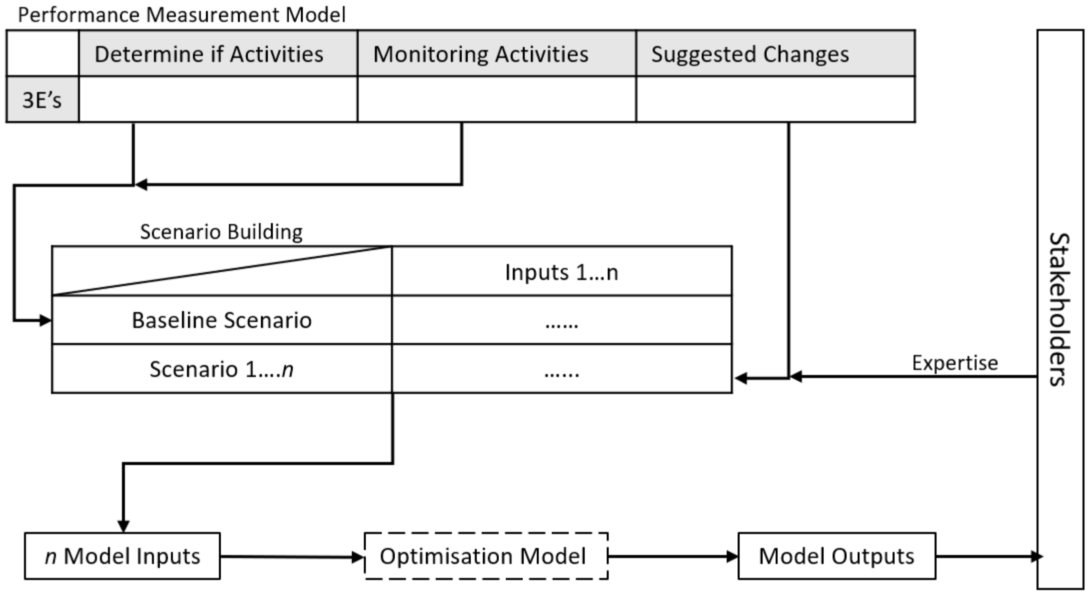

Figure 20: Scenario Building Process

For the implementation stages, we anticipated recommendation for future decision would derive from the optimisation model and from the overall application of the framework. When considering the tools for debating recommendation, adapting the Feasibility and Risks Scale tool (Figure 51, Appendix J) and Barriers to Change tool (Figure 50, Appendix J) for online implementation was the main concern. To address this, we explored several virtual application to identify the most suitable environment to host the tools.

6.4. Case Study

This approach was developed through a partnership with the Kent and Medway Mental Healthcare Trust (KMPT), a real-world PCMH service located in Kent, UK. KMPT collaborates with GP clinics and primary care partners to offer support to individuals with mild/moderate mental health conditions who don't require secondary care services. The service was conceived to address the growing emphasise on enhancing patient accessibility by integrating mental healthcare into primary care to promote better coordination with secondary, and tertiary mental healthcare services (NHS England, 2020).

The stakeholder group consisted of clinicians, service managers, executive members of the trust, personnel from the local Clinical Commissioning Groups (CCG) and a public health consultant from the local council. Table 32 provides a comprehensive list of workshop participants that took part in this study and their respective roles. The table also highlights which of the participants attended the three workshops that were conducted for the post-model coding stages described in this case study.

Table 32: Workshop Participants and Roles

Workshop Participants	Roles	Workshop 3a	Workshop 3b	Workshop 4
Project Champion (A)	Programme Manager Transformation and Improvement Team, KMPT	✓	✓	✓
Key Stakeholder (B)	Clinical Commissioning Group Project Lead	✓	✓	✓
Key Stakeholder (C)	Deputy Chief Operating Officer, KMPT	✓	✓	✓
Key Stakeholder (D)	PCMH Service Manager, KMPT	✓	✓	✓
Workshop Participant (E)	Head of Service West & North Kent	✓		✓
Workshop Participant (F)	Business Intelligence Analyst, KMPT	✓	✓	✓
Key Stakeholder (G)	Benefits Realisation Manager, KMPT	✓	✓	✓
Workshop Participant (H)	Service Manager of Maidstone Community Mental Health Team	✓		
Workshop Participant (I)	Research and Development, KMPT	✓	✓	✓
Key Stakeholder (J)	Primary Care Mental Health Specialist	✓	✓	✓
Workshop Participant (K)	Primary Care Mental Health Specialist	✓	✓	✓
Workshop Participant (L)	Primary Care Mental Health Clinicians	✓		
Workshop Participant (M)	Primary Care Mental Health Clinicians	✓		
Workshop Participant (N)	Consultant Psychiatrist & Assistant Medical Director (KMPT)			
Workshop Participant (0)	Public Health Consultant (West Kent County Council)			

During the pre-model coding stages described in Chapter 4, it was revealed that the PCMH service had originated to address immediate mental health concerns in primary care and to ease the in-flow of referrals to secondary care. In essence, the service had started as an ad-hoc experiment, and as such, lacked proper considerations for service design and operational specifications. The workshops revealed a lack of clarity regarding current and future service capacities, while demand was expected to rise. Stakeholders identified several problems including the absence of a standardised service model, limited information on service performance and clinical workforce utilisation, and the lack of benchmarking due to underutilisation of available data. Workforce distribution and efficiency were identified as key concerns, with specific emphasis on long discharge times, messy timetables, and the potential for improving patient accessibility. Stakeholders acknowledged that the existing service falls short and expressed a need for standard operating procedures.

Table 33 provides a description of the workshops that were conducted in this study across several dimensions. For each workshop, we first establish if there is any deviation from PartiSim. In this study, the experimentation stage was conducted over two workshops (3a and 3b) instead of one, as prescribed in PartiSim. Furthermore, the first two workshops were conducted in person before the onset of the COVID-19 pandemic. However, due to the unprecedented circumstances and subsequent restrictions, transitioning to a virtual environment became necessary. Each workshop had a duration of 2 hours, with a specific group of stakeholders participating. This chapter focuses on the stages spanning April 2020 to October 2020, that occurred after a six-month gap since workshop 2.

Table 33: Details of Workshops in this Study

	Maps to PartiSim?	Mode	When	Length	Workshop Participants
Workshop 1	✓	Face to Face	September, 2019	2 Hours	A, B, C, D, E, G, N, 0
Workshop 2	✓	Face to Face	September, 2019	2 Hours	A, B, C, D, E, G, N, 0
Workshop 3a	✓	Virtual	April, 2020	2 Hours	A, B, C, D, E, F, G, H, I, J, K, L, M
Workshop 3b	✗	Virtual	July, 2020	2 Hours	A, B, C, D, E, F, G, I, J, K
Workshop 4	✓	Virtual	October, 2020	2 Hours	A, B, C, D, E, F, G, I, J, K

We discovered that validating the initial optimisation model and its solution required a dedicated workshop of its own before experimentation through scenario generation. As a result, Stage 5 was split into two workshops instead of one. Details of the variations between PartiSim and the modified post-model coding stages for optimisation are described in the following sections. Also included are excerpts from the workshops highlighting stakeholder engagement with the tools, activities, and specific instances of learning.

Chapter 5 describes the technical implementation of the final validated model and the scenario analysis. Here, the focus is on presenting the process of validating the model, generating scenarios, and detailing the subsequent implementation process. During model development, the modelling team liaised with the project champion and service manager to acquire data for the model and for clarifications on model components. After developing the initial model, preliminary results from the scenario analysis including outputs from the data analysis were emailed to the stakeholder group in preparation for Workshop 3.

For the workshop, we extended and streamlined the Model Validation tool (Figure 47, Appendix I). Based on the premise discussed in Section 6.3, the original tool was extended for each of the five components. It was streamlined by omitting the middle column. And each validation form's first column was populated with the corresponding information from the "Conceptual Model Map" developed in the pre-model coding stages. Therefore, we prepared five validation forms, each for: Inputs, Objectives, Decisions, Constraints and Outputs. Similarly, presupposing successful validation of all model components, and particularly inputs, the Debating the Alternative Scenarios tool (Figure 47, Appendix I) was adapted to capture a combination of optimisation model inputs for each scenario. Specifically, the columns of the new adapted tool were populated with model inputs, as described in the "Optimisation Component Map". These tools were prepared on the virtual platform 'Google Docs' to be shared with stakeholders during virtual workshop (Appendix K).

6.4.1. Experiment with Model - Workshop 3a

In workshop 3a, we sought to validate the optimisation model and its results, and then develop alternate scenarios for experimentation. The validation forms, prepared on Google Docs were shared with stakeholders. These forms had editing access for participants to make changes to the document during the workshop.

In general, the process of validation followed these steps in sequence: 1) reviewing results of the descriptive and predictive analysis to highlight inefficiencies; 2) viewing conceptual model; 3) reviewing initial mathematical model and its components including inputs, decision variables, objective, and constraints; 4) explaining initial model assumptions; 5) reviewing the PMM and baseline scenario 6) assessing model output (schedule of which clinician allocated to which clinic for what appointment and in which week, day, & shift) and solution; 7) Considering the level of detail, scope, and accuracy of the model and solution; 8) reviewing other preliminary scenarios and the corresponding model results 9) generating alternative scenarios.

In the first step, stakeholders received a walkthrough of results from the data analysis which provided many critical insights about clinician capacity and utilisation. For instance, it showed a lack of consistency in clinician profiles relative to their bands, the number of clinicians allocated to each clinic, and challenged perceptions of appointment time durations. The analysis also highlighted the size of the waiting list, that was primarily perceived to be linked to the relatively small size of the clinician workforce. Additionally, the predictive analysis supported stakeholder intuition about a rising upward trend in demand for the service.

Next, stakeholders received a walkthrough of steps 2 to 5. When reviewing the scenarios, we started with the baseline scenario. Stakeholders were told how this scenario was built using information collected in previous workshops. For examples, the group was reminded that in the PMM developed in workshop 2, they had determined that for the PCMH system to be efficacious, clinician utilisation would need to be monitored, to determine if clinicians are conducting appropriate interventions. Stakeholders were then informed that to monitor this aspect of the service, the initial model was supplied with current clinician capacity data. That is, data from a recent four-week period of service operation was used for comparing the accuracy of the model's output with real service performance. The group was then told that this is done to determine how the model would allocate appointments to clinicians, under the specified constraints and to check the overall performance of the system, if appointments were allocated centrally.

The baseline scenario solution, generated by the model, was then assessed. There were several components to the solution, including the schedule and service performance measures. The schedule is a visual representation of which clinician is allocated to which clinic for what appointment and in which week, day, & shift. Additionally, stakeholders were also provided with levels of clinician utilisation based on current availabilities, and the number of unmet appointments.

The discussion began by examining clinician utilisation, which varied significantly in the model solution and was also confirmed by the data analysis. During model conceptualisation, stakeholders assumed that clinicians in the service, especially in the same skill group would have similar capacity utilisation. In other words, a degree of standardisation was assumed and the same was expected to be seen through the analysis. However, in assessing the model solution, supported by the results of the data analysis, stakeholders recognised the gap between practice and assumption. This challenged the general view that service operations were organised to a satisfactory degree, and a rise in demand could only be handled by hiring more clinicians. Given the mix within the stakeholder group, several perspectives of how the model could be used to address the gap emerged. For instance, in conveying what they understood about the model, some stakeholders were keen to confirm if the model could support the group in taking forward a particular operating strategy.

Stakeholder B: *I think we should use this information to set a benchmark for ourselves so we can monitor how we are doing and how many patients we are seeing or if we are meeting our target of seeing patients within a certain duration.*

Stakeholder D: *I agree. Also, so that we are appropriately positioned whilst we are working in the PCMHS network, we could direct our clinicians to where the demand is. Let's say we take the readily available data and do an analysis like the one you have conducted, we can see which GP's have the highest referrals, we should be able to calibrate and match those using the model, is that correct?*

A walkthrough of model solution in Step 6 demonstrated that there was room for improvement within the service, leading to then debating the accuracy of the mode and its solution. Here, the group stakeholders acknowledged how the allocation of clinicians is a direct result of model inputs and the constraints governing the system.

Stakeholders were supplied with the five model component validation tools and based on the information they had just received; the group was asked to consider whether the information is still accurate or if additions/changes were required. The group agreed that the objective, decisions, and outputs components of the model were satisfactory, and no changes were suggested for these components. However, stakeholders wanted to change the values of some inputs, add additional constraints and review some of the corresponding assumptions for these components. Table 5 is the validation tool for model inputs and Table 35 is the validation tool for constraints.

As seen in Table 34, direct updated values were provided for five of the six input values that were to be changed. For clinician availability (available capacity), stakeholders realised that an overview of this information was not currently available, and so they wanted us to collect data reflecting actual working patterns of clinicians to assess the impact of these availabilities on the service. It was recognised that the operating policy of the service did not explicitly state a standard for the division of working hours across activities. This feature was also identified as being crucial to the service and had previously been assumed to have a loose set of guidelines that clinicians were to follow. Now that this assumption was dispelled, the clinicians as well as the management personnel present at the workshop were keen to examine how clinicians were dividing their time, specifically for clinical activities and perhaps explore developing a generic template that stipulates or guides clinicians.

The validation of inputs was closely linked to gathering updates for constraints, as seen in Table 35. For instance, after agreeing to collect more data to determine clinician availability patterns, stakeholders also noted that the initial model assumes that clinicians' time is entirely dedicated to seeing patients, when this was not the case. The availability constraint needed to account for other clinical activities such as trainings, meetings, and supervision and part-time clinicians. Furthermore, as clinicians from the service were present at the workshop, they noted that in the schedule, although clinicians were being assigned to clinics within their designated geographic region, some clinicians were being allocated appointments in clinic locations that were not currently on their caseload. This was deemed to be disruptive to continuity of care. A decision was made to remove the geographical region constraint and replace that with a stricter constraint that would assign appointments to clinicians in specific locations that were currently on their caseload. Other updates to the constraints in the model were minor, where a constraint limiting the number of appointment per day was added.

Table 34: Inputs Validation Tool

Inputs Validation	Updates
From Data:	
- Set of clinicians, locations, appointment types, days, shifts	
- Demand per clinic location	
- Clinic locations assigned to clinicians	
- *Appointment durations*	Consider average and not median duration
- Clinician skills and appointment matrix	
- Clinician to clinic preference	
Stakeholder Supplied:	
- Max shifts per day	No more than two
- Max clinics per clinicians	
- Max clinicians per clinic	Between 4 and 8 per clinic
- *Clinicians' availability matrix*	Not standardised for all clinicians, collect data to determine availability based on clinician band.
- *Max travel distance between clinics*	Consider 7.5 miles
- *Shift Duration*	Give a one-hour window between each shift to account for lunch time and travel between locations.
Generated:	
- Distance between clinics	

Given that the model with its current input values, and constraints was not sufficiently reflective of their service, this workshop, as it went, became primarily about discussing updates to the inputs and constraints. It was agreed that scenario building would have to be an activity that is conducted in another workshop along with the validation of the updated model.

Table 35: Constraints Validation Tool

Constraints Validation	Updates/ New Constraints
- Assign demand from clinic locations based on available clinic capacities.	
- Assign appointments based on clinician skills	
- Assign appointments in clinics within designated geographical region	Remove and replace with continuity of care constraint
- Clinician travel constraints between clinic locations	
- Clinicians to be assigned less than or equal to utmost number of shifts	
- Clinics to be assigned less than or equal to utmost number of clinicians	
- Clinicians to have less than or equal to utmost number of clinics	
- Clinicians' availability	Clinician not available during training days, and community meeting days. More data needed to determine when not available.
- Travel distance between clinics to be limited by a threshold value	
	- Per day appointment allocation limit = 5 per day
	- Continuity of care constraint, clinicians with patients from a specific clinic on their caseload, need to continue seeing these patients.

In general, it was agreed that more data was needed for the model to improve stakeholder understanding of how resource capacity was being divvied up. Therefore, steps 8 and 9 were not conducted in this workshop and were shelved for further exploration in a second workshop. At the end of the workshop, feedback from an executive stakeholder was as follows:

Stakeholder C: *I think what you've given us today, we as a team need to really sit down and look at all of the data related to demand, caseloads and everything else. We need to think about how we want to develop the service going forward. I think the next workshop will be about bringing all of this together and then to have a look at the new data again to explore where we go next.*

6.4.2. Post - Experiment with Model Workshop 3a

After the completion of Workshop 3a, data collection was conducted to gather information on clinician availabilities. After data collection, it was found that each clinician's availability varied across the entire team as well as within their respective bands. When this information was relayed to the project champion, it was suggested that for the next workshop, model results should be presented for both perceived availability and with a loose standardisation, so stakeholders are able to compare the two and suggest other scenarios for experimentation. Hence, the model was updated and resolved with the new inputs and updated constraints in preparation for the next workshop. Preliminary scenarios included the baseline, and two other future scenarios where one had the combination of perceived clinician availability and high demand and the other with high demand and standardised availability.

With reference to tools, the Inputs Validation Tool (see Table 34) and Constraints Validation Tool (see Table 35) were prepared and pre-populated with the most up-to-date information. Additionally, the Scenario Parameter Combination Tool (Figure 56, Appendix K) that had previously been adapted from the Debating the Alternative Scenarios tool (Figure 47, Appendix I), in preparation for workshop 3a was also included.

6.4.3. Experiment with Model - Workshop 3b

The primary goal of this workshop was to validate the updated model and the corresponding solution. Results from the updated model were collated on a presentation and stakeholder were provided with a walkthrough by following the sequence of steps described in Section 6.4.1. In essence, this workshop was a continuation of workshop 3a.

The workshop began with a demonstration of how updates from the previous workshop were incorporated into the model. Stakeholders were then presented with the outputs of the model for the three preliminary scenarios, including the new baseline. Stakeholders were invited to discuss updates to the model and suggest any additional changes using the input and constraint validation tools. In this case, there was consensus within the group that the new model and the corresponding input parameter values were an appropriate representation of their service. Stakeholders recognised that many operational specification in the service could be improved, as evidenced by the results of the new data analysis, updated future demand predictions, as well as the persisting inefficiencies identified in the system through the updated optimisation model's outputs.

Once again, stakeholders were asked to view the model's results with a view to the PMM, to generate alternative scenarios through the process described in Figure 20, Section 6.3. Specifically, they were asked if the service, with its current operational specifications, was satisfying their desired performance measures. If not, stakeholders were asked to consider the **suggested changes** defined in the PMM, and consider which aspect of the service, that was under their control could be changed to improve the current situation.

Stakeholders were presented with a Scenario Parameter Combination Tool (Figure 56, Appendix K), made available on Google Docs. This tool was prepopulated with the model's input parameters. Table 36 is a condensed version of the final output from the scenario tool. Stakeholders were then asked to consider each model input and determine if the current value could be changed, either on its own, or in combination with other input values. Of all the model inputs available on the form, the group agreed that they would like to experiment with "appointment durations", "clinician availability", and "Demand from each clinician location for each appointment type". Additionally, the group acknowledged that the allocation of clinics to clinicians was not a suitable parameter for experimentation or change as continuity of care could not be disturbed. For the input "appointment durations", in round one of the validation, median values were considered while round two used average values. Stakeholders were not keen to move forward with either of these as they believed that clinicians often have appointments that last at least 60 minutes. Therefore, duration profiles derived from the data analysis were chosen as values for experimentation. Specifically, in profile 1, the durations for appointment types 'Assessments', 'Follow-Ups', 'Telephone', and 'Community' were set at 60, 60, 45, and 60 minutes respectively. Similarly, in profile 2, the durations considered were, 60, 45, 30, and 45 minutes, respectively.

Table 36: Scenario Parameter Combination Tool (condensed) Output

Scenarios	Clinician Available Hours	Appointment Demand	Appointment Duration
1	From Historical Data	Current	Profile 1
2	From Historical Data	Current	Profile 2
3	From Historical Data	15% Increase	Profile 1
4	From Historical Data	15% Increase	Profile 2
5	Standardised	Current	Profile 1
6	Standardised	Current	Profile 2
7	Standardised	15% Increase	Profile 1
8	Standardised	15% Increase	Profile 2

Clinician availability was a point of contention and led to a discussion between stakeholders on how to proceed. Eventually, stakeholders were directed to contemplate the patterns observed in clinicians within the same band as seen in the data collected post workshop 3a. Springboarding off this, a standardisation template based on clinician skills was agreed upon (described in Chapter 5). The study itself was conceived in anticipation of rising demand from primary mental healthcare service users. As such stakeholders wanted to examine the impact of higher demand values on clinician utilisation and the waiting list. Based on the predicted forecast, the group was asked what percentage of rise in demand they would like to be prepared for. It was agreed that as things stand, in terms of capacity and funding, they would like to be prepared to handle an increase of 15%. Therefore, values for input parameters and their corresponding combinations were gathered from stakeholders and a total of eight scenarios were generated using the Scenario Parameter Combination Tool (Figure 56, Appendix K). Table 36 is depicts the condensed version of the output, containing only the inputs that were chosen for scenario analysis.

In July 2020, it became apparent that an end to the pandemic was not in sight. In view of this fact, stakeholders were keen to understand how the service would perform under COVID induced PCMH service specifications. Following the discussion that resulted in Table 36, the project champion suggested using the model to inform the service on its capacity to continue operating in anticipation of an influx in referrals because of pandemic-related restrictions. They were also keen on understanding the impact of moving to virtual only appointments on clinician utilisation. The stakeholder group unanimously agreed to this proposal and suggested the development of a COVID-version of the model by stripping away travel and skill-based constraints and only consider one type of appointment. The same scenarios were to be utilised for experimentation with the COVID variant of the model.

6.4.4. Post – Experiment with Model Workshop 3b

Following workshop 3b, we developed the COVID-version of the model and ran scenario analysis for the two models. In addition to obtaining a planning schedule for each scenario, the results compare baseline performance measures of clinician utilisation and unmet demand values across each scenario. The stakeholders are sent a report containing a summary of the results and recommendations that draw from scenario results. An implementation workshop is then scheduled to discuss which of these scenarios would be taken forward and to develop an action plan.

The modelling team prepared for the workshop and considered how the prescribed tools could be adapted for online implementation. Several digital whiteboard platforms such as Padlet, Mentimeter, Canva, and Mural were explored to determine how paper based tools could be implemented virtually for stakeholders to collaborate and contribute to the workshop. Mural was selected as the preferred platform because of its versatile features including resizable canvas options, the ability to create mappings and diagrams, customisable templates, and facilitations aids such as timer and summoning participants to specific content. The Feasibility and Risks Scale tool (Figure 51, Appendix I) from PartiSim was explored and adapted for online implementation, and a template consisting of diagrams was designed on Mural. Any action items that emerged from the workshop would be recorded and incorporated onto the Action and Communication Plan (Figure 51, Section Chapter 7:Appendix I) that would be shared with stakeholders post workshop. Implementation tools manuals prescribed in the PartiSim framework would be used to facilitate the discussion in the workshops (www.partisim.org).

6.4.5. Implement Findings - Workshop 4

The final workshop of this study took place three months after workshop 3b – October 2020. Stakeholders were aware of the recommendation of the study following on form stage 5. The report also included key insights that were gained from the conceptual modelling stages, the model coding stage which included descriptive and predictive analytics whose results inform what goes into the optimisation model and help structure the scenario building. Not unlike the last two workshops which were conducted in 2020, this was organised virtually. The stakeholder group composition is presented in Table 32.

The workshop began with a presentation of the project timeline and key insights that were gained from each. Stakeholders were reminded of the study's inception with the exploration of root definition, service components, performance management model, and the systems model that depicted all the components of an ideal PCMHS. There was a general acknowledgement that the stages of model conceptualisation led to a common understanding of what was missing from the framing of the service and equally, what was in place to enable the delivery of primary mental healthcare to patients. Additionally, stakeholders were also quick to appreciate that there were several other transformations that were recognised in Workshop 1 that could warrant attention and perhaps be the focus of their own studies. At the same time, the group conceded that the Transformation addressed in this study was one that was a top priority for the continuation of the service, and one that was useful and transferrable to other community-based services provided by the mental health trust and commissioned by the local council.

Nex stakeholders were reminded of insights gained form the analytics conducted using historical data. There was agreement that the results revealed performance issues as well as patient backlog, they suspected but were unaware of and did not previously have evidence for. The group also recognised how the outputs of the analytics aided their understanding of what specifically needed attention. In other words, the analysis dispelled vagueness surrounding an otherwise opaque set of metrices. Some excerpts of this discussion are as follows:

Stakeholder D: *I think to answer your question of anything that has directly changed since we started the study. I think the work has highlighted certain areas that we are not covering. Especially, the data that you have explored and presented to us, we went back to the Business Intelligence (BI) team and said to them that these aspects are not being reported on and should be included in the report.*

Stakeholder J: *We also saw that there are certain things we are not capturing in the database that we should capture.*

Project Champion: *And also, raising our awareness about areas where we can do some efficiency savings in clinical time and that is a direct impact of this work.*

We then arrived at the recommendations that were the result of the optimisation model, and some emerged from the data collection activity that clinicians identified as warranting attention in the future. Specifically, the recommendations deriving from the scenarios were grouped as follows:

1. Standardise Service Delivery
   - Standardise length of a clinic slot and duration of appointments.
   - Extend intervention coding to standardise clinician availability across all clinicians and between clinicians of the same band.
2. Standardise Intervention Coding
   - For a given planning period, consider developing a standard coding—including frequency and duration—of each intervention across all clinicians.
3. Distribute Workload Equitably
   - Explore redistribution of clinics managed by clinicians, in proportion to the number of referrals.
   - Consider redistributing caseload for a more equitable sharing of workload.
4. Deploy Optimisation Model
   - Estimate effective service design and operating policy variables.
   - Evaluate ongoing performance levels.
   - Compare predicted performance with desired service goals or objectives.

Additionally, during both historical data analysis and subsequent data collection activities, stakeholders realised that they should be capturing one additional type of appointment that clinicians in higher bands were conducting. In particular, the database system did not have a provision to capture patient-related consultations with GP's conducted by band 8 clinicians. Seeing as these set of clinicians were hired recently as part of the service's ongoing expansion, the capture of all their duties was deemed an important future implementation that emerged from the study. As such, the recommendation was to:

5. Capturing all patient-related interventions
    - Consider expanding the list of direct patient-related activities to include GP consultations on the database management system.

All the above qualified as learning that emerged from the study. Additionally, as confirmed in the discussion snippet above, the reports that the service manager routinely requested form the in-house BI team had undergone a protracted transformation that included many of the key metrices that were reported in this study and as such, the clinicians had a wider and deeper view of their performance and that related to patient waiting time and length of stay. This was acknowledged as an improvement derived from this study that was not a mainstay in their daily operations.

In the next activity, stakeholders were invited to discuss and debate which of the recommendations were feasible for implementation, which of these were desirable but cannot be implemented and those that were both feasible and desirable to be carried forward. This activity was conducted on Mural where the adapted visual templates of the Feasibility and Risks Scale tool (Figure 51, Appendix I) were staged and prepopulated. The group were given editing access to the digital environment. The output of this discussion is depicted in Figure 21. Of the five recommendations, four were deemed to be both feasible and desirable.

To further the discussion, we then asked the stakeholders to discuss both benefits and potential barriers to each of these recommendations. The Feasibility and Risks Scale tool (Figure 51, Appendix I) adapted for online implementation was replicated for each recommendation to facilitate discussion, as seen in Figure 22. Each recommendation that was both desirable and feasible was debated in terms of associated benefits and barriers/risks. In general, we observed a pattern, where a recommendation would be desirable, but not feasible because it would not be practical to implement. Key points raised by a stakeholder would then be echoed by some others. At the same time, when prompted to consider how these practical issues could be dealt with, solutions would be put forward or attention to an overlooked detail would be called upon. This then led to a change in the opinion and a realisation of the feasibility as well as the desirability of a recommendation.

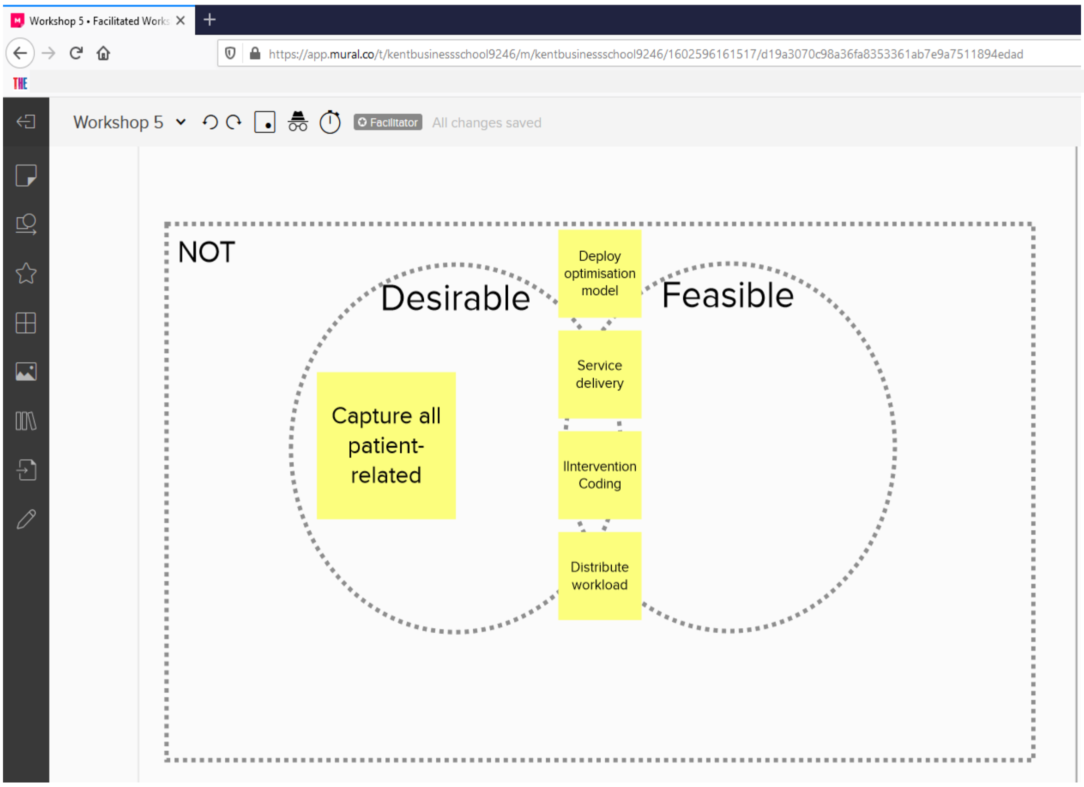

Figure 21: Feasibility and Risks Scale- Output 1

Recommendations 1 and 2 were deemed necessary to improve efficiency, reduce waiting times, improve patient access to care, prevent burnout among clinicians and prepare the service for increasing demand. It emerged through the discussion that both these activities had already been given the green light, where clinicians within the service had been identified to lead these changes. The force-field analysis was useful in identifying barriers that the implementation would need to account for. Given the variety of stakeholders present in the workshop, the benefits and barriers of each recommendation spanned the operational, tactical, strategic and policy levels. As seen in Figure 22, for the standardisation, the benefits outweighed the barriers, and furthermore, and an action trail emerged where collaborators to assist the lead for the implementation were identified.

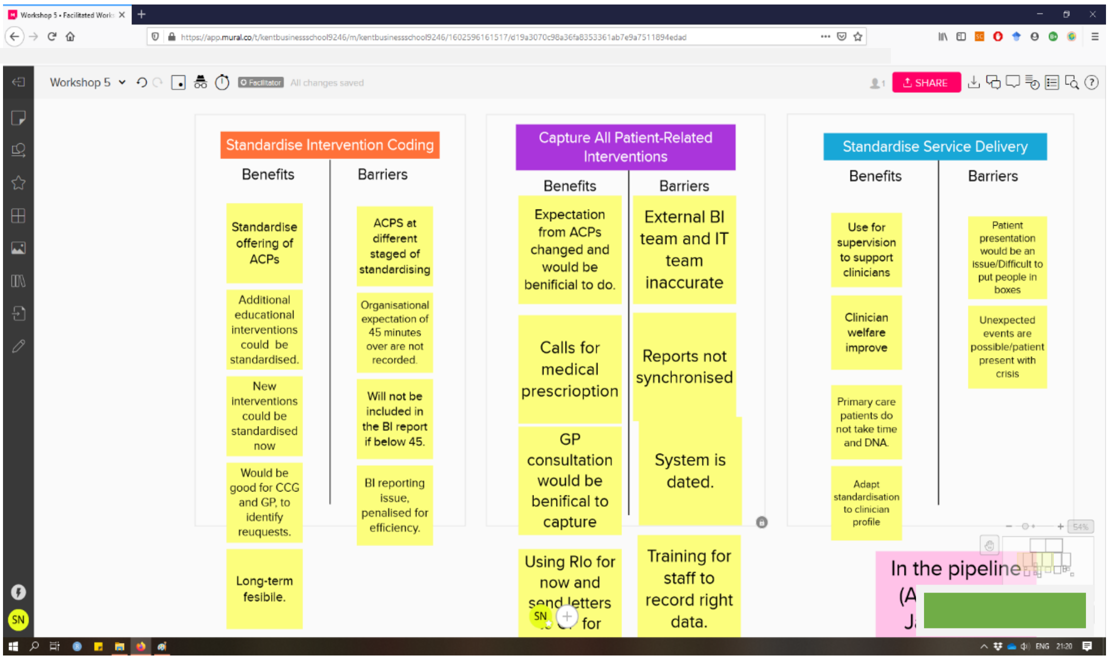

Figure 22: Feasibility and Risks Scale- Output 2

Next, we debated the equitable distribution of workload, as seen in Figure 23. Stakeholders recognised that workload distribution was not included as a decision variable in the model as continuity of care was not to be disturbed. However, in making a case for exploring this aspect of the service, stakeholders admitted that this was an activity that needed to be conducted to reduce the caseload of certain clinicians and identify training opportunities for other clinicians, allowing them to level up. However, the service manager admitted that moving forward, the allocation of new patients and new clinic locations would consider the existing caseload of the team (aided by the new robust reporting). The group also agreed that the model would again be called upon to support this activity. The following discussion is a demonstration of the patten described above:

Stakeholder D: *We know which clinics have a high referral rate and the clinicians that are currently assigned to these. Often these are high deprivation areas, and GPs are not keen on changing the clinicians that hold clinics there. But, because the clinicians were experiencing burn out, we did try to add more clinicians to these locations for screenings, but they would eventually be referred to the main clinicians. Basically, in practice this can cause issues.*

Project Champion: *We can take aspects of the recommendation forward. For clinicians who have allocated clinics that are in proximity, we can redistribute.*

Stakeholder B: *I wonder if the new restructuring and creating of Primary Care Networks, where clusters of GPs are now working together and providing services, does that have an impact or could that improve the distribution and reduce the travel load.*

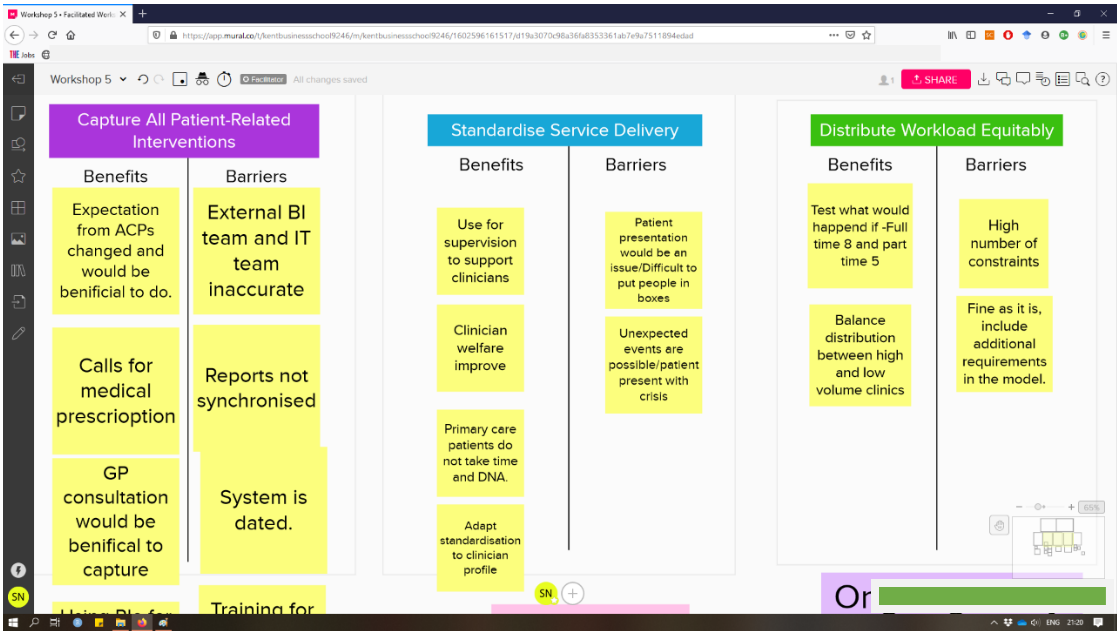

Figure 23: Feasibility and Risks Scale- Output 3 for Recommendations 3 & 4

Stakeholder D: *Yes! I did see that this made a huge positive impact, where I was seeing more patients, and it reduced my travel time.*

Stakeholder J: *Yes. What we have noticed is that some of the GP's have come forward and said that if you want to hold a clinic, because they are in close proximity, you can hold one clinic to see patients form several GPs. So yes, it's good. This is good. Albeit we haven't started seeing the true impact because of the pandemic.*

Stakeholder D: *I am actually looking at this right now. Trying to decide how many clinics a full-time and a part-time clinician should have respectively. Yeah, I am looking at this right now and its based on demand.*

The deployment of the model and the benefits of doing so outweighed the barriers. Stakeholders unanimously agreed of the insights gained throughout the study, and particularly appreciated the ability to experiment with the model that gave them insights into what can be expected from a particular change to the system. The executive members of the team highlighted several other services provided by the trust that could also benefit from the model and the overall framework. The group clearly situated confidence, value, and utility of the model in supporting them make informed decisions. However, the continually evolving nature of mental health services was identified as a barrier. Given the attention mental health is receiving globally and specifically in the UK, the ever-evolving policy environment would likely lead to changes in constraints or open doors to new data collection needs. In any case, it was recognised by the group that a considerable proportion of the groundwork conducted for this study would be transferrable, therefore making the adaptation relatively quicker and smoother. Leads for each of these actionable were re-confirmed and several other participants then volunteered to support the lead reach their goal. The following is an excerpt from the discussion and Figure 24 provides an overview of key themes that were discussed in this activity.

Stakeholder B: *I think it's very useful to have a comparison, as you've put there between the predicted and the actual. It's good to compare the variance of what is actually happening and investigating that.*

Stakeholder D*: It would useful if the team remains stable and if the restructuring settles down. As soon as we think we that it is a bit more stable and we have some clear processes, something new is being introduced.*

Project Champion: *So, I have connected with the director of performance, and I was really inspired by what **Stakeholder B** said, and so I am trying to facilitate wider conversations. And I think the role of this group now is to make a recommendation to the wider trust about the work that this study has conducted.*

Stakeholder J: *I completely agree, and I think with the new way of working having these structures in place will let everyone know what is expected of them and also help provide evidence about what they are doing. Especially when now an evidence base is the norm for any kind of quality improvement.*

Stakeholder G: *Personally, I would like to use the optimisation model with other services that I am working with but I realise this is outside the scope of this work but I would put it down as an aspiration.*

After workshop completion, an action plan was developed as per the Action and Communication Plan tool (Figure 51, Appendix J) and based on the agreed action trail in the workshop. The action plan was shared with stakeholders for approval post workshop.

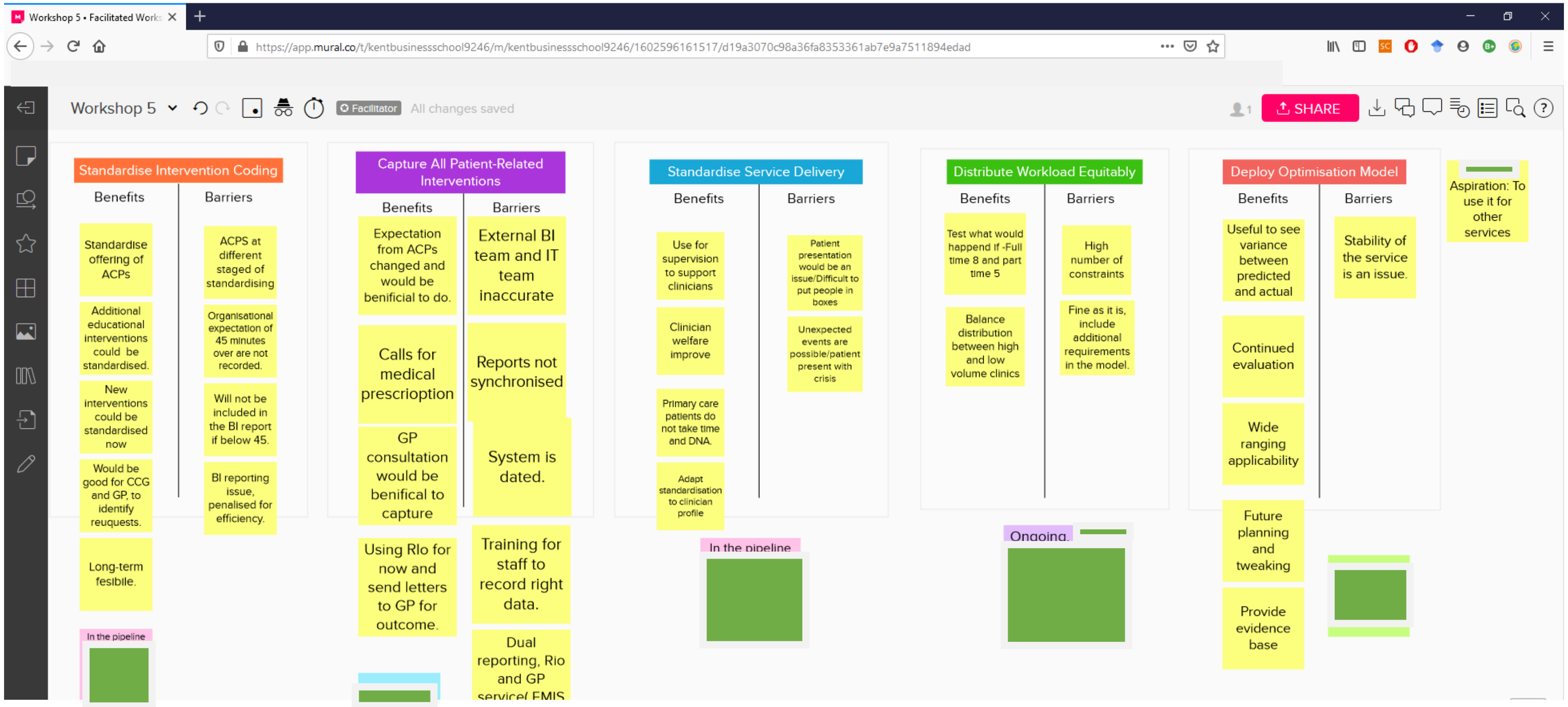

Figure 24: Feasibility and Risks Scale- Output Overview

6.5. Discussion

In this section, we begin by reflecting on the development and application of the facilitated post-modelling stages for optimisation. The framework along with the requisite activities, tools and prescribed outputs is proposed. We also discuss the implications of conducting virtual workshops for these stages, unlike the face-to-face workshops conducted for conceptual modelling (described in Chapter 4).

The case study presents a detailed description of the development of a facilitated framework for the validation, experimentation, and implementation of an optimisation model. In the next few sections, we reflect on each stage of the adaption and propose modification based on what was learnt from the application. Subsequently, we propose the Post-Model Coding stages for facilitated optimisation.

6.5.1. Reflections on Stage 5

Validation of the optimisation was to check whether the proposed model adequately predicts the behaviour of the system under study (Taha, 2017). In the proposed framework this involved checking if all the components of the optimisation model: decision variables, inputs, constraints, objective function, and outputs, were fit-for-purpose. The tools developed to validate each model component came directly from the 'Optimisation Component Map' developed during conceptualisation, described in Chapter 4. Each component and its contents were presented stakeholders to either confirm or suggest changes. During the first experimentation workshop described in Section 6.4.1, it transpired that the debate as well as the updates were concentrated around the model inputs and constraints. Given that constraints represent relationships between decision variables and input parameters, and that they are interchangeable with the objective, it does make logical sense for the two components to take centre stage. Therefore, based on the emphasis on two components, we settled on the "Model Constraints Validation Tool" and "Model Input Parameters Validation Tool" in the proposed framework.

A significant deviation from the PartiSim framework was the requirement of a second model validation workshop. As described in Section 6.4.3, in addition to significance of inputs and constraints, stakeholders identified several additions and changes that were required for the model and its outputs to be more representative of their service. Therefore, round one of model validation activity was conducted in Workshop 3a, followed by a sub-stage where the model was revised and run for new preliminary scenarios. Model validation was completed in Workshop 3b along with the generation of alternative future scenarios. We believe that the two workshops conducted for the Stage 5 were necessary for the successful validation of the optimisation model. However, on reflection, it could be possible to combine the two into a single workshop provided there is sufficient acceptance of model components and solution, and suggested changes do not require additional data collection. We also speculate whether for simulation models, the visual depiction of the model and the relatively faster model update time plays a positive role in quicker model validation. In comparison, an optimisation model requires more time to accommodate updates and perhaps will continue to require two workshops for satisfactory model validation.

The PartiSim framework does suggested the possibility of needing two workshops for model validation, should the situation call for it (Tako & Kotiadis, 2018). We found that once stakeholders fully grasped the model contents in Workshop 3.a, this new understanding gave way to a more perceptive and material discussion about the contents in workshop 3.b. Therefore, this stage could require two cycles of model validation, where the first dedicated round takes place in Workshop 3.a, followed by the second round in Workshop 3.b, which also includes scenario building. Therefore, in the final framework we divide Stage 5 into Stage 5.1, and an optional Stage 5.2. If validation is completed in Stage 5.1, scenario generation could be taken forward. However, if required, Stage 5.1 should entirely focus on model validation and scenario generation should be rescheduled for Stage 5.2, in workshop 3b. The addition of the optional stage also requires the introduction of an optional sub-stage 5.1.a, where the model is revised and material for workshop 3b is prepared. Future research could consider these factors when determining if model validation will occur over two or one workshop. This is likely dependent on the application area, the complexity of the model and the availability of data. We also note that in our case study, inputs and constraints were the model components that required updates during the validation. However, in other applications, it might be that the objective and decision variables will also need to be updated. Further studies are required to determine the need for tools to validate these specific components of an optimisation model.

In workshop 3b, the updated model is re-validated and validation procedure completed, leading to the scenario generation activity. Recent advances in healthcare optimisation are highlighting the relevance of scenario building with the involvement of experts that better know their reality (Amorim-Lopes et al., 2021). Our study contributes a structured process of involving stakeholders through a facilitated workshop in scenario-building for an optimisation model. Reflecting on the application to the case study, the scenario building process gave stakeholders the opportunity to imagine different futures for the service. In considering which of the inputs or service specification they would like to change in the future, the stakeholders grappled with evaluating several aspects of the service that needed changing for the better but could not be changed because of clinical priorities. This revealed to us that what might be operationally sound does not necessarily translate to clinically sound. Without the involvement of stakeholders, such rich perspectives would not otherwise be captured.

In terms of tools, we recognise that the "Scenario Parameter Combination Form" should have been preceded by another activity to narrow the scope of input parameters that would be taken forward for the scenarios. In other words, an equivalent to PartiSim's 'Rate the Performance Measures' tool (Figure 48, Appendix I) should have been prepared for narrowing the scope of input combinations to be considered for scenario generation. Specifically, in the workshop, stakeholders were presented with the form that included all inputs and were asked to consider each of these as an option for scenario analysis (Figure 56, Appendix K), while also putting forward potential values of the input. This activity could have been separated where first stakeholders are given a list of all inputs and asked to choose those they would like to experiment with. The "Scenario Parameter Combination Form" would then include the chosen inputs making its design much more streamlined, like that seen in Table 36. The form used in the workshop (Figure 56, Appendix K), was formatted after workshop 3b, to produce Table 36. We believe that an additional tool proposed here would be an improvement to the framework and conducive for stakeholder engagement. The proposed 'Scenario Input Selection' tool would be populated with the model's input values as presented in Table 48 (Appendix M). Stakeholders would then be asked to use symbols to indicate their choices. Specifically, ✖ would represent input variable that will not be chosen, while variable with the symbol ✓ would be chosen and those with the symbol * would be inputs that stakeholders recognised as impacting service performance but will not be chosen for experimentation for reasons unique to the system under consideration. In our case, these are variables that cannot be changed in favour of continuity of care for service users. Table 48 (Appendix M) is an example of how this tool might have been used in workshop 3b.

6.5.2. Reflections on Stage 6

In the implementation stage, the main challenge was adapting the prescribed activities and paper based tools for online implementation. As described in Section 6.4.4, the Feasibility and Risks Scale tool (Figure 51, Appendix I) from PartiSim was explored and adapted for online implementation, while the Barriers to Change tool (Figure 50, Appendix J)) was not explored for online implementation. This was done for several reasons. We determined that stakeholders would need to be provided with sufficient time to become familiar with the new environment and it would be unwise to overwhelm the team with multiple tools. Therefore, we reasoned that we would be better positioned to focus on adapting a single tool effectively rather than attempting to use both. Furthermore, we had concerns about how the recommendations would be received, given the implementation challenges highlighted in optimisation literature and the factors affecting implementation discussed in simulation literature. Therefore, to exercise caution, the Feasibility and Risks Scale tool was adapted and modified for online implementation.

This adaptation was driven by the move to a virtual environment and had this workshop taken place in an in-person setting, the Barriers to Change tool (Figure 50, Appendix J) would have been employed. Given the utility of other tools and activities that are prescribed in Stage 6 of PartiSim, most of which have been demonstrated in the adaptation, this specific tool is included in the proposed framework for future consideration. Additionally, the online adaptation of the Feasibility and Risks Scale tool offers an alternative to the paper-based tool and is a key contribution.

6.5.3. Reflections on Virtual Workshops

This study ended up having a hybrid workshop approach, more as a situational adaptation and less as an active choice. The first two workshop in this study were conducted face-to-face, in conjunction with site-visits, interviews, and informal conversations leading up to the study and in the initiation stage. We believe that these in-person interactions were crucial to instilling confidence, trust and secure a buy-in with the stakeholder group for the overall aims and objectives of the study. Additionally, the transition to the virtual environment was successful and although challenging, it was without a significant concern for a continuity in participation and interest of the stakeholders. Without this foundation, we believe ‘games’ as described by Tako & Kotiadis (2021) would need to be undertaken diligently to build the rapport and trust between the modelling team and the stakeholder group and within the stakeholder group.

We believe the advantages of moving to a virtual environment such as the elimination of travel and improved access and participation were reflected in our work presented here (Hofstädter-Thalmann et al., 2022). Specifically, with healthcare professionals, finding the space and more importantly a time that works for everyone is a significant challenge. Moreover, in our specific study, clinicians within the service were often at different locations, and these were challenges that led to longer durations between workshops in the conceptual modelling stages. However, we observed consistency in attendance, and sometimes even an increase in the number of participants during virtual workshops (Table 32). Equally, the duration between each meeting was relatively shorter. We agree that this is a descriptive analysis based on observation and perceptions and some empirical evidence (the duration between workshops, and to some degree the successful completion of all 5 workshops). As for the disadvantages, we can confirm that we experienced technical glitches such as bad network connection, inability to participate in third-party tools such as Mural (because of restrictions in the NHS firewall on stakeholder's computers). As suggested by Tako & Kotiadis (2021), we overcame these by taking over the note taking, while sharing the screen. These considerations were discussed prior to the workshops and contingency plans were agreed upon.

In workshops 3a and 3b, we used Google Docs for conducting our brainstorming sessions, and at the time, it served the purpose for collaboration as it allows for concurrent working on the same document. However, we subsequently found this platform to be insufficient as it lacked utilities for creating imaginative tools and the visualisations. A finding similar to that of adapting PartiSim to a virtual environment (2021). Therefore, for the implementation workshop, we used Mural. In fact, we found that Mural's features allowed us to create visually engaging templates that contained several tools on a single page. The ability to zoom in and out of each tool led to the creation of a mosaic that fit the chronology of activities. This utility was particularly useful in the implementation workshop which required more creative approaches to engaging stakeholders in a debate. In contrast, we did not find Google Docs lacking in workshops 3a and 3b, as the goal was more focused on validating the and building scenarios. In other words, the activities we investigative were procedural and did not require visual aids imagining the impacts of potential changes.

6.5.4. Towards Facilitated Post-Model Coding

In this chapter, we provide proof that it is possible to validate, build scenarios, and consider model implementation with stakeholders by following a structured facilitated approach. The proposed framework is adapted from PartiSim and modified for optimisation modelling by testing it in a real case study. The proposed framework including activities, tools and outputs are depicted in Table 37.

The activities highlighted in the table are new additions, while others are directly drawn from the PartiSim framework. Collectively, experimentation, and implementation are termed post-model coding, as these activities take place after a model has been developed (Kotiadis & Tako, 2018). In sub-stage 4.a, the new activity is the development of the baseline scenario that draws from the analytics elements described in Chapter 5. These aspects of the integrated optimisation model played a crucial role in this study in not only aiding the validation of the model, but also in building scenarios. That being said, the framework is focused on optimisation modelling, and should future studies not require such analytics, this can be an optional activity. However, with the recent push for data driven modelling and validation, particularly in a practice-based study, we believe the inclusion of this activity can be beneficial to both stakeholder engagement by providing a direction for scenario analysis and to making the optimisation model more realistic.

The main stages are 5.1 and the optional stage 5.2 for Experimentation along with stage 6. Implementation. The sub-stages either prepare for workshops or handle outputs following a workshop. Sub-stage 5.1.a is to be undertaken only if additional model validation is required, in the lead up to Stage 5.2 where model validation is completed.

We recognise that several aspects of validation were captured in the application, including operational, experimental, and data. Recall that operational validation is associated with the quality and comparability of the model's outputs to real world data (Gass, 1983; Landry et al., 1983; Sargent, 2020). In workshop 3a, following the walkthrough of the model components and the output, it was plain that for stakeholders, the operational validity was not at an acceptable level. In our case, this led to updates in the conceptual model and reformulation of the optimisation model. Closely linked is the experimental validation which in our case was not entirely in alignment with the stakeholder's perception of reality (Alkaabneh & Diabat, 2023; Kim & Mehrotra, 2015). In examining the data that was being supplied to the model as inputs, concerns relating to completeness, correctness and appropriateness of data emerged, particularly about clinician availability data were explored and additional data collection needs were identified. Taking note of these concerns, the scenario generation activity was abandoned in favour of incorporating the updates and data collection needs that were raised during validation. Since the model was being built for a real problem situation, these concerns warranted the attention they were given, to increase stakeholder confidence in the outputs. In healthcare, model building in isolation, with opaque model design approaches is associated with stakeholder resistance to trust outcomes and implement findings (Carter & Busby, 2022). The transparent validation approach described in this chapter negotiates and addresses these concerns. Furthermore, the incorporation of new perspectives from stakeholders enforced a reformulation and enhanced model specifications (Amorim-Lopes et al., 2021).

Table 37: Facilitated Post-Model Coding for Optimisation, adapted from PartiSim

Stage & Activities	Activities	Tools	Outputs
4.a Pre-workshop 3 sub-stage *<u>Purpose:</u>* Preparations for workshop 3a	Prepare preliminary materials for use in workshop 3a (stage 5): - Liaise with the project champion over correctness of model and its results (modeller and project champion) - Review preliminary scenarios with project champion **-Baseline scenario to depict current system performance (drawing from historical data analysis)** **-Future scenarios including uncertain inputs (generated using predictive analysis)** - Prepare preliminary materials for use in the next workshop		- Model validation and verification - Preliminary future scenarios
5.1 Experimentation stage ***(workshop 3a)*** ***<u>Purpose:</u>*** **Validate optimisation model & examine model solution**	Stakeholders are invited to: **- Validate the optimisation model components and its output** **- Debate constraints & input parameters**	- **Model constraints validation tool** - **Model input parameters validation tool**	- Model validation and verification
	- Choose model inputs for scenario building **- Identify input parameter combinations for scenario analysis**	- **Model input selection form** - **Scenario parameter combination form**	- Alternative future scenarios
(If required) **5.1.a Post-workshop 3a/Pre-workshop 3b sub-stage** ***<u>Purpose:</u>***	- Tweak or correct the **optimisation model** - **Data collection (if required)** - Implement additional scenarios suggested (based on stakeholder feedback from workshop 3a.)		- **Revised optimisation model**

Preparations for workshop 3b	- Liaise with the stakeholder team over correctness of model results - Prepare preliminary materials for use in workshop 3b		- **New preliminary future scenarios**
(If required) **5.2 Experimentation stage (Workshop 3b)** ***Purpose:*** **Complete validation & define alternative scenario to experiment with model**	 - **Validate Updated Model** **- Choose model inputs for scenario building** **- Identify input parameter combinations for scenario analysis**	- **Model input selection form** - **Scenario parameter combination form**	- Updated model validation and verification - Alternative future scenarios
5.2.a Post-workshop 3b/Pre-workshop 4 sub-stage *Purpose:* Refine alternative scenarios and prepare for workshop 4	Modelling team: - Tweak or correct **optimisation model** -Implement additional scenarios suggested (based on stakeholder feedback from workshop 3b.) - Liaise with the stakeholder team over correctness of model results - Prepare preliminary materials for use in workshop 4		- New alternative future scenarios - Revised optimisation model
6. Implementation stage (workshop 4) *Purpose:* Define an implementation plan	Stakeholders are invited to: - Review learning and changes implemented - Risk analysis and feasibility of change - Agree action trail	- Script for identifying changes in the system - Feasibility and Risks Scale tool with manual - Barriers to Change tool with manual - Action and Communication Plan tool with manual	- Feasible & desirable scenario(s) to be taken forward - Action plan with deliverables (including due date and person responsible)

Experimental, and operational validation are routinely conducted in optimisation literature as described in Section 6.2 (Humagain et al., 2020; Zamanifar & Hartmann, 2020). However, our case study is an example of how reconceptualisation and reformulation of the model is conducted when the model is conceptualised and built for a real-world problem situation. Additionally, deviating from majority of validation procedures in healthcare optimisation literature, computational efficiency, and the model's ability to improve the desired performance measures were not the most prioritised concerns of the validation process (Harris & Claudio, 2022; Samudra et al., 2016). Instead, the focus was on building a model that represented the PCMH service to an acceptable degree, to instil confidence in the model and its results. These types of validations are largely explicitly conducted in simulation modelling.

Many recommendations derived by the model and the study were agreed to be taken forward by the stakeholder group. Admittedly, the model was not implemented according to the traditional definition that involves the implementation of well-known mathematical algorithms or the development of a prototype (Ahmadi-Javid et al., 2017; Humagain et al., 2020). However, it does add to the growing literature of optimisation models utilising real data for testing (Samudra et al., 2016). A major factor contributing to the lack of sustained model application or integration of the analytics framework into the service was the lack of analytics capabilities within the service, as described in Chapter 5. Additionally, the IT systems did not have the ability to integrate the model into their overall system, as these were propriety, and accommodating new features would require significant redevelopment of a system that is not just used by the PCMH service, but by all the services within the trust. Add to that, we the modelling team did not have the expertise or the funding to develop a bespoke product that could be implemented without integrating into the existing IT system. Moreover, the project was conceived by stakeholders primarily to seek insights to support decision making. This is similar to a recent study that utilised participative approaches to develop an optimisation model (Abuabara et al., 2022). In examining these aspects of the implementation, this study contributes to healthcare optimisation literature by providing information on factors that intersect with actual implementation (Samudra et al., 2016; Zhu et al., 2019).

We argue that this study's implementation success is more appropriately viewed through the increasingly flexible definition from simulation modelling which values the process and context of the study, as much as tangible and measurable outcomes (Eldabi, 2009; Long et al., 2020; Pitt et al., 2008). We also assert that the collaborative model-building led to critical learning incidents for the stakeholders (Thompson et al., 2016). This study argues for optimisation literature to reframe what constitutes success for an intervention and to draw from the evolving research on implementation from simulation modelling literature.

6.5.5. Future research

In our case study, the optimisation model only has one objective. However, in many real-world problems, there is more than one objective. Even with two objectives, it is possible to have a multitude of Pareto optimal solutions. The proposed framework does not consider the possibility of engaging stakeholders when dealing with multiple-objectives or even when dealing with uncertainty. It is likely that new tools will have to be developed to help stakeholders identify the most desirable and feasible solution and is a key area of future research. Facilitated decision analysis could offer solutions for addressing these challenges. In particular, facilitated decision analysis includes a set of methods that aid modelling decision involving multiple objectives and/or uncertainty in. These include Multiple Criteria Decision Analysis (MCDA) methods, that are used to quantify benefits, risks, and uncertainties by explicitly considering criteria's along with their relative importance through a transparent process that incorporates stakeholder views outcomes (Durbach & Stewart, 2012; Tzeng & Huang, 2011). One of the main aims of MCDA methods is to enable decision-makers to reach a decision by facilitating an understanding of the problem, objectives, associated values, by organising and synthesising information that is complex and conflicting in nature (Belton & Stewart, 2002; Phillips & Bana e Costa, 2007). These frameworks have been successfully applied to solve decision problems in many areas, including energy management (Kumar et al., 2017), Transportation (Pamucar et al., 2021), budgeting and resource allocation (Zavadskas & Turskis, 2011) and health technology assessment (Oliveira et al., 2019), and healthcare commissioning decisions (Marsh et al., 2016).

For post-model coding, Kotiadis and Tako (2018) have utilised MCDA at an exploratory level to get an understanding of the performance of scenarios by utilising an existing MCDA computer software called VISA. The activity utilising The Rating Performance Measures tool was guided by MCDA tool to represent model results (performance measures) with stakeholders attaching corresponding weights to each one. More recently, in optimisation literature, Abuabara (2022) develop a novel participatory framework for a meal planning problem. In this study, the Rich Picture tool is deployed to survey the problem and build a linear programming model that maximised diet preferences. The model is provided with inputs from an MCDA tool, using where stakeholders rank the various meal alternatives. Clearly there is scope for MCDA methods to be utilised in situations where there are competing objectives and/or uncertainty in the system being modelled. For the proposed framework, the potential incorporation of MCDA tools is relevant to the post-model coding stages and is a key area of future research.

6.6. Conclusion

This study presents a facilitative framework for stakeholder participation, focusing on the validation, experimentation, and implementation of a mathematical optimisation model. The proposed approach was derived by applying it to a real case study in mental healthcare and subsequently modifying it by incorporating improvements. The case study illustrates how stakeholders engage and interact with the optimisation model after it has been coded. By conducting model validation in a facilitated setting, we demonstrate how the incorporation of stakeholder perspectives can enforce reformulation and enhance the model's specification. Furthermore, by describing the process of validation the study highlights how model validation is linked to stakeholder's perception of model usefulness. When considering model implementation, the study contributes to healthcare optimisation literature by providing information on factors that intersect with actual implementation. We argue that implementation success is more appropriately viewed through the increasingly flexible definition from simulation modelling which values the process and context as much as more tangible and measurable outcomes. We identify areas where the proposed framework can be improved in the path towards facilitated optimisation. We encourage researchers to conduct further investigations utilising real case studies to explore and expand on the advantages of applying participative approaches for optimisation model validation and implementation.

Chapter 7: Summary, Future Research and Conclusion

7.1. Introduction

The facilitated optimisation modelling framework, adapted from PartiSim, has been developed and evaluated using a case study in mental healthcare. This chapter reflects upon the adaptations and discusses the knowledge gained from its application with a view to discuss limitations and future research directions. In Section 7.1.1, the aims and objectives of this thesis are revisited. In Section 7.1.2, contributions of this research are presented. In Section 0, reflections on the adaptation and proposed framework and future research directions are presented in 7.1.4.

7.1.1. Aims and Objectives Revisited

As outlined in Chapter 1, this research intended to introduce a structured framework that can aid optimisation modellers to consider the key steps in the modelling lifecycle and involve stakeholders in the model development process. The aims of this research are: 1) To develop a comprehensive understanding of literature on the application of Operations Research (OR) methodologies in mental healthcare, with the objective of adapting and extending the PartiSim multi-methodology framework for optimisation modelling; and 2) To develop a facilitative multi-methodology framework that provides a pathway for developing optimisation models with stakeholder participation, particularly in the context of mental healthcare. Figure 25 depicts the structure of the thesis and a mapping of research objectives to thesis chapters.

Chapters 2 and 3 present a critical review of the application of DES and optimisation in mental healthcare. Both reviews helped in gaining an in-depth understanding of the mental healthcare landscape in the UK, which also shares several similarities with mental healthcare delivery systems in OECD (Organization for Economic Cooperation and Development) countries. In chapter 2, through the examination of the application of DES to mental healthcare, it is found that although DES is a popular tool for operations planning in healthcare, application to mental healthcare is limited. In chapter 3, through the meta-review, it is found that optimisation techniques have a rich history of application in healthcare to tackle a wide range of issues, including planning, scheduling, routing, and supply chain management in various healthcare settings. However, despite these advances, in mental healthcare, application of optimisation is found to be in its nascent stages. Several opportunities for transferability are highlighted.

Chapter 1: Introduction

Chapter 2: Application of Discrete-Event simulation for planning and operations issues in mental healthcare.

Chapter 3: Mind the gap: a review of optimisation in mental healthcare service delivery.

> **Objective 1:** Investigated existing knowledge on the mental healthcare services and identified characteristic of the system that are relevant to the study. (In relation to aim 1).
>
> **Objective 2:** Investigated the application of simulation and optimisation modelling techniques in mental healthcare and identified gaps and opportunities. (In relation to aim 1).

Chapter 4: Facilitative conceptual model development for mathematical optimisation: A case study in mental healthcare

> **Objective 3:** Iteratively adapted PartiSim for optimisation modelling through a case study in mental healthcare services. (In relation to aim 2).
>
> **Objective 5:** Proposed a Participative Optimisation (PartiOpt) multi-methodology framework for developing optimisation models (In relation to aim 2).

Chapter 5: An integrated optimisation and analytics approach for planning mental healthcare services.

> **Objective 4:** Investigated and identified OR/analytics techniques to support the development of an optimisation model for mental healthcare service provision and embedded within the overarching participative and facilitative multi-methodology framework. (In relation to aim 2).
>
> **Objective 5:** Proposed a Participative Optimisation (PartiOpt) multi-methodology framework for developing optimisation models (In relation to aim 2).

Chapter 6: How do stakeholders interact with optimisation models? A case study in mental healthcare

> **Objective 3:** Iteratively adapted PartiSim for optimisation modelling through a case study in mental healthcare services. (In relation to aim 2).
>
> **Objective 5:** Proposed a Participative Optimisation (PartiOpt) multi-methodology framework for developing optimisation models (In relation to aim 2).

Chapter 7: Summary and Conclusion

Figure 25: Chapters that meet the objectives of this thesis.

The exploration conducted within these two chapters provided providing an appreciation of the context of mental healthcare from a dual perspective of service provider and service user. The characteristics and salient features of service users and the care model they access helped frame the case study. This knowledge allowed the researcher to ask directional questions during initial problem exploration. The literature review helped recognise the collective influence of the social, clinical, political, and public health factors on the mental healthcare system. It became clear that these contextual factors would guide the modelling process and direct the outcomes of this study.

From a methodological perspective, the literature review highlighted the differences and similarities in the modelling lifecycles between optimisation and simulation. Optimisation literature is predominantly focused on model formulation and application of solution algorithms, while simulation modelling emphasises the entire modelling lifecycle starting from conceptual modelling and ending with implementation. It was recognised that these factors will have to be considered in the adaptation of PartiSim to optimisation.

Chapter 4 contributes to the realisation of objective 3 and 5. Here, the CM stages of the PartiSim framework are adapted for problem exploration and conceptualisation of an optimisation model. Through this adaptation Stages 1, 2 and 3 of the PartiOpt multi-methodology framework, are proposed, thereby contributing to Objective 5.

Chapter 5 presents the modelling approach that is employed for the case study, following model conceptualisation. Based on decision-support requirements gathered during the conceptualisation, descriptive and prescriptive analytics techniques are integrated to enhance the optimisation model. The conceptualisation also led to the development of a novel multi-skill multi-location optimisation model whose formulation is very specific to the problem put forward by the stakeholders. The analytics-driven optimisation modelling approach draws from the conceptualisation stages and influences the post-model coding stages of the proposed PartiOpt framework. Therefore, this chapter contributes to the realisation of objectives 4 and 5.

Chapter 6 contributes to the realisation of Objective 5 by considering the adaption of Stages 5 and 6 (post-model coding) of PartiSim to propose components of the PartiOpt framework. In particular, the validation, experimentation and implementation stages are tested through the case study and at the end of chapter 6, the remaining stages of PartiOpt are proposed, closing the loop on Objective 3 & 5.

7.1.2. Contributions of this research

7.1.2.1. Review of literature

Reviews in Chapters 2 and 3 highlight that people with mental health disorders have diverse and changing needs throughout the course of their illness. These needs are primarily addressed through community-based services in local settings in co-ordination with primary care, specialist care, social care, voluntary services, emergency services, education, housing, and the justice system. The mental healthcare system involves multiple stakeholders and interconnected components, resulting in complex interactions. Furthermore, there is a lack of uniformity in the delivery of services, with no single model of care available. Patients face various system-wide challenges including long waiting times, insufficient integration among services, bed shortages, and inadequate service provision. Inadequate funding, high workload pressure on mental health workers, and understaffing are barriers to adequate care provision. Given the escalating healthcare costs and persistent prevalence of mental health disorders, these challenges contribute to the urgency of making comprehensive decisions in service delivery and robust resource allocation.

In Chapter 3, the contribution is a systematic review to determine the extent to which studies have used DES within mental healthcare. This chapter builds on an existing review and contributes additional insights and a tailored roadmap for the future application of DES for planning and operations issues in Mental Healthcare (MH). In this chapter, several similarities are found between simulation application in social care, stroke care systems, and long-term care. However, the distinctive makeup of mental healthcare services necessitates further investigation for direct application of previous research. Opportunities for the combined use of simulation and optimisation are highlighted. In particular, simulation could be deployed to track system variability and scarce resources, while optimisation can be used to provide optimal resource configurations that best improve performance.

In Chapter 3, the contribution is threefold. First, a comprehensive overview of the application of optimisation in healthcare so far is provided. Through this, gaps in existing optimisation literature are highlighted and future research directions are examined. Second, the context of mental healthcare services is analysed to identify unique features and the consideration of similar features in healthcare literature is investigated. The primary contribution derived from results of the scoping review on the application of optimisation techniques in mental healthcare services, is the identification of issues for researchers to analyse, study and model.

Through the meta-review, it is found that optimisation techniques have a rich history of application in healthcare to tackle a wide range of issues, including planning, scheduling, routing, and supply chain management in various healthcare settings. These settings encompass operating rooms, outpatient systems, physician and nurse scheduling, home health care, emergency response planning, healthcare facility location, material logistics, inventory management, and surgical care planning.

However, despite these advances, in mental healthcare application of optimisation is found to be in its nascent stages. Existing research does not capture realistic model assumptions, variability in care models, and integrated services. Through the comparative analysis, several opportunities for transferability from existing healthcare literature and expansion of optimisation in mental healthcare are located.

7.1.2.2. Conceptual Modelling for Optimisation

Chapter 4 considers how a group of stakeholders can be involved in the conceptualisation of an optimisation model. Through the case study, an evaluation of how and if the conceptual modelling elements of the PartiSim framework can be adapted to optimisation is conducted. This chapter contributes a formalised facilitated modelling approaches for optimisation. To the best of the researcher's knowledge, the documentation or formalisation of a facilitated modelling approach for optimisation is not found in the scientific literature. Furthermore, the study complements existing research efforts in different ways. First, it investigates the adaptation of CM element of PartiSim to conceptualise an optimisation model. Second, it contributes to the understanding of facilitated modelling for optimisation. Third, it expands multimethodology literature by combining SSM with optimisation modelling, and last, it contributes a case study in mental healthcare.

The conceptualisation framework proposed in Chapter 4 is suitable for application in situations where a pragmatic and quick conceptualisation of an optimisation model is required, irrespective of problem type. The framework emphasises the practicalities of modelling new real and complex problems where conceptualisation is likely a necessity. Additionally, it is demonstrated through the case study that researchers can also use the facilitative and participative approach to develop a conceptual model and identify an existing problem in literature that can be adapted to fit the conceptualisation.

7.1.2.3. Analytics-driven optimisation modelling approach

Chapter 5 makes three contributions. First, a novel multi-skill multi-location optimisation model that schedules itinerant mental health clinicians to multiple geographical locations across a planning horizon is presented. Second, the model is applied to a real case study, adding to the limited pool of optimisation literature applied to mental healthcare. Third, an integrated three-stage optimisation-based analytics approach that combines descriptive, predictive, and prescriptive analytics, is proposed. Furthermore, the chapter highlights the utility of the analytics approach, demonstrated through the case study, which has the scope to be extended to other healthcare contexts that share similarities to mental healthcare.

From a PartiOpt perspective, the work presented in this chapter bridges the pre-model coding to the post-model coding stages. Specifically, outputs of the pre-model coding stage informed the model coding in three distinct ways: 1) To build a realistic optimisation model with the potential for implementation, the modelling process would require the combined use of multiple analytics tools; 2) Following the development of the conceptual model, a literature search led to the realisation that the resultant components of the formal model can be mapped to an existing type of optimisation model, namely a multi-skill multi-location model. Although the components were mapped to a specific model type, the model itself is novel and the formulation is very specific to the problem put forward by the stakeholders; 3) The outputs from pre-model coding laid the foundations for a scenario analysis that is subsequently conducted. Therefore, the direction for model coding is determined because of the preceding stages of the PartiOpt framework.

Additionally, the modelling activities within this chapter were executed without stakeholder participation. However, stakeholders were active participants in the model validation and experimentation process, described in Chapter 6. The flowchart in Figure 26 depicts the generalised process of how the coding of an optimisation model is embedded into the overarching framework and how the analytics option can be a choice informed from the pre-model coding stages. The flowchart also depicts how model validation and experimentation stages draw from the model coding.

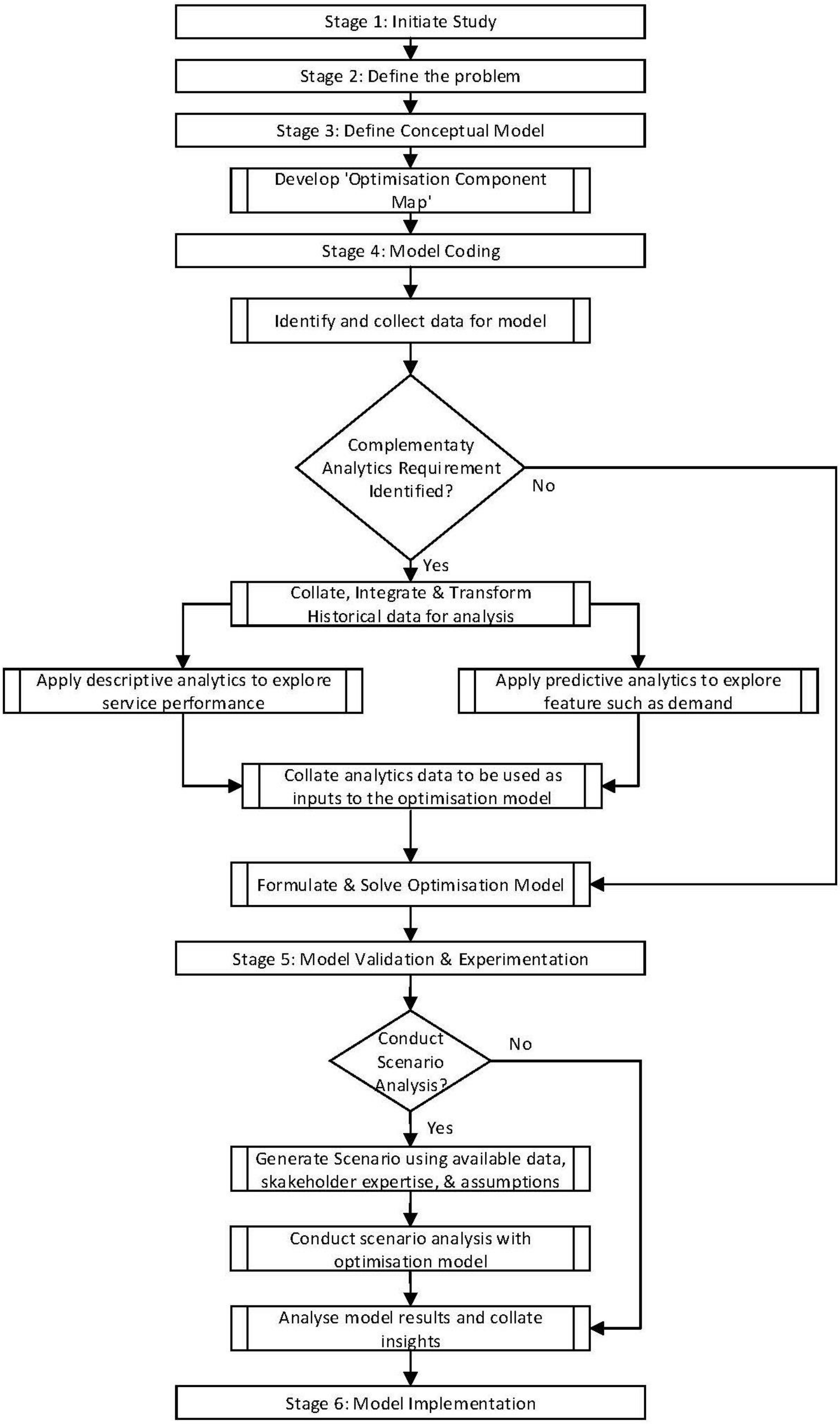

Figure 26: Model Coding Flowchart Linking Pre- and Post-Model Coding Stages

7.1.2.4. Facilitated post-modelling for optimisation.

This chapter contributes to the limited pool of studies that utilise Soft OR and participatory approaches in optimisation modelling. By adapting PartiSim, the study utilises existing work to explore opportunities offered by facilitation for involving stakeholders in optimisation modelling. The contributions are focused on the post-model coding stages of the PartiSim framework. Through the case study, the chapter presents evidence demonstrating the feasibility of validating, constructing scenarios, and considering model implementation in collaboration with stakeholders by following a structured facilitated approach. The chapter also offers insights into real-world factors that impact the actual implementation of optimisation models and emphasises how stakeholder engagement through workshops fosters acceptance and support for the model's recommendations. The chapter also contributes a case study where facilitated workshops were conducted in a virtual setting. The researcher argues for implementation success to viewed through the increasingly flexible definition from simulation modelling which values the process and context as much as measurable and tangible outcomes.

7.1.2.5. PartiOpt Multi-methodology framework

The proposed facilitated optimisation modelling framework adapted from PartiSim is presented in Table 38. Modifications and additions within the framework are presented in bold letters, specifically in Stages 3, 4, 5 and 6. Stages 1 and 2 were not modified and were used to initiate the study and define the system under consideration. Adaptations to Stage 3 were driven by the need to tailor tools to accommodate differences between the simulation and optimisation modelling components. The first new tool is the "Inputs Form", which captures all known specifications of the system under consideration, that were within the defined scope of the study and identifies the source of the information provided. Similarly, the "Constraint Form" uses the CATWOE output to draw a connection between the real system and the definition of constraints for the model. The "Optimisation Component Map" is an ad-hoc communicative model depicting the conceptualisation of the model components using descriptive text.

Model coding is an activity conducted by modellers independently of the stakeholders. Modifications to Stage 4 were informed by the results of the pre-model coding. Specifically, in addition to mapping the "Optimisation Component Map" to a formal optimisation model, additional requirements emerged from the pre-model coding stage. Therefore, the framework is structured to incorporate these analytics requirements, should the need arise.

Equally, the framework is easily modified to fit an intervention that only requires the development of an optimisation model, as depicted in the Flowchart in Figure 26.

Stage 5 is divided into two activities that are to be conducted over two successive workshops. In workshop 3a, it is identified that additional model input parameters would be required, and updates to the model were needed. Therefore, this stage could require two cycles of model validation, where the first dedicated round takes place in Workshop 3.a, followed by the second round in Workshop 3.b, which also includes scenario building. To support model validation, “Model Constraints validation tool” and “Model Input Parameters validation tool” are introduced. Similarly, for scenario generation, two new tools are introduced, namely “Model input selection form” and “Scenario parameter combination form”. In stage 6, the Feasibility and Risks Scale tool was adapted and modified for online implementation, while the Barriers to Change tool was not explored, but has been included in the proposed framework for future consideration.

7.1.3. Reflections on the adaptation and proposed PartiOpt Framework

In optimisation modelling literature, there is limited research on the modelling process and facilitation. This was challenging to navigate particularly given the nature of this thesis. PartiSim is positioned within simulation modelling, where there is ample literature on both the process and facilitation. In adapting PartiSim to optimisation, comparisons had to be made between conceptual modelling, model structure and components, model validation and experimentation, and model implementation. Information corresponding to each category was easily obtained for simulation while the same was not true for optimisation. These comparisons are most evident in Chapters 4 and 6. Information on modelling stages, besides model formulation and solution algorithm development, was mostly secured through older publications that were focused on gathering and framing the theoretical foundations of OR techniques including optimisation. Using these ideas, current optimisation literature was inspected for patterns that were echoes of these foundations or to identify current patterns of practice, which did and did not necessarily provide information on the process of model building. Additionally, the use of participative methods to engage with stakeholders is an important advancement in optimisation modelling literature. However, there is limited knowledge on facilitating stakeholder engagement for optimisation modelling. The thesis draws heavily on existing research on modelling processes and facilitation in simulation modelling, while also shedding some light into optimisation modelling processes and highlights the potential for facilitation.

Table 38: Proposed PartiOpt Framework

Stage and Purpose	***Activities***	***Tools***	***Outputs***
1. Initiate Study <u>*Purpose:*</u> Identify stakeholder team & key problem situation(s)	The modelling team undertake: - informal meetings and/or - on-site observations and/or - with project champion and key stakeholder(s), to address preliminary information needs.	Information Collection Tool	- Preliminary understanding of the problem situation.
1a. Pre-workshop stage <u>*Purpose*</u>: Preparations for Workshop 1	Modelling team prepare preliminary materials for tools to be used in workshop 1		
2. Define system (Workshop 1) <u>*Purpose*</u>: Agree on the problem situation and the wider system, within which it exists.	Participating stakeholders take part in a facilitated workshop process to: - Brainstorming problem area (s) to be addressed and identify study objectives - Define system boundaries	- Problem statement form - CATWOE and root definition - Care system model	- General study objective(s) - A bounded system within which the problem to be addressed exists
2a. Post-workshop1/Pre-workshop 2 stage <u>*Purpose*</u>: Disseminate workshop 1 outputs and prepare workshop 2	- Modelling team re-draw tools and disseminate workshop outputs to stakeholders - Prepare preliminary materials for tools used in workshop 2		
3. Specify conceptual model (Workshop 2) <u>*Purpose*</u>: Define specific elements of the conceptual model	Participating stakeholders take part in a facilitated workshop process to: - Put forward and agree on performance measures to address the problem identified in workshop 1 - **Identify inputs and decision variables of the model** - **Define the model constraints and objectives** - **Produce optimisation model component mapping** - Discuss responsibility for data collection.	- Performance measurement model (PMM) - **Inputs form** - **Constraints form** - **Optimisation Component Map**	- **Model inputs and decision variables** - **Model objectives and constraints** - A preliminary list of assumptions and simplifications - **A communicative model** - A list of data requirements

3a. Post-workshop 2 stage *Purpose*: Disseminate workshop 2 outputs and refine conceptual model	Modelling team prepare report detailing: - Refined CM outputs from stage 2.a and stage 3 - Data requirements		An agreeable to all (study participants) and feasible conceptual model describing an optimisation model
4. Model coding	- Data collection (modeller & stakeholder) - **Data analysis: descriptive and predictive (modeller)** - **Formulate, code, and solve optimisation model**		- Optimisation Model Solution - Model Results
4.a Pre-workshop 3a sub-stage *Purpose:* Preparations for workshop 3a	Prepare preliminary materials for use in workshop 3a - Liaise with the project champion over correctness of model and its results (modeller and project champion) - Review preliminary scenarios with project champion: - **Baseline scenario to depict current system performance (drawing from historical data analysis)** - **Future scenarios including uncertain inputs (generated using predictive analysis)** - Prepare preliminary materials for use in the next workshop		- Model validation and verification - Preliminary future scenarios
5.1 Experimentation stage *(workshop 3a)* ***Purpose:*** **Validate optimisation model & examine model solution**	Stakeholders are invited to: **- Validate the optimisation model components and its output** **- Debate constraints & input parameters**	- **Model constraints validation tool** - **Model input parameters validation tool**	- Model validation and verification
	- Choose model inputs for scenario building **- Identify input parameter combinations for scenario analysis**	- **Model input selection form** - **Scenario parameter combination form**	- Alternative future scenarios

(If required) **5.1.a Post-workshop 3a/Pre-workshop 3b sub-stage** ***<u>Purpose:</u>*** **Preparations for workshop 3b**	- Tweak or correct the **optimisation model** - **Data collection (if required)** - Implement additional scenarios suggested (based on stakeholder feedback from workshop 3a.) - Liaise with the stakeholder team over correctness of model results - Prepare preliminary materials for use in workshop 3b		- **Revised optimisation model** - **New preliminary future scenarios**
(If required) **5.2 Experimentation stage (Workshop 3b)** ***<u>Purpose:</u>*** **Complete validation & define alternative scenario to experiment with model**	- **Validate Updated Model** **- Choose model inputs for scenario building** **- Identify input parameter combinations for scenario analysis**	- **Model input selection form** - **Scenario parameter combination form**	- Updated model validation and verification - Alternative future scenarios
5.2.a Post-workshop 3b/Pre-workshop 4 sub-stage *<u>Purpose:</u>* Refine alternative scenarios and prepare for workshop 4	Modelling team: - Tweak or correct **optimisation model** -Implement additional scenarios suggested (based on stakeholder feedback from workshop 3b.) - Liaise with the stakeholder team over correctness of model results - Prepare preliminary materials for use in workshop 4		- New alternative future scenarios - Revised optimisation model
6. Implementation stage (workshop 4) *<u>Purpose:</u>* Define an implementation plan	Stakeholders are invited to: - Review learning and changes implemented - Risk analysis and feasibility of change - Agree action trail	- Script for identifying changes in the system - Feasibility and Risks Scale tool with manual - Barriers to Change tool with manual - Action and Communication Plan tool with manual	- Feasible & desirable scenario(s) to be taken forward - Action plan with deliverables (including due date and person responsible)

Kotiadis and Tako (2021) have acknowledged that workshop facilitation is an art that requires ongoing refinement and update of competencies though reading or practice activity. In this study, a total of five workshops were conducted. The first two were held in person and the remaining three were held virtually. The researcher, a novice facilitator, was guided by the expert facilitator (second supervisor), especially for the first two workshops. In particular, the organisation, goals and contents of the workshop were explored under the expert facilitators' guidance. Prior to the in-person workshops, scripts detailing activities, allocated time, and prompts were developed to aid the novice facilitator. On reflection, the researcher can say that the guidance and the preparations that were undertaken prior to the workshops were significant in laying the foundations of facilitation competencies. The guidance and subsequent practice in workshops acted as a bridge between theoretical knowledge and practical implementation.

The in-person workshops were much harder to conduct, in comparison to the virtual workshops. This could be due to several reasons. First, the in-person workshops were the researchers' (a novice facilitator) first and with no prior experience there were struggles managing unexpected surprises such as power imbalances and limited participation among stakeholders. The researcher did not have the flexibility to appropriately respond and adapt to these challenges which required the expert facilitator to intervene to stay on track to achieving workshop outcomes (Tavella & Papadopoulos, 2015). On reflection, watching the expert facilitator manage and manoeuvre these challenges in practice was a vital learning experience towards developing future competencies.

The virtual workshops were easier in comparison to the in-person workshops, perhaps because they were conducted in the second half of the study. A rapport had been established with the stakeholder group and there was a degree of buy-in for the study and its pursuits. This made navigating stakeholder interactions relatively easier. Additionally, the researcher had developed some competencies that provided a confidence boost in their ability to facilitate a workshop. More importantly, the researcher was able to actively refer to the script during each virtual workshop, which was not possible during in-person workshops. The level of preparedness that is required for in-person workshops was relatively less in a virtual setting. Furthermore, the virtual workshops were conducted following conceptualisation and model development. Meaning the scope and direction of each workshop was significantly narrow in comparison to the first two workshops where the boundaries were quite wide. The effect of virtual versus in-person workshops on a novice facilitator's skills and competencies were not examined to the extent that is potentially possible. Future research in facilitated optimisation could explore this aspect which has the potential to yield important guidance to both new and seasoned researchers.

In general, the pre-model coding stages of PartiSim were harder to adapt when compared to post-model coding. The level of difficulty was likely due to the wide scope, lack of literature on conceptual modelling in optimisation, and the researcher being a novice facilitator and modeller. And for the rather apparent reason that pre-model coding stages took place before post-model stages, allowing for experience, knowledge, and confidence to accrue in the later parts of the study. Tako and Kotiadis (2015) have highlighted that a key factor in using the PartiSim framework is the ability and preparedness to move from one paradigm to another or from the hard to soft paradigm and vice versa. As a novice modeller and facilitator, the researchers can attest to facing the challenge of overly identifying with the hard OR paradigm, given that the researchers' previous educational background and experience was predominantly in computer science and analytics. The researchers spent considerable time and energy into grasping the premise of Soft OR and its theoretical underpinnings to eventually begin to understand SSM tools. The theoretical knowledge led to an appreciation of the benefits of utilising SSM and since this effort was running in parallel to negotiations with the mental healthcare service, the importance of using Soft OR tool for the project became increasingly evident.

Admittedly, the researcher's understanding of Soft OR, and facilitated modelling practices is evolving to this day and is informed by retrospective reflection on the case study. The researcher concurs with the advice provided by the developers of PartiSim, of adapting a continuous improvement plan at a personal level to reflect on the knowledge and experience gained (Kotiadis & Tako, 2021).

The optimisation model developed for the case study was mapped to a specific model type and was formulated for the specific problem put forward by the stakeholders. Additionally, the model emphasises the incorporation of first-order hard constraints derived from clinical and cultural factors. The developed model could be coded and solved using a commercial software. In situations where the model requires the inclusion of additional complexities such as additional constraints, stochastic elements, and uncertainties, and for a larger system, it might become necessary to explore the deployment of novel heuristics, stochastic optimisation, and robust optimisation. This could change the way the proposed PartiOpt framework is utilised and could very well lead to further modifications to accommodate these methodological complexities. It is anticipated that the changes could be isolated to the post-model coding stages which predominantly deals with the results of the model solution.

It is possible that the pre-model coding stages can be used to conceptualise an optimisation model that will capture the complexities that require an enhanced solution strategy. However, it is true that if the model is too complex there may be the need for an additional stage/workshop to discuss simplifications to the conceptual model if the resulting optimisation model is intractable. These factors were not considered in this thesis and are an opportune area of research.

7.1.3.1. When to use the proposed framework?

Based on the experience of developing and implementing the proposed framework, the researcher believes that the building of an optimisation model using the proposed framework would be most suited for application and further development to situations with the following criteria:

Multiple Stakeholders: The framework can be applied for building optimisation models in contexts characterised by multiple stakeholders with diverse perspectives and concerns likely to influence decision-making; where the problem definition is not well defined or even absent, as was the case in the mental healthcare service that was the focus of our case study. Such an environment is also conducive to stakeholder leaning about the system of interest.

Improvement over Optimality is Preferred: Optimisation literature is primarily concerned with developing sophisticated model formulations and developing novel solution algorithms that are not always conducive to practical implementation. Often the goal is to produce an optimal or near-optimal solution. The proposed framework is suitable for practical and real-life applications where exploration of the problem situation and a sense of progress is preferred over optimality.

Simple to model: Following on from the last point, in preferring progress over optimality, a model that is simple, realistic, and intuitive to stakeholders will be developed. In healthcare, the complexity of the model and opaqueness of the model building process are deterrents to stakeholder uptake of model recommendations. In such an environment, it is advantageous to build simple optimisation models, gradually fostering trust and comprehension of the process, similar to the approach followed in simulation and system dynamics literature.

Visualisation: The framework has introduced a communicative model that is used to represent the conceptualisation of an optimisation model. In situations involving medical professionals or non-OR literate personnel, the researcher found that breaking down of complex concepts into digestible visual and descriptive representations was key to build trust and understanding of the model and the modelling process. In the proposed framework, a thread connects the initiation of the study all the way to the development of the conceptual model. The combined use of tools and visual depictions leading to the conceptual model make up the visualisation aspect of the framework. The framework is suitable for problem situations where this aspect is identified as desirable.

Multiple data sources: The PCMH service was keen to leverage the data that was available in the system. In fact, the project was commissioned on the premise of using this untouched data to evaluate the service and generate insights. The presence of unutilised data spread across several sources is a common feature in healthcare services, particularly in the UK. In situations where the utilisation of existing data for a modelling project is a key factor, the framework provides a so called "plug-and-go" structure to manage, analyse, and use the data. Presupposing relevant Ethics and GDPR approval are sought and received.

7.1.4. Future Research

The literature review on optimisation in mental healthcare highlighted several avenues for applying existing research from the healthcare field and outlined opportunities for growth that leverage the unique characteristics of mental healthcare, that are increasingly acknowledged as significant for future healthcare applications. Currently within optimisation literature, studies are emphasising the need to incorporate the inherent uncertainty and accessibility issues prevalent in healthcare systems such as cancer care. Future applications in mental healthcare can utilise these advances from cancer care to tackle issues related to risk and accessibility to care. Optimisation models in the future can model features such as continuity of care and planning multi-disciplinary patient care pathways by learning from application in community care, home healthcare and outpatient care, where such features are frequently explored.

Apart from identifying elements that can be incorporated into the models for future research in mental healthcare, there is a recognised requirement to develop holistic approaches that promote the involvement of multiple stakeholders. This requirement is applicable throughout the field of optimisation in healthcare. Specifically, the utilisation of advanced solution technologies like stochastic and robust optimisation can be beneficial in addressing planning issues in mental healthcare. However, the utilisation of modelling methodologies such as mixed-methods and multi-methodology can provide even greater benefits in addressing the challenges of mental healthcare planning and these methodologies can also be extended to other healthcare settings.

The case study in this thesis demonstrates the merits of shifting the focus of an optimisation model. In literature, most optimisation model, and by extension modellers are focused on computational efficiency and developing efficient algorithms that generate "the" optimal solution to problems that are often simplifications of reality. In contrast, the case study exemplifies reality wherein stakeholders are primarily concerned with having an accurate representation of the problem at hand and are particularly interested in identifying solutions that enhance current practices. The proposed framework was key in highlighting these factors that are otherwise not considered in the traditional mode of building optimisation models. This thesis puts forward a fundamental claim that the proposed framework has the potential to be used for developing optimisation models that can tackle problems not only within mental healthcare but also across the broader healthcare domain. The proposed framework has proved its value by offering crucial decision-support to a real-world problem. This assertion holds true for the healthcare domain and is reinforced by multiple existing reviews in healthcare optimisation literature, which emphasise the importance of incorporating stakeholder priorities, including realistic features, capture realistic assumptions, address real-life problems, and report on model implementation (Ahmadi et al., 2019; Ahmadi-Javid et al., 2017; Erhard et al., 2018; Marynissen & Demeulemeester, 2019; Samudra et al., 2016; Volland et al., 2017).

The proposed framework puts forward a structured approach to engage stakeholder's throughput the optimisation modelling lifecycle. The case study demonstrates how the framework supports the contextualise of the optimisation modelling through a thorough examination of the problem context by involving stakeholders. In the last two decades, six studies developed optimisation models for mental healthcare planning, as reviewed in Chapter 3 (Bester et al., 2007; Cohn et al., 2009; Hertz and Lahrichi, 2009; Pagel et al., 2012; Samorani and Laganga, 2015; Li et al., 2016). These application could have further enhanced their practical applicability if an approach like the one proposed in this thesis were implemented. For instance, most of these studies focused on developing optimisation models in a multi-care network, or in a real world practical context. However, limited information is provided on the influence of this type of setting on the problem situation and the model itself, aside from providing detailed information about the problem context relative to model formulation. Additionally, very little insights were provided about contextual factors like how the problem was structured, if there were any problems with data collection, how was technical information relative to the model and its results, communicated to stakeholders (Virtue et al., 2013). These systemic factors are inextricably linked to practical applicability and useability of the model. The case study reveals and highlights these factors, establishing a clear connection between the context and the optimisation model. It also provides insights into how these factors influenced the model and foregrounded the subsequent practicality and usefulness of the model's recommendations. Future research can utilise the proposed framework to capture these contextual factors within a problem of interest and develop optimisation models that are applicable in practice.

The framework can be applied to address problems in supply chain management, manufacturing, logistics, and disaster management, to address the dynamic nature of stakeholders' requirements. For instance, the framework can be particularly useful to collaboratively capture stakeholders' sustainability requirements and align sustainable practices to an organisations' strategies and capabilities (Chowdhury et al., 2019). In logistics and manufacturing, it could be deployed to provide context-specific insights to stakeholders and inform the design and implementation of innovative solutions (Malik et al., 2019; Petruzzelli et al., 2023). In disaster management, involving stakeholders in the model building process, particularly for conceptual modelling, experimentation and calibration is a key area of future research (Amideo et al., 2019; Çoban et al., 2021). It might be necessary to adapt the post-modelling stages for more comprehensive testing using random and realistic instances that is a characteristic requirement in the domain to establishing the optimisation model's applicability.

From a methodological view, it is likely that new tools will have to be incorporated into the proposed framework for engaging with stakeholders in situations with multiple-objectives or even when dealing with uncertainty. As highlighted before, the proposed framework does not consider these features, but it is argued that Facilitated decision analysis, and in particular, Multiple Criteria Decision Analysis (MCDA) methods, can be used to explicitly organise and synthesise information that is complex and conflicting in nature through a transparent process that incorporates stakeholder views outcomes (Belton & Stewart, 2002; Durbach & Stewart, 2012; Phillips & Bana, 2007; Tzeng & Huang, 2011). Additionally, given that the framework is an adaptation of the PartiSim, future researchers considering both DES and optimisation for their intervention can draw from the work presented here and the original framework. Researchers looking to integrate optimisation with hybrid simulation can draw from the representation method that can aid the modeller in defining the modelling frame (i.e., the combination of methods forming the hybrid model) (Jones et al., 2022).

In summary, the facilitated optimisation framework developed in this thesis is – to the best of the researcher's knowledge- the first instance of utilising a structured approach to involve stakeholders throughout the optimisation modelling lifecycle. The scope of the framework is broad and extends beyond healthcare, allowing for the construction of optimisation models in various domains. Admittedly, the title of the thesis serves as a signpost towards the trajectory of becoming an established approach for building optimisation models, while also indicating the position this work occupies on the journey towards establishing itself as well-defined area of research. Undoubtedly, the future of this framework will be supported by its sustained utilisation for optimisation modelling. Through application to real case studies, the framework will likely evolve and expand to include new tools and lead to improved guidance for optimisation modellers in addressing real world problems.

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Appendix A : PartiSim Tools – Initiate Study (Stage 1)

This section contains screenshots of tools prescribed by the PartiSim framework for Stage 1.

Problem situation	Source
Preliminary problem situation 1 …..	Source 1
Potential improvements/change	
Change 1 …..	Source 1

Figure 27: Screenshot of Study Initiation Tool 1: Situation of interest

Observation		
Persons involved in observing the situation of interest		
☐ *Key stakeholders*	☐ *Project champions*	☐ *Workshop participants*
☐ *Analysts*	☐ *Others*	*Name:*
Date when observation was made (DD-MM-YYYY)		
Place where the observation was made		
Duration of the observation		
Comments		

Figure 28: Screenshot of Initiate Study Tool 2: Recording Observations

Aspects to understand	A sample of potential questions
Problem situation	What is the purpose of the current system? How many types of services are provided? Are there any specific targets that need to be reached? In a normal day, what is the progression of activities followed? Are there any bottlenecks? What do you think is the cause? Is everyone happy with the service provided?
Improvement	Is there room for improvement? What do you think could potentially improve the situation? Has a change been implemented in the past? What results did it have?
Identify roles of people in the system	Who does the system serve? Who is involved in the provision of service? Who is affected? Who makes decisions? Who would object change? Can you identify any people who would not be happy with this change?

Figure 29: Screenshot of Study Script 2: Bank of Questions

ID	Name	Job title	Contact details	Role
		Job category: *Select job category ...* Comments: *Enter comments*	Address: Tel: Fax: Email: Web:	Role within project: *Select role* To participate in workshops? ❑ (tick if applicable)

Figure 30: Screenshot of Initiate Study Tool 3: Stakeholders' Contact details

Material	Material type	Material Suggested by	Material to be read by
Source A	Material type B	Suggested by... (Please enter the name and role of the person suggesting the material): Name: Role: C	❑ Key stakeholders ❑ Project champion ❑ Workshop participants ❑ Analysts ❑ Others (please specify below) D

Figure 31: Screenshot of Initiate Study Tool 4: List of reading materials

Appendix B : PartiSim Tools – Define the Problem Workshop (Stage 2)

This section contains screenshots of tools prescribed by the PartiSim framework for Stage 2.

CATWOE elements		Relevant Element chosen
Customers	*List examples of care system customers*	*Identify element Customers here*
Actors … Actors …	*List examples of Actors in the care system*	*Identify element Actors here*
Transformation process	*List the key inputs which are transformed to a specific output from care system activity* Input: Output:	*Out of the key inputs and outputs identify one pair (transformation) that would represent the key activity that is deemed to need improvement. Note: whatever goes through a transformation must come out in a transformed from.*
Weltanschauung (World view)	*Please list the key values that express the care system concerned*	*Identify one value that makes the transformation process defined above meaningful and give a definition of the world view taken.*
Owners	*List the people or groups who could stop the transformation process defined above*	*Identify the element Owners here:*
Environmental constraints	*List some of the care system constraints:*	*Define the element environmental constraints:*

Figure 32: Screenshot of Conceptual Modelling Tool 1: Define the System

Appendix C : PartiSim Tools – Define the Conceptual Model Workshop (Stage 3)

This section contains screenshots of tools prescribed by the PartiSim framework for Stage 3.

Stakeholder Form 1: Write study objectives

Please complete the form and return to facilitator

Name: ______________________

By the end of this study what do you hope to achieve?

Please attempt to write the study objectives that you think the study should explore. Try to follow the format provided, but you can deviate from this format if you feel you need to.

Example:

Purpose: *We want to increase the number of patients seen in the Surgical clinic*

Target performance: *by 20%*

Change: *by increasing the number of clinics run*

Constraints: *ranging between 8 and 20 clinics per week*

Objective 1:

Objective 2:

Figure 33: Screenshot of Stakeholder Form 1: Write Study Objectives

Stakeholder Form for Conceptual Modelling Tool 3 Please complete the form and return to facilitator			
	Monitoring activities	**Determine if activities**	**Suggested changes**
Efficacy (E1) - What? Ensure that the system provides the intended care.	What do you need to monitor (measure) to know that the system is providing the intended care?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think are needed to ensure that the system provides the intended care to obesity patients?
	I would like to monitor:	I would be able to determine if:	I would suggest:
Efficiency (E2) - How? Ensure that the system works efficiently, provides the best possible care, using the minimum resources.	What do you need to monitor (measure) to know that the system works efficiently?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think are needed to ensure that the system works efficiently?
	I would like to monitor:	I would be able to determine if:	I would suggest:
Effectiveness (E3)- Why? Ensure that the overall system provides a seamless patient journey.	What do you need to monitor (measure) to know that the system provides a seamless patient journey (the right thing is being done)?	By undertaking each monitoring activity, what would you be able to determine?	Based on each "Determine if" activity, what changes do you think are needed to be made to ensure that the system provides a seamless journey to obesity patients?
	I would like to monitor:	I would be able to determine if:	I would suggest:

Figure 34: Screenshot of Stakeholder Form for Conceptual Modelling Tool 3

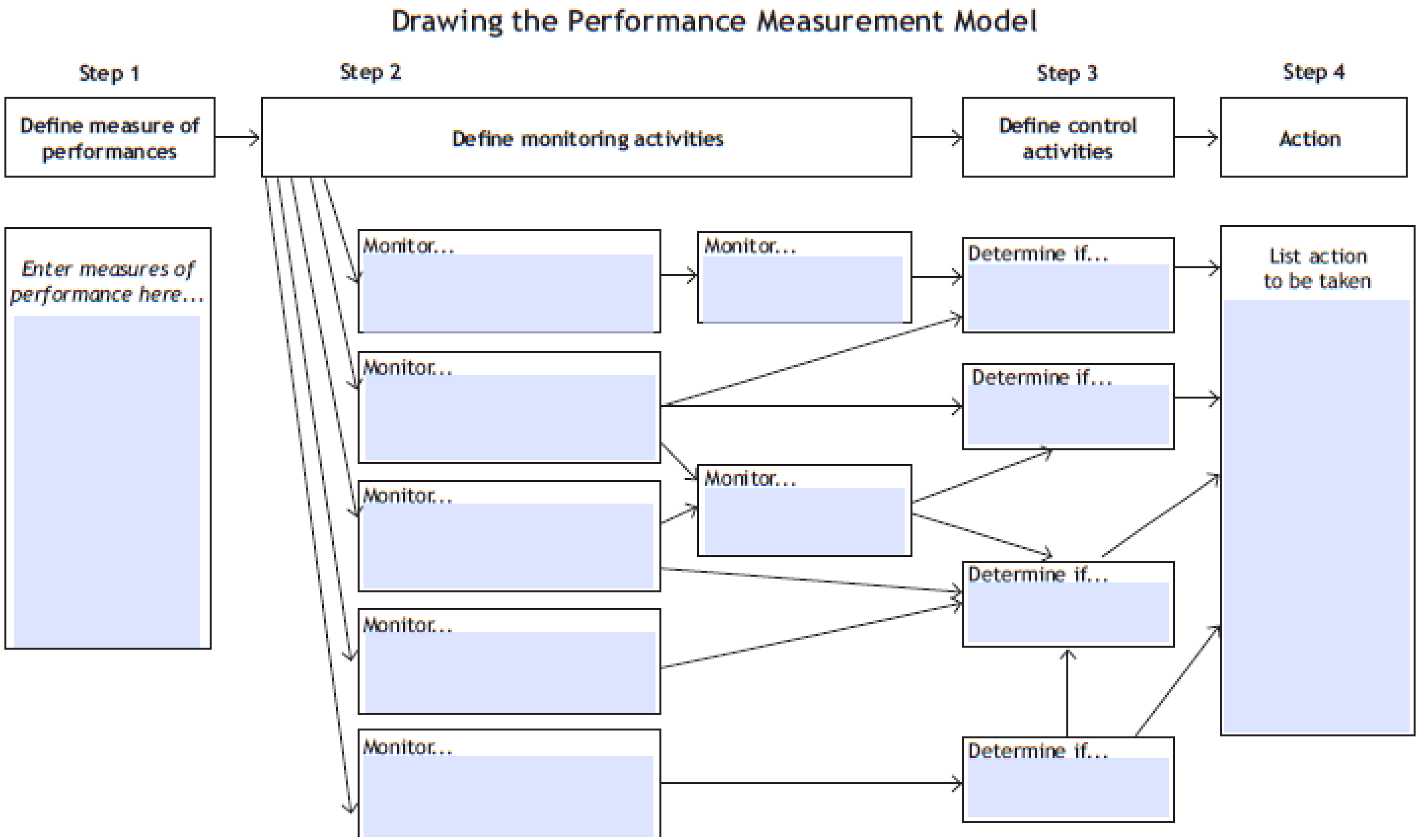

Figure 35: Screenshot of Conceptual Modelling Tool 3: Drawing the Performance Measurement Model

Suggested change in PMM	How can it be achieved	I: Identify range of variation	O: Identify good/bad performance
Change 1:			
Change 2:			
Change 3:			

Figure 36: Screenshot of Conceptual Modelling Tool 4: Identify model inputs and outputs.

	Purpose (What do we want to achieve?)	Target performance	Constraints
Objective 1			
Objective 2			
Objective 3			

Figure 37: Screenshot of Conceptual Modelling Tool 4: Determine Study Objectives

Appendix D : Case Study Outputs – Initiate Study (Stage 1)

This section contains screenshots of tools that were used directly from the PartiSim framework for the initiate study stage.

Table 39: Situation of Interest Form Output

Problem Situation	Source
Preliminary problem situation 1 A primary mental healthcare service model for the whole county Presently, trust executives acknowledge that there are a number of separate ideas floating around various CCG's for the service model.	Source 1 Stakeholder B
Preliminary problem situation 2 Lack of information on KPI's to check if the new service provides value for money or is having a clinical impact	Source 2 Stakeholder B, A, C, D, E.
Preliminary problem situation 3 How much time are clinicians spending in tasks and seeing patients. Is there an increase in productivity with all the recent changes (hiring of admin staff, CPA burden released, MDT team involvement.	Source 3 Stakeholder B, A, C, E.
Preliminary problem situation 4 GP referral data for the past 5 years is available and could be used as benchmark to capture service performance	Source 4 Stakeholder B, A, C.
Potential improvements/change	
Change 1 It would be beneficial to also have a primary care model for the whole county.	Source 1 Stakeholder B
Change 2 Empirical analysis of service operations to identify opportunities	Source 2 Stakeholder B, A, C, D, E.
Change 3	Source 3

Table 40: List of Reading Material Form Output

Material	Material Type	Material Suggested by	Material to be read by
Source MH LC WK presentration (6 sep 2018). (Sent over by Project Champion, after initial meeting on 29/04/2019)	Material type Presentation Slides (confidential)	Suggested by: (Please enter the name and role of the person suggesting the material): Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ☑ Analysts ☐ Others (Please specify below)
Source Recommendation for the WK MH LC clinical pathway. (Sent over by Project Champion, after initial meeting on 29/04/2019)	Material type Word document (confidential)	Suggested by: (Please enter the name and role of the person suggesting the material): Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ☑ Analysts ☐ Others (Please specify below)
Source Service specification west Kent LCMH Draft March 2019. Sent over by Project Champion, after initial meeting on 29/04/2019)	Material type Word document (confidential)	Suggested by: (Please enter the name and role of the person suggesting the material): Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ☑ Analysts ☐ Others (Please specify below)
Source WK MH LC draft evaluation framework. Sent over by MaProject Championrtine on 13/05/2019	Material type Presentation	Suggested by: (Please enter the name and role of the person suggesting the material): Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ☑ Analysts ☐ Others (Please specify below)
Source Local Care Monthly Meeting Action Log 9 May 2019.	Material type Word Document	Suggested by: (Please enter the name and role of the person suggesting the material): Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ☑ Analysts ☐ Others (Please specify below)

Source	Material type	Suggested by: (Please enter the name and role of the person suggesting the material)	
WK CCG Local Care Plan (July 2017) (Purpose: As an instance of patient data extraxted from KMPT and to faciltate inquiry into data sources)	Report (PDF)	Name: Sheema Noorain Role: Facilitator/ Analyst	✓ Key stakeholders ✓ Project champion ☐ Workshop participants ✓ Analysts ☐ Others (Please specify below)
Research proposal (Purpose: for methodology enumeration)	Proposal (PDF)	Name: Sheema Noorain Role: Facilitator/ Analyst	✓ Key stakeholders ✓ Project champion ☐ Workshop participants ✓ Analysts ☐ Others (Please specify below)
Implementation Guidance: A Framework for Community Mental Health Support. (sent by Martine on request)	Report (PDF)	Name: Project Champion Role: Program Manager Transformation (TaIT)	☐ Key stakeholders ☐ Project champion ☐ Workshop participants ✓ Analysts ☐ Others (Please specify below)

Table 41: Stakeholder Contact Details Output

ID	Name	Job title	Role	Contact Details
1	Project Champion	Job Category: Programme Manager Transformation and Improvement Team	Role within project: Champion To participate in workshops? Y	Email
2	Stakeholder F	Job Category: WK CCG Clinical Lead	Role within project: Key Stakeholder To participate in workshops? N	Email
3	Stakeholder C	Job Category: CCG Project Lead	Role within project: Key Stakeholder To participate in workshops? Y	Email
4	Stakholder A	Job Category: Deputy Chief Operating Officer, KMPT	Role within project: Key Stakeholder To participate in workshops? Y	Email
5	Stakeholder D	Job Category: Head of Service West & North Kent	Role within project: Key Stakeholder To participate in workshops? N	Email
6	Stakeholder E	Job Category: Service Manager of Maidstone Community Mental Health Team	Role within project: Key Stakeholder To participate in workshops? Y	Email
7	Stakeholder B	Job Category: Primary Care Mental Health Specialist & Service Manager	Role within project: Project Champion To participate in workshops? Y	Email

8	Stakeholder H	Job Category: Consultant Psychiactrist & Assistant Medical Director (KMPT)	Role within project: Stakeholder To participate in workshops? N	Email
9	Stakeholder F	Job Category: Business Intelligence Analyst	Role within project: Kay Stakeholder To participate in workshops? Y	Email
10	Stakeholder G	Job Category: Benefits Realisation Manager, KMPT	Role within project: Stakeholder To participate in workshops? Y	Email
11	Stakeholder I	Job Category: Research & Development	Role within project: Stakeholder To participate in workshops? N	Email
12	Stakeholder J	Job Category: Public Health Consultant (West Kent County Council)	Role within project: Stakeholder To participate in workshops? Y	Email

Table 42: Stakeholder Influence & Engagement Form Output

STAKEHOLDER INFORMATION								
ID	STAKEHOLDER	TITLE / ROLE	INTEREST: How much does this project affect them?	INFLUENCE: How much do they have?	What will the stakeholder hope to get out of this study?	HOW WILL HE/SHE CONTRIBUTE?	BEST WAY TO ENGAGE?	ADDITIONAL NOTES
1	Project Champion	Programme Manager Transformation and Improvement Team	Positively	High	Measure clinical impact	High contribution	direct engegement	
2	Stakeholder F	WK CCG Clinical Lead	Positively	High	If the new service provides value for money	Medium contribution	direct engegement	
3	Stakeholder C	CCG Project Lead	Positively	Medium	If the new service provides value for money	Medium contribution	direct engegement	
4	Staknolder A	Deputy Chief Operating Officer, KMPT	Positively	High	If the new service provides value for money	Medium contribution	direct engegement	
5	Stakeholder D	Head of Service West & North Kent	Positively	Medium	Clinical impact	Medium contribution	direct engegement	
6	Stakeholder E	Service Manager of Maidstone Community Mental Health Team	Positively	Medium	Clinical impact	High contribution	direct engegement	
7	Stakeholder B	Primary Care Mental Health Specialist: Nurse	Positively	Medium	Clinical impact	High contribution	direct engegement	
8	Stakeholder H	Consultane Psychiactrist & Assistant Medical Director (KMPT)	Positively	High	Clinical impact	High contribution	direct engegement	
9	Stakeholder F	Business Intelligence Analyst	Positively	Medium	Clinical impact	Very high contribution	direct engegement	
10	Stakeholder G	Benefits Realisation Manager, KMPT	Positively	Medium	Clinical impact	Medium contribution	direct engegement	
11	Stakeholder I	Research & Development	Positively	Medium	Clinical impact	Medium contribution	direct engegement	

Table 43: Data Availability & Accessibility Form Output

Data Availability & Accessibilty						
ID	Areas of Data Availability	Who is in charge/possession of this data?	Which data systems are currently in use to store this data?	How can it be accessed?	Patient Cohort	Where are the services based?
1	Primary Mental Healthcare Service	KMPT	RiO+Emis+Vision	Stakeholder F	People aged 18 years and over who have functional mental health illness. It will be provided to people who are not eligible for secondary care services but where IAPT services are also not appropriate.	Community + Specialist Base (Albion & Highlands house, sevenoaks)
2	Primary Care *(GP, PCN, Kinesis, West Kent Health/Federation)*	CCG	EMIS & Vision	Stakeholder E		GP practice + Patient's home
3	Mental Health Specialist Care (*Initial Intervention; Personality Disorder; Enduring Conditions*)	KMPT	RiO	Stakeholder F		Community + Specialist Base
3a	Community Mental Health Team	KMPT	RiO	Stakeholder F	Provide services to adults people living in the community who are experiencing mental health problems. Referrals to the CMHTs are usually made directly through a GP or A&E or SPoA	Community + Specialist Base
3b	Crisis Team	KMPT	RiO	Stakeholder F	Crisis teams support people who might otherwise need to go to hospital, for example due to psychosis, severe self-harm or suicide attempts. They usually include a number of mental health professionals, such as a psychiatrist, mental health nurses, social workers and support workers.	?
3c	Single Point of Access (SPOA)	KMPT	RiO	Stakeholder F	If urgent or emergency mental health help and support is needed and the patient is currently receiving secondary mental health care and treatment from Community Mental Health Teams	Phone Call

Appendix E : Case Study Outputs – Define the Problem Workshop (Stage 2)

This section contains screenshots of outputs from Stage 2 of the case study that were not taken forward.

Table 44: Problem Exploration Outputs that were not taken forward

- Liaison psychiatry referrals being discharged into GP and not LC
- Inequalities: Addressing and meeting needs
- Unmet needs: what, where?
    - People who do not require SC but need LC
    - People requiring care, but do not fit a box
    - No information on demand for digital technologies such as e-consultation.
    - No clinical outcomes to measure impact (PREM & PROM)
- Information about people referred by GP to other services
    - Electronic information sharing between services.
- Three different database's currently in use by the service.
- Information on how many are transitioning back to GP from LC
- How many are transitioning back to GP from LC
- How many avoided re-referrals
- How many avoided crisis (by CMHT or acute)
- No control over number of referrals (managing expectation)
- Staff-patient ratios.
- SI's and complaints on people being stepped down into PC (Is there enough support?)
- Effects of dual diagnosis: care pathway, demand.
- Effects on primary prevention: where does mental wellbeing present
- Unmet needs assessment

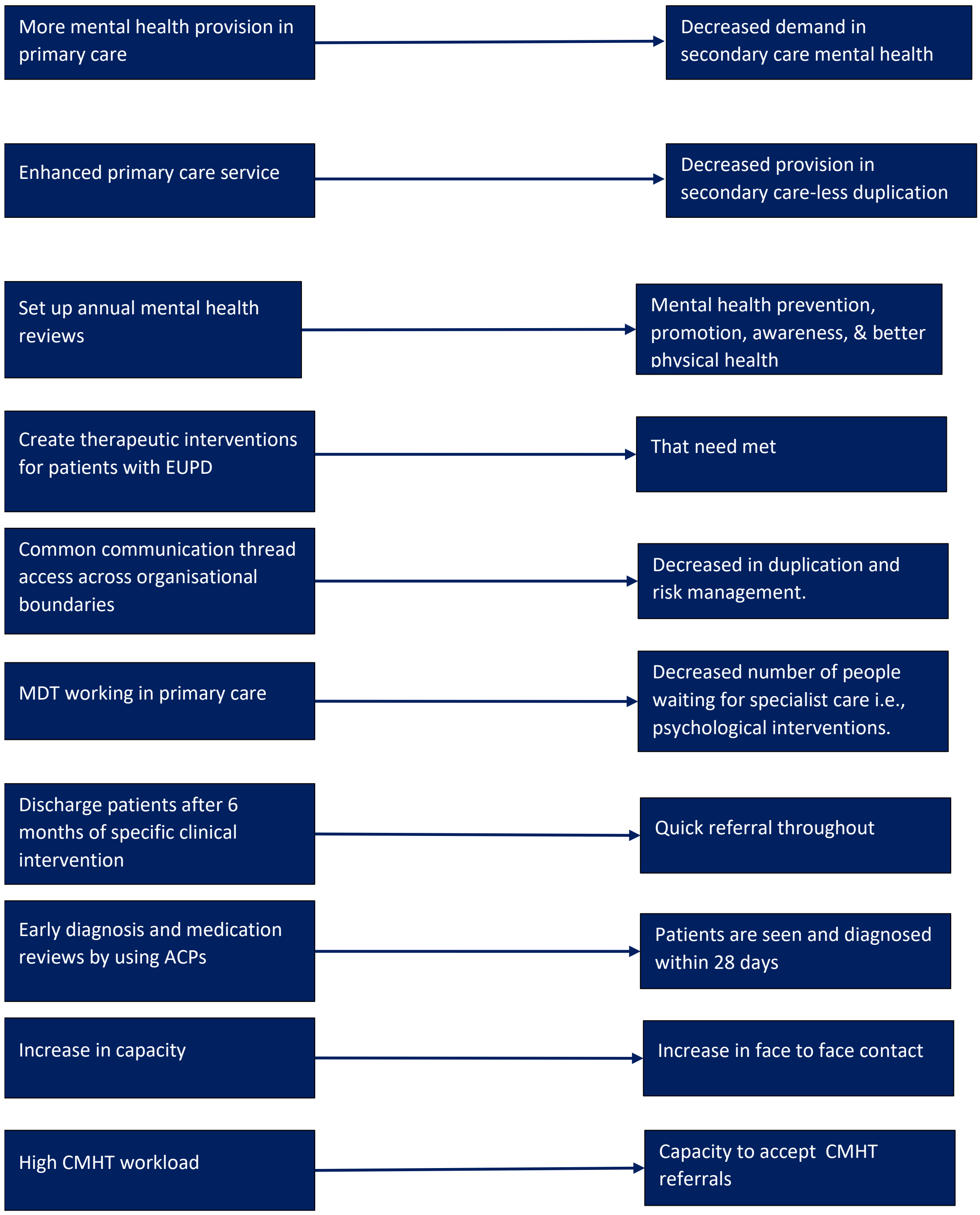

Figure 38: Transformations that were not explored in the study.

Appendix F : New Proposed Tools – Facilitated CM for Optimisation

This section contains new tools proposed for the CM stages of PartiOpt.

Table 45: Proposed 'Input Form'

Actors	Characteristics	Operational Details	Data Sources
	-	-	☐Known with certainty. ☐Derive using descriptive tools. ☐Collect new data. ☐Derive using predictive tools. ☐Stakeholder estimation ☐Scenario based data
Customers			
	-	-	☐Known with certainty. ☐Derive using descriptive tools. ☐Collect new data. ☐Derive using predictive tools. ☐Stakeholder estimation ☐Scenario based data

Table 46: Generic Constraints Form

Environmental Constraints (E)	
Budget E.g., at least X amount needs to be allocated for service A and at most Y amount for service B	
Human Resources E.g., balance health resource with patient load	
Physical Resources E.g., allocate appointments when consultation room available	
Time E.g., clinicians cannot do two consecutive night shifts	
Location/ Geographical E.g., clinicians in band X can only travel to locations A, B and C.	
Preference/ Utility E.g., clinician preference for night shifts	
Demand E.g., demand of severe patients must be satisfied	
Capacity E.g., limits on total number of beds or nurse work hours in a week	
Structural E.g., appointments in a given location cannot be assigned to a clinician who is not allocated to that location	

Appendix G : Images from Case Study – Workshop 1 & 2

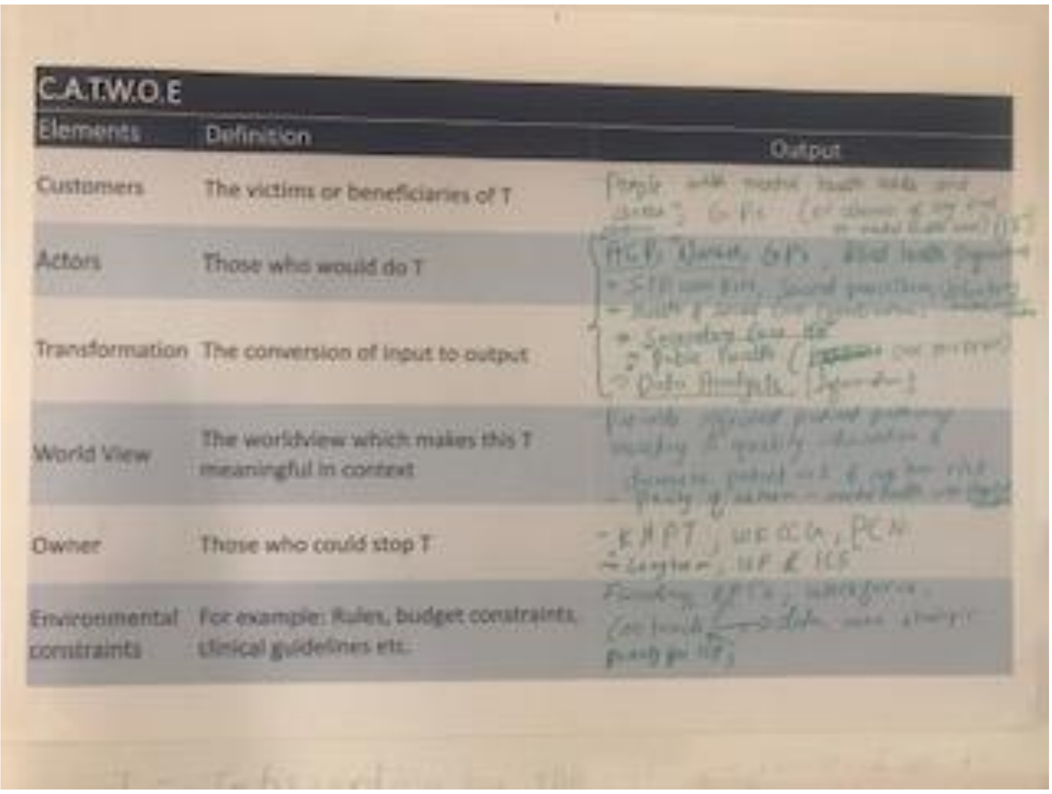

Figure 39: CATWOE Tool Output (Workshop 1)

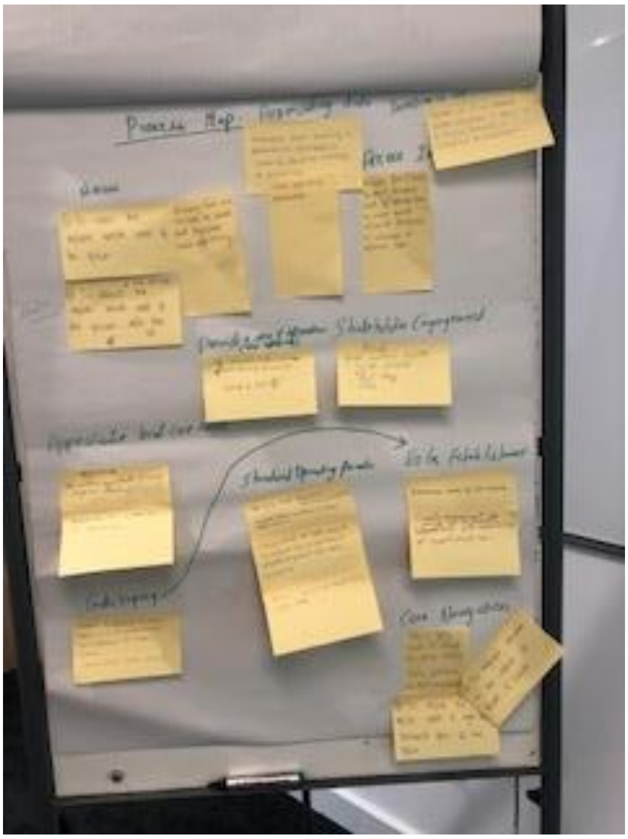

Figure 40: Care Systems Model Flipchart (Workshop 1)

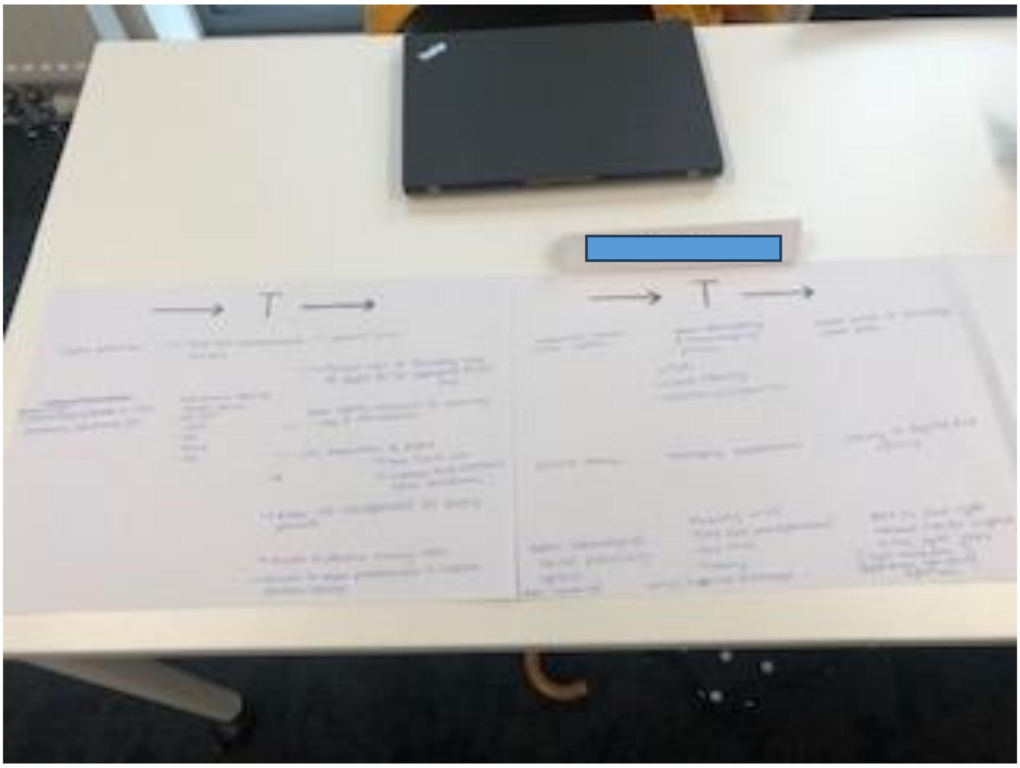

Figure 41: Stakeholder G Brainstorming Transformation (T)

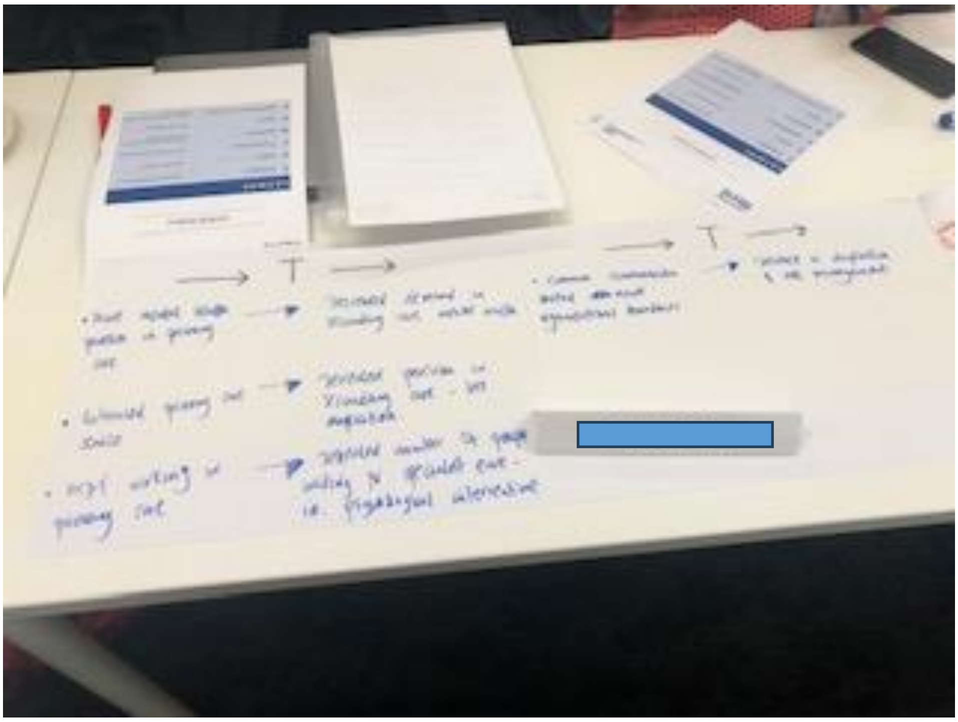

Figure 42: Stakeholder C Brainstorming Transformation (T) (Workshop 1)

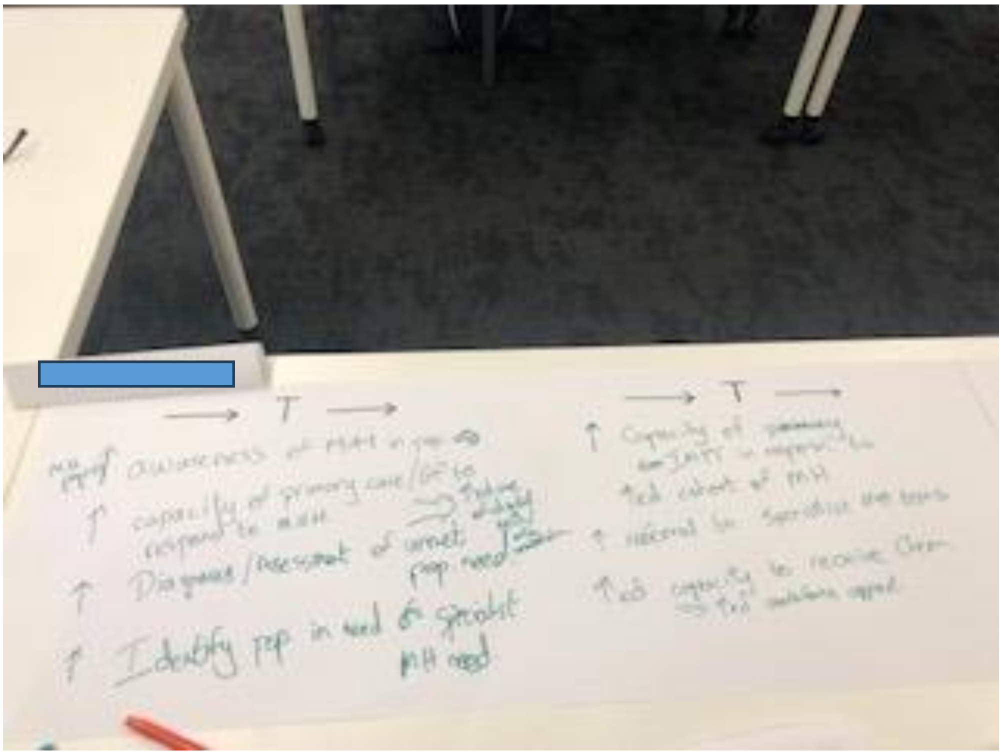

Figure 43: Project Champion Brainstorming Transformation (T) (Workshop 1)

Figure 44: Workshop 2 Setup - Angle 1

Figure 45: Workshop 2 Setup - Angle 2

Appendix H : CPLEX Code – Optimisation Model Coding (Stage 4)

```
int Fd=...; int Ad=...; int Td=...; int Cd=...; int L=...; int M=...;
int Smax=...; // maximum number of shifts per day
int Smin=...; //minimum number of shifts per day
float Tmax=...; // maximum travel distance between clinics
int shifts=...; int clinic=...; int nurses=...; int appointments=...;
int days=...; int Nc=...; int Nn=...; range Nurses=1..nurses;
range Shifts=1..shifts; range Clinics=1..clinic; range Days=1..days;
range Appointments=1..appointments; int Demand=...; int H[Nurses]=...;
int Followup[Clinics]=...; int Assessment[Clinics]=...;
int Telephone[Clinics]=...; int Community[Clinics]=...;
int Hns[Nurses][Shifts]=...;
float distance[Clinics][Clinics]=...; // distance between clinics
int Pnc[Nurses][Clinics]=...; // nurse preferences to clinics
int Bna[Nurses][Appointments]=...; // Appointment skill preferences to
clinics string stS[Days]=...; {int} shift1[Days]; {int} shiftAM=...;
{int} shiftPM=...;

execute convert_strings1
{
for(var b in Days)
{
   var ar=stS[b].split(",");
   for(var i=0;i<ar.length;i++) shift1[b].add(Opl.intValue(ar[i]));;}
}
execute
```

```
{
writeln(shift1;
}

dvar int+ Xf[Nurses][Clinics][Shifts];
dvar int+ Xa[Nurses][Clinics][Shifts];
dvar int+ Xt[Nurses][Clinics][Shifts];
dvar int+ Xc[Nurses][Clinics][Shifts];
dvar boolean U[Nurses][Appointments];
dvar boolean Y[Nurses][Clinics][Shifts];
dvar boolean w[Nurses][Clinics];
dvar boolean Q[Nurses][Shifts];
dvar boolean yy;
dvar int+ Z[Nurses];

dexpr int TotAssigned  =
  sum(n in Nurses, c in Clinics, s in Shifts)
(Xf[n][c][s]+Xa[n][c][s]+Xt[n][c][s]+Xc[n][c][s]);
dexpr int AAssigned  =
  sum(n in Nurses,c in Clinics, s in Shifts) Xa[n][c][s];
  dexpr int FAssigned  =
  sum(n in Nurses,c in Clinics, s in Shifts) Xf[n][c][s];
  dexpr int TAssigned  =
  sum(n in Nurses,c in Clinics, s in Shifts) Xt[n][c][s];
  dexpr int CAssigned  =
  sum(n in Nurses,c in Clinics, s in Shifts) Xc[n][c][s];
dexpr int UnmetDemand = Demand-TotAssigned;



minimize UnmetDemand;

subject to {

// assign followup appointments in clinics to shifts
forall(c in Clinics)
sum(n in Nurses,s in Shifts) Xf[n][c][s]<=Followup[c];

// assign assessetment appointments in clinics to shifts
forall(c in Clinics)
sum(n in Nurses, s in Shifts) Xa[n][c][s]<=Assessment[c];

 // assign telephone appointments in clinics to shifts
forall(c in Clinics)
sum(n in Nurses,s in Shifts) Xt[n][c][s]<=Telephone[c];

 // assign community appointments in clinics to shifts
forall(c in Clinics)
sum(n in Nurses,s in Shifts) Xc[n][c][s]<=Community[c];

//The sum of duration of all appointments assigned to a shift should not
exceed the length of each shift
forall(n in Nurses,s in Shifts,c in Clinics)
(Fd*Xf[n][c][s] +   Ad*Xa[n][c][s] + Td*Xt[n][c][s] +Cd*Xc[n][c][s])<=L;

//The sum of duration of all appointments assigned across shifts and
clinics should not exceed the length of available hours
forall(n in Nurses)
```

```
sum(s in Shifts,c in Clinics)(Fd*Xf[n][c][s] +   Ad*Xa[n][c][s] +
Td*Xt[n][c][s] +Cd*Xc[n][c][s])<=H[n];

 //Assign a clinic to a nurse in a given shift, only if that clinic and
shift have been assigned assesstment appointments
forall( n in Nurses, c in Clinics, s in Shifts)
M*Y[n][c][s]>= Xa[n][c][s];
 //Assign a clinic to a nurse in a given shift, only if that clinic and
shift have been assigned follow-up appointments
forall( n in Nurses, c in Clinics, s in Shifts)
M*Y[n][c][s]>= Xf[n][c][s];

//Assign a clinic to a nurse in a given shift, only if that clinic and
shift have been assigned telephone appointments
forall( n in Nurses, c in Clinics, s in Shifts)
M*Y[n][c][s]>= Xt[n][c][s];

//Assign a clinic to a nurse in a given shift, only if that clinic and
shift have been assigned community appointments
forall( n in Nurses, c in Clinics, s in Shifts)
M*Y[n][c][s]>= Xc[n][c][s];

// nurse can only be assigned to 1 or 0 shifts in a clinic.
forall(c in Clinics, s in Shifts)
sum(n in Nurses) Y[n][c][s]<= 1;

//Maximum number of shifts per day for each nurse
forall(n in Nurses, d in Days, s in shift1[d])
sum(c in Clinics) Y[n][c][s]<=Smax;

//second shift assignment for a nurse cannot be made to that clinic.
forall(n in Nurses, d in Days, s in shiftAM, c1,c2 in Clinics:c1!=c2)
(Y[n][c1][s]+Y[n][c2][s+1])-1 <=M*yy;

forall(c1,c2 in Clinics:c1!=c2)
distance[c1][c2] - Tmax <= M*(1-yy);

//Assign a clinic to a nurse in a given shift, only if that clinic is
assigned to that nurse
forall (c in Clinics, n in Nurses, s in Shifts)
Y[n][c][s]<= w[n][c];

//Each clinic can only be assigned utmost n nurse
forall (c in Clinics )
sum(n in Nurses) w[n][c]<=Nn;

//Each nurse can be assigned to utmost a threshold value of clinics
forall (n in Nurses)
sum(c in Clinics) w[n][c]<=Nc;

//Clinic Prefernce
 forall(n in Nurses,c in Clinics)
 w[n][c]<=Pnc[n][c];

//Assign a clinic to a nurse in a given shift, only if that clinic is
assigned to that nurse
forall (c in Clinics, n in Nurses, s in Shifts)
Y[n][c][s]<= Q[n][s];
```

```
//Shift Prefernce
forall(n in Nurses, s in Shifts)
Q[n][s]<=Hns[n][s];

forall (n in Nurses)
Z[n]-(H[n]-(sum(c in Clinics, s in Shifts)
(Fd*Xf[n][c][s]+Ad*Xa[n][c][s]+Td*Xt[n][c][s]+Cd*Xc[n][c][s])))>=0;
forall(n in Nurses, c in Clinics, s in Shifts, a in Appointments)
M*U[n][a]>= Xc[n][c][s];

forall(n in Nurses, c in Clinics, s in Shifts, a in Appointments)
M*U[n][a]>= Xf[n][c][s];

forall(n in Nurses, c in Clinics, s in Shifts, a in Appointments)
M*U[n][a]>= Xa[n][c][s];

forall(n in Nurses, c in Clinics, s in Shifts, a in Appointments)
M*U[n][a]>= Xt[n][c][s];

forall(n in Nurses, a in Appointments)
U[n][a]<= Bna[n][a];
};

execute writeoutput
{writeln("solution");
writeln(+AAssigned+","+FAssigned+","+TAssigned+","+CAssigned);
for(var n in Nurses){
writeln(+n+","+Z[n]);}
for(var n in Nurses){ for (var c in Clinics){ for (var s in Shifts){
if(Y[n][c][s]==1){\writeln(+n+","+c+","+s+","+Xa[n][c][s]+","+Xf[n][c][s]+
","+Xt[n][c][s]+","+Xc[n][c][s]);}}}}};
```

Appendix I : PartiSim Tools – Experiment with Model (Stage 5)

This section contains screenshots of tools prescribed by the PartiSim framework for Stage 5.

Name particular aspect/part of the model identified	Agree/Disagree	List suggested changes to the model

Figure 46: Screenshot of Experimentation Tool 1: Model Validation (Facilitator Form)

Name particular aspect of the model you would like to comment on	Agree/ Disagree	Suggested changes (change to model, data, etc.)

Figure 47: Experimentation Tool 1: Model Validation (Stakeholder Form)

Performance measure	Importance (Enter a value rating)
Number discharged / Throughput	
Number of referrals	
WL/ Number of people waiting for ...	
WL/ Number of people waiting for ...	
WL/ Number of people waiting for ...	
WL/ Number of people waiting for ...	
WL/ Number of people waiting for ...	
Average waiting time to X	
Target X	
Target X	
Target X	
Target X	

Figure 48: Screenshot of Experimentation Tool 2: Rate the Performance Measures

Scenario	Definition of Scenario	Performance Indicator 1		Performance Indicator 2		Performance Indicator 3		Performance Indicator 4		Performance Indicator 5		Performance Indicator 6	
		Average value	Ranking	Average value	Ranking	Average value	Ranking	Average value	Ranking	Average value	Ranking	Average value	Ranking
Current Scenario	1 Surgeon 1 Physician 100/month Referrals												
Scenario 1	+1 Surgeon-45 referrals/ month												
Scenario 2													
Scenario 3													
Scenario 4													
Scenario 5													

Figure 49: Screenshot of Experimentation Tool 3: Debate Alternative Scenarios (Form for Facilitator)

Scenario	Definition of scenario e.g. add 5 outpatient slots/ week	Comments

Figure 50: Experimentation Tool 3: Debate Alternative Scenarios (Form for Stakeholders)

Appendix J : PartiSim Tools – Implement Findings (Stage 6)

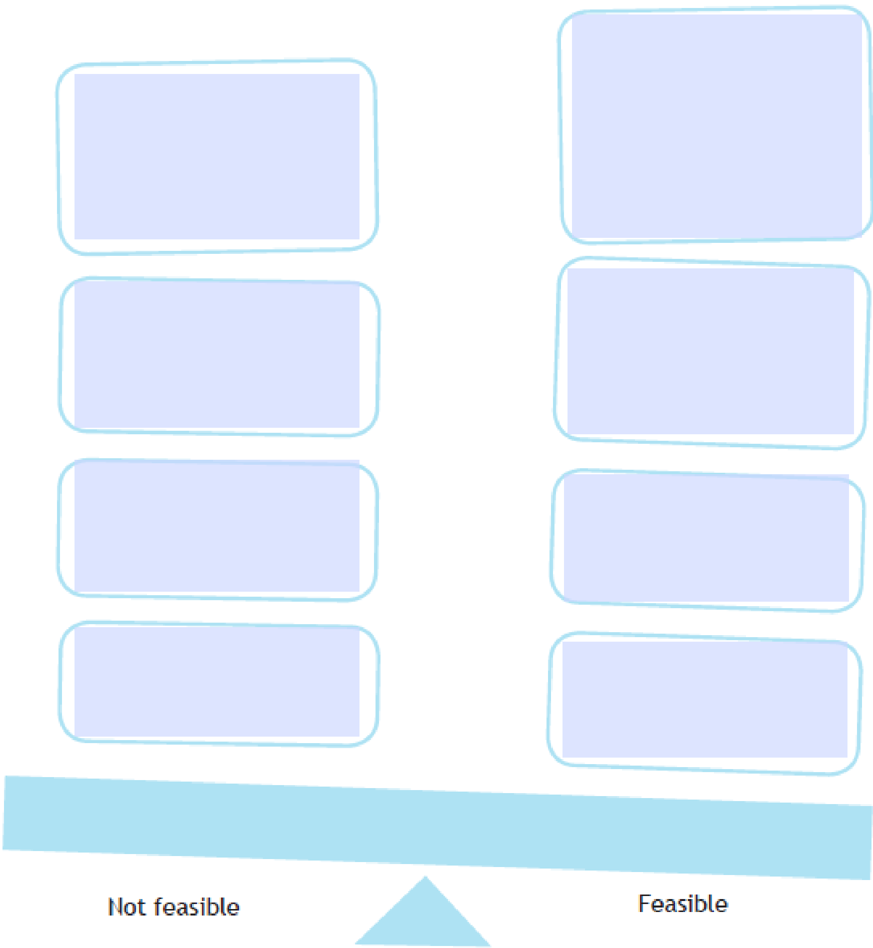

Figure 51: Screenshot of Implementation Tool 1: Feasibility and Risks Scale

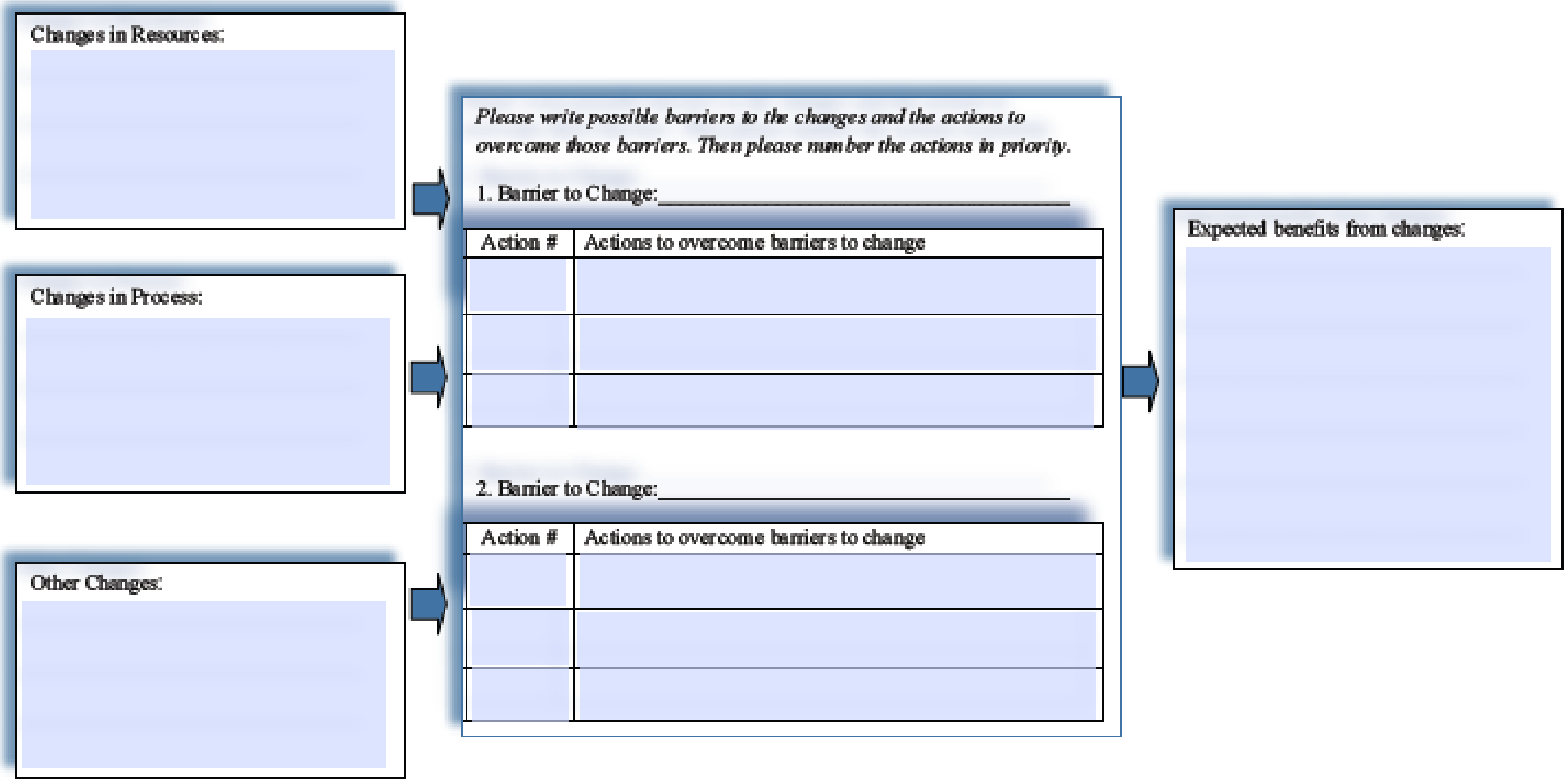

Figure 52: Screenshot of Implementation Tool 2: Barriers to Change

Action for change	Action and communication tasks	Action Leader	Stakeholders to communicate with	Date expected to complete Action	Date Achieved Action	Comments
1. e.g Hire Sugeon	Create report for HR	Mike Gray	Vanessa Brown Sara Halifax	12 May 2011	10 May 2011	HR approved report
2.						
3.						
4.						
5.						

Figure 53: Snapshot of Implementation Tool 3: Action and Communication Plan

Appendix K : Case Study Outputs: Experiment with Model (Stage 5)

This section contains screenshots of tools that were used directly from the PartiSim framework and presents new tools that were adapted for optimisation modelling.

Workshop 3

File Edit View Tools Extensions Help

Inputs	Changes/ Updates
Number of Clinicians, locations, appointment types, days, shifts	
Demand from Clinic Location	
Clinic locations assigned to clinicians	
Appointment durations	Consider average and not median duration
Clinician skills and appointment matrix	
Clinician to clinic preference	
Max shifts per day	No more than two
Max clinics per clinicians	
Max clinicians per clinic	Between 4 and 8 per clinic
Clinicians' availability matrix	Not standardised for all clinicians, collect data to determine availability based on clinician band.
Max travel distance between clinics	Consider 7.5 miles
Shift Duration	Give a one-hour window between each shift to account for lunch time and travel between locations.
Distance between clinics	

Figure 54: Google Docs Model Input Parameters Validation Tool (workshop 3a)

Workshop 3

File Edit View Tools Extensions Help

Constraints	Additions/Changes
Assign demand from clinics based on clinician capacity	
Assign Demand Based on clinician skill	
Assign appointments in clinics within designated geographical patch	Not accurate. Replace with new stricter clinic-location constraint
Max number of shifts per day	
Min number of shifts per day	
Clinician travel constraints between clinic locations	
Max number of clinician for each location	
Min number of clinician for each location	
Max number of locations for each clinician	
Min number of locations for each clinician	
Max clinician availability	Does not account for part-time staff, meetings, training days. Continuity of care not being considered. Annual Leave and Sick Leave
Min clinician availability	
	Daily limit on appointment allocation = 5 per day
	Continuity of care constraint, clinicians have to continue visiting clinics that are on their current caseload.

Figure 55: Google Docs Model Constrains Validation Tool (Workshop 3a)

Workshop 3

File Edit View Tools Extensions Help

	Number of clinicians	Number of Clinics	Clinics per clinician	Clinician Per Clinic	Distance between Clinics	Appointment Durations				Clinician to Location Preference	Max Clinician to Clinic Location	Max Clinic Location to Clinician	Shifts per day	Shift Length	Planning Horizon	Clinician Availability	Clinician to appointment skill requirement	Clinician to clinic caseload	Location based demand
Scenario 1						A= 60	F= 60	T= 45	C= 60							From Historical data			current
Scenario 2						A= 60	F= 60	T= 45	C= 60							From Historical data			Increase by 15%
Scenario 3						A= 60	F= 45	T= 30	C= 45							From Historical data			current
Scenario 4						A= 60	F= 45	T= 30	C= 45							From Historical data			Increase by 15%
Scenario 5						A= 60	F= 60	T= 45	C= 60							Standard			current
Scenario 6						A= 60	F= 60	T= 45	C= 60							Standard			Increase by 15%
Scenario 7						A= 60	F= 45	T= 30	C= 45							Standard			current
Scenario 8						A= 60	F= 45	T= 30	C= 45							Standard			Increase by 15%

Note:
Availability information can be obtained from Job Plans
Job plans 4 weeks based on Bands

Figure 56: Google Docs Scenario Parameter combination form (Workshop 3b)

Appendix L : Case Study Outputs: Implement Findings (Stage 6)

Table 47: Action and Communication Plan Output (Workshop 4)

Action for change	Action and communication tasks	Action Leader	Stakeholders to communicate with	Date expected to complete action	Data achieved action	Comments
Standardise intervention coding	In the pipeline, will be informed by study	Stakeholder J	Stakeholders K, L, and M	End of 2021		
Capture all patient related interventions		Stakeholder D	Stakeholders K, and L	Potentially mid 2022		
Standardise service delivery	In the pipeline, will be informed by study	Stakeholder J	Stakeholders K, L, and M	Potentially mid 2022		
Distribute workload equitably	Ongoing	Stakeholder D	Stakeholders J, K, L, and M	End of 2021		
Deploy optimisation model	To be put forward to wider trust	Project Champion (A)	Stakeholders C, D, J and Facilitator	Ongoing		

Appendix M : New Proposed Tool – Facilitated Post-Model Coding for Optimisation

Table 48: Scenario Input Selection Form

Model Inputs	Selection
Controllable Input Parameters	
Number of Appointment Types	✗
Appointment duration for each type	✓
Number of Clinicians	✗
Number of clinic locations	✗
Number of days in the planning horizon	✗
Number of available shifts in the planning horizon	✗
Number of shifts per day	✗
Length of each shift	✓
Maximum travel distance between clinics	✗
Clinician to appointment skill-based allocation.	✗
Clinician to clinic location allocation (& caseload)	*
Maximum number of shifts per day for each clinician	✗
Maximum number of clinic locations that can be assigned to a clinician	✗
Maximum number of clinicians that can be assigned to a clinic location	✗
Total available hours per clinician	✗
Clinician Availability	✓
Uncertain Input Parameters (generated)	
Demand from each clinician location for each appointment type	✓
Assumed/Generated Input Parameters	
Distance between clinic locations	✗