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Inspection Policy Optimization for Hierarchical Multistate Systems Under Uncertain Mission Scenarios: A Risk-Averse Perspective

Boyuan Zhang[a], Yu Liu[a,b1], Shaomin Wu[c]

[a]School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, PR China;

[b]Center for System Reliability and Safety, University of Electronic Science and Technology of China, Chengdu, Sichuan, PR China

[c]Kent Business School, University of Kent, Canterbury, Kent CT2 7FS, UK

Abstract

Most engineered systems intend to perform missions with a pre-specified target success probability to reduce undesirable failure risks. Before executing the next mission, inspection activities can be conducted across multiple physical levels of a system for assessing the probability of mission success. However, due to the randomness of a system's degradation behavior and the presence of measurement errors, inspection results inevitably contain uncertainty. Meanwhile, the mission duration and acceptable system states may also be uncertain due to uncontrollable factors, such as random operating environments and mission demands. In such a circumstance, it is of great significance to identify an optimal multilevel inspection policy to answer, as great confident as possible, the question that the system can complete the next mission with a target mission success probability. In this article, a novel metric is put forth to evaluate the benefit of a multilevel inspection policy to assess if the system can complete the next mission with the target success probability from a risk-averse perspective, based on which an optimization method is proposed to seek the optimal inspection policy under uncertain scenarios with the aim of minimizing the maximum regret of the proposed metric. A stochastic fractal search algorithm, along with two tailored local search rules, is designed to resolve the resulting optimization problem efficiently. Two cases, including a three-component system and a programmable logic controller control system, are used to illustrate the effectiveness of the proposed method. The results show that the proposed method is capable of effectively identifying the risk of mission failures by inspection policies.

Keywords: min-max regret; multilevel inspection; multistate systems; risk-averse; uncertain

[1] Corresponding author: email: yuliu@uestc.edu.cn

mission scenarios.

1 Introduction

The failure of an engineered system can cause potential hazards such as financial repercussions and significant safety accidents. To mitigate the risk of failures, the probability that a system successfully completes a mission can be required to be not less than a threshold, which is hereinafter referred to as the “target value”. For example, to guarantee the completion of a flight mission, an aircraft must satisfy a minimum mission success probability pre-specified by its airline company and/or aircraft manufacturer (Tiassou et al., 2013). It is therefore of great importance to assess if the mission success probability reaches the target value or not and then determine whether the subsequent mission should be continued or not. An interesting question arises: how can we ensure that the mission success probability meets the target value before starting the subsequent mission?

Inspections, enabling us to acquire a better understanding of a system’s and its components’ health status, can be one of the most effective methods to ensure if a target mission success probability can be satisfied or not (Alaswad and Xiang, 2017; Zhu and Xiang 2021; Zhu et al., 2021). In practice, an engineered system can be composed of multiple subsystems, each of which is composed of multiple components (Jiang and Liu, 2017; Li et al., 2014). Thanks to the hierarchical structure of engineered systems, we can inspect a system across multiple physical levels, such as system level, subsystem level, and component level, for which multilevel inspections can therefore adopted. Multilevel inspections can provide more details on a system’s and its constitutive components’ health status, and therefore accurately assess the mission success probability. Nevertheless, owing to insufficient resources such as time, manpower, and cost, it is often infeasible to collect inspection data from all system’s physical levels simultaneously (Liu et al., 2020). It is thus necessary to optimally distribute the constrained resources among various physical levels of the system.

Due to the discrepancy among manufacturing processes (Mondal et al., 2014) and the dynamicity of the operating environment and load (Jiang et al., 2023), the deteriorating processes of engineered systems inherently and inevitably contain uncertainty. On the other hand, the true states of the components, subsystems, and entire system may not be accurately observed by inspection activities because of various reasons such as low sensor accuracy, measurement errors, or poor diagnostic algorithms (Eben-Chaime, 2022; Liu et al., 2022). Additionally, the upcoming scenarios of the subsequent mission profiles, including mission durations and acceptable states that the system can complete the subsequent missions, may be uncertain owing to random operating environments and mission demands. For instance, the mission durations of a cooling/heating system are subject to uncertain weather conditions (Levitin et al., 2020), and the performance of a continuous production system needs to meet stochastic market demands (Li et al., 2014). The intrinsic imprecision of inspection techniques, compounded with the stochastic nature of the deteriorating process and the variability of the mission scenario, collectively engender unreliable and risky inspection results, and permeate through the subsequent decision-making predicated upon these results. Given the limitations of multilevel inspection optimization models in the existing literature in addressing the aforementioned issues, it is imperative to develop a risk-averse multilevel optimization framework that can ensure the mission success probability of the system meet the target value.

This article aims to propose methods to identify if the probability of mission success can reach the target value by multilevel inspection policies. The methods can address the uncertainty associated with inspection results and decision-making, which is developed for hierarchical multi-state systems (MSSs) from a risk-averse perspective. The unique contributions of this study are trifold:

1) A novel metric is put forth to evaluate the benefit of a particular multilevel inspection policy to answer if the system can complete the next mission with the pre-specified target success probability.
2) A risk-averse optimization model is formulated to take account of uncertain mission scenarios, including uncertain mission durations and uncertain acceptable system states. A regret function of the proposed metric is developed to evaluate the potential risk of conducting a specific inspection policy. The objective of the optimization model is to minimize the maximum regret of the proposed metric.
3) Two tailored local search rules are designed to resolve the resulting optimization model in a computationally efficient manner. In the proposed local search rules, the maximum regret of the proposed metric can be further used to enhance the global searching capability.

The remainder of this article is structured as follows. Section 2 conducts a literature review on existing work. Section 3 describes an inspection optimization problem for hierarchical MSSs under study. The formulation of the proposed metric is developed in Section 4. In Section 5, a risk-averse optimization model is put forth, and a stochastic fractal search (SFS) algorithm with two tailored local search rules is designed to resolve the optimization model. In Section 6, the effectiveness of the proposed method is examined via two illustrative examples. Section 7 is a closure and a summary of future work.

2 Literature review

The focus of this article is on optimizing multilevel inspection policies, which involves both the evaluation of system reliability with multilevel inspection data and the optimization of inspection policies. To identify research gaps, we undertake a literature review on data-driven dynamic reliability assessment and inspection policy optimization.

2.1 *Data-driven dynamic reliability assessment*

Conventionally, the reliability of an engineered system was estimated from the failure data of a set of systems (Jiang and Liu, 2017). Such reliability models ignored the heterogeneity of the systems, and the related reliability measure remains uniform for a population of the systems consistently (Lisnianski and Levitin, 2003; Kuo and Zuo, 2003; Natvig, 2010; Huang et al., 2021; Wang et al., 2022; Hong et al., 2018; Lu et al., 2021). For specific individual systems, their heterogeneity may be depicted by internal factors such as the degradation levels (Banjevic, 2009; Tian and Liao, 2011; Gorjian et al., 2010; Liu and Zio, 2017) or external factors such as loading history (Mathew et al., 2006; Pecht and Michael, 2009). Data reflecting such heterogeneity can be collected to dynamically update reliability of a system. This method, known as dynamic reliability assessment, has received significant attention in recent decades (Banjevic, 2009; Liu and Zio, 2017; Liu et al., 2015; Liu and Chen, 2017; Mathew et al., 2006). For simplicity of presentation, we refer data collected at the component level to as component level data hereinafter. Similarly, we have subsystem level data and system level data. Heterogeneous data is referred to data reflecting the heterogeneity of the systems.

The dynamic reliability assessment method uses data on internal and/or external factors from either single or multiple physical levels of a system to update reliability of the specific system. Single-level data is referred to as those data collected from the component level or the system level where the system is regarded as a single entity (Banjevic, 2009; Liu and Zio, 2017; Mathew et al., 2006). For example, Banjevic (2009) dynamically assessed the reliability of a binary-state system based on the condition that the system is functioning. (Banjevic, 2009). Liu and Zio (2017) updated the reliability and remaining useful life (RUL) of a system with the monitored degradation data of components. It is worth noting that the prediction of RUL can be regarded as a dynamic reliability assessment method with single-level data, as the reliability of a single-unit system can be deduced by the distribution of the RUL (Ghasemi et al., 2010; Ye et al., 2015; Si, 2015; Si et al., 2011; Kim and Liu, 2020). Multilevel data are collected among multiple physical levels of the system (Jiang and Liu, 2017; Li et al., 2014). High-level data can offer additional knowledge regarding the health status of its components, thereby facilitating the dynamic reliability assessment of a specific system (Liu and Chen, 2017). Liu et al. (2015) updated the reliability of a particular system dynamically with system-level data. Liu and Chen (2017) proposed a recursive Bayesian method to assess the reliability of a specific individual system with imperfect multilevel data.

All the above-discussed studies were dedicated to update the reliability function of a specific individual system with heterogeneous data. Nevertheless, none of the existing works has been done to evaluate the effectiveness of data and quantified the impact of uncertainty on the inferred results.

2.2 *Inspection policy optimization*

The optimization of inspection policies aims to seek the best decision variables for a given criterion such as the minimum expected cost of maintaining the availability of a system (Alaswad and Xiang, 2017). The decision variables in an inspection policy optimization can be categorized as: “when to inspect”, “what extent to inspect”, and “what to inspect”. “when to inspect” refers to determining the optimal inspection time interval, which can be periodic (Taghipour et al., 2010; Zhao et al., 2021; Huynh et al., 2011; Zhang et al., 2022; Chen et al., 2015; Shi et al., 2019) and nonperiodic inspections (Hajipour and Taghipour, 2016; Li et al., 2022; Castro and Landesa, 2019). “what extent to inspect” aims to determine the optimal inspection threshold to abort the inspection activity and implement the subsequent decision. The inspection thresholds conclude the system degradation level (Zhao et al., 2019; Truong-Ba et al., 2021; Guo et al., 2013; Do et al., 2015), the number of inspections (Zhao et al., 2019), and the risk indicator (Naderkhani ZG and Makis, 2015). “what to inspect” is a decision that selects appropriate objects and techniques of the system to be inspected (Jesus and Daniel, 2019; Liu et al., 2020). In this article, the optimization of inspection policies involves the determination of objects to be inspected and the corresponding inspection techniques, which is a typical “what to inspect” problem. We will therefore concentrate the literature review on “what to inspect”.

Jesus and Daniel (2019) optimized the regular inspection interval and, at each inspection time, further determined the number of inspected components and the collection of components to be inspected. Bismut and Straub (2021) determined the components that needed inspecting at each inspection time and studied an adaptive plan that the heuristic inspection plan is updated once new information through inspections is available. Several complementary methods were proposed by Chen et al. (2020) to seek optimal pipeline inspection routes considering platform limitations. Vereecken et al. (2020) developed an extended pre-posterior framework to optimize the time, object, and technique for the inspection using heuristics decisions. To adequately utilize multilevel inspection data, Liu et al. (2020) introduced a metric to evaluate the benefit of a multilevel inspection policy for revealing the true state of an MSS. They optimized limited inspection resources across multiple physical levels and the corresponding techniques based on the new metric.

Indeed, it is necessary to inspect a system to determine if the system can complete subsequent missions with the target mission success probability. The objective of aforementioned work is to identify the true state of the system more accurately, rather than to facilitate inspection policies from the perspective of mission completion.

3 Problem descriptions and model assumptions

This article aims to develop inspection policies to answer the question of whether the probability of mission success can satisfy the pre-specified target value or not. A new metric is proposed to assess the benefit of an individual inspection policy, and a framework using the proposed metric is delineated in Fig. 1. The system has executed k missions, and the historical inspection data are collected by executing inspection activities before the execution of each mission. We investigate if the success probability of the $(k+1)$th mission can meet the target value or not. The true system state can have two scenarios: the corresponding mission success probability is greater or equal to (Scenario 1) or less than (Scenario 2) the pre-specified target value. In the ideal case, the inference result would be perfectly consistent with the categorization of the true system state. However, the true system state is unknown after the completion of the kth mission, and it can be inferred by the initial system state, each component's deteriorating process, the historical inspection data, and the inspection policy. Meanwhile, due to the stochastic nature of the deteriorating behavior and the measurement error, the inferred system state is often unreliable and uncertain. How confident can we be in determining that the mission success probability can satisfy the target value by the inferred system state? The level of confidence is the metric that we need to assess, which will be elaborated in Section 4. The optimal inspection policy is selected by the proposed metric and the optimization model, as discussed in Section 5.

Without loss of generality, we encapsulate the fundamental assumptions underlying the studied system and its associated inspection problem as follows:

(1) A system is composed of M components, belonging to N subsystems, in a hierarchal fashion. The states of the system, subsystem h ($h \in \{1, \ldots, N\}$), and component l ($l \in \{1, \ldots, M\}$) at time t are denoted by $S(t) \in \{1, \ldots, N_S\}$, $S_h(t) \in \{1, \ldots, N_h\}$, and $C_l(t) \in \{1, \ldots, n_l\}$, respectively. A greater state suggests a state with better performance. As such, state 1 represents the worst state, whereas states N_S, N_h, and n_l represent the best of the system, subsystem h, and component l, respectively.

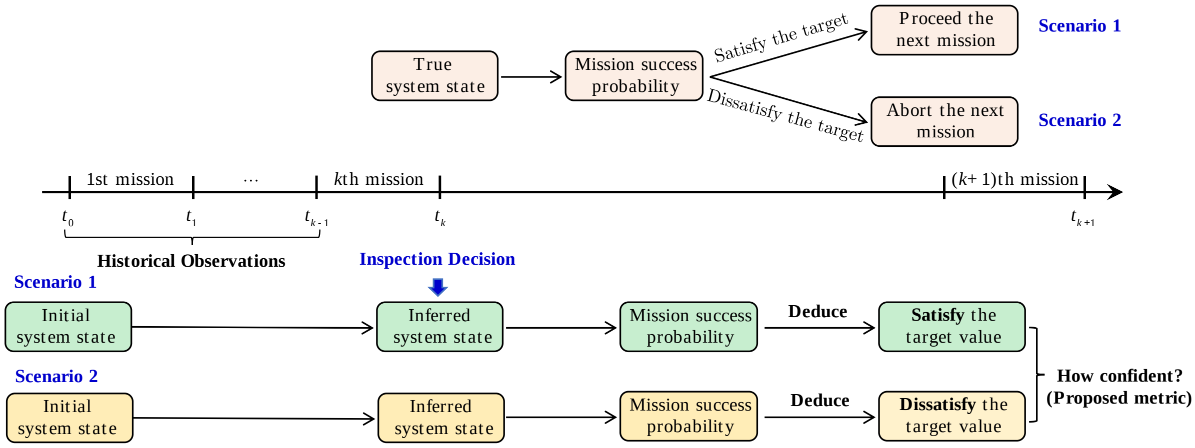

Figure 1. A framework of the proposed metric.

(2) The deteriorating process of each component follows a homogenous continuous-time Markov chain. All the components are *s*-independent of each other.

(3) The system state is a function of its components' states, $S(t) = f(C_1(t), \ldots, C_M(t))$, where $f(\cdot)$ is the system structure function. Similarly, the state of each subsystem is a function of its components' states as well.

(4) The state space of the system is denoted by $\mathbb{S} = \{\mathbf{s}_1, \ldots, \mathbf{s}_{N_S}\}$, where $\mathbf{s}_i = \{s(i,1), \ldots, s(i,L_i)\}$, $s(i,v)$ is a particular combination of components' states when the system stays at state i. $|\mathbb{S}|$ is equal to $\prod_{l=1}^{M} n_l$. Let $s(i,v,l)$ and $s(i,v,S_h)$ denote the states of component l and subsystem h when the components' states is $s(i,v)$, where $s(i,v,l) \in \{1, \ldots, n_l\}$, and $s(i,v,S_h) \in \{1, \ldots, N_h\}$, respectively.

(5) The status of an entity (which could be the system, a subsystem, or a component) can be mistakenly identified by an inspection at the end of the last mission. The classification matrices of the system, subsystem h, and component l are denoted as $\mathbf{A} = [a_{i,o}]_{N_S \times N_S}$, $\mathbf{B}_h = [b_{h,(i,o)}]_{N_h \times N_h}$,

$\mathbf{C}_l = [c_{l,(i,o)}]_{n_l \times n_l}$, respectively, qualifying the misclassification probabilities of multilevel inspections. The entries $a_{i,o}$, $b_{h,(i,o)}$, and $c_{l,(i,o)}$ represent the probabilities of identifying state i to be state o for various entities.

(6) The system has executed k missions and the inspection data are collected by executing inspection activities during each break between two adjacent missions, as shown in Fig. 1. The states of the components remain unchanged during the break. The completion time and the duration of the i th mission are denoted by t_i and τ_i, respectively, and we have $\tau_i = t_i - t_{i-1}$.

(7) The historical inspection data in the interval $(0, t_{k-1})$ are denoted by $\mathbf{O}_{k-1} = \{\mathrm{O}_0, \ldots \mathrm{O}_{k-1}\}$, where O_i represents the inspection data collected at time t_i, and $\mathrm{O}_i = \{O_{i,S}; O_{i,S_1}, \ldots, O_{i,S_N}; O_{i,1}, \ldots, O_{i,l}\}$. $O_{i,S}$, O_{i,S_h}, and $O_{i,l}$ are the inspection data of the system, subsystem h, and component l at time t_i, respectively. The predicted inspection data collected at time t_k are denoted by $\hat{\mathrm{O}}_k = \{\hat{O}_{k,S}; \hat{O}_{k,S_1}, \ldots, \hat{O}_{k,S_N}; \hat{O}_{k,1}, \ldots, \hat{O}_{k,M}\}$.

(8) Before the system executes the $(k+1)$th mission, an inspection policy is conducted at time t_k. The policy is denoted by $\mathbf{I}_k = (I_{k,S}; I_{k,S_1}, \ldots I_{k,S_N}; I_{k,1}, \ldots I_{k,M})$, where $I_{k,S}, I_{k,S_h}$, and $I_{k,l}$ denote the inspection techniques applied on the system, subsystem h, and component l, respectively. $I_{k,S} \in \{0,1,2,\ldots,|I_{k,S}|\}, I_{k,S_h} \in \{0,1,2,\ldots,|I_{k,S_h}|\}$, and $I_l \in \{0,1,2,\ldots,|I_{k,l}|\}$, where $|I_{k,S}|, |I_{k,S_h}|,$ and $|I_{k,l}|$ represent the numbers of available inspection techniques for the system, subsystem h, and component l, respectively. Inspection technique 0 represents the corresponding object is not to be inspected.

(9) $S(t_k)$ and $\hat{S}(t_k)$ denote the true state and the inferred state of the system at time t_k, respectively.

4 Proposed metric for assessing mission success probability

The mission success probability is defined as the probability that the system completes its pre-specified mission. w_{k+1} denotes as an acceptable threshold state of the $(k+1)$th mission. The degradation profile of the system is distinct when its components stay in different states, resulting in a diverse mission success probability. Meanwhile, the mission success probability is also affected by the mission duration and the acceptable threshold state. The success probability of the $(k+1)$th mission when the system stays in $s(i,v)$ is therefore formulated as:

$$R_{i,v}(w_{k+1},\tau_{k+1})=\sum_{u=w_{k+1}}^{N_S}\sum_{n=1}^{L_u}\Pr\{S(t_{k+1})=s(u,n)\mid S(t_k)=s(i,v)\}, \tag{1}$$

where $\tau_{k+1}=t_{k+1}-t_k$. $\Pr\{S(t_{k+1})=s(u,n)\mid S(t_k)=s(i,v)\}$ is the probability of the system state transiting from $s(i,v)$ to $s(u,n)$, and it can be deduced by:

$$\Pr\{S(t_{k+1})=s(u,n)\mid S(t_k)=s(i,v)\}=\prod_{l=1}^{M}\Pr\{C_l(t_{k+1})=s(u,n,l)\mid C_l(t_k)=s(i,v,l)\}, \tag{2}$$

where $\Pr\{C_l(t_{k+1})=s(u,n,l)\mid C_l(t_k)=s(i,v,l)\}$ is the transition probability of component l, and it can be evaluated by solving the Kolmogorov differential equations (Lisnianski and Levitin, 2003).

The target success probability of the $(k+1)$th mission is denoted by r_{k+1}. We aim to determine if the success probability of the $(k+1)$th mission can meet the target value, i.e., $R_{k+1}\geq r_{k+1}$. In fact, $R_{k+1}\geq r_{k+1}$ can be satisfied if the system is in a certain specific state at time t_k. All possible specific states are included in the set $\mathcal{A}_{k+1}=\{s(i,v)\mid R_{i,v}(w_{k+1},\tau_{k+1})\geq r_{k+1}\}$. Therefore, we only need to identify if $S(t_k)\in\mathcal{A}_{k+1}$ holds.

Apparently, executing different inspection policies would obtain different inspection data, potentially leading to various inference results. Therefore, we first need to propose a metric to quantify the effectiveness of inspection policies to select an appropriate inspection policy under limited resources. If $S(t_k)\in\mathcal{A}_{k+1}$, we wish that $\hat{S}(t_k)\in\mathcal{A}_{k+1}$ with the highest possible probability via an inspection strategy. Otherwise, if $S(t_k)\notin\mathcal{A}_{k+1}$, we wish that $\hat{S}(t_k)\notin\mathcal{A}_{k+1}$ with the highest possible probability via the inspection strategy. Therefore, the proposed metric can be formulated as:

$$P(\mathcal{A}_{k+1},\mathbf{I}_k)=\sum_{n=0}^{1}\Pr\left\{1_{\mathcal{A}_{k+1}}\left(\hat{S}(t_k)\right)=n\mid 1_{\mathcal{A}_{k+1}}\left(S(t_k)\right)=n,\mathbf{O}_{k-1},\mathbf{I}_k\right\}\times\Pr\left\{1_{\mathcal{A}_{k+1}}\left(S(t_k)\right)=n\mid\mathbf{O}_{k-1}\right\}, \tag{3}$$

where $1_{\mathcal{A}_{k+1}}(\cdot)$ is an indicator function to identify if the system state belongs to $\mathcal{A}_{k+1}$, that is,

$$1_{\mathcal{A}_{k+1}}\left(s(i,v)\right)=\begin{cases}1, & \text{if } s(i,v)\in\mathcal{A}_{k+1}\\ 0, & \text{if } s(i,v)\notin\mathcal{A}_{k+1}\end{cases}. \tag{4}$$

$\Pr\left\{1_{\mathcal{A}_{k+1}}\left(S(t_k)\right)=1\mid\mathbf{O}_{k-1}\right\}$ in Eq. (3) is the probability that the true system state belongs to $\mathcal{A}_{k+1}$. $\Pr\left\{1_{\mathcal{A}_{k+1}}\left(\hat{S}(t_k)\right)=1\mid 1_{\mathcal{A}_{k+1}}\left(S(t_k)\right)=1,\mathbf{O}_{k-1},\mathbf{I}_k\right\}$ in Eq. (3) represents the probability that given the historical inspection data $\mathbf{O}_{k-1}$ and the inspection policy $\mathbf{I}_k$, the inferred system state belongs to $\mathcal{A}_{k+1}$ when the true system state belongs to $\mathcal{A}_{k+1}$.

4.1 *Procedure of evaluating the proposed metric*

In order to proceed with the derivation of the proposed metric, several important notes need to be mentioned here: (1) Due to the stochastic deterioration process and imperfect inspection techniques, the predicted inspection data often are not certain values. $\hat{O}_k$ denotes the possible inspection data, and the set of all possible inspection data via the inspection policy $\mathbf{I}_k$ is denoted by $O(\mathbf{I}_k)$; (2) If $\hat{O}_k$ is missing, the inferred system state $\hat{S}(t_k)$ degenerates to $S(t_k)$; (3) $\hat{O}_k$ and $S(t_k)$ are independent of the inferred system state. Based on the above notes, Eq. (3) can be expanded to:

$$
\begin{aligned}
P(A_{k+1}, \mathbf{I}_k) &= \sum_{n=0}^{1} \Pr\left\{1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n \mid 1_{A_{k+1}}\left(S(t_k)\right) = n, \mathbf{O}_{k-1}, \mathbf{I}_k\right\} \times \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \Pr\left\{1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, \hat{O}_k \mid 1_{A_{k+1}}\left(S(t_k)\right) = n, \mathbf{O}_{k-1}\right\} \times \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \Pr\left\{1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, 1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid 1_{A_{k+1}}\left(S(t_k)\right) = n, \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n \mid 1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, \hat{O}_k, \mathbf{O}_{k-1}\right\} \\
&\qquad \times \Pr\left\{1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \frac{\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid 1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, \mathbf{O}_{k-1}\right\}}{\Pr\left\{\hat{O}_k \mid 1_{A_{k+1}}\left(\hat{S}(t_k)\right) = n, \mathbf{O}_{k-1}\right\}} \\
&\qquad \times \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \frac{\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\}}{\Pr\left\{\hat{O}_k \mid \mathbf{O}_{k-1}\right\}} \times \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{\hat{O}_k \in O(\mathbf{I}_k)} \sum_{n=0}^{1} \frac{\left(\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\}\right)^2}{\sum_{n=0}^{1} \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\}}
\end{aligned}
\tag{5}
$$

The proposed metric can be evaluated by $\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\}$ in Eq. (5). $\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\}$ can be further extended to:

$$
\begin{aligned}
\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, \hat{O}_k \mid \mathbf{O}_{k-1}\right\} &= \sum_{s(i,v) \in S} \Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n, S(t_k) = s(i,v), \hat{O}_k \mid \mathbf{O}_{k-1}\right\} \\
&= \sum_{s(i,v) \in S} \underbrace{\Pr\left\{1_{A_{k+1}}\left(S(t_k)\right) = n \mid S(t_k) = s(i,v), \hat{O}_k, \mathbf{O}_{k-1}\right\}}_{\text{Part 1}} \\
&\qquad \times \underbrace{\Pr\{\hat{O}_k \mid S(t_k) = s(i,v), \mathbf{O}_{k-1}\}}_{\text{Part 2}} \times \underbrace{\Pr\{S(t_k) = s(i,v) \mid \mathbf{O}_{k-1}\}}_{\text{Part 3}}
\end{aligned}
\tag{6}
$$

"Part 1" of Eq. (6) only depends on $S(t_k)$ and A_{k+1}. It can be, then, simplified by:

$$\Pr\left\{1_{\mathcal{A}_{k+1}}\left(S(t_k)\right) = n \mid S(t_k) = s(i,v), \hat{O}_k, \mathbf{O}_{k-1}\right\} = \Pr\left\{1_{\mathcal{A}_{k+1}}\left(s(i,v)\right) = n\right\}, \tag{7}$$

where $1_{\mathcal{A}_{k+1}}\left(s(i,v)\right)$ is a binary variable, taking value from 1 and 0 depending on the condition of $s(i,v) \in \mathcal{A}_{k+1}$. If $1_{\mathcal{A}_{k+1}}\left(s(i,v)\right)$ equates n, $\Pr\left\{1_{\mathcal{A}_{k+1}}\left(s(i,v)\right) = n\right\}$ is equal to 1. Otherwise, $\Pr\{1_{\mathcal{A}_{k+1}}\left(s(i,v)\right) = n\}$ is equal to 0.

"Part 2" of Eq. (6) is the occurrence probability of the predicted inspection data $\hat{O}_k$. The occurrence of $\hat{O}_k$ is dependent on the true system state and the inspected object. Meanwhile, inspection data among various levels are assumed to be mutually independent. Hence, "Part 2" of Eq. (6) can be evaluated by:

$$\begin{aligned}
&\Pr\{\hat{O}_k | S(t_k) = s(i,v), \mathbf{O}_{k-1}\} \\
&= \Pr\left\{\hat{O}_k = \left(\hat{O}_{k,S}; \hat{O}_{k,S_1}, \ldots, \hat{O}_{k,S_N}; \hat{O}_{k,1}, \ldots, \hat{O}_{k,M}\right) | S(t_k) = s(i,v)\right\} \\
&= \Pr\{\hat{O}_{k,S} \mid S(t_k) = s(i,v)\} \times \prod_{h=1}^{N} \Pr\{\hat{O}_{k,S_h} \mid S_h(t_k) = s(i,v,S_h)\} \cdot \\
&\quad \times \prod_{l=1}^{M} \Pr\{\hat{O}_{k,l} \mid C_l(t_k) = s(i,v,l)\}
\end{aligned} \tag{8}$$

Equation (8) can be further computed via the classification matrix (Liu et al. 2020).

"Part 3" of Eq. (6) is the true distribution of the system's components staying in certain states at time t_k, and it can be inferred by the deteriorating process of its components, that is:

$$\begin{aligned}
&\Pr\{S(t_k) = s(i,v) \mid \mathbf{O}_{k-1}\} \\
&= \sum_{s(j,m) \in \mathcal{S}} \underbrace{\Pr\{S(t_k) = s(i,v) \mid S(t_{k-1}) = s(j,m)\}}_{\text{Part 1}} \underbrace{\Pr\{S(t_{k-1}) = s(j,m) \mid \mathbf{O}_{k-1}\}}_{\text{Part 2}},
\end{aligned} \tag{9}$$

where "Part 1" in Eq. (9) is calculated in the way similar to Eq. (2). "Part 2" in Eq. (9) is the distribution of components staying in certain particular states at time t_{k-1}, given the historical inspection data $\mathbf{O}_{k-2}$ and the current inspection data at time t_{k-1}. It can be assessed by the dynamic reliability assessment method, which will be elaborated in the ensuing section.

In summary, "Part 1" to "Part 3" of Eq. (6) can be calculated by Eqs. (7) to (9), and the proposed metric can be further evaluated by Eq. (6).

4.2 *Dynamic reliability assessment method*

"Part 2" of Eq. (9) can be calculated by the dynamic reliability assessment method, that is:

$$\Pr\{S(t_{k-1}) = s(j,m) \mid \mathbf{O}_{k-1}\} = \frac{\Pr\{O_{k-1} \mid S(t_{k-1}) = s(j,m)\} \times \Pr\{S(t_{k-1}) = s(j,m) \mid \mathbf{O}_{k-2}\}}{\sum_{s(u,n) \in \mathcal{S}} \Pr\{O_{k-1} \mid S(t_{k-1}) = s(u,n)\} \times \Pr\{S(t_{k-1}) = s(u,n) \mid \mathbf{O}_{k-2}\}}, \tag{10}$$

where $\Pr\{\mathrm{O}_{k-1}|S(t_{k-1})=s(j,m)\}$ is the occurrence probability of O_{k-1} when the system stays in $s(j,m)$ at time t_{k-1}, and it can be evaluated by the same way as Eq. (6). $\Pr\{S(t_{k-1})=s(j,m)\,|\,\mathbf{O}_{k-2}\}$ can be computed by the deteriorating process of each component and the updated results at time t_{k-2} with the historical inspection data $\mathbf{O}_{k-2}$, that is:

$$\Pr\{S(t_{k-1})=s(j,m)\,|\,\mathbf{O}_{k-2}\} = \sum_{s(u,n)\in S} \underbrace{\Pr\{S(t_{k-1})=s(j,m)\,|\,S(t_{k-2})=s(u,n)\}}_{\text{Part 1}} \underbrace{\Pr\{S(t_{k-2})=s(u,n)\,|\,\mathbf{O}_{k-2}\}}_{\text{Part 2}}. \quad (11)$$

Equation (11) can be evaluated by the same way as Eq. (9). Based on Eqs. (9) to (11), $\Pr\{S(t_{k-1})=s(j,m)\,|\,\mathbf{O}_{k-1}\}$ can be, therefore, computed recursively by $\mathbf{O}_{k-1}$ and the initial system state $\Pr\{S(t_0)=s(j,m)\}$.

4.3 *Proposition of the proposed metric*

Proposition 1: If inspection activities are not implemented at t_k, that is, $\mathbf{I}_k=(0;0,\ldots,0;0,\ldots,0)$, the value of the proposed metric is equal to:

$$P\left(\mathrm{A}_{k+1},\mathbf{I}_k=(0;0,\ldots,0;0,\ldots,0)\right)=\left(\Pr\left\{1_{\mathrm{A}_{k+1}}\left(S(t_k)\right)=0\,|\,\mathbf{O}_{k-1}\right\}\right)^2+\left(\Pr\left\{1_{\mathrm{A}_{k+1}}\left(S(t_k)\right)=1\,|\,\mathbf{O}_{k-1}\right\}\right)^2.$$

The proof of Proposition 1 is given in Appendix A.

Proposition 2: The proposed metric lies in the range of [1/2,1].

The proof of Proposition 2 is shown in Appendix B.

Proposition 3: If inspections are non-informative, i.e., the elements within each classification matrix are equal, the proposed metric is equivalent to the case where inspections are absent (see Proposition 1), that is:

$$P(\mathrm{A}_{k+1},\mathbf{I}_k)=\left(\Pr\left\{1_{\mathrm{A}_{k+1}}\left(S(t_k)\right)=0\,|\,\mathbf{O}_{k-1}\right\}\right)^2+\left(\Pr\left\{1_{\mathrm{A}_{k+1}}\left(S(t_k)\right)=1\,|\,\mathbf{O}_{k-1}\right\}\right)^2.$$

The proof of Proposition 3 is given in Appendix C.

Proposition 4: If all the components are perfectly inspected, i.e., $I_{k,l}=|I_{k,l}|\ (l=1,\ldots,M)$ and all the diagonal elements of classification metrices are equal to one, the proposed metric equates 1, i.e., $\mathrm{P}\left(\mathrm{A}_{k+1},\mathbf{I}_k=(0;0,\ldots 0;|I_{k,1}|,\ldots,|I_{k,M}|)\right)=1$.

The proof of Proposition 4 is presented in Appendix D.

Proposition 5: If the inspected object with any inspection technique is added, the proposed metric of the new inspection policy is no less than that of the original inspection policy.

The proof of Proposition 5 is presented in Appendix E.

4.4 *Extension to uncertain mission scenarios*

The upcoming scenarios of the subsequent mission, including durations and acceptable threshold state, may be uncertain owing to random operating environments and unexpected events. Different mission scenarios will lead to different mission success probabilities, affecting the calculation of the proposed metric. Let $w_{k+1,m}$ and $t_{k+1,m}$ denote the acceptable threshold state and the duration of the subsequent mission under the m th mission scenario, respectively. $\mathrm{A}_{k+1,m}$ is a system state set and can be given by $\mathrm{A}_{k+1,m} = \{s(i,v) \mid R_{i,v}(w_{k+1,m}, t_{k+1,m}) \geq r_{k+1}\}$, where $R_{i,v}(w_{k+1,m}, t_{k+1,m})$ can be computed via Eqs. (1) and (2). The proposed metric $P(\mathrm{A}_{k+1,m}, \mathbf{I}_k)$ can be, then, calculated in the same manner as Section 4.1.

5 Risk-averse inspection policy modeling and optimization

The proposed metric is essentially a measure to quantify the effectiveness of an inspection policy. On the one hand, the magnitude of the metric varies among different inspection policies. On the other hand, the proposed metric of the same inspection policy may vary under different mission scenarios. Therefore, a specific mission scenario has its specific optimal inspection policy, but this policy may not accommodate other mission scenarios. We can, of course, take account of the uncertainty associated with mission scenarios and obtain a compromise and risk-neutral optimal inspection policy. However, if the potential consequences of misidentification are severe and undesirable, utilizing the compromise optimal inspection policy may entail a significant risk under the most extreme or unfavorable scenario. We therefore propose an optimization model of inspection policies from a risk-averse perspective.

In the risk-averse optimization model, the regret of an inspection policy under a specific mission scenario is formulated to evaluate the relative loss of the proposed metric between the inspection policy and the optimal inspection policy under the specific mission scenario. We wish to minimize the maximum regret of various mission scenarios to ensure the effectiveness of the selected inspection is acceptable even when the system is at the worst mission scenario. The solution algorithm with two tailored local search rules is put forth to resolve the optimization model in a computationally efficient manner. More details on the risk-averse optimization model and the solution methodology are shown as follows.

5.1 *Risk-averse inspection modeling*

The optimal inspection policy under a specific mission scenario can be obtained as follows. Various optional inspection techniques might be selected for each inspected object. Each inspection technique possesses its own accuracy to reflect the true state of the inspected object and its corresponding cost. By maximizing the proposed metric, a constrained optimization can be constructed to identify the optimal inspection policy under limited resources. The optimization problem is formulated as:

$$\mathbf{I}_{k,m}^{*} = \arg\max_{\mathbf{I}_k} P(\mathrm{A}_{k+1,m}, \mathbf{I}_k), \tag{12}$$

subject to:

$$C_{S,I_{k,S}} + \sum_{h=1}^{N} C_{S_h,I_{k,S_h}} + \sum_{l=1}^{M} C_{l,I_{k,l}} \le C_{\max}, \tag{13}$$

$$I_{k,S} \in \{0,1,2,\ldots,|I_{k,S}|\}, \tag{14}$$

$$I_{k,S_h} \in \{0,1,2,\cdots,|I_{k,S_h}|\}, \tag{15}$$

$$I_{k,l} \in \{0,1,2,\cdots,|I_{k,l}|\}. \tag{16}$$

where $\mathbf{I}_{k,m}^{*}$ denotes the optimal inspection policy under the *m*th mission scenario. $C_{S_h,I_{k,S_h}}$, $C_{S,I_{k,S}}$, and $C_{l,I_{k,l}}$ represent the cost of inspecting the system, subsystem h, and component l by the $I_{k,S}$ th, the I_{k,S_h} th, and the $I_{k,l}$ th inspection techniques, respectively. $C_{\max}$ is the inspection budget. Constraint (13) guarantees that the total cost cannot surpass the limited budget. Constraints (14) to (16) specify inspection techniques that can be selected.

The regret of an inspection policy under the *m*th mission scenario is denoted by $r(\mathbf{I}_k, m)$, and it is given by:

$$r(\mathbf{I}_k, m) = P(\mathrm{A}_{k+1,m}, \mathbf{I}_{k,m}^{*}) - P(\mathrm{A}_{k+1,m}, \mathbf{I}_k), \tag{17}$$

where $P(\mathrm{A}_{k+1,m}, \mathbf{I}_{k,m}^{*})$ denotes the proposed metric of the optimal inspection policy under the *m*th mission scenario. Based on $r(\mathbf{I}_k, m)$, the risk-averse optimization model can be constructed. The objective function is to minimize the maximum regret by accounting for uncertain mission scenarios, and it is given as Eq. (18). The constraints are the same as constrains (12) to (16):

$$\mathbf{I}_{k}^{*} = \arg\min_{\mathbf{I}_k} \max_{m} r(\mathbf{I}_k, m) \tag{18}$$

5.2 *Solution methodology*

The formulations in objective function (18) and constraints (12) to (16) provide a risk-averse inspection optimization model. The resulting optimization model is a constrained combinatorial optimization problem aiming to obtain a risk-averse inspection policy. The search space will dramatically magnify with the growth of inspected objects and their optional inspection techniques. The large search space makes the problem difficult to be resolved or find a feasible solution. To efficiently resolve the large-scale combinatorial optimization problem, many advanced meta-heuristic algorithms, such as the particle swarm optimization and the SFS, are often employed to seek the global or near global optimum. In this study, the SFS algorithm is utilized to resolve the resulting optimization problem because it can effectively explore the search space by the diffusion property.

The SFS algorithm is an evolutionary algorithm proposed by Salimi (2015) and consists of three main processes: the diffusing process and two updating processes. In this article, the SFS algorithm is implemented with an original penalty function and two local search rules for solving the above optimization model. More details on the procedure of solution methodology are expounded as follows.

5.2.1 *Penalty function*

A penalty function is conducted to exclude the infeasible solutions. The main procedure is to add penalty terms to the objective function value. The penalty function denotes is calculated as follows:

$$\psi(\mathbf{I}_k, C_{\max}) = \phi \cdot \max\left(0, C_{S,I_{k,S}} + \sum_{h=1}^{N} C_{S_h,I_{k,S_h}} + \sum_{l=1}^{M} C_{l,I_{k,l}} - C_{\max}\right), \tag{19}$$

where ϕ denotes the penalty parameter which is a positive constant. The fitness function can be, then, further evaluated by:

$$F(\mathbf{I}_k, C_{\max}) = \max_{m} r(\mathbf{I}_k, m) + \psi(\mathbf{I}_k, C_{\max}). \tag{20}$$

5.2.2 *Original SFS algorithm*

(1) Diffusing process: The new inspection policies group are created by the diffusing process according to Gaussian random walk as following:

$$\mathbf{I}_k' = \text{Gaussian}(\mathbf{I}_k, \boldsymbol{\sigma}), \tag{21}$$

where $\boldsymbol{\sigma}$ is the standard deviation determinant, and it can be calculated as following: $\boldsymbol{\sigma} = |\log(g)/g \cdot (\mathbf{I}_k - \mathbf{I}_{k,B})|$. g and $\mathbf{I}_{k,B}$ denote the number of iterations and the best inspection policy in the group, respectively. The optimization model is an integer programming problem, and therefore the entries of new inspection policy are randomly generated to their neighboring integer values. For example, if the value is equal to 1.3, it is a probability of 0.3 that changes to 1, and a probability of 0.7 that changes to 2.

(2) First updating process: All the inspection policies in the group are ranked from the largest to the smallest, based on the fitness value of the new inspection policies. $\mathrm{rank}\left(\mathbf{I}'_{k,i}\right)$ represents the rank of the i th inspection policy in the group, and its updating probability is calculated by:

$$P_i = \mathrm{rank}\left(\mathbf{I}'_{k,i}\right)/H\ , \tag{22}$$

where H is the number of inspection policies in the group. If $P_i > \varepsilon$, the inspection policy is updated by Eq. (23), otherwise it remains unchanged:

$$\mathbf{I}''_{k,i} = \mathbf{I}'_{k,r} - \varepsilon\left(\mathbf{I}'_{k,t} - \mathbf{I}'_{k,i}\right), \tag{23}$$

where $\mathbf{I}''_{k,i}$ denotes the i th inspection policy in the group after first updating. $\mathbf{I}'_{k,r}$ and $\mathbf{I}'_{k,t}$ are random selected inspection policies created by the diffusing process.

(3) Second updating process: All the inspection policies are ranked by Eq. (22). If $P_i > \varepsilon$, the inspection policy is updated by Eq. (24), otherwise it remains unchanged:

$$\mathbf{I}'''_{k,i} = \begin{cases} \mathbf{I}''_{k,i} - \hat{\varepsilon}\left(\mathbf{I}''_{k,t} - \mathbf{I}''_{k,B}\right), & \text{if } \varepsilon' \le 0.5 \\ \mathbf{I}''_{k,i} - \hat{\varepsilon}\left(\mathbf{I}''_{k,t} - \mathbf{I}''_{k,r}\right), & \text{if } \varepsilon' > 0.5 \end{cases} \tag{24}$$

where $\hat{\varepsilon}$ and ε' denote the standard normal distribution and the uniform distribution in the continues space [0,1]. $\mathbf{I}'''_{k,i}$ is selected to replace $\mathbf{I}''_{k,i}$ if and only $F(\mathbf{I}'''_{k,i}, C'''_{\max}) < F(\mathbf{I}''_{k,i}, C''_{\max})$.

5.2.3 *Two local search rules*

In the resulting optimization, the optimal inspection policy can be exactly on or sufficiently close to the budget constraint limit. Suppose that the remaining budget is not zero or does not tend to zero, the associated inspection policy is nonideal as the fitness function can be further reduced. Additionally, if the total cost of an inspection policy exceeds the budget constraint limit, the inspection policy is unfeasible. To address the above problems, we design two local search rules to ensure that the total cost approximates the budget.

We can control the total cost by two rules: implementing/removing inspection policies or improving/reducing the inspection accuracy.

Rule #1: Implementing/removing inspection activities. Based on Proposition 5, implementing an additional inspection activity with any inspection technique will increase the value of the proposed metric. Based on Eq. (18), the maximum regret will reduce due to the increased proposed metric. Therefore, when the budget is not exhausted, we can utilize the remaining budget to add the additional inspection activity for the reduction of the fitness function. Otherwise, when the budget exceeds the maximum, we can remove the inspection activity to reduce the penalty function.

Rule #2: Improving/reducing the inspection accuracy. The proposed metric will also increase by improving the accuracy of the original inspection policy. The maximum regret will also reduce due to the increased proposed metric. In the same way, one can improve or decrease the accuracy of the original inspection policy to control the total cost approximates the budget.

Based on these two policies, the two local search rules are shown as follows. If the residual budget does not approach zero, we wish to maximize the reduction of the fitness function per unit of cost. If the total cost of an inspection policy exceeds the budget, we wish to minimize the improvement of the fitness function per unit of cost. Let Θ denote the set of inspection policies of which corresponding cost is no more than the budget. Θ_i' is the set of inspection policies in which adding one more inspection activity than the i th inspection policy or increasing the accuracy of one of the original inspection activities. Θ_i'' is the set of inspection policies in which removing one more inspection activity from the i th inspection policy or reducing the accuracy of one of the original inspection activities. For each individual inspection policy, the local search probability is equal to p , and the new inspection policy is obtained as following:

$$\mathbf{I}_{k,i}^{\text{new}} = \begin{cases} \max\limits_{\mathbf{I}_k \in \Theta_i'} \left\{ \dfrac{F(\mathbf{I}_{k,i}', C_{\max}) - F(\mathbf{I}_k, C_{\max})}{C(\mathbf{I}_k) - C(\mathbf{I}_{k,i}')} \right\}, & \text{if } C(\mathbf{I}_{k,i}') < C_{\max} \\ \min\limits_{\mathbf{I}_k \in \Theta_i''} \left\{ \dfrac{F(\mathbf{I}_k, C_{\max}) - F(\mathbf{I}_{k,i}'', C_{\max})}{C(\mathbf{I}_{k,i}'') - C(\mathbf{I}_k)} \right\}, & \text{if } C(\mathbf{I}_{k,i}'') > C_{\max} \end{cases} . \tag{25}$$

The pseudocode of the SFS algorithm for resolving the optimal multilevel inspection policy is given in Appendix F.

6 Illustrative examples

A three-component system is exemplified to illustrate the step-by-step procedure of the metric and examine the accuracy of analytical solutions, and it is followed by a programmable logic controller (PLC) control system of a rocket propellant loading system.

6.1 *A three-component system*

A three-component system possesses five possible inspection sites, as shown in Fig. 2. The transition intensities and performance capacities of each component are tabulated in Tables 1 and 2, respectively. The performance capacity of the entire system, $G_S(t)$, and Subsystem 1,$G_{S_1}(t)$, are purely determined by the performance capacity of their components, $G_l(t)$ ($h = 1,2,3$). According to the physical structure of the specific system, one has $G_{S_1}(t) = G_1(t) + G_1(t)$, and $G_S(t) = \min\{G_{S_1}(t), G_3(t)\}$. Therefore, the entire system and Subsystem 1 have 4 and 6 states, respectively.

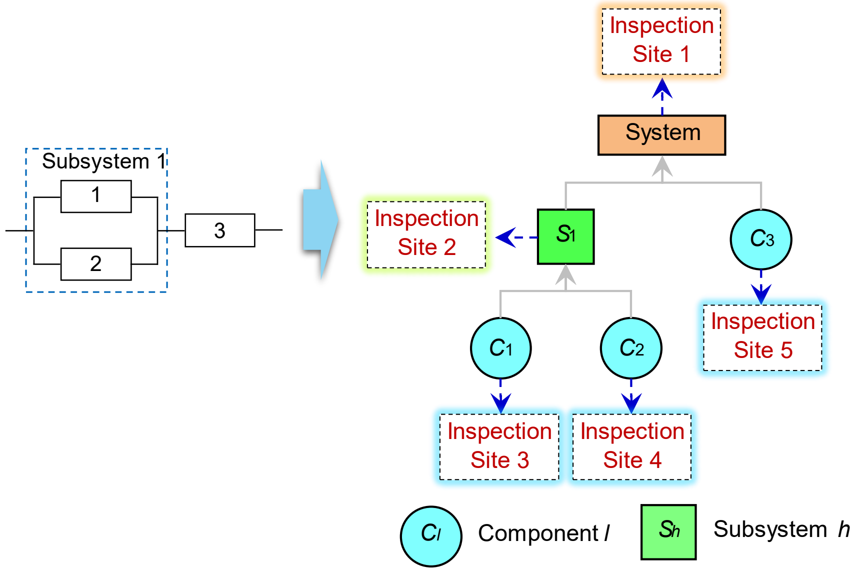

Figure 2. Arrangement of the three-component system and its possible inspection locations.

The residual parameters are set as follows. The initial state of each component is assumed to be in its best state, that is, $C_i(0) = 1, i = 1,2,3$. The system has already been operating for 0.4 months, and the subsequent mission duration is 0.1 months. The pre-specified target probability of the subsequent mission success is 0.9, i.e., $r_2 = 0.9$. The acceptable threshold state of the 2nd mission is state 2, i.e., $w_2 = 2$. More specifically, an inspection policy $\mathbf{I}_1 = \{1;0;1,0,0\}$, that is, inspecting the entire system and Component 1, is utilized to demonstrate the procedure of the metric. All the inspection activities are perfect, and thus their corresponding classification matrices are identity ones. The outlined procedures are elaborated in Appendix G.

Table 1. The transition intensities of each component (unit: month^{-1}).

Component ID	$l^{l}_{3,2}$	$l^{l}_{3,1}$	$l^{l}_{2,1}$
1	0.5	0.8	1.0
2	0.5	0.2	1.0
3	0.3	0.6	1.1

Table 2. The performance capacities of each component (unit: tons/hour).

Component ID	State 1	State 2	State 3
1	0	400	600
2	0	200	400
3	0	200	600

To further examine the accuracy of the metric, we use the Monte Carlo simulation as a benchmark for comparison. N_{sim} sets of deterioration profiles at $t_1 = 0.3$ months are randomly produced for each component by its deteriorating process. The inspection policy $\mathbf{I}_1 = \{1; 0; 1, 0, 0\}$ is conducted at t_1. The inspection data of each deterioration profile are the same as the true states of the system and Component 1, because all the classification matrices are unique matrices. Based on Eq. (11), the distribution of components staying in certain states can be updated with the inspection data. The probability that the system being in the set A_2 can be, then, obtained by the weighted sum of the updated probability of the system staying in $s(j,m)$ and the corresponding weight $\Pr\{1_{\mathrm{A}_2}(s(j,m)) = 1\}$, that is,

$$\Pr\{1_{\mathrm{A}_2}(S(t'_1)) = 1 \mid \hat{\Omega}_1\} = \sum_{i=1}^{N_S} \sum_{v=1}^{L_i} \Pr\{S(t'_k) = s(i,v) \mid \hat{\Omega}_1\} \Pr\{1_{\mathrm{A}_2}(s(i,v)) = 1\}.$$

In the same manner, the probability that the system's state is not in the set A_2 can be evaluated. Therefore, the probability of inferring accurately of one simulation system can be computed as following:

$$P_i = \begin{cases} \Pr\{1_{\mathrm{A}_2}(S(t'_1)) = 1 \mid \hat{\Omega}_1\}, & \text{if } 1_{\mathrm{A}_2}(s(i,v)) = 1 \\ \Pr\{1_{\mathrm{A}_2}(S(t'_1)) = 0 \mid \hat{\Omega}_1\}, & \text{if } 1_{\mathrm{A}_2}(s(i,v)) = 0 \end{cases}.$$

The simulation result is equal to the average of the probability of inferring accurately for all the simulation systems, that is,

$$P_{\text{sim}} = \sum_{i=1}^{N_{\text{sim}}} P_i / N_{\text{sim}}.$$

The relative error and running time of both the simulation result and analytic solution are tabulated in Table 3. The relative error of both two results is extremely close to zero when N_{sim} is set to 10^6. It takes 73.0711 seconds to obtain the simulation results via MATLAB 2020b on a PC with Intel (R) Core (TM) i7-8565U CPU @ 1.80 GHZ 1.99GHZ and 16-GB RAM, whereas the running time for the analytic solution is 2.281×10^{-3} seconds only.

Table 3. The running time and relative error of both two methods (time unit: seconds).

Method	Value of Proposed Metric	Running Time	Relative Error
Monte Carlo Simulation	0.9240638	73.0711	1.0688×10^{-4}
Analytic Solution	0.9241625	2.281×10^{-3}	

To demonstrate the impact of the elapsed mission time and the inspection accuracy on the proposed metric values, the elapsed mission time is set to be in the range of [0, 2] months, and a set of classification matrices with distinct accuracy are generated. Without loss of generality, to circumvent listing the classification matrix of each inspected object, each diagonal element of the matrix is given by:

$$d = \frac{a \times (n-1)+1}{n}, a = 0, 0.1, \ldots, 1$$

where a denotes the accuracy of the classification matrix. n is the number of the observable states of an inspection. Except for the diagonal elements, the rest of the matrix's elements are presumed to be identical. The results are delineated in Fig. 3. Some conclusions can be drawn from Fig. 3: (1) The possible value of the proposed metric lies in the range of [0.5, 1], which matches up with Proposition 2; (2) For the same elapsed mission time, the proposed metric have a mounting trend with the growth of the inspection accuracy; (3) For the same accuracy of inspection, the proposed metric decreases first, then ascends to one if elapsed mission time approaches infinity. The reason is that the uncertainty related to the states of the components and the uncertainty related to inspections affect the proposed metric values. The component state is deterministic at the initial time, that is, the initial state of each component is in its best states. Then, the uncertainty related to the states of the components will increase as the components deteriorate, and it reduces when these components gradually fall into their worst states with the increase of usage. Meanwhile, the uncertainty associated with inspections will reduce with the increase of inspection accuracy.

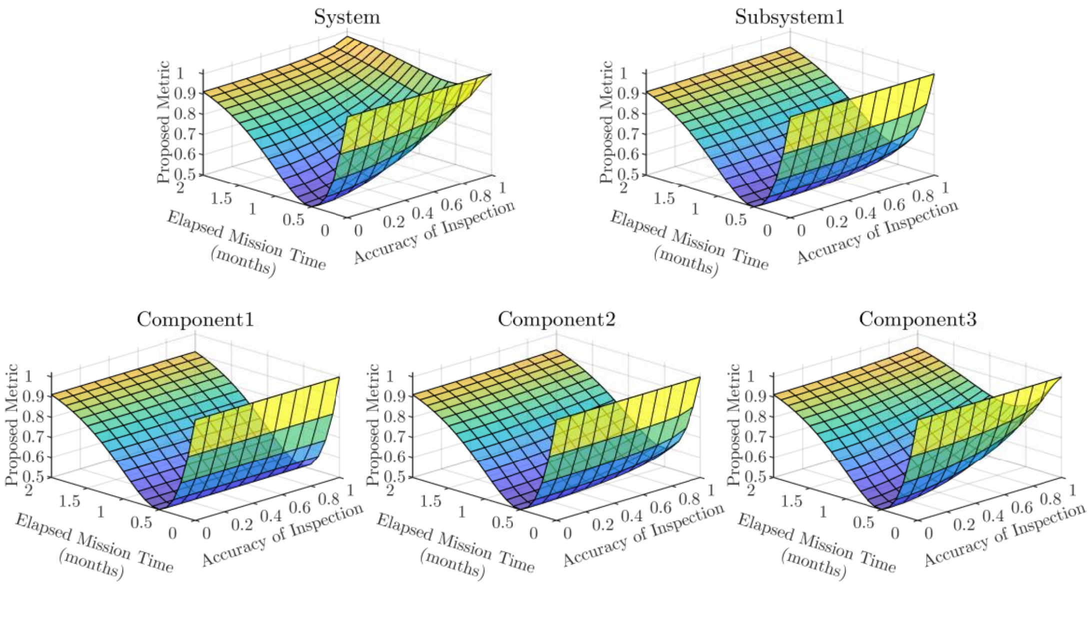

Figure 3. The proposed metric of inspecting one site with various setting.

To gain an in-depth understanding of the impact of the inspection accuracy on the proposed metric, we compare the values of the proposed metric under four scenarios: (1) inspections are perfect; (2) inspections are imperfect, and $a = 0.5$; (3) inspections are non-informative; (4) no inspection. The results are shown in Table 4: (1) The values of the proposed metric with noninformative inspections are the same as that with no inspection. It matches up with Propositions 1 and 3; (2) The value of the proposed metric is equal to 1 when all the components are perfectly inspected. It matches up with Propositions 4; (3) Implementing an additional inspection object with any inspection technique, the proposed metric is no less than that of the original inspection policy. It matches up with Propositions 5; (4) The value of the proposed metric of policy #6 with perfect inspections is equal to that of policies #13 and #14. This implies that perfectly inspecting Components 1 and 2 is equivalent to perfectly inspecting Subsystem 1 and Component 1 (or 2). It completely matches up with our intuition as determining the states of Subsystem 1 and one of its components will determine that of another component in this illustrative example. However, the state of a subsystem's component could be in multiple states while fixing the state of the subsystem and all other components. In this case, perfectly inspecting Components 1 and 2 is not equivalent to perfectly inspecting Subsystem 1 and Component 1 (or 2). The same conclusion can also be observed in policies #9 and #15; (5) The value of proposed metric of policy #11 with perfect inspections is equal to 1. In this case, it is sufficient to inspect only the system and Component 2 to accurately identify if the mission success probability can exceed 0.9, without the need of inspecting all components.

Table 4. Values of the proposed metric under four scenarios.

Policy No.	Inspection Objects	Scenario 1	Scenario 2	Scenario 3	Scenario 4
#1	System	0.9059	0.6140	0.5196	
#2	Subsystem 1	0.6126	0.5332	0.5196	
#3	Component 1	0.5235	0.5205	0.5196	
#4	Component 2	0.6100	0.5336	0.5196	
#5	Component 3	0.8305	0.5888	0.5196	
#6	Components 1 and 2	0.6379	0.5356	0.5196	0.5196
#7	Components 1 and 3	0.8362	0.5899	0.5196	
#8	Components 2 and 3	0.9600	0.6042	0.5196	
#9	System and Subsystem 1	0.9637	0.6291	0.5196	
#10	System and Component 1	0.9242	0.6170	0.5196	
#11	System and Component 2	1	0.6270	0.5196	

#12	System and Component 3	0.9059	0.6596	0.5196
#13	Subsystem 1 and Component 1	0.6379	0.5370	0.5196
#14	Subsystem 1 and Component 2	0.6379	0.5497	0.5196
#15	Subsystem 1 and Component 3	0.9637	0.6037	0.5196
#16	Components 1, 2 and 3	1	0.6064	0.5196

By changing the duration of the next mission, the proposed metric values of perfectly inspecting one site of the system are delineated in Fig. 4. As seen from Fig. 4, the proposed metric values will converge to one when the time duration of the next mission rises. The reason is that the mission success probabilities when components stay in any state are less than the target value. In this circumstance, we can identify the system cannot conduct the next mission with the target success probability without any inspections.

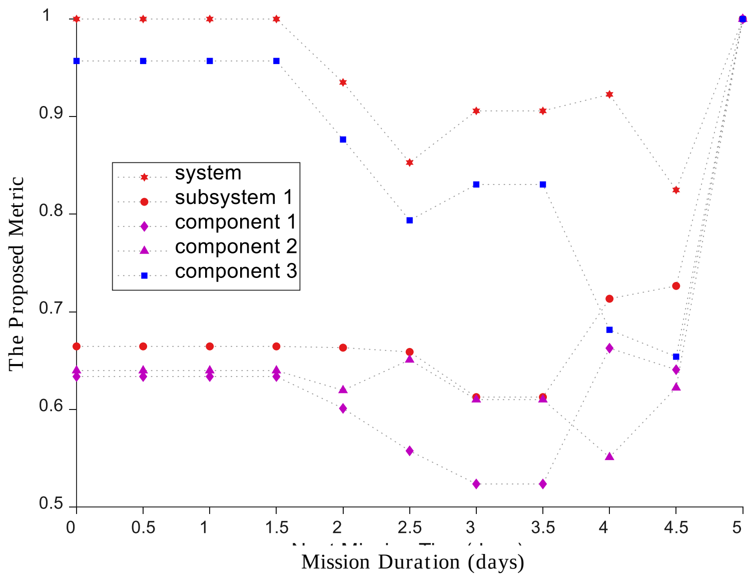

Figure 4. The proposed metric of perfectly inspecting one site with various mission time.

By varying the target mission success probability with a uniform interval of 0.1 within the range of 0 to 1, the proposed metric values of perfectly inspecting one site of the system are delineated in Fig. 5. Table 4 shows that the minimum, subminimum, and maximum mission success probabilities equal 0, 0.8106, and 0.9395, respectively. As observed from Fig.5: (1) The proposed metric values of all inspection policies are equal to 1 when the target mission success probability is 0. The reason is that all possible mission success probability is no less than the target value. Similarly, when the target value is set to 1, a similar conclusion can be drawn; (2) The proposed metric values of perfectly inspecting the system equate to 1 when the target value is less than the subminimum value. In this case, the mission success probability is equal to 0 when the system stays at the unacceptable state, which is less than the target value. The mission success probability is more than the target value when the system stays at the acceptable states. Therefore, perfectly inspecting the system can accurately identify whether the mission success probability exceeds the target value, that is, the proposed metric equates to 1.

Combining Figs. 4 and 5, we can draw the following conclusion. The time duration of the next mission will affect the next mission success probability for the system staying at various combinations of components' states. The set of possible specific states A_{k+1} is obtained by comparing the next mission success probability with the target value. When changing the mission duration and the target mission success probability, A_{k+1} will also undergo changes, leading to a different value of the proposed metric. Hence, the proposed metric demonstrates a discrete alteration pattern or a steplike variation only when A_{k+1} undergoes a transformation.

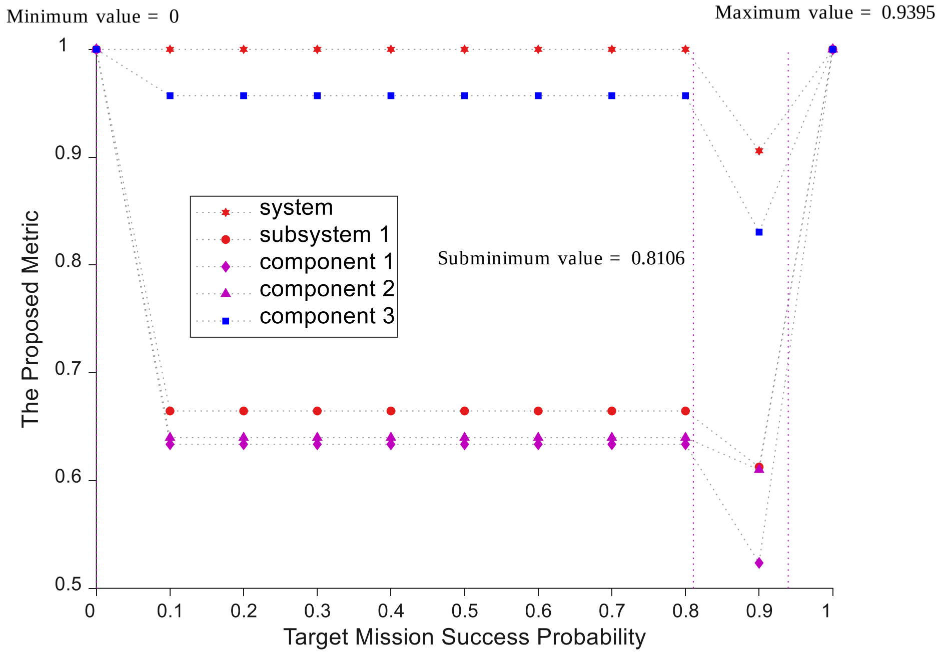

Figure 5. The proposed metric of perfectly inspecting one site with various target values.

6.2 *A PLC control system*

The PLC control system of a rocket propellant loading system, as shown in Figure 6, is employed to demonstrate the applicability of the proposed method. The system possesses four subsystems, which are connected in series. In the remote-control room, a propellant loading command is inputted into the remote-control subsystem. This subsystem is composed of two remote-control computers connected in parallel. The communication subsystem then transmits the command to the proximal-control subsystem in the base warehouse, consisting of two parallel switches. The proximal-control subsystem encompasses two proximal-control computers in parallel. The command is then outputted to the control subsystem to complete the rocket propellant loading mission. The control subsystem consists of two control cabinets connected in series that control the on-off of the electromagnetic valve and the propellant loading speed, respectively.

Based on the computer response speed, the states of the remote-control computer and the proximal-control computers can be classified into three states: unable to respond (State 1), response sluggishly (State 2), and perfectly functioning (State 3). On the basis of the transmission delay time, the states of the switches can be divided into three states: communication outage (State 1), communication delay (State 2), and communication normal (State 3). Furthermore, the states of the above three subsystems are determined by the best states of their components.

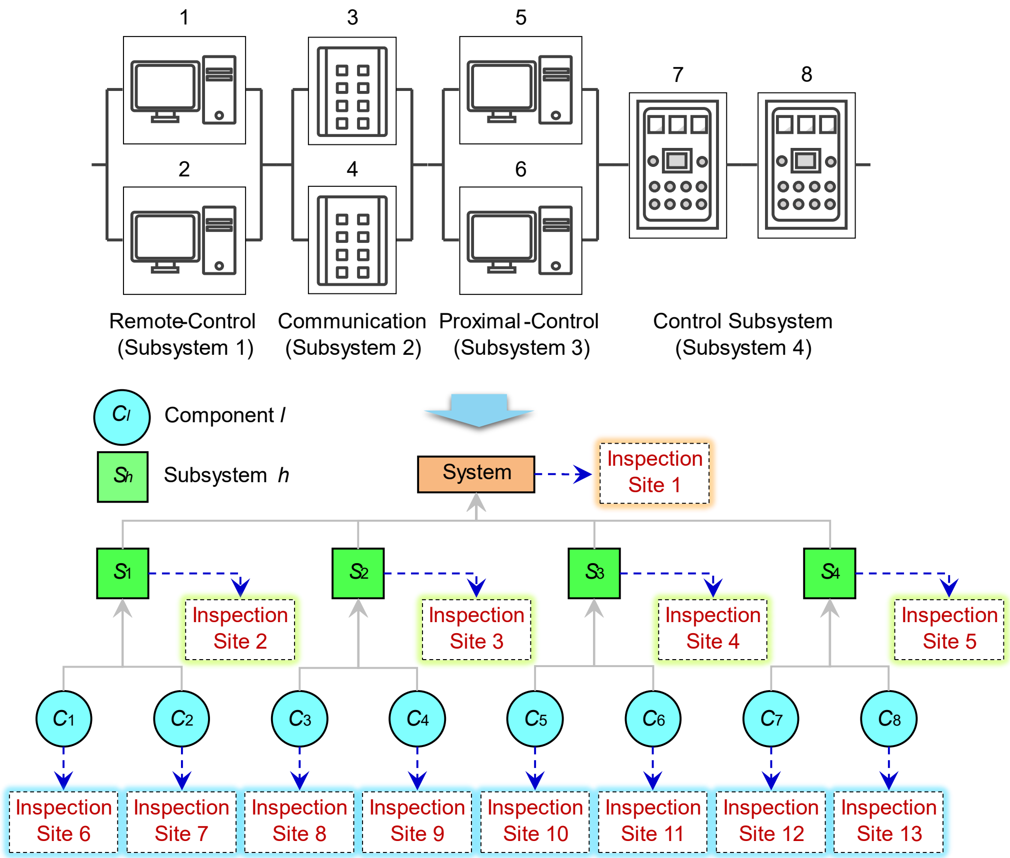

Figure 6. Configuration of the PLC control system and its potential inspection locations.

Depending on whether the valve is on, and the propellant loading speed can be appropriately controlled, the control cabinets can be viewed as a binary-state component, either failed (State 1) or functioning (State 2). The state of the entire system, which concludes in three states, i.e., failure to accomplish the mission (State 1), delay to accomplish the mission (State 2), and success to accomplish the mission (State 3), is determined by the states of all the components, that is,

$$S(t) = \begin{cases} 1, & C_7(t) = 1 \text{ or } C_8(t) = 1 \\ \min\{\max\{C_1(t),C_2(t)\}, \max\{C_3(t),C_4(t)\}, \max\{C_5(t),C_6(t)\}\}, & \text{otherwise} \end{cases}$$

The transition intensities of each component are tabulated in Table 5. The duration of the last mission is 0.3 months, i.e., $t_1 = 0.3$ months. The target probability of the subsequent mission success is set to be 0.9, that is, $r_2 = 0.9$. The inspection policy is conducted at t_1. Each inspection policy has four inspection techniques, and the corresponding cost are tabulated in Table 6. The maximum inspection cost is 600 US dollars. The subsequent mission has four possible scenarios, and their corresponding maximum proposed metric values are tabulated in Table 7.

Table 5. The transition intensity of each component (unit: month^{-1}).

Component ID	$l_{3,2}$	$l_{3,1}$	$l_{2,1}$
1	0.5	0.3	0.9
2	0.5	0.3	0.9
3	0.3	0.7	1.1
4	0.3	0.7	1.1
5	0.6	0.5	1.1
6	0.6	0.5	1.1
7	—	—	0.6
8	—	—	0.6

Table 6. Inspection activities and their corresponding inspection cost (unit: US dollars).

Technique ID	S	S_{h_1}	S_{h_2}	S_{h_3}	S_{h_4}	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	60	70	80	90	60	60	60	80	80	60	60	100	100
3	100	110	110	120	100	100	100	120	120	100	100	150	150
4	200	220	200	250	200	200	200	250	250	200	200	300	300

Table 7. Four possible mission scenarios and their corresponding maximum metric (time unit: months)

Scenario ID	T_2	w_2	$P(\mathbf{I}^*_{1,m}, \mathrm{A}_{2,m})$
1	0.05	1	0.9065
2	0.03	2	0.9439
3	0.08	1	0.8289
4	0.04	2	0.8166

To examine the effectiveness of the SFS algorithm, we compare the proposed method with the original SFS algorithm. The parameter settings of these algorithms are as follows: (1) The population N_p is 20; (2) The maximum number of iterations is 300; (3) The penalty parameter is 1; (4) The local search probability is 0.5. One hundred independent runs are conducted for each algorithm, and the mean value of the maximum regret function of the two algorithms are listed in Table 8. Even though the two algorithms execute the same number of calculating the fitness function, the mean value of the proposed SFS with two local search rules are less than that of the original SFS, indicating that the proposed two local search rules significantly enhance the global searching capability.

By changing the budget, the maximum values of the proposed metric under four mission scenarios are tabulated in Table 9, and the corresponding minimal maximum regret function is tabulated in Table 10. As observed in Table 9, the maximum values of the proposed metric gradually improve in tandem with the escalation of the inspection budget. It aligns with Proposition 5, as the increase in the budget can lead to additional inspections, thereby increasing the value of the metric. As observed in Table 10, the minimal maximum regret function is equal to zero when the budget is zero. At this point, the execution of any inspection policy is impossible. In various scenarios, the optimal and feasible policies are not to conduct inspections. The minimal maximum regret functions are also equal to zero when the inspection budgets equate to 100, 200, and 1900 US dollars. It means the optimal inspection policies for the four scenarios are the same. This minimal maximum regret function can somewhat reflect the uncertainty associated with inspection decision-making. When the budget is limited, there are few inspection policies available. With a surplus of budget, comprehensive component-level inspections are conducted. Conversely, the challenge lies in decision-making when the budget falls in the range between the two extremes. Therefore, the minimal maximum regret function ascends from zero before eventually receding to zero.

Table 8. The mean values of the maximum regret function for the two algorithms

Running Number of Fitness Functions	300	600	900	1200	1500	1800	2100	2400	2700	3000
Original SFS	0.0936	0.0802	0.0662	0.0588	0.0548	0.0505	0.0479	0.0453	0.0436	0.0426
Proposed SFS	0.0640	0.0573	0.0535	0.0487	0.0442	0.0411	0.0394	0.0383	0.0374	0.0371

Table 9. The maximum values of the metric for various inspection budget.

Inspection Budget (US Dollars)	Scenario 1	Scenario 2	Scenario 3	Scenario 4
0	0.5043	0.5026	0.6109	0.5336
100	0.7288	0.7703	0.6700	0.6616
200	0.8507	0.9319	0.7035	0.7388
300	0.8588	0.9339	0.7253	0.7548
400	0.8755	0.9361	0.7598	0.7821
500	0.8909	0.9397	0.7939	0.7969
600	0.9065	0.9439	0.8289	0.8166

1900	1	1	1	1

By fixing the inspection budget as 400 US dollars, the corresponding minimal maximum regret function for various elapsed mission time is tabulated in Table 11. The uncertainty associated with the system state is absent as it is in its best state at the initial time. Nevertheless, as the elapsed mission time increases, the uncertainty associated with the system state gradually rises. Subsequently, as the elapsed mission time further advances, the system state tends towards failure, leading to a reduction in its uncertainty. The observation presented in the table is consistent with the earlier description, where the minimal maximum regret function initially increases from zero and subsequently decreases.

Table 10. The minimal maximum regret function for various inspection budget.

Inspection Budget (US Dollars)	0	100	200	300	400	500	600	1900
The Minimal Maximum Regret Function	0	0	0	0.0076	0.0085	0.0183	0.0016	0

Therefore, the maximum regret function can effectively capture the uncertainty associated with the system state and inspection decision-making. Minimizing the maximum regret function can effectively mitigate uncertainty, thereby reducing the decision-related risks from such uncertainty.

Table 11. The minimal maximum regret function for various elapsed mission time.

Elapsed Mission Time (Months)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
The Minimal Maximum Regret Function	0	0.0026	0.0074	0.0085	0.0097	0.0072	0.0049	0.0035	0.0033	0.0029	0.0024

7 Conclusions and future work

In this article, inspection policies were developed to determine if the probability of mission success could meet a threshold, or the so-called “target value”. A novel metric was put forth to quantify the effectiveness of a specific inspection policy. A risk-averse optimization model was formulated through considering the uncertainty of mission durations and acceptable threshold states. In the risk-averse optimization model, the regret of an inspection policy was put forth to evaluate the relative loss of the proposed metric between the inspection policy and the optimal inspection policy under a specific mission scenario. The objective of the optimization model was to minimize the maximum regret of a particular inspection policy, under the constraint of the inspection budget. The stochastic fractal search algorithm with two customized local search rules was implemented to obtain the optimum solution. The results from the first illustrative example showed that the proposed metric depends on the uncertainty associated with system states and inspections. Additionally, the proposed metric demonstrates a discrete alteration pattern or a steplike variation when the mission duration and the target value change. Meanwhile, the programmable logic controller control system was analyzed to demonstrate the effectiveness of the proposed optimization model. The results showed that the proposed two local search rules significantly enhance the global searching capability, and the proposed method could reduce the decision-related risks from the uncertainty associated with the system state and inspection decision-making.

Some challenges will be addressed in our future work. First, this article assumed that the components' deteriorating processes are statistically independent. Nevertheless, in reality, the degradation behaviors of components may be correlated due to common cause failures, load sharing, and cascading failures. In such a circumstance, the inspection data of components may reveal the true state of other components. In addition to the stochastic dependence, structural dependence may exist among the components, implying that other components must be disassembled or taken apart for an inspection purpose. The multilevel inspection policy optimization problem with statistical and structural dependence will be explored in our future work. Second, in this article, we assumed that the subsequent mission success probability is no less than the target value by implementing inspection policies. Other methods, such as redundancy allocation policies in the design phase and maintenance policies in the operation phase, can also be implemented to ensure the system completing its missions with a desired success probability. The redundancy allocation, inspection, and maintenance policies will be jointly optimized in our future work.

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