Finance Research Letters 55 (2023) 103913

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Estimating the US trend short-term interest rate

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ARTICLE INFO

JEL classification:
C11
C32
E44
E52

Keywords:
Unobserved components model
Bayesian estimation

ABSTRACT

We estimate the trend short-term interest rate in the United States using an unobserved-components stochastic-volatility model with interest-rate and survey data from 1998Q2 to 2022Q4. Our results indicate that the trend short-term interest rate has drifted down during most of the sample and remains low in a historical perspective, despite the recent sharp increase in the short-term interest rate.

1. Introduction

Following the Global Financial Crisis of 2008–2009, nominal short-term interest rates in numerous countries stayed at low levels. However, the period of low short-term interest rates came to a halt during 2022 as many central banks started hiking policy interest rates to fight high inflation. The United States has been no exception; the target Federal Funds Rate has increased from 0–0.25 per cent in early 2022 to 4.75–5.00 per cent in early April 2023. The future evolution of short-term interest rates is uncertain, with disagreement as to whether they will remain at their new higher level in the medium-to-long run or whether a low inflation and low short-term interest-rate environment reasserts itself; see, for example, Caselli et al. (2022), Gopinath (2022) and International Monetary Fund (2023) for related discussions.

In this paper, we estimate the trend short-term interest rate in the United States using outcomes for the short-term interest-rate and survey data related to it, employing a model which permits time variation in the parameters as well as stochastic volatility. The trend short-term interest rate is a model-based concept which describes the level at which the model's forecasts will converge. The model sheds light on how much of the recent rise in short-term interest rates is attributed to a change in the trend short-term interest rate and thus where short-term interest rates are likely to stabilise in the future.

Specifically, we employ the unobserved-components stochastic-volatility model of Chan et al. (2018). This model has been applied to estimate trend inflation in the United States, Italy, Japan and the United Kingdom (Chan et al., 2018), a number of countries in the Asia-Pacific region (Garcia and Poon, 2022) and Sweden (Österholm and Poon, 2022). More generally, the model allows for the possibility that dynamics in the economy are evolving over time and that shocks that hit the economy may have time-varying volatility (that is, they are heteroskedastic). As such it provides a natural setting for the analysis of other variables where time-varying dynamics and heteroskedasticity are of concern. Jointly modelling these features has become increasingly common in the macroeconomic

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https://doi.org/10.1016/j.frl.2023.103913
Received 12 March 2023; Received in revised form 3 April 2023; Accepted 16 April 2023
Available online 17 April 2023

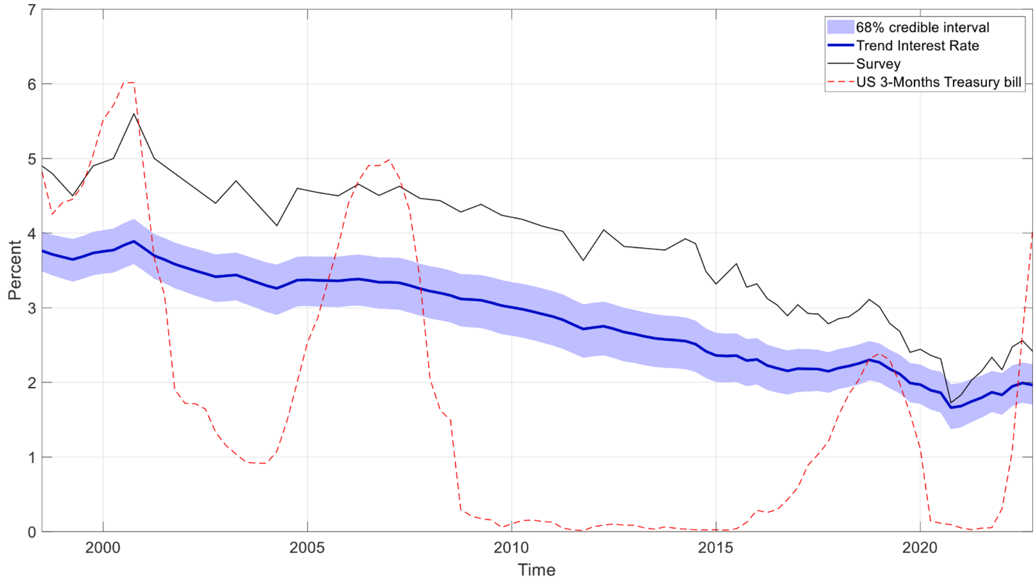

Fig. 1. Data and estimated trend short-term interest rate.
Note: Percent on vertical axis. The black solid line represents the long-run interest-rate expectation. The red dashed line represents the actual three-month Treasury bill rate. The blue solid line is the posterior mean of the trend interest rate and the shaded area is the associated the 68 percent credible interval.
Source: Consensus Economics, Macrobond and authors' own calculations.

literature during the last two decades, though it is typically done using vector autoregressions; see, for example, Cogley and Sargent (2005) and Primiceri (2005) for important early contributions, and Koop and Korobilis (2019), Akram and Mumtaz (2019) and Karlsson and Österholm (2020, 2023) for a few recent additional examples. In this paper, we apply the framework of Chan et al. (2018) to a nominal interest rate for the first time.

Because the model has time-varying parameters, the trend value can be seen as the local mean of the variable. Our study is hence related to studies which have estimated the local mean for other variables, such as inflation, the real interest rate and the unemployment rate; see, for example, Cogley and Sargent (2005), Clark and Doh (2014), Johannsen and Mertens (2021) and Fu (2023). In an environment in which the central bank targets inflation and the inflation target is credible, the question of where nominal short-term interest rates will stabilise is closely related to estimates of the trend short-term *real* interest rate or what the level of the *natural rate of interest*[1] is.[2] However, there is little research that focuses directly on the level at which *nominal* short-term interest rates will stabilise. Models of the nominal term structure implicitly model the long-run value of the nominal short-term interest rate; examples include applications of the parameterised model of the yield curve introduced by Nelson and Siegel (1987) and dynamic term-structure models such as those proposed by Duffee (2002) and Kim and Wright (2005). But these applications are generally aimed at modelling the shape and dynamics of the term structure rather than exploring the trend short-term nominal interest rate.

The rest of this paper is organised as follows: In Section 2, we present the model and the data. Our main results are reported in Section 3. We present results from sensitivity analysis in Section 4. Finally, Section 5 concludes.

2. Model and data

Eqs. (1) to (6) present the model. The short-term interest rate (i_t) is a first-order autoregressive process which moves around the time-varying trend short-term interest rate (i_t^*) – the variable of focus in this paper. The autoregressive parameter (b_t) is itself also a time-varying process. We also include the long-run expectation (z_t) of the short-term interest rate – as given by survey data – in the model to help identify the trend short-term interest rate. The long-run expectation is a function of the trend short-term interest rate; the coefficients ($d_{j,t}$) describing this relation are time varying (around constant means, μ_{dj}). The log volatilities ($h_{k,t}$) of the short-term interest rate and the trend short-term interest rate are assumed to follow random walks. Finally, ν_t, η_t, $\varepsilon_{b,t}$, $\varepsilon_{z,t}$, $\epsilon_{d_j,t}$ and $\gamma_{h_k,t}$ are disturbances.

[1] In line with, for example, Laubach and Williams (2003) and Holston *et al.* (2017), the natural rate of interest is often defined as the real interest rate consistent with output equal to its natural rate and stable inflation.

[2] Recent empirical contributions – from a very rich literature concerning the trend level of the short-term real interest rate and the natural rate of interest – include Holston *et al.* (2017), Benati (2023), Fu (2023) and Hirose and Sunakawa (2023).

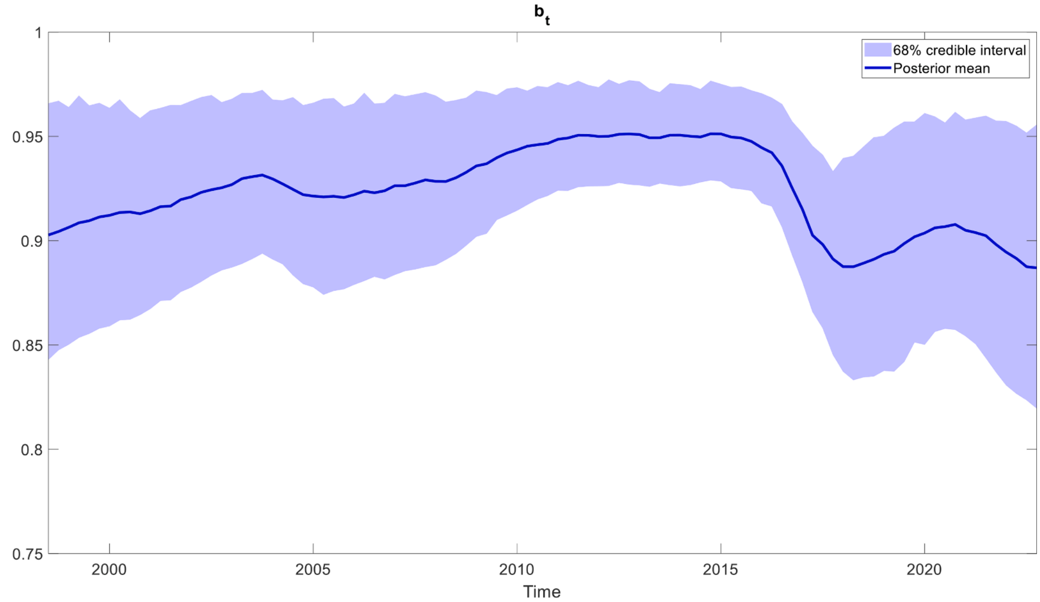

Fig. 2. Estimated persistence of interest-rate gap.
Note: The blue line is the posterior mean of the estimated persistence of the interest-rate gap (b_t) and the shaded area is the associated the 68 percent credible interval.
Source: Authors' own calculations.

$$i_t - i_t^* = b_t\left(i_t - i_{t-1}^*\right) + \nu_t,\ \ \nu_t \sim N\left(0, e^{h_{\nu,t}}\right) \tag{1}$$

$$i_t^* = i_{t-1}^* + \eta_t,\ \ \eta_t \sim N\left(0, e^{h_{\eta,t}}\right) \tag{2}$$

$$b_t = b_{t-1} + \epsilon_{b,t},\ \ \epsilon_{b,t} \sim TN_{(0,1)}\left(0, \sigma_b^2\right) \tag{3}$$

$$z_t = d_{0,t} + d_{1,t} i_t^* + \epsilon_{z,t} + \psi \epsilon_{z,t-1},\ \ \epsilon_{z,t} \sim N\left(0, \sigma_z^2\right) \tag{4}$$

$$d_{j,t} - \mu_{dj} = \rho_{d_j}\left(d_{j,t-1} - \mu_{dj}\right) + \epsilon_{d_j,t},\ \ \epsilon_{d_j,t} \sim N\left(0, \sigma_{d_j}^2\right),\ \ j = 0, 1 \tag{5}$$

$$h_{k,t} = h_{k,t-1} + \gamma_{h_k,t},\ \ \gamma_{h_k,t} \sim N\left(0, \sigma_{h_k}^2\right),\ \ k = \nu, \eta \tag{6}$$

We estimate the model on the three-month Treasury bill rate in the United States and survey expectations of this bill rate six-to-ten years ahead using quarterly data from 1998Q2 to 2022Q4. The three-month Treasury bill rate has been sourced from Macrobond. The data on long-run interest-rate expectations are from the survey *Consensus Forecasts* provided by Consensus Economics.[3] Both series are shown in Fig. 1. Worth noting are the large swings in the three-month Treasury bill rate between the start of the sample and the end of 2008, the extended period with a low rate that followed, and the sharp increase at the end of the sample. Long-run expectations of the Treasury bill rate trend down during the sample, declining approximately three percentage points between 2000 and 2020. This steady decline came to an end in late 2020; between 2020Q4 – when the bill rate reached its minimum – and 2022Q4, the long-run expectation increased by approximately 0.7 percentage point.

The model is estimated with Bayesian methods. Concerning priors, we impose an inverse-gamma prior on all the state variances stated in Eqs. (3), (4), (5) and (6) which govern the transition dynamics of the latent unobserved variables in our model. These are set to $\sigma_z^2 = \sigma_{d_0}^2 = \sigma_{h_\nu}^2 = \sigma_{h_\eta}^2 \sim IG(5, 0.04)$ and $\sigma_b^2 = \sigma_{d_1}^2 \sim IG(5, 0.004)$. Further details regarding estimation can be found in Chan et al. (2018).

[3] From 1998 to 2013, the survey was conducted twice per year – in April and October. In 2014, it was conducted three times (in April, July and October). Since 2015, it has been conducted four times per year (January, April, July and October). In the first part of our sample, we replace the missing observations for Q1 and Q3 with values that are averages of the neighbouring Q2 and Q4 values.

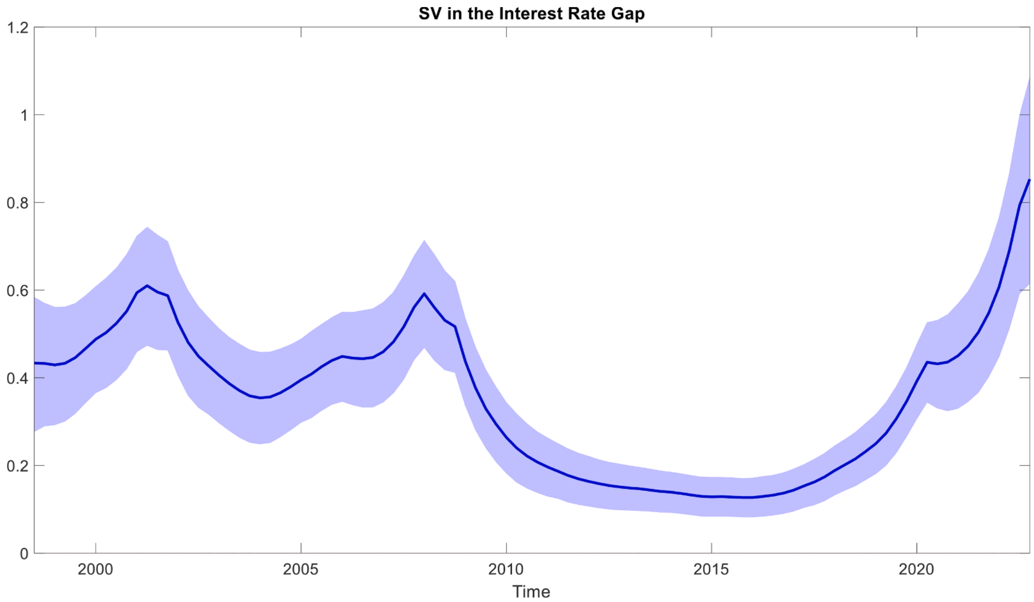

Fig. 3. Estimated stochastic volatility of the interest-rate gap.
Note: Stochastic volatility – given as standard deviation – in percent on vertical axis. Shaded area gives the 68 percent credible interval.
Source: Authors' own calculations.

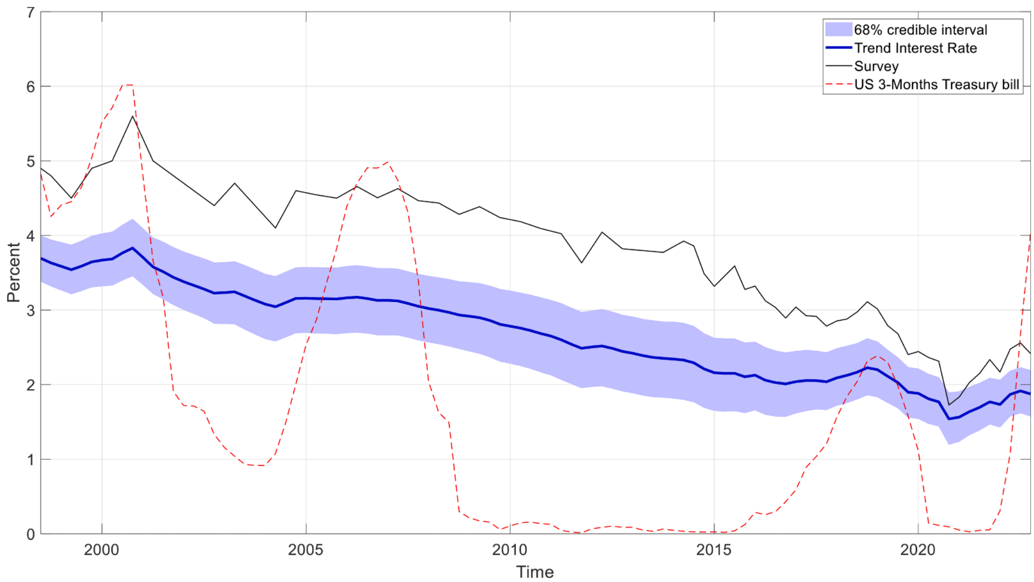

Fig. 4. Estimated trend short-term interest rate using horseshoe prior.
Note: Percent on vertical axis. The black solid line represents the long-run interest-rate expectation. The red dashed line represents the actual three-month Treasury bill rate. The blue solid line is the posterior mean of the trend interest rate and the shaded area is the associated the 68 percent credible interval.
Source: Consensus Economics, Macrobond and authors' own calculations.

3. Main results

Fig. 1 shows the estimated trend short-term interest rate. As can be seen, it has been on a downward trajectory during most of the sample and judging by point estimates, fell by over 2 percentage points from the late 1990s to late 2020 – from 3.8 per cent to 1.7 per

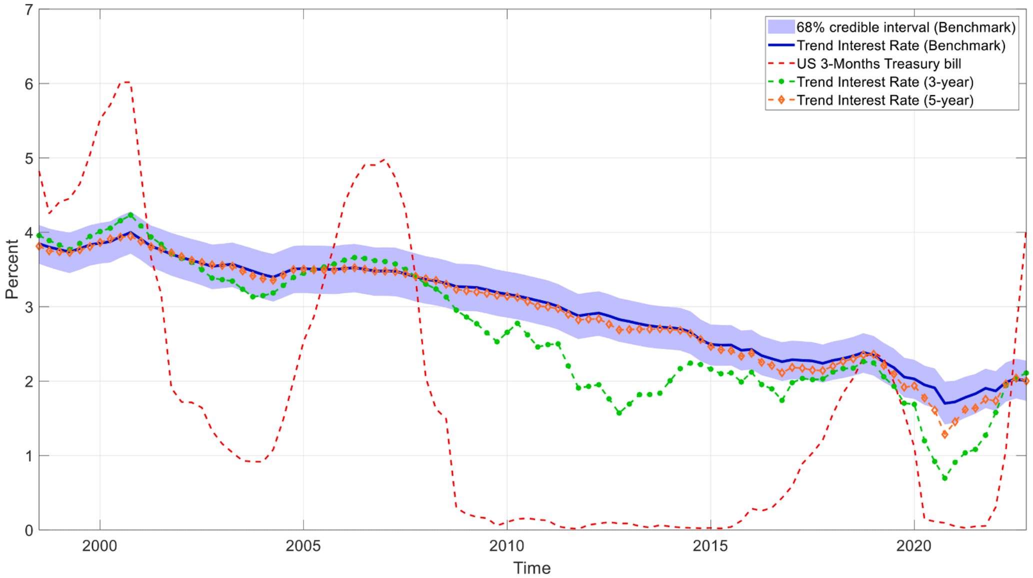

Fig. 5. Estimated trend short-term interest rate using alternative survey data.
Note: Percent on vertical axis. The red dashed line represents the actual three-month Treasury bill rate. The blue solid line is the posterior mean of the trend interest rate using the six-to-ten-year-ahead expectation (from the main analysis) and the shaded area is the associated the 68 percent credible interval. The green line with circles is the posterior mean of the trend interest rate using the three-year-ahead expectation. The orange line with diamonds is the posterior mean of the trend interest rate using the five-year-ahead expectation.
Source: Macrobond and authors' own calculations.

cent. However, since late 2020 the trend estimate has increased modestly, to 2 per cent in 2022Q4, as long-run survey expectations and the actual Treasury bill rate have risen.

The estimated trend short-term interest rate is much smoother than the actual three-month Treasury bill rate.[4] This means that deviations between the trend short-term interest rate and the actual three-month Treasury bill rate – what we call the interest-rate gap – are quite persistent. The point estimate of the autoregressive parameter of the model is approximately 0.9 (see Fig. 2).

The volatility of the interest-rate gap varies substantially over time (see Fig. 3) which supports the practice of explicitly modelling heteroskedasticity in macroeconomic time series, as discussed in the introduction (Section 1). Shock volatility increased considerably near the end of the sample. This reflects in part that the Federal Reserve's recent rate hike – and the accompanying increase in the three-month Treasury bill rate – was unanticipated by the model, but also that the model does not consider this increase to be permanent.

4. Sensitivity analysis

In this section, we assess how robust our findings are to two aspects of the estimation of the model. The first is the choice of prior for the variance of the stochastic volatility process of the trend interest rate; the second is the choice of data.

Since the variance of the error term for the stochastic volatility process of the trend short-term interest rate, $\sigma^2_{h_\eta}$, affects the smoothness with which the trend short-term interest rate is estimated to have evolved over time, it is a parameter of particular interest in this model. Following Österholm and Poon (2022), we in this sensitivity analysis estimate the model using the "horseshoe prior" of Prueser (2021) for $\sigma^2_{h_\eta}$; this allows for more abrupt changes in the estimated trend short-term interest rate than in our main analysis. In particular, the state variance $\sigma^2_{h_\eta} = \tau_t \lambda$ can be decomposed into a local component τ_t and a global component λ. Both the prior for the local and global component follows a half Cauchy distribution of $C^+(0,1)$.

The key result is provided in Fig. 4. As can be seen, changing the prior for $\sigma^2_{h_\eta}$ does not affect the estimated trend short-term interest rate much; the estimates in Fig. 4 are very similar to those shown in Fig. 1 for our main analysis.[5] The results appear to be robust to the choice of prior for this parameter and support the finding that the trend short-term interest rate has drifted down steadily during most of the sample.

Regarding data, we assess the sensitivity of our results with respect to the horizon of the expectations. In our main analysis we use the longest horizon available in the *Consensus Forecasts* survey – that is, six-to-ten years ahead. Using the longest horizon is reasonable

[4] The estimated stochastic volatility of the trend interest rate is also a smooth process; see Figure A1 in the Appendix.
[5] The estimated stochastic volatility of the trend interest rate is not affected much either; see Figure A2 in the Appendix.

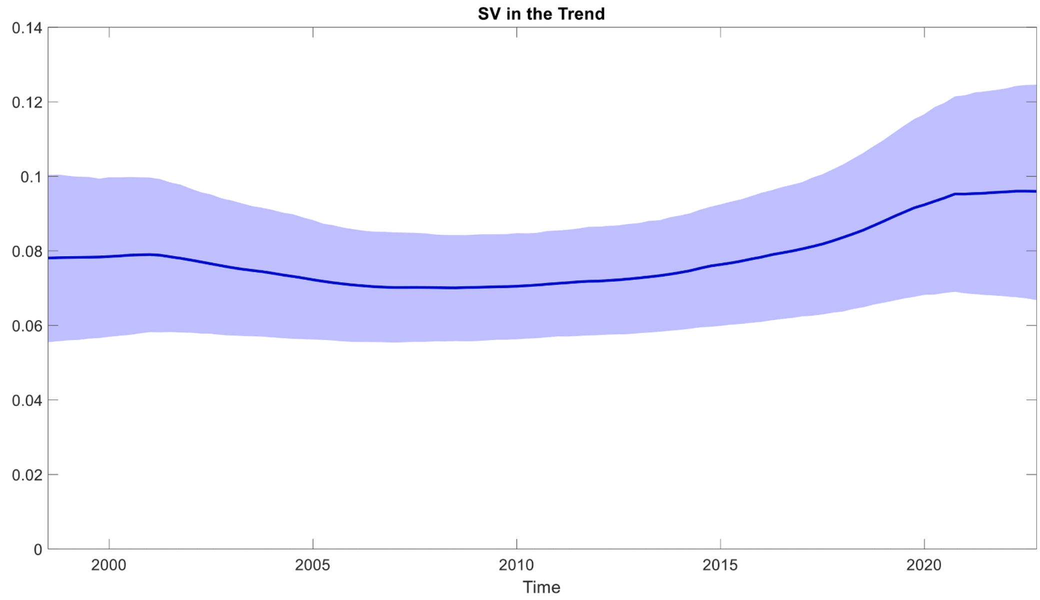

Fig. A1. Estimated stochastic volatility of trend interest rate.
Note: Stochastic volatility – given as standard deviation – in percent on vertical axis. Shaded area gives the 68 percent credible interval.
Source: Authors' own calculations.

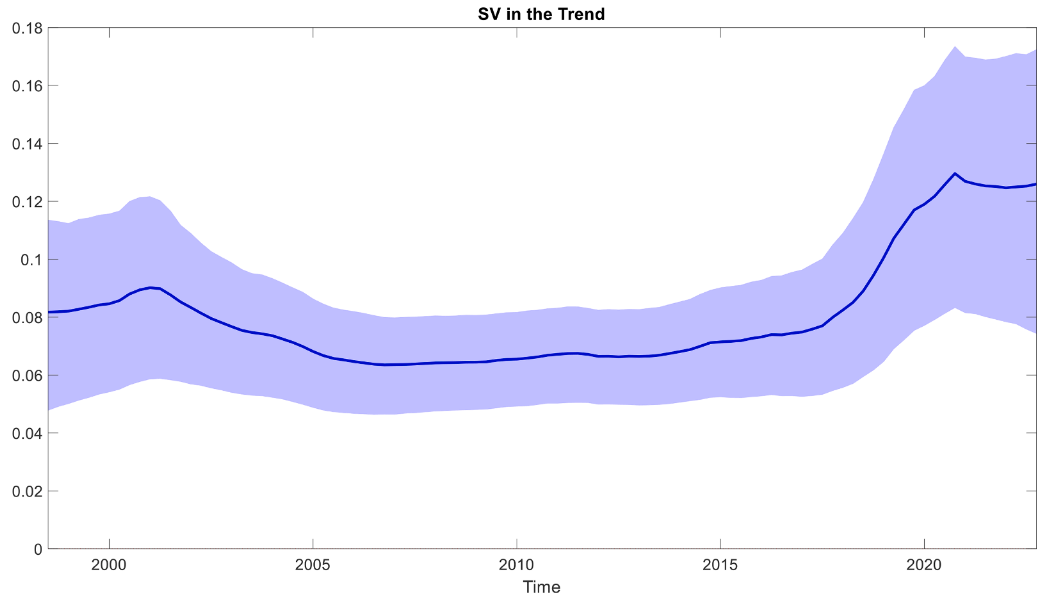

Fig. A2. Estimated stochastic volatility of trend interest rate using horseshoe prior.
Note: Stochastic volatility – given as standard deviation – in percent on vertical axis. Shaded area gives the 68 percent credible interval.
Source: Authors' own calculations.

given that our purpose is to estimate the trend short-term interest rate which – as we pointed out above – describes the level at which the model's forecasts will converge (and thereby the model's estimate of where the short-term interest rate is likely to stabilise in the future). It is a stylised fact that short-term interest rates are quite persistent and expectations at shorter horizons exhibit more co-

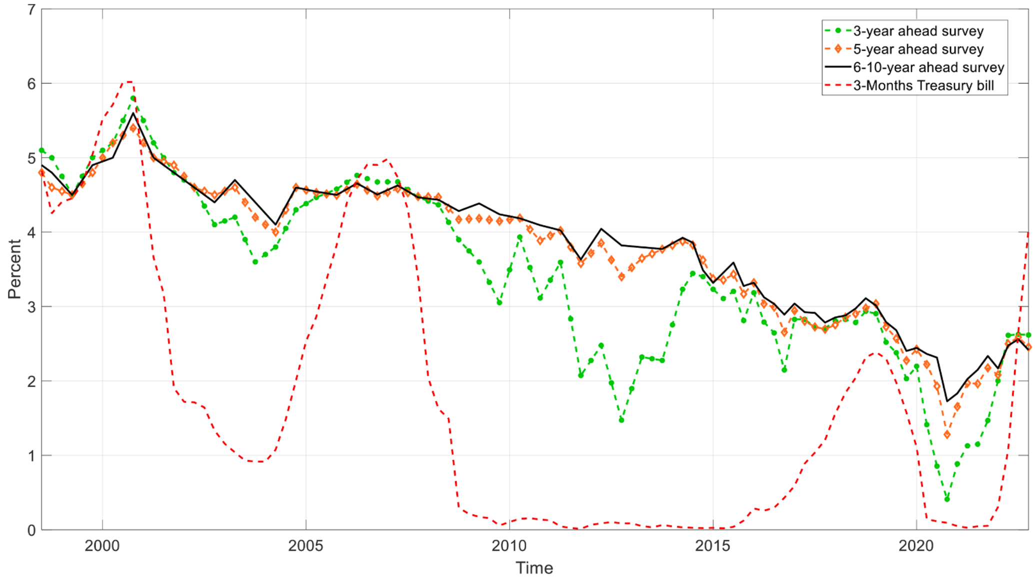

Fig. A3. Alternative survey data.
Note: Percent on vertical axis. The red dashed line represents the actual three-month Treasury bill rate. The black solid line is the six-to-ten-year-ahead expectation (from the main analysis). The green line with circles is the three-year-ahead expectation. The orange line with diamonds is the five-year-ahead expectation.
Source: Consensus Economics and Macrobond.

movement with the actual short-term interest rate.[6] The longer the horizon in the survey data, the more likely that the survey data capture the respondents' views of the level they think the short-term interest rate will stabilise at.

We conduct our sensitivity analysis using survey data for two additional horizons: three and five years.[7] The additional survey data are shown in Fig. A3 in the Appendix and behave largely as would be expected. Three-year-ahead expectations show the largest volatility and the most co-movement with the actual short-term interest rate. Five-year-ahead expectations are similar to the six-to-ten-year-ahead expectations, albeit marginally more volatile and more correlated with the actual short-term interest rate.

Results from estimating the model using the data for the two shorter horizons (one at a time) are given in Fig. 5. The estimated trend short-term interest rate from the model with the five-year-ahead expectations is very similar to the findings of our main analysis; the differences are typically negligible, although somewhat larger in 2020 and 2021. This similarity is to be expected given the similarity of the five-year-ahead and six-to-ten-year-ahead survey expectations.

The trend short-term interest rate estimated with three-year-ahead expectations shows substantially more co-movement with the actual short-term interest rate. This is also to be expected; the survey data are supposed to assist the model in estimating the trend short-term interest rate and if the survey data suggest that this trend shows a fair degree of co-movement with the actual short-term interest rate, the estimated trend should reflect this. We conclude that one should aim to use as long a forecast horizon as possible for survey expectations when estimating this type of model. While five years might be an acceptable horizon at which a respondent considers the short-term interest to have stabilised, this is clearly not the case for the three-year horizon.

5. Conclusions

Standing in early April 2023, there is fairly broad consensus that short-term nominal interest rates will remain high in the short run as high inflation is being combatted. However, it is less obvious where they are headed in the medium to long run. The estimates from our unobserved-components stochastic-volatility model indicate that while the trend short-term interest has risen slightly of late, it is still low from a historical perspective. This does not support the view that the three-month Treasury bill rate will stay at the current

[6] For example, it was pointed out already by Pesando (1979) that short-term interest rates might have a behaviour that is close to a random walk. Due to the high persistence of short-term interest rates, there has also been a debate on whether they are mean reverting or have a unit root; see, for example, Wu and Chen (2001), Romero-Ávila (2007) and Christiansen (2010).

[7] The *Consensus Forecasts* survey contains forecasts at seven horizons: *i)* three months, *ii)* twelve months, *iii)* at the end of the second calendar year, *iv)* at the end of the third calendar year, *v)* at the end of the fourth calendar year, *vi)* at the end of the fifth calendar year, and *vii)* six-to-ten years. In this sensitivity analysis, we use *iv* and *vi*. Three months and twelve months are fixed-horizon forecasts. However, the expectations that give a value for the end of the second, third, fourth or fifth calendar year have a time-varying horizon. For example, in the January survey, the expectation for the end of the second calendar year has a horizon which is close to two years; in October, on the other hand, this expectation has a horizon that is just approximately 14 months.

high levels. Rather, it is consistent with the view that the forces that have kept short-term interest rates low since the Global Financial Crisis – which include factors driving trend growth and productivity, inequality and demographic factors – will continue to put substantial downward pressure on short-term interest rates again in a not-too-distant future. The time-series analysis presented here is one approach to assessing the future level of the short-term nominal interest rate and complements analysis of structural determinants of trend short-term interest rates.

CRediT authorship contribution statement

Meredith Beechey: Conceptualization, Methodology, Investigation, Writing – review & editing. **Pär Österholm:** Conceptualization, Methodology, Investigation, Writing – review & editing. **Aubrey Poon:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – review & editing.

Data availability

The authors do not have permission to share data.

Acknowledgements

The views expressed in this paper are those of the authors and should not be attributed to the Reserve Bank of Australia or Sveriges Riksbank. Poon gratefully acknowledges financial support from Jan Wallanders och Tom Hedelius stiftelse (grant number B20–0020). We are grateful to two anonymous referees for valuable comments.

Appendix

Fig. A1, Fig. A2, Fig. A3

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