Competing risks-based resilience approach for multi-state systems under multiple shocks*

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Abstract: Effective measurement of system resilience provides a comprehensive understanding of the system's characteristics. However, little research has been devoted to the resilience of a technical system that is affected by both the degradation process of the system and the external shocks. In addition, existing studies have mainly evaluated the resilience of systems without considering competing risks, and rarely investigated the transient resilience evaluation subjected to the interaction of shocks and maintenance. In this paper, a new resilience model is proposed for the systems under competing risks. The paper first introduces a competing risk model to depict the failure modes of the single-component system, and then uses semi-Markov processes to describe the state transition process when the system suffers the attacks of multiple shocks. Then, according to the multi-state division of the system, a resistibility index, an absorbability index and a recoverability index are proposed and the overall resilience is then introduced. Considering that the system needs to meet the reliability requirement, constrained by limit budget for maintenance, a reliability and cost-based resilience model is proposed. Finally, the case of a radar system subjected to shocks of one type and multiple types of shocks is given to illustrate the concept developed in this paper.

Keywords: Competing risks; Resilience; System reliability; Condition-based maintenance; Semi-Markov process

1. Introduction

Engineered systems should be able to not only withstand external shocks such as natural or man-made disasters, but also recover quickly to the working condition from the disasters. High resilience has become an indispensable requirement for many modern systems in the face of threats posed by increasingly severe operating environment such as extreme weather. Resilience is often defined as the ability of a system to resist, mitigate, and recover quickly from potential disruptions [1]. Failures of single-component systems are usually due to a combination of internal causes (aging, wear) and

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external causes (shocks). Failures can be caused by internal factors such as deterioration and external shocks, which are competing risks [2], [3]. The presence of multiple failure causes on single-component systems make the relevant research a huge challenge and it therefore still an open research problem in quantifying the resilience of single-component systems.

Some research effort has been devoted towards the modeling the resilience for systems with under multiple failure modes, or competing risks. For example, Che et al. [4] developed a general reliability model for micro-engines affected by natural degradation and random shocks. Feng et al. [5] proposed a method based on a degradation-shock competing failure process to predict the life of a drill bit. Lyu et al. [6] believed that dependent competition failures include three types of failure, which are aging failure, and two types of sudden failure caused by random shocks. Wang et al. [7] proposed a competing failure model based on extreme shock damage. Ye et al. [8] proposed a new competing failure model for automated manufacturing systems to study the complex interactions of machine failure, product quality, etc. Wang et al. [9] addressed the problem of correlated probabilistic competing risks in system reliability analysis. Lyu et al. [10] proposed a new generalized surface wear model through combining the correlated competing failure processes.

Some research efforts have been made to the modeling of resilience. In terms of resilience for multi-state systems, Zeng et al. [11] developed a framework for resilience analysis of multi-state energy systems under extreme event shocks. Tan et al. [12] proposed four resilience metrics to evaluate the resilience of multi-state systems from different perspectives of resilience. Dhulipala et al. [13] proposed a Markov process-based model for multi-state systems under external shocks to model system resilience. Xu et al. [14] proposed a resilience-based component importance ranking method for multi-state systems from the perspective of the system recovery process after multi-shock events. Wang et al. [15] enhanced the overall resilience of multi-state interdependent infrastructure networks under uncertain disruption scenarios by solving the preparedness planning problem.

In terms of resilience for systems under multiple shocks, Cai et al. [16] proposed a resilience assessment method combining Markov models and dynamic Bayesian networks for systems under multiple shocks. Wu et al. [17] developed a Markov process-based resilience assessment framework considering internal degradation and external shocks. Tang et al. [18] investigated the resilience of rail transit systems under multiple disturbance events. Iannacone et al. [19] proposed a unified formulation for the degradation and recovery of infrastructure systems after multiple shocks to quantify its resilience over time. Yan and Dunnett [20] evaluated the resilience of nuclear power plants under aging and multiple shock events using the petri model. Cadini et al. [21] developed a Monte Carlo simulation program to evaluate the resilience of grid systems under extreme shock events. Liu et al. [22] proposed a few operational proactive strategies to improve system resilience under extreme shocks. Yodo et al. [23] developed a dynamic Bayesian network to evaluate the resilience of complex systems under shocks.

In terms of the application of resilience management, Dong et al. [24] proposed a comprehensive resilience index to evaluate transportation networks from the perspective of reliability and stability. Ferrario et al. [25] used resilience to analyze the characteristics of power networks to improve their recovery ability after multiple shocks. Taghizadeh et al. [26] proposed a probabilistic framework for evaluating the resilience of transportation networks during medical emergency situations. Panteli et al. [27] defined the key resilience characteristics that a power system should have and gave strategies to enhance resilience. Mahmoud et al. [28] developed a framework to comprehensively measure healthcare system resilience. Chen et al. [29] proposed a framework for assessing supply chain reliability and resilience. Magoua and Li [30] added the human factor to resilience research to provide accurate solutions for improving the resilience of critical infrastructure systems. Panteli and Mancarella [31] proposed a conceptual framework on resilience and evaluated the resilience of electrical power systems. Zhao et al. [32] developed a resilience assessment framework using the hidden Markov model and investigated the resilience of the water supply system in Shanghai. Gao et al. [33] proposed a resilience-oriented service restoration approach for microgrid systems. Talukder et al. [34] measured the resilience of the power system by quantifying the stability level of the power system. Ma et al. [35] used node protection strategies to improve the robustness and resilience of a network.

Existing literature has studied the resilience of systems after damage occurs, however, there are still knowledge gaps. First, the damage to systems in the existing literature is often assumed to be caused by external shocks (e.g., [16], [18], [21][23]), which does not apply to systems subjected to both internal and external shocks. Therefore, a competing risk model should be introduced to complement the causes of system damage, which aligns with the reality and makes the resilience model more complete. Second, existing literature has rarely investigated the transient resilience evaluation of systems subjected to the interaction of shocks and maintenance (e.g., [11], [14], [15]). These two knowledge gaps motivate us to conduct the current research work.

In this paper, we classify random shocks into four types and propose a condition-based maintenance strategy. Besides, semi-Markov processes are used to describe the state transition process of the system under shocks and maintenance. Then, an overall resilience index is proposed to measure the resilience of the system based on the ability of the system to resist, absorb and recover from external shocks. Third, the existing literature mainly evaluates system resilience and little has studied approaches to enhance system resilience (e.g., [24], [26], [28], [29], [31][34]). Therefore, a reliability and cost-based resilience optimization model is proposed. A solution is provided for improving the resilience under the available resources. The feasibility of the proposed resilience optimization model is demonstrated in the case study of a radar system subjected to shocks of one type and multiple types of shocks, respectively.

The remaining of this paper is as follows. In Section 2, multi-state systems are analyzed based on competing risks and condition-based maintenance. Section 3 describes semi-Markov models for multi-state systems under multiple shocks. The resilience optimization model under reliability and cost constraints is presented in Section 4. In Section 5, cases of a radar system subjected to shocks of one type and multiple types of shocks are given. Finally, Section 6 closes the paper.

Nomenclature	
S_i	Performance state, $i = 0,1,2,3,4$
$N(t)$	Number of shocks to the system in $[0, t]$
W_i	Shock intensity, $i = 1,2, \dots, N(t)$
D_i	Shock threshold, $i = 1,2,3$
MS_1	Preventive maintenance
MS_2	Corrective maintenance
MS_3	Special maintenance
$Z_Y(t)$	Total degradation
M	Degradation threshold
$Z(t)$	Natural degradation
α	Degradation volume at the initial moment
β_i	Degradation rate
$Y(t)$	Total degradation volume of the random shock
Y_j	Degradation value of the jth random shock
F_i	Performance value
$\boldsymbol{P_0}$	Initial state vector
$Time_i$	Moment of state transition probability
$p_{ij}(t)$	Probability of transition from S_i to S_j at moment t
$\bar{F}(t)$	Performance value at moment t
$\boldsymbol{P}(t)$	State transition probability matrix
$P_i(t)$	Probability of being in state S_i
$T_{\{Z(t) \le M\}}$	The time when the system is not fully aged in $[0, t]$

$R(t)$	System reliability
R_0	Minimum reliability to meet basic operating requirements
$m_{i,j}$	Direct loss, $0 \leq i < j \leq 3$
n_i	Unit time indirect loss
$C_R(t)$	Base cost when $R(t) = R_{min}$
$C_m(t)$	Direct cost
$C_n(t)$	Indirect cost
$C_k(t)$	Maintenance cost
$C(t)$	Total cost
C_{bd}	Maximum budget cost
P_{rt}	Resistibility
P_{an}	Absorbability
T_{ry}	Total time of maintenance
T_i	Cumulative time that the system stays in state S_i in $[0, t]$
T_{sd}	Maximum maintenance time
P_{ry}	Recoverability
P_{ol}	Overall resilience

2 Multi-state systems considering competing risks and maintenance strategies

The performance state of a single-component system is divided into five parts, which are the normal working state S_0, the slight damaged state S_1, the severe damaged state S_2, the fault state S_3, and the fully aged state S_4 [36], [37].

Suppose the system is subjected to a shock. When the shock intensity W_i is greater than the shock threshold D_i (D_1,D_2,D_3), the performance state of the system changes from the high-performance state to the low performance state (e.g., from S_0 to S_1). A shock with high intensity causes a jump of the performance state of the system to transition (e.g., from S_0 to S_3). With different thresholds of the shock, the shock is classified into four levels: Level I for $0 \leq W_i \leq D_1$; Level II for $D_1 \leq W_i \leq D_2$; Level III for $D_2 \leq W_i \leq D_3$; and Level IV for $W_i \geq D_3$. Shocks not only bring degradation to the system, but also accelerate the natural degradation of the system. The higher the shock intensity level is, the greater the impact on the natural degradation of the system, and the more obvious the effect of aggravating the system degradation.

The degradation of the system due to natural aging will only cause the system performance state to change to the fully aging state (i.e., S_4). The degradation caused by shocks will only make the system performance state to change to the low performance state (i.e., S_1, S_2, S_3), not to the fully aging state (i.e., S_4).

In this paper, ordinary maintenance is considered as imperfect maintenance, which cannot bring the system to the best working condition (i.e., S_0). Ordinary maintenance is divided into preventive maintenance MS_1 and corrective maintenance MS_2. When the system is at S_2 (severe damage state), preventive maintenance will be carried out to change its state from S_2 (severe damage state) to S_1 (slight damage state) after the maintenance. When the system is at S_3 (fault state), corrective maintenance will be performed to change its state from S_3 (fault state) to S_1 (slight damage state). For single-component systems, during the time when preventive maintenance or corrective maintenance is carried out, shocks do not attack the system but the system still ages. Special maintenance MS_3 is the replacement of parts that have fully aged. Special maintenance is the direct replacement of old parts with new parts, so that the single-component system can change from state S_4 (fully aged state) to S_0 (normal working state). Fig. 1 is the state transition process under shocks and maintenance.

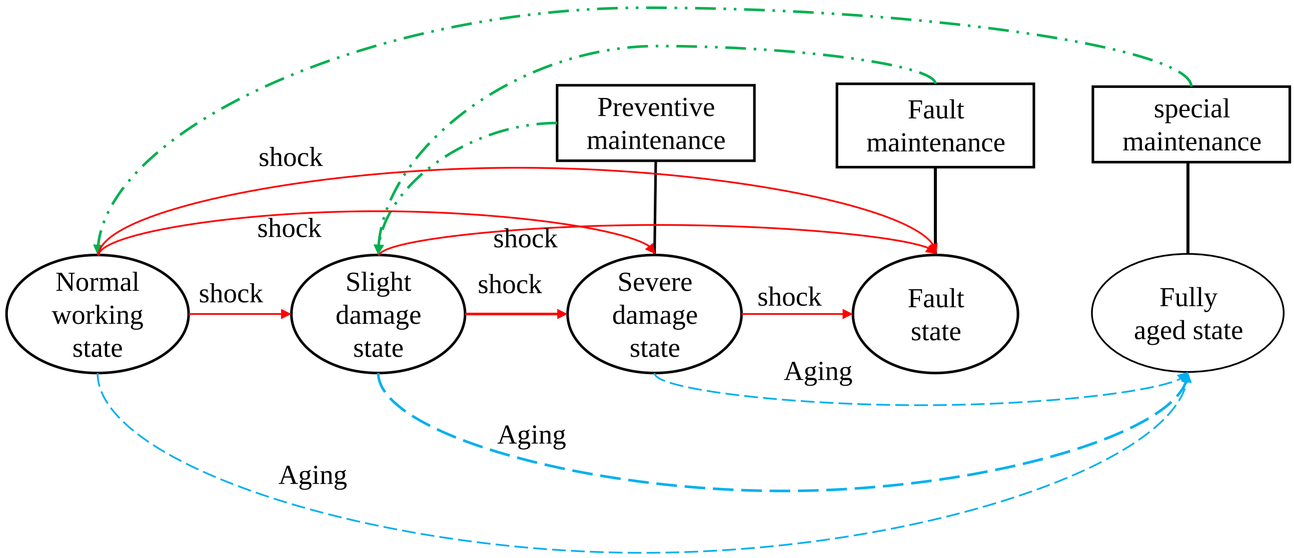

Fig. 1. State transition process of damage and maintenance.

Random shocks not only change the state of the system but also aggravate the performance of the system. Therefore, the total degradation $Z_Y(t)$ of the system during $[0, t]$ is the sum of the degradation due to natural aging and the degradation due to random shocks. When the system degradation volume is greater than the degradation threshold M, the system state changes to the fully aged state S_4, and the parts need to be replaced at this time. Fig. 2 gives the process and the corresponding state transition process of a system subjected to multiple shocks of different intensity, resulting in damage to the system.

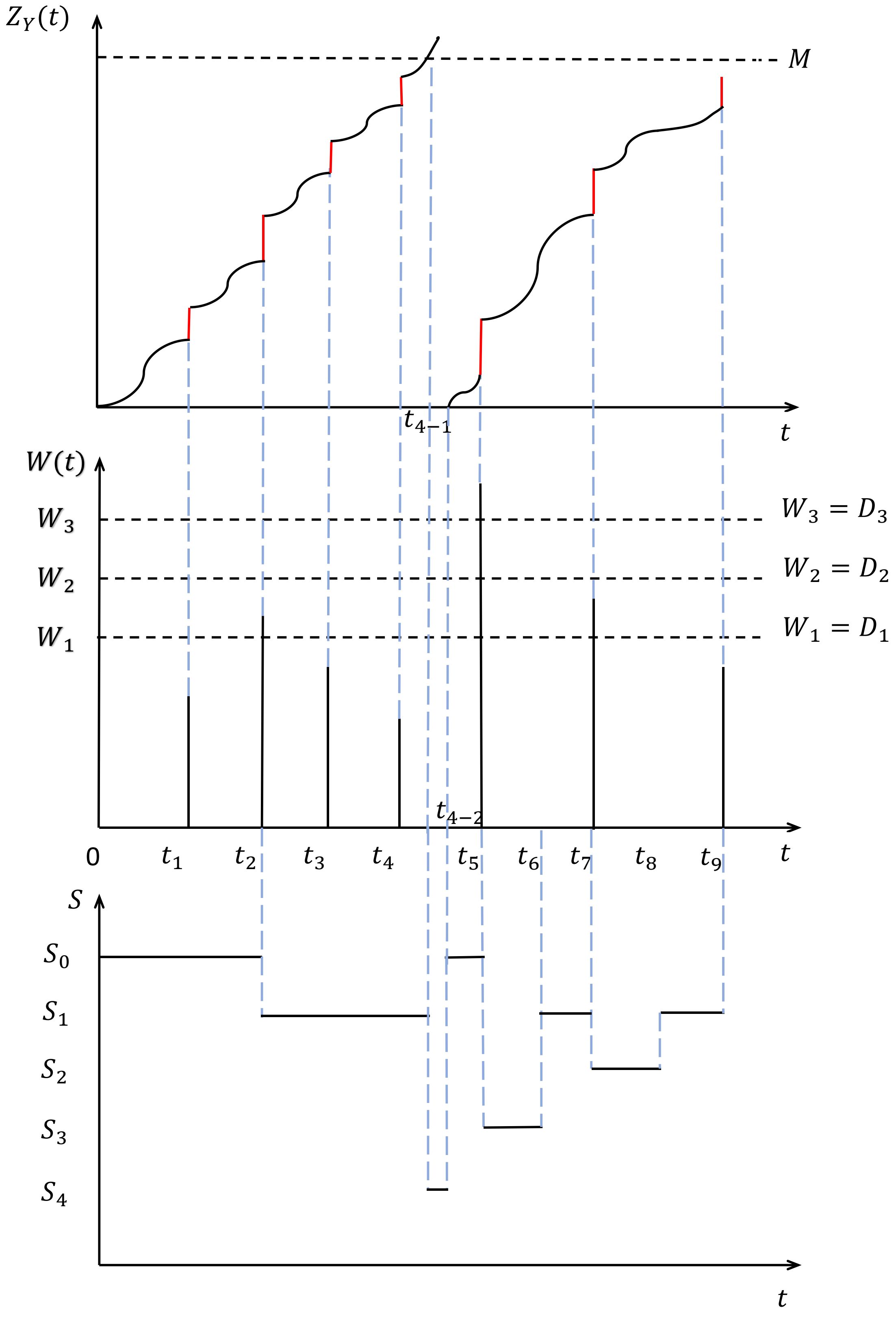

Fig. 2. System damage and state transition process under multiple shocks.

From Fig. 2, the system undergoes natural degradation within (0, t_1). At time t_1, the shock level is I, which will accelerate the system degradation, but the system state does not change. During (t_1, t_2), the system undergoes natural degradation. At time t_2, the shock intensity is at level II, at which a shock accelerates the degradation of the system, and the system state changes from S_0 (normal working state) to S_1 (slight damage state). At time t_{4-1}, the system degradation reaches the degradation threshold M, and the system state changes from S_1 (slight damage state) to S_4 (fully aged state), at which time it needs to be replaced with new parts, i.e., MS_3 (special maintenance). $(t_{4-2} - t_{4-1})$ is the time required to replace the new parts. At t_5, the shock intensity is level IV, causing the system state to change from S_0 (normal working state) to S_3 (fault state). $(t_6 - t_5)$ is the time to perform MS_2 (corrective maintenance), and the system state changes from S_3 (fault state) to S_1 (slight damage state) after the maintenance is completed. At t_7, the shock intensity is level II, at which a shock can cause the system state to change from S_1 (slight damage state) to S_2 (severe damage state). When the severe damage state is reached, MS_1 (preventive maintenance) is performed, causing the system state to change from S_2 (severe damage state) to S_1 (slight damage state). $(t_8 - t_7)$ is the time required for preventive maintenance.

To simplify the problem, the internal natural degradation is assumed to be a linear model [38], [39], and the same applies to other distribution models for calculation. The natural degradation of the system at time t is defined as

$$Z(t) = \alpha + \beta_i t, \tag{1}$$

where α is the degradation volume of the system at the initial moment and β_i is the rate of system degradation. Since shocks increase the rate of system degradation, β_i can be defined as

$$\beta_i = \left(1 + \frac{W_i}{\frac{D_1 + D_2 + D_3}{3}}\right)^{N(t)}, \tag{2}$$

where $N(t)$ is the number of shocks to the system in $[0, t]$, and W_i is the shock intensity of the ith shock.

The degradation of the random shock is $Y(t) = \sum_{j=1}^{N(t)} Y_j$, where Y_j denotes the degradation value of the jth random shock and is assumed to follow the normal distribution $\sim N(\mu_y, \sigma_y^2)$. The total degradation of the system at time t is defined as

$$Z_Y(t) = Z(t) + Y(t), \tag{3}$$

where $Z(t)$ is the natural degradation and $Y(t)$ is the degradation due to random shocks.

3 A series of semi-Markov models for multi-state systems under multiple shocks and maintenance

As aforementioned, the set of performance states of the single-component system is $S =$

$\{S_0, S_1, S_2, S_3, S_4\}$. Let F_i denote the corresponding performance value of the system in performance state S_i, then the corresponding performance level is denoted by $F = \{F_0, F_1, F_2, F_3, F_4\}$. The system in state S_0 is the normal operating state and the system performance F_0 is the best. The performance of the system in states S_3 (fault state) and S_4 (fully aged state) are zero. The performance values of the system in states S_1 and S_2 are between F_0 and F_3, and $F_1 > F_2$. The performance vector of the system is $\boldsymbol{F} = [F_0, F_1, F_2, F_3, F_4]$.

A new system starts working with its initial state S_0 $(t = 0)$, and the probability of the system sojourning in state i is denoted by E_i, then $P(X(0) = S_0) = 1$. Therefore, the initial state vector of the system can be defined as $\boldsymbol{P_0} = [P_0, P_1, P_2, P_3, P_4] = [1,0,0,0,0]$. $Time_i$ $(i = 1,2,3,4,5)$ denotes the moment of state transition. $Time_0$ indicates the moment of the system starting work, i.e., $Time_0 = 0$. The whole degradation process is divided into several time intervals, and each time interval represents the sojourn time of that state after a state transition. For example, $(Time_1 - Time_0)$ represents the sojourn time in state S_0. The sojourn time $(Time_3 - Time_2)$ in state S_2 represents the time of preventive maintenance. The sojourn time $(Time_4 - Time_3)$ in state S_3 indicates the time of corrective maintenance. The sojourn time $(Time_5 - Time_4)$ in state S_4 indicates the time required to replace new parts.

Since the level of the random shocks and the maintenance process are different, resulting in the time the system stays in each state may follow any distribution. Therefore, the semi-Markov process is used in this paper. Since the occurrence of a shock is random, the damage rate caused by each shock may be different. Besides, the diversity of maintenance ways leads to different maintenance effectiveness for each maintenance. Therefore, the effect of multiple external shocks and maintenance processes on the system can be described by semi-Markov processes. Based on the Markov characterization emphasized by the semi-Markov process, the next state is jointly determined by the current state, the corresponding sojourn time and transition probability, regardless of the past state or sojourn time [40] [41], i.e.,

$$\begin{aligned} &P\big(X_{n+1} = S_j,\, T_{n+1} \le t \mid X_n = S_i, \dots, X_2, X_1;\, T_n, \dots, T_2, T_1\big) \\ &\qquad = P\big(X_{n+1} = S_j,\, T_{n+1} \le t \mid X_n = S_i\big), \end{aligned} \tag{4}$$

where X_n denotes the last state of the system and X_{n+1} denotes the current state of the system.

Let the state transition probability $p_{ij}(t)$ represent the probability of the system changing from state S_i to state S_j at moment t, where $p_{ij}(t) = P\big(X_{n+1}(t) = S_j \mid X_n(0) = S_i\big)$.

Using the total probability formula and the definition of expectation, the performance $\bar{F}(t)$ of the system at moment t can be derived, and the calculation procedure is shown in Eq. (5).

$$\bar{F}(t) = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} P\big(X(0)=S_j\big)\,P\big(X(t)=S_i \big| X(0)=S_j\big)\right)F_i$$

$$= \boldsymbol{P_0}\boldsymbol{P}(t)\boldsymbol{F}^T, \tag{5}$$

where $\boldsymbol{P}(t)$ is the state transition probability matrix.

The state transition probability matrix $\boldsymbol{P}(t)$ is defined by the kernel matrix $\boldsymbol{\Phi}(t)$, $\Phi_{ij}(t) = P\big(X_{n+1}=S_j,\ T_{n+1} \le t \mid X_n = S_i\big)$. The mathematical calculation of the transition probability matrix is performed using the Markov-renew equation.

$$\boldsymbol{P}(t) = \boldsymbol{W}(t) + \int_0^t \boldsymbol{\Phi}'(\tau)\,\boldsymbol{P}(t-\tau)dt, \tag{6}$$

where $\boldsymbol{W}(t) = (W_{i,j}(t))$, $W_{i,j}(t) = \delta_{i,j}\big(1-\sum_{j=0}^{m} a_{i,j}(t)\big)$ and $\delta_{i,j}$ is a Kronecker function.

In a semi-Markov process, the kernel matrix $\boldsymbol{\Phi}(t)$ controls the next state of the system and the time is the sojourn time in the current state before making the next transition. The kernel matrix should satisfy the following conditions.

$$\lim_{t\to\infty}\sum_{j=1}^{N}\Phi_{ij}(t) = 1\,. \tag{7}$$

The transition model of system states based on the damage degradation and maintenance recovery is shown in Fig. 3.

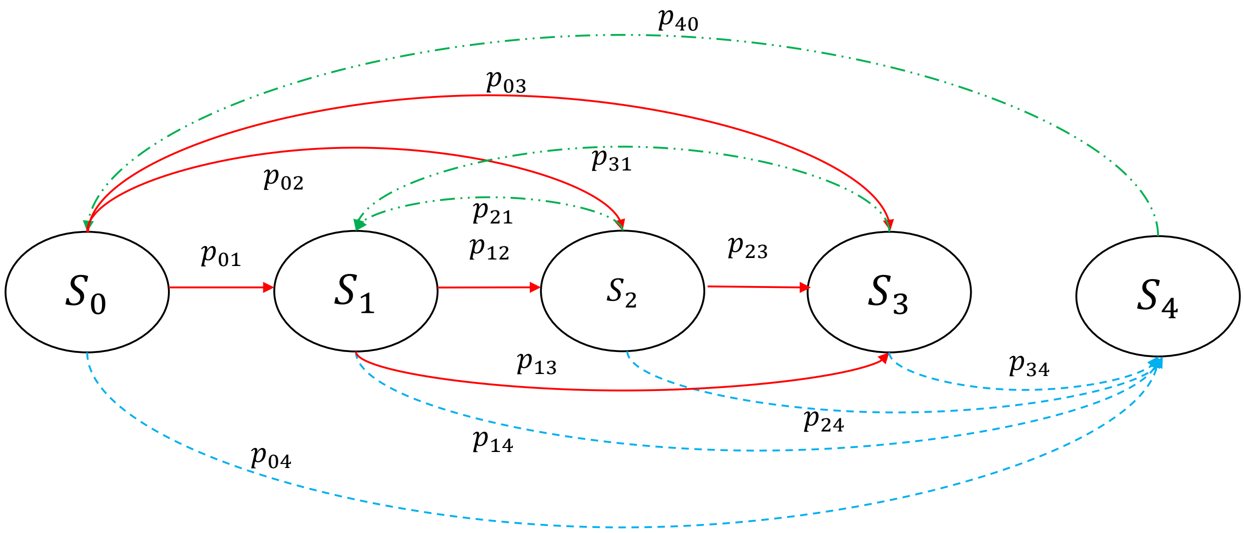

Fig.3. Multi-state transition model for degradation and recovery.

In Fig. 3, $p_{ij}(i=0,1,2; j=1,2,3; i<j)$ denotes the probability of state transition due to shock, and different levels of the shock intensity result in different state transition. $p_{i4}(i=0,1,2,3)$ denotes

the probability of the state transition due to aging. p_{21} denotes the probability of the state transition due to preventive maintenance. p_{31} denotes the probability of the state transition due to corrective maintenance, and p_{40} denotes the probability of the state transition due to special maintenance.

The system can be considered to be in the best performance state at the initial state, so the row vector of state probabilities $\boldsymbol{P_0} = [P_0, P_1, P_2, P_3, P_4] = [1,0,0,0,0]$. The probability of the system sojourning in each state at any moment is defined as

$$P_i(t) = \sum_{j=0}^{i} p_{ji}(t) - \sum_{j=i+1}^{5} p_{ij}(t) . \tag{8}$$

The sum of the probabilities of each state satisfies

$$\sum_{i=0}^{5} P_i(t) = 1 . \tag{9}$$

4 Resilience approach under the interaction of damage and maintenance

Improving system reliability to meet task requirements incurs reliability-associated costs. Degradation in system performance due to shocks and aging incurs performance-degradation-associated costs, and the maintenance process incurs maintenance-associated costs. In reality, the upper limit of each cost expenditure of the system is fixed. Therefore, the various cost of a single-component system is less than the upper limit of cost.

4.1 Reliability and cost constraints

Two conditions need to be satisfied for a system to remain in the working condition. First, the system is not fully aged, i.e., it is $Z(t) \le M$. The probability $P(Z(t) \le M)$ that the system is not fully aged on $[0, t]$ can be expressed as the ratio of the time that it is not fully aged to the total time t, as shown in Eqs. (10), (11) and (12).

$$T_{\{Z(t) \le M\}} = \int_0^t \beth\{Z(t) \le M\} \, du , \tag{10}$$

$$\beth\{Z(t) \le M\} = \begin{cases} 1, & if\ Z(t) \le M \\ 0, & otherwise \end{cases} , \tag{11}$$

$$P(Z(t) \le M) = \frac{T_{\{Z(t) \le M\}}}{t} . \tag{12}$$

In Eq. (12), $T_{\{Z(t) \le M\}}$ indicates the time when the system is not fully aged. Second, shocks did not cause the system to fail. For the second condition, we use the probability of the system sojourning in state S_0 (normal working state), state S_1 (slight damage state), and state S_2 (severe damage state) to measure. The system works with low performance even though it is damaged in states S_1 and S_2. Therefore, the reliability of the multi-state system is defined as

$$R(t) = P(Z(t) \leq M)\{P(X(t) = S_0) \cup P(X(t) = S_1) \cup P(X(t) = S_2)\}, \tag{13}$$

where $R(t)$ denotes the reliability of the system at moment t. To guarantee that the system can maintain the basic working needs, the reliability of the system needs to satisfy

$$R(t) \geq R_0, \tag{14}$$

where R_0 indicates the minimum value of reliability that needs to be achieved for a single-component system to meet basic operating requirements.

As discussed, costs include the cost of shock losses, maintenance costs, and the costs spent to meet basic reliability needs. Losses due to shocks include direct losses due to shocks and indirect losses due to low performance operation. The direct loss due to shocks relates the intensity of the shocks, and the indirect loss due to shocks is related to how long the system lasts in a low performance state. Maintenance costs include costs of preventive maintenance, corrective maintenance , and special maintenance.

The system undergoes a change from performance state S_i to state S_j $(0 \leq i < j \leq 3)$ during the shock damage phase, and the process incurs a direct loss of $m_{i,j}$.

$$\begin{cases} m_{i,j} > 0, if\ i < j \\ m_{i,j} = 0, if\ i \geq j \end{cases}. \tag{15}$$

The system also incurs maintenance costs during the maintenance phase, which are inversely proportional to the maintenance time by a factor of k_i $(i = 2,3,4)$.

In addition, there are periods during the damage and maintenance process when the system operates in the low performance state. The system operates in the low performance state S_i, compared to the optimal performance state operation in terms of reduced revenue, i.e., indirect losses, determined by the unit time loss n_i and the duration of the maintenance process T_i. The costs incurred by the maintainable multi-state system during the damage and maintenance processes are shown in Fig. 4.

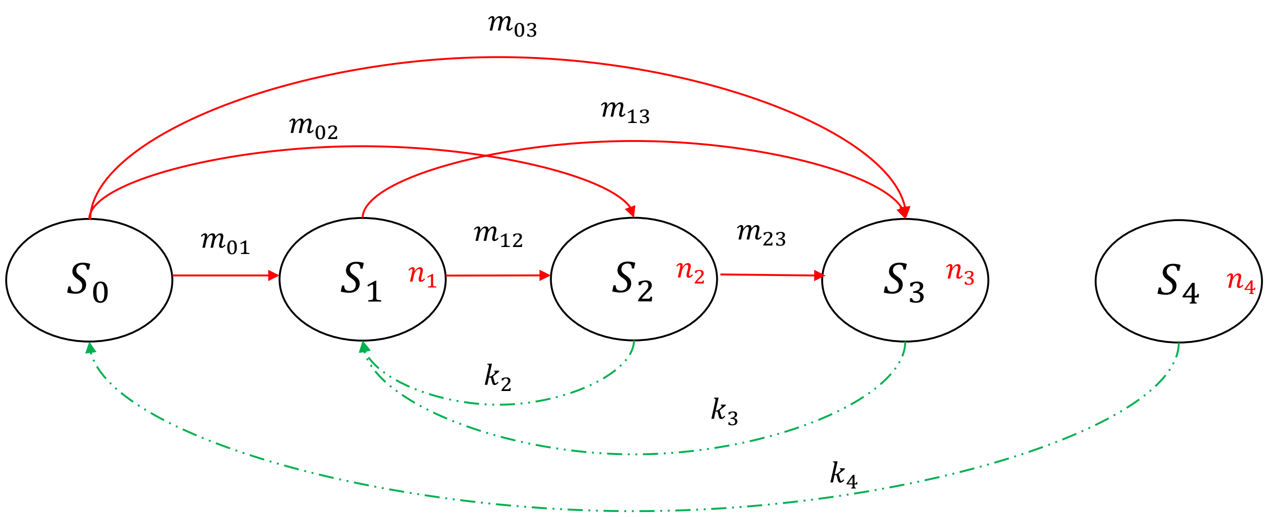

Fig. 4. Cost analysis diagram for multi-state system.

The relationship between the cost of a component and its reliability can be obtained from experience or from data on similar components. However, in many cases, such data are unreliable. In this paper, the cumulative distribution function (cdf) of cost with respect to reliability in [42] is assumed to give the cost spent to satisfy the basic reliability needs.

$$C_R(t) = c_R e^{\left[(1-f)\frac{R(t)-R_{min}}{R_{max}-R(t)}\right]}, \tag{16}$$

where c_R is the base cost when $R(t) = R_{min}$, and f denotes the feasibility that reliability can be improved, $f = [0,1]$. R_{min} is the minimum acceptable reliability and R_{max} is the maximum acceptable reliability.

Let $C(t)$ denote the total cost incurred by the system in $[0, t]$, which can be defined as

$$\begin{aligned} C(t) &= (C_m(t) + C_k(t) + C_n(t)) + C_R(t) \\ &= \left(\sum_{i=0}^{3}\sum_{j=0}^{3} m_{i,j} \cdot N_{i,j}(t) + \sum_{i=2}^{4} k_i \cdot \frac{1}{T_i} + \sum_{i=1}^{4} n_i \cdot T_i\right) \\ &\quad + c_R e^{\left[(1-f)\frac{R(t)-R_{min}}{R_{max}-R(t)}\right]}. \end{aligned} \tag{17}$$

In Eq. (17), $C_m(t), C_k(t),$ and $C_n(t)$ denotes the direct loss due to shocks, the maintenance cost of the system, and the indirect loss due to shocks, respectively. $N_{i,j}(t)$ denotes the number of the system moving from state S_i to state S_j from time 0 to time t. T_i denotes the cumulative time that the system stays in state S_j from 0 to t, which can be defined as

$$T_i = \int_0^t \zeta\{F(u) = F_i\}\, du, \tag{18}$$

$$\zeta\{F(u) = F_i\} = \begin{cases} 1, & if\ F(u) = F_i \\ 0, & otherwise \end{cases}, \tag{19}$$

where $F(u)$ denotes the performance value of the system at moment u and F_i denotes the performance value of the system at state S_i.

The total cost incurred during the operation of the system in $[0, t]$ cannot exceed the maximum budget cost C_{bd}, i.e.,

$$C(t) \le C_{bd}. \tag{20}$$

4.2 Resilience objectives

Resilience is a capability characteristic presented by a system in response to various perturbations and shocks, i.e., the ability of a system to resist, absorb and recover from external shocks from natural or man-made events [43]. In this paper, an overall resilience index will be proposed to measure the resilience of the system in three aspects: resistibility index, absorbability index, and recoverability index. Fig. 5 shows the correspondence between the three resilience indexes and the state of the system after multiple shocks in Fig. 2.

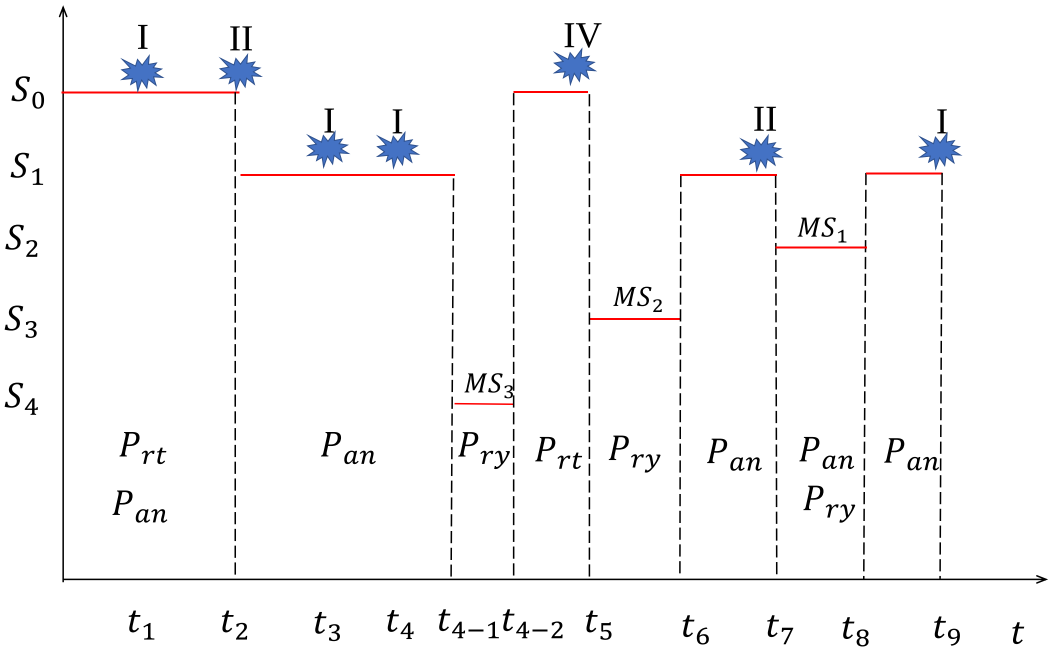

Fig. 5. Correspondence between resilience index and system state under multiple shocks.

1) Resistibility index

A system's resistibility is the probability that the system maintains its normal working state under random external shocks. In other words, a system with a high resistibility can operate at the best performance state after a random shock without the need for maintenance. The resistibility P_{rt} of a system, which is the probability that the system maintains the state S_0 on $[0, t]$, is defined by

$$P_{rt} = P\{X(\varphi) = S_0, \forall\varphi \in (0, t)\}, \tag{21}$$

where $X(\varphi)$ denotes the performance state of the system at the moment φ.

It is not hard to see that $0 \leq P_{rt} \leq 1$. When $P_{rt} = 0$, the system has zero resistibility and is in the critical collapse state. When $P_{rt} = 1$, the system can resist all external shocks and is in the operating state with the best performance. The larger the value of P_{rt}, the better the resistibility of the system.

2) Absorbability index

Absorbability refers to the ability of the system to absorb the impact of shocks, and after the shocks disappear, the system can return to its the best performance state without causing system failure or malfunction. In this paper, a shock is considered to have caused failure or malfunction to the system and to change the system state to a fault state or a fully aged state. The absorbability of a system should be reflected in the system state in $\{S_0, S_1, S_2\}$ after the shock, which keeps the system at an operating state, the absorbability is noted as P_{an}.

$$P_{an} = P\{X(\varphi) = S^a, \forall\varphi \in (0, t), S^a = \{S_0, S_1, S_2\}\}. \tag{22}$$

It is easy to see that $0 \leq P_{an} \leq 1$. When $P_{an} = 0$, the system has zero absorbability and a shock will make the system stop working. When $P_{an} = 1$, the system can absorb all external shocks and protect the continuous work. The larger the value of P_{an}, the better the absorbability of the system.

3) Recoverability index

Recoverability refers to the ability of a system to return to a high-performance operating state within a specified time after being damaged by a shock. The recovery time of the system is the preventive maintenance time in state S_2, the corrective maintenance time in state S_3, and the special maintenance time in state S_4. Therefore, the recovery time for high performance is defined as

$$T_{ry} = T_2 + T_3 + T_4, \forall \varphi \in (0, t), \tag{23}$$

where T_2, T_3 and T_4 denote the total time of the three maintenance durations, respectively, which are calculated according to Eqs. (18) and (19). The specified maximum maintenance time is T_{sd}, and T_{sd} is a function related to the number of impacts, impact strength, and damage rate.

$$T_{sd} = f(N(t), \boldsymbol{W}, \boldsymbol{\beta}), \tag{24}$$

where $N(t)$ is the number of impacts in $[0, t]$, $\boldsymbol{W}$ is the shock intensity matrix, and β is the degradation rate matrix, and the three correspond to each other. The recoverability is defined as

$$P_{ry} = \frac{T_{sd} - T_{ry}}{T_{sd}}. \tag{25}$$

It is easy to see that $0 \leq P_{ry} \leq 1$. When $P_{ry} = 0$, the system has zero recoverability. After shocks, the system maintenance time is longer. When $P_{ry} = 1$, the system has been in high performance operation, no maintenance is needed, The larger the value of P_{ry}, the better the recoverability of the system.

4) Overall Resilience

To better measure the overall resilience of the system, we propose an overall resilience index based on the established resistibility index, absorbability index, and recoverability index to comprehensively evaluate the resilience of the system. The overall resilience P_{ol} is defined as

$$P_{ol} = \sqrt[3]{P_{rt} \times P_{an} \times P_{ry}}, \tag{26}$$

where P_{rt}, P_{an}, and P_{ry} denote resistance resilience, absorption resilience, and recoverability, respectively. The overall resilience is obtained by utilizing the multiplication of the three resilience indexes which can reflect the level of coordination of the three resilience indexes.

It is easy to see that $0 \leq P_{ol} \leq 1$. When $P_{ol} = 0$, the system has no resilience. When $P_{ol} = 1$, the overall resilience of the system is very good, and all the resilience indexes are in the best condition. The larger the value of P_{ol}, the better the overall resilience of the system.

Therefore, the resilience optimization model of the single-component system under certain working capacity and limited cost constraints is defined as

$$\begin{cases} max : P_{ol} = \sqrt[3]{P_{rt} \times P_{an} \times P_{ry}} \\ R(t) \geq R_0 \\ C(t) \leq C_{bd} \end{cases}. \tag{27}$$

The overall resilience of the system for $P_{ol} = 1$ is the maximization objective. T_0, T_1, T_2, T_3, T_4 are the independent variables for system resilience optimization, and the constraints are reliability and cost constraints.

Therefore, the resilience framework for a single-component system is shown in Fig. 6.

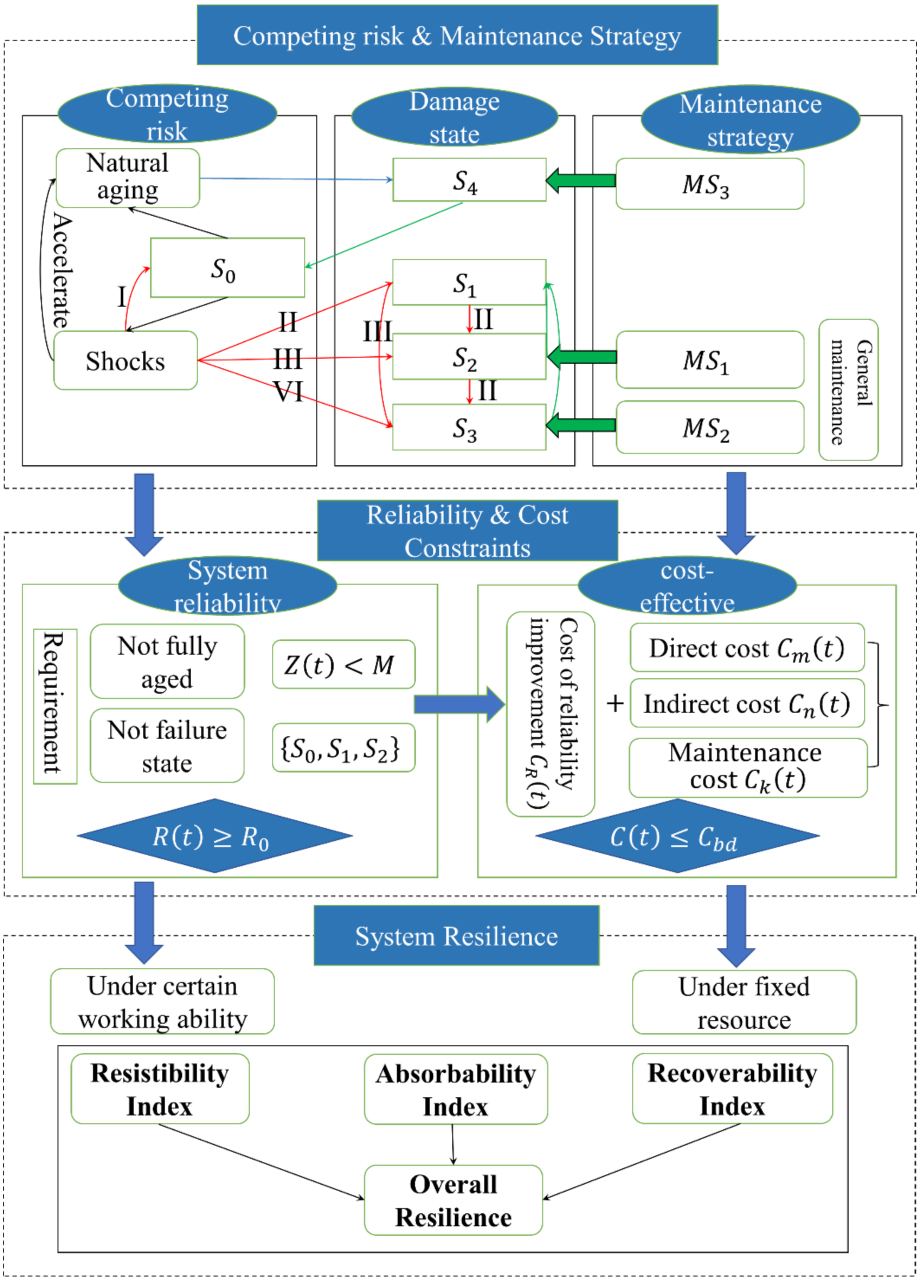

Fig. 6. System resilience optimization model based on a competing risk and condition-based maintenance.

5 Case study

A radar's main function is to monitor and detect enemy and enemy aircraft movements. Its failures are mainly caused by external shock failures and usage aging of the motor components within the system. Therefore, it can be viewed as a single-component equipment system subjected to competing risks.

The parameters of the radar system are set as follows. The performance state of the radar system and the corresponding performance values are shown in Table 1. The threshold values of the shock intensity are $D_1 = 5, D_2 = 15, D_3 = 30$. The radar system is intact at the initial moment, so the initial performance degradation of internal natural degradation $\alpha = 0$. It is assumed that the time of complete aging of the radar without any shock is 6 years, so the degradation threshold is $M = 365 \times 6 = 2190$. The frequency of shocks to the radar system obeys the Poisson distribution with rate parameter $\lambda = 8.22 \times 10^{-3}$. If the jth random shock type is level I, the amount of degradation is $Y_j \sim N(50, 10^2)$. If the jth random shock type is level II, the amount of degradation is $Y_j \sim N(100, 20^2)$. If the jth random shock type is level III, the amount of degradation is $Y_j \sim N(150, 30^2)$. If the jth random shock type is level IV, the amount of degradation is $Y_j \sim N(200, 40^2)$. The reliability of satisfying the basic operating capability is $R_0 = 0.75$. The cost of satisfying the minimum reliability is $c_R = 500$. The direct cost due to the shock is $m_{0,1} = 100$, $m_{0,2} = 250$, $m_{0,3} = 450$, $m_{1,2} = 150$, $m_{1,3} = 350$, $m_{2,3} = 200$. $k_2 = 1000$, $k_3 = 2000$, $k_4 = 3000$. The indirect cost per unit time due to low performance is $n_1 = 3, n_2 = 4, n_3 = 5, n_4 = 5$. The budget cost for one year is $C_{bd} = 4000$. The time interval between shocks satisfies $\Delta T \sim Exp(0.01)$. The transition time between the states obeys the Weibull distribution. When the shock type is at level II, the cumulative distribution function (cdf) of $S_0 \rightarrow S_1$ is $F_{0,1}^{\text{II}}(t) = 1 - e^{-(0.018t)^2}$, the cdf of $S_1 \rightarrow S_2$ is $F_{1,2}^{\text{II}}(t) = 1 - e^{-(0.015t)^3}$, and the cdf of $S_2 \rightarrow S_3$ is $F_{2,3}^{\text{II}}(t) = 1 - e^{-(0.01t)^{3.5}}$. When the shock type is at level III, the cdf of $S_0 \rightarrow S_2$ is $F_{0,2}^{\text{III}}(t) = 1 - e^{-(0.021t)^{2.3}}$, the cdf of $S_1 \rightarrow S_3$ is $F_{1,3}^{\text{III}}(t) = 1 - e^{-(0.02t)^{2.5}}$, the cdf of $S_2 \rightarrow S_3$ is $F_{2,3}^{\text{III}}(t) = 1 - e^{-(0.02t)^{3.5}}$. For the shock type level IV, the cdf of $S_0 \rightarrow S_3$ is $F_{0,3}^{\text{IV}}(t) = 1 - e^{-(0.03t)^{2.5}}$, the cdf of $S_1 \rightarrow S_3$ is $F_{1,3}^{\text{IV}}(t) = 1 - e^{-(0.03t)^3}$, the cdf of $S_2 \rightarrow S_3$ is $F_{2,3}^{\text{IV}}(t) = 1 - e^{-(0.03t)^{3.5}}$. For preventive maintenance, the cdf of $S_2 \rightarrow S_1$ is $F_{2,1}^{MS_1}(t) = 1 - e^{-(0.05t)^2}$. For corrective maintenance, the cdf of $S_3 \rightarrow S_1$ is $F_{3,1}^{MS_2}(t) = 1 - e^{-(0.031t)^2}$. For special maintenance, the cdf of $S_4 \rightarrow S_0$ is $F_{4,0}^{MS_3}(t) = 1 - e^{-(0.01t)^3}$. The initial state vector is $\boldsymbol{P_0} = [P_0, P_1, P_2, P_3, P_4] = [1,0,0,0,0]$. To simplify the calculation, we assume that $T_{sd} = 50 * n_{MS_1} + 80 * n_{MS_2} + 15 * n_{MS_3}$. n_{MS_1}, n_{MS_2}, and n_{MS_3} are the number of occurrences of preventive maintenance, corrective maintenance, and special maintenance, respectively. The time unit is one day.

Table 1. Performance values for each state of the radar system

Performance state	State meaning	The value of performance
S_0	Normal state	$F_0 = 1$
S_1	Slight damaged state	$F_1 = 0.7$
S_2	Severe damaged state	$F_2 = 0.3$
S_3	Fault state	$F_3 = 0$

S_4	Fully aged state	$F_4 = 0$

When there is only one type of shocks, we assume that the shock intensity level is II and the shock intensity $D = 10$. Assume that the system receives the first shock at $t = 30$ day. Therefore, the probability of the system in each state from 0~30 days is $\boldsymbol{P_0}(30) = [P_0, P_1, P_2, P_3, P_4] = [1,0,0,0,0]$. The degradation volume of the radar system under multiple shocks with maintenance in 1 year is simulated, as shown in Fig. 7. The resilience of the radar system is simulated. The resilience curves of multiple stages are stitched together to obtain the resilience of the radar system under shocks of one type with reliability and cost constraints in 1 year, as shown in Fig. 8.

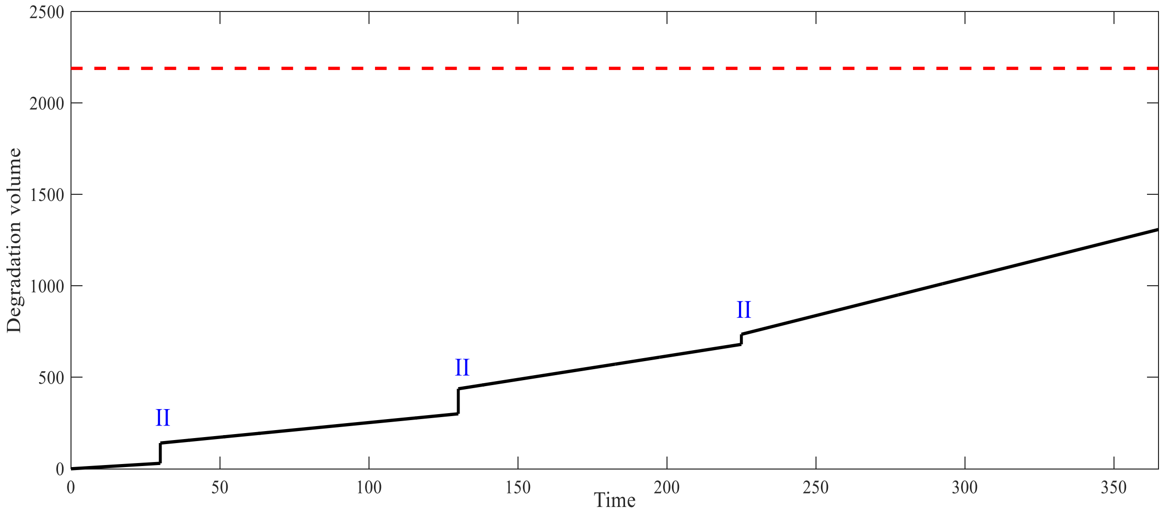

Fig. 7. Degradation volume of radar system under shocks of one type.

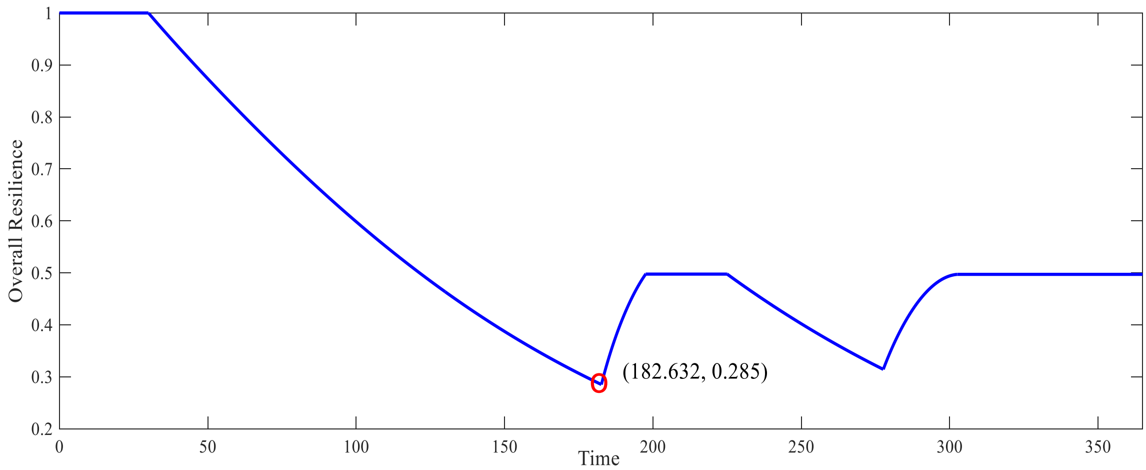

Fig. 8. Resilience of the radar system under shocks of one type.

According to Fig. 7, the radar system suffered three shocks in one year. The total degradation of the system did not exceed the degradation threshold M in 1 year, i.e., the radar system did not undergo fully aged. From Fig. 7 and Fig. 8, the radar system suffered two shocks in 30~182.632 days. The resilience of the system reached the minimum value of 0.285 at day 182.632. The radar system was in a severe damage state S_2 at 182.632 days, so preventive maintenance was carried out and the resilience of the system increased. On days 197.557 to 224.648, the system was not shocked and the resilience remained the same. On day 224.648, the system received a third shock, the system resilience decreased, and the system performance state changed from S_1 to S_2. Subsequently, the system underwent preventive maintenance again, the system resilience increased, and the system performance state improved to S_1. 302.556 days later, the system did not receive shocks, and the system resilience remained unchanged.

Under one type of shocks, the cost and reliability of the system's optimal resilience at the special points (the shock occurs, maintenance begins, and maintenance ends, etc.) are analyzed, as shown in Table 2.

Table 2. Cost and reliability at special points under shocks of one type

Time	Events	Reliability	Cost	Resilience
30	First shock occurs	1	100	1.000
128.435	Second shock occurs	0.896	495.305	0.463
182.632	End of the second shock	0.750	1512.093	0.285
182.632	First MS_1 occurs	0.750	1512.093	0.285
197.557	End of the first MS_1	0.873	1623.870	0.498
224.648	third shock occurred	0.873	1855.143	0.498
277.631	End of the third shock	0.754	2066.115	0.314
277.631	Second MS_1 occurs	0.754	2066.115	0.314
302.556	End of the second MS_1	0.906	2205.935	0.497
365	End	0.906	2393.267	0.497

When there are multiple types of shocks, it is assumed that the probability of occurrence of the three types of shocks II, III, and VI are the same, and the corresponding shock intensities are 10, 25, and 35, respectively. Assuming that the time when the system first suffers a shock is $t = 30$. The degradation volume of the radar system under multiple shocks with maintenance in 1 year is simulated, as shown in Fig. 9. The resilience of the radar system is simulated. The resilience curves of multiple stages are stitched together to obtain the resilience of the radar system under multiple types of shocks with reliability and cost constraints in 1 year, as shown in Fig. 10.

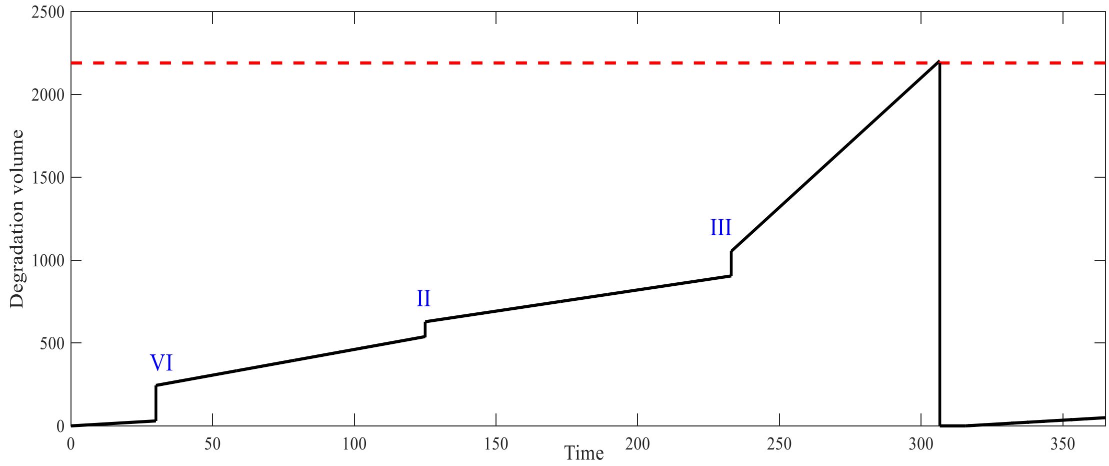

Fig. 9. Degradation volume of radar systems under multiple types of shocks.

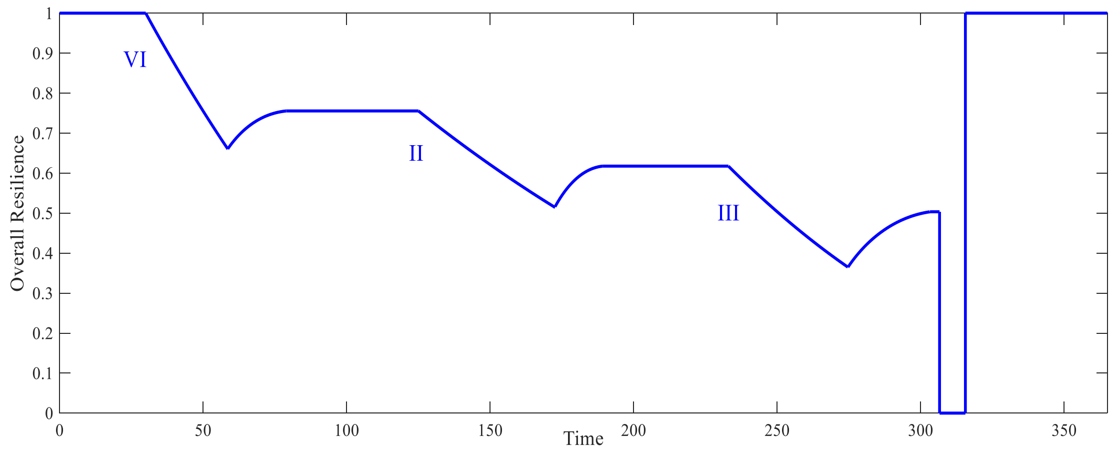

Fig. 10. Resilience of the radar system under multiple types of shocks.

According to Fig. 9, the radar system is subjected to shocks of three types and their levels are IV, II, and III. The radar system degrades to the degradation threshold on day 306.576, and special maintenance is carried out at this time. From Fig. 10, after the radar system suffers a shock of level IV, the system state changes from S_0 to S_3. Subsequently, corrective maintenance MS_2 is performed, and the system state changes to S_1 on day 78.980. Between days 78.980 and 126.833, the radar system does not receive any shock and the system's resilience does not change. On day 126.833, the radar system receives a level II shock, the system resilience decreases, and the performance state changes to S_2. Subsequently, preventive maintenance MS_1 occurs, and the system state returns to S_1. On day 232.546, the system receives a level III shock, the system performance decreases, and the performance state changes to the S_3, followed by failure maintenance MS_2. On day 306.576, the system degrades to the degradation threshold and the performance state changes to S_4. Subsequently, special maintenance is performed and the system performance state changes to S_0. The system resilience returns to 1.

The cost and reliability of the system optimal resilience under multiple types of shocks are analyzed at the special points (the shock occurs, maintenance begins, and maintenance ends, etc.), as shown in

Table 3.

Table 3. Cost and reliability at special points under multiple types of shocks

Time	Events	Reliability	Cost	Resilience
30	First shock occurs	1	450	1
58.571	End of the first shock	0.847	592.855	0.661
58.571	First MS_2 occurs	0.847	592.855	0.661
78.980	End of the first MS_2	0.926	792.896	0.756
126.833	Second shock occurs	0.926	942.896	0.756
172.619	End of the second shock	0.793	1126.040	0.514
172.619	MS_1 occurs	0.793	1126.040	0.514
189.286	End of MS_1	0.885	1252.707	0.618
232.546	Third shock occurs	0.885	1602.708	0.618
274.667	End of the third shock	0.75	2547.819	0.365
274.667	Second MS_2 occurs	0.75	2547.819	0.365
303.238	End of the second MS_2	0.871	2760.675	0.503
306.576	MS_3 occurs	0.871	3060.675	0
315.575	End of MS_3	0.856	3439.040	1
365	End	0.856	3439.040	1

6 Conclusions and future work

This paper developed a resilience optimization model for a single-component system, which differs from an ordinary system in that its failure modes considered in this paper originates from either external shocks or its own natural aging. Therefore, a competing risk model was introduced to describe the failure process of the single-component system. Considering that shocks of different intensities aggravate system aging to different degrees, random shocks were classified into four types of shocks, and condition-based maintenance strategies were proposed, i.e., preventive maintenance, corrective maintenance, and special maintenance. It also considered that the replacement of new parts as special maintenance, which takes a certain amount of time, is more in line with the realistic time delay caused by the re-purchase of new parts due to insufficient inventory. The system is assumed a multiple state system. Semi-Markov processes were used to describe the state transition process of the system under the interaction of multiple shocks and maintenance. The paper proposed a resistibility index, an absorbability index, and a recoverability index and then proposed an overall resilience index. Since the system needs to meet the reliability and cost constraints required for proper operation, a reliability and cost-based resilience optimization model was proposed. A solution was provided for the system to improve the resilience under the available resources. Finally, the feasibility of the proposed resilience optimization model was demonstrated by giving separate examples of radar system subjected to shocks of one type and multiple types of shocks.

There are several research directions worth exploring in future research. First, it is possible to investigate the occurrence of shocks during system maintenance. This paper assumed that no shocks occur during maintenance, but in real situations, this may be the case. Second, the role of maintenance can be specified, such as maintenance can improve the system's ability to resist shocks, or maintenance can accelerate the recovery of the system.

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References

[1] Hosseini S, Barker K, Ramirez-Marquez JE. A review of definitions and measures of system resilience. Reliability Engineering & System Safety 2016; 145:47-61. https://doi.org/10.1016/j.ress.2015.08.006.

[2] Zhao X, Xu J, Liu B. Accelerated degradation tests planning with competing failure modes. IEEE Transactions on reliability 2018; 67(1):142-155. https://doi.org/10.1109/TR.2017.2761025.

[3] Qin Y, Zhang X, Zeng J, Shi G, Wu B. Reliability analysis of mining machinery pick subject to competing failure processes with continuous shock and changing rate degradation. IEEE Transactions on reliability 2022;1-13. https://doi.org/10.1109/TR.2022.3192060.

[4] Che H, Zeng S, Guo J. A reliability model of micro-engines subject to natural degradation and dependent zoned shocks. IEEE Access 2019; 7:174951-174961. https://doi.org/10.1109/ACCESS.2019.2957305.

[5] Feng T, Li S, Guo L, Gao H, Chen T, Yu Y. A degradation-shock dependent competing failure processes based method for remaining useful life prediction of drill bit considering time-shifting sudden failure threshold. Reliability Engineering & System Safety 2023; 230:108951. https://doi.org/10.1016/j.ress.2022.108951.

[6] Lyu H, Qu H, Yang Z, Ma L, Lu B, Pecht M. Reliability analysis of dependent competing failure

processes with time-varying δ shock model. Reliability Engineering & System Safety 2023; 229: 108876. https://doi.org/10.1016/j.ress.2022.108876.

[7] Wang X, Li L, Chang M, Han K. Reliability modeling for competing failure processes with shifting failure thresholds under severe product working conditions. Applied Mathematical Modelling2020; 89(2):1747-1763. https://doi.org/10.1016/j.apm.2020.08.032.

[8] Ye Z, Cai Z, Si S, Zhang S, Yang H. Competing failure modeling for performance analysis of automated manufacturing systems with serial structures and imperfect quality inspection. IEEE Transactions on Industrial Informatics 2020; 16(10):6476-6486. https://doi.org/10.1109/TII.2020.2967030.

[9] Wang Y, Xing L, Wang H, Coit DW. System reliability modeling considering correlated probabilistic competing failures. IEEE Transactions on reliability 2018; 67(2):416-431. https://doi.org/10.1109/TR.2017.2716183.

[10] Lyu H, Zhang X, Yang Z, Wang S, Li C, Pecht M. Reliability analysis for the dependent competing failure with wear model and its application to the turbine and worm system. IEEE Access 2021; 9:50265-50280. https://doi.org/10.1109/ACCESS.2021.3062026.

[11] Zeng Z, Fang YP, Zhai Q, Du S. A Markov reward process-based framework for resilience analysis of multistate energy systems under the threat of extreme events. Reliability Engineering & System Safety 2021; 209:107443. https://doi.org/10.1016/j.ress.2021.107443.

[12] Tan Z, Wu B, Che A. Resilience modeling for multi-state systems based on Markov processes. Reliability Engineering & System Safety 2023; 235:109207. https://doi.org/10.1016/j.ress.2023.109207

[13] Dhulipala SL, Burton HV, Baroud H. A Markov framework for generalized post-event systems recovery modeling: From single to multihazards. Structural Safety 2021; 91:102091. https://doi.org/10.1016/j.strusafe.2021.102091.

[14] Xu Z, Ramirez-Marquez JE, Liu Y, Xiahou T. A new resilience-based component importance measure for multi-state networks. Reliability Engineering & System Safety 2020; 193:106591. https://doi.org/10.1016/j.ress.2019.106591.

[15] Wang K, Xu Z, Liu Y, Fang Y. Resilience Enhancement for Multistate Interdependent Infrastructure Networks: From a Preparedness Perspective. IEEE Transactions on reliability 2021; 1-15. https://doi.org/10.1109/TR.2021.3132774.

[16] Cai B, Zhang Y, Wang H, Liu Y, Ji R, Gao C, Kong X, Liu J. Resilience evaluation methodology

of Engineered systems with dynamic-Bayesian-network-based degradation and maintenance. Reliability Engineering & System Safety 2021; 209:107464. https://doi.org/10.1016/j.ress.2021.107464

[17] Wu J, Yu Y, Yu J, Chang X, Xu L, Zhang W. A Markov resilience assessment framework for tension leg platform under mooring failure. Reliability Engineering & System Safety 2023; 231:108939. https://doi.org/10.1016/j.ress.2022.108939.

[18] Tang J, Xu L, Luo C. Multi-disruption resilience assessment of rail transit systems with optimized commuter flows. Reliability Engineering & System Safety 2021; 214:107715. https://doi.org/10.1016/j.ress.2021.107715.

[19] Iannacone L, Sharma N, Tabandeh A, Gardoni P. Modeling time-varying reliability and resilience of deteriorating infrastructure. Reliability Engineering & System Safety 2022; 217:108074. https://doi.org/10.1016/j.ress.2021.108074.

[20] Yan R, Dunnett S. Resilience assessment for nuclear power plants using Petri nets. Annals of Nuclear Energy 2023; 176:109282. https://doi.org/10.1016/j.anucene.2022.109282.

[21] Cadini F, Agliardi G, Zio E. A modeling and simulation framework for the reliability/availability assessment of a power transmission grid subject to cascading failures under extreme weather conditions. Applied Energy 2017; 185:267-279. https://doi.org/10.1016/j.apenergy.2016.10.086.

[22] Liu H, Wang C, Ju P, Li H. A sequentially preventive model enhancing power system resilience against extreme-weather-triggered failures. Renewable and Sustainable Energy Reviews 2022; 156:111945. https://doi.org/10.1016/j.rser.2021.111945.

[23] Yodo N, Wang P, Zhou Z. Predictive resilience analysis of complex systems using dynamic Bayesian networks. IEEE Transactions on reliability 2017; 66(3):761-770. https://doi.org/10.1109/TR.2017.2722471.

[24] Dong S, Gao X, Mostafavi A, Gao J, Gangwal U. Characterizing resilience of flood-disrupted dynamic transportation network through the lens of link reliability and stability. Reliability Engineering & System Safety 2023; 232:109071. https://doi.org/10.1016/j.ress.2022.109071.

[25] Ferrario E, Poulos A, Castro S, de la Llera JC, Lorca A. Predictive capacity of topological measures in evaluating seismic risk and resilience of electric power networks. Reliability Engineering & System Safety 2022; 217:108040. https://doi.org/10.1016/j.ress.2021.108040.

[26] Taghizadeh M, Mahsuli M, Poorzahedy H. Probabilistic Framework for Evaluating the Seismic Resilience of Transportation Systems during Emergency Medical Response. Reliability

Engineering & System Safety 2023; 236:109255. https://doi.org/10.1016/j.ress.2023.109255.

[27] Panteli M, Trakas D, Mancarella P, Hatziargyriou N. Power systems resilience assessment: Hardening and smart operational enhancement strategies. Proceedings of the IEEE 2017; 105(7):1202-1213. https://doi.org/10.1109/JPROC.2017.2691357.

[28] Mahmoud H, Kirsch T, O'Neil D, Anderson S. The resilience of health care systems following major disruptive events: Current practice and a path forward. Reliability Engineering & System Safety 2023; 235:109264. https://doi.org/10.1016/j.ress.2023.109264.

[29] Chen X, Xi Z, Jing P. A unified framework for evaluating supply chain reliability and resilience. IEEE Transactions on reliability 2017; 66(4):1144-1156. https://doi.org/10.1109/TR.2017.2737822.

[30] Magoua JJ, Li N. The human factor in the disaster resilience modeling of critical infrastructure systems. Reliability Engineering & System Safety 2023; 232:109073. https://doi.org/10.1016/j.ress.2022.109073.

[31] Panteli M, Mancarella P. Modeling and evaluating the resilience of critical electrical power infrastructure to extreme weather events. IEEE Systems Journal 2017; 11(3):1733-1742. https://doi.org/10.1109/JSYST.2015.2389272.

[32] Zhao S, Liu X, Zhuo Y. Hybrid Hidden Markov Models for resilience metrics in a dynamic infrastructure system. Reliability Engineering & System Safety 2017; 164:84-97. https://doi.org/10.1016/j.ress.2017.02.009.

[33] Gao H, Chen Y, Xu Y, Liu C. Resilience-oriented critical load restoration using microgrids in distribution systems. IEEE Transactions on Smart Grid 2016; 7(6):2837-2848. https://doi.org/10.1109/TSG.2016.2550625.

[34] Talukder S, Ibrahim M, Kumar R. Resilience indices for power/cyberphysical systems. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2021; 51(4):2159-2172. https://doi.org/10.1109/TSMC.2020.3018706.

[35] Ma L, Zhang X, Li J, Lin Q, Gong M, Coello CAC, Nandi AK. Enhancing robustness and resilience of multiplex networks against node-community cascading failures. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2022; 52(6):3808-3821. https://doi.org/10.1109/TSMC.2021.3073212.

[36] Xiahou T, Zeng Z, Liu Y, Huang HZ. Fusing conflicting multisource imprecise information for reliability assessment of multistate systems: A two-stage optimization approach. IEEE

Transactions on reliability 2022; 1-16. https://doi.org/10.1109/TR.2022.3187075.

[37] Xiahou T, Zeng Z, Liu Y, and Huang HZ. Measuring conflicts of multisource imprecise information in multistate system reliability assessment. IEEE Transactions on reliability 2022; 71(4):1417-1434. https://doi.org/10.1109/TR.2021.3087531.

[38] Hao S, Yang J, Ma X, Zhao Y. Reliability modeling for mutually dependent competing failure processes due to degradation and random shocks. Applied Mathematical Modelling 2017; 51:232-249. https://doi.org/10.1016/j.apm.2017.06.014.

[39] Fan M, Zeng Z, Zio E, Kang R, Chen Y. A sequential Bayesian approach for remaining useful life prediction of dependent competing failure processes. IEEE Transactions on reliability 2018; 68(1):317-329. https://doi.org/10.1109/TR.2018.2874459.

[40] Wang K, Lukito H. Lifespan and medical expenditure prognosis for cancer metastasis - a simulation modeling using semi-Markov process. Computer Methods and Programs in Biomedicine 2023; 234:107509. https://doi.org/10.1016/j.cmpb.2023.107509.

[41] Dhulipala SLN, Flint MM. Series of semi-Markov processes to model infrastructure resilience under multihazards. Reliability Engineering & System Safety 2020; 193:106659. https://doi.org/10.1016/j.ress.2019.106659.

[42] Si S, Liu M, Jiang Z, Jin T, Cai Z. System reliability allocation and optimization based on generalized Birnbaum importance measure. IEEE Transactions on reliability 2019; 68(3):831-843. https://doi.org/10.1109/TR.2019.2897026.

[43] Goerger SR, Madni AM, Eslinger OJ. Engineered resilient systems: A DoD Perspective. Procedia Computer Science 2014; 28:865-872. https://doi.org/10.1016/j.procs.2014.03.103.