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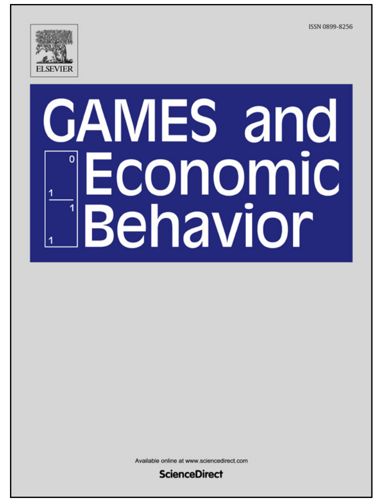

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Strategic default in financial networks

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Abstract. This paper investigates a model of default in financial networks where the decision by one agent on whether or not to default impacts the incentives of other agents to escape default. Agents' payoffs are determined by the clearing mechanism introduced in the seminal contribution of Eisenberg and Noe (2001). We first show the existence of a Nash equilibrium of this default game. Furthermore, we develop an algorithm to find all Nash equilibria and guide regulatory intervention that relies on the financial network structure. The algorithm provides a ranking for the set of Nash equilibria for specific financial network structures, which can serve as a measure of systemic risk. Finally, we show that introducing a central clearing counterparty achieves the efficient equilibrium at no additional cost.

JEL classification: C72, D53, D85, G21, G28, G33.

Keywords: systemic risk, default, financial networks, coordination games, ear decomposition, central clearing counterparty, financial regulation.

1. Introduction

Financial institutions carry out various transactions with each other, including risk–sharing and insurance. The architecture of the network of transactions between institutions can support financial stability because it enables them to share funding or transfer risk. But these linkages can also facilitate the diffusion of shocks through the system, due to chains of default and the domino effect. This is referred to as systemic risk. Systemic risk is costly for individuals, institutions and economies, as demonstrated by the last financial crisis. The obvious need for a stable financial system has led to a significant interest in policies that could reduce systemic risk and mitigate contagion.

This paper introduces a model of default in financial networks. We study a two-period economy where agents have a positive endowment in each period. The endowment represents agents' cash flows from outside the financial system. We assume that agents hold each other's financial liabilities and that this constitutes the network between them. These liabilities mature in the second period, and we assume that agents' second-period endowments are small and deterministic, so that they face a risk of default. More specifically, the liabilities structure results in cyclical payments interdependencies that are simultaneously computed according to the clearing mechanism described in the seminal contribution of Eisenberg and Noe (2001). The clearing vector satisfies three criteria:

- debt absolute priority, which stipulates that liabilities are paid in full in order to have positive asset;
- limited liability, which means that the payment made by each agent cannot exceed his inflows;
- equal seniority of all creditors, which implies pro rata repayments.

Agents can avoid default by storing part of their first-period endowment.

Due to complementarities in the payments, the decision taken by one agent to store part of his endowment exerts a positive externality on the other agents to whom he is connected.[1] We show that the strategic interactions in the financial system modelled here can be investigated as a coordination game, called the default game, where agents' decisions are simply whether to default or not. It is well known in the literature that

[1]The non-storage in our model can be equivalently interpreted as a bank run in the influential Diamond–Dybvig model.

coordination games will in general yield multiple pure–strategy Nash equilibria and that the set of pure–strategy Nash equilibria has a lattice structure—in particular, there are two extreme pure–strategy Nash equilibria. In our setting, the best equilibrium is the one where the largest number of agents choose the maximal action Non-Default and the worst equilibrium is the one where the largest number of agents choose the minimal action Default.

In the paper, we relate the multiplicity of Nash equilibria to the presence of a cycle of financial obligations.

Then, we develop a simple algorithm for finding all Nash equilibria of the default game. While there are easy algorithms for finding the maximal and minimal equilibria and relatively easy algorithms to compute all Nash equilibria in coordination games such as the default game (see Echenique, 2007), the advantage of the algorithm developed in this paper is that it relies on the financial network structure to inform the computation of Nash equilibria. By exploiting the network structure, our algorithm can quickly compute all Nash equilibria, and provide useful information on the strategic interactions between agents. In particular, the algorithm provides a ranking of the Nash equilibria in specific financial network structures. The ranking of the Nash equilibria is advantageous from a policy perspective since it can serve as a measure of systemic risk contribution of agents. More specifically, agents that default in all Nash equilibria will be called the *first wave of default*. Then, agents that default in all Nash equilibria except the highest Nash equilibrium will be called the *second wave of default* and so on.

In this paper, we show that the problem of inefficient coordination may arise in financial networks. Similar to other areas in economics, the strategic complementarities of payments due to the cyclical financial interconnections allow for the existence of multiple Nash equilibria. This gives rise to the question of which one of these equilibria will be the outcome of the underlying default game. From a policy perspective, given that inefficient coordination might pose a severe economic problem, there is a need for financial institutions fostering efficient coordination of agents' decisions. Recently, central clearing has become the cornerstone of policy reform in financial markets since it limits the scope of default contagion. Our analysis shows that introducing a central clearing counterparty (henceforth, CCP) also allows agents playing different actions at different Nash equilibria to coordinate on the efficient equilibrium at no additional cost. As a consequence, our result reinforces the key role CCP's play in stabilising financial markets.

This paper is structured as follows. In Section 2, we review the related literature. In Section 3 we present the model. We show the existence of a Nash equilibrium and develop an algorithm to find all Nash equilibria in Section 4, and Section 5 provides some policy implications of central clearing. Section 6 concludes the paper and Section 7 is an appendix devoted to the proofs.

2. Related Literature

The impact of the financial network structure on economic stability has been a subject of ongoing interest since the last financial crisis (of 2008). The seminal contributions of Allen and Gale (2000) and Eisenberg and Noe (2001) were first to acknowledge that the financial network structure determines default contagion, and would serve as a basis for many subsequent contributions.

Allen and Gale (2000) investigate how symmetric financial networks lead to contagion, where links represent sharing agreements. Their key finding is that incomplete financial networks are less resilient and more vulnerable to contagion than their complete counterparts. Eisenberg and Noe (2001) develop a static model of default contagion in a financial network where agents hold each other's financial liabilities and the activities and operations of each agent are condensed into one value: the operational cash flow. The repayment of liabilities will be interdependent, since whether an agent defaults or not is a result of his operational cash flow as well as the payments he receives from other agents. Eisenberg and Noe first prove the existence of a clearing payment vector that is unique under mild conditions. They also provide an algorithm to compute the clearing vector, which is important to predict chains of defaults.

Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) extend the Eisenberg–Noe model to accommodate agent exposure to outside shocks. They establish that up to a certain magnitude of shocks, the more connected the financial network is, the more stable it is; beyond this threshold, the connectedness of the network makes it more prone to contagion and thus more fragile. Elliott, Golub and Jackson (2014) introduce two concepts of cross-holdings that have distinctive and non-monotonic impact on default cascades.

Integration, which measures the dependence on counterparties, expands the extent of default contagion but reduces the probability of the first failure; while diversification, which measures the heterogeneity of cross-holdings, increases the propagation of failure cascades but decreases the exposure level among pairs of financial institutions. Cabrales, Gottardi and Vega-Redondo (2017) investigate the optimal network structure that maximizes risk-sharing benefits among interconnected firms while decreasing their risk exposure. Jackson and Pernoud (2020) investigate how the network structure impacts agents' investment strategies as well as optimal regulatory intervention. Other recent contributions include Teteryatnikova (2014) and Csóka and Herings (2016).

For a recent survey, see Jackson and Pernoud (2020). Several approaches have been investigated to mitigate the domino effect in the financial network, such as central clearing and identifying the most systemically relevant financial institutions and then targeting them through cash injections. For instance, Demange (2018), following a similar approach to Eisenberg and Noe (2001), develops a new measure, called the *threat index*, which identifies the most systemically relevant agents for optimal targeted cash injection.

3. The Model

Consider a two-period ($t = 1, 2$) economy with $N = \{1, 2, ..., n\}$ agents. Agent i's endowment in the first period is $z_i^1 \geq 0$ and in the second period is $z_i^2 > 0$. The endowment of agent i in each period denotes the cash flows arriving from outside the financial system. We assume that agents hold each other's liabilities, which mature in the second period. More specifically, given two agents $i, j \in N$, let $L_{ij} \in \mathbb{R}^+$ denote the liability that agent i owes agent j. Then, agent i's total liabilities are $L_i = \sum_{j \in N} L_{ij}$. Meanwhile, $\sum_{j \in N} L_{ji}$ is the total assets of agent i. Let $\boldsymbol{\alpha} = (\alpha_{ij})_{i,j \in N}$ denote the matrix of relative liabilities, with entries $\alpha_{ij} = \frac{L_{ij}}{L_i}$ representing the ratio of the liability agent i owes to agent j over the total amount of agent i's liabilities.

Each agent i can store an amount $x_i \in [0, z_i^1]$ from his first-period endowment and receives an interest rate $r > 0$ on his storage. Given the storage strategies of agents $\mathbf{x} = (x_i)_{i \in N}$, let $\boldsymbol{\pi}^{\mathbf{x}} = (\pi_i^{\mathbf{x}})_{i \in N}$ denote the clearing payment vector, uniquely defined as in

Eisenberg and Noe (2001), such that for each agent i it holds that

$$\pi_i^{\mathbf{x}} = \min\left\{ z_i^2 + (1+r)\,x_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}}; L_i \right\}.$$

This means that $z_i^1 - x_i$ denotes the assets of agent i in the first period and

$$z_i^2 + (1+r)x_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}}$$

denotes the assets of agent i in the second period.

The utility function of agent i is $U_i(e_i^1, e_i^2) = e_i^1 + e_i^2$, where e_i^1 represents assets withdrawn by agent i at $t = 1$ and e_i^2 is the asset of agent i remaining at $t = 2$ after receiving and making loan repayments. Therefore, the utility function of agent i, given the storage strategies of agents $\mathbf{x} = (x_i, x_{-i})$, can be expressed as

$$U_i\left(z_i^1 - x_i, z_i^2 + (1+r)\,x_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}} \right) = z_i^1 + z_i^2 + r x_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}}.$$

4. Nash Equilibria of the Default Game

First, we investigate further the economy introduced above. Observe that each agent will choose to store a positive amount of his first-period endowment if and only if he prefers (is better off) not to default; otherwise he will store nothing. If he prefers not to default, the combination of linear utility and the fixed interest rate implies that he will store his entire first-period endowment. Similarly, it is only the decision of an agent to default or not, rather than the amount of storage, that affects the other agents. This is because, if he defaults he will pay out his total second-period endowment and loan receipts, and if he does not default he will pay his total liability, neither of which is directly affected by his level of storage.

Therefore, the strategic interaction of agents in the economy can be investigated as a binary coordination game with two actions (Default) = 0 and (Non-Default) = 1 among which agents must choose. Now, define a threshold $\tau_i(\mathbf{a}_{-i})$ as the minimum amount agent i must pay in the second period to avoid default, given other agents' actions $\mathbf{a}_{-i}$.

Proposition 1. *The threshold $\tau_i(\mathbf{a}_{-i})$ is well-defined and decreasing in $\mathbf{a}_{-i}$.*

Proof. The proof of Proposition 1, together with all our other proofs, appears in the Appendix.□

Proposition 1 shows that the threshold $\tau_i(\mathbf{a}_{-i})$ is well-defined. Observe that agent i will choose to play 1 whenever

$$(1+r)z_i^1 - \tau_i(\mathbf{a}_{-i}) \geq z_i^1.$$

Therefore, the best reply function of agent i can be written as follows:

$$\Psi_i(\mathbf{a_{-i}}) = \begin{cases} 1 & \text{if } rz_i^1 - \tau_i(\mathbf{a}_{-i}) \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

A profile of actions $\mathbf{a}^* \in \{0,1\}^N$ is a Nash equilibrium if $a_i^* = \Psi_i\left(\mathbf{a^*_{-i}}\right)$.

The default game introduced above corresponds to a binary game of strategic complements. As defined in Topkis (1979), Milgrom and Roberts (1990), and Vives (1990) strategic complementarities arise if an increase in one agent's strategy increases the optimal strategies of the other agents.[2]

Theorem 1. *There exists a pure–strategy Nash equilibrium of the default game.*

Theorem 1 shows the existence of a pure–strategy Nash equilibrium. Understandably, the existence of a pure–strategy Nash equilibrium follows from the strategic complementarities between agents' actions, since the decision of an agent not to default makes it easier for other agents not to default too.

It is established in the literature that a binary game of strategic complements will in general have multiple pure–strategy Nash equilibria with a lattice structure. In particular, this class of games has two extreme equilibria: the best equilibrium is the equilibrium where the largest number of agents choose the maximal action (Non-Default) $= 1$; the worst equilibrium is the equilibrium where the largest number of agents choose the minimal action (Default) $= 0$.

For simplicity, for the remainder of this paper, we assume that at a Nash equilibrium of the default game, no agent is indifferent between (Non-Default) $= 1$ and (Default) $= 0$,

[2]See, Bulow, Geanakoplos and Klemperer (1985), Sobel (1988), Echenique and Sabarwal (2003), Amir (2005), Echenique (2007) and Barraquer (2013) for other economic applications of games of strategic complements.

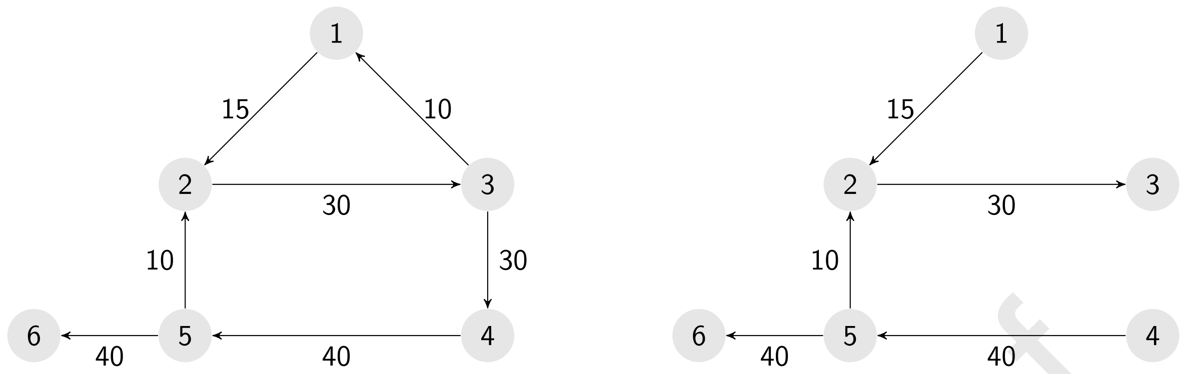

Figure 1. Cyclical obligations Unidirectional obligations

which is likely to be the case.[3] The following result highlights the connection between the multiplicity of equilibria and the structure of the financial network.

Proposition 2. *If the default game has multiple Nash equilibria then, the financial network has cyclical obligations.*

Proposition 2 shows that the presence of a cycle of financial obligations is necessary for the multiplicity of Nash equilibria as demonstrated in Figure 1. Eisenberg and Noe (2001) term this phenomenon cyclical interdependence and illustrate it as follows: "*A default by Firm A on its obligations to Firm B may lead B to default on its obligations to C. A default by C may, in turn have a feedback effect on A.*"

In a recent contribution Roukny et al. (2018) investigate a model where defaulting agents only recover a fraction of their assets and establish that multiple equilibria occur if and only if there is a cycle of financial liabilities. More specifically, their result shows that the contagion induced by an exogenous shock is not unique if and only if there exists a cycle composed of agents such that each agent's default depends on the default of his predecessor in the cycle. Interconnectedness, which is the main feature of the fabric of financial networks, provides therefore a feedback mechanism that can generate multiple equilibria. Similarly, our analysis highlights in a strategic setting that cyclical financial liabilities are the key condition for multiple equilibria. While in both settings multiple equilibria arise due to change in each agent's assets between Default and Non Default, partial recovery of assets in Roukny et al. (2018) or forgoing interest rate in this

[3]That is, this always holds except for a null set of first-period and second-period endowments.

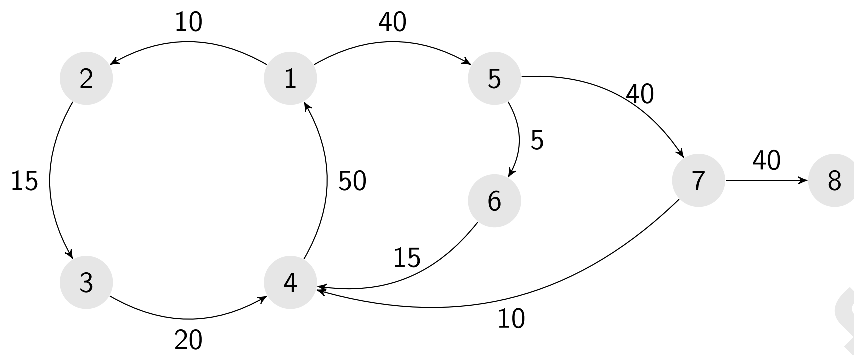

Figure 2. A financial network with eight agents

paper. In Roukny et al. (2018), agents are exposed to an exogenous shocks that propagate mechanically in the financial network whereas in this paper agents make strategic decisions on whether to default or not.

The next example illustrates the default game.

Example 1. Consider an economy of eight agents connected through their ownership of each other's liabilities, among which only the first seven agents are strategically relevant as illustrated in Figure 2. Agents' endowments in the first period are $\mathbf{z}^1 = (40, 45, 40, 25, 30, 75, 70)$ and in the second period are $\mathbf{z}^2 = (3, 3, 3, 3, 3, 3, 3)$ and the interest rate is $r = 0.1$. All agents have the same utility function $U_i(e_i^1, e_i^2) = e_i^1 + e_i^2$. This will result in three Nash equilibria $(0, 0, 0, 0, 0, 0, 0)$, $(0, 1, 1, 0, 0, 0, 0)$ (Fig. 4), $(1, 1, 1, 1, 1, 1, 1)$ (Fig. 5), for which computation will be provided at a later stage.

4.1. **A financial network with a unique SCC.** In the following, we will show that the close relationship between the multiplicity of Nash equilibria and the cyclical financial interconnections as shown in Proposition 2 is useful to solve for pure–strategy Nash equilibria of the default game. More specifically, we will provide an algorithm to find all pure–strategy Nash equilibria of the default game.

Recall that the financial network is strongly connected if there is a path of obligations between all pairs of agents. A strongly connected component (henceforth, SCC) of the financial network is a maximal[4] strongly connected subnetwork.

First, for simplicity, we consider the case of a financial network with a unique strongly connected component. We will use the following notion of *ear decomposition* of a network,

[4]In the sense that it is not properly contained in a larger strongly connected subnetwork.

which is useful given its close relationship to network connectivity. An ear decomposition of a network is a partition of the edges into directed paths, called ears. More precisely, an ear decomposition of a network is a partition of the edges into $E_p, \ldots, E_j, \ldots, E_1$ such that

- for each $j = p, \ldots, 1$ it holds that $E_j = \{(v_{j_1}, v_{j_2}), \ldots, (v_{j_{(k-1)}}, v_{j_k})\}$ is a directed path such that the start agent v_{j_1} and the end agent v_{j_k} are in $E_{j-1} \cup \ldots \cup E_1$ but the internal agents of E_j—that is, $v_{j_2}, \ldots, v_{j_{(k-1)}}$—are not in $E_{j-1} \cup \ldots \cup E_1$.
- E_1 is a cycle. That is, $v_{1_1} = v_{1_k}$.

A financial network is strongly connected if and only if it has an ear decomposition. In the following, we will rely on the ear decomposition to provide an algorithm to find all pure–strategy Nash equilibria of the default game of a financial network with a unique SCC.

Given an ear $E_j \in \{E_p, E_{p-1}, \ldots, E_1\}$, and an internal agent $v_{j_l} \in \{v_{j_2}, \ldots, v_{j_{(k-1)}}\}$, we define the *activation outflow* $A_j(v_{j_l})$ as the minimum outflow of the start agent v_{j_1} that is sufficient for v_{j_l} to escape default, conditional on the *activation outflows* of internal agents in preceding ears.

The algorithm, which we call USCCNE, builds on the above definitions and goes as follows:

Algorithm 1. (USCCNE)

(1) Compute an ear decomposition of the network $(E_p, \ldots, E_j, \ldots, E_1)$

(2) For each ear $E_j = E_p, E_{p-1}, \ldots, E_1$

  (a) Calculate the activation outflow from start agent v_{j_1} *that is sufficient for each internal agent* $u = v_{j_2}, \ldots, v_{j_{(k-1)}}$ *not to default, conditional upon the previous activation outflows of internal agents in* $\{E_p, E_{p-1}, \ldots, E_{j+1}\}$

  (b) Add calculated activation outflows to list $\mathcal{A}_j = \{(v_{j2}, A_j(v_{j2})), ...\}$.

(3) For each activation outflow profile in $\mathcal{A}_p \times \ldots \times \mathcal{A}_1$*, calculate repayment inflows into* $v_{p_1}, \ldots, v_{1_1}$ *and verify that the corresponding strategy profile is an equilibrium. Drop any strategy profiles that are not equilibria.*

The USCCNE algorithm makes the search for equilibria a recursive problem. More specifically, the algorithm traverses the network following the structure of the ear decomposition, starting from the final ear E_p and working backwards to E_1. At each ear E_j, the algorithm visits the internal agents outwards calculating their activation outflows from the start agent of the ear E_j, conditional upon the activation outflows of internal agents in preceding ears. At this point, we can eliminate some combinations of strategy profiles for agents in $E_j, E_{j+1}, ...E_p$. For example, the activation inflows for agents in E_{j+1} may be satisfied by the agents in E_j not defaulting. This would allow us to drop strategy profiles where the agents in E_j do not default, but the agents in E_{j+1} default. At the end, for each remaining strategy profile of activation outflows, the algorithm calculates repayment into each ear start agent and verifies whether the strategy profile is an equilibrium (that is, the repayment inflow is consistent with the activation outflow for each ear start agent).

Proposition 3. *(i) USCCNE identifies all equilibria.(ii) USCCNE returns only equilibria.*

USCCNE is particularly fast when there are fewer edges (liabilities) in the default game, and as a result, fewer ears. The number of ears in the network is equal to $|E| = m - n + 1$, where m is the number of edges. When the network has fewer, longer ears, the algorithm traverses the network more quickly. For example, given a cycle network, traversal of the network and calculation of consistent strategy profiles is the completed in linear time.

The key feature of the USCCNE is that it transforms the SCC into partition of ears (directed paths), where the strategy profiles of internal agents in each ear are computed based on the outflow of the start agent, conditional on strategy profiles in preceding ears.

We revisit Example 1 to illustrate the computation of Nash equilibria using the USCCNE.

Example 1. *(Revisited: Computing Nash equilibria)* Consider again the financial network in Figure 2. This network contains a unique SCC, $\{1, 2, 3, 4, 5, 6, 7\}$, which has three ears, $E_1 = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$, $E_2 = \{(1, 5), (5, 6), (6, 4)\}$ and $E_3 = \{(5, 7), (7, 4)\}$.

In order to compute the Nash equilibria, we apply USCCNE. Figure 3 shows the ears (directed paths) generated by the algorithm. Starting from E_3, we can compute the activation outflow of agent 7: $A_3(7)$. Then we move to E_2, calculate $A_2(5)$ and $A_2(6)$. Finally, the activation outflows for E_1 would be $A_1(2)$, $A_1(3)$ and $A_1(4)$ (conditional on previous

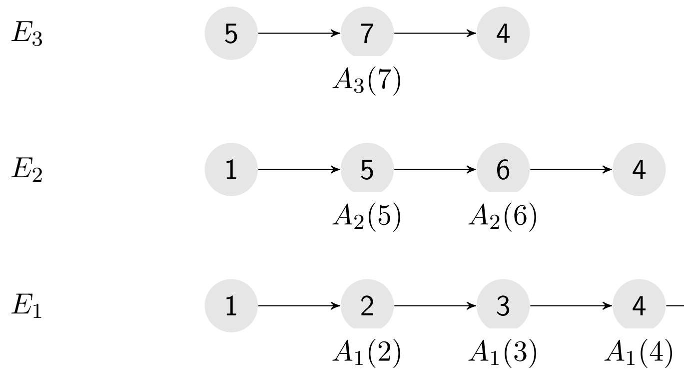

Figure 3. Ears and activation outflows

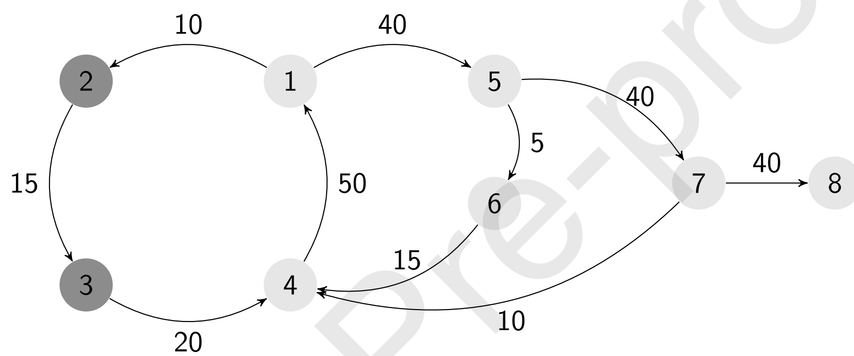

Figure 4. Intermediate Equilibrium

activation levels). We obtain the following $\mathcal{A}_3 = \{(7, 40)\}$, $\mathcal{A}_2 = \{(6, 37.5), (5, 39)\}$. The calculation of $\mathcal{A}_1$ is then conditional on $\mathcal{A}_3 \times \mathcal{A}_2$. Then, the algorithm checks the different remaining combinations $\mathcal{A}_3 \times \mathcal{A}_2 \times \mathcal{A}_1$ for potential equilibria. For instance, one possibility is that the outflow from agent 5 through ear 1 exceeds 40, the outflow from agent 1 through ear 2 exceeds 39 and the outflow from agent 1 through ear 1 to exceed 7.5. In this case, agents 7, 5, 6, 2, 3 and 4 do not default and pay their total liabilities. Finally we verify the repayment into each ear start agent (agents 1 and 5) are consistent with their outflows hence resulting in an equilibrium (the best equilibrium with no agent defaulting).

4.2. **Policy implications of the USCCNE algorithm.** The key feature of the USCCNE algorithm is that (based on the ear decomposition) it exploits the transformation of the SCC into ears (a partition of the edges of the network into directed paths).

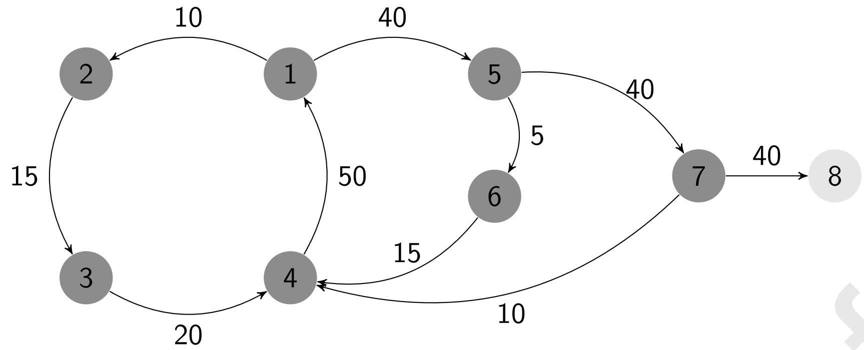

Figure 5. Best Equilibrium

Not only the USCCNE algorithm computes all the Nash equilibria of the default game, but it could also provide some concrete policy implications. Indeed, the USCCNE algorithm could guide regulatory interventions to achieve the best equilibrium by targeting the start agent of each ear. More specifically, if by relying on outside cash injection or regulation the outflow of the start agent of each ear is made equal to his best equilibrium outflow, then the best equilibrium is achieved. Note that both outside cash injection or regulation are budget neutral as policy interventions.

It is worth noting that the properties of the USCCNE algorithm can be even exploited further so that the best equilibrium could be achieved by targeting a smaller subset of ears' start agents as seed agents. In the following we will focus on a special case, where it is enough for the policy intervention to target just one seed agent to achieve the best equilibrium.

Definition 1. *We say that the start agent of the second ear is an overarching seed agent if his outflow determines the outflows of all other ears' start agents.*

In interpretation, an overarching seed agent belongs to every cycle of the network, which if targeted adequately would permit to achieve the best equilibrium. In the following, we revisit Example 1 to illustrate policy intervention with an overarching seed agent.

Example 1. *(Revisited: Policy intervention with an overarching seed agent)* Consider again the financial network in Figure 2. Using the USCCNE algorithm as described in Figures 3, 4 and 5, we observe that agent 1 is an overarching seed agent and hence can be potentially targeted by policy intervention. More specifically, if by relying on outside cash injection or regulation the outflow of agent 1 is made equal to his best equilibrium outflow, that is 50, then the best equilibrium $(1, 1, 1, 1, 1, 1, 1)$ is achieved.

Recall that the USCCNE algorithm ranks the internal agents in each ear by their order of non-default according to their activation outflows on each ear. Since in the presence of an overarching seed agent all Nash equilibria strategy profiles can be determined by his outflow of just one agent (the overarching seed agent) we have the following stronger prediction.

Corollary 1. *If there is an overarching seed agent, the USCCNE algorithm provides as well a ranking for the set of Nash equilibria.*

Corollary 1 shows that if there is an overarching seed agent the USCCNE algorithm developed in this paper based on the concept of ear decomposition provides the stronger property of ranking Nash equilibria within the SCC (as in Example 1). The ranking of Nash equilibria within the SCC follows from the fact that outflow of the overarching seed agent determines the outflows of each ear start agent, and consequently the outflows of all agents.

In this case, the ranking of Nash equilibria can be thought of as a measure of systemic risk based on waves of default. That is, the agents that default in all Nash equilibria will be called the *first wave of default*. Then, agents that default in all Nash equilibria except the highest Nash equilibrium will be called the *second wave of default* and so on.

Observe that, as illustrated in Figure 6, not all financial networks have an overarching seed agent. This is due to the presence of non-overlapping cycles, permitting independent coordination on the best equilibrium within each cycle.

Finally, it is worth noting that the subset of ears' start agents constitutes a feedback vertex set (FVS) of the financial network, which is a subset of agents S such that removing S makes the financial network acyclic (or equivalently, removes cyclical liabilities).

Proposition 4. *If there is an overarching seed agent, USCCNE has a worst-case time complexity of* $\mathcal{O}(n^4)$.

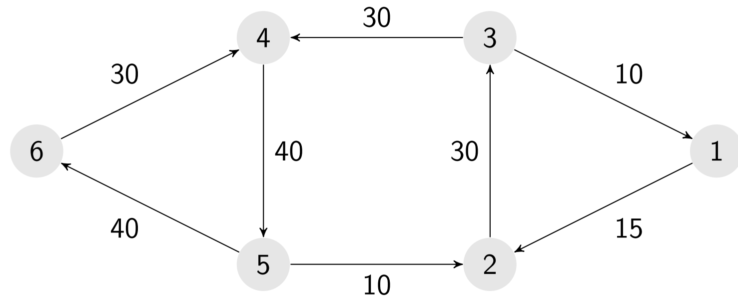

Figure 6. A financial network with with no overarching seed agent.

The key feature of USCCNE is that it allows us to traverse the network recursively, from outer ears to interior ears of the network. At each ear, we can collect information about the strategy profiles in outer ears that are consistent with some outflow from the current ear. If there is an overarching seed agent, then the set of strategy profiles consistent with equilibrium grows at worst linearly, as the strategies of outer ears can be determined straightforwardly from the repayments of interior ears. When we reach the overarching seed agent, we then need to check the remaining strategy profiles are consistent with equilibrium after accounting for cyclical obligations, for example by solving the linear system of repayments conditional upon each strategy profile.

4.3. **Welfare.** Now we investigate efficient outcomes among equilibrium and non equilibrium outcomes. Recall that the best equilibrium is the most efficient outcome only among equilibrium outcomes. To do so, we take a standard utilitarian approach and consider the social welfare function:

$$\mathcal{W}(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{i=1}^{n} U_i \left(z_i^1 - x_i, z_i^2 + (1+r)\, x_i + \sum_{j=1}^{n} \alpha_{ji} \pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}} \right),$$

which is the sum of utilities achieved by agents given their storage strategies $\mathbf{x}$.

Proposition 5. *The (possibly non-equilibrium) outcome where each agent stores his first period endowment is the most efficient outcome.*

Proposition 5 shows that the possibly non-equilibrium outcome where each agent stores his first period endowment is the most efficient outcome. Understandably, this result holds since paying out a dividend in the first period for any agent amounts to forgoing the opportunity for all agents to earn the risk-free return of r.

In order to achieve the set of efficient outcome where each agent stores his first period endowment, we can consider a two steps policy intervention through outside cash injection. In the first step use the policy intervention described in Section 4.1 to achieve the best equilibrium. Then as a second step target the remaining defaulting agents using the again maximal trees developed in the algorithm. It is worth noting that while the first intervention is budget neutral the second one might not be budget neutral.

4.4. **Arbitrary financial network.** Now we investigate the case of an arbitrary financial network. Recall that an arbitrary financial network can be transformed into a *directed acyclic graph* (henceforth, DAG)—that is, a network with no cycles–by contracting each SCC into a single large node (see Figures 6-7).

The algorithm described here MSCCNE is a generalisation of USCCNE. It consists of applying the USCCNE to each SCC in any given arbitrary network starting by the SCCs with no incoming link from any outside node or group of nodes, which are the SCCs that are not impacted by the other nodes in the network, and moving along the chain of SCCs.

In the following, we will rely on *transitive reduction*, which is a uniquely defined operation on a DAG, to compute the pure–strategy Nash equilibria of a financial network with multiple SCCs. A transitive reduction of a DAG is the network representation with the fewest possible links that preserves the chains of default of the original financial network(see Figure 8). It is hence constructed by removing all the links that are unnecessary for the chain of default to be realised and only the nodes which were connected by a path in the original network remain connected in the transitively reduced network. For instance, if A links to B, and B links to C, then the transitive reduction removes the link from A to C, if it exists.

Observe that, from the minimality of links in the transitive reduction, there exists a unique partition of the set of agents $\mathcal{W} = \{W_1, \dots, W_k\}$ such that W_1 corresponds to the SCCs with no incoming links, W_2 corresponds to the SCCs with only incoming links from W_1, W_3 corresponds to the SCCs with only incoming links from $W_1 \cup W_2$, and so on.

Then, the algorithm USCCNE can be easily extended to compute the Nash equilibria with multiple SCCs. The algorithm, which we call MSCCNE, goes as follows:

(1) Apply USCCNE to find all Nash equilibria for each SCC in W_1.

(2) For each p Nash equilibrium of SCCs in W_1, apply USCCNE to find all Nash equilibria for each SCC in W_2.

(3) For each Nash equilibrium of SCCs in $W_1 \cup W_2$, apply USCCNE to find all Nash equilibria for each SCC in W_3.

(4) Repeat the procedure until visiting all the elements of the partition $\mathcal{W}$.

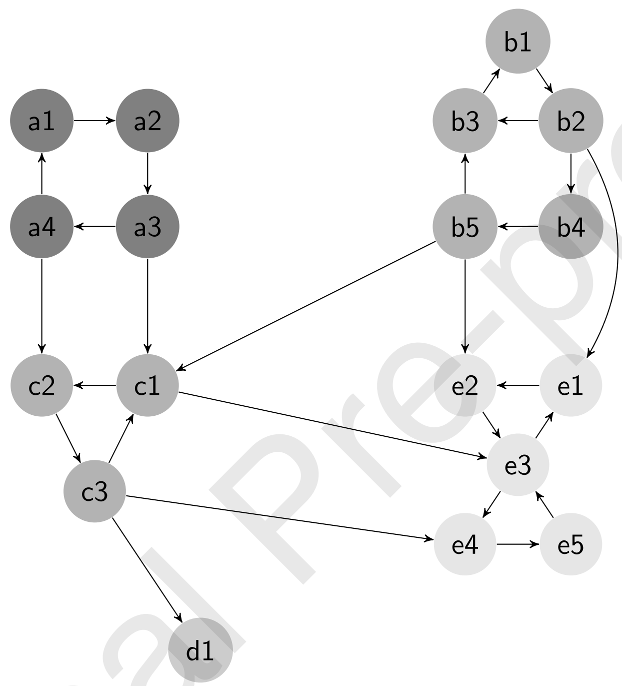

Figure 7. Example of a DAG

The MSCCNE algorithm is a simple algorithm that exploits a network decomposition technique to find all the pure–strategy Nash equilibria of a financial network. It is

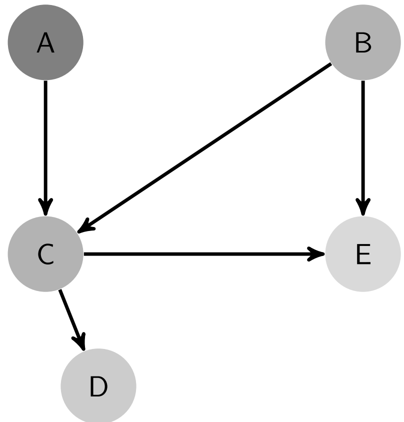

Figure 8. Condensation of the DAG

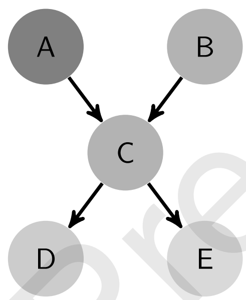

Figure 9. Transitive reduction of the DAG

worth noting that the MSCCNE algorithm can be easily adapted to compute the clearing payment vector of Eisenberg and Noe (2001).

Corollary 2. *Assume that the first-period endowment of each agent i is zero—that is, $z_i^1 = 0$. Then the MSCCNE algorithm computes the clearing payment vector in Eisenberg and Noe (2001).*

Recall that the clearing payment vector of Eisenberg and Noe (2001) is unique under mild conditions. Hence the existence of cyclical financial interconnections, while necessary for multiple equilibria, is not sufficient.

At the heart of the seminal contribution of Eisenberg and Noe (2001) lies the elegant *fictitious default algorithm* that computes the unique clearing payment vector. The fictitious default algorithm goes as follows. First, determine the set of agents who cannot fulfill their obligation, even when we assume that all agents receive their due payments. These agents will be called the *first wave of default*. Then, assume that the agents in the first wave of default pay their liabilities pro rata and the new defaulting agents will be called the *second wave of default* and so on until the algorithm terminates. In this way, the fictitious default algorithm produces a natural measure of systemic risk, which is the number of waves required to induce a given agent to default.

Echenique (2007) provides the most efficient algorithm for computing all pure–strategy Nash equilibria in the class of games of strategic complements, of which the default game is a special case. The algorithm elegantly checks whether there is another Nash equilibrium once the smallest and largest pure–strategy Nash equilibria are computed from classical algorithms (for example, Topkis (1979)).

While each of the above algorithms is clearly interesting in many aspects, arguably, the advantage of the MSCCNE algorithm developed in this paper is that it relies on the financial network architecture to compute the Nash equilibria. Generally, algorithms that exploit the financial network structure such as the algorithm developed in this paper, as well as having a clear computational advantage, provide valuable policy guidance to achieve the best equilibrium.

5. Policy Implications of central clearing

From a policy perspective, in view of the multiplicity of Nash equilibria of the default game, there is the central policy question of equilibrium selection. In particular, it may be desirable to implement the best equilibrium in order to achieve financial stability and minimise the cost of default.

Given the best and the worst equilibria, agents in the network can be classified into three types:[5]

(1) agents that choose 0 in the worst equilibrium and 1 in the best equilibrium;
(2) agents that choose 0 in the worst equilibrium and 0 in the best equilibrium;
(3) agents that choose 1 in the worst equilibrium and 1 in the best equilibrium.

Note that agents of type (2) and (3) are not strategically relevant since they play the same action in the worst and the best equilibrium. Actually, we could construct a *reduced financial network* containing only agents of type (1). To do so, we first eliminate all outgoing links emanating from agents of type (3) and, since none of them defaults, add

[5]Obviously, it is not possible for an agent to choose 1 in the worst equilibrium and 0 in the best.

their liabilities pro rata to the cash flow of the agents intercepting their outgoing links. As for agents of type (2), given that they default and pay their inflows—i.e. their cash flow and the payments they receive from their debtors—they can be eliminated from the network by adding their cash flow to the cash flow of their creditors pro rata and by extending their ingoing liabilities links to their creditors pro rata so that the new liabilities directly link between their debtors and their creditors.

Recently, CCP has become increasingly the cornerstone of policy reform in financial markets. Introducing a CCP in the financial network modifies the structure of the financial network: each liability between a debtor and a creditor is erased and replaced by two new liabilities—one liability between the debtor and the CCP, and another one between the CCP and the creditor. As a consequence, one of the key benefits of central clearing is that, by breaking down the cyclical connections of financial liabilities, it reduces the aggregate level of default exposure, which in turn reduces default contagion.

There is a growing literature which investigates the benefits of central clearing. Duffie and Zhu (2011) show that CCP's reduce significantly the counterparty risk even when clearing across multiple derivative classes. Zawadowski (2013) suggests that a CCP eliminates *ex ante* own default externalities by making banks contribute to the insurance of counterparty risk in the form of a guarantee fund. In other respect, Tirole (2011) argues that centralisation should be encouraged and CCP's enhance transparency and allow for multilateral netting. Acharya and Bisin (2014) study how the lack of transparency between agents sharing default risk produce counterparty risk externality and show that this externality disappears when introducing a centralized clearing mechanism which ensures transparency. They prove that the main advantage of central clearing is enhancing the aggregation of information.

The following proposition points out another potential benefit of introducing central clearing in financial markets.

Proposition 6. *Introducing a CCP in each SCC of the reduced financial network achieves the best equilibrium in the default game at no additional cost.*

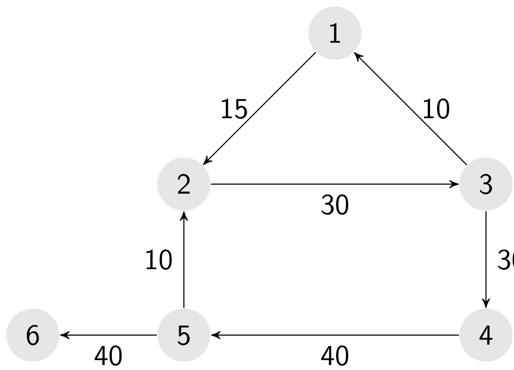

Figure 10. A financial network with five agents

Proposition 6 shows that when a CCP intermediates the liabilities of each SCC of the reduced financial network,[6] the best equilibrium is achieved and the CCP is budget neutral. As a consequence, in addition to reducing default contagion by eliminating the cyclical financial interconnections, central clearing can also serve as a coordination device that achieves the best equilibrium of the default game.

The following example illustrates this point.

Example 2 Consider an economy of six agents connected through their ownership of each other's liabilities, among which only the first five agents are strategically relevant. Agents' endowments in the first period are $\mathbf{z}^1 = (22, 22, 75, 180, 100)$ and in the second period are $\mathbf{z}^2 = (3, 3, 3, 3, 3)$ and the interest rate is $r = 0.1$. All agents have the same utility function $U_i(e_i^1, e_i^2) = e_i^1 + e_i^2$. The financial liabilities of agents to each other are illustrated in the network in Figure 10.

This financial network contains a unique SCC $\{1, 2, 3, 4, 5\}$. To compute the Nash equilibria, we apply the USCCNE algorithm described above. We find three Nash equilibria—the best equilibrium $(1, 1, 1, 1, 1)$, the intermediate equilibrium $(0, 0, 0, 1, 1)$, and the worst equilibrium $(0, 0, 0, 0, 0)$—which we illustrate in Figures 11-13.

[6]That is, the financial network with only strategic relevant agents.

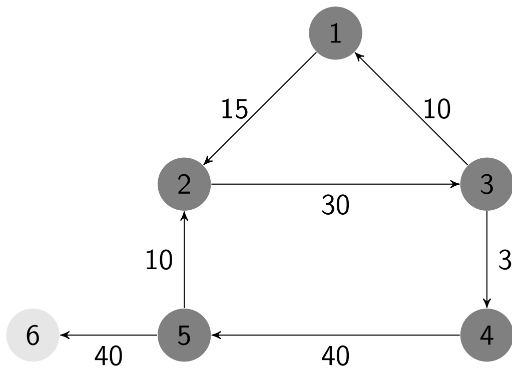

Figure 11. The best equilibrium

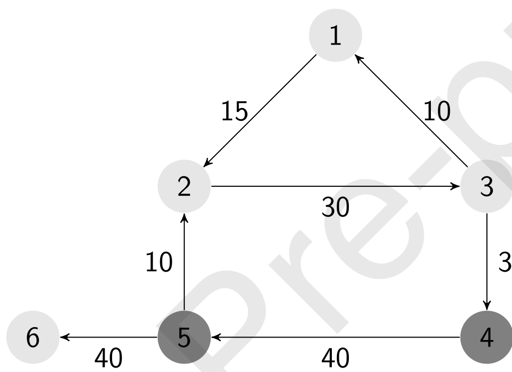

Figure 12. The intermediate equilibrium

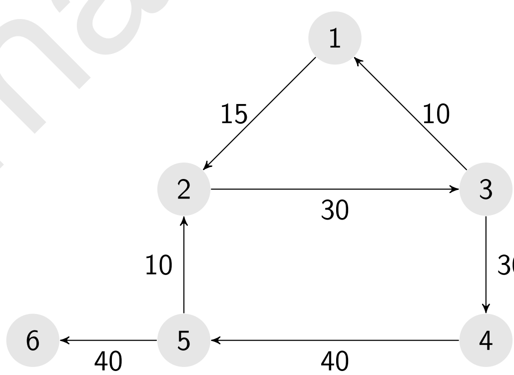

Figure 13. The worst Equilibrium

Adding a CCP will result in a new financial network as shown in Figure 14, with the following liabilities vector:

$$\tilde{\mathbf{L}} = (5, 5, 10, 10, 10, -40) \, .$$

Given that there are no feedback effects in the presence of the CCP, the minimum cash flow for an agent i to escape default is equal to the new liability $\tilde{L}_i$. Therefore, after the introduction of a CCP, it is easy to check that the best equilibrium is implemented at no additional cost since the inflows and outflows of CCP are equal.

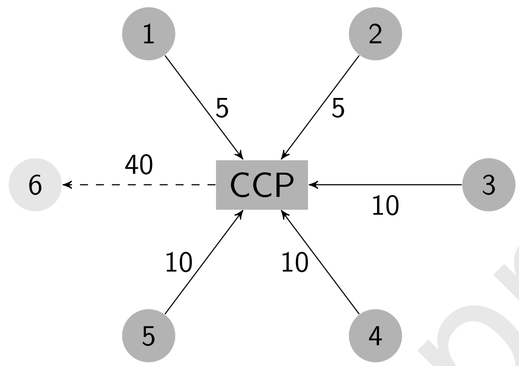

Figure 14. Adding a CCP

6. Conclusion

This paper shows that the introduction of a CCP allows agents playing different actions at different Nash equilibria to achieve the best equilibrium at no additional cost. As a consequence, central clearing can serve as a coordination device in financial markets. While our result reinforces the key role CCP plays in financial markets, as highlighted in several important contributions by Duffie and Zhu (2011), Tirole (2011), Zawadowski (2013) and Acharya and Bisin (2014), it remains to be seen whether other policies can be designed to minimise the number of defaults, such as identifying key agents and targeting them through either cash injection or minimum endowment requirement.

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8. Appendix

Proof of Proposition 1. Recall that the default game corresponds to a binary coordination game with two actions (Default) $= 0$ and (Non-Default) $= 1$ among which agents must choose.

First, for each agent i we will show that $\tau_i(\mathbf{a}_{-i})$ is well-defined given other agents' actions $\mathbf{a}_{-i} \in \{0,1\}^{N-1}$. To do so, for each agent i we consider an auxiliary economy with a modified network of liabilities, where we eliminate all outgoing links emanating from agent i and add his liabilities pro rata to the cash flow of the agents intercepting his outgoing links. Hence, the matrix of relative liabilities in the auxiliary economy is $\hat{\boldsymbol{\alpha}} = (\hat{\alpha}_{kj})_{k,j\in N}$, where $\hat{\alpha}_{kj} = \alpha_{kj}$ if $k \neq i$ and $\hat{\alpha}_{kj} = 0$ otherwise. Moreover, the (augmented) second-period endowment of agent j in the auxiliary economy is $\hat{z}_j^2 = z_j^2 + \alpha_{ij} L_i$.

Now, given other agents' actions $\mathbf{a}_{-i}$, let $\mathbf{x}^{\mathbf{a}_{-i}} = (x_j^{\mathbf{a}_{-i}})_{j\in N}$ denote the agents' storage strategies, where $x_j^{\mathbf{a}_{-i}} = z_j^1$ for each agent $j \neq i$ such that $a_j = 1$, and $x_j^{\mathbf{a}_{-i}} = 0$ otherwise. Let also $\boldsymbol{\pi}^{\mathbf{x}^{\mathbf{a}_{-i}}} = (\pi_j^{\mathbf{x}^{\mathbf{a}_{-i}}})_{j\in N}$ denote the clearing payment vector, uniquely defined as in Eisenberg and Noe (2001), such that for each agent j it holds that

$$\pi_j^{\mathbf{x}^{\mathbf{a}_{-i}}} = \min\left\{\hat{z}_j^2 + (1+r)\,x_j^{\mathbf{a}_{-i}} + \sum_{k=1}^{n} \hat{\alpha}_{kj}\pi_k^{\mathbf{x}^{\mathbf{a}_{-i}}}; L_j\right\}.$$

Therefore, since $x_i^{\mathbf{a}_{-i}} = 0$ it holds that

$$\tau_i(\mathbf{a}_{-i}) = \max\left\{L_i - z_i^2 - \sum_{j=1}^{n} \hat{\alpha}_{ji}\pi_j^{\mathbf{x}^{\mathbf{a}_{-i}}}; 0\right\}. \tag{8.1}$$

Hence, the threshold $\tau_i(\mathbf{a}_{-i})$ is well-defined.

Moreover, it follows from Lemma 5 in Eisenberg and Noe (2001) (see, also, Theorem 6 in Milgrom and Roberts (1990)) that $\boldsymbol{\pi}^{\mathbf{x}^{\mathbf{a}_{-i}}}$ is increasing in $\mathbf{x}^{\mathbf{a}_{-i}}$, which, in turn, is increasing in $\mathbf{a}_{-i}$. Hence, it follows from (8.1) that the threshold $\tau_i(\mathbf{a}_{-i})$ is decreasing in $\mathbf{a}_{-i}$.$\square$

Proof of Theorem 1. Since the threshold $\tau_i(\mathbf{a}_{-i})$ is decreasing in $\mathbf{a}_{-i}$ it follows that the best reply function of agent i

$$\Psi_i(\mathbf{a_{-i}}) = \begin{cases} 1 & \text{if } rz_i^1 - \tau_i(\mathbf{a}_{-i}) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is increasing in $\mathbf{a_{-i}}$. By the Knaster–Tarski Theorem, there exists a fixed point of the following map:

$$\Psi : \{0,1\}^N \longrightarrow \{0,1\}^N$$

$$\Psi(\mathbf{a}) = (\Psi_1(\mathbf{a_{-1}}), ..., \Psi_n(\mathbf{a_{-n}})),$$

which will be a Nash equilibrium of the default game.□

Proof of Proposition 2. Suppose not—that is, the default game has multiple equilibria and the financial network does not have cyclical obligations. Let R denote the set of agents who play 0 in the worst Nash equilibrium and 1 in the best Nash equilibrium. Then the subnetwork induced by R contains an agent i that does not have any ingoing link. As a consequence, the inflow of agent i does not change between the worst equilibrium and the best equilibrium, and as a result agent i will not change his choice in the worst equilibrium and the best equilibrium. This is a contradiction.□

Proof of Proposition 3. Part (i): Let γ be an equilibrium strategy profile. If γ is not in Γ^*, then it must be the case that the strategy profile γ was eliminated by the algorithm at some point, for example when the algorithm visited E_j. But the algorithm would only eliminate strategy profile γ if there were no outflow from v_{j1} consistent with the strategy profile γ, in which case, γ could not be an equilibrium strategy profile.

Part (ii): Part (ii) is ensured by Step 3 of USCCNE. If Step 2 generates any tree-consistent strategy profiles that are not equilibria of the default game, these will be identified upon the calculation of equilibrium repayments in Step 3 and subsequently removed.□

Proof of Proposition 4. Computing the ear decomposition in Step 1 can be performed in linear time. In step 2, each agent is visited once. When the algorithm visits ear E_j, the activation outflows for each interior agent in ear E_j are computed, along with the outflows that activate stratefy profiles carried backward from $E_{j+1}, ..., E_p$. By Corollary 1, when there is an overarching seed agent, these strategy profiles are bounded by the number of interior agents in $E_{j+1}, ..., E_p$. Therefore, the time complexity of Step 2 is $\mathcal{O}(n^2)$.

By Corollary 1, the set of strategy profiles generated in Step 2 of the algorithm form a total order over the space $\{0,1\}^n$, leaving at most n strategy profiles to check in Step 3. These strategy profiles can be checked in $\mathcal{O}(n^3)$ time, for example by solving a system of linear equations[7].$\square$

Proof of Proposition 5. Using clearing properties, it holds that

$$\begin{aligned}
\mathcal{W}(\mathbf{x}) &= \sum_{i=1}^{n} U_i(z_i^1 - x_i, z_i^2 + (1+r)\,x_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}})| \\
&= \sum_{i=1}^{n}[z_i^1 + z_i^2 + rx_i + \sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}} - \pi_i^{\mathbf{x}}] \\
&= \sum_{i=1}^{n}[z_i^1 + z_i^2 + rx_i] + \sum_{i=1}^{n}[\sum_{j=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}}] - \sum_{i=1}^{n}\pi_i^{\mathbf{x}} \\
&= \sum_{i=1}^{n}[z_i^1 + z_i^2 + rx_i] + \sum_{j=1}^{n}[\sum_{i=1}^{n} \alpha_{ji}\pi_j^{\mathbf{x}}] - \sum_{i=1}^{n}\pi_i^{\mathbf{x}} \\
&= \sum_{i=1}^{n}[z_i^1 + z_i^2 + rx_i] + \sum_{j=1}^{n}\pi_j^{\mathbf{x}} - \sum_{i=1}^{n}\pi_i^{\mathbf{x}} \\
&= \sum_{i=1}^{n}[z_i^1 + z_i^2 + rx_i].
\end{aligned}$$

Hence, $\mathcal{W}(\mathbf{x})$ is maximised when $x_i = z_i^1$, for each agent i.$\square$

Proof of Proposition 6. Adding a CCP in the middle of the financial network will net out the liabilities and will sort agents into two types: debtors and creditors to the CCP.

[7]Actually repayments can be calculated as follows:

$$\pi = (\mathbf{I} - \mathbf{D}\alpha')^{-1}((\mathbf{I}-\mathbf{D})\mathbf{L} + \mathbf{D}\mathbf{z}^2)$$

where $\mathbf{D}$ is a diagonal matrix with entry $D_{jj} = 1$ if and only if j is a defaulting agent.

Let node 0 represent the CCP, and $\tilde{L}_{i0}$ the liabilities to/from the CCP such that

$$\tilde{L}_{i0} = \sum_{j\in N} L_{ij} - \sum_{j\in N} L_{ji}.$$

Hence, if $\tilde{L}_{i0}$ is positive (resp. negative), agent i is a debtor (resp. creditor) to the CCP.

Since the best equilibrium can be reached, it follows that whenever agent i receives all the liabilities from his debtors, he will choose not default. Therefore, it holds that

$$z_i^2 + (1+r)\, z_i^1 + \sum_{j\in N} L_{ji} \geq \sum_{j\in N} L_{ij},$$

which implies

$$z_i^2 + (1+r)\, z_i^1 \geq \tilde{L}_{i0}.$$

Hence, the non-default condition is satisfied for each agent in the network with liabilities intermediated by the CCP and the best equilibrium is reached.□

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Declaration of interests

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☒ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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