Predicting preincubation parameters in goose eggs to reduce their hatching waste

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Abstract

The present study was aimed to seek algorithms for prior identification of goose eggs unsuitable for incubation that, otherwise, turn into to hatching waste. These included infertile eggs and those in which the embryo did not survive by the hatch time. The algorithm development was based on egg parameter measurements taken before and during incubation. As a result, a complex of egg's geometrical parameters, i.e., its length, breadth and diameter at a point 1/4 of the length from the pointed end, were established as the best predictors and used for producing four novel indicators. The above parameters were incorporated into the first indicator called the Emergency Geometrical Index (*EGI*). Its use prior to incubation enabled to correctly identify 14–26% of eggs within the hatching waste category. The second indicator was the ratio of air cell volume to egg weight. The respective preincubation measurements enable to correctly identify ~38% of unsuitable eggs. When combining the first two indicators, the third one was developed and called the Emergency Hatchability Index (*EHI*), with the correct identification rate being ~65% of unsuitable eggs. Egg density (*D*) during incubation was proposed as the fourth promising indicator. This was expressed as the tangent of the slope for a trend line based on *D* data calculated for successive days of incubation. Collectively, the proposed four indicators and few other new methodological approaches used for their derivation will be instrumental in predicting hatchability of goose and other poultry eggs before incubation to reduce hatching waste.

Keywords: Geese; Egg incubation; Egg fertility and hatchability; Egg sorting; Egg geometry parameters; Non-destructive testing

Nomenclature

a, b	Coefficients used for approximating the empiric dependences
B	Maximum egg breadth in cm
d	Air cell diameter in mm
d_1 to d_{10}	Air cell diameter on the respective day of the incubation in mm
D	Egg density in g cm^{-3}
D_1 to D_{10}	Egg densities on the appropriate day of the incubation in g cm^{-3}
$D_{L/4}$	Egg diameter at a distance of 1/4 of its length from the pointed end in cm
E	Margin of error
E_{max}	Admissible margin of error
h	Air cell height in cm
L	Egg length in cm
n	Number of eggs in the respective experimental sample
N	Total number of eggs involved in the experiment
S	Shell surface area in cm^2
V	Egg volume in cm^3
V_{ac}	Air cell volume in cm^3
V_{ac1} to V_{ac10}	Air cell volumes on the appropriate day of the incubation in cm^3
w	Distance between two vertical axes, one of which being B and the other intersecting the egg at $L/2$ in cm
W	Egg weight in g
W_1 to W_{10}	Egg weights on the respective day of the incubation in g
σ_y	Standard deviation of the parameter under study

1. Introduction

Domestic geese descended from the graylag goose [*Anser anser* (Linnaeus, 1758)] and swan goose [*A. cygnoides* (Linnaeus, 1758)] and have not undergone much genetic selective breeding as compared to chickens, turkeys, and ducks (to a lesser extent) (e.g., Jacob, 2023; Łukaszewicz, 2010; Romanov, 1995, 2018). In view of this, goose eggs are characterised by low hatchability and a high percentage of infertile eggs, especially at the beginning and end of the laying season (Karabulut, 2021; Łukaszewicz, 2010; Salamon, 2020). Evaluation of the hatchability of goose eggs based on their physical properties is instrumental in enhancing the reproduction, breeding and commercial use of geese (e.g., Ionov et al., 2023; Narushin, Romanov, Salamon, Kent, 2023; Romanov, 1997, 2018). Utilising and creating egg parameter indices that come from measuring, calculating, and analysing a variety of goose egg properties and their interactions seems crucial in this regard (Narushin, 2001; Narushin, Romanov, Salamon, Kent, 2023; Wilson, 1991).

Rearing geese in small farms is most often accompanied by the incubation of eggs in small hatcheries and individual incubators. In this scenario, the rational use of the incubator's working space is extremely important. This can be facilitated by the timely removal of infertile eggs and/or eggs with non-viable embryos. Technologically, such eggs can be identified between days 6 and 10 of incubation by transilluminating them with an ovoscope (Kucharska-Gaca, Adamski, Kuźniacka, Kowalska, 2016; Lukaszewicz, Lason, Rosenberger, Kowalczyk, Bakst, 2017; Salamon, 2020). However, this procedure is extremely difficult for goose eggs due to their thick shell and, often, the shell background colour. As a consequence, goose eggs are recommended to be scanned again on day 26 or 27 of incubation; however, this may or may not be executed (Biesiada-Drzazga, Banaszewska, Koncerewicz, Jozwik, Horbanczuk, 2015; Kucharska-Gaca et al., 2016; Salamon, 2020). At this point, candling goose eggs is challenging and requires a keen eye (Salamon, 2020).

The candling accuracy, however, leaves much to be desired even in chicken eggs that fit better to applications of related technological solutions. Not without reason, many studies are aimed at finding alternative methods for identifying unsuitable eggs in order to replace the operator-assisted candling labour with use of artificial intelligence, machine learning and other technological improvements. For example, Saifullah and Dreżewski (2022) proposed a special computer technology based on the Support Vector Machine enabling to assess the images produced by the egg candling process and determine whether an egg embryo is present. Çevik, Koçer, Boğa, and Taş (2023) developed a custom incubator by installing a camera (PI Camera), cold LEDs (Power LED) and a mini computer (Raspberry PI) to take and process egg images every 15 minutes during the first seven days of incubation. Based on the images obtained, a decision can be made about the fertility of a particular egg. A technological solution described by Fadchar and Dela Cruz (2020) involved the search for differences between fertilised and unfertilised eggs

using the colour segmentation process that allowed to extract the colour space parameters. A similar technology was assessed by Saifullah, Drężewski, Khaliduzzaman, Tolentino, and Ilyos (2022) and Çevik, Koçer, and Boğa (2022). An alternative to image recognition methods can be the convolutional neural network-based technology proposed by Geng, Hu, Xiao, and Xi (2019) for collecting the embryo's heartbeats during incubation, which can be used to draw a conclusion about the survival of hatching eggs.

If one admits that the above solutions can be acceptable and economically justified for large hatchery stations that sort hundreds of thousands of eggs daily, usage of such sophisticated technologies on small goose farms and hatcheries are barely profitable and realisable. If egg sorting is to be carried out before incubation, it should be simple, affordable, accurate and easily implemented using conventional methods of measurement and calculation.

In this regard, the goal of our study was to search for such a solution in the identification of goose eggs unsuitable for incubation under small hatchery conditions. Hereby, this investigation was relied on easily and non-destructively measurable linear egg parameters, and four novel mathematical models were tested and proposed as prior indicators of the unsuitability of eggs for incubation.

2. Materials and Methods

The Legarth goose flock was used to collect 80 goose eggs from Ballyrichard Farm in Arklow, Ireland (Arklow, Ireland; 52°50'5" N, 6°7'49" W). The proper flock management and housing requirements were followed, as previously stated (Kent and Murphy, 2003; Salamon and Kent, 2013a, 2016). Eggs were incubated in the local on-farm hatchery.

In order to obtain the largest possible sample of substandard eggs, the experiment was scheduled at the very end of the seasonal laying (June 2023). During this period, there was a high percentage of embryo mortality and the number of infertile eggs laid, which made it possible for us to obtain reliable data on three categories: (1) infertile eggs, (2) fertile eggs, and (3) eggs in which the embryos died on various stages of incubation.

The length (L) and maximum breadth (B) of each egg were measured with a vernier calliper to the nearest 0.1 mm, and their weight (W) was identified using an electronic scale with an accuracy of 0.01 g. All the eggs were photographed (Narushin, Lu, Cugley, Romanov, Griffin, 2020a), and from their photos, the distance w of the B axis shift from the egg's centre was determined to the nearest 0.1 mm (Narushin, Romanov, Lu, Cugley, Griffin, 2020b). Also, the diameter ($D_{L/4}$) was measured on the images of eggs at a point remote from the sharp end by a value of $L/4$.

The following computation formulae from Narushin et al. (2021b) were used to calculate the values of egg volume (*V*) and surface area (*S*):

$$V = \frac{0.5233B^2(L^2 + 0.0071Lw - 0.8565w^2)}{L}, \quad (1)$$

$$S = \frac{2.48B}{L} \cdot (L^2 - 0.34Lw - 4.27w^2), \quad (2)$$

where *V* is egg volume (in cm^3); *S* is its surface area (in cm^2); *L* is its length (in cm); *B* is its maximum breadth (in cm); and *w* is the distance between two vertical axes, one of which being *B* and the other of which intersecting the egg at *L*/2 (at point *O*). Scheme in Fig. 1 makes it easier to understand the measurements taken.

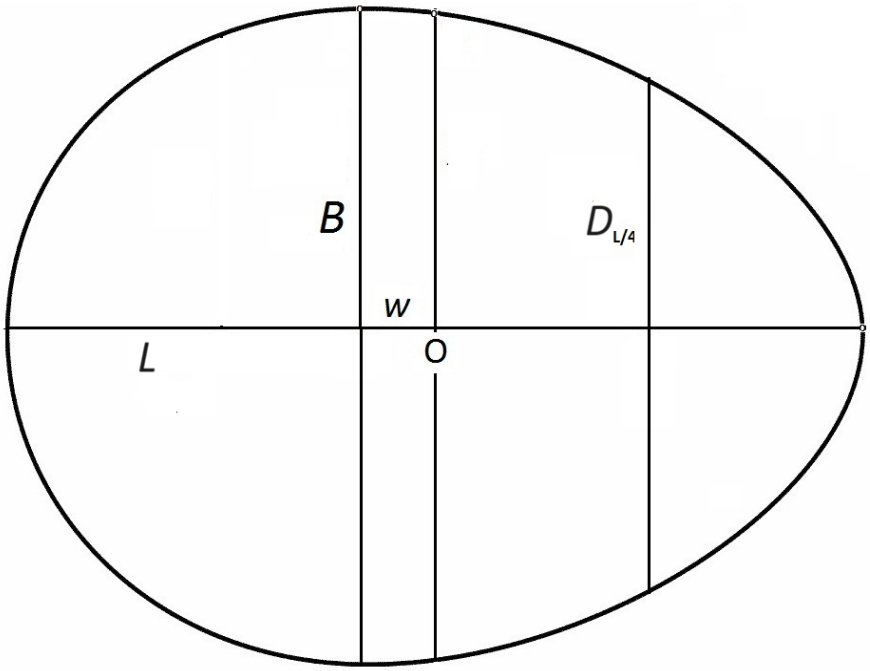


Fig. 1 – Schematic representation of the measured parameters of goose eggs. *B*, maximum egg breadth; $D_{L/4}$, egg diameter at a distance of *L*/4 from the pointed end; *L*, egg length; *O*, point of intersection between *L* and vertical axe at *L*/2; *w*, distance between *B* and vertical axe at *L*/2.

When candling the egg, the air cell diameter (*d*) was determined by measuring it twice (i.e., smallest and largest diameters) with a vernier calliper with an accuracy of 0.1 mm; the values were then averaged.

The following formulae from Narushin et al. (2021c) were used to compute the values of air cell height (*h*) and volume (V_{ac}):

$$h = \frac{LB^2 - 2d^2w - \sqrt{(2d^2w - LB^2)^2 - d^2B^2(L-2w)^2}}{2B^2}, \quad (3)$$

$$V_{ac} = \frac{1.32B^2h}{L^3}(h(L^2 + 3.79Lw - 4.08w^2) + 0.012L(L^2 - 5.92Lw + 172.58w^2)), \quad (4)$$

where h is the air cell's height (in cm), d is its diameter (in cm), and V_{ac} is its volume (in cm^3).

After these measurements, the eggs were placed in an incubator. During incubation, the eggs were removed daily at the same time from the setter, weighed and measured for d in the same way as described above. The procedure was repeated for 10 days, after which the eggs were candled and their fertility was determined. Infertile and mortal eggs were discarded. Eggs containing live embryos were further incubated, periodically observing and removing eggs containing dead embryos. As a result, the data obtained by the end of the incubation enabled to conditionally sort the initial egg sample into three categories, with two subgroups in each category:

(1) **Fertility**. This involved infertile eggs (I) and all fertile eggs (F). Subgroup F did not include eggs in which dead embryos were found by day 10 of incubation. In the case of their death at the early development stages, the egg parameters measured during the incubation process could change specifically, thereby introducing an error into the analysis process.
(2) **Mortality**. This included a subgroup of eggs with dead embryos and mortal eggs identified during the entire incubation period (M), and a subgroup of eggs from which healthy goslings subsequently hatched that was called alive embryo eggs (AE).
(3) **Hatchability**. This embraced alive embryo eggs (AE) and all other eggs were henceforth called hatching waste (HW) including eggs of subgroups I and M.

Since the total number of 80 eggs during the research was divided into the corresponding two subgroups, while the number of eggs in each could vary depending on the category (1) to (3) studied, it was necessary to assess the statistical representativeness of each sample. For this purpose, the computation formula for minimum sample from Cochran (1977) and the respective assumption on the margin of error (E) were used. Cochran (1977) left the choice of the allowable E_{max} value to the discretion of an investigator, however, recommending its value at the level of 5% (0.05). Thus, our task was reduced to calculating the E value and comparing it with the admissible value (E_{max}). If the computed E value does not exceed E_{max}, i.e., 0.05, the sample can be considered representative. In this case, the calculation formula for minimum sample from Cochran (1977) can be converted as follows:

$$E = \frac{(N-n)\sigma_y^2}{(N-1)n}, \quad (5)$$

where N is the total number of eggs involved in the experiment; n is the number of eggs in the respective experimental sample; σ_y is the standard deviation of the parameter under study, which in our case corresponded to the value of the function (y) for each of the three categories. That is, this was a numerical series for category (1) that reflected whether the egg being studied belongs to subgroup I or F; for category (2), respectively, whether a dead or living embryo was located inside the test egg; and for category (3), whether the egg can be characterised as hatching or not.

The STATISTICA 5.5 program (StatSoft, Inc./TIBCO, Palo Alto, CA, USA) and Microsoft Excel's computational tools were used to process the results.

3. Results and Discussion

Initially, using Eqn5, the representativeness of the entire experimental sample of 80 eggs was assessed. In our previous work (Narushin, Romanov, Salamon, Kent, 2023), using simulation methods, a database of 600 virtual eggs was created that included every possible set of basic parameters that can be found in nature. Using this database for our purposes, as well as conditionally accepting the fact that the amount of incubation defects in eggs from a commercial flock of geese can reach 50% (e.g., Salamon, 2020), the conditional value of σ_y was calculated. For these purposes, the conventions were adopted that were used in our further experiments in the form of corresponding estimates. Eggs unsuitable for incubation (categories 1, 2 and 3 indicated in Materials and Methods) were designated as '1', while suitable eggs were marked as '2'. Then, taking into account the possibility that half of all eggs may be unsuitable, i.e., correspond to the value 1, and the second half to 2, the main statistical values of such a series will be the average value of 1.5, with the standard deviation of 0.5. In this case, the calculation using Eqn5 gives the result of an E value equal to 0.008, which is by a huge margin below the limit value E_{max} = 0.05. Even taking a completely incredible result as an assessment, in which all the eggs will turn out to be unsuitable for incubation, or, conversely, completely suitable, i.e., σ_y = 1, the value of E is 0.011, which also satisfies our requirements. Thus, a sample of 80 goose eggs is quite representative for the conditions of our experiment and the possible results of its implementation.

3.1 Fertility

After candling on day 10 of incubation, it was determined that 52 of the 80 goose eggs involved in the experiment were infertile, thus forming subgroup I. Seven eggs with dead embryos were also detected and, accordingly, the remaining 21 eggs were classified as subgroup F. All eggs of subgroup I were assigned a

quantitative index of 1, and subgroup F, respectively, that of 2. Thus, the numerical series, which included 52 ones and 21 twos, reflected the value of the function (y) for the presence or absence of an embryo in a goose egg. For this series (y), the standard deviation (σ_y) was 0.46, which enabled us to compute E to be 0.007 using Eqn5. The obtained value was significantly lower than E_{max} = 0.05, thus, the studied samples were recognised as representative. Table 1 lists the appropriate measured and calculated I and F goose egg variables.

Table 1 – Data of measured and calculated goose egg variables for subgroups I and F.

Parameters	Mean		Standard deviation	
	I	F	I	F
Egg weight at the beginning of each incubation day, W (in g):				
W_1	154.6	154.2	21.3	16.8
W_2	153.2	153.0	21.2	16.8
W_3	151.8	151.7	21.2	16.8
W_4	150.5	150.5	21.2	16.7
W_5	149.2	149.3	21.2	16.7
W_6	148.1	148.3	21.2	16.7
W_7	146.7	147.0	21.2	16.7
W_8	145.5	145.9	21.2	16.6
W_9	144.2	144.8	21.2	16.7
W_{10}	143.0	143.7	21.2	16.7
Egg length, L (in cm)	8.72	8.69	0.42	0.42
Maximum egg breadth, B (in cm)	5.68	5.68	0.31	0.21
Shift of the B axis from the egg centre, w (in cm)	0.41	0.44	0.13	0.12
Egg diameter at a distance of 1/4 of its length from the pointed end, $D_{L/4}$ (in cm)	4.55	4.58	0.29	0.20
Egg volume, V (in cm^3)	145.0	145.1	20.1	15.7
Surface area, S (in cm^2)	119.8	119.1	11.6	9.5
Air cell diameter at the beginning of each incubation day, d (in mm):				
d_1	34.0 [a]	31.1 [a]	4.5	3.8
d_2	35.2 [a]	32.7 [a]	3.7	3.5
d_3	35.8 [a]	34.0 [a]	3.5	3.0
d_4	37.3 [a]	34.7 [a]	3.6	4.2
d_5	38.9 [a]	37.1 [a]	3.4	2.7
d_6	38.8 [a]	36.9 [a]	3.0	2.5
d_7	40.3 [a]	38.3 [a]	3.0	2.6
d_8	41.0 [a]	39.2 [a]	2.9	2.6
d_9	41.9 [a]	40.2 [a]	2.8	2.5

d_{10}	42.0 [a]	40.7 [a]	2.9	2.1

[a] $p < 0.05$; the values without any index are insignificant.

Among all the measured parameters, there were those that demonstrated significant differences between the two subgroups. These were the *d* values that, moreover, significantly differed between the subgroups during all 10 days of observation in the process of incubation. The recalculation of other air cell characteristics, i.e., *h* (following Eqn3) and V_{ac} (according to Eqn4), resulted in even more pronounced differences between the subgroups when comparing V_{ac} values. Graphical visualisation of the differences in the V_{ac} means between the subgroups on the days of measurements is shown in Fig. 2.

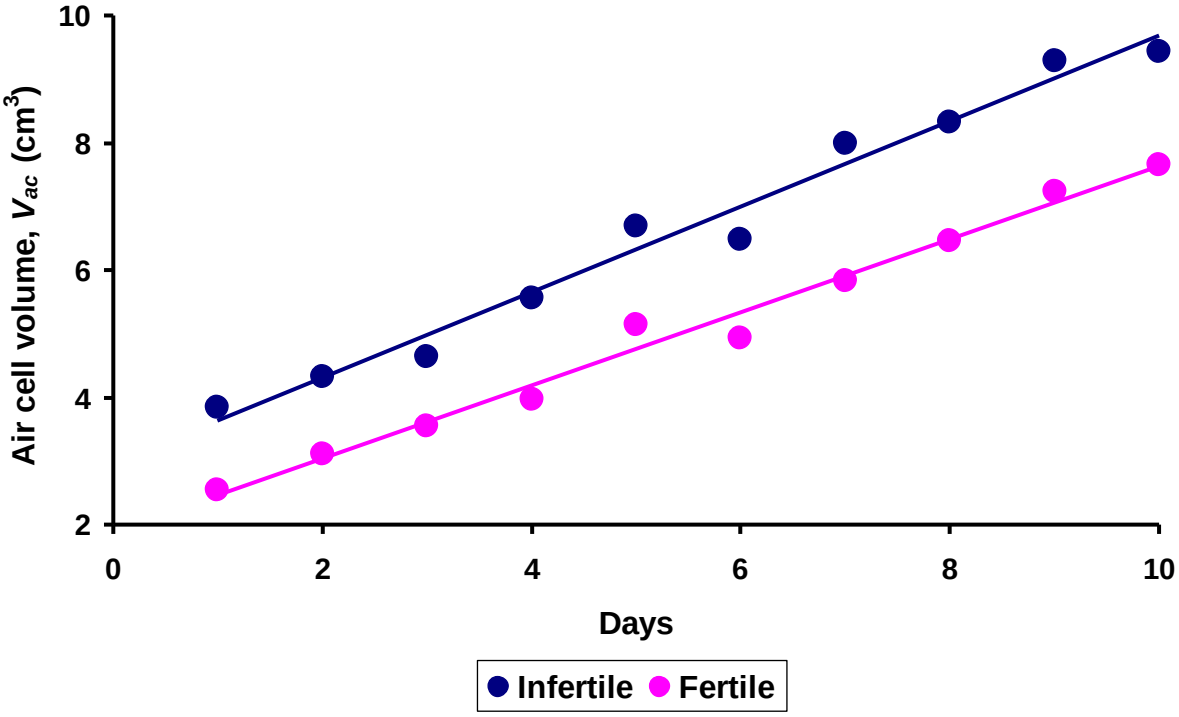


Fig. 2 – Graphs of changes in mean V_{ac} values for egg subgroups I (dark blue line) and F (purple line) during the first 10 days of incubation.

Note that the subgroup means cannot serve as a basis for developing computation methods for identifying specific eggs in terms of their belonging to a particular subgroup. In this regard, estimation of the degree of dispersion of the V_{ac} values in each subgroup in the general range of this parameter was undertaken for the entire sample of goose eggs. As an example, the values of V_{ac} on days 1 and 2 of incubation (V_{ac1} and V_{ac2}) were examined. Herewith, speaking of day 1 of incubation, the respective parameter values will further be considered as those identified before placing the eggs in the setter. The respective results are shown in Fig. 3.

a) b)

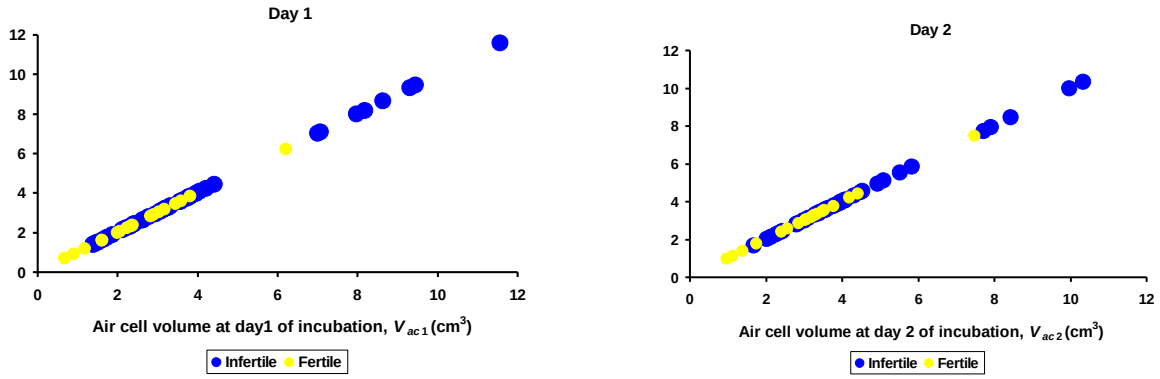


Fig. 3 – Visualisation of the distribution of V_{ac} values for subgroups I (blue dots) and F (yellow dots) in the total amount of studied goose eggs for day 1 (a) and day 2 of incubation (b).

Analysing Fig. 3, it can be argued that some localisation of subgroup F eggs on the graphs was noted in the region of low V_{ac} values. Nonetheless, a mathematical attempt was made to further enhance this localisation. The criterion for evaluating an efficiency of this attempt was a magnitude and significance of coefficients R for the correlation with the values of the function (y) consisting of ones and twos and characterising the respective presence or absence of an embryo in a goose egg. As a result of the appropriate mathematical manipulations with the V_{ac} value, taken alone or in combination with other measured parameters, the highest R value was established for the V_{ac}/W ratio. In this case, the R value ranged from −0.24 to −0.31 by day of incubation, with all values being significant ($p < 0.05$). As a comparison, the distribution of V_{ac}/W values for subgroups I and F in the total amount of the studied goose eggs for days 1 and 2 of incubation can be seen in Fig. 4.

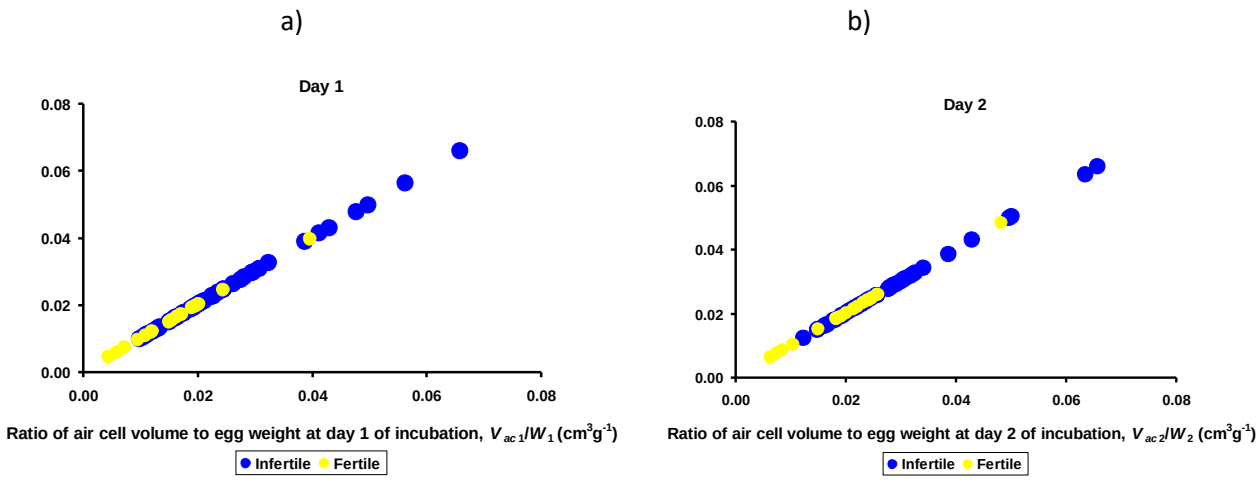


Fig. 4 – Visualisation of the distribution of V_{ac}/W values for subgroups I (blue dots) and F (yellow dots) in the total amount of studied goose eggs for day 1 (a) and day 2 of incubation (b).

The use of the V_{ac}/W indicator facilitated a somewhat greater shift in the subgroup F values to the area of lower values. Thus, it can be suggested that the larger V_{ac}/W value, the higher the probability that an egg is

fertile. Perhaps, the physiological meaning of this phenomenon may be that the presence of embryonic cells already in some way influences the intensity of gas exchange in the egg, as a result of which there might be a relative increase in V_{ac}/W.

The next step in developing the mathematical identification procedure for infertile eggs was to develop a formula that could be used to figure out which eggs were infertile and which could be put into the incubator. If the data in Fig. 3 is presented in the form of a functional dependence $y = f(V_{ac}/W)$, the latter, for example, on day 1 of incubation will look like in Fig. 5.

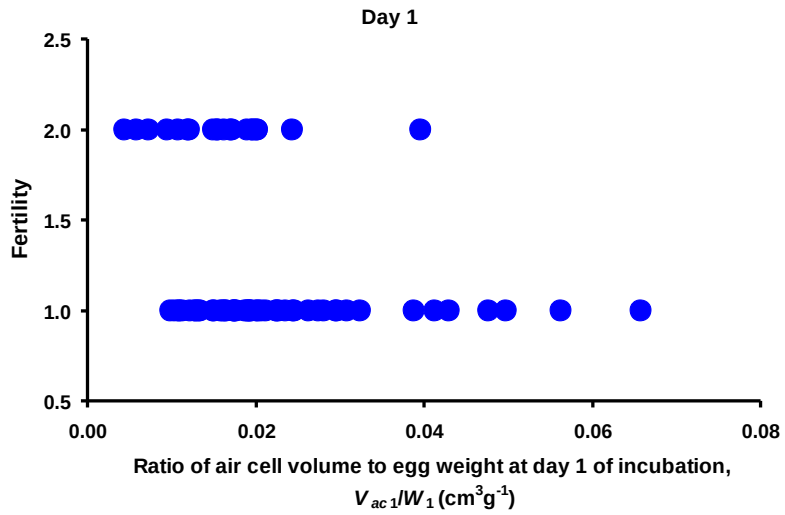


Fig. 5 – Functional dependence of the absence (y = 1) or presence (y = 2) of an embryo on the V_{ac}/W values on day 1 of incubation.

The only function that can accurately approximate data of this kind is sigmoid. In this case, depending on the shift of one of the horizontal lines called asymptotes towards larger or smaller values, the sigmoid can be considered normal or inverse. Using the theoretical background detailed in Supplementary Data A, the appropriate basic formulae were developed to describe both types of data series. As shown in Fig. 5, the upper horizontal line was shifted relative to the lower one toward smaller values. For this particular example, the inverse sigmoid variant should be used as follows:

$$y = 1 + \frac{1}{1 + e^{\frac{20}{|x_a|}(x - x_a)}}. \qquad (6)$$

Alternatively, if the upper horizontal line is shifted relative to the lower one in the direction of larger values, the approximation should be carried out using the classical sigmoid formula:

$$y = 1 + \frac{1}{1 + e^{-\frac{20}{|x_a|}(x - x_a)}}. \qquad (7)$$

For both formulae (6) and (7), x_a is the mean value of the independent variable.

Since, in our case (Fig. 5), the independent variable was V_{ac1}/W_1, with the mean value being 0.021, the inverse sigmoid formula (6) will take the following form:

$$y = 1 + \frac{1}{1 + e^{939\left(\frac{V_{ac1}}{W_1} - 0.021\right)}}, \qquad (8)$$

with $R = 0.296$ ($p < 0.05$).

The visualisation of the approximation results is presented in Fig. 6.

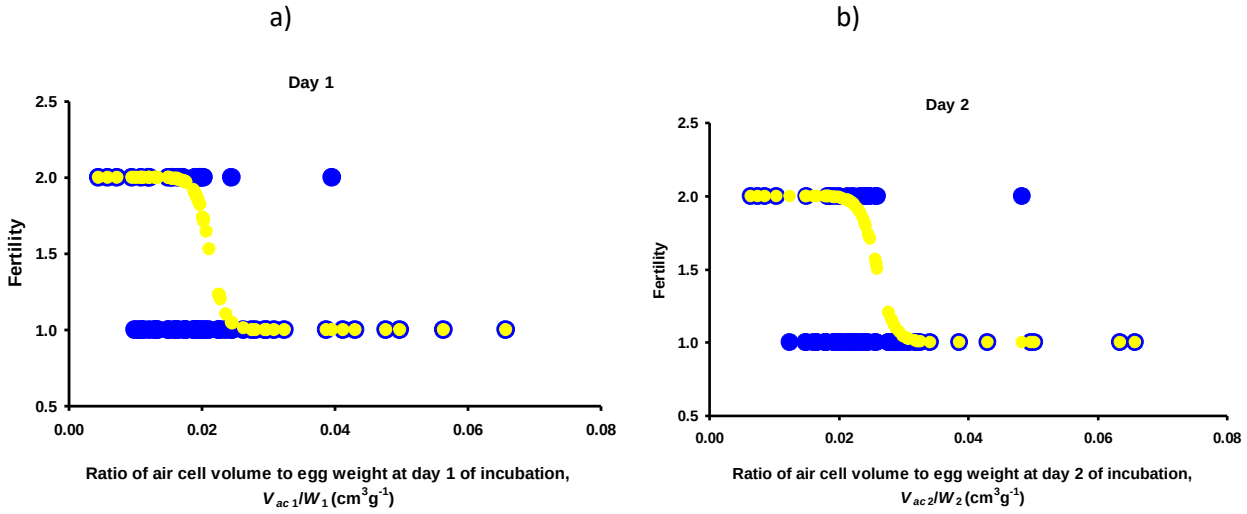


Fig. 6 – Visualisation of approximation results based on inverse sigmoid and using the respective data: (a) V_{ac1}/W_1 and function (8) (yellow line); (b) V_{ac2}/W_2 and function (9) (yellow line).

Rounding the obtained computed values to the nearest integer, a judgement was made regarding subgroup, i.e., I, if the result was 1, or F, if the rounding gave 2, respectively, this or that egg belonged to.

The application of formula (8) enabled to correctly identify 20 out of 52 subgroup I eggs. However, two subgroup F eggs containing goose embryos were misidentified and, unfortunately, were not sent for incubation. These two "unlucky" eggs that were numbered #6 and #32 in our experiment were clearly out of the aggregation area of their counterparts; in Fig. 6a, they conformed to two blue dots protruding to the right in the top row.

Here, the fundamental criterion was established for the implementation of our further analysis. It consisted in defining how important it was to preserve good eggs from random culling. In this situation, various approaches can be chosen. From the economic standpoint, there could be an equilibrium point when, for example, an accidentally removed good egg would be a justified waste, if there would be a larger number of correctly removed unsuitable eggs that will save space in the incubator. It is quite likely that such parity would be justified in our case when two fertile eggs would be lost, while ~38% of the infertile ones would be removed. Moreover, it is important that such removal will occur on day 1, i.e., before placing the entire subgroup of goose eggs in the setter. Nevertheless, further steps were undertaken, and other possible ways were tried to save every good egg.

Having solved this dilemma for ourselves, a test was executed as to how accurate the identification of fertile eggs during their incubation can be using the same V_{ac}/W ratio. The calculation showed that on day 2 of incubation, it was feasible to accurately identify already 22 infertile eggs, while losing, at the same time, the fertile egg #32 erroneously identified as unsuitable for incubation. However, egg #6 moved safely into the aggregation area of its fertile counterparts and, therefore, was "rescued" (Fig. 6b).

The resulting computation formula based on the inverse sigmoid (6) had the following form:

$$y = 1 + \frac{1}{1 + e^{770\left(\frac{V_{ac2}}{W_2} - 0.026\right)}}, \qquad (9)$$

with $R = 0.321$ ($p < 0.05$).

Our further attempts to correctly identify eggs using V_{ac}/W on incubation days 3, 4, etc. failed as the number of unfairly culled fertile eggs only grew. At the same time, the correlation coefficient of the calculation formulae decreased and was no longer significant. That being said, our "unlucky" eggs, #6 and #32, invariably occurred among the misidentified ones.

None of the other measured parameters showed significant differences between the means for subgroups I and F. However, a different approach to the mathematical transformation of the parameters was tested in order to obtain more significant differences. Let us consider this procedure using an example of changes in W during incubation (Fig. 7).

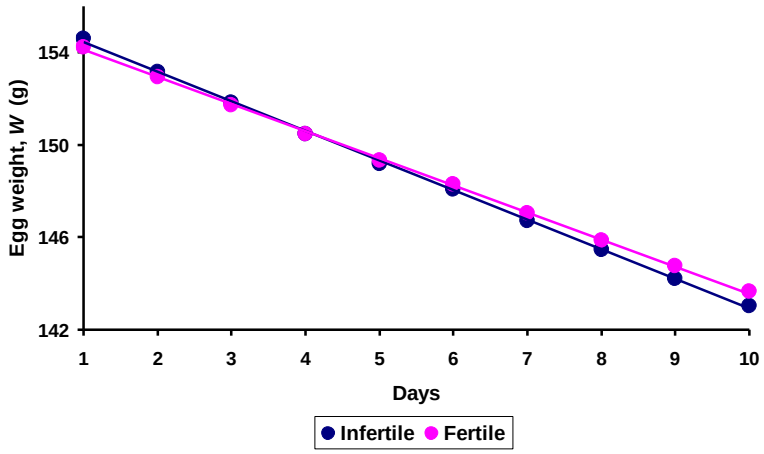


Fig. 7 – Graphical dependences of changes in mean *W* values for egg subgroups I (dark blue line) and F (purple line) during the first 10 days of incubation.

Visualisation of the measured changes in this parameter demonstrated how insignificant the differences between these two subgroups were. For instance, the computed value of differences between subgroups ranged from 0.03 to 0.13 by day 10 of incubation, while the tabular value of the Student's t-test for this sample and the 5% significance threshold was 1.993. However, based on the nature of the changes in *W*, it was suggested that infertile eggs lost more weight than their fertile counterparts. Therefore, the angle of inclination of their trend lines was different. It is most suitable to express this inclination angle in terms of a tangent that can be simply calculated using MS Excel tables, which was implemented in the course of further analysis. To compute this indicator, it was necessary to operate with at least two points, between which the trend line was drawn. Hence, nine parameters were calculated for each egg denoting them as TAN($W_{1–2}$) when the analysis involved parameters W_1 and W_2; TAN($W_{1–3}$) when the parameters W_1, W_2 and W_3 were involved in the analysis; and so on up to TAN($W_{1–10}$) when a trend line was drawn from the *W* measurements for each of the 10 incubation days. The outcome of comparing the mean TAN(*W*) values for egg subgroups I and F is presented graphically in Fig. 8.

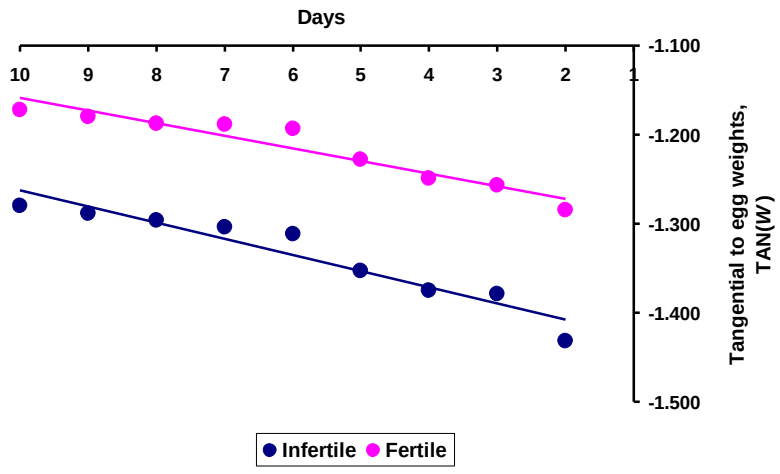


Fig. 8 – Graphic dependences of changes in the mean TAN(*W*) values for egg subgroups I (dark blue line) and F (purple line) during the first 10 days of incubation.

Although the differences were insignificant, the computed values of the Student's criterion increased by more than 10 times and fluctuated already between 1.2 and 1.6. It was even visually noticeable how much the differences between the two subgroups had grown. Despite the absence of significant differences, the possibility of applying the TAN(*W*) parameter was evaluated in the derivation and practical use of the approximation dependence. As in our previous trial with using air cell parameters, the degree of aggregation for eggs from different subgroups in the overall dataset was initially estimated. As a variable parameter, the TAN($W_{1–2}$) values resulted from *W* measurements on days 1 and 2 of incubation were chosen. Visualisation of the distribution of these values for subgroups I and F in the total amount of studied goose eggs is given in Fig. 9.

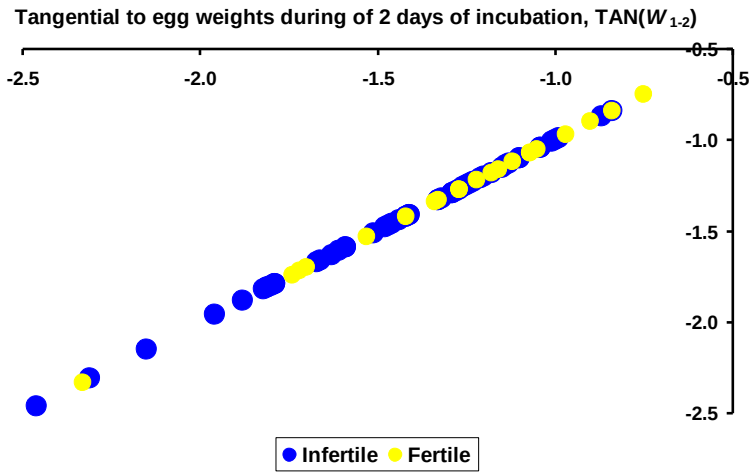


Fig. 9 – Visualisation of the distribution of TAN($W_{1–2}$) values for subgroups I (blue dots) and F (yellow dots) in the total amount of studied goose eggs.

Unlike the previous parameter V_{ac}/W (Fig. 4), the aggregation of subgroup F eggs was observed in the area of higher TAN($W_{1–2}$) values, albeit not so pronounced. In this case, the basic formula for data approximation was the classical sigmoid function (Eqn7). Its application resulted in the following calculation equation:

$$y = 1 + \frac{1}{1 + e^{-14.4(\mathrm{TAN}(W_{1-2}) + 1.39)}}, \qquad (10)$$

with $R = 0.186$.

Although *R* did not reach the threshold of 5% significance, the computation was carried out according to Eqn10 as visualised in Fig. 10.

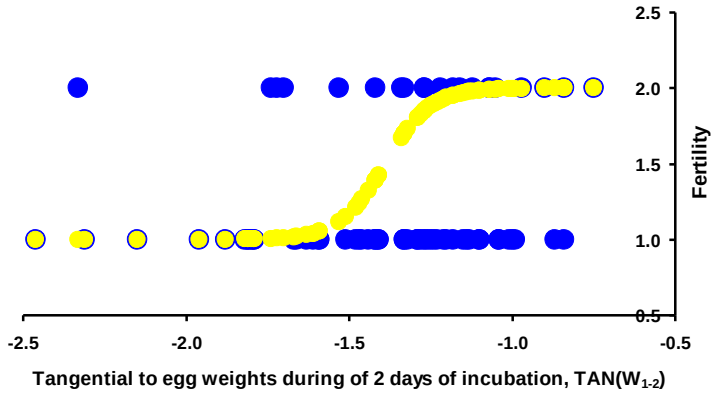


Fig. 10 – Approximation of TAN($W_{1–2}$) data by function (10) (yellow line).

The results of identification of eggs according to the formula (10) made it possible to accurately define 26 unfertilised eggs for removal. However, six eggs suitable for incubation were incorrectly assigned to subgroup I, which was recognised as completely unacceptable. Nevertheless, the experience of such transformations of egg parameters turned out to be successful and was used in the course of further stages of our analytical studies.

To maintain the intrigue, it should be noted that among the unfairly rejected fertile eggs according to the TAN($W_{1–2}$) criterion, #6 and #32 were again present. These two eggs did not visually stand out from the rest (Fig. 11), but with continued persistence did not fall into the right category, at least until the next stage of our research as will be laid out below.

Fig. 11 – Two exceptional goose eggs #6 (a) and #32 (b) used in the experiment.

Overall, as practical results of the possible identification of infertile eggs described in the section 3.1, the potential of such an indicator as V_{ac}/W was suggested that can be quite effective in culling eggs prior to incubation.

3.2 Mortality

The intrigue with the two eggs #6 and #32 was resolved quite logically in our experiment. These eggs contained live embryos at the time of their transillumination on day 10 of incubation, although the embryos died by day 18 of incubation. Thus, their culling, along with infertile eggs, would be quite appropriate. Their identification prompted us to consider it expedient to combine subgroups I and M into a single subgroup for their joint culling that, in principle, was planned in the course of further research. In the meantime, it was stated that subgroup M, numbering seven eggs by the tenth day, was supplemented with 10 more and, thus, already consisted of 17 eggs. Accordingly, subgroup F decreased by 10 eggs, and, in the amount of 11 eggs with viable embryos, turned into subgroup AE.

Subgroup M was assigned a quantitative index of one. For subgroup AE, the successor of F, the index, respectively, remained unchanged, being equal to two. Thus, the numerical data series of the function (y) included 11 twos and 17 ones and reflected the values of the function for the presence of a viable embryo in the goose egg.

For this series (y), the value of σ_y was 0.50. This enabled, using Eqn5, to calculate the E value that was 0.014. The obtained value was significantly lower than E_{max} = 0.05. Hence, the studied samples, despite the limited number of eggs in each, were recognised as representative.

The appropriate measured and computed variables for goose egg subgroups M and AE are provided in Table 2.

Table 2 – Data of measured and calculated goose egg variables for subgroups M and AE.

Parameters	Mean		Standard deviation	
	M	AE	M	AE
Egg weight at the beginning of each incubation day, W (in g):				
W_1	154.3	159.0	19.8	19.3
W_2	152.8	157.8	19.7	19.2
W_3	151.2	156.7	19.6	19.1
W_4	149.7	155.6	19.4	19.0
W_5	148.3	154.6	19.3	18.9
W_6	147.1	153.7	19.3	18.8
W_7	145.6	152.5	19.1	18.7
W_8	144.1	151.6	19.0	18.6
W_9	142.8	150.6	19.0	18.5
W_{10}	141.4	149.6	18.9	18.4
Egg length, L (in cm)	8.81	8.68	0.37	0.47

Maximum egg breadth, *B* (in cm)	5.65	5.76	0.25	0.22
Shift of the *B* axis from the egg centre, *w* (in cm)	0.44	0.40	0.14	0.12
Egg diameter at a distance of 1/4 of its length from the pointed end, $D_{L/4}$ (in cm)	4.53	4.67	0.25	0.18
Egg volume, *V* (in cm^3)	145.2	149.4	18.2	18.1
Surface area, *S* (in cm^2)	120.1	121.0	10.0	11.2
Air cell diameter at the beginning of each incubation day, *d* (in mm):				
d_1	34.0 [a]	30.0 [a]	3.8	4.1
d_2	35.8 [a]	31.5 [a]	4.2	3.7
d_3	36.8 [a]	32.8 [a]	3.2	3.4
d_4	37.6 [a]	33.6 [a]	3.8	4.9
d_5	39.7 [a]	35.7 [a]	2.5	2.8
d_6	39.5 [a]	35.7 [a]	2.8	2.4
d_7	40.3 [a]	37.5 [a]	3.6	2.6
d_8	41.6 [a]	38.2 [a]	2.8	2.7
d_9	42.6 [a]	39.1 [a]	2.9	2.3
d_{10}	42.9 [a]	39.8 [a]	2.4	2.0

[a] $p < 0.05$; the values without any index are insignificant.

The situation with differences between subgroups M and AE was similar to that observed for fertile and infertile eggs. The only measured parameter whose values differed significantly between the means of the two studied subgroups of goose eggs (M and AE) was *d* detected during all 10 days of incubation.

In the current and previous goose-egg related studies (e.g., Narushin, Romanov, Salamon, Kent, 2023), we, however, noticed the fact that the combination of few parameters into a complex one, or their determined mathematical transformation, helped to identify such differences. A similar effect was observed when egg subgroups M and AE were analysed. For example, single geometric parameters of the eggs did not differ significantly by subgroups. Yet, in the case of using the shape index (*B* to *L* ratio), the difference was already significant, and this situation was similar with $D_{L/4}$. In our previous studies (Narushin, Romanov, Lu, Cugley, Griffin, 2020b, 2021a), the high information content of this indicator was demonstrated that further increased when it was used in the form of the $D_{L/4}/B$ ratio. Narushin (2001) called this the index of conicity. In our case, the differences in the $D_{L/4}/B$ values were insignificant, but the calculated value of the Student's t-test was quite close to its tabular value. Since the use of geometric parameters for culling eggs with dead embryos has the highest priority and importance as it allows this procedure to be performed before incubation, the possibility of using the above two indices was analysed with all possible care.

Keeping in mind the principles of synergy, the aim here was to test the use of B/L and $D_{L/4}/B$ together when deriving the approximation formula. For this, the following basic equation was implemented:

$$y = \left(\frac{B}{L}\right)^{a} \left(\frac{D_{L/4}}{B}\right)^{b}, \quad (11)$$

where *a* and *b* are coefficients, and *y* are the function values for the presence of a viable embryo in a goose egg.

The coefficients in Eqn11 showed the maximum value of the correlation coefficient with the true values of *y* and approximately corresponded to $a \approx -2$ and $b \approx -1$. Since our target was not to accurately approximate the *y* values, but to obtain some kind of "symbiosis" of geometric parameters, it was considered more appropriate to give a specific name to this integrated indicator and called it Emergency Geometrical Index (*EGI*). The purpose of *EGI* will be to predict which ratios of the egg's geometric parameters may be threatening for the successful development of the embryo. As a result, Eqn11 was transformed into the following formula:

$$EGI = \frac{L}{B} \cdot \frac{L}{D_{L/4}}. \quad (12)$$

Such a transformation of Eqn12 was somewhat unexpected for us, since the usual ratio of indices took on an inverse form. In addition, *B* in $D_{L/4}/B$ was replaced by *L*. As for the relationship between *B* and *L* and, perhaps, due to the obtained integral indicator *EGI*, it was simultaneously possible to solve one philosophical question of oology: which index more adequately reflects the egg shape index, *B*/*L* or *L*/*B*? At first glance, the problem is akin to the insoluble contradictions of Swift's heroes on which side the egg should be peeled. Nonetheless, full consensus has not been reached. It so happens that in the study of poultry eggs, *B*/*L* is more traditional and became popular after the classic publication on the bird's egg by Romanoff and Romanoff (1949). However, Hamilton (2022) put the historical record straight by devoting an entire section on 'Who published it first?' to issues of this nature. According to his data, Dunn and Schneider (1923) were the founders of this shape index.

In contrast to poultry scientists, the reciprocal of the shape index (*L*/*B*) has been extensively used by ornithologists studying the eggs of wild bird species (e.g., Mänd, Nigul, Sein, 1986; Mytiai and Matsyura, 2017; Preston, 1968;). There were no works on determining the right of primacy for this type of ratio, which the authors call an elongation index. For the sake of fairness, it should be noted that in a number of works related to the study of the shape of goose and/or duck eggs, the authors (Salamon, 2015; Salamon and Kent, 2013b, 2014, 2017, 2020) used *L*/*B*, apparently tending to the fact that these species, rather more wild than fully domesticated and subjected to profound selection. Judging by the resulting relationship in

the *EGI* formula (Eqn12), Salamon and Kent (2013b, 2014, 2017, 2020) and Salamon (2015) were absolutely right in this respect.

The computation of *EGI* made it possible to obtain the functional dependence $y = f(EGI)$ that is visualised in Fig. 12.

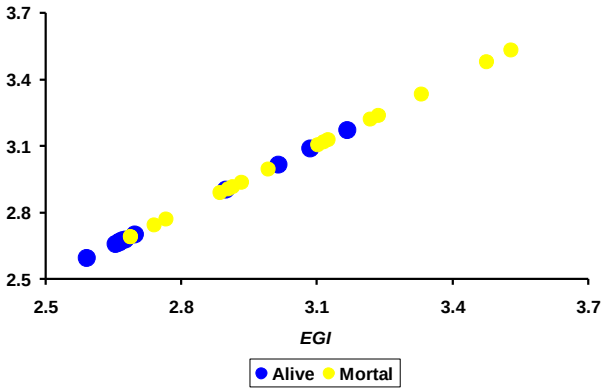


Fig. 12 – Visualisation of the distribution of the Emergency Geometrical Index (*EGI*) values for subgroups AE (blue dots) and M (yellow dots) in the total amount of studied goose eggs.

Analysing Fig. 12, it can be stated with a certain degree of confidence that eggs with higher *EGI* values have every reason for increased embryonic mortality during their incubation. In this case, the basic formula for data approximation will be the inverse sigmoid function (Eqn6), with some changes in the coefficient *c* (Supplementary Data A), resulting in the following calculation equation:

$$y = 1 + \frac{1}{1 + e^{37.8(EGI - 2.94)}}, \qquad (13)$$

with $R = 0.312$.

Albeit *R* did not reach the threshold of 5% significance, we, however, performed the computation according to Eqn13 as visualised in Fig. 13.

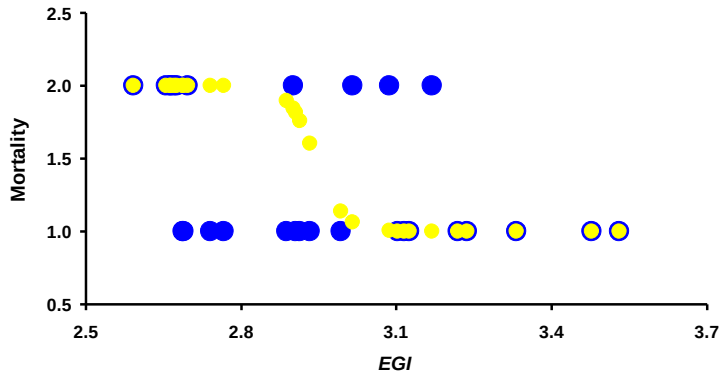


Fig. 13 – Approximation of the Emergency Geometrical Index (*EGI*) data by function (13) (yellow line).

The results of egg identification using formula (13) accurately detected 8 out of 17 eggs in which embryos would not survive during incubation. However, three eggs from subgroup AE were incorrectly classified as subgroup M. The rate of egg identification success using *EGI* exceeded those when using B/L and $D_{L/4}/B$ separately, so that the principle of synergy proved its effectiveness. Undoubtedly, the *EGI* indicator has the potential in timely culling eggs at the stage of their placing in the incubator, whereas its accuracy in terms of correct identification somewhat limits its use. Obviously, the use of indices using the geometric parameters of bird eggs is very promising. For example, based on the shape index of chicken eggs, Kayadan and Uzun (2023) were able to develop an algorithm for sorting them by the sex of the future embryo, which was previously considered practically impossible.

The next egg parameters that undeniably have the high potential are the air cell parameters. In the previous section (3.1), the best evidence for the identification of infertile eggs was demonstrated using V_{ac}/W. Its application made it possible to correctly identify ~38% of the eggs for removal prior to incubation. A similar situation existed for egg subgroups M and AE, suggesting the positive outcome in the earliest stages of observation. The distribution of eggs by subgroups using V_{ac}/W showed a higher degree of their localisation, which facilitated an accurate description of the function for embryo viability (y) using the inverse sigmoid formula (Fig. 14) as follows:

$$y = 1 + \frac{1}{1 + e^{968\left(\frac{V_{ac1}}{W_1} - 0.021\right)}}, \qquad (14)$$

with $R = 0.532$ ($p < 0.05$).

a) b)

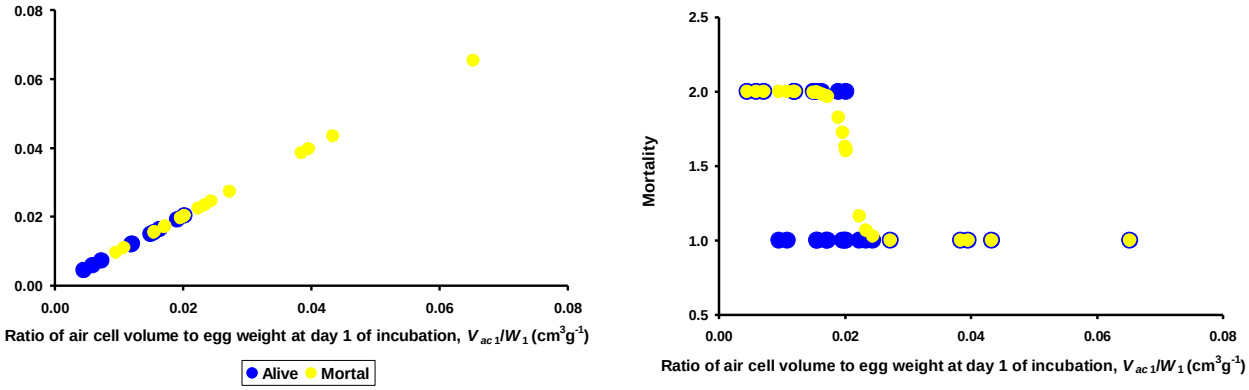


Fig. 14 – Visualisation of the distribution of V_{ac1}/W_1 values for subgroups AE (blue dots) and M (yellow dots) in the total range of studied goose eggs (a), and approximation of V_{ac1}/W_1 data by function (14) (yellow line) (b).

As a result, it was feasible before incubation to accurately identify eight out of 17 eggs (or 47%) that contained non-viable embryos. Most importantly, none of the subgroup AE eggs was mistakenly discarded. Because such division of eggs into subgroups was possible before the start of the incubation process, the results obtained clearly deserve the most positive consideration.

The analysis performed for revealing infertile eggs, as well as eggs with non-viable embryos, demonstrated the potential of using similar mathematical approaches and the same indices. Based on the results obtained, one can be even more confident in the potential success in identifying a joint, combined subgroup of eggs I and M, which probably is of the greatest commercial interest.

3.3 Hatchability

For this studied category, subgroups I and M were combined, with the name "hatching waste" (HW) being assigned to this joint subgroup. Thus, out of 80 goose eggs involved in the experiment, 69 eggs unsuitable for incubation were assigned to subgroup HW. As in the analysis of the previous category, the remaining 11 eggs that successfully completed the incubation process were referred to as subgroup of the alive embryo (AE) eggs. By analogy with the previous stages of our analytical studies, all eggs from subgroup HW were assigned a quantitative index of 1, and those of subgroup AE, respectively, that of 2. Thus, the numerical series that included 69 ones and 11 twos reflected the value of the function (y) for the presence of a viable embryo in a goose egg before or during the incubation.

For this series (y), the value of σ_y was 0.35 that enabled, using Eqn5, to calculate the E value that was equal to 0.01. The obtained value, as well as those for the previous categories, was significantly lower than E_{max} = 0.05. Thus, the studied samples were also recognised as representative.

The appropriate measured and computed HW and AE goose egg variables are shown in Table 3.

Table 3 – Data of measured and calculated goose egg variables for subgroups HW and AE.

Parameters	Mean		Standard deviation	
	HW	AE	HW	AE
Egg weight at the beginning of each incubation day, *W* (in g):				
W_1	154.5	159.0	20.8	19.3
W_2	153.1	157.8	20.7	19.2
W_3	151.7	156.7	20.7	19.1
W_4	150.3	155.6	20.7	19.0
W_5	149.0	154.6	20.6	18.9
W_6	147.8	153.7	20.6	18.8
W_7	146.4	152.5	20.6	18.7
W_8	145.1	151.6	20.6	18.6
W_9	143.9	150.6	20.5	18.5
W_{10}	142.6	149.6	20.5	18.4
Egg length, *L* (in cm)	8.74	8.68	0.41	0.47
Maximum egg breadth, *B* (in cm)	5.67	5.76	0.29	0.22
Shift of the *B* axis from the egg centre, *w* (in cm)	0.42	0.40	0.13	0.12
Egg diameter at a distance of 1/4 of its length from the pointed end, $D_{L/4}$ (in cm)	4.54 [a]	4.67 [a]	0.28	0.18
Egg volume, *V* (in cm^3)	145.0	149.4	19.5	18.1
Surface area, *S* (in cm^2)	119.9	121.0	11.1	11.2
Air cell diameter at the beginning of each incubation day, *d* (in mm):				
d_1	34.0 [a]	30.0 [a]	4.3	4.1
d_2	35.3 [a]	31.5 [a]	3.8	3.7
d_3	36.1 [a]	32.8 [a]	3.5	3.4
d_4	37.4 [a]	33.6 [a]	3.6	4.9
d_5	39.1 [a]	35.7 [a]	3.2	2.8
d_6	39.0 [a]	35.7 [a]	2.9	2.4
d_7	40.3 [a]	37.5 [a]	3.1	2.6
d_8	41.1 [a]	38.2 [a]	2.8	2.7
d_9	42.1 [a]	39.1 [a]	2.8	2.3
d_{10}	42.2 [a]	39.8 [a]	2.8	2.0

[a] $p < 0.05$; the values without any index are insignificant.

In connection with the paramount interest in identifying eggs from subgroup HW before incubation, those parameters and/or their derivatives were first of all evaluated and showed significant differences in the subgroups prior to incubation. Despite visible differences in *W* over 10 days of weighing during incubation,

the differences were not statistically significant. Perhaps, this fact may be explained by the high level of variability of this indicator in both subgroups HW and AE. The differences also turned out to be unreliable for the main geometric dimensions (*L* and *B*). However, the derivative of these parameters, *B*/*L*, already significantly differed between the two subgroups. The $D_{L/4}$ values were also initially significantly different. When interpreted as a conicity index ($D_{L/4}/B$), the significance of differences between the subgroups increased even more. Also, significant differences between the subgroups were demonstrated for *d*: increasing from day to day, the significance of their difference was invariably preserved.

Analysing the data in Table 3, a preliminary conclusion could be drawn that, on average, subgroup AE was characterised by a greater *W* during the 10 days of observation and a smaller air cell. Probably, the weight of egg contents played a key role in the process of embryonic development in geese at the earliest stages. Based on this preliminary analysis, a deeper analytical approach was taken to look for possible relationships to identify subgroups HW and AE using egg and air cell geometries.

In the section 3.2, a fairly informative integral indicator *EGI* was introduced that characterised the embryo viability depending on the ratios of various geometric parameters of the egg. The data of its calculation using Eqn12 showed significant differences in the values of this index between subgroups HW and AE. At the same time, the level of significance predictably exceeded that of *B*/*L* and $D_{L/4}/B$, which was quite logical since $D_{L/4}/B$ integrally contained both these egg characteristics. Visualisation of the distribution of *EGI* values for each of the two subgroups in the total amount of the studied eggs is provided in Fig. 15.

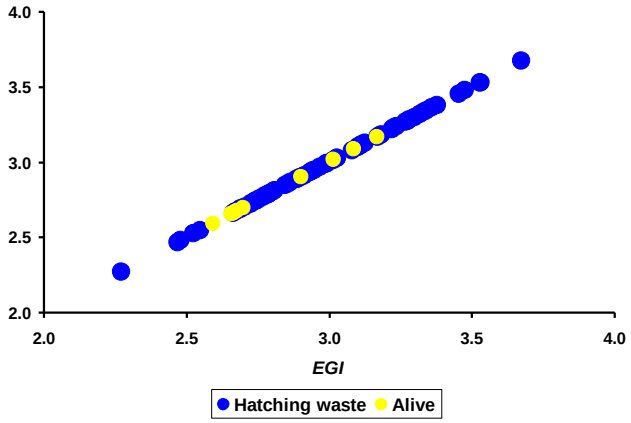


Fig. 15 – Visualisation of the distribution of the Emergency Geometrical Index (*EGI*) values for subgroups HW (blue dots) and AE (yellow dots) in the total amount of studied goose eggs.

In the case of *EGI*, a variant was encountered where subgroup AE, of our interest, was more or less closely concentrated not at the edges of the common line of eggs, but closer to its centre. This fact will be more evident (see Fig. 16) if, similarly to how it was presented earlier, the distribution of series would be

graphically established for those suitable eggs, whose function (*y*) values were equal to two (EGI_2), and that for eggs unsuitable for incubation with function (*y*) values being equal, respectively, to one (EGI_1).

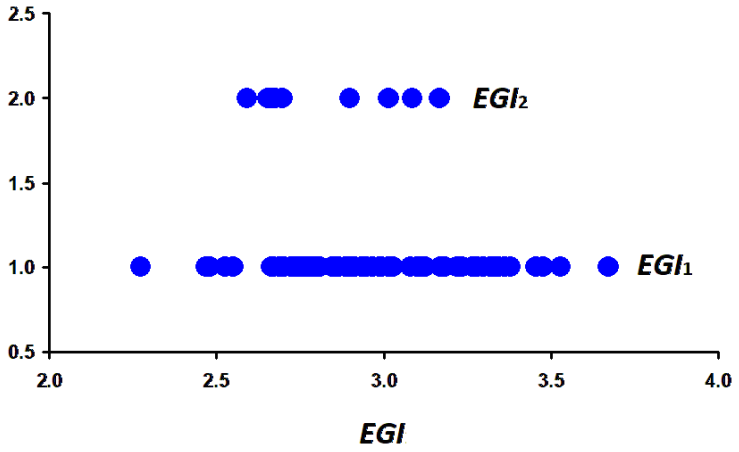


Fig. 16 – Functional dependence of embryo viability on the Emergency Geometrical Index (*EGI*) values.

Firstly, such a distribution of eggs by subgroups once again indicated that the extreme values of geometric indices, towards both the minimum and maximum, were highly undesirable for successful incubation. While this fact was quite logical for fertile eggs and was confirmed by various studies (Iqbal, Khan, Mukhtar, Ahmed, Pasha, 2016; Narushin and Romanov, 2002; Mitrovic et al., 2018; Salamon, 2020; Wilson, 1991), a similar relationship with infertile eggs was definitely unexpected. However, amongst the 69 eggs of subgroup HW, the infertiles (52 eggs) clearly dominated. Would it be possible that mother goose might be somehow selective about fertilisation and tried to retain eggs, the geometric parameters of which were far from perfect, infertile? Just in case, the significance between the previously analysed subgroups I and F was retested using the *EGI* parameter. However, the "miracle" did not happen, and this parameter did not have any significant differences between subgroups I and F.

Nevertheless, our objective was more applied in nature, and therefore, further trials were directed at the possibility of mathematical identification of subgroup AE eggs. Unlike the above sigmoid options already worked out, this case required a function with two sigmoid branches, ascending and descending. This can be a normal distribution function that, in general terms ad considering the previously accepted notation of variables and also the fact that, in our case, the function is lifted by *y* = 1, can be written in the following form (e.g. Zelen and Severo, 1964):

$$y = 1 + \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-x_a}{\sigma}\right)^2}, \qquad (15)$$

where the parameter x_a is the mean or expectation of the distribution (as well as its median and mode), while the parameter σ is its standard deviation.

In our particular case, it was necessary to describe the data conforming to two horizontal lines as depending on the *EGI* values. The top line corresponded to good eggs (EGI_2), whereas the lower one corresponded to HW (EGI_1). The computation of the results according to formula (15) specifically for our case made it possible to describe the data in Fig. 16 using a bell-shaped curve (Fig. 17). This enabled to identify all the eggs in subgroup AE absolutely correctly, meaning that none of the 11 hatchable eggs suffered from improper removal.

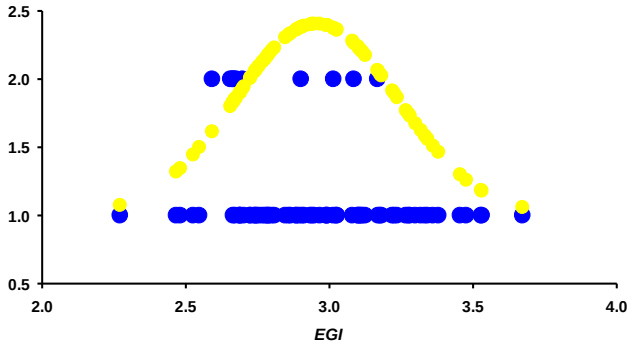


Fig. 17 – Approximation of the Emergency Geometrical Index (*EGI*) data by function (15) (yellow line).

In subgroup HW, only 10 eggs out of 69 were correctly identified. This accounted for only 14% eggs that were separated from the general sample. The remaining 59 eggs should have been sent for incubation, despite the deliberate failure in the final result. In this regard, formula (15) was required some improvement. First of all, the improvements related to limiting the extremum of the function to the value $y = 2$ that was achieved when $x = x_a$. In accordance with these prerequisites, formula (15) was modified:

$$y = 1 + e^{-\frac{1}{2}\left(\frac{x - x_a}{\sigma}\right)^2} \qquad (16)$$

Calculation by formula (16) gave a more adequate curve (Fig. 18).

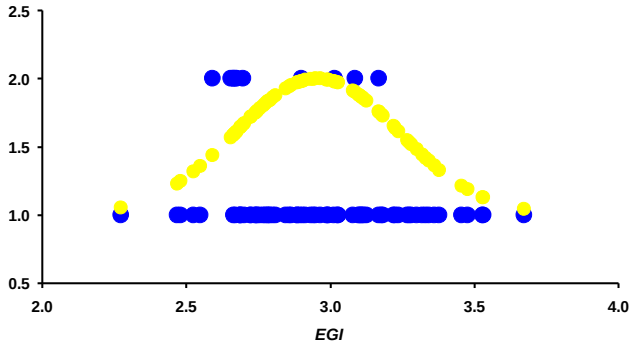


Fig. 18 – Approximation of the Emergency Geometrical Index (*EGI*) data by function (16) (yellow line).

As a result, 10 viable eggs were correctly identified, i.e., one of 11 hatchable eggs was misidentified due to improper selection. At the same time, the identification indicator in subgroup HW was slightly improved. Already 16 eggs out of 69 were correctly identified, which amounted to 23%. The remaining 53 eggs were ineffectively incubated.

Moreover, already at the very first stage, i.e., before incubation, it was possible to select more than 20% of all non-viable eggs, which can be considered a success. However, given that goose eggs with viable embryos have a considerably lower percentage of all laid eggs as compared to other poultry species (Bogenfürst, 1995, 2004; Łukaszewicz, 2010), the cost of mistakenly culling such an egg is incomparably higher than the possibility of identifying a few extra eggs of HW in order to save room in the incubator. Within the framework of the current experiment, the computation formula (15) and, accordingly, its visual representation in Fig. 17 may be more advantageous than Eqn16 (Fig. 18). After all, one good egg was otherwise lost and, under the conditions of our experiment, this was ~9% of the total number of good eggs. However, six unusable eggs were additionally removed from the incubator, which also made up ~9% of their total number, but the equality of these percentages was by no means equivalent. In this regard, it was considered appropriate to use Eqn15 as a baseline for identifying eggs localised in the centre of the numerical series (Figs. 15 and 16).

Thus, using the universal geometric index *EGI* and within the framework of our experiment, it was feasible to separate 10 eggs prior to incubation, which obviously failed during the incubation process. However, there was still 59 left that were also desirable to identify and remove as early as possible. In this regard, based on the data in Table 3, the air cell parameters had an undoubted potential. Using formulae (3) and (4), h and V_{ac} were recalculated. Also, other indices were analysed, including a set of air cell parameters, similar to those described above. The predicting potential of each of them was evaluated by the correlation coefficient (R) in comparison with the numerical series of egg viability where the value of function for eggs from subgroup HW was equal to one, and that for AE two.

Similar to our previous findings (sections 3.1 and 3.2), our results demonstrated the highest potential for V_{ac}/W that had an R value ranging from −0.28 to −0.39 over the first 10 days of incubation, with each being statistically significant. Graphical changes in V_{ac}/W for the mean values of each of subgroups HW and AE are shown in Fig. 19.

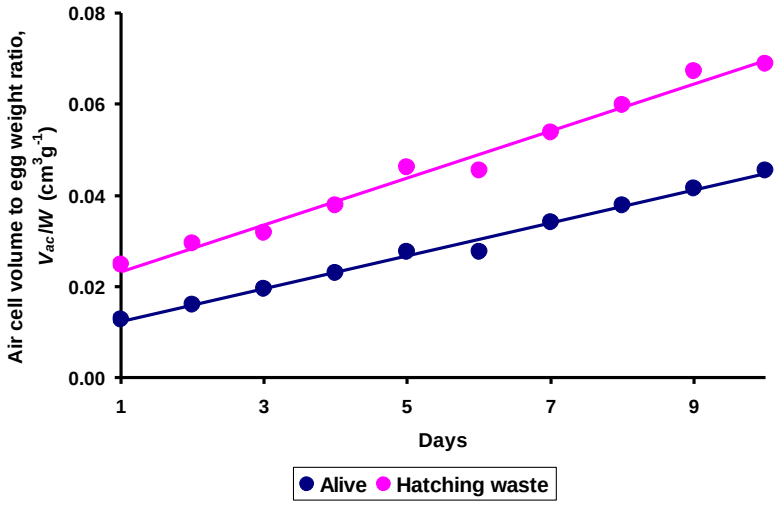


Fig. 19 – Graphical dependences of the change in V_{ac}/W for subgroups HW (purple line) and AE (dark blue line) during the first 10 days of incubation.

Since the difference between subgroups HW and AE was significant on day 1, i.e., before incubation, it was these values that were analysed in the first place. The respective preincubation measurements (V_{ac1}/W_1) are visualised in Fig. 20.

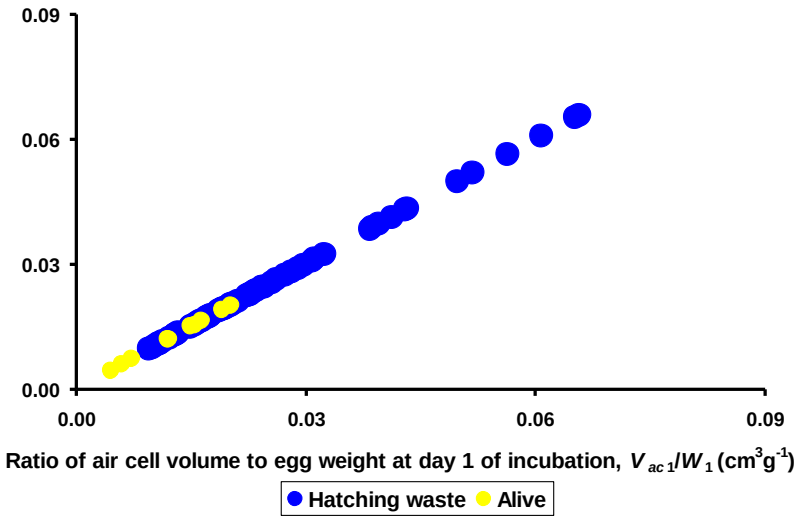


Fig. 20 – Visualisation of the distribution of V_{ac1}/W_1 values for subgroups HW (blue dots) and AE (yellow dots) in the total amount of studied goose eggs.

Using the V_{ac1}/W_1 index, it was possible to localise subgroup AE eggs in the area of smaller values on the general line of the studied eggs, as a result of which the best accuracy of data approximation was possible by sigmoid (6). For this particular case, the x_a value corresponded to the mean value of the V_{ac1}/W_1 numerical series, which was equal to 0.023 on day 1 of observation, as a result of which the computation formula had the following form:

$$y = 1 + \frac{1}{1 + e^{863\left(\frac{V_{ac1}}{W_1} - 0.023\right)}}, \quad (17)$$

with $R = 0.300$ ($p < 0.05$).

The visualisation of the approximation results is shown in Fig. 21.

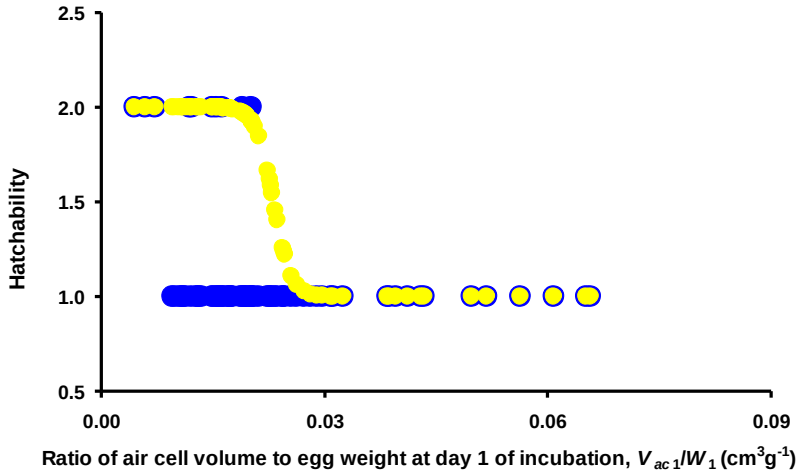


Fig. 21 – Approximation of V_{ac1}/W_1 data by function (17) (yellow line).

The use of formula (17) made it possible to correctly identify 26 eggs from subgroup HW, while all eggs of subgroup AE were misidentified.

Comparison of correctly identified eggs using *EGI* (ten eggs) and V_{ac1}/W_1 (26 eggs) revealed that four eggs from subgroup HW matched. In general, the use of these two predictors enabled to exclude 32 low-quality eggs before incubation. This number corresponded to ~46% of all eggs in subgroup HW, which was quite good for such an early stage. However, another possibility was tested to improve this result. Based on the principles of synergy, it was assumed that the combined use of *EGI* and V_{ac1}/W_1 values could drastically affect the identification results. Therefore, their combination obtained as a result of the most efficient mathematical operations with these indices was tested. To determine which mathematical operation is the

most effective, the values of the function (*y*) for the degree of embryonic viability were approximated that consisted of ones and twos using an equation of the following form:

$$y = EGI^{a}\left(\frac{V_{ac1}}{W_1}\right)^{b} \tag{18}$$

where *a* and *b* are some constant coefficients.

By analogy with the derivation of the *EGI* index, the coefficients in formula (18) were of our interest only from the viewpoint of mathematical relationships that provide the best approximation output.

The results of approximation by Eqn18 showed that both coefficients were negative, while their values approached one. In this respect, in order to evaluate a certain generalised index reflecting the synergistic effect of *EGI* and V_{ac1}/W_1, the following novel indicator was proposed conventionally called Emergency Hatching Index (*EHI*):

$$EHI = \frac{W_1}{EGI \cdot V_{ac1}} \tag{19}$$

Analysis of the new *EHI* index showed that the viability of goose embryos increased with increasing *EHI* value (Fig. 22a), and identification of eggs by subgroups HW and AE was possible using the sigmoid derived from Eqn7 (Fig. 22b) as follows:

$$y = 1 + \frac{1}{1 + e^{-(EHI - 19.75)}}, \tag{20}$$

with $R = 0.332$ ($p < 0.05$).

a) b)

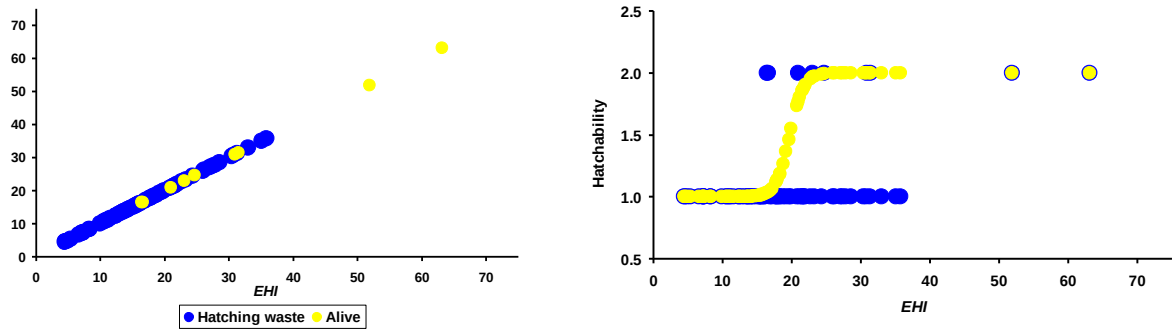


Fig. 22 – Visualisation of the distribution of the Emergency Hatching Index (*EHI*) values for subgroups HW (blue dots) and AE (yellow dots) in the total amount of studied goose eggs (a) and results of *EHI* data approximation by function (20) (yellow line) (b).

The calculation using Eqn20 demonstrated the possibility of correctly identifying 45 eggs from subgroup HW, which was almost 1.5 times higher than in the case of their separate identification. This number was 65% of the total number of eggs unsuitable for incubation, and preventing their setting in the incubator promised an undoubted economic effect. However, this euphoric success was somewhat mundane, as two eggs with perfectly viable embryos were incorrectly assigned to subgroup HW. Therefore, it was considered appropriate to keep the eggs from subgroup AE and continue further analysis for more accurate identification during their incubation. Anyhow, the *EHI* indicator turned out to be informative and may well be taken as a basis in other similar studies and/or related breeding programs.

Since the options for culling eggs before they were placed in the incubator were exhausted, the authors turned to the search for possible differences in the egg parameters that occur on day 2 of incubation. These were V_{ac2}/W_2 measured on day 2 of incubation and egg density (D), whose value will be considered in more detail below.

As was established earlier (Table 3), there were no significant differences in W between the two studied subgroups during all 10 days of incubation, with a similar situation being observed for V. However, when determining D, i.e., by examining the ratio of W to V, significant differences appeared on day 9 of incubation (Fig. 23).

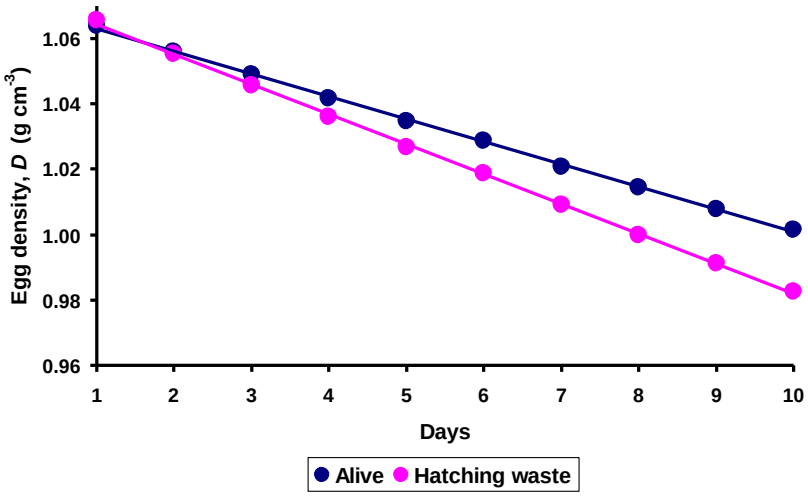


Fig. 23 – Graphs of egg density (*D*) change for subgroups HW (purple line) and AE (dark blue line) during the first 10 days of incubation.

Similarly to how it was performed earlier when analysing infertile eggs, it was assumed that a more convenient and informative indicator would be not the direct values of *D*, but the nature of their changes, i.e., the slope of the trend line, with its quantitative indicator being the tangent of the slope angle TAN(*D*). The change in this indicator during the first 10 days of incubation confirmed significant differences in subgroups HW and AE, starting from day 2 (Fig. 24). Similar to the previously used notation, TAN($D_{1–2}$) indicated that the slope of the trendline was computed from the two *D* values obtained on day 1 (D_1) and day 2 (D_2) of incubation, respectively. In the definition of TAN($D_{1–3}$), there were already three values involved (D_1, D_2, and D_3) and so on.

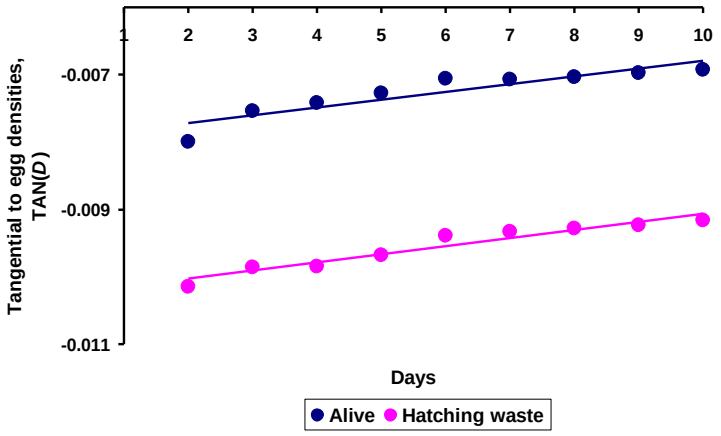
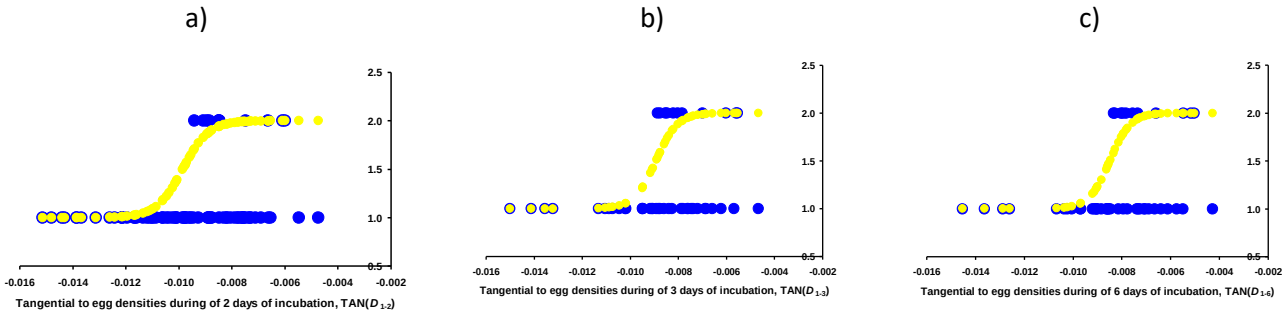


Fig. 24 – Graphic dependences of the change in the tangent of the slope for the density trend line, TAN(*D*), for subgroups HW (purple line) and AE (dark blue line) during the first 10 days of incubation.

Using the obtained calculated data for the corresponding intervals TAN(D_{1-2}) … TAN(D_{1-6}), it was feasible to approximate them with sigmoid in the form of Eqn7. Previously, 32 eggs were excluded from the examination because they were identified as incubation defects by using *EGI* and V_{ac1}/W_1 before incubation. Thus, further TAN(*D*) analysis was performed on a sample of 37 HW eggs and the remaining 11 AE eggs. As an example, a limit to three results from this series was set, and their visualisation, along with the respective equations, is presented in Fig. 25.



$$y = 1 + \frac{1}{1 + e^{-2172(\mathrm{TAN}(D_{1-2}) + 0.009)}} \quad (21)$$

with $R = 0.314$ ($p < 0.05$)

$$y = 1 + \frac{1}{1 + e^{-2241(\mathrm{TAN}(D_{1-3}) + 0.009)}} \quad (22)$$

with $R = 0.354$ ($p < 0.05$)

$$y = 1 + \frac{1}{1 + e^{-2357(\mathrm{TAN}(D_{1-6}) + 0.008)}} \quad (23)$$

with $R = 0.397$ ($p < 0.05$)

Fig. 25 – Approximation of data for TAN($D_{1\text{-}2}$) (a), TAN($D_{1\text{-}3}$) (b) and TAN($D_{1\text{-}6}$) (c) by the corresponding sigmoid Eqns 21 to 23 (yellow lines).

As a result of the corresponding computations using Eqns 21 to 23 and similar ones obtained for the TAN($D_{1–4}$) and TAN($D_{1–5}$) values, it was possible to achieve the correct identification of 19 HW eggs on day 2 of incubation. However, one egg from subgroup AE was, at the same time, misidentified. Furthermore, starting from day 3 of incubation, all 11 AE eggs were identified correctly, while the number of eggs for culling from subgroup HW was 18 pieces for day 3, 19 for day 4, 20 for day 5, and 21 for day 6. Subsequent results of the correct determination did not improve. Thus, taking into account our success in the timely removal of eggs (32 pieces) prior to incubation, measurement of their *W* values over 6 days of incubation allowed us to cull 21 more eggs, which altogether amounted to about 77% of the total number of incubation defects. Interestingly, out of these 53 eggs, 15 (of 17) eggs with non-viable embryos and, accordingly, 38 (out of 52) infertile eggs were promptly discarded. Thus, on a percentage basis, 88% of all mortal eggs and 73% of the infertile ones were correctly identified.

Also, the V_{ac}/W data respectively measured during the same period from day 2 to day 6 of incubation was used. However, the results obtained were clearly inferior to those for the TAN(*D*) indicators. For example, while 18 HW eggs correctly identified on day 2 of incubation, two AE eggs were undeservedly culled. Furthermore, the erroneous determination only increased to four eggs and remained so until day 6 of incubation. At the same time, the correct identification of HW eggs gradually declined from 18 to 12 eggs. Our attempts to apply the method of synergistic use of TAN(*D*) and V_{ac}/W also did not lead to positive results. The best try resulted in two good eggs lost due to improper culling on day 6 of incubation, although 27 HW eggs were identified correctly.

3.4 Generalisation of the investigation findings

Collectively, the authors believe that the most efficient algorithm for practical use might be as follows:

1. The culling of eggs according to *EGI* values and V_{ac}/W measured before incubation.
2. Egg culling in accordance with their measured *D* values for 6 days of incubation in terms of the tangent of the slope obtained for the trend line TAN(*D*).

This procedure will avoid incorrect culling of good eggs with viable embryos, while removing ~70% or more of HW eggs.

4. Conclusions

The studies carried out and the analytical interpretation of the results obtained made it possible to summarise here the main achievements of this experiment and recommendations for their practical use as follows.

1. It is expedient, both from commercial and technological viewpoints, to cull a joint (combined) subgroup of infertile and non-viable goose eggs that was defined as HW in this work.

2. Technologically, it is most effective to perform the identification of goose eggs unsuitable for incubation stepwise by the days of incubation, gradually identifying and removing HW day by day.

3. The best sole indicators were *EGI* that was introduced here and V_{ac}/W that enabled, within the framework of our experimental sample, to identify up to ~38% of the eggs before incubation. Herewith, *EGI* also represented a set of few geometric measurements of the egg combined into one single index. When using the combination of the two indicators, the *EHI* index was proposed, which included a complex of geometric, weight and volume characteristics of the entire egg and its air cell. When applying *EHI*, the number of correctly culled eggs increased to 65% of the total number of HW eggs.

4. A method of mathematical transformation was proposed and successfully tested for egg parameters, measured and changing during incubation, into the tangent value of the slope of the trend line formed on the basis of measured indicators in different periods of incubation. This method led to the increased significance degree of parameters between samples. The use of this approach enabled to effectively use *D* values measured in the periods from day 1 to day 2 and from day 1 to day 3 of incubation, resulting in correct identification of 26% of the rejected eggs.

5. Another methodological approach that can be extremely useful in conducting similar studies turned out to be the selection or mathematical transformations of one or a set of parameters in order to maximise the aggregation of the egg subgroup of our interest in a certain place of the general line of the studied sample. Depending on the localisation of these eggs on the respective graphs (in the centre or along the edges), a mathematical approach was developed to derive approximation expressions that can be used to calculate the match of a particular egg to a particular subgroup. This approach can be potentially used in other studies when their outcome resulted from only two indicators. Such studies, for example, may involve the

separation of eggs by sex of the embryo (male or female), the number of yolks in the egg (1 or 2), the heritability of a certain trait (yes or no), etc.

6. Our findings, along with the developed methodological, mathematical and analytical approaches, can be considered pioneering and represent a model not only for conducting similar studies in biology and agriculture, but also in a number of related disciplines. The resulting algorithms are fully adapted to the possibility of their implementation in appropriate automated equipment. This applies to both the ability to measure the required parameters and fairly simple software for calculations.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at

References

Biesiada-Drzazga, B., Banaszewska, D., Koncerewicz, A., Jozwik, A., & Horbanczuk, J. (2015). Examination of changes in selected external and internal egg traits during the geese laying season and their effect on gosling hatching results. *European Poultry Science*, *79*(February). https://doi.org/10.1399/eps.2015.77

Fadchar, N. A., & Dela Cruz, J. C. (2020). Prediction model for chicken egg fertility using artificial neural network. In *2020 IEEE 7th International Conference on Industrial Engineering and Applications (ICIEA)* (pp. 916–920). Bangkok, Thailand: IEEE. https://doi.org/10.1109/ICIEA49774.2020.9101966

Bogenfürst, F. (1995). The current state and future of incubation in waterfowl. In *Proceedings of the 10th European Symposium on Waterfowl* (pp. 241–256). Halle, Saale, Germany.

Bogenfürst, F. (2004). *A keltetés kézikönyve*. Budapest, Hungary: Gazda kiadó. ISBN: 9789637445507.

Çevik, K. K., Koçer, H. E., & Boğa, M. (2022). Deep learning based egg fertility detection. *Veterinary Sciences*, *9*(10), 574. https://doi.org/10.3390/vetsci9100574

Çevik, K. K., Koçer, H. E., Boğa, M., & Taş, S. M. (2023). Mask R-CNN approach for egg segmentation and egg fertility classification. In *The International Conference on Artificial Intelligence and Applied Mathematics in Engineering (ICAIAME 2021): Smart Applications with Advanced Machine Learning*

and Human-Centred Problem Design. Engineering Cyber-Physical Systems and Critical Infrastructures, vol. 1. Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-031-09753-9_36

Cochran, W. G. (1977). *Sampling techniques*. (3rd ed.). New York, NY, USA: John Wiley & Sons Inc.

Dunn, L. C., & Schneider, M. (1923). An approximate method of calculating the surface area of eggs. *Poultry Science*, *2*(3), 90–92. https://doi.org/10.3382/ps.0020090

Fadchar, N. A., & Dela Cruz, J. C. (2020). Prediction model for chicken egg fertility using artificial neural network. In *2020 IEEE 7th International Conference on Industrial Engineering and Applications (ICIEA)* (pp. 916–920). Bangkok, Thailand: IEEE. https://doi.org/10.1109/ICIEA49774.2020.9101966

Geng, L., Hu, Y., Xiao, Z., & Xi, J. (2019). Fertility detection of hatching eggs based on a convolutional neural network. *Applied Sciences*, *9*(7), 1408. https://doi.org/10.3390/app9071408

Hamilton, R. M. G. (2022). A comparison of the various equations published for the estimation of characteristics of hen's eggs, the importance of reporting the compression rate for shell strength measurements, and distinction between egg specific gravity and density. *World's Poultry Science Journal*, *78*(2), 447–466. https://doi.org/10.1080/00439339.2022.2026202

Iqbal, J., Khan, S. H., Mukhtar, N., Ahmed, T., & Pasha, R. A. (2016). Effects of egg size (weight) and age on hatching performance and chick quality of broiler breeder. *Journal of Applied Animal Research*, *44*(1). 54–64. https://doi.org/10.1080/09712119.2014.987294

Ionov, I. A., Katerinich, O. O., Kuchmistov, V. O., Anisimova, O. V., Griffin, D. K., Romanov, M. N., et al. (2023) Vitamin E and A availability in goose embryos and goslings and improvement of reproduction traits depending on the starting temperature regime of egg incubation. *Poultry*, *2*(2), 305–319. https://doi.org/10.3390/poultry2020023

Jacob, J. P. (2023). Selecting a goose breed for small or backyard poultry flocks. Small and Backyard Poultry. eXtension: United States Cooperative Extension System, University of Kentucky, Lexington, KY, USA. URL: https://poultry.extension.org/articles/getting-started-with-small-and-backyard-poultry/selecting-birds-to-get-for-a-small-or-backyard-poultry-flock/selecting-a-goose-breed-for-small-or-backyard-poultry-flocks/ (accessed on 5 August 5, 2023).

Karabulut, O. (2021). Relationship between weight, volume and specific gravity of goose eggs before incubation. *Journal of Advances in VetBio Science and Techniques*, *6*(2), 90–99. https://doi.org/10.31797/vetbio.929031

Kayadan, M., & Uzun, Y. (2023). High accuracy gender determination using the egg shape index. *Scientific Reports*, *13*, 504. https://doi.org/10.1038/s41598-023-27772-4

Kent, J. P., & Murphy, K. J. (2003). Synchronized egg laying in flocks of domestic geese (*Anser anser*). *Applied Animal Behaviour Science*, *82*, 219–228. https://doi.org/10.1016/S0168-1591(03)00063-7

Kucharska-Gaca, J., Adamski, M., Kuźniacka, J., & Kowalska, E. (2016). Goose eggs hatching technique improvement with the use of pre-incubation. *Acta Scientiarum Polonorum Zootechnica*, *15*(2), 37–46. https://doi.org/10.21005/asp.2016.15.2.04

Łukaszewicz, E. (2010). Artificial insemination in geese. *World's Poultry Science Journal*, *66*(4), 647–658. https://doi.org/10.1017/S0043933910000632

Lukaszewicz, E., Lason, M., Rosenberger, J., Kowalczyk, A., & Bakst, M. (2017). Goose embryonic development from oviposition through 16 h of incubation. *Poultry Science*, *96*(6), 1934–1938. https://doi.org/10.3382/ps/pew474

Mänd, R., Nigul, A., & Sein, E. (1986). Oomorphology: A new method. *The Auk*, *103*(3), 613–617. https://doi.org/10.1093/auk/103.3.613

Mitrovic, S., Mekic, C., Milojevic, M., Dimitrijevic, R. M., Đekic, V., & Đermanovic, V. (2018). Effect of egg mass of the white italian goose on fertilisation, loss of weight during the incubation period, hatchability and gosling quality. *Indian Journal of Animal Research*, *52*(12), 1803–1808. https://doi.org/10.18805/ijar.B-787

Mytiai, I. S., & Matsyura, A. V. (2017). Geometrical standards in shapes of avian eggs. *Ukrainian Journal of Ecology*, *7*(3), 264–282. https://doi.org/10.15421/2017_78

Narushin, V. G. (2001). Shape geometry of the avian egg. *Journal of Agricultural Engineering Research*, *79*, 441–448. https://doi.org/10.1006/jaer.2001.0721

Narushin, V. G, & Romanov, M. N. (2002). Egg physical characteristics and hatchability. *World's Poultry Science Journal*, *58*(3): 297–303. https://doi.org/10.1079/WPS20020023

Narushin, V. G., Lu, G., Cugley, J., Romanov, M. N., & Griffin, D. K. (2020a). A 2-D imaging-assisted geometrical transformation method for non-destructive evaluation of the volume and surface area of avian eggs. *Food Control*, *112*(June), 107112. https://doi.org/10.1016/j.foodcont.2020.107112

Narushin, V. G., Romanov, M. N., Lu, G., Cugley, J., Griffin, D. K. (2020b). Digital imaging assisted geometry of chicken eggs using Hügelschäffer's model. *Biosystems Engineering*, *197*(September) 45–55. https://doi.org/10.1016/j.biosystemseng.2020.06.008

Narushin, V. G., Romanov, M. N., & Griffin, D. K. (2021a). Egg and math: introducing a universal formula for egg shape. *Annals of the New York Academy of Sciences*, *1505*(1), 169–177. https://doi.org/10.1111/nyas.14680.

Narushin, V. G., Romanov, M. N., Lu G., Cugley J., & Griffin, D. K. (2021b). How oviform is the chicken egg? New mathematical insight into the old oomorphological problem, *Food Control*, *119*(January), 107484. https://doi.org/10.1016/j.foodcont.2020.107484

Narushin, V. G., Romanov, M. N., & Griffin, D. K. (2021c). Non-destructive evaluation of the volumes of egg shell and interior: Theoretical approach. *Journal of Food Engineering*, *300*(July), 110536. https://doi.org/10.1016/j.jfoodeng.2021.110536

Narushin, V. G., Romanov, M. N., Salamon, A., & Kent, J. P. (2023). Egg Quality Index: A more accurate alternative to the Haugh unit to describe the internal quality of goose eggs. *Food Bioscience*, *55*(October), 102968. https://doi.org/10.1016/j.fbio.2023.102968

Preston, F. W. (1968). The shapes of birds' eggs: Mathematical aspects. *The Auk*, *85*(3), 454–463. https://doi.org/10.2307/408329

Romanoff, A. L., & Romanoff, A. J. (1949). *The avian egg*. New York, NY, USA: John Wiley & Sons Inc.

Romanov, M. N. (1995). Evolution of domestic geese. Centres of domestication and dispersion. In A. N. Gudina (Ed.), New Investigations on Palearctic Geese (pp. 120–126). Zaporizhya, Ukraine: Zaporizhya Branch of the Ukrainian Ornithological Society, Zaporizhya State University.

Romanov, M. N. (1997). Impact of genotype, nutrition and management systems on goose production efficiency. In *Proceedings of 11th European Symposium on Waterfowl* (pp. 33–41). Nantes, France: Association Groupe Francais de la WPSA (French Branch).

Romanov, M. N. (2018). Geese. In C. K. Baer, D. E. Ullrey, & W. G. Pond (Eds.), *Encyclopedia of animal science* (Two-Volume Set, 2nd ed., eBook; vol. 1, pp. 487–489). London, UK: Taylor & Francis Inc.; Boca Raton, FL, USA: CRC Press Inc. https://doi.org/10.1081/E-EAS2-120019645

Saifullah, S., & Dreżewski, R. (2022). Non-destructive egg fertility detection in incubation using SVM classifier based on GLCM parameters. *Procedia Computer Science*, *207*(October), 3254–3263. https://doi.org/10.1016/j.procs.2022.09.383

Saifullah, S., Dreżewski, R., Khaliduzzaman, A., Tolentino, L. K., & Ilyos, R. (2022). K-means segmentation based-on lab color space for embryo detection in incubated egg. *Jurnal Ilmiah Teknik Elektro Komputer dan Informatika*, *8*(2), 175–185. https://doi.org/10.26555/jiteki.v8i2.23724

Salamon, A. (2015). Maternal investment – and its constraints – in the egg of domestic waterfowl. Ph. D. thesis, University College Dublin.

Salamon, A. (2020). Fertility and hatchability in goose eggs: A review. *International Journal of Poultry Science*, *19*(2), 51–65. https://doi.org/10.3923/ijps.2020.51.65

Salamon, A., & Kent, J. P. (2013a). Egg weight declines to baseline levels over the laying season in domestic geese (*Anser anser domesticus*). *International Journal of Poultry Science*, *12*, 509–516. https://doi.org/10.3923/ijps.2013.509.516

Salamon, A., & Kent, J. P. (2013b). Double and single yolked duck eggs: their contents and dimensions compared and the mechanical stimulation hypothesis for albumen secretion is supported. *International Journal of Poultry Science*, *12*(5), 254–260. https://doi.org/10.3923/ijps.2013.254.260

Salamon, A., & Kent, J. P. (2014). Changes during incubation within double-yolked duck (*Anas platyrhynchos domesticus*) eggs: yolk position, mortality, hatchability and the importance of an optimal egg size. *International Journal of Poultry Science*, *13*(12), 695–702. https://doi.org/10.3923/ijps.2014.695.702

Salamon, A., & Kent, J. P. (2016). Manual egg turning is necessary for optimal hatching in geese. *International Journal of Poultry Science*, *15*, 57–61. https://doi.org/10.3923/ijps.2016.57.61

Salamon, A., & Kent, J. P. (2017). Egg shape is constrained more by width than length, evidence from double-yolked duck eggs. *International Journal of Poultry Science*, *16*, 387–392. https://doi.org/10.3923/ijps.2017.387.392

Salamon, A., & Kent, J. P. (2020). The double-yolked egg: from the ‘miracle of packaging’ to nature’s ‘mistake’. *World's Poultry Science Journal*, *76*, 18–33. https://doi.org/10.1080/00439339.2020.1729671

Wilson, H. (1991). Interrelationships of egg size, chick size, posthatching growth and hatchability. *World's Poultry Science Journal*, *47*(1), 5–20. https://doi.org/10.1079/WPS19910002

Zelen, M., & Severo, N. C. (1964). Probability functions. In M. Abramowitz, & I. A. Stegun (Eds.), *Handbook of mathematical functions with formulas, graphs, and mathematical tables* (pp. 925–995). New York, NY, USA: National Bureau of Standards, U.S. Government Printing Office.