An inventory model of e-marketplace with promotional program

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Abstract

Purpose - The fast-growing e-commerce industry with its special characteristics brings new challenges to the optimization of the supply chain and inventory management. This study investigates inventory-related optimization of an e-marketplace platform when cashback promotion is used to attract more customers. This study proposes a new inventory model to maximize e-commerce profit by optimizing the cashback amount and delivery period.

Design/methodology/approach – The proposed model assumes that customer demand is a function of price and delivery time in which price is affected by the cashback amount. The e-commerce has a profit-sharing contract with an e-payment company that facilitates the payment. E-commerce also builds collaboration under a cost-sharing contract with a supplier to ensure product delivery. A mathematical model is developed and certain theories are investigated. A numerical example illustrates the model and further sensitivity analysis gives several insights.

Findings – A new inventory model for an e-commerce system has been introduced and shows the impact of cashback promotion in the business. This study shows that managers can optimize the cashback amount and delivery time to get maximum profit. In certain cases, the manager may set a high cashback amount (e.g. 100%) to attract customers to place more orders.

Originality/value – This study presents a new inventory model for today's fast-growing e-commerce business; therefore, the results contribute to the understanding of promotion program practices and inventory management and provide insights for e-commerce managerial decisions.

Keywords: e-commerce; e-marketplace, inventory; promotion; online payment

1. Introduction

The development of digital technologies has changed our lives and business systems. E-commerce has evolved over the years to enable business-to-business, business-to-customer, and customer-to-customer retail systems. E-commerce platform such as e-marketplace is a fast-growing business system in the world. On this virtual market, multiple sellers (manufacturer or retailer) and buyers trade via the same website or application. The e-commerce company may also have an official store to sell their products from several suppliers. This platform brings enormous transaction convenience that attracts both sellers and buyers and tends to be the best platform for online sales (Kawa & Wałęsiak, 2019; Merton, 2020). Various e-marketplaces established their positions with great impact such as Amazon, AliExpress, Zalando, Rakuten, Taobao, Shopee, etc. In many developing countries, local marketplaces such as Daraz, Pikkabo, Othoba, and Evaly in Bangladesh, are also growing rapidly. The marketplace provides a virtual space for retailers to place advertisements while they are physically or virtually stock the products. After the customer submits the order and pays through the payment system, the retailer sends the product to the customer. Legal agreements regulate the relationship of cooperation and transactions between the e-commerce company, retailer, and buyer.

The fast-growing e-commerce industry attracts many practitioners and researchers to study the behavior of all the e-commerce participants, the best practices of online business processes, also its trends and future challenges (Kawa & Wałęsiak, 2019; Wiengarten et al., 2020; etc). However, as far as the authors know, research on the supply chain and inventory management of e-commerce with its unique business systems is still limited. Patil & Divekar (2014) identified the challenges for an online retailer in maintaining its inventory due to demand variation, multi-channel system, stockless policy, etc. Jiang et al. (2015) developed an e-commerce retailer's decision model for pricing optimization. Alawneh & Zhang (2018) studied inventory management with a dual-channel warehouse for a retailer that works offline and online. None of this recent literature studied the inventory system under the effect of a cashback promotion initiated by the e-commerce company. Hasan et al. (2020a) have studied the inventory model of e-commerce retailers considering the effect of discounts but they focus on the discount during a preorder period. Our study will consider cashback promotion as another promotion program and incorporates cost-sharing between the e-commerce company and the supplier.

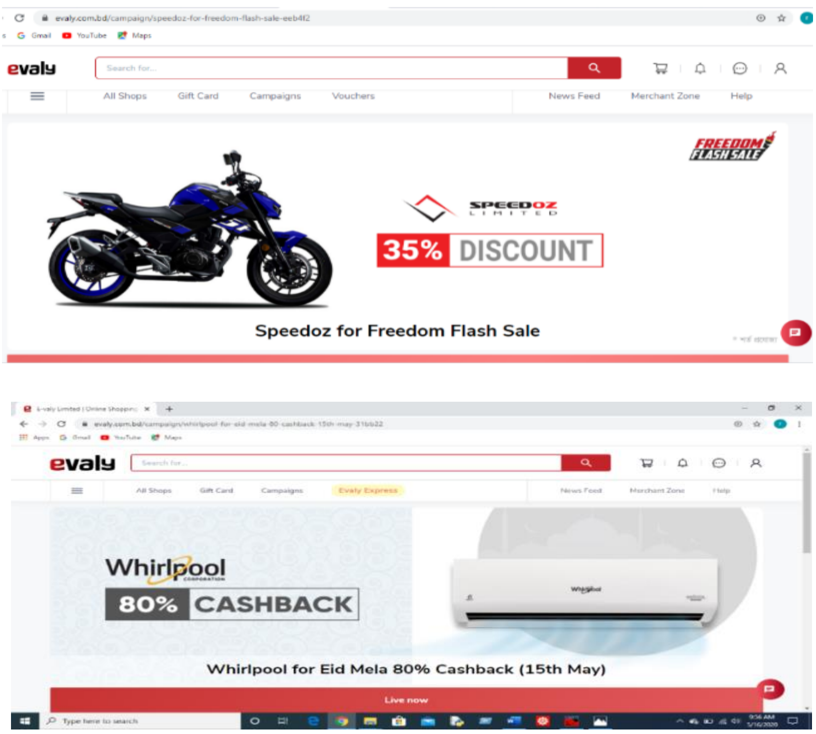

Figure 1. Discount and cashback promotion in Evaly

Promotion is a very common business strategy to attract customers. Various forms of promotion are also carried out on e-commerce (Jiang et al., 2015). Special programs such as flash sales, free delivery, lucky draw, bonus pack, discount, and cashback are among the common promotion activities. Yahya et al. (2019) and Hartini (2020) argued the effectiveness of those promotional tools. Promotion is frequently initiated by the retailer and the e-commerce company. For example, Figure 1 shows the examples of discount and cashback programs from an e-marketplace. The program will reduce the selling price, hence attract more customers. Such types of programs allow e-marketplaces to build a tremendous positive impact on market share. In addition, payment through an online system such as e-banking and e-wallet is a popular method in the e-commerce business. Many e-commerce companies relate this payment method to the promotion program. E-banking and e-wallet companies sign agreements with an e-commerce company and offering some benefits so that customers are attracted to use these banking for their payment. Considering the case in e-marketplace business, our study has two main questions:

a. How should e-marketplace design the promotion program to maximize the profit? How much are the optimal cashback and delivery periods?
b. How do the influencing factors of the system change the profit function?

The main contribution of our study is formulating the inventory decision model related to the cashback promotion initiated by the e-commerce company, accumulation of customer orders via the website or online application, customer online payments, and the inventory and delivery of products by the supplier. This study investigates the e-commerce company's profit maximization by optimizing the cashback amount and delivery period for a business system among an e-commerce company, a supplier, and the customers. It presents a new inventory model for today's fast-growing e-commerce business; therefore, the results contribute to the understanding of promotion program practices and inventory management and provide insights for e-commerce managerial decisions.

Section 2 represents some theoretical background by mentioning some of the main research that underlies the proposed approach. Section 3 describes the problem and the model assumptions. The study of the proposed model, including some current policies appear in section 4. A numerical example and its sensitivity analysis for the studied model have been computed in sections 5 and 6. Section 7 discussed the managerial implication of this work. Finally, the conclusion and future copes are directed in section 8.

2. Literature review

E-commerce with digital payment systems has a significant impact on the contemporary market around the world. The internet-based market impacts have been discussed by Bakos (1997, 1998) and Smith et al. (1999). It presents a good chance for buyers to choose the expected product at minimum cost within the shortest time. Brynjolfsson and Smith (2000) compared e-commerce and traditional retail in their work. The retailer and customers are more facilitated by the e-commerce marketing system. Apart from its advantages, the e-commerce business requires several adjustments to the logistics system to maintain performance and service levels (Yu et al., 2017). Information technology plays important roles in inventory, warehousing, and last-mile delivery management to maintain a good performance.

The impact of technology on retail business is discussed by Shankar et al. (2021). Hagberg et al. (2016) reviewed the development of digitalization in retail operations such as communications, transactions and distribution in the retail business. E-commerce includes some new features like mobile apps and social media connectivity (Menaka & Seethal, 2018). In this way, technologies are classified based on the influence on customer's buying manner (Roggeveen & Sethuraman, 2020). For example, online payments make it easy and encourage customer shopping (Garret et al., 2014). Upadhayaya (2012) investigated e-wallet as an easy-to-use, convenient, flexible, and secure payment system for e-commerce transactions. Uddin & Akhi (2014) explored the development of e-wallet for online transactions in Bangladesh and agreed with the advantages of e-wallet that support the growth of e-commerce business. The online payment through the e-wallet system for customer's preorder is addressed in our study. The advantage of the cashback promotion will be granted in form of an e-wallet balance.

Promotion for products has a positive impact on customer buying intention (Mashud et al., 2020). In an e-marketplace, the promotion is not only offered by the retailers but also can be initiated by the e-commerce company. For e-commerce businesses, Dye & Hsieh (2013) and He et al. (2019a) considered a preorder discount program and studied the impact on customer demand and inventory decision. Recently, a discount on online payment is incorporated by Hasan et al. (2020a) while Hasan et al. (2020b) proposed a discount for specific items to reduce the loss for defective inventories. Other forms of promotion are used by e-commerce such as flash sales, free delivery, lucky draw, bonus pack, discount, and cashback program (Jiang et al., 2015). The program lasts for certain periods and requires cooperation with the supplier. Our study takes a cashback program as the case in the proposed inventory model. The cashback program is initiated by the e-commerce company and affects the supplier. According to Christino et al. (2019), the cashback program has emerged as a key marketing tool in online shopping.

Logistics and supply chain management under the e-commerce system own special challenges. It involves the replenishment and stock level of products, fast and accurate preparation upon customer order, and fast delivery to customers (Yu et al., 2017). E-commerce may run the logistics by itself or outsource it to a logistics service provider (Yu et al., 2017; He et al., 2019b). Particularly, e-commerce faces many challenges in inventory management due to certain factors such as demand variation, complex delivery, and lost sales risks (Patil & Divekar, 2015). Dye & Hsieh (2013), He et al. (2019a), and Hasan et al. (2020a) studied the e-commerce inventory model with variable demand due to preorder and payment discounts. They incorporated a discount as a promotion program from an e-commerce retailer. Lei et al. (2018) also considered an e-commerce retailer inventory decision by optimizing the price and order fulfillment during a temporary selling period. Other research considered dual or omnichannel systems in which manufacturers or retailers sell via both online and offline means. Chiang & Monahan (2005) studied a two-echelon supply chain in which inventory is kept in both manufacturer and retailer's warehouses. Alawneh & Zhang (2018) developed an inventory model with dual-channel warehouses under stochastic demand and limited space. Zhou et al. (2018) incorporated a cost-sharing contract for the traditional retailer who sells their products under a dual-channel supply chain. Xie et al. (2018) and Saha et al. (2018) considered both revenue and cost-sharing contracts and optimize the price. Qi et al. (2020a, 2020b) used machine learning and deep learning to support inventory decisions for e-marketplace systems because very large digital data is available as multi sellers and multi customers are involved.

This study proposes a novel model for e-commerce inventory management. It addresses a new aspect in terms of promotion program initiated by the e-commerce company, in which previous research focus on promotion from the retailers (Dye & Hsieh, 2013; He et al., 2019a; and Hasan et al., 2020a; etc.). The model considers holding-cost sharing between the supplier and the e-commerce company by adapting Zhou et al. (2018), Xie et al. (2018), Saha et al. (2018), etc. The model also considers the order rate during the cashback promotion period and the delivery rate of the supplier in serving all the customers' online orders. The delivery time is optimized to maximize the profit. The level of cashback offered by the marketplace is also studied.

3. Problem description

E-commerce, such as e-marketplace, is a fast-growing business around the world. To attract more customers and increase customer awareness, the e-commerce company may offer a cashback program. Cashback programs have been implemented by many marketplaces (Evaly, Shopee, etc.). Our study presents the inventory system with this cashback promotional program under cooperation between the e-commerce company and the supplier.

When an e-commerce company offers a temporary cashback program, customer orders are accumulated until the end of the cashback period. Moreover, e-payment must be made via the customer's e-wallet account, and the cashback will top-up the balance. A supplier supports the e-commerce company by delivering the products directly to the customers. The supplier receives a payment based on the successful delivery, and the inventory holding cost is partially borne by the e-commerce company.

The cashback program can reduce a company's profits, even very large cashback offers (>75%) can result in losses. Therefore, this research focuses on: How the e-commerce company maintains its profit under the cashback program? This study derives the answer to the mentioned question by optimizing the cashback amount and delivery period. It proposes a novel mathematical model that extends the present e-commerce inventory model. Further, this study also investigates the influencing factors of the system through numerical examples and sensitivity analysis.

4. Model development

The proposed model is presented with the notations listed in Table 1.

Table 1. Notations list

Notation	Description
Parameters	
p	Selling price per unit
σ	Profit rate on the regular selling price

d	Average distance from supplier warehouse to the delivery point
t_1	Duration of the cashback offer
t_2	Time when the customers will get the cashback
b	Fixed time after delivery, when the e-commerce pays a regular purchase price to the supplier
k_b	Interest rate earned from e-payment provider (e.g. mobile banking company)
v	Service goodwill parameter related to the delivery time
C_h	Holding cost per unit per unit time
C_p	Purchase cost per unit
A	Setup cost for maintaining the website
D_1	Constant delivery rate
Decision variable	
T	Delivery period
μ	Cashback amount
Other variables	
Q	Maximum ordered products by customers within $0 \leq t \leq t_1$
$F(p, T)$	Order rate as a function of price and delivery time
$D(d)$	Delivery rate as a function of delivery distance
$TP(T, \mu)$	Total profit as a function of delivery time and the cashback amount

Further, the study works under the following assumptions:

a. An e-commerce company announces a cashback offer to the customer for buying a certain product in a certain period t_1. Online payment must be made before the end of the cashback period. The customer will get a cashback on his purchased product at time t_2 where $t_2 > t_1$, and the cashback amount will be added to his/her e-wallet balance corresponds to the marketplace platform. The cashback amount can only be used for future purchases from the same e-commerce platform with a regular price within a certain period.

b. The e-commerce company sets an agreement with a supplier or authorized agent of the product. After the cashback period, it shares the customer order information for supplier preparation. The supplier starts the delivery at t_2. The agent will get the payment for the products at time $T+b$ after successful delivery. E-commerce will also pay a part of the holding cost for these products under a cost-sharing contract.

c. The e-commerce company also set another agreement with an e-payment provider in which the provider will give a percentage of interest k_b of the amount paid through them (see Hasan et al., 2020a). The e-commerce company will also earn a percentage σ from the cashback amount when the customer uses his cashback to buy something at a regular price in the future. The order rate within the offered cashback period $0 \leq t \leq t_1$ is $F(p, T) = \alpha - \beta p(1-\mu)-vT$, where $\mu < 1$, $\alpha > 0$, $\beta > 0$, and $v > 0$. $L_1(t)$ is the number of ordered products by the customers within $0 \leq t \leq t_1$.

d. The delivery will be completed by the agent at T with $D(d) = \frac{D_1}{d}$ rate. The average delivery distance d is known and $L_2(t)$ is the inventory level during product delivery within $t_1 \leq t \leq T$ by the supplier. The inventory will be partially borne by the e-commerce company.

Figure 2 shows the ordering and delivery process of the e-commerce system with a cashback promotion.

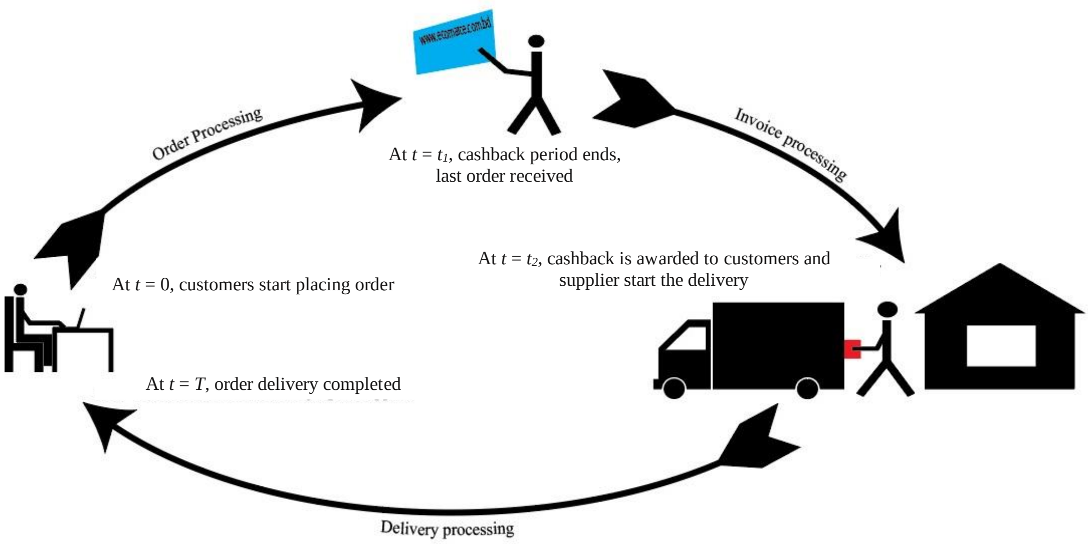

Figure 2. Graphical presentation of the ordering and delivery process

The e-commerce company offers a cashback program based on the selling price for a certain period $[0, t_1]$ and consequently, the customers do the preorder with a rate $F(p, T)$. The total amount of preordered products is Q at t_1. The e-commerce sends the total order data (invoice) to the supplier at $[t_1, t_2]$, i.e., invoice processing time and the supplier will deliver these products in $[t_2, T]$ with the delivery rate $D(d)$. So, all the preordered products Q are delivered by the supplier at time T. Figure 3 shows the graphical illustration of the inventory vs. time. This inventory is held by the supplier and will be partially borne by the e-commerce company.

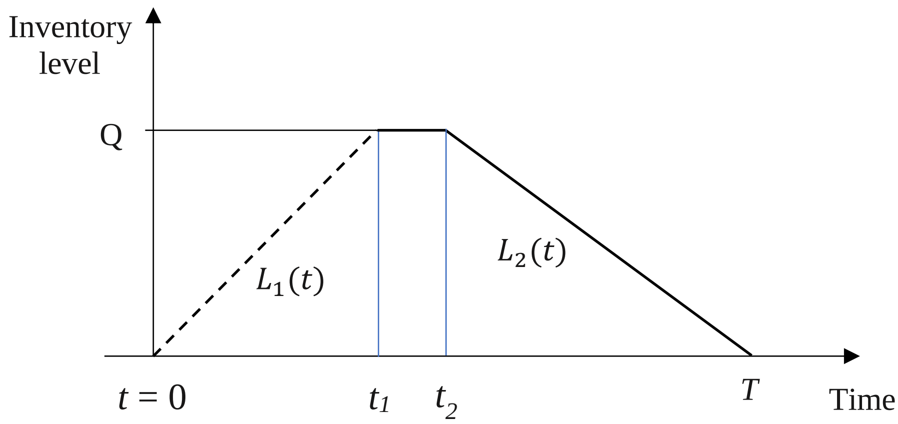

Figure 3. Inventory level vs time

The proposed inventory model for the e-commerce system can be represented by the following system of the ordinary differential equation for the preorder rate $F(p, T) = \alpha - \beta p(1\text{-}\mu)\text{-}vT$ and delivery rate $D(d) = \frac{D_1}{d}$.

$$\frac{dL}{dt} = \begin{cases} F(p,T) & ;0 \le t \le t_1 \quad with\ L_1(t) = 0\ at\ t = 0\ \ and\ L_1(t) = Q\ at\ t = t_1 \\ -D(d) & ;\ t_2 \le t \le T\ \ with\ L_2(t) = Q\ at\ t = t_2\ and\ L_2(t) = 0\ at\ t = T \end{cases} \quad (1)$$

$$L(t) = \begin{cases} L_1, & ;0 \le t \le t_1 \\ L_2 & ;t_2 \le t \le T \end{cases} \quad (2)$$

Solving equation (1), we get the preordered inventory level

$$L_1(t) = Ft = [\alpha - \beta p(1 - \mu) - vT]\mathrm{t} \quad (3)$$

Also, the inventory level during product delivery is

$$L_2(\mathrm{t}) = \frac{D_1}{d}(T - t) \quad (4)$$

The maximum preordered products is

$$Q = Ft_1 \quad (5)$$

and the delivered products will be the same with preordered products

$$Q = \frac{D_1}{d}(T - t_2) \tag{6}$$

So,

$$Ft_1 = \frac{D_1}{d}(T - t_2) \tag{7}$$

4.1. Inventory revenues and costs

The following revenues and costs for the e-commerce company (retail system) have been considered for this proposed work.

a. Sales Revenue

Revenue from selling the products Q at price p is

$$SR_1 = Qp \tag{8}$$

b. Revenue from e-payment provider

The e-payment provider (e.g., mobile banking company) will give k_b percentage on the cash flow through their banking system for the period from $[t_1, T + b]$. The preorder cash flow amount in $[0, t_2]$ is $Qp \int_{t_1}^{t_2} dt$ and if customers will get the cashback at the time t_2 then the amount of cash flow in $[t_2, T + b]$ is $Qp(1 - \mu) \int_{t_2}^{T+b} dt$.

So, the total amount of interest earned by e-commerce company is

$$SR_2 = k_b Q \left[p \int_{t_1}^{t_2} dt + p(1 - \mu) \int_{t_2}^{T+b} dt \right]$$

$$SR_2 = k_b Q[p(t_2 - t_1) + p(1 - \mu)(T + b - t_2)] \tag{9}$$

c. Revenue from cashback amount

According to the terms and conditions, customers have to spend their cashback amount to buy other products in the e-commerce from their e-wallet account at a regular price. If the minimum percentage of profit is σ from selling the product at regular price, then the profit gained from the total cashback amount *CB* is

$$SR_3 = CB.\sigma = \mu Qp\sigma \tag{10}$$

d. Holding cost paid by e-commerce company to the supplier

According to an agreement with the supplier, the e-commerce will pay half of the holding cost for products in the agent's warehouse in the period $[t_1, T]$; i.e., from send the invoice to the agent for confirming the order at the time t_1 by the retailer to deliver the products to respective customers at T by the agent.

Then the total cost for holding the products paid by the retailer to the supplier is

$$HC = \frac{C_h}{n}\int_{t_2}^{T} L_2(t)\, dt = \frac{C_h D_1}{2nd}(T - t_2)^2 \qquad n > 1 \tag{11}$$

e. Total cashback amount

When customers order for products in $[0, t_1]$, then he/she will get μ percentage of his/her purchase cost and this cashback amount will be confirmed by the e-commerce company. Then the total cashback amount that the e-commerce has to pay is

$$CB = \mu Q p \tag{12}$$

f. Purchase cost

The purchased products from the supplier for fulfilling customers' order is C_p per unit. Hence, the total purchase cost is

$$PC = QC_p \tag{13}$$

g. Setup cost

To maintain the web platform, the e-commerce company has to invest some money, which is termed as setup cost (*SC*) for this arrangement.

$$SC = A \tag{14}$$

4.2. Total profit

Total profit is the resultant of total revenue – total cost. Then the total profit for a cycle is

$$TP(T,\mu) = SR_1 + SR_2 + SR_3 - SC - HC - PC - CB \tag{15}$$

$$TP(T,\mu) = Qp + k_b Q[p(t_2 - t_1) + p(1-\mu)(T + b - t_2)] + \mu Q p \sigma - A$$
$$- QC_p - \mu Q p - \frac{C_h D_1}{2nd}(T - t_2)^2$$

$$TP(T,\mu) = Qp(1 + \mu\sigma - \mu) + k_b Q[p(t_2 - t_1) + p(1-\mu)(T + b - t_2)] - A$$
$$- QC_p - \frac{C_h D_1}{2nd}(T - t_2)^2 \tag{16}$$

The problem to find the optimal decisions can be summarized as follow:

Maximize $TP(T, \mu)$

Subject to $T > t_2 > t_1, \mu < 1$ and $Ft_1 = \frac{D_1}{d}(T - t_2)$

Lemma 1: Total profit function $TP(T, \mu)$ has a maximum at (T^*, μ^*) under the condition $\varphi_2(T^*, \mu^*) > \varphi_1(T^*, \mu^*)$.

where,

$$T^* = \frac{\theta_1 - \theta_2 {\mu^*}^2 + \mu^* \theta_3}{\theta_4 - \theta_5 \mu^*}\ ;\ \mu^* = \frac{\omega_2 {T^*}^2 + T^* \omega_3 - \omega_1}{\omega_5 * T^* + \omega_4}$$

Proof: See Appendix A.

Then, the profit function obtained from the proposed model will give the maximum benefit to the e-commerce if it makes policy in such a way that the above lemma will be satisfied.

4.3. Cashback

If the e-commerce earns more than the selling price per item from other revenue sources, it able to sell the products up to 100% cashback. For example, consider the regular selling price of a mobile phone is \$100. If anyone can earn more than \$100 to buy such a product by his/her progressive policies, then he/she must be able to sell this phone with a 100% discount or cashback offer.

To stimulate the customer's demand, cashback on selling price is an effective policy. The amount of cashback designated by the e-commerce depends on the revenue percentage from the online banking company (e-payment provider) k_b, start time (t_2) for the cashback to the customer, selling price (p), payment time $(T + b)$ for the product to supplier/agent, and average revenue percentage (σ) earned from using the cashback amount to buy products with a regular selling price for unit products.

Let the total earned revenue besides the regular selling price be W per item during the period, which is the sum of (i) Revenue for the unit product in $[t_2, T + b] = \frac{SR_2}{Q} = K_b p(t_2 - t_1) + K_b p(1 - \mu)(T + b - t_2)$ and (ii) Revenue earned from cashback amount $= \frac{SR_3}{Q} = \sigma p \mu$

Therefore,

$$W = K_b p(t_2 - t_1) + K_b p(1 - \mu)(T + b - t_2) + \sigma p \mu$$

Lemma 2: If $(t_2 - t_1)K_b + \sigma > 1$, then it is possible to offer up to 100% cashback on selling price p for the e-commerce company.

Proof: See Appendix B.

It is proven that if there are companies that meet Lemma 1, then they can make a 100% cashback offer on the selling price, avoiding losses. It will be more profitable and work to promote the company rapidly. Because when customers get such an offer, they will be more interested in buying. So, this type of policy helps expand the business to various market areas and stimulate customer demand.

5. Numerical Illustration

To present the numerical illustration of the proposed model, an example has been developed. From the following example, one can analyze the numerical result of the model.

Example: Consider, an e-commerce company invests \$100 for setting up a cashback program, with \$2 purchase cost per unit and \$0.5 for holding cost per unit. The value of k_b is 0.1 according to the agreement with the e-payment company. Cashback period t_1 is 4 weeks. Within this time, customers place the order. The time when the customer receives their cashback t_2 is 5 weeks. The invoice will be confirmed to the agent/supplier to process the ordered product for delivery on the 5th week. b is 2 weeks, i.e., the e-commerce company will pay for the purchase amount to the agent/supplier 2 weeks after completing the delivery. The selling price p is \$4 per unit. The constant delivery rate D_1 is 150. Customer distance from the supplier d is 3 miles. Finally, the values of $\sigma = 0.15$, $v = 6$, $\propto = 400$, and $n = 2$.

Then, our objective is to find the optimal value of T , μ, and the profit TP.

The computational work has been performed on a PC with an Intel Core i3 computer of 2.40 GHz Processor. The optimal solution was obtained with the help of the GRG method. The optimal values of T and μ, along with maximum profit, are displayed in Table 1.

Table 1: Solutions of example

Optimal results
T =14.52495
$\boldsymbol{\mu}$ =0.2546596
$\boldsymbol{TP}$ =1699.997

The concavity of $TP(T, \mu)$ concerning T and μ is examined by MATLAB software and is presented in Figure 4.

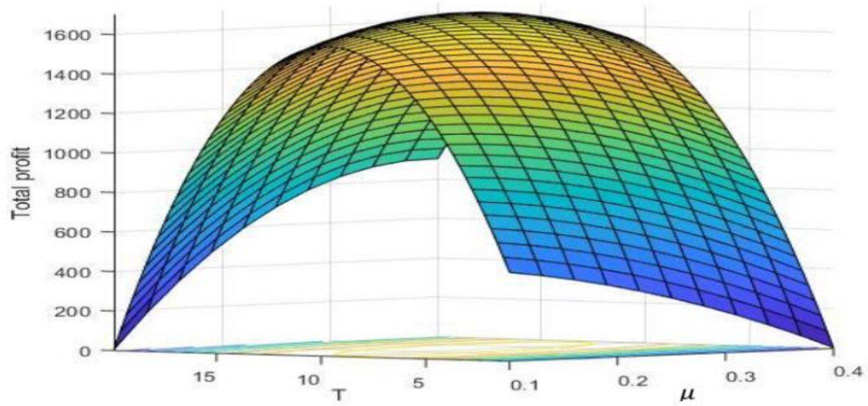

Figure 4. Concavity of $TP(T, \mu)$

6. Sensitivity Analysis

This study shows the validity range of the model for the variation of different parameters. The effect of parameters on the optimal decisions showed in Table 2 by changing the parameters' values from +20% to -20%. The outcomes are accomplished by transforming each setting in turn and keeping different parameters at their unique merits.

Table 2. Sensitivity investigation regarding various parameters

Parameter	Variations of parameters (-20% to 20%)	Variations in			
		TP^*	$\boldsymbol{\mu}^*$	T^*	Q^*
A	80	1719.98	0.25	14.52	476.25
	90	1710.00	0.25	14.52	476.25
	110	1690.00	0.25	14.52	476.25
	120	1680.00	0.25	14.52	476.25

$\propto$	320	756.92	0.37	11.84	341.78
	360	1172.89	0.31	13.17	408.56
	440	2350.88	0.20	15.89	544.64
	480	3138.23	0.14	17.27	613.58
C_p	1.6	1893.45	0.28	14.82	490.94
	1.8	1795.99	0.27	14.67	483.66
	2.2	1605.50	0.24	14.37	468.69
	2.4	1512.53	0.23	14.22	460.98
k_b	0.08	1345.24	0.21	13.92	446.00
	0.09	1519.81	0.23	14.24	462.09
	0.11	1884.92	0.27	14.78	488.76
	0.12	2073.85	0.29	15.00	499.85
C_h	0.4	1817.74	0.28	14.89	494.33
	0.45	1757.77	0.27	14.70	485.23
	0.55	1644.34	0.24	14.35	467.40
	0.60	1590.74	0.23	14.17	458.68
p	3.2	1605.50	0.05	14.37	468.69
	3.6	1652.56	0.17	14.45	472.49
	4.4	1747.81	0.33	14.60	479.97
	4.8	1795.99	0.39	14.67	483.66
β	52	2421.68	0.11	15.03	501.68
	58.5	2018.31	0.19	14.78	488.94
	71.5	1443.45	0.31	14.27	463.62
	78	1233.14	0.35	14.02	451.07

The following observations can be drawn from the above table:

a. Total profit TP increases with the increasing values of k_b, $\propto$, and p but it decreases with the increasing values of A, C_h, C_p, and β.

b. Optimal cashback μ increases when the parameters k_b, p and β increase and it decreases with the increasing values of $\propto$, C_h, β and C_p; meanwhile, it remains the same when changing the value of A.

c. Delivery period T is going along with the increase of k_b, $\propto$ and p and decreases for the increasing values of C_h and C_p. Again, A does not affect the delivery period.

d. The amount of ordered or delivered products Q is proportional to the change of k_b, p, and $\propto$. Also, the result is inversely proportional to the change of C_h, β and C_p but A has no impact on Q.

7. Discussion

The following observations can be obtained after going through this proposed work.

a. Lemma 1 confirms that the total profit obtained from the studied model has the maximum value for (T^*, μ^*). Therefore, managers can decide how much time they will take to deliver the products and what is the optimal cashback with maximum profit.

b. Proposition 1 shows an important result for the cashback offer in terms of how and when the managers can go through a higher cashback offer, even up to 100% on selling price, with maintaining a profit.

c. There are common factors for all the decision variables in which they are changing proportionally with the change of k_b and p. So, maximize the profit, cashback offer, order quantity, and the delivery time can be achieved for the highest possible of k_b and p.

d. Maximum delay to delivery is acceptable to customers for the highest value of k_b. On the other hand, all costs have a reverse impact on the optimal decision. For that reason, costs should be maintained at the lowest possible values to get more benefits from the business.

e. Finally, this work developed a model to interpret the collaborative retail system of e-commerce. This novel model has been studied by considering the delivery time issue, and consequently, it obtained a mathematical relation that can measure the possible maximum cashback offer. Hence, managers can forecast about his necessary decisions by studying this model.

8. Conclusion

The digitalization of the retail business enhances the speed of marketing processes. In this manner, e-commerce platforms are getting more attention from customers than the regular marketing process. This study proposes a mathematical model to deal with the challenges in the e-commerce inventory management policies considering a cashback promotion program. The cashback policy is very much efficient in stimulating the customer's demand and getting more profit. This study shows that managers can optimize the cashback amount and delivery time to get maximum profit. In certain cases, the manager may set a high cashback amount (e.g. 100%) to attract customers to place more orders. Further, to maintain maximum profit, the system also suggests collaboration with e-payment providers and suppliers through profit and cost-sharing.

This study considers several features associated with retail business in the emerging e-commerce market. The selling, ordering, and payment are performed online with a special cashback promotion. Future research may extend the model by allowing a certain situation in which the customer may cancel the confirmed order and the penalty for delivery failure. Also, the effect of different payment methods may be incorporated.

Appendix A.

Proof of Lemma 1:

We have the total profit function

$TP(T,\mu) =$

$$Qp(1+\mu\sigma-\mu)+k_bQ[p(t_2-t_1)+p(1-\mu)(T+b-t_2)]-A-QC_p-\frac{C_hD_1}{2nd}(T-t_2)^2 \qquad \text{(A.1)}$$

To find the optimality of $TP(T,\mu)$, we have to prove the following

(i) $\frac{\partial^2 TP(T,\mu)}{\partial T^2}, \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} < 0$ for any value of $(T^*,\mu^*) \in (T,\mu)$

(ii) $\frac{\partial^2 TP(T,\mu)}{\partial T^2} * \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} - \left[\frac{\partial^2 TP(T,\mu)}{\partial T\partial \mu}\right]^2 > 0$ at (T^*,μ^*)

Put $Q = Ft_1 = [\alpha - \beta p(1-\mu) - vT]t_1$in (A.1)

Then

$$TP(T,\mu) = \begin{bmatrix} [\alpha - \beta p(1-\mu) - vT]t_1 * p(1+\mu\sigma-\mu) \\ +k_b[\alpha - \beta p(1-\mu) - vT]t_1[p(t_2-t_1)+p(1-\mu)(T+b-t_2)] \\ -A-[\alpha - \beta p(1-\mu) - vT]t_1C_p-\frac{C_hD_1}{2nd}(T-t_2)^2 \end{bmatrix} \qquad \text{(A.2)}$$

Differentiating (A.2) with respect to T and μ

$$\frac{\partial TP(T,\mu)}{\partial T}=\begin{bmatrix}\frac{C_h D_1(-T+t_2)}{dn}+C_p t_1 v-pt_1(k_b p\alpha(-1+\mu)^2\\+v(1-\mu+b(k_b-\mathrm{kb}\mu)+k_b(2T-t_1-2T\mu+t_2\mu)+\mu\sigma))\end{bmatrix} \quad (A.3)$$

$$\frac{\partial TP(T,\mu)}{\partial \mu}=\begin{bmatrix}pt_1(k_b T^2 v+T(-2\mathrm{kb}p\alpha(-1+\mu)+v(1+bk_b-k_b t_2-\sigma))\\-\alpha(C_p+p(-2+2bk_b(-1+\mu)+2\mu+k_b(t_1+t_2-2t_2\mu)+\sigma-2\mu\sigma)))\end{bmatrix} \quad (A.4)$$

The necessary condition of maxima

$\frac{\partial TP(T,\mu)}{\partial T}=0$ and $\frac{\partial TP(T,\mu)}{\partial \mu}=0$

i.e.,

$$\begin{bmatrix}\frac{C_h D_1(-T+t_2)}{dn}+C_p t_1 v-pt_1(k_b p\alpha(-1+\mu)^2\\+v(1-\mu+b(k_b-\mathrm{kb}\mu)+k_b(2T-t_1-2T\mu+t_2\mu)+\mu\sigma))\end{bmatrix}=0 \quad (A.5)$$

$$\begin{bmatrix}pt_1(k_b T^2 v+T(-2\mathrm{kb}p\alpha(-1+\mu)+v(1+bk_b-k_b t_2-\sigma))\\-\alpha(C_p+p(-2+2bk_b(-1+\mu)+2\mu+k_b(t_1+t_2-2t_2\mu)+\sigma-2\mu\sigma)))\end{bmatrix}=0 \quad (A.6)$$

Solving equations (A.5) and (A.6) we gain

$T^*=\frac{\theta_1-\theta_2{\mu^*}^2+\mu^*\theta_3}{\theta_4-\theta_5\mu^*}$

$\mu^*=\frac{\omega_2{T^*}^2+T^*\omega_3-\omega_1}{\omega_5 * T^*+\omega_4}$

where,

$C_h D_1 t_2+dnt_1\left(C_p v-p(v+bk_b v-k_b t_1 v+k_b p\alpha)\right)=\theta_1$; $dk_b np^2 t_1\alpha=\theta_2$;

$dnpt1\left(2k_b p\alpha+v(1+bk_b-k_b t_2-\sigma)\right)=\theta_3$; $C_h D_1+2dk_b npt_1 v=\theta_4$;

$2dk_b npt_1 v=\theta_5$

$\alpha\left(C_p+p(-2-2bk_b+k_b(t_1+t_2)+\sigma)\right)=\omega_1$; $k_b v=\omega_2$;

$\left(2k_b p\alpha+v(1+bk_b-k_b t_2-\sigma)\right)=\omega_3$; $2p\alpha(1+bk_b-k_b t_2-\sigma)=\omega_4$; $2k_b p\alpha=\omega_5$

Differentiating (A.3) with regard to T

$$\frac{\partial^2 TP(T,\mu)}{\partial T^2}=-\frac{C_h D_1}{dn}-2k_b pt_1 v(1-\mu)<0 \quad (A.7)$$

Differentiating (A.4) with regard to μ

$$\frac{\partial^2 TP(T,\mu)}{\partial \mu^2}=2p^2 t_1\alpha(-1-k_b(b+T-t_2)+\sigma)$$

Since $\sigma<1$ and $k_b(b+T-t_2)>0$ then

$$\frac{\partial^2 TP(T,\mu)}{\partial \mu^2} = 2p^2 t_1 \alpha(-1 - k_b(b + T - t_2) + \sigma) < 0 \tag{A.8}$$

Thus $\frac{\partial^2 TP(T,\mu)}{\partial T^2}, \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} < 0$ for any value of $(T^*, \mu^*) \in (T, \mu)$

Hence condition (i) is proved.

Differentiating (A.3) with regard to μ and (A.4) with regard to T

$$\frac{\partial^2 TP(T,\mu)}{\partial T \partial \mu} = \frac{\partial^2 TP(T,\mu)}{\partial \mu \partial T} = pt_1(-2k_b p\alpha(-1 + \mu) + v(1 + bk_b + 2k_b T - k_b t_2 - \sigma)) \tag{A.9}$$

Now,

$$\frac{\partial^2 TP(T,\mu)}{\partial T^2} * \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} - \left[\frac{\partial^2 TP(T,\mu)}{\partial T \partial \mu}\right]^2 =$$

$$p^2 t_1 \begin{bmatrix} 2\alpha(-\frac{C_h D_1}{dn} + 2k_b p t_1 v(-1 + \mu))(-1 - k_b(b + T - t_2) + \sigma)) \\ -t_1(-2k_b p\alpha(-1 + \mu) + v(1 + bk_b + 2k_b T - k_b t_2 - \sigma))^2 \end{bmatrix} \tag{A.10}$$

At (T^*, μ^*) equation (A.10) becomes

$$\left[\frac{\partial^2 TP(T,\mu)}{\partial T^2} * \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} - \left[\frac{\partial^2 TP(T,\mu)}{\partial T \partial \mu}\right]^2\right]_{at\ (T^*,\mu^*)} =$$

$$p^2 t_1 \begin{bmatrix} 2\alpha(-\frac{C_h D_1}{dn} + 2k_b p t_1 v(-1 + \mu^*))(-1 - k_b(b + T^* - t_2) + \sigma)) \\ -t_1(-2k_b p\alpha(-1 + \mu^*) + v(1 + bk_b + 2k_b T^* - k_b t_2 - \sigma))^2 \end{bmatrix}$$

Let,

$$t_1(-2k_b p\alpha(-1 + \mu^*) + v(1 + bk_b + 2k_b T^* - k_b t_2 - \sigma))^2 = \varphi_1(T^*, \mu^*)$$

$$2\alpha(-\frac{C_h D_1}{dn} + 2k_b p t_1 v(-1 + \mu^*))(-1 - k_b(b + T^* - t_2) + \sigma)) = \varphi_2(T^*, \mu^*)$$

Now

$$\left[\frac{\partial^2 TP(T,\mu)}{\partial T^2} * \frac{\partial^2 TP(T,\mu)}{\partial \mu^2} - \left[\frac{\partial^2 TP(T,\mu)}{\partial T \partial \mu}\right]^2\right]_{at\ (T^*,\mu^*)} = \varphi_2(T^*, \mu^*) - \varphi_1(T^*, \mu^*) \tag{A.11}$$

According to condition (ii) $\varphi_2(T^*, \mu^*) - \varphi_1(T^*, \mu^*)$ should be positive, i.e.,

$$\varphi_2(T^*, \mu^*) - \varphi_1(T^*, \mu^*) > 0$$

Therefore

$$\varphi_2(T^*, \mu^*) > \varphi_1(T^*, \mu^*) \tag{A.12}$$

Hence the condition (ii) will be satisfied if the inequality (A.12) is true.

Appendix B.

Proof of Lemma 2:

A cashback up to 100% is only possible when W's value is greater than the selling price p, i.e., the online platform earns revenue from their designated policies is higher than the regular selling price of the unit product.

We have $W = K_b p(t_2 - t_1) + K_b p(1 - \mu)(T + b - t_2) + \sigma p \mu$

Now, 100% cashback is possible if the following mathematical condition satisfies $W > p$.

$\Rightarrow K_b p(t_2 - t_1) + K_b p(1 - \mu)(T + b - t_2) + \mu\sigma p > p$

$\Rightarrow K_b(t_2 - t_1) + K_b(1 - \mu)(T + b - t_2) + \mu\sigma > 1$

$\Rightarrow K_b(t_2 - t_1) + K_b(1 - \mu)(T + b - t_2) > 1 - \mu\sigma$

$\Rightarrow (t_2 - t_1) + (1 - \mu)(T + b - t_2) > \frac{1-\mu\sigma}{K_b}$

$$\Rightarrow (t_2 - t_1) + (1 - \mu)(T + b - t_2) > \frac{1-\mu\sigma}{K_b} \tag{B.1}$$

In that case, $\mu = 1$, then equation (B.1) gives:

$(t_2 - t_1)K_b + \sigma > 1$

Hence the Lemma 2 is proved.

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