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A Joint Demand and Supply Management Approach to Large Scale Urban Evacuation Planning: Evacuate or Shelter-in-Place, Staging and Dynamic Resource Allocation

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Abstract

Urban evacuation management is challenging to implement as it requires planning and coordination over a large geographical area. To address these challenges and to bolster evacuation planning and management, joint supply and demand management strategies should be considered. In this study, we explore and jointly optimize evacuate or shelter-in-place (SIP), dynamic resource allocation (DRA), and staging decisions for an efficient evacuation plan that minimizes total risk exposure of the population threatened by a sudden onset disaster. We introduce a Cell Transmission Model-based mathematical formulation and propose an exact solution methodology based on Benders decomposition. We further enhance the effectiveness of the algorithm by solving the Benders subproblem using a network flow based formulation on a time-expanded-network, and generating valid inequalities based on DRA decisions and for time-feasible solutions and develop an effective branch-and-cut algorithm to solve the master problem. We conduct extensive numerical experiments using realistic instances to test the effectiveness of the algorithm and to derive managerial insights. We find that considering evacuate or SIP, staging, and DRA decisions jointly contributes significantly to the efficiency of the evacuation operations. A zone-based approach where some zones are ordered to evacuate while others shelter-in-place is superior to other approaches where an evacuate or SIP decision is given for all population at risk.

Keywords: Humanitarian Logistics, Large Scale Urban Evacuation, Evacuate or Shelter-in-Place, Dynamic Resource Allocation, Benders Decomposition

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1. Introduction

The September 11, 2001 terrorist attacks in the United States (U.S.), were a no-notice event in a metropolitan setting that claimed thousands of lives. The 2004 Indian Ocean tsunami, the 2010 Haiti earthquake, the triple disasters that hit the Tohoku region of Japan in 2011, and the 2017 hurricane season in the U.S. were among the costliest and deadliest disasters in the history of mankind with tremendous operational challenges on governments and disaster management agencies, causing a massive level of

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destruction, and leaving millions of people homeless (Bayram, 2016, Halverson, 2018). Natural and man-made disasters are a great threat to humanity. Due to urbanization, climate change and population growth, the number of such disasters and their impact continue to increase (EM-DAT, 2020a,b), which illustrates the need to better prevent, prepare for and respond to disasters (EC, 2019).

To protect people from the impact of a disaster against the possibility of significant injury or death, evacuation of people from a threatened area to safer areas is the mostly used strategy. European Council (EC) states that an effective evacuation management is of utmost importance to reduce the risks citizens can face during such emergencies (EC, 2017). An efficient evacuation traffic management is reported as a critical capability that must be attained in the U.S. National Response Framework (USDHS, 2013). Inefficient management of evacuation operations may result in further losses (Thompson et al., 2017). Millions of people were evacuated due to hurricanes Floyd (1999), Katrina and Rita (2005), and Irma (2017) creating largest traffic jams in the U.S. history. Over eight million people across the U.S. were affected by evacuation orders in 2017 due to flooding, hurricanes and wildfires (FEMA, 2019). The U.S. Department of Homeland Security (DHS), Federal Emergency Management Agency (FEMA) and the U.S. Department of Transportation (DOT), Federal Highway Administration (FHWA) reports state that evacuations are not rare. The annual number of disasters that require an evacuation is about 45-75 and evacuations of 1,000 or more people occur every two to three weeks (FHWA, 2007).

Large scale urban evacuation management is challenging to implement as it requires planning and coordination over a large geographical area for an extended time span of hours and even days. The primary objective of evacuations is to save lives by moving evacuees out of threatened zone as safely and quickly as possible (Lindell et al., 2018). To address these challenges and to bolster evacuation planning and management, joint supply and demand management strategies should be considered. The U.S. National Response Framework (DHS, 2019) provides foundational emergency management doctrine and aligns key roles and responsibilities of responding agencies through emergency support functions (ESF). *ESF #1 - Transportation* is related to the management of evacuation traffic and *ESF #13 - Public Safety and Security* regulates the use of law enforcement personnel at critical intersections to control traffic (Matherly, 2013).

Evacuation demand is defined as the number of vehicles that are required to evacuate from the threatened area for each zone. Evacuation supply can be described as the ability of the road network structure to serve the evacuation demand and is related to the capacity of the road network to carry the evacuation flow. Effective supply and demand management strategies are required to manage the evacuation traffic as road network is not designed against such an unusual, sudden and wide spread surge in traffic demand. Supply management is about designing new road segments or enhancing the capacity of existing road network structure and utilizing it in the most efficient way by employing contraflow strategies, modifying/designing selected intersections by applying turn restrictions, lane/ramp closures, adding lane/ramp capacity and deploying law enforcement personnel and/or electronic devices to critical intersections to prevent/decrease the traffic congestion. Evacuate or shelter-in-place (SIP), staging, and shelter/route assignment decisions are used to organize the evacuation demand. To reduce traffic congestion by promoting a phased evacuation, evacuating only those at risk in the most vulnerable areas, a zone-based approach is preferred to manage evacuation demand (Wilmot and Meduri, 2005, FEMA, 2019). That way, jurisdictions can prioritize the evacuation orders and limit the need to evacuate a large population in different zones, which is not under direct threat of a hazard.

While intuitively one may think that evacuation of everyone from a threatened area is the best choice, this may not be a feasible or safe option. The evacuation of Houston metropolitan area against hurricane Rita in 2005 resulted in people getting stuck in traffic with a 100-mile long congestion on the highway and caused fatalities not by the hurricane itself (O'Driscoll et al., 2005). On the other hand, 34 patients of a nursing home were drowned since the nursing home management decided to shelter-in-place (Dosa et al., 2007). FEMA defines the goal of an evacuation as "*to move as few people as needed the shortest distance to safety*" and SIP as "*the use of a structure to temporarily separate individuals from a hazard or threat*" (FEMA, 2019) and advises SIP to be the first considered option when feasible to reduce costs, inefficient use of resources and the adverse affects of unnecessary mass evacuation. Depending on the type and impact of a disaster, the lead time from it, the protection level, the distance from the threat, the duration of exposure to the threat, and the level of congestion on the road network due to evacuation demand, the decision on whether to evacuate or SIP a zone may change (Lindell et al., 2018).

In case of a toxic gas release or in case of tornadoes, if there is not enough pre-evacuation time, SIP could be the preferred option (Sorensen et al., 2004). The only option after incidents such as terrorist attacks that involve chemical or biological hazards may be SIP due to further contamination risks of an evacuation (Zimmerman et al., 2007a). When a SIP order is recommended to a given zone in the threatened area, population in this zone will be mostly seeking shelter in their homes, apartments or workplaces. Among the weather related disasters, tornadoes have inflicted the third largest number of fatalities in U.S. over the last decade (NWS, 2019). Common response policy against tornadoes is to SIP. However, in the past ten years on average, around 38% of the fatalities caused by a tornado was inflicted on the people who sheltered in their permanent houses and around 30% of the fatalities was inflicted on the people who sheltered in their mobile homes/trailers (NWS, 2019). With the advance in weather forecasting capabilities and early warning systems, evacuation of threatened areas against tornadoes may also become an option (Simmons and Sutter, 2012). Depending on the structural properties of such buildings, the SIP decision may provide protection against wind, flood, and exposure to hazardous materials. In case of hurricanes or if there is enough time to clear a zone in case of toxic gas releases, however, evacuation may be the recommended option. The options against wildfires is to evacuate early, since late evacuation is very dangerous or to defend and SIP depending on expected fire intensity, the protection level provided by in-home/refuge shelters and other factors (Cova et al., 2009, McCaffrey et al., 2015, Kuligowski, 2021). Similarly, SIP or evacuate decisions during flash floods will depend on the depth of the flood, whether evacuation is possible to higher ground and the structure of the building to SIP (Haynes et al., 2009).

Evacuate or SIP decisions are given taking into consideration a risk or hazard caused by the disaster. A policy that minimizes the clearance time or total evacuation time in the network may not be the safest alternative to implement. For that reason it is of utmost importance to define the risk and employ it in planning considerations to be able to make these critical decisions.

Deployment of emergency management or law enforcement personnel and/or electronic devices, barriers, boards to critical intersections on the road network dynamically over time to enhance evacuation efficiency is defined as one of the supply management strategies (Pretorius et al., 2006, Zimmerman et al., 2007a,b, DHS, 2019). These traffic control and management resources can be reallocated to different intersections depending on the congestion levels in the road network (He and Peeta, 2014). Deployment of a resource at an intersection results in an enhancement in the capacities of the upstream road segments of the intersection and hence results in faster evacuation of the population at risk.

This study aims to contribute to the United Nations (U.N.) climate action and sustainable development goals (UNDESA, 2015) in building resilience and reducing vulnerability to climate-related disasters (e.g., heatwaves, floods, extreme weather events, etc.), and to the Sendai framework for disaster risk reduction (UNISDR, 2015) in that it offers a solution methodology for evacuation of urban areas in order to further prevent risk and to ensure safety of populations affected by such disasters. Our goal is to develop models and solution methodologies that jointly consider supply and demand management strategies for an efficient evacuation planning/management. We develop a Cell Transmission Model (CTM, Daganzo (1994, 1995))-based mixed integer programming (MIP) formulation that accounts for evacuate or SIP, staging and dynamic resource allocation (DRA) decisions simultaneously. Then, we propose an exact solution methodology based on Benders decomposition (Benders, 1962) to solve the problem effectively for realistic size instances. We further solve the Benders subproblem by using a network flow based formulation on a time-expanded network (TEN) and explore other strategies such as generating valid inequalities for the master problem to enhance the performance of proposed algorithm. We also derive valid inequalities that generally guarantee time-feasible solutions to speed up the solution procedure and develop an efficient branch-and-cut algorithm to solve the master problem. We conduct experiments to demonstrate the effectiveness of the proposed algorithm and derive managerial insights. In particular, we find that a zone-based approach where some zones are ordered to evacuate while others shelter-in-place is superior to other approaches where an evacuate or SIP decision is given for all population at risk. Considering joint evacuate or SIP, staging, and DRA demand and supply management strategies significantly contributes to the efficiency of the evacuation. Jointly considering these decisions generates evacuation plans that require less resources. When combined with demand management strategies evacuate or SIP and staging, DRA helps decrease total evacuation risk to a certain extent but it has more contribution for higher demand levels. It is of utmost importance to have early warning systems which help evacuation management authorities give timely evacuation orders to evacuate the population at risk to safety and if there is enough lead time, evacuation should be the preferred option for a minimal risk exposure.

The rest of the paper is organized as follows: In Section 2, we cover the literature on evacuation management and present our contributions. In Section 3, we define the problem setting and introduce a mathematical formulation for it. In Section 4, we propose an exact solution methodology based on Benders decomposition. In Section 5, we present results, analyses, and managerial insights based on an extensive computational study. Finally, we conclude in Section 6.

2. Literature Review

An efficient evacuation planning/management relies on efficient supply and demand management strategies. On the supply management side, studies in the literature generally focus on designing new road segments (Üster et al., 2018), contraflow or lane-based strategies (Cova and Johnson, 2003, Bretschneider and Kimms, 2011, Hasan and Van Hentenryck, 2020a, Liu et al., 2021), intersection design (Bretschneider and Kimms, 2012, Zhao et al., 2016), lane/ramp closures (Xie and Turnquist, 2009, Kimms and Maassen, 2012, Herrera-Restrepo et al., 2016), and resource allocation at critical intersections (Zhang et al., 2013, He and Peeta, 2014, He et al., 2015). The evacuation studies that explore demand management strategies are generally on staging (Lim et al., 2012, Bish and Sherali, 2013, Tuydes-Yaman and Ziliaskopoulos, 2014, Bish et al., 2014, Kimms and Maiwald, 2017), shelter/route assignment (Ben-Tal et al., 2011, Pillac et al., 2016, Esposito Amideo et al., 2021), and evacuate or shelter-in-place (Cova et al., 2011, Apivatanagul et al., 2012, Yi et al., 2017, Karabuk and Manzour, 2019, Davidson et al., 2020, Yang et al., 2019b,a). For more details on evacuation planning/management studies, we refer the reader to Wolshon et al. (2005a,b), Murray-Tuite and Wolshon (2013), and (Bayram, 2016).

The focus of this study is on resource allocation as a supply management strategy and evacuate or SIP and staging as demand management strategies. The number of studies in the literature that explore evacuate or SIP decisions is limited. Few studies exist that consider DRA problem during evacuations. To the best of our knowledge, there is no study in the literature that takes into consideration the combined effect of staging, evacuate or SIP, and DRA decisions. And most of these studies generally minimize clearance time or total evacuation time. There are few studies that minimize the total risk of an evacuation. Table 1 illustrates a comparative summary of the studies in the evacuation planning/management literature that are related to our study. In the table, we present whether a study is based on static/dynamic setting (S/D), whether it includes staging, evacuate or SIP, and resource allocation decisions, or any other decisions, whether resources are mobile or allocated once throughout the planning horizon, whether risk is included, solution methodology employed and finally whether the model and the solution methodology were applied to a realistic problem. Below, we will give more details related to studies on resource allocation, evacuate or SIP, and staging and the ones that consider risk.

2.1. Resource Allocation at Critical Intersections

The evacuation literature on allocation of resources at critical intersections is sparse. To the best of our knowledge, there exist only a few studies that explore the advantages of resource allocation. And in majority of these studies, resources are not mobile.

We could not spot any other study on resource allocation in evacuation literature that was conducted before that of Liu (2007). In the study by Liu (2007), the importance of determining critical intersections and deploying law enforcement personnel to these locations is pointed out. However, this perspective is only used as a resource limit in the constraints on total length of reversed road segments and identification of these critical intersections is not included in the proposed model. Similarly, Pillac et al. (2015) assign mobilization resources to evacuation zones to affect evacuee response behavior, modeled as response curves. Jabari et al. (2012) and Parr et al. (2015) name this problem as the officer deployment problem and manual traffic control, respectively, where police officers are deployed to critical intersections to manage the intersection right of way in response to over-saturated road network conditions. Hsu and Peeta (2014b,a) seek to determine evacuation zones with the highest risk in a stage-based approach. The resources in these studies are incorporated in the problem as a parameter rather than a decision variable and embedded in a resource availability constraint. Zhang et al. (2013) propose mixed integer nonlinear programming models, which aim to optimally determine the critical intersections and traffic

Table 1: A comparative illustration of large scale urban (vehicular) evacuation studies in the literature

Study	S/D	Decisions Included: Stag.	Decisions Included: Evac. or SIP	Decisions Included: RA	Decisions Included: Other Decisions	Mo. Res.	Risk	Solution Methodology	Real. App.
Liu (2007)	D	●	❍	❍	Contraflow Signal Timing CTM decisions	❍	❍	CTM-based modeling approach Resource limit as an input Heuristic, genetic algorithm	●
Pillac et al. (2015)	D	●	❍	❍	Res. All. at Zones Routing	❍	❍	NF-based modeling on a TEN Heuristic, column generation	❍
Jabari et al. (2012)	D	●	❍	●	CTM decisions Routing	❍	❍	CTM-based modeling approach Heuristic, genetic algorithm	❍
Parr et al. (2015)	D	❍	❍	●	Contraflow Routing	❍	❍	Evaluation with statistical techniques	●
Hsu and Peeta (2014b)	S	●	❍	❍	Zoning	❍	●	Stage-based approach Resource limit as an input Exact BB based algorithm	●
Hsu and Peeta (2014a)	D	●	❍	❍	Routing	❍	❍	Stage-based approach Resource limit as an input Heuristic, fuzzy logic	●
Zhang et al. (2013)	D	❍	❍	●	CTM decisions	❍	❍	Commercial solver	❍
He and Peeta (2014)	D	●	❍	●	CTM decisions	●	❍	CTM-based modeling approach Greedy heuristic	❍
He et al. (2015)	D	●	❍	●	Flow decisions	●	❍	EAF formulation on a TEN Exact, Benders Decomposition	●
Cova et al. (2011)	S	❍	●	❍		❍	●	Greedy heuristic Commercial solver	❍
Apivatanagul et al. (2012)	D	●	●	❍	Routing	❍	●	Bi-level model Iterative approach	●
Yi et al. (2017) Davidson et al. (2020) Yang et al. (2019b) Yang et al. (2019a)	D	●	●	❍	Routing	❍	●	Bi-level model Heuristic, progr. Hedging lagrangean relaxation	●
Karabuk and Manzour (2019)	D	●	●	❍	CTM decisions	❍	●	CTM-based modeling approach Commercial solver	❍
Chiu et al. (2007) Ng and Waller (2010) Tuydes-Yaman and Ziliaskopoulos (2014)	D	●	❍	❍	CTM decisions	❍	❍	CTM-based modeling approach Commercial solver	❍
Ben-Tal et al. (2011)	D	●	❍	❍	CTM decisions	❍	❍	CTM-based modeling approach Commercial solver	●
Bish and Sherali (2013)	D	●	❍	❍	CTM decisions Routing	❍	❍	CTM-based modeling approach Heuristic	●
Li and Ozbay (2015)	D	●	❍	❍	CTM decisions	❍	❍	CTM-based modeling approach Heuristic	❍
Bretschneider and Kimms (2011) Bretschneider and Kimms (2012)	D	●	❍	❍	Lane reversal Routing	❍	❍	NF-based modeling Heuristic	❍
Lim et al. (2012)	D	●	❍	❍	Routing	❍	❍	NF-based modeling on a TEN	●
Bish et al. (2014)	D	●	❍	❍	CTM decisions Routing	❍	❍	CTM-based modeling approach NF-based solution meth. on a TEN	●
Kimms and Maiwald (2017)	D	●	❍	❍	CTM decisions	❍	❍	CTM-based modeling approach NF-based solution meth. on a TEN	●
Hasan and Van Hentenryck (2020a) Hasan and Van Hentenryck (2020b)	D	●	❍	❍	Contraflow Routing	❍	❍	Network flow-based modeling Benders decomp. and col. gen.	●
Church and Cova (2000)	S	❍	❍	❍	Zoning	❍	●	Commercial solver Heuristic	●
Pyakurel et al. (2023)	S, D	●	❍	❍	Routing Interm. storage loc.	❍	❍	Network flow algorithms	❍
Stepanov and Smith (2009)	S	❍	❍	❍	Routing	❍	❍	Simulation-based optimization	❍
This Study	D	●	●	●	CTM decisions	●	●	CTM-based modeling approach Exact, Benders Decomposition MP: BC (feas. cuts, DRA cuts) SP: NF-based meth. on a TEN	●

control strategies at these intersections, i.e., a system optimal strategy at controlled intersections and a user equilibrium strategy at uncontrolled intersections. The studies mentioned so far, ignore the time dimension of the evacuation in making resource allocation decisions, i.e., resources are not mobile and stay in their allocated intersections throughout the evacuation.

There exist only two studies (He and Peeta, 2014, He et al., 2015) that address the dynamic allocation of mobile resources at critical intersections. He and Peeta (2014) propose a CTM-based mixed integer linear program to optimally decide on when, where and how long limited number of resources are allocated to improve the evacuation performance. The solution methodology proposed by He and Peeta (2014) is not exact due to the complexity caused by dynamic nature of the problem, discretization of original network into a cell network resulting in large number of variables even for modest network sizes and the discrete resource allocation decisions. To be able to solve the problem efficiently, they transform the problem into a two-stage optimization problem and employ a greedy heuristic algorithm. He et al. (2015) propose a DRA problem formulation based on the earliest arrival flow (EAF) formulation of Zheng et al. (2015). The advantage of using EAF formulation is that it allows to use a TEN structure to solve the problem as a minimum cost network flow problem. They employ a Benders decomposition algorithm to be able to solve large scale problem instances. The authors further use auxiliary variables to represent the start and end time steps of the resource assignment schedule, which allows them to generate some strong valid equalities and to relax integrality restrictions on binary resource allocation decision variables. Both studies minimize the total evacuation time and ignore the risk evacuees can be exposed to. He et al. (2015) assume that "at most one moveable resource can be assigned to an intersection once throughout the evacuation process". We relax this assumption and allow allocation of different resources and reallocation of the same resource to the same intersection at different time intervals at the expense of loss of efficiency of the proposed algorithm. Further, unlike in our study, evacuate or SIP decisions are not considered in these studies.

2.2. Evacuate or Shelter-in-Place

Although there is an increasing interest in studies that explore the benefits of evacuate versus SIP, the number of such papers is limited. Cova et al. (2011) present a framework for the decisions that can be made against wildfires: evacuate, shelter-in-refuge, or shelter-in-home to maximize the overall community protection, i.e., protection level of a shelter minus the fire threat level. While the model considers these three possible actions, it does not incorporate evacuation operations, i.e., routing of evacuees or other demand and supply management strategies. The problem is solved using a commercial solver.

Apivatanagul et al. (2012) introduce a bi-level model that minimizes the risk and travel time simultaneously and incorporate evacuate or SIP, staging and routing decisions considering the uncertainty regarding hurricane track, forward speed, and intensity. The upper level of the model decides who will evacuate, when and to which shelter. The lower level of the model is a dynamic user equilibrium (DUE) model. The problem is solved using an iterative approach passing information from the upper level model to lower level model and vice versa and a case study of eastern North Carolina is used for testing. Yi et al. (2017) propose a multi-stage stochastic programming formulation considering the uncertainty in hurricane evolution. The model they propose is also a bi-level optimization model, upper level of which creates an evacuation plan as to evacuate or shelter-in-place and staging. Further, discrete choice models are incorporated as to who will obey the evacuation orders. They also employ the risk levels defined by Apivatanagul et al. (2012) and use a heuristic solution procedure based on progressive hedging to solve the upper level model. The lower level of the problem is solved by a DUE algorithm.

To support hurricane evacuation decision making, Davidson et al. (2020) introduce an "integrated scenario based evacuation" (ISE) framework that incorporates the dynamic and uncertain nature of hurricanes and the human behavior, using a hazard modeling approach that considers coastal flooding, inland flooding, and wind (Blanton et al., 2020). They use a bi-level multi-stage stochastic programming formulation, which is an extension of the models proposed in Li et al. (2011) and Apivatanagul et al. (2012) and define risk as in Apivatanagul et al. (2012). While the upper level creates an evacuation plan minimizing total risk and travel time, lower level is a DUE traffic assignment model that evacuates threatened population to safe shelters. They use the same progressive hedging algorithm proposed in Yi et al. (2017) to solve the problem. Yang et al. (2019b) extend the hazard modeling of Davidson et al. (2020) and apply the framework to a case study of Hurricane Matthew (2016) near the North Carolina coast. Using the same ISE framework, Yang et al. (2019a) discuss the use of robust, adaptive, and repeated decision-making for an impending hurricane on a case study of Hurricane Isabel (2003) in North Carolina.

Karabuk and Manzour (2019) propose a multi-stage stochastic programming formulation based on CTM to discuss the use of evacuate or SIP decisions against convective weather events such as tornadoes. They discuss that the status-quo policy, i.e., SIP, against such events may not be the single or the best option with the advance of state-of-the-art weather prediction technology and compare the status-quo policy with evacuation as a response policy in terms of benefits, risks, and costs. They solve the problem using a commercial solver. Pyakurel and Dempe (2020) and Pyakurel et al. (2023) present abstract networks concept where intermediate storage of evacuees at potential nodes is possible whenever evacuees are not able to reach safe destinations due to capacity or time restrictions.

Except for Cova et al. (2011), Karabuk and Manzour (2019), and Apivatanagul et al. (2012), which use commercial solver and an iterative approach, respectively to solve their problems, none of the mentioned studies employ exact solution methodologies. Further, unlike in our study, these studies do not incorporate DRA decisions.

2.3. Staging

The literature on staging/phasing of evacuation operations considering departure times of evacuees is quite rich. The studies that we cite here is by no means comprehensive as we picked only representative ones. Among the studies that we cited under subsections 2.1 and 2.2, the studies by Apivatanagul et al. (2012), He and Peeta (2014), He et al. (2015), Yi et al. (2017), Yang et al. (2019b), Karabuk and Manzour (2019), Davidson et al. (2020) also consider staging.

Among other studies that consider staging/phasing or departure time decisions, Chiu et al. (2007), Ng and Waller (2010), Ben-Tal et al. (2011), Tuydes-Yaman and Ziliaskopoulos (2014), Bish and Sherali (2013), Li and Ozbay (2015) propose CTM-based models, Bretschneider and Kimms (2011, 2012), Lim et al. (2012), Bish et al. (2014), Kimms and Maiwald (2017), Hasan and Van Hentenryck (2020a) use network flow based modeling approaches on TENs.

Bish et al. (2014) address disaggragate level staging and routing strategies to disseminate household level instructions to evacuees using a CTM-based model. To reduce the computational complexities of a CTM-based formulation so that large scale realistic instances can be solved, they transform it to a TEN-based formulation. Finally, they design a sequential algorithm that requires solving a sequence of simpler linear programming models and test their algorithm on a realistic network of Virginia Beach for evacuations against hurricanes. Kimms and Maiwald (2017) also follow a similar approach and transform their CTM-based model into a minimum cost flow formulation defined over a space-time expanded network. They use a commercial solver to solve their problem.

Hasan and Van Hentenryck (2020a) introduce a network flow based formulation on a TEN that considers contraflow, staging and routing decisions for convergent and non-preemptive zone-based evacuation planning. They employ Benders decomposition and column generation techniques to solve their problem exactly. They test the efficiency of their algorithms and derive managerial insights on the mentioned strategies in a separate paper (Hasan and Van Hentenryck, 2020b).

Only a small number of these studies consider evacuation risk and none of them takes into consideration the joint effect of staging, evacuate or SIP, and DRA strategies.

2.4. Evacuation Risk

In evacuation planning/management literature the objectives of proposed models are generally minimizing the network clearance time, total/average evacuation time, earliest arrival flow or maximizing the number of evacuees reaching safety up to a specified time (Bayram, 2016). The number of papers considering minimization of total exposed risk is limited.

In disaster management literature, the risk of a location is generally defined by the estimated impact of the disaster on that location and its geographical features (Hsu and Peeta, 2014b). The risk caused by a hurricane to a location for instance, can be measured by the wind speed, rainfall, surge flooding, and the surge depth (Wolshon et al., 2016, Lindell et al., 2018, Blanton et al., 2020). The risk of a toxic chemical release depends on the weather and topographic conditions and has an inverse relationship with the distance of a neighborhood to the source of the toxic release (Lindell et al., 2018). Social vulnerability is another aspect of risk as it is related to the need for mobility and level of assistance required by a community, specifically in terms of evacuation. Some studies define the risk of a neighborhood during an evacuation operation as the ability of the neighborhood to clear the risk zone as quickly as possible, and for that reason it is linked to clearance time (Church and Cova, 2000), ignoring the impact of the disaster. Similarly, Hsu and Peeta (2014b) define risk of a location as whether that location can be cleared before disaster impacts it, i.e., as a function of the clearance time of location and the lead time to the impact of the disaster on that location. Kimms and Maiwald (2017, 2018) minimize total evacuation risk on a road network, where they create risk values randomly. These studies do not take into consideration the level of protection a given location provides.

Cova et al. (2011) consider evacuations against wildfires and define risk as a combination of fire threat level measured by flame length and the protection level of the shelter under five threat and protection levels. Considering evacuations against hurricanes, Apivatanagul et al. (2012), Davidson et al. (2020) define risk as the probability a person is killed, injured or had a traumatic experience, based on location and time, the protection level of the location, and the possible impact of the hurricane on that location. Karabuk and Manzour (2019) define a measure of risk of injury at a location, which depends on the relative position of the weather event and the protection level provided by a specific type of structure used as a shelter.

2.5. Our Contributions

- We propose a CTM-based formulation that jointly considers evacuate or SIP, DRA, and staging decisions.

- Rather than minimization of total evacuation time, we consider risk as a measure of the response policy, which depends on the impact of the disaster and the protection level provided by the location of the evacuees.

- We consider mobile resources and develop strategies about deploying these resources at critical intersections dynamically over time depending on the evolving risk and congestion levels.

- We relax the restriction employed in DRA literature and allow to assign different resources or reassign the same resource to the same location at different time intervals.

- We develop a Benders decomposition-based exact algorithm to solve large scale problem instances more effectively compared to a commercial solver and further enhance the effectiveness of the algorithm by deriving valid inequalities based on DRA decisions and for time-feasible solutions. While we solve the master problem through a branch-and-cut framework, we transform the CTM-based subproblem formulation into a minimum cost flow formulation on a TEN to be able to solve large scale realistic instances.

- Finally, we conduct numerical experiments on realistic instances to test the effectiveness of our algorithm and derive managerial insights. In particular we found that a zone-based approach where some zones are ordered to evacuate while others shelter-in-place is superior to other approaches where an evacuate or SIP decision is given for all population at risk. Jointly considering evacuate or SIP, staging, and DRA decisions significantly increases efficiency of the evacuation management, i.e., minimizes total risk and requires less resources. DRA helps reduce congestion and hence risk especially for higher levels of demand. As lead time increases the number of zones ordered to evacuate also increases.

The proposed model and solution methodology presented in this study aims to assist evacuation planning and management authorities in addressing the complexities associated with decision-making during disaster management, irrespective of the type of disaster. The fundamental objective of this research is to demonstrate that a coordinated approach that incorporates supply and demand management strategies and corresponding decisions, such as Evacuate or Shelter in Place (SIP), Staging, and Dynamic Resource Allocation (DRA), can result in more effective evacuation planning and management policies, which responsible authorities should adopt. The model and solution methodology we propose can be utilized by responsible agencies and authorities to develop baseline evacuation plans and procedures or to evaluate and enhance the existing ones. Although we propose an offline model/solution methodology working in a dynamic setting, it can also be implemented in a rolling horizon approach for real-time operations just before a disaster, for which there is sufficient lead time, such as a hurricane or tsunami.

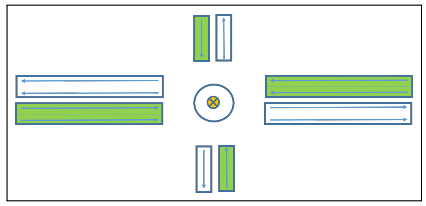
3. Assumptions, Problem Statement and Model Development

In this section, we define our problem and propose a formulation based on CTM to solve it. Cell Transmission Model is a DTA concept that was introduced by Daganzo (1994, 1995). Later on, that concept was transformed into a linear programming formulation by Ziliaskopoulos (2000), which attracted a lot of attention in evacuation planning/management literature as discussed in the Literature Review section. To obtain a CTM-based formulation, the original road network is transformed into a cell network, i.e., each arc in the original network is discretized into cells with a length that can be traveled in unit time interval using free flow speed. Cells are connected to each other using dummy arcs called connectors.

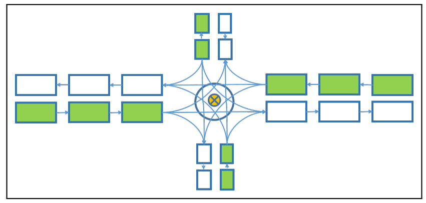
3.1. Assumptions

Before proceeding with the formal definition of our problem and formulation, we will explicate the assumptions that underlie our analysis. We posit that the evacuation of the population threatened by a disaster will be carried out by personal vehicles, implying that each individual has access to personal transportation and that these vehicles convey a typical number of passengers. It is possible to adjust the parameters of our problem to accommodate scenarios where a proportion of the zone's population is comprised of elderly, disabled, or otherwise immobilized persons who require evacuation via buses. However, incorporating motorbikes into the evacuation scheme may pose greater complexities since their traffic dynamics and interaction with conventional vehicles necessitate additional modeling considerations. Nonetheless, it may be reasonable to exclude motorbikes from our planning, considering their percentage in traffic in most countries. Our model and solution methodology operate on a zone-based approach, where each zone is assigned a unique evacuation directive. Zones are typically defined in a risk-homogenous manner, and their risk level may vary over time due to the evolving disaster, as reflected in our modeling strategy. Our modeling approach does not account for the first-mile problem, which entails sequential collection of immobilized individuals from their respective locations. Instead, we assume that evacuation vehicles commence their journeys from the corresponding zones and do not travel without passengers. This is a practical and valid assumption, as it simplifies the analysis while still providing useful insights into the evacuation plan's feasibility and effectiveness. This study does not account for the uncertainties resulting from the disaster's nature, including its evolution, the magnitude of the evacuation demand, and the conditions of the evacuation road network infrastructure. However, our model can be applied to a worst-case scenario or used to conduct experiments with various evacuation demand and supply scenarios, such as damage to the road network infrastructure. We make the additional assumption that communication channels, such as television, radio, cell phones, sirens, or social media, are reliable and operational, enabling the dissemination of information and evacuation directives to the public. Moreover, we assume that evacuees will fully comply with the evacuation orders issued to them.

The effective management of intersections poses a significant challenge for emergency managers during the development of evacuation plans (Parr et al., 2016). To enhance evacuation performance, it is benefi-



(a) Capacity Expansion in Road Segments (original network) due to DRA



(b) Capacity Expansion in Cells (cell network) due to DRA

Figure 1: DRA and Capacity Expansion

cial to allocate resources to critical intersections within a network. Our work builds upon the assumption put forth by He and Peeta (2014), which suggests that the capacity of a road segment can be increased by assigning a resource to its downstream intersection. This assumption is rooted in the idea that the presence of a resource, such as a law enforcement officer, facilitates the smooth passage of vehicles through an intersection. Consequently, the delay time experienced by road segments connected to this intersection is reduced, leading to an increase in their capacity. We model this effect by enhancing the capacity of upstream road segments, as the smoother traffic flow is reflected in an increased link volume with fewer conflicting movements at downstream nodes due to the presence of the assigned resource. Figure 6 shows that the green cells will experience increased capacity if a resource is allocated to the single intersection depicted.

We provide a detailed exposition of supplementary assumptions, including the road network's capacity, impact of locating resources on capacity, the speed of travel and the radius of the hazard as well as our approach to computing the potential risk of the disaster, taking into account factors such as the hazard's characteristics and infrastructure vulnerability under the Data Used and Experimental Design subsection and other relevant sections of the manuscript.

3.2. Problem Statement and Model Development

Given a graph $G^o = (V, A^o)$ of original evacuation road network, where V and A^o are the set of nodes and arcs (road segments, links) of the original network, we represent the transformed cell network with $G = (I, A)$, where I is the set of cells and A is the set of arcs (cell connectors) in the cell network. The set of cells I is the union of three disjoint subsets, i.e., the set of source cells S_o representing evacuation demand zones, the set of sink cells S_e representing safe shelters and the set of road segment cells R. The source cells have no incoming arcs and sink cells have no outgoing arcs. For cell $i \in S_o$, D_i denotes the number of vehicles to be evacuated. It is assumed that $D_i = 0$ for all $i \in I \setminus S_o$. Free flow speed is defined as the maximum speed at which vehicles can travel on a road segment with no congestion, delay or interruption and backward propagation speed is the speed at which a congestion wave caused by a sudden event propagates backward through the traffic stream. For cell $i \in R$, N_i denotes the maximum number of vehicles that can be accommodated in cell i, which is scaled down by δ_i, the ratio of the free-flow speed to the backward propagation speed, and Q_i denotes the maximum number of vehicles that can enter into or leave from cell i, i.e., the inflow/outflow capacity of cell i. Let T' be the set of disaster time periods, where $|T'|$ is the last time period for which the disaster continues to create risk over the disaster region and let $T \subset T'$ be the set of evacuation time periods, where $|T|$ is determined as a target time for evacuees to reach safe shelters, i.e., network clearance time. The parameter c_{it} represents the estimated hazard level for cell i in period $t \in T'$. The decision variable x_{it} is the number of vehicles at cell $i \in I$ in time $t \in T$ and y_{ijt} is the number of vehicles traveling from cell i to cell j in time $t \in T \setminus \{|T|\}$.

Let P be the set of resources and $V' \subseteq V$ be the set of potential nodes where resources can be allocated to increase capacity of the road segments connecting to these nodes. We define the decision variable z_{npt} to be 1 if resource $p \in P$ is allocated at node $n \in V'$ at time $t \in T$ and 0 otherwise. Let Δ_i be the increment in the inflow and outflow capacity of cell $i \in I_n \subset I$, in period $t \in T$ if resource $p \in P$ is allocated to this location at this time, where I_n is the set of cells associated with road segments for whom $n \in V'$ is a downstream node. We define $n(i)$ to be the node n associated with cell i, i.e., the junction which increases the capacity of cell i by Δ_i if a resource p is allocated to it. Let $\tau_{pnn'}$ be the time required for resource p to travel from node $n \in V'$ to node $n' \in V'$. In other words, if p is at node n at time t, it can be operational at node n' the earliest at time $t + \tau_{pnn'} + 1$. Resources can be deployed from a single or multiple depot nodes depending on the setting. We define $R' = \cup_{n \in V'} I_n \cap R$ to be the union of cells associated with road segments corresponding to different downstream nodes $n \in V'$.

Staging and SIP or evacuate are two demand management strategies that we employ in this study. Staging is accomplished by means of the decision variable y_{ijt}. In that sense, the term $\sum_{j:(i,j)\in A} y_{ijt}$ represents the number of evacuees starting their evacuation from resource cell $i \in S_o$ at time t. We include SIP or evacuate decisions as follows. Let e_i be 1 if we decide to evacuate source cell (zone) i and 0 if we make an SIP decision for that cell. A detailed descriptive notation table is presented in Apenndix A. The model formulation for joint evacuate or SIP, staging and DRA (JESDRA) is presented below:

JESDRA:

$$\min \sum_{i\in I}\sum_{t\in T} c_{it}x_{it} + \sum_{i\in S_o}\sum_{t\in T'} D_i c_{it}(1-e_i) \tag{1}$$

$$\text{s.t. } x_{it} = x_{i,t-1} + \sum_{j:(j,i)\in A} y_{ji,t-1} - \sum_{j:(i,j)\in A} y_{ij,t-1} \qquad i \in I, t \in T \setminus \{1\} \tag{2}$$

$$\sum_{i\in S_e} x_{i,|T|} = \sum_{i\in S_o} D_i e_i \tag{3}$$

$$\sum_{j:(j,i)\in A} y_{jit} \le \delta_i(N_i - x_{it}) \qquad i \in R, t \in T \setminus \{|T|\} \tag{4}$$

$$\sum_{j:(i,j)\in A} y_{ijt} \le x_{it} \qquad i \in I, t \in T \setminus \{|T|\} \tag{5}$$

$$\sum_{j:(i,j)\in A} y_{ijt} \le Q_i \qquad i \in R \setminus R', t \in T \setminus \{|T|\} \tag{6}$$

$$\sum_{j:(j,i)\in A} y_{jit} \le Q_i \qquad i \in R \setminus R', t \in T \setminus \{|T|\} \tag{7}$$

$$\sum_{j:(i,j)\in A} y_{ijt} \leq Q_i + \sum_{p\in P} \Delta_i z_{n(i)pt} \qquad i \in R', t \in T \setminus \{|T|\} \tag{8}$$

$$\sum_{j:(j,i)\in A} y_{jit} \leq Q_i + \sum_{p\in P} \Delta_i z_{n(i)pt} \qquad i \in R', t \in T \setminus \{|T|\} \tag{9}$$

$$\sum_{p\in P} z_{npt} \leq 1 \qquad n \in V', t \in T \tag{10}$$

$$\sum_{n\in V'} z_{npt} \leq 1 \qquad p \in P, t \in T \tag{11}$$

$$z_{npt_1} + \sum_{n'\in V':t_1+\tau_{pnn'}\geq t_2} z_{n'pt_2} \leq 1 \qquad n \in V', p \in P, t_1, t_2 \in T : t_2 - t_1 \leq \max_{n'\in V'} \tau_{pnn'} \tag{12}$$

$$x_{i1} = D_i e_i, \qquad i \in S_o \tag{13}$$

$$x_{i1} = 0 \qquad i \in I \setminus S_o \tag{14}$$

$$x_{it} \geq 0 \qquad i \in I, t \in T \tag{15}$$

$$y_{ijt} \geq 0 \qquad (i,j) \in A, t \in T \setminus \{|T|\} \tag{16}$$

$$z_{npt} \in \{0,1\} \qquad n \in V', p \in P, t \in T \tag{17}$$

$$e_i \in \{0,1\}, \qquad i \in S_o \tag{18}$$

Objective function (1) minimizes the total risk of people who are evacuating or sheltering-in-place. Flow balance is ensured by constraints (2). Constraints (3) guarantee that population of every zone for which "evacuate" decision was given is evacuated to a safe region by time T. Flow into each road segment cell i at every time interval t is limited by the capacity of the cell minus the number of vehicles in the cell and the inflow capacity of the cell due to constraints (4) and (7), respectively. The restriction that the flow out of a cell i at time interval t cannot be greater than the number of vehicles in that cell or the outflow capacity of the cell is enforced by constraints (5) and (6), respectively. Constraints (8) and (9) are the outflow and inflow capacity restrictions on cell i at time t with possible additional capacity provided by resource allocation. Constraints (10) - (12) are related to dynamic resource allocation. Constraints (10) prevent assigning more than one resource at the same location n at the same time t and constraints (11) do not allow assigning the same resource to more than one location at the same time interval t. Constraints (12) ensure that a resource p located at location n at time t_1 cannot be operational at a new location n' before $t_1 + \tau_{pnn'} + 1$. Constraints (13) define the starting conditions of each source cell i, i.e., there are D_i number of people to evacuate if an "evacuate" decision is given and 0 otherwise and constraints (14) define the initial conditions of other cells. Finally, constraints (15) - (17) define variable domains.

To illustrate the evacuate or SIP, DRA, and staging decisions, we conducted an experiment on a small-scale toy instance. Specifically, we presented the solution generated by our model in Appendix B.

4. A Benders Decomposition Approach

In this section, we propose an algorithm based on Benders decomposition (Benders, 1962) to solve realistic large scale instances. We project out continuous state and staging/routing decisions $\mathbf{x}$ and $\mathbf{y}$

to obtain a Master Problem (MP) that generates evacuate or SIP and DRA decisions. MP has fewer variables but a large number of constraints, i.e., Benders cuts, which are not all active at an optimal solution. An iterative procedure is pursued by sending fixed evacuate or SIP and DRA decisions to solve the subproblem (SP) and to obtain dual information from it to generate violated Benders cuts until all of them are satisfied at a relaxed MP solution. For fixed evacuate or SIP and DRA decisions obtained through solution of MP, the SP is an evacuation staging and routing problem.

We define the CTM-based SP and its transformation into a TEN-based network flow formulation in Section 4.1, and discuss MP, identify Benders optimality cuts, feasibility cuts and valid inequalities in Section 4.2.

4.1. The Subproblem

In our problem, the binary evacuate or SIP and DRA decision variables **e** and **z** are the complicating variables and are handled in the MP. Therefore, we can project out continuous state and staging/routing decision variables **x** and **y** in the SP and determine them based on the given values of $\bar{\mathbf{e}}$ and $\bar{\mathbf{z}}$ found in the MP. The SP determines staging and routing of evacuees from corresponding evacuation zones for which an evacuation order is given, to safe exits/shelters such that the total evacuation risk is minimized. The CTM-based SP (CTMSP) is presented below:

CTMSP:

$$\min \sum_{i\in I}\sum_{t\in T} c_{it}x_{it} \tag{19}$$

$$\text{s.t.} \quad \sum_{i\in S_e} x_{i,|T|} = \sum_{i\in S_o} D_i\bar{e}_i \tag{20}$$

$$\sum_{j:(j,i)\in A} y_{jit} \le Q_i + \sum_{p\in P} \Delta_i \bar{z}_{n(i)pt} \qquad i\in R', t\in T\setminus\{|T|\} \tag{21}$$

$$\sum_{j:(i,j)\in A} y_{ijt} \le Q_i + \sum_{p\in P} \Delta_i \bar{z}_{n(i)pt} \qquad i\in R', t\in T\setminus\{|T|\} \tag{22}$$

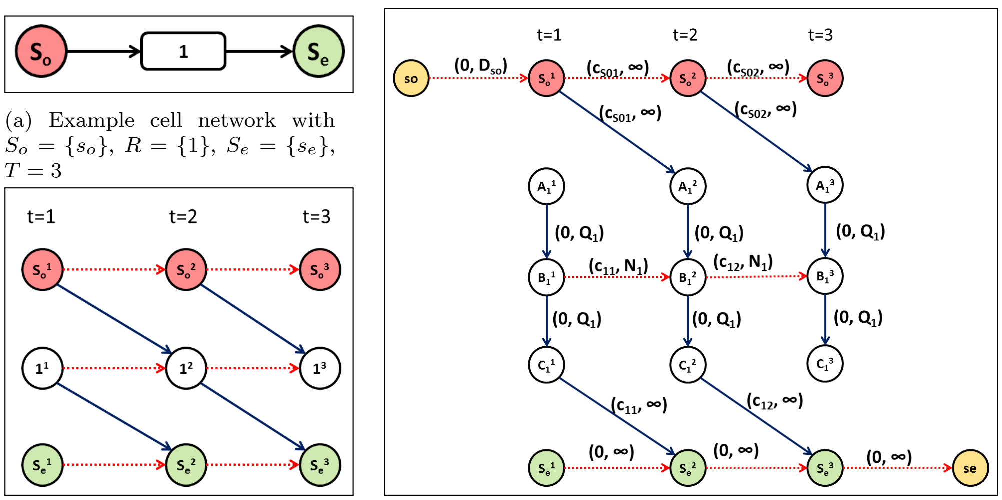
$$x_{i1} = D_i\bar{e}_i, \qquad i\in S_o \tag{23}$$

$$(2), (4)-(7), (14)-(16) \tag{24}$$

Due to complexities of working with a CTM-based model for realistic large scale networks, we transform CTM-based SP into a minimum cost flow formulation on a TEN denoted by $\hat{G} = (\hat{I}, \hat{A})$. Next, we discuss how we make this transformation.

We construct a new graph $G' = (I', A')$ by creating time copies of the cell set I, where $I' = \{(i,t) : i\in I, t\in T\}$ and arc set $A' = \{(i,t-1),(j,t) : (i,j)\in A \text{ or } i=j, t\in T\setminus\{1\}\}$. See Figure 2a and Figure 2b for an example cell network and its time-space expanded version, respectively, where dotted arcs are holdover arcs that represent the vehicles that do not leave a cell at a given time interval. Using $G' = (I', A')$, we transform the CTM-based SP into TEN-based formulation (TENSP) as in Bish et al. (2014) (See Appendix C). Bish et al. (2014) modify this formulation to remove some of the side constraints of the resulting formulation and we follow the same methodology. The modified time-space network-based formulation (TENSP) is based on the following modifications on the TEN:

A super-source node so is connected to the initial ($t = 1$) replication of each source node i via an arc of capacity D_i and a risk value of c_{i0}. Each source holdover arc from time t to $t+1$ has a risk value of c_{it} without any capacity restrictions. The final ($t = |T|$) replication of each sink is connected to a super-sink node se via an arc having infinite capacity and a risk value of 0. Each time-indexed replication (i,t) of roadway segment i is replaced by a three node sub-network, where all incoming movement arcs for it enter the first node (A), which is then connected to the second node (B) via an arc having a capacity of Q_i and a risk value of 0. Furthermore, the second node has an incoming holdover arc from its $(t-1)^{th}$ interval replication and an outgoing holdover arc to its $(t+1)^{th}$ interval replication, with these holdover arcs having a capacity of N_i and a risk value of c_{it}. The second node also has another outgoing arc of capacity Q_i and a risk value of 0, which connects it to the third node. All outgoing movement arcs for node (i,t) leave the third node (C) with risk value c_{it} and infinite capacity. Please see Figure 2c for the modified TEN. This modified network is denoted by $\hat{G} = (\hat{I}, \hat{A})$. Let $\tilde{A} \subset \hat{A}$ be the union of arcs



(a) Example cell network with $S_o = \{s_o\}$, $R = \{1\}$, $S_e = \{s_e\}$, $T = 3$

(b) Time-space Expanded Cell Network

(c) Modified Time-space Expanded Cell Network reducing side constraints

Figure 2: Transformation of cell network into a time-space expanded network

a in TEN, in the three-node-sub-network of road segment type cell $i \in R'$ over time replications $t \in T$, with capacity $q_a = Q_i$, i.e., the incoming/outgoing capacity of cell $i(a)$ and eligible to have an increased capacity if a resource $p \in P$ is allocated to $n(i)$ at time $t(a)$, and let $A^{so} = \Big\{(so, i^1) \in \hat{A} : i \in S_o\Big\}$. The subproblem formulation based on this time space network is as follows:

TENSP:

$$\min \sum_{a \in \hat{A}} c_a f_a \tag{25}$$

$$\text{s.t.} \quad \sum_{a \in \Gamma^+(k)} f_a - \sum_{a \in \Gamma^-(k)} f_a = \begin{cases} \sum_{i \in S_o} D_i \bar{e}_i & k = so \\ -\sum_{i \in S_o} D_i \bar{e}_i & k = se \\ 0 & \forall k \in \hat{I} \setminus \{so, se\} \end{cases} \tag{26}$$

$$f_a \le q_a + \sum_{p \in P} \Delta_{i(a)} \bar{z}_{n(i(a))pt(a)}, \qquad a \in \tilde{A}, \tag{27}$$

$$f_a \le D_{i(a)} \bar{e}_{i(a)}, \qquad a \in A^{so}, \tag{28}$$

$$f_a \leq q_a, \qquad a \in \hat{A} \setminus \left(\tilde{A} \cup A^{so}\right), \tag{29}$$

$$\sum_{a \in \Gamma^-(i_1^{t+1})} f_a \leq \delta_i \left(N_i - \sum_{a \in \Gamma^+(i_2^t)} f_a \right), \qquad i \in R, t \in T \setminus |T|, \tag{30}$$

$$f_a \geq 0, \qquad a \in \hat{A}. \tag{31}$$

where, c_a, q_a and f_a are risk, capacity and evacuation flow values on every arc $a \in \hat{A}$, and $i(a)$ and $t(a)$ are cell $i \in R'$ and time $t \in T$ associated with tail nodes of arc $a \in \hat{A}$, respectively. Side constraint (30) to that network flow based formulation ensures that the number of vehicles in cell i in period t plus the number of vehicles that enter that cell in period $t+1$ is restricted by N_i, where (i_k^t) corresponds to the k^{th} node, $k = A, B, C$, in the three-node-subnetwork of t^{th} copy of roadway segment cell $i \in R$.

We associate dual variables α_k, λ_a, and β_{it} with constraints (26)-(30), respectively to obtain Benders optimality and feasibility cuts used in MP. We assume that $\delta_i = 1$.

4.2. Benders Master Problem

The master problem of the HNDC includes evacuate or SIP, DRA decisions and a surrogate decision variable θ to represent total risk incurred due to evacuation of some zones of the subproblem. The MP is given below.

MP:

$$\min \sum_{i \in S_o} \sum_{t \in T'} D_i c_{it} (1 - e_i) + \theta \tag{32}$$

$$\text{s.t.} \sum_{p \in P} z_{npt} \leq 1, \qquad n \in V', t \in T \tag{33}$$

$$\sum_{n \in V'} z_{npt} \leq 1, \qquad p \in P, t \in T \tag{34}$$

$$z_{npt_1} + \sum_{n' \in V' : t_1 + \tau_{pnn'} \geq t_2} z_{n'pt_2} \leq 1, \qquad n \in V', p \in P, t_1, t_2 \in T \cup \{0\} : t_2 - t_1 \leq \max_{n' \in V'} \tau_{pnn'}, \tag{35}$$

$$\begin{aligned}(\bar{\alpha}^b_{so} - \bar{\alpha}^b_{se}) \sum_{i \in S_o} D_i e_i + \sum_{a \in \hat{A} \setminus \left(\tilde{A} \cup A^{so}\right)} \bar{\lambda}^b_a q_a + \sum_{a \in \tilde{A}} \bar{\lambda}^b_a \left(q_a + \sum_{p \in P} \Delta_{i(a)} z_{n(i(a))pt(a)} \right) \\ + \sum_{a \in A^{so}} \bar{\lambda}^b_a D_{i(a)} e_{i(a)} + \sum_{i \in R} \sum_{t \in T \setminus \{|T|\}} \bar{\beta}^b_{it} \delta_i N_i \leq \theta, \qquad b = 1, 2, \cdots, \mathscr{O}\end{aligned} \tag{36}$$

$$\begin{aligned}(\bar{\alpha}^f_{so} - \bar{\alpha}^f_{se}) \sum_{i \in S_o} D_i e_i + \sum_{a \in \hat{A} \setminus \left(\tilde{A} \cup A^{so}\right)} \bar{\lambda}^f_a q_a + \sum_{a \in \tilde{A}} \bar{\lambda}^f_a \left(q_a + \sum_{p \in P} \Delta_{i(a)} z_{n(i(a))pt(a)} \right) \\ + \sum_{a \in A^{so}} \bar{\lambda}^f_a D_{i(a)} e_{i(a)} + \sum_{i \in R} \sum_{t \in T \setminus \{|T|\}} \bar{\beta}^f_{it} \delta_i N_i \leq 0, \qquad f = 1, 2, \cdots, \mathscr{F}\end{aligned} \tag{37}$$

$$z_{npt} \in \{0, 1\}, \qquad n \in V', p \in P, t \in T \cup \{0\}, \tag{38}$$

$$e_i \in \{0, 1\}, \qquad i \in S_o. \tag{39}$$

where $(\overline{\alpha}^b, \overline{\lambda}^b, \overline{\beta}^b)$ is the vector of coefficients for an optimality cut and $(\overline{\alpha}^f, \overline{\lambda}^f, \overline{\beta}^f)$ is the vector of coefficients for a feasibility cut generated at an iteration of solving the SP and $\mathscr{O}$ and $\mathscr{F}$ are the index set of optimality and feasibility cuts, respectively. The objective function (32) minimizes the total risk incurred by the people ordered to shelter-in-place and the surrogate variable θ, which represents the value of the subproblem. Constraints (33) - (35) determine dynamic resource allocation decisions as explained previously. Constraints (36) and (37) are Benders optimality and feasibility cuts, respectively. And finally constraints (38) and (39) define variable domains.

4.3. Solving the Master Problem

In each iteration of BD, the MP (32)–(39) is solved to decide on the evacuate or SIP and DRA variables. Next, based on these decisions, the SP is solved to determine staging/routing of evacuees. Depending on the choice of a target evacuation time T, the solution generated by the MP at an iteration of the Benders decomposition algorithm may not be feasible for the TENSP, i.e., the evacuation demand at the source nodes for which an evacuation order were given may not be evacuated to safe shelters by time T even with the use of resources at the critical intersections. One way to address this issue is to look for Benders feasibility cuts first and check for optimality cuts when the subproblem is feasible. However, in BD, feasibility cuts are undesirable as they are usually much weaker compared to the optimality cuts and the need for adding a large number of them generally increases the solution time. To overcome this issue, we enhance the MP with a set of valid inequalities that generally guarantee feasible solutions in the TENSP, and add Benders feasibility cuts whenever necessary.

Next, we explain the details of this approach and introduce a branch-and-cut algorithm to solve the resulting MP with exponentially many constraints.

4.3.1. Guaranteeing SP Feasibility

To ensure that the solution generated by the MP is feasible for the TENSP, we first define time-feasible (T-feasible) solutions and then derive valid inequalities for them.

Definition 1. *Let $(\bar{e}, \bar{z})$ be a solution for the evacuate or SIP and resource allocation decisions in JESDRA. We call $(\bar{e}, \bar{z})$ a T-feasible solution, if there exists a feasible solution $(\bar{e}, \bar{z}, x, y)$ for JESDRA, i.e., the evacuation demand at the source nodes for which an evacuation decision was made can be evacuated to safe shelters by time T.*

The following proposition presents valid inequalities for the master problem.

Proposition 4.1. *Let $\bar{C} \subseteq \hat{A}$ be the set of arcs of a cut separating super-source node so and super-sink node se in TEN (Figure 2c). The following inequality is a valid inequality for the MP,*

$$\sum_{a \in \bar{C} \cap A^{so}} D_{i(a)} e_{i(a)} + \sum_{a \in \bar{C} \cap \bar{A}} q_a + \sum_{a \in \bar{C} \cap \tilde{A}} (Q_{i(a)} + \sum_{p \in P} \Delta_{i(a)} z_{n(i(a))pt(a)}) \geq \sum_{i \in S_o} D_i e_i \tag{40}$$

where $\bar{A} \subset \hat{A}$ is the set of arcs a in TEN, in the three-node-sub-network of road segment type cell $i \in R'$ over time replications $t \in T$, with capacity $q_a = N_{i(a)}$, i.e., the physical capacity of cell $i(a)$ and $q_a = Q_{i(a)}$, i.e., incoming/outgoing flow capacity of cell $i(a)$ not eligible to have an increased capacity if a resource $p \in P$ is allocated to $n(i)$ at time $t(a)$.

Proof: Let $(\bar{e}, \bar{z})$ be a solution for the MP. We define a feasibility graph G^f, with the same set of nodes and set of arcs defined in TEN $\hat{G}$, i.e., $G^f = (\hat{I}, \hat{A})$. Feasibility graph G^f is obtained from $\hat{G}$, by updating capacities of arcs $a \in A^{so}$ and $a \in \tilde{A}$. The capacity of each arc $a \in A^{so}$ corresponding to source cells (evacuation zones) $i \in S_o$ for which a SIP order is given, i.e., $e_i = 0$, is set to 0. Further, the capacity of arcs $a \in \tilde{A}$ corresponding to cells $i(a)$ for which a resource $p \in P$ is allocated to $n(i)$ at time $t(a)$, i.e., $z_{n(i(a))pt(a)} = 1$ is set to $Q_{i(a)} + \Delta_{i(a)}$. Then simply due to the well known maximum-flow minimum-cut duality theorem (Ahuja et al., 1988), the maximum-flow from super-source node so to super-sink node se in G^f cannot be bigger than the capacity of the the min-cut $\bar{C}$. $\square$

4.3.2. Valid Inequalities based on DRA Decisions

Next, we present the following proposition that characterizes a valid inequality based on DRA decisions in MP, which generalizes (35).

Proposition 4.2. *Let $t_1, t_2 \in T$ be such that $t_2 - t_1 \leq \max_{n' \in V'} \tau_{pnn'}$ and V_1 and V_2 be two disjoint subsets of V' such that $t_1 + \tau_{pnn'} \geq t_2$ for all $n \in V_1$ and $n' \in V_2$. The inequality*

$$\sum_{n \in V_1} z_{npt_1} + \sum_{n' \in V_2} z_{n'pt_2} \leq 1 \qquad V_1, V_2 \subset V' : V_1 \cap V_2 = \emptyset, p \in P, t_1, t_2 \in T : t_1 + \tau_{pnn'} \geq t_2 \tag{41}$$

is valid and generalizes (35).

Proof: Let resource p be deployed at location $n \in V_1$ at time step t_1. By definition of V_1 and V_2, for any of the locations $n' \in V_2$, $t_1 + \tau_{pnn'} \geq t_2$, i.e., resource p cannot be deployed at location n' until time step $t_2 + 1$. This proves the validity of the inequality. Constraint (35) is a special case of (41), where $V_1 = \{n\}$, i.e., it is a singleton and $V_2 = \{n' \in V' : t_1 + \tau_{pnn'} \geq t_2\}$. $\square$

We continue using constraints (35) in MP formulation and add (41) as valid inequalities. We denote the version of MP where we use T-feasible (40) and DRA (41) valid inequalities as MPF. We formally present MPF as follows:

MPF: Minimize (32)

$$\text{s.t.:} \sum_{a \in \bar{C} \cap A^{so}} D_{i(a)} e_{i(a)} + \sum_{a \in \bar{C} \cap \bar{A}} q_a + \sum_{a \in \bar{C} \cap \tilde{A}} (Q_{i(a)} + \sum_{p \in P} \Delta_{i(a)} z_{n(i(a))pt(a)}) \geq \sum_{i \in S_o} D_i e_i, \ \forall \bar{C} \in C \tag{42}$$

$$\sum_{n \in V_1} z_{npt_1} + \sum_{n' \in V_2} z_{n'pt_2} \leq 1, \ \forall V_1, V_2 \subset V' : V_1 \cap V_2 = \emptyset, p \in P, t_1, t_2 \in T : t_1 + \tau_{pnn'} \geq t_2 \tag{43}$$

$$(33) - (39)$$

Next, we describe how we solve MPF.

4.3.3. A Branch-and-Cut Algorithm

The number of inequalities in (42) grows exponentially with the number of arcs $|\hat{A}|$ in the TEN. Similarly, constraints (43) are exponential in size. Therefore, for realistic size problems it is not practical to solve MPF directly. We develop a branch-and-cut approach (B&C) to address this difficulty. We start solving the master problem without considering T-feasible (42) and DRA (43) valid inequalities. For every incumbent integer solution (found during the branch-and-bound search), we check whether there are any violated T-feasible valid inequalities to include in the model to cut off T-infeasible solutions. For a given integer solution $(\bar{e}, \bar{z})$ of the relaxed master problem we solve the separation (maximum flow) problem by using the feasibility graph G^f. If the maximum-flow value is less than $\sum_{i \in S_o} D_i e_i$, we find the arcs in the min-cut $\bar{C} \subseteq \hat{A}$ to detect the violated inequality (42). By itself, including inequalities (42) does not guarantee feasibility of TENSP. Therefore, when the algorithm cannot detect any violated inequalities (42), we check whether we should add Benders feasibility cuts (37) to ensure feasibility.

Similarly, valid inequalities (43) are added in an iterative manner finding violated ones at every iteration. Given a fractional solution $(\bar{z}, \bar{e})$, we solve the following separation problem for each p, t_1, and t_2 to detect a violated valid inequality (43):

$$\max \sum_{n \in V'} \overline{z}_{npt_1} a_n + \sum_{n' \in V'} \overline{z}_{n'pt_2} b_{n'} \tag{44}$$

$$\text{s.t.} \quad a_n + b_{n'} \leq 1 \qquad n, n' \in V' : t_1 + \tau_{pnn'} \leq t_2 - 1 \tag{45}$$

$$a_n \in \{0, 1\} \qquad n \in V' \tag{46}$$

$$b_n \in \{0, 1\} \qquad n \in V' \tag{47}$$

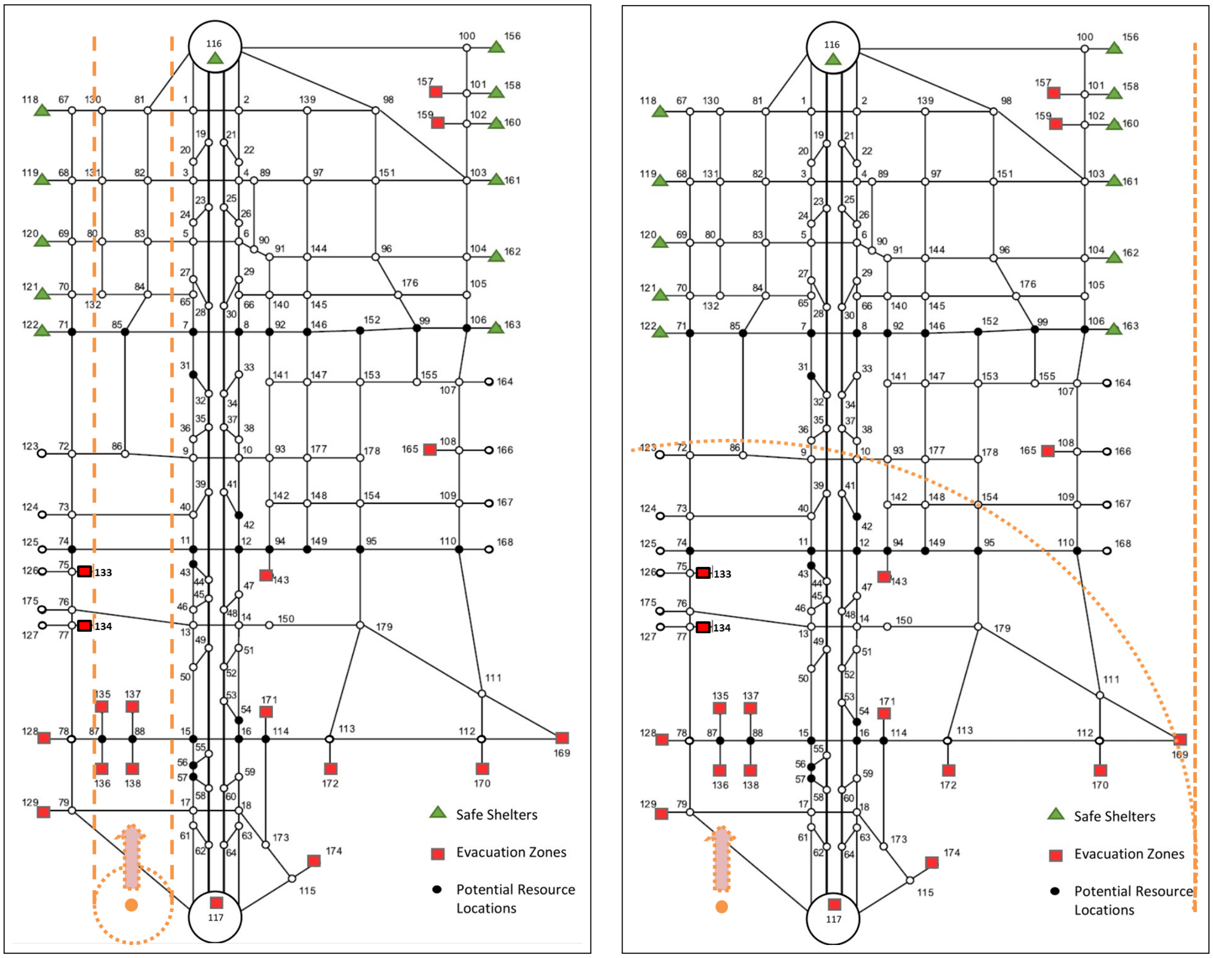
where variable a_n is 1 if $n \in V_1$ and b_n is 1 if $n \in V_2$. The constraint matrix of this separation problem is totally unimodular. Therefore it is sufficient to solve the LP relaxation of the separation problem.

5. Computational Study

In this section, we report results of extensive numerical experiments conducted to test the performance of the proposed algorithms and derive managerial insights based on the instances using a realistic case study. Before presenting the results of our experiments and the managerial insights derived, we first discuss the implementation details.

5.1. Data Used and Experimental Design

In our computational experiments, we used the Dallas-Fort Worth network data from He et al. (2015), shown in Figure 3. The network consists of 179 nodes, 445 arcs, 18 evacuation zones (source nodes), 12 safe shelters (exit nodes) and 27 potential resource locations. The network consists of a freeway between nodes 116 and 117, which is connected to arterial roads and local streets by on and off ramps. We assume that the freeway has a capacity 1,800 vehicles per hour per lane and a free flow speed of 65 miles per hour, arterial roads and local streets have a capacity of 900 vehicles per hour per lane and a free flow speed of 40 miles per hour and finally on and off-ramps have a capacity of 1,200 vehicles per hour per lane and a free flow speed of 30 miles per hour. To the best of our knowledge, no studies in the evacuation planning/management literature have utilized a higher temporal resolution than 30 seconds. A 30-second time step is considered appropriate as it leads to a realistic representation of traffic flows. In line with existing literature, we have adopted the same approach and used a time step length of 30 seconds in discretizing the time-expanded network. This translates into 584 cells (nodes) and 1,419 arcs in the cell network. We compute the capacity values N_i and Q_i for a cell i as follows: Maximum number



(a) Tornado case

(b) NBC type incident

Figure 3: Dallas Fort Worth Evacuation Road Network (He et al., 2015)

of vehicles that can be accommodated in cell i, N_i is computed by dividing cell length by sum of traffic jam distance and vehicle length. Cell length is a multiplication of time period length with the free flow speed at that road segment and traffic jam distance and vehicle length is assumed to be 1 meter and 4.77 meters, respectively (Kimms and Maiwald, 2017). The inflow and outflow capacity Q_i of a cell i is determined by dividing cell length with the sum of safety distance and vehicle length, where safety distance is the distance a vehicle will travel in one second using corresponding free flow speed of that road segment. These capacity values are multiplied by the number of lanes of a given road segment. If a resource is allocated to an intersection, we assume that the capacity of the cells corresponding to the upstream road segments of that intersection will be increased by a fraction (one third in the base case) of their original capacity.

Although the methodology we propose can be used for evacuation against any type of disasters, for the sake of illustration, we implement it for two types of disasters, a convective weather event such as a tornado and a nuclear/biological/chemical (NBC) event. We assume that if SIP order is given for a zone, population at this zone will comply with the order and that the risk of being in a structure such as home is smaller compared to being in a vehicle and that the risk exposed at safe shelters is zero. Dallas-Fort

Worth area is reported to be among the riskiest urban areas in U.S. in case of a large violent tornado (Rae and Stefkovich, 2000). For computation of risk values against tornado, we adopt a similar approach as in Karabuk and Manzour (2019) based on stepwise distances and protection levels "in residence" or "in vehicle" derived by means of tornado fujita scale damage mapping (NCTCoG, 2020). Speed of the tornado is assumed to be 30 miles per hour. For the nuclear/biological/chemical incident, we use risk as a function inversely proportional to the square of the distance from the source of the threat up to 10 miles (NRC, 2020) and the travel speed of the hazard is assumed to be 10 miles per hour in the base case. We assume the impact radius of the NBC incident is immediately in effect after its occurrence. In both cases, risk is valued between 0 and 1, i.e., it represents the probability of injury or possibly death at a given location and time. In this setting lead time (LT) is defined as the amount of time evacuees have to leave the risk zone before disaster hits. Disaster time period span T' is the time when tornado or NBC incident leaves the evacuation network.

We perform our computational tests on a workstation with 2 Xeon Silver 4114 2.20 GHz CPUs, 20 cores, 40 threads and 128 GB RAM by using Java ILOG CPLEX version 12.9. We employ a time limit of two hours to solve the instances. We employ the legacy (lazy constraint) and user cut callback features of Cplex to manage the Benders decomposition algorithm. We check the violation of Benders feasibility (37) and optimality (36) cuts at every fractional and integer point generated by the MPF. The violation of T-feasible valid inequalities (42) is checked at integer points, whereas the violation of DRA valid inequalities (43) is checked only at the root node of the branch and bound tree. At a fractional solution of MPF, if at the root node, first DRA valid inequalities, then Benders feasibility cuts are checked. If there is no violation of feasibility cuts, Benders optimality cuts are checked. Otherwise, if not at the root node, only Benders feasibility and optimality cuts are checked, as described. At an integer solution of MPF, first T-feasible valid inequalities are checked, and if none added, Benders feasibility cuts are checked to guarantee feasibility. Finally, Benders optimality cuts are added. We refer to this algorithm as BD.

We designed our experiments to test the computational efficiency of our algorithms as follows. There are two disaster types, 4 different lead times (0, 15, 30, 45), 6 uniformly distributed demand values between specified lower and upperbounds, three different hazard speeds and impact radii, four different Δ values, and finally 11 different number of resources used.

5.2. Computational Efficiency of the Proposed Algorithm

Next, we discuss the computational efficiency of the Benders decomposition algorithm we proposed in Section 4 and refer to as BD against a commercial solver (Cplex), which we use to solve the time-expanded network flow based evacuation model (NFBEM).

In Tables 2 and 3, we report the number of nodes searched in branch and bound tree not including the root node (# np), number of optimality cuts added at integer (# IntO) and fractional (# FrO) solutions, number of Benders feasibility cuts added at fractional (# FrF) and integer (# IntF) solutions, number of maximum-flow feasibility cuts (T-feasible valid inequalities) added (# MF), number of valid inequalities generated (# VI), and the solution time (Sol.T.) under different experimental settings, where "# Nodes" and "# Arcs" columns represent number of nodes and arcs in the TEN, "DT", "LT", "HTS", "HR", "DLB", "DD", "TD", $|P|$, and "CI" columns represent disaster type, lead time, hazard travel speed, hazard impact radius, demand lower bound and deviation from it to generate random evacuation demand uniformly for each evacuation zone, total evacuation demand (in number of vehicles), number of resources, and capacity index respectively. Capacity index determines the fraction of original capacity that can be used as additional capacity for the road segments if there is a deployed resource at the downstream node. For instance, if $CI = 3$ then Δ is one third of the original capacity of a cell. Under column "Sol.T.", we report the solution time if the instance is solved to optimality before the time limit, and report in parentheses the percent relative gap, otherwise.

In the first group of experiments presented in Table 2, evacuation time span (network clearance time) T is calibrated to a large enough number such that even if all demand from source cells $i \in S_o$ is evacuated with no resource allocation at intersections, the evacuation of everyone to safe shelters will be possible by T. For the second group of experiments presented in Table 3, on the other hand, the same instance is tested for various network clearance times T, and therefore may require adding maximum-flow (T-feasible) or Benders feasibility cuts.

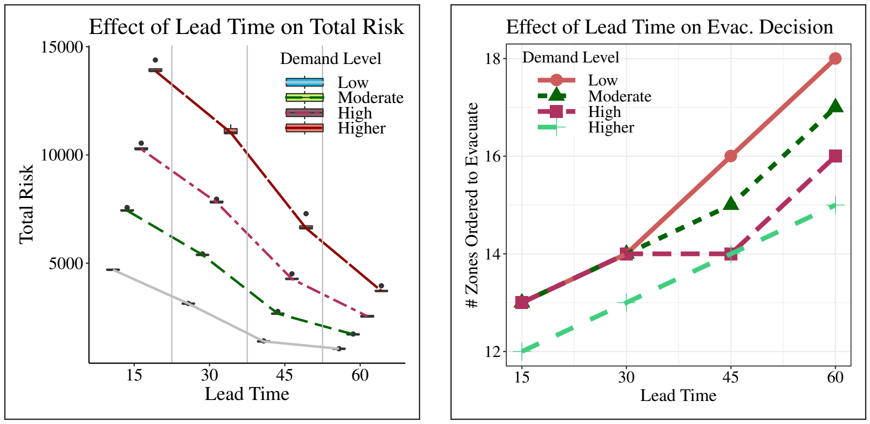
We observe that the setting of the evacuation (disaster type, hazard speed, risk, network clearance times, etc.) has an effect on the solution times. The instances related to the first type of disaster (tornado) are relatively easier and majority of them are solved to optimality by the two algorithms. On the other hand, for most of the disaster type 2 instances (NBC), algorithms hit the time limit. The instances that are solved to optimality, are generally solved at the root node, except for one instance for NFBEM and five instances for BD. This is due to including fractional optimality cuts and the valid inequalities at the root node for BD algorithm. As T and/or $|P|$ increase, the solution times increase, due to the fact that a larger T results in a larger size TEN and larger T and $|P|$ lead to larger number of binary resource allocation decision variables. However, due to the tradeoff between risk and evacuation target time (network clerance time), when T gets smaller for the same instance, solution times (or gaps) generally increase as the algorithm will try to find a solution that evacuates everyone by T at the expense of bigger total risk exposure, which results in a decrease in efficiency. For the NFBEM, out of a total of 66 instances, 5 instances cannot be solved (instances marked with NS - No Solution) and for 20 instances Cplex can only solve the linear programming relaxation at the root node but cannot retrieve an upper bound (integer solution) on the true optimal (instances with 100% relative gap). There are only two instances for BD, for which no solution (NS) is reported. Assuming a solution time of 7,200 seconds for the instances marked with NS or 100% relative gap, the average solution times over all instances are 4,699.16 and 4,815.38 for BD and NFBEM, respectively. Assuming a relative gap of 100% for the instances marked with NS, the average gaps over all instances are 6.49% and 65.33% for BD and NFBEM, respectively, i.e., BD is around 10 times superior compared to Cplex solver in terms of average gaps reported. Among 66 instances reported, there exist only five instances with a gap of 10% or higher for BD. This number is 33 for NFBEM. Actually, for NFBEM, 26 of the instances are reported with a relative gap of 90% or higher. As expected, we observe that as T and/or $|P|$ increase, total number of valid inequalities added increases. We also observe that, for the instances for which adding feasibility cuts may be required (Table 3), Benders feasibility cuts are added only at the fractional solutions. Although maximum-flow cuts (T-feasible valid inequalities) do not guarantee feasibility, no Benders feasibility cuts are added at integer solutions. Next, we present the managerial insights derived.

Table 2: Performance of the proposed algorithms I

Experiment Setting												Benders Decomposition								NFBEM	
T	# Nodes	# Arcs	DT	LT	HTS	HR	DLB	DD	TD	$\|P\|$	CI	# np	# IntO	# FrO	# FrF	# MF	# IntF	# VI	Sol. T.	# np	Sol. T.
160	270,722	495,787	1	0	48	966	1,500	300	30,351	5	3	0	1	0	0	0	0	0	220.26	0	(20.06)
160	270,722	495,787	2	0	16	16,934	1,500	300	30,351	5	3	0	12	17	0	0	0	730	(0.89)	0	(100.00)
160	270,722	495,787	1	15	48	966	1,500	300	30,351	5	3	0	3	0	0	0	0	9	518.05	0	(34.28)
160	270,722	495,787	2	15	16	16,934	1,500	300	30,351	5	3	0	8	8	0	0	0	456	(5.37)	0	(100.00)
160	270,722	495,787	1	30	48	966	1,500	300	30,351	5	3	0	1	0	0	0	0	0	168.99	0	428.49
160	270,722	495,787	2	30	16	16,934	1,500	300	30,351	5	3	0	8	25	0	0	0	1,059	(0.33)	0	(100.00)
160	270,722	495,787	1	45	48	966	1,500	300	30,351	5	3	0	0	0	0	0	0	0	155.28	0	268.53
160	270,722	495,787	2	45	16	16,934	1,500	300	30,351	5	3	0	14	27	0	0	0	635	(0.79)	0	(100.00)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	5	3	0	8	1	0	0	0	0	749.64	0	467.36
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	5	3	0	12	19	0	0	0	872	(0.98)	0	(100.00)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	0	3	0	1	0	0	0	0	0	293.10	0	90.10
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	0	3	0	10	5,379	0	0	0	0	(0.73)	0	142.42
180	304,562	558,007	1	30	48	966	1,800	350	34,851	1	3	686	4	116	0	0	0	16	(6.60)	337	(5.51)
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	1	3	0	12	121	0	0	0	130	(0.90)	0	804.89
180	304,562	558,007	1	30	48	966	1,800	350	34,851	2	3	0	7	7	0	0	0	26	969.35	0	289.50
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	2	3	0	21	54	0	0	0	68	(1.04)	0	(1.43)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	3	3	0	12	33	0	0	0	22	2,880.08	0	380.57
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	3	3	0	20	34	0	0	0	210	(0.97)	0	(1.2)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	4	3	0	9	1	0	0	0	0	665.21	0	450.02
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	4	3	0	16	26	0	0	0	260	(0.92)	0	(1.18)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	6	3	1	8	14	0	0	0	36	3,867.78	NS	NS
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	6	3	0	9	17	0	0	0	1,784	(1.37)	NS	NS
180	304,562	558,007	1	30	48	966	1,800	350	34,851	7	3	0	8	11	0	0	0	30	3,669.14	NS	NS
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	7	3	0	11	13	0	0	0	3,186	(2.07)	NS	NS
180	304,562	558,007	1	30	48	966	1,800	350	34,851	8	3	0	7	1	0	0	0	0	988.20	NS	NS
180	304,562	558,007	1	30	48	966	1,800	350	34,851	9	3	1	6	7	0	0	0	9	3,480.79	0	(100.00)
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	9	3	0	7	11	0	0	0	3,571	(5.30)	0	1,737.97
180	304,562	558,007	1	30	48	966	1,800	350	34,851	10	3	1	6	13	0	0	0	11	5,777.35	0	(1.08)
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	10	3	0	5	11	0	0	0	4,218	(5.25)	0	1,535.41
180	304,562	558,007	2	30	8	16,934	1,800	350	34,851	5	3	0	9	15	0	0	0	978	(0.40)	0	(100.00)
180	304,562	558,007	2	30	32	16,934	1,800	350	34,851	5	3	0	7	8	0	0	0	569	(12.33)	0	(100.00)
180	304,562	558,007	2	30	48	16,934	1,800	350	34,851	5	3	0	10	12	0	0	0	584	(8.32)	0	(100.00)
180	304,562	558,007	2	30	16	9,656	1,800	350	34,851	5	3	0	9	13	0	0	0	363	(1.21)	0	(100.00)
180	304,562	558,007	2	30	16	4,828	1,800	350	34,851	5	3	0	5	10	0	0	0	455	(14.54)	0	(0.87)
180	304,562	558,007	2	30	16	1,609	1,800	350	34,851	5	3	0	0	0	0	0	0	0	589.67	0	331.62
180	304,562	558,007	1	30	48	966	1,800	350	34,851	5	4	0	6	5	0	0	0	22	1,642.52	0	626.19
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	5	4	0	14	19	0	0	0	1,047	(0.78)	0	(100.00)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	5	2	0	7	16	0	0	0	14	3,001.52	0	450.50
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	5	2	0	13	18	0	0	0	1,274	(1.37)	0	(1.18)
180	304,562	558,007	1	30	48	966	1,800	350	34,851	5	1	5	12	4	0	0	0	0	1,638.62	0	529.57
180	304,562	558,007	2	30	16	16,934	1,800	350	34,851	5	1	0	9	10	0	0	0	1,793	(7.83)	0	(1.89)
200	338,402	620,227	1	30	48	966	2,000	400	39,551	5	3	0	3	6	0	0	0	43	1,626.18	0	(73.12)
200	338,402	620,227	2	30	16	16,934	2,000	400	39,551	5	3	0	8	17	0	0	0	1,017	(0.71)	0	(100.00)
220	372,242	682,447	1	30	48	966	2,200	450	43,551	5	3	0	4	10	0	0	0	73	2,839.71	0	(69.00)
220	372,242	682,447	2	30	16	16,934	2,200	450	43,551	5	3	0	10	8	0	0	0	835	(6.03)	0	(100.00)
250	423,002	775,777	1	30	48	966	2,500	500	49,951	5	3	0	3	5	0	0	0	27	1,941.60	0	(38.19)
250	423,002	775,777	2	30	16	16,934	2,500	500	49,951	5	3	0	6	8	0	0	0	264	(8.47)	0	(100.00)
300	507,602	931,327	1	30	48	966	3,000	600	60,351	5	3	0	5	7	0	0	0	42	3,521.87	0	(24.63)
300	507,602	931,327	2	30	16	16,934	3,000	600	60,351	5	3	0	5	4	0	0	0	304	(8.98)	0	(100.00)

Table 3: Performance of the proposed algorithms II

Experiment Setting												Benders Decomposition								NFBEM	
T	**# Nodes**	**# Arcs**	**DT**	**LT**	**HTS**	**HR**	**DLB**	**DD**	**TD**	$\|P\|$	**CI**	**# np**	**# IntO**	**# FrO**	**# FrF**	**# MF**	**# IntF**	**# VI**	**Sol. T.**	**# np**	**Sol. T.**
100	169,202	309,127	1	30	48	966	1,800	350	34,851	5	3	9	16	16	3	10	0	0	1,256.50	0	159.43
100	169,202	309,127	2	30	16	16,934	1,800	350	34,851	5	3	NS	NS	NS	NS	NS	NS	NS	NS	0	(98.16)
110	186,122	340,237	1	30	48	966	1,800	350	34,851	5	3	0	5	9	0	0	0	2	977.44	0	172.25
110	186,122	340,237	2	30	16	16,934	1,800	350	34,851	5	3	NS	NS	NS	NS	NS	NS	NS	NS	0	(100.00)
120	203,042	371,347	1	30	48	966	1,800	350	34,851	5	3	0	7	0	0	0	0	0	549.60	0	173.30
120	203,042	371,347	2	30	16	16,934	1,800	350	34,851	5	3	0	16	40	5	3	0	948	(2.10)	0	(2.02)
130	219,962	402,457	1	30	48	966	1,800	350	34,851	5	3	0	7	27	0	0	0	14	2,653.29	0	215.57
130	219,962	402,457	2	30	16	16,934	1,800	350	34,851	5	3	0	15	38	1	2	0	380	(0.86)	0	(100.00)
140	236,882	433,567	1	30	48	966	1,800	350	34,851	5	3	0	5	1	0	0	0	0	714.67	0	309.55
140	236,882	433,567	2	30	16	16,934	1,800	350	34,851	5	3	0	10	31	1	2	0	1,001	(0.98)	0	(100.00)
150	253,802	464,677	1	30	48	966	1,800	350	34,851	5	3	0	6	3	0	0	0	6	1,088.35	0	296.44
150	253,802	464,677	2	30	16	16,934	1,800	350	34,851	5	3	0	16	29	0	1	0	1,099	(0.99)	0	(100.00)
160	270,722	495,787	1	30	48	966	1,800	350	34,851	5	3	0	8	14	0	0	0	35	2,567.67	0	390.93
160	270,722	495,787	2	30	16	16,934	1,800	350	34,851	5	3	0	14	27	0	1	0	1,235	(1.04)	0	(100.00)
170	287,642	526,897	1	30	48	966	1,800	350	34,851	5	3	0	7	10	0	0	0	25	2,434.80	0	348.95
170	287,642	526,897	2	30	16	16,934	1,800	350	34,851	5	3	0	12	26	0	1	0	1,305	(0.91)	0	(0.93)



(a) Effect of lead time on total risk for various demand and resources levels

(b) Effect of lead time on # of zones ordered to evacuate, $|P| = 0$

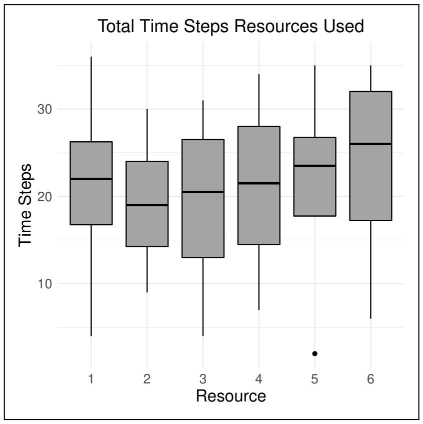
Figure 4: Effect of lead time, NBC case

5.3. Managerial Insights

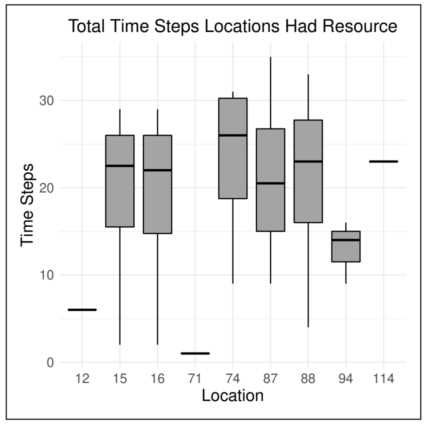
In this section, we investigate the individual and joint effect of dynamic resource allocation and evacuate or SIP decisions on the efficiency of evacuation management. For the tornado case, due to relatively smaller impact radius and the aggregate zone representation of the evacuation demand points, only zones 135, 136, 137, and 138 are evacuated and rest of the zones are ordered to SIP, in all of the instances.

Figure 4 illustrates the effect of lead time on evacuation management for the NBC case. Figure 4a shows how lead time affects the total risk evacuees are exposed to, i.e., as lead time increases the total risk evacuees are exposed to decreases. Naturally, total risk also increases with increasing demand and using resources at critical intersections has a decreasing effect on risk to some degree. As lead time increases, the number of zones ordered to evacuate also increases (see Figure 4b), that is, depending on the population size it becomes possible to evacuate people in more zones to safety with enough lead time from the impact of the disaster. When evacuation demand gets smaller, more zones are ordered to evacuate.

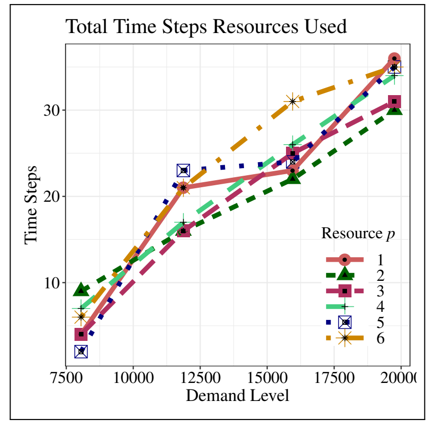
Figure 5 illustrates how the resources are used to manage evacuation for various levels of demand, for the NBC case. In Figures 5a and 5b, we present the number of time periods resources are used and the locations that are allocated a resource, respectively. On average, majority of the resources are used between 20-30 time steps (around 20-25% of T). The locations that receive a resource for at least one time step are: 12, 15, 16, 71, 74, 87, 88, 94, 114, and on average a location has a resource for around 20-25 time steps. The number of time steps resources are allocated to critical intersections highly depends on the demand level (see Figure 5c). As demand level increases resources are used for longer time periods in the same or different locations. In Figure 6, we illustrate dynamic allocation of different resources to the critical intersections of the evacuation road network for NBC case. We observe that different resources can cover the same location over different time intervals and the same resource can come back to the same location it covered in earlier time steps. This is also a good example to show the importance of dynamically moving resources over different locations. When an intersection becomes critical due to congestion, a resource located at a nearby intersection leaves its current location to move to that location,



(a) # of time steps resources are used



(b) # of time steps locations had a resource



(c) # of time steps resources are used

Figure 5: Resource usage, NBC case, LT = 15, $|P| = 6$

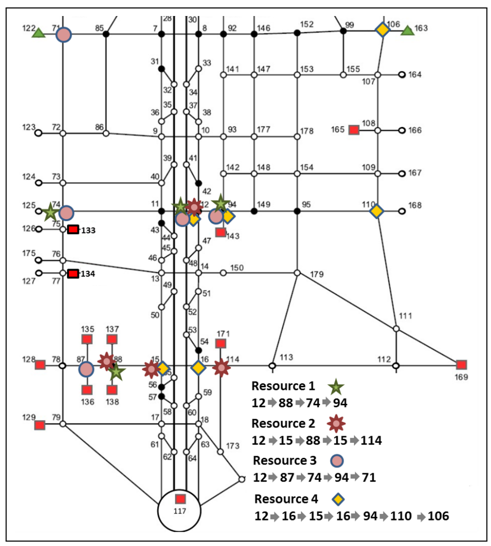
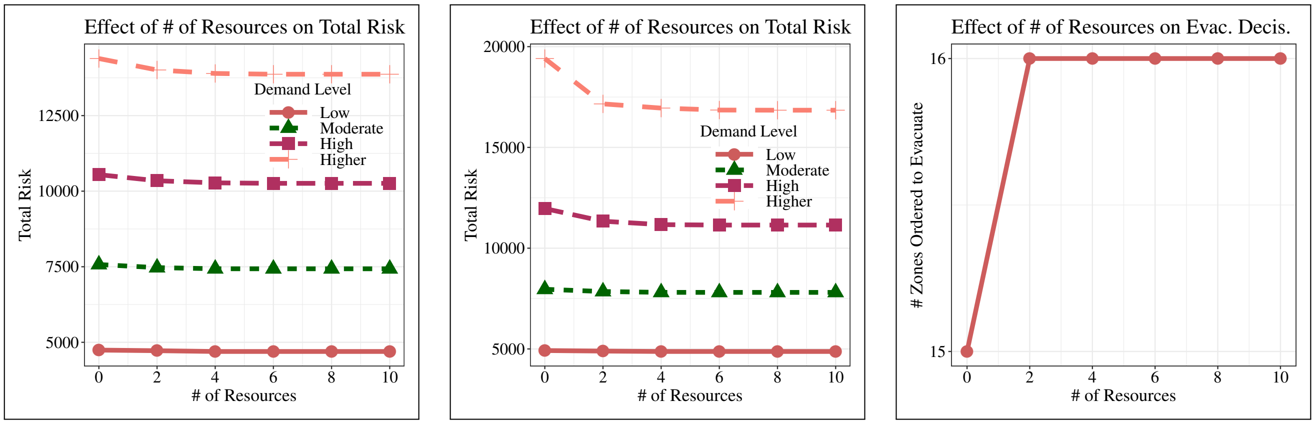


Figure 6: Movement of resources over different locations over time, NBC case, LT = 15, $|P| = 4$

while another resource may fill in the location of that resource that just moved.

Figure 7 demonstrates the effect of using dynamic resources at critical locations over time in the evacuation network. When "Evacuate or SIP" and dynamic resource allocation strategies are employed together, the amount of resources needed decreases compared to the policy where everyone is evacuated. Generally, for high levels of demand, there is a potential benefit of using up to four resources when "Evacuate or SIP" and dynamic resource allocation decisions are jointly optimized, and the benefit obtained from additional resources is marginal (see Figure 7a). There is an improvement in total risk value up to 5% due to DRA. For the policy, where SIP is not considered and all population is evacuated, there is a bigger need for and higher benefit from dynamic resource allocation (see Figure 7b). The improvement in the total risk due to DRA is up to 14% when all zones are evacuated. We observe that the total risk incurred by all population when "Evacuate or SIP" and dynamic resource allocation strategies are employed together with no resources is almost the same as the one incurred under "Evacuate" policy

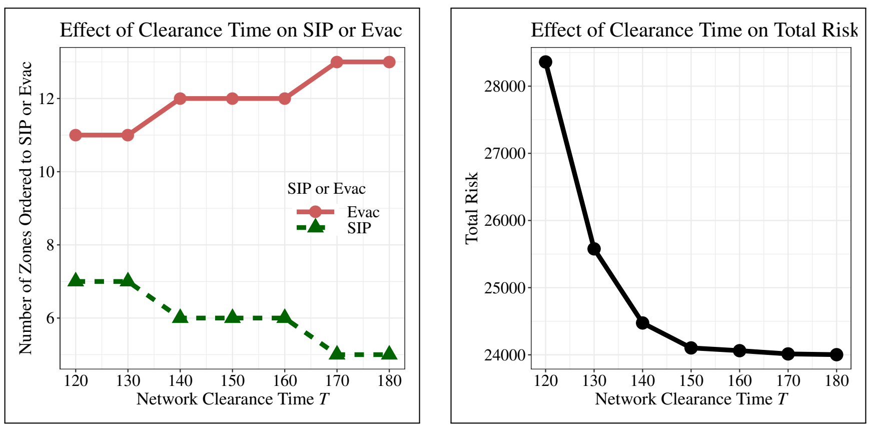


(a) The effect of # of resources on total risk for various demand levels, "Evacuate or SIP" policy, LT = 15

(b) The effect of # of resources on total risk for various demand levels, "Evacuate" policy, LT = 15

(c) The effect of # of resources on # of zones ordered to evacuate, LT = 60, TD = 19,751

Figure 7: Effect of # of resources, NBC case



(a) Effect of clearance time on SIP or evacuation decisions

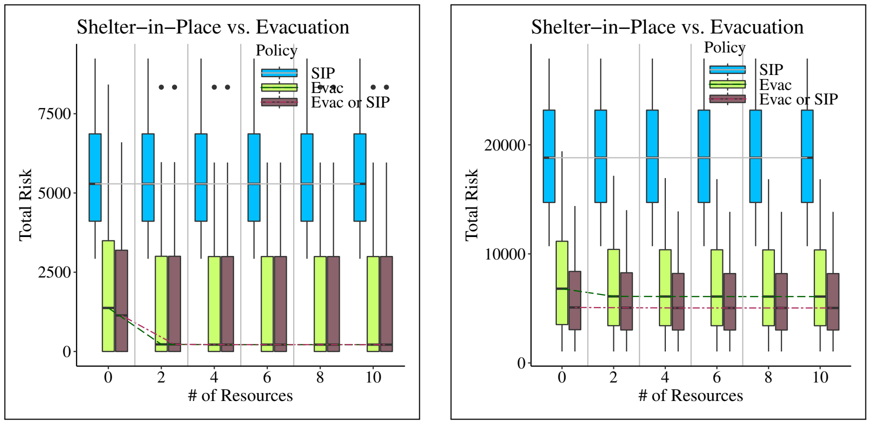
(b) Effect of clearance time on total risk incurred

Figure 8: Effect of network clearance time, NBC Case

with six resources. Generally, we did not observe an effect of number of resources used on the number of zones ordered to evacuate except for a single instance (see Figure 7c), where the number of zones ordered is 15 when there is no resource and becomes 16 when two resources are employed.

Selection of a network clearance time (evacuation target time) by evacuation management authorities is critical. This has a direct relationship with the lead time for the hazard (see Figure 4) and if the evacuation management authorities cannot determine network clearance times correctly, this may have further negative consequences on the evacuees. Figure 8 illustrates the effect of network clearance time (evacuation target time) on evacuate or SIP decisions (see Figure 8a) and total risk incurred (see Figure 8b). As target evacuation time gets larger, evacuation management authorities may tend to and should evacuate a bigger number of zones and should (will have to) order bigger number of zones to SIP, otherwise. Minimizing network clearance time does not necessarily mean to minimize total evacuation risk incurred by the evacuees.

Finally, Figure 9 compares the efficiency of three policies, i.e., "Evacuate", "SIP", and "Zone Based



(a) Policy comparisons, tornado case

(b) Policy comparisons, NBC case

Figure 9: Comparison of “Evacuate”, “SIP”, and “Zone Based Evacuate or SIP” policies

Evacuate or SIP”, for both NBC and tornado cases. It is clear from the figure that “Zone Based Evacuate or SIP” policy is the best policy that minimizes total risk. Dynamic resource allocation to critical intersections helps to reduce risk to some extent.

6. Conclusion

Large scale evacuation of urban areas is a difficult operation to manage and involves complexities that must be tackled. To address these complexities and to promote and effective evacuation plan, zone-based supply and demand management strategies must be jointly considered. Evacuation management agencies generally tend to implement or suggest SIP as the primary option. However, the decision on whether to evacuate or SIP must be carefully given as it requires considering various factors including risk.

In this paper, we introduced models and solution methodologies that jointly consider supply and demand management strategies for an efficient urban evacuation. We proposed a CTM-based based MIP formulation that jointly optimizes evacuate or SIP and staging decisions as demand management strategies and dynamic resource allocation as a supply management strategy. We developed a Benders decomposition based exact solution methodology to solve realistic instances in reasonable times. We used a network flow based formulation on a time-expanded network for the Benders subpoblem and enhanced the effectiveness of the algorithm through time-feasible and DRA valid inequalities. We further conducted extensive numerical experiments to test the effectiveness of the algorithm and to derive managerial insights.

In essence, we found that ordering whole population at risk either to evacuate or to SIP is not the safest approach. Rather, depending on the type and impact of the disaster, an approach that enables controlling and giving individual directions to the population at risk in different zones, as to whether evacuation or SIP is the safer option and if evacuation is the safer option as to the timing of the evacuation must be adopted. As a supply management strategy, dynamic resource allocation helps decreasing congestion in the evacuation network and hence enables faster and safer evacuation of the population at risk, but by itself is not enough. A joint optimization of zone-based evacuate or SIP and dynamic resource allocation decisions yields more efficient evacuation management solutions with minimal total risk incurred and less resources required. As evacuation demand gets bigger, the importance of implementing such a zone-based approach is emphasized. If an evacuation decision is necessary, it is important to give timely evacuation orders. Therefore, depending on the type of the disaster early warning systems play a crucial role for a safe evacuation operation. As lead time increases, the number of zones ordered to evacuate also increases. When there is enough lead time, evacuation should be the preferred option if disaster management agencies are seeking to minimize total risk rather than cost.

The problem setting can be extended in a variety of ways. In this study, we assumed a deterministic setting with perfect information on evacuation demand, road network capacity, and the track and impact of the disaster event. However, there is significant uncertainty that must be accounted for and a scenario-based two/multi-stage stochastic programming approach can be employed to accommodate for it. Also, other supply and demand management strategies can be incorporated in the mathematical formulation.

Acknowledgment

We thank the authors of He et al. (2015) for providing us with the data of Dallas Fort-Worth road network.

Appendices

A. Description of the Notation Used

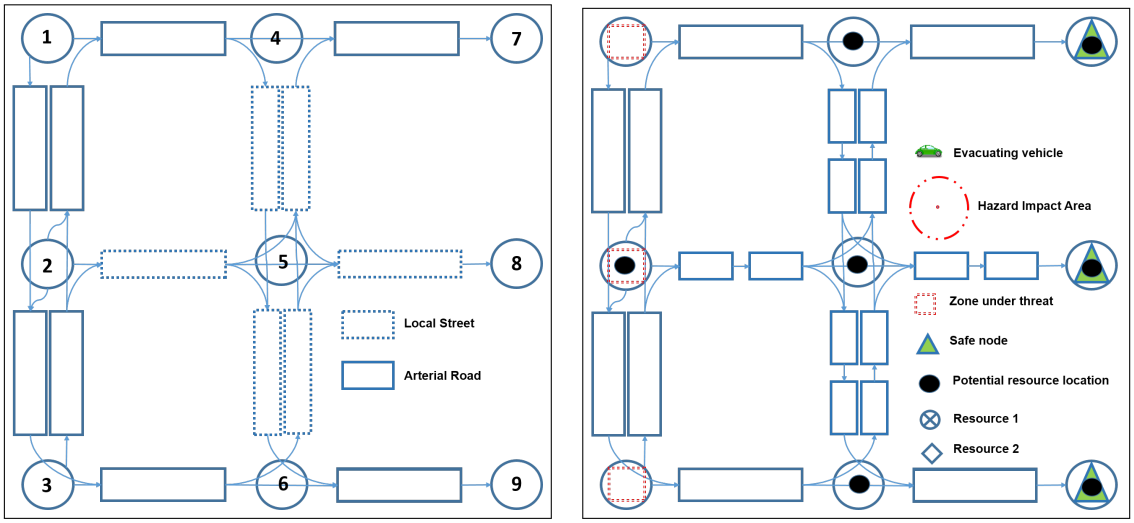
B. A small-scale illustrative toy instance

We conducted an experiment on a small-scale toy instance to illustrate the solution generated by our model. In this experiment, we assumed that each arterial road and local street in the road network has a speed limit, also known as the free flow speed, of 72 km/h and 36 km/h, respectively. We adopted a time step length of 2 seconds and an evacuation duration of $|T| = 10$ time steps. To avoid fractional capacity values and simplify the illustration, we used a vehicle length of 5 meters and a traffic jam distance of 0 meters. Consequently, the arterial road segments in the resulting cell network consisted of a single cell, each with a length of 40 meters, a physical capacity N_i of 8, and an outflow/inflow capacity Q_i of 2 vehicles. The local streets, on the other hand, were made up of two cells, each 20 meters in length, with a physical capacity N_i of 4 and an outflow/inflow capacity Q_i of 1 vehicle. The symbolic hazard used in the experiment had a radius of 50 meters and moved at a speed of 5 km/h in a west-east direction. We computed the risk values based on their inversely proportional distance squared from the hazard within the impact area, with no risk incurred outside the impact area. Figure 10 depicts the original road network and its corresponding cell network.

Figure 11 displays the temporal evolution of the solution to the toy instance over ten time steps, $t = 1$ through $t = 10$, when there are two deployable resources. While zones 1 and 2 are given evacuation orders, zone 3 is directed to shelter-in-place as there is no direct threat to it. Evacuation zone 2 has the highest evacuation demand (two times that of zone 1), however its evacuees cannot connect to the arterial road in the north since the capacity of these roads are already consumed by evacuees of zone 1. Instead, zone

Table 4: Description of the notation used

Sets	
$G^o = (V, A^o)$	Original evacuation road network
V	Set of nodes of the original network
A^o	Set arcs (road segments, links) of the original network
$G = (I, A)$	Transformed cell network
I	Set of cells in the cell network
A	Set of arcs (cell connectors) in the cell network
S_o	Set of source cells (evacuation demand zones)
S_e	Set of sink cells (safe shelters)
R	Set of road segment cells
$T^{'}$	Set of disaster time periods
$T \subset T^{'}$	Set of evacuation time periods
P	Set of resources
$V' \subseteq V$	Set of potential nodes where resources can be allocated
I_n	Set of cells associated with road segments for whom $n \in V'$ is a downstream node
R'	Union of cells associated with road segments corresponding to different downstream nodes $n \in V'$, i.e., $R' = \cup_{n \in V'} I_n \cap R$
$G' = (I', A')$	Time expanded network
I'	Set of nodes in time expanded network, i.e., $I' = \{(i,t) : i \in I, t \in T\}$
A'	Set of arcs in time expanded network, i.e., $A' = \{(i, t-1), (j, t) : (i, j) \in A \text{ or } i = j, t \in T \setminus \{1\}\}$
$\hat{G} = (\hat{I}, \hat{A})$	Modified time expanded network
$\hat{I}$	Set of nodes in modified time expanded network
$\hat{A}$	Set of arcs in modified time expanded network
$\tilde{A} \subset \hat{A}$	Union of arcs a in modified TEN, in the three-node-sub-network of road segment type cell $i \in R'$, over time replications $t \in T$, with capacity $q_a = Q_i$
$\bar{A} \subset \hat{A}$	Union of arcs a in modified TEN, in the three-node-sub-network of road segment type cell $i \in R'$, over time replications $t \in T$, with capacity $q_a = N_i$
A^{so}	$A^{so} = \left\{(so, i^1) \in \hat{A} : i \in S_o\right\}$
$n(i)$	Node n associated with cell i, i.e., the junction which increases the capacity of cell i by Δ_i if a resource p is allocated to it
$i(a)$	Cell $i \in R'$ associated with tail node of arc $a \in \hat{A}$
$t(a)$	Time $t \in T$ associated with tail node of arc $a \in \hat{A}$
i_k^t	k^{th} node in the three-node-subnetwork of t^{th} copy of roadway segment cell $i \in R$, $k = A, B, C$
Parameters	
D_i	Number of vehicles to be evacuated from source cell $i \in S_o$
N_i	Maximum number of vehicles that can be accommodated in cell $i \in R$
δ_i	Scaling factor, the ratio of the free-flow speed to the backward propagation speed for cell $i \in R$
Q_i	Maximum number of vehicles that can enter into or leave from, i.e., the inflow/outflow capacity of cell $i \in R$
c_{it}	Estimated hazard level for cell i in period $t \in T^{'}$
Δ_i	Increment in the inflow and outflow capacity of cell $i \in I_n \subset I$, in period $t \in T$ if resource $p \in P$ is allocated to this location
$\tau_{pnn'}$	Time required for resource p to travel from node $n \in V'$ to node $n' \in V'$
c_a	Risk level on arc $a \in \hat{A}$
q_a	Capacity value on arc $a \in \hat{A}$
$\Gamma^+(k)$	Set of outgoing arcs $a \in \hat{A}$ of node $k \in \hat{I}$ in TEN
$\Gamma^-(k)$	Set of incoming arcs $a \in \hat{A}$ of node $k \in \hat{I}$ in TEN
Variables	
x_{it}	Number of vehicles at cell $i \in I$ in time $t \in T$
x	Representation of x_{it}, $i \in I$, $t \in T$ in vector form
y_{ijt}	Number of vehicles traveling from cell $i \in I$ to cell $j \in I$ in time $t \in T \setminus \{\lvert T \rvert\}$
y	Representation of y_{ijt}, $i \in I$, $j \in I$, $t \in T$ in vector form
z_{npt}	Binary decision variable that takes the value of 1, if resource $p \in P$ is allocated at node $n \in V'$ at time $t \in T$ and 0 otherwise
$\bar{z}_{npt}$	Fixed value of z_{npt} in Benders subproblem generated by a given solution of MP
z	Representation of z_{npt}, $p \in P$, $n \in V'$, $t \in T$ in vector form
$\bar{z}$	Fixed value of z in Benders subproblem generated by a given solution of MP
e_i	Binary decision variable that takes the value of 1, if we decide to evacuate source cell (zone) i and 0, if we make an SIP decision for that cell
$\bar{e}_i$	Fixed value of e_i in Benders subproblem generated by a given solution of MP
e	Representation of e_i, $i \in S_o$ in vector form
$\bar{e}$	Fixed value of e in Benders subproblem generated by a given solution of MP
f_a	Number of vehicles on arc $a \in \hat{A}$



(a) Original Road Network

(b) Cell Network

Figure 10: Toy Example Road Network and Symbols

2 evacuees can use local streets towards the east or the arterial road in the south, alternatively. However, the limited outflow/inflow capacity of the local streets poses a hindrance to the evacuation process. To overcome this, dynamic resources 2 and 1 are deployed to critical intersections 5 and 8 at time steps 4 and 6, respectively, to increase outflow/inflow capacities of corresponding upstream cells of local streets.

C. A Time Expanded Network Flow-Based Formulation for Benders Subproblem

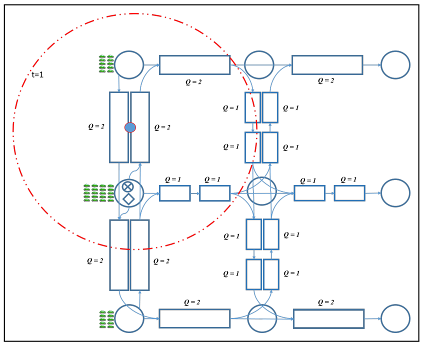
Let $f_{(i,t),(j,t+1)}$ denote the flow on arc $(i,t),(j,t+1) \in A'$. We use the equivalences $y_{ijt} = f_{(i,t),(j,t+1)}$, for $(i,j) \in A$ and $t \in T \setminus \{|T|\}$ and $x_{it} = \sum_{j:((i,t),(j,t+1)) \in A'} f_{(i,t),(j,t+1)}$, for $i \in I$ and $t \in T \setminus \{|T|\}$, to obtain TEN-based formulation. Below is a description of how we make this transformation for each constraint.

For $i \in I$ and $t \in T \setminus \{1\}$, the right hand side of the flow conservation constraint is :

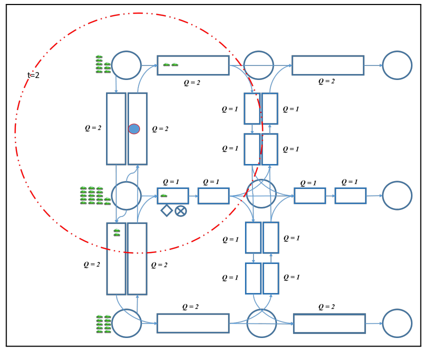
$$
\begin{aligned}
x_{i,t-1} + \sum_{j:(j,i)\in A} y_{ji,t-1} - \sum_{j:(i,j)\in A} y_{ij,t-1} &= \sum_{j:((i,t-1),(j,t))\in A'} f_{(i,t-1),(j,t)} + \sum_{j:(j,i)\in A} f_{(j,t-1),(i,t)} - \sum_{j:(i,j)\in A} f_{(i,t-1),(j,t)} \\
&= f_{(i,t-1),(i,t)} + \sum_{j:(j,i)\in A} f_{(j,t-1),(i,t)} \\
&= \sum_{j:((j,t-1),(i,t))\in A'} f_{(j,t-1),(i,t)}.
\end{aligned}
$$

Consequently, for $i \in I$ and $t \in T \setminus \{1, |T|\}$, the flow conservation constraint becomes:

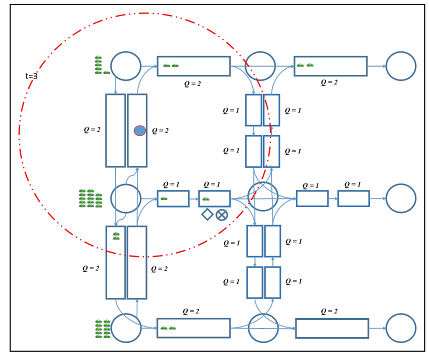
$$
\sum_{j:((i,t),(j,t+1))\in A'} f_{(i,t),(j,t+1)} = \sum_{j:((j,t-1),(i,t))\in A'} f_{(j,t-1),(i,t)}
$$



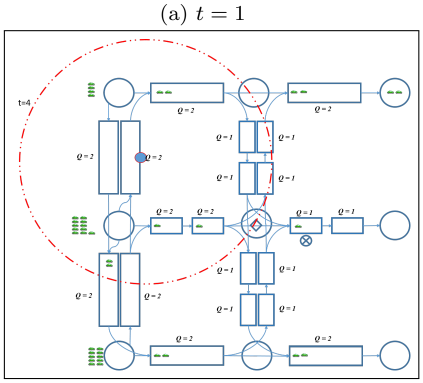
(a) $t = 1$



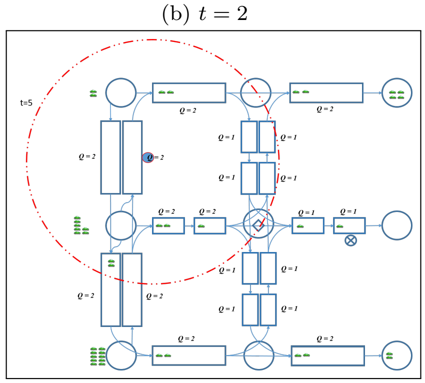
(b) $t = 2$



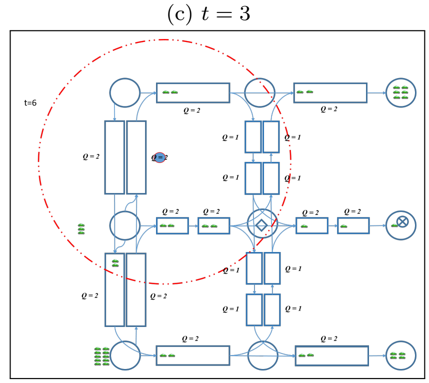
(c) $t = 3$



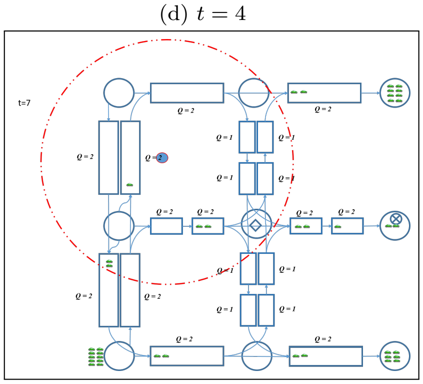
(d) $t = 4$



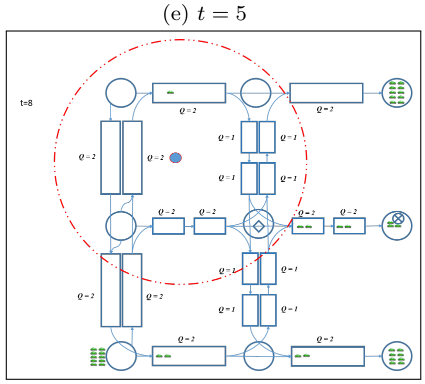
(e) $t = 5$



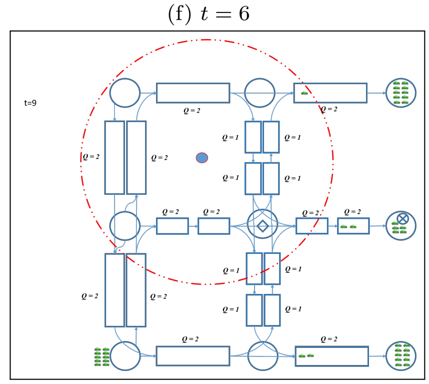
(f) $t = 6$



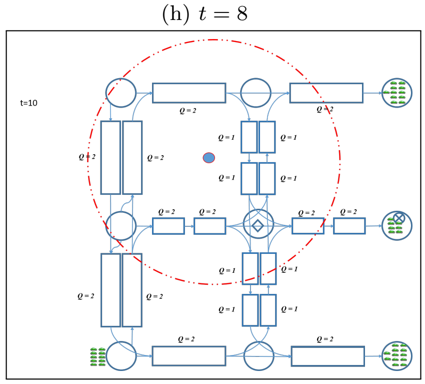
(g) $t = 7$



(h) $t = 8$



(i) $t = 9$



(j) $t = 10$

Figure 11: Toy Example Solution

For $i \in I \setminus S_e$, $x_{i,|T|}$ must be zero for feasibility. Hence the flow conservation constraint for $t = |T|$ is

$$0 = \sum_{j:((j,|T|-1),(i,|T|)) \in A'} f_{(j,|T|-1),(i,|T|)}.$$

For $i \in S_e$, we have $x_{i,|T|} = \sum_{j:((j,|T|-1),(i,|T|)) \in A'} f_{(j,|T|-1),(i,|T|)}$. We substitute this in constraint (20), which becomes

$$\sum_{i \in S_e} \sum_{j:((j,|T|-1),(i,|T|)) \in A'} f_{(j,|T|-1),(i,|T|)} = \sum_{i \in S_o} D_i \bar{e}_i$$

Finally, constraint (5) for $i \in I$ and $t \in T \setminus \{|T|\}$ becomes

$$\sum_{j:(i,j) \in A} f_{(i,t),(j,t+1)} \leq \sum_{j:((i,t),(j,t+1)) \in A'} f_{(i,t),(j,t+1)}$$

which is the same as $f_{(i,t),(i,t+1)} \geq 0$. Below we present TEN-based formulation we obtain:

$$\min \sum_{i \in I} \sum_{t \in T \setminus \{|T|\}} c_{it} \sum_{j:((i,t),(j,t+1)) \in A'} f_{(i,t),(j,t+1)} \tag{48}$$

$$\text{s.t.} \quad \sum_{j:((i,t),(j,t+1)) \in A'} f_{(i,t),(j,t+1)} = \sum_{j:((j,t-1),(i,t)) \in A'} f_{(j,t-1),(i,t)} \qquad i \in I, t \in T \setminus \{1, |T|\} \tag{49}$$

$$\sum_{j:((j,|T|-1),(i,|T|)) \in A'} f_{(j,|T|-1),(i,|T|)} = 0 \qquad i \in I \setminus S_e \tag{50}$$

$$\sum_{i \in S_e} \sum_{j:((j,|T|-1),(i,|T|)) \in A'} f_{(j,|T|-1),(i,|T|)} = \sum_{i \in S_o} D_i \bar{e}_i \tag{51}$$

$$\sum_{j:(j,i) \in A} f_{(j,t),(i,t+1)} \leq \delta_i \left(N_i - \sum_{j:((i,t),(j,t+1)) \in A'} f_{(i,t),(j,t+1)} \right) \qquad i \in R, t \in T \setminus \{|T|\} \tag{52}$$

$$\sum_{j:(j,i) \in A} f_{(j,t),(i,t+1)} \leq Q_i \qquad i \in R \setminus R', t \in T \setminus \{|T|\} \tag{53}$$

$$\sum_{j:(i,j) \in A} f_{(i,t),(j,t+1)} \leq Q_i \qquad i \in R \setminus R', t \in T \setminus \{|T|\} \tag{54}$$

$$\sum_{j:(j,i) \in A} f_{(j,t),(i,t+1)} \leq Q_i + \sum_{p \in P} \Delta_i \bar{z}_{n(i)pt} \qquad i \in R', t \in T \setminus \{|T|\} \tag{55}$$

$$\sum_{j:(i,j) \in A} f_{(i,t),(j,t+1)} \leq Q_i + \sum_{p \in P} \Delta_i \bar{z}_{n(i)pt} \qquad i \in R', t \in T \setminus \{|T|\} \tag{56}$$

$$\sum_{j:((i,1),(j,2)) \in A'} f_{(i,1),(j,2)} = D_i \bar{e}_i \qquad i \in I \tag{57}$$

$$f_{(i,t),(j,t+1)} \geq 0 \qquad (i,t),(j,t+1) \in A' \tag{58}$$

References

Ahuja, R.K., Magnanti, T.L., Orlin, J.B., 1988. Network flows. Cambridge, Mass.: Alfred P. Sloan School of Management, Massachusetts ....

Apivatanagul, P., Davidson, R.A., Nozick, L.K., 2012. Bi-level optimization for risk-based regional hurricane evacuation planning. Natural Hazards 60, 567–588.

Bayram, V., 2016. Optimization models for large scale network evacuation planning and management: A literature review. Surveys in Operations Research and Management Science 21, 63–84.

Ben-Tal, A., Do Chung, B., Mandala, S.R., Yao, T., 2011. Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains. Transportation Research Part B: Methodological 45, 1177–1189.

Benders, J., 1962. Partitioning procedures for solving mixed-variables programming problems. Numerische Mathematik 4, 238–252.

Bish, D.R., Sherali, H.D., 2013. Aggregate-level demand management in evacuation planning. European Journal of Operational Research 224, 79–92.

Bish, D.R., Sherali, H.D., Hobeika, A.G., 2014. Optimal evacuation planning using staging and routing. Journal of the Operational Research Society 65, 124–140.

Blanton, B., Dresback, K., Colle, B., Kolar, R., Vergara, H., Hong, Y., Leonardo, N., Davidson, R., Nozick, L., Wachtendorf, T., 2020. An integrated scenario ensemble-based framework for hurricane evacuation modeling: Part 2−hazard modeling. Risk Analysis 40, 117–133.

Bretschneider, S., Kimms, A., 2011. A basic mathematical model for evacuation problems in urban areas. Transportation Research Part A: Policy and Practice 45, 523–539.

Bretschneider, S., Kimms, A., 2012. Pattern-based evacuation planning for urban areas. European Journal of Operational Research 216, 57–69.

Chiu, Y.C., Zheng, H., Villalobos, J., Gautam, B., 2007. Modeling no-notice mass evacuation using a dynamic traffic flow optimization model. IIE Transactions 39, 83–94.

Church, R.L., Cova, T.J., 2000. Mapping evacuation risk on transportation networks using a spatial optimization model. Transportation Research Part C: Emerging Technologies 8, 321–336.

Cova, T.J., Dennison, P.E., Drews, F.A., 2011. Modeling evacuate versus shelter-in-place decisions in wildfires. Sustainability 3, 1662–1687.

Cova, T.J., Drews, F.A., Siebeneck, L.K., Musters, A., 2009. Protective actions in wildfires: evacuate or shelter-in-place? Natural Hazards Review 10, 151–162.

Cova, T.J., Johnson, J.P., 2003. A network flow model for lane-based evacuation routing. Transportation Research Part A: Policy and Practice 37, 579–604.

Daganzo, C.F., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. Transportation Research Part B: Methodological 28, 269–287.

Daganzo, C.F., 1995. The cell transmission model, part ii: Network traffic. Transportation Research Part B: Methodological 29, 79–93.

Davidson, R.A., Nozick, L.K., Wachtendorf, T., Blanton, B., Colle, B., Kolar, R.L., DeYoung, S., Dresback, K.M., Yi, W., Yang, K., et al., 2020. An integrated scenario ensemble-based framework for hurricane evacuation modeling: Part 1−decision support system. Risk Analysis 40, 97–116.

DHS, 2019. National response framework. Department of Homeland Security URL: `https://www.fema.gov/sites/default/files/2020-04/NRF_FINALApproved_2011028.pdf`.

Dosa, D.M., Grossman, N., Wetle, T., Mor, V., 2007. To evacuate or not to evacuate: lessons learned from louisiana nursing home administrators following hurricanes katrina and rita. Journal of the American Medical Directors Association 8, 142–149.

EC, 2017. Overview of natural and man-made disaster risks the european union may face. European Commission URL: https://ec.europa.eu/echo/sites/echo-site/files/swd_2017_176_overview_of_risks_2.pdf.

EC, 2019. European disaster risk management. European Commission, European Civil Protection and Humanitarian Aid Operations URL: https://ec.europa.eu/echo/files/aid/countries/factsheets/thematic/disaster_risk_management_en.pdf.

EM-DAT, 2020a. Cred crunch 58 - disaster year in review (2019). Centre for Research on the Epidemiology of Disasters (CRED) .

EM-DAT, 2020b. Natural disasters trends, the international disaster database. Centre for Research on the Epidemiology of Disasters (CRED) .

Esposito Amideo, A., Scaparra, M.P., Sforza, A., Sterle, C., 2021. An integrated user-system approach for shelter location and evacuation routing. Networks .

FEMA, 2019. National engagement - planning considerations: Evacuation and shelter-in-place. Federal Emergency Management Agency URL: https://www.fema.gov/media-library-data/1564165488078-09ab4aac641f77fe7b7dd30bad21526b/Planning_Considerations_Evacuation_and_Shelter-in-Place.pdf.

FHWA, 2007. Using highways for no-notice evacuations. Federal Highway Administration, Routes to Effective Evacuation Planning Primer Series FHWA-HOP-08-003 URL: https://www.fema.gov/media-library-data/1564165488078-09ab4aac641f77fe7b7dd30bad21526b/Planning_Considerations_Evacuation_and_Shelter-in-Place.pdf.

Halverson, J.B., 2018. The costliest hurricane season in us history. Weatherwise 71, 20–27.

Hasan, M.H., Van Hentenryck, P., 2020a. Large-scale zone-based evacuation planning–part i: Models and algorithms. Networks 77, 127–146.

Hasan, M.H., Van Hentenryck, P., 2020b. Large-scale zone-based evacuation planning, part ii: Macroscopic and microscopic evaluations. Networks .

Haynes, K., Coates, L., Leigh, R., Handmer, J., Whittaker, J., Gissing, A., McAneney, J., Opper, S., 2009. 'shelter-in-place'vs. evacuation in flash floods. Environmental Hazards 8, 291–303.

He, X., Peeta, S., 2014. Dynamic resource allocation problem for transportation network evacuation. Networks and Spatial Economics 14, 505–530.

He, X., Zheng, H., Peeta, S., 2015. Model and a solution algorithm for the dynamic resource allocation problem for large-scale transportation network evacuation. Transportation Research Part C: Emerging Technologies 59, 233–247.

Herrera-Restrepo, O., Triantis, K., Trainor, J., Murray-Tuite, P., Edara, P., 2016. A multi-perspective dynamic network performance efficiency measurement of an evacuation: A dynamic network-dea approach. Omega 60, 45–59.

Hsu, Y.T., Peeta, S., 2014a. Behavior-consistent information-based network traffic control for evacuation operations. Transportation Research Part C: Emerging Technologies 48, 339–359.

Hsu, Y.T., Peeta, S., 2014b. Risk-based spatial zone determination problem for stage-based evacuation operations. Transportation Research Part C: Emerging Technologies 41, 73–89.

Jabari, S.E., He, X., Liu, H.X., 2012. Heuristic solution techniques for no-notice emergency evacuation traffic management, in: Network Reliability in Practice. Springer, pp. 241–259.

Karabuk, S., Manzour, H., 2019. A multi-stage stochastic program for evacuation management under tornado track uncertainty. Transportation Research Part E: Logistics and Transportation Review 124, 128–151.

Kimms, A., Maassen, K.C., 2012. Cell-transmission-based evacuation planning with rescue teams. Journal of Heuristics 18, 435–471.

Kimms, A., Maiwald, M., 2017. An exact network flow formulation for cell-based evacuation in urban areas. Naval Research Logistics (NRL) 64, 547–555.

Kimms, A., Maiwald, M., 2018. Bi-objective safe and resilient urban evacuation planning. European Journal of Operational Research 269, 1122–1136.

Kuligowski, E., 2021. Evacuation decision-making and behavior in wildfires: Past research, current challenges and a future research agenda. Fire Safety Journal 120, 103129.

Li, A.C., Xu, N., Nozick, L., Davidson, R., 2011. Bilevel optimization for integrated shelter location analysis and transportation planning for hurricane events. Journal of Infrastructure Systems 17, 184–192.

Li, J., Ozbay, K., 2015. Evacuation planning with endogenous transportation network degradations: a stochastic cell-based model and solution procedure. Networks and Spatial Economics 15, 677–696.

Lim, G.J., Zangeneh, S., Baharnemati, M.R., Assavapokee, T., 2012. A capacitated network flow optimization approach for short notice evacuation planning. European Journal of Operational Research 223, 234–245.

Lindell, M.K., Murray-Tuite, P., Wolshon, B., Baker, E.J., 2018. Large-scale evacuation: the analysis, modeling, and management of emergency relocation from hazardous areas. CRC Press.

Liu, J., Jiang, R., Li, X., Jia, B., Liu, Z., Bouadi, M., 2021. Lane-based multi-class vehicle collaborative evacuation management. Transportmetrica B: Transport Dynamics , 1–23.

Liu, Y., 2007. An integrated optimal control system for emergency evacuation. Ph.D. thesis.

Matherly, D., 2013. A Transportation Guide for All-Hazards Emergency Evacuation. volume 740. Transportation Research Board.

McCaffrey, S., Rhodes, A., Stidham, M., 2015. Wildfire evacuation and its alternatives: perspectives from four united states' communities. International Journal of Wildland Fire 24, 170–178.

Murray-Tuite, P., Wolshon, B., 2013. Evacuation transportation modeling: An overview of research, development, and practice. Transportation Research Part C: Emerging Technologies 27, 25–45.

NCTCoG, 2020. Tornado damage risk assessment. North Central Texas Council of Governments URL: https://www.nctcog.org/regional-data/regional-data-center/tornado-damage-risk-assessment.

Ng, M., Waller, S.T., 2010. Reliable evacuation planning via demand inflation and supply deflation. Transportation Research Part E: Logistics and Transportation Review 46, 1086–1094.

NRC, 2020. Emergency preparedness and response, about emergency preparedness. U.S. Nuclear Regulatory Commission URL: https://www.nrc.gov/about-nrc/emerg-preparedness/about-emerg-preparedness/planning-zones.html.

NWS, 2019. Weather related fatality and injury statistics. National Weather Service URL: https://www.weather.gov/hazstat/.

O'Driscoll, P., Wolf, R., Hampson, R., 2005. The evacuation worked, but created a highway horror. USA Today URL: https://usatoday30.usatoday.com/news/nation/2005-09-25-evacuation-cover_x.htm.

Parr, S.A., Wolshon, B., Dixit, V., 2015. Selection and allocation of manual traffic control points and personnel during emergencies. Journal of Emergency Management (Weston, Mass.) 13, 121–133.

Parr, S.A., Wolshon, B., Murray-Tuite, P., 2016. Unconventional intersection control strategies for urban evacuation. Transportation research record 2599, 52–62.

Pillac, V., Cebrian, M., Van Hentenryck, P., 2015. A column-generation approach for joint mobilization and evacuation planning. Constraints 20, 285–303.

Pillac, V., Van Hentenryck, P., Even, C., 2016. A conflict-based path-generation heuristic for evacuation planning. Transportation Research Part B: Methodological 83, 136–150.

Pretorius, P., Anderson, S., Akwabi, K., Crowther, B., Ye, Q., Houston, N., Vann Easton, A., et al., 2006. Evacuation transportation management. Task five, Operational concept. Technical Report. United States. Federal Highway Administration.

Pyakurel, U., Dempe, S., 2020. Network flow with intermediate storage: models and algorithms, in: SN Operations Research Forum, Springer. pp. 1–23.

Pyakurel, U., Khanal, D.P., Dhamala, T.N., 2023. Abstract network flow with intermediate storage for evacuation planning. European Journal of Operational Research 305, 1178–1193.

Rae, S., Stefkovich, J., 2000. The tornado damage risk assessment predicting the impact of a big outbreak in dallas–fort worth, texas, in: Preprints, 20th Conf. on Severe Local Storms, Orlando, FL, Amer. Meteor. Soc, Citeseer.

Simmons, K.M., Sutter, D., 2012. The 2011 tornadoes and the future of tornado research. Bulletin of the American Meteorological Society 93, 959–961.

Sorensen, J.H., Shumpert, B.L., Vogt, B.M., 2004. Planning for protective action decision making: evacuate or shelter-in-place. Journal of Hazardous Materials 109, 1–11.

Stepanov, A., Smith, J.M., 2009. Multi-objective evacuation routing in transportation networks. European Journal of Operational Research 198, 435–446.

Thompson, R.R., Garfin, D.R., Silver, R.C., 2017. Evacuation from natural disasters: a systematic review of the literature. Risk Analysis 37, 812–839.

Tuydes-Yaman, H., Ziliaskopoulos, A., 2014. Modeling demand management strategies for evacuations. Annals of Operations Research 217, 491–512.

UNDESA, 2015. Sustainable development goals. United Nations Department of Economic and Social Affairs URL: `https://sdgs.un.org/goals`.

UNISDR, 2015. Sendai framework for disaster risk reduction 2015–2030. United Nations Office for Disaster Risk Reduction URL: `https://www.unisdr.org/files/43291_sendaiframeworkfordrren.pdf`.

USDHS, 2013. National response framework. Department of Homeland Security, W.D. .

Üster, H., Wang, X., Yates, J.T., 2018. Strategic evacuation network design (send) under cost and time considerations. Transportation Research Part B: Methodological 107, 124–145.

Wilmot, C.G., Meduri, N., 2005. Methodology to establish hurricane evacuation zones. Transportation Research Record 1922, 129–137.

Wolshon, B., Pande, A., et al., 2016. Traffic engineering handbook. John Wiley & Sons.

Wolshon, B., Urbina, E., Wilmot, C., Levitan, M., 2005a. Review of policies and practices for hurricane evacuation. i: Transportation planning, preparedness, and response. Natural Hazards Review 6, 129–142.

Wolshon, B., Urbina Hamilton, E., Levitan, M., Wilmot, C., 2005b. Review of policies and practices for hurricane evacuation. ii: Traffic operations, management, and control. Natural Hazards Review 6, 143–161.

Xie, C., Turnquist, M.A., 2009. Integrated evacuation network optimization and emergency vehicle assignment. Transportation Research Record 2091, 79–90.

Yang, K., Davidson, R.A., Blanton, B., Colle, B., Dresback, K., Kolar, R., Nozick, L.K., Trivedi, J., Wachtendorf, T., 2019a. Hurricane evacuations in the face of uncertainty: Use of integrated models to support robust, adaptive, and repeated decision-making. International Journal of Disaster Risk Reduction 36, 101093.

Yang, K., Davidson, R.A., Vergara, H., Kolar, R.L., Dresback, K.M., Colle, B.A., Blanton, B., Wachtendorf, T., Trivedi, J., Nozick, L.K., 2019b. Incorporating inland flooding into hurricane evacuation decision support modeling. Natural Hazards 96, 857–878.

Yi, W., Nozick, L., Davidson, R., Blanton, B., Colle, B., 2017. Optimization of the issuance of evacuation orders under evolving hurricane conditions. Transportation Research Part B: Methodological 95, 285–304.

Zhang, X., He, R., Shi, Q., Ban, J., Ran, B., 2013. Critical traffic control locations for emergency evacuation. Journal of Transportation Engineering 139, 1030–1038.

Zhao, X., Ren, G., Huang, Z.f., 2016. Optimizing one-way traffic network reconfiguration and lane-based non-diversion routing for evacuation. Journal of Advanced Transportation 50, 589–607.

Zheng, H., Chiu, Y.C., Mirchandani, P.B., 2015. On the system optimum dynamic traffic assignment and earliest arrival flow problems. Transportation Science 49, 13–27.

Ziliaskopoulos, A.K., 2000. A linear programming model for the single destination system optimum dynamic traffic assignment problem. Transportation Science 34, 37–49.

Zimmerman, C., Brodesky, R., Karp, J., et al., 2007a. Using highways for no-notice evacuations: routes to effective evacuation planning primer series. Technical Report. United States. Federal Highway Administration. Office of Operations.

Zimmerman, C., Robert, B., Jordan, K., 2007b. Using Highways During Evacuation Operations for Events with Advance Notice. Technical Report. Publication FHWA-HOP-08-003. FHWA, US Department of Transportation.