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Decentralised sliding mode control for nonlinear interconnected systems with uncertainties

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ABSTRACT

With the advances in science and technology, nonlinear large-scale interconnected systems have widely appeared in the real life. Traditional centralised control methods have inevitable disadvantages when they are used to deal with complex nonlinear interconnected systems with uncertainties. In connection with this, people desire to develop the novel control strategy which can be applied to complex interconnected systems. Therefore, decentralised sliding mode control (SMC) for interconnected systems has attracted great attention in related fields due to its advantages, for instance, simple structure, low cost of calculation, fast response, reduced-order sliding mode dynamics and insensitivity to matched variation of parameters and disturbances in systems.

This thesis focuses on the development of decentralised SMC for nonlinear interconnected systems with uncertainties under certain assumptions. Several methods and different techniques have been considered in design of the controller to improve the robustness. The main contributions of this thesis include:

- The state feedback decentralised SMC is developed for nonlinear interconnected systems with matched uncertainty and mismatched unknown interconnections. A state feedback decentralised SMC strategy, under the assumption that all system states are accessible, is proposed to attenuate the impact of the uncertainties by using bounds on uncertainties and interconnections. The bounds used in the design are fully nonlinear which provide higher applicability for different complex interconnected systems. Especially, for this fully nonlinear system, the proposed method does not need to use the technique of linearisation, which is widely used in existing work to deal with nonlinear interconnected systems with uncertainties.

- The dynamic observer is applied to complex nonlinear interconnected systems with matched and mismatched uncertainties. This dynamic observer can estimate the system states which can not be achieved during the controller design. The proposed method has great identification ability with small estimated errors for the states of nonlinear interconnected systems with matched and mismatched uncertainties. It should be pointed out that the considered uncertainties of nonlinear interconnected

systems have general forms, which means that the proposed method can be effectively used in more generalised nonlinear interconnected systems.

- A variable structure observer-based decentralised SMC is proposed to control a class of nonlinear interconnected systems with matched and mismatched uncertainties. Based on the designed dynamic observer, a dynamic decentralised output feedback SMC using outputs and estimated states is presented to control the interconnected systems with matched and mismatched uncertainties. The nonlinear interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties are relaxed which are nonlinear and take more general forms. Moreover, the limitation for the interconnected system is reduced when compared with the existing results in which the proposed strategies adopt the full-order observer. Besides that, the presented method improves the robustness of nonlinear interconnected systems to be against the effects of uncertainties.

This thesis also provides several numerical and practical simulations to demonstrate the effectiveness of the proposed decentralised SMC for nonlinear interconnected systems with matched uncertainty, mismatched uncertainty and nonlinear interconnections.

NOTATION AND SYMBOLS

$\forall$	For all
$\in$	Belongs to
$\Rightarrow$	Implies
R	The set of real numbers
R^+	The set of nonnegative real numbers
R^n	The n dimensional Euclidean space
$R^{n\times m}$	The set of $n \times m$ matrices with elements in R
$\lvert a\rvert$	The absolute value of a scaler a
$\lVert \cdot \rVert$	The Euclidean norm or its induced norm
I_n	The unit matrix with dimension n
A^T	The transpose of matrix A
$A > 0$	A is a symmetric positive definite matrix
$A < 0$	A is a symmetric negative definite matrix
$\lambda_{min}(A)$	The minimum eigenvalue of the squar matrix A
$\lambda_{max}(A)$	The maximum eigenvalue of the squar matrix A
L_f	The Lipschitz constant of the function $f(\cdot)$
$J_f(x)\ or\ \frac{\partial f(x)}{\partial x}$	The Jacobian matrix of the function $f(x)$
$\dot{y}$	The first derivative of y with respect to time
col$(x_1, x_2, \cdots, x_n)$	The column vector $[x_1, x_2, \cdots, x_n]^T$ where $x_i \in R$ for $i = 1, 2, \cdots, n$
$A := B$	A is defined by B
$A \Rightarrow B$	A implies B
$sgn(\cdot)$	The signum function

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CHAPTER. 1

INTRODUCTION

With the development of society, the structure of actual controlled systems with different functions becomes more and more complex. Therefore, the requirements of the related controllers need to be improved. Especially, for complex interconnected systems which widely exist in the practical world, in order to enhance system performance, it needs to design controllers using different modern control technologies due to the existence of disturbances and interconnections [1, 2, 3]. Modern control theory has been widely concerned and deeply studied in recent decades. In modern control theory, the control problem of nonlinear interconnected systems with uncertainties is very important. For example, industrial robots, aerospace vehicles and modern power systems are all typical nonlinear interconnected control systems. In these control objects, there are usually various complex uncertainties, such as modelling error, parameter perturbation, external disturbance and input nonlinearity. These factors may not only reduce the control accuracy, but also may destroy stability of systems [4]. Therefore, how to design a reasonable and effective robust controller to make the controlled object achieve the desired performance index has been one of the main topics of general interests in the fields of modern control theory. This thesis will be focused on decentralised SMC for nonlinear interconnected systems with uncertainties which has been carried out with a detailed background.

1.1. RESEARCH BACKGROUND

The interconnected system is a composite large-scale system composed of low dimensional subsystems interconnected in specific ways [5]. In many practical control applications, system models can be described in the form of interconnected systems, such as power systems, chemical production systems, intelligent transportation systems, computer network communication systems and economic operation systems. Therefore, the research on control problems of interconnected systems has attracted great attention of researchers. In a complex interconnected system, subsystems are interconnected together which result in high dimensions, strong coupling, strong uncertainty and other complex characteristics. Traditional control methods for centralised systems are difficult to be applied to analyse and control interconnected systems.

Interconnected systems exist in many fields of practical applications regarding industry and real life. With the strong expectations to analyse and control this kind of complex systems, the study of interconnected systems has become essential and imperative for modern control theory and practical engineering. People need to improve the performance of interconnected systems to achieve desired control results. Therefore, the related control strategies tend to be increasingly sophisticated. This topic has motivated a great number of modern control methods to be applied with aiming at high-performance of interconnected systems [6, 7].

There are many different characteristics in the various kinds of interconnected systems. Several inherent characteristics of interconnected systems have been pointed out summarised by many researchers, including nonlinearity, high dimensionality and uncertainty [8]. Due to these reasons, the complexity of traditional control methods for interconnected systems always depends on the structure of interconnected systems and specific working environments. Then, most of traditional control approaches for interconnected systems are inefficient and unfeasible. In connection with these, the proposed approaches in this thesis mainly concentrate on solving the problems of these inherent characteristics to improve the performance of the whole interconnected system.

Firstly, nonlinearity is a very common characteristic and widely exists in real world. In practical applications, most of interconnected systems have the characteristic of non-linearity due to the complex structure and unknown external disturbances [9]. Linear control strategies which have developed and achieved lots of great results in the long-time research are usually the first choice, but traditional linear control methods sometimes can not be adopted to complex nonlinear interconnected systems. So, control strategies for nonlinear interconnected systems are needed to further develop. For example, the most common and convenient technology is named as the linearisation technique which can transform the nonlinear form into the linear form [10]. Nevertheless, for many specific situations, linearisation is not available because of the existence of finite time escape, multiple isolated equilibria, limit cycles and harmonic oscillation in the nonlinear systems [11]. Based on these reasons, linearisation is not an appropriate selection in these specific situations above. Thus, it needs to develop more effective and general methods for nonlinear interconnected systems.

Secondly, the characteristic of dimensionality is obvious in interconnected systems, specifically in large-scale systems. Large-scale interconnected systems with different functions mean that this kind of systems distributed over a large range sometimes has complex structure and high dimension. In this situation, centralised control which is realised as a common control method can cost great calculation. Besides that the subsystems of interconnected systems can be distributed in different spaces, the centralised controller is hard to be implemented due to poor data transmission between subsystems [12]. So such problems of efficiency and robustness often occur, and they usually have negative impacts on the performance of interconnected systems in the centralised strategy. To solve this kind of problems and ensure the high-performance of objects, many modern control approaches have had great development in recent years, but many existing methods are very complex and inefficient which may result in computation burden, specifically for large-scale interconnected systems. Therefore the problem of dimensionality is still very thorny for interconnected systems.

Last but not the least, uncertainty is an inevitable problem in the modern control design for interconnected systems, though some previous methods under ideal conditions do not consider it. In actual systems, it is very hard to establish an accurate mathematical model which can describe the dynamics of the object accurately due to the existence of various uncertain factors [13]. Most of the uncertainties are without rules and unpredictable. This thorny issue stimulates the development of the robust control theory which can guarantee the performance index of the practical system in the presence of uncertainties. Different complex uncertainties widely exist in practical systems. Robust control can not have the general synthesis to deal with them. Therefore, uncertainty is a very challenging issue for the control of interconnected systems, specifically when it appears in interconnections.

Due to the ubiquitous existence of nonlinearities and uncertainties, robust control has been an attractive issue of research in many years. The research on robust control began in the 1950s. The so-called robustness refers to the characteristic that the control system maintains certain performance under certain parameter perturbation and uncertainties. According to different definitions of system performance, it can be divided into stability robustness and performance robustness. The fixed controller designed with the robustness of the closed-loop system as the objective becomes a robust controller. Some technologies, for instance, mode-free adaptive control [14, 15], variable structure control (VSC) [16], $H\infty$ control [17, 18] and minimum gain control [19], have great progress to enhance the robustness of interconnected systems with uncertainties. There are two main different uncertainties according to the structure, the first one is included in the input channel and named as matched uncertainty, and the other one which is out of the input channel is named as mismatched uncertainty [20]. Based on the sources of the uncertainties, they can be classified as external uncertainty and internal uncertainty. The main difference between external uncertainty and internal uncertainty is that the first one usually does not depend on the system variables, such as states, inputs or outputs. Disturbance observer-based control which is widely adopted in practical systems can only eliminate the impact of the external uncertainty. So, disturbance observer-based control can not achieve great results when the system has both external and internal uncertainties [21].

SMC, which is a special kind of VSC, is realised as the high robustness to solve the problem of nonlinear systems with matched uncertainty. SMC is a typical discontinuous control which can drive the system into a pre-defined sliding surface and keeps the sliding motion on it. When selecting an appropriate surface, the object system will be forced to the designed sliding surface in time. Besides that, the system can be transferred as a reduced-order system when it keeps on the sliding motion. SMC due to its high robustness against uncertainties, has been recognised as an effective method for controlling systems with matched uncertainties, and thus many SMC strategies have been developed by combining different techniques. [22] proposed a modified SMC which was able to deal with mismatched uncertainty, where dynamic feedback was employed. [23] presented an innovative SMC to stabilise a kind of under-actuated systems in the cascade case. [24] proposed a novel dynamic integral sliding mode controller for state-dependent matched and mismatched uncertainties. State output feedback SMC was designed for time delay systems in [25], where the designed controller was independent of the time delay. [26] focused on designing an H∞ SMC for neutral-type stochastic systems with Markovian switching parameters and nonlinear uncertainties. [27] used the SMC method to uncertain multi-input multi-output linear Markovian jump systems. [28] imposed a SMC combined with the backstepping method to control a cascade of equation-ODE system with matched and mismatched disturbances. [29] presented the second-order SMC to improve the closed-loop performance of nonlinear affine systems with quantised uncertainty. A neural network fuzzy SMC presented by [30] was applied to pneumatic muscle actuators, where an adaptive training used neural network was able to establish a fuzzy SMC controller, and an integrator could minimise the tracking error. [31] adopted the terminal SMC and the full-order terminal SMC to improve the performance of multiple-input multiple-output systems with mismatched uncertainty, respectively. [32] proposed a SMC for discrete-time switched systems via an event-triggered strategy. [33] adopted incremental nonsingular terminal SMC for multi-input multi-output nonlinear systems with uncertainties, disturbances, and sudden actuator faults. [34] used chattering-free model-free adaptive SMC to gas collection process with data dropout. It should be mentioned that all of the achievements above are only for centralised systems.

There are two main different strategies defined as centralised control and decentralised control for interconnected systems. Decentralised control only uses local information while centralised control can employ all the information of the whole system in control design. Therefore, decentralised control does not need data transmission from the other subsystems and can reduce the complexity of calculation in practical applications [12]. Although centralised control has low conservatism, it will not work when the channels of data transferring between subsystems are blocked, in this case, decentralised control can still work well. Moreover, the structure of decentralised controllers is usually simpler than that of centralised controllers. In general, decentralised control is easy to implement in the complex and harsh industrial environments. Due to these advantages of decentralised control, lots of researchers devoted themselves to this field and got many dazzling results [35, 36, 37]. The novel decentralised control strategy with finite-time convergence was developed for the trajectory tracking of a space manipulator in [38]. The decentralised voltage control strategy based on the gradient projection method was presented for the wind farm in [39]. The decentralised composite suboptimal control strategy was presented to solve the optimal control problem for a class of two-time-scale networks in [40]. [41] investigated a decentralised event-triggered adaptive control problem of uncertain interconnected lower-triangular nonlinear systems using corrupted local state feedback. [42] presented a decentralised output-feedback control scheme for a class of nonstrict-feedback nonlinear large-scale systems with input delay, saturation and unknown virtual control gains. [43] presented a decentralised adaptive neural asymptotic tracking for switched nonlinear interconnected systems with unknown strong interconnections and predefined transient performance. The acceleration feedback-based finite time platoon control for the interconnected vehicular system was proposed in [44]. [45] put forward a decentralised adaptive control for uncertain interconnected systems with triggering state signals. [46] proposed a neural network decentralised observer-based fault-tolerant control for the nonlinear interconnected fractional-order systems. [47] investigated the problems of stability and decentralised control for a class of interconnected fractional-order systems. [48] presented the fault-tolerant load frequency control for the interconnected wind power systems. Although, there are many great achievements related to interconnected systems, there are just a few results using the technology of SMC.

Motivated by the existing problems in traditional control methods and practical applications, this thesis focuses on the topic of the decentralised SMC for nonlinear interconnected systems with uncertainties. The considered interconnected systems are nonlinear with matched uncertainty, mismatched uncertainty and interconnections. Moreover, the situations of interconnected systems with known states and unknown states are both considered. Furthermore, the proposed approaches have general forms, which can improve the robustness of performance for many kinds of nonlinear interconnected systems with uncertainties.

1.2. Contributions and Thesis Organisation

This thesis focuses on the development of decentralised SMC for nonlinear interconnected systems with uncertainties. Several methods and different techniques based on SMC have been provided to design the controllers to improve the robustness. The main contributions of this thesis are summarised as follows:

- A decentralised SMC scheme for nonlinear interconnected systems is proposed under the assumption that all system states are accessible. For systems with matched uncertainty and mismatched uncertainty, a state feedback decentralised SMC strategy is proposed to eliminate the effect of the uncertainties by using the bounds on uncertainties and interconnections. The bounds used in the design are fully nonlinear which has higher applicability for different complex interconnected systems. Especially, for fully nonlinear systems, the proposed method does not need to use the technique of linearisation, which is widely used in the existing works to deal with nonlinear interconnected systems with uncertainties.
- The dynamic observer is presented to complex nonlinear interconnected systems in the presence of matched and mismatched uncertainties. This dynamic observer can estimate the states which can not be measured in the system. The presented approach has excellent identification ability with small estimated errors for states of nonlinear interconnected systems with matched and mismatched uncertainties. It should be pointed out that the considered uncertainties of nonlinear interconnected

systems have general forms, which mean that the proposed method can be effectively used in more generalised nonlinear interconnected systems.

- A variable structure observer-based decentralised SMC is developed to control nonlinear interconnected systems with matched and mismatched uncertainties. Based on the designed dynamic observer, a decentralised output feedback SMC using outputs and estimated states is given to control the interconnected systems in the presence of matched and mismatched uncertainties. The nonlinear interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties has the variable range, which have more general structures. Moreover, the limitation for this interconnected system is reduced when compared with the existing achievements in which the previous approaches use the full-order observer. Besides that, the proposed approach improves the robustness of nonlinear interconnected systems in the presence of uncertainties.

This thesis, for readers' convenience in understanding the developed methodology, also provides several numerical and practical simulations to demonstrate the effectiveness of the proposed decentralised SMC for nonlinear interconnected systems with matched uncertainty, mismatched uncertainty and nonlinear interconnections.

The rest of this thesis is structured as follows:

Chapter 2 gives some mathematical preliminaries needed for the following chapters. Especially, the essential concepts and main results of the Lipschitz function, the existence and uniqueness of system solution, the Lyapunov stability theory, and the converse Lyapunov theory are presented in this chapter.

Chapter 3 provides basic concepts and fundamental knowledge to help readers to understand this thesis. First of all, the fundamental concepts of observability and controllability of linear systems are explained. Then, basic knowledge of feedback control, state observer and SMC are described. Moreover, some important knowledge related to interconnected systems is introduced, such as the architecture and different control methods for interconnected systems. Two different practical systems are also presented in this part. The necessity and superiority of the decentralised SMC are given in the summary of this chapter.

Chapter 4 proposes a novel decentralised robust state feedback SMC which can be used to stabilise a class of nonlinear interconnected systems with matched and mismatched uncertainties. A composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. Then, a decentralised state feedback sliding mode control is proposed to drive the interconnected system to the designed sliding surface in finite time, and a sliding motion is maintained thereafter. The bounds on the uncertainties and interconnections are allowed to have more general nonlinear forms, which are employed in control design to reject the effects of uncertainties and unknown interconnections and enhance the robustness. It is not required either the isolated nominal subsystems linearisable or the interconnections linearisable. At last, a numerical example and a practical simulation of the two coupled inverted pendula on carts are presented to demonstrate the effectiveness of the proposed control strategy.

Chapter 5 presents a dynamic observer which is applied to the complex interconnected systems with matched and mismatched uncertainties. This dynamic observer can identify all state variables which are required in the design of the controller. The proposed method has great identification ability with small state estimated errors for interconnected systems. A numerical simulation and a practical example of the lateral flight control system are presented to demonstrate the effectiveness of the proposed observer.

Chapter 6 proposes a variable structure observer-based decentralised SMC which can be applied to control a class of nonlinear interconnected systems with matched and mismatched uncertainties. Based on the estimated states from the dynamic observer, a composite sliding surface is designed, and the stability of the sliding motion is analysed based on the regular form of the interconnected system. Using the pre-designed observer, a dynamic decentralised output feedback SMC is presented to drive the interconnected systems to the designed sliding surface in finite time, and then the sliding motion is maintained thereafter. The nonlinear interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds on the uncertainties are relaxed which are nonlinear and take more general forms. Finally, a numerical simulation example is presented to illustrate the developed strategy.

Chapter 7 presents a summary of the main conclusions of this thesis. Discussion for the potential future work is also provided in this chapter.

CHAPTER. 2

MATHEMATICAL PRELIMINARIES

In this chapter, fundamental mathematical concepts and definitions are introduced which will be used in the following chapters. To be specific, the Lipschitz function is explained in section 2.1. The existence and uniqueness of system solution are described in section 2.2. Definitions related to the Lyapunov stability theory are given in section 2.3, and section 2.4 provides the converse Lyapunov theory.

2.1. LIPSCHITZ FUNCTION

This section will explain the concepts of the Lipschitz condition and the generalised Lipschitz condition.

2.1.1. LIPSCHITZ CONDITION

Firstly, two important concepts are introduced which are named as the time-invariant system and the time-variant system. The time-invariant system means that the characteristic of the system does not change with time, and the dynamics of the system which is independent of time only depends on the input signal and the structure of the system [49]. The time-variant system is defined as a system that its characteristic is changing over time [49].

Theorem 2.1. ***(Lipschitz condition of time-invariant function [50])****. Consider a time-invariant function $f(x)$. If any two points x_1 and x_2 in a domain D satisfy*

$$\|f(x_1) - f(x_2)\| \leq L\|x_1 - x_2\| \tag{2.1}$$

where L is a nonnegative constant, then, the function (2.1) is called as the Lipschitz condition, L and $f(x)$ are named as Lipschitz constant and Lipschitz function, respectively. If $D = R^n$, it is said that $f(x)$ satisfies the global Lipschitz condition. If $D \subset R^n$, it is said that $f(x)$ satisfies the local Lipschitz condition.

Theorem 2.2. ***(Lipschitz condition of time-variant function [50])****. Consider a time-variant function $f(t, x)$, if $\forall t \in [a, b]$, and $\forall x_1, x_2 \in D$, there is a constant L satisfying*

$$\|f(t, x_1) - f(t, x_2)\| \leq L\|x_1 - x_2\| \tag{2.2}$$

it is said that $f(t, x)$ satisfies the Lipschitz condition related to $t \in [a, b]$ in the domain D, where a and b are nonnegative constants. L and $f(t, x)$ are named as Lipschitz constant and Lipschitz function, respectively. If $D = R^n$, it is said that $f(t, x)$ satisfies the global Lipschitz condition. If $D \subset R^n$, it is said that $f(t, x)$ satisfies the local Lipschitz condition.

2.1.2. Generalised Lipschitz Condition

The Lipschitz condition presented in the previous subsection can be extended to the vector function. Another important concept called generalised Lipschitz condition is presented in this subsection.

Theorem 2.3. ***(Generalised Lipschitz condition [20])****. Consider a function $f(x_1, x_2, x_3)$, where $x_1 \in D_1 \subset R^{n_1}$, $x_2 \in D_2 \subset R^{n_2}$ and $x_3 \in D_3 \subset R^{n_3}$, $D_1 \times D_2 \times D_3 \mapsto R^n$, if there exists the nonnegative continuous functions $L_1(x_3)$ and $L_2(x_3)$ in the domain D_3 such that the inequality*

$$\|f(x_1, x_2, x_3) - f(\bar{x}_1, \bar{x}_2, x_3)\| \leq L_1(x_3)\|x_1 - \bar{x}_1\| + L_2(x_3)\|x_2 - \bar{x}_2\| \tag{2.3}$$

holds for $\forall x_3 \in D_3$*, where* $\bar{x}_1 \in D_1$ *and* $\bar{x}_2 \in D_2$*, it is said that* $f(x_1, x_2, x_3)$ *satisfies the generalised Lipschitz condition related to* x_1 *and* x_2 *uniformly for* x_3*. Moreover, if* $D_1 = R^{n_1}$ *and* $D_2 = R^{n_2}$*,* $f(x_1, x_2, x_3)$ *is said to satisfy the global generalised Lipschitz condition related to* x_1 *and* x_2 *uniformly for* x_3*. If* $D_1 \subset R^{n_1}$ *and* $D_2 \subset R^{n_2}$*,* $f(x_1, x_2, x_3)$ *is said to satisfy the local generalised Lipschitz condition related to* x_1 *and* x_2 *uniformly for* x_3*.*

In (2.3), $L_1(x_3)$ and $L_2(x_3)$ are named as the generalised Lipschitz bounds which are nonnegative continuous functions. This is different from the Lipschitz condition that L in (2.1) or (2.2) is the constant. Therefore, the generalised Lipschitz condition is an extension of the Lipschitz condition discussed in the subsection 2.1.1.

2.2. EXISTENCE AND UNIQUENESS OF SYSTEM SOLUTION

This subsection presents some concepts and definitions related to the existence and uniqueness of system solution.

Consider a system described as follows

$$\dot{x} = f(t, x), \quad x(t_0) = x_0 \tag{2.4}$$

where $x \in R^n$ denotes the system state, and t is time.

Definition 2.1. (Existence and uniqueness of the solution [51]). For the system (2.4), if there exists a continuous differentiable function $x(t)$ satisfying (2.4) in $[t_0, t_1)$, then this function $x(t)$ is named as a solution of the system (2.4). In the other words, this situation is called as the existence of the solution of the system (2.4). If there is only one solution $x(t)$ satisfying by the system (2.4), this $x(t)$ has the characteristic of uniqueness, and the system (2.4) is said to have the unique solution.

In the practical application, many mathematical models similar to the system (2.4) are used to describe the dynamic behaviour of the object from t_0 to t_1. These mathematical models have a unique solution to predict the dynamics of systems in future. If solutions of mathematical models are multiple, these systems will be unpredictable in the future. This class of unpredictable systems is very hard to control in practical application.

If $f(t, x)$ is a continuous function related to time t and state x, then the solution $x(t)$ of $f(t, x)$ is continuously differentiable. In practical apects, people usually use computers which only give control signals by sampling. So the closed-loop system is piecewise continuous related to time t. Therefore, the research about continuous systems sometimes focuses on the time segment continuous situation.

It is very important to make the judgement about the existence and uniqueness of system solution.

Theorem 2.4. ***(Local existence and uniqueness of the solution [51])***. *Consider the system (2.4), if the function $f(t, x)$ is piecewise continuous related to the time t for $\forall x_1, x_2 \in D = \{x \in R^n | \|x - x_0\| \leq \delta\}$ in $[t, t_1]$ and satisfies the Lipschitz condition*

$$\|f(t, x_1) - f(t, x_2)\| \leq L\|x_1 - x_2\| \tag{2.5}$$

Then, there exists a positive constant ϵ such that the state equation $\dot{x} = f(t, x)$ with $x(t_0) = x_0$ has a unique solution over $[t_0, t_0 + \epsilon]$.

Theorem 2.4 presents conditions under which the system (2.4) has the unique solution. During the process of the proof for Theorem 2.4, it is easy to get that $\epsilon \leq t_1 - t_0$, therefore, $[t_0, t_0 + \epsilon] \subset [t_0, t_1]$ and the solution $x(t) \in D$.

Theorem 2.5. ***(Global existence and uniqueness of the solution [51])***. *If the function $f(t, x)$ in (2.4) is piecewise continuous related to the time t for $\forall x_1, x_2 \in R^n$ in $[t_0, t_1]$ and satisfies the Lipschitz condition (2.5). Then, the state equation $\dot{x} = f(t, x)$ with $x(t_0) = x_0$ has a unique solution over $[t_0, t_1]$.*

Compared with the two Theorems 2.4 and 2.5, if the function $f(t, x)$ in (2.4) satisfies the local Lipschitz condition related to x and t, there exists a locally unique solution for this system. If the function (2.4) meets the global Lipschitz condition related to x and t, there exists a globally unique solution for this system. For the local Lipschitz condition in Theorem 2.4, it is easy to realise in practical applications. Theorem 2.5 is usually very conservative. In practical applications, some nonlinear systems which may not satisfy the condition of Theorem 2.5 still have a unique solution. Therefore, it is necessary to develop some results with more relaxed conditions to determine the global existence and uniqueness of system solution. Then, another theorem is given to use the local Lipschitz condition to determine the global existence and uniqueness of system solution.

Theorem 2.6. ***(Global existence and uniqueness of the solution [51])***. *If the function $f(t, x)$ in (2.4) is piecewise continuous in t and locally Lipschitz in x for all $t \geq t_0$ and all x in a domain $D \subset R^n$. Let W be a compact subset of D, $x_0 \in W$, and suppose it is known that every solution of the system (2.4) lies entirely in W. Then, there is a unique solution that is defined for all $t \geq t_0$.*

Theorem 2.6 is a more general result which can help people to determine the existence and uniqueness of the solution of continuous systems. This theorem needs all signals of closed-loop to be bounded. So the unique solution $x(t)$ is bounded. In this case, we just need to ensure that the systems satisfy the local Lipschitz condition related to x to guarantee that the systems have a unique solution.

2.3. LYAPUNOV STABILITY THEORY

The Lyapunov stability theory used to analyse the stability of the system was established by a Russian mathematician and mechanist named as A.M. Lyapunov in 1892 [51]. With the development of the system theory, the stability theory is very important in the analysis and the design of controllers for systems. For control design, stability is an essential issue which needs to be considered as unstable systems can not work in reality. In the study of linear time-invariant systems, there are many criteria such as algebraic stability criteria, Nyquist stability criteria and so on, which can be used to determine the stability of systems. In this thesis, the Lyapunov stability theory is used as one of the main methods. Therefore, some main results of the Lyapunov stability theory are presented in this part.

Take consideration of a general time-invariant system

$$\dot{x} = f(x) \tag{2.6}$$

where $x \in D \subset R^n$, and $f(\cdot)$ is a continuous function satisfying Lipschitz conditions.

Definition 2.2. (Stability, uniformly stability and asymptotically stability [20]). The equilibrium point $x = 0$ of the system (2.6) is considered to be

(i) stable if there is $\zeta = \zeta(\varrho, t_0) > 0$ for $\forall \varrho > 0$,

$$\|x(t_0)\| < \zeta \Rightarrow \|x(t)\| < \varrho, \qquad \forall t \geq t_0 \geq 0 \tag{2.7}$$

(ii) unstable if it does not satisfy the condition (2.7).

(iii) uniformly stable if there is the $\zeta = \zeta(\varrho) > 0$ for $\forall \varrho > 0$ which is independent of t_0 to make (2.7) hold.

(iv) asymptotically stable if it satisfies (2.7), there exists a positive constant $m = m(t_0)$ such that $x(t) \to 0$ when $t \to \infty$ for all $\|x(t_0)\| < m$.

Now, take consideration of a general linear system as follows

$$\dot{x} = Ax \tag{2.8}$$

where $x \in R^n$, A is a matrix with appropriate dimensions.

Theorem 2.7. ***(Stability of linear system [20]).*** *The linear system (2.8) is*

(i) stable if all real parts of eigenvalues of the matrix A are negative.

(ii) unstable if there exists the positive real part of eigenvalues of the matrix A.

Definition 2.3. (Lyapunov equation [20]). The coefficient matrix A in system (2.8) is stable if for the given positive definite symmetric matrix Q, there is an unique positive definite symmetric matrix P such that

$$PA + A^T P = -Q \tag{2.9}$$

Then, the function (2.9) is called as the Lyapunov equation.

Theorem 2.7 and Definition 2.3 are usually used for linear systems. The following theorems of Lyapunov stability are for nonlinear systems.

Theorem 2.8. ***(Lyapunov stability theorem of nonlinear systems [20]).*** *For the system (2.6) defined in $x \in D \subset R^n$. The function V is a continuously differentiable function satisfying*

$$V(0) = 0 \tag{2.10}$$

$$V(x) > 0, \quad x \in D \backslash \{0\} \tag{2.11}$$

Then, the system (2.6) is

(i) stable if for $\forall x \in D$

$$\dot{V}(x) \leq 0 \tag{2.12}$$

(ii) asymptotically stable if for $\forall x \in D \backslash \{0\}$

$$\dot{V}(x) < 0 \tag{2.13}$$

(iii) unstable if for $\forall x \in D \backslash \{0\}$

$$\dot{V}(x) > 0 \tag{2.14}$$

Consider a time-varying nonlinear system as follows

$$\dot{x} = f(t, x) \tag{2.15}$$

where $x \in D \subset R^n$, and D includes the origin $x_0 = 0$ and $f(\cdot)$ in the system (2.15) is piecewise continuous related to time t and locally Lipschitz regard of x.

Theorem 2.9. ***(Expanded Lyapunov asymptotically stability theorem [20]).*** *For the system (2.15) defined in $x \in D \subset R^n$, $\forall t \geq 0$, if there exists a continuously differentiable function V satisfying*

$$M_1(x) \leq V(t, x) \leq M_2(x) \tag{2.16}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -M_3(x) \tag{2.17}$$

where $M_i(x)$ for $i = 1, 2, 3$ are continuous positive definite functions in domain D, then the system (2.15) is uniformly asymptotically stable.

2.4. Converse Lyapunov Theory

In this subsection, the converse Lyapunov theory is to be introduced.

Definition 2.4. (Class K functions [20]). A continuous function $\alpha : [0, a) \longrightarrow [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Definition 2.5. (Class KC^1 functions [20]). A class $\mathcal{K}$ function is said to belong to class $\mathcal{KC}^1$ if it is continuously differentiable.

Theorem 2.10. ***(Converse Lyapunov theorem [20]).*** *Consider a time-varying nonlinear system as follows*

$$\dot{x} = f(t, x) \tag{2.18}$$

where $x \in H \subset R^n$, and $H = \{x \in R^n | \|x\| < \eta, \eta > 0\}$. $f(\cdot)$ is a continuously differentiable function, and the Jacobian matrix $\partial f / \partial x$ is bounded in the domain H uniformly in t. Choose m, ϵ and η_0 as the positive constants, and $\eta_0 < \eta/m$, $H_0 = \{x \in R^n | \|x\| < \eta_0\}$. If the system meets the following condition

$$\|x\| \leq m \|x(t_0)\| e^{-\epsilon(t-t_0)}, \quad \forall x(t_0) \in H_0, \quad \forall t \geq t_0 \geq 0$$

then, there is a continuously differentiable equation $V(t, x)$ satisfying

$$\beta_1(\|x\|) \leq V(\cdot) \leq \beta_2(\|x\|) \tag{2.19}$$

$$\frac{\partial V(\cdot)}{\partial t} + \frac{\partial V(\cdot)}{\partial x} f(\cdot) \leq -\beta_3(\|x\|) \tag{2.20}$$

$$\left\| \frac{\partial V(\cdot)}{\partial x} \right\| \leq \beta_4(\|x\|) \tag{2.21}$$

where $\beta_i(\cdot)$ for $i = 1, 2, 3, 4$ are class K functions in the domain $[0, r_0]$. If the system is autonomous, V is independent of time t.

Theorem 2.10 shows that there exists a Lyapunov function for the system (2.18) if all the conditions of Theorem 2.10 are satisfied.

CHAPTER. 3

FUNDAMENTAL CONTROL THEORY AND BASIC CONCEPTS

In this chapter, fundamental control theory and basic concepts are to be presented for readers' convenience. The basic knowledge of SMC theory as one of the main techniques used in this thesis will be introduced in this chapter as well. The basic definitions of stability, observability and controllability of the linear system are described in section 3.1. The concepts of state space and feedback control are presented in section 3.2. A brief description of the state observer is written in section 3.3. Some basic definitions and popular control methods of SMC are introduced in Section 3.4. The review related to interconnected systems is presented in section 3.5. In section 3.6, two practical systems which will be analysed in the following chapters are described. At last, the summary of the previous fundamental knowledge is given in section 3.7.

3.1. OBSERVABILITY AND CONTROLLABILITY OF LINEAR SYSTEM

This section presents the related analysis based on the linear model to describe observability and controllability. The basic knowledge of observability and controllability is essential for late analysis and design. Observability and controllability of a control system need to be considered before the control design [49].

Consider a simple time-variant linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.1}$$

$$y(t) = Cx(t) \tag{3.2}$$

where $A \in R^{n \times n}$, $B \in R^{n \times 1}$ and $C \in R^{1 \times n}$, $x(\cdot) \in R^n$, $u(\cdot) \in R$ and $y(\cdot) \in R$ are state, input and output, respectively.

Definition 3.1. (Controllability) [52]. A time-variant linear system (3.1)-(3.2) is said to be controllable at $t_0 \in T$ if it possible to find the input $u(t)$ defined over $t \in T$ (T is time interval), which will transfer the initial state $x(t_0)$ to the origin at the finite time $t_1 \in T$ and $t_1 > t_0$. If this is true for all initial time $t_0 \in T$ and all initial states $x(t_0)$, the system (3.1)-(3.2) is completely controllable.

Controllability means that the control signal can control the states of the linear system from initial value to final value in finite time. The input signal can not have an impact on the whole system when the system is uncontrollable. A system (3.1)-(3.2) is controllable when the controllability matrix K is full rank [49], where K is defined as

$$K = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B] \tag{3.3}$$

Definition 3.2. (Observability) [52]. A time-variant linear system (3.1)-(3.2) is said to be observable at t_0 if for any $x(t_0) \neq 0$, it can be determined from the output $y(t)$, where $t \in [t_0, t_1] \subset T$, and $t_1 \geq t_0$ is the finite time belonging to the domain T. If this is true for all $t_0 \in T$ and $x(t_0) \neq 0$, the system (3.1)-(3.2) is said to be completely observable.

Observability means that states of the linear system can be observed via measured output. If the system is not observable, it can be stable when the unobservable linear system is stable. Then, the output feedback controller can not be applied to stabilise the linear system when this unobservable system is not stable. The system (3.1)-(3.2) is observable when the observability matrix E is full rank [49], where E is described as

$$E = [C \quad CA \quad CA^2 \quad \cdots \quad CA^{n-1}]^T \tag{3.4}$$

It is essential to analyse the controllability and observability of the object system before design of the control law. When partial system is neither controllable nor observable, then the dynamics of these parts of the system usually can not be changed. A linear system can achieve high performance when the whole system is controllable and observable.

3.2. STATE SPACE AND FEEDBACK CONTROL

Feedback control is realised as one of the fundamental and important control strategies in control fields. This classic control method is widely applied to practical systems, such as intelligent robots, mechanical arms and transport networks. Feedback control is a control mechanism which employs information/data from measurements to manipulate variables to achieve the desired performance. The control law can be designed based on the comparison between the actual values and related desired values of system variables [53]. However, this working mechanism of feedback control is affected by the disturbances which are inevitable in practical applications. Disturbances and uncertainties must be considered in the design of controller for feedback control. Many researchers have been working on this field for a long time.

State space is widely used to describe control systems. State refers to an ordered set that the minimum number of variables in the system, which can determine the dynamics of the system [53]. The so-called state space denotes the set of all possible states of the system. In short, the state space can be regarded as a space with the state variable as the coordinate axis. So states of the system can be expressed as a vector in this space. State space is a mathematical model including a set of inputs, outputs and states. In the state space, the relationship between inputs, outputs and states is expressed by a set of first-order differential functions. State space provides a convenient and simple method to analyse the multi-input and multi-output systems. An n-th order system with m inputs and p outputs is presented in Fig 3.1.

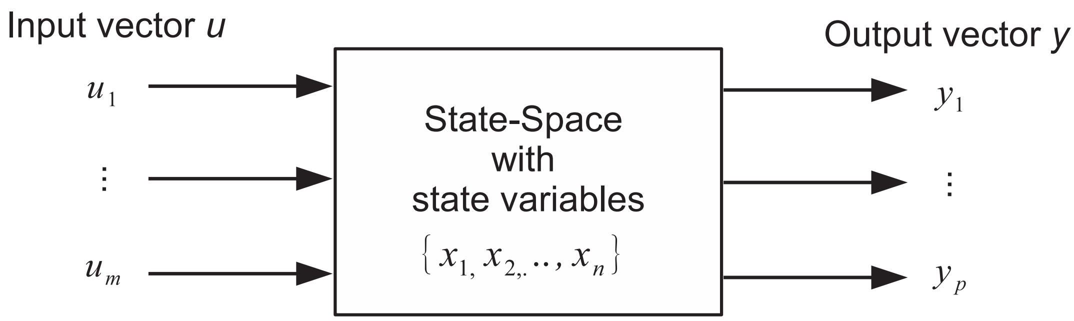

Figure 3.1: State space.

Consider the mathematical formula, which can describe the state space in Fig 3.1, as follows

$$\dot{x}_i(t) = f_i(t, x, u), \qquad i = 1, 2, 3, \cdots, n \tag{3.5}$$

where $\dot{x}_i(t) = \frac{dx_i}{dt}$, and $x(t) = \text{col}(x_1, x_2, \cdots, x_n) \in R^n$, $u \in R^m$ and $t \in R^+$ denote the state, input and time, respectively. The function $f_i(\cdot)$ can describe the system dynamics related to x_i for $i = 1, 2, 3, \cdots, n$. Equation (3.5) is a typical expression about the $n-$dimensional state space.

The system output in Fig 3.1 can be described by

$$y_i(t) = g_i(t, x), \qquad i = 1, 2, 3, \cdots, p \tag{3.6}$$

Let

$$y(t) = \text{col}(y_1(t), y_2(t), \cdots, y_p(t))$$
$$f(\cdot) = \text{col}(f_1(\cdot), f_2(\cdot), \cdots, f_n(\cdot))$$
$$g(\cdot) = \text{col}(g_1(\cdot), g_2(\cdot), \cdots, g_p(\cdot))$$

Then the system (3.5) with the output (3.6) can be written in a compact form as

$$\dot{x}(t) = f(t, x, u) \tag{3.7}$$

$$y(t) = g(t, x) \tag{3.8}$$

If $m, p = 1$ in equations (3.5) and (3.6), the system is defined as the single-input single-output system. If $m, p > 1$ in equations (3.5) and (3.6), the system is called as the multi-input multi-output system.

Take consideration of the time-invariant linear system,

$$\dot{x} = Ax + Bu \tag{3.9}$$

$$y = Cx \tag{3.10}$$

where the matrices A, B and C have the appropriate dimensions. (A, B) and (A, C) are controllable and observable, respectively. If the control law $u = -K_s x$ is designed based on the accessible states of the system (3.9)-(3.10), where K_s is the constant matrix with appropriate dimensions. This kind of control is defined as state feedback control. The related closed-loop dynamics can be written as

$$\dot{x} = (A - BK_s)x \tag{3.11}$$

Then the main objective is to design an appropriate feedback gain matrix K_s to make the matrix $(A - BK_s)$ become stable. The block diagram in Fig 3.2 shows the structure of state feedback control framework.

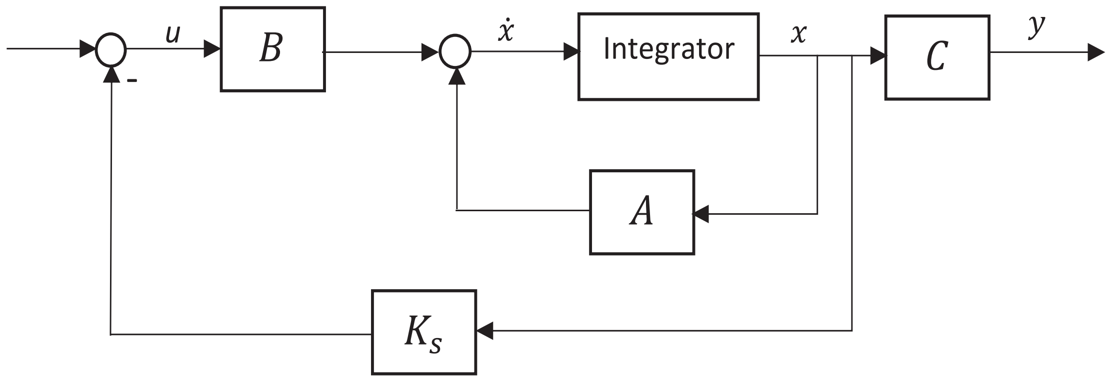

Figure 3.2: Structure of state feedback control.

In the actual situation, because of the harsh operating environment or intrinsic complexity of the system, values of states for the object system may be unavailable. In this case, the state feedback control is unsuitable for this class of systems. Instead of state feedback control, output feedback control which is only based on the measured outputs is an effective approach used in systems with unknown states. The control law of output feedback control can usually be expressed as $u = -K_o y$, where K_o is the gain matrix. Based on output feedback control, the corresponding closed-loop dynamics can be described as

$$\dot{x} = (A - BK_oC)x \tag{3.12}$$

This strategy needs to guarantee the stability of the matrix $(A - BK_oC)$ by designing an appropriate gain matrix K_o. The detailed structure is presented in Fig 3.3.

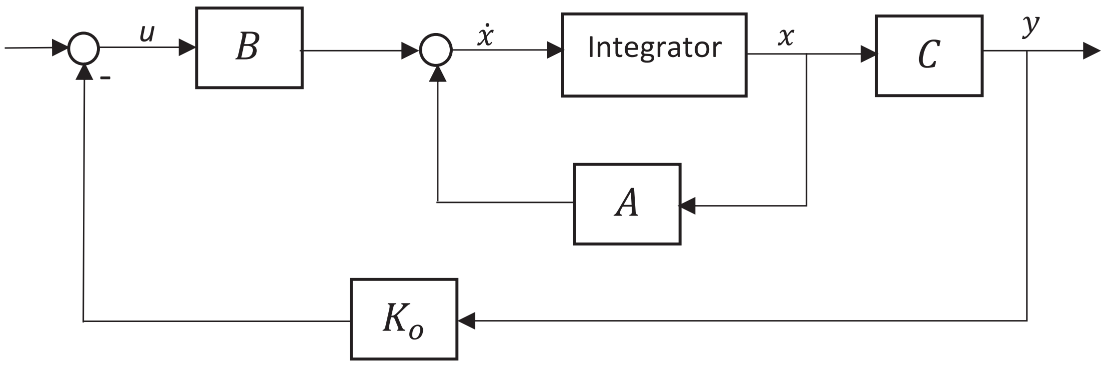

Figure 3.3: Structure of output feedback control.

Both state feedback control and output feedback control are widely applied in many practical systems. Lots of researchers have obtained great achievements and make indelible contributions to this field [54, 55].

3.3. State Observer

In this subsection, the concept of state observer is to be introduced. The state observer is a class of dynamical systems in which the estimated states are obtained from

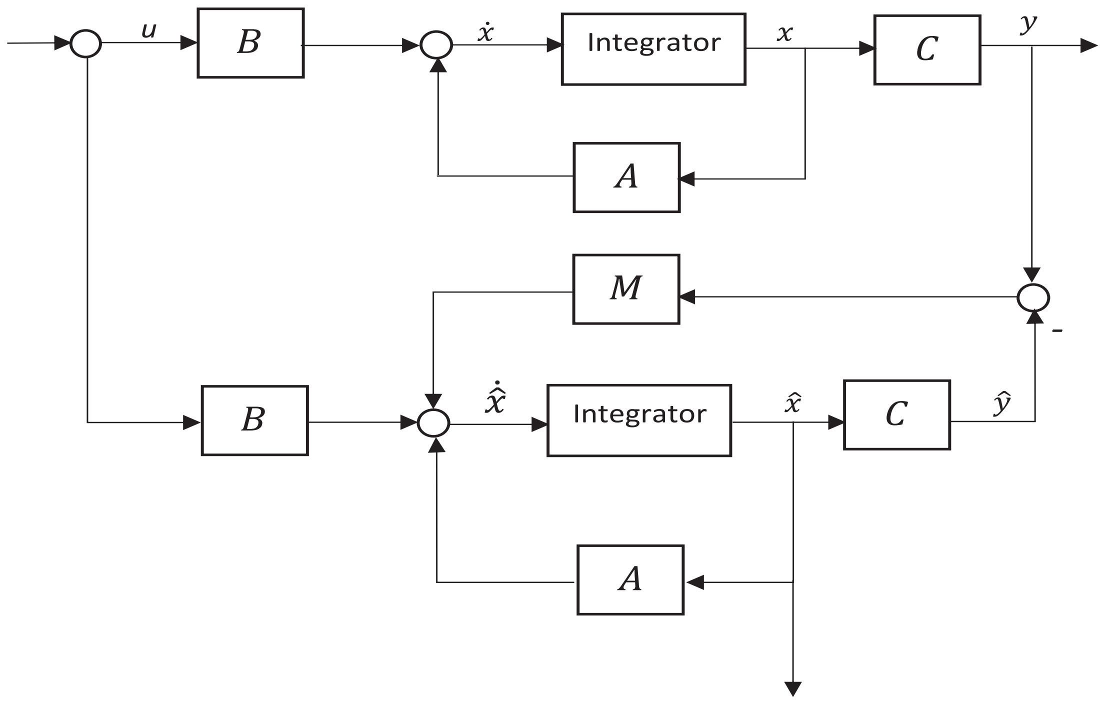

Figure 3.4: Structure of the state observer.

the measured values of the external variables including input variables and output variables of the original system, which is also known as the state configurator. In the early 1960s, in order to implement state feedback control for systems with inaccessible states, D. G. Luenberg, R. W. Bass and J. E. Bertrand proposed the concept of state observer and constructed the corresponding method [56]. This control strategy solved the problem that states can not be directly measured through sensors. With development of science and technology, the state observer not only gives a practical possible approach for state feedback control; but also has been adopted in modern control theories [57, 58, 59].

Now, consider the state observer design. The state observer can be designed for the system (3.9)-(3.10) as

$$\dot{\hat{x}} = A\hat{x} + Bu + M(Cx - C\hat{x}) \tag{3.13}$$

where M is the designed gain matrix. The structure of the state observer is presented by the block chart in Fig 3.4.

According to equations (3.9) and (3.13), the estimated error e_s of states can be given as

$$\dot{e_s} = (A - MC)e_s \tag{3.14}$$

The suitable selection of M lets the matrix $(A - MC)$ be stable if (A, C) is observable, and then the value of e_s tends to zero when the time tends to infinity.

$$\lim_{t \to \infty} e_s(t) \to 0 \tag{3.15}$$

State observers can be classified as full-order state observers and reduced-order state observers according to the dimensions of observers. If dimensions of state observers and original systems are equal, this class of state observers is named as full-order state observers. If the dimensions of state observers are less than that of the original system, this kind of state observers is called as reduced-order state observers. In other words, the reduced-order state observer just estimated the partial states of original systems [60]. These two different kinds of state observers have their advantages and disadvantages, respectively. People usually choose the appropriate kind of state observer based on system structures and characteristics of the object systems as well as the practical requirements.

3.4. Sliding Mode Control

This part is to describe the background and important concepts related to the SMC.

3.4.1. Research Background of Sliding Mode Control

In daily life, people will meet various control systems, for instance, water circulation systems, waste recycling systems and air purification systems. These control systems may belong to different research areas, such as sociology, economics, biology, mechanical engineering, electrical engineering, etc. In the control theory, it is essential to build mathematical models for specific control systems. For different mathematical models, various control concepts and approaches have been developed, such as optimal control [61], fuzzy logic control [62], intelligent control [63], etc. However, both the earlier classical control theory and the popular modern control theory require that the mathematical model of the control system is known more accurately. In the practical process of modelling, there usually exists a deviation between the actual system and the mathematical model. This deviation may come from unmodelled dynamics, perturbation and approximation of the considered systems. Therefore, the designed controller must ensure the control system work with the desired performance under the existence of modelling deviation [64]. This class of problems is the well-known robust control problem.

SMC is a typical VSC. In the 1950s, a VSC method was proposed for the second-order linear systems for the first time [64]. Different feedback gains were set for different systems. The control system could obtain certain characteristics which could not be achieved if the controller's structure was not variable. In the early 1960s, the term named as variable structure first appeared. The second-order nonlinear system with multiple unstable equilibrium points was asymptotically stable where the system states moved on a hyperplane in the state space to approach zero [65]. The hyperplane was the so-called sliding surface, and this method was called as SMC. Traditional VSC requires detailed information about the structure of the system. It is very difficult to be implemented in high-order systems. In view of this, Emelyanov did some research on the controllable single-input system with high-order based on the technique of SMC. According to the sign of the sliding function, the different control gains are selected to change the direction of the sliding function which is opposite to the sign of the sliding function [66]. Some characteristics of interference suppression and chattering are also revealed during the research, and many useful conclusions for controllable systems are summarised in related books [64]. From the 1970s to the early 1980s, this kind of control strategy was gradually extended to the multi-input systems. However, it could only achieve very conservative results with the respect to the multi-input systems [66]. Additionally, the characteristics of the robustness of SMC had not been systematically researched, and thus, in this period, SMC did not attracted more attention of researchers.

With the publication of a famous review article written by Utkin [67], SMC entered a new stage of development that ideas of VSC and SMC were known by many researchers around the world. Especially in the late 1980s, robust control became a hot topic in the control field, and SMC was developed rapidly because of its inherent characteristics of robustness against matched disturbances [68]. From the late 1980s to nowadays, many great results of SMC have been achieved [64, 69, 70, 71, 72].

3.4.2. Existence of Sliding Motion

Traditional SMC has two phases exhibiting different responses which are the sliding phase and the reaching phase. Sliding phase is defined as the period when the object system enters to the designed sliding surface and keeps the motion on it. The order of the system will be reduced when the system moves on the sliding surface. This kind of dynamics is defined as sliding mode dynamics. The reaching phase means the period that the system starts from the initial point driven by the control law until it arrives at the sliding surface.

Take consideration of the nonlinear control system written as follows

$$\dot{x} = f(t,x) + q(t,x)u \tag{3.16}$$

where $x \in R^n$ and $u \in R^m$ are system states and inputs, respectively. $f(\cdot) \in R^n$ and $q(\cdot) \in R^{n\times m}$ are continuous. Select the control law written by

$$u = \begin{cases} u^+(t,x), & \sigma(x) > 0 \quad (3.17) \\ u^-(t,x), & \sigma(x) < 0 \quad (3.18) \end{cases}$$

where $\sigma(x)$ is the sliding function defined by

$$\sigma(x) = \mathrm{col}(\sigma_1(x), \sigma_2(x), \cdots, \sigma_m(x)) \tag{3.19}$$

Definition 3.3. (Sliding mode domain [67]). The domain S in the manifold $\sigma(x) = 0$ can be defined as the sliding mode domain, if for any $\epsilon > 0$, there is a $\delta > 0$ such that any motion from the n dimensional ϵ vicinity of the domain S can only leave there via its boundary.

Fig 3.5 shows the detailed explanation about the sliding domain.

Theorem 3.1. (Existence of sliding motion [67]). For the $(n-m)$ dimensional sliding mode domain S, it is sufficient that in the n dimensional domain D, $S \subset D$, there is

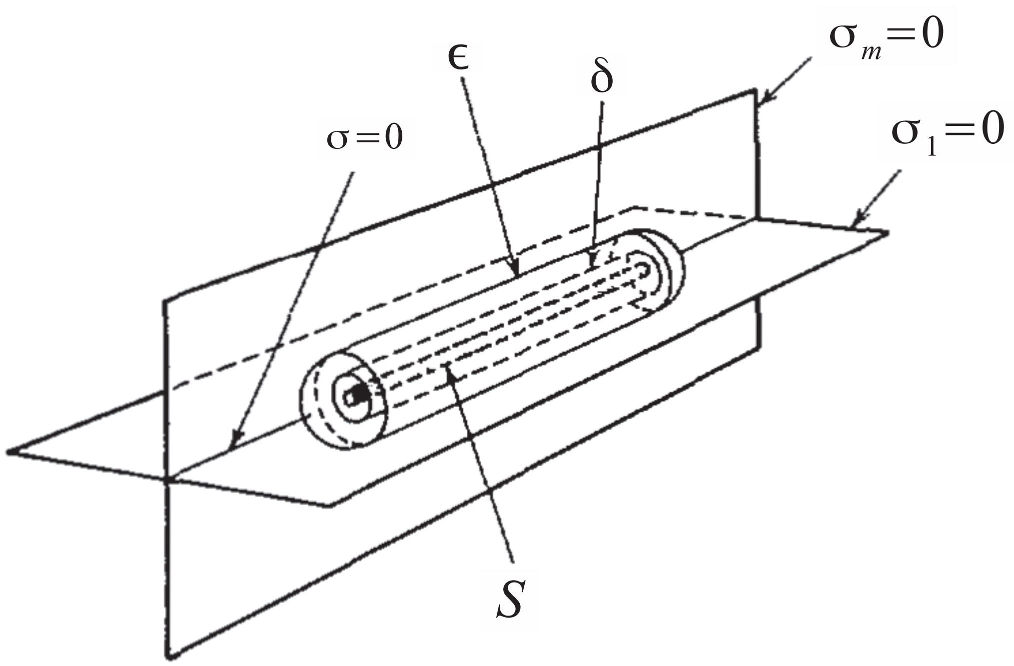

Figure 3.5: Sliding mode domain [20].

a continuously differential function V for (3.16) which is satisfied with conditions as follows

(i) The continuously differential function V is positive definite with respect to σ and for any $x \in S$ and t,

$$\inf_{\|\sigma\|=\tau} V = \xi \tag{3.20}$$

$$\sup_{\|\sigma\|=\tau} V = \varpi \tag{3.21}$$

where $\xi \neq 0$ if $\tau > 0$, and ξ and ϖ depend on τ.

(ii) Time derivative of the function V has negative supremum on small enough spheres $\|\sigma\| = \tau$ with removed points on the discontinuity surfaces where this derivative does not exist.

After the confirmation of the existence of sliding motion, then, an appropriate control law needs to be designed to satisfy the reachability condition to guarantee the system to be driven to the sliding surface. This condition can be described as

$$\sigma^T(x)\sigma(x) < 0 \tag{3.22}$$

where $\sigma(x)$ is the pre-designed sliding function. This initial reachability condition can only ensure the system move to the sliding surface asymptotically. So, several researchers modified this original reachability condition in (3.22) as follows

$$\sigma^T(x)\sigma(x) < -\eta\|\sigma(x)\| \tag{3.23}$$

This modified condition is called as η reachability condition which can ensure the system move to the sliding surface in finite time. The constant η is named as reach gain which can be determined in the design of the SMC controller.

3.4.3. Equivalent Control Approach

There are two main methods which can be used to analyse the stability of sliding motion. One is named as the equivalent control, and the other one is called as the regular form. Firstly, we briefly introduce the approach of equivalent control.

Consider a system described as follows

$$\dot{x} = Ax + B(u + \phi(x)) \tag{3.24}$$

where $x \in R^n$ is system state, and $u \in R^m$ is input, $m \leq n$, the matrices A and B are known with appropriate dimensions. $\phi(x)$ is the unknown matched uncertainty.

If the system (3.24) is driven to the pre-designed sliding function $\sigma(x)$ defined as

$$\sigma(x) = Fx \tag{3.25}$$

Based on the fact that in the sliding surface $\sigma(x) = 0$, then $\dot{\sigma}(x) = 0$, it is easy to get the related equivalent control as follows

$$u_{eq} = -(FB)^{-1}FAx - \phi(x) \tag{3.26}$$

where the matrix F is selected satisfying that FB has full rank. It ensures that this equivalent control has a unique solution. Then, the sliding mode dynamics can be presented as

$$\dot{x} = (I_n - B(FB)^{-1}F)Ax \tag{3.27}$$

$$Fx = 0 \tag{3.28}$$

This control strategy is widely used in many systems in which states are all accessible in the ideal situation. In actual projects, the equivalent control technique is sometimes used to practical systems combined with the identification technology to overcome the problem of unknown states [66].

3.4.4. Regular Form Approach

The other strategy is the regular form method which can also be applied to analyse the dynamics of the sliding motion.

Consider a simple system as follows

$$\dot{x} = Ax + Bu \tag{3.29}$$

where $x \in R^n$ and $u \in R^m$ are state and input, respectively, $m \leq n$. the matrices A and B are known with appropriate dimensions. Assume that rank$(B) = m$. Then, there exists a coordinate transformation $z = Tx$ such that

$$TB = \begin{bmatrix} 0 \\ \tilde{B} \end{bmatrix} \tag{3.30}$$

where $\tilde{B} \in R^{m\times m}$ is a nonsingular matrix. The coordinate transformation matrix T is nonsingular. The system (3.29) can be transferred to a new system via the coordinate transformation. In the new coordinates z, the system (3.29) can be described by

$$\dot{z}_1 = \tilde{A}_{11}z_1 + \tilde{A}_{12}z_2 \tag{3.31}$$

$$\dot{z}_2 = \tilde{A}_{13}z_1 + \tilde{A}_{14}z_2 + \tilde{B}u \tag{3.32}$$

where $z_1 \in R^{n-m}$, $z_2 \in R^m$, $z = \text{col}(z_1, z_2)$ and

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{13} & \tilde{A}_{14} \end{bmatrix} = TAT^{-1} \tag{3.33}$$

and $\tilde{B}$ is given in (3.30).

There is no control signal u in the equation (3.31), the control signal u is only in the equation (3.32). The equations (3.31)-(3.32) are called as the regular form of the system (3.29).

Then, select a suitable sliding function as follows

$$\sigma(x) = F_1 z_1 + F_2 z_2 \tag{3.34}$$

where $F_1 \in R^{m\times(n-m)}$ and $F_2 \in R^{m\times m}$ are design matrices with F_2 being invertible. $\sigma(x) = 0$ is the sliding surface. On the sliding surface,

$$z_2 = -F_2^{-1} F_1 z_1 \tag{3.35}$$

Substitute the equation (3.35) to (3.31), the sliding mode dynamics can be described by

$$\begin{aligned}\dot{z}_1 &= \tilde{A}_{11} z_1 - \tilde{A}_{12} F_2^{-1} F_1 z_1 \\ &= (\tilde{A}_{11} - \tilde{A}_{12} F_2^{-1} F_1) z_1\end{aligned} \tag{3.36}$$

The only objective is that find the suitable F_1 and F_2 to ensure the stability of $(\tilde{A}_{11} - \tilde{A}_{12} F_2^{-1} F_1)$. Then, the whole system is asymptotically stable. The matrix pair (A_{11}, A_{12}) is controllable if and only if the pair (A, B) is controllable. It is easy to see that the dynamics of the sliding mode is reduced-order. Therefore, sliding mode dynamics (3.36) is a reduced-order system when compared with the original system (3.29).

Insensitivity property is another inherent characteristic of the regular form method [66]. Consider a system with matched and mismatched uncertainties described by

$$\dot{x} = Ax + B(u + \phi(x)) + \varphi(x) \tag{3.37}$$

where $x \in R^n$ and $u \in R^m$ are state and input, respectively, $m \leq n$, the matrices A and B are known with appropriate dimensions. $\phi(x)$ is the matched uncertainty, and $\varphi(x)$ is the mismatched uncertainty. Use the transformation $z = Tx$ given above to get the following system

$$\dot{z}_1 = \tilde{A}_{11} z_1 + \tilde{A}_{12} z_2 + \tilde{\varphi}_1(z_1, z_2) \tag{3.38}$$

$$\dot{z}_2 = \tilde{A}_{13}z_1 + \tilde{A}_{14}z_2 + \tilde{B}(u + \tilde{\phi}(z_1, z_2)) + \tilde{\varphi}_2(z_1, z_2) \tag{3.39}$$

where $z = \text{col}(z_1, z_2)$ with $z_1 \in R^{n-m}$ and $z_2 \in R^m$, and

$$TB = \begin{bmatrix} 0 \\ \tilde{B} \end{bmatrix} \tag{3.40}$$

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{13} & \tilde{A}_{14} \end{bmatrix} = TAT^{-1} \tag{3.41}$$

$$\tilde{\varphi}_1(z_1, z_2) = T\varphi(T^{-1}z) = \begin{bmatrix} \tilde{\varphi}_1(z_1, z_2) \\ \tilde{\varphi}_2(z_1, z_2) \end{bmatrix} \tag{3.42}$$

$$\tilde{\phi}(z_1, z_2) = \phi(T^{-1}z) \tag{3.43}$$

where $\tilde{\varphi}_1(z_1, z_2) \in R^{(n-m)}$ and $\tilde{\varphi}_2(z_1, z_2) \in R^m$. Then, choose the sliding function $\sigma(x) = F_1z_1 + F_2z_2$, where $F_1 \in R^{m\times(n-m)}$ and $F_2 \in R^{m\times m}$ are design matrices with F_2 being invertible. It is easy to get the dynamics of sliding motion given by

$$\dot{z}_1 = \tilde{A}_{11}z_1 + \tilde{A}_{12}F_2^{-1}F_1z_1 + \tilde{\varphi}_{1s}(z_1) \tag{3.44}$$

where $\tilde{\varphi}_{1s}(z_1) = \tilde{\varphi}_1(z_1, z_2)|_{z_2=-F_2^{-1}F_1z_1}$.

From the equation (3.44), the dynamic of the sliding motion is independent of the matched uncertainty $\tilde{\phi}(\cdot)$ and uncertainty $\tilde{\varphi}_2(\cdot)$. This kind of phenomenon is called as the insensitive characteristic of sliding motion to matched uncertainties.

3.5. Interconnected System

A detailed introduction related to interconnected systems is presented in this section, including research background, description and main control methods for interconnected systems.

3.5.1. Background of Interconnected Systems

In practical applications, ideal linear systems are very rare because of the wide existence of nonlinearity in actual control systems. If the degree of nonlinearity in systems is not very high to some extent, the nonlinear system can be approximated or simplified as a linear system using the linearisation technique, and then this nonlinear system can be analysed and controlled via the control theory for linear systems. However, with the great development of science and technology, especially, in high-tech fields such as aircrafts and robots, the requirements of the performance for control systems are increasing. In order to achieve better control results, the nonlinear system control theory must be established to solve the problem of nonlinear systems which can be well modelled for real systems [51].

At the same time, in various fields of industry and daily life, a variety of large-scale complex systems widely exist. In order to describe and analyse these systems, people put forward a concept of interconnected system [73]. The interconnected system is a composite large-scale system composed of low dimension subsystems interconnected in a specific way, which generally has the following basic characteristics [74, 75, 76]:

(i) **Large scale**. Interconnected systems tend to have high dimensions and a large number of subsystems and variables.

(ii) **Complex structure**. Interconnected systems usually can be decomposed into several isolated subsystems, and dynamics of each subsystem can be integrated to form the dynamics of the whole system.

(iii) **Multiple controllers**. Interconnected systems are generally controlled by more controllers.

(iv) **Suboptimum**. It can be controlled suboptimally. In control of interconnected systems, there may be multiple objectives. These multiple objectives cause that the controllers usually can not achieve the optimal results.

(v) **Various influencing factors**. Interconnected systems are complex systems with multiple inputs and outputs with strong uncertainties. The interconnected system may have strong interconnections between subsystems, and there may be different control strategies for each subsystem. It makes the analysis and design of interconnected systems become more difficult.

Interconnected systems usually represent actual complex systems, and their hierarchical or multi-level structure is suitable for describing systems related to economic management, aviation, aerospace, navigation, robotics, etc [77]. The characteristics of interconnected systems determine that their theories and methods have various applications. The control strategies of many practical problems of interconnected systems are based on the inherent characteristics of systems themselves. According to the continuity of states in interconnected systems, they can be classified as continuous interconnected systems and discrete interconnected systems. According to the expression of states in interconnected systems, they can be classified as linear interconnected systems and nonlinear interconnected systems. According to the time term in the interconnected system, it can be classified as time-delay interconnected systems and non-delay interconnected systems [78, 76]. In essence, the most important task in studying interconnected systems is to analyse the interconnections, which describe the influence of each subsystem on the other subsystems. Therefore, for interconnected systems, the structure of interconnection comes very naturally to become the main topic of research to distinguish interconnected systems from the traditional centralised systems.

3.5.2. Description of Interconnected Systems

This subsection gives a brief description of interconnected systems.

Consider the mathematical model of a complex nonlinear interconnected system which is composed of several subsystems described by

$$\dot{x}_i = \epsilon_i(t, x_i) + \varsigma_i(t, x_i)(u_i + \varrho_i(t, x_i)) + H_i(t, x), \qquad i = 1, 2, \cdots, n \tag{3.45}$$

where $x_i \in \Omega_i \subset \mathcal{R}^{n_i}$ (Ω_i denotes a neighbourhood of the origin), and $u_i \in \mathcal{R}^{m_i}$ are, respectively, state variables and inputs of the $i\text{-}th$ subsystem with $m_i < n_i$, $x := \text{col}(x_1, x_2, \cdots, x_n) \in \Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$. The matrix function $\varsigma_i(\cdot) \in \mathcal{R}^{n_i \times m_i}$ and the nonlinear vector $\epsilon_i(\cdot) \in \mathcal{R}^{n_i}$. The term $\varrho_i(\cdot)$ denotes matched disturbance, and $H_i(\cdot)$ represents the interconnection.

Some definitions for the system (3.45) are to be introduced as follows.

Definition 3.4. (Isolated subsystem [79]). Consider the system (3.45). The following system

$$\dot{x}_i = \epsilon_i(t, x_i) + \varsigma_i(t, x_i)(u_i + \varrho_i(t, x_i)), \qquad i = 1, 2, \cdots, n \tag{3.46}$$

is called the i-th isolated subsystem of the system (3.45).

Definition 3.5. (Nominal isolated subsystem [79]). From the system (3.45), the system

$$\dot{x}_i = \epsilon_i(t, x_i) + \varsigma_i(t, x_i)u_i, \qquad i = 1, 2, \cdots, n \tag{3.47}$$

is called the i-th nominal isolated subsystem of the system (3.45).

Each subsystem has its own control law u_i, and the interconnection denotes the mutual influence of each other.

3.5.3. Main Control Methods for Interconnected Systems

Control methods for interconnected systems mainly include centralised control, distributed control, hierarchical control and decentralised control, among which the decentralised control method is widely used in the control of interconnected systems. These methods are to be briefly discussed in this subsection.

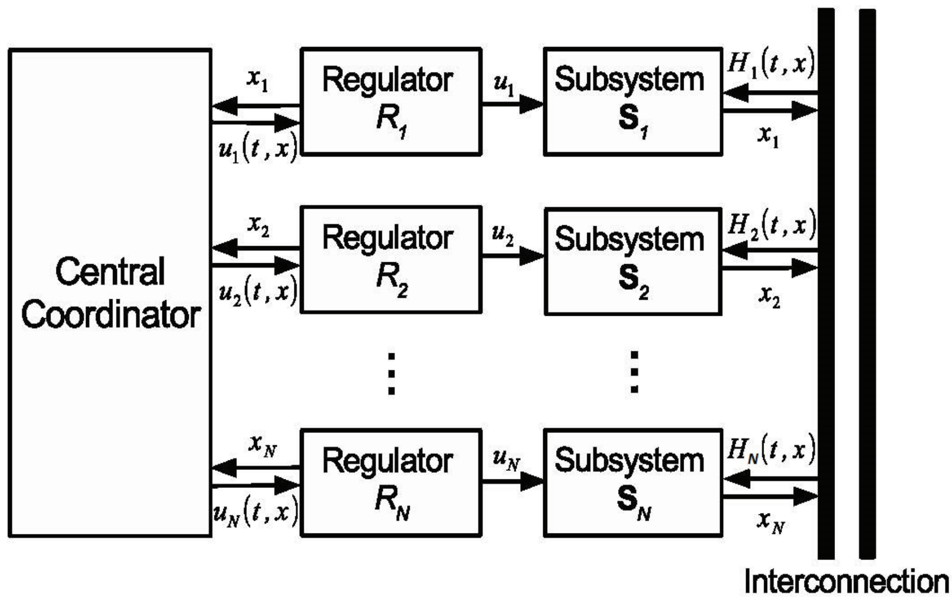

Figure 3.6: Centralised control method [80].

Centralised control: Firstly, centralised control is introduced. Fig 3.6 gives an explanation of centralised control for an interconnected system [80]. In the design of centralised control, based on the traditional control theory and methods, people usually consider the overall interconnected system as the research object, then they analyse the characteristics of the overall interconnected system and adopt the overall modelling, order reduction to analyse and calculate the whole system before proposing the control strategy [80]. Common control methods include feedback control [81], optimal control [82], PID control [83], robust control [84], intelligent control [63], neural network control [85], fuzzy control [86], etc. These control strategies are fully developed and widely used. Lots of practical applications have proved that centralised control has satisfactory performance for centralised systems with low dimensions. However, for interconnected systems with high dimensions and complex structures, there are many constraints which should be considered in the process of control design. In some cases, the dimension of the designed controller is very high which will cause the problem of dimension disaster. For large-scale interconnected systems, centralised control makes the information exchange of the whole system become extremely complex, which leads to an increase in the calculation of integration and operation for the control system. Moreover, when local faults occur in one or several subsystems of the interconnected system, it is necessary to analyse the dynamics of the whole system, which leads to the poor reliability and low fault tolerance of the entire interconnected system [87].

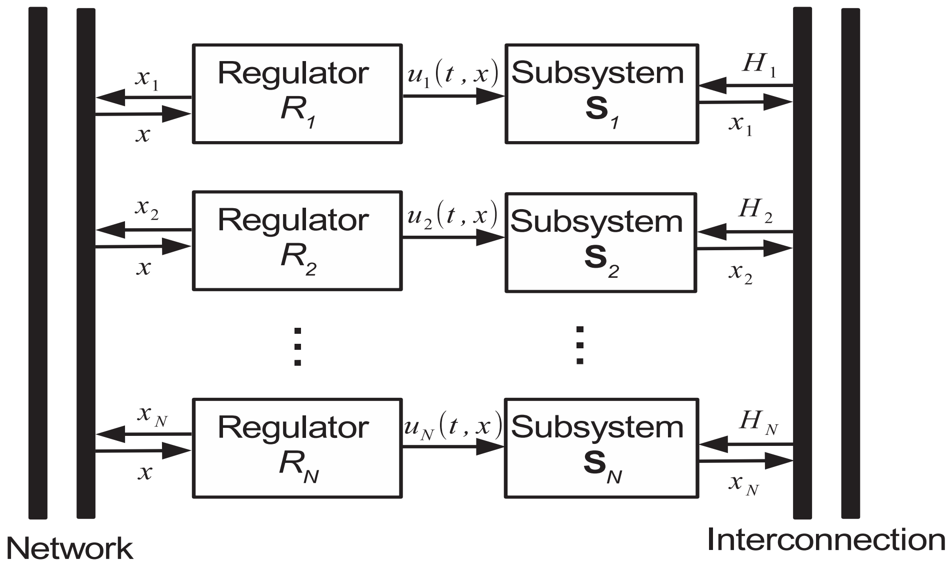

Figure 3.7: Distributed control method [88].

Distributed control: In the distributed control method, controllers of subsystems in the interconnected system are distributed according to the detailed situation of subsystems. The control objectives of the interconnected system are allocated to the subsystems in a certain way, and there is limited information exchange between them [88, 89]. This distributed layout improves the reliability and flexibility of the related controllers. Moreover, it reduces the communication cost between subsystems. In essence, distributed control is a control method between centralised control and decentralised control. Fig 3.7 presents the structure of the common distributed control for interconnected systems [88].

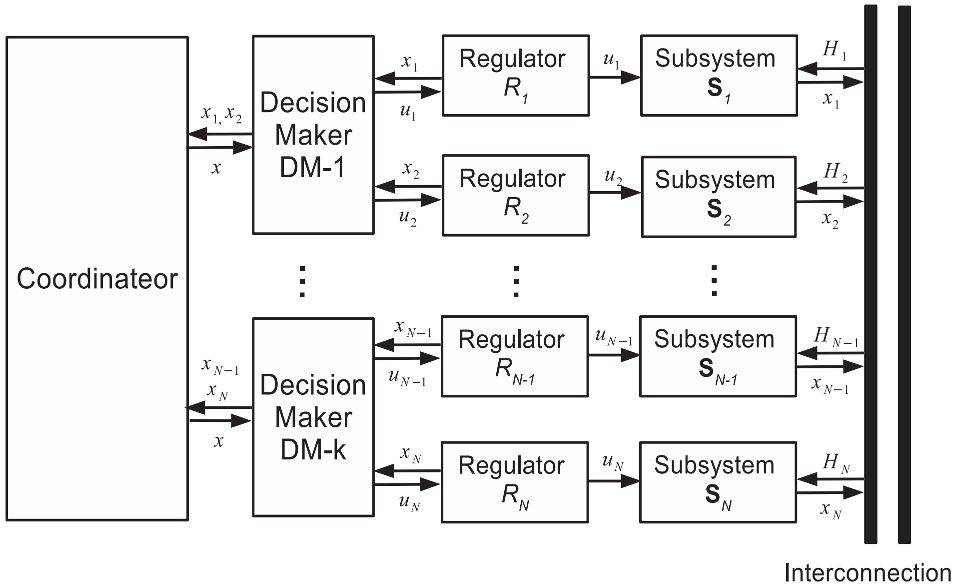

Figure 3.8: Hierarchical control method [90].

Hierarchical control: The fundamental idea of hierarchical control is to let each controller of the interconnected system not only construct its own dynamic compensator independently; but also establish a high-level dynamic coordinator to determine the related control strategy of the subsystem, this characteristic enhances the performance of each controller of the subsystem [90]. Fig 3.8 gives an example of hierarchical control [90]. Interconnected systems usually have two general structures. One is called as the multi-layer structure, which divides the interconnected system into multiple layers according to its functions. The lowest layer is the direct control layer, which directly controls the subsystem of the interconnected system. The related parameters of the direct control layer are determined by its upper layer named as the layer of optimization.

The parameters related to the environment are set in the layer of optimization, which is calculated by the adaptive function layer. In the multi-layer structure, the lower layer is only directly controlled by the corresponding upper layer, and there is no information exchange between the same layers. The second structure is called the multi-level structure. This structure is to add a coordinator to the local controller of subsystems to solve the problem of cooperation between subsystems. The coordinator is designed to provide necessary information for each controller of the local subsystem [91]. The hierarchical control, which can get global optimization via achieving local optimization, has a lot of information exchange between the upper and lower layers. In some cases, the complexity of the hierarchical system may lead to the decline of the reliability of interconnected systems, and it is mainly used to the open-loop control system. So the applications of the hierarchical control are limited [92]. In addition, the hierarchical control law requires to collect the information from all subsystems. In a sense, hierarchical control is a generalised centralised control.

Decentralised control: Complex characteristics of interconnected systems inevitably lead to the appearance of decentralised control. Decentralised control theory as an important branch of large-scale system control theory has achieved great development since it was proposed at the end of the 1960s [93]. Currently, decentralised control is still one of the hot topics in control theory research of interconnected systems.

Most interconnected systems are multi-input multi-output systems. For example, there are many control substations in the power system, and each substation is responsible for its own function. In the process of control system design, this situation is called as decentralisation. Fig 3.9 describes the structure of decentralised control [94]. When designing a controller for the subsystem, it is necessary to determine a structure and allocate the input of the subsystem to a local controller. This controller only uses the state or output of the subsystem, and such a kind of control methods is called as decentralised control [94]. In other words, in a decentralised control system, the interconnected system is decomposed into several subsystems which are controlled by independent local controllers. The cooperation of these local controllers helps the interconnected system meet the requirements of the performance. Each subsystem is only controlled based on

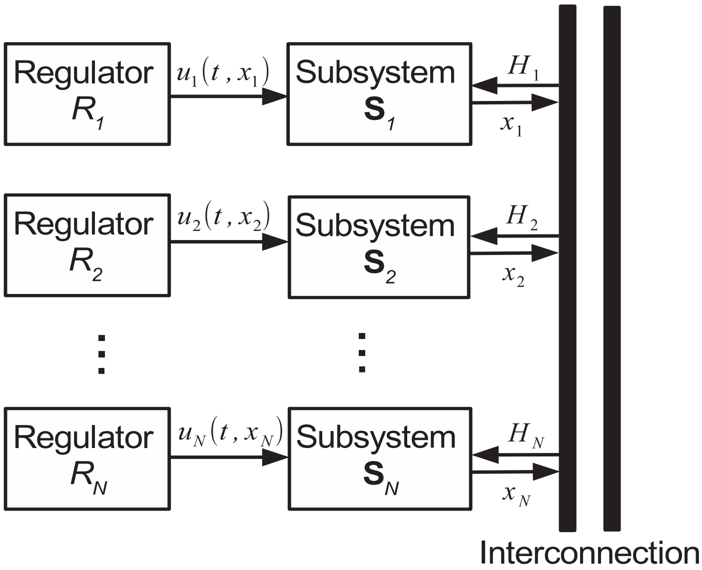

Figure 3.9: Decentralised control method [94].

its own information. Therefore, in decentralised control, local controllers do not need to exchange information with other subsystems, which can save transmission cost [95]. It can also improve the reliability of the entire interconnected system, and overcome the problem caused by the collection of information of storage [96]. In practical engineering applications, the idea of decentralised control can be effectively used to deal with the problems of dimension, uncertainty and structure constraints.

3.6. Several Practical Systems

This section introduces several practical systems which are typical interconnected systems.

3.6.1. Coupled Inverted Pendula on Carts

Take consideration of a coupled inverted pendulum connected via a spring on two carts which is a typical nonlinear interconnected system described in Fig 3.10 [20]. In this system, the pivot position of the spring can be changed with the variation of the length l of two pendulums due to the equation regarding time t. The input signal u_i of two pendulums is the torque applied at the pivot [97]. In the common situation, this input signal is produced by the external force F_i.

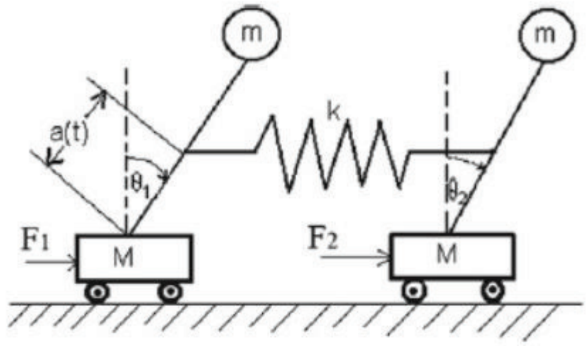

Figure 3.10: The general structure of coupled inverted pendula on carts [20].

Consider the dynamic mathematical model for this system as follows

$$\dot{x_1} = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{cml^2} \end{bmatrix} u_1 + \begin{bmatrix} 0 & 0 \\ \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{m}{M} x_{12}^2 \sin x_{11} + \frac{ka(t)(a(t)-cl)}{cml^2}(s_1 - s_2) \end{bmatrix} \tag{3.48}$$

$$\dot{x_2} = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{cml^2} \end{bmatrix} u_2 + \begin{bmatrix} 0 & 0 \\ \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{m}{M} x_{22}^2 \sin x_{21} + \frac{ka(t)(a(t)-cl)}{cml^2}(s_2 - s_1) \end{bmatrix} \tag{3.49}$$

where $x_1 = \text{col}(x_{11}, x_{12})$ and $x_2 = \text{col}(x_{21}, x_{22})$, x_{11} and x_{12} denote the angle and angular velocity of θ_1, respectively. x_{21} and x_{22} denote the angle and angular velocity of θ_2, respectively. s_1 and s_2 are the positions of carts,

$$c = \frac{M}{M+m} \tag{3.50}$$

k and g are spring and gravity constants, respectively.

3.6.2. LATERAL FLIGHT CONTROL SYSTEM

This part has the simple introduction of the lateral flight control system for aircrafts, which is widely used in civil airliners (see Fig 3.11). For easy implementation of the control of lateral flight control systems, linearisation technology is adopted to this kind of complex flight control systems. The dynamics of the aileron and the rudder actuator need to be modelled in this progress. The yaw rate which can be applied as the input to deal with the problem of damping can be measured by a class of high pass filters [98]. The dynamics of aileron and rudder actuators for aero craft are generally expressed as

Figure 3.11: The civil airliner [99].

$$\text{Aileron:} \quad \frac{\varrho(s)}{\varrho_u(s)} = \frac{25}{s+25} \tag{3.51}$$

$$\text{Rudder:} \quad \frac{\varpi(s)}{\varpi_u(s)} = \frac{20}{s+20} \tag{3.52}$$

where $\varrho(s)$, $\varrho_u(s)$, $\varpi(s)$ and $\varpi_u(s)$ are aileron deflection, the perturbed aileron deflection command, rudder deflection and the perturbed rudder deflection command, respectively.

The high-frequency yaw manoeuvring components of the yaw rate can be achieved via the high pass filter named as washout filter, which is described as follows

$$\frac{\zeta(s)}{\dot{\upsilon}(s)} = \frac{\sigma s}{\sigma s + 1} \tag{3.53}$$

where $\zeta(s)$ and $\upsilon(s)$ denote the output of washout filter and yaw rare, respectively. σ denotes a time constant.

Based on the equations (3.51)-(3.53), the lateral flight control system at the cruising flight condition can be described as linear equations via the linearisation technology,

$$\dot{x} = Ax + Bu \tag{3.54}$$

$$y = Cx \tag{3.55}$$

where

$$x = \text{col}(\alpha, \dot{\delta}, \dot{\upsilon}, \delta, \zeta, \varrho, \varpi) \tag{3.56}$$

$$u = \text{col}(\varrho_u, \varpi_u) \tag{3.57}$$

$$y = \text{col}(\delta, \zeta, \varrho, \varpi) \tag{3.58}$$

The variables α, $\dot{\delta}$, $\dot{\upsilon}$, δ, ζ, ϱ and ϖ are sideslip angle, roll rate, yaw rate, bank angle, washout filter output, aileron deflection, and rudder deflection, respectively. ϱ_u and ϖ_u are the perturbed aileron deflection command and the perturbed rudder deflection command, respectively. The matrices A, B and C, which are dependent on the structure of the aircraft, are constants with appropriate dimensions.

According to [20], the lateral flight control system can also be described in the form of interconnected systems as follows

$$\dot{x}_1 = A_1x_1 + B_1u_1 + H_1(x) \tag{3.59}$$

$$y_1 = C_1x_1 \tag{3.60}$$

$$\dot{x}_2 = A_2x_2 + B_2u_2 + H_2(x) \tag{3.61}$$

$$y_2 = C_2x_2 \tag{3.62}$$

where $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \text{col}(x_{21}, x_{22}, x_{23}, x_{24})$ denote roll rate, bank angle, aileron deflection, sideslip angle, yaw rate, washout filter output and rudder deflection, respectively. Input signals u_1 and u_2 are the perturbed aileron deflection command and the perturbed rudder deflection command, respectively. Functions $H_1(\cdot)$ and $H_2(\cdot)$ are interconnections. Outputs $y_1 = \text{col}(y_{11}, y_{12})$ and $y_2 = \text{col}(y_{21}, y_{22})$ denote bank angle, aileron deflection, washout filter output and rudder deflection, respectively. The matrices A_1, B_1, C_1, A_2, B_2 and C_2, which are dependent on the structure of the aircraft, are constants with appropriate dimensions.

The lateral flight control system is just related to the lateral movement of the aircraft. The complete flight control system includes pitch, roll and yaw control [99], lift and drag increase control [100], manual trim [101], direct force control [102] and other configuration change control of aircraft [103, 104]. Different control systems composed of many subsystems have the cooperation with each other to realise the objective of controlling aircraft.

3.7. SUMMARY

In this chapter, the fundamental knowledge of observability, controllability, feedback control, state observer, SMC and interconnected systems has been presented. SMC is an effective method for interconnected systems due to its advantages, such as the fast response, reduced-order dynamics and insensitivity to variation of parameters and disturbances in systems. After a preliminary understanding of the four main control strategies for interconnected systems, decentralised control due to its simple structure, low cost of calculation and reliability is a better choice compared with the other three methods for interconnected systems. Besides to overcome the problem of unknown states in interconnected systems, the technique of state observer is also essential. Based on these reasons, SMC, state observer and decentralised control are considered as the main methodology for interconnected systems in the following chapters.

CHAPTER. 4

DECENTRALISED STATE FEEDBACK SLIDING MODE CONTROL FOR INTERCONNECTED SYSTEMS

In this chapter, a novel decentralised robust state feedback SMC is presented to stabilise a class of nonlinear interconnected systems with matched uncertainty and mismatched unknown interconnections. In section 4.1, the introduction of the research background is presented. Section 4.2 is about the system description and problem formulation. In section 4.3, a composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. Then, a decentralised state feedback SMC is proposed to drive the interconnected systems to the designed sliding surface in finite time, and a sliding motion is maintained thereafter. In section 4.4, a numerical simulation example and a practical simulation example of two coupled inverted pendula on carts are presented to demonstrate the effectiveness of the proposed control strategy. The summary is given in section 4.5.

4.1. INTRODUCTION

With the advance of scientific technology, many industrial and commercial systems are modelled by interconnected systems with high complexity and large-scale. These systems usually consist of a set of composite objects through interactions, which are possibly different sorts of physical, natural, and artificial dynamics. Such a class of systems widely exist in the real world, for instance, modern power systems, transportation systems, aircrafts and robots [20, 105, 106]. In reality, the existence of nonlinearities, uncertainties and interconnections increases the difficulty of analysis and design for interconnected systems. Besides that, practical systems are affected by internal and external disturbances including modelling errors, parameter variation, temperature change, pressure and mechanical loss etc. Therefore, the exploration of complex interconnected systems with unknown uncertainties and disturbances is full of challenges.

Due to the advantages of SMC and decentralised control, many researchers have focused on the decentralised SMC of interconnected systems recently. [107] applied the decentralised SMC to the distributed simulation of differential-algebraic equation systems. But the proposed method could not be applied to general nonlinear systems. An adaptive decentralised SMC for a class of non-affine stochastic nonlinear interconnected systems was presented in [108], which was used to estimate one adaptive parameter of each subsystem. Moreover, uncertainties were not considered in [108]. A kind of novel decentralised state-feedback adaptive SMC was proposed in [109] for large-scale interconnected systems with nonlinear interconnections and time-delay. The global decentralised discrete SMC for interconnected systems based on output feedback was employed by [110]. These two strategies achieved good results for specified interconnected systems, though all isolated subsystems are linear. A decentralised integral SMC combined with PID was proposed in [111] for unmanned aerial vehicles, where the control sensitivity with respect to the network topology was analysed, but the mismatched uncertainties were not considered. [112] investigated a sliding variable-based decentralised static output feedback SMC for interconnected descriptor systems, where it required that the interconnected systems was linear. [113] presented a strategy of SMC for load frequency problems in two area interconnected power systems, though SMC was only applied to specific interconnected systems without considering mismatched uncertainties. Although many researchers have obtained the remarkable achievements of decentralised SMC, few people concentrated on the nonlinear interconnected systems with mismatched uncertainties and unknown interconnections at the same time. Due to the complexity of nonlinear systems, the technology of SMC combined with decentralised control for nonlinear interconnected systems with unknown interconnections is challenging and significant.

In this chapter, a state feedback decentralised SMC scheme is proposed to stabilise a class of nonlinear interconnected systems. The considered interconnected systems possess both nonlinear interconnections and nonlinear isolated subsystems. A coordinate transformation is applied to transform all the isolated subsystems into the regular form to facilitate the controller design as well as the interconnected system analysis. Then, for the transformed system, a composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable based on the Lyapunov theory. A state feedback SMC law is established to drive the system to the sliding surface in finite time and maintain the sliding motion after that. The bounds on all uncertainties and interconnections have general nonlinear forms related to system states, which are employed in decentralised control design to reduce the effects of uncertainties. It is shown that under certain conditions, the effect of the unknown interconnections can be completely cancelled by an appropriately designed decentralised controllers with regard to the reachability analysis. At last, a numerical simulation and a practical example are provided to demonstrate the effectiveness of the proposed control strategy.

4.2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider nonlinear time-varying interconnected systems with matched disturbances and unknown interconnections consisted of n interconnected subsystems,

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)(u_i + \varphi_i(t, x_i)) + h_i(t, x), \qquad i = 1, 2, \cdots, n \tag{4.1}$$

where $x_i \in \Omega_i \subset \mathcal{R}^{n_i}$ (Ω_i denotes a neighbourhood of the origin), and $u_i \in \mathcal{R}^{m_i}$ are, respectively, state variables and inputs of the i-th subsystem with $m_i < n_i$, $x :=$ col$(x_1, x_2, \cdots, x_n) \in \Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$. It is assumed that the matrix function $g_i(\cdot) \in \mathcal{R}^{n_i \times m_i}$ is known and has full column rank; the nonlinear vector $f_i(\cdot) \in \mathcal{R}^{n_i}$ is known. The term $\varphi_i(\cdot)$ denotes matched disturbance, and $h_i(\cdot)$ represents the unknown interconnection. All nonlinear functions are assumed to be continuous in their arguments in the considered domain to guarantee the existence of system solutions.

Now, consider a nonlinear transformation,

$$z_i = T_i(x_i), \qquad i = 1, 2, \cdots, n \tag{4.2}$$

which is a diffeomorphism. The Jacobian matrices $\partial T_i / \partial x_i$ are nonsingular in the considered domain for $i = 1, 2, \cdots, n$. Then, the transformation (4.2) defines a new coordinate $z =$ col$(z_1, z_2, \cdots, z_n)$. In the new coordinate z, system (4.1) can be described by

$$\begin{aligned} \dot{z}_i =& \left[\frac{\partial T_i}{\partial x_i} \dot{x}_i \right]_{x_i = T_i^{-1}(z_i)} \\ =& \left[\frac{\partial T_i}{\partial x_i} \big(f_i(t, x_i) + g_i(t, x_i) \cdot (u_i + \varphi_i(t, x_i)) + h_i(t, x) \big) \right]_{x_i = T_i^{-1}(z_i)} \\ & \qquad\qquad i = 1, 2, \cdots, n \end{aligned} \tag{4.3}$$

It is assumed that system (4.1) in the new coordinate z can be described by

$$\dot{z}_{i1} = F_{i1}(t, z_{i1}, z_{i2}) + H_{i1}(t, z) \tag{4.4}$$

$$\dot{z}_{i2} = F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2}) \cdot (u_i + \Phi_i(t, z_{i1}, z_{i2})) + H_{i2}(t, z) \tag{4.5}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$, $z_{i2} \in \Omega_{z_{i2}} \subset \mathcal{R}^{m_i}$, $z = \mathrm{col}(z_1, z_2, \cdots, z_n)$, $z_i = \mathrm{col}(z_{i1}, z_{i2}) \in \Omega_{T_i} \subset \mathcal{R}^{n_i}$,

$$\Omega_{T_i} := \Omega_{z_{i1}} \times \Omega_{z_{i2}} := \{(z_{i1}, z_{i2}) \mid (z_{i1}, z_{i2}) = T_i(x_i), x_i \in \Omega_i\}$$

and

$$\begin{bmatrix} F_{i1}(\cdot) \\ F_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} f_i(t, x_i)\right]_{x_i = T_i^{-1}(z_i)} \tag{4.6}$$

$$H_i(\cdot) := \begin{bmatrix} H_{i1}(\cdot) \\ H_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} h_i(t, x)\right]_{x_i = T_i^{-1}(z_i)} \tag{4.7}$$

$$\begin{bmatrix} 0 \\ G_i(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} g_i(t, x_i)\right]_{x_i = T_i^{-1}(z_i)} \tag{4.8}$$

$$\Phi_i(\cdot) := [\varphi_i(t, x_i)]_{x_i = T_i^{-1}(z_i)} \tag{4.9}$$

where $G_i(\cdot) \in \mathcal{R}^{m_i \times m_i}$ is nonsingular in the considered domain Ω_{T_i} for $i = 1, 2, \ldots, n$.

It should be noted that systems (4.4)−(4.5) are in the traditional regular form, which is very useful for the constructive application of the sliding mode paradigm.

Remark 4.1. It should be pointed out that there is no systematic method to find a coordinate transformation (4.2) to transfer system (4.1) to the regular form (4.4)-(4.5). But the work in [114] and [115] can be referred to construct the corresponding transformation in certain cases.

In the following, the nonlinear interconnected system (4.4)-(4.5) is focused on. The objective of this chapter is to develop a state feedback decentralised SMC scheme, such that the controlled systems (4.4)−(4.5) are uniformly asymptotically stable irrespective of disturbances and unknown interconnections. It should be emphasised that the results developed in this chapter can be easily extended to all interconnected systems (4.1) which can be transformed to the systems (4.4)−(4.5) by a known nonsingular transformation.

4.3. Sliding Motion Analysis and Control Design

In this section, the sliding surface will be designed and the corresponding sliding motion is to be analysed. Then, a novel decentralised SMC strategy is to be proposed under the assumption that all system states are accessible.

4.3.1. Stability of Sliding Motion

Based on the specific structure of the system (4.4)-(4.5), the switching function for the i-th subsystem can be selected as

$$s_i(z_i) = z_{i2}, \qquad i = 1, 2, \cdots, n \tag{4.10}$$

Then, the composite sliding function for the interconnected system (4.4)$-$(4.5) is given as

$$\begin{aligned} S(z) &= \text{col}\,(s_1(z_1), s_2(z_2), \cdots, s_n(z_n)) \\ &= \text{col}\,(z_{12}, z_{22}, \cdots, z_{n2}) \end{aligned} \tag{4.11}$$

So, the composite sliding surface is written by

$$\{\text{col}(z_1, z_2, \dots, z_n) \mid z_{i2} = 0 \quad \text{for} \quad i = 1, 2, \dots, n\} \tag{4.12}$$

When the interconnected system is limited to moving on the sliding surface (4.12), $z_{i2} = 0$ for $i = 1, 2, \dots, n$. It follows from the structure of the system (4.4)-(4.5) that the corresponding sliding mode dynamics can be described by

$$\dot{z}_{i1} = F_{i1s}(t, z_{i1}) + H_{i1s}(t, z_{11}, z_{21}, \cdots, z_{n1}), \qquad i = 1, 2, \dots, n \tag{4.13}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$ denotes the state of the sliding mode dynamics, and

$$F_{i1s}(\cdot) := F_{i1}(t, z_{i1}, z_{i2})|_{z_{i2}=0} \tag{4.14}$$

$$H_{i1s}(\cdot) := H_{i1}(t, z)|_{z_{12}=0, \cdots, z_{n2}=0} \tag{4.15}$$

where $F_{i1}(\cdot)$ and $H_{i1}(\cdot)$ are defined in (4.6) and (4.7), respectively. From (4.7), it is clear to see that the term $H_{i1s}(\cdot)$ comes from $h_i(t, x)$, which represents the unknown interconnection of the i-th subsystems in (4.13) for $i = 1, 2, \dots, n$.

In order to analyse the sliding motion governed by interconnected system (4.13) and related to the composite sliding surface (4.12), the following assumptions are needed.

Assumption 4.1. *There exists the continuously differentiable functions* $V_i(t, z_{i1}) : \mathcal{R}^+ \times \mathcal{R}^{n_i - m_i} \mapsto \mathcal{R}^+$ *for* $i = 1, 2, \ldots, n$*, such that for any* $z_{i1} \in \Omega_{z_{i1}}$ *the following inequalities hold:*

(i) $p_{i1}^2(\|z_{i1}\|) \leq V_i(t, z_{i1}) \leq p_{i2}^2(\|z_{i1}\|)$*;*

(ii) $\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T F_{i1s}(t, z_{i1}) \leq -p_{i3}^2(\|z_{i1}\|)$*;*

(iii) $\left\| \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T \right\| \leq p_{i4}(\|z_{i1}\|)$*,*
where the functions $p_{il}(\cdot)$ *for* $l = 1, 2, 3, 4$ *are class* $\mathcal{KC}^1$ *functions.*

There are continuous functions $\varsigma_{il}(\cdot)$ such that for any $z_{i1} \in \Omega_{z_{i1}}$, $p_{il}(\cdot)$ can be decomposed as

$$p_{il}(\|z_{i1}\|) = \varsigma_{il}(\|z_{i1}\|)\|z_{i1}\|, \qquad l = 1, 2, 3, 4 \tag{4.16}$$

where $\varsigma_{il}(\cdot)$ are continuous functions in $\mathcal{R}^+$ for $i = 1, 2, \ldots, n$ and $l = 1, 2, 3, 4$.

Remark 4.2. Assumption 4.1 implies that all nominal isolated subsystems of the interconnected system (4.13) are uniformly asymptotically stable. It is worth clarifying that Assumption 4.1 is usually required when the nominal sliding mode dynamics are fully nonlinear [115, 116]. Moreover, if the nominal system is exponentially stable, then Assumption 4.1 will be satisfied. It should be mentioned that, the fact that $\dot{z}_{i1} = F_{i1s}(t, z_{i1})$ is uniformly asymptotically stable does not mean that the nominal system (4.4) is uniformly asymptotically stable. It should be pointed out that in [110, 112, 116], the whole interconnected systems need to satisfy the constraint conditions to ensure all nominal isolated subsystems of (4.4)-(4.5) are asymptotically stable, while only (4.13) needs to satisfy the related conditions to guarantee that nominal isolated subsystems of the reduced-order subsystems (4.13) are asymptotically stable. Therefore, the approach proposed in this chapter can reduce the constraint conditions and calculate burden compared with the work mentioned above in this regard.

Assumption 4.2. *The interconnection term* $H_{i1s}(\cdot)$ *in the system (4.13) satisfies*

$$\|H_{i1s}(t, z_{11}, z_{21}, \cdots, z_{n1})\| \leq \beta_i(t, z_{11}, z_{21}, \cdots, z_{n1}) \sum_{j=1}^{n} \|z_{j1}\| \tag{4.17}$$

where $\beta_i(\cdot)$ are known continuous functions for $i = 1, 2, \cdots, n$.

Remark 4.3. Assumption 4.2 ensures that the interconnections in (4.13) are bounded by known functions related to states of the system (4.13). However, the method developed in this chapter can be applied to a wider class of interconnections, for example, (4.17) can be replace by

$$\|H_{i1s}(\cdot)\| \leq \beta_{1i}(\cdot)\|z_{11}\| + \beta_{2i}(\cdot)\|z_{21}\| + \cdots + \beta_{ni}(\cdot)\|z_{n1}\|$$

It is required that β_{ji} are the constants for $i, j = 1, 2, \cdots, n$ in [110]. Note, in this chapter, $\beta_{ji}(\cdot)$ are known continuous functions which include the interconnections considered in [110] as a special case in this regard. In reality, the bounds on uncertainties for a specific practical system usually can be obtained/estimated based on the prior knowledge and engineering experiences as well as statistical/historical data collected for the considered system. It should be noted that under certain conditions, the method proposed in [117] can be applied if the bounds on uncertainties are unknown.

The following result is ready to be presented.

Theorem 4.1. *Under Assumptions 4.1 and 4.2, the sliding motion associated with the sliding surface (4.12) of the system (4.4)–(4.5) is uniformly asymptotically stable if the function matrix $M^T(\cdot) + M(\cdot) > 0$ in the considered domain $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$, where $M = (m_{ij}(\cdot))_{n\times n}$ is a $n \times n$ function matrix with its entries defined by*

$$m_{ij} = \begin{cases} \varsigma_{i3}^2(\cdot) - \varsigma_{i4}(\cdot)\beta_i(\cdot), & i = j \\ -\varsigma_{i4}(\cdot)\beta_i(\cdot), & i \neq j \end{cases} \tag{4.18}$$

where $\varsigma_{i3}(\cdot)$ and $\varsigma_{i4}(\cdot)$ are given in (4.16) and $\beta_i(\cdot)$ is defined in (4.17) for $i, j = 1, 2, \ldots, n$.

Proof. From the analysis above, it is clear to see that system (4.13) is the sliding mode dynamics related to the composite sliding surface (4.12). The remain is to show that the system (4.13) is uniformly asymptotically stable.

Under the condition that $p_{il}(\cdot)$ is class $\mathcal{KC}^1$ function, the equations in (4.16) hold. For system (4.13), consider the candidate Lyapunov function

$$V(t, z_{11}, z_{21}, \cdots, z_{n1}) = \sum_{i=1}^{n} V_i(t, z_{i1}) \tag{4.19}$$

where $V_i(\cdot)$ is defined in Assumption 4.1. The time derivative of $V(\cdot)$ along the trajectory of system (4.13) is described as

$$\begin{aligned}
&\dot{V}(t, z_{11}, z_{21}, \cdots, z_{n1}) = \sum_{i=1}^{n} \dot{V}_i(t, z_{i1}) \\
&= \sum_{i=1}^{n} \left(\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}} \right)^T (F_{i1s}(\cdot) + H_{i1s}(\cdot)) \right) \\
&\leq \sum_{i=1}^{n} \left(\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}} \right)^T F_{i1s}(\cdot) + \left\| \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}} \right)^T \right\| \cdot \|H_{i1s}(\cdot)\| \right)
\end{aligned} \tag{4.20}$$

From Assumptions 4.1 and 4.2, the equation (4.20) can be written as follows

$$\dot{V}(t, z_{11}, z_{21}, \cdots, z_{n1}) \leq \sum_{i=1}^{n} \Big(- p_{i3}^2(\|z_{i1}\|) + p_{i4}(\|z_{i1}\|) \cdot \beta_i(\cdot) \sum_{j=1}^{n} \|z_{j1}\| \Big) \tag{4.21}$$

According to equation (4.16), it follows that

$$\begin{aligned}
&\dot{V}(t, z_{11}, z_{21}, \cdots, z_{n1}) \\
&\leq \sum_{i=1}^{n} \left(-\varsigma_{i3}^2(\|z_{i1}\|)\|z_{i1}\|^2 + \varsigma_{i4}(\|z_{i1}\|)\|z_{i1}\|\beta_i(\cdot) \sum_{j=1}^{n} \|z_{j1}\| \right) \\
&= -\sum_{i=1}^{n} \left(\varsigma_{i3}^2(\|z_{i1}\|) - \varsigma_{i4}(\|z_{i1}\|)\beta_i(\cdot) \right) \|z_{i1}\|^2 \\
&\quad - \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \varsigma_{i4}(\|z_{i1}\|) \cdot \beta_i(\cdot)\|z_{i1}\| \cdot \|z_{j1}\| \\
&= -\frac{1}{2} Z^T \left(M^T + M \right) Z
\end{aligned} \tag{4.22}$$

where $Z := \text{col}(\|z_{11}\|, \|z_{21}\|, \cdots, \|z_{n1}\|)$ and M is the $n \times n$ matrix with entries defined in (4.18). Hence, the result follows from $M^T + M > 0$. □

Remark 4.4. Theorem 4.1 provides a set of sufficient conditions under which the sliding mode is uniformly asymptotically stable. The function matrix M in Theorem 4.1 only depends on $\varsigma_{i3}(\cdot)$, $\varsigma_{i4}(\cdot)$ and $\beta_i(\cdot)$, which are determined by the given system. The condition that $M^T + M > 0$ with M defined in (4.18) implies the limitation to the mismatched interconnections.

4.3.2. Reachability Analysis

A set of conditions have been developed in Theorem 4.1 above to guarantee the sliding motion stability of the considered interconnected systems (4.4)−(4.5). The objective now is to design a decentralised state feedback SMC such that the interconnected system is driven to the sliding surface (4.12) in finite time.

For the interconnected system (4.4)−(4.5), the corresponding reachability condition based on the composite sliding surface is described by

$$S^T(z)\dot{S}(z) \leq -\eta\|S(z)\| \tag{4.23}$$

where $S(z)$ is defined by (4.11), and η is a positive constant.

Consider the system (4.4)−(4.5), the following assumption is introduced for further analysis and control design.

Assumption 4.3. *The uncertainty $\Phi_i(t, z_{i1}, z_{i2})$ and the interconnection $H_{i2}(t, z)$ in (4.5) satisfy*

$$\|\Phi_i(t, z_{i1}, z_{i2})\| \leq \xi_{i1}(t, z_{i1}, z_{i2}) \tag{4.24}$$

$$\|H_{i2}(t, z)\| \leq \sum_{j=1}^{n} \epsilon_{ij}(t, z_j) \tag{4.25}$$

where $\xi_{i1}(t, z_{i1}, z_{i2})$ and $\epsilon_{ij}(t, z_j)$ are known continuous functions.

It should be noted that Assumption 4.3 is the limitation to system uncertainties as well as interconnections. It is clear to see that the bounds on the uncertainties and interconnections are fully nonlinear, which are to be employed in the control design to reject the effects of them on system performance. Construct the control law

$$\begin{aligned} u_i = &- G_i^{-1}(t, z_{i1}, z_{i2})F_{i2}(t, z_{i1}, z_{i2}) - G_i^{-1}(t, z_{i1}, z_{i2})k_i \cdot \text{sgn}(z_{i2}) \\ &- G_i^{-1}(t, z_{i1}, z_{i2})\Big(\|G_i(t, z_{i1}, z_{i2})\|\xi_{i1}(t, z_{i1}, z_{i2})\text{sgn}(z_{i2}) \\ &+ \frac{n}{2}z_{i2} + \frac{1}{2}\frac{z_{i2}}{\|z_{i2}\|^2}\sum_{j=1}^{n}\epsilon_{ji}^2(t, z_i)\Big), \qquad i = 1, 2, \ldots, n \end{aligned} \tag{4.26}$$

where $F_{i2}(\cdot)$ is given in (4.6), $\xi_{i1}(\cdot)$ and $\sum_{j=1}^{n}\epsilon_{ij}(t, z_j)$ are given in (4.24) and (4.25), respectively, sgn$(\cdot)$ is the usual signum function, and k_i is the control gain which is a positive constant.

Remark 4.5. From the control structure in (4.26), it follows that the functions $\epsilon_{ij}(t, z_j)$ need to be vanished at $z_j = 0$ for $i, j = 1, 2, \ldots, n$. This implies that the unknown interconnections $H_{i2}(t, z)$ must be vanished at the origin $z_{i2} = 0$ for $i = 1, 2, \ldots, n$. Otherwise, it may result in infinite control due to the term $\frac{z_{i2}}{\|z_{i2}\|^2}$.

Theorem 4.2. *Under Assumption 4.3, the nonlinear interconnected system (4.4)–(4.5) can be driven to the sliding surface (4.12) in finite time by the designed controller in (4.26) and maintains a sliding motion on it thereafter.*

Proof. From the definition of $S(z)$ in (4.11) and the system (4.5), it follows that

$$\begin{aligned} S^T(z)\dot{S}(z) &= \sum_{i=1}^{n} z_{i2}^T \dot{z}_{i2} \\ &= \sum_{i=1}^{n} z_{i2}^T \Big(F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2}) \Big(u_i + \Phi_i(t, z_{i1}, z_{i2}) \Big) + H_{i2}(t, z) \Big) \end{aligned} \tag{4.27}$$

Substituting the control u_i in (4.26) into equation (4.27),

$$\begin{aligned} S^T(z)\dot{S}(z) = \sum_{i=1}^{n} z_{i2}^T \Bigg(& F_{i2}(\cdot) + G_i(\cdot) \Bigg(- G_i^{-1}(\cdot) F_{i2}(\cdot) \\ & - G_i^{-1}(\cdot) \Big(\|G_i(\cdot)\| \xi_{i1}(\cdot) \text{sgn}(z_{i2}) + \frac{n}{2} z_{i2} + \frac{1}{2} \frac{z_{i2}}{\|z_{i2}\|^2} \sum_{j=1}^{n} \epsilon_{ji}^2(\cdot) \Big) \\ & - G_i^{-1}(\cdot) k_i \cdot \text{sgn}(z_{i2}) + \Phi_i(\cdot) \Bigg) + H_{i2}(\cdot) \Bigg) \end{aligned} \tag{4.28}$$

Rearrange the associated terms in (4.28), it follows that

$$\begin{aligned} & S^T(z)\dot{S}(z) \\ = \; & \sum_{i=1}^{n} \Bigg(z_{i2}^T G_i(\cdot) \Phi_i(\cdot) - \|G_i(\cdot)\| \xi_{i1}(\cdot) z_{i2}^T \text{sgn}(z_{i2}) \\ & + z_{i2}^T H_{i2}(\cdot) - \Big(\frac{n}{2} z_{i2}^T z_{i2} + \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \sum_{j=1}^{n} \epsilon_{ji}^2(\cdot) \Big) - k_i z_{i2}^T \text{sgn}(z_{i2}) \Bigg) \\ = \; & \sum_{i=1}^{n} \Big(z_{i2}^T G_i(\cdot) \Phi_i(\cdot) - \|G_i(\cdot)\| \xi_{i1}(\cdot) z_{i2}^T \text{sgn}(z_{i2}) \Big) \\ & + \Big(\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n} \frac{n}{2} z_{i2}^T z_{i2} - \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \epsilon_{ji}^2(\cdot) \Big) \\ & - \sum_{i=1}^{n} k_i z_{i2}^T \text{sgn}(z_{i2}) \end{aligned} \tag{4.29}$$

Based on (4.24), (4.25) and the fact that $s^T \text{sgn}(s) \geq \| s \|$ for any vectors s (see Lemma 1 in [118]), it follows that

$$\begin{aligned}
&\sum_{i=1}^{n} \left(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\| \xi_{i1}(\cdot) z_{i2}^T \text{sgn}(z_{i2}) \right) \\
\leq &\sum_{i=1}^{n} \left(\|z_{i2}\| \cdot \|G_i(\cdot)\| \cdot \|\Phi_i(\cdot)\| - \|z_{i2}\| \cdot \|G_i(\cdot)\| \cdot \xi_{i1}(\cdot) \right) \leq 0
\end{aligned} \tag{4.30}$$

Then, by similar reasoning as in (4.30), and from (4.25)

$$\begin{aligned}
&\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n} \frac{n}{2} z_{i2}^T z_{i2} - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \epsilon_{ji}^2(\cdot) \\
\leq &\sum_{i=1}^{n} \|z_{i2}\| \cdot \|H_{i2}(\cdot)\| - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2}\|z_{i2}\|^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \epsilon_{ji}^2(t, z_i) \\
= &\sum_{i=1}^{n} \|z_{i2}\| \cdot \|H_{i2}(\cdot)\| - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2}\|z_{i2}\|^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \epsilon_{ij}^2(t, z_j)
\end{aligned} \tag{4.31}$$

From the fact that $\frac{a^2+b^2}{2} \geq |a|\,|b|$, it follows that

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2}\|z_{i2}\|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \epsilon_{ij}^2(t, z_j) \\
= &\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \left(\|z_{i2}\|^2 + \epsilon_{ij}^2(t, z_j) \right) \\
\geq &\sum_{i=1}^{n}\sum_{j=1}^{n} \|z_{i2}\| \epsilon_{ij}(t, z_j) \\
= &\sum_{i=1}^{n} \|z_{i2}\| \sum_{j=1}^{n} \epsilon_{ij}(t, z_j) \\
\geq &\sum_{i=1}^{n} \|z_{i2}\| \cdot \|H_{i2}(\cdot)\|
\end{aligned} \tag{4.32}$$

From (4.32) and (4.31),

$$\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n} \frac{n}{2} z_{i2}^T z_{i2} - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{2} \frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2} \epsilon_{ji}^2(\cdot) \leq 0 \tag{4.33}$$

Substituting (4.30) and (4.33) into (4.29) yields

$$S^T(z)\dot{S}(z) \leq -\sum_{i=1}^{n} k_i z_{i2}^T \text{sgn}(z_{i2}) \leq -\eta \sum_{i=1}^{n} z_{i2}^T \text{sgn}(z_{i2}) \leq -\eta \|S\| \tag{4.34}$$

where η is chosen such that $\eta \leq \min\{k_1, k_2, \cdots, k_n\}$.

The inequality (4.34) shows that the reachability condition (4.23) is satisfied, and thus the interconnected system (4.4)−(4.5) can be driven to the sliding surface (4.12) in finite time and maintain a sliding motion on it thereafter. Hence, the result follows. □

According to SMC theory, Theorems 4.1 and 4.2 together show that the closed-loop system formed by applying control law (4.26) to interconnected system (4.4)−(4.5) is uniformly asymptotically stable.

Remark 4.6. From the proof of Theorem 4.2 above, it is clear to see that both the matched uncertainties and the mismatched interconnection terms can be cancelled by the designed decentralised controllers in the reachability analysis, which is one of the main contributions in this chapter. Such controllers can enhance the robustness against unknown interconnections even in the framework of decentralised scheme. Moreover, the developed decentralised controllers can guarantee that the interconnected systems are driven to the composite sliding surfaces in finite time. As for how to estimate the finite reaching time, refer to the recent work in [119]. It should be noted that the effect of the mismatched interconnections can not be rejected by designing controller. Actually the limitation to the mismatched interconnection is necessary for sliding phase, which can be seen from the comments in Remark 4.4.

Remark 4.7. It should be emphasised that in this chapter, the considered systems are fully nonlinear with nonlinear disturbances and nonlinear interconnections. It is not required that the nominal subsystems are linear, or the nominal subsystems are linearizable or partial linearizable. This is in comparison with the most of existing work [120]. Therefore, the methodology developed in this chapter can be applied to a wide class of interconnected systems.

4.4. Simulation Examples

This section presents the simulation results of a numerical example and a practical example to prove the effectiveness of the proposed method.

4.4.1. NUMERICAL SIMULATION

Consider the nonlinear interconnected system which is composed of two third-order subsystems

$$\dot{x_1} = \underbrace{\begin{bmatrix} -6x_{12}^2 x_{13}^2 - 4x_{12}^2 - 2x_{11} \\ -3x_{12}x_{13}^2 - 3x_{12} + \frac{1}{16}(x_{12}^2 - x_{11})^2 \\ 3x_{12}^2 x_{13} - 3x_{13} - \frac{1}{4}(x_{12}^2 - x_{11})\exp\{-t\}\cos(x_{13}t) \end{bmatrix}}_{f_1(\cdot)}$$

$$+ \begin{bmatrix} -4(x_{13}^2 \sin^2 t + 1) \\ 0 \\ 0 \end{bmatrix} (u_1 + \varphi_1(t, x_1)) + h_1(t, x) \tag{4.35}$$

$$\dot{x_2} = \underbrace{\begin{bmatrix} -8x_{21} + x_{23} \\ -7x_{22} + x_{23} \\ x_{21} \end{bmatrix}}_{f_2(\cdot)} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (u_2 + \varphi_2(t, x_2)) + h_2(t, x) \tag{4.36}$$

where $x_i = \text{col}(x_{i1}, x_{i2}, x_{i3}) \in \mathcal{R}^3$ and $u_i \in \mathcal{R}$ are, respectively, the state variables and inputs of the i-th subsystem for $i = 1, 2$. The terms $\varphi_i(\cdot)$ and $h_i(\cdot)$ for $i = 1, 2$ are matched disturbances and unknown interconnections, respectively.

Consider the transformation T_1 and T_2 defined by

$$T_1 : \begin{cases} z_{11}^a = x_{12} \\ z_{11}^b = x_{13} \\ z_{12} = \dfrac{1}{4}(x_{12}^2 - x_{11}) \end{cases} \quad \text{and} \quad T_2 : \begin{cases} z_{21}^a = x_{21} \\ z_{21}^b = x_{21} + x_{22} \\ z_{22} = x_{23} \end{cases}$$

It is easy to find that the Jacobian matrices of T_1 and T_2 are given by

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(1/4) & (1/2)x_{12} & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which are nonsingular in whole state space. By direct calculation, the system (4.35)-(4.36) in the new coordinates is given by

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a \left(z_{11}^b\right)^2 - 3z_{11}^a + z_{12}^2 \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b - z_{12}\exp\{-t\}\cos\left(z_{11}^b t\right) \end{bmatrix} + H_{11}(\cdot) \tag{4.37}$$

$$\dot{z}_{12} = -2z_{12} + \frac{1}{2}z_{11}^a z_{12}^2 + (1 + \left(z_{11}^b\right)^2 \sin^2 t)(u_1 + \Phi_1(\cdot)) + H_{12}(\cdot) \tag{4.38}$$

$$\dot{z}_{21} = \left[-8z_{21}^a + z_{22} - 7z_{21}^b - z_{21}^a + 2z_{22}\right] + H_{21}(\cdot) \tag{4.39}$$

$$\dot{z}_{22} = z_{21}^a + (u_2 + \Phi_2(\cdot)) + H_{22}(\cdot) \tag{4.40}$$

where $H_{i1}(\cdot) \in \mathcal{R}^2$ and $H_{i2}(\cdot) \in \mathcal{R}^1$ for $i = 1, 2$.

In order to demonstrate the theoretical results obtained in this chapter, it is assumed that the uncertainties in (4.37)-(4.40) satisfy

$$|\Phi_1(\cdot)| \le (\|z_{11}^b\| + 1)\exp\{-t\} \tag{4.41}$$

$$\|H_{11}\| \le \|z_{11}^a\|\sin^2 t + \|z_{12}\| + \|z_{22}\| \tag{4.42}$$

$$\|H_{12}\| \le \sum_{j=1}^{2} \epsilon_{1j}(t, z_j) \le 0.25(\|z_{11}^a\|\sin^2 t + \|z_{12}\| + \|z_{22}\|) \tag{4.43}$$

$$\sum_{j=1}^{2} \epsilon_{1j}^2(t, z_j) \le 0.06(\|z_{11}^a\|\sin^2 t + \|z_{12}\| + \|z_{22}\|)^2 \tag{4.44}$$

$$|\Phi_2(\cdot)| \le \|z_{21}^b\|\sin^2 z_{22} \tag{4.45}$$

$$\|H_{21}\| \le 1.618(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22} \tag{4.46}$$

$$\|H_{22}\| \le \sum_{j=1}^{2} \epsilon_{2j}(t, z_j) \le 0.40(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22} \tag{4.47}$$

$$\sum_{j=1}^{2} \epsilon_{2j}^2(t, z_j) \le 0.32(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^4 \sin^4 z_{22} \tag{4.48}$$

For (4.37)-(4.40), select the switching function $S(z) := \mathrm{col}\,(z_{12}, z_{22})$. When the sliding motion occurs, $z_{12} = z_{22} = 0$. It can be obtained by direct calculation that the sliding mode dynamics are written as follows

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a \left(z_{11}^b\right)^2 - 3z_{11}^a \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b \end{bmatrix} + H_{11s}(\cdot) \tag{4.49}$$

$$\dot{z}_{21} = \begin{bmatrix} -8z_{21}^a \\ -7z_{21}^b - z_{21}^a \end{bmatrix} + H_{21s}(\cdot) \tag{4.50}$$

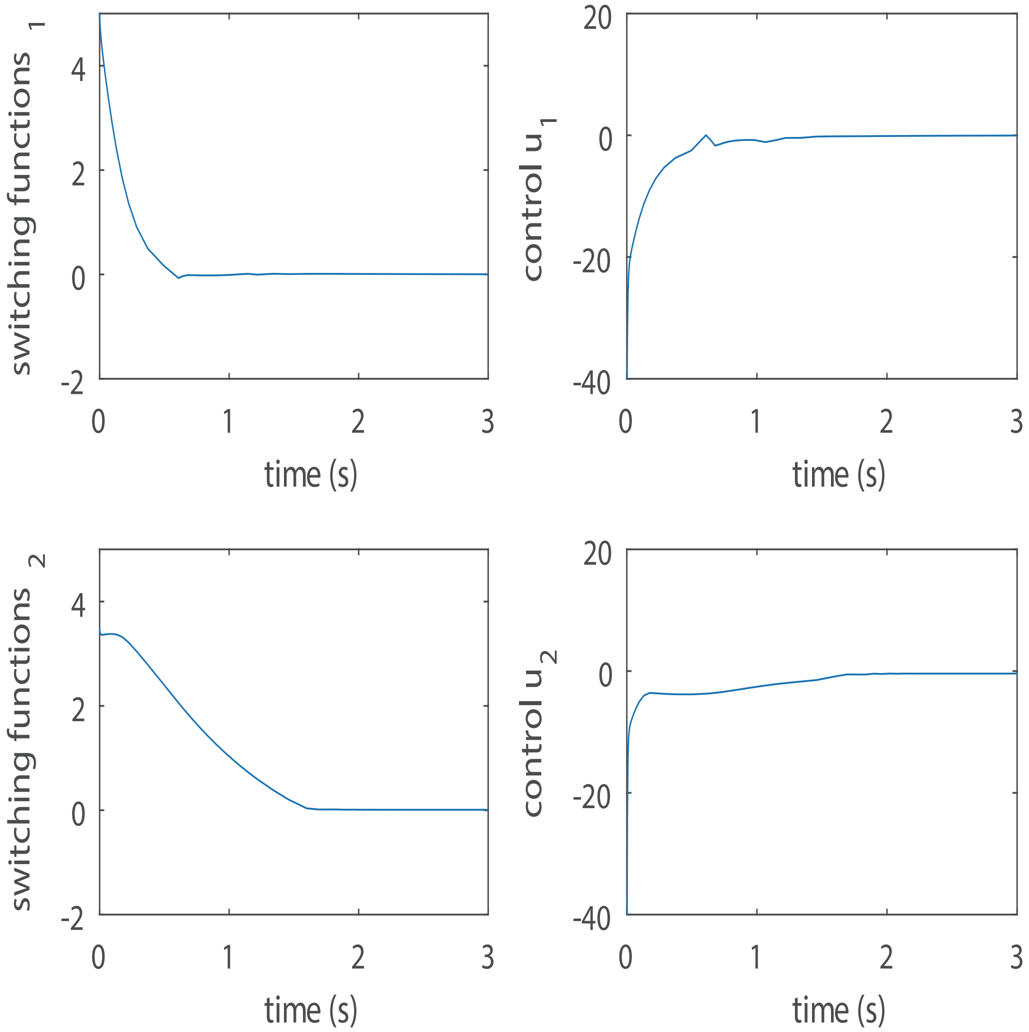

Figure 4.1: Time responses of the switching function s_1 and control signal u_1 of the subsystem (4.35) (Upper); time responses of the switching function s_2 and control signal u_2 of the subsystem (4.36) (Bottom) for $k_1 = 0.2$ and $k_2 = 1.5$.

It is clear to see from (4.42) and (4.46) that

$$\|H_{11s}(\cdot)\| \leq \|z_{11}^a\| \sin^2 t \leq \|z\| \tag{4.51}$$

$$\|H_{21s}(\cdot)\| = 0 \tag{4.52}$$

and thus $\beta_1 = 1$ and $\beta_2 = 0$.

For the system (4.37)-(4.40), consider the candidate Lyapunov function as

$$V(\cdot) = V_1(\cdot) + V_2(\cdot)$$

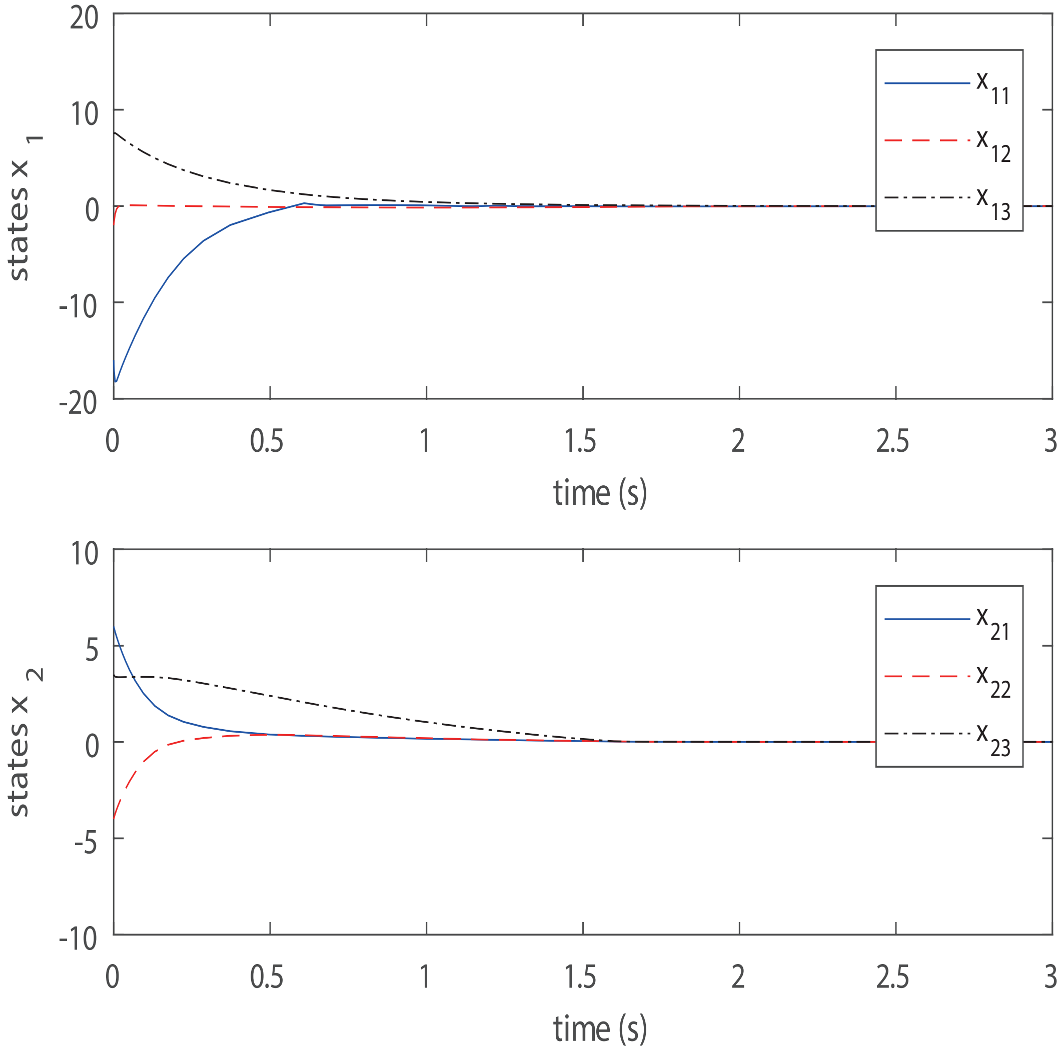

Figure 4.2: Time responses of the state variables of the subsystem (4.35) (Upper); time responses of the state variables of the subsystem (4.36) (Bottom) for $k_1 = 0.2$ and $k_2 = 1.5$.

where $V_1 = (z_{11}^a)^2 + \left(z_{11}^b\right)^2$ and $V_2 = (z_{21}^a)^2 + \left(z_{21}^b\right)^2$. By direct calculation,

$$p_{il}(\|z_{i1}\|) = \tau_{il}\|z_{i1}\|, \quad i = 1, 2, \quad l = 1, 2, 3, 4 \tag{4.53}$$

where τ_{il} for $i = 1, 2$, $l = 1, 2, 3, 4$ are the positive constants. It is easy to find that

Assumption 4.1 holds and the $p_{il}(\cdot)$ satisfy (4.53) with

$$\begin{aligned}\tau_{11} = \tau_{12} = 1, \quad \tau_{13} = \sqrt{6}, \quad \tau_{14} = 2\\ \tau_{21} = \tau_{22} = 1, \quad \tau_{23} = \sqrt{13}, \quad \tau_{24} = 2\end{aligned}$$

Then from (4.18), it follows by direct calculation that

$$M^T + M > 0 \tag{4.54}$$

According to Theorem 4.1, the designed sliding mode is asymptotically stable.

Based on (4.26), the designed control is given by

$$\begin{aligned}u_1(\cdot) = {} & \frac{-3z_{12} + 0.5z_{11}^a z_{12}^2}{1 + \left(z_{11}^b\right)^2 \sin^2 t} - \frac{k_1 \mathrm{sgn}(z_{12})}{1 + \left(z_{11}^b\right)^2 \sin^2 t}\\ & - \left((\|z_{11}^b\| + 1) \exp\{-t\} \mathrm{sgn}(z_{12}) + \frac{0.03z_{12}(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|)^2}{\|z_{12}\|^2 \left(1 + \left(z_{11}^b\right)^2 \sin^2 t\right)} \right)\end{aligned} \tag{4.55}$$

$$\begin{aligned}u_2(\cdot) = {} & z_{21}^a - k_2 \mathrm{sgn}(z_{22}) - z_{22}\\ & - \left(\|z_{21}^b\| \sin^2 z_{22} \mathrm{sgn}(z_{22}) + \frac{0.16z_{22}}{\|z_{22}\|^2} \left(z_{11}^a + (z_{11}^a)^2 - 4z_{12}\right)^4 \cdot \sin^4 z_{22} \right)\end{aligned} \tag{4.56}$$

where constants k_1 and k_2 are chosen as

$$k_1 = 0.2 \quad \text{and} \quad k_2 = 1.5$$

From Theorems 4.1 and 4.2, it follows that the controller (4.55)-(4.56) can stabilise the interconnected system (4.37)-(4.40) uniformly asymptotically.

For simulation purposes, the initial states are chosen as $x_{10} = (-2, 7.5, 5)$ and $x_{20} = (6, 2, 3.5)$, and the uncertainties and interconnections are chosen as

$$H_{11} = \begin{bmatrix} -0.5\left(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|\right)\\ 0.7\left(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|\right)\end{bmatrix} \tag{4.57}$$

$$H_{12} = 0.05(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|) \tag{4.58}$$

$$\Phi_1(\cdot) = 0.9 \cdot (\|z_{11}^b\| + 1) \exp\{-t\} \tag{4.59}$$

$$H_{21} = \begin{bmatrix} -0.647(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22}\\ 0.323(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22}\end{bmatrix} \tag{4.60}$$

$$H_{22} = 0.32(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22} \tag{4.61}$$

$$\Phi_2(\cdot) = 0.7\|z_{21}^b\| \sin^2 z_{22} \tag{4.62}$$

Fig 4.1 shows the control signals and the sliding functions with respect to time. The simulation results in Fig 4.2 show that the closed-loop system formed by applying control (4.55)-(4.56) to the interconnected system (4.37)-(4.40) is uniformly asymptotically stable which is in consistence with the obtained theoretical results.

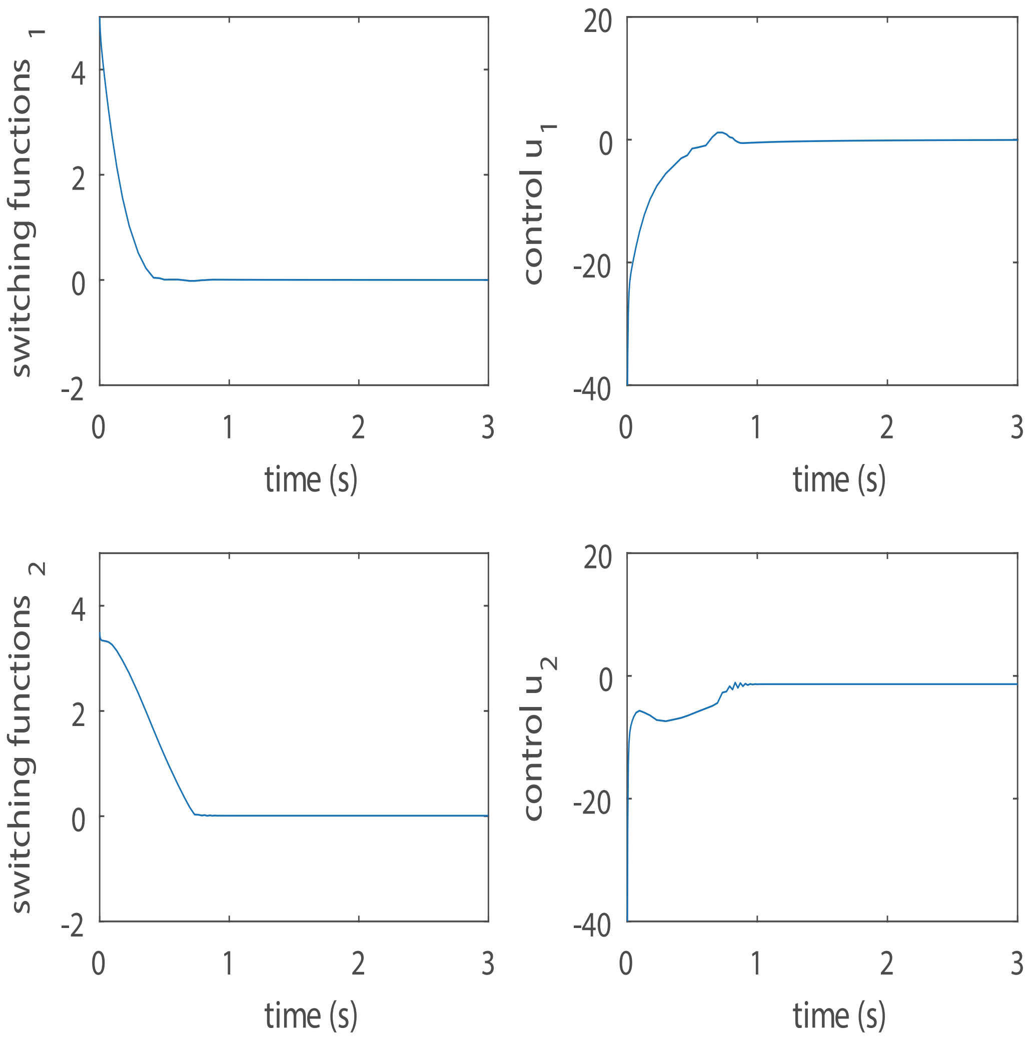

Figure 4.3: Time responses of the switching function s_1 and control signal u_1 (Upper); time responses of the switching function s_2 and control signal u_2 (Bottom) for $k_1 = 2.5$ and $k_2 = 5$.

It should be noted that the reachability constant depends on the parameters k_1 and k_2

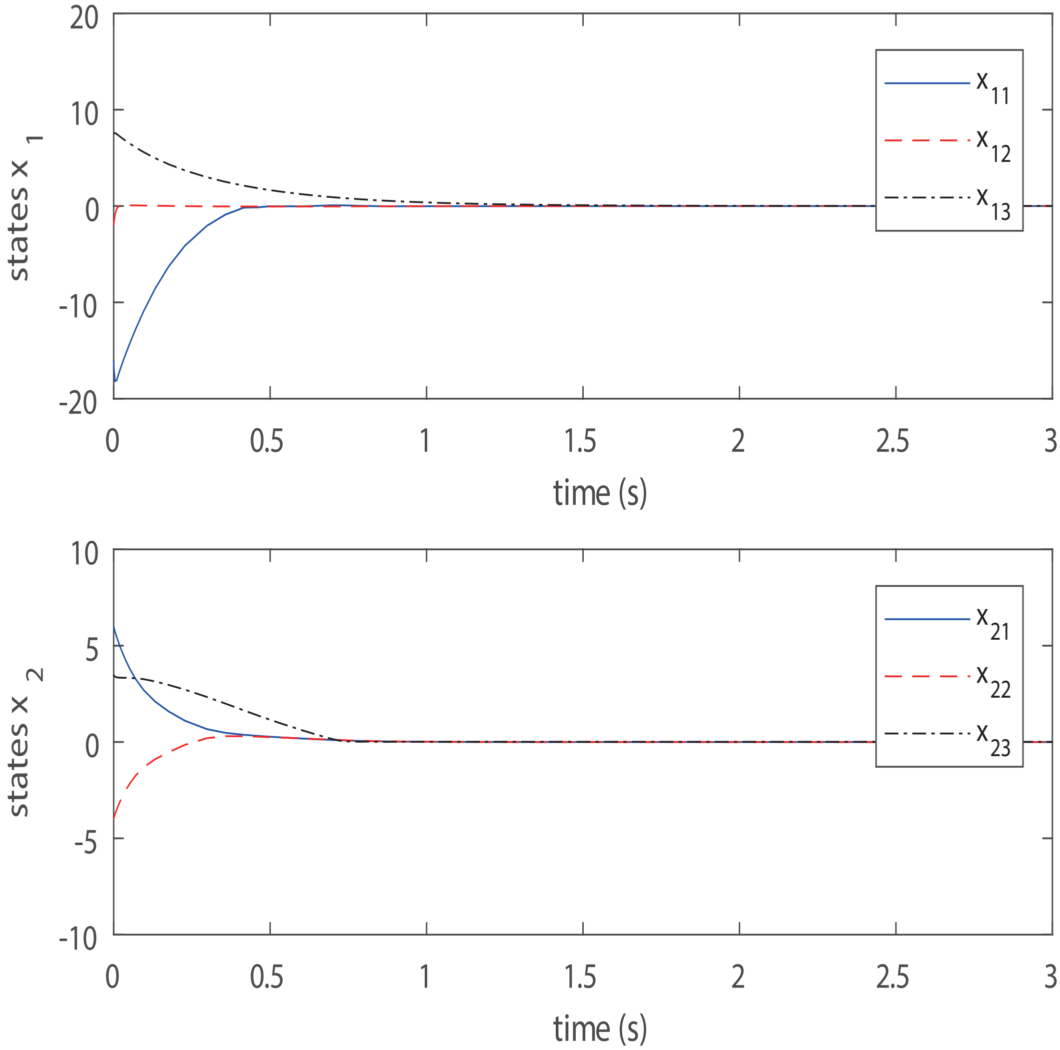

Figure 4.4: Time responses of the state variables of the subsystem (4.35) (Upper); time responses of the state variables of the subsystem (4.36) (Bottom) for $k_1 = 2.5$ and $k_2 = 5$.

which affect the convergent rates of sliding functions as well as system state variables. In order to demonstrate this, keep all the other parameters the same but increase k_1 and k_2 to $k_1 = 2.5$ and $k_1 = 5$. The simulation results are presented in Figs 4.3 and 4.4. It is clear to see, by comparing Figs 4.1 and 4.2 with Figs 4.3 and 4.4, that the bigger the values of k_1 and k_2 are, the faster the sliding functions and system state variables converge. In the practical situation, k_i is selected according the actual systems, and values of k_i are not infinity. Excessive values of k_i can cause the problem of the chattering.

Remark 4.8. It should be noted that the interconnected system (4.35)-(4.36) are fully nonlinear where both matched uncertainties and unmatched interconnections are involved. Therefore, the methods proposed in the recent work [121, 122] cannot be applied to system (4.35)-(4.36). Although the considered interconnected systems are nonlinear in [121, 122], it is required that the nominal subsystems have a triangle structure and the uncertainties/interconnections have a linear growth rate in [121]. Moreover, it is required that the interconnections are matched in [122].

4.4.2. Simulation Results of Two Coupled Inverted Pendula on Carts

Consider a nonlinear interconnected system of the two coupled inverted pendula on carts [20] as follows

$$\dot{x_1} = \underbrace{\begin{bmatrix} x_{12} \\ 0.25\sin(3t) - 0.25\sin(2t) + 0.75x_{11} - 0.02x_{12}^2 \sin x_{11} \end{bmatrix}}_{f_1(\cdot)} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ h_1(t,x) \end{bmatrix} \tag{4.63}$$

$$\dot{x_2} = \underbrace{\begin{bmatrix} x_{22} \\ 0.25\sin(2t) - 0.25\sin(3t) + 0.75x_{21} - 0.02x_{22}^2 \sin x_{21} \end{bmatrix}}_{f_2(\cdot)} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ h_2(t,x) \end{bmatrix} \tag{4.64}$$

where $x_i = \text{col}(x_{i1}, x_{i2}) \in \mathcal{R}^2$ and $u_i \in \mathcal{R}$ are the state variables and inputs of the i-th subsystem for $i = 1, 2$, respectively. x_{i1} and x_{i2} denote the angle θ_i and angular velocity $\dot{\theta}_i$ for $i = 1, 2$. $h_i(\cdot) \in \mathcal{R}$ for $i = 1, 2$ are unknown interconnections. Fig 4.5 shows the structure of this nonlinear interconnected system.

It is easy to find that the system (4.63)-(4.64) has the regular form. So choose the transformation $T_1 = I_2$ and $T_2 = I_2$. The system (4.63)-(4.64) is rewritten in the regular

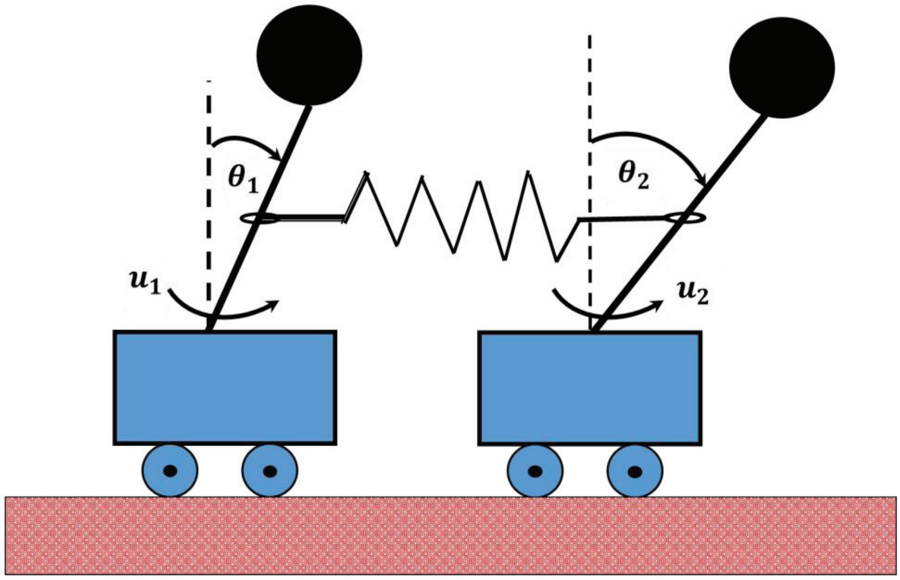

Figure 4.5: The structure of the two coupled inverted pendula on carts [20].

form as follows

$$\dot{x}_{11} = x_{12} \tag{4.65}$$

$$\dot{x}_{12} = 0.25\sin(3t) - 0.25\sin(2t) + 0.75x_{11} - 0.02x_{12}^2 \sin x_{11} + 0.5u_1 + h_1(t,x) \tag{4.66}$$

$$\dot{x}_{21} = x_{22} \tag{4.67}$$

$$\dot{x}_{22} = 0.25\sin(2t) - 0.25\sin(3t) + 0.75x_{21} - 0.02x_{22}^2 \sin x_{21} + 0.5u_2 + h_2(t,x) \tag{4.68}$$

It is assumed that the interconnections $h_i(\cdot)$ for $i = 1, 2$ in (4.65)-(4.68) satisfy

$$\begin{aligned}\|h_1\| &\le \sum_{j=1}^{2} \epsilon_{1j}(t, z_j) \\ &\le (0.25\|x_{11}\| + 0.5\|x_{12}\| + 0.5\|x_{21}\| + 0.5\|x_{22}\|)^{0.5}(2x_{11} + x_{12})^{0.5}\sin t \qquad (4.69)\\ \|h_2\| &\le \sum_{j=1}^{2} \epsilon_{2j}(t, z_j) \\ &\le (0.7\|x_{11}\| + 0.6\|x_{12}\| + 0.25\|x_{21}\| + 0.4\|x_{22}\|)^{0.5}(2x_{21} + x_{22})^{0.5}\sin t \qquad (4.70)\end{aligned}$$

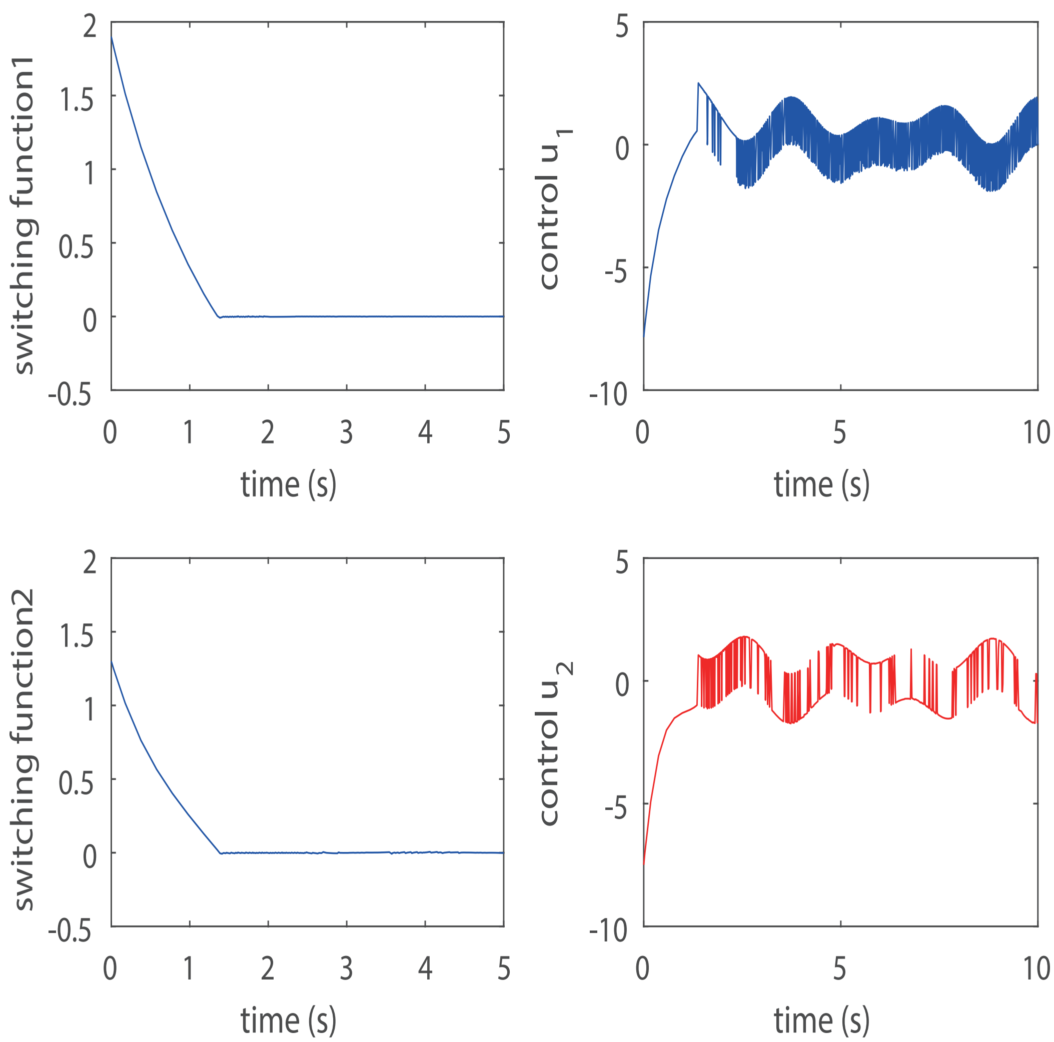

Figure 4.6: Time responses of the switching function s_1 and control signal u_1 for the subsystem (4.63) (Upper); time responses of the switching function s_2 and control signal u_2 for the subsystem (4.64) (Bottom).

For (4.65)-(4.68), select the switching function $S(z) := \text{col}\,(s_1, s_2)$, where $s_i = 2x_{i1} + x_{i2}$ for $i = 1, 2$. When the sliding motion occurs,

$$2x_{11} + x_{12} = 2x_{21} + x_{22} = 0 \tag{4.71}$$

It can be obtained by direct calculation that the sliding mode dynamics are written as follows

$$\dot{x}_{11} = -2x_{11} \tag{4.72}$$

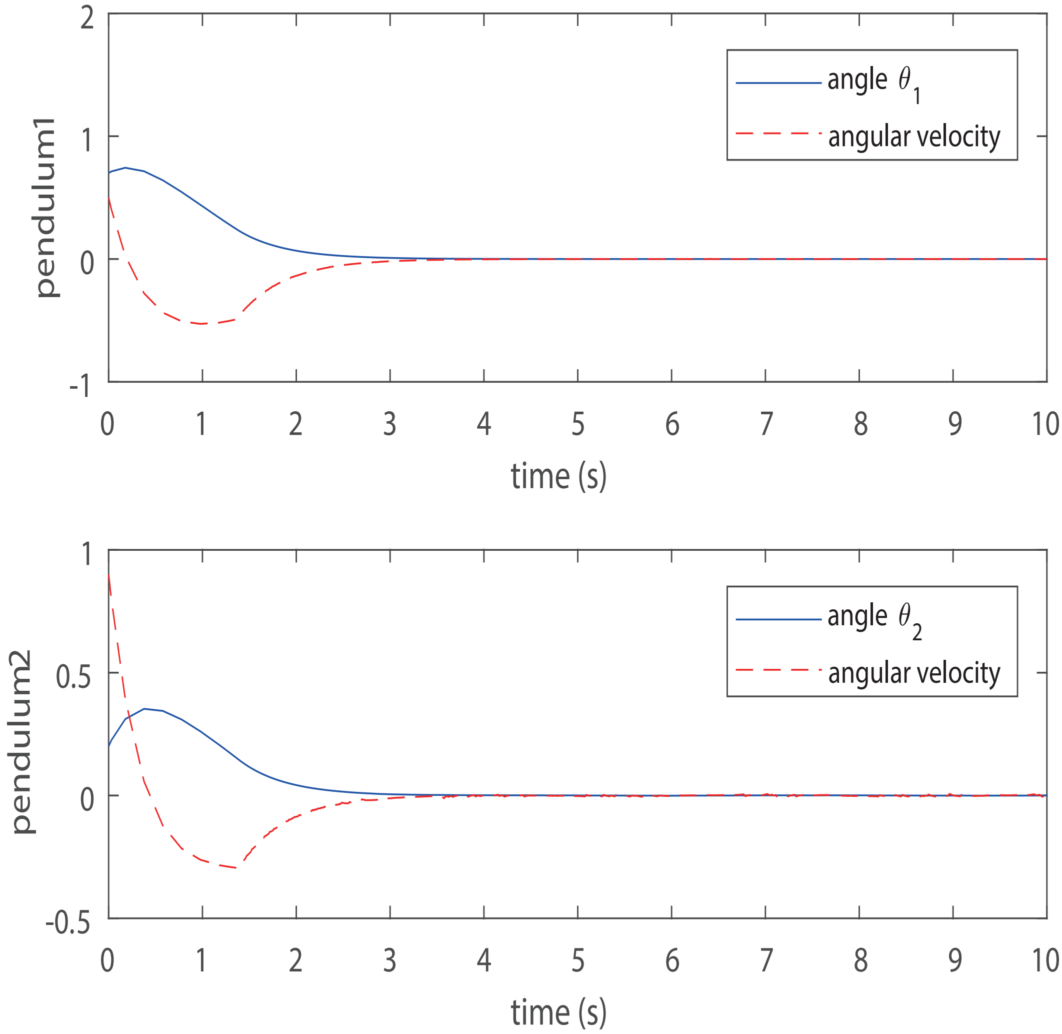

Figure 4.7: Time responses of angle θ_1 and angular velocity $\dot{\theta}_1$ (Upper); time responses of angle θ_2 and angular velocity $\dot{\theta}_2$ (Bottom).

$$\dot{x}_{21} = -2x_{21} \tag{4.73}$$

It is easy to achieve that $\beta_1 = 0$ and $\beta_2 = 0$.

For the system (4.65)-(4.68), consider the candidate Lyapunov function as

$$V(\cdot) = V_1(\cdot) + V_2(\cdot)$$

where $V_1 = x_{11}^2$ and $V_2 = x_{21}^2$. By direct calculation,

$$p_{il}(\|x_{i1}\|) = \tau_{il}\|x_{i1}\|, \quad i = 1, 2, \quad l = 1, 2, 3, 4 \tag{4.74}$$

where τ_{il} for $i = 1, 2,\ l = 1, 2, 3, 4$ are the positive constants. It is easy to find that Assumption 4.1 holds and the $p_{il}(\cdot)$ satisfy (4.74) with

$$\tau_{11} = \tau_{12} = 1, \quad \tau_{13} = \tau_{14} = 2$$
$$\tau_{21} = \tau_{22} = 1, \quad \tau_{23} = \tau_{24} = 2$$

Then from (4.18), it follows by direct calculation that

$$M^T + M > 0 \tag{4.75}$$

According to Theorem 4.1, the designed sliding mode is asymptotically stable.

Based on (4.26), the designed control is given by

$$\begin{aligned} u_1(\cdot) =& -0.5 \sin 3t + 0.5 \sin 2t - 5.5x_{11} - 6x_{12} + 0.04x_{12}^2 \sin x_{11} - 2k_1 \mathrm{sgn}(2x_{11} + x_{12}) \\ & - \frac{(2x_{11} + x_{12})^2 (0.25\|x_{11}\| + 0.5\|x_{12}\| + 0.5\|x_{21}\| + 0.5\|x_{22}\|) \sin^2 t}{2\|2x_{11} + x_{12}\|^2} \end{aligned} \tag{4.76}$$

$$\begin{aligned} u_2(\cdot) =& -0.5 \sin 2t + 0.5 \sin 3t - 5.5x_{21} - 6x_{22} + 0.04x_{22}^2 \sin x_{21} - 2k_2 \mathrm{sgn}(2x_{21} + x_{22}) \\ & - \frac{(2x_{21} + x_{22})^2 (0.7\|x_{11}\| + 0.6\|x_{12}\| + 0.25\|x_{21}\| + 0.4\|x_{22}\|) \sin^2 t}{2\|2x_{21} + x_{22}\|^2} \end{aligned} \tag{4.77}$$

where constants k_1 and k_2 are chosen as

$$k_1 = 0.5 \quad \text{and} \quad k_2 = 0.6$$

According to Theorems 4.1 and 4.2, it follows that the controller (4.76)-(4.77) can stabilise the interconnected system (4.65)-(4.68) uniformly asymptotically.

For simulation purposes, the initial states are chosen as $x_{10} = (0.7, 0.5)$ and $x_{20} = (0.2, 0.9)$, and interconnections h_1 and h_2 are selected as

$$h_1 = 0.2(0.25\|x_{11}\| + 0.5\|x_{12}\| + 0.5\|x_{21}\| + 0.5\|x_{22}\|)^{0.5}(2x_{11} + x_{12})^{0.5} \sin t \tag{4.78}$$
$$h_2 = 0.3(0.7\|x_{11}\| + 0.6\|x_{12}\| + 0.25\|x_{21}\| + 0.4\|x_{22}\|)^{0.5}(2x_{21} + x_{22})^{0.5} \sin t \tag{4.79}$$

Fig 4.6 shows the time responses of control signals and the sliding functions. The simulation results in Fig 4.7 show that the closed-loop system formed by applying control (4.76)-(4.77) to the interconnected system (4.65)-(4.68) is uniformly asymptotically stable. Simulation results for the two coupled inverted pendula on carts prove the effectiveness and feasibility of the proposed method. It is mentioned that there is the chattering in Fig 4.6, which is a general phenomena in SMC. It will not cause the unstabitily of systems, when the amplitude of the chattering is within a reasonable range. The future work of this chapter will be focused on solving the problem of chattering.

Remark 4.9. For this nonlinear interconnected system of the two coupled inverted pendula on carts (4.65)-(4.68), the sliding surface is selected as $2x_{i1} + x_{i2}$ instead of x_{i2} in (4.10) due to the specific structure of this system. The proposed method in this chapter can adopt any sliding surface related to the states, because all states of object system are available. In the common situation, choosing $s_i = x_{i2}$ can reduce the cost of calculation.

4.5. Summary

A class of fully nonlinear interconnected systems with unknown nonlinear interconnections has been considered in this chapter. A composite sliding surface has been designed, and a set of conditions has been developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. A novel decentralised state feedback control law is designed for the nonlinear interconnected systems to ensure that the interconnected system is driven to the designed sliding surface in finite time. The proposed strategy supplies an approach to improve the robustness of nonlinear interconnected systems in that effects of all matched uncertainties and mismatched interconnections can be rejected by the designed decentralised control regarding the reaching phase using fully nonlinear bounds of uncertainties and interconnections. This chapter focuses on fully nonlinear systems, the proposed method does not need to use the technique of linearisation, which is widely used in existing works to deal with nonlinear interconnected systems with uncertainties. Finally, numerical and practical simulation results have been presented to show the effectiveness of the proposed methods.

CHAPTER. 5

DYNAMIC OBSERVER FOR NONLINEAR INTERCONNECTED SYSTEMS WITH UNCERTAINTIES

In this chapter, a dynamic observer for a class of complex nonlinear interconnected systems with matched and mismatched uncertainties is presented. The research background and the system description are given in sections 5.1 and 5.2, respectively. In section 5.3, the dynamic observer is applied to estimate the states of the nonlinear interconnected systems. The simulation results of a numerical example and a practical example called as the lateral flight control system are presented to demonstrate the effectiveness of the proposed strategy in section 5.4. Section 5.5 concludes this chapter.

5.1. RESEARCH BACKGROUND

In recent years, systems in both industry and daily life have become larger and more complex. They are usually not simple which just has a single function. Most of these complex systems are composed of several subsystems, and all subsystems are interacted with each other. Some of these complex systems can be named as interconnected systems. Interconnected systems which widely exist in practical world consist of many subsystems with various functions and structures. For instance, quadrotors, smart cars and electronic monitoring systems are all typical interconnected systems which are popular in daily life. Because of nonlinearity, uncertainties, high dimensions and complex components in these interconnected systems, it is very hard to get the accurate values of system states. So, it is difficult to analyse and control the interconnected systems effectively.

In practical cases, some states of practical systems are unavailable due to inaccurate modelling or poor operating environment. Many classic control theories based on the accurate values of states in systems can not achieve high performance in this situation. In the case when system states are not available, one way is to establish an observer to estimate the system states, and then the estimated states are used to form feedback loop if possible. The observer is a kind of dynamic system which is dependent on the inputs and outputs of the original system. With the development of control theory, there are many different kinds of observers which are applied in the practical systems, such as unknown input observers, deadbeat observers, SMC observers and backstepping observers etc. A novel state observer for nonlinear systems with non-negligible and different time delays was proposed in [123]. Cascade high-gain observer was applied to output feedback control which overcame the problem of high dimension in traditional high-gain observers in [124]. An actuator fault and disturbance estimation scheme using sliding mode observer based on TS fuzzy system model was proposed in [125]. State observers whose convergence rate was faster than the standard asymptotic observers for reaction systems were described in [126]. A sensor-less speed estimator based on an adaptive non-linear high gain observer which only used the measured stator currents and control voltages was presented to estimate the speed of an induction motor in [127]. Event-triggered observers were designed for output-sampled nonlinear state affine systems in [128]. A strategy related to the reduced-order observer of the Boolean control networks for fault diagnosis using the semi-tensor product of matrices was proposed in [129]. State estimation using a network of distributed observers with unknown inputs for a class of linear time-invariant systems was presented in [130]. The output feedback SMC based on dynamic gain observer for an uncertain linear system with unstable zeros was considered in [131]. It should be noted that these observers mentioned above are only for centralised control systems.

It is a common situation that values of states are not available for complex nonlinear interconnected systems. Therefore, the observer is required to identify/estimate state variables for interconnected systems. A full-order nonlinear observer-based control for interconnected power systems was proposed in [132]. The approach of feedback linearisation was used to design the nonlinear observer when the power system was fully linearised. However, the linearisation of the nonlinear interconnected systems could greatly reduce the accuracy as well as the resulting performance. A novel distributed observer for interconnected multi-rate systems was presented and achieved great results in [133]. But it ignored the existence of matched uncertainty, and the interconnection of the system was in linear form. The new method for designing distributed reduced-order functional observers of a class of interconnected systems with time delays was considered in [134]. In this strategy, interconnected systems without matched uncertainty were not fully nonlinear. Besides that, it had several restrictive conditions due to using the reduced-order observer. Observer-based fuzzy adaptive optimal stabilization control for completely unknown nonlinear interconnected systems was presented in [135], this strategy did not consider the matched uncertainty, and the interconnections had the specific form which needed to satisfy the conditions of fuzzy logic. [136] investigated a decentralised tracking control problem for a class of strict-feedback interconnected systems with unknown parameters, the interconnected systems needed to meet several conditions which might not be used in the complex interconnected systems. Based on these reasons above, the dynamic observer design for nonlinear interconnected systems with matched and mismatched uncertainties is full of challenges and meaningful.

In this chapter, the dynamic observer is applied to complex nonlinear interconnected systems in the presence of both matched and mismatched uncertainties. This dynamic observer can estimate the states which may not be available for control design. The proposed method has great identification ability with small estimated state errors for nonlinear interconnected systems. It is pointed out that the uncertainties of nonlinear interconnected systems considered in this chapter have general structures, which indicates that the presented strategy can be effectively used in generalised nonlinear interconnected systems with uncertainties.

5.2. SYSTEM DESCRIPTION

Consider the system

$$\dot{x}_i = A_i x_i + B_i(u_i + G_i\varphi_i(x_i,t)) + E_i f_i(x_i,t) + \sum_{j=1}^{n} H_{ij}(x_j,t) \tag{5.1}$$

$$y_i = C_i x_i, \qquad i = 1,2,\cdots,n \tag{5.2}$$

where $x_i \in \Omega_i \subset R^{n_i}$ (Ω_i is a neighbourhood of the origin), $u_i \in R^{m_i}$ and $y_i \in R^{q_i}$ with $m_i \leq q_i < n_i$ are the state, input and output of the i-th subsystem, respectively. A_i, B_i, C_i, E_i and G_i are known constant matrices with appropriate dimensions, where B_i is of full row rank, and C_i is of full column rank. $E_i f_i(x_i,t)$ and $G_i\varphi_i(x_i,t)$ are mismatched uncertainty and matched uncertainty, respectively, where $f_i(x_i,t)$ and $\varphi_i(x_i,t)$ are unknown functions with appropriate dimensions, and the matrices G_i and E_i are used to describe the structure of matched and mismatched uncertainties, respectively. $\sum_{j=1}^{n} H_{ij}(x_j,t)$ is the known nonlinear interconnection of the i-th subsystem. It is assumed that all nonlinear terms involved in this chapter are continuous in the considered domain to guarantee the existence of the solutions of the system (5.1)-(5.2).

Assumption 5.1. *The unknown functions $f_i(x_i,t)$ and $\varphi_i(x_i,t)$ satisfy*

$$\|f_i(x_i,t)\| \leq \epsilon_i(x_i,t) \tag{5.3}$$

$$\|\varphi_i(x_i,t)\| \leq \vartheta_i(x_i,t) \tag{5.4}$$

where $\epsilon_i(x_i,t)$ and $\vartheta_i(x_i,t)$ are known Lipschitz functions with respect to x_i in the domain $\Omega_i \subset R^{n_i}$ and uniformly about t. The known nonlinear interconnection $H_{ij}(\cdot)$ is Lipschitz with respect to x_j in the domain $\Omega_j \subset R^{n_j}$ and uniformly about t for $i,j = 1,2,\cdots,n$.

From Assumption 5.1, it follows that for any x_i, $\hat{x}_i$, x_j and $\hat{x}_j$ in the considered domain,

$$\|\epsilon_i(x_i,t) - \epsilon_i(\hat{x}_i,t)\| \leq \mathcal{L}_{\epsilon_i}(t)\|x_i - \hat{x}_i\| \tag{5.5}$$

$$\|\vartheta_i(x_i,t) - \vartheta_i(\hat{x}_i,t)\| \le \mathcal{L}_{\vartheta_i}(t)\|x_i - \hat{x}_i\| \tag{5.6}$$

$$\|H_{ij}(x_j,t) - H_{ij}(\hat{x}_j,t)\| \le \mathcal{L}_{H_{ij}}(t)\|x_j - \hat{x}_j\| \tag{5.7}$$

where $\mathcal{L}_{\epsilon_i}(t)$, $\mathcal{L}_{\vartheta_i}(t)$ and $\mathcal{L}_{H_{ij}}(t)$ are nonnegative functions in $R^+ = \{t|t \ge 0\}$.

Remark 5.1. The matrices E_i and G_i are employed to describe the structural characteristics of the nonlinear mismatched and matched uncertainties, respectively. $f_i(x_i,t)$ and $\varphi_i(x_i,t)$ are unknown functions with known bounds which will be used in analysis and observer design later.

Assumption 5.2. *The matrix pair (A_i, C_i) is observable for $i = 1, 2, \cdots, n$.*

Assumption 5.2 is a basic limitation for the matrix pair (A_i, C_i). A dynamic observer is to be designed in next section.

5.3. DYNAMIC OBSERVER DESIGN

In view of the observability of the pair (A_i, C_i) in Assumption 5.2, there exists a matrix L_i such that $(A_i - L_iC_i)$ is stable and thus for any $Q_i > 0$ the following Lyapunov equation has a unique solution $P_i > 0$,

$$(A_i - L_iC_i)^T P_i + P_i(A_i - L_iC_i) = -Q_i \tag{5.8}$$

Assumption 5.3. *There exists known matrices F_i, J_i and P_i such that $[E_i \quad B_iG_i]^T P_i = [F_i \quad J_i]C_i$ holds, where P_i satisfies (5.8), and the matrices B_i, C_i, E_i and G_i are given in equations (5.1) and (5.2).*

Consider the following dynamic system

$$\dot{\hat{x}}_i = A_i\hat{x}_i + B_i(u_i + \Upsilon_i(\hat{x}_i, y_i, t)) + L_i(y_i - C_i\hat{x}_i) + \Psi_i(\hat{x}_i, y_i, t) + \sum_{j=1}^{n} H_{ij}(\hat{x}_j, t) \tag{5.9}$$

where $\hat{x}_i \in R^{n_i}$, $L_i \in R^{n_i \times q_i}$ satisfy (5.8), and

$$\Psi_i(\hat{x}_i, y_i, t) = \begin{cases} E_i \dfrac{F_i(y_i - C_i\hat{x}_i)}{\|F_i(y_i - C_i\hat{x}_i)\|}\epsilon_i(\hat{x}_i, t), & F_i(y_i - C_i\hat{x}_i) \ne 0 \quad (5.10) \\ 0, & F_i(y_i - C_i\hat{x}_i) = 0 \quad (5.11) \end{cases}$$

$$\Upsilon_i(\hat{x}_i, y_i, t) = \begin{cases} G_i \dfrac{J_i(y_i - C_i\hat{x}_i)}{\|J_i(y_i - C_i\hat{x}_i)\|}\vartheta_i(\hat{x}_i, t), & J_i(y_i - C_i\hat{x}_i) \neq 0 \quad (5.12) \\ 0, & J_i(y_i - C_i\hat{x}_i) = 0 \quad (5.13) \end{cases}$$

where E_i, F_i, G_i and J_i satisfy Assumption 5.3, the known functions $\epsilon_i(\cdot)$ and $\vartheta_i(\cdot)$ are given in Assumption 5.1.

Let state estimated error $e_i = x_i - \hat{x}_i$. It follows from (5.1) and (5.9) that the error dynamic is given by

$$\begin{aligned} \dot{e}_i =&(A_i - L_iC_i)e_i + (E_if_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) + B_i(G_i\varphi_i(x_i, t) - \Upsilon_i(\hat{x}_i, y_i, t)) \\ &+ \sum_{j=1}^{n}(H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \end{aligned} \tag{5.14}$$

The following results are presented to underpin subsequent analysis.

Lemma 5.1. *Suppose that Assumptions 5.1-5.3 are satisfied, the following results hold:*

(i) $e_i^TP_i(E_if_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) \leq \mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|\|e_i\|^2$

(ii) $e_i^TP_iB_i(G_i\varphi_i(x_i, t) - \Upsilon_i(\hat{x}_i, y_i, t)) \leq \mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|\|e_i\|^2$

(iii) $e_i^TP_i\sum_{j=1}^{n}(H_{ij}(x_j, t) - H_{ij}(\hat{x}_j, t)) \leq \sum_{j=1}^{n}\bar{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t)\|e_j\|\|e_i\|$

where $\mathcal{L}_{\epsilon_i}(t)$, $\mathcal{L}_{\vartheta_i}(t)$ *and* $\mathcal{L}_{H_{ij}}(t)$ *are satisfied (5.5), (5.6) and (5.7), respectively.* $\bar{\lambda}(P_i)$ *is the maximum eigenvalue of the matrix* P_i*.*

Proof. From Assumptions 5.1 and 5.3, combining with equations (5.10) and (5.11), if $F_iC_ie_i \neq 0$,

$$\begin{aligned} &e_i^TP_i(E_if_i(x_i, t) - \Psi_i(\hat{x}_i, y_i, t)) \\ &= (F_iC_ie_i)^Tf_i(x_i, t) - \frac{(F_iC_ie_i)^TF_iC_ie_i}{\|F_iC_ie_i\|}\epsilon_i(\hat{x}_i, t) \\ &\leq \|F_iC_ie_i\|\epsilon_i(x_i, t) - \|F_iC_ie_i\|\epsilon_i(\hat{x}_i, t) \\ &\leq \mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|\|e_i\|^2 \end{aligned} \tag{5.15}$$

Otherwise if $F_iC_ie_i = 0$, then from $E_i^TP_i = F_iC_i$ in Assumption 5.3, there is

$$e_i^TP_iE_i = (E_i^TP_ie_i)^T = (F_iC_ie_i)^T = 0 \tag{5.16}$$

Therefore, from analysis above,

$$e_i^T P_i(E_i f_i(x_i,t) - \Psi_i(\hat{x}_i, y_i, t)) = 0 \le \mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|\|e_i\|^2 \tag{5.17}$$

Hence the conclusion (i) follows.

From Assumptions 5.1 and 5.3, combining with the equations (5.12) and (5.13), if $J_iC_ie_i \neq 0$,

$$\begin{aligned}
&e_i^T P_i B_i(G_i\varphi_i(x_i,t) - \Upsilon_i(\hat{x}_i, y_i, t)) \\
&= (J_iC_ie_i)^T\varphi_i(x_i,t) - \frac{(J_iC_ie_i)^T J_iC_ie_i}{\|J_iC_ie_i\|}\vartheta_i(\hat{x}_i,t) \\
&\le \|J_iC_ie_i\|\vartheta_i(x_i,t) - \|J_iC_ie_i\|\vartheta_i(\hat{x}_i,t) \\
&\le \mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|\|e_i\|^2
\end{aligned} \tag{5.18}$$

Otherwise if $J_iC_ie_i = 0$, then from $(B_iG_i)^T P_i = J_iC_i$ in Assumption 5.3, there is

$$e_i^T P_i B_i G_i = ((B_iG_i)^T P_i e_i)^T = (J_iC_ie_i)^T = 0 \tag{5.19}$$

Therefore, from analysis above,

$$e_i^T P_i B_i(G_i\varphi_i(x_i,t) - \Upsilon_i(\hat{x}_i, y_i, t)) \le \mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|\|e_i\|^2 \tag{5.20}$$

Hence the conclusion (ii) follows.

Based on (5.7), it follows

$$\begin{aligned}
&e_i^T P_i \sum_{j=1}^{n}(H_{ij}(x_j,t) - H_{ij}(\hat{x}_j,t)) \\
&\le \|e_i\|\bar{\lambda}(P_i)\sum_{j=1}^{n}(\mathcal{L}_{H_{ij}}(t)\|x_j - \hat{x}_j\|) \\
&= \sum_{j=1}^{n}\bar{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t)\|e_j\|\|e_i\|
\end{aligned} \tag{5.21}$$

Hence conclusion (iii) follows. □

Theorem 5.1. *Suppose that Assumptions 5.1-5.3 are satisfied. Then, there exists positive constants* α_1 *and* α_2 *such that*

$$\|e_i\| \le \alpha_2 exp\{-\alpha_1 t\} \tag{5.22}$$

if $(M+M^T)$ *is positive definite with* $M=(m_{ij})_{n\times n}$ *defined by*

$$m_{ij}=\begin{cases}\underline{\lambda}(Q_i)-2\mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|-2\mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|-2\overline{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t), i=j & (5.23)\\ -2\overline{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t), \qquad i\neq j & (5.24)\end{cases}$$

where Q_i *is satisfied in equation (5.8),* F_i *and* J_i *are defined in Assumption 5.3.* $\underline{\lambda}(Q_i)$ *is the minimum eigenvalue of the matrix* Q_i*,* $\overline{\lambda}(P_i)$ *is the maximum eigenvalue of the matrix* P_i*.*

Proof. For the system (5.14), consider a Lyapunov function candidate $V_1=\sum_{i=1}^n e_i^TP_ie_i$. Then, the time derivative of V_1 along the trajectories of system (5.14) is given by

$$\begin{aligned}\dot{V}_1=&\sum_{i=1}^n\Bigg(-e_i^TQ_ie_i+2e_i^TP_i(E_if_i(x_i,t)-\Psi_i(\hat{x}_i,y_i,t))+2e_i^TP_iB_i(G_i\varphi_i(x_i,t)\\&-\Upsilon_i(\hat{x}_i,y_i,t))+2e_i^TP_i\sum_{j=1}^n(H_{ij}(x_j,t)-H_{ij}(\hat{x}_j,t))\Bigg)\end{aligned}$$

where equation (5.8) is used above. From conclusions (i)-(iii) of Lemma 5.1, it follows that

$$\begin{aligned}\dot{V}_1\leq&\sum_{i=1}^n(-\underline{\lambda}(Q_i)\|e_i\|^2+2\mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|\|e_i\|^2\\&+2\mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|\|e_i\|^2+2\sum_{j=1}^n\bar{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t)\|e_i\|\|e_j\|)\\=&-\sum_{i=1}^n(\underline{\lambda}(Q_i)-2\mathcal{L}_{\epsilon_i}(t)\|F_iC_i\|-2\mathcal{L}_{\vartheta_i}(t)\|J_iC_i\|-2\overline{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t))\|e_i\|^2\\&+\sum_{i=1}^n\sum_{j=1,j\neq i}^n2\overline{\lambda}(P_i)\mathcal{L}_{H_{ij}}(t)\|e_i\|\|e_j\|\\=&-\frac{1}{2}[\|e_1\|\|e_2\|\ldots\|e_n\|](M+M^T)[\|e_1\|\|e_2\|\ldots\|e_n\|]^T\\\leq&-\frac{1}{2}\underline{\lambda}(M+M^T)\sum_{i=1}^n\|e_i\|^2\end{aligned}\tag{5.25}$$

Given that

$$\sum_{i=1}^n e_i^TP_ie_i\leq \max_i\{\overline{\lambda}(P_i)\}\sum_{i=1}^n\|e_i\|^2\tag{5.26}$$

then, according to (5.25) and (5.26), it follows that

$$\begin{aligned}\dot{V}_1 \leq & -\frac{\underline{\lambda}(M+M^T)}{2\max_i\{\overline{\lambda}(P_i)\}}\sum_{i=1}^{n} e_i^T P_i e_i \\ = & -\frac{\underline{\lambda}(M+M^T)}{2\max_i\{\overline{\lambda}(P_i)\}} V_1 \\ = & -\alpha_1 V_1\end{aligned}$$

where

$$\alpha_1 \equiv: \frac{\underline{\lambda}(M+M^T)}{2\max_i\{\overline{\lambda}(P_i)\}} > 0 \tag{5.27}$$

Based on the analysis above, it follows that

$$V_1(t) \leq \bar{V}_1(0)\exp\{-\alpha_1 t\}$$

Since $\min_i\{\underline{\lambda}(P_i)\}\|e_i\|^2 \leq e_i^T P_i e_i \leq \sum_{i=1}^{n} e_i^T P_i e_i = V_1$, the conclusion $\|e_i\| \leq \alpha_2\exp\{-\alpha_1 t\}$ follows by letting

$$\alpha_2 > \sqrt{\bar{V}_1(0)/\min_i\{\underline{\lambda}(P_i)\}} \tag{5.28}$$

Hence, the result is obtained. □

Remark 5.2. Theorem 5.1 shows that the dynamic observer in (5.9) is an exponential observer for the system (5.1)-(5.2). This can be seen from the inequality (5.22). The proof is also constructive and provides a method to determine the values of α_1 and α_2.

5.4. SIMULATION RESULTS

This part shows the results of both numerical and practical simulations to demonstrate the effectiveness of the presented observer design above.

5.4.1. NUMERICAL EXAMPLE

Take consideration of the interconnected system composed of two subsystems as follows

$$\dot{x_1} = \begin{bmatrix} -7 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix}$$

$$+\begin{bmatrix}0\\0\\1\end{bmatrix}(u_1+\varphi_1(t,x_1))+\begin{bmatrix}0.9126 & 1.1776\\1 & 1.2\\0.5 & 1\end{bmatrix}f_1(x_1,t)+\begin{bmatrix}0.14(\|x_{21}\|+\sin(x_{13}))\\0\\0.07(\sin(x_{21})+\|x_{11}\|)\end{bmatrix} \tag{5.29}$$

$$y_1=\begin{bmatrix}1 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_{11}\\x_{12}\\x_{13}\end{bmatrix} \tag{5.30}$$

$$\dot{x_2}=\begin{bmatrix}-2 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}x_{21}\\x_{22}\end{bmatrix}$$

$$+\begin{bmatrix}0\\1\end{bmatrix}u_2+\begin{bmatrix}4.5 & 2.25\\1 & 0.5\end{bmatrix}f_2(x_2,t)+\begin{bmatrix}0.05(\|x_{11}\|+\|x_{22}\|)\\0.04\sin(x_{12})\end{bmatrix} \tag{5.31}$$

$$y_2=\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_{21}\\x_{22}\end{bmatrix} \tag{5.32}$$

where $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \text{col}(x_{21}, x_{22})$ denote states of subsystems. The uncertainties are assumed to satisfy

$$\|f_1(\cdot)\| \le \epsilon_1(\cdot) = 0.2(\sin(x_{12}) + \|x_{13}\|) \tag{5.33}$$

$$\|f_2(\cdot)\| \le \epsilon_2(\cdot) = 0.02\|x_{22}\| \tag{5.34}$$

$$\|\varphi_1(\cdot)\| \le \vartheta_1(\cdot) = 0.13\sin(x_{11}) \tag{5.35}$$

$$\mathcal{L}_{\epsilon_1}(\cdot) = 0.4 \tag{5.36}$$

$$\mathcal{L}_{\epsilon_2}(\cdot) = 0.02 \tag{5.37}$$

$$\mathcal{L}_{\vartheta_1}(\cdot) = 0.13 \tag{5.38}$$

and

$$\mathcal{L}_{H_{1j}}(\cdot) = 0.313 \tag{5.39}$$

$$\mathcal{L}_{H_{2j}}(\cdot) = 0.108 \tag{5.40}$$

where

$$H_{1j}(\cdot)=\begin{bmatrix}0.14(\|x_{21}\|+\sin(x_{13}))\\0\\0.07(\sin(x_{21})+\|x_{11}\|)\end{bmatrix} \tag{5.41}$$

$$H_{2j}(\cdot) = \begin{bmatrix} 0.05(\|x_{11}\| + \|x_{22}\|) \\ 0.04\sin(x_{12}) \end{bmatrix} \tag{5.42}$$

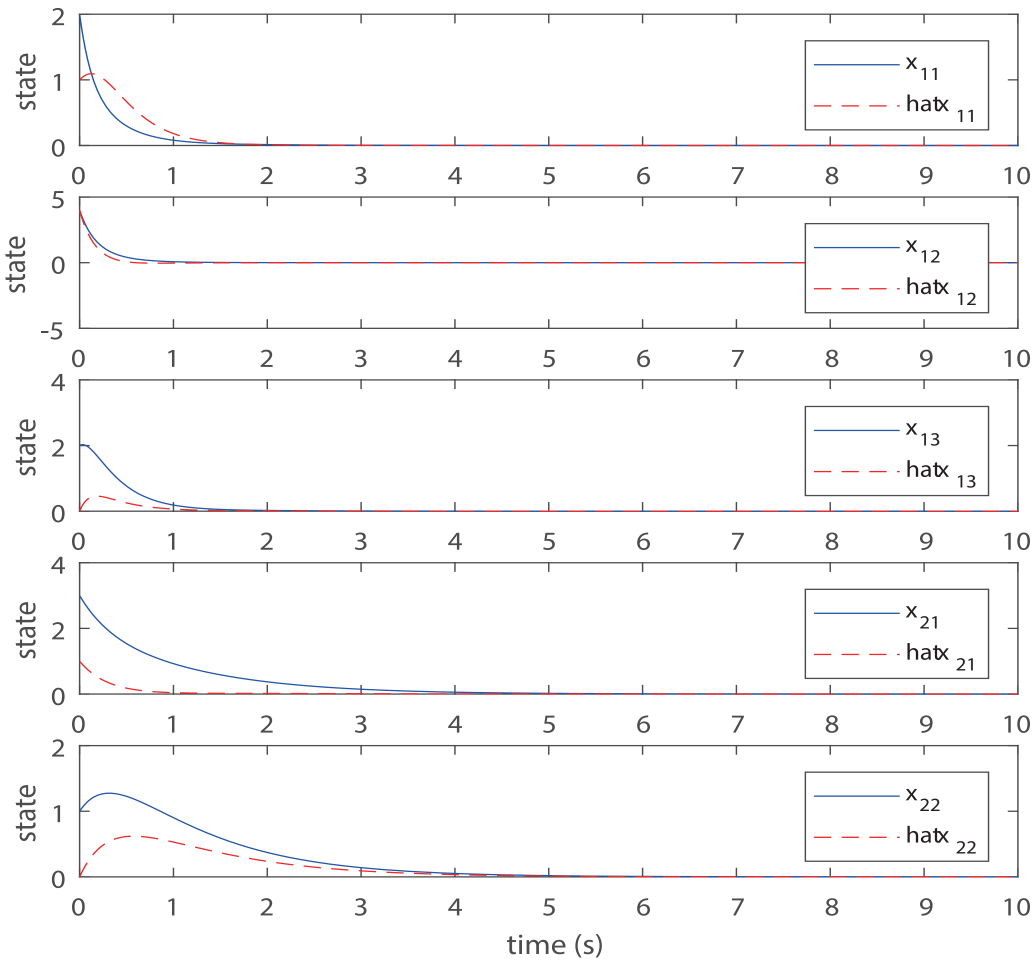

Figure 5.1: Time responses of the estimated states of the system (5.29)-(5.32).

In the system (5.29)-(5.32),

$$A_1 = \begin{bmatrix} -7 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{5.43}$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \tag{5.44}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{5.45}$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5.46}$$

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.47}$$

$$C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{5.48}$$

By direct verification, (A_1, C_1) and (A_2, C_2) are observable, thus Assumption 5.2 is satisfied.

Choose

$$L_1 = \begin{bmatrix} -3.5979 & 5.3943 \\ 0.2988 & -1.7078 \\ 1.3056 & 2.2992 \end{bmatrix} \tag{5.49}$$

$$L_2 = \begin{bmatrix} -0.5 \\ 1.0000 \end{bmatrix} \tag{5.50}$$

By calculation, $(A_1 - L_1C_1)$ and $(A_2 - L_2C_2)$ are stable. For $Q_1 = \mathrm{I}_3$ and $Q_2 = \mathrm{I}_2$, the solutions of Lyapunov equation (5.8) are

$$P_1 = \begin{bmatrix} 0.1497 & 0.0668 & -0.0960 \\ 0.0668 & 0.1339 & -0.0789 \\ -0.0960 & -0.0789 & 0.3081 \end{bmatrix} \tag{5.51}$$

$$P_2 = \begin{bmatrix} 0.3333 & 0.2000 \\ 0.2000 & 0.8000 \end{bmatrix} \tag{5.52}$$

Let

$$E_1 = \begin{bmatrix} 0.9126 & 1.1776 \\ 1 & 1.2 \\ 0.5 & 1 \end{bmatrix} \tag{5.53}$$

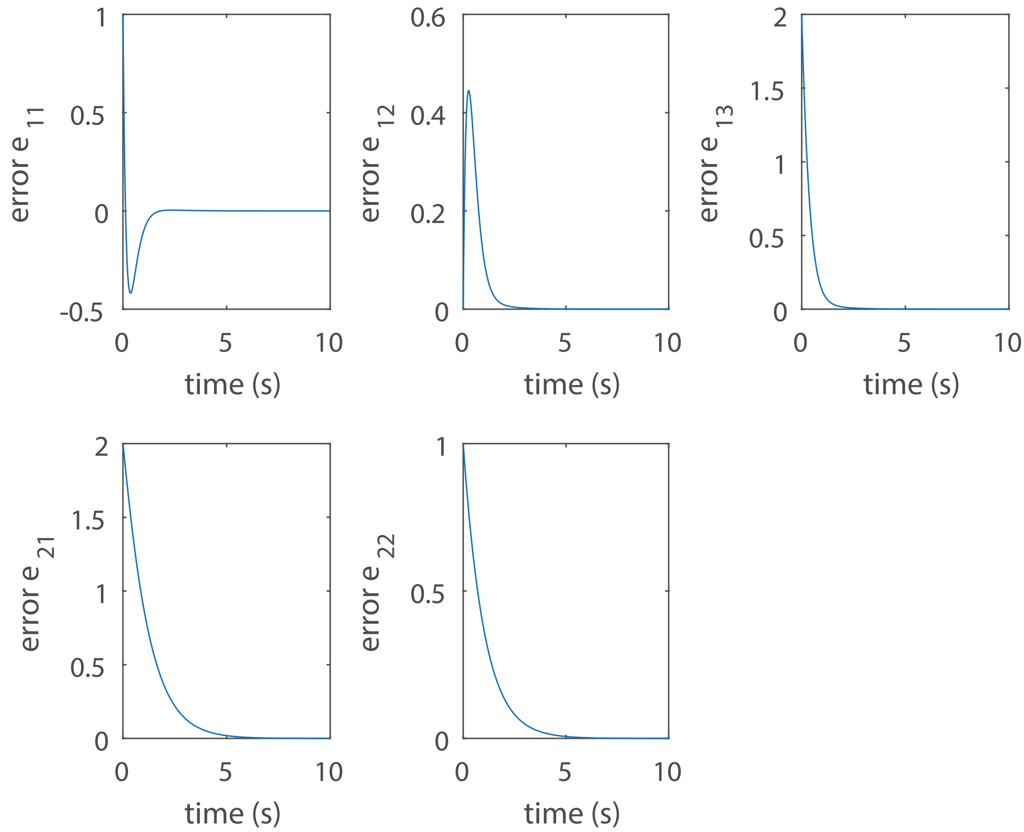

Figure 5.2: Time responses of observation errors.

$$F_1 = \begin{bmatrix} 0.1554 & -0.0125 \\ 0.1604 & 0.1004 \end{bmatrix} \tag{5.54}$$

$$E_2 = \begin{bmatrix} 4.5 & 2.25 \\ 1 & 0.5 \end{bmatrix} \tag{5.55}$$

$$F_2 = \begin{bmatrix} 1.7 \\ 0.85 \end{bmatrix} \tag{5.56}$$

$$J_1 = \begin{bmatrix} -0.0875 & 0.3081 \end{bmatrix} \tag{5.57}$$

$$G_1 = I_1 \tag{5.58}$$

According to (5.23)-(5.24),

$$(M + M^T) = \begin{bmatrix} 1.0658 & -0.3111 \\ -0.3111 & 1.4084 \end{bmatrix} \tag{5.59}$$

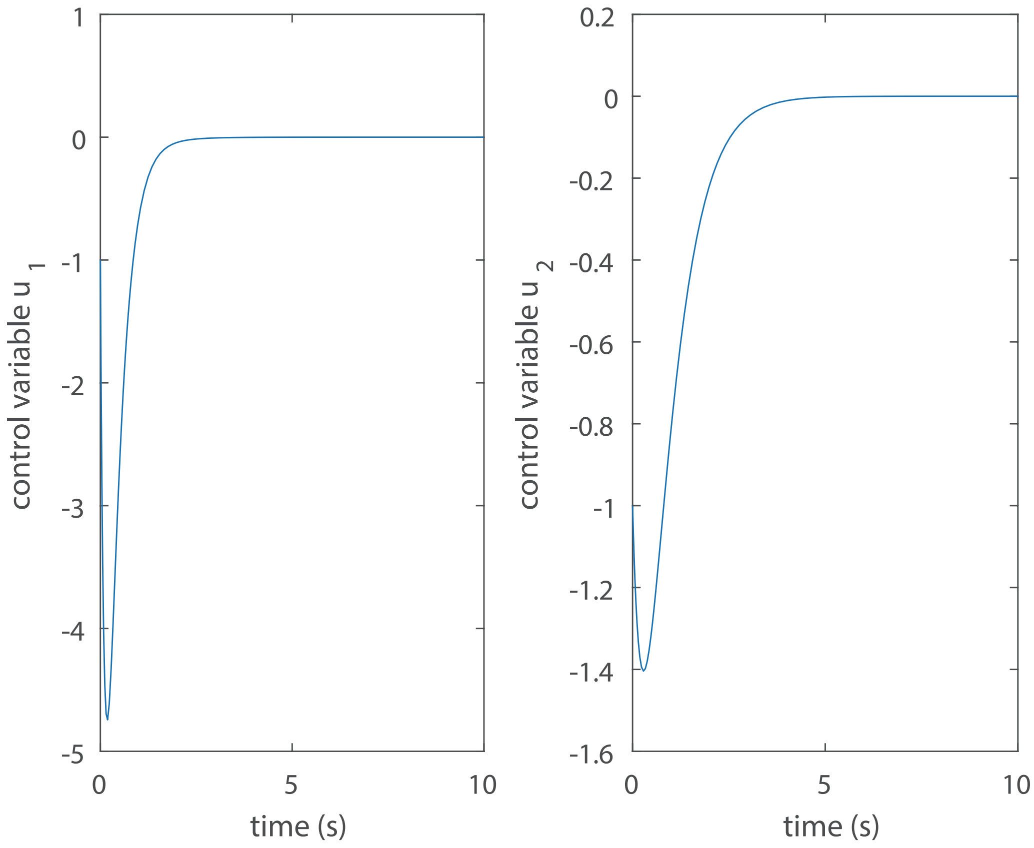

Figure 5.3: Time responses of control laws.

where $(M + M^T)$ is positive definite, thus Theorem 5.1 is satisfied. Based on (5.27) and (5.28), $\alpha_1 = 1.1$ and $\alpha_2 = 0.15$.

Then, for simulation purpose, the control law is selected as follows

$$u_1 = -(\hat{x}_{11} + 8\hat{x}_{13}) \tag{5.60}$$

$$u_2 = -(\hat{x}_{21} + 3\hat{x}_{22}) \tag{5.61}$$

The initial conditions are selected as $x_1 = \text{col}(2, 4, 2)$, $x_2 = \text{col}(3, 1)$, $\hat{x}_1 = \text{col}(1, 4, 0)$ and $\hat{x}_2 = \text{col}(1, 0)$. $f_1(\cdot)$, $f_2(\cdot)$ and $\varphi_1(\cdot)$ are selected as

$$f_1(\cdot) = 0.05(\sin(x_{12}) + \|x_{13}\|) \tag{5.62}$$

$$f_2(\cdot) = 0.01\|x_{22}\| \tag{5.63}$$

$$\varphi_1(\cdot) = 0.13\sin(x_{11}) \tag{5.64}$$

Fig 5.1 shows the time responses of the estimated states, Fig 5.2 shows the time responses of observation errors, and Fig 5.3 presents the time responses of control laws. The results of this numerical simulation show the effectiveness of the presented observer.

5.4.2. Observer Design for Lateral Flight Control System

Consider a lateral flight control system. The nominal aircraft lateral mode at the cruising flight condition can be presented as (see [98])

$$\dot{x_1} = \begin{bmatrix} -1.588 & 0 & -0.883 \\ 1 & 0 & 0 \\ 0 & 0 & -25 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 25 \end{bmatrix} u_1 + \begin{bmatrix} -0.2164 & -0.1625 \\ 1 & 0.75 \\ 2 & 1.4 \end{bmatrix} f_1(x_1, t) + \begin{bmatrix} 0.07x_{21} + 0.045x_{22} + 0.037x_{24} \\ 0 \\ 0 \end{bmatrix} \tag{5.65}$$

$$y_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \tag{5.66}$$

$$\dot{x_2} = \begin{bmatrix} -0.161 & 1 & 0 & -0.052 \\ -5.446 & -0.386 & 0 & -2.185 \\ -5.446 & -0.386 & -0.5 & -2.185 \\ 0 & 0 & 0 & -20 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20 \end{bmatrix} u_2 + \begin{bmatrix} 1 & 0.4 \\ -3.685 & -1.474 \\ -5.741 & -2.2964 \\ 1.2 & 1.9 \end{bmatrix} f_2(x_2, t) + \begin{bmatrix} 0.02x_{11} + 0.01x_{12} \\ 0.01x_{11} \\ 0.01x_{11} \\ 0 \end{bmatrix} \tag{5.67}$$

$$y_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \end{bmatrix} \tag{5.68}$$

where $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \text{col}(x_{21}, x_{22}, x_{23}, x_{24})$ denote roll rate, bank angle, , sideslip angle, yaw rate, washout filter output and rudder deflection, respectively. It is assumed that bank angle, aileron deflection, washout filter output and rudder deflection are available which take as the outputs of the system (5.65)-(5.68). Input signals u_1 and u_2 are the perturbed aileron deflection command and the perturbed rudder deflection command, respectively. The dynamic coefficients represent a Boeing 707 aircraft cruising at the specific speed [98]. The uncertainties are assumed to satisfy

$$\|f_1(\cdot)\| \leq \epsilon_1(\cdot) = 0.045(\sin(x_{11}) + \|x_{12}\|) \tag{5.69}$$

$$\|f_2(\cdot)\| \leq \epsilon_2(\cdot) = 0.02(\sin(x_{21}) + \sin(x_{22}) + \|x_{23}\|) \tag{5.70}$$

$$\mathcal{L}_{\epsilon_1}(\cdot) = 0.09 \tag{5.71}$$

$$\mathcal{L}_{\epsilon_2}(\cdot) = 0.06 \tag{5.72}$$

and

$$\mathcal{L}_{H_{1j}}(\cdot) = 0.2 \tag{5.73}$$

$$\mathcal{L}_{H_{2j}}(\cdot) = 0.07 \tag{5.74}$$

where

$$H_{1j}(\cdot) = \begin{bmatrix} 0.07x_{21} + 0.045x_{22} + 0.037x_{24} \\ 0 \\ 0 \end{bmatrix} \tag{5.75}$$

$$H_{2j}(\cdot) = \begin{bmatrix} 0.02x_{11} + 0.01x_{12} \\ 0.01x_{11} \\ 0.01x_{11} \\ 0 \end{bmatrix} \tag{5.76}$$

In the lateral flight control system (5.65)-(5.68),

$$A_1 = \begin{bmatrix} -1.588 & 0 & -0.883 \\ 1 & 0 & 0 \\ 0 & 0 & -25 \end{bmatrix} \tag{5.77}$$

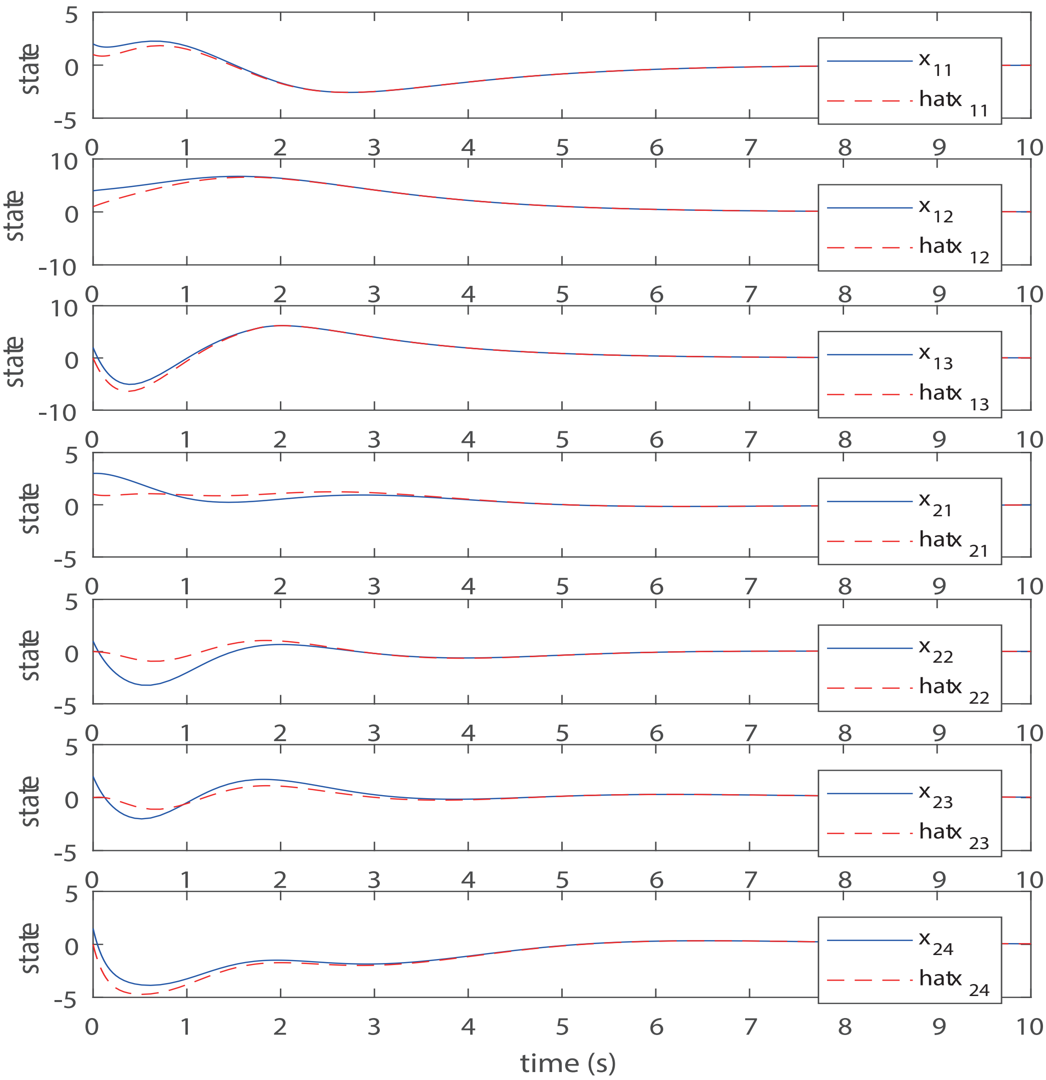

Figure 5.4: Time responses of the estimated states of the lateral flight control system.

$$A_2 = \begin{bmatrix} -0.161 & 1 & 0 & -0.052 \\ -5.446 & -0.386 & 0 & -2.185 \\ -5.446 & -0.386 & -0.5 & -2.185 \\ 0 & 0 & 0 & -20 \end{bmatrix} \tag{5.78}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 25 \end{bmatrix} \tag{5.79}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20 \end{bmatrix} \tag{5.80}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.81}$$

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.82}$$

By direct verification, (A_1, C_1) and (A_2, C_2) are observable, thus Assumption 5.2 is satisfied.

Choose

$$L_1 = \begin{bmatrix} -0.0363 & -0.8830 \\ 1.9120 & 0 \\ 0 & -24.0000 \end{bmatrix} \tag{5.83}$$

$$L_2 = \begin{bmatrix} -0.8884 & -0.0520 \\ 4.0914 & -2.1850 \\ 4.9530 & -2.1850 \\ 0 & -19.0000 \end{bmatrix} \tag{5.84}$$

By calculation, $(A_1 - L_1C_1)$ and $(A_2 - L_2C_2)$ are stable. For $Q_1 = \mathrm{I}_3$ and $Q_2 = \mathrm{I}_4$, the solutions of Lyapunov equation (5.8) are

$$P_1 = \begin{bmatrix} 0.3646 & 0.0789 & 0 \\ 0.0789 & 0.2630 & 0 \\ 0 & 0 & 0.5000 \end{bmatrix} \tag{5.85}$$

$$P_2 = \begin{bmatrix} 1.1772 & -0.4134 & 0.4704 & 0 \\ -0.4134 & 0.8277 & -0.6033 & 0 \\ 0.4704 & -0.6033 & 0.6210 & 0 \\ 0 & 0 & 0 & 0.5000 \end{bmatrix} \tag{5.86}$$

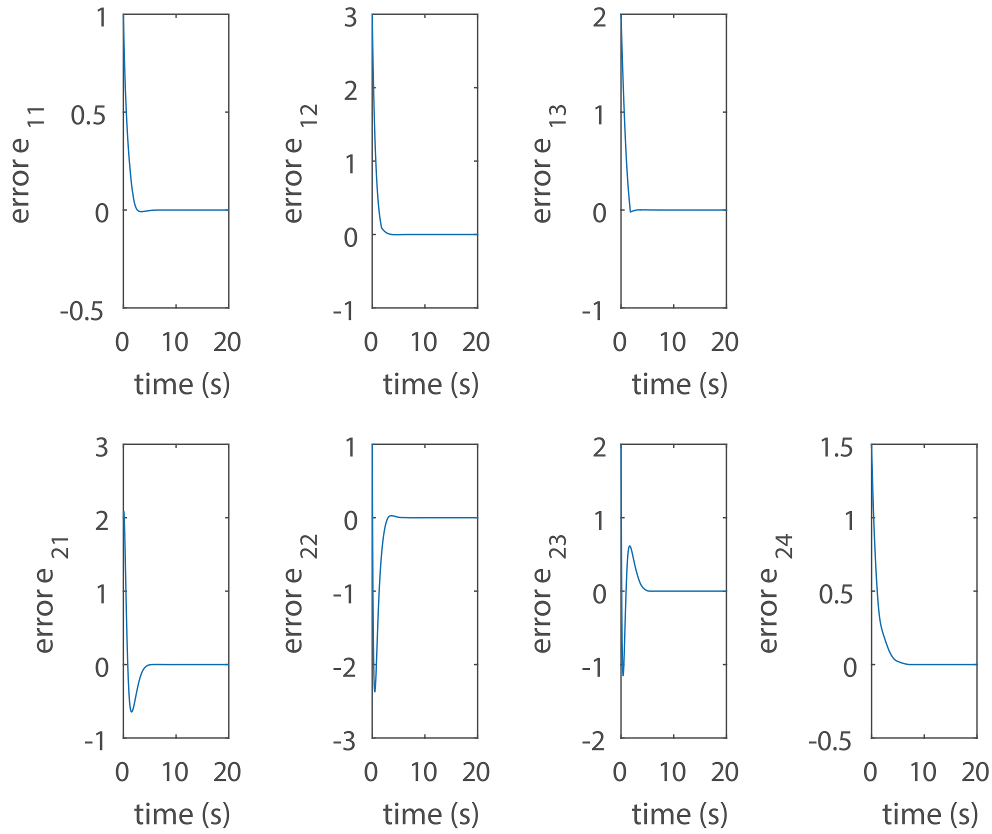

Figure 5.5: Time responses of observation errors of the lateral flight control system.

Let

$$E_1 = \begin{bmatrix} -0.2164 & -0.1625 \\ 1 & 0.75 \\ 2 & 1.4 \end{bmatrix} \tag{5.87}$$

$$F_1 = \begin{bmatrix} 0.2459 & 1 \\ 0.1844 & 0.7 \end{bmatrix} \tag{5.88}$$

$$E_2 = \begin{bmatrix} 1 & 0.4 \\ -3.685 & -1.474 \\ -5.741 & -2.2964 \\ 1.2 & 1.9 \end{bmatrix} \tag{5.89}$$

$$F_2 = \begin{bmatrix} -0.8716 & 0.6 \\ -0.3486 & 0.95 \end{bmatrix} \tag{5.90}$$

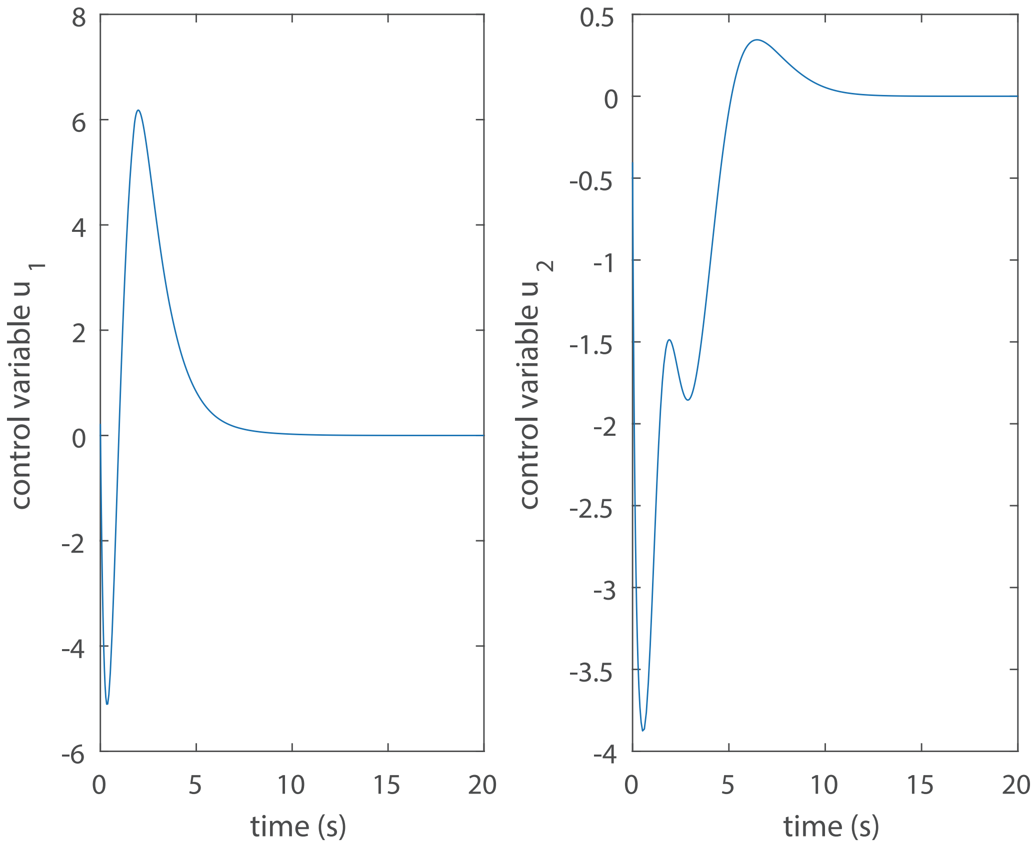

Figure 5.6: Time responses of control laws of the lateral flight control system.

According to (5.23)-(5.24),

$$(M + M^T) = \begin{bmatrix} 1.1468 & -0.4637 \\ -0.4637 & 1.1378 \end{bmatrix} \tag{5.91}$$

where $(M + M^T)$ is positive definite, thus Theorem 5.1 is satisfied. Based on (5.27) and (5.28), $\alpha_1 = 0.18$ and $\alpha_2 = 0.3$.

Then, choose the control law for simulation as follows

$$u_1 = -(-0.0850\hat{x}_{11} - 0.1359\hat{x}_{12} - 0.8835\hat{x}_{13}) \tag{5.92}$$

$$u_2 = -(0.4032\hat{x}_{21} + 0.0057\hat{x}_{22} - 0.1465\hat{x}_{23} - 0.7024\hat{x}_{24}) \tag{5.93}$$

The initial values of simulation are selected as $x_1 = \text{col}(2, 4, 2)$, $x_2 = \text{col}(3, 1, 2, 1.5)$, $\hat{x}_1 = \text{col}(1, 1, 0)$ and $\hat{x}_2 = \text{col}(1, 0, 0, 0)$. $f_1(\cdot)$ and $f_2(\cdot)$ are selected as

$$f_1(\cdot) = 0.0112(\sin(x_{11}) + \|x_{12}\|) \tag{5.94}$$

$$f_2(\cdot) = 0.005(\sin(x_{21}) + \sin(x_{22}) + \|x_{23}\|) \tag{5.95}$$

Fig 5.4 describes the time responses of the estimated states of the lateral flight control system, Fig 5.5 shows the time responses of observation errors related to states of the lateral flight control system, and Fig 5.6 presents time responses of the control laws for the lateral flight control system. The simulation results of the lateral flight control system demonstrate that the proposed dynamic observer is effective.

5.5. SUMMARY

In this chapter, the dynamic observer is presented for complex nonlinear interconnected systems with matched and mismatched uncertainties. This dynamic observer can estimate the values of states which can not be accessed for controller design. The proposed method has great identification ability with small estimated state errors for nonlinear interconnected systems. It should be mentioned that the uncertainties of nonlinear interconnected systems have general structures, which means that the proposed method can be effectively used in generalised nonlinear interconnected systems.

CHAPTER. 6

ROBUST DECENTRALISED SLIDING MODE CONTROL FOR NONLINEAR INTERCONNECTED SYSTEMS USING A DYNAMIC OBSERVER

In this chapter, a variable structure observer-based decentralised SMC will be proposed, which can be applied to control a class of nonlinear interconnected systems with matched and mismatched uncertainties. The observer developed in chapter 5 will be used in this chapter to form the decentralised dynamical controller to stabilise the considered nonlinear interconnected systems. The background is introduced in section 6.1. Section 6.2 gives the problem formulation and basic concepts. In section 6.3, a composite sliding surface is designed, and the stability of the sliding motion is analysed based on the regular form of the interconnected system. Using the pre-designed observer, a dynamic decentralised output feedback SMC is presented to drive the interconnected systems to the designed sliding surface in finite time, and then the sliding motion is maintained thereafter. In section 6.4, the simulation of a numerical example is given to demonstrate the effectiveness of the presented approach. The summary is described in section 6.5.

6.1. INTRODUCTION

In recent decades, nonlinear large-scale interconnected systems have widely appeared in the real life. This class of complex systems consists of many subsystems with different functions and structures. However, it is difficult to analyse and control the interconnected systems because of the existence of nonlinearity, uncertainties and interconnections between subsystems [20, 137].

In practical systems, states of dynamic systems are sometimes not available, and in this case, state feedback can not be applied. Therefore, an observer is required to estimate the value of states for dynamic systems. Observers, such as unknown input observers, deadbeat observers, reduce-order observers etc [138], are the special useful tools for control design when system states are not available. A unified $H\infty$ dynamic observer was presented for linear systems with unknown inputs and disturbances in [139]. [140] proposed a variable structure observer-based control design which could guarantee asymptotic convergence of the trajectory of the object to the equilibrium in the presence of both matched and mismatched uncertainties. [141] designed a linear observer for a multi-link flexible manipulator to overcome the difficulty of parameter uncertainties, unknown nonlinearities, and exogenous disturbances input. [142] adopted an extended state observer and adaptive dynamic programming approach to solve the problem of spacecraft output feedback attitude control. [143] presented four sliding mode observers based on Takagi-Sugeno fuzzy modelling of multi-input multi-output nonlinear systems with non-differentiable operating points. All of these observers mentioned above are for centralised control systems.

SMC, because of its outstanding advantages, for instance, fast response and insensitivity to variation of parameters and disturbances in systems, is a preferred selection for many complex systems with uncertainties [20]. Especially, decentralised control based SMC including the merits of both SMC and decentralised control has widely been applied to many fields. An observer-based fuzzy neural SMC scheme was designed for interconnected unknown chaotic systems though they ignored the matched uncertainty [144]. Global decentralised SMC of interconnected systems using only output information was considered in [145], where neither matched nor mismatched uncertainties were considered. A decentralised control was designed to stabilise a class of fully time delay nonlinear interconnected systems in [146] where strong conditions were imposed on the considered systems and interconnections due to the static output feedback employed. A new decentralised SMC strategy for the complex interconnected systems subjected to non-smooth nonlinearities was presented in [147]. This strategy could achieve the results only under the condition that the interconnection had linear structure. A reduced order observer-based integral SMC was proposed for the interconnected descriptor systems where the mismatched uncertainty was not considered, and the interconnected system needed to have the specific structure [148]. Decentralised SMC for nonlinear interconnected systems was designed using static state feedback, and thus all states of the interconnected systems needed to be known [149, 79]. Although the great results of decentralised SMC have been achieved, the results for nonlinear interconnected systems with unknown states, matched and mismatched uncertainties are few. So decentralised SMC for nonlinear interconnected systems with unknown states, matched and mismatched uncertainties deserves to be studied.

In this chapter, a dynamic observer-based decentralised SMC strategy is specifically proposed to stabilise a class of nonlinear interconnected systems with matched and mismatched uncertainties. The dynamic observer proposed in chapter 5 is used to estimate the states of interconnected systems. A coordinate transformation is used to transform each isolated subsystem into the regular form which is convenient for the design of SMC. Using the pre-designed observer, a composite sliding surface is designed, and sufficient conditions are developed to ensure that the sliding motion is uniformly asymptotically stable via the Lyapunov theory. A dynamic decentralised output feedback SMC law is designed to drive the object to the sliding surface in finite time and maintain the sliding motion on it. The impact of matched and mismatched uncertainties can be completely removed during the reachability analysis under the certainty conditions, and the presented method can reduce the conservatism of the developed results. The bounds of the uncertainties are relaxed which are nonlinear and take more general forms. Finally, the results of the numerical simulation are used to demonstrate the effectiveness of the proposed approach.

6.2. PROBLEM FORMULATION AND DYNAMIC OBSERVER DESIGN

Consider the system

$$\dot{x}_i = A_i x_i + B_i(u_i + G_i\varphi_i(x_i,t)) + E_i f_i(x_i,t) + \sum_{j=1}^{n} H_{ij}(x_j,t) \tag{6.1}$$

$$y_i = C_i x_i, \qquad i = 1, 2, \cdots, n \tag{6.2}$$

where $x_i \in \Omega_i \subset R^{n_i}$ (Ω_i is a neighbourhood of the origin), $u_i \in R^{m_i}$ and $y_i \in R^{q_i}$ with $m_i \leq q_i < n_i$ are the state, input and output of the i-th subsystem for $i = 1, 2, \cdots, n$, respectively. A_i, B_i, C_i, E_i and G_i are known constant matrices with appropriate dimensions, where B_i is of full row rank, and C_i is of full column rank. $E_i f_i(x_i,t)$ and $G_i\varphi_i(x_i,t)$ are mismatched uncertainty and matched uncertainty, respectively, where $f_i(x_i,t)$ and $\varphi_i(x_i,t)$ are unknown functions with appropriate dimensions, and the matrices G_i and E_i are used to describe the structure of matched and mismatched uncertainties, respectively. $\sum_{j=1}^{n} H_{ij}(x_j,t)$ is the known nonlinear interconnection of the i-th subsystem. It is assumed that all the nonlinear terms involved in this chapter are continuous in the considered domain to guarantee the existence of the solutions of system (6.1)-(6.2).

Assumption 6.1. *The matrix pair* (A_i, B_i) *is controllable for* $i = 1, 2, \cdots, n$.

Assumption 6.1 is a basic limitation for the matrix pair (A_i, B_i). The related assumptions about the unknown functions $f_i(x_i,t)$ and $\varphi_i(x_i,t)$ are described in Assumption 5.1 in chapter 5. The conditions of matrix A_i and C_i are presented in Assumption 5.2 in chapter 5. The detailed design and results of dynamic observer is presented in section 5.3 in chapter 5. This chapter focuses on the decentralised SMC based on the previous dynamic observer for nonlinear interconnected systems with uncertainties.

6.3. Sliding Motion Analysis and Control Synthesis

In this section, the dynamic observer (5.9) is used to form the dynamic feedback control to stabilise the system (6.1)-(6.2). The stability of sliding motion is analysed at first. Then, the controller is designed to satisfy the reachability condition.

6.3.1. System Transformation

Consider the system (6.1)-(6.2), since B_i is full row rank, there exists a coordinate T_i such that

$$T_iB_i = \begin{bmatrix} 0 \\ \tilde{B}_i \end{bmatrix} \tag{6.3}$$

where $\tilde{B}_i \in R^{m_i \times m_i}$ is a nonsingular matrix for $i = 1, 2, \cdots, n$.

For systems (5.9), (6.1) and (6.2), consider a transformation

$$\begin{bmatrix} z_i \\ \hat{z}_i \end{bmatrix} := \begin{bmatrix} T_i & 0 \\ 0 & T_i \end{bmatrix} \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix} \tag{6.4}$$

where T_i is the nonsingular matrix satisfying (6.3). Let $\tilde{e}_i = z_i - \hat{z}_i$, from (6.4) and $e_i = x_i - \hat{x}_i$, it follows that

$$\tilde{e}_i = T_ix_i - T_i\hat{x}_i = T_ie_i \tag{6.5}$$

In the new coordinates, the system (5.14), (6.1) and (6.2) can be rewritten as

$$\dot{z}_{i1} = \tilde{A}_{i1}z_{i1} + \tilde{A}_{i2}z_{i2} + \tilde{f}_{i1}(z_{i1}, z_{i2}, t) + \sum_{j=1}^{n} \tilde{H}_{ij1}(z_{j1}, z_{j2}, t) \tag{6.6}$$

$$\begin{aligned} \dot{\tilde{e}}_i &= (\tilde{A}_i - \tilde{L}_i\tilde{C}_i)\tilde{e}_i + (\tilde{f}_i(z_{i1}, z_{i2}, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \\ &\quad + \tilde{B}_i(\tilde{\varphi}_i(z_{i1}, z_{i2}, t) - \tilde{\Upsilon}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^{n} (\tilde{H}_{ij}(z_{j1}, z_{j2}, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \end{aligned} \tag{6.7}$$

$$\dot{z}_{i2} = \tilde{A}_{i3}z_{i1} + \tilde{A}_{i4}z_{i2} + \tilde{B}_i(u_i + \tilde{\varphi}_i(z_{i1}, z_{i2}, t)) + \tilde{f}_{i2}(z_{i1}, z_{i2}, t) + \sum_{j=1}^{n} \tilde{H}_{ij2}(z_{j1}, z_{j2}, t) \tag{6.8}$$

$$y_i = \tilde{C}_i z_i \tag{6.9}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset R^{n_i - m_i}$, $z_{i2} \in \Omega_{z_{i2}} \subset R^{m_i}$, $z_i = \text{col}(z_{i1}, z_{i2}) \in \Omega_{T_i} \subset R^{n_i}$, and

$$\Omega_{T_i} = \Omega_{z_{i1}} \times \Omega_{z_{i2}} = \{(z_{i1}; z_{i2}) | (z_{i1}; z_{i2}) = T_i x_i, x_i \in \Omega_i \subset R^{n_i}\}$$

$\hat{z}_{i1} \in \Omega_{\hat{z}_{i1}} \subset R^{n_i - m_i}$, $\hat{z}_{i2} \in \Omega_{\hat{z}_{i2}} \subset R^{m_i}$, $\hat{z}_i = \text{col}(\hat{z}_{i1}, \hat{z}_{i2}) \in \Omega_{\hat{T}_i} \subset R^{n_i}$, and

$$\Omega_{\hat{T}_i} = \Omega_{\hat{z}_{i1}} \times \Omega_{\hat{z}_{i2}} = \{(\hat{z}_{i1}; \hat{z}_{i2}) | (\hat{z}_{i1}; \hat{z}_{i2}) = T_i \hat{x}_i, \hat{x}_i \in R^{n_i}\}$$

and

$$\tilde{C}_i = C_i T_i^{-1} \tag{6.10}$$

$$\tilde{L}_i = T_i L_i \tag{6.11}$$

$$\tilde{\Psi}_i(\hat{z}_i, y_i, t) = T_i \Psi_i(T_i^{-1}\hat{z}_i, y_i, t) \tag{6.12}$$

$$\tilde{\varphi}_i(z_i, t) = G_i \varphi_i(T_i^{-1} z_i, t)) \tag{6.13}$$

$$\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) = \Upsilon_i(T_i^{-1}\hat{z}_i, y_i, t) \tag{6.14}$$

$$\tilde{E}_i = T_i E_i \tag{6.15}$$

$$\bar{f}_i(z_i, t) = f_i(T_i^{-1} z_i, t) \tag{6.16}$$

$$\tilde{A}_i = \begin{bmatrix} \tilde{A}_{i1} & \tilde{A}_{i2} \\ \tilde{A}_{i3} & \tilde{A}_{i4} \end{bmatrix} = T_i A_i T_i^{-1} \tag{6.17}$$

$$\tilde{f}_i(z_i, t) = \begin{bmatrix} \tilde{f}_{i1}(\cdot) \\ \tilde{f}_{i2}(\cdot) \end{bmatrix} = \tilde{E}_i \bar{f}_i(z_i, t) \tag{6.18}$$

$$\tilde{H}_{ij}(z_j, t) = \begin{bmatrix} \tilde{H}_{ij1}(\cdot) \\ \tilde{H}_{ij2}(\cdot) \end{bmatrix} = T_i H_{ij}(T_j^{-1} z_j, t) \tag{6.19}$$

where $\tilde{A}_{i1} \in R^{(n_i - m_i)\times(n_i - m_i)}$, $\tilde{A}_{i2} \in R^{(n_i - m_i)\times m_i}$, $\tilde{A}_{i3} \in R^{m_i \times (n_i - m_i)}$, $\tilde{A}_{i4} \in R^{m_i \times m_i}$, $\tilde{f}_{i1}(\cdot) \in R^{n_i - m_i}$, $\tilde{f}_{i2}(\cdot) \in R^{m_i}$, $\tilde{H}_{ij1}(\cdot) \in R^{n_i - m_i}$ and $\tilde{H}_{ij2}(\cdot) \in R^{m_i}$.

In the following part, the interconnected system (6.6)-(6.9) is to be focused using the sliding mode technique.

6.3.2. SLIDING SURFACE DESIGN AND STABILITY OF SLIDING MOTION

From the Assumption 6.1, the matrix pair (A_i, B_i) is controllable. Therefore, from [150], the matrix pair (A_{i1}, A_{i2}) is controllable. Then, there exists matrix $\tilde{K}_i$ such that $(\tilde{A}_{i1} - \tilde{A}_{i2}\tilde{K}_i)$ is stable, and thus for any $\bar{Q}_i > 0$, the following Lyapunov equation has a unique solution $\bar{P}_i > 0$,

$$(\tilde{A}_{i1} - \tilde{A}_{i2}\tilde{K}_i)^T \bar{P}_i + \bar{P}_i(\tilde{A}_{i1} - \tilde{A}_{i2}\tilde{K}_i) = -\bar{Q}_i \tag{6.20}$$

Based on the specific structure of the system (6.6)-(6.9), the switching function for the system (6.6)-(6.9) can be selected by

$$\sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2}, \qquad i = 1, 2, \cdots, n \tag{6.21}$$

where $S_{i1} \in R^{m_i \times (n_i - m_i)}$ and $S_{i2} \in R^{m_i \times m_i}$ are designed parameters with S_{i2} being nonsingular. The matrix S_{i1} is selected as

$$S_{i1} = S_{i2}\tilde{K}_i \tag{6.22}$$

where S_{i2} is a nonsingular matrix to be designed later, and $\tilde{K}_i$ satisfies (6.20). Then, from (6.20) and (6.22),

$$(\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1})^T \bar{P}_i + \bar{P}_i(\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1}) = -\bar{Q}_i \tag{6.23}$$

Then, the composite sliding function for the interconnected system (6.6)-(6.9) is given as

$$\sigma(\hat{z}) = \text{col}(\sigma_1(\hat{z}_1), \sigma_2(\hat{z}_2), \cdots, \sigma_n(\hat{z}_n)) \tag{6.24}$$

where $\sigma_i(\cdot)$ for $i = 1, 2, 3, \cdots, n$ are defined in (6.21). So, the composite sliding surface is written as

$$\{\text{col}(\hat{z}_1, \hat{z}_2, \cdots, \hat{z}_n) | \sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2} = 0, \qquad i = 1, 2, \cdots, n\} \tag{6.25}$$

Let $\tilde{e}_{i1} = z_{i1} - \hat{z}_{i1}$ and $\tilde{e}_{i2} = z_{i2} - \hat{z}_{i2}$, where $\tilde{e}_{i1} \in R^{n_i - m_i}$, $\tilde{e}_{i2} \in R^{m_i}$, $\tilde{e}_i = \text{col}(\tilde{e}_{i1}, \tilde{e}_{i2}) \in R^{n_i}$. The sliding surface can be rewritten as

$$\sigma_i(\hat{z}_i) = S_{i1}\hat{z}_{i1} + S_{i2}\hat{z}_{i2} = S_{i1}z_{i1} + S_{i2}z_{i2} - S_{i1}\tilde{e}_{i1} - S_{i2}\tilde{e}_{i2}, \qquad i = 1, 2, \cdots, n \tag{6.26}$$

When the system is limited on the sliding surface (6.25), $\sigma_i(\hat{z}_i) = 0$. Thus, from (6.26),

$$z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}, \qquad i = 1, 2, \cdots, n \tag{6.27}$$

Therefore, the sliding mode dynamics of the system (6.6)-(6.9) corresponding to the sliding surface (6.25) can be described by

$$\begin{aligned} \dot{z}_{i1} =& (\tilde{A}_{i1} - \tilde{A}_{i2}S_{i2}^{-1}S_{i1})z_{i1} + \tilde{A}_{i2}S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{A}_{i2}\tilde{e}_{i2} \\ &+ \tilde{f}_{i1s}(z_{i1}, \tilde{e}_i, t) + \sum_{j=1}^{n} \tilde{H}_{ij1s}(z_{j1}, \tilde{e}_j, t) \end{aligned} \tag{6.28}$$

$$\begin{aligned} \dot{\tilde{e}}_i =& (\tilde{A}_i - \tilde{L}_i\tilde{C}_i)\tilde{e}_i + (\tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \tilde{B}_i(\tilde{\varphi}_{is}(z_{i1}, \tilde{e}_i, t) \\ &- \tilde{\Upsilon}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^{n}(\tilde{H}_{ijs}(z_{j1}, \tilde{e}_j, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \end{aligned} \tag{6.29}$$

where

$$\tilde{f}_{i1s}(\cdot) = \tilde{f}_{i1}(z_{i1}, z_{i2}, t)|_{z_{i2}=-S_{i2}^{-1}S_{i1}z_{i1}+S_{i2}^{-1}S_{i1}\tilde{e}_{i1}+\tilde{e}_{i2}} \tag{6.30}$$

$$\tilde{H}_{ij1s}(\cdot) = \tilde{H}_{ij1}(z_{j1}, z_{j2}, t)|_{z_{j2}=-S_{j2}^{-1}S_{j1}z_{j1}+S_{j2}^{-1}S_{j1}\tilde{e}_{j1}+\tilde{e}_{j2}} \tag{6.31}$$

$$\tilde{\varphi}_{is}(\cdot) = \tilde{\varphi}_i(z_{i1}, z_{i2}, t)|_{z_{i2}=-S_{i2}^{-1}S_{i1}z_{i1}+S_{i2}^{-1}S_{i1}\tilde{e}_{i1}+\tilde{e}_{i2}} \tag{6.32}$$

$$\tilde{f}_{is}(\cdot) = \tilde{f}_i(z_{i1}, z_{i2}, t)|_{z_{i2}=-S_{i2}^{-1}S_{i1}z_{i1}+S_{i2}^{-1}S_{i1}\tilde{e}_{i1}+\tilde{e}_{i2}} \tag{6.33}$$

$$\tilde{H}_{ijs}(\cdot) = \tilde{H}_{ij}(z_{j1}, z_{j2}, t)|_{z_{j2}=-S_{j2}^{-1}S_{j1}z_{i1}+S_{j2}^{-1}S_{j1}\tilde{e}_{j1}+\tilde{e}_{j2}} \tag{6.34}$$

From Assumption 5.2 and equations (5.8), (6.10), (6.11), (6.15) and (6.17), it is easy to get

$$\begin{aligned} &(T_i^{-1}\tilde{A}_iT_i - T_i^{-1}\tilde{L}_i\tilde{C}_iT_i)^TP_i + P_i(T_i^{-1}\tilde{A}_iT_i - T_i^{-1}\tilde{L}_i\tilde{C}_iT_i) = -Q_i \\ &\tilde{A}_i^T(T_i^{-1})^TP_iT_i^{-1} - \tilde{C}_i^T\tilde{L}_i^T(T_i^{-1})^TP_iT_i^{-1} + (T_i^{-1})^TP_iT_i^{-1}\tilde{A}_i - (T_i^{-1})^TP_iT_i^{-1}\tilde{L}_i\tilde{C}_i \\ &= -(T_i^T)^{-1}Q_iT_i^{-1} \end{aligned} \tag{6.35}$$

Let

$$(T_i^{-1})^TP_iT_i^{-1} = \tilde{P}_i \tag{6.36}$$

$$(T_i^T)^{-1}Q_iT_i^{-1} = \tilde{Q}_i \tag{6.37}$$

Then, equations (6.35)-(6.37) imply that for any $\tilde{Q}_i > 0$, the Lyapunov equation

$$(\tilde{A}_i - \tilde{L}_i\tilde{C}_i)^T\tilde{P}_i + \tilde{P}_i(\tilde{A}_i - \tilde{L}_i\tilde{C}_i) = -\tilde{Q}_i \tag{6.38}$$

has a unique solution $\tilde{P}_i > 0$.

Assumption 6.2. *It assumed that*

$$\| \tilde{f}_{i1s}(z_{i1}, \tilde{e}_i, t) \| \leq \chi_{i1}(z_{i1}, \tilde{e}_i, t) \| z_{i1} \| + \chi_{i2}(z_{i1}, \tilde{e}_i, t) \| \tilde{e}_i \| \tag{6.39}$$

$$\sum_{j=1}^{n} \|\tilde{H}_{ij1s}(z_{j1}, \tilde{e}_j, t)\| \leq \sum_{j=1}^{n} (\varrho_{ij1}(z_{j1}, \tilde{e}_j, t) \| z_{j1} \| + \varrho_{ij2}(z_{j1}, \tilde{e}_j, t) \| \tilde{e}_j \|) \tag{6.40}$$

where $\chi_{i1}(\cdot)$, $\chi_{i2}(\cdot)$, $\varrho_{ij1}(\cdot)$ *and* $\varrho_{ij2}(\cdot)$ *are known functions.*

Theorem 6.1. *Suppose Assumptions 5.1-5.3 and Assumption 6.2 are satisfied. Then, the sliding motion dynamics (6.28)-(6.29) are asymptotically stable if* $W + W^T$ *is positive definite with* W *defined by*

$$W = \begin{bmatrix} w_{ij}^a & w_{ij}^b \\ w_{ij}^b & w_{ij}^c \end{bmatrix} \tag{6.41}$$

$$w_{ij}^a = \begin{cases} \underline{\lambda}(\bar{Q}_i) - 2\overline{\lambda}(\bar{P}_i)\chi_{i1}(\cdot) - 2\overline{\lambda}(\bar{P}_i)\varrho_{ij1}(\cdot), & i = j \quad (6.42) \\ -2\overline{\lambda}(\bar{P}_i)\varrho_{ij1}(\cdot), & i \neq j \quad (6.43) \end{cases}$$

$$w_{ij}^b = \begin{cases} -\big(2\|\bar{P}_i\bar{D}_i\| + 2\overline{\lambda}(\bar{P}_i)\chi_{i2}(\cdot) + 2\overline{\lambda}(\bar{P}_i)\varrho_{ij2}(\cdot)\big), & i = j \quad (6.44) \\ -2\overline{\lambda}(\bar{P}_i)\varrho_{ij2}(\cdot), & i \neq j \quad (6.45) \end{cases}$$

$$w_{ij}^c = \begin{cases} \underline{\lambda}(\tilde{Q}_i) - 2\mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\| - 2\mathcal{L}_{\tilde{\vartheta}_i}(t)\|J_i\tilde{C}_i\| - 2\overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), i = j & (6.46) \\ -2\overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t), & i \neq j \quad (6.47) \end{cases}$$

where $\tilde{\epsilon}_i(z_i, t) = \epsilon_i(T_i^{-1}z_i, t)$, $\tilde{\vartheta}_i(z_i, t) = \vartheta_i(T_i^{-1}z_i, t)$ *and* $\bar{D}_i = [\tilde{A}_{i2}S_{i2}^{-1}S_{i1}, \tilde{A}_{i2}]$. $\bar{P}_i$ *and* $\bar{Q}_i$ *are defined in (6.23).* $\tilde{P}_i$ *and* $\tilde{Q}_i$ *satisfy (6.38).* $\underline{\lambda}(\bar{Q}_i)$ *and* $\underline{\lambda}(\tilde{Q}_i)$ *are the minimum eigenvalues of* $\bar{Q}_i$ *and* $\tilde{Q}_i$*, respectively.* $\overline{\lambda}(\bar{P}_i)$ *and* $\overline{\lambda}(\tilde{P}_i)$ *are the maximum eigenvalues of* $\bar{P}_i$ *and* $\tilde{P}_i$*, respectively.* $\chi_{i1}(\cdot)$, $\chi_{i2}(\cdot)$, $\varrho_{ij1}(\cdot)$ *and* $\varrho_{ij2}(\cdot)$ *are given in Assumption 6.2.* $\mathcal{L}_{\tilde{\epsilon}_i}(t) = \mathcal{L}_{\epsilon_i}(t)\|T_i^{-1}\|$, $\mathcal{L}_{\tilde{\vartheta}_i}(t) = \mathcal{L}_{\vartheta_i}(t)\|T_i^{-1}\|$ *and* $\mathcal{L}_{\tilde{H}_{ij}}(t) = \|T_i\|\mathcal{L}_{H_{ij}}(t)\|T_i^{-1}\|$ *are nonnegative continuous Lipschitz functions, where* $\mathcal{L}_{\epsilon_i}(t)$, $\mathcal{L}_{\vartheta_i}(t)$*, and* $\mathcal{L}_{H_{ij}}(t)$ *given in (5.5)-(5.7), respectively.*

Proof. For the system (6.28)-(6.29), consider the Lyapunov function candidate $V_2 = \sum_{i=1}^{n}(z_{i1}^T\bar{P}_iz_{i1} + \tilde{e}_i^T\tilde{P}_i\tilde{e}_i)$. Then, the time derivative of V_2 along the trajectories of the dynamic system (6.28)-(6.29) is given as

$$\dot{V}_2 = \sum_{i=1}^{n}(\dot{z}_{i1}^T\bar{P}_iz_{i1} + z_{i1}^T\bar{P}_i\dot{z}_{i1} + \dot{\tilde{e}}_i^T\tilde{P}_i\tilde{e}_i + \tilde{e}_i^T\tilde{P}_i\dot{\tilde{e}}_i)$$

$$= \sum_{i=1}^{n} \Big(- z_{i1}^T \bar{Q}_i z_{i1} + 2 z_{i1}^T \bar{P}_i (\tilde{A}_{i2} S_{i2}^{-1} S_{i1} \tilde{e}_{i1} + \tilde{A}_{i2} \tilde{e}_{i2} + \tilde{f}_{i1s}(\cdot) + \sum_{j=1}^{n} \tilde{H}_{ij1s}(\cdot))$$
$$- \tilde{e}_i^T \tilde{Q}_i \tilde{e}_i + 2 \tilde{e}_i^T \tilde{P}_i (\varpi_1 + \varpi_2 + \varpi_3) \Big) \tag{6.48}$$

where

$$\varpi_1 = \tilde{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t) \tag{6.49}$$
$$\varpi_2 = \tilde{B}_i(\tilde{\varphi}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{\Upsilon}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \tag{6.50}$$
$$\varpi_3 = \sum_{j=1}^{n} (\tilde{H}_{ijs}(z_{j1}, \tilde{e}_j, t) - \tilde{H}_{ij}(\hat{z}_{j1}, \hat{z}_{j2}, t)) \tag{6.51}$$

From $\tilde{e}_i = \mathrm{col}[\tilde{e}_{i1}, \tilde{e}_{i2}]$

$$z_{i1}^T \bar{P}_i (\tilde{A}_{i2} S_{i2}^{-1} S_{i1} \tilde{e}_{i1} + \tilde{A}_{i2} \tilde{e}_{i2}) = z_{i1}^T \bar{P}_i \bar{D}_i \tilde{e}_i \tag{6.52}$$

where $\bar{D}_i = [\tilde{A}_{i2} S_{i2}^{-1} S_{i1}, \tilde{A}_{i2}]$. Then,

$$z_{i1}^T \bar{P}_i (\tilde{A}_{i2} S_{i2}^{-1} S_{i1} \tilde{e}_{i1} + \tilde{A}_{i2} \tilde{e}_{i2}) \le \|\bar{P}_i \bar{D}_i\| \|z_{i1}\| \|\tilde{e}_i\| \tag{6.53}$$

According to (6.39), it follows

$$z_{i1}^T \bar{P}_i \tilde{f}_{i1s}(\cdot) \le \overline{\lambda}(\bar{P}_i) \chi_{i1}(\cdot) \|z_{i1}\|^2 + \overline{\lambda}(\bar{P}_i) \chi_{i2}(\cdot) \|\tilde{e}_i\| \|z_{i1}\| \tag{6.54}$$

From (6.40),

$$z_{i1}^T \bar{P}_i \sum_{j=1}^{n} \tilde{H}_{ij1s}(\cdot) \le \|z_{i1}\| \overline{\lambda}(\bar{P}_i) \sum_{j=1}^{n} \varrho_{ij1}(\cdot) \|z_{j1}\| + \|z_{i1}\| \overline{\lambda}(\bar{P}_i) \sum_{j=1}^{n} \varrho_{ij2}(\cdot) \|\tilde{e}_j\| \tag{6.55}$$

where $\tilde{H}_{ij1s}(\cdot)$ is the first $n_i - m_i$ rows of $\tilde{H}_{ij}(z_j, t)|_{z_{j2} = -S_{j2}^{-1} S_{j1} z_{j1} + S_{j2}^{-1} S_{j1} \tilde{e}_{j1} + \tilde{e}_{j2}}$.

Based on equations (5.10), (5.11) and (6.12),

$$\tilde{\Psi}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t) = \tilde{\Psi}_i(\hat{z}_i, y_i, t)$$

$$\tilde{\Psi}_i(\hat{z}_i, y_i, t) = \begin{cases} \tilde{E}_i \dfrac{F_i \tilde{C}_i \tilde{e}_i}{\|F_i \tilde{C}_i \tilde{e}_i\|} \tilde{\epsilon}_i(\hat{z}_i, t), & F_i \tilde{C}_i \tilde{e}_i \neq 0 \quad (6.56) \\ 0, & F_i \tilde{C}_i \tilde{e}_i = 0 \quad (6.57) \end{cases}$$

where $\tilde{\epsilon}_i(\hat{z}_i, t) = \epsilon_i(T_i^{-1} \hat{z}_i, t)$.

From equations (5.12), (5.13) and (6.14),

$$\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) = \begin{cases} G_i \dfrac{J_i\tilde{C}_i\tilde{e}_i}{\|J_i\tilde{C}_i\tilde{e}_i\|}\tilde{\vartheta}_i(\hat{z}_i, t), & J_i\tilde{C}_i\tilde{e}_i \neq 0 \qquad (6.58) \\ 0, & J_i\tilde{C}_i\tilde{e}_i = 0 \qquad (6.59) \end{cases}$$

where $\tilde{\vartheta}_i(\hat{z}_i, t) = \vartheta_i(T_i^{-1}\hat{z}_i, t)$.

According to Assumption 5.3, and equations (6.10), (6.15) and (6.36),

$$\begin{aligned} (T_i^{-1}\tilde{E}_i)^T T_i^T \tilde{P}_i T_i &= F_i\tilde{C}_i T_i \\ \tilde{E}_i^T \tilde{P}_i &= F_i\tilde{C}_i \end{aligned} \tag{6.60}$$

By the similar reasoning as (6.60),

$$G_i^T \tilde{B}_i^T \tilde{P}_i = J_i\tilde{C}_i \tag{6.61}$$

From (6.56) and (6.60), if $F_i\tilde{C}_i\tilde{e}_i \neq 0$,

$$\begin{aligned} \tilde{e}_i^T \tilde{P}_i \varpi_1 &= \tilde{e}_i^T \tilde{P}_i \left(\tilde{E}_i \bar{f}_{is}(z_{i1}, \tilde{e}_i, t) - \tilde{E}_i \frac{F_i\tilde{C}_i\tilde{e}_i}{\|F_i\tilde{C}_i\tilde{e}_i\|}\tilde{\epsilon}_i(\hat{z}_i, t) \right) \\ &= (F_i\tilde{C}_i\tilde{e}_i)^T \bar{f}_{is}(z_{i1}, \tilde{e}_i, t) - \frac{(F_i\tilde{C}_i\tilde{e}_i)^T F_i\tilde{C}_i\tilde{e}_i}{\|F_i\tilde{C}_i\tilde{e}_i\|}\tilde{\epsilon}_i(\hat{z}_i, t) \\ &\leq \|F_i\tilde{C}_i\tilde{e}_i\|\|\bar{f}_{is}(z_{i1}, \tilde{e}_i, t)\| - \|F_i\tilde{C}_i\tilde{e}_i\|\tilde{\epsilon}_i(\hat{z}_i, t) \\ &\leq \|F_i\tilde{C}_i\tilde{e}_i\|\|\bar{f}_i(z_i, t)\| - \|F_i\tilde{C}_i\tilde{e}_i\|\tilde{\epsilon}_i(\hat{z}_i, t) \\ &\leq \|F_i\tilde{C}_i\tilde{e}_i\|\tilde{\epsilon}_i(z_i, t) - \|F_i\tilde{C}_i\tilde{e}_i\|\tilde{\epsilon}_i(\hat{z}_i, t) \\ &\leq \mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\|\|\tilde{e}_i\|^2 \end{aligned} \tag{6.62}$$

where $\bar{f}_{is}(z_{i1}, \tilde{e}_i, t) = \bar{f}_i(z_{i1}, z_{i2}, t)|_{z_{i2} = -S_{i2}^{-1}S_{i1}z_{i1} + S_{i2}^{-1}S_{i1}\tilde{e}_{i1} + \tilde{e}_{i2}}$. If $F_i\tilde{C}_i\tilde{e}_i = 0$,

$$\tilde{e}_i^T \tilde{P}_i \varpi_1 = (F_i\tilde{C}_i\tilde{e}_i)^T \bar{f}_{is}(z_{i1}, \tilde{e}_i, t) = 0 \leq \mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\|\|\tilde{e}_i\|^2 \tag{6.63}$$

So, from (6.62) and (6.63),

$$\tilde{e}_i^T \tilde{P}_i \varpi_1 \leq \mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\|\|\tilde{e}_i\|^2 \tag{6.64}$$

Following the similar reasoning as (6.64), it follows from (6.58), (6.59) and (6.61),

$$\tilde{e}_i^T \tilde{P}_i \varpi_2 \leq \mathcal{L}_{\tilde{\vartheta}_i}(t)\|J_i\tilde{C}_i\|\|\tilde{e}_i\|^2 \tag{6.65}$$

According to (5.7), (6.19) and (6.51), similar as (6.65),

$$\tilde{e}_i^T \tilde{P}_i \varpi_3 \leq \sum_{j=1}^{n} \overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t)\|\tilde{e}_i\|\|\tilde{e}_j\| \tag{6.66}$$

Substituting (6.53)-(6.55) and (6.64)-(6.66) into (6.48), $\dot{V}_2$ can be described as

$$\begin{aligned}
\dot{V}_2 \leq & \sum_{i=1}^{n}\bigg(-\underline{\lambda}(\bar{Q}_i)\|z_{i1}\|^2 + 2\|\bar{P}_i\bar{D}_i\|\|z_{i1}\|\|\tilde{e}_i\| + 2\overline{\lambda}(\bar{P}_i)\chi_{i1}(\cdot)\|z_{i1}\|^2 \\
& + 2\overline{\lambda}(\bar{P}_i)\chi_{i2}(\cdot)\|\tilde{e}_i\|\|z_{i1}\| + 2\|z_{i1}\|\overline{\lambda}(\bar{P}_i)\sum_{j=1}^{n}\varrho_{ij1}(\cdot)\|z_{j1}\| \\
& + 2\|z_{i1}\|\overline{\lambda}(\bar{P}_i)\sum_{j=1}^{n}\varrho_{ij2}(\cdot)\|\tilde{e}_j\| - \underline{\lambda}(\tilde{Q}_i))\|e_i\|^2 + 2\mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\|\|\tilde{e}_i\|^2 \\
& + 2\mathcal{L}_{\tilde{\vartheta}_i}(t)\|J_i\tilde{C}_i\|\|\tilde{e}_i\|^2 + 2\sum_{j=1}^{n}\overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t)\|\tilde{e}_i\|\|\tilde{e}_j\|\bigg) \\
= & -\sum_{i=1}^{n}\Big(\underline{\lambda}(\bar{Q}_i) - 2\overline{\lambda}(\bar{P}_i)\chi_{i1}(\cdot) - 2\overline{\lambda}(\bar{P}_i)\varrho_{ij1}(\cdot)\Big)\|z_{i1}\|^2 \\
& -\sum_{i=1}^{n}\Big(\underline{\lambda}(\tilde{Q}_i) - 2\mathcal{L}_{\tilde{\epsilon}_i}(t)\|F_i\tilde{C}_i\| - 2\mathcal{L}_{\tilde{\vartheta}_i}(t)\|J_i\tilde{C}_i\| - 2\overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t)\Big)\|\tilde{e}_i\|^2 \\
& + \sum_{i=1}^{n}\big(2\|\bar{P}_i\bar{D}_i\| + 2\overline{\lambda}(\bar{P}_i)\chi_{i2}(\cdot) + 2\overline{\lambda}(\bar{P}_i)\varrho_{ij2}(\cdot)\big)\|\tilde{e}_i\|\|z_{i1}\| \\
& + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} 2\overline{\lambda}(\bar{P}_i)\varrho_{ij1}(\cdot)\|z_{i1}\|\|z_{j1}\| + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} 2\overline{\lambda}(\bar{P}_i)\varrho_{ij2}(\cdot)\|z_{i1}\|\|\tilde{e}_j\| \\
& + \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n} 2\overline{\lambda}(\tilde{P}_i)\mathcal{L}_{\tilde{H}_{ij}}(t)\|\tilde{e}_i\|\|\tilde{e}_j\| \\
= & -\frac{1}{2}[\|z_{11}\|\|z_{21}\|\cdots\|z_{n1}\|\|\tilde{e}_1\|\|\tilde{e}_2\|\cdots\|\tilde{e}_n\|](W+W^T) \\
& \cdot[\|z_{11}\|\|z_{21}\|\cdots\|z_{n1}\|\|\tilde{e}_1\|\|\tilde{e}_2\|\cdots\|\tilde{e}_n\|]^T
\end{aligned} \tag{6.67}$$

Hence, the conclusion follows from $W + W^T > 0$. □

Theorem 6.1 shows that the sliding motion of the system (6.6)-(6.9) relating to the sliding surface (6.25) is asymptotically stable. In the next section, sliding mode controller will be designed to drive the system to the sliding surface (6.25).

6.3.3. Dynamic Sliding Mode Controller Design

The objective now is to design a control law such that the reachability condition [20]

$$\sum_{i=1}^{n} \frac{\sigma_i^T(\hat{z}_i)\dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} < 0 \tag{6.68}$$

is satisfied where $\sigma_i(\hat{z}_i)$ is the composite sliding function given in (6.25). If condition (6.68) is satisfied by some controllers, the system (6.6)-(6.9) will be driven to the sliding surface and maintained a slding motion on it.

Based on the estimated state $\hat{z}_i = \text{col}(\hat{z}_{i1}, \hat{z}_{i2})$ and $\tilde{e}_i = \text{col}(\tilde{e}_{i1}, \tilde{e}_{i2})$ given by (6.6)-(6.9), the following decentralised SMC is proposed

$$\begin{aligned} u_i = &-(S_{i2}\tilde{B}_i)^{-1}\Bigg((S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})\hat{z}_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})\hat{z}_{i2} \\ &+ \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\big(\|S_{i1}\|\|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\| + \|S_{i2}\|\|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\| + \sum_{j=1}^{n}\|S_{j1}\|\|\tilde{H}_{ji1}(\hat{z}_i, t)\| \\ &+ \|S_{i2}\tilde{B}_i\|\|\tilde{\Upsilon}_i(\hat{z}_i, y_i, t)\| + \sum_{j=1}^{n}\|S_{j2}\|\|\tilde{H}_{ji2}(\hat{z}_i, t)\| + k_i(\hat{z}_i, t)\big)\Bigg) \end{aligned} \tag{6.69}$$

where $k_i(\hat{z}_i, t)$ is the control gain to be determined later. $\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)$ is the first $n_i - m_i$ rows of $\tilde{\Psi}_i(\hat{z}_i, y_i, t)$, and $\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)$ is the last m_i rows of $\tilde{\Psi}_i(\hat{z}_i, y_i, t)$.

Theorem 6.2. *Suppose that Assumptions 5.1-5.3 and Assumptions 6.1-6.2 are satisfied. The control law in (6.69) drives the system (6.6)-(6.9) to the sliding surface (6.25) and maintains a sliding motion on it thereafter if*

$$\begin{aligned} k_i(\hat{z}_i, t) >&\|T_i\|\alpha_2 exp\{-\alpha_1 t\}\Bigg(\|S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3}\| + \|S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4}\| \\ &+ (\|S_{i1}\| + \|S_{i2}\|)\|\tilde{A}_i - \tilde{L}_i\tilde{C}_i\|\Bigg) \end{aligned} \tag{6.70}$$

where α_1 and α_2 are given in (5.27) and (5.28), respectively.

Proof. From the definition of $\sigma_i(\hat{z}_i)$ in (6.21) and the system (6.6)-(6.8), it follows that

$$\begin{aligned} &\sum_{i=1}^{n} \frac{\sigma_i^T(\hat{z}_i)\dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\ =&\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\Bigg(S_{i1}\big(\tilde{A}_{i1}z_{i1} + \tilde{A}_{i2}z_{i2} + \tilde{f}_{i1}(\cdot) + \sum_{j=1}^{n}\tilde{H}_{ij1}(\cdot)\big) \end{aligned}$$

$$
\begin{aligned}
&+ S_{i2}\big(\tilde{A}_{i3}z_{i1} + \tilde{A}_{i4}z_{i2} + \tilde{B}_i(u_i + \tilde{\varphi}_i(\cdot)) + \tilde{f}_{i2}(\cdot) + \sum_{j=1}^{n}\tilde{H}_{ij2}(\cdot)\big) \\
&- S_{i1}\big(D_{i1}\tilde{e}_i + (\tilde{f}_{i1}(\cdot) - \tilde{\Psi}_{i1}(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \\
&+ \sum_{j=1}^{n}(\tilde{H}_{ij1}(\cdot) - \tilde{H}_{ij1}(\hat{z}_{j1}, \hat{z}_{j2}, t))\big) \\
&- S_{i2}\big(D_{i2}\tilde{e}_i + (\tilde{f}_{i2}(\cdot) - \tilde{\Psi}_{i2}(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) \\
&+ \tilde{B}_i(\tilde{\varphi}_i(\cdot) - \tilde{\Upsilon}_i(\hat{z}_{i1}, \hat{z}_{i2}, y_i, t)) + \sum_{j=1}^{n}(\tilde{H}_{ij2}(\cdot) - \tilde{H}_{ij2}(\hat{z}_{j1}, \hat{z}_{j2}, t))\big)\bigg) \\
= &\sum_{i=1}^{n}\frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\bigg((S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})z_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})z_{i2} \\
&+ S_{i1}\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) + S_{i1}\sum_{j=1}^{n}\tilde{H}_{ij1}(\hat{z}_j, t) + S_{i2}\tilde{B}_i u_i + S_{i2}\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)) \\
&+ S_{i2}\tilde{B}_i\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) + S_{i2}\sum_{j=1}^{n}\tilde{H}_{ij2}(\hat{z}_j, t) - (S_{i2}D_{i2} + S_{i1}D_{i1})\tilde{e}_i\bigg)
\end{aligned} \tag{6.71}
$$

where D_{i1} is the first $n_i - m_i$ rows of $(\tilde{A}_i - \tilde{L}_i\tilde{C}_i)$. D_{i2} is the last m_i rows of $(\tilde{A}_i - \tilde{L}_i\tilde{C}_i)$. Substituting the control law (6.69) into (6.71),

$$
\begin{aligned}
&\sum_{i=1}^{n}\frac{\sigma_i^T(\hat{z}_i)\dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\
= &\sum_{i=1}^{n}\frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\bigg((S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})\tilde{e}_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})\tilde{e}_{i2} \\
&+ (S_{i1}\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i1}\|\|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\|) \\
&+ (S_{i1}\sum_{j=1}^{n}\tilde{H}_{ij1}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\sum_{j=1}^{n}\|S_{j1}\|\|\tilde{H}_{ji1}(\hat{z}_i, t)\|) \\
&+ (S_{i2}\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i2}\|\|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\|) \\
&+ (S_{i2}\tilde{B}_i\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i2}\tilde{B}_i\|\|\tilde{\Upsilon}_i(\hat{z}_i, y_i, t)\|) \\
&+ (S_{i2}\sum_{j=1}^{n}\tilde{H}_{ij2}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\sum_{j=1}^{n}\|S_{j2}\|\|\tilde{H}_{ji2}(\hat{z}_i, t)\|) \\
&- (S_{i1}D_{i1} + S_{i2}D_{i2})\tilde{e}_i - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}k_i(\hat{z}_i, t)\bigg)
\end{aligned} \tag{6.72}
$$

where $\tilde{e}_{i1} = z_{i1} - \hat{z}_{i1}$ and $\tilde{e}_{i2} = z_{i2} - \hat{z}_{i2}$.

From (6.4)

$$\|T_i^{-1}(z_i - \hat{z}_i)\| = \|T_i^{-1}\tilde{e}_i\| = \|e_i\| \tag{6.73}$$

Then, from (5.22) and (6.73),

$$\|\tilde{e}_i\| \leq \|T_i\|\|T_i^{-1}\tilde{e}_i\| \leq \|T_i\|\alpha_2 \exp\{-\alpha_1 t\} \tag{6.74}$$

In equation (6.72),

$$\begin{aligned}
&\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}((S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3})\tilde{e}_{i1} + (S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4})\tilde{e}_{i2}) \\
&\leq \sum_{i=1}^{n}(\|S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3}\| + \|S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4}\|)\|\tilde{e}_i\|
\end{aligned} \tag{6.75}$$

From (6.56) and (6.57),

$$\begin{aligned}
&\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\left(S_{i1}\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i1}\|\|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\|\right) \\
=&\sum_{i=1}^{n}\left(\frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}S_{i1}\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t) - \|S_{i1}\|\|\tilde{\Psi}_{i1}(\hat{z}_i, y_i, t)\|\right) \leq 0
\end{aligned} \tag{6.76}$$

Based on equation (6.19), and from the fact that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} S_{i1}\tilde{H}_{ij1}(\hat{z}_j, t) = \sum_{i=1}^{n}\sum_{j=1}^{n} S_{j1}\tilde{H}_{ji1}(\hat{z}_i, t) \tag{6.77}$$

It follows that

$$\begin{aligned}
&\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\left(S_{i1}\sum_{j=1}^{n}\tilde{H}_{ij1}(\hat{z}_j, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\sum_{j=1}^{n}\|S_{j1}\|\|\tilde{H}_{ji1}(\hat{z}_i, t)\|\right) \\
=&\sum_{i=1}^{n}\left(\frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}S_{i1}\sum_{j=1}^{n}\tilde{H}_{ij1}(\hat{z}_j, t) - \sum_{j=1}^{n}\|S_{j1}\|\|\tilde{H}_{ji1}(\hat{z}_i, t)\|\right) \leq 0
\end{aligned} \tag{6.78}$$

Take consideration of (6.56) and (6.57),

$$\begin{aligned}
&\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\left(S_{i2}\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i2}\|\|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\|\right) \\
=&\sum_{i=1}^{n}\left(\frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}S_{i2}\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)) - \|S_{i2}\|\|\tilde{\Psi}_{i2}(\hat{z}_i, y_i, t)\|\right) \leq 0
\end{aligned} \tag{6.79}$$

Following similar reasoning as used to obtain (6.79), it follows that

$$\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\left(S_{i2}\tilde{B}_i\tilde{\Upsilon}_i(\hat{z}_i, y_i, t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\|S_{i2}\tilde{B}_i\|\|\tilde{\Upsilon}_i(\hat{z}_i, y_i, t)\|\right) \leq 0 \tag{6.80}$$

$$\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}\left(S_{i2}\sum_{j=1}^{n}\tilde{H}_{ij2}(\hat{z}_j,t) - \frac{\sigma_i(\cdot)}{\|\sigma_i(\cdot)\|}\sum_{j=1}^{n}\|S_{j2}\tilde{H}_{ji2}(\hat{z}_i,t)\|\right) \leq 0 \tag{6.81}$$

$$\sum_{i=1}^{n} \frac{\sigma_i^T(\cdot)}{\|\sigma_i(\cdot)\|}(S_{i1}D_{i1} + S_{i2}D_{i2})\tilde{e}_i \leq (\|S_{i1}\| + \|S_{i2}\|)\|\tilde{A}_i - \tilde{L}_i\tilde{C}_i\|\|\tilde{e}_i\| \tag{6.82}$$

Based on (6.75)-(6.76) and (6.78)-(6.82),

$$\begin{aligned}
&\sum_{i=1}^{n} \frac{\sigma_i^T(\hat{z}_i)\dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} \\
\leq &\sum_{i=1}^{n}\Bigg((\|S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3}\| + \|S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4}\|)\|\tilde{e}_i\| \\
&+ (\|S_{i1}\| + \|S_{i2}\|)\|\tilde{A}_i - \tilde{L}_i\tilde{C}_i\|\|\tilde{e}_i\| - k_i(\hat{z}_i,t)\Bigg) \\
= &\sum_{i=1}^{n}\Bigg(\big(\|S_{i1}\tilde{A}_{i1} + S_{i2}\tilde{A}_{i3}\| + \|S_{i1}\tilde{A}_{i2} + S_{i2}\tilde{A}_{i4}\| \\
&+ (\|S_{i1}\| + \|S_{i2}\|)\|\tilde{A}_i - \tilde{L}_i\tilde{C}_i\|\big)\|\tilde{e}_i\| - k_i(\hat{z}_i,t)\Bigg)
\end{aligned} \tag{6.83}$$

Hence, from (6.70), (6.74) and (6.83),

$$\sum_{i=1}^{n} \frac{\sigma_i^T(\hat{z}_i)\dot{\sigma}_i(\hat{z}_i)}{\|\sigma_i(\hat{z}_i)\|} < 0 \tag{6.84}$$

Then, the composite reachability condition is satisfied. □

Remark 6.1. In this section, a dynamic decentralised output feedback SMC is presented to drive an interconnected system to the designed sliding surface in finite time using the pre-designed observer. The sliding motion is then maintained thereafter. The interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties, which are nonlinear and take more general forms, are relaxed. This strategy improves the robust of the whole system and manages the impact of matched and mismatched uncertainties by appropriate reachability analysis.

6.4. SIMULATIONS

Take consideration of the interconnected system composed of two subsystems

$$\begin{aligned}\dot{x_1} &= \begin{bmatrix}-7 & 0 & 1\\ 0 & -6 & 1\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_{11}\\ x_{12}\\ x_{13}\end{bmatrix}\\ &+ \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}(u_1+\varphi_1(t,x_1)) + \begin{bmatrix}0.9126 & 1.1776\\ 1 & 1.2\\ 0.5 & 1\end{bmatrix} f_1(x_1,t) + \begin{bmatrix}0.14(\|x_{21}\|+\text{sin}(x_{13}))\\ 0\\ 0.07(\text{sin}(x_{21})+\|x_{11}\|)\end{bmatrix}\end{aligned} \tag{6.85}$$

$$y_1 = \begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_{11}\\ x_{12}\\ x_{13}\end{bmatrix} \tag{6.86}$$

$$\begin{aligned}\dot{x_2} &= \begin{bmatrix}-2 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_{21}\\ x_{22}\end{bmatrix}\\ &+ \begin{bmatrix}0\\ 1\end{bmatrix}u_2 + \begin{bmatrix}4.5 & 2.25\\ 1 & 0.5\end{bmatrix} f_2(x_2,t) + \begin{bmatrix}0.05(\|x_{11}\|+\|x_{22}\|)\\ 0.04\text{sin}(x_{12})\end{bmatrix}\end{aligned} \tag{6.87}$$

$$y_2 = \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_{21}\\ x_{22}\end{bmatrix} \tag{6.88}$$

where $x_1 = \text{col}(x_{11}, x_{12}, x_{13})$ and $x_2 = \text{col}(x_{21}, x_{22})$ denote the states of subsystems. The uncertainties are assumed to satisfy

$$\|f_1(\cdot)\| \leq \epsilon_1(\cdot) = 0.2(\text{sin}(x_{12}) + \|x_{13}\|) \tag{6.89}$$

$$\|f_2(\cdot)\| \leq \epsilon_2(\cdot) = 0.02\|x_{22}\| \tag{6.90}$$

$$\|\varphi_1(\cdot)\| \leq \vartheta_1(\cdot) = 0.13\text{sin}(x_{11}) \tag{6.91}$$

$$\mathcal{L}_{\epsilon_1}(\cdot) = 0.4 \tag{6.92}$$

$$\mathcal{L}_{\epsilon_2}(\cdot) = 0.02 \tag{6.93}$$

$$\mathcal{L}_{\vartheta_1}(\cdot) = 0.13 \tag{6.94}$$

and

$$\mathcal{L}_{H_{1j}}(\cdot) = 0.313 \tag{6.95}$$

$$\mathcal{L}_{H_{2j}}(\cdot) = 0.108 \tag{6.96}$$

where

$$H_{1j}(\cdot) = \begin{bmatrix} 0.14(\|x_{21}\| + \sin(x_{13})) \\ 0 \\ 0.07(\sin(x_{21}) + \|x_{11}\|) \end{bmatrix} \tag{6.97}$$

$$H_{2j}(\cdot) = \begin{bmatrix} 0.05(\|x_{11}\| + \|x_{22}\|) \\ 0.04\sin(x_{12}) \end{bmatrix} \tag{6.98}$$

In the interconnected system (6.85)-(6.88),

$$A_1 = \begin{bmatrix} -7 & 0 & 1 \\ 0 & -6 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{6.99}$$

$$A_2 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} \tag{6.100}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{6.101}$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{6.102}$$

$$C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6.103}$$

$$C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \tag{6.104}$$

By direct verification, (A_1, B_1) and (A_2, B_2) are controllable, thus Assumption 6.1 is satisfied. (A_1, C_1) and (A_2, C_2) are observable, thus Assumption 5.2 is satisfied.

Choose

$$L_1 = \begin{bmatrix} -3.5979 & 5.3943 \\ 0.2988 & -1.7078 \\ 1.3056 & 2.2992 \end{bmatrix} \tag{6.105}$$

$$L_2 = \begin{bmatrix} -0.5 \\ 1.0000 \end{bmatrix} \tag{6.106}$$

By calculation, $(A_1 - L_1C_1)$ and $(A_2 - L_2C_2)$ are stable. For $Q_1 = \mathrm{I}_3$ and $Q_2 = \mathrm{I}_2$, the solutions of Lyapunov equation (5.8) are

$$P_1 = \begin{bmatrix} 0.1497 & 0.0668 & -0.0960 \\ 0.0668 & 0.1339 & -0.0789 \\ -0.0960 & -0.0789 & 0.3081 \end{bmatrix} \tag{6.107}$$

$$P_2 = \begin{bmatrix} 0.3333 & 0.2000 \\ 0.2000 & 0.8000 \end{bmatrix} \tag{6.108}$$

Let

$$E_1 = \begin{bmatrix} 0.9126 & 1.1776 \\ 1 & 1.2 \\ 0.5 & 1 \end{bmatrix} \tag{6.109}$$

$$F_1 = \begin{bmatrix} 0.1554 & -0.0125 \\ 0.1604 & 0.1004 \end{bmatrix} \tag{6.110}$$

$$E_2 = \begin{bmatrix} 4.5 & 2.25 \\ 1 & 0.5 \end{bmatrix} \tag{6.111}$$

$$F_2 = \begin{bmatrix} 1.7 \\ 0.85 \end{bmatrix} \tag{6.112}$$

$$J_1 = \begin{bmatrix} -0.0875 & 0.3081 \end{bmatrix} \tag{6.113}$$

$$G_1 = I_1 \tag{6.114}$$

According to (5.23)-(5.24),

$$(M + M^T) = \begin{bmatrix} 1.0658 & -0.3111 \\ -0.3111 & 1.4084 \end{bmatrix} \tag{6.115}$$

where $(M + M^T)$ is positive definite, thus Theorem 5.1 is satisfied. Based on (5.27) and (5.28), $\alpha_1 = 1.1$ and $\alpha_2 = 0.15$.

The system (6.85)-(6.88) satisfies the equation (6.3), so consider the transformation T_1 and T_2 selected by

$$T_1 = I_3 \tag{6.116}$$

$$T_2 = I_2 \tag{6.117}$$

Then,

$$\begin{bmatrix} z_1 \\ \hat{z}_1 \end{bmatrix} := \begin{bmatrix} T_1 & 0 \\ 0 & T_1 \end{bmatrix} \begin{bmatrix} x_1 \\ \hat{x}_1 \end{bmatrix} \tag{6.118}$$

$$\begin{bmatrix} z_2 \\ \hat{z}_2 \end{bmatrix} := \begin{bmatrix} T_2 & 0 \\ 0 & T_2 \end{bmatrix} \begin{bmatrix} x_2 \\ \hat{x}_2 \end{bmatrix} \tag{6.119}$$

where $z_1 = \text{col}(z_{11}^a, z_{11}^b, z_{12}) = \text{col}(x_{11}, x_{12}, x_{13})$, $z_2 = \text{col}(z_{21}, z_{22}) = \text{col}(x_{21}, x_{22})$, $\hat{z}_1 = \text{col}(\hat{z}_{11}^a, \hat{z}_{11}^b, \hat{z}_{12}) = \text{col}(\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13})$ and $\hat{z}_2 = \text{col}(\hat{z}_{21}, \hat{z}_{22}) = \text{col}(\hat{x}_{21}, \hat{x}_{22})$. From (6.116)-(6.119), it is straightforward to determine that, for $i = 1, 2$,

$$\tilde{C}_i = C_i \tag{6.120}$$

$$\tilde{L}_i = L_i \tag{6.121}$$

$$\tilde{E}_i = E_i \tag{6.122}$$

$$\tilde{f}_i(\cdot) = E_i f_i(\cdot) \tag{6.123}$$

$$\tilde{A}_i = A_i \tag{6.124}$$

$$\tilde{B}_i = B_i \tag{6.125}$$

$$\tilde{H}_{ij}(\cdot) = H_{ij}(\cdot) \tag{6.126}$$

$$\mathcal{L}_{\tilde{\epsilon}_i}(\cdot) = \mathcal{L}_{\epsilon_i}(\cdot) \tag{6.127}$$

$$\mathcal{L}_{\tilde{\vartheta}_i}(\cdot) = \mathcal{L}_{\vartheta_i}(\cdot) \tag{6.128}$$

$$\mathcal{L}_{\tilde{H}_{ij}}(\cdot) = \mathcal{L}_{H_{ij}}(\cdot) \tag{6.129}$$

Based on equations (6.56)-(6.59),

$$\tilde{\Psi}_1(\cdot) = \begin{cases} \tilde{E}_1 \dfrac{F_1 \tilde{C}_1 \tilde{e}_1}{\|F_1 \tilde{C}_1 \tilde{e}_1\|} \tilde{\epsilon}_1(\cdot), & F_1 \tilde{C}_1 \tilde{e}_1 \neq 0 \qquad (6.130) \\ 0, & F_1 \tilde{C}_1 \tilde{e}_1 = 0 \qquad (6.131) \end{cases}$$

$$\tilde{\Psi}_2(\cdot) = \begin{cases} \tilde{E}_2 \dfrac{F_2 \tilde{C}_2 \tilde{e}_2}{\|F_2 \tilde{C}_2 \tilde{e}_2\|} \tilde{\epsilon}_2(\cdot), & F_2 \tilde{C}_2 \tilde{e}_2 \neq 0 \quad (6.132) \\ 0, & F_2 \tilde{C}_2 \tilde{e}_2 = 0 \quad (6.133) \end{cases}$$

$$\tilde{\Upsilon}_1(\cdot) = \begin{cases} G_1 \dfrac{J_1 \tilde{C}_1 \tilde{e}_1}{\|J_1 \tilde{C}_1 \tilde{e}_1\|} \tilde{\vartheta}_1(\cdot), & J_1 \tilde{C}_1 \tilde{e}_1 \neq 0 \quad (6.134) \\ 0, & J_1 \tilde{C}_1 \tilde{e}_1 = 0 \quad (6.135) \end{cases}$$

Consider (6.39)-(6.40),

$$\chi_{11}(\cdot) = \chi_{12}(\cdot) = 0.4444 \tag{6.136}$$

$$\chi_{21}(\cdot) = 0.1006 \tag{6.137}$$

$$\chi_{22}(\cdot) = 0.2012 \tag{6.138}$$

$$\varrho_{1j1}(\cdot) = \varrho_{1j2}(\cdot) = 0.14 \tag{6.139}$$

$$\varrho_{2j1}(\cdot) = \varrho_{2j2}(\cdot) = 0.1 \tag{6.140}$$

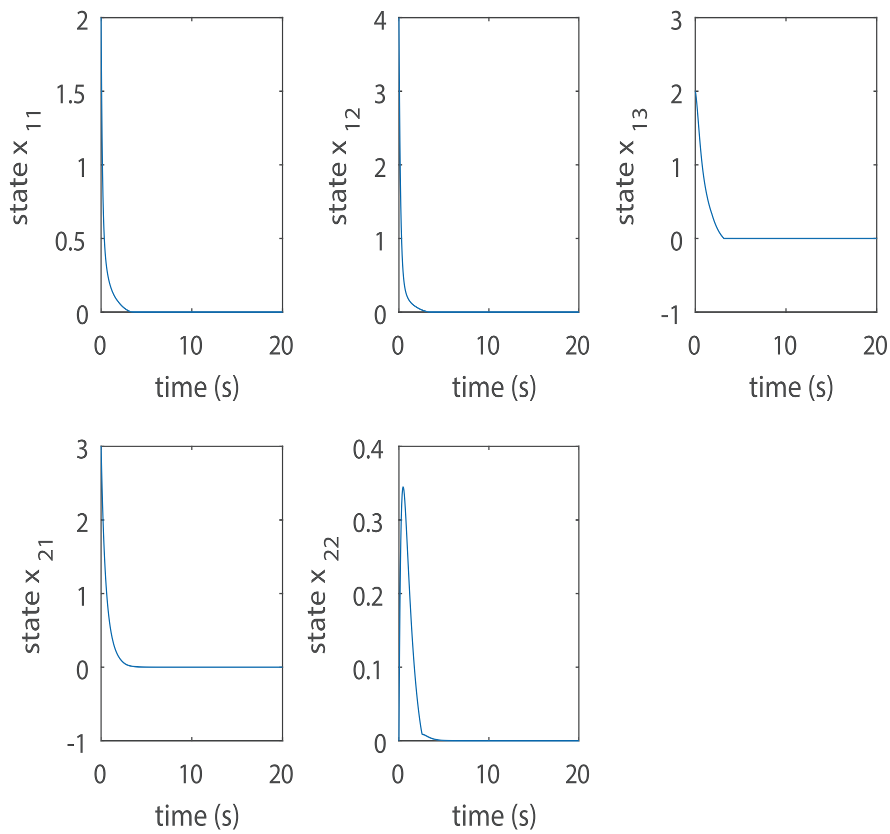

Figure 6.1: Time responses of the states of the interconnected system (6.85)-(6.88).

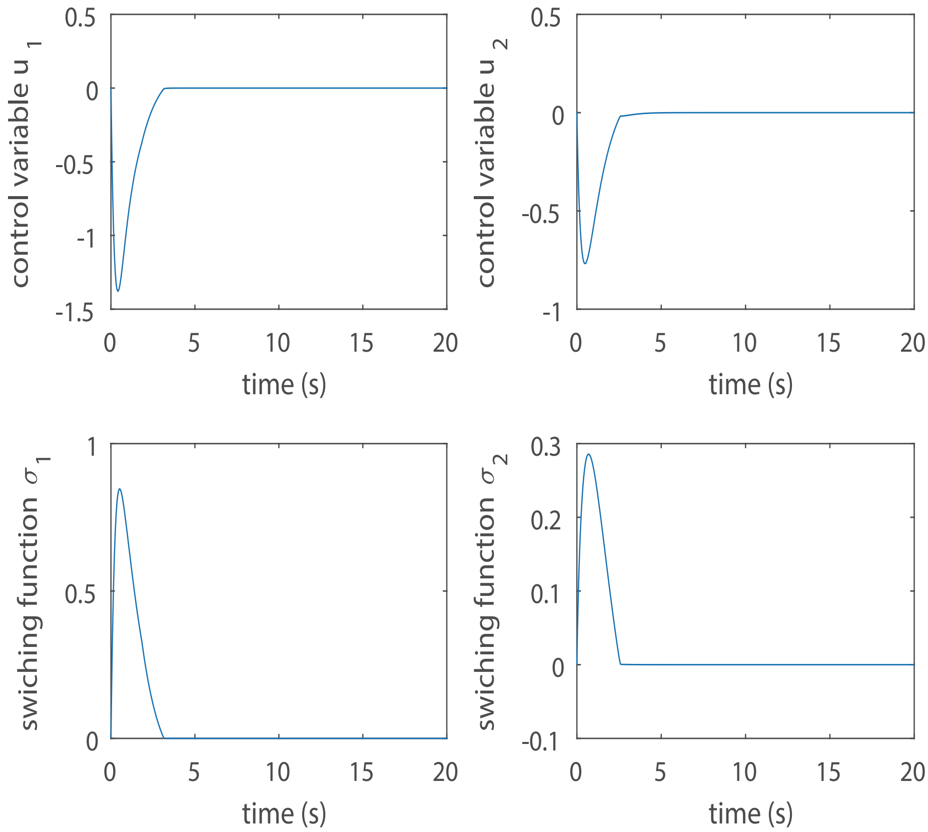

Figure 6.2: Time responses of the control laws u_1 and u_2 (Upper); time responses of the switching functions σ_1 and σ_2 (Bottom).

Choose $\bar{Q}_1 = \mathrm{I}_2$ and $\bar{Q}_2 = \mathrm{I}_1$, $S_{11} = 0$, $S_{12} = 1$, $S_{21} = 2$, $S_{22} = 2$. According to equation (6.23)

$$\bar{P}_1 = \begin{bmatrix} 0.0714 & 0 \\ 0 & 0.0833 \end{bmatrix} \tag{6.141}$$

$$\bar{P}_2 = 0.1667 \tag{6.142}$$

And based on the equations (6.36) and (6.37), it can be seen that $\tilde{P}_1 = P_1$, $\tilde{P}_2 = P_2$, $\tilde{Q}_1 = Q_1$ and $\tilde{Q}_2 = Q_2$.

Based on the equations (6.41)-(6.47), it follows that

$$W + W^T = \begin{bmatrix} 1.8052 & -0.0567 & -0.6659 & -0.0567 \\ -0.0567 & 1.8662 & -0.0567 & -1.1436 \\ -0.6659 & -0.0567 & 0.8201 & -0.4339 \\ -0.0567 & -1.1436 & -0.4339 & 1.4084 \end{bmatrix} \tag{6.143}$$

where $W + W^T$ is positive definite. Then, Theorem 6.1 is satisfied. From (6.70), $k_1(\cdot)$ and $k_2(\cdot)$ are selected as

$$k_1(\cdot) = 1.97952e^{-1.1t} + b_1 \tag{6.144}$$

$$k_2(\cdot) = 2.59864e^{-1.1t} + b_2 \tag{6.145}$$

where b_1 and b_2 are selected as 0.1 and 0.1, respectively. Then, the control law can be described as

$$u_1 = -\left(\hat{z}_{11}^a + \frac{\sigma_1(\cdot)}{\|\sigma_1(\cdot)\|}(\|\tilde{\Psi}_{12}(\cdot)\| + \|\tilde{\Upsilon}_1(\cdot)\| + \|0.07(\sin(\hat{z}_{21}) + \|\hat{z}_{11}^a\|)\| + k_1)\right) \tag{6.146}$$

$$\begin{aligned} u_2 = &- 0.5\Bigg(- 2\hat{z}_{21} + 2\hat{z}_{22} + \frac{\sigma_2(\cdot)}{\|\sigma_2(\cdot)\|}(2\|\tilde{\Psi}_{21}(\cdot)\| + 2\|\tilde{\Psi}_{22}(\cdot)\| + 0.1(\|\hat{z}_{11}^a\| + \|\hat{z}_{22}\|) \\ &+ 0.08\|\sin(\hat{z}_{11}^b)\| + k_2)\Bigg) \end{aligned} \tag{6.147}$$

where $\sigma_1(\cdot) = \hat{z}_{12}$, $\sigma_2(\cdot) = 2\hat{z}_{21} + 2\hat{z}_{22}$, $\tilde{\Psi}_{12}(\cdot)$ is the last row of $\tilde{\Psi}_1(\cdot)$, $\tilde{\Psi}_{21}(\cdot)$ is the first row of $\tilde{\Psi}_2(\cdot)$, $\tilde{\Psi}_{22}(\cdot)$ is the last row of $\tilde{\Psi}_2(\cdot)$.

For simulation, the initial conditions are selected as $z_1 = \text{col}(2, 4, 2)$, $z_2 = \text{col}(3, 0)$, $\hat{z}_1 = \text{col}(0, 4, 0)$ and $\hat{z}_2 = \text{col}(1, 0)$. Fig 6.1 presents time responses of the states of the interconnected system (6.85)-(6.88), and Fig 6.2 presents time responses of the control laws and sliding functions. The results of this numerical simulation show the effectiveness of the presented approach.

6.5. SUMMARY

A dynamic observer-based decentralised SMC strategy is proposed to stabilise a class of nonlinear interconnected systems in the presence of both matched and mismatched uncertainties. The dynamic observer is designed to estimate the states of interconnected systems. A coordinate transformation is used to transform each isolated subsystem into the regular form which is to facilitate the decentralised SMC design. Then, a composite sliding surface is proposed incorporating the states from the dynamic observer. A decentralised feedback SMC law based on the estimated states and outputs of the system is designed to drive the interconnected system to the sliding surface in finite time and maintain sliding motion on it. The nonlinear interconnections are employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties, which are nonlinear and take more general forms, are relaxed. The presented method improves the robustness of nonlinear interconnected systems in the presence of uncertainties.

CHAPTER. 7

CONCLUSIONS AND FUTURE WORK

This chapter is to summarise the contributions of this thesis and give brief discussions of relevant potential future work.

7.1. CONCLUSIONS

In this thesis, the research background of the whole thesis has been presented in Chapter 1. The survey of research background gave some reasonable motivations to make the research on decentralised SMC for nonlinear interconnected systems with uncertainties. Some necessary mathematical concepts, definitions and theorems which were used in the following chapters were given in Chapter 2. After that, many fundamental control theories and concepts which played an important role in control analysis and design have been introduced in Chapter 3. Besides that, a simple introduction of some practical systems which were analysed in the following chapters was described in Chapter 3 at the same time. Chapters 2 and 3 have built the fundamental knowledge for this thesis.

In Chapter 4, decentralised state feedback stabilisation for nonlinear interconnected systems using SMC has been presented. For systems with matched uncertainty, mismatched uncertainty and unknown interconnections, a state feedback decentralised SMC strategy, under the assumption that all system states were accessible, was proposed to eliminate the effect of uncertainties by using the bounds on uncertainties and interconnections. The bounds used in the design were fully nonlinear which could have a higher applicability for complex interconnected systems. Especially, for fully nonlinear systems, the proposed method did not need to use the technique of linearisation, which was widely used in the existing work to deal with nonlinear interconnected systems with uncertainties. The results of numerical and practical simulations related to coupled inverted pendula on carts demonstrated the effectiveness of the proposed method.

In Chapter 5, the dynamic observer has been applied to the complex nonlinear interconnected systems with matched and mismatched uncertainties. This dynamic observer could estimate the system states which could not be achieved during the design of the controller. The proposed method had great identification ability with small estimated state errors for nonlinear interconnected systems. It should be pointed out that the considered uncertainties of nonlinear interconnected systems had general forms, which meant that the proposed method could be effectively used in more generalised nonlinear interconnected systems. The results of numerical and practical simulations related to the lateral flight control system proved the superiority of the presented strategy.

The research in Chapter 6 was based on the achievement of Chapter 5. A variable structure observer-based decentralised SMC has been proposed to control a class of nonlinear interconnected systems with matched and mismatched uncertainties. Based on the designed dynamic observer, a dynamic decentralised output feedback SMC using outputs and estimated states was presented to control the interconnected systems. The nonlinear interconnections were employed in the control design to reduce the conservatism of the developed results. The bounds of the uncertainties were relaxed which were nonlinear and took more general forms. Moreover, the limitation for the interconnected system was reduced when compared with the existing results. In this chapter, the proposed strategies adopted the full-order observer. Besides that, the presented method improved the robustness of nonlinear interconnected systems to be against the effects of uncertainties.

The results of the numerical simulation example proved the availability of the proposed approach.

7.2. Future Work

In practical systems, due to the error of measurement, time-space lag switch, system inertia, system delay and other factors, the sliding motion can travel back and forth around the ideal sliding surface, instead of accurately occurring on the ideal sliding surface. This situation can produce chattering. The chattering of SMC actually comes from the discontinuous switching characteristics of SMC. The problem of chattering can reduce the control accuracy and performance, result in unnecessary wear and tear on the actuator components, increase energy consumption, stimulate the unmodelled high-frequency dynamics of the system, or even destroy the stability of the system resulting in unpredictable serious disasters sometimes. Therefore, one of the important issues of SMC is to eliminate the effect of chattering. There are two main methods to deal with the problem of chattering. One method of overcoming this drawback is to introduce a boundary layer about the discontinuous surfaces which may affect the control accuracy. Another strategy is to use higher order sliding mode techniques, but it requires the considered system to have a certain structure.

For uncertain nonlinear systems, SMC is robust and adaptive to uncertainties and external disturbances on the premise that the uncertainties are bounded. In this thesis, all uncertainties of nonlinear interconnected systems are assumed to be bounded and have certain bounds. However, there are many nonlinear systems with uncertainties which have unknown bounds due to the special structure or tough operating environment. Methods proposed in this thesis are hard to be applied to such a kind of systems. In this situation, the neural network can be used to approach the uncertain dynamics, which does not need to know the bound of uncertainty. The theory of the neural network has developed for many years. It combines the research results of modern neurobiology, which simulates the biological evolution process to reflect the computing structure of some characteristics of the human brain. The neural network can change the weight or topology to make the network output constantly close to the expected output. The obvious merits of neural networks are abilities of strong learning and highly parallel computing, which can fully approximate any complex nonlinear systems and enhance strong robustness. Besides that, fuzzy logic control is also an effective method to deal with the unknown bound of uncertainties, it can use a fuzzy system to estimate the uncertainties of the system to construct an equivalent control dynamically. The decentralised SMC for nonlinear interconnected systems combines with the neural network or fuzzy logic control can improve the performance of the whole control strategy and overcome the problem of unknown bounds of uncertainties.

In this thesis, the selection of the sliding surface can be optimised so that the dynamic sliding surface can be applied. It adds an additional integral term to the sliding surface, which can provide one more degree of freedom. Moreover, time delay which widely exists in practical systems can be considered in the design of decentralised SMC for nonlinear interconnected systems. In general, there is still a lot of work which can improve the proposed methods in this thesis, I will continue to modify and improve the strategies of decentralised SMC for nonlinear interconnected systems with uncertainties in future.

LIST OF PUBLICATION

[1] N. Ji, X. G. Yan, Z. H. Mao, D. Y. Zhao, and B. Jiang, "Decentralised Sliding Mode Control for Nonlinear Interconnected Systems with Unknown Interconnections," *IFAC-PapersOnLine,*, 2020, 53(2): 4064-4069.

[2] N. Ji, X. G. Yan, Z. H. Mao, D. Y. Zhao, and B. Jiang, "Decentralised State Feedback Stabilisation for Nonlinear Interconnected Systems using Sliding Mode Control," *International Journal of Systems Science,* 2022, 53(5): 1017-1030.

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