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# Firm revenue elasticity and business cycle sensitivity ☆

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ABSTRACT

We show that it is not necessary to have price and quantity data separately in order to study firm responses to business cycle shocks. We explain that *revenue elasticities*, which measure the response of firm revenue to input changes and combine price and quantity data, are sufficient to understand business cycle amplification. We present theory to show that higher revenue elasticity firms generate greater business cycle amplification. We use US data to measure revenue elasticities at the firm level, and we show that higher revenue elasticity firms respond more to business cycle shocks, consistent with our theory. We conclude that trends towards lower revenue elasticity firms implies weaker business cycle amplification over time.

## 1. Introduction

What can revenue elasticities teach us about business cycle behaviour? We present theoretical and empirical evidence that higher revenue elasticity firms are more responsive to business cycle shocks. We document a decline in revenue elasticities across US firms since the 1980s. This observation is consistent with a well-known decline in business cycle volatility over the same period, e.g., Kim and Nelson (1999); Galí and Gambetti (2009).

Revenue elasticities measure the response of firm revenue to input changes. They combine information about markups and returns to scale. These components are well-established determinants of business cycle dynamics in theory (Rotemberg and Woodford, 1999). But in practice, identifying between the two is difficult because it requires separate price and quantity data. On the other hand, revenue elasticities are simple to construct from firm datasets with revenue and cost data. We show that they are insightful for business cycle analysis, even without identifying between markups and returns to scale.

In our theoretical analysis, we present a model of production with imperfect competition and non-constant returns to scale. The model shows that firm revenue elasticities are equal to the ratio of variable costs to revenue. This provides a measurement equation for us to take to the data. The model implications show that a higher revenue elasticity firm has a greater increase in revenue in response to (i) positive productivity or demand shocks, (ii) factor price decreases, and (iii) operating profit decreases. The intuition is that higher revenue elasticity firms have either lower markups or higher output elasticity (returns to scale). Consequently, they have lower profit shares to cushion shocks, and revenues respond more.

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In our empirical analysis, we measure firms' revenue elasticities using Compustat data. We calculate revenue elasticities at the firm-level as the ratio of variable costs to revenue. In our data, this is the ratio of Cost of Goods Sold (COGS) to turnover. We document four data facts: (i) our firm-level revenue elasticity measures correlate strongly with estimated elasticities from revenue production functions, (ii) revenue elasticities are decreasing on average from 1985 to 2015, (iii) revenue elasticity dispersion across firms is increasing because low revenue elasticity firms experience a higher decline than high revenue elasticity firms, and (iv) high revenue elasticity firms are more procyclical (comove more with GDP) than low revenue elasticity firms.

After documenting the stylized facts, we test the theoretical result that higher revenue elasticity firms are more sensitive to various shocks. We use local projection estimation to generate impulse responses to (i) firm-level productivity shocks, (ii) aggregate-level productivity shocks, and (iii) aggregate business cycle shocks (policy uncertainty shocks). Consistent with our theory, we find that shocks are amplified when revenue elasticities are higher and dampened when they are lower. Overall, we conclude that trends towards lower revenue elasticity firms implies firm-level amplification mechanisms are declining since the 1980s. This is consistent with evidence of an overall decline in macroeconomic volatility over the period.

There is a long literature which studies how the separate components of revenue elasticity (markups and returns to scale) determine business cycle amplification and propagation (e.g., Hall, 1986; Hornstein, 1993; Rotemberg and Woodford, 1993; Burnside et al., 1995; Devereux et al., 1996; Basu and Fernald, 2001; Jaimovich and Floetotto, 2008; Kim, 2021). However, these components are challenging to measure in practice. Our contribution is to show that easily-measured revenue elasticities are insightful for business cycle analysis, even though they conflate markups and returns to scale.

Revenue elasticities have gained prominence due to their role in measuring markups. De Loecker et al. (2020) document rising markups in the U.S. over recent decades. We use the same dataset and cleaning procedures as them. But, we construct revenue elasticities at the firm level with an accounting approach, rather than using production function estimation which yields industry estimates. Bond et al. (2021) explain that using revenue elasticities to proxy output elasticities in markup estimation procedures is problematic, whilst De Ridder et al. (2022) temper this result by characterising the bias from revenue data. We stress that revenue elasticities are declining, using an independent methodology, and this trend is consistent with rising markups, though does not identify them specifically. Our point is that secular trends in revenue elasticity are robust—whether driven by markups or other wedges—and should be considered independently as they are insightful for macroeconomic inference.

In addition to the amplification mechanism we study, other literature shows that the ratio of output elasticity to markups (our revenue elasticity) is important. Hopenhayn (2014) shows that the ratio affects allocation across firms with heterogeneous productivity, and Atkeson and Kehoe (2005) propose a popular calibration of the ratio in an analysis of organizational capital. The underlying intuition is the same across this diverse set of topics. Revenue elasticities determine profit shares. And, profit shares are important for many mechanisms: allocation through firm entry and exit; division of organizational capital; and resilience to shocks over the business cycle.

Lastly, our paper is related to literature on heterogeneous firm behaviour over the business cycle. It is well-documented that large and small firms make different financing decisions over the business cycle (e.g., Covas and Den Haan, 2011; Begenau and Salomao, 2018), and recently Burstein et al. (2020) show that they also make different pricing decisions: large firms have procyclical markups, but small firms have counter-cyclical markups. Crouzet and Mehrotra (2020) find that large firms' sales are less sensitive to the business cycle than small firms. According to our paper, less sensitive sales for large firms implies lower revenue elasticities for large firms. This is consistent with declining trends in average revenue elasticity if large firms are gaining market share, as has been documented for the US (Grullon et al., 2019).

Section 2 presents a theoretical framework with revenue and output elasticity as well as markups. Then, Section 3 derives revenue responses to shocks conditional on revenue elasticity. Section 4 covers data and revenue elasticity measurement. It presents descriptive analysis of revenue elasticity trends. Section 5 presents an empirical model that tests the theory.

## 2. Theoretical framework

### 2.1. Consumer problem

The representative consumer's utility is an additively-separable function of a continuum of goods. The consumer solves the following problem:

$$\max_{Q_j \geq 0} U(Q_j) = H\left(\int_0^N \xi_j u(Q_j) dj\right), \quad \text{s.t.} \quad \int_0^N P_j Q_j \leq I, \tag{1}$$

where the functions satisfy $H' > 0$, $u' > 0$, and $u'' < 0$. The variable $P_j$ is price, $Q_j$ is quantity, $\xi_j$ is a preference shock, and $I$ is aggregate income. The first-order condition yields the following inverse demand function for good $j$:

$$P_j = \xi_j \frac{1}{\eta} \frac{\partial u(Q_j)}{\partial Q_j}. \tag{2}$$

The variable $\eta$ is the budget constraint's Lagrange multiplier (divided by $H'$) which represents the marginal utility of income.[1] The preference shock affects the marginal utility directly. We re-express the first order condition as

$$P_j = \xi_j \mathcal{P}_j(Q_j). \tag{3}$$

We denote the corresponding demand function by

$$Q_j = \mathcal{D}_j(P_j/\xi_j), \tag{4}$$

where $\mathcal{D}_j(\cdot)$ is an inverse function of $\mathcal{P}_j(\cdot)$ that is strictly deceasing and twice continuously differentiable. In Appendix B, we present an example demand system.

### 2.2. Producer problem

Firms solve a two-stage problem in each period. First, a firm indexed by $j = 1, 2, \ldots, N$ chooses inputs to minimize variable costs subject to a production function. Second, given their cost-minimizing input choices, the firm chooses output to maximize profits subject to a demand constraint which follows from the consumer problem. There is no factor market power (monoposony power), so cost-minimizing firms take input costs as given.

Firm $j$'s production function is given by

$$Q_j = \mathcal{F}_j(A_j X_j). \tag{5}$$

The variables $Q_j$, $X_j$, and $A_j$ denote firm $j$'s output, variable input, and factor-augmenting productivity.[2] The production function $\mathcal{F}_j(\cdot)$ is twice continuously differentiable and strictly concave.

#### 2.2.1. Cost minimization

Firms choose inputs to minimize variable costs subject to the production constraints.

$$C_j(Q_j; W/A_j) := \min_{X_j} \quad W X_j \qquad \text{s.t.} \quad Q_j = \mathcal{F}_j(A_j X_j) \tag{6}$$

The first-order condition is $W = \lambda_j \partial\mathcal{F}_j/\partial X_j$. This implies that the cost function is

$$W X_j = \lambda_j \frac{\partial \mathcal{F}_j}{\partial X_j} X_j = \frac{\partial C_j}{\partial Q_j} \times Q_j \times \frac{\partial \mathcal{F}_j}{\partial X_j} \frac{X_j}{Q_j}, \tag{7}$$

where the Lagrange multiplier $\lambda_j$ is equivalent to marginal cost: $\lambda_j = \partial C_j/\partial Q_j \equiv \mathrm{MC}_j$ by the envelope theorem.

#### 2.2.2. Profit maximization

Firms maximize profits by choosing price and output subject to the inverse demand function. $\mathrm{FC}_j$ is a fixed cost.

$$\max_{Q_j} \quad P_j Q_j - C_j(Q_j; W/A_j) - \mathrm{FC}_j \qquad \text{s.t.} \quad P_j = \xi_j \mathcal{P}_j(Q_j). \tag{8}$$

The first-order condition rearranges to give the following optimality condition of operating firms:

$$P_j \left(1 + \frac{\partial P_j}{\partial Q_j} \frac{Q_j}{P_j}\right) = \mathrm{MC}_j. \tag{9}$$

### 2.3. Price, revenue and output elasticity

We define the price elasticity of demand as follows and we assume it is greater than one:

$$-\frac{\partial \mathcal{D}_j}{\partial P_j} \frac{P_j}{Q_j} = -\left(\frac{\partial P_j}{\partial Q_j} \frac{Q_j}{P_j}\right)^{-1} > 1. \tag{10}$$

From the production function (5), the output elasticity is

$$\frac{\partial \mathcal{F}_j}{\partial X_j} \frac{X_j}{Q_j} = \frac{\partial \mathcal{F}_j}{\partial (A_j X_j)} \frac{A_j X_j}{Q_j}. \tag{11}$$

From the production function (5) and inverse demand function (3), the revenue function is given by

[1] The first-order condition states that—apart from $\eta$—the demand price of good $j$ only depends on good $j$ variables. The marginal utility of income $\eta$ is a sufficient statistic to represent the rest of the economy from the good $j$ perspective. In sector $j$, firms consider $\eta$ as given exogenously, but in general equilibrium it is endogenous. Consequently, when $\eta$ is treated as fixed it refers to a firm's perceived demand curve.

[2] We assume one input. With $M$ inputs, we should introduce an aggregating function $\mathcal{G}_j : \mathbb{R}_+^M \to \mathbb{R}_+$ satisfying $Q_j = \mathcal{F}(\mathcal{G}(X_j^1, \ldots, X_j^M))$ where $\mathcal{G}_j(\cdot)$ is homogeneous of degree one.

$$P_j Q_j = \xi_j \mathcal{P}_j(Q_j) Q_j = \xi_j \mathcal{P}_j(\mathcal{F}_j(A_j X_j)) \mathcal{F}_j(A_j X_j) = \mathcal{R}_j(A_j X_j; \xi_j). \tag{12}$$

Therefore, revenue elasticity is given by

$$\frac{\partial \mathcal{R}_j}{\partial X_j} \frac{X_j}{P_j Q_j} = \left[ \frac{\partial P_j}{\partial Q_j} \frac{\partial \mathcal{F}_j}{\partial X_j} Q_j + P_j \frac{\partial \mathcal{F}_j}{\partial X_j} \right] \frac{X_j}{P_j Q_j} = \left[ \left( \frac{\partial \mathcal{D}_j}{\partial P_j} \frac{P_j}{Q_j} \right)^{-1} + 1 \right] \frac{\partial \mathcal{F}_j}{\partial X_j} \frac{X_j}{Q_j}. \tag{13}$$

The right-hand side is the ratio of the output elasticity to the markup, where the (inverse) markup is in square brackets. Hence, using the following notation for revenue elasticity, output elasticity and the markup:

$$\zeta_j \equiv \frac{\partial \mathcal{R}_j}{\partial X_j} \frac{X_j}{P_j Q_j} \quad \text{and} \quad \gamma_j \equiv \frac{\partial \mathcal{F}_j}{\partial X_j} \frac{X_j}{Q_j} \quad \text{and} \quad \mu_j \equiv \left[ \left( \frac{\partial \mathcal{D}_j}{\partial P_j} \frac{P_j}{Q_j} \right)^{-1} + 1 \right]^{-1}, \tag{14}$$

we get

$$\zeta_j = \frac{\gamma_j}{\mu_j}. \tag{15}$$

Revenue elasticity and output elasticity are equivalent when price is independent of output such that there is a perfectly elastic demand curve, $[(\partial \mathcal{D}_j / \partial P_j)(P_j / Q_j)]^{-1} = 0$. Using that $\partial \mathcal{R}_j / \partial X_j = W$, we can write that a firm's revenue elasticity is equal to the ratio of firm variable costs to firm revenue:

$$\zeta_j = \frac{W X_j}{P_j Q_j}. \tag{16}$$

Firm-level variable cost and revenue data are widely available, so the result implies that firm-level revenue elasticities are simple to construct. This result assumes there is no wedge in variable input choices.[3] Additionally, we can show that revenue elasticity is inversely related to the profit and fixed cost share. A decline in revenue elasticity implies a rise in profit and/or fixed cost share in revenue (see Appendix C).

This section shows that revenue elasticity is simple to construct, but given revenue elasticity we cannot identify output elasticity and markups separately. Therefore, in the next section we ask: what can we learn from revenue elasticity without decomposing between the output elasticity and markup?

## 3. Theoretical results

This section shows that higher revenue elasticity firms respond more to the business cycle than lower revenue elasticity firms. We provide full derivations in Appendix A.

### 3.1. *Business cycle framework*

The following total (log) derivative of revenue shows that changes in revenue arise from changes in factor inputs, productivity, and the demand shifter.[4]

$$\mathrm{d} \ln P_j Q_j = \left( \frac{\partial \ln \mathcal{R}_j}{\partial \ln A_j X_j} \right) \mathrm{d} \ln A_j X_j + \mathrm{d} \ln \xi_j = \zeta_j (\mathrm{d} \ln A_j + \mathrm{d} \ln X_j) + \mathrm{d} \ln \xi_j \tag{17}$$

We define the total factor revenue productivity (TFPR) as:

$$\mathrm{d} \ln \mathrm{TFPR}_j = \mathrm{d} \ln P_j Q_j - \zeta_j \, \mathrm{d} \ln X_j = \zeta_j \, \mathrm{d} \ln A_j + \mathrm{d} \ln \xi_j. \tag{18}$$

TFPR changes capture revenue changes that are not explained by changing factor inputs weighted by revenue elasticity (cost share). A firm with higher TFPR growth generates more revenue growth from a change in input growth weighted by revenue elasticity level. The second equality shows that a change in TFPR for a firm can occur through a change in productivity $d \ln A_j$ or a change in demand $d \ln \xi_j$. And, the same change in productivity will differ across firms depending on their revenue elasticity $\zeta_j$.

Taking the total (log) derivative of the firm's implicit cost function, $W X_j = C_j(Q_j; W / A_j)$, we obtain:

$$\mathrm{d} \ln W X_j = \left( \frac{\partial \ln C_j}{\partial \ln W / A_j} \right) \mathrm{d} \ln W / A_j + \left( \frac{\partial \ln C_j}{\partial \ln Q_j} \right) \mathrm{d} \ln Q_j = \mathrm{d} \ln W / A_j + \frac{1}{\gamma_j} \mathrm{d} \ln Q_j. \tag{19}$$

[3] As discussed in Hsieh and Klenow (2014); Eslava and Haltiwanger (2020); Hashemi et al. (2022), the revenue elasticity of an input is only equivalent to its cost share in revenue if the input is not distorted. Therefore, estimated revenue elasticity coefficients on inputs that differ from their input cost share in revenue are evidence of a distortion in the input. Our Fig. 2 shows a strong positive relationship between the two measures, but it is not one-to-one, suggesting the presence of input wedges. Our analysis exploits dynamic variation in revenue elasticity, which is equivalent to assuming constant input wedges over time, rather than the stricter assumption of zero input wedges.

[4] Equation (17) is similar to Decker et al. (2020) equation (3), though the functional form differs because of the shock formulation. In Section 3.4.1, we discuss alternative shock representations.

We have simplified the coefficients using equations (A.1) and (A.2). The result shows that a change in firm costs depends on a change in efficiency-adjusted input cost and output changes weighted by returns to scale. If there were constant returns $\gamma_j = 1$, there would be a proportional relationship between cost changes and output changes, whereas with decreasing returns $\gamma_j < 1$ costs increase more than proportionally with output increases, and with increasing returns $\gamma_j > 1$ costs increase less than proportionally with output increases.

Similarly, the total (log) derivative of a firm's implicit demand function, $Q_j = \mathcal{D}_j(P_j/\xi_j)$, yields:

$$\mathrm{d}\ln Q_j = \left(\frac{\partial \ln \mathcal{D}_j}{\partial \ln P_j}\right)(\mathrm{d}\ln P_j - \mathrm{d}\ln \xi_j) = \left(\frac{\mu_j}{\mu_j - 1}\right)(\mathrm{d}\ln P_j - \mathrm{d}\ln \xi_j). \tag{20}$$

Demand is decreasing in price and increasing in the demand shock. The responsiveness of demand to these variables depends on the markup. When the markup is lower demand responses are amplified and they are dampened if the markup is higher.

Lastly, from the definition of the markup as the price to marginal cost ratio and output elasticity as the average cost to marginal cost ratio (equation (A.1)), we obtain that

$$\mathrm{d}\ln P_j = \mathrm{d}\ln \mu_j + \mathrm{d}\ln \mathrm{MC}_j, \tag{21}$$

$$\mathrm{d}\ln \mathrm{MC}_j = \mathrm{d}\ln W X_j - \mathrm{d}\ln Q_j - \mathrm{d}\ln \gamma_j. \tag{22}$$

The two equations show that changes in price can be decomposed into changes in the markup and changes in marginal cost. Similarly, changes in the marginal cost can be decomposed into changes in input costs, changes in output, and changes in returns to scale (output elasticity).

### 3.2. Propagation mechanism

Aggregating individual firms' real revenue responses indicate aggregate business cycle fluctuations. This is because, by definition, real GDP (aggregate output) is equal to the sum of firms' real value-added, which is revenues less material costs.[5] From equations (20)–(22), we can express firm revenue responses as a function of economic conditions:

$$\mathrm{d}\ln P_j Q_j = \frac{\zeta_j}{1-\zeta_j}(\mathrm{d}\ln A_j - \mathrm{d}\ln W + \mathrm{d}\ln \zeta_j) + \frac{1}{1-\zeta_j}\,\mathrm{d}\ln \xi_j. \tag{23}$$

We provide full derivations in Appendix A. Four shocks affect a firm's revenue change: productivity, factor price, revenue elasticity (inversely related to operating profitability), and demand changes. Revenue elasticity $\zeta_j$ determines the coefficient which captures the responsiveness of firm revenue to different shocks. A higher revenue elasticity firm has a greater increase in revenue in response to (i) positive productivity and demand shocks, (ii) factor price decrease, and (iii) operating profit decreases (i.e., revenue elasticity increases).

We can re-write equation (23) in terms of TFPR which we defined in equation (18):

$$\mathrm{d}\ln P_j Q_j = \frac{1}{1-\zeta_j}(\mathrm{d}\ln \mathrm{TFPR}_j - \zeta_j\,\mathrm{d}\ln W + \zeta_j\,\mathrm{d}\ln \zeta_j). \tag{24}$$

This shows that revenue elasticity is helpful to understand the response of firm revenue to revenue productivity shocks $\mathrm{d}\ln \mathrm{TFPR}_j$. A high revenue elasticity firm's revenue increases more than a low revenue elasticity firm when TFPR increases or/and factor prices decrease.[6] Here, the revenue elasticity change component $\mathrm{d}\ln \zeta_j$ can represent market structure changes, such as factor market power, as well as the inverse of the operating profitability changes $\mathrm{d}\ln[1 - (W X_j)/(P_j Q_j)]$. This can be seen through equation (16) which equates revenue elasticity to factor shares in revenue.

In summary, we conclude that a higher revenue elasticity firm has greater cyclical sensitivity to economic conditions.

### 3.3. Model discussion

The revenue elasticity is the ratio of output elasticity to the markup and this ratio determines the revenue response to shocks. Low revenue elasticities imply weaker revenue responses to (revenue) productivity shocks. In other words, a large markup relative to output elasticity or a low output elasticity relative to markup. Consider a decrease in revenue productivity. Firms with low revenue elasticity are cushioned as their scale falls. Low revenue elasticity occurs because output elasticity (production side) is small relative to markup (demand side). On the production side, low output elasticity dampens the negative shock because firms produce more efficiently at a lower scale, so the output component of revenue is cushioned. On the demand side, a high markup dampens the negative shock because the price component of revenue dominates the output component of revenue, hence revenues are less sensitive to output change.

The main insight is that rather than having to identify the markup or output elasticity separately, we can study their ratio—the revenue elasticity—in order to make macroeconomic inference.

[5] For example, we commonly specify a 'final goods producer' profit maximization problem as $\max_{y_j} PY - \int_0^N P_j Q_j \,\mathrm{d}j$ subject to an aggregate output constraint which aggregates individual goods $Y = G(Q_j)$. Under perfect competition, optimality conditions yield equivalence between aggregate output and the sum of firm revenues, which implies zero profits.

[6] Related to the response to TFPR changes, the factor price change component can account for general equilibrium channels in reacting to shocks. In Appendix D we elaborate on the possibility of general equilibrium effects.

### 3.4. Theory extensions & robustness

#### 3.4.1. Alternative shock formula

Even though we do not specify the production and (inverse) demand functions, we do assume both factor augmenting productivity and a marginal utility shifting demand shock. Here, we compare our previous results to alternative common assumptions. In particular, we use Hicks neutral productivity and Marshallian demand shocks as supply and demand shocks.

Denote the alternative functional forms by tilde as follows. The production and variable cost functions are

$$Q_j = \tilde{A}_j \tilde{\mathcal{F}}_j(X_j) \quad \text{and} \quad W X_j = \tilde{C}_j(Q_j/\tilde{A}_j; W). \tag{25}$$

Similarly, the demand and inverse demand functions are

$$Q_j = \tilde{\xi}_j \tilde{D}_j(P_j) \quad \text{and} \quad P_j = \tilde{\mathcal{P}}_j(Q_j/\tilde{\xi}_j). \tag{26}$$

Then, the revenue function with changes is given by

$$\mathrm{d}\ln P_j Q_j = \zeta_j \left(\frac{1}{\mu_j}\mathrm{d}\ln \tilde{A}_j + \mathrm{d}\ln X_j\right) + \left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\ln \tilde{\xi}_j. \tag{27}$$

Define TFPR as

$$\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j = \mathrm{d}\ln P_j Q_j - \zeta_j \,\mathrm{d}\ln X_j = \frac{\zeta_j}{\mu_j}\mathrm{d}\ln \tilde{A}_j + \left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\tilde{\xi}_j, \tag{28}$$

in which the measurement of TFPR ($\mathrm{d}\ln P_j Q_j - \zeta_j \,\mathrm{d}\ln X_j$) is the same as TFPR with our benchmark shock formula in equation (18).

As in the previous subsection, we can write firm revenue responses as a function of economic conditions.

$$\mathrm{d}\ln P_j Q_j = \frac{\zeta_j}{1-\zeta_j}\left(\frac{1}{\mu_j}\mathrm{d}\ln \tilde{A}_j - \mathrm{d}\ln W + \mathrm{d}\ln \zeta_j\right) + \left(\frac{1}{1-\zeta_j}\right)\left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\ln \tilde{\xi}_j. \tag{29}$$

Equation (29) shows that the revenue elasticity is not sufficient to understand the response of revenues to Hicks neutral productivity and Marshallian demand shocks described in equations (25) and (26). We need the markup and revenue elasticity to understand the response of revenues to supply and demand shocks $\mathrm{d}\ln \tilde{A}_j$ and $\mathrm{d}\ln \tilde{\xi}_j$. More specifically, we need any two of the three variables related in equation (15) as $\zeta_j = \gamma_j/\mu_j$. In contrast, revenue elasticity was enough in equation (23) with the factor augmenting productivity and marginal utility shifting demand shocks, $\mathrm{d}\ln A_j$ and $\mathrm{d}\ln \xi_j$.

Although we need to separate output elasticity and markups in equation (29) in order to understand Hicks neutral productivity and Marshallian demand shocks, we can still use the revenue elasticity to understand the revenue response to TFPR shocks:

$$\mathrm{d}\ln P_j Q_j = \frac{1}{1-\zeta_j}(\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j - \zeta_j \,\mathrm{d}\ln W + \zeta_j \,\mathrm{d}\ln \zeta_j). \tag{30}$$

This is identical to equation (24) except for $\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j$. Equation (30) shows that higher revenue elasticity strengthens a firm's revenue response to economic conditions such as TFPR, factor price, and profitability.

#### 3.4.2. Input wedges and adjustment costs

When input wedges or adjustment costs exist, the ratio of firm variable costs to revenue might deviate from revenue elasticity in equation (16). To illustrate this, we introduce input wedges/adjustment costs: $\kappa_j(X_j) \geq 0$, where $\kappa_j(\cdot)$ is twice continuously differentiable and convex. Our approach is similar to the appendix of Bond et al. (2021). In the presence of input wedges/adjustment costs the cost minimization problem in equation (6) becomes:

$$C_j(Q_j; W/A_j) := \min_{X_j} \quad W X_j + W\kappa_j(X_j) \qquad \text{s.t.} \quad Q_j = \mathcal{F}_j(A_j X_j). \tag{31}$$

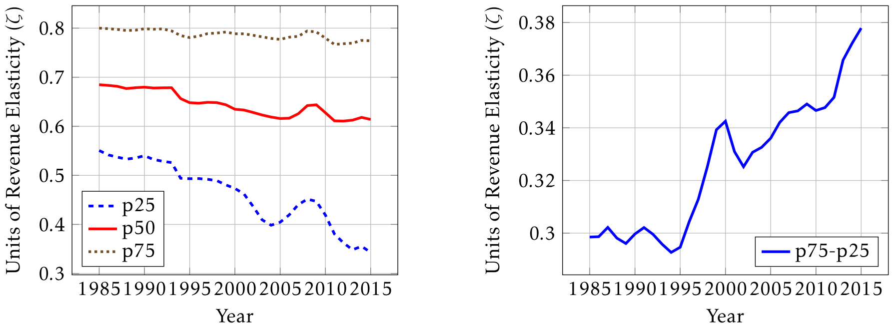
The first order condition implies

$$\zeta_j = \frac{W X_j}{P_j Q_j}\left[1 + \kappa_j'(X_j)\right], \tag{32}$$

where $\kappa_j'(X_j) = \partial\kappa/\partial X_j$. Firm revenue elasticity remains equal to the ratio of output elasticity to markups as in equation (15). In the data, we observe variable costs which incorporate adjustment costs: $W X_j[1 + \kappa(X_j)/X_j]$. Thus, the ratio of firm variable costs to revenue measures the revenue elasticity with distortion:

$$\zeta_j \left[\frac{1 + \kappa_j(X_j)/X_j}{1 + \kappa_j'(X_j)}\right] = \frac{W X_j \left[1 + \kappa_j(X_j)/X_j\right]}{P_j Q_j}. \tag{33}$$

The result shows that the ratio of variable cost to revenue from the right-hand side can be higher either due to a higher revenue elasticity or because of a higher average wedge to marginal wedge ratio. If the average to marginal wedge ratio is close to one, $\kappa_j'(X_j) \approx \kappa_j(X_j)/X_j$, the problem of wedges biasing the revenue elasticity measure are mitigated.




(a) Quartiles (b) Inter-Quartile Range

**Fig. 1.** Revenue elasticity quartile trends.

The formulation helps us to understand potential biases. If the adjustment cost were decreasing in size $\kappa' < 0$, then our measure of revenue elasticity on the right-hand side would overestimate the true revenue elasticity. If adjustment costs are linear, for example $\kappa_j(X_j) = (a_j X_j^{b_j-1} - 1)X_j$, where $b_j = 1$ and $W\kappa_j(X_j) = (a_j - 1)WX_j$, then there will be no distortion.

Lastly, even if the levels measures of revenue elasticity are biased, the ratio of firm variable costs to revenue will successfully represent firm heterogeneity in revenue elasticities as well as in cyclical sensitivity, if the gap between the average and marginal wedges is similar across firms and over time. In this case, the bias becomes a fixed effect which disappears with time series or firm variation.

## 4. Data and measurement

We measure firm-level revenue elasticities using the *Compustat North America Fundamentals Annual* database. We use data from 1984 to 2016. The database covers publicly listed firms in the US. We follow De Loecker et al. (2020) for data cleaning and variable construction. From the database, we use firm-level data on sales growth, productivity growth, revenue elasticity, market share, employment, cash holdings, short-term debt, long-term debt and working-capital ratio. We present summary statistics in Appendix E.

### *4.1. Revenue elasticity measurement and trends*

We construct revenue elasticity as the ratio of firm costs to firm revenue, respectively $WX_j$ and $P_jQ_j$ in equation (16). We use *cost of goods sold* (COGS) for costs and *net-sales/turnover* (SALE) for revenue. This yields a firm-specific measure of revenue elasticity. In our sample, we have some outliers. For instance, the maximum value is around 950. Out of 178,332 observations, 75 and 997 had ratios greater than 100 and 10, respectively. These values are remarkably higher than the ratios of the majority of observations: the median, top 75%, and 95% values are 0.64, 0.79, and 1.00, respectively. It is possible that observations with extreme values solely determine regression outcomes. To avoid it, we replace the ratio by 1.5 when it exceeds 1.5 (5,123 observations).[7] A ratio exceeding 1 implies negative operating profits, and capping at 1.5 means we adjust observations that have an operating profit share of below −50%. Then we calculate the three-year centred moving average, hence the plots are from 1985–2015. *Firm-level* revenue elasticity is important for us because our interest is the relationship in equation (23). That is, we estimate a model of firm-level revenue responses conditional on firm-level revenue elasticities.

Fig. 1 shows the trends in revenue elasticity based on our cost-share approach. The trends show decreasing revenue elasticity over the sample period. As shown in the previous section, decreasing revenue elasticity is equivalent to an increasing profit share. Barkai (2020) documents a rising profit share in the U.S. over the same period. Similarly, as described above, decreasing revenue elasticity is consistent with increasing markups or decreasing output elasticity. De Loecker et al. (2020) document rising markups. Revenue elasticity is decreasing for high, low and medium revenue elasticity firms. That is, the upper quartile, lower quartile and median revenue elasticity firms all observe declining revenue elasticity as shown in Fig. 1. However, the decline among the high revenue elasticity firms is weaker than the decline among the low revenue elasticity firms. This causes an increase in the inter-quartile range which indicates an increase in *revenue elasticity dispersion*. In addition to the long-run trends, Fig. 1 indicates that revenue elasticity is countercyclical in the short run. That is, revenue elasticity and GDP are negatively correlated. In recession average revenue elasticity increases. In Appendix E, we plot the cyclical behaviour of revenue elasticity relative to GDP.

### *4.2. Other measures of revenue elasticity*

#### *4.2.1. Cost share and estimation approaches*

There are two methods to measure revenue elasticity:

[7] The winsorization with a higher value such as 5 has negligible impacts on our regression results.

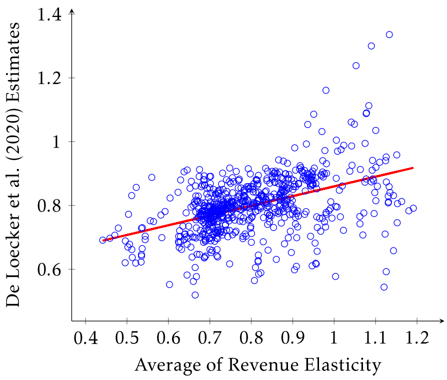



**Fig. 2.** Revenue elasticities and revenue function estimates: two-digit NAICS.

Notes: The x-axis is the average of our firm-level revenue elasticity for each industry in each year. The y-axis is the (Online Appendix pp. 18, De Loecker et al., 2020, Figure 12.2), estimated coefficient (labelled PF2) on inputs (variable input, capital, and overhead inputs). The results without overhead inputs (PF1) yield similar results.

(i) *Cost-share approach:* Construct revenue elasticity as the ratio of variable costs to revenue, as in equation (16).
(ii) *Revenue function estimation approach:* Estimate elasticity from a revenue function.

The cost-share approach is firm-specific. It is the ratio of firm costs to firm revenue, respectively $WX_j$ and $P_jQ_j$ in equation (16). We use *cost of goods sold* (COGS) for costs and *net-sales/turnover* (SALE) for revenue. We take both directly from Compustat. The revenue function estimation approach is industry level. One should estimate a panel regression on all firms in an industry in order to get a coefficient corresponding to revenue elasticity for that industry. In this paper, we prefer to use the cost-share approach because it is independent of estimation methods and functional forms of the production function.

Fig. 2 shows the cost-share approach and (revenue) production function approach are highly correlated. We plot the De Loecker et al. (2020) estimates of industry-level elasticities (from revenue data) against the industry averages from our cost-share approach. The data is pooled across years (1985–2015). Each scatter point represents a two-digit industry in a year. Appendix E provides plots by year and industry.

### 4.2.2. *Alternative cost-share measures of revenue elasticity*

Our measure of revenue elasticity is costs divided by revenue as in equation (16). We present a *benchmark* and two *alternative* measures of revenue elasticity depending on our measure of variable cost. Since variable costs are vital for our revenue elasticity measures, these robustness checks ensure other plausible measures or variable cost do not affect our results. For the alternative measures, we perform the same cleaning of revenue elasticity as in the benchmark case. We consider three cases:

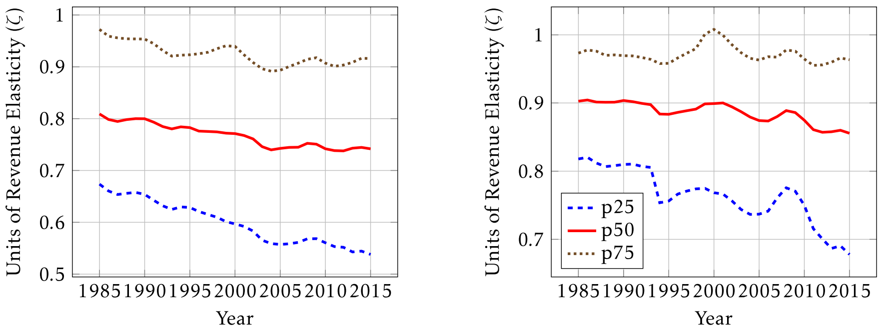
(i) Benchmark: We measure variable costs as *cost of goods sold* (COGS).
(ii) Alternative I: We measure variable costs as *cost of goods sold* (COGS) plus capital costs.
(iii) Alternative II: We measure variable costs as operating expenses, i.e., *cost of goods sold* (COGS) plus *selling, general, and administrative expense* (SGA).

Benchmark corresponds to theory that treats capital as pre-determined. That is, capital is not a variable input for a firm in the short run. This is more common in industrial organization literature (e.g., De Loecker et al., 2020).

Alternative I corresponds to theory that treats capital as a variable input. This is common in macroeconomics, particularly business cycle literature. Capital is not fixed in the firm's profit maximization problem. We measure capital costs using the same method as De Loecker et al. (2020, p.8). The capital rental rate is $(I - \Pi) + \delta$ where $I$, $\Pi$, and $\delta$ are the nominal interest rate, the inflation rate, and a depreciation rate. Capital is measured by gross capital (PPEGT) adjusted by the Relative Price of Investment Goods from FRED.

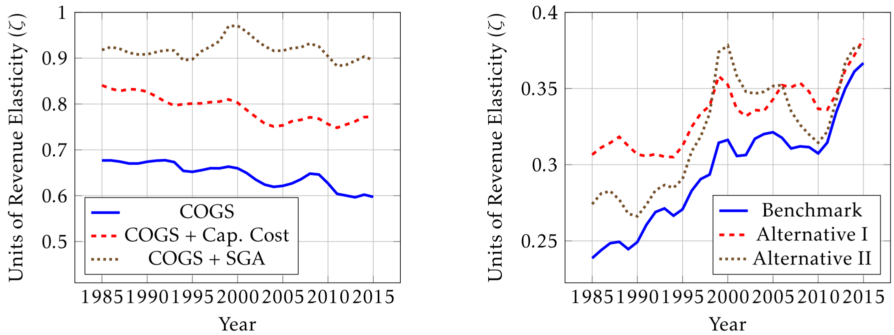
Alternative II considers operating expenses (OPEX), which are costs of goods sold (COGS) plus selling, general, and administrative expenses (SGA). COGS measure direct inputs in production, such as materials and most labour, whereas SGA measures indirect inputs in production, most commonly marketing and management expenses. Traina (2018) argues that OPEX are a better measure of variable costs than COGS alone. However, it is unclear whether variable costs should include SGA because SGA can represent fixed costs. According to Compustat, SGA accounts for all operating expenses (other than those directly related to production) incurred in the regular course of business.

Fig. 3 shows that the alternative measures of revenue elasticity have similar distributional trends to the benchmark measure in Fig. 1. For all measures the inter-quartile range (p75–p25) increases over time and median (p50) revenue elasticity decreases over time. The increasing IQR (p75–p25), represents greater cross-sectional heterogeneity, and is caused by low elasticity firms. There is a strong downward trend in the lower quartile (p25) for all measures of revenue elasticity, whereas the upper quartile (p75) is more stable.




(a) Alternative I (b) Alternative II

**Fig. 3.** Alternative revenue elasticity quartile trends.




(a) Mean (b) Standard Deviation

**Fig. 4.** Revenue elasticity mean and standard deviation trends.

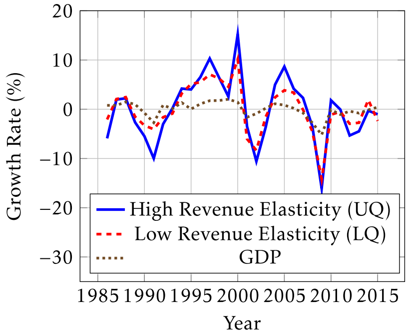
Fig. 4 shows that the benchmark and alternative measures of revenue elasticity have similar trends in mean revenue elasticity and similar trends in dispersion. The first panel shows that the mean level of revenue elasticity is decreasing in the long run, and all measures are inversely related to the business cycle in the short run. That is, revenue elasticity is counter-cyclical. The second panel shows that the standard deviation of revenue elasticity across firms is increasing in the long run. This implies a greater dispersion in firm-level revenue elasticity. In the short-run, dispersion in revenue elasticity across firms is procyclical. The standard deviation of revenue elasticities across firms decreases in recession, implying that on average firms have revenue elasticities closer to the mean than in normal times.

As a final robustness check, we compare our accounting-approach measures of revenue elasticity to common measures in the literature. Our levels of revenue elasticity are all centred around 0.85, which is a common *implied* revenue elasticity in macroeconomic models, and widely used in calibration exercises. For example, Atkeson and Kehoe (2005) specify the markup as $\mu \approx 1.11$, based on estimates of the underlying elasticity of substitution, and they specify output elasticity of variable factors as $\gamma = 0.95$ based on surveying production function estimation literature. Therefore the implied revenue elasticity is $\zeta = \gamma/\mu \approx 0.85$ which is consistent with earlier work by Atkeson et al. (1996). A number of subsequent papers adopt this calibration (e.g., Restuccia and Rogerson, 2008; Barseghyan and DiCecio, 2011). Notably the calibration is used regardless of the presence of a markup, so it is interpreted in the broader sense of the factor revenue share, whilst being agnostic on the division between markups and output elasticity. Recent work by Ruzic and Ho (2021) finds that in U.S. manufacturing weighted, industry-level, revenue elasticities have declined from 0.84 in 1982 to 0.64 in 2007. They use an accounting approach to gain revenue elasticities for labour and a GMM estimation approach to get revenue elasticities for capital, then they sum the two elasticities to get revenue elasticity. Lastly, a revenue elasticity $\zeta = 0.85$ implies that the economic profit share in revenue is 15%, which is consistent with recent evidence (Barkai, 2020).

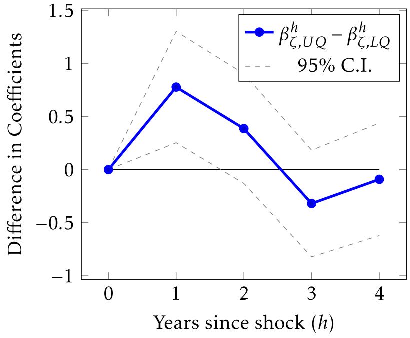
To conclude our discussion of alternative measures, all measures show a significant downward trend of in revenue elasticity among low elasticity firms, a downward trend in average elasticity, and an upward trend in inter-quartile range (cross-sectional heterogeneity). The mean and median levels of revenue elasticity are within a range of existing estimates acquired by econometric estimation.

### 4.3. Revenue elasticity descriptive statistics: cyclical sensitivity

Fig. 5a shows a time series for the average growth rate of real sales for high and low revenue elasticity firms. We have demeaned these series to highlight fluctuations. High-revenue elasticity firms are firm-year observations in the upper quartile of the revenue elasticity distribution and low-revenue elasticity firms are in the lower quartile.




(a) Average Firm-Level Sales Growth




(b) IRFs to GDP

**Fig. 5.** Revenue elasticity and cyclical sensitivity.

The series for the upper and lower quartile firms (blue solid and red dashed lines, respectively) strongly comove with the demeaned GDP growth (brown short-dashed line). Their correlation coefficients are 0.82 and 0.74, respectively. In Fig. 5a, the average sales growth of the high revenue elasticity group is more responsive to GDP fluctuations than the low revenue elasticity group.

Similar to Crouzet and Mehrotra (2020), we estimate the difference of a firm's revenue response to aggregate GDP between the two groups ($UQ_{j,t}$ and $LQ_{j,t}$) to investigate a link between a firm's revenue elasticity and its cyclical behaviour. We use the local projections methodology of Jordà (2005) to estimate:

$$\begin{aligned}\Delta^h \ln P_{j,t}Q_{j,t} &= \beta^h_{1,UQ}(\Delta^1 \ln \mathrm{GDP}_t \times UQ_{j,t}) + \beta^h_{1,LQ}(\Delta^1 \ln \mathrm{GDP}_t \times LQ_{j,t}) \\ &\quad + \beta^h_{2,UQ}UQ_{j,t} + \beta^h_{2,LQ}LQ_{j,t} + \mathbf{traits}_{j,t}\mathbf{b}^h_2 + \delta^h_{j,t} + \varepsilon^h_{j,t}. \end{aligned} \tag{34}$$

We index a firm with $j$ and $h \geq 1$ represents the shock horizon. The dummy variable $UQ_{j,t}$ is 1 if firm $j$ is in the upper quartile of revenue elasticities and is 0 otherwise. The dummy variable $LQ_{j,t}$ is 1 if firm $j$ is in the lower quartile of revenue elasticities and is 0 otherwise. Vectors are in bold font and $\mathbf{X}^\intercal$ represents the vector transpose. The delta operator $\Delta^h$ represents the difference between $t+h$ and $t+h-1$. The variable $\mathbf{traits}_{j,t}$ is a vector of controls (firm traits). The controls are sales share in industry, employment, cash holding to asset ratio, short-term debt to asset ratio, long-term debt to asset ratio, and working capital ratio all in logs. We control time-invariant firm effects and time-variant industry effects by including the firm-level fixed effects and industry-by-year fixed effects. Standard errors are clustered at the firm-level.

The estimated coefficients represent the elasticity of firm-level revenue to aggregate GDP. When using GDP changes, we capture whether a firms' revenue growth is procyclical or countercyclical in response to aggregate GDP growth. The difference between the upper and lower quartile coefficients, $\beta^h_{1,UQ} - \beta^h_{1,LQ}$ for $h = 1, 2, 3, 4$, captures the difference in revenue response of high and low revenue elasticity firms to GDP changes. When the difference is positive, as we find in Fig. 5b, it suggests that high revenue elasticity firms are more procyclical than low revenue elasticity firms. Appendix Table A.2 lists the estimated coefficients and standard errors.

### *4.4. Shocks*

We investigate the effect of firm-level revenue elasticity on cyclical sensitivity, in particular revenue amplification for firm-level revenue productivity shocks, aggregate productivity shocks, and GDP shocks.

#### *4.4.1. Firm-level revenue productivity shocks*

Equation (23) formalizes our hypothesis: high revenue elasticity firms react more to the supply and demand shocks than low revenue elasticity firms. Equation (24) allows us to test the hypothesis in terms of total factor revenue productivity (TFPR) shocks. First, we measure a *firm-level* revenue productivity shock series. We define the series as firm-level labour productivity annual growth rate denoted by $\Delta^1 LP_{j,t} \equiv \ln LP_{j,t+1} - \ln LP_{j,t}$ where $LP_{j,t} = \mathrm{sales}_{j,t}/\mathrm{employees}_{j,t}$ represents labour productivity.[8] This measure is a proxy for the difference between revenue and factor growth rates (i.e., $\Delta^1 P_{j,t}Q_{j,t} - \Delta^1 X_{j,t}$). When the revenue elasticity equals one (zero operating profit), the difference measures the productivity changes $\Delta^1 \ln A_{j,t} + \Delta^1 \ln \xi_{j,t}$ well. However, the simple difference is biased with non-unit elasticity. According to equation (18), we obtain that

$$\Delta^1 \ln \mathrm{TFPR}_{j,t} \approx \Delta^1 \ln P_j Q_{j,t} - \Delta^1 \ln X_{j,t} + (1 - \zeta_{j,t})\Delta^1 \ln X_{j,t}, \tag{35}$$

[8] In Compustat, we use *the actual number of people employed by the company and its consolidated subsidiaries* (EMP) for the number of employees. Some firms report an average number of employees, and some firms report the number of employees at year-end. (If both are given, the year-end number is used.) The item includes (i) all employees of consolidated subsidiaries, both domestic and foreign, (ii) all part-time and seasonal employees, (iii) full-time equivalent employees, and (iv) officers, however, excludes (i) consultants, (ii) contract workers, (iii) directors, and (iv) Employees of unconsolidated subsidiaries.

where we use $\Delta^1$employees$_{j,t}$ as a proxy for $\Delta^1 \ln X_{j,t}$. For a firm with positive factor growth, the difference between revenue and factor growth underestimates TFPR growth, and the bias is larger when revenue elasticity is low. Thus, we consider labour productivity growth $\Delta^1 LP_{j,t}$ and corrected labour productivity growth $\Delta^1 LP_{j,t} + (1-\zeta_{j,t})\Delta^1 \ln X_{j,t}$ as representing TFPR growth. To minimize the impact of outliers, we winsorize the shock measurements at the 1% and 99% levels. To calculate idiosyncratic parts, we de-mean the shocks by using the 3-digit NAICS industry's cross-sectional average productivity growth rate (weighted by size and prior-year sales) for each period.

We note that our labour productivity growth measure is an imperfect proxy for the difference between revenue and factor growth rates since it omits other factors. Increasing a factor that we omit, such as tangible or intangible capital, will increase revenue, but should be offset by subtracting the input increase from revenue.[9] Whereas, with our proxy, revenue will increase without the offsetting increase in the factor. Consequently, our measure of firm-level revenue productivity will increase more than if the factor increase were included. Therefore, the proxy could overestimate true revenue productivity. A mitigating factor is that if factor inputs are correlated then the degree of bias will be constant over time because omitted factors will bias the revenue elasticity level the same each period as labour and other factors move in harmonisation. This will not affect results that affect shock variation.

### 4.4.2. Aggregate productivity and uncertainty shocks

In addition to revenue responses to firm-level productivity shocks, we measure the sensitivity of firm-level revenue to aggregate economic conditions. We introduce an *aggregate-level* productivity shock that directly affects $\Delta^1 A_{j,t}$ and $\Delta^1 \xi_{j,t}$. The aggregate productivity shock series also allow us to test our hypothesis from equation (23).

We first construct the aggregate productivity shock by aggregate total factor productivity growth from Penn World Table 9.1 (RTFPNA: TFP at constant national prices, 2011 = 1), i.e., $\text{shock}_{j,t} = \Delta^1 \text{RTFPNA}_t$. Additionally, we consider uncertainty shocks. We use the news-based policy uncertainty index from the Measuring Economic Policy Uncertainty database by Scott Baker, Nicholas Bloom, and Steven J. Davis.[10] We take into account two categorical uncertainty indices on monetary policy and fiscal policy (taxes or spending). Specifically, we make the monthly index series annual by averaging it, i.e., $\text{shock}_{j,t} = (1/12)\sum_{m=1}^{12}(\text{Index}_{t,m}/100) - 1$, where $t$ and $m$ index year and month, respectively. In our regression analysis, these uncertainty shocks function as exogenous, negative shocks on firm revenue productivity.

## 5. Empirical analysis

### 5.1. Empirical methodology

This section outlines a reduced-form model to quantify the effect of shocks on firm revenues conditional on firm revenue elasticity. In order to estimate the dynamics of differential responses across firms, we use local projection estimation following Jordà (2005).

### 5.1.1. Specification with continuous measure of revenue elasticity

To test our hypothesis from equation (23), we interact shocks with a firm's pre-existing traits.[11] Therefore, we estimate:

$$\begin{aligned}\Delta^h \ln P_{j,t}Q_{j,t} = {} & \beta_0^h \text{shock}_{j,t} + \beta_{1,\zeta}^h(\text{shock}_{j,t} \times \ln \zeta_{j,t}) + (\text{shock}_{j,t} \times \mathbf{traits}_{j,t}^{\intercal})\mathbf{b}_1^h \\ & + \beta_2^h \ln \zeta_{j,t} + \mathbf{traits}_{j,t}^{\intercal}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h. \end{aligned} \tag{36}$$

The dependent variable is the difference between log revenue in period $t+h$ and log revenue in the previous period $t+h-1$. As an example, $\beta_0^2$ represents the effect of a shock in period $t$ on revenue growth after two periods. The variable $\zeta_{j,t}$ represents firm $j$'s revenue elasticity at $t$. The variable '$\text{shock}_{j,t}$' represents a shock. The variable $\mathbf{traits}_{j,t}$ is a vector of controls. The vector $\mathbf{b}_1^h$ contains coefficients that represent the effect of the shock on firm revenue conditional on firm traits. The vector $\mathbf{b}_2^h$ contains coefficients that represent the effect of firm traits on firm revenue. We control for lagged revenue elasticity $\beta_2^h \ln \zeta_{j,t}$ and firm-level fixed effects $\delta_{j,t}^h$. The firm-level fixed effects control for (i) time-invariant firm characteristics that generate firm-specific trends in revenue growth, and (ii) time-varying and time-invariant industry differences that might affect firms' reactions to the shocks, for example general equilibrium effects. Standard errors are clustered at the firm-level.

The main coefficients of interest are $\beta_{1,\zeta}^h$ for $h = 1,2,3,4$. The coefficient $\beta_{1,\zeta}^h$ represents a firm's percentage change (log difference) in revenue after $1,2,3$ and $4$ years following a shock in $t$ relative to a firm with a (log) unit lower revenue elasticity. Hence a positive coefficient means a shock has a greater effect on revenue for firms with higher revenue elasticity.

### 5.1.2. Specification with discrete measure of revenue elasticity

As an alternative to regression equation (36), we consider a discrete measure of revenue elasticity as in equation (34). Our re-specified equation is

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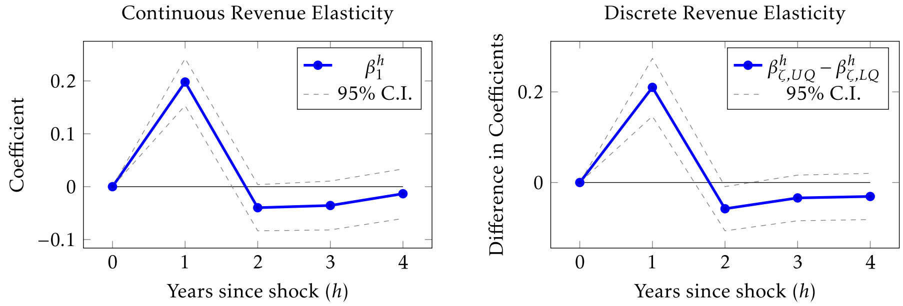
[9] This proxy is similar to Decker et al. (2020) who use revenue labour productivity (RLP) when they analyse the whole economy, rather than only manufacturing which has cleaner measures.

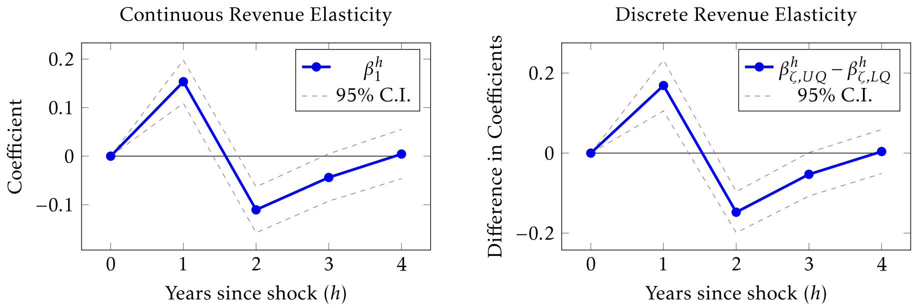
[10] The database is available at https://policyuncertainty.com/.

[11] We only consider the pre-existing traits, $\mathbf{traits}_{j,t}$. Jordà (2005)'s local projection allows high-order lagged exogenous control variables ($\mathbf{traits}_{j,t-k}$ for $k = 1,2,3\ldots$ in our framework). Adding the more lagged variables in the regression equations (36) and (37) does not affect our main results.




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 6.** IRFs to firm-level labour productivity shock.




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 7.** IRFs to firm-level corrected labour productivity shock.

$$\Delta^h \ln P_{j,t}Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,UQ}^h(\text{shock}_{j,t} \times UQ_{j,t}) + \beta_{1,LQ}^h(\text{shock}_{j,t} \times LQ_{j,t}) + \beta_{2,UQ}^h UQ_{j,t} + \beta_{2,LQ}^h LQ_{j,t} + \textbf{traits}_{j,t}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h. \quad (37)$$

The coefficient $\beta_{1,UQ}^h$ captures the effect of a shock on revenue conditional on being a high revenue elasticity firm, and $\beta_{1,LQ}^h$ captures the effect of a shock on revenue conditional on being a low revenue elasticity firm. The difference between the upper and lower quartile coefficients, $\beta_{1,UQ}^h - \beta_{1,LQ}^h$ for $h = 1,2,3,4$, represents the difference in revenue response of high and low revenue elasticity firms to shocks. When the difference is positive, it implies that high revenue elasticity firms respond more to shocks than low revenue elasticity firms.

## 5.2. *Empirical results*

We find evidence that a firm's revenue elasticity increases its revenue response to firm- and aggregate-level shocks. In this section, we report results with our benchmark revenue elasticities. Results with alternative elasticity measures are in Appendix F.

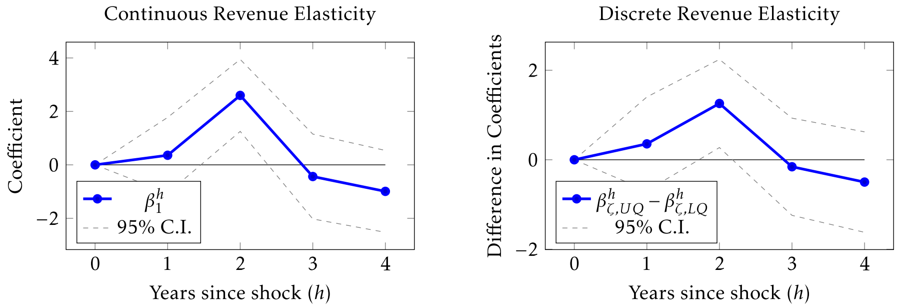
### 5.2.1. *Firm-level revenue productivity shocks*

Figs. 6 and 7 show how revenue elasticity shapes the impulse response function (IRF) following a productivity change. All results with firm's labour revenue productivity shocks in Fig. 6 are robust after considering productivity biases arising from non-unit revenue elasticities. Fig. 7 plots the IRF gap to a labour productivity shocks with corrections described in equation (35). See Appendix Tables A.3 and A.4 for the number of estimated coefficients (IRFs) and their standard errors.

In both figures, panel (a) shows the effect of the shock on impact ($h = 1$) and after one, two and three years ($h = 2,3,4$). More specifically, the plots capture the effect of a productivity shock on revenue conditional on a firm's revenue elasticity. All the plots show that firms with higher revenue elasticity adjust revenues more in response to a productivity shock than firms with lower revenue elasticity. The effect is large on impact, but dissipates after one year.

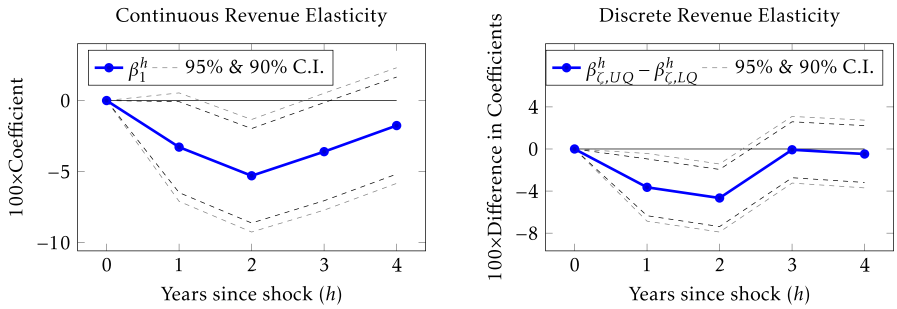
Panel (a) displays the estimate of $\beta_{1,\zeta}^h$ on the y-axis for x-axis values $h = 1,2,3,4$. These follow from the regression equation (36) where we use a continuous measure of revenue elasticity. The coefficient $\beta_{1,\zeta}^h$ represents a firm's percentage change (log difference) in revenue following a technology shock compared to a firm with a (log) unit lower revenue elasticity. For example, if we take the first point $\beta_{1,\zeta}^1 \approx 0.15$, this implies that following a productivity shock, on impact, a firm increases revenue by 15% more than a firm with one (log) unit lower revenue elasticity

Panel (b) plots the differential response of productivity shocks on firms in the upper and lower quartiles $\beta_{1,UQ}^h - \beta_{1,LQ}^h$ on the y-axis for x-axis values $h = 1,2,3,4$ year since shocks. These follow from the regression equation (37) where we use a discrete measure of revenue elasticity. The value $\beta_{1,UQ}^h - \beta_{1,LQ}^h$ represents a firm's percentage change (log difference) in revenue following a technology




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 8.** IRFs to aggregate TFP shock.




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 9.** IRFs to news based economic policy uncertainty index.

shock for a high revenue elasticity firm (upper quartile) compared to a low revenue elasticity firm (lower quartile). For example Panel (b) of Fig. 6, if we take the first point $\beta^1_{1,UQ} - \beta^1_{1,LQ} \approx 0.2$ implies that the upper quartile of revenue elasticity firms increase their revenue 0.2% more than the lower quartile of revenue elasticity firms following a one percent productivity shock.

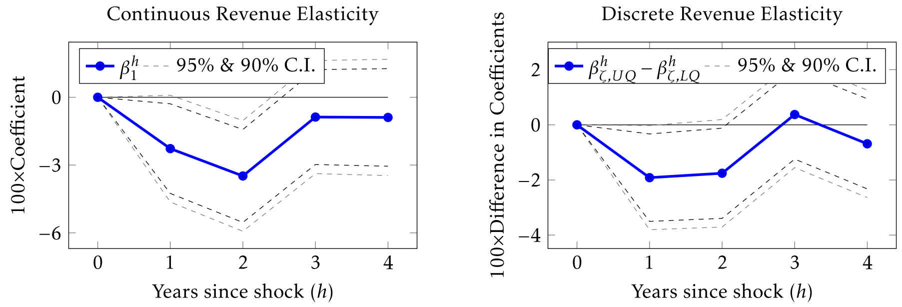
In Figs. 6 and 7, the IRFs oscillate. In response to the labour productivity shock, revenues rise on impact $h = 1$. But by year two $h = 2$, the IRFs of high revenue elasticity firms are lower than low revenue elasticity firms, i.e., $\beta^2_{1,\zeta} < 0$ and $\beta^2_{1,UQ} - \beta^2_{1,LQ} < 0$. Appendix Tables A.3 and A.4 present the numerical values of the coefficient over time. Following a one percent shock to labour productivity, a firm's revenue with mid-range elasticity ($UQ_{j,t} = 0$ and $LQ_{j,t} = 0$) rises 0.6% initially but declines 0.2% in the next period. Specifically, $\beta^1_0 = 0.557$ and $\beta^2_0 = -0.242$ in regression equation (37). When firms lower their revenue after two years following positive productivity shocks, a high revenue elasticity firm reduces its revenue more than a low revenue elasticity firm, which results in the negative coefficient of $h = 2$ in Figs. 6 and 7. The dynamic shows that high revenue elasticity firms are more sensitive in both positive and negative directions. And, a labour productivity shock – proxying TFPR – leads to an initial burst in revenues followed by a contraction in revenues, which is marginally less severe. This could occur if firms bring-forward sales following the revenue productivity shock, raising capacity utilization and consequently depreciation, leading to weaker sales in the following period.

### 5.2.2. *Aggregate productivity and uncertainty shocks*

Figs. 8–10 represent how revenue elasticity shapes the impulse response function (IRF) following aggregate changes: total factor productivity and various economic policy uncertainty shocks.[12] See Appendix Tables A.5 and A.6 for the number of estimated coefficients (IRFs) and their standard errors.

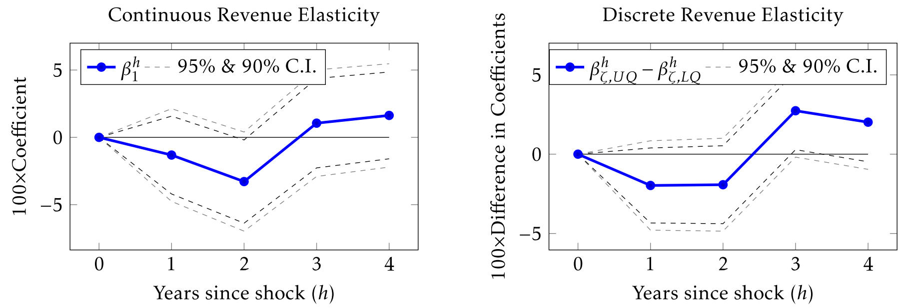
Fig. 8 displays how the impulse response functions (IRFs) to the aggregate productivity shocks depend on a firm's revenue elasticity from estimating regression equations (36) and (37) with a continuous and discrete measure of revenue elasticity, respectively. Consistent with our hypothesis, high revenue elasticity firms' sales respond more to aggregate productivity shocks than lower revenue elasticity firms. Notably, the coefficients in Fig. 8 are higher than the coefficients for the firm-level shocks in Figs. 6 and 7. In equation (23), each firm's de-meaned labour productivity shock has a negligible effect on factor prices $\Delta W$ because we present a partial equilibrium framework and an individual firm has no factor market power. However, aggregate productivity changes will have general equilibrium effects on factor prices that are not present with firm-level idiosyncratic shocks. This general equilibrium channel could account for the higher firm responses to aggregate shocks compared with its responses to idiosyncratic firm-level shocks.

[12] Because the aggregate shocks are identical across firms and the regressions include year-by-industry fixed effects, we eliminate $\beta^h_0 \text{shock}_t$ in regression equations (36) and (37).




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 10.** IRFs to fiscal policy (taxes/spending) uncertainty index.




(a) Regression Equation (36) (b) Regression Equation (37)

**Fig. 11.** IRFs to monetary policy uncertainty index.

Figs. 9–11 show how the IRFs following a change in economic policy uncertainty vary with revenue elasticity. An increase in uncertainty index serves as a negative revenue productivity shock because elevated uncertainty might diminish customer demands and make firm operations challenging.

The estimated coefficients in equations (36) and (37) show the effect on revenue conditional on a firm's revenue elasticity following an increase in economic policy uncertainty on impact ($h = 1$) and after one, two and three years ($h = 2, 3, 4$). The plots demonstrate that, compared to firms with lower revenue elasticity, those with higher revenue elasticity experience a greater decline in revenues as a reaction to a negative aggregate shock. After two years, the impact normally disappears. Additionally, the results of the specific policy uncertainty in Figs. 10 and 11 are typically less substantial and less statistically significant than the results of the overall economic policy uncertainty in Fig. 9.

## 6. Conclusion

We analyse the effect of firm-level revenue elasticities on business cycle dynamics. We focus on revenue elasticities because they are simple to obtain at the firm level, but are understudied relative to the related concepts of price markups and output elasticities. Firm profit can increase through market power or declining output elasticity (returns to scale). These channels are from the demand (price) and supply (output) side. Revenue fluctuations are closely related to profitability. This implies that the business cycle properties of firms and their heterogeneity could be accounted for by one metric (a combination of markups and output elasticity).

We present theory which shows that higher revenue elasticity firms are more responsive to business cycle shocks than low revenue elasticity firms. We test this theoretical relationship on U.S. data and find evidence in support of the theory. Furthermore, we present empirical results on the behaviour of revenue elasticities of U.S. firms over the last three decades. In particular, we show a secular decline in average revenue elasticity. According to our theory this implies declining amplification of business cycle shocks. Overall, our paper stresses that the complexities of identifying markups from revenue data need not be an obstacle to making macroeconomic inference, and empirical trends in revenue data are consistent with existing analyses of markups and market power.

## Appendix A. Mathematical appendix

### A.1. Derivation of returns to scale and cost function homogeneity

The inverse variable cost elasticity from (7) is equal to the ratio of average variable cost to marginal cost, which in turn is equal to the output elasticity. This is commonly defined as *returns to scale*. Denote the average variable cost by $\mathrm{AVC}_j = WX_j/Q_j$. Then, we obtain

$$\frac{\mathrm{AVC}_j}{\mathrm{MC}_j} = \left(\frac{\partial C_j}{\partial Q_j}\frac{Q_j}{WX_j}\right)^{-1} = \frac{\partial \mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j}. \tag{A.1}$$

The cost function is homogeneous of degree one in efficiency unit factor prices ($W/A_j$):

$$\frac{\partial C_j}{\partial (W/A_j)}\frac{(W/A_j)}{WX_j} = A_j X_j \frac{1}{A_j X_j} = 1. \tag{A.2}$$

The result follows from Shephard's lemma that factor demand equals the derivative of the cost function with respect to factor price $X_j = \partial C_j/\partial W_j = A_j^{-1}\partial C_j/\partial(W/A_j)$. Equations (7) and (A.2) inform our log-linearization.

#### A.1.1. Alternative derivation without cost minimization

We can derive Equations (A.1) and (A.2) directly from the production function and cost definition, without using the first-order condition from cost-minimization. From $Q_j = \mathcal{F}_j(A_j X_j)$, write the cost function as $C_j(Q_j) = WX_j = (W/A_j)\mathcal{F}_j^{-1}(Q_j)$. Then, $\partial C_j/\partial Q_j = (W/A_j)\partial \mathcal{F}_j^{-1}/\partial Q_j$. Because $\partial \mathcal{F}_j^{-1}/\partial Q_j = A_j(\partial \mathcal{F}_j/\partial X_j)^{-1}$, we obtain the second equality of equation (A.1). Similarly, $\partial C_j/\partial(W/A_j) = \mathcal{F}_j^{-1}(Q_j) = A_j X_j$, which gives equation (A.2).

### A.2. Derivation of demand and cost functions

We derive equations (20) and (19). Denote price elasticity of demand as $\theta_j \equiv -\partial \ln \mathcal{D}_j/\partial \ln P_j$. Then, the total derivative of demand function yields the demand change varies with the price change and demand shock as $\mathrm{d}\ln Q_j = -\theta_j(\mathrm{d}\ln P_j - \mathrm{d}\ln \xi_j)$. The optimal condition links the elasticity and markup as $\theta_j = (\theta_j - 1)\mu_j$. Replacing $\theta_j$ by $\mu_j$, we obtain equation (20).

Similarly, the total derivative of cost function approximation, $\gamma_j \,\mathrm{d}\ln C_j = \gamma_j \,\mathrm{d}\ln W\hat{X}_{j,t} = \mathrm{d}\ln Q_j + \gamma_j \,\mathrm{d}\ln W/A_j$, yields equation (19). Here, the variable cost function, $C_j(\cdot)$, is a homogeneous function of the first and $1/\gamma_j$-th degree with respect to $W/A_j$ and $Q_j$, respectively, where $\gamma_j = (\partial \mathcal{F}_j/\partial X_j)(X_j/Q_j)$.

### A.3. Derivation of revenue responses

We derive equations (23) and (24). The firm cost function under optimality implies that output elasticity is the ratio of average cost to marginal cost; $\gamma_j = (WX_j/Q_j)/\mathrm{MC}_j$. Therefore, the total derivative gives

$$\mathrm{d}\ln \mathrm{MC}_j = -\mathrm{d}\ln\gamma_j - \mathrm{d}\ln Q_j + \mathrm{d}\ln WX_j \tag{A.3}$$

$$= -\mathrm{d}\ln\gamma_j + \mathrm{d}\ln W - \mathrm{d}\ln A_j + \left(\frac{1}{\gamma_j} - 1\right)\mathrm{d}\ln Q_j. \tag{A.4}$$

In the second, line we use equation (19). Our markup definition is $\mu_j \equiv P_j/\mathrm{MC}_j$, therefore:

$$\mathrm{d}\ln P_j = \mathrm{d}\ln\mu_j + \mathrm{d}\ln \mathrm{MC}_j\,. \tag{A.5}$$

Combining the marginal cost change of equation (A.4) with the price change of equation (A.5), and using the revenue elasticity change ($\mathrm{d}\ln\zeta_j = \mathrm{d}\ln\gamma_j - \mathrm{d}\ln\mu_j$), we can express the price change as follows.

$$\mathrm{d}\ln P_j = -\mathrm{d}\ln\zeta_j + \mathrm{d}\ln W - \mathrm{d}\ln A_j + \left(\frac{1}{\gamma_j} - 1\right)\mathrm{d}\ln Q_j \tag{A.6}$$

Putting equation (20) into equation (A.6) and rearranging this expression yields

$$-\frac{1}{\mu_j - 1}\mathrm{d}\ln P_j = \frac{\gamma_j}{\mu_j - \gamma_j}(\mathrm{d}\ln\zeta_j - \mathrm{d}\ln W + \mathrm{d}\ln A_j) + \frac{\gamma_j - 1}{\mu_j - \gamma_j}\frac{\mu_j}{\mu_j - 1}\mathrm{d}\ln\xi_j. \tag{A.7}$$

Using the demand function of equation (20), we can rewrite the revenue change, $\mathrm{d}\ln P_j Q_j = \mathrm{d}\ln P_j + \mathrm{d}\ln Q_j$, in terms of price:

$$\mathrm{d}\ln P_j Q_j = -\frac{1}{\mu_j - 1}\mathrm{d}\ln P_j + \left(\frac{\mu_j}{\mu_j - 1}\right)\mathrm{d}\ln\xi_j \tag{A.8}$$

Putting equation (A.7) into the above equation, the revenue change becomes

$$\mathrm{d}\ln P_j Q_j = \frac{\gamma_j}{\mu_j - \gamma_j}(\mathrm{d}\ln\zeta_j - \mathrm{d}\ln W + \mathrm{d}\ln A_j) + \frac{\mu_j}{\mu_j - \gamma_j}\mathrm{d}\ln\xi_j. \tag{A.9}$$

Substituting $\zeta_j = \gamma_j/\mu_j$, we obtain the main theoretical result of our paper in equation (23). From TFPR in equation (18), equation (23) yields equation (24).

## Appendix B. An example demand system with continuum-pollak preferences

Consider the case of Neary (2016). Aggregation is simplified such that $H(x) = x$. There are quadratic sub-utilities over a unit continuum of $j$ consumption goods, called continuum-Pollak preferences: $\int_0^1 \xi_j(aQ_j - 0.5bQ_j^2)\,\mathrm{d}j$. The corresponding inverse demand function for $j$ is $P_j(Q_j) = (\xi_j/\eta)(a - bQ_j)$, thus $\mathcal{D}_j(P_j) = Q_j = b^{-1}(a - \eta P_j/\xi_j)$. Treating the Lagrange multiplier $\eta$ as an exogenous constant, yields the perceived demand curve which is linear in output. If the firm considered the true demand curve then $\eta$ is endogenous and the demand curve is not linear. The price elasticity of demand of the perceived demand curve is $-\partial \ln \mathcal{D}_j/\partial \ln P_j = (\eta/b)P_j/(\xi_j Q_j)$. Revenue elasticity is $\partial \ln \mathcal{R}_j/\partial \ln X_j = [1 - (b/\eta)\xi_j Q_j/P_j](\partial \ln \mathcal{F}_j/\partial \ln X_j)$, where the markup is the inverse of the term in square brackets.

## Appendix C. Revenue elasticity and profit share

We can also interpret revenue elasticity in terms of profits. Define profits ($\Pi$) as revenue ($P_jQ_j$) less variable costs ($WX_j$) less fixed costs ($FC_j$), so $\Pi_j = P_jQ_j - WX_j - FC_j$. Divide profits by revenue which gives the profit share in revenue as a function of variable cost and fixed cost shares in revenue: $s_{\Pi,j} = 1 - s_{C,j} - s_{FC,j}$, where $s_{x,j}$ denotes the revenue share if $x$ for firm $j$. Since revenue elasticity is the variable cost share in revenue, then revenue elasticity is equal to the profit and fixed cost share remainder: $\zeta_j = 1 - s_{\Pi,j} - s_{FC,j}$. Therefore a decline in revenue elasticity implies a rise in profit and/or fixed cost share in revenue.

## Appendix D. General equilibrium channels

Equations (23) and (24) have firm-level revenue responses on the left-hand side determined by input prices on the right-hand side. Our analysis makes a partial equilibrium assumption that input prices are given exogenously. However, it is possible that if all firms, or a significant share of firms, change their revenue this would feedback to factor prices. Hence our results could change in the presence of such general equilibrium feedback effects.

The factor price change component $\mathrm{d}\ln W$ can account for general equilibrium channels in reacting to shocks. To understand these channels, rewrite equation (24) as

$$\mathrm{d}\ln P_jQ_j = \frac{1}{1-\zeta_j}\left(1 - \zeta_j \frac{\mathrm{d}\ln W}{\mathrm{d}\ln \mathrm{TFPR}_j}\right)\mathrm{d}\ln \mathrm{TFPR}_j + \frac{\zeta_j}{1-\zeta_j}\,\mathrm{d}\ln \zeta_j. \tag{A.10}$$

With monopolistic competition all firms charge the same constant markup, an idiosyncratic revenue productivity shock disappears at the aggregate level. Therefore, factor market prices do not respond $\mathrm{d}\ln W/\mathrm{d}\ln \mathrm{TFPR}_j = 0$. However, if large firms have more market power in goods or factor markets, for example under oligopolistic competition, then idiosyncratic shocks will not disappear with aggregation. Thus, if the general equilibrium effect is $\mathrm{d}\ln W/\mathrm{d}\ln \mathrm{TFPR}_j > 0$, then the firm – which has significant market power – will adjust revenue less to a firm-specific TFPR shock. In the US economy, we expect that one firm or a group cannot change factor prices significantly in the short-run following revenue changes. Additionally, the high market power firm tends to have low revenue elasticity due to high markups. Hence, we conclude that a firm's response to idiosyncratic TFPR changes is close to $(1-\zeta_j)^{-1}$. That is, the term in brackets is one because the second term tends to zero. Given these reasons we consider the partial equilibrium assumption to be a reasonable first approximation for the US economy, but future research with richer quantitative models could determine the size of these GE feedback effects more precisely.

We predict that revenue elasticity will affect a firm's response to aggregate shocks less/more than it will affect idiosyncratic shocks due to general equilibrium channels. In contrast to idiosyncratic shocks, aggregate productivity shocks can affect factor prices. The role of revenue elasticities in propagation can therefore be written as follows.

$$\frac{\partial}{\partial \zeta_i}\left[\frac{1}{1-\zeta_j}\left(1 - \zeta_j \frac{\mathrm{d}\ln W}{\mathrm{d}\ln \mathrm{TFPR}}\right)\right] = \left(\frac{1}{1-\zeta_j}\right)^2\left(1 - \frac{\mathrm{d}\ln W}{\mathrm{d}\ln \mathrm{TFPR}}\right) \tag{A.11}$$

When a aggregate shock has a negative impact on factor prices ($\mathrm{d}\ln W/\mathrm{d}\ln \mathrm{TFPR} < 0$), the coefficient of shocks still increases with a firm's revenue elasticity, $\zeta_j$, in equation (A.10). When a aggregate shock has a positive impact on factor prices ($\mathrm{d}\ln W/\mathrm{d}\ln \mathrm{TFPR} > 0$), the negative link between the coefficient and revenue elasticity is reduced. A firm's revenue impulse response could turn negative, which is an unlikely scenario, if factor prices rise more than an aggregate productivity.

## Appendix E. Additional figures and tables

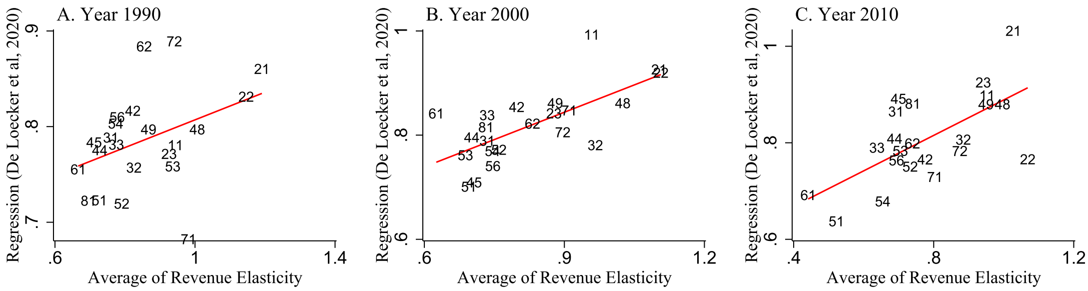



**Fig. A.1.** 1990, 2000, 2010 revenue elasticities and revenue function estimates: two-digit NAICS.

Notes: We use (Online Appendix pp. 18 De Loecker et al., 2020, Figure 12.2)'s estimated coefficients (labelled PF2) of variable input bundle where there are three inputs (variable input, capital, and overhead inputs). The results with the other specification (PF1) without overhead inputs yield similar results.

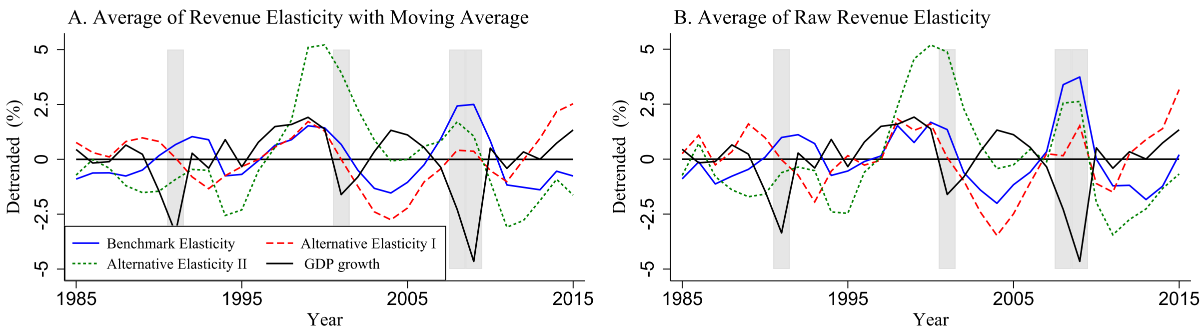



**Fig. A.2.** Detrended average revenue elasticities and GDP growth.

Notes: The detrend series are residuals of the regression of each variable on constant and time. In Panel A, we calculate an individual firm's revenue elasticity with three-year centred moving average within firms. Then, we detrend their cross-sectional average. In Panel B, we use the detrended cross-sectional average of contemporaneous firms' revenue elasticities. The shaded areas are NBER recession years.

**Table A.1**
Summary statistics.

| | Count | Mean | SD | p10 | p25 | p50 | p75 | p90 |
|---|---|---|---|---|---|---|---|---|
| **Panel A. Period: 1985–1995** | | | | | | | | |
| Sales growth (%) | 64343 | 8.58 | 43.74 | -21.62 | -4.94 | 5.40 | 19.24 | 42.94 |
| Labour productivity growth (%) | 57463 | 0.97 | 33.27 | -27.03 | -9.98 | -0.08 | 10.31 | 29.68 |
| Revenue elasticity: Benchmark | 64343 | 0.66 | 0.23 | 0.38 | 0.53 | 0.67 | 0.79 | 0.89 |
| Revenue elasticity: Alternative I | 59517 | 0.80 | 0.26 | 0.49 | 0.64 | 0.79 | 0.94 | 1.12 |
| Revenue elasticity: Alternative II | 64342 | 0.89 | 0.22 | 0.65 | 0.80 | 0.90 | 0.97 | 1.12 |
| Market share (%) | 64343 | 1.52 | 6.04 | 0.00 | 0.02 | 0.12 | 0.69 | 2.97 |
| Employment (thousand, log) | 56616 | -0.51 | 2.37 | -3.51 | -2.06 | -0.40 | 1.14 | 2.45 |
| Cash holding/Asset | 64126 | 0.13 | 0.16 | 0.01 | 0.02 | 0.07 | 0.17 | 0.34 |
| Short-term debt/Asset | 63788 | 0.23 | 34.54 | 0.00 | 0.01 | 0.04 | 0.10 | 0.23 |
| Long-term debt/Asset | 64120 | 0.20 | 0.50 | 0.00 | 0.03 | 0.15 | 0.30 | 0.45 |
| Working capital ratio | 54111 | 2.84 | 8.27 | 0.79 | 1.22 | 1.89 | 2.94 | 4.79 |
| **Panel B. Period: 1996–2005** | | | | | | | | |
| Sales growth (%) | 66235 | 12.52 | 48.65 | -20.77 | -3.62 | 7.25 | 23.04 | 52.20 |
| Labour productivity growth (%) | 58246 | 2.49 | 37.15 | -28.28 | -9.86 | 0.78 | 12.81 | 35.39 |
| Revenue elasticity: Benchmark | 66235 | 0.63 | 0.26 | 0.31 | 0.46 | 0.64 | 0.78 | 0.90 |
| Revenue elasticity: Alternative I | 56706 | 0.77 | 0.28 | 0.41 | 0.59 | 0.76 | 0.92 | 1.10 |
| Revenue elasticity: Alternative II | 66235 | 0.90 | 0.26 | 0.62 | 0.76 | 0.89 | 0.98 | 1.30 |
| Market share (%) | 66235 | 1.25 | 5.24 | 0.00 | 0.01 | 0.08 | 0.53 | 2.29 |
| Employment (thousand, log) | 57624 | -0.43 | 2.27 | -3.19 | -1.96 | -0.46 | 1.17 | 2.48 |
| Cash holding/Asset | 66136 | 0.16 | 0.20 | 0.01 | 0.03 | 0.07 | 0.23 | 0.48 |
| Short-term debt/Asset | 65971 | 0.11 | 1.59 | 0.00 | 0.00 | 0.03 | 0.08 | 0.18 |
| Long-term debt/Asset | 65920 | 0.21 | 0.43 | 0.00 | 0.02 | 0.12 | 0.30 | 0.49 |
| Working capital ratio | 52838 | 2.87 | 8.54 | 0.78 | 1.20 | 1.91 | 3.14 | 5.30 |

(*continued on next page*)

**Table A.1** (*continued*)

| | Count | Mean | SD | p10 | p25 | p50 | p75 | p90 |
|---|---|---|---|---|---|---|---|---|
| **Panel C. Period: 2006–2015** | | | | | | | | |
| Sales growth (%) | 47754 | 6.18 | 41.83 | -21.23 | -5.97 | 3.61 | 15.24 | 35.33 |
| Labour productivity growth (%) | 43989 | 2.73 | 32.25 | -21.92 | -7.58 | 1.22 | 10.76 | 28.19 |
| Revenue elasticity: Benchmark | 47754 | 0.60 | 0.28 | 0.22 | 0.41 | 0.62 | 0.78 | 0.89 |
| Revenue elasticity: Alternative I | 39574 | 0.74 | 0.30 | 0.35 | 0.56 | 0.74 | 0.91 | 1.09 |
| Revenue elasticity: Alternative II | 47754 | 0.87 | 0.26 | 0.58 | 0.73 | 0.87 | 0.96 | 1.21 |
| Market share (%) | 47754 | 1.65 | 6.18 | 0.00 | 0.01 | 0.11 | 0.75 | 3.27 |
| Employment (thousand, log) | 43594 | -0.27 | 2.48 | -3.32 | -1.89 | -0.17 | 1.51 | 2.82 |
| Cash holding/Asset | 47717 | 0.17 | 0.20 | 0.01 | 0.03 | 0.09 | 0.24 | 0.48 |
| Short-term debt/Asset | 47688 | 0.12 | 2.16 | 0.00 | 0.00 | 0.02 | 0.05 | 0.13 |
| Long-term debt/Asset | 47449 | 0.22 | 0.59 | 0.00 | 0.02 | 0.13 | 0.31 | 0.51 |
| Working capital ratio | 36577 | 3.37 | 56.85 | 0.79 | 1.21 | 1.90 | 3.08 | 5.23 |

Notes: The market share is in the three-digit NAICS industry. We replace employment by one for zero. We keep the following observations in the Compustat database. (a) No major mergers flag: Comparability status (COMPST) does not equal to AB, (b) Country ISO 3 digit code (LOC) equals to USA, (c) Currency ISO 3 digit code (CURCD) equals USD, (d) Real sales are higher than 0.1M.

**Table A.2**
Estimated coefficients in regression Equation (34).

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t}Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h =$ | 1 | 2 | 3 | 4 |
| **Aggregate GDP Growth—Fig. 5b** | | | | |
| $\text{Shock}_t \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | -0.284 | -0.179 | 0.095 | 0.030 |
| | (0.177) | (0.166) | (0.165) | (0.161) |
| $\text{Shock}_t \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | $0.493^{**}$ | 0.207 | -0.224 | -0.062 |
| | (0.243) | (0.245) | (0.238) | (0.260) |
| Observations | 87484 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.287 | 0.257 | 0.233 |

Notes: All regressions include other controls, firm and time-industry fixed effects. Standard errors are clustered at the firm-level and are presented in parentheses. $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$.

**Table A.3**
Estimated coefficients in regression equation (36) with firm-level shocks.

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t}Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h =$ | 1 | 2 | 3 | 4 |
| **Panel A. Firm-Level Labour Productivity Shock—Fig. 6a** | | | | |
| $\text{Shock}_{j,t}$ | $0.575^{***}$ | $-0.211^{***}$ | -0.097 | -0.069 |
| | (0.067) | (0.059) | (0.062) | (0.059) |
| $\text{Shock}_{j,t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | $0.198^{***}$ | $-0.040^{*}$ | -0.036 | -0.013 |
| | (0.023) | (0.022) | (0.024) | (0.024) |
| Observations | 87481 | 78206 | 70116 | 63001 |
| R-squared | 0.488 | 0.328 | 0.265 | 0.240 |
| **Panel B. Firm-Level Corrected Labour Productivity Shock—Fig. 7a** | | | | |
| $\text{Shock}_{j,t}$ | $0.611^{***}$ | $-0.184^{***}$ | -0.091 | -0.079 |
| | (0.066) | (0.063) | (0.064) | (0.061) |
| $\text{Shock}_{j,t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | $0.153^{***}$ | $-0.111^{***}$ | $-0.044^{*}$ | 0.004 |
| | (0.023) | (0.024) | (0.025) | (0.026) |
| Observations | 83155 | 74509 | 66915 | 60175 |
| R-squared | 0.505 | 0.321 | 0.265 | 0.242 |

Notes: All regressions include other controls, firm and time-industry fixed effects. Standard errors are clustered at the firm-level and are presented in parentheses. $^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$.

**Table A.4**
Estimated coefficients in regression equation (37) with firm-level shocks.

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t}Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h =$ | 1 | 2 | 3 | 4 |
| **Panel A. Firm-Level Labour Productivity Shock—Fig. 6b** | | | | |
| $\text{Shock}_{j,t}$ | $0.557^{***}$ | $-0.242^{***}$ | $-0.038^{***}$ | $-0.030^{***}$ |
| | (0.017) | (0.012) | (0.011) | (0.011) |
| $\text{Shock}_{j,t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | -0.028 | $0.037^{*}$ | 0.002 | 0.020 |
| | (0.030) | (0.022) | (0.019) | (0.021) |
| $\text{Shock}_{j,t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | $0.182^{***}$ | -0.021 | -0.033 | -0.011 |
| | (0.025) | (0.020) | (0.022) | (0.022) |
| Observations | 87484 | 78206 | 70116 | 63001 |
| R-squared | 0.466 | 0.325 | 0.264 | 0.239 |

**Table A.4** (*continued*)

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t} Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h=$ | 1 | 2 | 3 | 4 |
| **Panel B. Firm-Level Corrected Labour Productivity Shock—Fig. 7b** | | | | |
| $\text{Shock}_{j,t}$ | 0.620*** | -0.166*** | -0.029*** | -0.051*** |
| | (0.016) | (0.012) | (0.011) | (0.011) |
| $\text{Shock}_{j,t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | -0.017 | 0.054** | 0.020 | 0.015 |
| | (0.029) | (0.022) | (0.019) | (0.020) |
| $\text{Shock}_{j,t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | 0.152*** | -0.094*** | -0.033 | 0.019 |
| | (0.025) | (0.022) | (0.024) | (0.025) |
| Observations | 83158 | 74509 | 66915 | 60175 |
| R-squared | 0.487 | 0.316 | 0.264 | 0.242 |

Notes: All regressions include other controls, firm and time-industry fixed effects. Standard errors are clustered at the firm-level and are presented in parentheses. $^{*}$ $p<0.10$, $^{**}$ $p<0.05$, $^{***}$ $p<0.01$.

**Table A.5**
Estimated coefficients in regression equation (36) with aggregate-level shocks.

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t} Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h=$ | 1 | 2 | 3 | 4 |
| **Panel A. Aggregate TFP Shock—Fig. 8a** | | | | |
| $\text{Shock}_{t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | 0.359 | 2.598*** | -0.436 | -0.992 |
| | (0.723) | (0.687) | (0.810) | (0.781) |
| Observations | 87481 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.289 | 0.258 | 0.233 |
| **Panel B. News Based Economic Policy Uncertainty—Fig. 9a** | | | | |
| $\text{Shock}_{j,t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | -0.033* | -0.053*** | -0.036* | -0.018 |
| | (0.019) | (0.020) | (0.021) | (0.021) |
| Observations | 87481 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.289 | 0.258 | 0.233 |
| **Panel C. Fiscal Policy (Taxes/Spending) Uncertainty Index—Fig. 10a** | | | | |
| $\text{Shock}_{t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | -0.023* | -0.035*** | -0.009 | -0.009 |
| | (0.012) | (0.013) | (0.013) | (0.013) |
| Observations | 87481 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.289 | 0.258 | 0.233 |
| **Panel D. Monetary Policy Uncertainty Index—Fig. 11a** | | | | |
| $\text{Shock}_{t} \times \ln$ Revenue $\text{elasticity}_{j,t}$ | -0.013 | -0.033* | 0.011 | 0.016 |
| | (0.018) | (0.019) | (0.020) | (0.020) |
| Observations | 87481 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.289 | 0.258 | 0.233 |

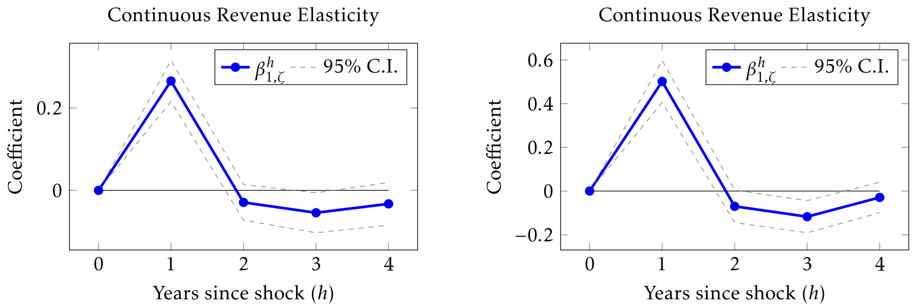
Notes: All regressions include other controls, firm and time-industry fixed effects. Standard errors are clustered at the firm-level and are presented in parentheses. $^{*}$ $p<0.10$, $^{**}$ $p<0.05$, $^{***}$ $p<0.01$.

**Table A.6**
Estimated coefficients in regression equation (37) with aggregate-level shocks.

| | Revenue Growth b/w $t+h$ and $t$: $\Delta^h \ln P_{j,t} Q_{j,t}$ | | | |
|---|---|---|---|---|
| Years since shock: $h=$ | 1 | 2 | 3 | 4 |
| **Panel A. Aggregate TFP Shock—Fig. 8b** | | | | |
| $\text{Shock}_{t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | 0.219 | -0.602* | -0.278 | -0.079 |
| | (0.344) | (0.329) | (0.344) | (0.343) |
| $\text{Shock}_{t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | 0.574 | 0.656 | -0.434 | -0.578 |
| | (0.493) | (0.473) | (0.528) | (0.543) |
| Observations | 87484 | 78851 | 71195 | 64390 |
| R-squared | 0.232 | 0.287 | 0.257 | 0.233 |
| **Panel B. News Based Economic Policy Uncertainty—Fig. 9b** | | | | |
| $\text{Shock}_{t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | 0.011 | 0.015 | 0.002 | -0.002 |
| | (0.011) | (0.011) | (0.011) | (0.010) |
| $\text{Shock}_{t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | -0.026* | -0.031** | 0.001 | -0.007 |
| | (0.015) | (0.015) | (0.014) | (0.016) |
| Observations | 87484 | 78851 | 71195 | 64390 |
| R-squared | 0.233 | 0.287 | 0.257 | 0.233 |
| **Panel C. Fiscal Policy (Taxes/Spending) Uncertainty Index—Fig. 10b** | | | | |
| $\text{Shock}_{t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | -0.003 | 0.003 | -0.002 | 0.001 |
| | (0.006) | (0.007) | (0.007) | (0.007) |
| $\text{Shock}_{t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | -0.022** | -0.014 | 0.002 | -0.006 |
| | (0.009) | (0.009) | (0.009) | (0.010) |
| Observations | 87484 | 78851 | 71195 | 64390 |
| R-squared | 0.232 | 0.287 | 0.257 | 0.233 |
| **Panel D. Monetary Policy Uncertainty Index—Fig. 11b** | | | | |
| $\text{Shock}_{t} \times$ Lower Quartile Revenue $\text{elasticity}_{j,t}$ | 0.020** | -0.001 | -0.016 | -0.013 |
| | (0.009) | (0.010) | (0.010) | (0.010) |
| $\text{Shock}_{t} \times$ Upper Quartile Revenue $\text{elasticity}_{j,t}$ | 0.001 | -0.020 | 0.012 | 0.008 |
| | (0.013) | (0.013) | (0.013) | (0.014) |
| Observations | 87484 | 78851 | 71195 | 64390 |
| R-squared | 0.232 | 0.287 | 0.257 | 0.233 |

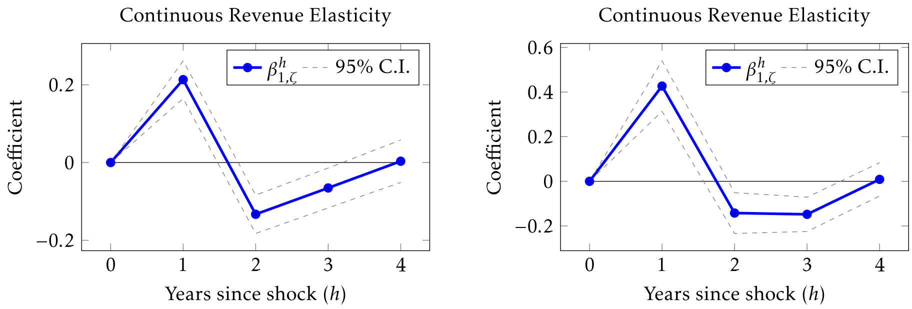
Notes: All regressions include other controls, firm and time-industry fixed effects. Standard errors are clustered at the firm-level and are presented in parentheses. $^{*}$ $p<0.10$, $^{**}$ $p<0.05$, $^{***}$ $p<0.01$.

## Appendix F. IRFs with alternative revenue elasticity measures



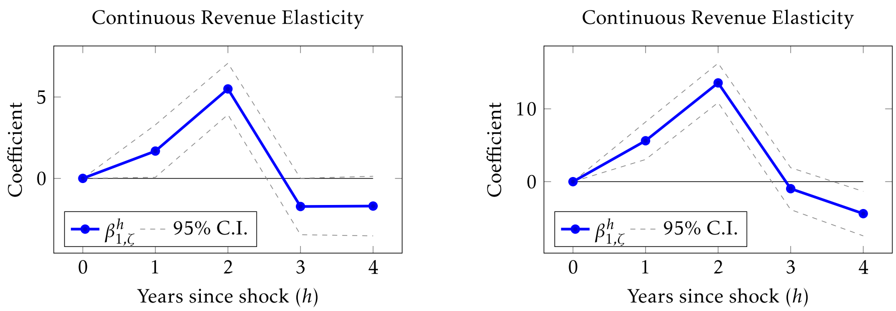

(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.3.** IRFs to firm-level labour productivity shock with alternative elasticities.



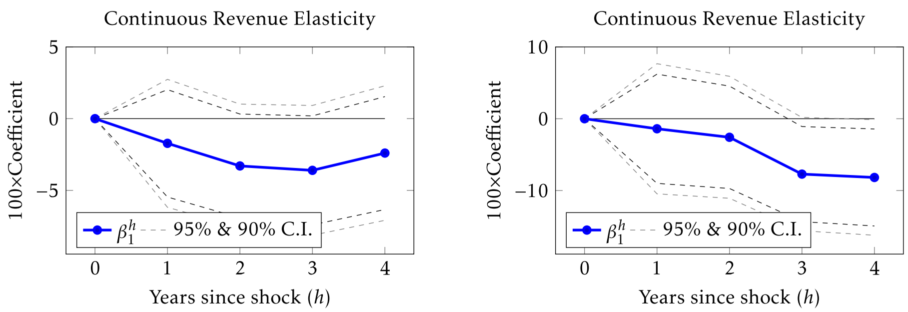

(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.4.** IRFs to firm-level corrected labour productivity shock with alternative elasticities.



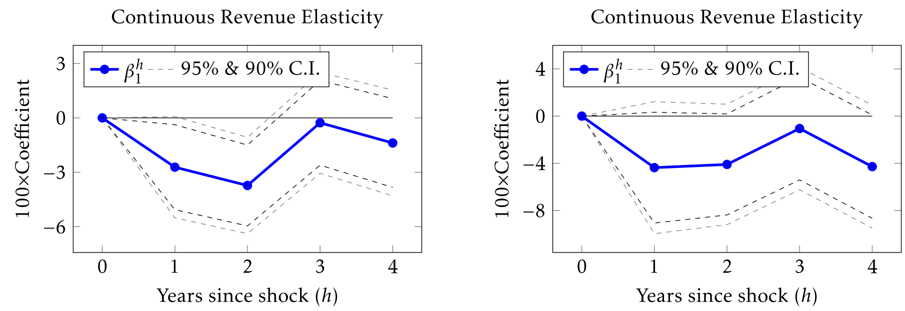

(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.5.** IRFs to aggregate TFP shock with alternative elasticities.



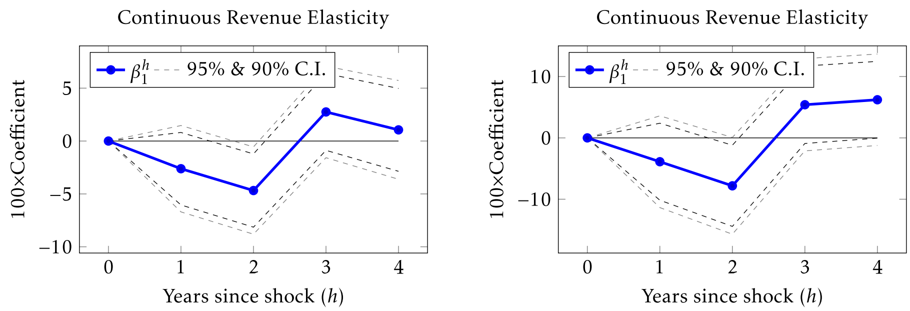

(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.6.** IRFs to news based economic policy uncertainty index with alternative elasticities.




(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.7.** IRFs to fiscal policy (taxes/spending) uncertainty index with alternative elasticities.




(a) Alternative Elasticity I (b) Alternative Elasticity II

**Fig. A.8.** IRFs to monetary policy uncertainty index with alternative elasticities.

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