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A Multi-Objective q-Rung Orthopair Fuzzy Programming Approach to Heterogeneous Group Decision Making

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Abstract

In allusion to heterogeneous multi-criteria group decision making (MCGDM) problems with incomplete weights and q-rung orthopair fuzzy (q-ROF) truth degrees, where many kinds of criteria values, i.e., crisp values, intervals, trapezoidal fuzzy values, hesitant fuzzy values and q-ROF values (q-ROFVs), and multiple types of interactions exist, i.e., positive synergetic interactions, negative synergetic interactions and independence, a novel multi-objective q-ROF programming approach is proposed. In particular, in order to globally capture the interactions among criteria, Choquet-based relative closeness degrees are developed based on the technique for order performance by similarity to ideal solution (TOPSIS) and the Choquet integral. Then, the q-ROF Choquet-based group consistency index (q-ROFCGCI) and the q-ROF Choquet-based group inconsistency index (q-ROFCGII) are defined. Next, to derive optimal 2-additive fuzzy measures on the criteria set and optimal experts' weights, a new multi-objective q-ROF mathematical programming model is established by minimizing the q-ROFCGII and maximizing the q-ROFCGCI. Subsequently, an algorithm based on the adaptive non-dominated sorting genetic algorithm III (A-NSGA-III) is designed to solve the established model. Afterwards, the Choquet-based overall relative closeness degrees of the alternatives is used to obtain their preferred ordering. Finally, the effectiveness and advantage of the proposed approach is verified using four real cases concerning the evaluation of social commerce.

Key words: Heterogeneous information; Evolutionary computation; 2-additive fuzzy measure; q-Rung orthopair fuzzy values; Multi-criteria group decision making.

1. Introduction

Social commerce (s-commerce) refers to the use of social media and Web 2.0 tools to enhance the interactions of individuals on the Internet in order to support consumers' acquisition of products and services (Doha et al., 2019). In recent years, with the increasing maturity of s-commerce, increasingly more consumers, especially the new generation, have taken part in s-commerce (Hu et al., 2019). However, due to the virtual and anonymous nature of s-commerce, consumers cannot see merchants and commodities (Scarle et al., 2012). There is also a time asymmetry between payment and shipment, which makes it possible for merchants to commit fraud during transactions (Chen et al., 2008). For these reasons, consumers may suffer from high credit risks, which would negatively affect their purchase intention. With the help of the s-commerce credit evaluation of merchants, consumers can analyse a merchant's credit status effectively to select a desirable merchant and reduce credit risks. Therefore, it is necessary and meaningful to evaluate the credit level of merchants. The s-commerce credit evaluation of merchant problem commonly involves several experts and multiple assessment criteria (Yong, 2012), such as product quality, information quality, service quality, delivery time, product price and consumer identity; thus, it is essentially a multi-criteria group decision making (MCGDM) problem (Qiyas et al., 2021) . Because the credit evaluation problem includes both qualitative and quantitative criteria, the assessments of these criteria might cover various types of information, such as crisp values, intervals, trapezoidal fuzzy values (TrFVs), intuitionistic fuzzy values (IFVs), Pythagorean fuzzy values (PFVs), hesitant fuzzy values (HFVs) and q-rung orthopair fuzzy values (q-ROFVs). Therefore, the MCGDM to address in this context is a heterogeneous MCGDM.

There are usually two different methodology approaches to tackle heterogeneous MCGDM problems: accounting for the preference information over pairwise comparisons of alternatives (Wan et al., 2017, 2020; Zhang et al., 2016) and not accounting for such preference information (Liang et al., 2020; Tang et al., 2022b). This second methodology approach uses only the multiple kinds of criteria information and omits pairwise comparisons of alternatives, whereas the first one takes advantage of both. Thus, the first methodology approach is now gaining increased attention in the area of heterogeneous decision making, with extensive linear programming technique for multidimensional analysis of preference

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(LINMAP) approaches being most representative for managing heterogeneous decision-making problems (Chen, 2019; Herrera-Viedma et al., 2021; Li & Wan, 2013, 2014; Wan & Dong, 2015; Wan & Li, 2013a,b, 2015; Wan et al., 2017, 2020; Wu et al., 2018; Zhang et al., 2016).

LINMAP was proposed by Srinivasan & Shocker (1973), and it is one of the most well-known typical heterogeneous MCGDM methods present in modern decision theory. Utilizing decision information on the pairwise comparisons of alternatives and on the criteria values, it can objectively derive the optimal weights and accurately generate the best compromise alternative. Based on the traditional LINMAP, Zhang et al. (2016) and Li & Wan (2013, 2014) developed fuzzy mathematical programming approaches to deal with heterogeneous decision-making problems; Wan & Li (2013a,b) investigated IF programming methods for heterogeneous MCGDM with IF truth degrees; Wan & Dong (2015) and Wan & Li (2015) explored interval-valued IF (IVIF) programming methods for heterogeneous decision making with IVIF truth degrees; Wan et al. (2017) presented an HF mathematical programming method for hybrid MCGDM with HF truth degrees; and Wan et al. (2020) proposed a prospect theory-based approach to handle heterogeneous MCGDM with hybrid truth degrees of alternative comparisons.

The existing LINMAP approaches, although effective and applicable for solving heterogeneous MCGDM problems, suffer from the following limitations.

- Although the above referenced studies have extended LINMAP to a variety of fuzzy environments, including IFVs, IVIFVs, PFVs and HFVs, they cannot handle some special cases. For instance, a decision maker (DM) assessment value $\langle 0.7, 0.9 \rangle$ is not covered by IFV and PFV because $0.7 + 0.9 = 1.6 > 1$ and $0.7^2 + 0.9^2 = 1.3 > 1$, respectively. However, this value is covered by q-ROFV when $q = 4$ since $0.7^4 + 0.9^4 = 0.8962 < 1$. It is obvious that q-ROFVs include extra uncertainties that IFVs and PFVs do not, which means that they are able to adapt to higher levels of uncertainty. Therefore, extending the LINMAP to accommodate q-ROF information is of great significance for scientific research and practical applications.

- These existing LINMAP approaches assume independence of criteria and, therefore, they cannot capture the interactive characteristics among dependent criteria. There may be a situation where complementary, redundant, or independent features exist among criteria. For example, when assessing the s-commerce credit of merchants, consumer approval and delivery time can be considered as redundant criteria, while product price and product quality can be regarded as complementary criteria. Consequently, the existing LINMAP approaches cannot address this kind of decision-making problem. Fortunately, the Choquet integral (Grabisch, 1996) can capture the complex relationship among criteria since it is based on the 2-additive fuzzy measure (2AFM) (Grabisch, 1997) or the Banzhaf index (Marichal, 2000), which is a powerful tool to model the interaction or independence of criteria (Liu et al., 2021). Therefore, we shall combine the LINMAP with Choquet integral to overcome this issue.

- The following approaches (Chen, 2019; Wan & Dong, 2015; Wan & Li, 2013a; Wan et al., 2017, 2020; Zhang et al., 2016) are based on the assumption that the weights of experts are neglected, completely unknown or already known. Due to the complexity and uncertainty of decision-making problems and the inherent subjective nature of human thinking, It is common that experts' weights are partially known in real heterogeneous MCGDM (Zhang et al., 2015). In such a case, the above methods are inconsistent with the real importance of experts because they do not capture their valuable subjective judgement. Additionally, it is difficult for these methods to provide a clear illustration of the outcomes (Liu & Hu, 2015). To guarantee the accuracy and interpretability of a decision result, it is necessary to take advantage of the provided partial information on experts' weights.

- The following approaches (Chen, 2019; Li & Wan, 2013, 2014; Wan & Dong, 2015; Wan & Li, 2013a,b, 2015; Wan et al., 2017, 2020) transform multiple objectives into a single objective by dimension reduction methods and then optimize the single objective. The shortcomings of these approaches are various: the subjectivity involved in dividing the hierarchy; the assignment of different weights to each objective and the trade-off relationships among the objectives are neglected, which result in less reliable optimization outcomes (Deb, 2001). In fact, these multi-objective programming models can be directly solved using an intelligent optimization algorithm to derive the Pareto set including many non-dominant optimal solutions. Accordingly, DMs select the desirable solution according to their preferences. It is worth mentioning that the adaptive non-dominated sorting genetic algorithm III (A-NSGA-III) (Jain & Deb, 2013) is one of most representative intelligent optimization algorithms. It can simultaneously optimize many objectives using optimal search in the objective space (Jain & Deb, 2013). Consequently, to derive a more reasonable solution, A-NSGA-III (Jain & Deb, 2013) will be used to directly solve the many-objective programming model.

All of the aforementioned deficiencies limit the application of the extensive LINMAP methods. To address these issues, based on the above analysis, a novel multi-objective q-ROF programming approach is created for heterogeneous MCGDM with q-ROF truth degrees and incomplete weight information. First, q-ROFVs are used to express fuzzy truth degrees of alternatives comparisons; the q-ROFVs, HFVs, TrFVs, intervals and crisp values are used to describe criteria values. Then, borrowing the ideas of the Choquet integral (Xu et al., 2021) and technique for order performance by similarity to ideal solution (TOPSIS) (Liu & Zhang, 2013), the Choquet-based relative closeness degree is developed. Next, the q-ROF Choquet-based group consistency index (q-ROFCGCI) and q-ROF Choquet-based group inconsistency index (q-ROFCGII) are defined. Meanwhile, by minimizing the q-ROFCGII and maximizing the q-ROFCGCI, a new multi-objective q-ROF mathematical programming model is constructed to accurately derive weights of experts and criteria. Based on the score and accuracy functions of q-ROFVs, the q-ROF programming model is transformed into a many-objective programming model. Afterwards, the A-NSGA-III (Jain & Deb, 2013) is applied to solve the established model to derive the Pareto set. Accordingly, the desirable solution, as the weighting result from the last Pareto set, is selected using TOPSIS (Liu & Zhang, 2013). Furthermore, the preferred ordering of alternatives is derived by computing the Choquet-based overall relative closeness degrees. In this way, a new multi-objective q-ROF programming method is put forward for heterogeneous MCGDM problems.

The main contributions and novelty of this study are:

- Formulating the truth degrees of alternatives comparisons using q-ROFVs for the first time, which accurately and flexibly addresses MCGDM problems that involve a high degree of uncertainty.
- Proposing a Choquet-based relative closeness degree based on the Choquet integral (Grabisch, 1996) and TOPSIS (Liu & Zhang, 2013), which globally captures the positive synergetic interactive, negative synergetic interactive and independent characteristics of the criteria.
- Introducing a new multi-objective q-ROF programming model based on the LINMAP (Srinivasan & Shocker, 1973) and then designing its solution algorithm using the A-NSGA-III (Jain & Deb, 2013), which accurately derives the optimal weights of criteria and experts.
- Developing a multi-objective q-ROF programming method to handle heterogeneous MCGDM problems concerning the evaluation of s-commerce that overcomes the weaknesses of previous decision-making approaches (Khan, 2019; Liu et al., 2020; Peng & Yang, 2016; Wan & Dong, 2015; Wan & Li, 2013a; Wan et al., 2020). Relative to these previous methods, the main advantages of the developed MCGDM approach are that it flexibly expresses decision information, globally captures the interactions among criteria, and also objectively determines weighting vectors.

The remainder of this article is presented as follows. Section 2 reviews some concepts regarding the q-ROF set (q-ROFS), the HF set (HFS), TrFVs and the Choquet integral. Section 3 formulates the heterogeneous MCGDM problem with incomplete and interactive conditions and describes a resolution process for it. In Section 4, according to the resolution process, a multi-objective q-ROF programming model is constructed, and then its solution is obtained based on the A-NSGA-III. In Section 5, the applicability and advantages of the proposed method are explained using four application cases. Section 6 concludes this article.

2. Preliminaries

In this section, we will recall some basic concepts regarding q-ROFSs, HFSs, TrFVs and the Choquet integral that shall be used later in the article.

2.1. q-ROFS

Definition 2.1 (Yager (2016)). *A q-ROFS Q in a fixed set Y is denoted by*

$$Q = \{\langle y, (g_Q(y), f_Q(y))\rangle | y \in Y\}, \tag{1}$$

where $g_Q(y)$ and $f_Q(y)$ represent the membership degree (MD) and the non-membership degree (NMD) of element $y \in Y$ to the q-ROFS Q, respectively, with the restrictions that $g_Q(y) : Y \longrightarrow [0, 1]$, $f_Q(y) : Y \longrightarrow [0, 1]$, $0 \le (g_Q(y))^q + (f_Q(y))^q \le 1$ and $q \ge 1$. The degree of indeterminacy of element $y \in Y$ belonging to the q-ROFS Q is $\pi = (1 - (g_Q(y))^q - (f_Q(y))^q)^{1/q}$. For convenience, $Q = \langle g, f\rangle$ is called a q-ROFV.

IFV and PFV (Lin et al., 2021a; Meng et al., 2022) are special cases of q-ROFV, and their space ranges are displayed in Fig.1.

Definition 2.2 (Liu & Wang (2018)). *It is assumed that $Q_1 = \langle t_1, f_1\rangle$ and $Q_2 = \langle t_2, f_2\rangle$ are two q-ROFVs. Their algebraic operations are*

$$Q_1 \oplus Q_2 = \langle((g_1)^q + (g_2)^q - (g_1 g_2)^q)^{1/q}, f_1 f_2\rangle, \tag{2}$$

$$\lambda Q_1 = \langle(1 - (1 - (g_1)^q)^\lambda)^{1/q}, (f_1)^\lambda\rangle. \tag{3}$$

To rank q-ROFVs, the score and accuracy functions of q-ROFV $Q_1 = \langle g_1, f_1\rangle$ were defined as $S(Q_1) = (g_1)^q - (f_1)^q$ and $A(Q_1) = (g_1)^q + (f_1)^q$, respectively (Liu & Wang, 2018).

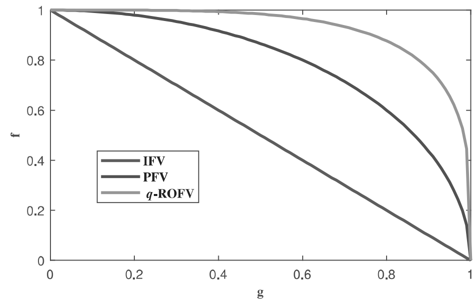

Fig. 1. Comparison of space of IFV, PFV and q-ROFV

Definition 2.3 (Liu & Wang (2018)). *It is assumed that Q_1 and Q_2 are two q-ROFVs. Then,*

$$\begin{aligned}&1)\ \textit{If}\ S(Q_1) > S(Q_2), \textit{then}\ Q_1 > Q_2;\\ &2)\ \textit{If}\ S(Q_1) = S(Q_2), \textit{then}\\ &\quad a)\ \textit{if}\ A(Q_1) > A(Q_2), \textit{then}\ Q_1 > Q_2;\\ &\quad b)\ \textit{if}\ A(Q_1) = A(Q_2), \textit{then}\ Q_1 = Q_2.\end{aligned}$$

Based on the Euclidean distance of IFVs (Szmidt & Kacprzyk, 2000), the Euclidean distance between q-ROFVs $Q_1 = \langle g_1, f_1\rangle$ and $Q_2 = \langle g_2, f_2\rangle$ is defined as

$$d_q(Q_1, Q_2) = \sqrt[2]{\frac{1}{2}(|g_1^q - g_2^q|^2 + |f_1^q - f_2^q|^2 + |\pi_1^q - \pi_2^q|^2)}. \tag{4}$$

2.2. HFS

Definition 2.4 (Xia & Xu (2011)). *An HFS D in a fixed set Y is denoted by*

$$D = \{\langle y, h_D(y)\rangle | y \in Y\}, \tag{5}$$

where $h_D(y)$ represents a set of values in $[0, 1]$, denoting the possible membership degrees of element $y \in Y$ to the HFS D. For convenience, $h = \{\gamma_1, \gamma_2, \cdots, \gamma_\iota\}$ is called an HFV, where $0 \le \gamma_1 \le \gamma_2 \le \cdots \le \gamma_\iota \le 1$.

The number of values in different HFVs may be different. To deal with this case, Xu & Xia (2011) put forward the following method to normalize HFVs: let h be an HFV; it can be normalized by adding values. The added values are defined as $\overline{\gamma} = \varphi\gamma_1 + (1 - \varphi)\gamma_\iota$, where φ $(0 \le \varphi \le 1)$ is a parameter provided by the DM according to his/her risk preference. Let $\varphi = 0.5$ in this article. To derive the Euclidean distance between HFVs h_1 and h_2, Xu & Xia (2011) further propose the expression

$$d_h(h_1, h_2) = \sqrt[2]{\frac{1}{\iota}\sum_{\kappa=1}^{\iota} |\gamma_{\kappa 1} - \gamma_{\kappa 2}|^2}, \tag{6}$$

where $\#h_1$ and $\#h_2$ denote the cardinalities of the sets h_1 and h_2, respectively, $\iota = \max\{\#h_1, \#h_2\}$; $\gamma_{\kappa 1}$ and $\gamma_{\kappa 2}$ denote the $\kappa-$th values in h_1 and h_2, respectively. It should be noted that HFVs need to be normalized in advance for the computation of their distance measure.

2.3. TrFV

It is assumed that $S = (s_1, s_2, s_3, s_4)$ is a TrFV, whose membership function (Delgado et al., 1998) can be written as

$$u(y) = \begin{cases} (y - s_1)/(s_2 - s_1), & \text{if } s_1 \le y < s_2\\ 1, & \text{if } s_2 \le y \le s_3\\ (s_4 - y)/(s_4 - s_3), & \text{if } s_3 < y \le s_4\\ 0, & \text{if } y > s_4 \text{ or } y < s_1\end{cases} \tag{7}$$

where $s_1 \le s_2 \le s_3 \le s_4$. When $s_1 = s_2 = s_3 = s_4$, the TrFV S becomes a crisp value. When $s_1 = s_2$ and $s_3 = s_4$, the TrFV S becomes an interval value.

It is assumed that $S = (s_1, s_2, s_3, s_4)$ and $\zeta = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ are two TrFVs. Their Euclidean distance (Wan & Li, 2013a) is

$$d(S, \zeta) = \sqrt[2]{\frac{1}{6}((s_1 - \varsigma_1)^2 + 2(s_2 - \varsigma_2)^2 + 2(s_3 - \varsigma_3)^2 + (s_4 - \varsigma_4)^2)}. \tag{8}$$

2.4. 2AFM and Choquet integral

Grabisch (1997) proved the following theorem to obtain a 2AFM.

Theorem 2.1 (Grabisch (1997)). *Given the set $C = \{c_1, c_2, \cdots, c_n\}$, μ is a 2AFM on C if the following restrictions are verified:*

1) $\mu(\{c_j\}) \geq 0, \forall c_j \in C$;

2) $\sum_{\{c_j,c_q\}\subseteq C} \mu(\{c_j, c_q\}) - (n-2) \sum_{\{c_j\}\subseteq C} \mu(\{c_j\}) = 1$;

3) $\sum_{\{c_j\}\subseteq B \backslash c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \geq (|B|-2)\mu(\{c_q\}), \forall B \subseteq C$ *with* $c_q \in B$ *and* $|B| \geq 2$.

Although $\mu(B)$ $(B \subseteq C)$ could be regarded as the importance of decision criteria set B, it only models the interactions between two criteria, which may result in information loss. To overall capture the interaction among criteria, the following generalized Banzhaf function with 2AFM was provided.

Theorem 2.2 (Marichal (2000)). *It is assumed that μ is a 2AFM on the set C. The generalized Banzhaf index with 2AFM μ is*

$$\psi(B) = \sum_{\{c_j,c_q\}\subseteq B} \mu(\{c_j, c_q\}) + \sum_{c_j\in B, c_q\in C\backslash B} \frac{1}{2}\left(\mu(\{c_j, c_q\}) - |B|\mu(c_j)\right) - \frac{|C|+|B|-4}{2} \sum_{c_j\in B} \mu(\{c_j\}), \tag{9}$$

where B represents any subset of C, $C\backslash B$ represents the difference set between C and B, and $|B|$ and $|C|$ are the cardinality of B and C, respectively. If there is only one element c_j in the set B, namely, $B = \{c_j\}$, then (9) becomes the Banzhaf function with 2AFM:

$$\psi(\{c_j\}) = \frac{3-n}{2}\mu(\{c_j\}) + \sum_{c_q\in C\backslash c_j} \frac{1}{2}\left(\mu(\{c_j, c_q\}) - \mu(\{c_q\})\right). \tag{10}$$

Definition 2.5 (Grabisch (1996)). *Let μ be an FM on the set C and y be a function on the set C. The Choquet integral of y is*

$$C_\mu(y(c_{(1)}), y(c_{(2)}), \cdots, y(c_{(n)})) = \sum_{j=1}^{n} y(c_{(j)})(\mu(C_{(j)}) - \mu(C_{(j+1)})), \tag{11}$$

where the subscript $(\cdot)$ is the permutation of the elements in the set C such that $y(c_{(1)}) \leq y(c_{(2)}) \leq \cdots \leq y(c_{(n)})$, and $C_{(j)} = \{c_{(j)}, c_{(j+1)}, \cdots, c_{(n)}\}$ with $c_{(n+1)} = \emptyset$ $(j = 1, 2, \cdots, n)$.

3. Framework for Heterogeneous MCGDM with Incomplete Weights and Interactive Criteria

In this section, we first describe the heterogeneous MCGDM problem with incomplete weight information and interactive criteria. Then, we present its resolution framework.

3.1. Description of the Heterogeneous MCGDM Problem

For simplicity, we use the following notations to model the heterogeneous MCGDM problem with incomplete and interactive conditions:

$Z = \{z_1, z_2, \cdots, z_m\}$: the set of m alternatives, where $z_i (i = 1, 2, \cdots, m)$ denotes ith alternative.

$C = \{c_1, c_2, \cdots, c_n\}$: the set of n criteria, where $c_j (j = 1, 2, \cdots, n)$ denotes the jth criterion. These criteria are classified into five types: $C_1 = \{c_1, c_2, \cdots, c_{j_1}\}$, $C_2 = \{c_{j_1+1}, c_{j_1+2}, \cdots, c_{j_2}\}$, $C_3 = \{c_{j_2+1}, c_{j_2+2}, \cdots, c_{j_3}\}$, $C_4 = \{c_{j_3+1}, c_{j_3+2}, \cdots, c_{j_4}\}$ and $C_5 = \{c_{j_4+1}, c_{j_4+2}, \cdots, c_n\}$, where $1 \leq j_1 \leq j_2 \leq j_3 \leq j_4 \leq n$, $C_g \bigcap C_f = \emptyset$ $(g, f = 1, 2, \cdots, 5; g \neq f)$ and $\bigcup_1^5 C_g = C$. The sets C_1, C_2, C_3, C_4 and C_5 are criteria sets, where the criteria values are denoted by crisp values, intervals, TrFVs, HFVs and q-ROFVs, respectively. These criteria are classified into two categories: costs and benefits. It is assumed that C_d and C_b are the sets of cost and benefit criteria, respectively, verifying $C_d \cup C_b = C$ and $C_d \cap C_b = \emptyset$. Here, the criteria are interactive.

$\boldsymbol{W} = (\mu(\{c_1\}), \mu(\{c_2\}), \cdots, \mu(\{c_{n-1}, c_n\}))$: the vector of $n(n+1)/2$ 2AFMs on the criteria set, where $\mu(\{c_j\})(j = 1, 2, \cdots, n)$ denotes the importance degree of criterion c_j, such that $0 \leq \mu(\{c_j\}) \leq 1$, and $\mu(\{c_j, c_q\})(j, q = 1, 2, \cdots, n; j \neq q)$ represents the importance degree of criteria set $\{c_j, c_q\}$, such that $0 \leq \mu\{c_j, c_q\} \leq 1$. Due to the complexities and uncertainties of the actual MCGDM problem and the DM's limited experience in the problem, the information on the weights of criteria is commonly incomplete.

$E = \{e_1, e_2, \cdots, e_o\}$: the set of o experts, where $e_\tau (\tau = 1, 2, \cdots, o)$ denotes the τth expert.

$\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_o)$: the weighting vector of o experts, where $\omega_\tau (\tau = 1, 2, \cdots, o)$ denotes the weight of expert e_τ, verifying $\omega_\tau \geq 0$ and $\sum_{\tau=1}^{o} \omega_\tau = 1$. In this paper, the information on the weights of experts is partially known.

$\widetilde{\boldsymbol{A}}^\tau = [\widetilde{A}_{ij}^\tau]_{m \times n}$: heterogeneous decision matrices, where $\widetilde{A}_{ij}^\tau$ is the outcome of alternative z_i concerning criterion c_j provided by expert e_τ.

$\Omega^\tau = \{\langle (k, i), t_{k,i}^\tau \rangle | z_k \geq_\tau z_i \text{ with } t_{k,i}^\tau (k, i = 1, 2, \cdots, m)\}$: experts' preference relations between alternatives, where (k, i) denotes an ordered pair of alternatives z_k and z_i of which expert e_τ prefers z_k to z_i (expressed by $z_k \geq_\tau z_i$) with the q-ROF truth degree $t_{k,i}^\tau$.

The research question addressed in this article is how to choose the optimal alternative(s) by making full use of three different types of decision information, namely, the criteria information, pairwise comparisons of alternatives and weight information, while capturing the interactions among criteria.

3.2. Constructed Framework

Figure 2 depicts the below described four phases of the resolution framework of the the above problem:

Acquiring information phase. In this phase, a committee made up of a set of experts and DMs is formed. The set of alternatives and set of criteria are determined. Experts provide the information on criteria values and on pairwise comparisons of alternatives. DMs provide the weight information on criteria and experts.

Calculating Choquet-based relative closeness degree phase. In this phase, crisp, interval, TrFVs, HFVs and q-ROFVs decision data is normalized. Positive and negative ideal solutions are identified. A new formula is provided to calculate the Choquet-based relative closeness degrees of alternatives based on 2AFMs.

Establishing q-ROF programming model phase. In this phase, q-ROFCGCI and q-ROFCGII are defined. A new multi-objective q-ROF programming model based on LINMAP is established to derive optimal 2AFMs on the criteria set and optimal experts' weights.

Solving the model and ranking alternative phase. In this phase, the established q-ROF programming model is transformed into a many-objective mathematical programming model. Then, the A-NSGA-III-based optimization algorithm is designed to solve the many-objective programming model and to derive the Pareto set. Subsequently, TOPSIS is used to select the desirable solution from the Pareto set as the weighting result. Accordingly, the Choquet-based overall relative closeness degrees of alternates are computed. Finally, a preferred ordering of alternatives is obtained.

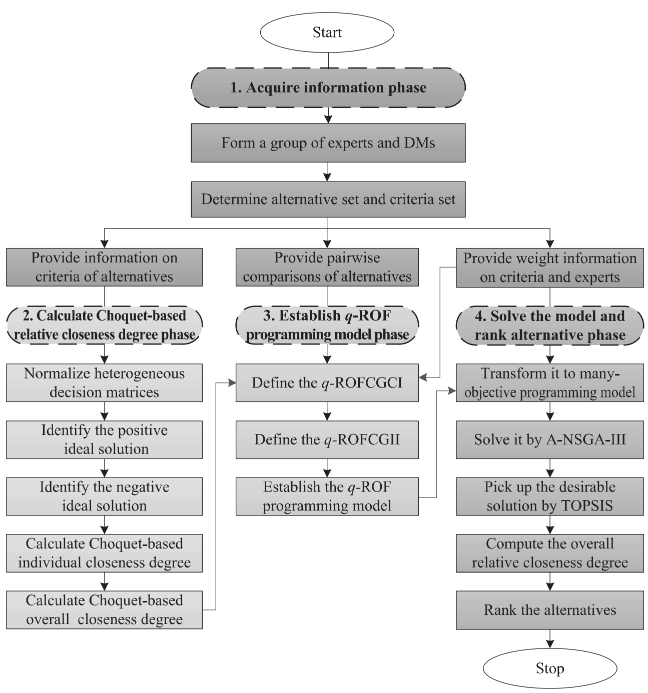

Fig. 2. Resolution framework for heterogeneous MCGDM

4. The Developed q-ROF Programming Method

In this section, a detailed description of the proposed q-ROF programming method will be presented on the basis of the resolution process depicted in Section 3.

4.1. Information Acquisition

To make a scientific evaluation and decision, a committee composed of a set of experts and DMs is formed. Then, the set of alternatives Z and set of criteria C are identified. The three types of decision information the committee members provide are described below.

4.1.1. The information on criteria value

Expert $e_\tau(\tau = 1, 2, \cdots, o)$ provides evaluation information $\widetilde{A}^\tau_{ij}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ for the alternative z_i concerning criterion c_j. It should be noted that if $c_j \in C_1$, $\widetilde{A}^\tau_{ij}$ should be given in the form of a crisp value, denoted by $\widetilde{A}^\tau_{ij} = \widetilde{\alpha}^\tau_{ij}$; if $c_j \in C_2$, $\widetilde{A}^\tau_{ij}$ should be given in the form of an interval value, denoted by $\widetilde{A}^\tau_{ij} = [\widetilde{\beta}^\tau_{ij1}, \widetilde{\beta}^\tau_{ij2}]$; if $c_j \in C_3$, $\widetilde{A}^\tau_{ij}$ should be given in the form of a TrFV, denoted by $\widetilde{A}^\tau_{ij} = (\widetilde{s}^\tau_{ij1}, \widetilde{s}^\tau_{ij2}, \widetilde{s}^\tau_{ij3}, \widetilde{s}^\tau_{ij4})$; if $c_j \in C_4$, $\widetilde{A}^\tau_{ij}$ should be given in the form of an HFV, denoted by $\widetilde{A}^\tau_{ij} = \{\widetilde{\gamma}^\tau_{ij1}, \widetilde{\gamma}^\tau_{ij2}, \cdots, \widetilde{\gamma}^\tau_{ij\iota^\tau_{ij}}\}$; if $c_j \in C_5$, $\widetilde{A}^\tau_{ij}$ should be given in the form of a q-ROFV, denoted by $\widetilde{A}^\tau_{ij} =< \widetilde{g}^\tau_{ij}, \widetilde{f}^\tau_{ij} >$, where $0 \leq \widetilde{\alpha}^\tau_{ij}$, $0 \leq \widetilde{\beta}^\tau_{ij1} \leq \widetilde{\beta}^\tau_{ij2}$, $0 \leq \widetilde{s}^\tau_{ij1} \leq \widetilde{s}^\tau_{ij2} \leq \widetilde{s}^\tau_{ij3} \leq \widetilde{s}^\tau_{ij4} \leq 1$, $0 \leq \widetilde{\gamma}^\tau_{ij1} \leq \widetilde{\gamma}^\tau_{ij2} \leq \cdots \leq \widetilde{\gamma}^\tau_{ij\iota^\tau_{ij}} \leq 1$, $0 \leq \widetilde{g}^\tau_{ij} \leq 1$, $0 \leq \widetilde{f}^\tau_{ij} \leq 1$ and $(\widetilde{g}^\tau_{ij})^q + (\widetilde{f}^\tau_{ij})^q \leq 1$, where $q \geq 1$.

When all the experts give evaluation information for the alternatives concerning the criteria, the heterogeneous decision matrices $\widetilde{\boldsymbol{A}}^\tau = [\widetilde{A}^\tau_{ij}]_{m\times n}(\tau = 1, 2, \cdots, o)$ are derived.

4.1.2. The information on alternatives' pairwise comparisons

Experts provide truth degrees of alternatives' pairwise comparisons using q-ROFVs due to their powerful capability to describe uncertain information. Taking an ordered pair of the alternatives z_1 and z_2 as an example, if expert e_1 prefers alternative z_1 to z_2 with the truth degree $\langle 0.9, 0.7\rangle$, then the preference relation between the alternatives z_1 and z_2 provided by expert e_1 is represented by $\langle (1, 2), t^1_{1,2}\rangle$, where $t^1_{1,2} = \langle 0.9, 0.7\rangle$. In this way, all the preference relations between ordered pairs of alternatives provided by the experts are derived, and denoted by $\Omega^\tau = \{\langle (k, i), t^\tau_{k,i}\rangle | z_k \geq_\tau z_i \text{ with } t^\tau_{k,i}(k, i = 1, 2, \cdots, m)\}(\tau = 1, 2, \cdots, o)$, where $t^\tau_{k,i} = \langle g_{t^\tau_{k,i}}, f_{t^\tau_{k,i}}\rangle$ is a q-ROFV, verifying the restrictions that $0 \leq g_{t^\tau_{k,i}} \leq 1$, $0 \leq f_{t^\tau_{k,i}} \leq 1$ and $(g_{t^\tau_{k,i}})^q + (f_{t^\tau_{k,i}})^q \leq 1$, where $q \geq 1$. An $(\hbar, \curlyvee)$ cut set of Ω^τ is denoted by $\Omega^\tau_{\langle\hbar,\curlyvee\rangle} = \{(k, i) | g_{t^\tau_{k,i}} \geq \hbar, f_{t^\tau_{k,i}} \leq \curlyvee (k, i = 1, 2, \cdots, m)\}$, where $0 \leq \hbar \leq 1$, $0 \leq \curlyvee \leq 1$ and $\hbar^q + \curlyvee^q \leq 1$, where $q \geq 1$. Then, the support of Ω^τ is $\Omega^\tau_{\langle 0,1\rangle} = \{(k, i) | g_{t^\tau_{k,i}} \geq 0, f_{t^\tau_{k,i}} \leq 1 (k, i = 1, 2, \cdots, m)\}$. The number of alternative pairs in $\Omega^\tau_{\langle 0,1\rangle}$ is denoted by $|\Omega^\tau_{\langle 0,1\rangle}|$, which is at most $m(m-1)/2$. Usually, the preference relations provided by $\Omega^\tau_{\langle\hbar,\curlyvee\rangle}$ are of partial order (Wan & Li, 2013a). In most cases, the experts cannot provide all preference relations and only specify some pairwise comparisons between alternatives, namely, $|\Omega^\tau_{\langle 0,1\rangle}| < m(m-1)/2$.

4.1.3. The information on weights of criteria and experts

It is assumed that the weight information provided in this article takes the form of intervals. For example, when assessing the weight of c_1, DMs may provide different weights, say 0.15, 0.2, 0.2, and 0.25. Then, the value range of the weight of c_1 is expressed by $0.15 \leq \mu(\{c_1\}) \leq 0.25$. In this way, the interval weights of all criteria $c_j(j = 1, 2, \cdots, n)$ and criteria sets $\{c_j, c_q\}$ $(j, q = 1, 2, \cdots, n; j \neq q)$ are derived. For convenience, let Λ be the set of all interval weights of c_j $(j = 1, 2, \cdots, n)$ and $\{c_j, c_q\}$ $(j, q = 1, 2, \cdots, n; j \neq q)$ provided by DMs. Similarly, the set of all interval weights of experts are derived and denoted by Γ.

4.2. Calculation of Choquet-based Relative Closeness Degree

4.2.1. Normalization of the decision-making information

It may be necessary to normalize criteria values to guarantee their compatibility since criteria are often classed into benefit criteria and cost criteria, and different criteria have different dimensions and measurements. This is the case in this study, and therefore the following is applied to normalise criteria values $\widetilde{A^{\tau}_{ij}}$ into A^{τ}_{ij}:

$$A^{\tau}_{ij} = \begin{cases} \alpha^{\tau}_{ij} = \frac{\widetilde{\alpha}^{\tau}_{ij}}{\overline{\alpha}_j}, & \text{if } c_j \in C_1 \bigcap C_b \\ \alpha^{\tau}_{ij} = 1 - \frac{\widetilde{\alpha}^{\tau}_{ij}}{\overline{\alpha}_j}, & \text{if } c_j \in C_1 \bigcap C_d \\ [\beta^{\tau}_{ij1}, \beta^{\tau}_{ij2}] = [\frac{\widetilde{\beta}^{\tau}_{ij1}}{\overline{\beta}_j}, \frac{\widetilde{\beta}^{\tau}_{ij2}}{\overline{\beta}_j}], & \text{if } c_j \in C_2 \bigcap C_b \\ [\beta^{\tau}_{ij1}, \beta^{\tau}_{ij2}] = [1 - \frac{\widetilde{\beta}^{\tau}_{ij2}}{\overline{\beta}_j}, 1 - \frac{\widetilde{\beta}^{\tau}_{ij1}}{\overline{\beta}_j}], & \text{if } c_j \in C_2 \bigcap C_d \\ (s^{\tau}_{ij1}, s^{\tau}_{ij2}, s^{\tau}_{ij3}, s^{\tau}_{ij4}) = (\frac{\widetilde{s}^{\tau}_{ij1}}{\overline{s}_{j4}}, \frac{\widetilde{s}^{\tau}_{ij2}}{\overline{s}_{j4}}, \frac{\widetilde{s}^{\tau}_{ij3}}{\overline{s}_{j4}}, \frac{\widetilde{s}^{\tau}_{ij4}}{\overline{s}_{j4}}), & \text{if } c_j \in C_3 \bigcap C_b \\ (s^{\tau}_{ij1}, s^{\tau}_{ij2}, s^{\tau}_{ij3}, s^{\tau}_{ij4}) = (1 - \frac{\widetilde{s}^{\tau}_{ij4}}{\overline{s}_{j4}}, 1 - \frac{\widetilde{s}^{\tau}_{ij3}}{\overline{s}_{j4}}, 1 - \frac{\widetilde{s}^{\tau}_{ij2}}{\overline{s}_{j4}}, 1 - \frac{\widetilde{s}^{\tau}_{ij1}}{\overline{s}_{j4}}), & \text{if } c_j \in C_3 \bigcap C_d \\ \{\gamma^{\tau}_{ij1}, \gamma^{\tau}_{ij2}, \cdots, \gamma^{\tau}_{ij\iota_j}\} = \{\widetilde{\gamma}^{\tau}_{ij1}, \widetilde{\gamma}^{\tau}_{ij2}, \cdots, \widetilde{\gamma}^{\tau}_{ij\iota_i}\}, & \text{if } c_j \in C_4 \bigcap C_b \\ \{\gamma^{\tau}_{ij1}, \gamma^{\tau}_{ij2}, \cdots, \gamma^{\tau}_{ij\iota_j}\} = \{1 - \widetilde{\gamma}^{\tau}_{ij\iota_j}, 1 - \widetilde{\gamma}^{\tau}_{ij\iota_j - 1}, 1 - \widetilde{\gamma}^{\tau}_{ij1}\}, & \text{if } c_j \in C_4 \bigcap C_d \\ \langle g^{\tau}_{ij}, f^{\tau}_{ij} \rangle = \langle \widetilde{g}^{\tau}_{ij}, \widetilde{f}^{\tau}_{ij} \rangle, & \text{if } c_j \in C_5 \bigcap C_b \\ \langle g^{\tau}_{ij}, f^{\tau}_{ij} \rangle = \langle \widetilde{f}^{\tau}_{ij}, \widetilde{g}^{\tau}_{ij} \rangle, & \text{if } c_j \in C_5 \bigcap C_d \end{cases} \tag{12}$$

where $\overline{\alpha}_j = \max_{i,\tau}\{\alpha^{\tau}_{ij}\}$, $\overline{\beta}_j = \max_{i,\tau}\{\beta^{\tau}_{ij2}\}$ and $\overline{s}_j = \max_{i,\tau}\{s^{\tau}_{ij4}\}$; $\{\gamma^{\tau}_{ij1}, \gamma^{\tau}_{ij2}, \cdots, \gamma^{\tau}_{ij\iota_j}\}$ is derived using Xu & Xia (2011)'s normalization method, where $\iota_j = \max_{i,\tau}\{\iota^{\tau}_{ij}\}$.

4.2.2. Identification of positive and negative ideal solutions

Based on the central value of TrFV (Delgado et al., 1998), the score function of HFV (Xia & Xu, 2011), and score and accuracy functions of q-ROFV (Liu & Wang, 2018), we derive the optimal ideal solution $\overline{\boldsymbol{A}} = (\overline{A}_1, \overline{A}_2, \cdots, \overline{A}_n)$ and negative ideal solution $\underline{\boldsymbol{A}} = (\underline{A}_1, \underline{A}_2, \cdots, \underline{A}_n)$ as below:

$$\overline{A}_j = \begin{cases} \overline{\alpha}_j, & \text{if } c_j \in C_1 \\ [\overline{\beta}_{j1}, \overline{\beta}_{j2}] & \text{if } c_j \in C_2 \\ (\overline{s}_{j1}, \overline{s}_{j2}, \overline{s}_{j3}, \overline{s}_{j4}) & \text{if } c_j \in C_3 \\ \{\overline{\gamma}_{j1}, \overline{\gamma}_{j2}, \cdots, \overline{\gamma}_{j\iota_j}\}, & \text{if } c_j \in C_4 \\ < \overline{g}_j, \underline{f}_j >, & \text{if } c_j \in C_5 \end{cases} \tag{13}$$

and

$$\underline{A}_j = \begin{cases} \underline{\alpha}_j, & \text{if } c_j \in C_1 \\ [\underline{\beta}_{j1}, \underline{\beta}_{j2}] & \text{if } c_j \in C_2 \\ (\underline{s}_{j1}, \underline{s}_{j2}, \underline{s}_{j3}, \underline{s}_{j4}) & \text{if } c_j \in C_3 \\ \{\underline{\gamma}_{j1}, \underline{\gamma}_{j2}, \cdots, \underline{\gamma}_{j\iota_j}\}, & \text{if } c_j \in C_4 \\ < \underline{g}_j, \overline{f}_j >, & \text{if } c_j \in C_5 \end{cases} \tag{14}$$

where $\overline{\alpha}_j = \max_{i,\tau}\{\alpha^{\tau}_{ij}\}$, $\overline{\beta}_j = \max_{i,\tau}\{\beta^{\tau}_{ij\kappa}\}(\kappa = 1, 2)$, $\overline{s}_{j\kappa} = \max_{i,\tau}\{s^{\tau}_{ij\kappa}\}(\kappa = 1, 2, 3, 4)$, $\overline{\gamma}_{j\kappa} = \max_{i,\tau}\{\gamma^{\tau}_{ij\kappa}\}(\kappa = 1, 2, \cdots, \iota_j)$, $\overline{g}_j = \max_{i,\tau}\{g^{\tau}_{ij}\}$, $\underline{f}_j = \min_{i,\tau}\{f^{\tau}_{ij}\}$, $\underline{\alpha}_j = \min_{i,\tau}\{\alpha^{\tau}_{ij}\}$, $\underline{\beta}_{j\kappa} = \min_{i,\tau}\{\beta^{\tau}_{ij\kappa}\}(\kappa = 1, 2)$, $\underline{s}_{j\kappa} = \min_{i,\tau}\{s^{\tau}_{ij\kappa}\}(\kappa = 1, 2, 3, 4)$, $\underline{\gamma}_{j\kappa} = \min_{i,\tau}\{\gamma^{\tau}_{ij\kappa}\}(\kappa = 1, 2, \cdots, \iota_j)$, $\underline{g}_j = \min_{i,\tau}\{g^{\tau}_{ij}\}$ and $\overline{f}_j = \max_{i,\tau}\{f^{\tau}_{ij}\}$.

4.2.3. Calculation of Choquet-based overall relative closeness degree

From (4), (6) and (8), the squares of Euclidean distances $d^2(A^{\tau}_{ij}, \overline{A}_j)$ and $d^2(z_i, \underline{A}_j)$ between A^{τ}_{ij} and $\overline{A}_j$ and between A^{τ}_{ij} and $\underline{A}_j$ are derived, respectively.

$$d^2(A^{\tau}_{ij}, \overline{A}_j) = \begin{cases} (\alpha^{\tau}_{ij} - \overline{\alpha}_j)^2, & \text{if } c_j \in C_1 \\ \frac{1}{2}((\beta^{\tau}_{ij1} - \overline{\beta}_{j1})^2 + (\beta^{\tau}_{ij2} - \overline{\beta}_{j2})^2), & \text{if } c_j \in C_2 \\ \frac{1}{6}((s^{\tau}_{ij1} - \overline{s}_{j1})^2 + 2(s^{\tau}_{ij2} - \overline{s}_{j2})^2 + 2(s^{\tau}_{ij3} - \overline{s}_{j3})^2 + (s^{\tau}_{ij4} - \overline{s}_{j4})^2), & \text{if } c_j \in C_3 \\ \frac{1}{\iota_j} \sum_{\kappa=1}^{\iota_j} (\gamma^{\tau}_{ij\kappa} - \overline{\gamma}_{j\kappa})^2, & \text{if } c_j \in C_4 \\ \frac{1}{2}(((g^{\tau}_{ij})^q - (\overline{g}_j)^q)^2 + ((f^{\tau}_{ij})^q - (\underline{f}_j)^q)^2 + ((\pi^{\tau}_{ij})^q - (\overline{\pi}_j)^q)^2), & \text{if } c_j \in C_5 \end{cases} \tag{15}$$

where $\pi^\tau_{ij} = \sqrt[q]{1-(g^\tau_{ij})^q-(f^\tau_{ij})^q}$ is the hesitant degree of q-ROFV A^τ_{ij} and $\overline{\pi}_j = \sqrt[q]{1-\overline{g}^q_j-\underline{f}^q_j}$ is the hesitant degree of $\overline{A}_j$.

$$d^2(A^\tau_{ij},\underline{A}_j) = \begin{cases} (\alpha^\tau_{ij}-\underline{\alpha}_j)^2, & \text{if } c_j \in C_1 \\ \frac{1}{2}((\beta^\tau_{ij1}-\underline{\beta}_{j1})^2+(\beta^\tau_{ij2}-\underline{\beta}_{j2})^2), & \text{if } c_j \in C_2 \\ \frac{1}{6}((s^\tau_{ij1}-\underline{s}_{j1})^2+2(s^\tau_{ij2}-\underline{s}_{j2})^2+2(s^\tau_{ij3}-\underline{s}_{j3})^2+(s^\tau_{ij4}-\underline{s}_{j4})^2), & \text{if } c_j \in C_3 \\ \frac{1}{\iota_j}\sum_{\kappa=1}^{\iota_j}(\gamma^\tau_{ij\kappa}-\underline{\gamma}_{j\kappa})^2, & \text{if } c_j \in C_4 \\ \frac{1}{2}(((g^\tau_{ij})^q-(\underline{g}_j)^q)^2+((f^\tau_{ij})^q-(\overline{f}_j)^q)^2+((\pi^\tau_{ij})^q-(\underline{\pi}_j)^q)^2), & \text{if } c_j \in C_5 \end{cases} \tag{16}$$

where $\underline{\pi}_j = \sqrt[q]{1-\underline{g}^q_j-\overline{f}^q_j}$ is the hesitant degree of $\underline{A}_j$.

Based on TOPSIS (Liu & Zhang, 2013), the relative closeness degree cd^τ_{ij} of A^τ_{ij} with respect to $\overline{A}_j$ and $\underline{A}_j$ is derived:

$$cd^\tau_{ij} = \frac{d^2_p(A^\tau_{ij},\underline{A}_j)}{d^2_p(A^\tau_{ij},\overline{A}_j)+d^2_p(A^\tau_{ij},\underline{A}_j)}. \tag{17}$$

Under the assumption independence of the criteria set C, and its weighting vector being $\boldsymbol{W} = (w_1, w_2, \cdots, w_n)$, the relative closeness degree cd^τ_i of the alternative z_i concerning expert e_τ is derived:

$$cd^\tau_i = \sum_{j=1}^{n} w_j cd^\tau_{ij}. \tag{18}$$

As mentioned before, in practice, the criteria may be dependent (Liu et al., 2021), making (18) unsuitable for this kind of decision-making problem. Nevertheless, the 2AFM is a powerful tool to describe the interaction among criteria (Grabisch, 1997), and the Choquet integral (Grabisch, 1996) concerning 2AFM can effectively aggregate decision information with complex relationship. Thus, the following Choquet-based individual relative closeness degree cd^τ_i of the alternative z_i concerning expert e_τ is proposed

$$cd^\tau_i = \sum_{j=1}^{n}(\mu(C_{(j)})-\mu(C_{(j+1)}))cd^\tau_{i(j)}, \tag{19}$$

where $cd^\tau_{i(j)}$ represents the j-th smallest value in the set $\{cd^\tau_{i1}, cd^\tau_{i2}, \cdots, cd^\tau_{in}\}$, $c_{(j)}$ represents the criterion corresponding to $cd_{i(j)}$, $C_{(j)} = \{c_j, c_{j+1}, \cdots, c_n\}$ with $C_{(n+1)} = \emptyset$, and μ denotes a 2AFM on the criteria set C.

Note that (19) only captures the interaction between adjacent combinations of criteria, $C_{(j)}$ and $C_{(j+1)}$. In fact, other combinations of criteria should also be reflected, which means that (19) may not always be suitable. In the following, to globally reflect the various combinations of criteria, we extend the generalized Banzhaf index to (19) and then derive the below improved Choquet-based individual relative closeness degree cd^τ_i concerning expert e_τ

$$cd^\tau_i = \sum_{j=1}^{n}(\psi(C_{(j)})-\psi(C_{(j+1)}))cd^\tau_{i(j)}, \tag{20}$$

where $\psi(C_{(j)})$ denotes the generalized Banzhaf index with 2AFM on the criteria set $C_{(j)}$. Since $\psi(C_{(j)}) - \psi(C_{(j+1)}) = \psi(\{c_{(j)}\})$ (Tang et al., 2020a), (20) can be equivalently transformed into the following form:

$$cd^\tau_i = \sum_{j=1}^{n}\psi(\{c_j\})cd^\tau_{ij}, \tag{21}$$

where $\psi(\{c_j\})(j = 1, 2, \cdots, n)$ denotes the Banzhaf function with 2AFM on the criterion c_j. It is apparent that the larger the value of cd^τ_i is, the better the alternative z_i for expert e_τ is. Thus, the improved Choquet-based individual relative closeness degree cd^τ_i could be regarded a type of objective criteria for identifying the preferred order of alternatives.

Finally, by integrating all individual relative closeness degrees cd^1_i, cd^2_i, $\cdots$, and cd^o_i, we derive the Choquet-based overall relative closeness degree cd_i of the alternative z_i

$$cd_i = \sum_{\tau=1}^{o}\omega_\tau cd^\tau_i = \sum_{\tau=1}^{o}\sum_{j=1}^{n}\omega_\tau\psi(\{c_j\})cd^\tau_{ij}, \tag{22}$$

where ω_τ represents the weight of expert e_τ.

4.3. Multi-Objective q-ROF Mathematical Programming Model

4.3.1. Definitions of q-ROF Choquet-based consistency and inconsistency indices

As mentioned above, the preference relations Ω^τ provided by experts are pairwise comparisons of alternatives with no criteria involved, which captures experts' subjective views on ordered pairs of alternatives. Thus, it could be considered as a type of subjective information for identifying the preferred order of alternatives. Additionally, the improved Choquet-based individual relative closeness degree cd_i^τ could be regarded a type of objective information for identifying the preferred order of alternatives. To make a fit decision, the subjective and the objective information should be as consistent as possible.

It is assumed that the criteria weighting vector $\boldsymbol{W}$ is provided in advance. For the ordered pair of alternatives (k, i) and expert e_τ, we can compute the improved Choquet-based individual relative closeness degree cd_k^τ and cd_i^τ using (21). Assuming that expert e_τ prefers z_k to z_i, $(k, i) \in \Omega^\tau_{\langle 0,1\rangle}$, if $cd_k^\tau > cd_i^\tau$, then the improved Choquet-based individual relative closeness degree of alternative z_k is larger than that of alternative z_i for expert e_τ. Thus, the objective preferred order of alternatives z_k and z_i obtained by cd_k^τ and cd_i^τ based on criteria weighting vector $\boldsymbol{W}$ is consistent with the subjective preference relation provided by the expert e_τ. Conversely, if $cd_k^\tau \leq cd_i^\tau$, then the objective ranking order based on criteria weighting vector is inconsistent with the subjective preference relation, and the criteria weighting vector W is not chosen properly.

To measure the degree of consistency between the ranking order of alternatives z_k and z_i obtained by cd_k^τ and cd_i^τ, and the preference provided by expert e_τ (who prefers z_k to z_i), the following q-ROF Choquet-based consistency for the expert e_τ is provided:

$$(cd_k^\tau - cd_i^\tau)^+ = \begin{cases} (cd_k^\tau - cd_i^\tau)t_{k,i}^\tau, & cd_k^\tau > cd_i^\tau \\ \langle 0, 1\rangle, & cd_k^\tau \leq cd_i^\tau \end{cases} \tag{23}$$

It is obvious that the objective preferred order of alternatives z_k and z_i obtained by Choquet-based individual relative closeness degrees cd_k^τ and cd_i^τ is consistent with the subjective preference provided by the expert e_τ if $cd_k^\tau > cd_i^\tau$. In such a case, $(cd_k^\tau - cd_i^\tau)^+$ is defined to be $(cd_k^\tau - cd_i^\tau)t_{k,i}^\tau$. On the other hand, the preferred order of alternatives z_k and z_i obtained by cd_k^τ and cd_i^τ is inconsistent with the subject preference provided by the expert e_τ if $cd_k^\tau \leq cd_i^\tau$. In such a case, $(cd_k^\tau - cd_i^\tau)^+$ is defined to be $\langle 0, 1\rangle$. The consistency index can be rewritten as:

$$(cd_k^\tau - cd_i^\tau)^+ = \max\{0, cd_k^\tau - cd_i^\tau\}t_{k,i}^\tau. \tag{24}$$

The collective consistency index for the expert e_τ is defined as:

$$G^\tau = \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} (cd_k^\tau - cd_i^\tau)^+ = \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} \max\{0, cd_k^\tau - cd_i^\tau\}t_{k,i}^\tau. \tag{25}$$

Definition 4.1. *The q-ROFCGCI is the weighted average of the set of collective consistency indices* $\{G^1, G^2, \ldots, G^o\}$*:*

$$G = \sum_{\tau=1}^{o} \omega_\tau G^\tau = \sum_{\tau=1}^{o} \omega_\tau \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} (cd_k^\tau - cd_i^\tau)^+ = \sum_{\tau=1}^{o} \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} \omega_\tau \max\{0, cd_k^\tau - cd_i^\tau\}t_{k,i}^\tau. \tag{26}$$

Analogously, the q-ROF Choquet-based inconsistency for the expert e_τ is:

$$(cd_k^\tau - cd_i^\tau)^- = \begin{cases} (cd_i^\tau - cd_k^\tau)t_{k,i}^\tau, & cd_k^\tau \leq cd_i^\tau \\ \langle 0, 1\rangle, & cd_k^\tau > cd_i^\tau \end{cases} \tag{27}$$

This can be expressed as:

$$(cd_k^\tau - cd_i^\tau)^- = \max\{0, cd_i^\tau - cd_k^\tau\}t_{k,i}^\tau. \tag{28}$$

The collective inconsistency index for the expert e_τ is defined as:

$$B^\tau = \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} (cd_k^\tau - cd_i^\tau)^- = \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} \max\{0, cd_i^\tau - cd_k^\tau\}t_{k,i}^\tau. \tag{29}$$

Definition 4.2. *The q-ROFCGII is the weighted average of the set of collective inconsistency indices* $\{B^1, B^2, \ldots, B^o\}$*:*

$$B = \sum_{\tau=1}^{o} \omega_\tau B^\tau = \sum_{\tau=1}^{o} \omega_\tau \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} (cd_k^\tau - cd_i^\tau)^- = \sum_{\tau=1}^{o} \sum_{(k,i)\in\Omega^\tau_{\langle 0,1\rangle}} \omega_\tau \max\{0, cd_i^\tau - cd_k^\tau\}t_{k,i}^\tau. \tag{30}$$

Remark 4.1. *Notably, G and B are q-ROFVs since truth degrees are expressed by q-ROFVs. Despite many studies having extended consistency and inconsistency indices to a variety of fuzzy settings (Chen, 2019; Li & Wan, 2013, 2014; Wan & Dong, 2015; Wan & Li, 2013a,b, 2015; Wan et al., 2017, 2020; Zhang et al., 2016), no attempt has been made to use* *q-ROFVs to describe fuzzy truth degrees. This is the main distinction between our method and the previous methods.*

4.3.2. Establishment of the multi-objective q-ROF programming model

Both q-ROFCGCI G and q-ROFCGII B capture the total consistency and inconsistency between the objective preferred order and the subjective preference relations provided by the group of experts, the greater the value of G and the smaller the value of B are, the more the weight information is reasonably derived by the model. Based on this and Theorems 2.1 and 2.2, the following multi-objective q-ROF programming model is established to identify the optimal weighting vectors of criteria $\boldsymbol{W}^*$ and experts $\boldsymbol{\omega}^*$.

$$
\begin{aligned}
&\max\left\{G=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau \max\{0, cd_k^\tau - cd_i^\tau\}t^\tau_{k,i}\right\}\\
&\min\left\{B=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau \max\{0, cd_i^\tau - cd_k^\tau\}t^\tau_{k,i}\right\}\\
&s.t.\begin{cases}
\psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum\limits_{c_q\in C\backslash c_j}\frac{1}{2}(\mu(\{c_j,c_q\})-\mu(\{c_q\}),\forall c_j\in C;\\
\sum\limits_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum\limits_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\
\sum\limits_{\{c_j\}\subseteq S\backslash c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\}),\forall S\subseteq C \text{ with } c_q\in S \text{ and } |S|\geq 2;\\
\mu(\{c_j\})\geq 0,\forall c_j\in C;\\
\mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda,\forall\{c_j\},\{c_j,c_q\}\subseteq C;\\
0\leq\omega_\tau\leq 1(\tau=1,2,\cdots,o);\ \sum\limits_{\tau=1}^{o}\omega_\tau=1;\\
\omega_\tau\in\Gamma(\tau=1,2,\cdots,o).
\end{cases}
\end{aligned}
\tag{31}
$$

Denoting $\xi^\tau_{ki}=\max\{0,cd^\tau_k-cd^\tau_i\}$ and $\lambda^\tau_{ki}=\max\{0,cd^\tau_i-cd^\tau_k\}$, model (31) can be rewritten as:

$$
\begin{aligned}
&\max\left\{G=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau \xi^\tau_{ki}t^\tau_{k,i}\right\}\\
&\min\left\{B=\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau \lambda^\tau_{ki}t^\tau_{k,i}\right\}\\
&s.t.\begin{cases}
\xi^\tau_{ki}=\max\{0,cd^\tau_k-cd^\tau_i\},(k,i)\in\Omega^\tau_{\langle 0,1\rangle}(\tau=1,2,\cdots,o);\\
\lambda^\tau_{ki}=\max\{0,cd^\tau_i-cd^\tau_k\},(k,i)\in\Omega^\tau_{\langle 0,1\rangle}(\tau=1,2,\cdots,o);\\
\psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum\limits_{c_q\in C\backslash c_j}\frac{1}{2}(\mu(\{c_j,c_q\})-\mu(\{c_q\}),\forall c_j\in C;\\
\sum\limits_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum\limits_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\
\sum\limits_{\{c_j\}\subseteq S\backslash c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\}),\forall S\subseteq C \text{ with } c_q\in S \text{ and } |S|\geq 2;\\
\mu(\{c_j\})\geq 0,\forall c_j\in C;\\
\mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda,\forall\{c_j\},\{c_j,c_q\}\subseteq C;\\
0\leq\omega_\tau\leq 1(\tau=1,2,\cdots,o);\ \sum\limits_{\tau=1}^{o}\omega_\tau=1;\\
\omega_\tau\in\Gamma(\tau=1,2,\cdots,o).
\end{cases}
\end{aligned}
\tag{32}
$$

The detailed discussion on the boundedness and monotonicity of the q-ROF programming in (32) is provided in Appendix I.

Remark 4.2. *Model (32) is a q-ROF programming model because its objective functions involve q-ROFVs, while Zhang et al. (2016)'s method is an ordinary programming model, Wan et al. (2017)'s method is an HF programming model, Wan et al. (2020)'s method is a hybrid programming model, Wan & Li (2013b)'s method and Wan & Li (2013a)'s method are IF programming models, Li & Wan (2013)'s method and Li & Wan (2014)'s method are TrF programming models, Wan & Dong (2015)'s method and Wan & Li (2015)'s method are IVIF programming models and Chen (2019)'s method is a PF programming model. This is another prominent difference between tour model and he existing models.*

Remark 4.3. *Since model (32) is based on 2AFMs, it considers the interactions among the criteria, which is a new and key contribution that the existing models (Chen, 2019; Li & Wan, 2013, 2014; Wan & Dong, 2015; Wan & Li, 2013a,b, 2015; Wan et al., 2017, 2020; Zhang et al., 2016) are unable to offer.*

4.4. A Solving Method for the Multi-Objective q-ROF Programming Model

In what follows, a novel approach is proposed to solve model (32). According to (2) and (3), the objective functions of model (32) are both the following q-ROFVs:

$$\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau\xi^{\tau}_{ki}t^{\tau}_{k,i}=\left\langle\sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\xi^{\tau}_{ki}}},\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\xi^{\tau}_{ki}}\right\rangle \tag{33}$$

and

$$\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}\omega_\tau\lambda^{\tau}_{ki}t^{\tau}_{k,i}=\left\langle\sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\lambda^{\tau}_{ki}}},\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\lambda^{\tau}_{ki}}\right\rangle. \tag{34}$$

Using the score and accuracy functions of q-ROFVs, model (32) can be transformed into the following many-objective mathematical programming model:

$$\begin{aligned}
&\min\left\{Z_1=-1+\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\xi^{\tau}_{ki}}+(\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\xi^{\tau}_{ki}})^q\right\}\\
&\min\left\{Z_2=-1+\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\xi^{\tau}_{ki}}-(\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\xi^{\tau}_{ki}})^q\right\}\\
&\min\left\{Z_3=1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\lambda^{\tau}_{ki}}-(\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\lambda^{\tau}_{ki}})^q\right\}\\
&\min\left\{Z_4=1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau\lambda^{\tau}_{ki}}+(\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}}(f_{t^{\tau}_{k,i}})^{\omega_\tau\lambda^{\tau}_{ki}})^q\right\}\\
&s.t.\begin{cases}
\xi^{\tau}_{ki}=\max\{0,cd^{\tau}_k-cd^{\tau}_i\},(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}(\tau=1,2,\cdots,o);\\
\lambda^{\tau}_{ki}=\max\{0,cd^{\tau}_i-cd^{\tau}_k\},(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}(\tau=1,2,\cdots,o);\\
\psi(\{c_j\})=\frac{3-n}{2}\mu(\{c_j\})+\sum_{c_q\in C\backslash c_j}\frac{1}{2}(\mu(\{c_j,c_q\})-\mu(\{c_q\}),\forall c_j\in C;\\
\sum_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum_{\{c_j\}\subseteq C}\mu(\{c_j\})=1;\\
\sum_{\{c_j\}\subseteq S\backslash c_q}(\mu(\{c_j,c_q\})-\mu(\{c_j\}))\geq(|S|-2)\mu(\{c_q\}),\forall S\subseteq C\text{ with }c_q\in S\text{ and }|S|\geq 2;\\
\mu(\{c_j\})\geq 0,\forall c_j\in C;\\
\mu(\{c_j\}),\mu(\{c_j,c_q\})\in\Lambda,\forall\{c_j\},\{c_j,c_q\}\subseteq C;\\
0\leq\omega_\tau\leq 1(\tau=1,2,\cdots,o);\ \sum_{\tau=1}^{o}\omega_\tau=1;\\
\omega_\tau\in\Gamma(\tau=1,2,\cdots,o).
\end{cases}
\end{aligned}\tag{35}$$

Remark 4.4. *Model (35) involves* 4 *objective functions,* $2|\Omega^{\tau}_{\langle 0,1\rangle}|+(n^2+n)/2+o$ *unknown variables,* $n+2+2|\Omega^{\tau}_{\langle 0,1\rangle}|$ *equalities and* $n\times 2^{n-1}+o$ *inequalities (excluding the constraints in* Λ *and* Γ*). In general, the greater the* $|\Omega^{\tau}_{\langle 0,1\rangle}|$ *is, the more fit the derived weighting vectors are.*

Since model (35) has four nonlinear objective functions and many constraints, it is hard to solve using classical exact algorithms, and a considerable amount of computational time is needed. To solve this model, evolutionary many-objective optimization algorithms are used. A very competitive method to solving constrained many-objective optimization problems with acceptable computational requirements is the NSGA-III (Jain & Deb, 2013). NSGA-III uses reference points and niche technology to select the novel parent population, which greatly improve the diversity of the population and the ability to solve many-objective optimization problems. However, NSGA-III may not end up distributing all population members uniformly over the entire Pareto-optimal front because not every extended reference line may intersect with the Pareto-optimal front. To address this issue, the NSGA-III is made adaptive (A-NSGA-III) (Jain & Deb, 2013) in deleting and including new reference points on the fly. Therefore, the A-NSGA-III-based optimization algorithm is proposed here to solve model (35) and further determine the optimal weighting vectors of criteria $\boldsymbol{W}^*$ and experts $\boldsymbol{\omega}^*$. Its procedure comprises an initialization process, the creation of offspring population (Deb & Agrawal, 1995; Deb & Goyal, 1996), the non-dominated sorting, the selection mechanism (Das & Dennis, 1998; Yuan et al., 2015) and the adaption of reference points. The detailed procedure description is given in Appendix I, while its algorithm flowchart is depicted in Fig. 3. It yields a set of Pareto non-dominant solutions that denote different weighting results. Any objective of these solutions cannot be improved without degrading other objectives. However, under the weighting context, a desirable solution from the non-dominant solution set should be selected to denote the weighting result. In this paper, TOPSIS (Liu & Zhang, 2013) is employed to select the desirable solution from the last Pareto non-dominant set as the weighting result. Based on (22) and the optimal weighting vectors $\boldsymbol{W}^*$ and $\boldsymbol{\omega}^*$, the Choquet-based overall relative closeness degree cd_i of each alternative z_I $(i = 1, 2, \cdots, m)$ can be computed to derive the ranking order of alternatives.

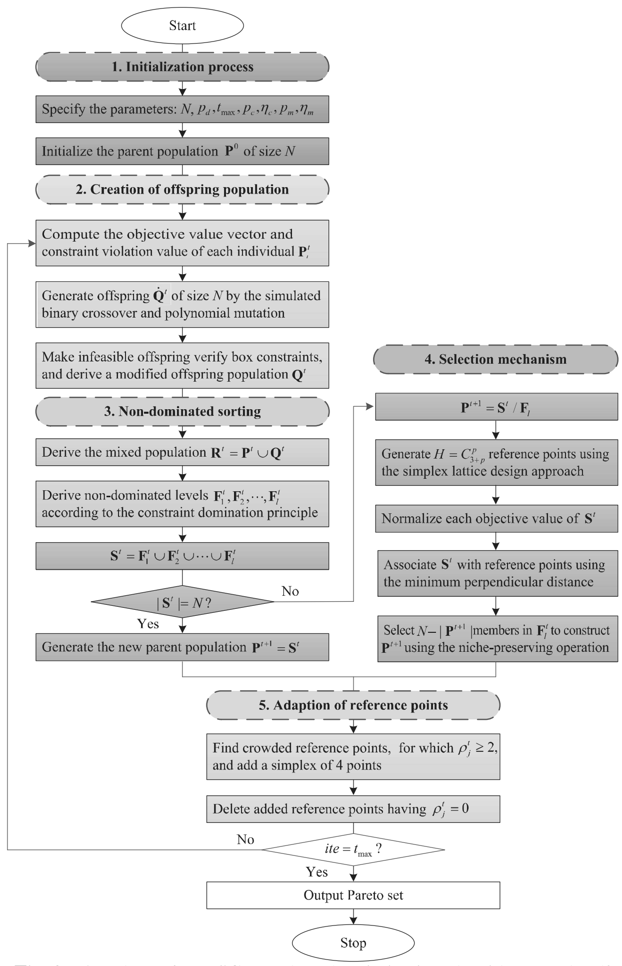

Fig. 3. Flowchart of A-NSGA-III-based optimization algorithm to solve (35)

Remark 4.5. *We first apply evolutionary many-objective optimization to solve the multi-objective q-ROF programming model. It can simultaneously optimize many objectives using optimal search in the objective space without predefined weighting coefficients (Konak et al., 2006). However, the existing methods (Chen, 2019; Li & Wan, 2013, 2014; Wan & Dong, 2015; Wan & Li, 2013a,b, 2015; Wan et al., 2017, 2020) need to subjectively divide the hierarchy or assign different weights to each objective and neglect the trade-off relationships among the objectives, which may result in less reliable optimization outcomes (Deb, 2001). This is the remarkable distinction between our method and the existing methods.*

Remark 4.6. *Our proposed multi-objective q-ROF programming method can simultaneously determine the weights of criteria and experts. However, the existing methods (Chen, 2019; Wan & Dong, 2015; Wan & Li, 2013a; Wan et al., 2017, 2020; Zhang et al., 2016) can determine the weights of criteria but fail to identify the weights of the experts directly. This is a sharp difference between our model and the existing models.*

4.5. Decision-making process of the proposed multi-objective q-ROF programming approach

Based on the aforesaid analyses, the procedure of the proposed q-ROF programming approach is outlined as follows.

Phase 1. Information Acquisition

Step 1: Form a committee composed of a set of experts $E = \{e_1, e_2, \cdots, e_o\}$ and DMs.

Step 2: Determine the alternative set $Z = \{z_1, z_2, \cdots, z_m\}$ and criteria set $C = \{c_1, c_2, \cdots, c_n\}$.

Step 3: The experts $e_\tau(\tau = 1, 2, \cdots, o)$ construct the heterogeneous decision matrices $\widetilde{\boldsymbol{A}^\tau} = [\widetilde{A_{ij}^\tau}]_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$.

Step 4: The experts $e_\tau(\tau = 1, 2, \cdots, o)$ provide truth degrees of alternatives' pairwise comparisons using q-ROFVs, represented by Ω^τ.

Step 5: The DMs provide the incomplete weight information on criteria and experts, expressed by Λ and Γ, respectively.

Phase 2. Calculation of Choquet-based Relative Closeness Degree

Step 6: Normalize the heterogeneous decision matrices $\widetilde{\boldsymbol{A}^\tau} = [\widetilde{A_{ij}^\tau}]_{m\times n}(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; \tau = 1, 2, \cdots, o)$ into $\boldsymbol{A}^\tau = [A_{ij}^\tau]_{m\times n}$ using (12).

Step 7: Identify the positive ideal solution $\overline{\boldsymbol{A}} = (\overline{A}_1, \overline{A}_2, \cdots, \overline{A}_n)$ and negative ideal solution $\underline{\boldsymbol{A}} = (\underline{A}_1, \underline{A}_2, \cdots, \underline{A}_n)$ using (13) and (14), respectively.

Step 8: Calculate the improved Choquet-based individual relative closeness degree $cd_i^\tau(i = 1, 2, \cdots, m; \tau = 1, 2, \cdots, o)$ and Choquet-based overall relative closeness degree cd_i of each alternative z_i using (21) and (22), respectively.

Phase 3. Multi-Objective q-ROF Programming Model

Step 9: Compute the q-ROFCGCI and q-ROFCGII using (26) and (30), respectively.

Step 10: Establish the multi-objective q-ROF programming model using (32).

Phase 4. A Solving Method for the Multi-Objective q-ROF Programming Model

Step 11: Transform the multi-objective q-ROF programming model into the many-objective programming model using (35).

Step 12: Solve the many-objective programming model using the A-NSGA-III-based optimization algorithm to derive Pareto non-dominant solutions.

Step 13: Select the desirable solution from the last Pareto non-dominant set as the optimal 2AFMs on criteria set $\boldsymbol{W}^*$ and optimal weights of experts $\boldsymbol{\omega}^*$ using the TOPSIS.

Step 14: Compute the Banzhaf index on each criterion using (10).

Step 15: Compute the Choquet-based overall relative closeness degree cd_i of each alternative $z_i(i = 1, 2, \cdots, m)$ using (22) and optimal weight information.

Step 16: Rank the alternatives $z_1, z_2, \cdots$, and z_m and select the best alternative based on $cd_1, cd_2, \cdots$, and cd_m.

5. Application Examples

This section provides four examples to explain the proposed multi-objective q-ROF programming approach. A comparative analysis is also performed to illustrate the superiority of the proposed approach.

5.1. Decision-Making Steps

Example 1. *It is assumed five well-known s-commerce merchants, anonymously denoted as z_1, z_2, z_3, z_4 and z_5, and three experts e_1, e_2 and e_3 assessing their credit status to help the s-commerce consumer select proper transaction partners based on six criteria: c_1–product price, c_2–delivery time, c_3–product quality, c_4–service quality, c_5–information quality and c_6–consumer approval.*

- *The two criteria c_1 and c_2 are cost and quantitative criteria, respectively, while criteria c_3, c_4, c_5 and c_6 are benefit and qualitative criteria.*
- *The evaluation for product price c_1 is denoted by crisp real numbers.*
- *Because of the uncertainty of the time required to complete delivery, it is better to use intervals to denote the criterion c_2.*
- *The evaluation for product quality c_3 is denoted by TrFVs because the experts prefer to provide upper, lower limits and the most intervals for c_3.*
- *The evaluation for service quality c_4 involves multiple parts, such as service attitude, complaint rate and refund rate. HFVs can represent different degrees of satisfaction by several MDs, which are fit for evaluating criterion c_4.*
- *The evaluations for information quality c_5 and consumer approval c_6 are divided into two parts, namely, dissatisfaction and satisfaction, which correspond to the NMD and MD of q-ROFVs, respectively. Thus, q-ROFVs are used to evaluate criteria c_5 and c_6.*
- *It is assumed that the criteria are independent, and their importance given by experts is $\Lambda = \{0.1 \le \mu(\{c_1\}) \le 0.15, 0.05 \le \mu(\{c_2\}) \le 0.1, 0.15 \le \mu(\{c_3\}) \le 0.25, 0.25 \le \mu(\{c_4\}) \le 0.3, 0.2 \le \mu(\{c_5\}) \le 0.3, 0 \le \mu(\{c_6\}) \le 0.1\}$.*

- *It is assumed that the uncertain weight information of the experts is given by* $\Gamma = \{0.1 \leq \omega_1 \leq 0.2, 0.4 \leq \omega_2 \leq 0.7, 0.3 \leq \omega_3 \leq 0.5\}$.
- *The heterogeneous decision matrices* $\tilde{\boldsymbol{A}}^{\tau} = [\tilde{A}^{\tau}_{ij}]_{5\times 6} (\tau = 1, 2, 3)$ *are established according to experts' views, which are listed in Table 1.*
- *With experts' comprehension and judgements, they offer q-ROF preference relations between merchants, which are:*
$\Omega^1 = \{\langle(1,2), t^1_{1,2}\rangle, \langle(2,3), t^1_{2,3}\rangle, \langle(4,3), t^1_{4,3}\rangle, \langle(4,5), t^1_{4,5}\rangle, \langle(5,3), t^1_{5,3}\rangle\}$,
$\Omega^2 = \{\langle(1,2), t^2_{1,2}\rangle, \langle(1,5), t^2_{1,5}\rangle, \langle(3,2), t^2_{3,2}\rangle, \langle(4,3), t^2_{4,3}\rangle, \langle(4,5), t^2_{4,5}\rangle\}$,
$\Omega^3 = \{\langle(1,3), t^3_{1,3}\rangle, \langle(2,3), t^3_{2,3}\rangle, \langle(3,4), t^3_{3,4}\rangle, \langle(4,2), t^2_{4,2}\rangle, \langle(5,4), t^3_{5,4}\rangle\}$,
where the corresponding q-ROF truth degrees are:
$t^1_{1,2} = \langle 0.6, 0.4\rangle$, $t^1_{2,3} = \langle 0.5, 0.4\rangle$, $t^1_{4,3} = \langle 0.6, 0.3\rangle$, $t^1_{4,5} = \langle 0.7, 0.3\rangle$, $t^1_{5,3} = \langle 0.5, 0.4\rangle$;
$t^2_{1,2} = \langle 0.7, 0.2\rangle$, $t^2_{1,5} = \langle 0.5, 0.4\rangle$, $t^2_{3,2} = \langle 0.6, 0.4\rangle$, $t^2_{4,3} = \langle 0.6, 0.3\rangle$, $t^2_{4,5} = \langle 0.5, 0.3\rangle$;
$t^3_{1,3} = \langle 0.4, 0.5\rangle$, $t^3_{2,3} = \langle 0.4, 0.6\rangle$, $t^3_{3,4} = \langle 0.4, 0.6\rangle$, $t^2_{4,2} = \langle 0.7, 0.1\rangle$, $t^3_{5,4} = \langle 0.3, 0.5\rangle$.
- *The supports of* Ω^1, Ω^2 *and* Ω^3 *are:*
$\Omega^1_{\langle 0,1\rangle} = \{(1,2), (2,3), (4,3), (4,5), (5,3)\}$,
$\Omega^2_{\langle 0,1\rangle} = \{(1,2), (1,5), (3,2), (4,3), (4,5)\}$,
$\Omega^3_{\langle 0,1\rangle} = \{(1,3), (2,3), (3,4), (4,2), (5,4)\}$.
It is apparent that some of preferences relations of merchants provided by expert e_1 *contrast with those provided by expert* e_2. *For instance,* e_1 *prefers* z_2 *to* z_3, *whereas* e_2 *prefers* z_3 *to* z_2.

Table 1
Decision Making Matrices for Example 1

Expert	Merchants	Criteria					
		c_1	c_2	c_3	c_4	c_5	c_6
	z_1	5	[3, 5]	(5, 6, 7, 9)	{0.5, 0.7}	⟨0.6, 0.3⟩	⟨0.5, 0.2⟩
	z_2	5.5	[2, 4]	(2, 5, 6, 9)	{0.6, 0.9}	⟨0.4, 0.5⟩	⟨0.6, 0.3⟩
e_1	z_3	7	[4, 7]	(3, 5, 7, 8)	{0.3, 0.5, 0.6}	⟨0.7, 0.1⟩	⟨0.4, 0.5⟩
	z_4	6.5	[2, 3]	(4, 6, 8, 9)	{0.3, 0.4, 0.8}	⟨0.8, 0.1⟩	⟨0.7, 0.3⟩
	z_5	8	[1, 3]	(5, 7, 8, 9)	{0.5, 0.7, 0.9}	⟨0.4, 0.5⟩	⟨0.3, 0.5⟩
	z_1	6	[2, 4]	(4, 5, 7, 8)	{0.3, 0.5, 0.6}	⟨0.5, 0.3⟩	⟨0.7, 0.3⟩
	z_2	6	[4, 6]	(3, 4, 7, 8)	{0.5, 0.7, 0.8}	⟨0.5, 0.4⟩	⟨0.6, 0.2⟩
e_2	z_3	7.5	[5, 7]	(4, 5, 6, 7)	{0.4, 0.5, 0.7}	⟨0.8, 0.2⟩	⟨0.5, 0.4⟩
	z_4	7	[2, 4]	(5, 6, 7, 9)	{0.6, 0.8}	⟨0.6, 0.3⟩	⟨0.8, 0.2⟩
	z_5	8	[3, 5]	(5, 7, 8, 9)	{0.4, 0.6}	⟨0.6, 0.4⟩	⟨0.5, 0.4⟩
	z_1	5.5	[5, 7]	(3, 5, 6, 8)	{0.4, 0.5, 0.7}	⟨0.5, 0.3⟩	⟨0.6, 0.3⟩
	z_2	8	[5, 6]	(2, 3, 4, 6)	{0.4, 0.8}	⟨0.7, 0.3⟩	⟨0.3, 0.4⟩
e_3	z_3	6.5	[3, 4]	(4, 6, 7, 9)	{0.3, 0.4, 0.5}	⟨0.4, 0.6⟩	⟨0.6, 0.3⟩
	z_4	6	[4, 5]	(6, 7, 8, 9)	{0.4, 0.5, 0.7}	⟨0.5, 0.3⟩	⟨0.8, 0.2⟩
	z_5	7	[2, 4]	(3, 6, 7, 8)	{0.2, 0.3, 0.5}	⟨0.6, 0.4⟩	⟨0.7, 0.2⟩

To select the optimal merchant, the proposed multi-objective q-ROF programming method is used (see Appendix II for detailed process), and the final ranking of merchants obtained is: $z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$, which makes merchant z_4 the best.

To explain the effectiveness of the proposed multi-objective q-ROF programming method, the validity verification approach introduced by Wang & Triantaphyllou (2008) is utilized, which consists of the following three test criteria.

Test criterion 1. An effective MCGDM method should not change the optimal alternative when substituting a non-optimal alternative for another non-optimal alternative without changing the criteria weights.

Using test criterion 1, the worse alternative $\bar{z}_1$ (see Table 2) is substituted for the non-optimal alternative z_1 in the initial decision matrix. The criteria weights obtained by the initial decision matrix are also utilized in this changed MCGDM problem to maintain the relative importance of criteria unchanged. By using the proposed method (or the developed model in equation 31), the following aggregated results of alternatives are obtained: $cd_1 = 0.5500, cd_2 = 0.5038, cd_3 = 0.4181, cd_4 = 0.7019, cd_5 = 0.4302$,. Thus, the ranking ordering is $z_4 \succ \bar{z}_1 \succ z_2 \succ z_5 \succ z_3$, i.e., the best s-commerce merchant is still z_4. For other non-optimal s-commerce merchants, such as z_2, z_3 and z_5, the same conclusion holds. Therefore, the proposed method (or the developed model in equation 31) meets test criterion 1.

Test criterion 2. An effective MCGDM method needs to meet transitive property.

Test criterion 3. When a MCGDM problem is decomposed into several sub-problems, and the same method is utilized to solve these sub-problems to derive the ranking of the alternatives, the combined ranking of the alternatives should be the same as the initial MCGDM problem.

Using test criteria 2 and 3, we decompose the original MCGDM problem into two smaller MCGDM problems $\{z_1, z_2, z_3, z_5\}$ and $\{z_1, z_2, z_4, z_5\}$. According to the procedure of the proposed method (the developed model in equation

Table 2
Evaluation values of s-commerce merchant z_1 for different DMs.

	c_1	c_2	c_3	c_4	c_5	c_6
e_1	5	$[3,4]$	$(5,6,7,9)$	$0.5, 0.7$	$<0.6, 0.3>$	$<0.5, 0.2>$
e_2	6	$[2,4]$	$(4,5,7,8)$	$0.3, 0.5, 0.6$	$<0.5, 0.3>$	$<0.7, 0.3>$
e_3	5.5	$[5,7]$	$(3,5,6,8)$	$0.4, 0.5, 0.7$	$<0.4, 0.3>$	$<0.6, 0.3>$

31), we obtain preferred orders $z_1 \succ z_5 \succ z_2 \succ z_3$ and $z_4 \succ z_1 \succ z_5 \succ z_2$ for the two sub-problems, respectively. If the orderings of the sub-problems are combined together, we derive the integrated preferred order as $z_4 \succ z_1 \succ z_5 \succ z_2 \succ z_3$, which is the same as the ranking in the initial MCGDM problem and consistent with the transitive property. If the initial MCGDM problem is decomposed into other sub-problems, the same conclusion holds. Hence, the proposed method (or the developed model in equation 31) verifies test criteria 2 and 3. Therefore, for equation 31, the aggregated result is effective and valid.

5.2. Comparative Analysis

5.2.1. Verifying the effectiveness of the proposed method

To justify the effectiveness of our proposed q-ROF programming method, experimental results of our method are compared with methods by Wan & Li (2013a), Wan & Dong (2015) and Wan et al. (2020). It is noteworthy that there are some minor errors in the definition and models in Wan & Li (2013a). According to (Chen & Tan, 1994; He et al., 2014), the score function and accuracy function of IFV $A = \langle \mu_A, \nu_A \rangle$ on [page 301] $S(A) = \mu_A + \nu_A$ and $H(A) = \mu_A - \nu_A$ should be revised to $S(A) = \mu_A - \nu_A$ and $H(A) = \mu_A + \nu_A$, respectively. Accordingly, the objective functions on [page 305, models (21)-(23)] are revised to the below ones:

$$\min\left\{ z_1 = 1 - \prod_{p=1}^{Q} \prod_{(k,j)\in\widetilde{\Omega}_p^{<0,1>}} (1-\mu_{\widetilde{C}_p(k,j)})^{\lambda_{kj}^p} - \prod_{p=1}^{Q} \prod_{(k,j)\in\widetilde{\Omega}_p^{<0,1>}} (\nu_{\widetilde{C}_p(k,j)})^{\lambda_{kj}^p} \right\}$$

$$\min\left\{ z_2 = 1 - \prod_{p=1}^{Q} \prod_{(k,j)\in\widetilde{\Omega}_p^{<0,1>}} (1-\mu_{\widetilde{C}_p(k,j)})^{\lambda_{kj}^p} + \prod_{p=1}^{Q} \prod_{(k,j)\in\widetilde{\Omega}_p^{<0,1>}} (\nu_{\widetilde{C}_p(k,j)})^{\lambda_{kj}^p} \right\}$$

where the meanings of notations are the same as those in Wan & Li (2013a). In addition, the methods by Wan & Li (2013a) and Wan & Dong (2015) do not admit HFVs, but IFVs. Thus, before applying these methods to solve Example 1, it is necessary to transform c_4 HFVs into IFVs using envelopes of HFVs (Torra, 2010). Furthermore, in our proposed method, positive and negative ideal solutions are provided in advance according to (13) - (14). Again, to obtain a fair comparison, for the method by Wan & Li (2013a) and Wan & Dong (2015), the positive and negative ideal solutions are also derived according to (13) - (14); for the method by Wan et al. (2020), it is assumed that the positive ideal solution is derived according to (13). Moreover, as suggested in (Wan & Li, 2013a), their threshold $\tilde{h} = \langle \mu_{\tilde{h}}, \nu_{\tilde{h}} \rangle$ is set to be $\langle 0.001, 0.9 \rangle$. As suggested in (Wan & Dong, 2015), their proportion parameter η is set to be 0.9; the preemptive priorities P_1, P_2, P_3 and P_4 are set to be $P_1 = P_4 = P_3 = P_4$. As recommended by Tversky & Kahneman (1979), for the method by Wan & Dong (2015), the risk-seeking coefficient α, risk-averse coefficient β and loss-averse coefficient ρ are set to be 0.88, 0.88 and 2.25, respectively; while the risk attitude parameter δ and threshold ε are set to be 0.5 and 0.01, respectively, as suggested in (Wan et al., 2020). The preferred orderings of merchants in Example 1 for the different methods are given in Table 3.

As one can see from Table 3, the methods by Wan & Dong (2015) and Wan et al. (2020) and our proposed method result in the same merchants preferred ordering ($z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$), while a slightly merchants preferred ordering ($z_4 \succ z_1 \succ z_5 \succ z_2 \succ z_3$) is obtained with the method by Wan & Li (2013a). However, all methods rank the same merchant (z_4) the best. One reason for these methods to obtain different preferred orderings of merchants is that the methods by Wan & Dong (2015) and Wan et al. (2020) and our proposed method all consider inconsistency and consistency relative to the positive and negative ideal solutions, while the method by Wan & Li (2013a) only captures inconsistency relative to the positive ideal solution and ignores the consistency and negative ideal solution. In fact, consistency is as important as inconsistency in group decision (Wan et al., 2017). Furthermore, according to (Liu & Zhang, 2013), the positive and negative ideal solutions are of equal importance during the procedure of decision making. Thus, both minimizing the inconsistency relative to positive and negative ideal solutions and maximizing consistency relative to positive and negative ideal solutions should be considered. Therefore, compared with the method by Wan & Li (2013a), the preferred ordering derived with the methods by Wan & Dong (2015) and Wan et al. (2020) and our proposed method is more comprehensive and reasonable. This finding verifies that our method is effective and can overcome the limitation of the method by Wan & Li (2013a).

In the above part, we have verified the effectiveness of our proposed method. However, because Example 1 assumes that the criteria are independent, it is hard to illustrate the validity and effectiveness of our method for handling the

Table 3
Ranking Results from Different Methods for Example 1

Method	Outcome	Preferred order
Wan & Li (2013a)'s approach	$S_1^1 = 0.0302, S_2^1 = 0.0669, S_3^1 = 0.0675,$ $S_4^1 = 0.0251, S_5^1 = 0.0651; S_1^2 = 0.0548,$ $S_2^2 = 0.0649, S_3^2 = 0.0678, S_4^2 = 0.0254,$ $S_5^2 = 0.0612; S_1^3 = 0.0774, S_2^3 = 0.1019,$ $S_3^3 = 0.1082, S_4^3 = 0.0458, S_5^3 = 0.0765.$	$z_4 \succ z_1 \succ z_5 \succ z_2 \succ z_3$
Wan & Dong (2015)'s approach	$D_1^1 = 0.7416, D_2^1 = 0.5535, D_3^1 = 0.4875,$ $D_4^1 = 0.7975, S_5^1 = 0.5468; D_1^2 = 0.5196,$ $D_2^2 = 0.5333, D_3^2 = 0.5429, D_4^2 = 0.7669,$ $D_5^2 = 0.4711; D_1^3 = 0.5179, D_2^3 = 0.4044,$ $D_3^3 = 0.3539, D_4^3 = 0.6689, D_5^3 = 0.4069.$	$z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$
Wan et al. (2020)'s approach	$V_1 = -0.1226, V_2 = -0.1460, V_3 = -0.1954,$ $V_4 = 0.0057, V_5 = -0.1661.$	$z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$
Our proposed method	$cd_1 = 0.5539, cd_2 = 0.5068, cd_3 = 0.4209,$ $cd_4 = 0.7048, cd_5 = 0.4301.$	$z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$

*__Note:__ $S_i^\tau (i = 1, 2, 3, 4, 5; \tau = 1, 2, 3)$ is the square of the distance of the merchant z_i from the ideal solution for the expert e_τ; D_i^τ is the comprehensive relative closeness degree of the merchant z_i for the expert e_τ; V_i is the collective overall prospect value of the merchant z_i.

MCGDM problem with interactive criteria. Thereby, a different example with an interactive condition is provided below to further justify the validity and effectiveness of our proposed method.

Example 2. *Like in Example 1, assume five well-known s-commerce websites, anonymously denoted as z_1, z_2, z_3, z_4 and z_5, and three experts e_1, e_2 and e_3 assessing their service quality to facilitate and support customers' choices of the optimal s-commerce website based on five criteria: c_1–visual aesthetics, c_2– customization, c_3–security, c_4–entertainment, c_5–community drivenness and c_6–user friendliness.*

- *It is assumed that the criteria have an inherent complex relationship, and their importance given by experts is* $\Lambda = \{0.1 \le \mu(\{c_1\}) \le 0.2, 0.1 \le \mu(\{c_2\}) \le 0.2, 0.05 \le \mu(\{c_3\}) \le 0.15, 0.05 \le \mu(\{c_4\}) \le 0.1, 0.2 \le \mu(\{c_5\}) \le 0.3, 0.25 \le \mu(\{c_6\}) \le 0.35, 0.1 \le \mu(\{c_1, c_2\}) \le 0.2, 0.4 \le \mu(\{c_4, c_5\}) \le 0.5\}$.
- *It is assumed that the uncertain weight information of the experts is given by* $\Gamma = \{0.3 \le \omega_1 \le 0.4, 0.3 \le \omega_2 \le 0.35, 0.3 \le \omega_3 \le 0.35\}$.
- *It is assumed that the assessment of the s-commerce website $z_i(i = 1, 2, 3, 4, 5)$ concerning the criterion $c_j(j = 1, 2, 3, 4, 5, 6)$ given by the expert $e_\tau(\tau = 1, 2, 3)$ is a q-ROFV $\tilde{A}_{ij}^\tau$. The decision matrices $\tilde{\boldsymbol{A}}^\tau = [\tilde{A}_{ij}^\tau]_{5\times 6}(\tau = 1, 2, 3)$ are tabulated in Table 4.*
- *In addition, experts offer the following q-ROF preference relations between websites:* $\Omega^1 = \{\langle(1, 2), t_{1,2}^1\rangle, \langle(5, 2), t_{5,2}^1\rangle\}$, $\Omega^2 = \{\langle(1, 3), t_{1,3}^2\rangle, \langle(4, 5), t_{4,5}^2\rangle\}$, $\Omega^3 = \{\langle(2, 1), t_{2,1}^3\rangle, \langle(5, 3), t_{5,3}^3\rangle\}$, *where the corresponding q-ROF truth degrees are:* $t_{1,2}^1 = \langle 0.5, 0.3\rangle$, $t_{5,2}^1 = \langle 0.7, 0.4\rangle$; $t_{1,3}^2 = \langle 0.5, 0.3\rangle$, $t_{4,5}^2 = \langle 0.8, 0.3\rangle$; $t_{2,1}^3 = \langle 0.4, 0.7\rangle$, $t_{5,3}^3 = \langle 0.6, 0.5\rangle$. *Thus, the supports of* Ω^1, Ω^2 *and* Ω^3 *are* $\Omega_{\langle 0,1\rangle}^1 = \{(1, 2), (5, 2)\}$, $\Omega_{\langle 0,1\rangle}^2 = \{(1, 3), (4, 5)\}$ *and* $\Omega_{\langle 0,1\rangle}^3 = \{(2, 1), (5, 3)\}$, *respectively.*

Table 4
Decision Making Matrices for Example 2

Expert	S-commerce	Criteria					
	website	c_1	c_2	c_3	c_4	c_5	c_6
e_1	z_1	⟨0.5, 0.4⟩	⟨0.6, 0.4⟩	⟨0.3, 0.6⟩	⟨0.4, 0.5⟩	⟨0.9, 0.1⟩	⟨0.7, 0.1⟩
	z_2	⟨0.7, 0.3⟩	⟨0.5, 0.5⟩	⟨0.4, 0.5⟩	⟨0.6, 0.2⟩	⟨0.5, 0.4⟩	⟨0.6, 0.2⟩
	z_3	⟨0.5, 0.4⟩	⟨0.7, 0.2⟩	⟨0.4, 0.6⟩	⟨0.6, 0.2⟩	⟨0.4, 0.6⟩	⟨0.8, 0.3⟩
	z_4	⟨0.7, 0.4⟩	⟨0.7, 0.3⟩	⟨0.7, 0.1⟩	⟨0.6, 0.3⟩	⟨0.8, 0.1⟩	⟨0.9, 0.2⟩
	z_5	⟨0.8, 0.2⟩	⟨0.5, 0.3⟩	⟨0.6, 0.5⟩	⟨0.4, 0.6⟩	⟨0.3, 0.6⟩	⟨0.7, 0.3⟩
e_2	z_1	⟨0.5, 0.4⟩	⟨0.5, 0.2⟩	⟨0.6, 0.3⟩	⟨0.7, 0.3⟩	⟨0.7, 0.1⟩	⟨0.6, 0.4⟩
	z_2	⟨0.7, 0.3⟩	⟨0.4, 0.7⟩	⟨0.5, 0.5⟩	⟨0.4, 0.5⟩	⟨0.4, 0.5⟩	⟨0.7, 0.2⟩
	z_3	⟨0.3, 0.4⟩	⟨0.7, 0.1⟩	⟨0.6, 0.5⟩	⟨0.5, 0.5⟩	⟨0.7, 0.2⟩	⟨0.7, 0.1⟩
	z_4	⟨0.6, 0.3⟩	⟨0.6, 0.2⟩	⟨0.4, 0.6⟩	⟨0.7, 0.2⟩	⟨0.9, 0.1⟩	⟨0.8, 0.1⟩
	z_5	⟨0.5, 0.4⟩	⟨0.5, 0.4⟩	⟨0.6, 0.2⟩	⟨0.5, 0.4⟩	⟨0.8, 0.3⟩	⟨0.8, 0.2⟩
e_3	z_1	⟨0.3, 0.6⟩	⟨0.7, 0.1⟩	⟨0.5, 0.3⟩	⟨0.6, 0.3⟩	⟨0.4, 0.4⟩	⟨0.7, 0.2⟩
	z_2	⟨0.5, 0.3⟩	⟨0.3, 0.5⟩	⟨0.4, 0.5⟩	⟨0.7, 0.2⟩	⟨0.5, 0.4⟩	⟨0.6, 0.2⟩
	z_3	⟨0.4, 0.4⟩	⟨0.5, 0.4⟩	⟨0.8, 0.1⟩	⟨0.5, 0.4⟩	⟨0.3, 0.7⟩	⟨0.4, 0.5⟩
	z_4	⟨0.8, 0.1⟩	⟨0.5, 0.5⟩	⟨0.8, 0.3⟩	⟨0.6, 0.4⟩	⟨0.7, 0.2⟩	⟨0.6, 0.2⟩
	z_5	⟨0.6, 0.1⟩	⟨0.6, 0.4⟩	⟨0.4, 0.5⟩	⟨0.4, 0.4⟩	⟨0.9, 0.1⟩	⟨0.7, 0.3⟩

To further illustrate the effectiveness of our proposed method to solve the MCGDM problem with the interactive condition, the ranking orders of our proposed method for Example 2 is compared with those of the following state-of-the-art methods: Liu et al. (2020)'s approach based on the q-ROF power weighed Maclaurin symmetric mean (q-ROFPWMSM) operator; Peng & Yang (2016)'s approach based on the PF Choquet integral average (PFCIA) operator; and Khan (2019)'s approach based on the PF Einstein Choquet integral averaging (PFECIA) operator. The prominent feature of these three methods (Khan, 2019; Liu et al., 2020; Peng & Yang, 2016) is that they can effectively solve MCGDM problems with complex relationship because the q-ROFPWMSM operator, PFCIA operator and PFECIA operator all capture complex relationship among the criteria. It should be noted that Wan & Li (2013a)'s approach, Wan & Dong (2015)'s approach and Wan et al. (2020)'s approach all assume that the criteria are independent and fail to deal with q-ROFVs, where $q > 1$, so they are not chosen for this comparison. Because Liu et al. (2020)'s approach, Peng & Yang (2016)'s approach, and Khan (2019)'s approach assume that the weights of criteria and experts are known, they cannot directly cope with MCGDM problems with incomplete weights. To apply Liu et al. (2020)'s approach to solve Example 2, we first determine experts' weights using the extended TOPSIS method (Wan et al., 2018) and then identify weights of criteria using the multi-objective parametric comprehensive deviation programming model (Wan et al., 2018). To apply Peng & Yang (2016)'s approach and Khan (2019)'s approach to solve Example 2, we first determine experts' weights using the extended TOPSIS method (Wan et al., 2018) and then identify optimal fuzzy measures on the criteria set using the entropy-based method (Liang et al., 2019). Moreover, a minor error is discovered in the definition of the entropy-based method (Liang et al., 2019). According to (Xia & Xu, 2012), the formula of cross-entropy $CE(\alpha_1, \alpha_2)$ of two q-ROFVs $\alpha_1 = \langle u_1, v_1 \rangle$ and $\alpha_2 = \langle u_2, v_2 \rangle$ on [page 3282, formula (9)] is wrong, and it should be revised to the below:

$$\begin{aligned} CE(\alpha_1, \alpha_2) = \frac{1}{1 - 2^{1-p}}\{ & \frac{(u_1)^{pq} + (u_2)^{pq}}{2} - (\frac{(u_1)^q + (u_2)^q}{2})^p + \frac{(v_1)^{pq} + (v_2)^{pq}}{2} \\ & - (\frac{(v_1)^q + (v_2)^q}{2})^p + \frac{(\pi_1)^{pq} + (\pi_2)^{pq}}{2} - (\frac{(\pi_1)^q + (\pi_2)^q}{2})^p\}, \end{aligned}$$

where the meanings of notations are the same as those in (Liang et al., 2019). Furthermore, according to the characteristic of Example 2, the parameters q and k of Liu et al. (2020)'s approach are both set to be 2; for the multi-objective parametric comprehensive deviation programming model (Wan et al., 2018), the parameter λ is set to be 0.5, and the weighting vector ζ is set to be $\zeta = (0.2, 0.2, 0.2, 0.2, 0.2)$; the parameter p of the entropy-based method (Liang et al., 2019) is set to be 1.5; and for our proposed method, the parameter q is set to be 2, with the other parameters being the same used in Example 1. The preferred orderings of merchants in Example 2 for the different methods are given in Table 5.

Table 5
Ranking Results from Different Methods for Example 2

Method	Outcome	Preferred order
Liu et al. (2020)'s approach	$S_1 = 0.2144, S_2 = 0.0844, S_3 = 0.1519,$ $S_4 = 0.4310, S_5 = 0.2563.$	$z_4 \succ z_5 \succ z_1 \succ z_3 \succ z_2$
Peng & Yang (2016)'s approach	$S_1 = 0.1658, S_2 = 0.0419, S_3 = 0.0711,$ $S_4 = 0.3752, S_5 = 0.1882.$	$z_4 \succ z_5 \succ z_1 \succ z_3 \succ z_2$
Khan (2019)'s approach	$S_1 = 0.2453, S_2 = 0.0788, S_3 = 0.1513,$ $S_4 = 0.4479, S_5 = 0.2798.$	$z_4 \succ z_5 \succ z_1 \succ z_3 \succ z_2$
Our proposed method	$cd_1 = 0.4742, cd_2 = 0.2893, cd_3 = 0.4390,$ $cd_4 = 0.7524, cd_5 = 0.5458.$	$z_4 \succ z_5 \succ z_1 \succ z_3 \succ z_2$

***Note:** $S_i (i = 1, 2, 3, 4, 5)$ is the score value of the merchant z_i.

As one can see from Table 5, Liu et al. (2020)'s approach, Peng & Yang (2016)'s approach, Khan (2019)'s approach, and our proposed method obtain the same preferred ordering ($z_4 \succ z_5 \succ z_1 \succ z_3 \succ z_2$), which verifies that our method is effective and can validly solve MCGDM problems with interactive criteria.

5.2.2. Verifying the superiority of the proposed method

It can be easily concluded that Examples 1 and 2 are evidence of the effectiveness and validity of our proposed method. Nevertheless, our method derives the same ranking order than previous approaches (Wan & Dong, 2015; Wan et al., 2020) and (Khan, 2019; Liu et al., 2020; Peng & Yang, 2016) in Examples 1 and Example 2, respectively, which makes it hard to explain the superiority of our proposed approach well. Therefore, two illustrative cases are given below to further compare our method with the existing approaches (Khan, 2019; Liu et al., 2020; Peng & Yang, 2016; Wan & Dong, 2015; Wan et al., 2020) regarding the merits of our proposed approach.

Example 3. *In Example 1, independence of criteria is assumed independent. However, in a real situation, the considered criteria exhibit heterogeneous relationship, ranging from a negative synergetic interaction to a positive synergetic interaction. For instance, generally speaking, the longer the delivery time c_2 is, the lower the consumer approval degree c_6 is; the better the product quality c_3 is, the higher the product price c_1 is. As a consequence, it is nor realistic to assume independence of criteria. Therefore, criteria interaction is added to Example 1 by assuming the following incomplete* *weight information:*

$$0.1 \le \mu(\{c_1\}) \le 0.15, 0.05 \le \mu(\{c_2\}) \le 0.1, 0.15 \le \mu(\{c_3\}) \le 0.25, 0.25 \le \mu(\{c_4\}) \le 0.3,$$
$$0.2 \le \mu(\{c_5\}) \le 0.3, 0 \le \mu(\{c_6\}) \le 0.1, 0.5 \le \mu(\{c_1, c_4\}) \le 1, 0.05 \le \mu(\{c_2, c_6\}) \le 0.15,$$
$$0.4 \le \mu(\{c_3, c_5\}) \le 1, 0.4 \le \mu(\{c_5, c_6\}) \le 1.$$

The results obtained with the compared different methods is given in Table 6 with the parameter q of q-ROFVs set to be 1, *and the rest of parameters for the different methods are those used in the previous examples.*

Table 6
Ranking Results from Different Methods for Example 3

Method	Outcome	Preferred order
Wan & Li (2013a)'s approach	$S_1^1 = 0.0302, S_2^1 = 0.0669, S_3^1 = 0.0675,$ $S_4^1 = 0.0251, S_5^1 = 0.0651; S_1^2 = 0.0548,$ $S_2^2 = 0.0649, S_3^2 = 0.0678, S_4^2 = 0.0254,$ $S_5^2 = 0.0612; S_1^3 = 0.0774, S_2^3 = 0.1019,$ $S_3^3 = 0.1082, S_4^3 = 0.0458, S_5^3 = 0.0765.$	$z_4 \succ z_1 \succ z_5 \succ z_2 \succ z_3$
Wan & Dong (2015)'s approach	$D_1^1 = 0.7416, D_2^1 = 0.5535, D_3^1 = 0.4875,$ $D_4^1 = 0.7975, D_5^1 = 0.5468; D_1^2 = 0.5196,$ $D_2^2 = 0.5333, D_3^2 = 0.5429, D_4^2 = 0.7669,$ $D_5^2 = 0.4711, D_1^3 = 0.5179, D_2^3 = 0.4044,$ $D_3^3 = 0.3539, D_4^3 = 0.6689, D_5^3 = 0.4069.$	$z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$
Wan et al. (2020)'s approach	$V_1 = -0.1226, V_2 = -0.1460, V_3 = -0.1954,$ $V_4 = 0.0057, V_5 = -0.1661.$	$z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$
Liu et al. (2020)'s approach	Cannot be computed	Cannot be derived
Peng & Yang (2016)'s approach	Cannot be computed	Cannot be derived
Khan (2019)'s approach	Cannot be computed	Cannot be derived
Our proposed method	$cd_1 = 0.6037, cd_2 = 0.5107, cd_3 = 0.3977,$ $cd_4 = 0.6922, cd_5 = 0.3807.$	$z_4 \succ z_1 \succ z_2 \succ z_3 \succ z_5$

***Note:** The meanings of $S_i^\tau (i = 1, 2, 3, 4, 5; \tau = 1, 2, 3)$, D_i^τ and V_i are the same as those in Table 3.

From Table 6, one can notice that when independent criteria of the heterogeneous MCGDM problem is modified to interactive criteria, Wan & Li (2013a)'s approach still retains the same ranking order ($z_4 \succ z_1 \succ z_5 \succ z_2 \succ z_3$) because it fails to capture interactions among criteria. Example 3 exhibits a complex relationship. For instance, delivery time c_2 and consumer approval c_6 exhibit a negative synergetic interaction. In other words, the comprehensive weight of these two criteria considered together is lower than the sum of the weight of the two criteria when considered separately. Therefore, the ranking order obtained by Wan & Li (2013a)'s approach is not reasonable for this purpose because it omits the interaction among criteria. Additionally, one can notice that when independent criteria of the heterogeneous MCGDM problem is modified to interactive criteria, Wan & Dong (2015)'s approach and Wan et al. (2020)'s approach also obtain the same ranking because they are based on the following equalities: $\mu(\{c_j, c_q\}) = \mu(\{c_j\}) + \mu(\{c_q\})$ $(j, q = 1, 2, 3, 4, 5, 6; j \neq q)$, which are invalid in Example 3. Therefore, the ranking results obtained by Wan & Dong (2015)'s approach and Wan et al. (2020)'s approach are also not reasonable for this purpose. One can further notice that Liu et al. (2020)'s approach, Peng & Yang (2016)'s approach and Peng & Yang (2016)'s approach fail to handle this example and a ranking result is not obtained. This is because Example 3 is a type of heterogeneous MCGDM problem, whereas these methods can only solve homogeneous MCGDM problems. Moreover, one can notice that when the independent criteria of the heterogeneous MCGDM problem is modified into dependent criteria, the preferred order obtained by our approach changes from $z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$ to $z_4 \succ z_1 \succ z_2 \succ z_3 \succ z_5$, although the optimal merchant remains the same. It is apparent that the ranking result of merchants obtained by our proposed method fits the new dependence assumption because it is based on the following inequalities: $\mu(\{c_1, c_3\}) = 0.4128 > \mu(\{c_1\}) + \mu(\{c_3\}) = 0.3395$, $\mu(\{c_2, c_6\}) = 0.0927 < \mu(\{c_2\}) + \mu(\{c_6\}) = 0.1228$, $\mu(\{c_3, c_4\}) = 0.3927 < \mu(\{c_3\}) + \mu(\{c_4\}) = 0.4578$. That is, our approach captures the positive synergetic interaction among the criteria c_1 and c_3, the negative synergetic interaction among the criteria c_2 and c_6, and the negative synergetic interaction among the criteria c_3 and c_4. Therefore, our approach derives a realistic and convincing ranking result, which is not the case with the existing approaches it was compared with.

Example 4. *In Example 3, all q-ROFVs* A_{ij}^τ $(i = 1, 2, 3, 4, 5;\ j = 5, 6; \tau = 1, 2, 3)$ *and* t_{ki}^τ $(k, i = 1, 2, 3, 4, 5; \tau = 1, 2, 3)$ *verify the restriction that the sum of MD and NMD is* $[0, 1]$. *In reality, specially for high ambiguous decision-making problems, experts may provide data with sum of MD and NMD greater than* 1. *For instance, when the expert* e_3 *assesses consumer approval* c_6 *of merchant* z_2 *according to information sharing behaviour and information collection status, he/she may believe the consumer approval is very good all in all and provides an assessment value of 0.95 as its MD. However, sometimes he/she may believe the consumer is unwilling to share information to some extent due to some influencing factors, such as environmental factors, technical factors and personal factors and provides an assessment value of* 0.4 *as its NMD. Accordingly, the criterion value* A_{26}^3 *would be* $\langle 0.95, 0.4 \rangle$. *Analogously, expert* e_3 *may believe the value* $t_{2,3}^3$ *is* $\langle 0.95, 0.6 \rangle$. *Assuming that the other decision information remains identical to that of Example 3, the ranking results of* *the merchants obtained by all compared methods are provided in Table 7, with the parameter q of q-ROFVs set to be* 4*, and the rest of parameters for the different methods are those used in the previous examples.*

Table 7
Ranking Results from Different Methods for Example 4

Method	Outcome	Preferred order
Wan & Li (2013a)'s approach	Cannot be computed	Cannot be derived
Wan & Dong (2015)'s approach	Cannot be computed	Cannot be derived
Wan et al. (2020)'s approach	Cannot be computed	Cannot be derived
Liu et al. (2020)'s approach	Cannot be computed	Cannot be derived
Peng & Yang (2016)'s approach	Cannot be computed	Cannot be derived
Khan (2019)'s approach	Cannot be computed	Cannot be derived
Our proposed method	$cd_1 = 0.4831, cd_2 = 0.4836, cd_3 = 0.3527,$ $cd_4 = 0.5868, cd_5 = 0.3459.$	$z_4 \succ z_2 \succ z_1 \succ z_3 \succ z_5$

From Table 7, one can observe that Wan & Li (2013a)'s approach, Wan & Dong (2015)'s approach, Wan et al. (2020)'s approach, Liu et al. (2020)'s approach, Peng & Yang (2016)'s approach, and Khan (2019)'s approach fail to solve the above revised example, whereas our proposed method is able to obtain a preferred ordering of the five merchants ($z_4 \succ z_2 \succ z_1 \succ z_3 \succ z_5$). Wan & Li (2013a)'s approach, Wan & Dong (2015)'s approach, and Wan et al. (2020)'s approach are based on IFVs, whose MD g and NMD f need to verify the restriction $g + f < 1$. However, the revised values $A^3_{26} = \langle 0.95, 0.4\rangle$ and $t^3_{2,3} = \langle 0.95, 0.6\rangle$ are not IFVs because $0.95 + 0.4 > 1$ and $0.95 + 0.6 > 1$. Thus, they are unable to derive a decision result. As mention before, Liu et al. (2020)'s approach, Peng & Yang (2016)'s approach are not able to cope this example since it is a type of heterogeneous MCGDM problem. Thus, they are also unable to derive a decision result. Our proposed method can effectively cope with the heterogeneous MCGDM problem with q-ROFVs. The prominent feature of q-ROFVs is that their MD g and NMD f verify the restriction that $(g)^q + (f)^q < 1$, where $q \geq 1$. The revised values $A^3_{26} = \langle 0.95, 0.4\rangle$ and $t^3_{2,3} = \langle 0.95, 0.6\rangle$ are q-ROFVs for $q \geq 4$ because $0.95^4 + 0.4^4 < 1$ and $0.95^4 + 0.6^4 < 1$. It is apparent that q-ROFVs are more general and flexible than IFVs as the value of parameter q can be adjusted dynamically to control the scope of the fuzzy information expression. This finding implies that our proposed method can overcome the shortcoming of existing methods. Therefore, our proposed method is more practical and feasible to address real complex decision-making problems.

5.2.3. Sensitivity analysis

To explore the influence of distinct values of parameter q on the results of Example 4, we set the parameter q to be 4, 5, 6, 7, 8, 9, 10 and 20 in our proposed method, to assess the derived ranking of the merchants. The decision results are depicted in Fig. 4, from which the following conclusions are drawn.

- Slightly different preferred orderings of merchants are derived for distinct values of q, though the optimal and worst merchants remain consistent. This observation implies that the decision results are sensitive to the values of q to a certain extent.
- The relative closeness degrees (for the same merchant) derived by our proposed method on the whole decrease as the parameter q increases.
- The parameter q can be regarded as the DM's "attitude": the larger the value of parameter q becomes, the more optimistic the DM is; the smaller the value of q becomes, the more pessimistic the DM is.

There are two ways to determine the value of q. One way is to determine the most desirable value for a particular problem based on the DM's preferences and/or specific requirements (Tang et al., 2020b). The other way is to choose the minimum integer q verifying the restriction $(t)^q + (f)^q \leq 1$ (Liu et al., 2020). For example, if we assume the assessment value $\langle 0.95, 0.6\rangle$, then since $0.95^3 + 0.6^3 > 1$ and $0.95^4 + 0.6^4 < 1$ the parameter q could be set to be 4.

5.2.4. Summary of the proposed method

From the above analysis, features of all compared methods are summarized in Table 8. It can be concluded that our proposed method is a strong alternative to the state-of-the-art approaches to MCGDM problems, and its unique features make it highly attractive for heterogeneous MCGDM problems with incomplete weights and interactive criteria.

6. Conclusion

This study proposed a multi-objective q-ROF programming method for heterogeneous MCGDM with incomplete weights and q-ROF truth degrees based on 2AFMs, the LINMAP and the A-NSGA-III. Four parts were involved: an information acquisition process, the calculation of the Choquet-based relative closeness degree, the establishment of the q-ROF programming model process, and the resolution of the established model and determination of the ranking of alternatives.

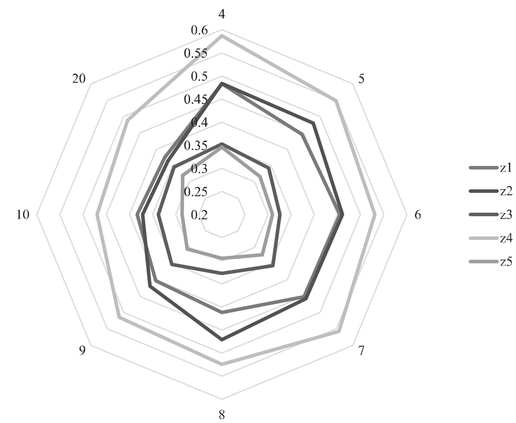

Fig. 4. Radar plot showing the effect of parameter q on the decision results

Table 8
Comparisons of Different Approaches

Features / Methods	Can depict the heterogeneous information	Can consider the interaction among the criteria	Can express a wilder range of information
Wan & Li (2013a)'s approach	Yes	No	No
Wan & Dong (2015)'s approach	Yes	No	No
Wan et al. (2020)'s approach	Yes	No	No
Liu et al. (2020)'s approach	No	Yes	Yes
Peng & Yang (2016)'s approach	No	Yes	No
Khan (2019)'s approach	No	Yes	No
Our proposed method	Yes	Yes	Yes

Features / Methods	Can simultaneously optimize many objectives without any predefined weighting coefficients	Can objectively determine the weight information
Wan & Li (2013a)'s approach	No	Yes
Wan & Dong (2015)'s approach	No	Yes
Wan et al. (2020)'s approach	No	Yes
Liu et al. (2020)'s approach	No	No
Peng & Yang (2016)'s approach	No	No
Khan (2019)'s approach	No	No
Our proposed method	Yes	Yes

- In the first part, q-ROFVs were introduced to model and represent criteria values and pairwise comparisons of alternatives. The outstanding characteristic of q-ROFVs is its parameter $q \geq 1$, so that they can describe the range of fuzzy information in a more flexible manner.
- In the second part, a formulation of the Choquet-based relative closeness degree based on 2AFMs, which can capture the interaction among criteria, was obtained.
- In the third part, to objectively determine weighting vectors of criteria and experts, a multi-objective q-ROF programming model minimizing the q-ROFCGII and maximizing the q-ROFCGCI was established.
- In the fourth part, an algorithm was designed based on the A-NSGA-III to solve the established model. Then, TOPSIS was employed to select the desirable solution as the weighting result from the last Pareto set. Finally, the collective ranking order of alternatives was generated.

To illustrate the effectiveness and superiority of our proposed method, four application cases were used to conduct a comparative analysis between our proposed method and existing MCGDM methods (Khan, 2019; Liu et al., 2020; Peng & Yang, 2016; Wan & Dong, 2015; Wan & Li, 2013a; Wan et al., 2020). The experimental results provided evidence of our proposed method outperforming the existing MCGDM methods. In future, we will focus our research efforts on the identification of some parameters involved in our proposed method using machine learning. Furthermore, we shall apply the proposed approach to solve some real decision making problems, such as investment evaluation (Huang et al., 2020; Tang et al., 2022a), medical diagnosis (Lin et al., 2021b) and selection of cloud service productions(Lin et al., 2020).

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Appendix I

(1) We first explain the monotonicity of objective functions as follows:

The first objective function is:

$$\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} \omega_\tau \xi^{\tau}_{ki} t^{\tau}_{k,i} = \left\langle \sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}}, \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (f_{t^{\tau}_{k,i}})^{\omega_\tau \xi^{\tau}_{ki}} \right\rangle.$$

In the following, its monotonicity is proved. If $g_{t^{\tau}_{k,i}} = 1$, we can derive $\sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}} = 0$. Suppose that $g_{t^{\tau}_{k,i}} \neq 1$ and let $h(g_{t^{\tau}_{k,i}}) = \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}$, then we derive

$$ln\left(h(g_{t^{\tau}_{k,i}})\right) = ln\left(\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}\right) = \omega_\tau \xi^{\tau}_{ki}\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} ln\left(1-(g_{t^{\tau}_{k,i}})^q\right).$$

In addition, the partial derivative of $ln\left(h(g_{t^{\tau}_{k,i}})\right)$ with respect to $g_{t^{\tau}_{k,i}}$, where ω_τ, ξ^{τ}_{ki} and q are constant, is:

$$\frac{\partial ln\left(h(g_{t^{\tau}_{k,i}})\right)}{\partial g_{t^{\tau}_{k,i}}} = \omega_\tau \xi^{\tau}_{ki}\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} \frac{1}{(1-(g_{t^{\tau}_{k,i}})^q)}(-q(g_{t^{\tau}_{k,i}})^{q-1}).$$

Since $0 \leq g_{t^{\tau}_{k,i}} < 1$, $q \geq 1$ and $\xi^{\tau}_{ki} \geq 0$, we derive $1-(g_{t^{\tau}_{k,i}})^q > 0$ and $-q(g_{t^{\tau}_{k,i}})^{q-1} < 0$. Thus, $\frac{\partial ln\left(h(g_{t^{\tau}_{k,i}})\right)}{\partial g_{t^{\tau}_{k,i}}} \leq 0$. In other words, $h(g_{t^{\tau}_{k,i}})$ is non-monotonic increasing with respect to $g_{t^{\tau}_{k,i}}$. Therefore, $\sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}}$ is non-monotonic decreasing with respect to $g_{t^{\tau}_{k,i}}$.

Similarly, we can derive that $\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (f_{t^{\tau}_{k,i}})^{\omega_\tau \xi^{\tau}_{ki}}$ is non-monotonic increasing with respect to $f_{t^{\tau}_{k,i}}$.

The second objective function is:

$$\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} \omega_\tau \lambda^{\tau}_{ki} t^{\tau}_{k,i} = \left\langle \sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \lambda^{\tau}_{ki}}}, \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (f_{t^{\tau}_{k,i}})^{\omega_\tau \lambda^{\tau}_{ki}} \right\rangle.$$

Similar to the proof of the first objective function, we can derive that $\sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \lambda^{\tau}_{ki}}}$ is non-monotonic decreasing with respect to $g_{t^{\tau}_{k,i}}$ and $\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (f_{t^{\tau}_{k,i}})^{\omega_\tau \lambda^{\tau}_{ki}}$ is non-monotonic increasing with respect to $f_{t^{\tau}_{k,i}}$.

(2) The boundedness of objective functions is explained as follows:

Since $0 \leq g_{t^{\tau}_{k,i}} \leq 1$, $q \geq 1$, $0 \leq \omega_\tau \leq 1$, we can derive $0 \leq 1-(g_{t^{\tau}_{k,i}})^q \leq 1$ and $0 \leq \sqrt[q]{1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}}} \leq 1$.

Then, since $0 \leq f_{t^{\tau}_{k,i}} \leq 1$, $q \geq 1$, $0 \leq \omega_\tau \leq 1$, we can easily derive $0 \leq \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (f_{t^{\tau}_{k,i}})^{\omega_\tau \xi^{\tau}_{ki}} \leq 1$.

In addition, we derive

$$\begin{aligned} &1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} (1-(g_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}} + \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} ((f_{t^{\tau}_{k,i}})^q)\omega_\tau \xi^{\tau}_{ki} \\ \leq &1-\prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} ((f_{t^{\tau}_{k,i}})^q)^{\omega_\tau \xi^{\tau}_{ki}} + \prod_{\tau=1}^{o}\prod_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} ((f_{t^{\tau}_{k,i}})^q)\omega_\tau \xi^{\tau}_{ki} = 1. \end{aligned}$$

Thus, $\sum_{\tau=1}^{o}\sum_{(k,i)\in\Omega^{\tau}_{\langle 0,1\rangle}} \omega_\tau \xi^{\tau}_{ki} t^{\tau}_{k,i}$ is a q-ROFV. Therefore, the first objective function meets the boundedness.

Similarly, we can derive that the second objective function meets the boundedness.

(3) The boundedness and monotonicity solution of the q-ROF programming is explained as follows:

By solving the established q-ROF programming in (32), we can identify the optimal fuzzy measures on criteria set $\boldsymbol{W}^*$ and the optimal experts' weights $\boldsymbol{\omega}^*$. Because the experts' weights meet the condition $\sum_{\tau=1}^{0}\omega_\tau = 1$. Thus, experts' weights do not verify monotonicity. Meanwhile, because the optimal fuzzy measures on criteria set meet the condition $\sum_{\{c_j,c_q\}\subseteq C}\mu(\{c_j,c_q\})-(n-2)\sum_{\{c_j\}\subseteq C}\mu(\{c_j\}) = 1$. Obviously, the optimal fuzzy measures on criteria set also do not verify monotonicity. Therefore, the solutions of the q-ROF programming do not satisfy the monotonicity. As for the boundedness of solution, because the fuzzy measures on criteria set satisfy boundedness and experts' weights also verify boundedness, the solutions of the q-ROF programming composed of fuzzy measures on criteria set and experts' weights meet boundedness.

Appendix II

The detailed procedure description of the A-NSGA-III-based optimization algorithm is described as follows.

Stage 1. Initialization Process

Step 1: Specify population size N, reference point divisions p_d, maximum iteration number t_{max}, simulated binary crossover (SBX) possibility p_c, SBX recombination parameter index η_c, polynomial mutation (PM) possibility p_m and PM recombination parameter index η_m.

Step 2: Initialize the population $\boldsymbol{P}^0$, which comprises of N individuals $\boldsymbol{P}^0 = [\boldsymbol{P}_1^0, \boldsymbol{P}_2^0, \cdots, \boldsymbol{P}_N^0]$. The ιth individual $\boldsymbol{P}_\iota^0(\iota = 1, 2, \cdots, N)$ is denoted by a vector $(\boldsymbol{W}_\iota^0, \boldsymbol{\omega}_\iota^0)^T$, where $\boldsymbol{W}_\iota^0 = (\mu_\iota^0(\{c_1\}), \mu_\iota^0(\{c_2\}), \cdots, \mu_\iota^0(\{c_{n-1}, c_n\}))^T$ and $\boldsymbol{\omega}_\iota^0 = (w_{\iota 1}^0, w_{\iota 2}^0, \cdots, w_{\iota o}^0)^T$, where $\mu_\iota^0(\{c_j\})(j = 1, 2, \cdots, n)$, $\mu_\iota^0(\{c_j, c_q\}(q = 1, 2, \cdots, n; j \neq q)$ and $w_{\iota\tau}^0(\tau = 1, 2, \cdots, o)$ are randomly generated, and they verify the following box constraints: $0 \leq \mu_\iota^0(\{c_j\}) \leq 1$, $0 \leq \mu_\iota^0(\{c_j, c_q\} \leq 1$, $0 \leq w_{\iota\tau}^0 \leq 1$, $\mu_\iota^0(\{c_j\}) \in \Lambda$, $\mu_\iota^0(\{c_j, c_q\} \in \Lambda$ and $w_{\iota\tau}^0 \in \Gamma$. Additionally, $\mu_\iota^0(\{c_n, c_{n-1}\})$ is determined by $\mu_\iota^0(\{c_n, c_{n-1}\}) = (n-2) \sum_{\{c_j\} \subseteq C} \mu_\iota^0(\{c_j\}) - 1 - \sum_{\{c_j, c_p\} \subseteq C \setminus \{c_n, c_{n-1}\}} \mu_\iota^0(\{c_j, c_p\})$, and $w_{\iota o}^0$ is determined by $w_{\iota o}^0 = 1 - w_{\iota o}^0 - \cdots - w_{\iota o-1}^0$.

After the initialization process, set the number of iterations *ite* to 0. Repeatedly create offspring population, perform non-dominated sorting, conduct selection mechanism and make it adaptive in the reference points, and then increase the number of iterations *ite* by 1 for each iteration, until the number of iterations *ite* reaches the maximum iteration number t_{max}. Let us consider the tth generation of the A-NSGA-III-based optimization algorithm.

Stage 2. Creation of Offspring Population

Step 3: Calculate the objective value vector $\boldsymbol{Z}_\iota^t = (Z_1(\boldsymbol{P}_\iota^t), Z_2(\boldsymbol{P}_\iota^t), Z_3(\boldsymbol{P}_\iota^t), Z_4(\boldsymbol{P}_\iota^t))$ and constraint violation value CV_ι^t of each individual $\boldsymbol{P}_\iota^t$, where $\boldsymbol{Z}_\iota^t$ and CV_ι^t are derived by objective functions and constraints in (35), respectively. According to the formula of constraint violation measure (Jain & Deb, 2013), one can observe that $CV_\iota^t \geq 0$. For individual $\boldsymbol{P}_\iota^t$, when $CV_\iota^t > 0$, it is deemed as an infeasible solution; otherwise, it is a feasible solution.

Stage 4: Select two members $\boldsymbol{P}_\iota^t$ and $\boldsymbol{P}_\nu^t$ from $\boldsymbol{P}^t$ at random, and then, if $D_\iota^t = D_\nu^t = 0$, randomly choose a solution $\bar{\boldsymbol{P}}_\varepsilon^t$ between them; otherwise, choose a better solution $\bar{\boldsymbol{P}}_\varepsilon^t$ with a smaller constraint violation value. In this way, the second solution $\bar{\boldsymbol{P}}_\epsilon^t$ is selected. Next, the SBX (Deb & Agrawal, 1995) and PM (Deb & Goyal, 1996) are applied on $\bar{\boldsymbol{P}}_\varepsilon^t$ and $\bar{\boldsymbol{P}}_\epsilon^t$ to generate two offspring solutions $\dot{\boldsymbol{Q}}_\varepsilon^t$ and $\dot{\boldsymbol{Q}}_\epsilon^t$. This process is continued until N offspring are generated to form the population $\dot{\boldsymbol{Q}}^t$.

Stage 5: Because (35) involves many box constraints, some offspring (let their number be $\dot{N}$, where $0 \leq \dot{N} \leq N$) produced after crossover and mutation no longer meet box constraints. This situation increases the difficulty for the algorithm to converge. To improve the evolution efficiency, a correction mechanism is applied to handle this issue: randomly generate $\dot{N}$ individuals that meet box constraints to replace the above offspring, so a modified offspring population $\boldsymbol{Q}^t$ of N individuals is derived.

Stage 3. Non-dominated Sorting

Step 6: Merge $\boldsymbol{P}^t$ and $\boldsymbol{Q}^t$ to obtain a mixed population $\boldsymbol{R}^t = \boldsymbol{P}^t \cup \boldsymbol{Q}^t$.

Step 7: Sort the population $\boldsymbol{R}^t$ according to the constraint domination principle. Let $\boldsymbol{R}_\iota^t$ and $\boldsymbol{R}_\nu^t$ be two individuals from the population $\boldsymbol{R}^t$. Then,

- If $\boldsymbol{R}_\iota^t$ and $\boldsymbol{R}_\nu^t$ are feasible and infeasible, respectively, then $\boldsymbol{R}_\iota^t$ dominates $\boldsymbol{R}_\nu^t$.
- If $\boldsymbol{R}_\iota^t$ and $\boldsymbol{R}_\nu^t$ are both infeasible, then the solution with a smaller constraint violation value dominates the other one.
- If $\boldsymbol{R}_\iota^t$ and $\boldsymbol{R}_\nu^t$ are feasible, the following domination principle is used: if $\forall i(i = 1, 2, 3, 4)$, $Z_i(\boldsymbol{R}_\iota^t) \leq Z_i(\boldsymbol{R}_\nu^t)$, and $\exists i$, $Z_i(\boldsymbol{R}_\iota^t) < Z_i(\boldsymbol{R}_\nu^t)$, then $\boldsymbol{R}_\iota^t$ dominates $\boldsymbol{R}_\nu^t$.

The individuals that are not dominated by others in the population are classified into the non-dominated level $\boldsymbol{F}_1^t$. These individuals are then removed to find the novel level $\boldsymbol{F}_2^t$. Repeatedly perform this procedure until individuals in $\boldsymbol{R}^t$ are classified into l non-dominated levels $\boldsymbol{F}_1^t, \boldsymbol{F}_2^t, \cdots, \boldsymbol{F}_l^t$, where $|\boldsymbol{F}_1^t| + |\boldsymbol{F}_2^t| + \cdots + |\boldsymbol{F}_{l-1}^t| < N$ and $|\boldsymbol{F}_1^t| + |\boldsymbol{F}_2^t| + \cdots + |\boldsymbol{F}_l^t| \geq N$.

Stage 4. Selection Mechanism

Step 8: All population members from non-dominated level $\boldsymbol{F}_1^t$ to $\boldsymbol{F}_l^t$ are involved in $\boldsymbol{S}^t$. If $|\boldsymbol{S}^t| = N$, the next generation is started with $\boldsymbol{P}^{t+1} = \boldsymbol{S}^t$; if $|\boldsymbol{S}^t| > N$, then $\boldsymbol{P}^{t+1} = \cup_1^{l-1} \boldsymbol{F}_i^t$, and the remaining $N - |\boldsymbol{P}^{t+1}|$ population members are selected from the last front $\boldsymbol{F}_l^t$ according to the reference points-based individual selection mechanism. This mechanism is of paramount importance to many-objective optimization problems because it effectively maintains the diversity of solutions. It includes the following four steps.

Step 9: Generate uniformly distributed reference points on the hyperplane using the simplex lattice design approach Das & Dennis (1998). For a 4-dimensional objective space, when p divisions are selected for each objective axis, this approach produces $H = C_{3+p}^p$ reference points, where C is combination.

Step 10: Normalize each objective value of $\boldsymbol{S}^t$ using $\dot{Z}_i(\boldsymbol{S}) = Z_i(\boldsymbol{S}) - Z_i^{\min}(\boldsymbol{S})(i = 1, 2, 3, 4)$, where $Z_i(\boldsymbol{S})$ is the ith objective value of solution $\boldsymbol{S} \in \boldsymbol{S}^t$, $Z_i^{\min}(\boldsymbol{S})$ is the ith dimension of the ideal point that is determined by the minimum value of each objective function in $\cup_0^t \boldsymbol{S}^\varsigma$. Then, $\dot{Z}_i(\boldsymbol{S})$ is further normalized using $\ddot{Z}_i(\boldsymbol{S}) = \frac{\dot{Z}_i(\boldsymbol{S})}{Z_i^{nadir}(\boldsymbol{S}) - Z_i^{\min}(\boldsymbol{S})}$, where $Z_i^{nadir}(\boldsymbol{S})$ is the ith dimension of the nadir point, which is derived using the approach of (Yuan et al., 2015).

Step 11: Associate each population member of $\boldsymbol{S}^t$ with one of these H reference points using the minimum perpendicular distance (Jain & Deb, 2013).

Step 12: Select $N - |\boldsymbol{P}^{t+1}|$ members in $\boldsymbol{F}_l^t$ using the following niche-preserving operation. Select the reference point with the least number of associated individuals from $\boldsymbol{P}^{t+1}$. Then, there are three cases:

- If this reference point has no associated individual in $\boldsymbol{F}_l^t$, it is removed.
- If this reference point has associated individuals in $\boldsymbol{F}_l^t$ and no associated individual in previous levels, select the nearest individual associated with this reference point and add it into population $\boldsymbol{P}^{t+1}$.
- If this reference point both has associated individuals in $\boldsymbol{F}_l^t$ and previous levels, select one of the associated individuals in $\boldsymbol{F}_l^t$ at random and add it into population $\boldsymbol{P}^{t+1}$.

This process is continued until $N - |\boldsymbol{P}^{t+1}|$ members in $\boldsymbol{F}_l^t$ are selected.

Stage 5. Adaptation of Reference Points

Step 13: For the addition of reference points, we first find crowded reference points for which $\rho_j^t \geq 2$, where ρ_j^t is the number of population members that are associated with j-th reference point. Then, for each of these reference points, we simply introduce a simplex of 4 points having a distance between them that is the same as the distance between two consecutive reference points on the original simplex.

Step 14: After the addition operation is conducted, the niche counts of all reference points are updated. Next, all added reference points (excluding the original reference points) having $\rho_j^t = 0$ are deleted. Thus, the original reference points are always kept, as are all those added reference points that have a niche count of exactly one.

Appendix III

To select the optimal merchant, the proposed q-ROF programming method is used and the process is summarized below:

Steps 1-5: See the detailed description of Example 1.

Step 6: (12) is used to normalize the heterogeneous decision matrices $\tilde{\boldsymbol{A}}^\tau = [\tilde{A}_{ij}^\tau]_{5\times 6} (\tau = 1, 2, 3)$ into the normalized decision matrices $\boldsymbol{A}^\tau = [A_{ij}^\tau]_{5\times 6}$, as shown in Table 1a.

Table 1a
Normalized Heterogeneous Decision Making Matrices for Example 1

Expert	Merchants	Criteria					
		c_1	c_2	c_3	c_4	c_5	c_6
e_1	z_1	0.375	[0.2857, 0.5714]	(0.5556, 0.6667, 0.7778, 1)	{0.5, 0.6, 0.7}	⟨0.6, 0.3⟩	⟨0.5, 0.2⟩
	z_2	0.3125	[0.4286, 0.7143]	(0.2222, 0.5556, 0.6667, 1)	{0.6, 0.75, 0.9}	⟨0.4, 0.5⟩	⟨0.6, 0.3⟩
	z_3	0.125	[0, 0.4286]	(0.3333, 0.5556, 0.7778, 0.8889)	{0.3, 0.5, 0.6}	⟨0.7, 0.1⟩	⟨0.4, 0.5⟩
	z_4	0.1875	[0.5714, 0.7143]	(0.4444, 0.6667, 0.8889, 1)	{0.3, 0.4, 0.8}	⟨0.8, 0.1⟩	⟨0.7, 0.3⟩
	z_5	0	[0.5714, 0.8571]	(0.5556, 0.7778, 0.8889, 1)	{0.5, 0.7, 0.9}	⟨0.4, 0.5⟩	⟨0.3, 0.5⟩
e_2	z_1	0.25	[0.4286, 0.7143]	(0.4444, 0.5556, 0.7778, 0.8889)	{0.3, 0.5, 0.6}	⟨0.5, 0.3⟩	⟨0.7, 0.3⟩
	z_2	0.25	[0.1429, 0.4286]	(0.3333, 0.4444, 0.7778, 0.8889)	{0.5, 0.7, 0.8}	⟨0.5, 0.4⟩	⟨0.6, 0.2⟩
	z_3	0.0625	[0, 0.2857]	(0.4444, 0.5556, 0.6667, 0.7778)	{0.4, 0.5, 0.7}	⟨0.8, 0.2⟩	⟨0.5, 0.4⟩
	z_4	0.125	[0.4286, 0.7143]	(0.5556, 0.6667, 0.7778, 1)	{0.6, 0.7, 0.8}	⟨0.6, 0.3⟩	⟨0.8, 0.2⟩
	z_5	0	[0.2857, 0.5714]	(0.5556, 0.7778, 0.8889, 1)	{0.4, 0.5, 0.6}	⟨0.6, 0.4⟩	⟨0.5, 0.4⟩
e_3	z_1	0.3125	[0, 0.2857]	(0.3333, 0.5556, 0.6667, 0.8889)	{0.4, 0.5, 0.7}	⟨0.5, 0.3⟩	⟨0.6, 0.3⟩
	z_2	0	[0.1429, 0.2857]	(0.2222, 0.3333, 0.4444, 0.6667)	{0.4, 0.6, 0.8}	⟨0.7, 0.3⟩	⟨0.3, 0.4⟩
	z_3	0.1875	[0.4286, 0.5714]	(0.4444, 0.6667, 0.7778, 1)	{0.3, 0.4, 0.5}	⟨0.4, 0.6⟩	⟨0.6, 0.3⟩
	z_4	0.25	[0.2857, 0.4286]	(0.6667, 0.7778, 0.8889, 1)	{0.4, 0.5, 0.7}	⟨0.5, 0.3⟩	⟨0.8, 0.2⟩
	z_5	0.125	[0.4286, 0.7143]	(0.3333, 0.6667, 0.7778, 0.8889)	{0.2, 0.3, 0.5}	⟨0.6, 0.4⟩	⟨0.7, 0.2⟩

Step 7: (13) - (14) are used to identify the optimal ideal solution $\overline{\boldsymbol{A}}$ and negative ideal solution $\underline{\boldsymbol{A}}$, respectively:

$$\begin{aligned}\overline{\boldsymbol{A}} &= (0.375, [0.5714, 0.8571], (0.6667, 0.7778, 0.8889, 1), \{0.6, 0.75, 0.9\}, \langle 0.8, 0.1\rangle, \langle 0.8, 0.2\rangle);\\ \underline{\boldsymbol{A}} &= (0, [0, 0.2857], (0.2222, 0.3333, 0.4444, 0.6667), \{0.2, 0.3, 0.5\}, \langle 0.4, 0.6\rangle, \langle 0.3, 0.5\rangle).\end{aligned}$$

Step 8: (21) is used to calculate the Choquet-based relative closeness degree $cd_i^\tau (i = 1, 2, 3, 4, 5; \tau = 1, 2, 3)$ of each merchant z_i concerning expert e_τ (let $q = 1$):

$$\begin{aligned}cd_1^1 &= 1\psi(\{c_1\}) + 0.5\psi(\{c_2\}) + 0.9153\psi(\{c_3\}) + 0.7521\psi(\{c_4\}) + 0.6364\psi(\{c_5\}) + 0.4375\psi(\{c_6\}),\\ cd_2^1 &= 0.9615\psi(\{c_1\}) + 0.9\psi(\{c_2\}) + 0.4386\psi(\{c_3\}) + 1\psi(\{c_4\}) + 0.0588\psi(\{c_5\}) + 0.7\psi(\{c_6\}),\\ cd_3^1 &= 0.2\psi(\{c_1\}) + 0.0385\psi(\{c_2\}) + 0.6078\psi(\{c_3\}) + 0.1983\psi(\{c_4\}) + 0.95\psi(\{c_5\}) + 0.0714\psi(\{c_6\}),\\ cd_4^1 &= 0.5\psi(\{c_1\}) + 0.9615\psi(\{c_2\}) + 0.913\psi(\{c_3\}) + 0.3308\psi(\{c_4\}) + 1\psi(\{c_5\}) + 0.9231\psi(\{c_6\}),\\ cd_5^1 &= 0\psi(\{c_1\}) + 1\psi(\{c_2\}) + 0.988\psi(\{c_3\}) + 0.9704\psi(\{c_4\}) + 0.0588\psi(\{c_5\}) + 0\psi(\{c_6\});\end{aligned}$$

$$cd_1^2 = 0.8\psi(\{c_1\}) + 0.9\psi(\{c_2\}) + 0.6939\psi(\{c_3\}) + 0.1983\psi(\{c_4\}) + 0.5\psi(\{c_5\}) + 0.9231\psi(\{c_6\}),$$
$$cd_2^2 = 0.8\psi(\{c_1\}) + 0.1\psi(\{c_2\}) + 0.4545\psi(\{c_3\}) + 0.9379\psi(\{c_4\}) + 0.25\psi(\{c_5\}) + 0.6923\psi(\{c_6\}),$$
$$cd_3^2 = 0.0385\psi(\{c_1\}) + 0\psi(\{c_2\}) + 0.4667\psi(\{c_3\}) + 0.4571\psi(\{c_4\}) + 0.9412\psi(\{c_5\}) + 0.3\psi(\{c_6\}),$$
$$cd_4^2 = 0.2\psi(\{c_1\}) + 0.9\psi(\{c_2\}) + 0.9153\psi(\{c_3\}) + 0.9704\psi(\{c_4\}) + 0.6364\psi(\{c_5\}) + 1\psi(\{c_6\}),$$
$$cd_5^2 = 0\psi(\{c_1\}) + 0.5\psi(\{c_2\}) + 0.988\psi(\{c_3\}) + 0.3186\psi(\{c_4\}) + 0.3636\psi(\{c_5\}) + 0.3\psi(\{c_6\});$$
$$cd_1^3 = 0.9615\psi(\{c_1\}) + 0\psi(\{c_2\}) + 0.4468\psi(\{c_3\}) + 0.4571\psi(\{c_4\}) + 0.5\psi(\{c_5\}) + 0.7\psi(\{c_6\}),$$
$$cd_2^3 = 0\psi(\{c_1\}) + 0.0385\psi(\{c_2\}) + 0\psi(\{c_3\}) + 0.7521\psi(\{c_4\}) + 0.75\psi(\{c_5\}) + 0.05\psi(\{c_6\}),$$
$$cd_3^3 = 0.5\psi(\{c_1\}) + 0.7222\psi(\{c_2\}) + 0.8596\psi(\{c_3\}) + 0.0510\psi(\{c_4\}) + 0\psi(\{c_5\}) + 0.7\psi(\{c_6\}),$$
$$cd_4^3 = 0.8\psi(\{c_1\}) + 0.2778\psi(\{c_2\}) + 1\psi(\{c_3\}) + 0.4571\psi(\{c_4\}) + 0.5\psi(\{c_5\}) + 1\psi(\{c_6\}),$$
$$cd_5^3 = 0.2\psi(\{c_1\}) + 0.9\psi(\{c_2\}) + 0.7455\psi(\{c_3\}) + 0\psi(\{c_4\}) + 0.3636\psi(\{c_5\}) + 0.9286\psi(\{c_6\}).$$

Then, (22) is used to calculate the Choquet-based overall relative closeness degree cd_i of each merchant z_i:

$$cd_i = \omega_1 cd_i^1 + \omega_2 cd_i^2 + \omega_3 cd_i^3. \tag{36}$$

Steps 9-10: (26) and (30) are used to compute the q-ROFCGCI and q-ROFCGCC, respectively. Then, (32) is used to establish the following q-ROF programming model:

$$\begin{aligned}
&\max\Big\{G = \omega_1\xi_{12}^1 t_{1,2}^1\langle 0.6, 0.4\rangle + \omega_1\xi_{23}^1 t_{2,3}^1\langle 0.5, 0.4\rangle + \omega_1\xi_{43}^1 t_{4,3}^1\langle 0.6, 0.3\rangle + \omega_1\xi_{45}^1 t_{4,5}^1\langle 0.7, 0.3\rangle \\
&\quad + \omega_1\xi_{53}^1 t_{5,3}^1\langle 0.5, 0.4\rangle + \omega_2\xi_{12}^2 t_{1,2}^2\langle 0.7, 0.2\rangle + \omega_2\xi_{15}^2 t_{1,5}^2\langle 0.5, 0.4\rangle + \omega_2\xi_{32}^2 t_{3,2}^2\langle 0.6, 0.4\rangle \\
&\quad + \omega_2\xi_{43}^2 t_{4,3}^2\langle 0.6, 0.3\rangle + \omega_2\xi_{45}^2 t_{4,5}^2\langle 0.5, 0.3\rangle + \omega_3\xi_{13}^3 t_{1,3}^3\langle 0.4, 0.5\rangle + \omega_3\xi_{23}^3 t_{2,3}^3\langle 0.4, 0.6\rangle \\
&\quad + \omega_3\xi_{34}^3 t_{3,4}^3\langle 0.4, 0.6\rangle + \omega_3\xi_{42}^2 t_{4,2}^2\langle 0.7, 0.1\rangle + \omega_3\xi_{54}^3 t_{5,4}^3\langle 0.3, 0.5\rangle\Big\} \\
&\min\Big\{B = \omega_1\lambda_{12}^1 t_{1,2}^1\langle 0.6, 0.4\rangle + \omega_1\lambda_{23}^1 t_{2,3}^1\langle 0.5, 0.4\rangle + \omega_1\lambda_{43}^1 t_{4,3}^1\langle 0.6, 0.3\rangle + \omega_1\lambda_{45}^1 t_{4,5}^1\langle 0.7, 0.3\rangle \\
&\quad + \omega_1\lambda_{53}^1 t_{5,3}^1\langle 0.5, 0.4\rangle + \omega_2\lambda_{12}^2 t_{1,2}^2\langle 0.7, 0.2\rangle + \omega_2\lambda_{15}^2 t_{1,5}^2\langle 0.5, 0.4\rangle + \omega_2\lambda_{32}^2 t_{3,2}^2\langle 0.6, 0.4\rangle \\
&\quad + \omega_2\lambda_{43}^2 t_{4,3}^2\langle 0.6, 0.3\rangle + \omega_2\lambda_{45}^2 t_{4,5}^2\langle 0.5, 0.3\rangle + \omega_3\lambda_{13}^3 t_{1,3}^3\langle 0.4, 0.5\rangle + \omega_3\lambda_{23}^3 t_{2,3}^3\langle 0.4, 0.6\rangle \\
&\quad + \omega_3\lambda_{34}^3 t_{3,4}^3\langle 0.4, 0.6\rangle + \omega_3\lambda_{42}^2 t_{4,2}^2\langle 0.7, 0.1\rangle + \omega_3\lambda_{54}^3 t_{5,4}^3\langle 0.3, 0.5\rangle\Big\} \\
&s.t.\begin{cases}
\xi_{1,2}^1 = \max\{0, cd_1^1 - cd_2^1\}, \xi_{2,3}^1 = \max\{0, cd_2^1 - cd_3^1\}, \xi_{4,3}^1 = \max\{0, cd_4^1 - cd_3^1\}, \\
\xi_{4,5}^1 = \max\{0, cd_4^1 - cd_5^1\}, \xi_{5,3}^1 = \max\{0, cd_5^1 - cd_3^1\}, \xi_{1,2}^2 = \max\{0, cd_1^2 - cd_2^2\}, \\
\xi_{1,5}^2 = \max\{0, cd_1^2 - cd_5^2\}, \xi_{3,2}^2 = \max\{0, cd_3^2 - cd_2^2\}, \xi_{4,3}^2 = \max\{0, cd_4^2 - cd_3^2\}, \\
\xi_{4,5}^2 = \max\{0, cd_4^2 - cd_5^2\}, \xi_{1,3}^3 = \max\{0, cd_1^3 - cd_3^3\}, \xi_{2,3}^3 = \max\{0, cd_2^3 - cd_3^3\}, \\
\xi_{3,4}^3 = \max\{0, cd_3^3 - cd_4^3\}, \xi_{4,2}^3 = \max\{0, cd_4^3 - cd_2^3\}, \xi_{5,4}^3 = \max\{0, cd_5^3 - cd_4^3\}; \\
\lambda_{1,2}^1 = \max\{0, cd_2^1 - cd_1^1\}, \lambda_{2,3}^1 = \max\{0, cd_3^1 - cd_2^1\}, \lambda_{4,3}^1 = \max\{0, cd_3^1 - cd_4^1\}, \\
\lambda_{4,5}^1 = \max\{0, cd_5^1 - cd_4^1\}, \lambda_{5,3}^1 = \max\{0, cd_3^1 - cd_5^1\}, \lambda_{1,2}^2 = \max\{0, cd_2^2 - cd_1^2\}, \\
\lambda_{1,5}^2 = \max\{0, cd_5^2 - cd_1^2\}, \lambda_{3,2}^2 = \max\{0, cd_2^2 - cd_3^2\}, \lambda_{4,3}^2 = \max\{0, cd_3^2 - cd_4^2\}, \\
\lambda_{4,5}^2 = \max\{0, cd_5^2 - cd_4^2\}, \lambda_{1,3}^3 = \max\{0, cd_3^3 - cd_1^3\}, \lambda_{2,3}^3 = \max\{0, cd_3^3 - cd_2^3\}, \\
\lambda_{3,4}^3 = \max\{0, cd_4^3 - cd_3^3\}, \lambda_{4,2}^3 = \max\{0, cd_2^3 - cd_4^3\}, \lambda_{5,4}^3 = \max\{0, cd_4^3 - cd_5^3\}; \\
0.1 \le \mu(\{c_1\}) \le 0.15, 0.05 \le \mu(\{c_2\}) \le 0.1, 0.15 \le \mu(\{c_3\}) \le 0.25, \\
0.25 \le \mu(\{c_4\}) \le 0.3, 0.2 \le \mu(\{c_5\}) \le 0.3, 0 \le \mu(\{c_6\}) \le 0.1; \\
\mu(\{c_j, c_q\} = \mu(\{c_j\} + \mu(\{c_q\}(j, q = 1, 2, 3, 4, 5, 6; j \ne q); \\
0.1 \le \omega_1 \le 0.2, 0.4 \le \omega_2 \le 0.7, 0.3 \le \omega_3 \le 0.5; \\
\psi(\{c_j\}) = -1.5\mu(\{c_j\}) + \sum_{c_q \in C\setminus c_j} \frac{1}{2}(\mu(\{c_j, c_q\}) - \mu(\{c_q\})(j = 1, 2, 3, 4, 5, 6); \\
\sum_{\{c_j, c_q\}\subseteq C} \mu(\{c_j, c_q\}) - (n-2)\sum_{\{c_j\}\subseteq C} \mu(\{c_j\}) = 1; \\
\sum_{\{c_j\}\subseteq S\setminus c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \ge (|S| - 2)\mu(\{c_q\}), \forall S \subseteq C \text{ with } c_q \in S \text{ and } |S| \ge 2.
\end{cases}
\end{aligned} \tag{37}$$

Step 11: (35) is used to transform (37) into the following many-objective programming model:

$$
\begin{aligned}
\min \{Z_1 = & -1 + (1-0.6)^{\omega_1\xi^1_{1,2}}(1-0.5)^{\omega_1\xi^1_{2,3}}(1-0.6)^{\omega_1\xi^1_{4,3}}(1-0.7)^{\omega_1\xi^1_{4,5}}(1-0.5)^{\omega_1\xi^1_{5,3}}(1-0.7)^{\omega_2\xi^2_{1,2}}\\
& (1-0.5)^{\omega_2\xi^2_{1,5}}(1-0.6)^{\omega_2\xi^2_{3,2}}(1-0.6)^{\omega_2\xi^2_{4,3}}(1-0.5)^{\omega_2\xi^2_{4,5}}(1-0.4)^{\omega_3\xi^3_{1,3}}(1-0.4)^{\omega_3\xi^3_{2,3}}\\
& (1-0.4)^{\omega_3\xi^3_{3,4}}(1-0.7)^{\omega_3\xi^3_{4,2}}(1-0.3)^{\omega_3\xi^3_{5,4}} + (1-0.4)^{\omega_1\xi^1_{1,2}}(1-0.4)^{\omega_1\xi^1_{2,3}}(1-0.3)^{\omega_1\xi^1_{4,3}}\\
& (1-0.3)^{\omega_1\xi^1_{4,5}}(1-0.4)^{\omega_1\xi^1_{5,3}}(1-0.2)^{\omega_2\xi^2_{1,2}}(1-0.4)^{\omega_2\xi^2_{1,5}}(1-0.4)^{\omega_2\xi^2_{3,2}}(1-0.3)^{\omega_2\xi^2_{4,3}}\\
& (1-0.3)^{\omega_2\xi^2_{4,5}}(1-0.5)^{\omega_3\xi^3_{1,3}}(1-0.6)^{\omega_3\xi^3_{2,3}}\ (1-0.6)^{\omega_3\xi^3_{3,4}}(1-0.1)^{\omega_3\xi^3_{4,2}}(1-0.5)^{\omega_3\xi^3_{5,4}}\Big\}\\
\min \{Z_2 = & -1 + (1-0.6)^{\omega_1\xi^1_{1,2}}(1-0.5)^{\omega_1\xi^1_{2,3}}(1-0.6)^{\omega_1\xi^1_{4,3}}\ (1-0.7)^{\omega_1\xi^1_{4,5}}(1-0.5)^{\omega_1\xi^1_{5,3}}(1-0.7)^{\omega_2\xi^2_{1,2}}\\
& (1-0.5)^{\omega_2\xi^2_{1,5}}(1-0.6)^{\omega_2\xi^2_{3,2}}(1-0.6)^{\omega_2\xi^2_{4,3}}(1-0.5)^{\omega_2\xi^2_{4,5}}(1-0.4)^{\omega_3\xi^3_{1,3}}(1-0.4)^{\omega_3\xi^3_{2,3}}\\
& (1-0.4)^{\omega_3\xi^3_{3,4}}(1-0.7)^{\omega_3\xi^3_{4,2}}(1-0.3)^{\omega_3\xi^3_{5,4}} - (1-0.4)^{\omega_1\xi^1_{1,2}}(1-0.4)^{\omega_1\xi^1_{2,3}}(1-0.3)^{\omega_1\xi^1_{4,3}}\\
& (1-0.3)^{\omega_1\xi^1_{4,5}}(1-0.4)^{\omega_1\xi^1_{5,3}}(1-0.2)^{\omega_2\xi^2_{1,2}}(1-0.4)^{\omega_2\xi^2_{1,5}}(1-0.4)^{\omega_2\xi^2_{3,2}}(1-0.3)^{\omega_2\xi^2_{4,3}}\\
& (1-0.3)^{\omega_2\xi^2_{4,5}}(1-0.5)^{\omega_3\xi^3_{1,3}}(1-0.6)^{\omega_3\xi^3_{2,3}}\ (1-0.6)^{\omega_3\xi^3_{3,4}}(1-0.1)^{\omega_3\xi^3_{4,2}}(1-0.5)^{\omega_3\xi^3_{5,4}}\Big\}\\
\min \{Z_3 = & 1 - (1-0.6)^{\omega_1\lambda^1_{1,2}}(1-0.5)^{\omega_1\lambda^1_{2,3}}(1-0.6)^{\omega_1\lambda^1_{4,3}}\ (1-0.7)^{\omega_1\lambda^1_{4,5}}(1-0.5)^{\omega_1\lambda^1_{5,3}}(1-0.7)^{\omega_2\lambda^2_{1,2}}\\
& (1-0.5)^{\omega_2\lambda^2_{1,5}}(1-0.6)^{\omega_2\lambda^2_{3,2}}(1-0.6)^{\omega_2\lambda^2_{4,3}}(1-0.5)^{\omega_2\lambda^2_{4,5}}(1-0.4)^{\omega_3\lambda^3_{1,3}}(1-0.4)^{\omega_3\lambda^3_{2,3}}\\
& (1-0.4)^{\omega_3\lambda^3_{3,4}}(1-0.7)^{\omega_3\lambda^3_{4,2}}(1-0.3)^{\omega_3\lambda^3_{5,4}} - (1-0.4)^{\omega_1\lambda^1_{1,2}}(1-0.4)^{\omega_1\lambda^1_{2,3}}(1-0.3)^{\omega_1\lambda^1_{4,3}}\\
& (1-0.3)^{\omega_1\lambda^1_{4,5}}(1-0.4)^{\omega_1\lambda^1_{5,3}}(1-0.2)^{\omega_2\lambda^2_{1,2}}(1-0.4)^{\omega_2\lambda^2_{1,5}}(1-0.4)^{\omega_2\lambda^2_{3,2}}(1-0.3)^{\omega_2\lambda^2_{4,3}}\\
& (1-0.3)^{\omega_2\lambda^2_{4,5}}(1-0.5)^{\omega_3\lambda^3_{1,3}}(1-0.6)^{\omega_3\lambda^3_{2,3}}\ (1-0.6)^{\omega_3\lambda^3_{3,4}}(1-0.1)^{\omega_3\lambda^3_{4,2}}(1-0.5)^{\omega_3\lambda^3_{5,4}}\Big\}\\
\min \{Z_4 = & 1 - (1-0.6)^{\omega_1\lambda^1_{1,2}}(1-0.5)^{\omega_1\lambda^1_{2,3}}(1-0.6)^{\omega_1\lambda^1_{4,3}}\ (1-0.7)^{\omega_1\lambda^1_{4,5}}(1-0.5)^{\omega_1\lambda^1_{5,3}}(1-0.7)^{\omega_2\lambda^2_{1,2}}\\
& (1-0.5)^{\omega_2\lambda^2_{1,5}}(1-0.6)^{\omega_2\lambda^2_{3,2}}(1-0.6)^{\omega_2\lambda^2_{4,3}}(1-0.5)^{\omega_2\lambda^2_{4,5}}(1-0.4)^{\omega_3\lambda^3_{1,3}}(1-0.4)^{\omega_3\lambda^3_{2,3}}\\
& (1-0.4)^{\omega_3\lambda^3_{3,4}}(1-0.7)^{\omega_3\lambda^3_{4,2}}(1-0.3)^{\omega_3\lambda^3_{5,4}} + (1-0.4)^{\omega_1\lambda^1_{1,2}}(1-0.4)^{\omega_1\lambda^1_{2,3}}(1-0.3)^{\omega_1\lambda^1_{4,3}}\\
& (1-0.3)^{\omega_1\lambda^1_{4,5}}(1-0.4)^{\omega_1\lambda^1_{5,3}}(1-0.2)^{\omega_2\lambda^2_{1,2}}(1-0.4)^{\omega_2\lambda^2_{1,5}}(1-0.4)^{\omega_2\lambda^2_{3,2}}(1-0.3)^{\omega_2\lambda^2_{4,3}}\\
& (1-0.3)^{\omega_2\lambda^2_{4,5}}(1-0.5)^{\omega_3\lambda^3_{1,3}}(1-0.6)^{\omega_3\lambda^3_{2,3}}\ (1-0.6)^{\omega_3\lambda^3_{3,4}}(1-0.1)^{\omega_3\lambda^3_{4,2}}(1-0.5)^{\omega_3\lambda^3_{5,4}}\Big\}
\end{aligned}
$$

$$
s.t.\begin{cases}
\xi^1_{1,2} = \max\{0, cd^1_1 - cd^1_2\}, \xi^1_{2,3} = \max\{0, cd^1_2 - cd^1_3\}, \xi^1_{4,3} = \max\{0, cd^1_4 - cd^1_3\},\\
\xi^1_{4,5} = \max\{0, cd^1_4 - cd^1_5\}, \xi^1_{5,3} = \max\{0, cd^1_5 - cd^1_3\}, \xi^2_{1,2} = \max\{0, cd^2_1 - cd^2_2\},\\
\xi^2_{1,5} = \max\{0, cd^2_1 - cd^2_5\}, \xi^2_{3,2} = \max\{0, cd^2_3 - cd^2_2\}, \xi^2_{4,3} = \max\{0, cd^2_4 - cd^2_3\},\\
\xi^2_{4,5} = \max\{0, cd^2_4 - cd^2_5\}, \xi^3_{1,3} = \max\{0, cd^3_1 - cd^3_3\}, \xi^3_{2,3} = \max\{0, cd^3_2 - cd^3_3\},\\
\xi^3_{3,4} = \max\{0, cd^3_3 - cd^3_4\}, \xi^3_{4,2} = \max\{0, cd^3_4 - cd^3_2\}, \xi^3_{5,4} = \max\{0, cd^3_5 - cd^3_4\};\\
\lambda^1_{1,2} = \max\{0, cd^1_2 - cd^1_1\}, \lambda^1_{2,3} = \max\{0, cd^1_3 - cd^1_2\}, \lambda^1_{4,3} = \max\{0, cd^1_3 - cd^1_4\},\\
\lambda^1_{4,5} = \max\{0, cd^1_5 - cd^1_4\}, \lambda^1_{5,3} = \max\{0, cd^1_3 - cd^1_5\}, \lambda^2_{1,2} = \max\{0, cd^2_2 - cd^2_1\},\\
\lambda^2_{1,5} = \max\{0, cd^2_5 - cd^2_1\}, \lambda^2_{3,2} = \max\{0, cd^2_2 - cd^2_3\}, \lambda^2_{4,3} = \max\{0, cd^2_3 - cd^2_4\},\\
\lambda^2_{4,5} = \max\{0, cd^2_5 - cd^2_4\}, \lambda^3_{1,3} = \max\{0, cd^3_3 - cd^3_1\}, \lambda^3_{2,3} = \max\{0, cd^3_3 - cd^3_2\},\\
\lambda^3_{3,4} = \max\{0, cd^3_4 - cd^3_3\}, \lambda^3_{4,2} = \max\{0, cd^3_2 - cd^3_4\}, \lambda^3_{5,4} = \max\{0, cd^3_4 - cd^3_5\};\\
0.1 \le \mu(\{c_1\}) \le 0.15, 0.05 \le \mu(\{c_2\}) \le 0.1, 0.15 \le \mu(\{c_3\}) \le 0.25,\\
0.25 \le \mu(\{c_4\}) \le 0.3, 0.2 \le \mu(\{c_5\}) \le 0.3, 0 \le \mu(\{c_6\}) \le 0.1;\\
\mu(\{c_j, c_q\} = \mu(\{c_j\} + \mu(\{c_q\}(j, q = 1, 2, 3, 4, 5, 6; j \ne q);\\
0.1 \le \omega_1 \le 0.2, 0.4 \le \omega_2 \le 0.7, 0.3 \le \omega_3 \le 0.5;\\
\psi(\{c_j\}) = -1.5\mu(\{c_j\}) + \sum\limits_{c_q \in C \setminus c_j} \frac{1}{2}(\mu(\{c_j, c_q\}) - \mu(\{c_q\})(j = 1, 2, 3, 4, 5, 6);\\
\sum\limits_{\{c_j, c_q\} \subseteq C} \mu(\{c_j, c_q\}) - (n-2) \sum\limits_{\{c_j\} \subseteq C} \mu(\{c_j\}) = 1;\\
\sum\limits_{\{c_j\} \subseteq S \setminus c_q} (\mu(\{c_j, c_q\}) - \mu(\{c_j\})) \ge (|S| - 2)\mu(\{c_q\}), \forall S \subseteq C \text{ with } c_q \in S \text{ and } |S| \ge 2.
\end{cases}
$$

Step 12: It is assumed the parameters involving the A-NSGA-III-based optimization algorithm are set as follows: population size $N = 300$, maximum iteration number $ite = 1000$, reference point divisions $p = 10$, SBX possibility $p_c = 20$, SBX recombination parameter index $\eta_c = 20$, PM possibility $p_m = 1$ and PM recombination parameter index $\eta_m = 20$. Then, after applying the optimization algorithm with MATLAB R2017b to solve above model, the final Pareto set is derived and depicted on a value path plot in Fig. 1a.

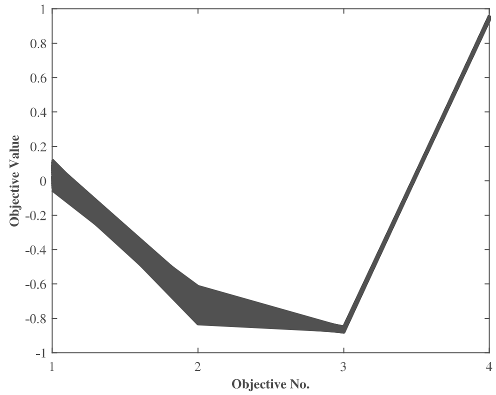

Fig. 1a. Pareto set using 4-objective value path format for Example 1

Step 13: The TOPSIS [26] is used to select a desirable solution from the Pareto set as the weighting result:

$$\mu(\{c_1\}) = 0.1450, \mu(\{c_2\}) = 0.0574, \mu(\{c_3\}) = 0.2386, \mu(\{c_4\}) = 0.2964, \mu(\{c_5\}) = 0.2372,$$
$$\mu(\{c_6\}) = 0.0254, \mu(\{c_1, c_2\}) = 0.2024, \mu(\{c_1, c_3\}) = 0.3836, \mu(\{c_1, c_4\}) = 0.4414,$$
$$\mu(\{c_1, c_5\}) = 0.3822, \mu(\{c_1, c_6\}) = 0.1704, \mu(\{c_2, c_3\}) = 0.2960, \mu(\{c_2, c_4\}) = 0.3538,$$
$$\mu(\{c_2, c_5\}) = 0.2946, \mu(\{c_2, c_6\}) = 0.0828, \mu(\{c_3, c_4\}) = 0.5350, \mu(\{c_3, c_5\}) = 0.4758,$$
$$\mu(\{c_3, c_6\}) = 0.2640, \mu(\{c_4, c_5\}) = 0.5336, \mu(\{c_6, c_4\}) = 0.3218, \mu(\{c_5, c_6\}) = 0.2626;$$
$$\omega_1 = 0.1119, \omega_2 = 0.4308, \omega_3 = 0.4573.$$

Step 14: (10) is used to compute Banzhaf value $\psi(\{c_j\})(j = 1, 2, 3, 4, 5, 6)$ on each criterion c_j:

$$\psi(\{c_1\}) = 0.1450, \psi(\{c_2\}) = 0.0574, \psi(\{c_3\}) = 0.2386, \psi(\{c_4\}) = 0.2964,$$
$$\psi(\{c_5\}) = 0.2372, \psi(\{c_6\}) = 0.0254.$$

Step 15: Based on $\psi(\{c_j\})(j = 1, 2, 3, 4, 5, 6)$ and $\omega_\tau(\tau = 1, 2, 3)$, (36) is used to compute Choquet-based overall relative closeness degree $cd_i(i = 1, 2, 3, 4, 5)$ of each merchant z_i:

$$cd_1 = 0.5539, cd_2 = 0.5068, cd_3 = 0.4209, cd_4 = 0.7048, cd_5 = 0.4301.$$

Step 16: Because $cd_4 > cd_1 > cd_2 > cd_5 > cd_3$, the final ranking of merchants is: $z_4 \succ z_1 \succ z_2 \succ z_5 \succ z_3$.