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Child care costs, household liquidity constraints, and gender inequality

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Abstract

In a model with endogenous female labour supply and wages, we show that liquidity constraints that prevent households from buying child care generate an inefficiency and amplify gender gaps in the labour market. We evaluate the relative merits of paid maternity leave, child care subsidies, and government loans in mitigating liquidity constraints and promoting gender equality. While an extension in the duration of the leave has ambiguous effects, child care subsidies and loans in the form of child care vouchers remove the liquidity constraints and reduce gender gaps in participation and wages. We illustrate the mechanisms at play in a numerical example using Spanish data.

Keywords Liquidity constraints · Gender wage and participation gaps · Statistical discrimination · Numerical example

JEL Classification J16 · J18 · J13

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1 Introduction

Despite progress, gender gaps in the labour market are still wide. The average gender participation gap in EU countries is around 10 percentage points and the unadjusted gender wage gap is around 15%, with large variation across countries. According to Bertrand (2020) and Cortés and Pan (2020), having children remains a key source of gender inequality in the labour market. There is increasing evidence that, while parenthood is almost a non-event in fathers' labour market outcomes, mothers reduce labour force participation, the number of hours worked, and experience a reduction in hourly wages (Angelov et al. 2016; Kleven et al. 2019a; De Quinto et al. 2021; Casarico and Lattanzio 2021; Herrarte and Urcelay 2022). These negative effects persist throughout women's lives rather than being short-term, and are common to many countries irrespective of differences in family policies (Kleven et al. 2019b).

Affordability of care is often mentioned in surveys as one of the reasons why mothers do not work or quit their jobs. According to the Eurostat Database, between 0.6% (in Czechia) and 7.5% (in Romania) of mothers do not work because child care is too expensive. Indeed, across European countries, female labour force participation is negatively correlated with the share of mothers not working because child care services are too expensive. In Italy, 46% of mothers who quit their job in 2020 gave as reason the difficulties of combining work and care (Ispettorato Nazionale del Lavoro 2021), due to lacking care support. In Spain, the share of mothers who do not work because child care is too expensive is 6.4%.

Child care costs are certainly critical in mothers' labour supply decisions. If the household has enough resources to buy child care, both parents can work and accumulate experience, granting the household a higher lifetime income. In the impossibility to borrow from future earnings, some households with children may be unable to pay child care costs and face a liquidity constraint that forces one of the parents, typically the mother, to quit working. Quits impose adjustment costs on firms, that reduce the wage of young women. In addition, since women who quit accumulate less experience, also their future wages will be lower.

Developing a model that allows for endogenous labour force participation and wages, this paper studies the impact of liquidity constraints on gender gaps in the labour market and evaluates the relative merits of an extension in paid maternity leave duration, a child care subsidy, and a government loan in mitigating liquidity constraints and reducing gender inequality. We illustrate the mechanisms at work with a numerical example using data from Spain.

To the best of our knowledge, we are the first to isolate the role of liquidity constraints related to the purchase of child care on gender inequality.[1] Clearly, not all households face the same market child care costs and are equally likely to be liquidity constrained, even at similar income levels. In fact, market child care costs can show considerable heterogeneity. Many households rely on friends and relatives, and,

[1] Guner et al. (2020) analyse the effects on household labour supply and welfare of different types of child care policies in a life-cycle model where households cannot borrow, but they do not deal with the effect of these policies on gender inequality.

therefore, face zero market child care costs. Others only need a few hours of babysitting. Among households needing to place their children in a nursery, some may live near a public facility, while others may have to use (more expensive) private institutions. Some may have neither. In some households with multiple children, the older one can help take care of the younger, or children may be all taken care of at once. Finally, some households may require special care for one or more of their children. We rely on this heterogeneity to illustrate that women in liquidity constrained households may be willing to, but unable to work.

We set up a simple unitary model, where households are composed of a man and a woman of given identical productivity. Some households will have children. Following Bjerk and Han (2007), the market cost of child care is randomly distributed across households, to account for the aforementioned heterogeneity in needs for child care in a setup with exogenous fertility.[2] Differently from Bjerk and Han (2007), we assume that mothers have the right to a paid maternity leave, as it is the case in most developed countries: both the leave and the decision to quit in order to take care of the child generate an adjustment cost for the firm, which is reflected in lower wages for women compared to men. In addition, we account for a second period of work, when all men and women work, wages depend on productivity and accumulated experience, and there is no cost related to child care.

Firms meet workers at the beginning of the first period, when a contract is signed. When offering a work contract, firms form beliefs on the probability that a household will have children, know that mothers will be on maternity leave, and form expectations on whether they will return to work after the leave period. Households are formed immediately after a work contract is signed and a share of them have children. Mothers are on paid leave for a fraction of the first period, at the end of which the household has to decide whether to buy child care in the market or take care of the child at home. In the latter case, one parent must quit working. As long as lifetime income when both parents work is higher than lifetime income when only one of them does, the household will prefer to buy child care in the market. However, child care costs need to be paid in the first period of work, and—in the impossibility to borrow from future earnings—there may be households that cannot afford to pay them out of first period income. Since firms penalise women ex ante for their period on leave and for the threat that they will quit, they earn less than men with the same productivity and end up being the ones to quit when the household cannot afford to pay for child care costs.

We show that the presence of liquidity constraints generates an inefficiency and increases gender wage and participation gaps in equilibrium, compared to a situation in which all households interested in buying child care can afford to do so. As a result, enabling women in constrained households to return to work when young reduces gender gaps in the labour market.

[2]Dzhumashev and Tursunalieva (2023) provide a model with endogenous fertility and child care costs to explain the decline in fertility rates during the demographic transition and the fertility rebound observed in recent decades in high-income countries.

Regarding the effectiveness of different policy instruments in addressing the liquidity constraint and the ensuing gender inequality, we find that an extension in the duration of maternity leave has an ambiguous impact on gender gaps in participation and wages. On the one side, a longer leave reduces child care costs and may make it more likely that mothers return to work after the leave, increasing their participation, with a positive effect on their wages when young and on average female wages when old. On the other side, a longer leave is more costly to firms and this has a negative effect on young women's wages, reducing their participation. We do not know a priori which effect will prevail. In the case of Spain, our numerical exercise shows that when we increase the duration of maternity leave from 4 to 12 months per child, the gender gap in participation declines, whereas that in wages increases.

The introduction of a child care subsidy reduces child care costs, allowing women in liquidity constrained households to return to work. This increases female labour force participation as well as wages, thanks to the lower adjustment costs firms face. Thus, gender inequality in the labour market is reduced. A loan in the form of a child care voucher has the same effect on gender inequality as the subsidy, but it entails no tax cost. The numerical exercise confirms that both policies can remove the liquidity constraint and increase labour force participation and wages of young women in Spain. Removing the liquidity constraint with a loan has slightly larger effects on gender equality than doing so with a proportional subsidy, given that the latter requires increasing the tax.

Our paper is related to contributions studying the role of statistical discrimination in generating gender gaps and how policy can address them (e.g. Bjerk and Han 2007; Dolado et al. 2013; Lommerud et al. 2015).[3] Unlike these previous works, we emphasise the contribution of liquidity constraints to amplifying gender gaps and explore the role of loans among other policies. Chapman and Higgins (2009) propose household loans to help women with children, but they do not look at gender inequality. Student loans have been used and discussed for a long time to address liquidity constraints preventing the payment of education costs, which deliver returns in the future. Findeisen and Sachs (2016) and Stantcheva (2018) have recently underlined the role of income contingent student loans as part of the optimal tax policy.

Ho and Pavoni (2020) have studied the optimal design of child care subsidies in a setting where agents are heterogenous and have private information on their productivity. Bastani et al. (2020) have investigated the subsidisation of child care expenditure in an optimal taxation framework, where parents can choose the quality and quantity of care, and the latter affects children's human capital (see also Casarico et al. 2015).

[3] A vast empirical literature investigates the effects of family policies on maternal labour supply and health, on fertility and time allocation decisions, on children's human capital and health, with a view on the overall impact in terms of reduction in gender gaps in the labour market and in household production. See Olivetti and Petrongolo (2017) and Rossin-Slater (2018) for exhaustive surveys. Österbacka and Räsänen (2022) provide evidence on the effects of child home care and private day care allowances on mothers' return to employment after childbirth in Finland. Bergemann and Riphahn (2023) study the employment effects of a change in parental leave benefits in Germany.

Parental leaves are a central element of family policies in most OECD countries, and a few papers study their effects in a theoretical setup. Barigozzi et al. (2018) focus on the endogenous formation of social norms and show that parental leave can reduce social welfare. Bastani et al. (2019) show that a mandatory parental leave can be part of the socially optimal policy when firms are not allowed to offer differentiated contracts due to anti-discrimination legislation. Del Rey et al. (2017) underline the role of the relative bargaining power of firms and workers in determining the effect of leave duration on unemployment and wages. Finally, Del Rey et al. (2021) explore the impact of maternity leave duration on female labour supply in a model with endogenous fertility. Their model allows for non-monotonic effects of leave duration on female labour supply.

The rest of the paper is organised as follows. Section 2 presents the model and Section 3 the equilibrium. Section 4 shows the effects of the policies. Section 5 presents the numerical example based on Spanish data. Section 6 offers a discussion and Section 7 concluding remarks.

2 The model

To analyse how household liquidity constraints influence gender gaps in participation and wages, and study the role of policy, we build on Bjerk and Han (2007). Starting from their basic framework, we add a paid maternity leave for women with children and a second period of work, when earnings depend on productivity and accumulated experience. In this setting, we allow for the presence of liquidity constraints for households with children. In the next sections we describe the building blocks of the model, starting from the behaviour of workers and firms, to then determine the equilibrium and its properties.

2.1 Workers

In period t, there is a continuum of young individuals of identical productivity x and gender $g = \{m, f\}$. The total measure of males [resp. females] is normalised to one. Young individuals coexist with an equal mass of children of type x and gender g, that make no economic decision, and an equal mass of old individuals of type x, gender g, and labour market experience ϵ. We neglect time indices because all periods are the same. Population growth rate is zero, as implied from above.

Individuals live for three periods during which they are children, young and old, respectively. From the perspective of individual lifetime, we use first and second period to refer to the periods in which agents are active in the labour market. They are young in the first period and old in the second period. If individuals work during the whole first period, they accumulate high experience h. If they work only during part of the first period, they accumulate intermediate experience i. If they do not work during the first period they accumulate nil experience n. Therefore, experience is $\epsilon = \{h, i, n\}$, with $h > i > n > 0$.

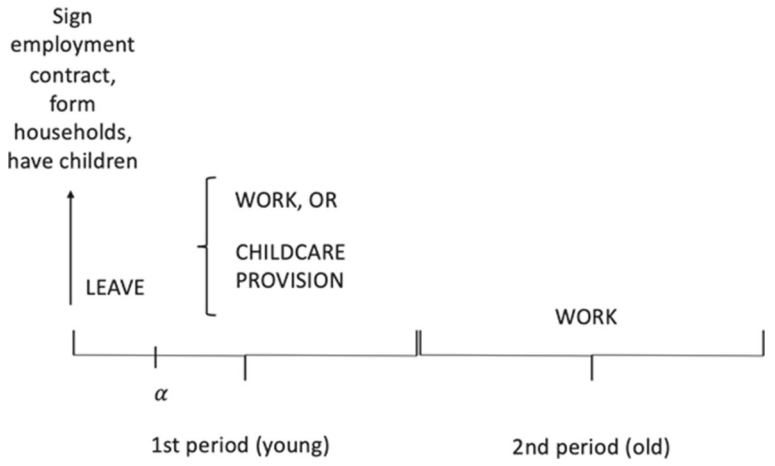

Fig. 1 Mothers' timeline

At the beginning of the first period, young individuals sign a work contract involving a wage $w_g(x)$. Immediately after signing the contract, households are formed by a woman and a man, with a proportion ρ of these households having children.[4] Mothers take a paid maternity leave of total length $0 < \alpha \leq 1$, after which they may return to work or remain at home for the rest of the first period. The length of the paid leave is set by the government and cannot be chosen by the households. The government finances the leave with a lump-sum tax τ levied on all workers, young and old. The paid leave, instead, is exempt from taxation by assumption.[5] The interest rate is zero.[6] If mothers stay at home, they will take care of the children. If the mother returns to work, the household has to buy child care in the market at cost $\eta > 0$, where η is a random variable with an increasing and continuous distribution function F on $(0, \infty)$, with $F' = f > 0$. As discussed previously, the cost of buying child care η can take different values depending on the availability of relatives, of child care facilities, the number of hours of care, or special needs.

Old individuals earn a wage that depends only on the observable productivity and experience $w^{\epsilon}(x)$, with $\epsilon = \{h, i, n\}$. There is no unemployment. Finally, there are no capital markets where households can borrow.

Figure 1 represents mothers' timeline. Men and women without children are assumed to work during the whole first and second periods.

[4]To guarantee constant population, a proportion ρ of households will have $2/\rho$ children.

[5]This is a peculiarity of the Spanish paid maternity leave system, on which we focus in the numerical example and which we consider also in the theoretical part. The assumption does not affect the results.

[6]This assumption is discussed in footnote 8.

2.2 Firms

There is a continuum of competitive, profit maximising firms, that offer wages $w_g(x)$ to young workers of type x and gender $g = \{m, f\}$, and wages $w^{\epsilon}(x)$ to old workers of type x, and experience $\epsilon = \{h, i, n\}$. The female partner takes a maternity leave, which imposes on the firm a cost $q(x) > 0$ during her absence. This cost can be interpreted in terms of adjustment, reallocation of tasks to cover for the missing worker, or administrative costs. If a worker quits her job during the first period, the firm incurs a cost $p(x) > 0$, which also reflects the presence of adjustment costs related to turnover. To simplify, we assume that $q(x) = p(x)$. When workers are young, they have no experience. When they are old, they no longer need to purchase child care on the market. Wages offered are those, which set profits to zero. Profits made when hiring a young male worker of productivity x are

$$\pi_m(x) = x - w_m, \text{ then } w_m = x \equiv w_m(x) \tag{1}$$

When hiring young women, firms know that they will have children with probability ρ, take a leave of duration α and return to work with probability λ. Then, *expected* profits when hiring a young woman of productivity x are

$$\pi_f(x) = (1-\rho)(x - w_f) + \rho\,(1-\alpha)\,\lambda(x - w_f) - \rho(1-(1-\alpha)\lambda)q(x) \tag{2}$$

where $q(x)$ is the cost imposed on the firm when a (female) worker of productivity x is absent, either because she is on maternity leave, or because she quits.

To better understand Eq. (2), Table 1 summarises the proportion of young female workers in different situations, the associated surplus and costs for the firm. The $(1-\rho)$ women who do not have children produce x and are paid w_f. Since they will work for the entire first period after having signed a contract, they impose no cost on the firm. Women who have children (ρ) take a leave of duration α, which costs the firm $\alpha q(x)$. Among those ρ who have children, firms expect a proportion λ to return to work after the leave and generate a surplus $(1-\alpha)(x - w_f)$. Finally, the firm expects $(1-\lambda)$ female workers with children not to return to work; this implies an additional cost $(1-\alpha)q(x)$. The last part of Eq. (2) captures the total expected costs that women impose on firms, given by the sum of $\rho\alpha q(x)$ during the maternity leave, and $\rho(1-\alpha)(1-\lambda)q(x)$, for women who quit.

Using Eq. (2), and setting profits equal to zero, we obtain the wage offered to young women of productivity x, given firm's beliefs λ:

$$w_f = x - \frac{\rho(1-(1-\alpha)\lambda)}{(1-\rho) + \rho\,(1-\alpha)\,\lambda} q(x) \equiv w_f(x, \alpha) \tag{3}$$

Table 1 Proportion of young female workers across different states, surplus and cost to the firm

Proportion	Surplus	Cost
$(1-\rho)$	$x - w_f$	0
$\rho\lambda$	$(1-\alpha)\left(x - w_f\right)$	$\alpha q(x)$
$\rho\,(1-\lambda)$	0	$\alpha q(x) + (1-\alpha)q(x)$

which implies $w_f(x,\alpha) < x$. Note that this wage is the smallest if all mothers are expected to leave and never work, i.e. $\lambda = 0$. Women are willing to sign a work contract before entering the stage of household formation as long as $w_f(x,\alpha) > 0$. We now impose a condition that guarantees that all young women are willing to sign a work contract before forming a household, even when offered the lowest possible wage $w_f(x,\alpha) = x - \frac{\rho}{1-\rho}q(x)$. This ensures that the female participation rate is positive.

Assumption 1

$$(1-\rho)x - \rho q(x) > 0 \tag{4}$$

Since there is no compulsory leave for men, $w_m(x) > w_f(x,\alpha)$, and all men are willing to sign a work contract too.

Finally, when hiring an old worker of productivity x, and experience ϵ, which are both observable characteristics, firms' profits are:

$$\pi^{\epsilon} = \epsilon x - w^{\epsilon} \text{ then } w^{\epsilon} = \epsilon x \equiv w^{\epsilon}(x) \tag{5}$$

2.3 Households' lifetime income

Members of households without children work both periods and their net lifetime income is

$$w_m(x) - \tau + w_f(x,\alpha) - \tau + w^h(x) - \tau + w^h(x) - \tau \tag{6}$$

where τ stands for the lump-sum tax paid by each worker in each period to finance the maternity leave.

In households with children, both adult members are active in the labour market at the beginning of the first period. Then, mothers take the paid leave, which is not subject to taxation, for a portion α of the period. If the mother goes back to work when the leave is over, the household has to buy market child care at price η during the period $1-\alpha$. In the second period children are grown up and they no longer impose a cost of care on the parents. Both members of the household continue working since they have more experience and hence higher wages. Fathers work the whole time when young and have experience $\epsilon = h$. Also mothers work in the first period but are on leave a fraction α of it, hence, they accumulate less experience ($\epsilon = i$). Thus, net lifetime income of households where young mothers work is

$$w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - (1-\alpha)\eta + w^h(x) - \tau + w^i(x) - \tau \tag{7}$$

If the mother does not work when the leave is over, the household does not buy child care in the market and the woman accumulates no experience ($\epsilon = n$). Assumption 2 states that all wages are larger than taxes and implies, in particular, that old women always work irrespective of experience.[7]

[7] Accounting also for the case where inexperienced old women do not work complicates the exposition without adding insight.

Assumption 2 $w_g(x) - \tau > 0$ for $g = \{m, f\}$, $w^{\epsilon}(x) - \tau > 0$ for $\epsilon = \{h, i, n\}$

Net lifetime income of households where young mothers do not work is:

$$w_m(x) - \tau + \alpha w_f(x, \alpha) + w^h(x) - \tau + w^n(x) - \tau \tag{8}$$

Note that children affect women's wages in two distinct ways. First, the compulsory leave α and the fact that some women quit their jobs to take care of children increase firms' expected costs and, hence, reduce wages for all young women, whether they are mothers or not, because of statistical discrimination. Second, there is a penalty children impose only on mothers, through a reduction in second period wages due to lower experience ($w^n < w^i < w^h$).

3 Equilibrium

The choice to return to work after the leave by mothers and the simultaneous setting of wages by firms, together with a balanced government budget constraint determine the equilibrium.

3.1 Decision to return to work after maternity leave

Households with children decide on whether the mother goes back to work after the leave by comparing household lifetime income when she does (and the household buys child care) and when she does not (and the mother stays at home to provide care). The return to work after the leave period affects mothers' experience and their wage when old.

Comparing household lifetime income if young mothers go back to work after the leave period and if they do not, i.e. Eqs. (7) and (8), it will be optimal for the household that the mother goes back to work if:

$$(1 - \alpha)\left(w_f(x, \alpha) - \tau - \eta\right) + w^i(x) - w^n(x) > 0 \tag{9}$$

This condition allows to identify a threshold level of child care costs η^*, below which households are better off if women return to work after the leave period, for given wages:

$$\eta < w_f(x, \alpha) - \tau + \frac{w^i(x) - w^n(x)}{(1 - \alpha)} = \eta^*(x, \alpha). \tag{10}$$

If Eq. (10) is satisfied, households want to buy child care on the market. Otherwise, it is better if the mother stays at home. By Assumption 2, $\eta^*(x, \alpha)$ defined by Eq. (10) is positive. At equilibrium, the number of mothers who wish to return to work at the end of the paid leave for given wages is $F(\eta^*(x, \alpha))$.

Note that η^* is also the threshold of child care costs below which it is efficient for women to return to work after the leave, given taxes and paid leave duration. If child care costs are larger than η^*, the additional income earned by mothers by going back to work and accumulating more experience is smaller than the costs incurred.

So far, we have not considered whether households have enough income when young to pay for child care costs. All that mattered was lifetime income as if there

were perfect credit markets where households could borrow. When households cannot use their future earnings as collateral for a loan, for the mother to go back to work at the end of the paid leave it must hold that two earner households can afford to buy child care in the first period.

To consider the question of affordability, let c denote unavoidable household consumption (food, housing, etc). The following assumption guarantees that households can always pay for minimum consumption in the first period, even if mothers do not work.

Assumption 3 $w_m - \tau + \alpha w_f(x, \alpha) > c$.

This assumption also guarantees that households can pay for minimum consumption in the first period when the mother goes back to work at the end of the leave.[8] However, for them to be able to pay for child care costs we further need that:

$$w_m(x) - \tau + w_f(x, \alpha) - (1 - \alpha)\tau - c \geqslant (1 - \alpha)\eta \tag{11}$$

We can now identify a threshold η^c, above which households *cannot afford* the purchase of child care, i.e. Eq. (11) is *not* satisfied:

$$\eta > \frac{w_m(x) - \tau + w_f(x, \alpha) - (1 - \alpha)\tau - c}{1 - \alpha} = \eta^c(x, \alpha) \tag{12}$$

where $\eta^c > 0$. If $\eta^c(x, \alpha) \geq \eta^*(x, \alpha)$, all those households willing to buy child care are able to. If $\eta^c(x, \alpha) < \eta^*(x, \alpha)$, or, by Eqs. (10) and (12):

$$w_m(x) - \tau + \alpha w_f(x, \alpha) - c < w^i(x) - w^n(x) \tag{13}$$

some households will be liquidity constrained, i.e. unable to buy child care and let the mother return to work, in spite of this choice generating more net lifetime income. That is, in spite of this being the efficient choice. In this case, the equilibrium number of mothers that return to work is $F(\eta^c(x, \alpha))$. In households with $\eta \in (\eta^c(x, \alpha), \eta^*(x, \alpha))$ women would like to go back to work after the leave but cannot afford to do so. Hence, the number of liquidity constrained households is $F(\eta^*(x, \alpha)) - F(\eta^c(x, \alpha))$. Note that the number of liquidity constrained households depends both on how many households find it optimal to buy child care in the market and how many of them are able to pay for it. Eliminating this liquidity constraint is efficient because it increases aggregate income. As we will see below, it also reduces gender inequality. Figure 2 represents the relevant thresholds and preferences over/affordability of market child care.

3.2 Participation and wages

With perfect competition (firm's zero profits at equilibrium), young and old male and old female workers' wages coincide with their respective marginal productivities,

[8] In households where mothers work, earnings in the first period is $w_m - \tau + w_f(x, \alpha) - (1 - \alpha)\tau$. Since all wages exceed taxes by Assumption 2, households where mothers work can also afford minimum consumption c.

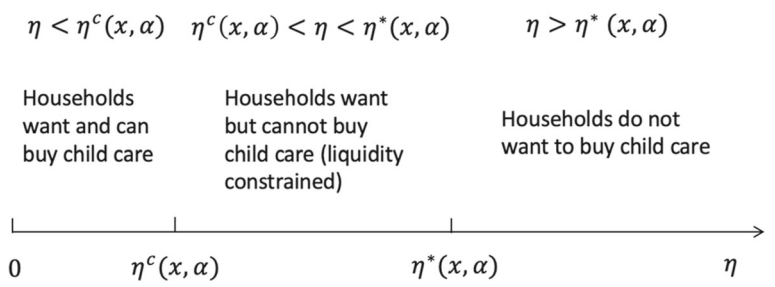

Fig. 2 Thresholds and types of households with children

because the firm observes gender and experience (see Eqs. (1) and (5)). The wage paid to a young woman is given by Eq. (3). In equilibrium, firm's beliefs on how many mothers will return to work at the end of the paid leave coincide exactly with how many do, i.e.

$$\lambda = F\left(\hat{\eta}\right) \tag{14}$$

where

$$F\left(\hat{\eta}\right) = \min\left\{F(\eta^*(x,\alpha)), F(\eta^c(x,\alpha))\right\}. \tag{15}$$

Then, the equilibrium wage of a young woman is:

$$w_f(x,\alpha) = x - \rho\frac{1-(1-\alpha)F\left(\hat{\eta}\right)}{1-\rho+\rho(1-\alpha)F\left(\hat{\eta}\right)}q(x) \tag{16}$$

For the existence of the equilibrium, it is *necessary and sufficient* that,

$$\frac{\rho(1-\alpha)f(\eta^*(x,\alpha))q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^*(x,\alpha)))^2} < 1 \tag{17}$$

if all households willing to buy child care can afford to do so, i.e. $\hat{\eta} = \eta^*(x,\alpha)$, and

$$\frac{\rho f(\eta^c(x,\alpha))q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^c(x,\alpha)))^2} < 1 \tag{18}$$

if some households are constrained. Appendix 1 provides the formal proof.

We conclude this section with a proposition characterising gender inequality at equilibrium. To this end, first, we compute labour force participation of young men and women. All men participate for the entire first period, that is, $MLF = 1$. Women participate the whole first period if they have no children or, if they have children and return to work at the end of the leave, since women on maternity leave are also part of the labour force. Labour force participation of young women at equilibrium is, hence:

$$FLF = (1-\rho) + \rho\alpha + \rho(1-\alpha)F(\hat{\eta}) \tag{19}$$

Second, we calculate the gender gap in wages of old workers who, by assumption, always work. Note that wages of old workers depend on accumulated experience and this is different on average for men and women. In particular, we denote the average wage of old workers of productivity x and gender $g = \{m, f\}$, $\bar{\omega}_g(x)$. It

holds that $\bar{\omega}_m(x) = w^h(x)$, since all men have high experience. The average wage of old women writes:

$$\bar{\omega}_f(x) = (1-\rho)w^h(x) + \rho F(\hat{\eta})w^i(x) + \rho(1-F(\hat{\eta}))w^n(x) \tag{20}$$

The first term on the right hand side captures wages of old women without children, who all have high experience. The term $\rho F(\hat{\eta})$ refers to women who have children, return to work after the leave, and earn $w^i(x)$ in the second period of work; $\rho(1-F(\hat{\eta}))$ are women who have children and go back to work only when the children are grown-ups and earn $w^n(x)$ in the second period of work.

Proposition 1 *The equilibrium exhibits gender gaps in labour force participation and wages. In particular,*

1. *The ratio of male to female labour force participation for young workers is:*

$$\frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\left(\hat{\eta}\right)} > 1 \tag{21}$$

2. *The ratio of male to female wages for young workers is:*

$$\frac{w_m(x)}{w_f(x,\alpha)} = \frac{x}{x-\rho\frac{1-(1-\alpha)F(\hat{\eta})}{1-\rho+\rho(1-\alpha)F(\hat{\eta})}q(x)} > 1 \tag{22}$$

3. *There is no gender participation gap among old workers by assumption.*
4. *The ratio of male to female average wages for old workers is:*

$$\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)} = \frac{w^h(x)}{(1-\rho)w^h(x)+\rho F\left(\hat{\eta}\right)w^i(x)+\rho(1-F\left(\hat{\eta}\right))w^n(x)} > 1 \tag{23}$$

Proof By construction, all men, women without children, and older women work. Participation of young mothers is given by Eq. (19). Wages are given by Eqs. (1), (5) and (16). □

We now investigate the effects of enabling women in constrained households to return to work. This amounts to raising the equilibrium threshold from $\eta^c(x)$ to $\eta^*(x)$. In the equilibrium in which some households cannot afford to pay child care costs, i.e. $\hat{\eta} = \eta^c(x)$, gender gaps in participation and wages result from a combination of statistical discrimination and liquidity constraints in young age, and lower accumulated experience by mothers, with negative repercussions on female wages in old age. Lifting the liquidity constraint, the labour force participation of young mothers increases and the ratio in Eq. (21) goes down. In addition, the wage of young women increases. Indeed, differentiating Eq. (16) with respect to η^c we get:

$$\frac{\partial w_f(x,\alpha)}{\partial \eta^c} = \frac{\rho\,(1-\alpha)\,f\,(\eta^c)\,q(x)}{(1-\rho+\rho\,(1-\alpha)\,F\,(\eta^c))^2} > 0 \tag{24}$$

Then, the gender wage gap in Eq. (22) goes down. Finally, more women accumulate labour market experience and the average wage of old women increases. Hence, the gender wage gap in Eq. (23) is reduced. This allows us to write the following

Corollary 1 *Enabling women in constrained households to return to work when young increases efficiency and reduces gender gaps in participation and wages:*

$$\frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\left(\eta^{*}\right)} < \frac{1}{(1-\rho)+\rho\alpha+\rho(1-\alpha)F\left(\eta^{c}\right)}, \tag{25}$$

$$\left.\frac{w_m(x)}{w_f(x,\alpha)}\right|_{\eta=\eta^{*}} < \left.\frac{w_m(x)}{w_f(x,\alpha)}\right|_{\eta=\eta^{c}} \tag{26}$$

and

$$\left.\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)}\right|_{\eta=\eta^{*}} < \left.\frac{\bar{\omega}_m(x)}{\bar{\omega}_f(x)}\right|_{\eta=\eta^{c}}. \tag{27}$$

In Appendix 2, we study how gaps in participation and wages change with individual productivity x through a comparative statics exercise.

3.3 Balanced government budget constraint

The government funds the benefits accruing to mothers on leave by levying a lump-sum tax τ on all workers. Letting $F(\hat{\eta})$ denote the number of households with child care costs smaller than $\hat{\eta}$, where $\hat{\eta}$ is the child care cost born by the last household where the mother goes back to work at equilibrium, the government budget constraint reads:

$$\rho\alpha w_f(x,\alpha) = \left(3+(1-\rho)+(1-\alpha)\rho F(\hat{\eta})\right)\tau \tag{28}$$

4 Policy

In this section we explore the effect of alternative policies on gender inequality when some households are liquidity constrained. We first discuss the effects of increasing the duration of the maternity leave. Then, we explore the role of child care subsidies to dual earner households. Finally, we consider a government loan. To study this instrument, we assume that, unlike households, the government can borrow in international markets to obtain the funds required to cover child care expenses. We also assume that the government has the power to seize incomes directly, in case households do not repay the loan.

4.1 Extending the duration of paid maternity leave

We first consider the impact of changes in the duration of the paid maternity leave on gender gaps when some households are liquidity constrained. In principle, longer periods of paid maternity leave reduce the market cost of child care, because households in which mothers return to work will have to pay it for a shorter period of time. However, they also affect wages directly, because longer leave periods are more costly to firms. This has repercussions on participation, which may feed back into wages. We state the following proposition.

Proposition 2 *If some households are liquidity constrained, increasing the duration of the paid maternity leave α has the following effects on labour market outcomes:*

a) *More young women of productivity x return to work after maternity leave and their wages increase iff*

$$\frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)} > \frac{F(\eta^c(x,\alpha))}{f(\eta^c(x,\alpha))} \tag{29}$$

Then, the participation of young women in the labour market increases. As a result, gender gaps in participation and wages for young workers decrease, and so does the gender wage gap of old workers.

b) *More young women of productivity x return to work after maternity leave and their wages decrease iff*

$$\frac{F(\eta^c(x,\alpha))}{f(\eta^c(x,\alpha))} > \frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)}$$

$$> \frac{\rho F(\eta^c(x,\alpha))q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^c(x,\alpha)))^2} \tag{30}$$

Then, the participation of young women in the labour market increases, and the gender gap in participation for young workers and in wages for old workers decrease, whereas that in wages of young workers increases.

c) *Fewer young women of productivity x return to work after maternity leave and their wages decrease iff*

$$\frac{F(\eta^c(x,\alpha))}{f(\eta^c(x,\alpha))} > \frac{\rho F(\eta^c(x,\alpha))q(x)}{(1-\rho+\rho(1-\alpha)F(\eta^c(x,\alpha)))^2}$$

$$> \frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-\alpha)} \tag{31}$$

Then, the effect on the participation of young women in the labour market is ambiguous and so is the effect on the gender gap in participation for young workers. The gender wage gap increases both for young and old workers.

Proof See Appendix 2. □

The intuition of the proposition is as follows. Increasing the duration of the maternity leave has two different effects on the number of women returning to work. First, longer duration reduces child care costs and incentivises women to go back to work. Second, wages can increase or decrease, with a further impact on the number of women who return to work.

In fact, with a longer leave, mothers are less likely to quit—which reduces costs for firms—but are also absent from work for a longer period, which increases costs for firms. Depending on which of the two effects dominates, wages can increase or decrease. If wages increase, the incentive to go back to work is stronger. This is case a in the Proposition. If wages decrease, this weakens the incentives to return to work. In case b in the Proposition, more women return to work in spite of the decrease in wages. In case c, the negative effect on wages dominates the reduction in child care

costs and fewer women go back to work after the leave. This reinforces the negative effect on wages further.

With respect to female labour force participation, given by Eq. (19), a longer duration of the leave keeps women attached to the labour force for longer, and can increase or decrease the number of mothers going back to work after the leave. If more women return to work after the leave, participation increases (cases a and b). If fewer women return to work (case c), the effect of a longer leave on female labour force participation is ambiguous. Finally, the impact on the average wage of old women hinges on the proportion of women returning to work after childbirth. Hence, the average wage of old women increases in cases a and b, and decreases in case c when the duration of the leave is extended.

To conclude this analysis, note that funding a longer maternity leave will require adjusting the government budget constraint. We can show that increasing taxes has a negative effect on gender inequality (see Appendix 2). In particular, it holds that

$$sign\frac{dw_f}{d\tau} = sign\left(-\frac{\rho\,(2-\alpha)\,f\,(\eta^c)\,q(x)}{(1-\rho+\rho\,(1-\alpha)\,F(\eta^c))^2}\right) < 0 \tag{32}$$

$$sign\frac{d\eta^c}{d\tau} = sign\left(-\frac{1+(1-\alpha)}{1-\alpha}\right) < 0 \tag{33}$$

Hence, increasing taxes limits the positive effects of extending paid leave duration in cases a and b, and exacerbates the negative effects in cases b and c.

4.2 Child care subsidies to dual earner households

The government could subsidise households with child care costs $\eta \in (\eta^c, \eta^*)$, that is, households for which it is optimal that the mother returns to work, but cannot afford it. However, since η is not observed, the government does not know the child care needs of one particular family and, thus, cannot subsidise constrained households only. Under these circumstances, we assume that the government subsidises a proportion s of all child care bought in the market. The first period income of a constrained household would then become: $w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c - (1-s)(1-\alpha)\eta$. Hence, the households that can now afford child care are those with

$$\eta < \frac{w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - c}{(1-s)\,(1-\alpha)} = \eta^s(x,\alpha,s) \tag{34}$$

Clearly, $\eta^s(x,s) > \eta^c(x,\alpha)$: more households can afford for the woman to work after childbirth, given α. From *Corollary 1*, this reduces gender gaps in participation and wages. Subsidising child care, however, requires higher taxes. The government budget constraint becomes:

$$\rho\alpha w_f(x,\alpha) + s(1-\alpha)\int_0^{\eta^s} \eta dF(\eta) = \left(3 + (1-\rho) + (1-\alpha)\rho F(\eta^s)\right)\tau \tag{35}$$

As before, taxes limit the positive effects of the subsidy since, with subsidies,

$$sign\frac{dw_f}{d\tau} = sign\left(-\frac{\rho\,(2-\alpha)\,f\,(\eta^s)\,q(x)}{(1-s)\,(1-\rho+\rho\,(1-\alpha)\,F(\eta^s))^2}\right) < 0 \tag{36}$$

$$sign\frac{d\eta^s}{d\tau} = sign\left(-\frac{1+(1-\alpha)}{(1-s)(1-\alpha)}\right) < 0 \tag{37}$$

The details of these calculations are available in Appendix 2.

Summing up, subsidising child care costs in dual earner households can mitigate liquidity constraints and reduce gender inequality in the labour market, but the required taxes will hinder their effectiveness in doing so.

We now assume that the government can borrow in the international capital market to lend constrained households what they need to buy child care. Since the government will not aim to make a profit on this loan, we assume that it lends at the same rate at which it borrows. This justifies our assumption that interest rates are zero for simplicity.[9]

4.3 Loans

In this section we characterise a simple loan programme run by the government to mitigate liquidity constraints and, by *Corollary 1*, reduce gender inequality. We show that only constrained households have incentives to apply for a loan that can be used exclusively to pay for child care services. Our assumption is that the government can borrow in international markets, and that it can directly seize household income so that non-repayment is not an option.

Proposition 3 *Let* $0 < \eta^c(x,\alpha) < \eta^*(x,\alpha)$. *If the government provides loans in the form of child care vouchers:*

a) *Households with* $\eta < \eta^c(x,\alpha)$ *do not borrow.*
b) *Households with* $\eta \in (\eta^c(x,\alpha), \eta^*(x,\alpha))$ *borrow, and repay, the amount*

$$B(x,\eta) = (1-\alpha)(\eta - \eta^c(x,\alpha)) \tag{38}$$

c) *Households with* $\eta > \eta^*(x,\alpha)$ *do not borrow.*

Proof First, note that over-borrowing and default are not relevant options. On the one hand, no household has an interest in borrowing more than it needs, since borrowing can only be used to pay for child care services and has to be paid back. This prevents over-borrowing. On the other hand, the government can seize an amount of income that could even be larger than the amount owed in case of non-repayment. This eliminates incentives for default. Let us now look at each type of households in turn:

[9] In particular, let r denote the cost of borrowing for the government. This is also both the interest households would obtain from lending (opportunity cost of waiting) and the interest households would pay for a government loan (since the government will not intend to make a profit). Then, with $R = 1+r$, the present value of lifetime income of a household where the mother goes back to work after borrowing B and repays it in the second period is: $w_m(x) - \tau + w_f(x,\alpha) - (1-\alpha)\tau - (1-\alpha)\eta + B + \frac{w^h(x)-\tau+w^i(x)-\tau}{R} - \frac{RB}{R}$. Assuming $r = 0$ in this context is innocuous.

a) For households with $\eta < \eta^c(x, \alpha)$, first period income is larger than child care costs, hence they do not need to borrow and borrowing would not lead to an increase in lifetime income.
b) For households with $\eta \in (\eta^c(x, \alpha), \eta^*(x, \alpha))$, first period income $w_m(x) - \tau + w_f(x, \alpha) - (1 - \alpha)\tau - c$ is lower than child care costs $(1 - \alpha)\eta$. They need to borrow that difference, which we can write $(1 - \alpha)(\eta - \eta^c(x, \alpha))$. If they borrow, women in these households will go back to work and the additional income earned will be larger than their loan repayment since $\eta < \eta^*(x, \alpha)$.
c) In households with $\eta > \eta^*(x, \alpha)$, since Eq. (9) is not satisfied, it holds that $w_f(x, \alpha) - (1-\alpha)\tau - (1-\alpha)\eta + w^i(x) < \alpha w_f(x, \alpha) + w^n(x)$, i.e. households attain higher lifetime income if mothers do not go back to work after the leave. Hence, they are better off staying at home and providing care themselves, instead of buying child care to return to work.

□

Clearly, more complex environments (e.g. the inclusion of uncertainty, different attitudes towards risk, or asymmetric information) provide additional challenges to the design of a loan programme. Chapman and Higgins (2009) were the first to propose household loans to help women with children to return to work. A very similar tool, that of student loans, has, however, been discussed for a long time. Like higher education investments, child care can be seen as an investment that improves women's future earning prospects. Hence, all the insights gained about the implementation of student loans can be applied to child care loans. Income contingent loans, in particular, have gained prominence as a way to deal with asymmetric information and uncertain future outcomes.[10]

We now propose a numerical example to compare the effects on gender inequality of the three policies, when some households are liquidity constrained.

5 A numerical example

The theoretical model presented before shows that some households' inability to afford child care, besides generating an inefficiency, amplifies gender gaps in participation and wages. It also demonstrates how different policies affect the extent of gender inequality, by altering households' constraints. In particular, the model illustrates that a longer paid maternity leave has unclear effects on female labour force participation and wages, and that the effects of subsidies and loans differ due to the role played by taxes. In this section, we calibrate and simulate the model using Spanish data. Since there are many aspects of the real world that are currently not captured by our model, our goal is not to reach quantitative conclusions. Instead, we

[10] See Barr et al. (2019), Britton et al. (2019) and Van Long (2019), for some practical lessons from income contingent loan design around the world. Quiggin (2014) shows the advantages of income contingent loans under asymmetric information.

wish to provide an example of how the different policies affect gender inequality at equilibrium when some households are liquidity constrained.

5.1 Calibration

We calibrate the model in yearly terms for households with average earnings in the Spanish economy in 2018. Table 2 presents the calibrated parameters and variables. Next, we describe the calibration procedure.

Households and benchmark leave duration Young individuals are between 30 and 49 years old. Old individuals are 50 and above. We set the proportion of households with children at $\rho = 0.704$, which reflects the percentage of women aged 30 to 49 who are mothers in 2018 according to the Spanish National Institute of Statistics. In the benchmark calibration we consider a scenario of young households with two adults (a man and a woman) and $2/\rho$ children. Mothers receive 4 months of fully paid maternity leave per child. Thus, we set $\alpha = (4\ months \times 2children)/(19\ years \times 12\ months \times \rho) = 0.0496$, implying that a woman aged between 30 and 49 years spends 5% of her available time on leave. Older households consist of two adult members.

Wages We use the 2018 Spanish Wages Structure Survey to calibrate the wage distribution of full time workers. Our model has young and old women and men. Old men have high job experience, while old women may have high, intermediate or no job experience. We consider that male and female workers with high job experience are those with more than 11 years of job seniority. In contrast, female workers with no job experience have less than one year of job seniority. For female workers with intermediate experience we want to focus on those who only stopped working during maternity leave. For this reason, for intermediate experience, we consider women with 10 to 11 years of job seniority.[11]

Using a total sample of 28,500 establishments with around 220,000 employees, we compute the average annual wage of old experienced men, which is equal to 42,953 euros, and normalise it to $w^h(x) = 1$. In the model, women without children work for the entire youth period, which gives them high experience when old. Thus, we assume that they earn the same wage as old men. This assumption will only affect the computation of the average wage of old women, which will be higher than that observed in the data, without any other implication.[12] We express the other average wages as ratios of $w^h(x)$. Thus, the wage of a young man is set to

[11] Unfortunately, the Spanish Wages Structure Survey has no direct information on accumulated years of experience across different jobs. We assume that male and female workers with job experience have more than 11 years of job seniority, because this value corresponds to the average of job seniority for both groups of workers. In turn, we assume that workers without job experience have less than one year of job seniority since, due to the intensive use of temporary contracts, the average duration of a contract in Spain is 49 days.

[12] Our main aim is to isolate the effect of career interruptions on the average earnings of old women; we thus neglect other possible sources of gender differences in wages in old age.

Table 2 Calibrated parameters and variables for Spain

	Value	Source/target
Parameters:		
Proportion of women with children, ρ	0.704	National Institute of Statistics 2018
Average duration of parental leave, α	0.050	(4 months $\times$ 2 children) / (12 months $\times$ 19 years $\times \rho$)
Wage of old men and women without children, $w^h(x)$	1	Normalised
Wage of young men, $w_m(x)$	0.349	2018 Spanish Wages Structure Survey
Wage of old women without job experience, $w^n(x)$	0.295	2018 Spanish Wages Structure Survey
Wage of old women with intermediate job experience, $w^i(x)$	0.749	2018 Spanish Wages Structure Survey
Firm's cost when mothers are on leave or quit, q	0.147	Solves (16)
Minimum consumption level c	0.312	Spanish Institute of Statistics (INE)
Shape of the Weibull distribution cost function, η_{shape}	0.229	Matches $F(\eta^*)$ and $F(\eta^c)$
Scale of the Weibull distribution cost function, η_{scale}	0.112	Matches $F(\eta^*)$ and $F(\eta^c)$
Variables:		
Labour force participation of young women, (19)	0.810	2018 Spanish Labour Force Survey
Wages of young women, $w_f(x)$	0.307	2018 Spanish Wages Structure Survey
Average wage of old women, $\bar{\omega}_f(x)$	0.732	Solves (20)
Proportion of constrained households with children, $F(\eta^*) - F(\eta^c)$	0.064	2010 Spanish Labour Force Survey
Proportion of mothers returning to work after childbirth, $F(\eta^c)$	0.717	Solves (19)
Unconstrained threshold level of child care costs η^*	0.782	Solves (10)
Constrained threshold level of child care costs η^c	0.357	Solves (12)
Lump-sum tax, τ	0.0029	Solves (28)

$w_m(x) = 15,021/42,953 = 0.349$. The wage of an old woman with intermediate experience is $w^i(x) = 32,158/42,953 = 0.749$. As to the remaining wages, that of an old woman without job experience is $w^n(x) = 12,657/42,953 = 0.295$. The wage of a young woman is $w_f(x) = 13,186/42,953 = 0.307$. Note that the average unadjusted gender wage gap of young workers $(w_m(x) - w_f(x))/w_f(x)$ is equal to 13.9%.

Minimum consumption level Using data from the Spanish Institute of Statistics 2018, we set the minimum level of consumption equal to the average expenditure in food, housing, water and energy per household, which equals 13,403 euros in 2018. Thus, $c = 13,403/42,953 = 0.312$.

Labour supply and proportion of mothers returning to work after leave By assumption, all young men, old men, and old women work. Only young women can be inactive, if they have children and do not go back to work after the leave. Using data from the Spanish Labour Force Survey, we target the labour force participation rate of young women aged between 30 and 49 in Spain in 2018 at 81.0%. In the model, the share of women with children who return to work is $F(\eta^c)$. Then, using Eq. (19) and the female labour force participation rate (81.0%), we obtain the proportion of mothers who go back to work after the leave is over $F(\eta^c) = 0.717$. Plugging this value in Eq. (20), we also obtain the calibrated average wage of an old woman: $\bar{\omega}_f(x) = 0.732$.

Taxes We calibrate the lump-sum tax by calculating the revenues necessary to cover the cost of paid maternity leave per taxpayer as a fraction of $w^h(x) = 1$. Using Eq. (28), the tax τ required to finance the maternity leave is equal to $(\rho\alpha w_f(x))/(3+(1-\rho)+(1-\alpha)\rho F(\eta^c)) = 0.0029$. This implies an annual amount of 124 euros per taxpayer, which is not far from the average expenditure in parental leave per employee observed in Spain in 2018 (94 euros).

Liquidity constrained households The 2010 Spanish special module on reconciliation between work and family life from the Labour Force Survey shows that 6.4% of the mothers with children below 15 years of age do not work because child care services are too expensive. We assume that this percentage matches that of households who are liquidity constrained. Then, $F(\eta^*) - F(\eta^c) = 0.064$. Thus, we obtain $F(\eta^*) = 0.781$.

Child care costs Each household needs to spend a different amount on child care for the mother to be able to return to work. These costs depend on a large variety of elements, for example: whether the household can get help from relatives, and how much; availability of public or private child care facilities nearby; working schedules; commuting time; whether the child gets sick often (needing a different arrangement, like a baby sitter who takes care of him/her at home); the age distribution of children, as older children can take care of younger ones, or other special needs.

Calibrating the distribution of these costs is not an easy task. The distribution of actual expenditure on child care can be a good measure of the distribution of child care costs only for those households that buy child care on the market, but it is not informative of the costs faced by those households, that decide to rely on household provision of child care. Since the costs of the latter type of households are not observed, we assume that the overall distribution of child care costs is of the Weibull type and calibrate the parameters of the distribution to match the values of

$F(\eta^*)$ and $F(\eta^c)$ that we have obtained before.[13] To calibrate this distribution we first need the thresholds η^* and η^c, which are calibrated using Eqs. (10) and (12). We obtain $\eta^* = 0.782$ and $\eta^c = 0.357$. Then, having two parameters to calibrate in each case (the scale parameter and the shape parameter), and using the targets $F(\eta^*)$ and $F(\eta^c)$ as well as the Weibull distribution function, we obtain $\eta_{shape} = 0.229$ and $\eta_{scale} = 0.112$. The median of this distribution is 0.029, approximatively 10% of the young woman wage at the benchmark scenario corresponding to a fully paid leave of 4 months per child ($\alpha = 0.050$).

Firm's adjustment costs The costs incurred by the firm when mothers are on leave or quit are obtained using the wage Eq. (16) with $\hat{\eta} = \eta^c$. We get $q = 0.147$.

5.2 Simulations

We first explore the effect on gender inequality of increasing the duration of paid maternity leave when some households are liquidity constrained. We then study the effects of a proportional subsidy and a loan. We know that both instruments reduce gender inequality, and that a loan can eliminate the liquidity constraint. Therefore, in the simulation, we calculate the subsidy rate that eliminates the liquidity constraint so that the subsidy and the loan can be compared on equal terms.

5.2.1 Modifying the length of fully paid maternity leave

We change the duration of the fully paid maternity leave α when households are liquidity constrained. We maintain the assumption that households have $2/\rho$ children. Besides the benchmark scenario ($\alpha = 0.050$), we consider two additional scenarios. The first one assumes that there is no paid leave and mothers work for the entire first period. Thus, we set $\alpha = 0$. In the second, the paid leave increases to 12 months per child ($\alpha = 0.15$), which is near to the average paid leave duration in OECD countries in 2020 according to Table PF2.1.A in the OECD Family Database. Note that, according to our strategy of calibration, all these scenarios imply adjusting the lump-sum tax to finance the change in the leave duration. Table 3 shows the simulated scenarios.

If we start from the benchmark calibration with $\alpha = 0.05$—see column 2—and eliminate maternity leave by setting $\alpha = 0$—see column 1—the female labour force participation rate decreases from 81.0% to 79.9%, while $w_f(x)$ increases from 0.307 to 0.3127. As a result, the gender wage gap of young workers falls from 13.91% to 11.83%. In contrast, when maternity leave duration increases from four months to one year ($\alpha = 0.15$)—see column 3—the female labour force participation rate increases from 81.0% to 83.22%, while $w_f(x)$ falls from 0.307 to 0.2943. Thus, the gender wage gap of young workers increases from 13.91% to 18.84%. Since participation of

[13] We use the Weibull distribution because it is flexible and can capture the characteristics of many different types of distributions without further assumptions.

Table 3 Simulated effects of changes to paid maternity leave duration α when households are constrained (Benchmark $\alpha = 0.05$)

Variable	1. $\alpha = 0.00$	2. $\alpha = 0.05$	3. $\alpha = 0.15$
Young female participation rate (%)	79.90	81.00	83.22
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3127	0.3070	0.2943
$\tilde{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.731	0.732	0.733
Constrained households (%)	6.40	6.40	6.41
Gender wage gap (%), young	11.83	13.91	18.84
Lump-sum tax τ as fraction of $w^h(x) = 1$	0.0000	0.0029	0.0084

young women increases when the duration of the leaves goes up, more old women have intermediate experience and the average wage of old women increases.

According to our model, the reduction in the wage of young mothers takes place because the negative effect of a higher α on $w_f(x)$ dominates the positive one due to a higher labour force participation (case b in Proposition 2). Note that the share of constrained households does not fall in response to higher α. In fact, while more households can afford to pay child care (η^c shifts to the right) it is also the case that more households find it optimal to do so (η^* shifts to the right). For example, the percentage of constrained households increases from 6.40% to 6.41% when the paid leave parameter increases from $\alpha = 0.05$ to $\alpha = 0.15$. Finally, note that the increase in the maternity leave duration from four months ($\alpha = 0.05$) to one year ($\alpha = 0.15$) increases the lump-sum tax from 124 ($\tau = 0.0029$) to 360 ($\tau = 0.0084$) euros.

5.2.2 Introducing child care subsidies

In Section 4.2 we saw that a proportional subsidy on child care costs can reduce the proportion of households that are liquidity constrained and, thus, gender inequality. We now compare two different scenarios of child care subsidies. The first one corresponds to our benchmark scenario where the proportion of the child care cost subsidised by the government is equal to $s = 0$. In the second scenario, we introduce a proportional subsidy s and set the rate so that the percentage of households that cannot pay child care costs but would be better off if they could is set to zero. This happens when $s = 0.543$. We adjust the lump-sum tax to finance the change in s, which implies increasing τ from 0.0029 (124 euros) to 0.0088 (378 euros).

As expected, the female labour force participation increases and so do wages, with an ensuing reduction of gender wage gaps. This happens because more households can afford for women to participate in the labour market, thus reducing the firm's expected cost of quitting, with positive effects on female wages and labour force participation. Specifically, the participation rate of women increases from 81.0% to 85.27% and the wage of young women increases from 0.3070 to 0.3169. As a result, the average wage of old women goes up too, reducing the gender wage gap in old age (Table 4).

Table 4 Simulated effects of removing liquidity constraints with a proportional child care subsidy

Variable	1. $s = 0$	2. $s = 0.543$
Young female participation rate (%),	81.00	85.27
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3070	0.3169
$\bar{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.732	0.752
Constrained households (%)	6.40	0.00
Gender wage gap (%), young	13.91	10.35
Lump-sum tax τ as fraction of $w^h(x) = 1$	0.0029	0.0088

5.2.3 Introducing loans

We now explore the effect of removing the liquidity constraint through the provision of a loan. The loan (see (38)) covers the difference between the child care cost the household faces if the mother returns to work once the maternity leave is over, $(1-\alpha)\eta$, and household income in the first period $w_m(x) - c - \tau + (1-\alpha)(w_f(x) - \tau)$. In other words, the child care cost $(1-\alpha)\eta^c$, which they can afford with this income. In our simulated scenario, the average loan provided by the government amounts to 0.173 as a fraction of $w^h(x) = 1$ (7,430 euros, 2,653 per child).

Table 5 shows the benchmark calibration with constrained households ($F(\eta^*) - F(\eta^c) > 0$, column 1) and the results of simulating the removal of household liquidity constraints ($F(\eta^*) - F(\eta^c) = 0$, column 2). Removing liquidity constraints increases female labour force participation and reduces gender wage gaps for both young and old. Specifically, the participation rate of women increases from 81.0% to 85.3%. This effect is slightly larger than the one obtained with the proportional subsidy because taxes remain unchanged in this case. Young women's wages increase from 0.3070 to 0.3170 and old women's wages increase from 0.732 to 0.752.

Table 5 Simulated effects of removing liquidity constraints with loans

Variable	1. $B = 0$	2. $B > 0$
Young female participation rate (%)	81.00	85.30
$w_f(x)$ as fraction of $w^h(x) = 1$	0.3070	0.3170
$\bar{\omega}_f(x)$ as fraction of $w^h(x) = 1$	0.732	0.752
Constrained households (%)	6.40	0.00
Gender wage gap (%), young	13.91	10.32
Loan provided by the government as a fraction of $w^h(x) = 1$	–	0.173
Lump-sum tax τ as fraction of $w^h(x) = 1$	0.0029	0.0029

6 Discussion

Child care is expensive. Some households cannot afford it, and this forces them to have one parent staying at home until (pre-)school is free. Typically, it is the mother who stays home, and this amplifies gender gaps in labour market participation and wages. For these households the outcome is inefficient: their lifetime income net of child care costs would be larger if the mother went back to work right after maternity leave, because of the positive effect of accumulated experience on wages. We show that allowing mothers in these households to remain employed reduces gender gaps in the labour market.

The inefficiency studied here is similar to that arising in (tertiary) education, where liquidity constraints can prevent the young from making investments that would yield positive net returns. To address these liquidity constraints many countries use student loans as part of their education policy. One of the advantages of government-led loan programmes is that repayments can be embedded in the income tax, like in the optimal student financial aid formulas developed in Colas et al. (2021). In the context of child care policy a similar idea has been advanced by Chapman and Higgins (2009), but to the best of our knowledge no country has implemented a policy of this kind to date. Our numerical example in Section 5 (see Table 5) shows that, in 2018, child care related liquidity constraints could have been removed with an average loan of 7,430 euros (2,653 per child) in Spain.

In contrast, child care subsidies and maternity leaves are very common instruments around the world to support maternal employment. Their effect on gender inequality when some households are liquidity constrained have, however, not been considered before. Child care subsidies reduce child care costs and allow women in liquidity constrained households to return to work. Firms then face lower adjustment costs related to hiring women and female wages go up. In our numerical example, a subsidy of 54.3% of child care expenses eliminates the liquidity constraint and increases young female labour force participation rate and wages almost by the same extent as loans (see Table 4).

Also, maternity leave policies are a potentially good policy tool to address liquidity constraints, because longer maternity leaves expand the time the mother is at home, thus making child care less expensive. However, longer leave periods impose adjustment costs on firms, which may not only lower young women's wages but also offset the positive effect of the leave on participation. From proposition 2, we can see that increasing the duration of paid maternity leave is more likely to reduce gender gaps when, for instance, the right hand side of Eq. (29) is small. This happens if $f(\eta^c)$—the number of women who return to work thanks to the extended duration—is large relative to $F(\eta^c)$—the number of women that do so before the change in the policy. Also, an extension in maternity leave duration is more likely to have positive effects on women's labour market outcomes when firm adjustment costs $q(x)$ are lower, and therefore the wage of young women is higher (left hand side of Eq. (29) larger through Eq. (16)). In our numerical example, the extended duration of the maternity leave from 4 to 12 months increases the female labour force participation rate but not as much as the other policies, and reduce the wage of young women slightly. Thus,

it is not the best policy to address gender inequality in the labour market, at least in Spain.

7 Concluding comments

Maternity leaves and child care subsidies are widely used around the world to guarantee mothers a job-protected leave and promote work-life balance. They are also a sizeable fraction of overall family policy expenditure in OECD countries. The potential benefits of loans in the context of family policy, instead, have been put forward by Chapman and Higgins (2009), but their role in addressing gender inequality in the labour market has not been considered in the literature, as far as we know.

In this paper we show that mitigating liquidity constraints that prevent households from buying child care reduce gender gaps in participation and wages. In this context, we evaluate the relative merits of an extension in paid maternity leave duration, a child care subsidy, and a government loan. We find that increasing the duration of paid maternity leave has ambiguous effects on gender inequality because, on the one hand, this policy reduces child care costs and liquidity constraints but, on the other hand, it imposes higher adjustment costs on firms that then pay women lower wages. Subsidising child care costs mitigates liquidity constraints and unambiguously reduces gender inequality because these subsidies do not impose costs on firms. The same happens with a loan given out in the form of a child care voucher. The subsidy requires higher taxes but our numerical example shows that the tax per worker required to fund it is relatively small.

Future work can assess the effectiveness of these policies in reducing gender inequality in more complex environments, where uncertainty about future earnings plays a role. Note also that we have studied the effects of these policies on gender gaps in participation and wages rather than on overall welfare, taking a positive rather than a normative approach. We leave the analysis of welfare effects for future research.

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