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# **Essays on Labour Market Effects of Fiscal Policy**

*Thesis submitted in fulfillment of the requirements for the degree of*

## **Doctor of Philosophy**

*by*

**Goher Fatima**

School of Economics, University of Kent

**December, 2022**

## DECLARATION

I hereby declare that the thesis entitled “Essays on Labour Market Effects of Fiscal Policy” submitted by me, for the award of the degree of *Doctor of Philosophy* to School of Economics, University of Kent is a work carried out by me under the supervision of Dr. Andrey Launov, School of Economics, University of Kent, Canterbury.

I also declare that the work presented in the thesis has not been submitted and will not be submitted for the award of any other degree in any other institute.

**(Goher Fatima)**

## **CERTIFICATE**

This is to certify that the thesis entitled “Essays on Labour Market Effects of Fiscal Policy” submitted by Mrs. Goher Fatima, School of Economics, University of Kent, Canterbury for the award of the degree of *Doctor of Philosophy*, is work carried out by her under my supervision, as per the University of Kent regulations.

**(Dr. Andrey Launov)**

## ABSTRACT

This thesis aims to contribute to the existing literature of fiscal policy with search and matching frictions using Dynamics Stochastic General Equilibrium (DSGE) modeling. This thesis consists of three essays (chapters) and the respective abstracts are as follows:

Title of first chapter is ***Labour Market Effects of Government Spending***. In this chapter, I contributed to the Search and Matching Frictions (SMF) theory by examining the effects of fiscal multiplier shock on output and labour market for the US economy. I estimated US quarterly data and showed that government spending has positive effect on output, employment, labour force participation, normalized vacancies, job finding probability and labour market tightness and negative effect on unemployment and real wages. This study contributes to the literature of SMF theory by estimating the model after inclusion of total job separation (a. as a combination of quits and layoffs, b. as segregated quits and layoffs) in the model. Positive government spending shock has a positive effect on total job separation and quits and negative effect on layoffs and average wages, which results in dispersion in wage distribution. This dispersion in wage distribution leads to income inequality. I also found that income inequality (gini) responded negatively to government spending shock, whereas P90/10 showed a positive response in short run and negative response in long run.

In the second essay, '***Fiscal Stimulus, On-The-Job-Search and Labor Market Dynamics***', I theoretically examined the effects of government spending shock and aggregate productivity shock on the labour market using search and matching frictions. For this purpose, I developed a DSGE model using search and matching frictions and introduced two endogenous job separation margins (i.e., quits and Layoffs). I introduced quits as on-the-job-search with search effort and layoffs as operating cost. Results show that positive government spending shock has a positive effect on wages in good sector and a negative effect on wages in bad sector. Results also show that government spending increases job creation in good sector which leads to increase in job-to-job movements from bad to good sector. Fiscal policy shock has a negative effect on layoffs in bad sector. Using simulation, I calculated average wages by using both types of wages and both types of employment. Results show that initially there is a sharp decrease in average wage and subsequently showed increasing trend indicating

wage inequality in labor market which results in movement of workers from bad to good sector.

In the third essay, '**Fiscal Policy with Labor Market Frictions**', I investigated the effects of hiring subsidy (subsidy to the cost of posting vacancy) on US labor market in a DSGE model using search and matching frictions with two distinct margins of endogenous job separation i.e., quits through on-the-job-search and layoffs through operating cost. The results show that hiring subsidy has a positive effect on job creation in both good and bad sectors. Hiring subsidy affects unemployment negatively in good sector and positively in bad sector. Hiring subsidy significantly affects wages in both sectors. All these results show that hiring subsidy has a positive effect on workers' welfare through wages and job creation. Results further show that firm's operating cost responded negatively to the hiring subsidy. Other variables also responded to positive hiring subsidy shock, but the effects are trivial.

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# Labour Market Effects of Government Spending

## 1.1 Introduction

Post-recession academic research, in the field of macroeconomics, is largely dominated by the literature on how to avoid or overcome any un-anticipated economic crises <sup>1</sup>. These concerns have shifted the focus to study multi-dimensional effects of fiscal multipliers, such as government spending Monacelli, Perotti, and Trigari (2010), tax cuts and reforms Alesina and Ardagna (2009) and other recurring expenditures such as transfer payments Hyunseung and Reis (2011) on the output of the country. Global financial crisis was considered to be the main reason of the vast popularity of this important topic because this crisis had long lasting adverse effects on the economies all over the world and it was the time when this concept gained more popularity among both the researchers and the policymakers.

Fiscal multipliers are considered very effective tools to stabilize the economy and has a strong effect on output Clemens and Miran (2012). However, seemingly, it was not easy for the policy makers to find the solution of economic crisis by using only monetary and expansionary fiscal policies and proved to be insufficient to increase the output in general and the level of employment in particular.

The global recession of 2008 also had an adverse effect on the output and employment which triggered the government spending. Soon after, different fiscal stimulus packages were announced in different countries to reduce unemployment. In US, American Job Act (AJA) was passed in 2008 to facilitate more job creation. Academic researchers also contributed to the fiscal policy and unemployment literature using search and matching theory of unemployment of Mortensen and Pissarides (1994),

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<sup>1</sup>see Mittnik and Semmler (2012); Parker (2011)

and contributed to improve the labour markets.

Labour markets are largely characterized by two important distinct features i.e., searching job and matching job, both of which are performed by the workers and the firms, respectively. There are different important variables in the labour market, such as; vacancy announcement, job creation, job destruction etc., which are considered necessary for developing search and matching models in the broad domain of labour market Mortensen and Pissarides (1994).

Monacelli, Perotti, and Trigari (2010) conducted a study, both empirically as well as theoretically, to investigate unemployment multipliers of government spending for US economy. Results show that government-spending multipliers has positive effect on employment, job vacancy and labour force participation and negative effect on unemployment. Later on, Brückner and Pappa (2012) shows expansionary fiscal policy has a positive effect on both employment and unemployment through higher labour force participation in selected OECD economies.

Nakamura and Steinsson (2014) conducted a study to estimate fiscal multipliers in a monetary union by using changes in military spending in different region of US. Furthermore, they also calculated effects of government spending on unemployment and showed that when unemployment was high, effect of government spending was large and vice versa. Rendahl (2016), using DSGE model, estimated the fiscal multiplier for unemployment and output. They calibrated the model using quarterly frequency and showed that increase in government spending has positive effect on output and negative effect on unemployment.

From the above arguments and after analysing literature, it is evident that the policy makers have shown more interest on the government spending increases on one hand and the tax cuts on the other hand to achieve the optimal output and to alleviate the unemployment. However, empirical literature shows that research community conducted more studies to examine effects of government spending on different macroeconomic variables. Most of current literature looks only at employment response at the level of the stock and inflows but less attention is given to examine the effects of fiscal policy on output and labour market specifically in the context of outflow from employment such as total job separation (i.e., quits and layoffs).

Another research gap which I identified from literature is that disregarding outflows from employment misses important insights about dynamics of employment, earnings and inequality and there is no such study where income inequality is added in the

model with search and matching theory to find out impact of fiscal policy shocks not only on output and unemployment but also on income inequality. Brinca et al. (2016) considered inequality but do not look into the link between quits, layoffs and the dynamics of the wage distribution.

This study addresses these gaps in the literature and examines the effects of fiscal policy shock on job separation (quits and layoffs) and income inequality for US economy. There are three research objectives of this study. First objective of the study is to examine the effect of fiscal policy shock on output and labour market using search and matching theory. Second objective of the study is to investigate employment response to government spending shock at the level of the outflows from employment and third objective of the study is to find out effect of government spending shock on unemployment, total job separation, quits and layoffs and on income inequality.

This research contributed to the existing literature of effects of fiscal multipliers on labour market in two ways: firstly, to the best of available literature, the study is the first of its kind to investigate the impact of fiscal policy shock on output and labour market using job separation (i.e., quits and layoffs) with other variables, such as labour market tightness (ratio of vacancy to unemployment), unemployment rate, total employment, labour force participation, job finding rate and real wage; secondly, this study is also a first to examine the impact of fiscal multiplier on income inequality (differential in wages) using Gini coefficient and P90/10. There is a dire need of such work after the financial crises to estimate fiscal multiplier of output and unemployment using job separation (quits and layoffs) with other labour market variables. This study further contributes to the existing literature by enhancing the understanding about the nature of the relationships between government spending multiplier and job separation.

To achieve the objectives of this study, data for the US economy of selected variables was collected from different sources (detail is available in data collection section) for the period from 2001 to 2018 and estimated the model using Structural Vector Autoregressive (SVAR) approach. Main results are summarised as follows:

I started with the simple search and matching model to see the effect of positive government spending shock on output and employment. Results show that positive government spending shock has positive and significant effect on output, consumption, labour force participation and employment and negative effect on unemployment and real wages. Labour force participation and employment showed increasing trend which indicates that higher employment and labour force participation reduce unemployment

in the economy due to positive government spending shock. Vacancies, job finding probability and labour market tightness positively responded to positive government spending shock.

To examine the behaviour of labour market more deeply in response to positive government spending shock, I included total job separation in the model as it is the relevant and important dimension of the labour market. Government spending shock has a positive impact on the total job separation, employment and labour force participation and negative effect on real wage. This implies that addition of total job separation variable in the model has some important association with these labour market variables. On one hand there is an increase in job separation i.e., there is job destruction and on the other hand there is increase in participation and employment which indicates that there is job-to-job movement. To see either there is job destruction or there is job-to-job movement, I segregated total job separation into quits and layoffs.

I found that government spending shock has a positive and significant effect on quits and negative effect on layoffs, which confirm the above results that workers are still in the labour market and employed and will go for the higher wage ladder. On the other side, there is a job creation due to positive government spending shock, this means some workers will go for the jobs created in government sector which are on average low paid jobs, hence, vacancy creation is at the lower tail of earnings distribution. This all implies that there is no involuntary unemployment.

Job creation at both higher and lower end of wage distribution results in a spread in the wage distribution which may subsequently lead to increase in income inequality. More spread in wage distribution means more unequal society, therefore, to see either these dynamics in wages lead to increase in income inequality, I used two measures of income inequality i.e., Gini coefficient and P90/10. I found that there is no significant increase in income inequality which implies job-to-job movement did not go too far to the right of wage distribution and government sector jobs are not as low paid as one could think. As GINI is the measure of entire wage distribution therefore to see the changes only on the tails of the distribution, I replaced GINI with P90/10 (IIE) and estimated the model. I found that Government spending has an instantaneous and short-term impact on IIE. This implies that there is initial spread in wage distribution then contraction and in the short run, government spending may increase income inequality due to dynamics in the wage distribution but overall effect of government spending on income inequality is insignificant with a decreasing trend.

Finally, for the robustness analysis, I estimated the model with total job separation which is constructed using Shimer (2005) formula and found that effect of positive government spending shock on total job separation is negative and this is opposite to the total job separation which I constructed by the addition of quits and layoffs. This implies that Shimer (2005) only include layoffs while constructing total job separation.

Rest of the chapter is organized as follows. Section 1.2 discusses literature review, section 1.3 explains empirical model, methodology and data sources. Section 1.4 is about empirical findings and section 1.5 presents robustness analysis. Section 6 discusses summary and conclusion of the chapter.

## 1.2 Literature Review

Government spending is considered one of the most important determinants of fiscal policy <sup>2</sup> and this important component has multiple effects on any economy in different ways. Different researchers analysed effect of government spending on economy empirically as well theoretically. This review of literature begins with the effects of government spending on overall economy and afterwards I reviewed literature of effects of government spending on labour market. I also presented theoretical as well empirical evidences from past studies and at the end I presented few studies which contributed to the literature on effect of fiscal multiplier on income distribution.

### 1.2.1 Effects of Government Spending on Economy

Academic researchers used different models and identification assumptions to investigate the dynamic effects of government spending on macroeconomic variables. Literature shows that researchers introduced different approaches to calculate fiscal multipliers using VAR by imposing long run and short run restrictions O. J. Blanchard, Fischer, et al. (1989); Clarida and Gali (1994). There was confusion about the size and nature of fiscal multipliers V. A. Ramey (2011b), Leeper, Walker, and Yang (2010). Even so that these results are very different according to the development degree of the countries such as they are different in under-developed, emerging and developed economies. Empirical literature shows that fiscal multipliers is small for open economies as compared to closed economies. For example, Ilzetzki, Mendoza, and Végh (2013) conducted a panel study using SVAR to examine effects of macroeconomic fiscal multi-

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<sup>2</sup>see Barro (2009), Auerbach and Gorodnichenko (2012), Clemens and Miran (2012)

pliers on output and showed that the effect of government consumption on output is larger in industrial economies as compared to developing countries. They used different determinants of fiscal multipliers (e.g., exchange rate regime, public debt, openness to trade) and showed that economies operating under predetermined exchange rates has relatively large fiscal multiplier and it is zero in economies who are operating under flexible exchange rates. Born, Juessen, and Müller (2013) estimated panel VAR by using time series data for OECD countries to examine the impact of government spending under different exchange rate regimes and found that fiscal multiplier is large under fixed exchange rate regime compared to flexible exchange rate regime.

W. Miyamoto, Nguyen, and Sergeyev (2018) recently conducted a study for Japanese economy and found that government spending has positive effect on both consumption and investment with Zero Lower Bound (ZLB) but without it there is crowding out of investment. O. Blanchard and Perotti (2002) employed SVAR to estimate the dynamic effects of shocks in government spending and taxes on US economy and demonstrated that positive government spending shock has a positive effect on output, whereas positive tax shocks has a negative effect on output. They further concluded that government-spending multipliers are close to 1 for US economy.

In a recent study, Fève and Sahuc (2017) used standard DSGE-VAR techniques to measure the size of fiscal multiplier. Their study is different to other studies and used hand-to-mouth consumers and Edgeworth Complementarity as government spending shocks and that former transmission is better than the latter in examining the effect of government spending shocks.

## **1.2.2 Effects of Government Spending on Labour Market**

Pissarides (1985), Mortensen and Pissarides (1994) and Mortensen and Pissarides (1999) presented search and matching model and gained popularity as a theory of unemployment. These models explained, in detail, the functions of labour market. Later, Shimer (2005) used this friction in DSGE model and showed that, by using calibrated parameters, this model fails to explain cyclical properties of unemployment and vacancies in US economy. Mortensen and Pissarides (1994) explained the concepts of job creation and job destruction not only theoretically but also demonstrated their empirical results. They considered the non-cooperative wage behaviour while modelling the job-specific shock processes through the matching models of the unemployment, such as endogenous job creation and job destruction processes. They further demonstrated

that there is a negative correlation between the job creation and job destruction, which is induced by the aggregate shock; however, the positive correlation between these two is induced through the dispersion shock. Further segregating their effects, they showed that the job destruction process is attributed to unpredictable dynamics rather than the job creation process. Baldwin, Dunne, and Haltiwanger (1998) compared data of job creation and job destruction for Canada and USA and found that industry level data of both variables is same for both economies because of same technology used in industries. Furthermore, they found that job destruction is more cyclical and unpredictable than job creation.

Merz (1995) argues that economic performance of the US economy, specifically in the context of labour market, is largely explained through the stylized facts. These stylized facts explain empiric of labour market. Merz further argues that US economy is largely influenced by the fact that the labour productivity is more volatile as compared to the real wages. She further argues that all the above observations cannot be explained simply in growth model, hence, used the search and matching frictions for the first time in Real Business Cycles (RBC). Later, Andolfatto (1996) also contributed in the literature and showed that inclusion of RBC in Search and Matching frictions improved the empirical results in many ways. Results revealed that there is more fluctuations in working hours as compared to wages and there is a negative relationship between working hours and labour productivity.

Gomes (2009) estimated search and matching frictions with two-sector DSGE model to find out the labour market effects of public sector employment and wages. For this purpose, public sector wages are used and results showed that public sector wage shock has a positive effect on unemployment, whereas employment shocks has a negative effect. Michailat (2014) used search and matching theory and developed New Keynesian model to calculate public employment multiplier. Results showed that multiplier and unemployment are moving in same direction when unemployment increases from 5 to 8 percent multiplier increases from 0.2 to 0.4.

### **1.2.3 Theoretical Analysis**

Theoretically, Dynamic Stochastic General Equilibrium (DSGE) models have been used to estimate fiscal multipliers either through calibration method or Bayesian estimation techniques e.g., Galí, López-Salido, and Vallés (2007), Parker (2011) estimated the effect of government spending on consumption by extending the New Key-

nesian DSGE model with the inclusion of *rule of thumb* consumers and found positive relationship between consumption and government spending. Drautzburg and Uhlig (2015) estimated medium scale new Keynesian model using Bayesian estimation technique and found that using posterior mean (0.53) short run multipliers were positive but for the long run the effect was negative. Leeper, Traum, and Walker (2017) recently conducted a study using Bayesian estimation technique to estimate government spending multiplier for US economy. By using fiscal-monetary regime, they found that this regime could play an important role to calculate the size and persistence of fiscal multipliers. Their result further showed that multipliers are higher and long lasting in both regimes. American Recovery and Reinvestment Act (2009) also attracted research community to investigate the impact of fiscal multipliers on output.

## 1.2.4 Empirical Evidence

Empirical literature shows that using quarterly data for different time period for the US economy and applying SVAR approach, the effect of fiscal multiplier on output is positive (Mountford and Uhlig (2009), Cogan et al. (2010), Fisher and Peters (2010)). C. D. Romer and D. H. Romer (2010) calculated fiscal multiplier (tax changes on output) for US economy using narrative approach and found that multiplier effect is contractionary. V. A. Ramey (2011b) used VAR for quarterly data of US economy for the period 1939–2008 to estimate govt. spending multiplier for consumption and real wage. Their results showed that consumption and real wage increased due to increase in government spending. Furthermore, the results also showed that range of government spending multipliers was from 0.6 to 1.2. Holden and Sparrman (2018) used panel data of OECD countries to investigate effects of government spending on unemployment under exchange rate regime, recession and expansion. Results show that effect of government spending on unemployment is positive and higher under fixed exchange rate compared to floating exchange rate and same results for recession compared to expansion.

## 1.2.5 Fiscal Multiplier and Income Inequality

Empirical literature shows that fiscal policy has a strong effect not only on labour market but also on income distribution in the economy. Benabou (2000) and Brinca et al. (2016) conducted panel studies for 15 OECD countries using SVAR and showed

that fiscal multiplier and wealth inequality are positively related. They also corroborated these findings theoretically through development of a neo-classical heterogeneous agents model. Results showed that there is a positive association between fiscal multiplier and fraction of credit-constrained agents and showed that fiscal multiplier is also very sensitive to wealth distribution. Recently, Gunasinghe et al. (2019) conducted a study to investigate impact of fiscal shocks on output and income inequality for Australian economy. They used three different types of fiscal shocks including government spending, direct taxes and indirect taxes shocks and estimated SVAR and structural vector error correction (SVEC) model. Results show that decrease in direct taxes lead to decrease income inequality and increase in per capita real GDP, whereas government spending and indirect taxes has adverse effect on income inequality.

## 1.3 Model, Methodology and Data

### 1.3.1 Model Development and Methodology

Structural model is developed to examine the impact of fiscal multiplier on output and employment.

$$A_0 Y_t = A(L) Y_{t-1} + B \varepsilon_t \quad (1.1)$$

$A_0$  is a matrix of contemporaneous relations between variables.

$$Y_t \equiv [GOV_t, OUTPUT_t, CON_t, UN_t, LFP_t, EM_t, RW_t, VAC_t, JFP_t, \\ LMT_t, SEP_t, QT_t, LO_t, GINI_t, IIE_t]$$

$A(L)$  is a polynomial in the lag operator and  $B$  is  $(n \times n)$  matrix which captures the linear relation between structural shock and reduced form shocks and  $\varepsilon_t$  is vector of structural shocks of the variables.

Solving equation 1 to get the reduced form VAR.

$$Y_t = A_0^{-1} A(L) Y_{t-1} + A_0^{-1} B \varepsilon_t$$

$$Y_t = B_1 + U_t$$

(reduced form VAR representation)

where,

$$B_1 = A_0^{-1}A(L)Y_{t-1}$$

$$U_t = A_0^{-1}B\epsilon_t$$

$U_t = (e_t^g, \dots, e_t^{IE})$  is a K- dimensional vector of reduced form residuals with *zero mean white noise process*.

There are two main approaches in existing literature which are considered by different economists to identify fiscal multiplier Ilzetzki, Mendoza, and Végh (2013). First approach is "Natural Experiment" approach, derived by Barro (1981) and further elaborated by V. A. Ramey and Shapiro (1998) and V. A. Ramey (2011a) using military spending shocks. In this approach military data is used to estimate the impact of defense spending on output V. A. Ramey (2011b). Second, structural vector autoregressive (SVAR) approach pioneered by O. Blanchard and Perotti (2002) which is most famous and most commonly used to examine the impact of fiscal shock on GDP and other macroeconomic variables. Perotti (2007) and Perotti (2005) further extended this approach.

O. Blanchard and Perotti (2002) used quarterly data and estimated Structural Vector Autoregressive (SVAR). They used institutional aspects to include more restrictions to examine impact of government spending and tax multiplier on output. They further stated that it is quite complicated to examine contemporaneous relationship between taxes and GDP. Furthermore, they argued that there is no contemporaneous relationship of government spending with GDP and taxes. Keeping in mind this, they identified government-spending shocks using Choleski decomposition where government spending is modeled before the other variables. Since government spending does not respond contemporaneously to output, therefore, I used SVAR (O. Blanchard and Perotti (2002)) to examine the response of government spending to output using quarterly data. In Choleski Decomposition, ordering of the variables is very important therefore, I followed Monacelli, Perotti, and Trigari (2010) and the economic theory to order the variables. In this study, I used Choleski decomposition to identify government spending as predetermined variable and used it as a first variable.

### **1.3.2 Data Collection**

I used US quarterly data from the period 1970:Q1 to 2018:Q4 of 15 macroeconomic variables i.e. log of real per capita government spending (GOV), log of real GDP (OUT-

PUT), log of real consumption (CON), unemployment rate (UN), labour force participation (LFP), log of total civilian employment (EM), log of real wages (RW), normalized vacancies (VAC), job finding probability (JFP), labour market tightness (LMT), total job separation (SEP), total nonfarm quits rate (QT), layoffs and discharge rate (LO), gini coefficient (GINI) and income inequality P90/10 (IIE).

It is important to use quarterly data for the identification of fiscal shock because in SVAR, it is assumed that fiscal policy needs at least one quarter to respond to shock in output (Ilzetzki, Mendoza, and Végh (2013)). Brief description of all variables is provided below.

Data for OUTPUT and CON (addition of private consumption of non durable goods and services) is collected from Bureau of Economic Analysis (BEA). I divided these two variables by implicit price deflator to make them real and finally took their natural log. Data of GOV (addition of government expenditures and gross investment) is also collected from BEA. I divided it by implicit price deflator to make it real and then divided it by non institutional population to make it per capita and finally took natural log of the series. I collected data EM, UN, LFP, RW (median usual weekly real earnings), QT, LO (2001:Q1 to 2018:Q4) from Bureau of Labor Statistics (BLS).

I constructed data for SEP by adding QT and LO in level and divided it by the total employment. Data of GINI and IIE is collected from US Census Board. Data of JFP and Shimer's Job separation is constructed by following Shimer (2005)<sup>3</sup>. Data of VAC is constructed.<sup>4</sup> I also constructed data of LMT by taking ratio of normalized vacancies to normalized unemployment.<sup>5</sup>

It is important to test stationarity of the data before estimation. I used unit root test for the purpose and all data is stationary at first difference. Lag-2 is selected for the estimation using Akaike information criterion (AIC) lag length criteria..

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<sup>3</sup>Job finding probability is constructed by  $f_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t}$  where  $f_t$  is Job finding probability,  $u_{t+1}$  is unemployment level,  $U_{t+1}^s$  = is unemployment less than 5 weeks and Shimer's job separation is constructed by  $S_t = \frac{u_{t+1}^s}{e_t(1 - \frac{1}{2}f_t)}$  where  $S_t$ = separation rate and  $e_t$  = civilian employment

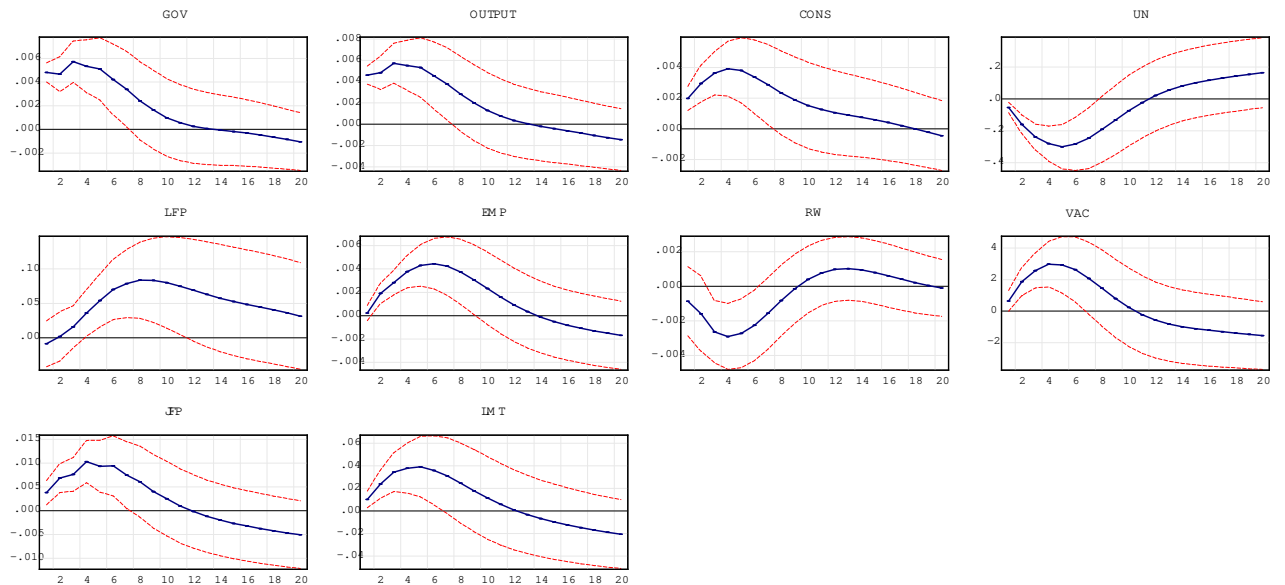
<sup>4</sup>Data of job opening (vacancy) is collected from Job Openings and Labor Turnover Survey (JOLTS) from the period of 2001 as data is not available prior to the this period. I normalized the job opening by taking its 2001:Q1=100 value. Then I collected data of vacancies from Conference Board Help Wanted Index (HWI). I normalized HWI by its 2001:Q1=100 value and then I used the series of the normalized HWI till 2000Q4. To get the series of normalized vacancies I used series of normalized HWI from period of 1970Q-2000Q4 and after that I used series of normalized job opening from the period of 2001Q1-2018Q4 to get the series of normalized vacancies.

<sup>5</sup>To calculate normalized unemployment I did the same for the unemployment. I collected data of Unemployment in level and normalized it by taking 2001:Q1=100 value and the get the quarterly series of normalized unemployment.

## 1.4 Empirical Findings

### 1.4.1 Results of the Model with Different Dimensions

To achieve the objectives of the study I conducted the estimation in different steps. I applied SVAR in the above model in five steps, each step with incrementally modified model. In first step, I estimated the model with all important variables of search and matching theory except the variable of total job separation to examine response of output and unemployment without total Job separation to the positive government spending shock (Figure 1.1). In the second step, to find out how positive government spending shock affect output and unemployment by adding total job separation, I estimated the model after including the variable of total job separation (Figure 1.2). In the third step, I segregated total job separation in two distinct dimensions (quits and layoffs) to examine how positive government spending shock affect quits and layoffs, separately (Figure 1.3). In these three steps, I examined the effect of government spending shocks on the labour market variables, therefore, it is important to examine whether changes in the labour market affect income inequality, and how the positive government spending shock affects the income inequality. In the fourth step, I estimated the two models separately, each includes one measure of income inequality, such as Gini coefficient and P90/10, which are specifically used to measure income inequality (Figure 1.4 and Figure 1.5 for Gini coefficient and P90/10, respectively). Finally, for the robustness check in the fifth step, I estimated the extended model after replacing the total job separation - composite of quits and layoffs - (Figure 1.2), with the other measure of total job separation - only quits - constructed by Shimer (2005) to examine and compare the responses of these two differently constructed variables of total job separation to positive government spending shock (Figure 1.6). Accordingly, the results provided in Figure 1.2 and Figure 1.6 are compared for these two different measures of the total job separation. The size of the shock is used as one standard deviation increase in government spending. Dotted lines on the graphs show 90 percent confidence interval (CI) based on Monte Carlo simulations and solid lines in the graphs show responses of the variables to the shock i.e., Impulse Response Function (IRF)

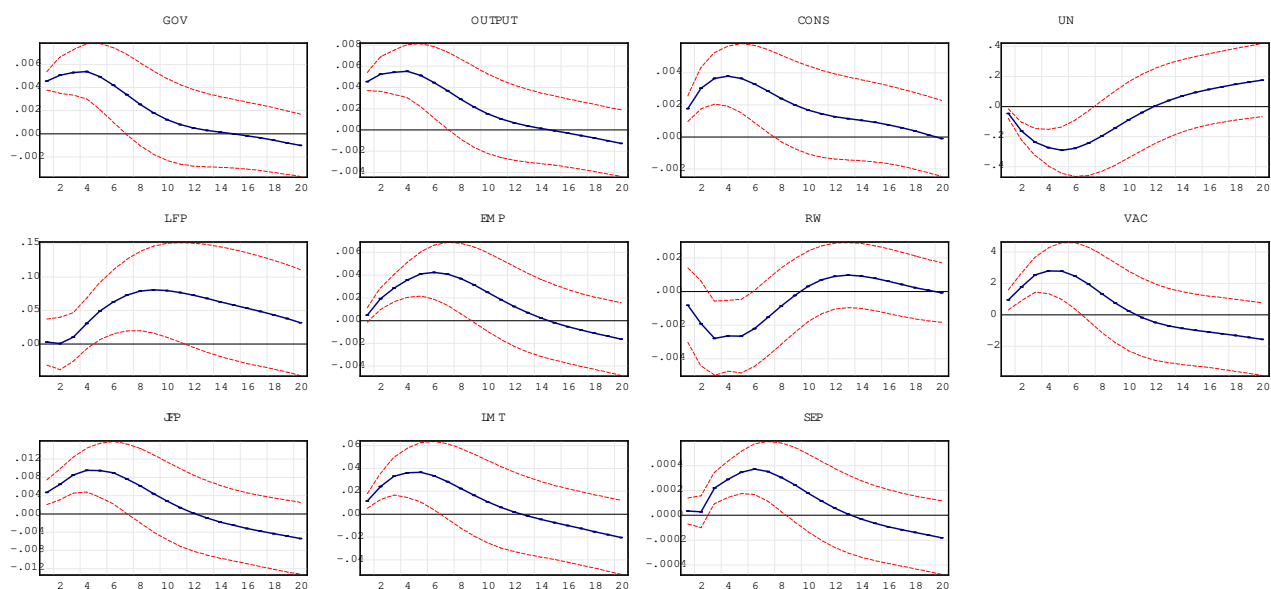


**Fig. 1.1** Impulse Response Functions Without Total Job Separation

Results, provided in the Figure 1.1, show that positive government spending shock has positive and significant effects on output and consumption. Both are positive and significant at the 95 percent confidence interval. Positive government spending shock raises the output and consumption till third and fourth year, respectively, and in the subsequent periods they decrease and then gradually converge to the steady state. Unemployment decreases with increase in government spending and reaches at peak of -0.3 percent between 0.2 to 0.4 percents and it is negative till 12th quarter. This shows that positive government spending has a negative and significant impact on unemployment. Labour force participation and employment are increasing which indicate that higher employment and labour force participation lead to decrease in unemployment in the economy due to positive government spending shock. Though, impact of shock on employment is insignificant in the first quarter but after that it is positive till 14th quarter. Real wage has shown a decreasing trend till the 10th quarter and afterwards started an increasing trend due to one standard deviation shock in the government spending.

Normalized vacancies also rise till the 10th quarter with the peak of 3 percent. This implies that there is job creation due to increase in government spending and positive trend in normalized vacancies lead to decrease in unemployment and increase in labour force participation and employment. Job finding probability and labour market tightness are also increased substantively with peak of 0.010 percent and 0.04 percent, respectively. All of the above impulse responses show that positive government

spending shock has positive and significant impact on all variables mentioned above and negative but insignificant impact on unemployment and real wage. These results also indicate that government spending multiplier has a positive and significant impact on labour market variables. These results are consistent with earlier works of Monacelli, Perotti, and Trigari (2010), and W. Miyamoto, Nguyen, and Sergeyev (2018).



**Fig. 1.2** Impulse Response Functions With Total Job Separation

I have just shown that responses of the labour market variables are significant. In the basic model of equilibrium unemployment Mortensen and Pissarides (1994) rising tightness (consistent with rising vacancies and falling unemployment) implies higher job finding probability and higher employment; together with increasing participation, this is consistent with matches going up. However, higher tightness should also lead to higher wages, which I show is not the case. This urges me to look at inflows and outflows simultaneously, adding total job separations to my analysis.

Total job separation is very important component of search and matching theory <sup>6</sup> along with other dimensions of the labour market, which will be examined after getting insights about the effects of the government spending shocks on the total job separation.

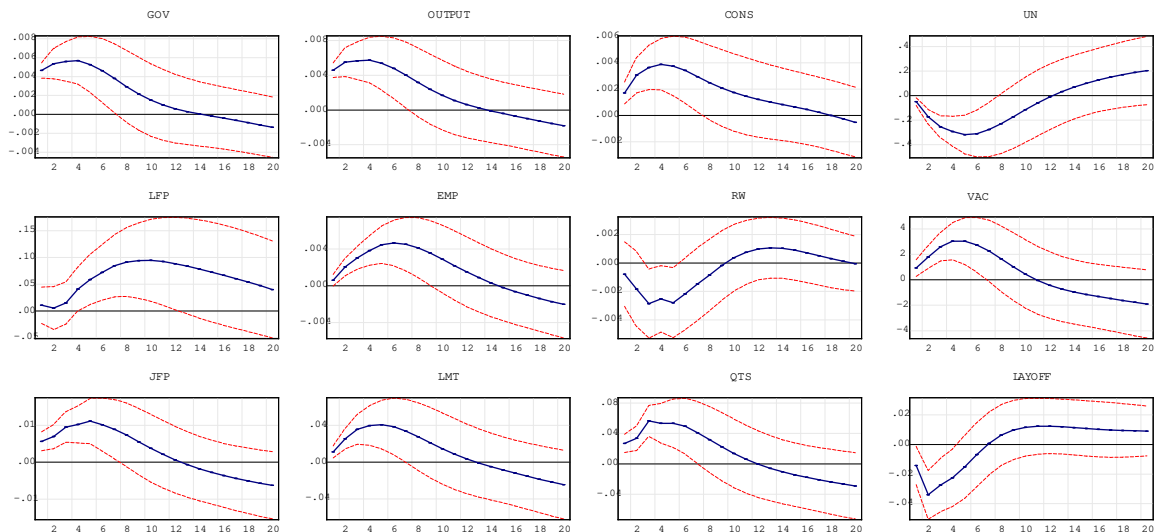
I started the extended model after including the total job separation not only to examine the effects of the government spending shocks on the total job separation but also its association with the other variables which are considered important in the perspective of the search and matching theory Pissarides (2000).

<sup>6</sup>Pissarides (2000); Shimer (2005); Hall (2005); Fujita and G. Ramey (2009); Shimer (2012).

In Figure 1.2, the empirical results show that one standard deviation positive shock to the government spending has a positive impact on the total job separation till the 14th quarter, afterwards it becomes negative. The trend of real wage is opposite to what it is for the total job separation, for example, for the first 10 quarters real wages are decreasing, while afterwards started increasing. I have shown that separations go up and this implies that increasing separations reduce value of employment, which is reflected in the fall of the real wage.

I have shown here that separation goes up. Where do these workers go? Is there any job destruction? From the graph I can say yes, there is job destruction but there is also possibility that no, there is not, because reduction in unemployment shows that workers cannot go to unemployment or exit labour force, since both of these variables are positively responding to the shock. The only place to go is a different job, so there must be an increase in job-to-job transitions. It is also evident from the employment graph, which is also consistent with the previous research Shimer (2006) Fallick and Fleischman (2004).

To check either there is Increment in job-to-job transition or there is increase in job destruction makes me breaking the total separation rate into quits and layoffs . This segregation is helpful in understanding the dynamics of the total job separation and their association with the other variables in the economy. It is also important to segregate, because if the separation is voluntary (quits) then total job separation will be upward slope and if it is involuntary (layoffs) then this is job reallocation or transition with wage cut then decrease in layoffs is expected. Therefore, based on this segregation, the results of the quits and layoffs are provided in Figure 1.3.



**Fig. 1.3** Impulse Response Functions Total Job Separation (Quits and Layoffs)

The results in Figure 1.3 show the impulse response functions of segregated total job separation, i.e., quits and layoffs. The results are quite interesting, as one standard deviation positive shock to government spending has a positive and significant effect on quits at peak of 0.06 percent till the 12th quarter, and afterwards it starts decreasing and then converges to the steady state after 20 quarters.

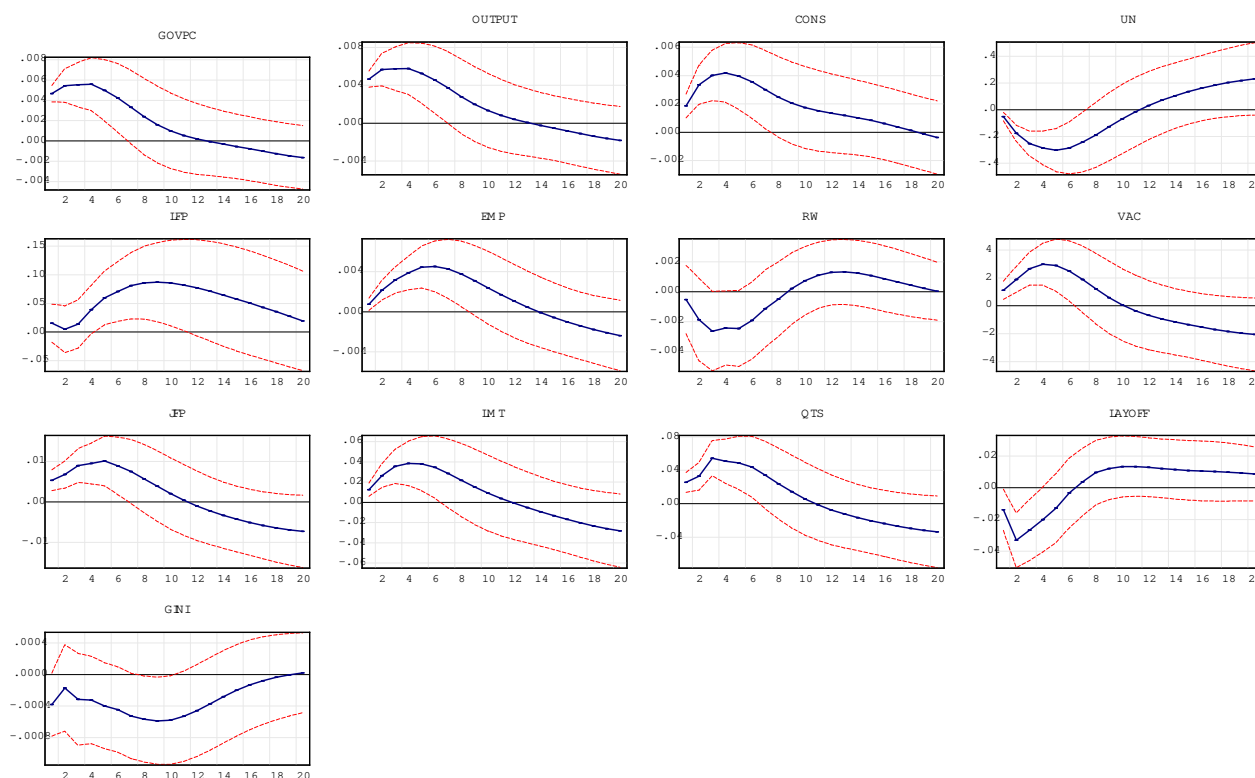
Increase in quits coupled with increase in participation and decrease in unemployment means that workers are in the state of employment and there is job-to-job transition. Workers are quitting one job and are still in employment which means workers go for higher wage jobs climbing the wage ladder, resulting in an increase in the job-to-job transitions. However, the response of the layoffs to the government spending shock is negative which is opposite to that of the total job separation. For example, the layoffs show a decreasing trend, such as -0.03 till 8th quarter and afterwards it starts increasing and converges to steady state. These results clearly show that workers are in the state of employment. These results further clarify the situation, as they stay in the labour force and there is no voluntary unemployment. Furthermore, the downward slope of layoffs also indicates that firms are not ready to layoff their workers and want to retain them because, on average, the wage cost is low for the firm.

These separations in quits creates more unfilled jobs and hence increasing trend in the vacancies. Furthermore, positive government spending shock also has a positive effect on vacancies which means government creates lower paid (on average) public sector jobs, hence, there is vacancy creation at the lower tail of earnings distribution. Borjas (2002) argued that there is a difference in wages between public and private

sector in US.

One important thing from above result reveals the fact that when I divided the total job separation into quits and layoffs then results show that job-to job movement to higher wage ladder and job creation in government sector on lower segment due to positive government spending shock, creates two opposing forces. First, they are pushed toward upward tail of wage distribution and second, there is increase in the mass on bottom and this spread in wage distribution may lead to increase in income inequality.

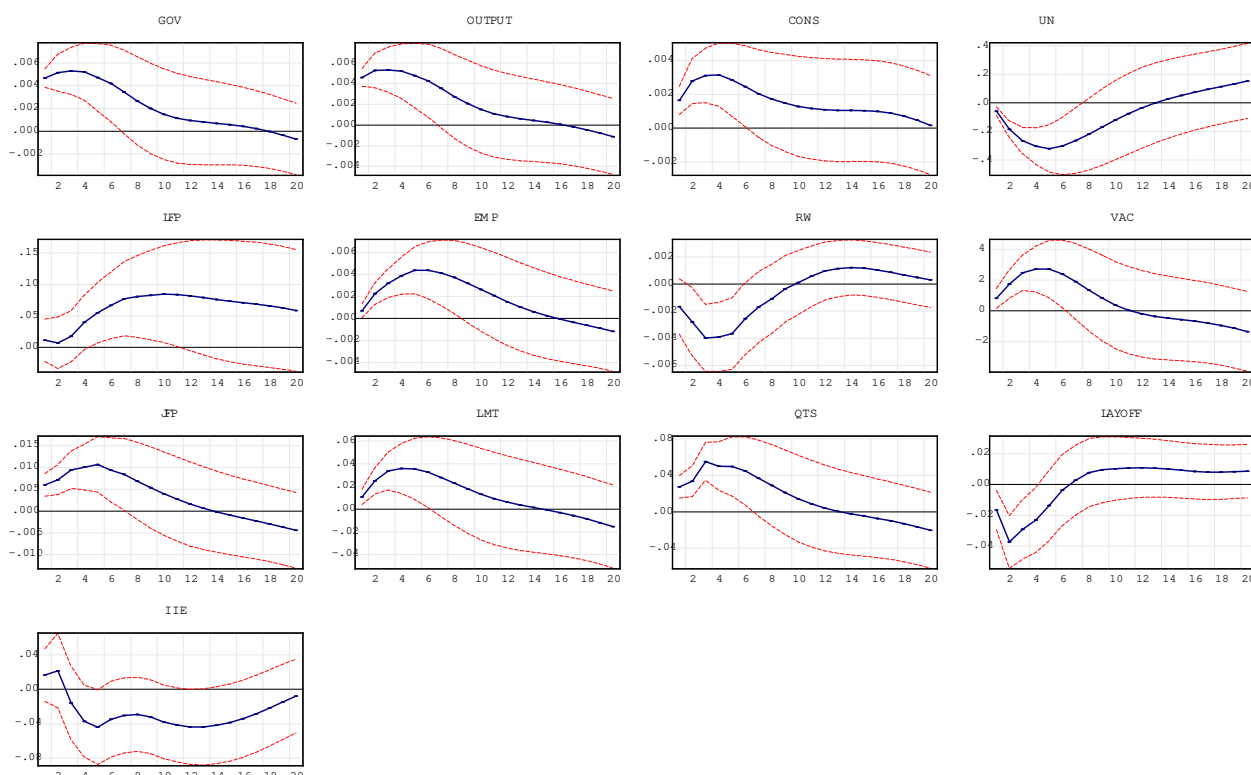
In this context, to examine how these movements in the job separation (such as quits and layoffs) and spread in wage distribution increase the income inequality. Two measures of income inequality (such as Gini Coefficient, and P90/10) are important here Burkhauser, Feng, and Jenkins (2009). Therefore, I considered these two distinct measures of income inequality in the model with the total job separation. Gini coefficient (GINI) is overall measure of income inequality, while P90/10 (IIE) is specifically used to measure the tails of the distribution. The results for the GINI and IIE are provided in the Figure 1.4 and Figure 1.5, respectively.



**Fig. 1.4** Impulse Response Functions Total Job Separation (Quits and Layoffs) (Gini)

The GINI Graph in the Figure 1.4 shows that positive government spending shock

has a negative and insignificant effect on the GINI, which is negative and decreasing throughout time period and start increasing in 20th quarter. This means that government spending shock has an effect on income distribution not only in terms of real wages but it also affects the income inequality. It can also be seen that there is no significant increase in income inequality. This implies that spread in wage distribution is still consistent that job-job movement is not very far to the right and not fall too much to the bottom and it is concluded that there is decrease in inequality. Hence government spending does not compromise inequality. Goñi, López, and Servén (2008), Martinez-Vazquez, Moreno-Dodson, and Vulovic (2012), stated that increase in government spending leads to decrease in income inequality. Moldogaziev, Monogan, and Witko (2018) empirically found that greater redistributive spending leads to increase in income inequality and Ball et al. (2013) also found that fiscal consolidation has a significant and positive effect on income inequality. As GINI is the measure of entire wage distribution and to see the changes only on the tails of the distribution, I examined the P90/10 (IIE) and estimated the model again by replacing P90/10 with the GINI. The results are provided in the Figure 1.5.

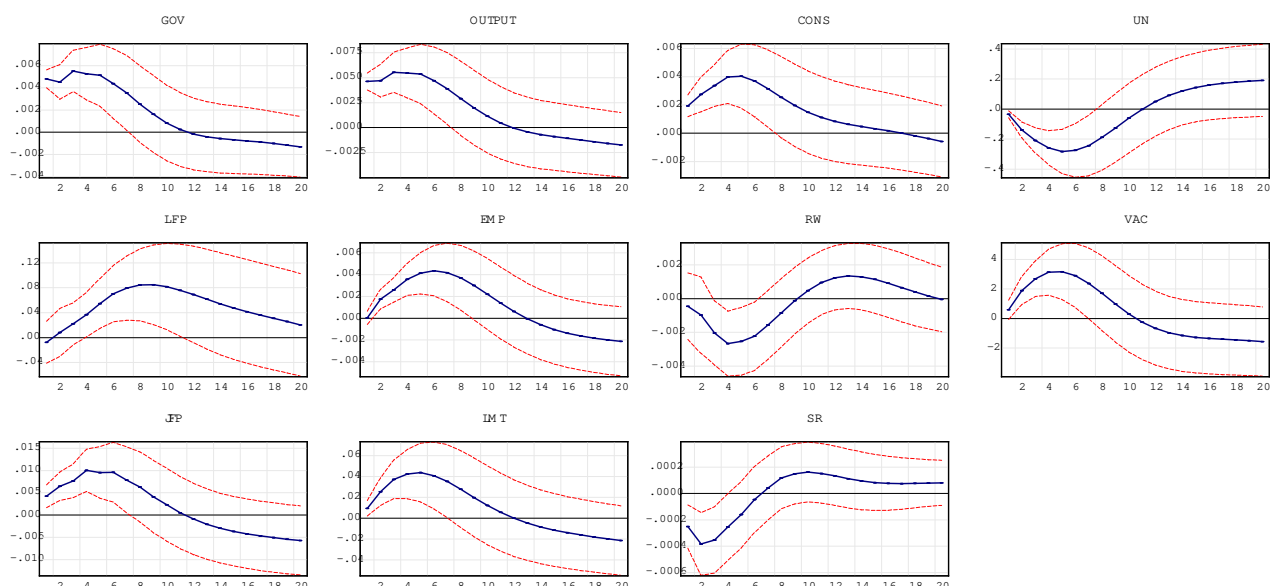


**Fig. 1.5** Impulse Response Functions Total Job Separation (Quits and Layoffs) (P90/10)

The IIE graph in the Figure 1.5 shows the response of IIE to the government spending shocks. P90/10 is the measure of income inequality, which is used to see the response of positive government spending shock specifically on the tails of the wage distribution. Results show that the immediate response of IIE exhibits slight increase in inequality till second quarter and started decreasing afterwards. It is clear that the Government spending has an instantaneous and short term impact on IIE. This implies that in the short run, government spending may increase income inequality due to dynamics in the labour market but as time goes by these jobs are concentrating closer to the mean and hence there is initial spread in wage distribution) (as can be seen through the spread in wages which is higher than the spread in IIE) and then there is contraction. In conclusion, overall effect of government spending on income inequality is insignificant with a decreasing trend.

### 1.4.2 Comparison of Total Job separation with Shimer's Total Job Separation

For the robustness check, I compared total job separation (i.e., calculated by adding quits and layoffs) with total job separation constructed by Shimer (2005). Purpose of this comparison is to see the responses of both types of separations to the positive government spending shock.



**Fig. 1.6** Impulse Response Functions Shimer's Total Job Separation Rate

I replaced total job separation with the Shimer's total job separation and estimated the model with other variables. Shimer's total job separation was constructed using

formula derived by (Shimer 2005) which I already explained in section 4.3 (Data Collection). Due to the difference in the construction of total job separation, results of the Figure 1.2 and Figure 1.6 are quite different such that the graph of the total separation in the Figure 1.6 clearly shows that government spending shock has a negative but insignificant effect on total job separation and this result is opposite to Figure 1.2 where government spending shock has a positive effect on total job separation. However, graph of total job separation (i.e., Figure 1.6) is similar to the graph of layoffs (i.e., Figure 1.3). These differences are due to the fact that Shimer (2005) did not include quits while constructing the total job separation. Since total job separation (Figure 1.2) is constructed by adding quits and layoffs, therefore, addition quits while constructing total job separation has changed the response of total job separation to positive government shock. In this backdrop, it is concluded that addition of new variable (quits) has a very strong effect on labour market through positive government spending shock.

## 1.5 Conclusion

Using all the important variables of the search and matching theory, I estimated the effects of fiscal policy shock on output and employment and on income inequality for the US economy. I found very interesting results by applying SVAR. The results showed that positive government spending shock has positive and significant effect on output, consumption, labour force participation, employment, vacancies, job finding probability and labour market tightness and negative effect on unemployment and real wages.

To examine the behaviour of labour market more deeply in response to positive government spending shock I included total Job Separation in the model. Government spending shock has a positive impact on the total job separation, employment and labour force participation and negative effect on real wage. This implies that including the total job separation variable in the model has some important association with these labour market variables. On one hand, there is increase in job separation which reveals that there is job destruction and on the other hand, there is increase in participation and employment. This implies that there is a job-to-job movement.

To see either there is job destruction or there is job-to-job movement, I segregated total job separation into quits and layoffs. I found that government spending has a positive and significant effect on quits and negative effect on layoffs, which confirm the above results that workers are still in the labour market and are in the state of

employment and there is job-to-job movement. Increase in quits together with falling unemployment and increasing participation means workers go for higher wage jobs as job-to-job transitions are climbing. Results also show that positive government spending shock increases job creation in public sector which are on average lower paid jobs hence vacancy creation is at the lower tail of wage distribution.

Job creation, both at higher and lower end of wage distribution, leads to create the spread in wage distribution which may lead to increase in income inequality. More dispersion in wage distribution means more unequal society. To see either these dynamics in wages are affecting income inequality, I used two measures of income inequality i.e. Gini coefficient and P90/10. I found that there is no significant increase in income inequality which implies job-to-job movement did not go too far to the right of wage distribution and government sector jobs are not as low paid as one could think. As GINI is the measure of entire wage distribution, therefore, to see the changes only on the tails of the distribution, I replaced GINI with P90/10 (IIE) and estimated the model. I found that government spending has an instantaneous and short-term impact on IIE. This implies that there is an initial spread in wage distribution then contraction and in the short run, government spending may increase income inequality due to dynamics in the labour market, but overall effect of government spending on income inequality is insignificant with a decreasing trend.

# Fiscal Stimulus, On-the-Job-Search and Labor Market Dynamics

## 2.1 Introduction

During past decade, several studies examined the effect of government spending shock on unemployment using Dynamic Stochastic General Equilibrium (DSGE) models (e.g., Monacelli, Perotti, and Trigari [2010](#), Campolmi, Faia, and Winkler ([2011](#)), Brückner and Pappa ([2012](#)), Shoag ([2013](#)), Serrato and Wingender ([2016](#)) and Kim ([2019](#))). Most of these studies examined the exogenous job separation in their models. Their underpinning assumption was that labor is getting employment through matching function which is endogenously determined, while their transition from employment to unemployment is only due to exogenous separation. Much of the current literature looks at employment response only at the level of the stock and inflows (job finding rate). Disregarding outflows from employment misses important insights about dynamics of employment, earnings and inequality. However, many empirical studies exhibited that labor's transition from employment to unemployment is not only due to exogenous separation but also due to endogenous job separation. This endogenous job separation, among other factors, gives rise to the on-the-job search.

The importance of on-the-job search in the labour market is widely discussed in the literature (Pissarides ([1994](#)), Nagypál ([2005](#)), Burdett and Mortensen ([1998](#)), Krause and T. A. Lubik ([2006](#)), Nagypál ([2007](#)) Tasci ([2007](#)), Menzio and Shi ([2010](#)), Epstein ([2012](#)), Gertler, Huckfeldt, and Trigari ([2020](#)), among others). Pissarides ([1994](#)) demonstrated that using on-the-job-search in search and matching model improves the results and found that productivity shocks has a negative effect on unemployment rate. Barlevy ([2002](#)) also examined on-the-job-search in the model and found that, in recessions, there is a decrease in the job-to-job transition which leads to movement

of workers to less productive jobs. Fallick and Fleischman (2004) and Nagypál (2005) also explained that job-to-job transitions are an important part of search and matching theory and play an important role to explain the dynamics of employment.

These studies incorporated on-the-job-search in the standard search and matching theory, however, recent studies show that on-the-job-search plays an important role to improve the performance of DSGE models. For example, Pissarides (1994), Barlevy (2002), T. A. Lubik, Krause, et al. (2004), Nagypál (2005), Krause and T. A. Lubik (2006). These studies use different shocks including productivity shock on these variables but none of them examined the effect of government spending shock in their studies.

Drawing on the above discussion, it is evident that there are very few studies which examined the effect of fiscal policy shock on the employment, particularly in the context of the DSGE framework using search and matching theory. Most of the studies used exogenous job separation while developing the model, with very limited studies added endogenous job separation. This study contributes to the existing literature by developing a DSGE model using search and matching theory where I added two endogenous job separations in the model i.e., quits via on-the-job-search with effort  $s_t$ , and layoffs via operating cost  $K_t$ . By developing theoretical DSGE model using search and matching frictions I examined the responses of these variables to the fiscal policy shock along with productivity shock. These relationships have already been examined empirically in the US Economy (Chapter 1), which are summarized below:

I empirically examined the employment response to government spending shock at the level of outflows from employment for the US economy using 15 macroeconomic variables including variables of search and matching theory. Results showed that government spending positively affects employment and participation and negatively affects unemployment. Furthermore, I segregated total job separation into quits and layoffs and found that quits and layoffs move in opposite directions after the positive government spending shock. This movement implies increase in wage inequality. To examine the effects of government spending on wage inequality, I used two variables i.e. Gini coefficient and P90/10 and found that predicted increase in inequality is short-lived and negligible and inequality falls in longer term.

The above results warranted development of theoretical model for fiscal multipliers by incorporating search and matching frictions in a DSGE model. There are three objectives of this study. First objective is to corroborate the empirical findings of the

chapter 1. For this purpose I developed a theoretical DSGE model using search and matching frictions with responses of endogenous job separation to the fiscal policy and productivity shocks. Second objective is to examine the effects of productivity shock and government spending shock on a) employment and b) job-to-job transition in bad as well as good jobs. Third objective of the study is to examine the effects of fiscal policies on wages in bad and good jobs, and on two endogenous job separations. To achieve these objectives of this chapter/study theoretical model was developed and calibrated for US economy by giving productivity shock and government spending shock.

Results show that on impact, aggregate output and output for bad type of jobs are responding negatively to the aggregate productivity shock with increasing trend. Productivity shock has positive effect on output for good job in short run and start decreasing afterwards. Wages in good and bad jobs are moving in opposite directions in response to productivity shock. Employment in good type of jobs responded positively to shock in short run. Responses of vacancies in good sector to productivity shock are positive whereas it is negative to vacancies in bad sector. This implies there is job creation in good sector. Search intensity and on-the-job search are showing same responses to the productivity shock i.e. both are negative but show increasing trend. The reason behind this increasing trend in both graphs can be related to downward slope of unemployment in both sectors. As unemployment decreases, employed searchers start increasing the job in good sector as it is expected that there will be vacancy creation after the positive productivity shock.

Effects of positive government spending shock on labor market are quite interesting and according to the expectations. Positive government spending has negative effect on output in good sector in the short run but it is increasing in the long run, whereas it is opposite for the bad sector. Wages are considered to be most important variables, as they affect the job-to-job transitions in both the sectors. Results also support the proposition used in the paper that positive government spending shock has positive effect on wages in good sector. Wages are positive throughout time while it is opposite for the wages in the bad sector. Firms working in good sector are paying higher wages and are not employing more workers therefore, employment in good sector is decreasing as compared to employment in bad sector. This situation is favorable for the firms in bad sector to create more vacancies and hire more workers because they have to pay less wages. From the worker's perspective, until there is an increase in

vacancy creation in good sector, they will prefer to stay in the employment in bad job. The results also showed that positive government spending shock has a positive effect on both the search intensity and on-the-job-search. This indicates that there is an endogenous job separation and due to this, there is a job-to-job transition from bad to good sector because of the differential in wages in both sectors. These results have a very strong contribution in the search theory because these results verify the results of chapter 1. Higher wages in good sector attract those worker who are already employed with less wages. There is an increase in quits in bad sector but decrease in layoffs as can be seen in the results that employment in bad sector increases but employment in good sector is decreasing. This increase in employment in bad sector suggests that there is a decrease in layoffs of employees from firms, however, workers are not willing to work on current wages, therefore, as soon as they get job in good sector they would most probably quit the bad sector and will be employed in good sector.

Theoretical model with two distinct wages and employment helps us to do different kind of simulations. Therefore, keeping in mind the above findings it is important to calculate the weighted average of both types of wages. Because, from the above discussion, I can say that there is a job-to-job transition and reason of this transition is wage dispersion and dispersion in wages leads to create wage inequality. I calculated the weighted average of both good and bad jobs by using employment in both sectors. Results show that there is a sharp decrease in average wage till third quarter, afterwards wages are increasing and due to increase in wages, labor are moving from bad to good jobs. Higher increase in average wages also indicates that there is an increase in wage inequality but the increment in inequality lasts only for five periods, afterwards there is sharp decrease in wage inequality.

Rest of the chapter is organized as follows. In section 2.2, I provide review of important and relevant literature, section 2.3 includes theoretical model, section 2.4 contains equilibrium equations, section 2.5 contains quantitative analysis and calibration and steady state values, section 2.6 presents model analysis, section 2.7 provides inequality analysis, section 2.8 provides the summary and conclusion of the chapter.

## 2.2 Literature Review

Merz (1995) in her seminal work introduced exogenous job separation in the Search and Matching theory of Diamond Mortensen-Pissarides by employing DSGE frame-

work. Merz pointed out the shortcomings of the Real Business Cycle in the US economy and explained the nature of association between vacancies and unemployment through the lens of search and matching models. Later, Den Haan, G. Ramey, and Watson (2000) further developed the model by using search and matching frictions in DSGE model with endogenous job separation.

Shimer (2005) further developed and showed that search and matching model explained the relationship of unemployment and vacancies in the USA economy and also suggested further developments in the model. Following the suggestions of Shimer (2005), different authors developed and tested the theory of unemployment in different dimensions of the labor market, for example, sticky wage (Hall 2005) and on-the-job search (Mortensen and Nagypal 2007).

Krause and T. Lubik (2010) used on-the-job-search in search and matching theory under DSGE framework. They introduced two different types of jobs namely, “good” and “bad” to explain on-the-job-search and job-to-job transition in the model. They argued that both the employed and unemployed workers search for jobs. The former search for good jobs, while the latter search for both the bad as well good jobs. They further demonstrated that on-the-job-search plays vital role in explaining observed cyclical up-gradation of workers to better employment opportunities, particularly in periods of booms. They argued that workers always look for the better opportunities, therefore, number workers searching for good jobs are always high as compared to those looking for bad jobs. Their model is consistent with other features of search and matching model i.e., instability of vacancies and unemployment, and pro-cyclical nature of the job-to-job transition.

Nagypál (2007) found that on-the-job-search can play an important role to explain the cyclical behavior of search and matching models. He further argued that it is important for the firms to offer higher wages to already employed workers viz-a-viz unemployed workers if they want to attract more employed worker.

Tasci (2007) and Mortensen and Nagypal (2007) used on-the-job search in a quite different way and introduced matching quality for the on-the-job-search. They argued that employed workers will only switch to new employment if the new job would offer better terms and conditions as compared to their current job.

There are very few studies which explained the effect of government spending on unemployment using search and matching theory under DSGE framework Monacelli, Perotti, and Trigari (2010), Campolmi, Faia, and Winkler (2011), Brückner and

Pappa (2012), Shoag (2013), Serrato and Wingender (2016) and Chun-Hung and Hiroaki (2019) used subsidies to the cost of posting vacancies as a fiscal policy shock along with government spending shock and showed that positive government spending shock has a negative effect on vacancies with endogenous job separation.

Elsby, Michaels, and Solon (2009) and Fujita and G. Ramey (2009) found that unemployment inflow rates contribute more than half of cyclical changes in unemployment. However, Shimer (2012) found that the effect of outflow rate is larger as compared to inflow rate and termed the inflow rate as non-cyclical in nature.

## 2.3 The Model

I begin with DSGE model with search and matching frictions in which quits and layoffs are determined endogenously i.e., endogenous job separation. Purpose of developing this theoretical model is to examine the effect of government spending shock and the effects aggregate productivity shock on unemployment and on the economy to corroborate the empirical results of chapter 1. The model presentation, the symbols and notations of the variables are standard as provided in Pissarides (2000) book.

### 2.3.1 Environment

In a standard DSGE model, an economy consists of three actors i.e. households, firms and the government. Following Krause and T. A. Lubik (2006) the basic assumption is that in each household there is a continuum of infinitely-lived homogeneous workers and from each firm there is a continuum of infinitely-lived heterogeneous firms, each of measure one in a discrete time. According to them firms are categorized in two types i.e. good and bad and they are different in terms of costs of creating new vacancies. Since labor market is frictional, therefore, the costs of creating new vacancies have some rents which ultimately lead to increase the differences across vacancies for the employed worker and as a result this differential impel workers who are in low wage job to search for job in high wage jobs.

It is assumed that only low wage employees do on-the job search subject to the search intensity which is determined endogenously. Those worker who are already in good job will not search for job as they are already getting those benefits which low wage workers are looking for and also there is no incentive for them to search for job. Firms and workers are working in frictional labor market. Firms are working in perfectly

competitive market and by using labour, firms are producing their output and sell their goods to the household. Low wage workers supply their services to the labor market if they are employed and earn wages and will do on-the-job search until they find high wage job.

In the model, employment is determined through search and matching and real wage, while labor supply is determined through bargaining process. Once the firm and worker meets and a job is created, production continues until a negative idiosyncratic operating cost arrives and job is destroyed. In this model, job destruction is determined endogenously through quits and layoffs. This means, both the workers and firms can make decision about job destruction. Two endogenous job separations are added in the model i.e., quits via on-the-job-search with effort  $s_t$ , and layoffs via operating cost  $K_t$ . By following Chun-Hung and Hiroaki (2019), it is assumed that firm and worker has to face an operating cost  $K_t$  and it is idiosyncratic to each match in the firm. It is also assumed that  $K_t$  is independent and has identical distribution (i.i.d) across firms with a range between 0 and 1. The range can be written in the form of the cumulative distribution of  $\mathbb{Z} : [\underline{K}, \bar{K}] \rightarrow [0, 1]$

This suggests that the existing match decides, on the basis of the resultant new unique cost, either to continue the employment relationship and keep producing or to simply terminate it. So each match in the firm selects a reservation value of operating cost i.e.  $\tilde{K}$ . Firm will continue to produce till the point where  $K_t$  of job is below the  $\tilde{K}$ . Otherwise firm will destroy the job.

Since there are two types of jobs in this model i.e. good and bad. Firm in good sector pay higher wages and face low risk of loosing employees as compared to bad sector. Therefore, profitability of both firms are different and at the same draw from the distribution of operating cost may have different effects on the job destruction at both types of firms. This motivates that reservation value of operating cost  $\tilde{K}$  for good and bad jobs are different. Therefore, I added  $\tilde{K}_t^g$  and  $\tilde{K}_t^b$  and considered different operating cost for both firms. By adding these two distinct reservation values, I added two equations that would pin down threshold and by virtue of reservation property, solve the equations of value of job filled at the level of threshold equal to zero.

$$J(\tilde{K}_t^g) = 0$$

$$J(\tilde{K}_t^b) = 0$$

It is also assumed that there is probability of exogenous job separation which is independent to idiosyncratic operating cost and denoted by  $\rho^K$ .

## 2.3.2 Household's Problem

Following Merz (1995), a representative household consists of a continuum of individuals of mass one. It is assumed that a member of the household is either employed or unemployed. Employed member of household is denoted by  $n_t^i$  and unemployed is denoted by  $u_t = 1 - n_t$ . The household's expected life time utility

$$\sum_{i=1}^{\infty} \beta^t \left[ \frac{C_t^{1-\tau}}{1-\tau} - \Psi_0 \frac{n_t^{1+\Upsilon}}{1+\Upsilon} \right] \quad (2.1)$$

$0 < \beta < 1$  is the subjective discount factor,  $C_t$  denote Consumption of HH,  $n_t =$  Number of workers,  $\Psi_0$  is Disutility of working,  $\Upsilon =$  Inverse of the Frisch elasticity of labor supply,  $\frac{1}{\tau} =$  Intertemporal elasticity of substitution. Household budget constraint is defined as:

$$C_t + T_t = w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b + u_t z + \Pi_t \quad (2.2)$$

Above equation states that employed household members whose source of earnings are the incomes  $W$  generated through their employment and the other household members who are unemployed and are on the benefits  $z$ . In addition, these households receive profits  $\pi$  after paying the lump sum taxes  $T_t$  to the government and both types of firms face aggregate hiring cost  $H_t = \sigma^g v_t^g + \sigma^b v_t^b$ . Since aggregate production functions are CRS, therefore, profits, income and unemployment benefits are equal to zero, **HH optimization:**

$$\sum_{i=1}^{\infty} \beta^t \left[ \frac{C_t^{1-\tau}}{1-\tau} - \Psi_0 \frac{n_t^{1+\Upsilon}}{1+\Upsilon} \right]$$

subject to:

$$C_t + T_t = w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b + u_t z + \Pi_t$$

$\lambda_t$  is the Lagrange multiplier

$$L = E \left[ \sum_{i=1}^{\infty} \beta^t \left[ \frac{C_t^{1-\tau}}{1-\tau} - \Psi_0 \frac{n_t^{1+\Upsilon}}{1+\Upsilon} \right] - \lambda_t \{ C_t + T_t - w_t^g(K_t)n_t^g - w_t^b(K_t)n_t^b - u_t z - \Pi_t \} \right]$$

Differentiate w.r.t.  $c_t$  to get the first order condition

$$\frac{\partial \mathcal{L}}{\partial C_t} = \partial C_t : 0 = \frac{(1 - \tau)C_t^{1-\tau-1}}{1 - \tau} - \lambda_t$$

$$C_t^{-\tau} = \lambda_t \quad (2.3)$$

### 2.3.3 Firm's Problem

Following Acemoglu (2001) and Krause and T. A. Lubik (2006) Firm's problem is defined as follows: Production of a relevant firm of both types  $i = g, b$  is give by

$$y_{i,t} = A_t(n_t)^i \quad (2.4)$$

$A_t$  = Aggregate Productivity,  $n_t^i$  = no of workers in sector  $i$ . As there are two types of jobs, I would have two types of output produced by two different firms. Therefore, aggregate output is the combination of output of good firm and output of bad firm i.e.

$$y_t = [\alpha y_{bt}^v + (1 - \alpha)y_{gt}^v]^{1/v} \quad (2.5)$$

where  $y_{gt}$  and  $y_{bt}$  shows aggregate production of first input and aggregate production of second input respectively.  $v < 1$  and elasticity of substitution between both inputs is  $1/(1 - v)$ .  $\alpha$  shows relative share of  $y_{bt}$  in the final production. Since, I assumed the perfectly competitive market so prices of two goods i.e.  $y_{bt}, y_{gt}$  are determined as follows:

$$P_{gt} = (1 - \alpha)y_{gt}^{v-1}y_t^{1-v}$$

$$P_{gt} = (1 - \alpha) \left( \frac{y_{gt}}{y_t} \right)^{-(1-v)} \quad (2.6)$$

$$P_{bt} = \alpha y_{bt}^{v-1}y_t^{1-v}$$

$$P_{bt} = \alpha \left( \frac{y_{bt}}{y_t} \right)^{-(1-v)} \quad (2.7)$$

Total profits of both firms are as follows:

$$\Pi_t = \int_{\underline{K}}^{\bar{K}} (P_{gt}y_{gt} + P_{bt}y_{bt}) - (w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b) - (\sigma^g v_t^g + \sigma^b v_t^b)$$

## 2.3.4 Labour Market

Labor market is designed by following the standard model of search and matching developed by Mortensen and Pissarides (1994). I added on-the-job-search as a variable by following Krause and T. A. Lubik (2006) with endogenous job separation. They argued that there is a frictional labor market and there is no instantaneously match of firm and worker. Firm and worker will have to face a time consuming search process. On-the-job-search is incorporated by defining two types of jobs i.e. good job and bad job. As I have two types of jobs, therefore, keeping in mind this I created two types of matching functions. I assumed that both matching functions are continuous, increasing both in its arguments, concave and homogeneous of degree one.

- **Matching Functions**

Matching functions for good job and bad job are described as follows:

$$m_t^g = m_g(v_t^g)^{1-\zeta}(u_t^g + e_t)^\zeta \quad (2.8)$$

$$m_t^b = m_b(v_t^b)^{1-\zeta}(u_t^b)^\zeta \quad (2.9)$$

$m_i$  measure efficiency of matching process of good jobs  $u_{it,i=g,b}$  unemployed workers are searching for both types of jobs.  $v_{it,i=g,b}$  number of vacancies.  $\zeta$  is matching elasticity with respect to unemployment in both types of jobs.

$$e_t = s_t n_t^b \quad (2.10)$$

$e_t$  measure the efficiency of employed worker in bad job search for good job.  $s_t =$  search intensity. Search intensity will be same for unemployed worker for the time being.

$n_t^b$  is measure of all workers employed at time  $t$  in bad jobs by across all firms that heterogeneous with respect to operating cost  $K_t$ . This implies,  $n_t^b$  is aggregate measure of firm size across all heterogeneous firm. Given that,  $n_t^g$  and  $n_t^b$  are aggregate measure of employment in good and bad sector in the economy respectively Following Krause and T. A. Lubik (2006) stocks of employment are described as follows:

$$n_{t+1}^g = (1 - \rho^K)[n_t^g + m_t^g] \quad (2.11)$$

$$n_{t+1}^b = (1 - \rho^K)[n_t^b + m_t^b - p_t^g s_t n_t^b] \quad (2.12)$$

and Aggregate unemployment is given by

$$u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b = 1 - n_t \quad (2.13)$$

Labor market tightness for good job is defined as  $\theta_t^g = \frac{v_t^g}{u_t^g + e_t}$  and same like labor market tightness for bad job is defined as  $\theta_t^b = \frac{v_t^b}{u_t^b}$ . Job finding probability of each types of job is given by

$$p_t^g \equiv \frac{m_t^g}{u_t^g + e_t} = m_b(\theta_t^g)^{1-\zeta} \quad (2.14)$$

$$p_t^b \equiv \frac{m_t^b}{u_t^b} = m_b(\theta_t^b)^{1-\zeta} \quad (2.15)$$

The current probability that a firm fill a vacancy in good and bad job is given by

$$q_t^g \equiv \frac{m_t^g}{v_t^g} = m_g(\theta_t^g)^{-\zeta} \quad (2.16)$$

$$q_t^b \equiv \frac{m_t^b}{v_t^b} = m_b(\theta_t^b)^{-\zeta} \quad (2.17)$$

I assumed that if firms and workers are matched in current time period i.e.  $t$ , then this match will also continue at the beginning of next period i.e.  $t + 1$ .

#### • Value Function of Job Filled (GOOD)

In search and matching model, choices of firms and workers are defined by the Bellman equations. As here are two types of jobs, therefore, I have two types of value functions of jobs filled with worker. The values of the two types (good and bad) of jobs filled with an idiosyncratic operating cost  $K_t$ ,  $J_t(K_t)$ , satisfies for good job

$$J_t^g(K_t) = A_t P_{gt} - w_t^g(K_t) - K_t + E_t \beta_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - (1 - \rho^K)\Phi(\tilde{K}_{t+1}) \right] V_{t+1}^g \right\} \quad (2.18)$$

Where  $\beta_t = \beta \frac{\lambda_{t+1}}{\lambda_t}$  is the discount factor.  $E_t$  is expectation operator at time  $t$ . Above value function states that in current time period firm will produce output and pay wages  $w_t^g$  and pay operating cost  $K_t$ . In next time period, if any exogenous shock is not destroying the match and if  $K_t$  is below the  $\tilde{K}_{t+1}^g$  then match will continue and firm will

get  $J_{t+1}^g(K_{t+1})$ , and if the match will not continue then the firm will post a new vacancy in good job.

- **Value Functions of Job Filled (BAD)**

$$J_t^b(K_t) = A_t P_{bt} - w_t^b(K_t) - K_t + E_t \beta_t \left\{ (1 - \rho^K)(1 - p_t s_t) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + [1 - (1 - \rho^K)p_t^g s_t \Phi(\tilde{K}_{t+1})] V_{t+1}^b \right\} \quad (2.19)$$

Value of filled job with bad type of job states that, in current time period firm will produce output and pay wages  $w_t^b$  and bear operating cost  $K_t$ . For bad job I can say that worker will continue to work in the next time period at the rate of  $(1 - \rho^K)$ . Bad job has to face an additional risk that workers will leave and they can go to the good jobs. It depends on the search effort  $s_t$  of the worker, such as higher search effort/intensity with higher probability  $p_t^g$  that matched with good job results in movement of workers to good job and reduces the probability to retain within bad job in the following period.

- **Value Function of Vacant Job**

Value of a vacancy for both types of jobs  $i = g, b$ , is:

$$V_t^i = -\sigma^i + E_t \beta_t \left\{ q_t^i (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^i} J_{t+1}^i(K_{t+1}) d\Phi(K_{t+1}) + [(1 - (1 - \rho^K)q_t^i)\Phi(\tilde{K}_{t+1}^i)] V_{t+1}^i \right\} \quad (2.20)$$

Where  $\sigma$  representing Vacancy cost. A vacancy is created and filled in the following period with the probability  $q_t^i (1 - \rho^K)$  and the probability that job will remain unfilled is  $(1 - q_t^i (1 - \rho^K))$ . Free entry condition states that these values of vacant jobs are equal to zero at any point in time,  $V_t^g = V_t^b = 0$  for all  $t$ . By solving value functions of both unfilled jobs give the following job creation conditions for both types of jobs.

Job Creation Conditions

$$\frac{\sigma^g}{q_t^g} = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}), \quad (2.21)$$

and

$$\frac{\sigma^b}{q_t^b} = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \quad (2.22)$$

This above expression states that expected cost of posting vacancy is equal to the expected benefits of job filled in good and bad jobs respectively.

- **Value Function of Employed Worker (GOOD)**

Value of employed worker of two types of job i.e. good and bad are as follows:

$$W_t^g(K_t) = w_t^g(K_t) + E_t \beta_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \quad (2.23)$$

The value of employed worker for good job includes wages of good job and the value of staying in employment if the match is not destroyed or the value of not staying in the employment if the match is destroyed.

- **Value Function of Employed Worker (BAD)**

$$W_t^b(K_t) = \max_{s_t} \left\{ w_t^b - I(s_t) + E_t \beta_t \left\{ (1 - \rho^K)(1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \\ \left. \left. + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + 1 - (1 - \rho^K) \Phi(\tilde{K}_{t+1}) U_{t+1} \right\} \right\} \quad (2.24)$$

The value of employed worker for bad job consists of wages of bad job, cost of search, search effort of worker and probability that worker is matched with the good job and the value of employment become good.  $s_t$  is search intensity and  $I(s_t)$  is the cost of search intensity and it is strictly convex. As Krause and T. A. Lubik (2006) stated that higher search intensity means there are more chances of worker to match with good job. Optimal Search intensity is derived as:

$$I'(s_t) = \frac{\mu}{1 - \mu} p_t^g \left( \frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b} \right) \quad (2.25)$$

- **Value Function of Unemployed Worker**

Values of unemployed worker for two types of jobs i.e. good and bad are

$$U_t^i = z + E_t \beta_t \left\{ p_t^i (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^i(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^i (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1}^i \right\} \quad (2.26)$$

$U_t^g = U_t^b = U_t$  for all  $t$ . Above value function for unemployed worker states that unemployed workers get the unemployment benefit  $z$  in current period and start search for job. With  $p_t^i$ , is the probability of matching with the job. If  $K_t$  is below the  $\tilde{K}_{t+1}$  and match is continue without any exogenous shock, then the workers will remain in the employment in next period and get the value to remain in the employment, otherwise,

workers will be unemployed and get the unemployment benefits of being unemployed.

$$U_t^g = z + E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \quad (2.27)$$

$$U_t^b = z + E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \quad (2.28)$$

By solving both value functions simultaneously and using job creation conditions above I get

$$\frac{\sigma^g}{q_t^g} p_t^g = \frac{\sigma^b}{q_t^b} p_t^b \quad (2.29)$$

Further solving the above equations

$$\theta_t^g \sigma^g = \theta_t^b \sigma^b \quad (2.30)$$

This implies that labor market tightness for good job and bad job is proportional to the cost of job creation.

### • Wage Determination

Wages are determined as a result of a mutual bargaining process between firms and workers. Negotiation between firms and workers take place through Nash bargains and choose wages to divide the surplus of match between firms and workers depending on the relative bargaining power of both. Surplus for workers is  $(W_t^i(K_t) - U_t)^\mu$  and surplus for firms is  $(J_t^i - V_t^i)^{1-\mu}$ . Using free entry condition  $V_t^i = 0$  the surplus of firm =  $(J_t^i)^{1-\mu}$ . where  $\mu \in (0, 1)$  = Worker's bargaining power. Total surplus of a match is  $(S_j^i = J_t^i + W_t^i - U_t)$  and worker's surplus from total surplus is  $W_t^i - U_t = \mu S_t^i$  whereas firm's surplus  $J_t^i = (1 - \mu) S_t^i$ . As there are two types of jobs, therefore, I derive wage equation for both types of job one by one. Complete derivation is available in appendix.

Wage Determination for Good Job

$$w_t^g(K_t) = \mu P_{gt} A_t - \mu K_t + \mu \sigma^g \theta_t^g + z(1 - \mu) \quad (2.31)$$

Wage Determination for Bad job

$$w_t^b(K_t) = \mu (P_{bt} A_t - K_t + (1 - s_t) \sigma^g \theta_t^g) + (1 - \mu) (z + I(s_t)) \quad (2.32)$$

Equation (2.31) and (2.32) states that wage is sum of marginal revenue product, operating cost, and income of being unemployed ( unemployment benefits).

### 2.3.5 The Government

Lump sum taxes are imposed to household to finance government spending  $G_t$  and to pay unemployment benefits  $u_t z$  to the unemployed workers. Hence the budget constraints is given as follows:

$$T_t = G_t + u_t z \quad (2.33)$$

Government spending  $G_t$ , is added in the model as a shock. I investigated the effect of positive government spending shock on the labour market variables keeping in mind that there is job-to-job transition from bad to good type of job. I also investigated how these movements from bad to good affect unemployment and employment.  $G_t$ , follows the exogenous stochastic processes:

$$\log G_t = \rho_G \log G_{t-1} + \epsilon_{G,t} \quad (2.34)$$

where  $\rho_G =$  Persistency Coefficient,  $\epsilon_{G,t} =$  i.i.d innovation. Combining household budget constraint with government budget constraint I get

$$y_t = C_t + G_t - \sigma^g v_t^g - \sigma^b v_t^b \quad (2.35)$$

## 2.4 Equilibrium

Four blocks are added in the model i.e. Household, Firm and Labour market. Equilibrium of the model is attained by using all equilibrium equations with the all endogenous variables. There are 28 equilibrium equation and 28 variables.

$$\left( y_t, y_t^g, y_t^b, C_t, P_{g,t}, P_{b,t}, A_t, G_t, m_t^g, m_t^b, n_t^g, n_t^b, q_t^g, q_t^b, \theta_t^g, \theta_t^b, u_t^g, u_t^b, v_t^g, v_t^b, w_t^g, w_t^b, \tilde{K}_t^g, \tilde{K}_t^b, e_t, s_t, p_t^g, p_t^b \right)$$

and the parameters of the model are as follows:

$$\left( \alpha, \beta, \delta, \Upsilon, \tau, \Psi, m_g, m_b, \zeta, \sigma^g, \sigma^b, \rho_K, \eta, \mu, \gamma, \Omega, z, \sigma, \rho_g, \rho_A, v \right)$$

## 2.5 Set of Equilibrium Equations

In this section I summarize all equations of the model. Steady State equations and log-linearized model is presented in Appendix A.3.

### Output for Good Job

$$y_{g,t} = A_t n_t^g \quad (2.1)$$

### Output for Bad Job

$$y_{b,t} = A_t n_t^b \quad (2.2)$$

### Aggregate Output

$$y_t = [\alpha y_{bt}^v + (1 - \alpha) y_{gt}^v]^{1/v} \quad (2.3)$$

### Prices in Good Job

$$P_{g,t} = (1 - \alpha) \left( \frac{y_{gt}}{y_t} \right)^{-(1-v)} \quad (2.4)$$

### Prices in Bad Job

$$P_{b,t} = \alpha \left( \frac{y_{bt}}{y_t} \right)^{-(1-v)} \quad (2.5)$$

### Employment in Good Job

$$n_{t+1}^g = (1 - \rho^K) [n_t^g + m_t^g] \quad (2.6)$$

### Employment in Bad Job

$$n_{t+1}^b = (1 - \rho^K) [n_t^b + m_t^b - p_t^g s_t n_t^b] \quad (2.7)$$

### Employed Searchers

$$e_t = s_t n_t^b \quad (2.8)$$

### Aggregate Unemployment

$$u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b = 1 - n_t \quad (2.9)$$

### Matching Functions Good Job

$$m_t^g = m(v_t^g, u_t^g + e_t) = m_g (v_t^g)^{1-\zeta} (u_t^g + e_t)^\zeta \quad (2.10)$$

### Matching Functions Bad Job

$$m_t^b = m(v_t^b, u_t^b) = m_b(v_t^b)^{1-\zeta}(u_t^b)^\zeta \quad (2.11)$$

### Firm Match Probability in Good job

$$q_t^g = \frac{m_t^g}{v_t^g} = m^g(\theta_t^g)^{-\zeta} \quad (2.12)$$

### Firm Match Probability in Bad job

$$q_t^b = \frac{m_t^b}{v_t^b} = m^b(\theta_t^b)^{-\zeta} \quad (2.13)$$

### Worker Match Probability in Good job

$$p_t^g = \frac{m_t^g}{(u_t^g + e_t)} = m^g(\theta_t^g)^{1-\zeta} \quad (2.14)$$

### Worker Match Probability in Bad job

$$p_t^b = \frac{m_t^b}{u_t^b} = m^b(\theta_t^b)^{1-\zeta} \quad (2.15)$$

### Operating Cost Reservation Threshold for Good job

$$J(\tilde{K}_t^g) = 0 \quad (2.16)$$

### Operating Cost Reservation Threshold for Bad job

$$J(\tilde{K}_t^b) = 0 \quad (2.17)$$

### Labour Market Tightness for Good job

$$\theta_t^g = \frac{v_t^g}{(u_t^g + e_t)} \quad (2.18)$$

### Labour Market Tightness for Bad job

$$\theta_t^b = \frac{v_t^b}{(u_t^b)} \quad (2.19)$$

### Job Creation Condition for Good job

$$\frac{\sigma^g}{q_t^g} = \beta_t (1 - \rho^K) (1 - \mu) \tilde{K}_{t+1}^g \left[ \tilde{K}_{t+2}^g - \frac{\tilde{K}_{t+1}^g}{2} \right] \quad (2.20)$$

### Job Creation Condition for Bad job

$$\frac{\sigma^b}{q_t^b} = \beta_t (1 - \rho^K) (1 - \mu) (1 - s_t) p_t^g \tilde{K}_{t+1}^b \left[ \tilde{K}_{t+2}^b - \frac{\tilde{K}_{t+1}^b}{2} \right] \quad (2.21)$$

### Wage Determination in Good Job

$$w_t^g(K_t) = \mu [P_{g,t} A_t - K_t + \sigma^g \theta_t^g] + z(1 - \mu) \quad (2.22)$$

### Wage Determination in Bad Job

$$w_t^b(K_t) = \mu [P_{b,t} A_t - K_t + (1 - s_t) \sigma^g \theta_t^g] + (1 - \mu)(z + I(s_t)) \quad (2.23)$$

### Optimal Search Intensity

$$\gamma \Omega(s_t)^{\Omega-1} = \frac{\mu}{1 - \mu} p_t^g \left( \frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b} \right) \quad (2.24)$$

### Resource Constraint Condition

$$c_t = y_t - \sigma^g v_t^g - \sigma^b v_t^b - G_t \quad (2.25)$$

### Aggregate Consumption

$$c_t^\tau = c_{t+1}^{-\tau} \quad (2.26)$$

### Government Spending Shock

$$\log G_t = \rho_G \log G_{t-1} + \epsilon_{Gt} \quad (2.27)$$

### Aggregate Productivity Shock

$$\log A_t = \rho_A \log A_t + \epsilon_{At} \quad (2.28)$$

## 2.6 Quantitative Analysis and Calibration

In this section I examined the dynamic responses of the model to the positive government spending shock and productivity shock. First, I approximated the equilibrium equations around the non stochastic steady state, and then I estimated the model by assigning the numerical values to the parameters. Values of all parameters and sources are available in [2.1](#).

I calibrated the model by using parameters values commonly used in literature for US<sup>1</sup>. Parameter values of matching elasticity  $\zeta = 0.4$ , vacancy cost of good job and vacancy cost of bad job  $\sigma^g = 0.16$ ,  $\sigma^b = 0.04$ , respectively, level parameters  $m_g = m_b = 0.6$  and relative risk aversion parameter  $\tau = 1$  are used by following Krause and T. A. Lubik ([2006](#)). Values of some parameters are standard in literature for the US economy e.g. subjective discount factor  $\beta = 0.99$ , separation rate  $\rho^K = 0.1$ . Bargaining share is set to  $\mu = 0.5$  which is also standard for US in the literature. I assume that range of idiosyncratic cost distribution is  $[0, \eta]$  where  $\Phi(K) = K/\eta$ .  $\eta$  represents endogenous job separation and  $\eta = 1$  as set by Kuo and H. Miyamoto ([2019](#)). Value of Frisch elasticity is set  $\Upsilon = 1.0$  and value of unemployment benefits is  $z = 0.24$  as set by Shimer ([2009](#)) and disutility of worker  $\Psi_0 = 0.40$  set by Shimer ([2012](#)). Following Krause and T. A. Lubik ([2006](#)) search cost of on the job is assumed to convex and increasing in search intensity. For the model, I assumed that  $I(s) = \gamma s^\Omega$  where  $\gamma > 0$  and  $\Omega > 1$ . Values of these parameters also set by following Krause and T. A. Lubik ([2006](#)) i.e. Search cost function parameter is set to  $\gamma = 0.04$  and search elasticity is set to  $\Omega = 1.1$ . For the calibration of the shock process government spending shock follow AR(1) process with the common parametric value in literature i.e.  $\rho_G = 0.9^{1/3}$ .

---

<sup>1</sup>Shimer ([2009](#)), Shimer ([2012](#)), Krause and T. A. Lubik ([2006](#)). Most of the values used here are based on the studies which conducted for US

**Table 2.1** Calibration and Parameter Values

Parameter	Description	Values
$\beta$	Discount Factor	0.99
$\alpha$	bad job share in Production	0.333
$\Upsilon$	Frisch Elasticity	1.0
$\tau$	Relative Risk Aversion	1.0
$\Psi_0$	Disutility of Worker	0.40
$m_g$	Level Parameter	0.66
$m_b$	Level Parameter	0.66
$\zeta$	Matching Elasticity	0.4
$\sigma^g$	Vacancy Cost of good job	0.16
$\sigma^b$	Vacancy Cost of bad job	0.04
$\rho^K$	Separation Rate	0.1
$\eta$	Upper support of $\Phi$	1
$\mu$	Bargaining share	0.5
$\gamma$	Search Cost Function Parameter	0.04
$\Omega$	Search Elasticity	1.1
$\nu$	CES Elasticity Parameter	0
$z$	Unemployment Benefits	0.24
$\rho_A$	Agg. Productivity Autoregressive parameter	0.90
$\rho_G$	Govt. Spending Autoregressive parameter	$0.9^{1/3}$
$\epsilon_A$	Standard Deviation of Innovation	1
$\epsilon_G$	Standard Deviation of Govt. Spending	1

Steady state values are calculated numerically and few are taken from Krause and T. A. Lubik (2006) for estimation. Probabilities of filling vacancies are set to  $q_{ss}^g = 0.59$  and  $q_{ss}^b = 0.57$  for good and bad job respectively. Likewise probabilities of job finding from good and bad are set as  $p_{ss}^g = 0.43$ ,  $p_{ss}^b = 0.67$ . Steady state value of vacancy for good and bad job is  $V_{ss}^g = 0.020$  and  $v_{ss}^b = 0.025$ . Employment level for good and bad job is calculated numerically  $n_{ss}^g = 0.60$ ,  $n_{ss}^b = 0.35$ . Wages for good and bad sector are calculated numerically  $w_{ss}^g = 0.46$  and  $w_{ss}^b = 0.45$ . Matching efficiency for both types of job is  $m_{ss}^g = 0.057$  and  $m_{ss}^b = 0.092$ . Steady state search effort is set as  $S_{ss} = 0.48$  with the steady state value of employed searcher is calculated as  $e_{ss} = S_{ss}n_{ss}^b = 0.168$ . Steady state values of Aggregate output  $Y_{ss} = 0.25$ , Output (good and bad)  $Y_{ss}^g = 0.51$ ,  $Y_{ss}^b = 0.19$  are also calculated numerically. Reservation costs for good and bad job is calculated numerically i.e.  $\tilde{K}_{ss}^g = 1.1$ ,  $\tilde{K}_{ss}^b = 0.53$ . Labor market tightness for good job and bad job are  $\theta_{ss}^g = 0.186$ ,  $\theta_{ss}^b = 0.89$ . Lastly steady state value of  $A_{ss} = 1$ . Steady state value of prices in good sector  $P_{gss} = 0.24$  and prices in bad sector  $P_{bss} = 0.54$  is calculated numerically.

**Table 2.2** Steady State Values

Parameter	Description	Values
$A$	Aggregate Productivity	1
$n^g$	Employment (good)	0.60
$n^b$	Employment (bad)	0.35
$w^g$	Wages (good)	0.46
$w^b$	Wages (bad)	0.45
$q^g$	Probability of filling vacancy (good)	0.59
$q^b$	Probability of filling Vacancy (bad)	0.57
$p^g$	Probability of job finding (good)	0.43
$p^b$	Probability of job finding (bad)	0.67
$v^g$	Vacancy (good)	0.020
$v^b$	Vacancy (bad)	0.025
$m^g$	Matching Efficiency (good)	0.057
$m^b$	Matching Efficiency (bad)	0.092
$S$	Search Effort	0.48
$E$	Employed Searcher	0.168
$Y$	Aggregate Output	0.25
$Y^g$	Output (good)	0.51
$Y^b$	Output (bad)	0.19
$\theta^g$	LMT (good)	0.186
$\theta^b$	LMT (bad)	0.89
$P^g$	Prices (good)	0.24
$P^b$	Prices (bad)	0.54
$\tilde{K}_{ss}^g$	Reservation cost (good Job)	1.1
$\tilde{K}_{ss}^b$	Reservation cost (bad Job)	0.53

## 2.7 Model Analysis

I analyzed the model through impulse responses. I used the aggregate productivity and government spending shocks to estimate the model of the labor market. I analyzed the theoretical model with the introduction of two separation margins i.e. quits (through on-the-job-search) and layoffs (through operating cost). As the objective of the study is to examine the effect of fiscal policy shock and aggregate productivity shock on labour market through these two separation margin, the results show the importance of theoretical model which i developed to confirm the results of chapter 1.

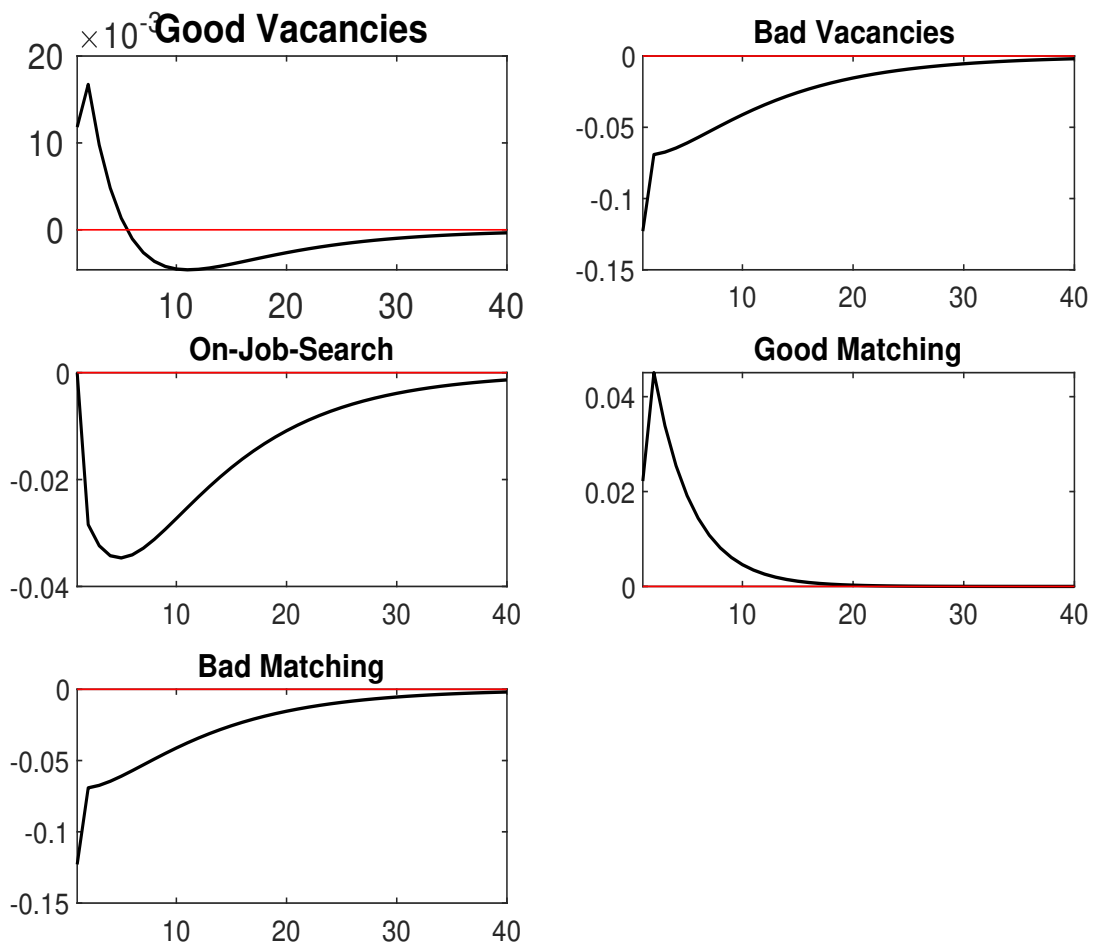
### 2.7.1 Effects of Aggregate Productivity Shocks on Labour Market

In the below figure 2.1 the results show the dynamic behavior of core variables of model in two types of the jobs i.e. (good and bad job) in response to positive aggregate productivity shock.



**Fig. 2.1** Impulse response Functions to 1 Percent Productivity Shock

Results show that on impact, aggregate output and output in bad sector are responding negatively to the aggregate productivity shock till the 40 period but the trend is increasing which means it will be positive in the following periods. Whereas, output for good job is positive throughout the time period and increasing till the 10th period and starts decreasing afterwards. Wages in good and bad job are moving in opposite directions. Wages in bad sector are positive and increasing till 8 periods and after that it start decreasing. Positive productivity shock has positive effect on employment in good sector but trend is declining and converges to steady states. Results also show that there is a slight increase in unemployment in good sector and decrease in bad sector.

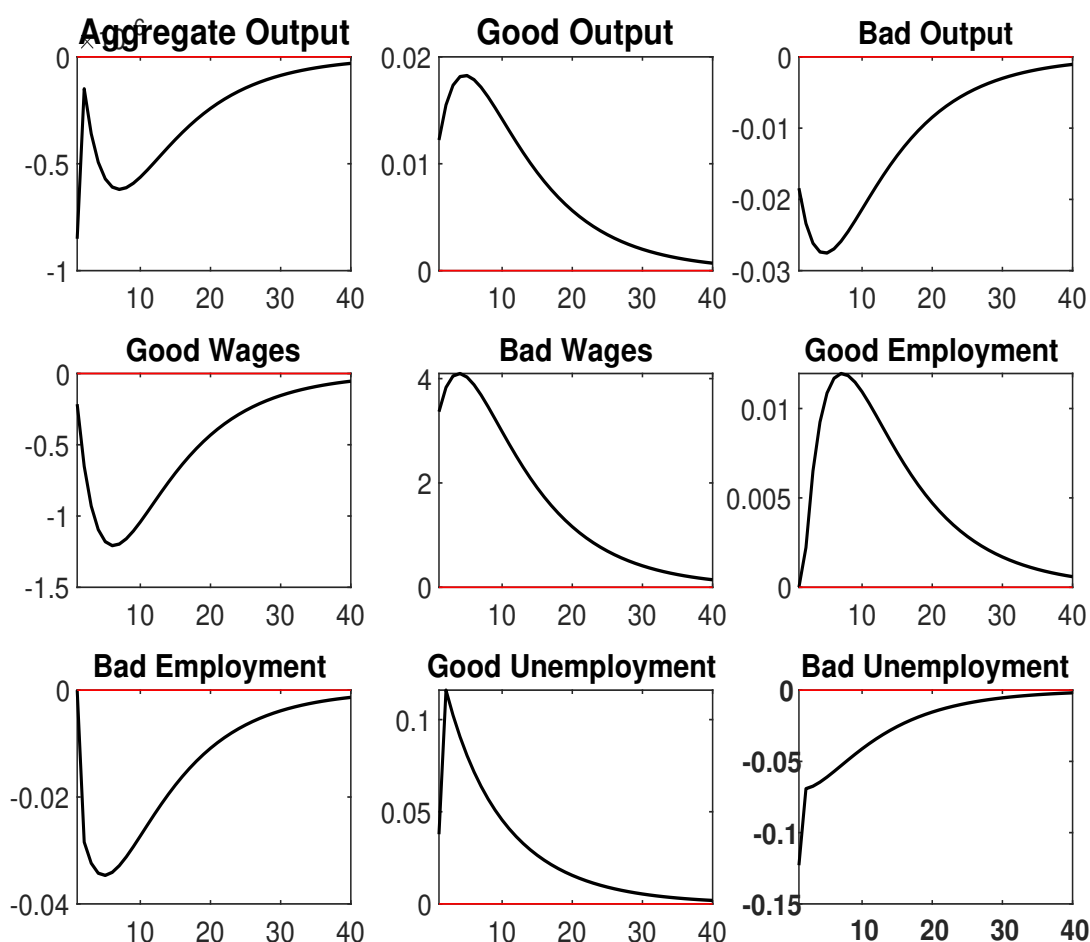


**Fig. 2.2** Impulse response Functions to 1 Percent Productivity Shock

The results in figure 2.2 show that responses of vacancies in both sectors are quite different. For good sector, it is increasing i.e., aggregate productivity has a positive impact on vacancy creation in good sector but it has negative impact on vacancies in bad sector. Search intensity and on the job search are showing same responses to productivity shock i.e. on-the-job-search is declining and below zero till 40 period and converging to steady state in long run, similarly search intensity is positive throughout the period but showing declining trend. The reason behind this negative trend in both graphs can be related to downward slope of unemployment in both sectors. As unemployment decreases employed searchers start increasing searching the job in good sector as they expect vacancy creation after the positive productivity shock. Furthermore, productivity shock has a negative impact on labor market tightness of both sectors. In contrast to the existing literature, the impulse responses to productivity shock with on-the-job search are different e.g. Krause and T. A. Lubik (2006). This

difference is due to the extension of the model of Krause and T. A. Lubik (2006) by including endogenous job separation in the model with two distinct margins. In addition to this extension, I also added government spending shock with productivity shock. Therefore, the impulse responses of this study are different from those available in the existing literature.

## 2.7.2 Effects of Government Spending Shock on Labour Market



**Fig. 2.3** Impulse Response Functions to 1 Percent Govt. Spending Shock

Figure 2.3 reporting the effect of positive fiscal policy shock on labour market. Results show that the aggregate output is above zero but showing downward trend and approaches to zero in the long run. Whereas output in good sector is negative but responded positively to the government spending shock, i.e., the effect is increasing after the 10th period. Government spending shock has a positive impact on the output

in bad sector as it is positive throughout the periods but trend is slightly decreasing. Wages, the actual reason of job-to-job transition, is one of the most important variables of this model in both sectors. As good sector is paying higher wages, therefore, workers who are employed in bad sector are looking for jobs in good sector because they want to get higher wages. Results also support the proposition used in the paper that positive government spending shock has positive effect on wages in good sector. Wages are positive throughout time period till 40 periods and increasing from period 0 to 10 periods and opposite for the wages in the bad sector. Firms, operating in the good sector and paying higher wages, are not employing more workers, therefore, employment in good sector is decreasing as compared to employment in bad sector. It is beneficial for the firms working in bad sector to increase vacancy creation and employ more labor because they have to pay less wages. It was not possible to find these results in an empirical study which I conducted in chapter 1. Responses of these variables to the positive government spending shock and addition of two separation margin in a different way in theoretical model clearly show that the development of theoretical model with two separation margins not only improve the results but also show job-to-job transition of worker from bad to good sector and responses of workers to both types of wages.



**Fig. 2.4** Impulse Response Functions to 1 Percent Govt. Spending Shock

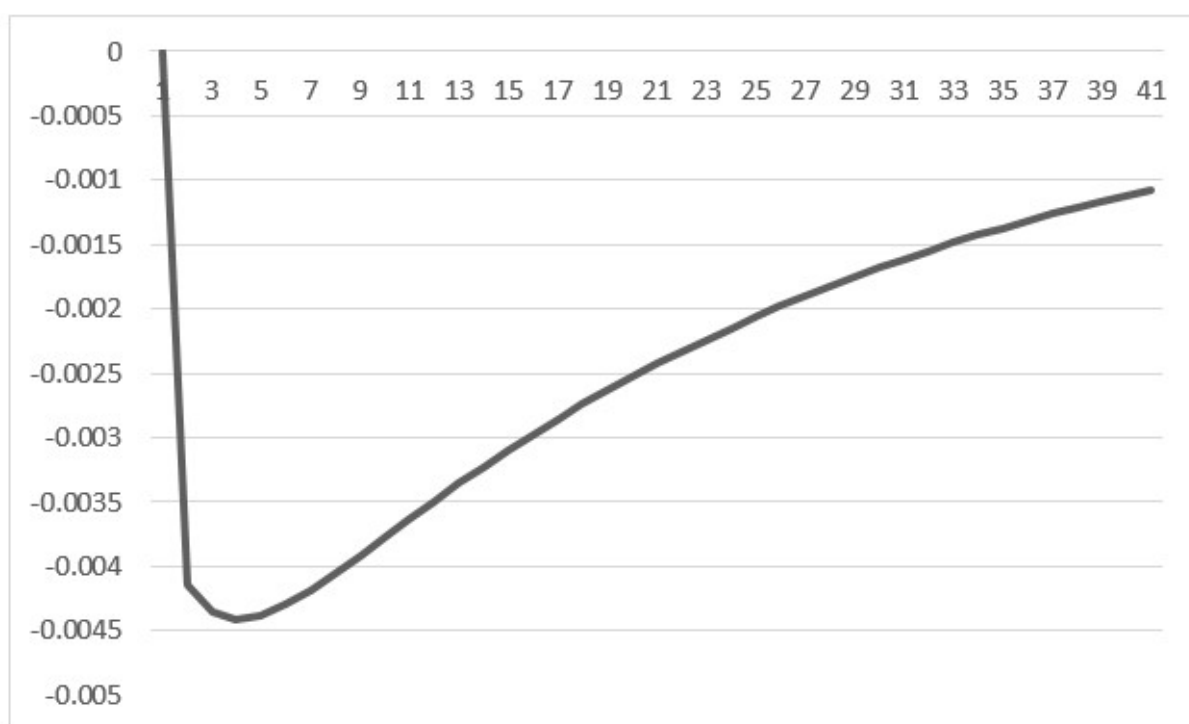
Figure 2.4 explains that from the worker's perspective until there is an increase in vacancy creation in good sector, they will prefer to stay in the employment in bad job. Operating cost is  $K_t$  in the model and firm will produce output while paying wages  $w_t^g$  while bearing operating cost  $K_t$  in the current time period. However, in the next time period, if any exogenous shock is not destroying the match and if  $K_t$  is below the  $\tilde{K}_{t+1}^g$ , the match will continue and firm will get  $J_{t+1}^g(K_{t+1})$ , and if the match will not continue the firm will post a new vacancy in good job. I can see from the results that as match in good job is decreasing there is an increase in the vacancy creation in good sector. Positive government spending shock has a positive effect on both the search intensity and on-the-job-search. This shows that there is a job separation in the economy which results in the job-to-job transition from good to bad sector and reason is the differential in wages in both sector.

These results have a very strong contribution in the search and matching theory because these results are corroborated with the empirical evidence provided in the chapter 1. The IRFs of chapter 1 also presenting effect of government spending shock on labour market but the results of this study show that theoretical model using search and matching frictions specifically by adding quits (through on-the-job-search) and layoffs (through operating cost) in the model give us strong evidence that positive government spending shock has effect on the operating cost, vacancy creation in good sector, vacancy creation in bad sectors and more importantly on wages in both sectors. It was not possible to find all these effects in an empirical study. Therefore, findings of chapter 1 not only explore new ways of thinking for me in the form of developing a DSGE model but also help me to find effect of government spending shock on labour market in different perspective. The results of the second chapter, i.e., the results produced by using a theoretical model under the DSGE framework using search and matching theory with the on-the-job-search not only support results in the chapter 1 but also showing detailed analysis of the labour market.

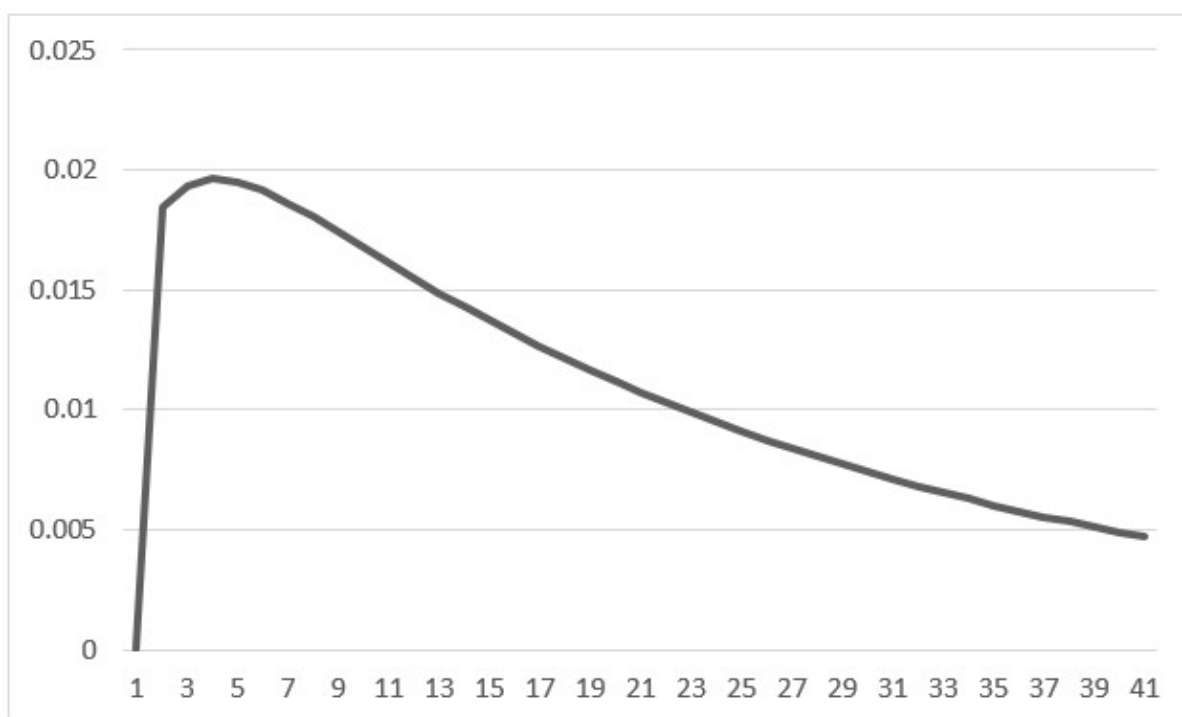
## 2.8 Inequality Analysis

Theoretical model with two distinct wages and employment helps us to do different kind of simulations. Therefore, keeping in mind the above findings it is important to calculate the weighted average of both types of wages. Because, from above discussion, I can say that there is a job-to-job transition and reason of this transition is wage dis-

person and dispersion in wages leads to create wage inequality. By using the results I calculated the weighted average of both good and bad jobs by using employment in both sectors. Below graphs shows that there is sharp decrease in average wage till third quarter and afterwards wages are increasing and due to increase in wages labor are moving from bad to good jobs. Higher increase in average wage also indicates that there is increase in wage inequality but the increment in inequality is only last for five periods and then there is sharp decrease in wage inequality. These findings are consists with chapter 1 where I have used extensive secondary data of US labor market and found that job creation at both higher and lower end of wage distribution lead to create spread in the wage distribution and this spread in wage distribution may lead to increase in income inequality. I used Gini coefficient and P90/10 to measure the income inequality and found that Government spending has an instantaneous and short-term impact on income inequality. This implies that there is initial spread in wage distribution then contraction and in the short run, government spending may increase income inequality due to dynamics in the wage distribution but overall effect of government spending on income inequality is insignificant with a decreasing trend.



**Fig. 2.5** Aggregate Wage



**Fig. 2.6** Wage Inequality

## 2.9 Conclusion

In this chapter I examined the effect of fiscal stimulus on labor market dynamics for US Economy. For this purpose, I developed a DSGE model using search and matching frictions and introduced two endogenous job separations margins (quits and layoffs) in the model. Quits are introduced through on-the-job-search with search effort and layoffs through operating cost. I also used aggregate productivity shock with government spending shock. Results show that positive government spending shock has positive effect on wages in good sector and a negative effect on wages in bad sector. Results also show that government spending increases job creation in good sector which increases job-to-job movements from bad to good sector. Fiscal policy shock has a negative effect on layoffs in bad sector, as due to decrease in wages in bad sector firms want to retain their workers however, workers will stay only until they get job offer from good sector. The results further show that, once there is a vacancy creation in good sector, there is increase in quits in the bad sector, while decrease in unemployment in good sector. These results suggest that there is always a trade-off between the employment generation in the good sector would almost always increase unemployment in the bad sector. This would further increase the job creation in the bad sector and more opportunities for the new workers in the job market.

I also did simulation by calculating average wages by using both types of wages and both types of employments. The results show that there is sharp decrease in average wage and it start increasing afterward. This indicates that there is wage inequality in labor market. Worker move from bad to good sector due to this wage inequality. These movement creates more job vacancies in the bad sector and firms in the bad sectors struggle to fill these vacancies, unless there are fresh unemployed workers in the labour market. The aim of the fresh workers is only securing a job regardless of it being good or bad. This implies that government spending creates more job opportunities in the good sector in first instance, and movement of workers from bad sector to the good sector ultimately provides more job opportunities in the bad sector for new workers.

## CHAPTER 3

# Fiscal Policy with Labor Market Frictions

## 3.1 Introduction

This chapter is the extension of the chapter (chapter 2) and is motivated by the interesting results. In chapter 2, I contributed to the existing body of knowledge by adding endogenous job separation with two distinct forms of job separation i.e., quits through on-the-job search and layoffs via operating cost in DSGE model using search and matching theory. I empirically examined these relationships for the US economy (see chapter 1). In chapter 2, I theoretically examined the responses of these variables to the fiscal policy (i.e., government spending) and productivity shocks.

Following Krause and T. A. Lubik (2006), in chapter 2, I introduced two types of firms named as "good" and "bad" and they are different in terms of costs of creating new vacancies. Since labor market is frictional, therefore, the costs of creating new vacancies have some rents which ultimately lead to increase the differences across vacancies for the employed worker and as a result this differential impel workers who are in low wage job (bad sector) to search for job in high wage jobs (good sector). I assumed that workers in bad sector do on-the-job search as they want to get higher wages.

Results of chapter 2 show that positive government spending shock has positive effect on wages in good sector whereas negative effect on wages in bad sector. Results also show that government spending increases job creation in good sector which leads to increase in job-to-job movements from bad to good sector. Fiscal policy shock has a negative effect on layoffs in bad sector. Firms, in bad sector, want to retain the workers in the long run, however, workers will stay in the firm until they get job offer from good sector. Results further show that the vacancy creation in good sector increases quits

in bad sector and decreases unemployment in good sector. Furthermore, on average, aggregate wages are decreasing in the short run and increasing in the long run, which is an indication of the wage inequality in the labor market and causing workers' movement from bad to good sector.

From the above discussion, it is evident that there is a wage inequality in US labor market which leads to increase in income inequality in the economy and hence workers' welfare decreases in the long run. Therefore, there is a need of government intervention specifically for the workers to decrease the wage inequality. This motivated me to introduce such fiscal policy instrument in the existing model of chapter 2 which can be helpful to reduce wage inequality, improve welfare of the workers and create job vacancies in the US economy. The most relevant fiscal policy instrument in this case is *hiring subsidy*. Hiring subsidy can be defined as 'to reduce the cost of posting vacancy government provide subsidy to all firms for more job creation'. Thus, the main aim of this study is to develop DSGE model with endogenous job separation considering on-the-job-search by introducing fiscal policy tool i.e., hiring subsidy and to propose some recommendations on the basis of results to reduce the wage inequality. I want to corroborate the results of the empirical study (chapter 1) and theoretical study (chapter 2) with responses of endogenous job separation.

To achieve the objectives of this study, I devised three research questions. First, what are the effects of hiring subsidy on US labor market? Second, which sector (good or bad) of the labor market is affected by hiring subsidy? and Third, does hiring subsidy reduce wage inequality?

Theoretical as well as empirical literature examined the effect of different employment subsidies and other labor policy instruments on unemployment (e.g., firing tax Tilli et al. (2019), wage subsidy Almeida, Orr, and Robalino (2014), hiring subsidy Boockmann et al. (2012): Chugh, Lechthaler, and Merkl (2018); Neumark and Grijalva (2017)).

Very few studies have examined the effects of hiring subsidy on employment framework (Cagnolmi, Faia, and Winkler (2011), Kitao, Şahin, and Song (2011), Neumark (2013), Chugh, Lechthaler, and Merkl (2018)). Hiring subsidy provided to private sector always lead to increase employment and are more effective compared to other fiscal policy instruments (Martin and Grubb (2001)), Stromback and Dockery (2000), (Carling and Richardson (2004), Sianesi (2008)).

Literature also shows that hiring subsidy has different effects on different economies.

Yashiv (2004) conducted study for Israel and considered different fiscal policy instruments e.g., hiring subsidies, employment subsidies, unemployment benefits and wage tax reductions in search and matching friction theory to find out which policy is more effective on labor market. Results show that among all these instruments only hiring subsidy has a negative effect on unemployment. Jahn and Wagner (2008) conducted study for European labor market by using four types of unemployed workers. Passive job seeker, active job seeker, short term unemployed and long term unemployed. Results show that hiring subsidy has positive effect on job destruction and duration of unemployment for passive job seekers, and found that number of active job seekers is reduced not only in short term unemployed but also in long term unemployed. All these effects lead to increase employment in the economy and hence hiring subsidy has a positive effect on employment.

Neumark (2013) conducted study for US economy and showed that by comparison hiring subsidies are as effective as public job creation programs. Kitao, Şahin, and Song (2011) used typical search and matching model to examine effect of hiring subsidy on job creation and job destruction for US economy and found that hiring subsidy has positive effect on job creation but in the long run there is increase in unemployment. Hiring subsidy policy is successful in terms of increasing employment in the case of those firms where wages are quite low with wage rigidity (Cahuc, Carcillo, and Le Barbanchon (2014)). Brown and Koettl (2015) argued that hiring subsidy is considered to be best tool among other labor policy tools to reduce unemployment.

Based on the findings of chapter 1 and chapter 2 and from the literature, it is important to introduce a fiscal policy instrument in the search and matching framework with on-the-job search to examine the effect of this policy instrument on US labor market. This study will contribute to existing literature by examining the hiring subsidy in DSGE model using search and matching frictions with endogenous job separation with two distinct margins of job separation quits through on-the-job search and layoffs via operating cost.

Hiring subsidy is most relevant fiscal policy tool to reduce the firm's cost of posting vacancies and to increase more job creation Pissarides (2000). Therefore, this policy instrument is added in the existing theoretical model to examine the effects of this shock to the labor market, specifically on wages and employment.

Results show that hiring subsidy shock has a positive effect on output in bad sector and negative effect on output in good sector. In bad sector, trend is positive till quarter

10 and then it starts decreasing but positive throughout the time period. However, in good sector the trend is positive after quarter five. Response of employment in both sectors is opposite to each other, i.e., in good sector, it is negative and decreasing till quarter five and increasing afterwards and in bad sector it is positive till quarter five and starts decreasing afterwards. Employment depends on the vacancy creation in both sectors and when firm gets subsidy to the cost of posting vacancy, job creation takes place in both sectors as can be seen in the trend of both type of vacancies. Job creation in good sector motivates workers in bad sector for job-to-job movement from bad to good sector.

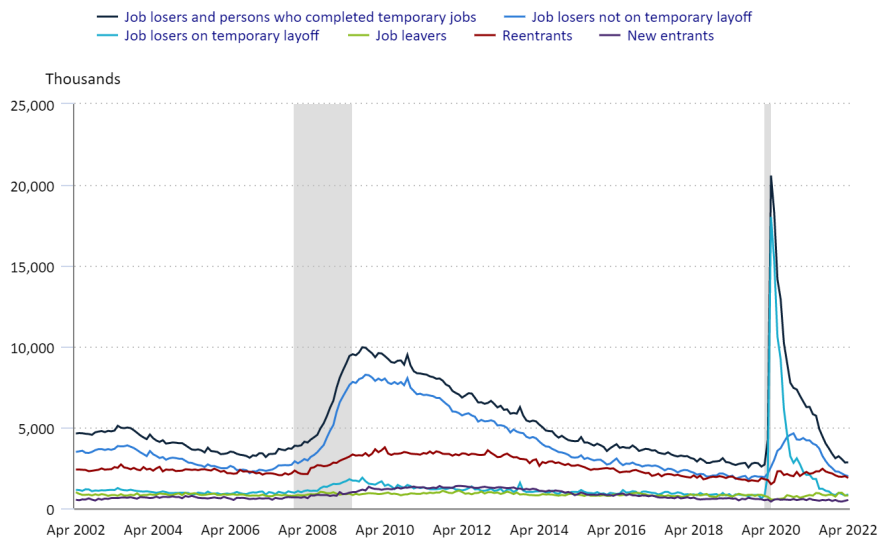
Hiring subsidy has a strong and positive effect on wages in both sectors. Given the high waging nature of the firms in good sector, they tend to hire less workers. This indicates that separation is less costly for the workers in good sector due to hiring subsidy, therefore, there will be an increase in separation in good sector ( i.e., unemployment in good sector). Workers in bad sector continue to search job in good sector as can be seen from the graph that effect of hiring subsidy on wages in good sector is positive and increasing but negative and increasing in bad sector. Vacancy creation in good sector also pushes unemployment upward in bad sector due to on-the-job search i.e., workers will start searching jobs in good sector and hence there will be job-to-job movement from bad to good, which shows that hiring subsidy leads to reduce income inequality through transition of workers from bad to good sector.

Increase in hiring subsidy reduces cost of posting vacancy and job creation will be less costly for firms. Hiring subsidy has positive effect on vacancies in bad sector, on-the-job search and also on search intensity. This indicates that hiring subsidy increases employment i.e., due to differences in wages, workers always prefer to get employed in good sector. This can be seen from unemployment in bad sector which shows positive trend throughout time period. Hiring subsidy has negative effect on the operating cost of the firms, as they are in a better position to operate well. Therefore, there is a reduction in operating cost of the firm due to hiring subsidy.

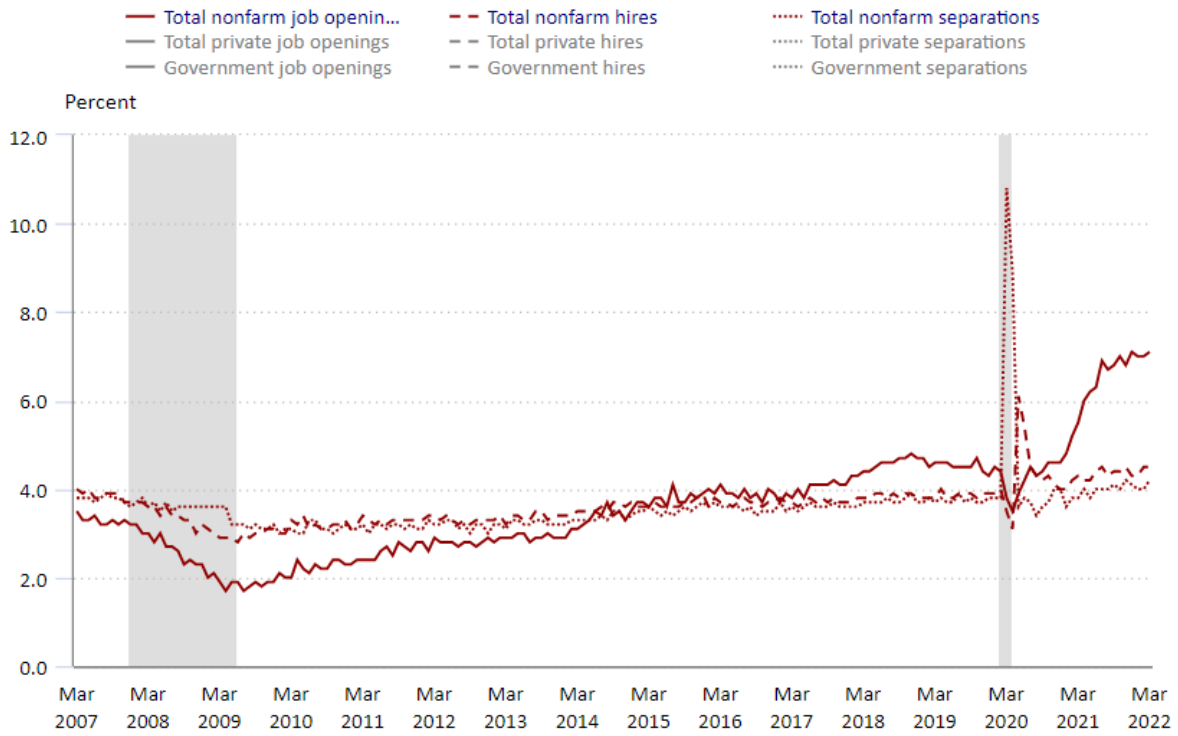
Rest of the paper is organized as follows. Section 3.2 presents some recent facts and figures of US labor market. Theoretical DSGE model is presented in section 3.3. Equilibrium of the model is explained in section 3.4, and quantitative analysis along with model calibrations and steady state are presented in sections 3.5. Section 3.6 explains model analysis, and finally, conclusion and recommendations are presented in section 3.7.

## 3.2 US Labour Market

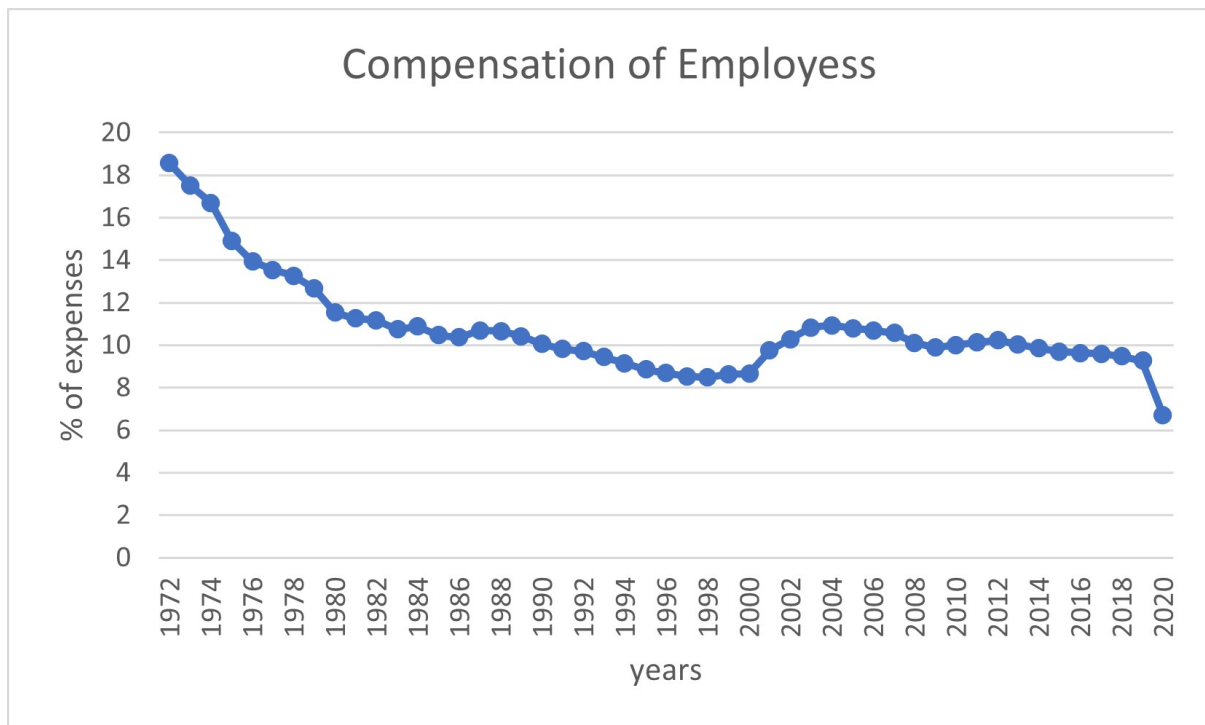
In this section, I showed recent facts and figures of US labor market to assess the need of fiscal intervention. From the figure 3.1, it can be seen that there are different reasons of unemployment but the curve of job losers and persons who completed temporary jobs and job losers on temporary layoff are at highest level reaching 20,000. In figure 3.2, there is fluctuation in total non-farm job opening, hires and separation, however, more fluctuation with increasing trend can be observed in total non-farm separation. These figures indicate that there are more separation than hires. Figure 3.3 shows compensation of employees as a percentage of government expenses from the period of 1972-2020 and overall trend is downward sloping and minimum in 2020. Considering these facts, there is a need of some fiscal policy instrument to reduce the separation and to increase the ratio of hiring. In this case, hiring subsidy is best tool to increase hires and to reduce the job separation.



Source: Bureau of Labor Statistics  
**Fig. 3.1** Reasons for Unemployment



Source: Bureau of Labor Statistics  
**Fig. 3.2** Total Nonfarm Hires and Separation



Source: Bureau of Labor Statistics  
**Fig. 3.3** Compensation of Employees

### 3.3 The Model

I extend the model from the Chapter 2 with same firm and household behavior for the US labor market by adding hiring subsidy  $\chi_t$ . Subsidies are given to firms for vacancy creation and increase employment in both good and bad sectors. I included these two sectors by including good and bad jobs in the model. I assume that both types of jobs are getting same fraction of subsidy. Hiring subsidy is commonly used in the search and matching literature (Mortensen and Pissarides (2001), Campolmi, Faia, and Winkler (2011), Arseneau and Chugh (2012) and Tilli et al. (2019)). I added hiring subsidy to examine the effect of exogenous fiscal policy shock in the US labor market. The underlying assumption is that, in each household there is a continuum of infinitely-lived homogeneous workers and there is a continuum of infinitely-lived heterogeneous firms, both are measured in a discrete time. Firms and workers are working in frictional labor market. Firms are working in perfectly competitive market and by using labour, firms are producing their output and sell their goods to the household. Low wage workers supply their services to the labor market if they are employed and earn wages and will do on-the-job search until they find high wage job.

#### 3.3.1 Household's Problem

The representative household optimization problem is same as in chapter 2 (see page 29), but the budget constraint is changed now as I added hiring subsidy in the household budget constraint.

New budget constraint is as follows:

$$C_t + T_t = w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b + u_t z + \Pi_t \quad (3.1)$$

Above equation states that employed household members whose source of earnings are the incomes  $W$  generated through their employment and the other household members who are unemployed and are on the benefits  $z$ . In addition, these households receive profits  $\Pi_t$  after paying the lump sum taxes  $T_t$  to the government and both types of firms face aggregate hiring cost  $H_t = \chi\sigma^g v_t^g + \chi\sigma^b v_t^b$ . Since aggregate production functions are CRS, therefore profits, income and unemployment benefits are equal to zero,

**HH optimization:**

$$\sum_{i=1}^{\infty} \beta^t \left[ \frac{C_t^{1-\tau}}{1-\tau} - \Psi_0 \frac{n_t^{1+\Upsilon}}{1+\Upsilon} \right]$$

subject to:

$$C_t + T_t = w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b + u_t z + \Pi_t$$

$\lambda_t$  is the Lagrange multiplier

$$L = E \left[ \sum_{i=1}^{\infty} \beta^t \left[ \frac{C_t^{1-\tau}}{1-\tau} - \Psi_0 \frac{n_t^{1+\Upsilon}}{1+\Upsilon} \right] - \lambda_t \{ C_t + T_t - w_t^g(K_t)n_t^g - w_t^b(K_t)n_t^b - u_t z - \Pi_t \} \right]$$

By differentiating w.r.t.  $C_t$ , I get two first order conditions which are presented in chapter 2 (see page 30).

### 3.3.2 Firm's Problem

Firm's problem is same as in chapter 2. The only difference is in the total profits of the firm as i added hiring subsidy in the model.

Total profits of firms is changed now with the addition of hiring subsidy,

$$\Pi_t = \int_{\underline{K}}^{\bar{K}} (P_{gt}y_{gt} + P_{bt}y_{bt}) - (w_t^g(K_t)n_t^g + w_t^b(K_t)n_t^b) - (\chi\sigma^g v_t^g + \chi\sigma^b v_t^b)$$

Total profit comprises output of both firms and wage which are paid to workers by both firms. Both firms face operating cost and and last term of profit function defining that both types of firms face aggregate hiring cost and also getting subsidy to cover the hiring cost.

### 3.3.3 Labour Market

Considering frictional labour market as in chapter 2, firm receives a subsidy for the posting vacancy when a job is created in the firm, therefore, I included subsidy to the cost of posting of vacancy  $\chi_t$  in the value functions of vacant job for good and bad by following Pissarides (2000). Rest of the value functions are same as provided in chapter 2 therefore, I added only value functions of vacant job for good and bad.

- **Value Function of Vacant Job**

Value of a vacancy for both types of jobs i.e.,  $i = g, b$ , in addition of hiring subsidy are as follows:

$$V_t^i = -(1-\chi_t)\sigma^i + E_t\beta_t \left\{ q_t^i(1-\rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^i(K_{t+1})d\Phi(K_{t+1}) + [(1-(1-\rho^K)q_t^i)\Phi(\tilde{K}_{t+1})]V_{t+1}^i \right\} \quad (3.2)$$

Where  $\sigma$  is representing vacancy cost with subsidy  $\chi_t^i$ . A vacancy is created and filled in the following period with the probability  $q_t^i(1-\rho^K)$  and the probability that job will remain unfilled is  $(1-q_t^i(1-\rho^K))$ . Free entry condition states that these values of vacant jobs are equal to zero at any point in time,  $V_t^g = V_t^b = 0$  for all  $t$ . By solving value functions of both unfilled jobs give the following job creation conditions for both types of jobs.

### Job Creation Conditions

$$\frac{(1-\chi_t)\sigma^g}{q_t^g} = (1-\rho^K)E_t\beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1})d\Phi(K_{t+1}), \quad (3.3)$$

and

$$\frac{(1-\chi_t)\sigma^b}{q_t^b} = (1-\rho^K)E_t\beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1})d\Phi(K_{t+1}) \quad (3.4)$$

This above expression states that expected cost of posting vacancy in addition of hiring subsidy is equal to the expected benefits of job filled in good and bad jobs, respectively.

Value functions of employed worker (good and bad) are same as provided in chapter 2. By solving these value functions and using above job creation condition, I derived optimal search intensity which is different from the optimal search intensity of chapter 2. Complete derivation is available in Appendix.

$$I'(s_t) = \frac{\mu}{1-\mu}p_t^g \left( \frac{(1-\chi_t)\sigma^g}{q_t^g} - \frac{(1-\chi_t)\sigma^b}{q_t^b} \right) \quad (3.5)$$

Value function of unemployed worker (good and bad) is also same as it is provided in chapter 2. By solving these value functions and using job creation condition, I get following:

$$\frac{\sigma^g}{q_t^g}p_t^g = \frac{\sigma^b}{q_t^b}p_t^b \quad (3.6)$$

Further solving the above equations

$$\theta_t^g(1 - \chi_t)\sigma^g = \theta_t(1 - \chi_t)^b\sigma^b \quad (3.7)$$

This implies that labor market tightness for good job and bad job is proportional to the cost of job creation with subsidy. Complete derivation is available in Appendix.

#### • Wage Determination

Wages are determined as a result of a mutual bargaining process between both agents of labor market. Negotiation between firms and workers take place through Nash bargaining and choose wages to divide the surplus of match between firms and workers depending on the relative bargaining power of both. As there are two types of jobs, therefore, I derived wage equations for both types of jobs one by one with hiring subsidy. Complete derivation is available in Appendix.

Wage Determination for Good Job

$$w_t^g(K_t) = \mu P_{gt}A_t - \mu K_t + \mu(1 - \chi_t)\sigma^g\theta_t^g + z(1 - \mu) \quad (3.8)$$

Wage Determination for Bad job

$$w_t^b(K_t) = \mu (P_{bt}A_t - K_t + (1 - s_t)(1 - \chi_t)\sigma^g\theta_t^g) + (1 - \mu)(z + I(s_t)) \quad (3.9)$$

### 3.3.4 Government

Government finances an exogenous stream of spending  $g_t$  by collecting lump sum taxes  $T_t$  which are imposed on household. Government also pays unemployment benefits  $u_t z$  to the unemployed workers. In addition, government also provides hiring subsidies to both sectors (i.e., good and bad) and is financed by lump sum taxes. Government spending is determined exogenously. Hence, the budget constraints are given as follows:

$$T_t = e_t^g + (\chi_t)\sigma^g v_t^g + (\chi_t)\sigma^b v_t^b + u_t z \quad (3.10)$$

I investigated the effect of positive hiring subsidy shock on the labour market assuming there is job-to-job transition from bad to good type of job. I also investigated how these movements from bad to good affect unemployment and employment.

### 3.3.5 Resource Constraint

Combining household budget constraint with government budget constraint, I get

$$y_t = C_t + e^g + (\chi_t)\sigma^g v_t^g + (\chi_t)\sigma^b v_t^b \quad (3.11)$$

### 3.3.6 Stochastic Process

$\chi_t$  follows the exogenous stochastic process:

$$\log(\chi_t) = (1 - \rho_\chi)\log(\chi) + \rho_\chi\log(\chi_{t-1}) + \epsilon_{\chi_t} \quad (3.12)$$

where  $\rho_\chi$  is Persistency Coefficients,  $\epsilon_{\chi,t}$  i.i.d innovation.

## 3.4 Equilibrium

I extended the existing model, which I derived in chapter 2, by adding fiscal policy instrument i.e.,  $\chi_t$  and giving shock of this variable to the US labor market to assess its response. Four blocks are added in the model i.e., household, firm and labour market and government. Equilibrium of the model is attained by using all equilibrium equations with all endogenous variables. There are 28 equilibrium equations and 28 variables i.e.,

$$\{y_t, y_t^g, y_t^b, C_t, A_t, P_{g,t}, P_{b,t}, m_t^g, m_t^b, n_t^g, n_t^b, q_t^g, q_t^b, \theta_t^g, \theta_t^b, u_t^g, u_t^b, v_t^g, v_t^b, w_t^g, w_t^b, \tilde{K}_t^g, \tilde{K}_t^b, e_t, s_t, \chi_t, p_t^g, p_t^b\}$$

and the parameters of the model are as follows:

$$\alpha, \beta, \delta, \Upsilon, \tau, \Psi, m_g, m_b, \zeta, \sigma^g, \sigma^b, \rho_K, \mu, \eta, \gamma, \Omega, z, \sigma, \rho_\chi, v$$

## 3.5 Model Calibration and Steady State

To examine the dynamic responses of the model to the subsidy shock. I solved the model by following the process, first, I approximated the equilibrium equations around the non stochastic steady state, second, I estimated the model by assigning the numerical values to the parameters. I calibrated the model by using parameters values commonly used in literature for US<sup>1</sup>. Brief description of parameter values and steady state values is provided in Table 2.1 of chapter 2. Steady state values are calculated

<sup>1</sup>Shimer (2009), Shimer (2012), Krause and T. A. Lubik (2006). Most of the values used here are based on the studies which conducted for US

numerically by using steady state model. I set steady state value of hiring subsidy  $\chi_{ss} = 0.1$ . All parameters and steady state values are provided in Table 3.1, 3.2

**Table 3.1** Model Parameters

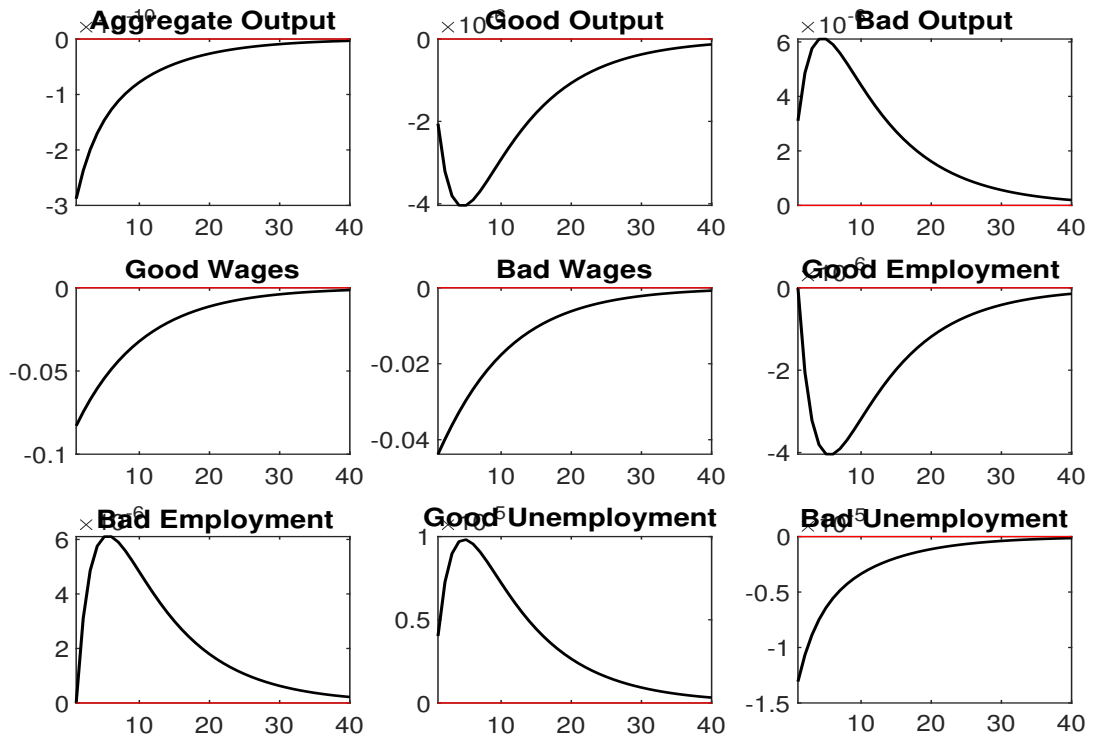
Parameter	Description	Values
$\beta$	Discount Factor	0.99
$\alpha$	bad job share in Production	0.333
$\Upsilon$	Frisch Elasticity	1.0
$\tau$	Relative Risk Aversion	1.0
$\Psi_0$	Disutility of Worker	0.40
$m_g$	Level Parameter	0.66
$m_b$	Level Parameter	0.66
$\zeta$	Matching Elasticity	0.4
$\sigma^g$	Vacancy Cost of good job	0.16
$\sigma^b$	Vacancy Cost of bad job	0.04
$\rho^x$	Separation Rate	0.1
$\eta$	Upper support of $\Phi$	0.67
$\mu$	Bargaining share	0.5
$\gamma$	Search Cost Function Parameter	0.04
$\Omega$	Search Elasticity	1.1
$v$	CES Elasticity Parameter	0
$z$	Unemployment Benefits	0.24
$\rho_\chi$	Subsidy Autoregressive parameter	0.96
$\epsilon_\chi$	Standard Deviation of subsidy	1

**Table 3.2** Steady State Values

Parameter	Description	Values
$A$	Aggregate Productivity	1
$n^g$	Employment (good)	0.60
$n^b$	Employment (bad)	0.35
$w^g$	Wages (good)	0.46
$w^b$	Wages (bad)	0.45
$q^g$	Probability of filling vacancy (good)	0.59
$q^b$	Probability of filling Vacancy (bad)	0.57
$p^g$	Probability of job finding (good)	0.43
$p^b$	Probability of job finding (bad)	0.67
$v^g$	Vacancy (good)	0.020
$v^b$	Vacancy (bad)	0.025
$m^g$	Matching Efficiency (good)	0.057
$m^b$	Matching Efficiency (bad)	0.092
$S$	Search Effort	0.48
$E$	Employed Searcher	0.168
$Y$	Aggregate Output	0.25
$Y^g$	Output (good)	0.51
$Y^b$	Output (bad)	0.19
$\theta^g$	LMT (good)	0.186
$\theta^b$	LMT (bad)	0.89
$P^g$	Prices (good)	0.24
$P^b$	Prices (bad)	0.54
$\chi$	Hiring subsidy	0.1
$\tilde{K}_{ss}^g$	Reservation cost (good Job)	1.1
$\tilde{K}_{ss}^b$	Reservation cost (bad Job)	0.53

## 3.6 Model Analysis

I analyzed the model through impulse responses. I used hiring subsidy shock to estimate the model of the labor market. Following two figures i.e., 3.4 and 3.5 show the effects of hiring subsidy shock on labor market with on-the-job search.



**Fig. 3.4** Impulse response Functions to 1 Percent Hiring Subsidy Shock



**Fig. 3.5** Impulse Response Functions to 1 Percent Hiring subsidy Shock

### 3.6.1 Effects of Hiring Subsidy Shocks on Labour Market

Figures 3.4 and 3.5 show dynamic response of labor market to hiring subsidy. Hiring subsidy has a positive effect on aggregate output and output in bad sector and negative effect in good sector. In bad sector, trend is positive till quarter 10 and then it starts decreasing but positive throughout the time period. However, in good sector trend is positive after quarter five. Response of employment in both sectors is opposite to each other. In good sector, it is negative and decreasing till quarter five and increasing afterwards and in bad sector, it is positive till quarter five and start decreasing afterward. Employment depends on the vacancy creation in both sectors and when firm gets subsidy to the cost of posting vacancy, job creation takes place in both sectors as can be seen in the trend of both types of vacancies. Job creation in good sector motivates workers in bad sector for job-to-job movement from bad to good sector. It is not easy for the worker to get employed in good sector, therefore, when there will be job creation in both sectors, unemployed people will search job in both sectors but employed worker in bad sector starts searching for employment in good sector.

Hiring subsidy has a strong positive effect on wages in both sectors (see figure 3.4). Given the high waging nature of the firms in good sector, they tend to hire less workers. This indicates that separation is less costly for the workers in good sector due to hiring subsidy, therefore, there will be an increase in separation in good sector (figure 3.5 i.e., unemployment in good sector). Workers in bad sector continue to search job in good sector as can be seen from the graph that effect of hiring subsidy on wages in good sector is positive and increasing but negative and increasing in bad sector. Vacancy creation in good sector also pushes upward pressure on unemployment in bad sector due to on-the-job-search i.e., worker will start searching jobs in good sector, hence, there will be job-to-job movement from bad to good. This shows that hiring subsidy leads to reduce income inequality through transition of worker from bad to good sector. These results are very important as purpose of the study was also to see whether hiring subsidy leads to decrease wage inequality or not. From the above discussion, it can be argued that hiring subsidy has a positive effect on vacancy creation in good sector and movements of worker from bad to good indicates that there is reduction, not only in unemployment in good sector but also in wage inequality and support the findings of Brown and Koettl (2015).

Increase in hiring subsidy reduces cost of posting vacancy and job creation will be

less costly for firms and this is good sign for unemployed workers. In Figure 3.5, hiring subsidy has positive effect on vacancies in bad sector, on-the-job search and also on search intensity. This indicates that hiring subsidy increases employment i.e., due to differences in wages, workers in bad sector always prefer to get employed in good sector. This can be seen from unemployment in bad sector, which shows increasing trend throughout time period. This indicates that vacancy creation in good sector attracts the employed workers in bad sector and there will be job-to-job movement from bad to good sector. Hiring subsidy has negative effect on the operating cost of the firms, as they are in a better position to operate well. Therefore, there will be reduction in operating cost of the firm due to hiring subsidy. These results are consistent with the findings of Martin and Grubb (2001) and Sianesi (2008). It is argued that provision of hiring subsidy to both sectors creates more employment opportunities in bad sector but as it has positive effect on wages in good sector, there will job-to-job movement from bad to good, hence wage inequality will decrease. High wages lead to decrease income inequality, and alleviate overall societal inequality.

### **3.7 Conclusion**

This study is the extension of chapter 2 and triangulation of the results of the chapter 1. Results of chapter 1, with the addition of quits and layoffs with other macroeconomic variables in the study, gave me opportunity to develop a theoretical DSGE model with search and matching frictions by introducing these two separation margins to verify the results of chapter 1. In both chapters, I found that positive government spending has a positive effect on employment, vacancy creation and wages. I also found that there is a job-to-job movement which is due to wage dispersion which creates income inequality in the economy. Although, it is short lived but there is income inequality. On the basis of these results, it was important to introduce a fiscal policy instrument specifically for the labor market to reduce income inequality in the economy. For this purpose, I introduced hiring subsidy in the theoretical DSGE model and results show that provision of subsidy to the firms (bad or good) to the cost of posting vacancy reduces income inequality.

I used hiring subsidy as a fiscal policy instrument under DSGE model using search and matching frictions with two distinct margins of job separation i.e., quits through on-the-job-search and layoffs through operating cost. Results show that hiring sub-

sidy has substantial effect on wages and operating cost in both sectors. Response of wages in both sectors is increasing but positive in good sector and negative in bad sector. As a result, welfare of the workers in both sectors will improve due to hiring subsidy. Workers in bad sector continue to search job in good sector as wages are higher in good sector. Vacancy creation in good sector also puts upward pressure on unemployment in bad sector due to on-the-job search i.e., workers will start searching jobs in good sector.

Based on the findings of this study, I suggest that government should introduce hiring subsidy for the firms to cover their cost of posting vacancy to increase the job creation. As the results show that inequality in wages contributes to the general income inequality in economy, therefore, government should provide subsidies to firms to reduce inequality in wages and to improve the welfare of workers.

For future directions, it will be interesting to do welfare analysis by using optimal policy analysis. Another analysis would be interesting to include firing tax along with hiring subsidy to examine their effects in the same model.

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## Appendix A

### Mathematical Derivations

## A.1 Derivation of Theoretical Model (Chapter 2)

- **Optimal Search Intensity**

I differentiated value function of employed workers (bad job) (equation 2.24) w.r.t.  $s_t$  and put it equal to zero to find the optimal search intensity.

$$24 \Rightarrow W_t^b(K_t) = \max_{s_t} \left\{ w_t^b(K_t) - I(s_t) + E_t \beta_t \left\{ (1 - \rho^K)(1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \right. \right. \\ \left. \left. (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right\} \right\}$$

$$\frac{\partial \mathcal{W}(\mathcal{X})}{\partial s_t} = -I'(s_t) + E_t \beta_t \left\{ (1 - \rho^K)(-p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \right.$$

$$\left. (1 - \rho^K) p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right\} = 0$$

$$I'(s_t) = E_t \beta_t (1 - \rho^K) \left\{ p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right\}$$

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right\}$$

add and subtract  $U_{t+1}(\tilde{K}_{t+1})$  on R.H.S we get

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}(\tilde{K}_{t+1}) - \right.$$

$$\begin{aligned}
& \left. \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + U_{t+1}(\tilde{K}_{t+1}) \right\} \\
I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g & \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(\tilde{K}_{t+1}) \Phi(\tilde{K}_{t+1}) - U_{t+1}(\tilde{K}_{t+1}) \right. \\
& \left. - \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}(\tilde{K}_{t+1}) \right) \right\} \quad (\text{A.1})
\end{aligned}$$

we know that  $U_t^g = U_t^b = U_t$  and from equations (2.27) and (2.28) we also know that

$$\begin{aligned}
\frac{\mu}{1 - \mu} J_{t+1}^g(K_t) &= \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}^g \\
\frac{\mu}{1 - \mu} J_{t+1}^b(K_t) &= \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}^b
\end{aligned}$$

after substituting in equation (A.1)

$$\begin{aligned}
I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g & \left\{ \left( \frac{\mu}{1 - \mu} \right) J_{t+1}^g - \left( \frac{\mu}{1 - \mu} \right) J_{t+1}^b(K_t) \right\} \\
I'(s_t) = \frac{\mu}{1 - \mu} E_t \beta_t (1 - \rho^K) p_t^g & \left\{ J_{t+1}^g(K_t) - J_{t+1}^b(K_t) \right\}
\end{aligned}$$

from equation 2.21 and 2.22 we know

$$\frac{\sigma^g}{q_t^g} = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}),$$

and

$$\frac{\sigma^b}{q_t^b} = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1})$$

inserting in above equation we get optimal search intensity

$$I'(s_t) = \frac{\mu}{1 - \mu} p_t^g \left( \frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b} \right) \quad (\text{A.2})$$

### • Derivation of Value Functions of Unemployed Workers

Setting both value functions equal and using bargaining equations.

$$\begin{aligned}
& z + E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \\
= & z + E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}
\end{aligned}$$

$z$  will be cancel out

$$\begin{aligned}
& E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} = \\
& E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}
\end{aligned}$$

multiplying  $U_{t+1}$  inside the bracket

$$\begin{aligned} & E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ U_{t+1} - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \\ & = E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ U_{t+1} - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \end{aligned}$$

Cancelling  $U_{t+1}$  and taking  $p_t^g (1 - \rho^K)$  common, we get

$$\begin{aligned} & E_t \beta_t p_t^g (1 - \rho^K) \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right\} \\ & = E_t \beta_t p_t^b (1 - \rho^K) \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right\} \\ & E_t \beta_t p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = E_t \beta_t p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \end{aligned}$$

By solving both value functions simultaneously and using job creation conditions above we get

$$\frac{\sigma^g}{q_t^g} p_t^g = \frac{\sigma^b}{q_t^b} p_t^b \quad (\text{A.3})$$

Further solving the above equations

$$\theta_t^g \sigma^g = \theta_t^b \sigma^b \quad (\text{A.4})$$

This implies that labor market tightness for good job and bad job is proportional to the cost of job creation

## A.2 Derivation of Wage Determination

### • Wage Determination for Good Job

$$\left( \frac{\mu}{W_t^g(K_t) - U_t^g} \right) \frac{\partial W_t^g(K_t)}{\partial w^g(K)} = - \left( \frac{1 - \mu}{J_t^g(K_t) - V_t^g} \right) \frac{\partial J_t^g(K_t)}{\partial w^g(K)}$$

$$\mu (J_t^g(K_t) - V_t^g) = -(1 - \mu) (W_t^g(K_t) - U_t^g) \frac{\frac{\partial J_t^g(K_t)}{\partial w^g(K)}}{\frac{\partial W_t^g(K_t)}{\partial w^g(K)}}$$

where  $\frac{\frac{\partial J_t^g(K_t)}{\partial w^g(K)}}{\frac{\partial W_t^g(K_t)}{\partial w^g(K)}} = -1$  and as indicated  $V_t^g = 0$  we have

$$\mu (J_t^g(K_t)) = (1 - \mu) (W_t^g(K_t) - U_t^g) \quad (\text{A.1})$$

and substituting in for  $J_t^g(K)$  and  $W_t^g(K)$  from (2.18) and (2.23) in (A.5)

$$2.18 \Rightarrow J_t^g(K_t) = P_{gt} A_t - w_t^g(K_t) - K_t + E_t \mu_t [(1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) +$$

$$\left[1 - (1 - \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] V_{t+1}^g \Big\}$$

$$\Rightarrow J_t^g(K_t) = P_{gt}A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g}$$

2.23  $\Rightarrow$

$$W_t^g(K_t) = w_t^g(K_t) + E_t \beta_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[1 - (1 - \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} \right\}$$

Substitute value functions of  $J_t^g(K_t)$ ,  $W_t^g(K_t)$  and  $U_t^g$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) + E_t \mu_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \right. \right.$$

$$\left. \left. \left[1 - (1 - \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} - z - \left( p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[1 - p_t^g (1 - \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} \right) \right\} \right]$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ \left( (1 - \rho^K) - p_t^g (1 - \rho^K) \right) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[1 - (1 - \rho^K) - (1 - p_t^g (1 - \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} \right\} \right]$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ (1 - \rho^K - p_t^g + \rho^K p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ -\rho^K - (1 - p_t^g + p_t^g \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} \right\} \right]$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ (1 - \rho^K - p_t^g + \rho^K p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ -\rho^K - (1 - p_t^g + p_t^g \rho^K) \Phi \left( \tilde{K}_{t+1} \right) \right] U_{t+1} \right\} \right]$$

$$\begin{aligned}
& (K_{t+1}) d\Phi(K_{t+1}) + \left[ (1 - \rho^K - p_t + p_t^g \rho^K) \Phi(\tilde{K}_{t+1}) U_{t+1} \right] \\
& \mu \left( P_{gt} A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = \\
(1-\mu) & \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ \left( (1 - \rho^K)(1 - p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right) U_{t+1} \right\} \right] \\
& \mu \left( P_{gt} A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g} \right) = \\
(1-\mu) & w_t^g(K_t) - (1-\mu)z + (1-\mu)E_t \beta_t (1-\rho^K)(1-p_t^g) \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right)
\end{aligned}$$

Multiply and divide by  $\mu$  in the last term on L.H.S

$$\begin{aligned}
& \mu(P_{gt} A_t - w_t^g(K_t) - K_t + \frac{\sigma^g}{q_t^g}) = \\
(1-\mu) & w_t^g(K_t) - (1-\mu)z + (\mu) \frac{(1-\mu)}{\mu} E_t \beta_t (1-\rho^K)(1-p_t^g) \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right) U_{t+1}
\end{aligned}$$

as we know

$$\frac{\sigma^g}{q_t^g} = E_t \beta_t (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = \frac{(1 - \mu)}{\mu} E_t \beta_t (1 - \rho^K)$$

$$\left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right) U_{t+1}$$

by substituting in above we get

$$\mu P_{gt} A_t - \mu w_t^g(K_t) - \mu K_t + \mu \frac{\sigma^g}{q_t^g} = (1 - \mu) w_t^g(K_t) - (1 - \mu) z + (\mu) (1 - p_t^g) \frac{\sigma^g}{q_t^g}$$

$$\mu P_{gt} A_t - \mu w_t^g(K_t) - \mu K_t + \mu \frac{\sigma^g}{q_t^g} = w_t^g(K_t) - \mu w_t^g(K_t) - (1 - \mu) z + (\mu) \frac{\sigma^g}{q_t^g} - \mu \frac{\sigma^g p_t^g}{q_t^g}$$

after cancelling some term and using  $\frac{p_t^g}{q_t^g} = \theta^g$  we get

$$\mu P_{gt} A_t - \mu w_t^g(K_t) - \mu K_t + \mu \frac{\sigma^g}{q_t^g} = w_t^g(K_t) - \mu w_t^g(K_t) - (1 - \mu) z + (\mu) \frac{\sigma^g}{q_t^g} - \mu \sigma^g \theta^g$$

$$(K_t) - z(1 - \mu) - \mu \sigma^g \theta^g$$

after simplification final Wage equation is as follows:

$$w_t^g(K_t) = \mu P_{gt} A_t - \mu K_t + \mu \sigma^g \theta_t^g + z(1 - \mu) \quad (\text{A.2})$$

### – Wage Determination for Bad Job

$$\begin{aligned} & \max_{w_t^b(K_t)h_t} (W_t^b(K_t) - U_t^b)^\mu (J_t^b(K_t) - V_t^b)^{1-\mu} \\ w_i &= \max_{w_t^b(K_t)h_t} (W_t^b(K_t) - U_t^b)^\mu (J_t^b(K_t) - V_t^b)^{1-\mu} \end{aligned}$$

Taking natural log we get

$$\ln(w_i) = \mu \ln (W_t^b(K_t) - U_t^b) + (1 - \mu) \ln (J_t^b(K_t) - V_t^b)$$

Differentiate w.r.t  $w^b(K)$  to get wage equation

$$\frac{\partial w_i}{\partial w^b(K)} = \left( \frac{\mu}{W_t^b(K_t) - U_t^b} \right) \frac{\partial W_t^b(K_t)}{\partial w^b(K)} + \left( \frac{1 - \mu}{J_t^b(K_t) - V_t^b} \right) \frac{\partial J_t^b(K_t)}{\partial w^b(K)} = 0$$

$$\left( \frac{\mu}{W_t^b(K_t) - U_t^b} \right) \frac{\partial W_t^b(K_t)}{\partial w^b(K)} = - \left( \frac{1 - \mu}{J_t^b(K_t) - V_t^b} \right) \frac{\partial J_t^b(K_t)}{\partial w^b(K)}$$

$$\mu (J_t^b(K_t) - V_t^b) = -(1 - \mu) (W_t^b(K_t) - U_t^b) \frac{\frac{\partial J_t^b(K_t)}{\partial w^b(K)}}{\frac{\partial W_t^b(K_t)}{\partial w^b(K)}}$$

where  $\frac{\frac{\partial J_t^b(K_t)}{\partial w^b(K)}}{\frac{\partial W_t^b(K_t)}{\partial w^b(K)}} = -1$  remember  $V_t^b = 0$  we have

$$\mu (J_t^b(K_t)) = (1 - \mu) (W_t^b(K_t) - U_t^b) \quad (\text{A.3})$$

and substituting in for  $J_t^b(K)$  and  $W_t^b(K)$  from (2.19) and (2.24) in (A.6)

$$\begin{aligned} 2.19 \Rightarrow J_t^b(K_t) &= P_{bt} A_t - w_t^b(K_t) - K_t + E_t \beta_t [(1 - \rho^K) (1 - p_t^g s_t) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \\ &+ [1 - (1 - \rho^K) p_t^b s_t \Phi(\tilde{K}_{t+1})] V_{t+1}^b] \end{aligned}$$

Since  $\frac{\sigma^b}{q_t^b} = \frac{\sigma^g}{q_t^g}$  and also using job creation condition of bad job 16 will become  $\Rightarrow$

$$J_t^b(K_t) = P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g}$$

$$2.24 \Rightarrow W_t^b(K_t) = \max_{s_t} \left\{ w_t^b(K_t) - I(s_t) + E_t \beta_t \left[ (1 - \rho^K) (1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \\ \left. \left. + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right] \right\}$$

Substitute value functions of  $J_t^b(K_t)$ ,  $W_t^b(K_t)$  and  $U_t^b$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b - I(s_t) + E_t \beta_t \left[ (1 - \rho^K) (1 - s_t p_t^g) \right. \right.$$

$$\left. \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right] - \\ \left. z - E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$\left. \left[ (1 - \rho^K) (1 - s_t p_t^g) - p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \\ \left. \left. + \left[ \rho^K - (1 - p_t^b (1 - \rho^K)) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t (1 - \rho^K) \left[ (1 - \right. \right. \\ \left. \left. s_t p_t^g) - p_t^b \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - \left[ (1 - p_t^b) \Phi(\tilde{K}_{t+1}) U_{t+1} + s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right] \right] \right]$$

From (2.28) we know that

$$\int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1})$$

inserting in above we get

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$(1 - \rho^K) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \cancel{s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1})} \right.$$

$$\left. - p_t^b \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - \left[ \Phi(\tilde{K}_{t+1}) U_{t+1} - p_t^b \Phi(\tilde{K}_{t+1}) U_{t+1} + \cancel{s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1})} \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right. \\ \left. (1 - \rho^K) \left[ (1 - p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - (1 - p_t^g) \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right] \right]$$

Taking common  $(1 - p_t^g)$  from L.H.S

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right. \\ \left. (1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) w_t^b(K_t) - (1 - \mu) I(s_t) - (1 - \mu) z + (1 - \mu) E_t \beta_t$$

$$(1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right]$$

multiply and divide by  $\mu$  in last term of L.H.S

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) w_t^b(K_t) - (1 - \mu) I(s_t) - (1 - \mu) z + E_t \beta_t$$

$$\mu \frac{\mu}{(1 - \mu)} (1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right]$$

As we know

$$\frac{\sigma^g}{q_t^g} = E_t \beta_t p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g$$

Inserting above we get

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu) w_t^b(K_t) - (1 - \mu) I(s_t) - (1 - \mu) z + (\mu) (1 - p_t^g) \frac{\sigma^g}{q_t^g}$$

after cancelling some terms we get

$$\mu P_{bt}A_t - \cancel{\mu w_t^b(K_t)} - \mu K_t + \cancel{\mu \frac{\sigma^g}{q_t^g}} - \mu s_t p_t^g \frac{\sigma^g}{q_t^g} = w_t^b(K_t) - \cancel{\mu w_t^b(K_t)} - (1 - \mu) I(s_t) - (1 - \mu) z + \cancel{\mu \frac{\sigma^g}{q_t^g}} - \mu p_t^g \frac{\sigma^g}{q_t^g}$$

$$\mu P_{bt}A_t - \mu K_t - \mu s_t \sigma^g \theta_t^g = w_t^b - (1 - \mu) I(s_t) - (1 - \mu) z - \mu \sigma^g \theta_t^g$$

$$\mu (P_{bt}A_t - K_t + (1 - s_t) \sigma^g \theta_t^g) \mu + (1 - \mu) (z + I(s_t)) = w_t^b(K_t)$$

$$w_t^b(K_t) = \mu (P_{bt}A_t - K_t + (1 - s_t) \sigma^g \theta_t^g) + (1 - \mu) (z + I(s_t)) \quad (\text{A.4})$$

## A.3 Set of Equilibrium Equations

In this section we summarize all equations of the model. Equilibrium equations are denoted with (Eq), steady states equations as (SS) and log linearized equations as (LL). For the SS we assume that  $A_{ss} = 1$  and we know that  $\Phi(K) = \frac{K}{\mu}$ ,  $c_t^{-\tau} = \lambda_t$ . Matching rates of the firm and worker are presented in term of labor market tightness using the log linear approximation, i.e.  $\hat{q}_{it} = -\zeta \hat{\theta}_{it}$  and  $\hat{p}_{it} = (1 - \zeta) \theta_{it}$ .

### Output for Good Job

$$Eq : y_{g,t} = A_t n_t^g$$

$$SS : y_{g,ss} = n_{ss}^g$$

$$LL : \hat{y}_t^g = \hat{A}_t + \hat{n}_t^g$$

### Output for Bad Job

$$Eq : y_{b,t} = A_t n_t^b$$

$$SS : y_{b,ss} = n_{ss}^b$$

$$LL : \hat{y}_t^b = \hat{A}_t + \hat{n}_t^b$$

### Aggregate Output

$$Eq : y_t = [\alpha y_{bt}^v + (1 - \alpha) y_{gt}^v]^{1/v}$$

$$SS : y_{ss} = [\alpha y_{b,ss}^v + (1 - \alpha) y_{g,ss}^v]^{1/v}$$

$$LL : \hat{y}_t = \frac{\alpha y_{b,ss}^v}{\alpha y_{b,ss}^v + (1 - \alpha) y_{g,ss}^v} \hat{y}_{bt} + \frac{(1 - \alpha) y_{g,ss}^v}{\alpha y_{b,ss}^v + (1 - \alpha) y_{g,ss}^v} \hat{y}_{gt}$$

### Prices in Good Job

$$Eq : P_{g,t} = (1 - \alpha) \left( \frac{y_{gt}}{y_t} \right)^{-(1-v)}$$

$$SS : P_{g,ss} = (1 - \alpha) \left( \frac{y_{g,ss}}{y_{ss}} \right)^{-(1-v)}$$

$$LL : \hat{P}_{gt} = (1 - v) \hat{y}_t - (1 - v) \hat{y}_{gt}$$

## Prices in Bad Job

$$Eq : P_{b,t} = \alpha \left( \frac{y_{bt}}{y_t} \right)^{-(1-\nu)}$$

$$SS : P_{b,ss} = \alpha \left( \frac{y_{b,ss}}{y_{ss}} \right)^{-(1-\nu)}$$

$$LL : \hat{P}_{bt} = (1-\nu)\hat{y}_t - (1-\nu)\hat{y}_{bt}$$

## Employment in Good Job

$$Eq : n_{t+1}^g = (1-\rho^K)[n_t^g + m_t^g]$$

$$SS : n_{ss}^g = (1-\rho^K)[n_{ss}^g + m_{ss}^g]$$

$$LL : \hat{n}_t^g = (1-\rho^K)\hat{n}_{t-1}^g + (\rho)\hat{m}_{t-1}^g$$

## Employment in Bad Job

$$Eq : n_{t+1}^b = (1-\rho^K)[n_t^b + m_t^b - p_t^g s_t n_t^b]$$

$$SS : n_{ss}^b = (1-\rho^K)[n_{ss}^b + m_{ss}^b - p_{ss}^g s_{ss} n_{ss}^b]$$

$$LL : \hat{n}_t^b = (1-\rho^K)(1-p_{ss}^g s_{ss})\hat{n}_{t-1}^b + (1-\rho^K)\frac{m_{ss}^b}{n_{ss}^b}m_{t-1}^b - (1-\rho^K)s_{ss}p_{ss}^g\hat{s}_{t-1}^g - (1-\rho^K)s_{ss}p_{ss}\hat{p}_{t-1}^g$$

## Employed Searchers

$$Eq : e_t = s_t n_t^b$$

$$SS : e_{ss} = s_{ss} n_{ss}^b$$

$$LL : \hat{e}_t = \hat{s}_t + \hat{n}_t^b$$

## Aggregate Unemployment

$$Eq : u_t = u_t^g + u_t^b = 1 - n_t^g - n_t^b = 1 - n_t$$

$$SS : u_{ss} = u_{ss}^g + u_{ss}^b = 1 - n_{ss}^g - n_{ss}^b = 1 - n_{ss}$$

$$LL : \hat{u}_t^g u_{ss}^g + \hat{u}_t^b u_{ss}^b + \hat{n}_t^g n_{ss}^g + \hat{n}_t^b n_{ss}^b = 0$$

## Matching Functions Good Job

$$Eq : m_t^g = m(v_t^g, u_t^g + e_t) = m_g(v_t^g)^{1-\zeta}(u_t^g + e_t)^\zeta,$$

$$SS : m_{ss}^g = m_g(v_{ss}^g)^{1-\zeta}(u_{ss}^g + e_{ss})^\zeta$$

$$LL : \hat{m}_t^g = \zeta(\hat{u}_t^g + e_t) + (1 - \zeta)\hat{v}_t^g,$$

### Matching Functions Bad Job

$$Eq : m_t^b = m(v_t^b, u_t^b) = m_b(v_t^b)^{1-\zeta}(u_t^b)^\zeta$$

$$SS : m_{ss}^b = m_b(v_{ss}^b)^{1-\zeta}(u_{ss}^b)^\zeta$$

$$LL : \hat{m}_t^b = \zeta\hat{u}_t^b + (1 - \zeta)\hat{v}_t^b,$$

### Firm Match Probability in good job

$$Eq : q_t^g = \frac{m_t^g}{v_t^g} = m^g(\theta_t^g)^{-\zeta},$$

$$SS : q_{ss}^g = m^g(\theta_{ss}^g)^{-\zeta}$$

$$LL : \hat{q}_t^g = (-\zeta)\hat{\theta}_t^g$$

### Firm Match Probability in Bad job

$$Eq : q_t^b = \frac{m_t^b}{v_t^b} = m^b(\theta_t^b)^{-\zeta}$$

$$SS : q_{ss}^b = m^b(\theta_{ss}^b)^{-\zeta}$$

$$LL : \hat{q}_t^b = (-\zeta)\hat{\theta}_t^b$$

### Worker Match Probabilities in Good job

$$Eq : p_t^g = \frac{m_t^g}{(u_t^g + e_t)} = m^g(\theta_t^g)^{1-\zeta},$$

$$SS : p_{ss}^g = m^g(\theta_{ss}^g)^{1-\zeta},$$

$$LL : \hat{p}_t^g = (1 - \zeta)\hat{\theta}_t^g$$

### Worker Match Probabilities in Bad job

$$Eq : p_t^b = \frac{m_t^b}{u_t} = m^b(\theta_t^b)^{1-\zeta}$$

$$SS : p_{ss}^b = m^b(\theta_{ss}^b)^{1-\zeta}$$

$$LL; \hat{p}_t^b = (1 - \zeta) \hat{\theta}_t^b$$

### 17. Operating Cost Reservation threshold for Good job

$$J(\tilde{K}_t^g) = 0$$

### Operating Cost Reservation threshold for Bad job

$$J(\tilde{K}_t^b) = 0$$

### Labour Market Tightness for Good job

$$Eq : \theta_t^g = \frac{v_t^g}{(u_t^g + e_t)}$$

$$SS : \theta_{ss}^g = \frac{v_{ss}^g}{(u_{ss}^g + e_{ss})}$$

$$LL : \hat{\theta}_t^g = \hat{v}_t^g - \left( \frac{u_{ss}^g}{u_{ss}^g + e_{ss}} \right) \hat{u}_t^g - \left( \frac{e_{ss}}{u_{ss}^g + e_{ss}} \right) \hat{e}_t^g$$

### Labour Market Tightness for Bad job

$$Eq : \theta_t^b = \frac{v_t^b}{(u_t^b)}$$

$$SS : \theta_{ss}^b = \frac{v_{ss}^b}{(u_{ss}^b)}$$

$$LL : \hat{\theta}_t^b = \hat{v}_t^b - \hat{u}_t^b$$

### Job Creation Condition for Good job

$$Eq : \frac{\sigma^b}{q_t^b} = \beta_t (1 - \rho^K) (1 - \mu) \tilde{K}_{t+1}^g \left[ \tilde{K}_{t+2}^g - \frac{\tilde{K}_{t+1}^g}{2} \right]$$

$$SS : \frac{\sigma^b}{q_t^b} = \beta_t (1 - \rho^K) (1 - \mu) \left( \frac{\tilde{K}_{ss}^g}{2} \right)^2$$

$$LL : \zeta \hat{\theta}_t^b = \beta_t (1 - \rho^K) (1 - \mu) (\tilde{K}_{t+2}^g - \tilde{K}_{t+1}^g)$$

## Job Creation Condition for Bad job

$$Eq : \frac{\sigma^b}{q_t^b} = \beta_t (1 - \rho^K) (1 - \mu) (1 - s_t) p_t^g \tilde{K}_{t+1}^b \left[ \tilde{K}_{t+2}^b - \frac{\tilde{K}_{t+1}^b}{2} \right]$$

$$SS : \frac{\sigma^b}{q_t^b} = \beta_t (1 - \rho^K) (1 - \mu) (1 - s_t) p_t^g \left( \frac{\tilde{K}_{ss}^b}{2} \right)^2$$

$$LL : \zeta \hat{\theta}_t^b = \beta_t (1 - \rho^K) (1 - \mu) (1 - s_t) p_t^g \left( \tilde{K}_{t+2}^b - \tilde{K}_{t+1}^b \right)$$

## 24. Wage Determination in Good Job

$$Eq : w_t^g(K_t) = \mu [P_{g,t} A_t - K_t + \sigma^g \theta_t^g] + z(1 - \mu)$$

$$SS : w_{ss}^g(K_t) = \mu [P_{g,ss} A_{ss} - K_{ss} + \sigma^g \theta_{ss}^g] + z(1 - \mu)$$

$$LL : w_{ss}^g \hat{w}_t^g = \mu \left[ A_{ss} P_{gss} \hat{P}_{gt} + A_{ss} P_{gss} \hat{A}_t - K_{ss} \hat{K}_t + p_{ss}^g \hat{p}_t^g \left( \frac{\sigma^g}{q_{ss}^g} \right) + p_{ss}^g \hat{q}_t^g \left( \frac{\sigma^g}{q_{ss}^g} \right) \right]$$

## Wage Determination in Bad Job

$$w_t^b(K_t) = \mu [P_{b,t} A_t - K_t + (1 - s_t) \sigma^g \theta_t^g] + (1 - \mu)(z + I(s_t))$$

$$SS : w_{ss}^b(K_{ss}) = \mu [P_{b,ss} A_{ss} - K_{ss} + (1 - s_{ss}) \sigma^g \theta_{ss}^g] + (1 - \mu)(z + I(s_{ss}))$$

$$LL : w_{ss}^b \hat{w}_t^b = \mu \left[ A_{ss} P_{bss} \hat{P}_{bt} + A_{ss} P_{bss} \hat{A}_t - K_{ss} \hat{K}_t + (1 - s_{ss}) p_{ss}^g \hat{p}_t^g \left( \frac{\sigma^g}{q_{ss}^g} \right) - (1 - s_{ss}) p_{ss}^g \hat{q}_t^g \left( \frac{\sigma^g}{q_{ss}^g} \right) \right]$$

$$-s_{ss} \hat{s}_t p_{ss}^g \left( \frac{\sigma^g}{q_{ss}^g} \right) + (1 - \mu) (\gamma \Omega s_{ss}^\Omega \hat{s}_t)$$

## Optimal Search Intensity

$$Eq : \gamma \Omega (s_t)^{\Omega-1} = \frac{\mu}{1 - \mu} p_t^g \left( \frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b} \right)$$

$$SS : \gamma \Omega (s_{ss})^{\Omega-1} = \frac{\mu}{1 - \mu} p_{ss}^g \left( \frac{\sigma^g}{q_{ss}^g} - \frac{\sigma^b}{q_{ss}^b} \right)$$

$$LL : (\Omega - 1) \hat{s}_t = \hat{p}_t^g + \frac{\frac{\sigma^g}{q_t^g}}{\frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b}} \hat{q}_t^g - \frac{\frac{\sigma^b}{q_t^b}}{\frac{\sigma^g}{q_t^g} - \frac{\sigma^b}{q_t^b}} \hat{q}_t^b$$

## Resource Constraint Condition

$$c_t = y_t - \sigma^g v_t^g - \sigma^b v_t^b - G_t$$

## Aggregate Consumption

$$Eq : c_t^\tau = c_{t+1}^{-\tau}$$

$$SS : C_{ss}^{-\tau} = \lambda_{ss}$$

$$LL : \tau \hat{c}_t = -\tau \hat{c}_{t+1}$$

## Government Spending Shock

$$\log G_t = \rho_G \log G_{t-1} + \epsilon_{Gt}$$

## Aggregate Productivity Shock

$$\log A_t = \rho_A \log A_t + \epsilon_{At}$$

# A.4 Mathematical Derivations with Hiring Subsidy (Chapter 3)

### – Optimal Search Intensity

I differentiated value function of employed workers (bad job) (equation 2.24) w.r.t.  $s_t$  and put it equal to zero to find the optimal search intensity.

$$2.24 \Rightarrow W_t^b(K_t) = \max_{s_t} \left\{ w_t^b(K_t) - I(s_t) + E_t \beta_t \left\{ (1 - \rho^K)(1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \right. \right. \\ \left. \left. (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right\} \right\}$$

$$\frac{\partial \mathcal{W}(\mathcal{K})}{\partial s_t} = -I'(s_t) + E_t \beta_t \left\{ (1 - \rho^K)(-p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \right.$$

$$(1 - \rho^K) p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \Big\} = 0$$

$$I'(s_t) = E_t \beta_t (1 - \rho^K) \left\{ p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right\}$$

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right\}$$

add and subtract  $U_{t+1}(\tilde{K}_{t+1})$  on R.H.S we get

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}(\tilde{K}_{t+1}) \right.$$

$$\left. - \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + U_{t+1}(\tilde{K}_{t+1}) \right\}$$

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}(\tilde{K}_{t+1}) - \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right. \right.$$

$$\left. - U_{t+1}(\tilde{K}_{t+1}) \right\} \text{(A.1)}$$

we know that  $U_t^g = U_t^b = U_t$  and from equations (2.27) and (2.28) we also know that

$$\frac{\mu}{1 - \mu} J_{t+1}^g(K_t) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}^g$$

$$\frac{\mu}{1 - \mu} J_{t+1}^b(K_t) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - U_{t+1}^b$$

after substituting in equation A.9

$$I'(s_t) = E_t \beta_t (1 - \rho^K) p_t^g \left\{ \left( \frac{\mu}{1 - \mu} \right) J_{t+1}^g - \left( \frac{\mu}{1 - \mu} \right) J_{t+1}^b(K_t) \right\}$$

$$I'(s_t) = \frac{\mu}{1 - \mu} E_t \beta_t (1 - \rho^K) p_t^g \left\{ J_{t+1}^g(K_t) - J_{t+1}^b(K_t) \right\}$$

from equation 3.11 and 3.12 we know

$$\frac{(1 - \chi_t) \sigma^g}{q_t^g} = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}),$$

and

$$\frac{1 - \chi_t}{q_t^b} \sigma^g = (1 - \rho^K) E_t \beta_t \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1})$$

inserting in above equation we get optimal search intensity

### – Derivation of Value functions of Unemployed worker

Setting both value functions equal and using bargaining equations.

$$z + E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\} =$$

$$z + E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$z$  will be cancel out

$$E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$= E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

multiplying  $U_{t+1}$  inside the bracket

$$E_t \beta_t \left\{ p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ U_{t+1} - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$= E_t \beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + \left[ U_{t+1} - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

Cancelling  $U_{t+1}$  and taking  $p_t^g (1 - \rho^K)$  common, we get

$$E_t \beta_t p_t^g (1 - \rho^K) \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right\}$$

$$= E_t \beta_t p_t^b (1 - \rho^K) \left\{ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right\}$$

$$E_t \beta_t p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = E_t \beta_t p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1})$$

By solving both value functions simultaneously and using job creation conditions above we get

$$\frac{(1 - \chi_t) \sigma^g}{q_t^g} p_t^g = \frac{(1 - \chi_t) \sigma^b}{q_t^b} p_t^b \quad (\text{A.2})$$

Further solving the above equations we get final equation.

$$\theta_t^g (1 - \chi_t) \sigma^g = \theta_t^b (1 - \chi_t) \sigma^b \quad (\text{A.3})$$

## A.5 Derivation of Wage Determination

### – Wage Determination for good job with Hiring Subsidy

$$\max_{w_t^g(K_t)} (W_t^g(K_t) - U_t^g)^\mu (J_t^g(K_t) - V_t^g)^{1-\mu}$$

$$w_i = \max_{w_t^g(K_t)h_t^g} (W_t^g(K_t) - U_t^g)^\mu (J_t^g(K_t) - V_t^g)^{1-\mu}$$

the function in the log form is:

$$\ln(w_i) = \mu \ln(W_t^g(K_t) - U_t^g) + (1 - \mu) \ln(J_t^g(K_t) - V_t^g)$$

Differentiate w.r.t  $w^g(K)$  to get wage equation

$$\frac{\partial w_i}{\partial w^g(K)} = \left( \frac{\mu}{W_t^g(K_t) - U_t^g} \right) \frac{\partial W_t^g(K_t)}{\partial w^g(K)} + \left( \frac{1 - \mu}{J_t^g(K_t) - V_t^g} \right) \frac{\partial J_t^g(K_t)}{\partial w^g(K)} = 0$$

$$\left( \frac{\mu}{W_t^g(K_t) - U_t^g} \right) \frac{\partial W_t^g(K_t)}{\partial w^g(K)} = - \left( \frac{1 - \mu}{J_t^g(K_t) - V_t^g} \right) \frac{\partial J_t^g(K_t)}{\partial w^g(K)}$$

$$\mu (J_t^g(K_t) - V_t^g) = -(1 - \mu) (W_t^g(K_t) - U_t^g) \frac{\frac{\partial J_t^g(K_t)}{\partial w^g(K)}}{\frac{\partial W_t^g(K_t)}{\partial w^g(K)}}$$

where  $\frac{\frac{\partial J_t^g(K_t)}{\partial w^g(K)}}{\frac{\partial W_t^g(K_t)}{\partial w^g(K)}} = -1$  and as indicated  $V_t^g = 0$  we have

$$\mu (J_t^g(K_t)) = (1 - \mu) (W_t^g(K_t) - U_t^g) \tag{A.1}$$

and substituting in for  $J_t^g(K)$  and  $W_t^g(K)$  from (2.18 and (2.23) in (A.12)

$$2.18 \Rightarrow J_t^g(K_t) = P_{gt}A_t - w_t^g(K_t) - K_t + E_t\beta_t[(1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^g} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1})$$

$$+ [1 - (1 - \rho^K) \Phi(\tilde{K}_{t+1})] V_{t+1}^g]$$

$$\Rightarrow J_t^g(K_t) = P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g}$$

2.23  $\Rightarrow$

$$W_t^g(K_t) = w_t^g(K_t) + E_t\beta_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^g} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + [1 - (1 - \rho^K) \Phi(\tilde{K}_{t+1})] U_{t+1} \right\}$$

Substitute value functions of  $J_t^g(K_t)$ ,  $W_t^g(K_t)$  and  $U_t^g$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) + E_t \beta_t \left\{ (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) \right. \right.$$

$$\left. d\Phi(K_{t+1}) + \left[ 1 - (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} - z - \left( p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \\ \left. \left. + \left[ 1 - p_t^g (1 - \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right) \right]$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ ((1 - \rho^K) - p_t^g (1 - \rho^K)) \right. \right.$$

$$\left. \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ 1 - (1 - \rho^K) - (1 - p_t^g (1 - \rho^K)) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ (1 - \rho^K - p_t^g + \rho^K p_t^g) \right. \right.$$

$$\left. \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ -\rho^K - (1 - p_t + p_t^g \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ (1 - \rho^K - p_t^g + \rho^K p_t^g) \right. \right.$$

$$\left. \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \left[ (1 - \rho^K - p_t + p_t^g \rho^K) \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) =$$

$$(1 - \mu) \left[ w_t^g(K_t) - z + E_t \beta_t \left\{ ((1 - \rho^K)(1 - p_t^g)) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right] U_{t+1} \right\}$$

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) =$$

$$(1 - \mu) w_t^g(K_t) - (1 - \mu) z + (1 - \mu) E_t \beta_t (1 - \rho^K) (1 - p_t^g) \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right)$$

Multiply and divide by  $\mu$  in the last term on L.H.S

$$\mu \left( P_{gt}A_t - w_t^g(K_t) - K_t + \frac{(1 - \chi_t) \sigma^g}{q_t^g} \right) =$$

$$(1 - \mu)w_t^g(K_t) - (1 - \mu)z + (\mu) \frac{(1 - \mu)}{\mu} E_t \beta_t (1 - \rho^K) (1 - p_t^g) \left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right) U_{t+1}$$

as we know

$$\frac{(1 - \chi_t) \sigma^g}{q_t^g} = E_t \beta_t (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = \frac{(1 - \mu)}{\mu} E_t \beta_t (1 - \rho^K)$$

$$\left( \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) \right) U_{t+1}$$

by substituting in above we get

$$\mu P_{gt}A_t - \mu w_t^g(K_t) - \mu K_t + \mu \frac{(1 - \chi_t) \sigma^g}{q_t^g} = (1 - \mu)w_t^g(K_t) - (1 - \mu)z + (\mu)(1 - p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}$$

$$\mu P_{gt}A_t - \mu w_t^g - \mu K_t + \mu \frac{(1 - \chi_t) \sigma^g}{q_t^g} = w_t^g - \mu w_t^g - (1 - \mu)z + (\mu) \frac{(1 - \chi_t) \sigma^g}{q_t^g} - \mu \frac{(1 - \chi_t) \sigma^g p_t^g}{q_t^g}$$

after cancelling some term and using  $\frac{p_t^g}{q_t^g} = \theta^g$  we get

$$\mu P_{gt}A_t - \mu w_t^g(K_t) - \mu K_t + \mu \frac{(1 - \chi_t) \sigma^g}{q_t^g} = w_t^g(K_t) - \mu w_t^g(K_t) - (1 - \mu)z + (\mu) \frac{(1 - \chi_t) \sigma^g}{q_t^g} - \mu \sigma^g \theta^g$$

$$(K_t) - z(1 - \mu) - \mu(1 - \chi_t) \sigma^g \theta^g$$

after simplification final Wage equation is as follows:

$$w_t^g(K_t) = \mu P_{gt}A_t - \mu K_t + \mu(1 - \chi_t) \sigma^g \theta^g + z(1 - \mu) \quad (\text{A.2})$$

### – Wage Determination for Bad Job with Hiring Subsidy

$$\max_{w_t^b(K_t)h_t} (W_t^b(K_t) - U_t^b)^\mu (J_t^b(K_t) - V_t^b)^{1-\mu}$$

$$w_i = \max_{w_t^b(K_t)h_t} (W_t^b(K_t) - U_t^b)^\mu (J_t^b(K_t) - V_t^b)^{1-\mu}$$

Taking natural log we get

$$\ln(w_i) = \mu \ln(W_t^b(K_t) - U_t^b) + (1 - \mu) \ln(J_t^b(K_t) - V_t^b)$$

Differentiate w.r.t  $w^b(K)$  to get wage equation

$$\frac{\partial w_i}{\partial w^b(K)} = \left( \frac{\mu}{W_t^b(K_t) - U_t^b} \right) \frac{\partial W_t^b(K_t)}{\partial w(K)^b} + \left( \frac{1 - \mu}{J_t^b(K_t) - V_t} \right) \frac{\partial J_t^b(K_t)}{\partial w^b(K)} = 0$$

$$\left( \frac{\mu}{W_t^b(K_t) - U_t^b} \right) \frac{\partial W_t^b(K_t)}{\partial w^b(K)} = - \left( \frac{1 - \mu}{J_t^b(K_t) - V_t} \right) \frac{\partial J_t^b(K_t)}{\partial w^b(K)}$$

$$\mu (J_t^b(K_t) - V_t) = -(1 - \mu) (W_t^b(K_t) - U_t^b) \frac{\frac{\partial J_t^b(K_t)}{\partial w^b(K)}}{\frac{\partial W_t^b(K_t)}{\partial w^b(K)}}$$

where  $\frac{\frac{\partial J_t^b(K_t)}{\partial w^b(K)}}{\frac{\partial W_t^b(K_t)}{\partial w^b(K)}} = -1$  remember  $V_t^b = 0$  we have

$$\mu (J_t^b(K_t)) = (1 - \mu) (W_t^b(K_t) - U_t^b) \quad (\text{A.3})$$

and substituting in for  $J_t^b(x)$  and  $W_t^b(K)$  from (2.19) and (2.24) in (A.14)

$$2.19 \Rightarrow J_t^b(K_t) = P_{bt}A_t - w_t^b(K_t) - K_t + E_t\beta_t[(1 - \rho^K)(1 - p_t^g s_t) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} J_{t+1}^b(K_{t+1}) d\Phi(K_{t+1})$$

$$+ [1 - (1 - \rho^K) p_t^b s_t \Phi(\tilde{K}_{t+1})] V_{t+1}^b - \Phi(\tilde{K}_{t+1}) f_t^b]$$

Since  $\frac{(1 - \chi_t^b) \sigma^b}{q_t^b} = \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}$  and also using job creation condition of bad job 19 will be-

$$\text{come} \Rightarrow J_t^b(K_t) = P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}$$

$$2.24 \Rightarrow W_t^b(K_t) = \max_{s_t} \left\{ w_t^b(K_t) - I(s_t) + E_t\beta_t [(1 - \rho^K) (1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right.$$

$$\left. + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}^b} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right\}$$

Substitute value functions of  $J_t^b(K_t)$ ,  $W_t^b(K_t)$  and  $U_t^b$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \Phi^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b - I(s_t) + E_t\beta_t [(1 - \rho^K) (1 - s_t p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) \right.$$

$$\left. + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}^b} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) + \rho^K \Phi(\tilde{K}_{t+1}) U_{t+1} \right] - z - E_t\beta_t \left\{ p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}^b} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + [1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1})] U_{t+1} \right\}$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$\left. \left[ (1 - \rho^K) (1 - s_t p_t^g) - p_t^b (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) + (1 - \rho^K) s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \\ \left. \left. + [\rho^K - (1 - p_t^b (1 - \rho^K) \Phi(\tilde{K}_{t+1}))] U_{t+1} \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t (1 - \rho^K) \left[ (1 - s_t p_t^g) - p_t^b \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - [(1 - p_t^b) \Phi(\tilde{K}_{t+1})] U_{t+1} + s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right] \right]$$

we also know that

$$\int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1})$$

inserting in above we get

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$\left. \left[ (1 - \rho^K) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right. \right. \right. \\ \left. \left. - p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^b(K_{t+1}) d\Phi(K_{t+1}) - [\Phi(\tilde{K}_{t+1})] U_{t+1} - p_t^g \Phi(\tilde{K}_{t+1}) U_{t+1} + s_t p_t^g \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$\left. \left[ (1 - \rho^K) \left[ (1 - p_t^g) \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - (1 - p_t^g) \Phi(\tilde{K}_{t+1}) U_{t+1} \right] \right] \right]$$

Taking common  $(1 - p_t^g)$  from L.H.S

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu) \left[ w_t^b(K_t) - I(s_t) - z + E_t \beta_t \right.$$

$$\left. \left[ (1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1} \right] \right] \right]$$

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu)w_t^b(K_t) - (1 - \mu)I(s_t) - (1 - \mu)z + (1 - \mu)E_t\beta_t$$

$$(1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right]$$

multiply and divide by  $\mu$  in last term of L.H.S

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{\sigma^g}{q_t^g} \right) = (1 - \mu)w_t^b(K_t) - (1 - \mu)I(s_t) - (1 - \mu)z + E_t\beta_t$$

$$\mu \frac{\mu}{(1 - \mu)} (1 - \rho^K) (1 - p_t^g) \left[ \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g \right]$$

As we know

$$\frac{\sigma^g}{q_t^g} = E_t\beta_t p_t^g (1 - \rho^K) \int_{\underline{K}}^{\tilde{K}_{t+1}} J_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) = \int_{\underline{K}}^{\tilde{K}_{t+1}} W_{t+1}^g(K_{t+1}) d\Phi(K_{t+1}) - \Phi(\tilde{K}_{t+1}) U_{t+1}^g$$

Inserting above we get

$$\mu \left( P_{bt}A_t - w_t^b(K_t) - K_t + (1 - s_t p_t^g) \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} \right) = (1 - \mu)w_t^b(K_t) - (1 - \mu)I(s_t) - (1 - \mu)z +$$

$$(\mu)(1 - p_t^g) \frac{\sigma^g}{q_t^g}$$

after cancelling some terms we get

$$\mu P_{bt}A_t - \cancel{\mu w_t^b(K_t)} - \mu K_t + \mu \frac{(1 - \chi_t^g) \sigma^g}{q_t^g} - \cancel{\mu s_t p_t^g \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}} = w_t^b(K_t) - \cancel{\mu w_t^b(K_t)} - (1 - \mu)I(s_t) - (1 - \mu)z$$

$$+ \cancel{\mu \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}} - \mu p_t^g \frac{(1 - \chi_t^g) \sigma^g}{q_t^g}$$

$$\mu P_{bt}A_t - \mu K_t - \mu s_t (1 - \chi_t^g) \sigma^g \theta_t^g = w_t^b - (1 - \mu)I(s_t) - (1 - \mu)z - \mu (1 - \chi_t^g) \sigma^g \theta_t^g$$

$$\mu (P_{bt}A_t - K_t + (1 - s_t) (1 - \chi_t^g) \sigma^g \theta_t^g) \mu + (1 - \mu) (z + I(s_t)) = w_t^b(K_t)$$

$$w_t^b(K_t) = \mu (P_{bt}A_t - K_t + (1 - s_t)(1 - \chi_t)\sigma^g\theta_t^g) + (1 - \mu)(z + I(s_t)) \quad (\text{A.4})$$