INFLATION AND THE PERSONAL DISTRIBUTION OF INCOME IN PORTUGAL

by

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Abstract

This thesis is about differential inflation in Portugal during the period 1971-81 and its influence on the personal distribution of income. The analysis is carried out separately for four regions, using different commodity price information in each of them. Price indices are computed for each of the 15,914 households of the 1973/74 Household Expenditure Survey, and their distributions analysed. Cost-of-Living indices are calculated from the estimated parameters of the Linear Expenditure System, and in order to characterize the personal income distributions, several empirical equivalence scales are computed.

The results emphasize the inadequacy of a single Consumer Price Index (CPI) to represent the experienced rates of inflation of all households in each of the regions. As a rule we find a significant, indirect correlation between the rate of inflation and the household's income. The existence of different commodity price information for each region is of considerable importance in explaining the contrasting differential inflation between some of the regions.

To my wife Branca and
my daughter Ana.

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Introduction

This thesis is about the consequences of inflation in Portugal in the period 1971-1981. It is a partial study in that it only deals with and investigates the different rates of inflation experienced by households in that period - the so called 'differential inflation' - and its influence on the personal distribution of inoomes.

The rates of inflation are usually measured by indices such as the CPI (consumer price index) or the RPI (retail price index), and it is assumed that they adequately represent the rates of inflation faced by all the households in a country. Averages as they are, it is recognized the inevitability of deviations from their values, but it is supposed that these deviations are random and are not systematically related to any household characteristic.

These ideas have been challenged, and some recent studies have shown that there may exist some significant relationships between the household's characteristics and the rate of inflation it experiences, namely that low income households may suffer substantially higher rates of inflation than high income ones. This fact led, for instance, to the reconstruction of a separate price index for pensioners in the U.K. However this conclusion does not seem to be valid for all countries in all time periods, so that a specific study has to be done for each particular case.

Since, as far as I know, the studies on differential inflation have only been carried out in developed and industrialized countries where the rates of inflation have been relatively low, the present thesis broadens their scope to a semi-industrialized country with much higher inflation rates (averaging 20% per year). This study has some distinctive features in that it is carried out separately for four regions in Portugal and, above all, because it uses different price information for each of them.

The methodology adopted will be clear from the structure of this thesis, which is as follows:

In chapter 1 (The Consequences of Inflation) some of the most important empirical studies related to the consequences of inflation are reviewed. This enables us not only to examine the results obtained in similar inflation studies, but also to evaluate them in relation to studies concerned with other inflationary effects (e.g. on growth and on the redistribution of income and wealth).

As the construction of price indices for different households is central in this thesis, the theoretical and practical aspects of such a task are dealt with in chapter 2 (The Price Index). In this chapter we not only make clear the distinction between a price index and a cost-of-living index, but also an examination of the price information existent in Portugal.

In chapter 3 (Differential Inflation in Portugal) price indices are computed for each of 15,915 households living in the four regions. The information on the households' expenditures and demographic composition comes from the raw data of the Household Expenditure Survey (HES) carried out in Portugal in 1973/74. The formula used for the computation of the price indices assumes the constancy of the expenditure weights for each household. Price indices are calculated for yearly periods as well as for longer ones. The longer periods are 1971-76 and 1977-81, which are closely related to substantial price information modifications which occurred in 1977.

The distribution of the computed price indices are then tabulated by household income class and by age of the head of the household.

Finally, a regression analysis is carried out where the dependent variable is the price index and the explanatory variables relate to household income, age and occupation of the head of the household, number of children and location of the household. This analysis is of course made for each of the four regions, but it also focuses on the households living in and near the cities ('concelhos') where the price information directly applies.

The assumption of a constant expenditure pattern over the years in response to price changes may be unreasonable if there are appreciable substitution effects. To allow for these effects, cost-of-living indices for several income classes are computed from the parameters of a Linear Expenditure System (chapter 4 - The Cost-of-Living Index and the Linear Expenditure System), and the results are compared.

It should be said, that in the initial plan for this thesis, there was the intention of using not only the household expenditure survey carried out in 1973/74 but also the equivalent survey carried out in 1980. The results of the latter survey are considered to be important because the Portuguese Revolution in April 1974 may have changed the households' expenditure patterns. However successive postponements in the availability of this survey have made it impossible to take it into account. At the time of writing, (Sept.1983), this information is still not available. Despite this, the fact that the CPI's published by the Portuguese Statistical Office still rely on the expenditure weights drawn from the 1973/74 HES means that, for all practical purposes the results obtained in this study can be directly compared to any of the official indices.

One of the objectives of this thesis is to characterize the personal distribution of incomes and the influence of differential inflation on it.

In chapter 5 (Equivalence Scales) it is argued that the distribution of total household income may be misleading because it does not take into account different household compositions. The theoretical and practical problems of estimating equivalence scales are dealt with and several empirical scales are estimated for Portugal, using various methodologies.

Chapter 6 (Inequality and Differential Inflation in Portugal) analyses income distribution in all the regions in terms of inequality. Not only the distribution of the total household income but also the distribution of income per consumption unit (using a nutritional scale) and the income per adult equivalent (using an empirical scale estimated in chapter 5).

If we compute the inequality measures for the distributions of those variables, say income y, and compare their values with those obtained for the distributions y/P, where P is the price index experienced by each household, we are able to measure the effects of differential inflation on inequality. In the same chapter we simulate and study the effects of a 20% price increase in each of the broad expenditure aggregates.

Chapter 1

The Consequences of Inflation

Inflation and unemployment are two of the major concerns of economic policy and governments have alternately emphasized the resolution of either, depending not only on their actual magnitudes, but also on the political philosophy and economic theories prevalent at the time.

The recent upsurge (after 1973) in the rates of inflation that has happened almost worldwide, in conjunction with the slowdown of economic growth has meant a renewed interest in the phenomenon of inflation. Top priority has been given to its elimination or at least reduction, even if it means, for instance an increased unemployment rate.

Inflation has never been popular amongst people. It is more or less correctly perceived as a general and sustained increase in the price level and it is associated with reductions in real disposable incomes. Although nominal incomes have in general more than kept pace with the inflation rate so that real incomes have at least not decreased, people feel that inflation is biting their deserved rise in nominal incomes. The disturbing fact that prices are rising in the local shop may force individuals to search more thoroughly of what happens elsewhere, leading to extra costs in time and money. With a pay rise only once a year inflation will make one feel poorer 364 days out of 365. Moreover inflation is change, and there is always a certain psychological resistance to change.

Economists have always dedicated a great deal of their attention to inflation. The literature on the causes and cures of inflation is vast and burgeoning. Several types of inflation have been detected (demand-pull, cost-push, structural, imported, etc.), and associated with these are many ways of reducing or eliminating it by fiscal or monetary methods.

However, all this effort, which arises from the fears caused by the consequences of inflation has not had a counterpart in the systematic study of these consequences. Most books devoted to inflation only "en passant" mention the inflationary effects and there are also very few articles specifically related to this subject. Why is this? There are basically three possible explanations: the first is that the effects of inflation are very well known and can be summarized in a few paragraphs; the second is that there is almost complete ignorance about and even these few paragraphs are perhaps too many; the third and most likely explanation is that although the effects of inflation are known in broad terms, very few permanent, definite and detailed conclusions exist. The consequences are generally recognized to be bad, in particular the increased uncertainty introduced into the economy, but the costs involved and the effort of trying to measure empirically those effects are thought to be quite substantially. If it is accepted that inflation is bad, then perhaps all our energy should be directed to reducing it, instead of bothering with the empirical details. After all, it may be impossible to isolate the effects of inflation from other "real" effects.

Although there is no doubt about the complexity of the task of empirically measuring the effects of inflation, its necessity should never be minimized, not only because otherwise economic knowledge would be incomplete, but also because the recent high levels of inflation make it more urgent that any effects are properly quantified. The few studies which have already been done on the subject have significantly contributed to dismantling much of the conventional wisdom, as will be shown later.

The conventional wisdom of the effects of inflation can be briefly summarized as follows: inflation introduces a significant, arbitrary and regressive redistribution of income, leads to unfavourable external balances and to inefficient resource allocations.

The implications on the redistribution of incomes are most often mentioned, in particular that those with fixed incomes (e.g. pensioners, old people) lose with inflation (which is true by definition); the debtors gain from creditors (with the implication that nominal interest rates do not keep pace with inflation rates); and sometimes that the profit share increases at the expense of the wage share (wage-lag hypothesis).

These and some other statements will be analysed in depth in the following sections where some of the most important empirical studies related to the consequences of inflation are reviewed. They will be classified under these headings: "Inflation and Growth", "Inflation and Redistribution" and "Differential Inflation". In "Inflation and Growth" I will analyse the claim that "mild" inflation may be favourable to economic growth. The perspective for this is the long run macroeconomic level, and belongs to the literature of economic development. The section on "Inflation and Redistribution" examines most of the studies on the consequences of inflation. A microeconomic approach is adopted and consists essentially on the study and empirical estimation of the influence of inflation on the price of assets and their yields. The transformation from this asset by asset approach to an agent approach is easy if the portfolio of the economic agents is known. There are a multitude of possible interesting classifications of economic agents but they are grouped either by income, by age, or by socioeconomic status. Both sections assume a unique rate of inflation faced by all households in the economy, whose value is given either by the Consumer Price Index (CPI), the Retail Price Index (RPI) or even the GNP deflator. This is challenged by the "Differential Inflation" studies where it is shown that households may experience considerable differences in price index rises.

1.1 INFLATION AND GROWTH

The idea that inflation may be favourable to growth is an old one. David Hume (1752) could write:[1]

> "The good policy of the magistrate, consists only in keeping it [money], if possible, still increasing; because by that means, he keeps a spirit of industry alive in the nation, and increases the stock of labour, wherein consists all real power and riches...When any quantity of money is imported into a nation, it is not at first dispersed into many hands; but it is confined to the coffers of a few persons, who immediately seek to employ it to the best advantage. Here are a set of manufacturers or merchants, we shall suppose...[who] are thereby enabled to employ more workmen than formerly, who never dream of demanding higher wages, but who are glad of employment from such good paymasters."

And Thornton:

> "It must be admitted that, provided we assume an excessive issue of paper to lift up, as it may for a time, the costs of goods tho' not the price of labour, some augmentation of stock [i.e. capital] will be the consequence."

Hamilton (1952) stressed that inflation was a powerful stimulant to growth through its effects on aggregate demand, profits and investment and detected some periods where that happened, such as in England and France in the sixteenth and seventeenth centuries and in England in the latter half of the eighteenth century. Rostow (1960) also highlighted the importance and correlation of inflation in industrial take-offs.

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1. This and the following citations are taken from Thirlwall (1974).

This basic idea is taken up by Keynes, Kaldor and in general by the neo-Keynesians: development can be financed by inflationary means. Inflation redistributes income from wage-earners with a low propensity to save to profit-earners with a high propensity to save, this effect being more pronounced the more close to full employment the economy is. Investment will then increase and with it the necessary conditions for increased growth. Investment will also be stimulated because inflation raises the nominal rate of return on investment and reduces the real rate of interest. It is stressed that these beneficial aspects of inflation are likely to come about if there is inflation caused by excess demand. Supply-side inflation (e.g. by costs of raw materials or wages) will have depressing effects on investment. Three other assumptions are implicit in the assertion that demand inflation stimulates growth: 1) Wages lag behind prices (the wage-lag hypothesis); 2) the income transferred from wages to profits due to inflation is significant and 3) the propensity to save and to invest on the part of the profit-earners is high. The wage-lag hypothesis will be thoroughly studied in the next section, but even if it is true some doubts can be cast on the latter two propositions inasmuch as inflationary finance is often proposed as a solution for LDC countries, with their desperate need of new investments and ways of financing them. The probability that increased profits are employed in sumptuous and very often imported goods is not low, and the magnitude of the transfers may not be important, bearing in mind that the beneficial rates of inflation need to be "mild".

Finally, even if mild rates of inflation are important for growth, how much "mild" is "mild"? Is there an optimal rate of inflation that maximizes the growth rate?

The empirical evidence presents conflicting results. Bathia (1962) studies the historical relation between inflation and growth in five countries in order to test Rostow's thesis that inflation was associated with take-offs in industry. The countries were Germany, Sweden, Canada, Japan and the United Kingdom. No evidence was found of such a relation in Sweden, Canada and Japan. Of all the countries the relation between inflation and growth was significantly positive only in Sweden.

Eckstein (1958)[1] after considering the performance of eight industrialized countries over nearly a century concluded that:

> "periods of rapid growth occurred with and without inflation, and that periods of stagnation also saw a very wide range of price changes. Thus, as a long-run phenomenon, there is no historical association between growth and inflation."

Thirlwall and Barton (1971) in a cross-section study of fifty-one countries over the period 1958-67 found no significant relation between inflation and growth over the whole sample. When they split up the sample by grouping the countries by their per capita incomes and rates of inflation they arrived at the following results: the relation between g - the growth rate and p - the rate of inflation is given by $g = 2.79 + 0.61p$; $r^2 = 0.48$, for seventeen developed countries with per capita incomes above $800 per annum. That relation appeared to be negative for developing countries with rate of inflation higher than 10 per cent, and over all countries "it was found that mild inflation tends to be associated with the highest rate of investment to income, compared with countries with relative price stability and inflation in excess of 10 per cent." This was more or less confirmed by Tun Wai (1959) who estimated an optimum rate of inflation of 12.8 per cent per annum, with a positive relation between inflation and growth up to that point and an inverse one afterwards.

1. Cited in Thirlwall (1974)

Harberger (1964) suggested that the optimum rate of inflation should be between 5 and 10 per cent, after considering the Latin American experience. But Wallich (1969) in another cross-section study for forty-three countries over the period 1956-65 concludes that there exists a negative relation between inflation and growth. Little, Scitovsky and Scott (1970) also found no evidence that inflation increased the flow of saving, so that no important effect of inflation on growth would exist in the seven developing countries studied. What can be concluded from all these empirical studies? Mainly that there is no unanimity that inflation, even if mild will lead to or provide a stimulating environment to growth. It should be borne in mind, however, that the argument is mainly for demand-pull inflation and that there is always the practical impossibility of differentiating it from the other types of inflation. In cross-section studies, for instance, countries with mainly cost-push or structural inflations are put in the same basket alongside others with mainly demand-pull inflations so that the results may not prove much. The historical evidence for each country is also not conclusive. The only safe conclusion is to say that empirical studies provide conflicting evidence on the possible beneficial effects of (mild) inflations to growth, and that the more specific argument for demand-pull inflation is and probably will remain essentially untested.

However, this is not to say that the process of growth may not generate (mild?) inflation. Increased purchasing power distributed during the mature-time of investments, rapidly shifting demands, imported goods, etc., all may lead to (mild?) inflations that may constitute a necessary cost of growth.

On the other hand, inflation may be harmful to growth through other routes, namely through its effects on the balance of payments and on investment. It is well known that different rates of inflation will lead to changes in the terms of trade among countries. There will be the tendency for countries possessing higher rates of inflation to have their currency devalued in relation to lower inflation countries. Although their exports will now be cheaper for the trading partners, giving scope for a rise in the volume of exports, and at the same time a decrease in the volume of imports, there is a possibility due to "ill-behaved" elasticities, that the end result is not an improvement in the balance of payments, but a higher deficit. This is very much so in developing countries, where imports of manufactured goods are essential for development, exhibiting as a result, rigid demands, and where the exports mainly of raw materials, do not depend much on their price. As imported goods will be more expensive, they will represent additional costs, so that higher prices are likely to follow. This refuelling of inflation by imported goods may represent an additional and important danger.

In more developed countries, possessing strong and important financial markets, the different rates of inflation may not lead to devaluations but even to appreciation of their currencies in the short run. This is due to the fact that higher rates of interest generally follow higher rates of inflation, so that the influx of short term and speculative capital from abroad may originate a more favourable exchange rate. In spite of this, in the long run the prevailing effect will be devaluation.

The harmful effects of inflation on investment may come about either through the erosion of the real value of depreciation made by firms (accounting methods have been remarkably slow to adjust to inflation), resulting in the need to apply for new external (to the firms) and costly credit; or through the increased uncertainty introduced in contracts that may lead to a preponderance of short-term contracts and short-term investments in detriment to otherwise perfectly possible and important long-term investments. These disincentives to investment may, at least in part, offset some of the stimulus provided by eventual increased profits. It could be pointed out that these disincentives are not likely to happen, or at least to be very important if there is a situation of "mild" inflation. But if by "mild" inflation it is thought of the already mentioned demand-pull inflation with increased profits and investment and no important secondary effects, then by definition the point made is always valid, although not a very interesting one.

1.2. INFLATION AND REDISTRIBUTION[1]

A distinction has to be made between the vulnerability of an asset to inflationary depreciation and redistribution.[2] Inflation exerts two depreciative effects on an asset: a capital effect and an income effect. The capital effect is due to the fact that any asset valued in money terms will have its real value reduced through inflation; the income effect is related to the monetary income yielded by the asset, which will also decrease in real terms due to inflation.

For redistribution to exist it is necessary that both inflationary depreciative effects (capital and income effects) are not accounted for by a variation in the nominal valuation of the asset and/or an increase in the nominal interest income yielded by the asset. That is, if adequate compensation exists, inflation will not necessarily imply any redistributive effects. The problem now is to consider how compensation can occur and what determines the extent of this

1. The basic framework for grouping the empirical studies in this section closely follows that of Foster (1976).

2. As Foster (1976) stresses.

compensation. The scope for compensation varies from asset to asset (claims, bonds, equities, physicals and employment). Money (non-interest bearing money) offers no scope for compensation and leisure is inflation proof. With no inflation bonds and physicals are assets with a known real return but uncertain capital valuation. Claims (e.g. savings, bank deposits) and employment have a certain capital value, with a negotiable return. Inflation will increase uncertainty: the real return on bonds is now uncertain as is the capital value of claims.

This increased uncertainty can only be removed by full indexation of all assets so that in real terms both their capital and income value are the same. However this would imply that interest should be paid on money. Even in this case, and as is stressed by Foster (1976), full indexation is not a sufficient condition for the removal of inflation, because

> "there remains the problem of estimating what inflation would be at the stage of asset price formation. This is so because indexation is an ex-post compensation device. Consequently, there remains scope for redistribution in the form of capital gains and losses simply because indexation would not succeed in removing the problem of inflation uncertainty from these assets [bonds, physicals and equities]."

Only if inflation rates are perfectly predictable (e.g. in a "steady state" inflation) would full indexation work, with the notable exception of money.[1]

Indexation schemes have however an additional danger: by reducing the costs of inflation, that represent an important constraint to substantial increases in prices, the way is open to an auto-refuelling of inflation with price-wage or wage-price spirals, leading eventually to hyperinflations.

1. A fuller account of the costs of "anticipated inflation" will be given later.

If compensation to inflationary depreciation can occur either by adjustments in the market prices of the assets and/or in the valuation of the nominal yields, the extent of that compensation will depend on the behaviour of those who hold the assets. Most of the times it is only mentioned the necessity of accurate expectations in relation to future inflations. But accurate expectations are not enough to having full compensation: the actions of the economic agents taken in response to those expectations are also important. And even in the case where accurate expectations exist and appropriate actions are taken the result of all the interactions between all the agents may not lead to full compensation. As an example let us suppose that in a given moment both employers and employees can accurately foresee that the rate of inflation will run at 6% higher in the next six months. That accurate forecast will not necessarily lead to an increase in nominal wages by 6% unless appropriate action is taken. That action may not be possible if, for instance, both parties are legally bound to some previous contracts covering those months. If actions can be taken (talks, strikes, lower productivity, lock-outs, etc.) the end result may not be a 6% increase: a stronger bargaining power and inflexible attitude on the part of the employers may settle the dispute say, at a 4% increase in nominal wages.

Accurate forecasts of inflation and appropriate action by the economic agents need to be supplemented by consistent market results for full compensation to happen.

1.2.1 Asset Approach Studies

The literature in this area has long since been preoccupied with the empirical test of the two most often mentioned redistributive effects of inflation: during an inflationary process wages lag behind prices (the wage-lag hypothesis) and creditors lose to debtors (the debtor-creditor hypothesis).

The wage-lag hypothesis

The wage-lag hypothesis plays a crucial role in explaining why inflation may encourage the growth rate of a country. As wages lag behind prices during inflation, profits will rise and with them savings and investment, implying a rise in capital formation and consequently the formation of all the necessary conditions for an increased rate of growth. The empirical studies which examine the relationship between inflation and growth periods are an indirect test of the wage-lag hypothesis. As we concluded in that section, the evidence is inconclusive, and there is no firm indication that inflation (even if it is mild) leads to an increased growth. Other alternative tests have been devised, also related to some implications of the hypothesis: if data on prices, wages and profits is collected, then real wages should decrease with rising prices, turning points in time-series data on prices should precede the corresponding turning points in nominal wages; firms with a higher wage ratio should exhibit higher profits; finally, data on the functional distribution of incomes should show an increased profit-share at the expense of the wage-share.

There are however important empirical difficulties. The level of real wages is not only dependent on prices, but also on "real" forces such as the relative supplies of capital and labour, the level of instruction, skills and training of the labour force, the available technology, shifts in the pattern of final demands, etc. All this implies that for time-series data it is practically impossible to impute changes of real wages to inflation,unless the effects of the "real" forces are considered constant.

Several theoretical reasons have been given to explain the wage-lag hypothesis such as "inertia" or "sluggishness" of wages to keep path with prices, due either to custom, weak bargaining power of labour or imperfect foresight of the rates of inflation. These imperfections of the labour market were stressed by Mitchell (1908) for the U.S. economy of the late nineteenth century, where due to the then prevailing individual bargaining procedure

> "the individual labourer is a poor bargainer. He is ignorant of the possibilities of his situation, exposed to the competition of others with the same disabilities, more anxious to sell than the employers to buy. Moreover, custom in the form of rooted ideas about what is a "fair wage" has a peculiarly tenacious hold upon the minds of both parties in the labour market, weakening the wage-earner's aggression and strengthening the employer's resistance."

Bresciani-Turroni (1937) in his study of the German hyperinflation argued that real wages decline during inflation because employees are almost invariably creditors of their employers and in inflations debtors lose to creditors. As wages are paid after they are earned workers are giving credit to the employers. In practical terms it is doubtful that this loss is important in a non-hyperinflationary environment.

Fisher (1926), using the same debtor-creditor hypothesis, and insisting on the fact that the relations between employers and employees are contractual ones, makes a parallel with the money market: as interest rates (nominal) do not respond in a way as to match inflation rates, due to a lack of foresight, this same lack of foresight would lead to a fixation of nominal wages below the marginal product of labour when prices increase. Real wages fall between each consecutive contract, and only at the time of signature are real wages equal to the marginal product of labour.

All these theoretical reasons give support to the wage-lag hypothesis and early empirical studies seemed to verify it. As Kessel and Alchian (1960) in a review of those earlier empirical studies point out, "E.J. Hamilton probably has contributed more to the acceptance of the hypothesis that inflation causes real wages to fall than has any other single economist." Hamilton (1934, 1936, 1942) used time series data on wages and prices for Spain from 1350 to 1800. Although aware of the possible influence of real factors, he implicitly considered their effects as constant, so that the wage-lag hypothesis is tested by observing the movements in real wages during inflationary periods. From 1350 to 1500, and in the three regions studied, Valencia, Aragon and Navarre, only the latter presented inflation, but here wages advanced much faster than prices in the last decade of the fourteenth century. In the period 1501-1650 he concludes that "with few interruptions, thr trend in [real wages] was downward from 1520 to 1600." Kessel and Alchian challenge this conclusion by pointing out the importance of the years of comparison: if real wages in 1522 are compared with real wages in 1602 then real wages rose. They also challenged the idea that the decline in real wages from 1651 to 1800 in the urban areas of Madrid and Valencia was due to higher prices: as the Spanish population was increasing rapidly, doubling in the eighteenth century with the consequent migration from rural to urban areas, the fall in real wages would have been the result mainly of an increased supply of labour.

Hamilton's (1942) finding that real wages fell in London between 1729-1800 could not be generalized to the rest of England for the same period, because, as shown by Gilboy (1934), real wages rose in the North of England. For France, Hamilton (1929) gave evidence, without any later criticism, that prices had increased more than wages from 1500-1700.

Two studies on the inflationary process of the U.S. Civil War also had an important impact on the wage-lag debate. Mitchell (1903) studied the data on wages and prices for the North and found that real wages fell during that period whichever of the two price indices were used. Alchian and Kessel argue that this fall in real wage rates was mainly due to the destruction of the triangular trading relationship among the North, the South and England, with the consequent rise in import prices in addition to the turnover taxes and tariffs employed as a means of war finance. Real wages would probably have fallen anyway even without inflation. For the South, Lerner (1954, 1955) also concluded that prices rose much faster than wages in the Confederacy, and Southern businessmen made large profits. The blockade by the North of the cotton produced in the South, and the taxes and payments in kind that were important means of war finances are some of the real factors that Kessel and Alchian suggest to explain that evidence.

Hansen (1925) also found that real wages had fallen in the U.S. during the first world war but this conclusion would be reversed if slightly different reference dates were chosen.

All these earlier studies led Kessel and Alchian to conclude that:

> "In general it appears that a highly selective sampling from the population of all inflations has produced two unambiguous cases of a fall in real wages for individual economies, those of the North and the South during the Civil War. For these cases, the wage-lag hypothesis has to compete with price theory. For the one case that has been studied in great detail, that of the North during the Civil War, price theory offers a more satisfactory explanation."

Alchian and Kessel themselves tested the wage-lag hypothesis through one of its implications: if the wage-lag hypothesis is valid, then the greater the wage bills paid by firms the greater should be the increase in profits (wealth) in relation to firms with smaller annual wage bills. By implication, "the ratio of wages to equity is an indicator of the relative rise in stock prices attributable to a lag of wages behind prices", so that the greater the wage-to-equity ratio, the larger should be the rise in equity values as a result of inflation. They considered data for 113 firms over the period 1940-1952, and concluded that the average equity rise was greater the lower the wages-to-equity ratio, what is contrary to the hypothesis. In order to exclude effects of variables such as the net debtor status of the firm, they employed multiple and partial correlation analysis, but the same conclusion prevailed: there was a negative relationship between the wage to equity ratio and the rise in equity values, so that there was no support to the wage-lag hypothesis.

It should be borne in mind that the test devised by Alchian and Kessel, besides being an indirect one, supports the existence of a strict relationship between equity values and profits. The interpretation of their results is not strictly a rejection of the wage-lag hypothesis, but rather a quite different conclusion that firms did not gain with inflation.

Bathia (1962) investigated the relationship between profits and money earnings in the United States for the years 1935 to 1959. He was particularly interested in testing the hypothesis put forward by Kaldor (1959) that the rise in money wages depends on the increase in profits of the previous year. Two sub-periods were considered, 1935-1948 and 1948-1959 and the data refers to the manufacturing sector. For the latter period, profits are defined as the percentage rate of return on equity capital and an index of hourly earnings is constructed as the earnings variable. Several linear regressions were estimated, with the two variables taken either in levels, in rates of change, or both, and with or without the presence of lags. The highest correlation coefficient was obtained with a wage-adjustment lag of two months. In the period 1935-48, due to lack of data, profits were expressed as a proportion of total wage payments. The same lack of data restricted substantially the experimentation with lags. The correlation coefficients were generally low, but a lag of six months was found to exist between the profit ratio and the rate of increase in hourly earnings.

Although not directly comparable, the results for both periods seem to indicate a reduction in the earnings lag from six to two months, and in either case substantially less than the lag postulated by Kaldor (one year).

As Bathia points out,

> "these findings carry with them one of two implications. One is that the so-called cost inflation in the United States has been of the profit-push rather than of the wage-push variety. The other is that the entire post-war period has been characterized by demand-pull inflation which first increases prices and profits and then wages."

This study would then confirm the wage-lag hypothesis.

Bearing in mind that the widespread belief in the existence of a wage-lag had only been severely shaken by Alchian and Kessel and that Bathia's conclusions were in line with the mainstream economists' thought, Cargill (1969) decided to test the hypothesis by using spectral methods on time-series of wages and prices. The use of regression analysis and visual graphic comparisons were deemed to be unsuitable, the former because, due to data limitations, would allow only a small number of lags, and the latter because it lacks statistical reliability as it is limited to the comparisons of turning points, and without adjustments for trends in the variables. He used three sets of data for the United States covering the periods 1791-1932, 1820-1965 and 1860-1965, and two for England (1729-1935 and 1806-1954). The definition of wages differ generally for each data set as well as their recipients and respective locations. In order to reduce the influence of changes in real variables he divided the main periods into three sub-periods, arriving at fifteen data sets where the spectral methods were applied. His results are not particularly favourable to the existence of a wage-lag, especially in the short run, while in the long run and especially in England about half the cases show a significant wage-lag. There are many instances where wages and prices are coincident; when they are not, then a wage-lag tends to exist.

In parallel with the studies based on time-series data of wages, prices and profits, the wage-lag hypothesis has also been tested by studying the behaviour of the factor shares, namely the wage and the profit shares.

An example is the article by Bach and Ando (1957), which represents the first "modern" attempt at measuring the redistributional effects of inflation and has profoundly influenced the methodology of most of the papers under roughly the same title. It is, like some others, a mixture of an asset approach and agent approach study and as such will be considered in both sections. Its aim is mainly to inquire on the effects of moderate inflation on different groups in periods of substantially full employment. They studied the 1939-52 U.S. inflation, and tested the wage-lag hypothesis in a very crude way: if there is a wage-lag, then the factor share going to labour (wages) would lose relatively to the share going to capital (profits) in an inflationary period. This would be a necessary implication in terms of the functional distribution of income, so that the empirical observation of the behaviour of the wage and profit shares during the inflationary period would provide the test for the wage-lag hypothesis. The problem with this procedure is, as before, that the functional shares do not depend only on inflation but also on some other "real" factors, so that the empirical conclusions arrived at cannot be solely ascribed to inflation.[1] Without attempting to isolate the effects of inflation nothing definite can be said by using this method. It may even happen that "real" factors will more than compensate the eventual time tendency for inflation to induce income movements in favour of profits. Bach and Ando are aware of this fact, but after saying that "We know no satisfactory way to estimate what portion of the observed changes can be attributed to inflation", they proceeded to showing what was the evolution of the factor shares in the U.S. in their reference period: labour share rose by 6% of total personal income, unincorporated business excluding farming kept a stable share of total personal income and the farm share declined by 1%. If it is taken into account the overstatement of profits due to understated depreciations due to inflation, then the loss of the profit share was important. They broke down the entire period into three sub-periods and obtained some evidence supporting the wage-lag hypothesis only in one of them, but overall it was rejected.

Bach and Stephenson (1974) also noticed that in the U.S. and for the inflationary periods 1950-52 (Korean War), 1955-57 and 1965-71 (Vietnam War) the shifts in national income to wages and salaries from

1. This would be the case if neutral technical progress is admitted along with a unitary elasticity of substitution between capital and labour in a production function. However this very restrictive case has been consistently contradicted in empirical studies.

business profits were "dramatic". As they point out, these shifts do not demonstrate that the changes in income shares were due to inflation, but that if inflation would tend to shift incomes in the opposite direction then this effect was very weak because it was overwhelmed by other forces.

Nilda (1978) in a recent article related to the inflationary experience of Japan (1955-75) and in line with the approach of Bach and Ando arrive at similar conclusions:

> "the intermittent inflation period studied here caused a drastic shift in the distribution of current income from unincorporated business to wages and salaries and corporate business profits over the entire 1955-75 period."

What conclusions can be drawn from all these studies? Is there empirical support to the wage-lag hypothesis?

The conclusion arrived at by the "functional distribution" studies that the profit share has in general been falling in relation to the wage share, is not in itself proof against the existence of wage-lags. Rising capital/labour ratios and an elasticity of substitution between capital and labour less than unity are some of the overall "real" and long-term causes that could explain the observed tendency. Inflation would be incapable to counteract this powerful background movement. Nevertheless, if the analysis is scaled down to short-term periods, the influence of inflation should be felt in stronger terms, particularly during sudden short bursts of inflation and there is no substantial evidence for this to have happened.

For time-series studies there is also conflicting evidence, and the always present problem of isolating the "real" effects from the inflationary ones, although the findings are much more favourable to the existence of a lag in wages in relation to profits.

The type of experienced inflation will also be important. In a profit-push inflation, by definition profits will lead wages. It is surprising that most of the more frequently cited studies relate to old or even very old experiences (16th to 19th centuries and early 20th century). As Bathia's results suggest, the wage response to rising prices may have evolved in time, namely through a faster adjustment. That at least should be expected on an "a priori" basis, as no possible comparison can be made between the individual bargaining power of a worker in the 19th or earlier centuries and the union's power of the 20th century.

Since the wage-lag hypothesis depends on the institutional background of an economy (e.g. the existence or not of trade unions and their legal powers, the length of contracts and/or wage payments, etc.) it is questionable whether any valid generalisations can be made in this area. The findings obtained for the U.S. or the U.K. economies may not be universally valid. But even if it is accepted that, globally and for a certain economy, the wage-lag hypothesis is not a valid one, that does not imply the absence of leads or lags in specific sectors, in specific areas of activity or in specific professions. It is widely observed that in an inflationary process unionized workers of more "belligerant" and important unions end up in a much better shape than non-unionized workers or workers belonging to more "peaceful" unions.

The main objective of wage-lag studies should be the detection of permanent and significant lags in wages in specific areas of activity, rather than the existence of a global wage-lag.

The Debtor-Creditor Hypothesis

The idea that creditors lose to debtors in inflationary periods is an old and well established one in economics. Implicit in it is a certain behaviour of interest rates in times of inflation, that is, nominal interest rates not keeping pace with the inflation rates. As inflation is usually unanticipated, future expectations are downward-biased, resulting in lower interest rates than otherwise. Long and relatively steady periods of inflation will mean that expectations are more correctly formed, so that creditors will be able to introduce clauses into their contracts counteracting the depreciative effects of inflation. Even in this case some legal constraints may appear, such as usury laws impeding a proper indexation of debts in practical terms. As a result credit may tend to diminish or at least to be only of the short-term type, with possible serious consequences to investment and growth, as already mentioned.

Few studies exist on the empirical verification of the debtor-creditor hypothesis, but all of them agree that debtors gain from creditors. Keynes (1923) and Fisher (1920) were the first economists to enunciate the debtor-creditor hypothesis in relation to business firms: as firms are mainly debtors they will increase substantially their gains with inflation, in addition to the gains obtained because wages lag behind prices.

Kessel (1956) was the first to test empirically the Keynes-Fisher proposition using data for large firms in the U.S. As he pointed out, there are two implications of the debtor-creditor hypothesis: one for absolute and the other for relative changes in real wealth positions. The absolute change implication means that debtors will gain and creditors lose in relation to their pre-inflation wealth positions. As firms are thought to be mainly debtors, then they will be gainers. If it is possible to find creditor business firms, then they will be worse off with inflation. The relative change implication is that large debtors will profit more from inflation than small debtors. Both implications should be verified in order to confirm the hypothesis. To measure the gains and losses of the firms he observed the increases or decreases in the prices of their stocks in the Stock Exchange.

Due to their particular characteristics Kessel studied first a sample of 16 bank stocks. Their prices rose by 47% in the period 1942-48 while the wholesale price index increased by 60% in the same period. It would seem there that the debtor-creditor hypothesis is rejected because, as is conventionally believed, banks are large debtors, the ratio of debt to equity being generally several times larger than for industrial firms. However almost all bank assets are monetary, in the sense that their value does not depend on the price level, such as bonds, notes, loans, discounts and deposits with other banks (the exceptions are real assets such as buildings and accounting machines). It is then not that clear that banks, although being large debtors are not on balance net creditors. Indeed, he concluded that for each of the 16 banks monetary assets exceeded monetary liabilities, a result confirming an implication of the debtor-creditor hypothesis.

The second implication was also verified: a negative correlation existed between relative stock price changes and a measure of the intensity of creditor status (the ratio of the difference between monetary assets and monetary liabilities to total assets).

Kessel also examined a random sample of thirty industrial corporations from a population of one thousand, whose stock was traded on the New York Stock Exchange. In 1939 about 40% of the observed firms were creditors, raising doubts about the claim that almost all companies are net debtors. The firms were split up into two groups, the net monetary debtors and the net monetary creditors and the changes in share prices between 1939 and 1946 were examined. A significant difference was detected between the rise of share prices in the two groups: the share prices of the firms with a net debtor status rose significantly more than the share prices of the net creditor firms. In order to check that this phenomenon happens due to inflation and not because, for instance, the net debtor firms tend to be the most dynamic ones, with better performances, the same analysis for a deflationary period was carried out (1929-33). It was found that the share prices of net monetary creditors fell significantly less than the share prices of net monetary debtors, in total accordance with the debtor-creditor hypothesis.

This type of study was much enlarged by Alchian and Kessel (1959) who considered all the industrial firms whose common stock was traded on the New York Stock Exchange at any time between 1914 and 1952. For 1933-1952 the American Stock Exchange was also included. In addition, four separate industries were investigated in order to hold constant any industry differences. The total number of observations analysed amounted to 14000 during the whole period. An interesting conclusion was drawn: there was a marked shift in the debtor-creditor status of the firms - from predominantly net debtors around the time of World War I to a ratio of approximately 50:50 in 1952. Again the net monetary status was correlated with stock price changes in the directions predicted by the debtor-creditor hypothesis, either in inflationary or deflationary periods.

A study on British data was carried out by De Alessi (1963) using basically the methodology adopted by Alchian and Kessel, but employing more sophisticated econometric and statistical techniques. This data refers to two samples of firms whose shares were quoted in the Stock Exchange in 1948 and the period of analysis was between 1949 and 1957. The testing of the debtor-creditor hypothesis was associated with the test that inflation is unanticipated.

He argues that if creditors lose to debtors it is because nominal interest rates failed to keep pace with inflation and this can only happen if inflation is not perfectly anticipated. As already discussed, perfect knowledge does not imply perfect actions by economic agents and/or consistent market results, so that from the validity of the debtor-creditor hypothesis cannot be inferred that inflation is unanticipated.

He concluded that although the sizes of his samples were too small relative to the observed rates of inflation and to the observed degree of net monetary status, the results obtained were not inconsistent with the debtor-creditor hypothesis, so that his study for the U.K. corroborated the results obtained by Alchian and Kessel for the U.S.

A completely different methodology was adopted by Bach and Ando (1957), who tried to estimate the total amount transferred from creditors to debtors due to inflation in the period 1939-52 in the U.S. They arrived at the figure of $500 billion in 1952 prices, but the procedure adopted was extremely crude: as "fixed dollar value intangible assets (deposits and currency, life insurance reserves, pension and retirement funds, receivables, mortgages, corporate bonds, government securities, and other miscellaneous items)... provide a measure of the total debts owed to creditors, susceptible to erosion through inflation", and if to the total value of those assets in a year is applied the Consumer Price Index for that year and the same is done in successive years, then a measure of the total loss to creditors is obtained. The problem of applying the CPI to the outstanding debt in the economy is that really what is being estimated is the degree of vulnerability to inflation and not the effects of inflation. As we know, the existence of compensatory aspects in inflation should be taken account of, namely the possible increase in the interest payments which may at least partially offset the erosion provoked in monetary assets by inflation. Bach and Stephenson (1974) acknowledged this fact. But, even allowing for interest rate adjustments, and assuming that no inflation adjustment was made on currency and demand deposits and that full inflation adjustment occurred in all the other assets, the total transferred in the period 1946-71 in the U.S. due to inflation would still be about $0.4 trillion.

Niida (1978), without allowing for interest rate adjustments estimated in 220 trillion yen the amount of creditor's claims wiped out by inflation in three years (1973-75), which is larger than the national income figure of 125 trillion yen in 1975 for Japan.

To sum up, the analysis of the redistributional effects of inflation through the debtor-creditor effect was mainly done for business firms whose shares were traded on the Stock Exchange and no evidence was found to reject it. In addition, the idea was challenged that business firms are predominantly debtors, and it was observed that the proportion between net debtor and net creditor firms may substantially change with time.

Other type of studies mainly estimate the potential and not actual transfers of income from creditors to debtors, by estimating the degree of asset vulnerability to inflation.

The consensus obtained as concerns the debtor-creditor type thesis may explain the nonexistence of any further substantial attempts to empirically measuring the losses of creditors. However, although not labelled under the debtor-creditor heading, a great deal of economic literature has been dealing with an intimate and related subject: the behaviour of interest rates under inflation. It has been shown that even under anticipated inflation, the global macro functioning of the economy might prevent nominal interest rates to rise at the same rate as the rate of inflation: Mundell (1963) demonstrated that inflation by reducing the value of real money balances and hence wealth, would raise saving and reduce the real interest rate. Tobin (1965) in his analysis of economic growth obtained a similar result.

Sargent (1972,1973, 1976) concluded that in the context of a general equilibrium economic model the magnitude of the response of the nominal interest rate to anticipated inflation depended upon various structural parameters of the model, and that only under certain extreme conditions would an increase in the anticipated rate of inflation produce an immediate equivalent rise in the nominal interest rate.

1.2.2 Agent Approach Studies

To this category belong the studies predominantly interested in the redistributive effects of inflation among economic agents. In contrast with the ease with which assets may be classified in a unique and definite way in respect to their vulnerability and scope for compensation to inflation, there is a multitude of possible interesting classifications of economic agents.

Normally they are classified as in the national accounts (households, firms and Government), but particular attention can be focused on to households (which are then sub-divided by income class, type of income, by age of the head, by occupation, etc.), or on firms (corporated or unincorporated, grouped by sector of activity, etc.) Whatever the adopted agent classification, the methodology used is almost always the same: the composition of each economic agent's portfolio is estimated and the total effect of inflation on the valuation and yields of the portfolio are calculated as the sum of the effects of inflation on each asset. Given that attention in the asset type studies has been mainly related with the debtor-creditor (often only at the level of the vulnerability of the outstanding debt) and the wage-lag hypothesis, an enormous task is faced by those who intend to measure the effects of inflation at a disaggregated level. Inevitably some simplifications have to be made which sometimes cast serious doubts on the results achieved.

Agent type studies will be divided into two categories, depending on whether they use simulation models. These simulation models are rather complex, but most of them have the great advantage of broadening the scope of the effects of inflation by including, for instance, the effects of inflation on unemployment. The consequences of inflation are then studied not only at the micro, but also at the macro level.

Non-simulation studies

The already mentioned study by Bach and Ando (1957) on the effects of three inflationary periods (1939-46, 1946-49 and 1949-52) in the United States was based upon the debtor-creditor status of the economic agents. In reality, and as has been repeatedly stressed only the vulnerability of the outstanding debt to inflation was measured since no consideration was given to compensation devices. Within this framework they were able to conclude that inflation had transferred real purchasing power primarily from the household sector to the government and that net transfers involving business were much smaller. Special attention was given to the household sector (the major net creditor in the U.S. economy): households were grouped by money income before taxes, by occupation, by net-worth and by age of head of household. The only definite conclusion was that older families and retired persons suffer more than others the burden of inflation. Bach and Stephenson (1974) for the period 1950-71 also concluded that households were the major net creditors and that massive transfers of wealth went from households to the federal government and "more recently to business firms." Older people lost to young people, but they also identified a transfer of purchasing power from the very poor and the very rich to the middle and upper middle groups.

Brimmer (1971) corroborated the finding that the aged lost significantly with inflation in the U.S. in the period 1961-69, pointing out that they were the predominantly affected agents.

With a substantially different and innovative approach, Niehans (1962) found for the U.S. (in the moderate inflation of the period 1947/48 to 1957/58) that "the burden of inflation per household tends to increase with increasing income, with increasing age (except for the very youngest groups) and with increasing wealth." The novelty of his approach lies in the way he tried to remove the effects of inflation from the real effects. He defined the income of a group as the sum of wages, rents, profits, and income from interest and bonds. He then calculates by differentiation the percentage rate of change of income. This variable is then differentiated in relation to the rate of inflation and we are left with the change of income due only to the change in the inflation rate. The main problem with this approach is that it requires large amounts of (unavailable) data and his "tentative" conclusions can only be reached after several parameter values are postulated based on some a priori reasoning.

Rather similar results were obtained by Niida (1978) for Japan and Foster (1976b)and Piachaud (1978) for the U.K.: the younger and higher income groups tend to gain, and the elderly and low income groups to lose in wealth with inflation.

The type of studies (and conclusions) which were very influenced by the Bach and Ando's methodology, were severely criticized by Pesek (1960). In his own words:

> "The conclusion that everybody loses by inflation in proportion to his net money assets is not fully satisfactory because it rests on the implicit assumption that the alternative to the burden of inflation is no burden at all. Actually, the real alternative facing the public is whether to bear the cost of inflation or the cost of some economic policy designed to achieve equilibrium through other means: direct controls, or monetary policy, or fiscal policy, or some combination of all."

He then compares the distributional burden of inflation with the distributional burden of taxation, using data for the U.S. Three different types of taxes are considered, and as he himself concedes, some quite "heroic" assumptions had to be made, among them the implicit one that inflation is fully anticipated. Although the available statistical evidence was unsatisfactory he found that the three lowest income groups suffered more from inflation than they would do from the three alternative types of taxation. A movement from inflation to the three types of taxation would make the tax system more progressive. If this is true it means that a statement such as "the income distribution was not affected by inflation" would be misleading because with taxation as an alternative to inflation the income distribution would be more equalitarian. Of course this would only hold if there is in fact a tax-inflation tradeoff, which is doubtful. But the point made is important: the costs of inflation should be weighed against the costs of no or reduced inflation, e.g. against eventual higher unemployment costs.

Simulation Studies

There are three simulation studies of the impact of inflation on the distribution of income and wealth, all of them related to the U.S. economy. A main feature of their methodology is the investigation and measurement of the effects of inflation in the valuation and yields of the asset and on income and debt types, in order to arrive at individual adjustment coefficients to inflation. Some of these adjustment coefficients are the result of empirical investigation, whereas others are theoretical and "plausible" assumptions. In all these studies a very disaggregated data set on income, wealth and expenditure of individual households is utilized.

Once the adjustment coefficients are obtained it is a relatively easy task to observe the effects of inflation: households are ranked before inflation, by income, net worth, or some other variable; inflation is then simulated and the adjustment coefficients applied to all assets; the sum of all these effects is computed for all the individual cases, so that a final distribution is obtained. Comparison of the initial and final distributions will show the consequences of inflation.

Sometimes the simultation models are inserted in a more global macroeconomic model as in Nordhaus (1973), where, for instance, the inflation-unemployment tradeoff can be analysed.

Although the simulation models provide a very detailed, and at the same time global analysis, where all the effects of inflation on the assets are taken account of in the adjustment coefficients, the reliability of their conclusions is dependent on the reliability of the computed adjustment coefficients. As we have already mentioned, some of the procedures used to arrive at these values are far from being satisfactory. In addition, these adjustment coefficients, even if accurately calculated, represent the responses given by the economic agents to past inflation. Their reactions in the future may change, either because inflation becomes more (less) correctly anticipated or because the leads (lags) in the adjustment process may vary. Although these criticisms are common to every economic model (even to those allowing dynamic structural parameters) and are inherent to the unpredictability of the future, the problems here are relatively more acute as expectations and the reactions to them play a central role.

The first of the simulation studies is due to Budd and Seiders (1971) who used a file on the financial characteristics of consumers in the U.S. with data on income, asset holdings and liabilities for 1962. Their objective was to detect the effects of a change in the rate of inflation in the first (annual) period in which it occurs, rather than the long-term effects.

The rates of inflation simulated are 2 and 5% and they abstract possible implications on employment or output due to inflation. The distributions considered are always before tax so that indirect redistributional effects of inflation via taxation are excluded. From time series data for the past World War II period they calculated price adjustment coefficients for several net worth and income items. Some of them are hypothesized (the coefficients for claims and debts were set equal to zero, and for tangible assets were set equal to unity - implying the existence of perfect markets). Other coefficients are the result of empirical analysis, such as the coefficient of wage and salary income which was arrived at after testing the wage-lag hypothesis. Using regression analysis, they tested the existence of an inverse relationship between the rate of change of prices and the share of wages and salaries in total income, with real factors (real income and productivity) held constant. The hypothesis was rejected in separate tests for the total economy and different sectors even with the inclusion of varying lags. Consequently the wage price adjustment coefficient was set to unity. There is no need to go through all the assumptions made to obtain the adjustment coefficients, but it needs to be said that the results are very sensitive to the assumed coefficients.

The results reveal that the redistributive effects of inflation are very small for the two simulated inflations(2 and 5%). If in the definition of income, net wealth is included, inflation is progressive, in that it will tend to reduce inequality(the top income groups are the main losers.

Minarik (1979) updated this study and introduced some refinements: a better and larger sample data set for the year 1970 was used representing the entire population and exhausting national income, tax liabilities and the adjustment of transfer payments to inflation were modelled; two types of inflation were simulated (again at 2% or 5%)

and by assuming a uniform rate of change in all prices, or an inflation concentrated only on food and fuel, both the short and long run effects of inflation were investigated.

As in the study of Budd and Seiders, the major problem is to arrive at values for the adjustment coefficients, and again many doubts can be cast on the validity of some of the assumptions made. Curiously, the adjustment coefficient for wages is again set to unity, with the rejection of the wage-lag hypothesis. This is done by regressing the labour share in the corporate sector on the rate of change and change in the rate of change of the price level, with simultaneous inclusion of the "real" variables - unemployment rate and the ratio of actual to potential gives national product. As the coefficients of the price variables were not statistically significant Minarik concludes that the labour share is not influenced by changes in the price level and that wages can be assumed to be perfectly indexed to the price level. The data is quarterly and refers to the period 1948-74.

Setting aside some other assumptions (e.g. on the behaviour of long and short term interest rates), some interesting conclusions were reached. Firstly, that the income concept is important; with a narrow income concept, inflation was found to redistribute income to those at the top of the distribution, while a broader concept including the change in net worth revealed that the costs were concentrated at the extremes of the distribution, with no losses in middle incomes. Secondly, the losses of the low income households were always very modest, whatever the income concept.

The overall conclusion is that the redistributional effects of inflation are so negligible that they can be totally disregarded.

Although written earlier than Minarik's article, Nordhaus' (1973) simulation study is unique in its approach; it attempts to determine the effects of inflation not on the distribution of income and wealth (as generally considered), but on the distribution of lifetime incomes, or more precisely on the per capita lifetime consumption annuity. This is done by using the observed income and wealth distribution for 1962 (the same data set as Budd and Seiders). Given wealth and current labour income, future labour income and so total lifetime wealth can be predicted. By postulating individual utility functions the potential consumption path is derived. It should be noted that it is implicitly assumed that households will behave over time exactly like the observed households with the same characteristics.

As the effects of inflation on lifetime consumption are envisaged, only the long-run effects of inflation matter. Another interesting and innovative feature is that the redistributive effects are seen within the framework of three distinct models. As Nordhaus (1973) points out

> "the analysts of inflation do not often agree on the economic mechanism which generates and transmits inflation. For this reason it is crucial to distinguish the effects of inflation in different economic systems. In this study we analyse three different kinds of economies, which we call classical, neoclassical and Keynesian."

The classical system is defined as one in which there is a unique equilibrium for all real variables, whatever the level of prices or the rate of inflation. Inflation only alters the distribution of income and wealth according to the relative holdings of money-fixed assets. In the neoclassical model the level of capacity utilization is taken as constant and production and savings functions together with a mechanism for income (factors remunerated by their marginal productivity) are explicitly included. Inflation will then affect capital accumulation, real wages and real interest rates through the neoclassical production and distribution systems with consequent changes in income, wealth and consumption. The Keynesian system is considered as a neoclassical system with a very simple cyclical mechanism: over the cycle, inflation varies inversely with unemployment as in the Phillips curve.

Two types of inflation were simulated: a once and for all burst of inflation of 10% in the first year with inflation returning to the usual rate subsequently; and an increase in the rate of inflation of one percent for all future periods above the usual rate. Furthermore, the expected rate of inflation is thought to be determined by an adaptive mechanism. More specifically, the expected rate of inflation adjusts to the actual rate according to a geometric distributed lag pattern. In all cases the inequality comparisons between pre and post inflation distributions were made taking into consideration, one at a time, five different social welfare functions.

The results obtained for all the models were very similar: inflation was found to be progressive, the equalitarian effect being much more pronounced in the Keynesian case due to the employment effect associated with higher inflation. However inflation was also found to be an inefficient way of redistributing economic welfare, if compared with a progressive tax.

Many crucial and debatable assumptions and simplifications had to be made in order to accomplish this study. Although, as Nordhaus himself concedes, the technique used was quite complicated, sensitive to the assumptions, and made severe demands on the survey data employed, this study is very important in what it combines both the micro and macro approaches: the effects of inflation are modelled at the micro level (the household) which possess certain behaviour and certain reactions, and at the same time all those reactions are inserted in a macro model to assess the impact on other variables that may be crucial in assessing the costs of inflation (e.g. the unemployment rate). The type of macroeconomic model and its embodied assumptions are also important: if instead of the postulated Phillips curve Nordhaus had assumed a direct relationship between inflation and unemployment, his results might have been completely different.

1.2.3 Inflation and the Government. Anticipated Inflation

The government is one of the major gainers from inflation. It gains from inflation in three ways: it reduces the real value of government interest bearing debt, increases the real volume of tax payments and finally because even "anticipated" inflation levies a tax on the holders of non-interest bearing money stock.

Inflation reduces the real value of government interest bearing debt because interest rates do not keep pace with the inflation rates. The government, a debtor, has the gains that debtors in general obtain from creditors. However as inflation tends to persist and to become more anticipated there will be pressure on the government to raise the interest rates paid on new borrowings, otherwise new issues of government securities will not be subscribed in any substantial proportion. Government resistance to raise interest rates may lead it to shift its borrowing away from the public towards the central bank, with an inevitable expansion of the money supply, and additional pressure on inflation. Higher rates of inflation will mean higher interest rates, so that, as "monetarists"tend to emphasize it is very difficult, if not impossible to keep interest rates down at low levels during inflation.

Another important gain to the government coming out of the inflationary process is related to the tax system. The tax system was set up for a non-inflationary world. The regulations relating to nominal rather than real variables. For instance, as income tax is concerned, allowable deductions are defined in nominal terms and the marginal tax rates are progressive with respect to nominal rather than real income; tax on capital gains relates to nominal rather than real gains; appreciation of inventories due to inflation are often considered as an additional profit liable to taxation, etc. Government gains in all these cases because, with higher price levels the real burden of taxation is increased even though the real base upon which the tax is levied is the same.

For empirical studies, particular attention has been given to the income tax and to what is called "fiscal drag": with inflation taxpayers will tend to "jump" to heavier taxed nominal income brackets and more people will begin to pay taxes. Although governments have been making some corrections (e.g. by raising allowances and setting up new income brackets), they have by no means offset the inflationary effects. Although it is always impossible to predict the reactions and modifications to the tax system governments would do in the absence of inflation, studies for Britain by Allen and Savage (1975) and the Royal Commission on the Distribution of Income and Wealth (1976) have shown that the heaviest burden has fallen on taxpayers at the lower and upper end of the scale. Trinder (1975), also for Britain, concluded that the largest increases in average tax rates have occurred at relatively low income levels.

The third way government gains from inflation is related to the tax levied by "anticipated" inflation on holders of non-interest bearing money. A large and disproportionate attention has been devoted to this redistributive effect of inflation, mainly because it is a by-product of the modern quantity theory of money in its study of the demand for money and the impact of monetary policy. Anticipated inflation is assumed to isolate the losses caused by inflation on holders of high powered money (the only redistributive effect in the world). In this mythical world, a given and unchanged rate of inflation is expected by everyone, "because say, of the announcement by the government of the rate at which it expects to increase the money supply"[1]; everyone acts in conformity to those expectations, that is, contracts are rewritten in such a way that their real values remain constant; nominal interest rates are adjusted so that real interest rates are kept constant; salaries, wages and pensions are regularly adjusted in accordance to cost-of-living indices; bank deposits are not used as money or are negligible in amount. In this world, the effect of inflation on the private economy "would not be any different from that of a direct tax on the holding of cash balances."[1]

Given an anticipated inflation rate there is a given demand for real balances by the private sector, so that in order to maintain their real balances constant, the nominal balances must increase at the rate of inflation. The government will obtain real resources in exchange for the issue of additional nominal balances.[2] The value of the resources is equal to the product of the inflation rate and the

1. Bailey (1956).

2. This analysis assumes zero real growth and zero nominal interest rate in the absence of monetary expansion.

stock of real high powered money balances held. This is the way of getting finance through inflationary means. Also the expansion of the money supply has important social welfare costs: inflation induces a substitution of real resources for money services and so additional time is spent on increased frequency of purchases, on barter transactions, increased frequency of payments, additional administrative and accounting costs, the need to rescale price lists, etc.

Bailey (1956) studied these social welfare costs, by using the estimates obtained by Cagan (1956) for the parameters of the demand for money functions in seven post world war hyperinflations, and Marshallian consumer surplus techniques. By comparing the revenues with the social welfare costs, he concluded that an inflationary tax on cash balances was inefficient.

If it is assumed that the inflationary finance is employed by the government to finance development programs the conclusions might be quite different. Mundell (1965) and Marty (1967) studied this possibility by using models with perfect anticipated inflation but again the conclusion was that the extra growth generated by the inflationary finance was too small in relation to the extra costs involved.

1.3. DIFFERENTIAL INFLATION[1]

All the previous discussion on the effects of inflation was based on the assumption that households face the same rate of inflation in a given period, represented either by the Consumer Price Index (CPI), the Retail Price Index (RPI) or even by the GNP deflator.[2]

1. As the present thesis belongs to this type of studies,only a brief discussion of the methodology and results already obtained in different countries will be presented here,as all these matters will be further elaborated in the subsequent chapters.

2. The GNP deflator is rarely used in studies of the consequences of inflation,so that the following discussion will be based on the CPI or RPI concepts.

The CPI (or RPI), published in most countries by a government agency usually on a monthly basis, is an index of a weighted average of price rises, the weights being the proportion of total consumption expenditure spent on the different consumption items by what are called the "index" households.

The index households and their weights are drawn from one or more Household Expenditure Surveys (HES), whose main finality is precisely to provide the basic information to the construction of a price index. The type of households chosen as "index" households may and have differed from country to country or from time to time within the same country: urban wage-earners households, households within certain income brackets (usually with the exclusion of the highest income households), all households except pensioners, etc. The criteria for that choice vary widely, although it always prevail the intention to pick up the most representative households and households that on an a priori basis are thought to suffer most with inflation. Having selected the "index households", the weight for a certain item, say cereals, is arrived at by adding up the expenditures on cereals made by each household, by calculating the sum of the total expenditures made by all the households on all the items and then dividing the former quantity by the latter. Of course, the sum of all weights must be one.

The prices considered in the CPI are those for the items most often consumed by the "index households" and must be very detailed described, either by their physical qualities, by the brands commonly bought, or by some other identifying information.

The resultant index is deemed to be representative of the inflation rate in a country, so that all households are assumed to experience the same rate of price increases. Since the CPI is an average of prices and an average of weights it is recognized that individual cases (households) will have different price indices, as is true in all averages. Nevertheless, those deviations from the mean are assumed to be randomly distributed, so that no household classification will show up in any systematic and statistically significant manner.

In other words, unless we are prepared to consider each individual case, the CPI will exhaust all the relevant information on the price indices. This idea has sometimes been challenged in a surprisingly thin literature on the subject of differential price indices or differential inflations.

With the usual methodology employed in the calculation of the CPI, the price index for different households may differ for three main reasons: firstly, because households allocate differently their total expenditures to distinct consumption items (different weights); secondly because households pay a different price for the same item; thirdly because, even assuming that the same prices are paid and initial equality of weights, price increases may lead to unequal substitution effects, with different quantities being bought of some items, implying different final weights.

The literature on differential inflation has mainly investigated the existence of distinct price indices due to initial different weights, although the evolution of weights in time is sometimes taken into account by the examination of successive surveys, or by employing the estimated parameters of complete systems of demand equations. No allowance is given to the possibility that households may pay different prices for the same item, although for instance, Caplovitz (1963) for the U.S. and Piachaud (1974) for Britain provided evidence that for a given basket of goods the better off paid a lower price than the lower income groups.[1]

1. However, what is important as concerns differential inflations is to know whether the eventual higher prices paid by the lower income groups rise faster than the lower prices paid by the well off. Only the rates of increase, not the price levels really matter.

Most of the studies on differential inflations have been carried out for the U.S. and the U.K. economies, and it should be said at the outset that they provide conflicting evidence: while in the U.K. it has generally been found substantial differences among household class price indices, no such conclusion can be drawn for the U.S.

In the U.K. the first major study was done by Allen (1958). He investigated the expenditure weights of pensioner families of one or two persons, of manual and clerical employees' families and of high income families, using the 1953-54 HES and distinguishing twenty-four sub-groups of expenditure. With the 1949-57 prices he concluded that the price index for one-person pensioner families rose slightly faster than for the others but he could state that "the all-items index numbers for various consumer groups are found to be very little different". However Lynes (1962) based on the same survey, but considering three income groups near the national assistance level and the 1948-61 prices discovered that the official price index substantially understated the price index of those groups. This fact may have led the Department of Employment to introduce separate quarterly indices for one- and two-person pensioner households in 1968, with indices calculated since 1962. These pensioner indices have in general shown a slightly faster rise in prices than the all households index.

Tipping (1970) ranked the households by income magnitude and computed the correspondent expenditure weights for ten groups of expenditure as being an average of the weights drawn from the surveys carried out in the years 1959, 1962, 1963 and 1965. The analysis was for the period 1956-66, and the conclusion was that price indices decreased as we went up the income scale, namely prices for the 5th percentile had risen by 6% more than for the 95th percentile.

Piachaud (1978) using a far greater disaggregation (93 groups of expenditure), and for the same period (1956-66) estimated that difference as being 10.9 percentage points. This may indicate the importance of disaggregation in the final results, although the method employed by Piachaud was not strictly identical to that of Tipping (he allowed weights to vary - in 1956, 1961 and 1966).

Piachaud extended the analysis until 1974, and over the whole period (1956-74) prices rose 30.9 percentage points more for the 5th percentile (low incomes) than for the 95th percentile (high incomes).

Muellbauer (1974) using cost-of-living indices for adult equivalent units instead of price indices (allowing so for price substitution effects and family composition), and within the framework of the Linear Expenditure System corroborated all the previous conclusions: in the period 1964-72 there had been an inegalitarian trend in prices. Muellbrauer (1976) updated this study for the years 1975 and 1976 and found the inegalitarian trend to be even more pronounced.

Muellbauer (1974c)has also shown that the RPI in the U.K. represented more or less accurately the expenditure pattern of households (corrected for size) situated around the 70th percentile. The reason for this is that, since the weights are calculated by adding up the expenditures of all households, the final values will be more influenced by the higher expenditures made, or what is the same, by the higher income classes. The weights used in the RPI will be closer to the weights of the higher income classes - in the U.K. case, those situated at the 70th percentile.

Using the weights taken from the 1977 HES, and prices for 40 expenditure groups during the period 1974-80, Smith (1980) investigated the price indices for four family types (families with no, or with one, two or three children) and for different income classes within each family type. An econometric analysis was also carried out by regressing the price indices on income (or total expenditure corrected by family size) and dummy variables for the number of children. He again found an inverse relation between inflation and income (for each £100 of additional expenditure,- 3.7% in the rate of inflation), and a direct relationship between the number of children and the additional rise in the price index.

There is such overwhelming support in the U.K. as to the existence of well marked differential rates of inflation, and not only by income classes (as Smith has shown). Unfortunately these differential rates are always in the wrong direction: those with low incomes and/ or with larger families face the higher price rises.

For the U.S. the picture is completely different. Hollister and Palmer (1972) in a study on the impact of inflation on the poor examined the effects of inflation on their expenditure (differential inflation), income and wealth, in an attempt to exhaust all possible effects. As concerns the differential inflation, they used fixed weights drawn from the 1960/61 HES and for several categories of consumers (the aged poor, the urban nonaged poor, all poor, all wealthy and a middle income group). Constructing price indices for all those categories for the period 1953-67, they could conclude:

> "the expenditure effects of the type of inflation we have experienced since World War II, in general, have not been adverse for the poor. Particularly in the 1960's the expenditure effects of rising price levels have fallen somewhat less heavily on the poor than on other income groups."

A slightly different conclusion was reached when Palmer and Barth (1974) updated that study for the years 1967-74; the ratio of the rise in the poor person's price index to the rise in the rich person's price index was 1.03 from 1967 to 1972 and 1.09 from 1972 to 1974.

But the main study on the "variation across households in the rate of inflation" for the U.S. was done by Michael (1979). He also used the 1960-61 Survey but with the difference that he calculated price indices for each of the 11000 households considered, a unique feature among the published literature. He considered several time periods varying in length from a five-year period to a one-month period, all of them included between the reference years 1967 and 1974. He found out a considerable dispersion among households in the computed price indices, but

> "When the households are combined into relatively homogeneous groups defined by income, education, age, city size, marital status, race, etc., the within-group dispersion in price indices is still very substantial. The dispersion within groups tends to dominate the differences in group means."

Some significant relationships between price indices and certain household characteristics, valid for specific periods were not stable over time. The main implication of these findings is that no gain in information would exist in the substitution of group specific price indices for the published consumer Price Index.

Conclusions

Ideally the consequences of inflation should be appreciated by taking into account simultaneously its microeconomic effects (on expenditure, on income and on wealth), and macroeconomic ones (e.g. on unemployment and growth).

Due to the extreme complexity of that task, no single study has ever accomplished it, some simulation models being the closest to that desideratum.

Of these partial studies, most of the evidence relates to the U.S. or U.K. economies, and some of the contradictory results obtained are a reminder that each country's experience should be carefully studied. The reason is that inflationary effects may differ with the type of inflation observed or with the country's institutional background.

This is not in contradition with my personal conviction that top priority should be given to the theoretical research of the inflationary effects, for instance by trying to incorporate them in current descriptive and planning models of economic activity.

But some qualified conclusions can be drawn from the empirical work already carried out:

1 - Mild rates of inflation do not necessarily provide a stimulant environment to growth. That is not to say that growth will not generate some degree of inflation.

2 - If there is an inflation-unemployment trade off, some of the harmful aspects of inflation will be considerably mitigated or overcome by the increased employment in the economy.

"Acceptable"inflation rates will then be dependent on the political, sociological and humanitarian views prevalent in the society in question.

3 - The inflation rates experienced by households may differ considerably (differential inflation) and be in the "wrong" direction, that is, low income and larger families may face higher price indices than high income and small families.

4 - Setting aside the previous case, the effects of inflation on income and wealth may also lead to the conclusion that low income and old households are the main losers. An enlarged income definition, including net worth may also show high income households as major losers due to the depreciative effects of inflation on the assets held.[1]

5 - The most established inflationary effect is that debtors gain from creditors, or what is the same, interest rates do not keep pace with inflation rates.
The government, as a major debtor is a major gainer and households as major creditors are major losers.
It is doubtful that firms are in general net debtors.

6 - Government also gains through "fiscal drag" and through the holdings by the private sector of non-interest bearing money, so that it may be considered as a primary beneficiary of inflation (at least in financial terms).

7 - It is not clear that for the economy as a whole wages lag behind prices, although there may be and certainly are lags or leads in specific sectors of activity or specific professions.

8 - The identification of the groups or classes of individuals who bear the major burden of inflation should always be one of the major priorities of studies on the consequences of inflation.

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1. It should be noted, however that this conclusion is reached after consideration of these effects on a given and fixed portfolio. There is some evidence (see Tait (1967)) that high income households are more successful than low income ones in reacting to inflation by changing the composition of their portfolios in order to make them more inflation proof.

Chapter 2

The Price Index

The theory of index numbers is based not only on statistical techniques but also on economic theory. Although the construction of index numbers can be considered in abstract, applications for them are found in many disparate fields of science. However, most of the theoretical and practical developments were due to the necessity of calculating a price index for the whole economy. More precisely, the index number theory owes a great deal to the efforts of early economists in trying to determine the value or purchasing power of money, or what is equivalent, the general price level (one being the reciprocal of the other).

The index number problem arises because we want a measure for a variable that has no physical existence (an abstraction) and for which no common physical unit exists. For instance, the price level can not be directly measured, only the prices of the thousands of goods and services transactioned in an economy. There is no single physical unit common to all the commodities: some are expressed in kilos, others in litres, others in tonnes or even in individual units. This means that it is impossible to measure "the" price level in the economy: should we take the prices of the litre or of the pint, of the ton or of the kilo? In spite of this, the observations of all the prices in the economy may lead us to form the idea of a general level of prices, and economic theories make abundant use of this abstraction.

Index numbers do not intend to measure the level of the abstracted variables (it is impossible), but only their changes:

in the case of the price index it is measured by the change in the price level from one period (base period) to a final period.[1] By considering changes and not levels one of the basic indeterminacies is removed, provided we take the prices of the same quantities of the goods in both periods.

If there are N commodities and the reference periods are designated as o (the base period) and 1 (the final period), we can obtain N price ratios or price relatives p_1/p_o which are manifestations of the variations (increases or decreases) in the price level.

But being in possession of the N price relatives does not mean the end of all the problems. How can the change in the overall price level be calculated from them? By using averages (arithmetic, harmonic, geometric) or by estimating the median or the mode? Should we weight the price relatives with the quantities transactioned, and if so with the base or the final period quantities? Should they be referred to the economy as a whole or to some specified group or groups? Should we consider the prices and quantities in the two periods as two sets of independent variables or are there some characteristic relations between them? Finally, we should always bear in mind that we probably have available only a sample n of all the N prices in the economy.

Frisch (1936) pointed out that there are fundamentally two different ways of approaching the problem of price index numbers: one which he called the atomistic approach, where the prices $(p_1,\ldots,p_n)$ and quantities $(q_1,\ldots,q_n)$ of the goods in the two periods are taken as two sets of independent variables, and where the objective

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1. Without loss of generality, we shall consider comparisons between periods (the most frequent) and not for instance comparisons between regions.

is to define a certain function of the 2n variables which will give the best representation of the price level changes; the other he called the functional approach and where certain characteristic relations are assumed to exist between prices and quantities. In this functional approach the demand and utility theories play a crucial role and it is sometimes called the preference field approach[1] or the economic theory of index numbers.[2]

2.1 The Atomistic or Statistic Approach

The notion that an increase in the money supply must lead to a proportionate increase in the price level and that this increase "ought" to be manifested in a proportional change of all prices was central in some of the earlier works on the price level.[3] According to this hypothesis, the deviations from that proportionate change observed in individual prices are due to non-monetary factors and can be regarded as errors of observation. This implies that any of the price relatives p_{1i}/p_{oi} $(i = 1,\dots,n)$ is, in a first approximation, just as good an estimate of the change in the price level as any other ratio. If so, there is a strong case for taking the simple average of the price relatives as an estimate of the price level change. There would be no special need, for instance, in using weighted averages with the quantities as weights, unless it is thought that some of the price relatives are less reliable than others. This type of

1. Wold (1953).
2. Samuelson and Swamy (1974).
3. See Bowley (1926) and Edgeworth (1925).

analysis leads almost inevitably to the study of the statistical distribution of the individual price ratios, namely to tests of its normality. Detection of skewed distributions (such as the lognormal distribution) may indicate that different types of averages should be preferred, such as the geometric mean. This stochastic approach, mainly adopted by Edgeworth and Bowley is untenable in theoretical terms, although it is an ingenious way of eliminating the indeterminacy problem which the atomistic approach must inevitably face. In fact, even if the price level rises due only to monetary factors, such as an increase in the money supply, the end result is not a proportional increase in all prices: the additional money balances will originate increased demands for some goods and very small or negligible ones for other goods.

It is curious that Keynes adopted the stochastic approach in some of his earlier studies,[1] but later rejected it[2] even in the case where the main objective is to measure changes in the value of money. He argued that the purchasing power of money is the power of money to buy goods and services. Individuals buy goods and services to satisfy their consumption needs so that the appropriate index number is the consumption index. It follows that the purchasing power or the consumption index have no clear meaning unless they are referred to a particular and well defined set of individuals or groups and that they will probably be different for different groups.

The idea that we should construct differential price indices for different groups because only then will we get meaningful results is emphasized by almost all the authors writing on index numbers, but unfortunately it is seldom followed in practice.

1. Keynes (1921).
2. Keynes (1930).

Let us suppose then that we want to calculate price index numbers for a certain group of people. What formula or formulae should we use? Is there a best formula representing with exactitude changes in the price levels?

Let us take the n price ratios p_1/p_o of the n items consumed by that group and calculate the arithmetic, harmonic or geometric means, their median or mode, whether or not weighted by the base or final year quantities consumed (or a combination of the two); and if to this already vast number of formulae we add some new formulae that are the result of the 'crossing'[1] of the previous ones, we can imagine that there is an infinite variety of algebraic expressions as possible candidates to measuring changes in price levels. This was the methodology followed by Fisher who, in his classical book "The Making of Index Numbers" carefully studied and catalogued 134 of such formulae.

Before discussing Fisher's method for discerning the best or "ideal" formula from the multitude of possible candidates we must present the most common expressions for index numbers used nowadays, both in theoretical and in practical applications. The reasons for this popularity will become apparent as we proceed.

Let us again consider two periods, the base period (o) and the final period (1) and suppose that the prices (p_o and p_1) and the quantities (q_o and q_1) of the n goods bought by a well defined class of people are known. That is, we know the vectors

$$p_r = (p_{r1}, \ldots, p_{ri}, \ldots, p_{rn}) \qquad \text{with } r = o,1$$

$$q_r = (q_{r1}, \ldots, q_{ri}, \ldots, q_{rn}) \qquad i = 2, \ldots, n$$

1. See Fis her (1927). 'Crossing' formulae is to average antitheses. The antithesis of a given formula is found by interchanging the two reference times, (o,1) or the two factors (p, q) and by dividing the resulting expression into unity.

To simplify the notation, let the symbol p.q denote $\sum_i p_i . q_i$, that is, the sum of the expenditures made on each item (total expenditure) in a given period. So $p_o . q_o$ will represent total expenditure made on the bundle of goods consumed in period o, valued at prices of the same period; $p_1 . q_o$ the total expenditure on the same bundle of goods, but valued at period 1 prices.

The Laspeyres index is given by the formula:

$$L_{o1} = \frac{p_1 . q_o}{p_o . q_o}$$

The prices in both periods are weighted by the quantities purchased in the base period. Later on it will be shown that this formula is equivalent to a weighted average of price relatives, the weights being the base period expenditure proportions devoted to each item. The Laspeyres index has an immediate economic interpretation: it is the ratio of the total expenditure necessary to buy a fixed bundle of goods in two periods. It is important to note that this fixed bundle is not any bundle, but it consists of the items and quantities purchased in the base period (o) by the reference people.

The index may be said to represent the ratio of the cost of living in period 1 relative to the cost of living in period o.[1]

If the quantities purchased in period 1 are deemed to be more representative and are used as weights, the relevant expression is called the Paasche price index:

$$P_{o1} = \frac{p_1 . q_1}{p_o . q_1}$$

1. With some important qualifications, as discussed in the next section.

There is always the problem of choosing between period o or period 1 quantities, so that sometimes a compromise is proposed where both quantities are considered. It is easy to think of an index where prices are weighted by the sum of the two quantities, such as in the formula

$$\frac{p_1 \cdot (q_o + q_1)}{p_o \cdot (q_o + q_1)}$$

which is called the Marshall-Edgeworth formula. Indices can also be constructed using quantity weights that are the arithmetic, geometric or harmonic means of the quantities in both periods.

Another possibility is to take simultaneously into account the Laspeyres and Paasche indices, e.g. by averaging them. The geometric average of the Laspeyres and Paasche indices was called by Fisher the "ideal index"[1]:

$$F_{o1} = (L_{o1} \cdot P_{o1})^{\frac{1}{2}} = \left(\frac{p_1 \cdot q_o}{p_o \cdot q_o} \cdot \frac{p_1 \cdot q_1}{p_o \cdot q_1} \right)^{\frac{1}{2}}$$

Instead of trying to obtain a compromise between the quantities purchased in both periods we could use fixed weights q_a calculated say, as an average of the quantities consumed over a number of years and to suppose them constant through time. The expression would then be

$$\frac{p_1 \cdot q_a}{p_o \cdot q_a}$$

It is important to realize that this formula is neither a Laspeyres nor a Paasche index, as the quantities q_a do not relate to any of the two periods (o or 1).

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1. For reasons to be explained later.

A price index gives a comparison between two and only two time periods, as Fisher continually stressed. What if we want to compare periods that are very distant in time? In this case it is very likely that the consumption patterns will be different even with the possibility that some of the goods purchased in one period be not available in the other. The problem of what quantities to choose as weights becomes even more critical. The best theoretical solution is to construct a chain index.

The idea is to divide the long period into shorter successive periods and to have price indices for each of them. A chain of comparisons is obtained, each between successive periods and of course each time on a changed base. The comparison between the initial and final periods will be made by taking into account not only the information related to those two points in time but also all the information available for the intermediate points.

If I_{ik} is an index where the period k is compared with a base period i, then the series

$$I_{o1}, I_{12}, I_{23}, \ldots, I_{k-1,k}$$

constitutes a chain of indices. Each of these indices is a link in the chain, the period of comparison in one index being the base of the succeeding one.

The chain index between period o and period k $(k > o)$, is given by

$$C_{ok} = I_{o1} \cdot I_{12} \cdot I_{23} \cdots I_{k-1,k} \qquad (1)$$

The generalised definition of a chain index is[1]

$$C_{st} = \frac{I_{o1} \cdot I_{12} \cdot \cdots I_{t-1,t}}{I_{o1} \cdot I_{12} \cdot \cdots I_{s-1,s}} \qquad (2)$$

1. See Frisch (1936)

Where the period t is compared with the period s. This expression is valid for s greater or less than t.

It is easy to show that

$$C_{rs} \cdot C_{st} = C_{rt}$$

or $$C_{rs} = \frac{1}{C_{sr}}$$

which implies that

$$C_{rr} = C_{ss} = 1$$

and that when s and t are consecutive periods the chain index C_{st} is equal to the elementary index I_{st}.

It should be noticed that no restriction was imposed on the individual indices I_{st} of a chain index. It is interesting to compare the chain index with the fixed base indices, e.g. with the fixed-weight formula and with the Laspeyres index.

Let us suppose we want to compare period o (the base) with period t. By using the fixed-base method, with the fixed weight formula, the expression for the price index is

$$\frac{p_t \cdot q_a}{p_o \cdot q_a}$$

The same price index obtained with the chain method is

$$C_{ot} = I_{o1} \cdot I_{12} \cdot I_{23} \cdot \ldots \cdot I_{t-1,t}$$

that is

$$C_{ot} = \frac{p_1 \cdot q_a}{p_o \cdot q_a} \cdot \frac{p_2 \cdot q_a}{p_1 \cdot q_a} \cdot \frac{p_3 \cdot q_a}{p_2 \cdot q_a} \cdots \frac{p_t \cdot q_a}{p_{t-1} \cdot q_a} = \frac{p_t \cdot q_a}{p_o \cdot q_o}$$

Which means that, with the fixed-weight formula, the fixed-base method and the chain method give identical results.

The same does not happen with the Laspeyres formula:

$$L_{ot} = \frac{p_t \cdot q_o}{p_o \cdot q_o}$$

$$C_{ot} = \frac{p_1 \cdot q_o}{p_o \cdot q_o} \cdot \frac{p_2 \cdot q_1}{p_1 \cdot q_1} \cdots \frac{p_t \cdot q_{t-1}}{p_{t-1} \cdot q_{t-1}} \neq \frac{p_t \cdot q_o}{p_o \cdot q_o}$$

In this case the fixed-base and the chain methods provide different results. As already stated, the chain method should be preferred on theoretical grounds, apart from the practical problems like the eventual unavailability of data or the difficulty of computation.

A theoretical and logical justification for the chain index was given by Divisia [1]. His main objective was to obtain a monetary index, expressing the purchasing power of money.

Let us suppose that we can split the total transactions value Σpq into a product of two factors,

$$PQ = \Sigma pq \tag{3}$$

Where P represents the general price level and Q the total physical volume. The problem now is to obtain expressions for the general price level and for the total quantity, only as a function of the individual prices and quantities. To do so, Divisia considers the historical evolution of these prices and quantities. As the change in total value depends both on the price and quantity changes, the above equation can be written in differential form as,

$$PdQ = QdP = \Sigma(pdq + qdp) \tag{4}$$

Dividing (4) by (3), we get

$$\frac{dQ}{Q} + \frac{dP}{P} = \frac{\Sigma pdq}{\Sigma pq} + \frac{\Sigma qdp}{\Sigma pq} \tag{5}$$

1. Divisia (1925). His work was further elaborated by Roy (1927).

Separating (5) into two differentials, the differential in price and the differential in quantity,

$$\frac{dP}{P} = \frac{\Sigma q dp}{\Sigma pq} \quad , \quad \frac{dQ}{Q} = \frac{\Sigma p dq}{\Sigma pq} \qquad (6)$$

Which may also be written as

$$d \log P = \Sigma \alpha \, d \log p \qquad (7)$$

and $d \log Q = \Sigma \alpha \, d \log q$ where $\alpha = \frac{pq}{\Sigma pq}$

In either (6) or (7) we have a differential definition for the price index and a similar one for the quantity index.

If we integrate numerically the expressions for the price index we are led to the expression for the chain index given by (1) or more generally by (2). We may get the Laspeyres, Paasche, or indeed nearly any other formula, according to the approximation principle for the steps of the numerical integration.

The Divisia price index is of course a continuous price index, whose exact formula may be deduced as follows:

$$d \log P = \frac{\Sigma q dp}{\Sigma pq}$$

and since both components are a function of time, let

$$\phi(t) = \frac{\Sigma q(t) dp(t)}{\Sigma p(t) q(t)} \quad \text{and}$$

$$f(t) = \int_0^t \phi(t) dt$$

We can now write

$$d \log P(t) = \phi(t)$$

so that

$$\log P(t) - \log P_0 = \int_0^t \phi(t) \, dt = f(t)$$

and finally

$$P(t) = P_o e^{f(t)} \quad \text{with } P_o = 100 \text{ in base year o}$$

which gives the expression for the Divisia Integral index.

Fisher's tests

As mentioned before, the statistical or atomistic approach to the price index problem leads to the existence of an infinite possible variety of formulae and to the necessity of finding criteria for selecting the most appropriate ones. This is the origin of Fisher's tests.[1]

The basic idea underlying some of the tests is simple: if a price index is a ratio of two price levels, then it ought to possess the same properties held by that ratio. Let r, s, t be any three periods and P_r, P_s and P_t the price levels in each of them. Let also I_{rs} be the price index between the periods r and s where the period r is taken as the base. Then $I_{rs} = P_s/P_r$, $I_{st} = P_t/P_s$, $I_{rt} = P_t/P_r$, so that

$$I_{rr} = 1$$

$$I_{rs} = \frac{1}{I_{sr}}$$

$$I_{rs} . I_{st} = I_{rt}$$

The above expressions are a formalized way of stating simple and intuitive reasonings: if, by using a certain formula we discover that the price level in period s is double the price level in period r, when r is the base, then we should conclude that the price level in period r is half the price level in period s if s is now the base and the same formula is used; also, if the price level in s is twice the one in r (with r the base), and twice in t when s is the base, then the direct comparison between r and t should lead to the conclusion that the price level in t is four times the price level in r.

1. See Fisher, op.cit.

The Fisher's tests for index number formulae are then the following:

<u>The Identity Test:</u> $I_{rr} = 1$. The price index for a period compared with the same period should be one. It is easily shown that all the indices previously referred to satisfy this test.

<u>The Time (Point) Reversal Test:</u> $I_{rs} \cdot I_{sr} = 1$. The formula should work forward and backward in time, so that reciprocal results are obtained in each direction. Fulfillment of this test implies the fulfillment of the Identity Test:

If $I_{rs} \cdot I_{sr} = 1$, then $I_{rr} \cdot I_{rr} = 1$ so that $I_{rr} = 1$

The Laspeyres formula does not satisfy this test, as

$$\frac{p_s \cdot q_r}{p_r \cdot q_r} \cdot \frac{p_r \cdot q_s}{p_s \cdot q_s} \neq 1$$

The same happens with the Paasche formula:

$$\frac{p_s \cdot q_s}{p_r \cdot q_s} \cdot \frac{p_r \cdot q_r}{p_s \cdot q_r} \neq 1$$

Fisher's formula, however satisfies the test:

$$\left(\frac{p_s \cdot q_r}{p_r \cdot q_r} \cdot \frac{p_s \cdot q_s}{p_r \cdot q_s} \right)^{\frac{1}{2}} \cdot \left(\frac{p_r \cdot q_s}{p_s \cdot q_s} \cdot \frac{p_r \cdot q_r}{p_s \cdot q_r} \right)^{\frac{1}{2}} = 1$$

as does the chain index.

If a formula does not pass the time reversal test an "error" is made and it can be measured by the expression $I_{rs} \cdot I_{sr} - 1$.

<u>The Factor Reversal Test</u>: $P_{rs} \cdot Q_{rs} = V_{rs}$ (Where P, Q and V are the price, quantity and value indices, respectively).

It was clearly evident in the deduction of the Divisia index that for each price index there is a matching quantity index. The test requires that the product of the price and quantity indices be exactly equal to the Value index, that is, equal to the ratio of the total expenditures in both periods valued at each period's prices.

It is easily demonstrated that both the Laspeyres and Paasche formulae do not satisfy this test, but that Fisher's formula does. In this latter case

$$P_{rs} = \left(\frac{p_s \cdot q_r}{p_r \cdot q_r} \cdot \frac{p_s \cdot q_s}{p_r \cdot q_s} \right)^{\frac{1}{2}}$$

$$Q_{rs} = \left(\frac{q_s \cdot p_r}{q_r \cdot p_r} \cdot \frac{q_s \cdot p_s}{q_r \cdot p_s} \right)^{\frac{1}{2}}$$

so that

$$V_{rs} = P_{rs} \cdot Q_{rs} = \frac{p_s \cdot q_s}{p_r \cdot q_r}$$

<u>The Circular Test</u>: $I_{rs} \cdot I_{st} = I_{rt}$. Fulfillment of this test implies the fulfillment of the time reversal and of the identity tests:

If $t = s$,

$$I_{rs} \cdot I_{ss} = I_{rs}$$

or $I_{ss} = 1$ (identity test).

If $t = r$

$$I_{rs} \cdot I_{sr} = I_{rr} = 1$$ (time reversal test).

The circularity of the test can be appreciated when the expression

$$I_{rs} \cdot I_{st} = I_{rt}$$

is substituted by the equivalent one

$$I_{rs} \cdot I_{st} \cdot I_{tr} = 1$$

The circular test is not satisfied by the Laspeyres, Paasche or Fisher formulae, but is satisfied by the fixed weight formula or by the chain index.

Some other tests are often suggested, such as

The Base Test: The ratio I_{tr}/I_{sr} should be independent of the base r.

The Commensurability Test: The unit of measurement should not affect the index number.

The Determinateness Test: The index number should not become zero, infinite or indeterminate if an individual price or quantity becomes zero.

The Proportionality Test: If all prices have changed in the same proportion from period r to period s, the price index should be equal to the common factor of proportionality.

Eichhorn(1976) and Swamy (1965)[1] have proved that Fisher's tests are inconsistent, in the sense that no single formula can satisfy all the tests at the same time. However, it should be said that Fisher (1927) utilizes mainly the time and factor reversal tests to select the best formula. In fact his "ideal" formula was chosen because it satisfied those two tests and was simpler in comparison to some other (few) competitors. Namely, he rejected the circular

1. Swamy also demonstrated that earlier proofs by Frisch and Wald were incorrect.

test on the grounds that it can only be fulfilled if the weights are constant in all periods (or places), a kind of imposition that is economically absurd.

2.2. The Functional Approach, or the Economic Theory of Index Numbers

The statistical approach cannot lead to a unique price index formula liable to represent the "true" rise in the price level between two periods. The tests set up for judging the formulae, although based on some intuitive notions of the price level properties may not be the most adequate ones (e.g. Fisher's criticism of the circular test), they are inconsistent, and even if a consistent subset is taken up more than one formula will satisfy them.

In fact, it is always possible to construct new price index expressions possessing some interesting empirical and statistical properties, the problem being that they are not well founded in economic theory. On the contrary, economic theory should not only provide the framework for criticising the existing statistical indices but also set up the guidelines for constructing new and more adequate ones. These are the objectives of the "economic theory of index numbers". In this approach the prices and quantities of the goods purchased are not thought to be independent, but linked by the behaviour of rational consumers. While in the stochastic approach individual prices ought to change in the same proportion so that a meaningful index number might be obtained, in the functional approach it is precisely those deviations from proportionality that are important. They reflect and allow the detection of eventual systematic relationships that shed some light on the behaviour and preferences of the consumers. Demand and utility theories must enter into the picture, and once more the analysis will only have any sense for well defined and homogeneous classes of consumers.

If the preference order or utility functions are known or assumed (with the proviso that they are not incompatible with the data) the index problem turns out to be the construction of a cost-of-living index.[1]

The cost of living index measures the relative costs of reaching a certain level of utility (standard of living) under two different situations. One way of getting the same level of utility is to buy a fixed bundle of goods, say q_a, so that the cost-of-living index between period o and period 1 is given by the already familiar expression

$$\frac{p_1 \cdot q_a}{p_o \cdot q_a} \qquad (8)$$

However this is a very restrictive expression for the cost-of-living index (CLI) because there are generally several bundles of goods that provide the same level of utility (i.e. lie on the same indifference curve). A more general definition for the true CLI is then the ratio of the minimum costs necessary to reach a specific indifference curve (utility level or standard of living) when the prices are different in the two periods:

$$CLI = \frac{C(u_a, p_1)}{C(u_a \cdot p_o)}$$

Where $C(u,p)$ is a cost function, that is, it represents the minimum cost of attaining utility level u at prices p (price vector). It should then be borne in mind that the cost-of-living index depends not only on the two periods price vectors, p_o and p_1, but also on the utility level u_a.

1. This is the case where a complete demand system is estimated such as the Linear Expenditure system. Chapter 4 constructs cost-of-living indices for Portugal from the estimated parameters of such a system.

When only two periods are compared, the natural choices for u_a and q_a are, of course q_o, u_o or q_1, u_1.

As we already know, when q_a in expression (8) is substituted by q_o or q_1 we get respectively the Laspeyres and the Paasche indices. It is then easy to deduce their relationships with the true cost of living indices.[1] To be more specific, the Laspeyres index (with weights q_o) is to be compared with the true index at utility level u_o and the Paasche one (with weights q_1) with the true index at utility level u_1.

Figures 1 and 2 show graphically those relationships for a simplified situation where there are only two goods (A and B) and two periods (o and 1), period 1 differing from period o only because the price of B is higher.

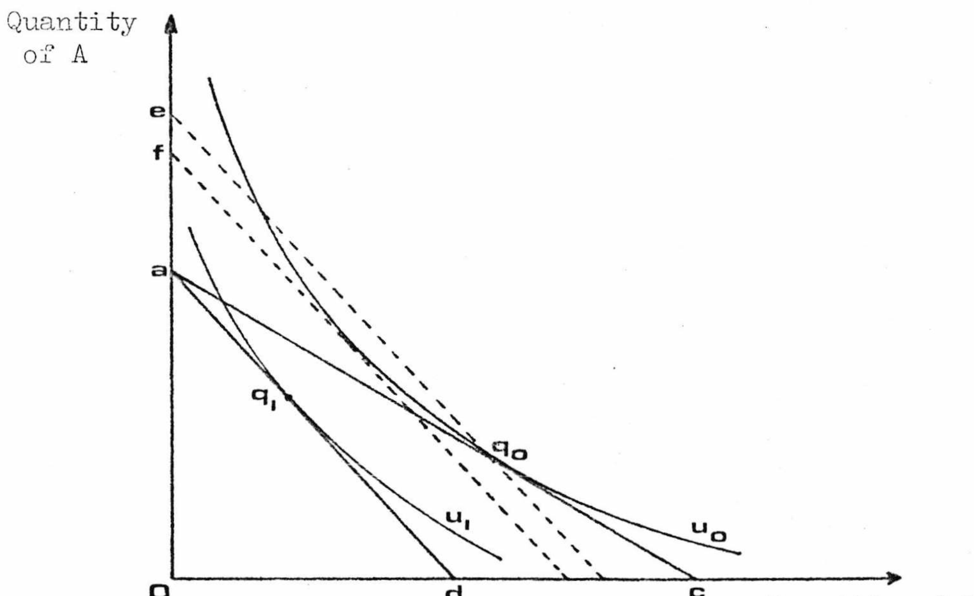

Fig.1 The Laspeyres and the Base Weighted True Cost-of-Living Index.

1. See Deaton and Muellbauer (1980).

In fig.1 the initial equilibrium point is at q_0, the point of tangency of the indifference curve u_0 with the budget line ac. When the price of B increases in period 1, the budget line rotates to ad and the new equilibrium point is q_1 where the new budget line is tangent to the indifference curve u_1. The budget line necessary to buy q_0 at the new price intersects the vertical axis at point e. As the price of A is constant, distances along the vertical axis are proportional to total expenditure, so we can observe that a lower level of expenditure of would provide the same level of original utility. Hence, the Laspeyres index is given by oe/oa and the base-weighted true index by of/oa, the former being greater than the latter.

Fig. 2 depicts the relationship between the Paasche and the current weighted true index. The two figures are very similar, only

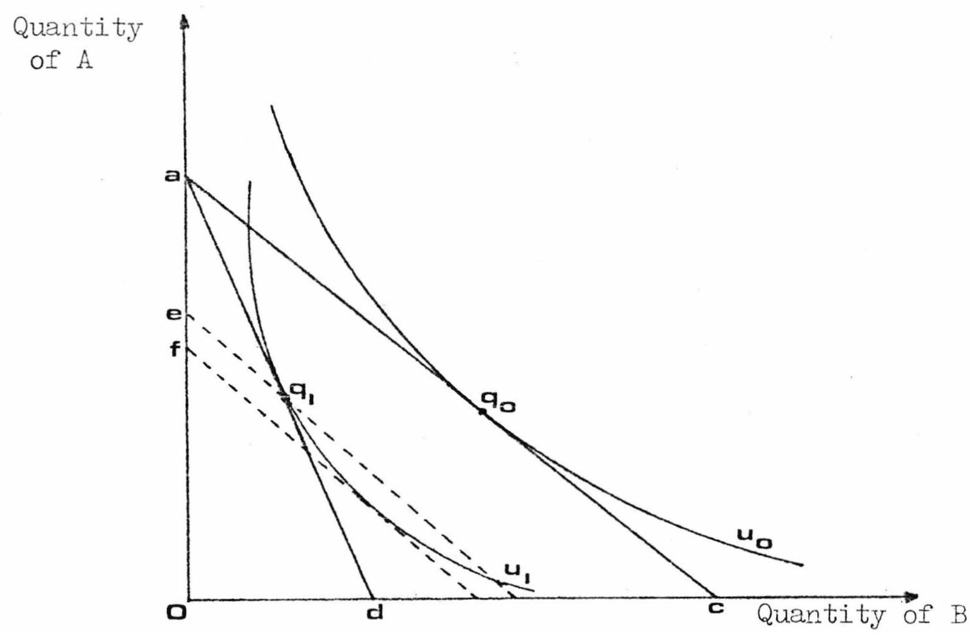

Fig.2 The Paasche and the Current-Weighted True Cost-of-Living Index.

now we represent the costs of purchasing the bundle q_1 in period 1 (oa) and in period o (oe), and the reference utility level is u_1 instead of u_o. The Paasche index is then given by the ratio oa/oe and the true cost-of-living index by the ratio oa/of. As the former ratio represents a lower quantity than the latter, we conclude that the Paasche index is less than the current utility true index.

These inequalities, illustrated diagrammatically can be shown to hold in general. Utility level u_o can be reached with the bundle q_o, but this is not necessarily the cheapest way, when prices are p_1. It follows that $p_1 \cdot q_o \geqslant c(u_o, p_1)$, that is, the cost of q_o at p_1 prices $(p_1 \cdot q_o)$ is greater than or equal to the minimum cost of u_o at p_1. Since by definition, $p_o \cdot q_o$ is equal to $c(u_o, p_o)$, then

$$L_{o1}(\text{Laspeyres}) = \frac{p_1 \cdot q_o}{p_o \cdot q_o} \geqslant \frac{c(u_o, p_1)}{c(u_o, p_o)} = \text{True index (base weighted utility index)}$$

Similarly, as q_1 is only one way of reaching utility level u_1 at p_o, $p_o \cdot q_1 \geqslant c(u_1, p_o)$, and since $p_1 \cdot q_1 = c(u_1, p_1)$ then

$$P_{o1}(\text{Paasche}) = \frac{p_1 \cdot q_1}{p_o \cdot q_1} \leqslant \frac{c(u_1, p_1)}{c(u_1, p_o)} = \text{True index (current weighted utility index).}$$

The above inequalities, which are sometimes called the Konüs[1] inequalities do not imply, as is frequently thought, that the true index has as lower and upper bounds the Paasche and the Laspeyres indices respectively. They only indicate that the Paasche index is a lower bound to the current weighted utility index, and the Laspeyres index is an upper bound to the base weighted utility index. It is even possible for the Paasche index to be greater than the Laspeyres index.

1. Konüs (1924).

As the true cost of living index depends on the utility level u_a, a single individual will generally have more than one index, depending on the reference utility. When comparing different individuals with different expenditure levels, the same price changes will represent different cost-of-living indices, even if they have identical tastes. Only in the very special case where preferences are homothetic would this not happen. When preferences are homothetic the cost function is proportional to utility (all indifference curves are the same shape and expenditure patterns are independent of the expenditure level. The cost function can be written as $c(u,p) = u\ b\ (p)$ where $b\ (p)$ is some function of p, so that the above inequalities can be expressed as follows:

$$P_{01}(\text{Paasche}) = \frac{p_1 \cdot q_1}{p_o \cdot q_1} \leqslant \frac{b(p_1)}{b(p_o)} \leqslant \frac{p_1 \cdot q_o}{p_o \cdot q_o} = L_{01}(\text{Laspeyres})$$

Only now can we talk of a unique true index lying between the Paasche and Laspeyres indices. As can be seen in figs.1 and 2 the Paasche and Laspeyres indices are different from the true index due to the substitution effects. Rising prices will induce reductions in the quantities consumed that cannot be captured by the fixed quantities (q_o and q_1) assumed in the Laspeyres or Paasche indices.[1]

Recent literature[2] on the economic theory of index numbers have shown how well-known index number formulae, such as Fisher's ideal index, can be derived from specific preference structures. At the same time new indices have been suggested which are the result of other postulated preference structures.

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1. However, some studies, e.g.Braithwait (1980) have suggested that the deviations between the true index and the Laspeyres and Paasche indices may be small.

2. See Diewert (1981).

2.3. The Price Index in Practice.

The Consumer Price Index in Portugal

The Laspeyres and Paasche are by far the most utilized indices in practice. The reasons for this are much more related to their ease of computation and clear economic meaning than to their theoretical properties.[1] As the weights are usually derived from household expenditure surveys (HES) which may not be carried out very often due to cost, the Laspeyres index tends to be used more frequently. In the case where the necessary data exists the Paasche index requires much more computations (as the base is continually shifting) so that the Laspeyres index is again generally prefered.

The penalties paid for using the Laspeyres index are mainly related to the assumption of constant expenditure patterns along time, but they are likely to be considerably reduced if the weights are changed, say, every five years (carrying out new expenditure surveys).

In fact, the information for the weights drawn from the HES do not refer to the quantities consumed in the base period[2] but to the proportion of the total expenditure spent on them. In effect, the Laspeyres formula can be expressed as

$$L_{o1} = \frac{p_1 \cdot q_o}{p_o \cdot q_o} = \frac{\Sigma p_1 q_o}{\Sigma p_o q_o} \qquad (9)$$

$$= \frac{\Sigma p_o q_o \dfrac{p_1}{p_o}}{\Sigma p_o q_o} \qquad (10)$$

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1. Though these are important, as seen in the Konüs inequalities.
2. It would require very detailed information, and even so some aggregative problems would arise.

$$= \sum \frac{p_o q_o}{\Sigma p_o q_o} \cdot \frac{p_1}{p_o}$$

or

$$L_{o1} = \sum w_o \frac{p_1}{p_o} \qquad (11)$$

where

$$w_o = \frac{p_o q_o}{\Sigma p_o q_o}$$

that is, w_o is the expenditure weight, or the proportion of total expenditure spent on good i. Expression (11) shows the Laspeyres index as a weighted average of price relatives where the weights are the expenditure proportions. Knowing the weights w_o from the HES and the price relatives from period o to period 1 it is straightforward to calculate the price index.

There is however, a neglected point that was stressed by Ruderman (1954): In expression (11) the p_o in the ratio p_1/p_o and the prices implicit in the weights w_o are not exactly the same when the actual computations are done. The prices p_1 and p_o in p_1/p_o are market prices actually paid for a well defined good i, whereas the other p_o are taken from family budget studies and are implicit in the expenditure weights. These latter prices represent an average of the prices paid by the class of consumers being analysed, on the group of goods which are related to and defined as good i, so that they are different from the former prices. However, Banerjee (1975) has shown that some current practices in obtaining the price relatives p_1/p_o lead to unbiased estimates of the p_o in family living studies.

Another problem frequently arises when calculating the price index between two periods, say 1 and 2, when the base period is o. In this case it is common to employ the formula

$$\sum w_o \cdot \frac{p_2}{p_1} \qquad (12)$$

and to say that a Laspeyres index was calculated. This is not so because in a Laspeyres index only the quantities q_o purchased in the base period are assumed constant, not the expenditure weights w_o. The correct Laspeyres formula for this situation would be

$$L_{12(o)} \doteq \frac{\sum w_o \frac{p_2}{p_o}}{\sum w_o \frac{p_1}{p_o}} \qquad (13)$$

which is equal to

$$L_{12(o)} = \frac{\Sigma p_2 q_o}{\Sigma p_1 q_o} \qquad (14)$$

and is obviously different from (12)[1].

In Portugal, the first Consumer Price Indices began to appear in 1948/49. They were based on the Laspeyres formula and the weights were drawn from special "enquiries to the living conditions of the families" carried out in six cities. They were sample surveys with an annual duration and the cities were: Lisbon (July 1948/June 1949), Oporto (July 1950/June 1951), Coimbra (July 1953/June 1954), Evora (June 1955/May 1956), Viseu (July 1955/July 1956) and Faro (July 1961/June 1962).

The reference population consisted, for each of them, in households with three or more people residents in the respective city and where the heads of the households were unionized or were civil servants with a category equal to or below "first officer".

1. Michael (1979) called the price index given by (12) the Expenditure Price Index (EPI) and found that for a large disaggregation the EPI did not differ appreciably from the Laspeyres index.

These indices calculated on a monthly basis, were available until 1979 and from now on will be called the "old" indices.

From January 1977 the INE (National Statistical Institute) has been publishing monthly Consumer Price Indices for the Continent (the whole country except the islands of Azores and Madeira), and for the two major cities, Lisbon and Oporto. The weights for the Laspeyres formula were drawn from a sample survey carried out from July 1973 to July 1974 and called Household Expenditure Survey 1973/74. The households were observed during one week and the total number of households were 15,921 in the Continent, 707 in the Azores and 652 in Madeira.

The reference population is the subset of households with one to five consumption units, an annual income lying between 30 and 180 thousand escudos, and whose head was an employee (except if he belonged to the Services, was an executive or a senior civil servant). This subset represents 70% of the total households.

The equivalent consumption units were calculated from the following table[1], which is used by the ILO for international comparisons:

Individuals with less than 2 years	0.2
" from 2 to 3 "	0.3
" " 4 to 5 "	0.4
" " 6 to 7 "	0.5
" " 8 to 9 "	0.6
" " 10 to 11 "	0.7
" " 12 to 13 "	0.8
Females from 14 to 59 "	0.8
Males from 14 to 59 "	1.0
Individuals with more than 60 years	0.8

1. This table will be analyzed in Chapter 5 on equivalence scales.

This scale is based on nutritional requirements and the reference is a male aged between 14 and 59 years.

The price indices for the Continent and for the cities of Lisbon and Oporto (based on the 1973/74 HES and published since 1977) will be called the "new" indices.

Chapter 3

Differential Inflation in Portugal

3.1 Introduction

Several empirical studies have shown that published inflation rates may be misleading, in the sense that they implicitly assume the same rates of price increase to be faced by everybody. In fact it has been shown that low income and older households may experience considerably higher rates of inflation than high income and younger ones, and that some statistically significant differences may also arise if households are grouped by other socio-economic variables.[1] The question now is: what has happened in Portugal? Is there a case for the Statistical Office to start publishing more than one price index?

To answer these questions the methodology adopted was the following: firstly obtain the raw data of the last available Household Expenditure Survey (1973/74); secondly construct price indices for each of the households using the published information on prices with the maximum possible disaggregation; and finally to verify whether there are any significant differences among households when they are grouped in accordance to one or more socio-economic variables.

The period of analysis was from January 1971 to January 1982 for a regional study. Four regions in the Continent were considered, North, Centre, Lisbon and South[2] (see map) and they correspond to the planning regions used until 1974 which are still widely used in regional studies for Portugal.

The HES 1973/74 provides information for 4338 households situated in the North region, 4217 in the Centre, 5632 in Lisbon and 1728 in the South.

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1. See chapter 1, namely the studies related to the U.K.cited in the Section "Differential Inflation".
2. The regions comprise the following "distritos": North-Braga, Braganca, Oporto, Viana do Castelo, Vila Real: Centre-Aveiro, Castelo Branco, Coimbra, Guarda, Leiria, Viseu; Lisbon-Lisbon, Santarem, Setubal; South-Beja, Evora, Faro, Portalegre.

The formula for the price indices was

$$P_{12(o)} = \sum W_o \frac{p_2}{p_1}$$

Where W_o are the expenditure weights drawn from the HES, p_2/p_1 are the price relatives between periods 2 and 1 and o is the base period. This formula has already been discussed in the previous chapter (see expression 12), where it was shown that it is based on, but not equivalent to a Laspeyres index. Michael (1979) called it the Expenditure Price Index (EPI) and found that for large samples it did not differ appreciably from the Laspeyres formula. Its main advantage is that it is simpler to use and avoids some of the enormous amount of calculations this type of study requires.

Fig.1 Map of Portugal with the four regions.

To link the time periods, say from period 1 to period 6, the expression used is

$$P_{1t} = P_{12} \cdot P_{23} \cdots P_{t-1,t}$$

that is,

$$P_{12} = \left[\sum W_o \frac{p_2}{p_1}\right] \cdot \left[\sum W_o \frac{p_3}{p_2}\right] \cdots\cdots \left[\sum W_o \frac{p_t}{p_{t-1}}\right]$$

which is different from $\sum W_o \frac{p_t}{p_1}$.

The above expression is not of course a chain index, as the base is not changed in each period, but it uses all the information on the prices observed in that period.

3.2 The Data

The information on the households' demographic characteristics and expenditure patterns was taken from the raw data of the Household Expenditure Survey 1973/74, available on tapes and obtained from the National Statistical Institute (INE). The data on prices is published by the INE in its Monthly Statistical Review. The "old"[1] prices for the cities Lisbon, Oporto, Coimbra, Viseu and Faro are available from 1971 to 1979. Their base was 1948/49 (Lisbon), 1950/51 (Oporto), 1953/54 (Coimbra), 1955/56 (Evora and Viseu) and 1961/62 (Faro), corresponding to the years in which a specific HES for each city were carried out. In 1974 the base year for the index was changed for all cities to 1963. There was no index for the Continent as a whole.

From 1977 onwards "new" price indices for the Continent and the cities of Lisbon and Oporto have been published, with weights drawn from the HES 1973/74 and base prices referred to 1976.

After some minor aggregation for the sake of comparability, 28 expenditure items were identified in the "old" indices. That number has been increased to 73 for the new indices, but for the Continent it was possible to get additional information, so that in this case the disaggregation amounts to 78 items. The full lists of both sets of items are included in the appendix.

The existence from 1971 to 1977 of price indices for the six cities and not for the Continent as a whole is in a way a bonus rather than a nuisance, since in this study a comparison is made among regions. In each region the price indices for the cities therein were used, that is[2], in the North the prices for Oporto, in the Centre the prises for Viseu and Coimbra, in Lisbon the prices for Lisbon, and in the South the prices for

1. See chapter 2, section 2.3.
2. See fig.1.

Evora and Faro. In order to work with only one set of prices in each region, new price indices were constructed for the Centre and the South by weighting the cities' prices by their populations. That is, in the Centre a price index was constructed by weighting the prices for Coimbra by 3.2 and for Viseu by 1. In the South, the weights were 1.16 for Evora and 1 for Faro.[2]

Given that there were two sets of expenditure items in the old and new indices with very different degrees of aggregation and sometimes definition, the next task was to obtain for each household, information on its separate expenditures. That was a difficult task for the following reasons:

1) There were separate tapes for the households' demographic characteristics and for the expenditure details. 2) The records were ranked by expenditure item and not by household. 3) The level of expenditure disaggregation exceeded 90,000 items!

Since each record had a field with an expenditure classification similar to that used in the "old" indices it was relatively easy to get the expenditures on those items. However for the 78 items corresponding to the new indices a correspondence table had to be constructed relating each one of the 9,000 items with one of the 78.

Next there was the question of what month or months to choose to reflect the annual rate of inflation. Traditionally, the rate of inflation in any one year is computed by averaging the twelve months price indices and then comparing that average with a similar average for the previous year. This way of proceeding is misleading, as can be seen in the case where the inflation rate has been accelerating. If in the first year inflation had been running at a steady 10% and in the second year at 30%,

1. From the 1970 Population Census, the cities populations were: Coimbra- 56,568, Viseu- 17,636, Evora- 24,003 and Faro- 20,687.

the above procedure would lead to the conclusion that inflation in the second year was 21% instead of the real 30%. An equivalent result (21%) could be reached by comparing the rates of inflation in June/July of both years. Comparison of the price increases between the end and the beginning of the second year would provide the right answer. Of course this is not always true, but in situations of rapid acceleration and decelerations, comparisons between two months may give a better picture than comparisons between annual averages.

The following table, taken from Gomes (1982) shows, for the published price indices in Portugal, during the period 1970-80, the kind of differences that may arise when calculating the inflation rates by the above two methods.

Table 1
Inflation rates in Portugal
The influence of the computing methods

Years	Average Year X Year X - 1	December Year X Year X - 1	Difference
1970	6.2	4.6	-1.6
1971	8.4	11.0	2.6
1972	8.4	6.3	-2.1
1973	11.9	18.7	6.8
1974	29.2	29.3	0.1
1975	20.4	17.5	-2.9
1976	19.2	25.7	6.5
1977	25.8	20.8	-5.0
1978	22.6	25.2	2.6
1979	23.6	22.4	-1.2
1980	16.6	13.1	-3.5

In the same article, Gomes argues that both ways of measuring are arbitrary and that inflation has its own calendar, that is usually incompatible with the traditional one. He detected cycles of accelerating and decelerating prices in the inflationery process in Portugal of around 20 months. The peaks were in February 1969, November 1971, July 1974 and July 1977, and troughs in May 1970, November 1972, October 1975 and September 1980. He then argued that inflation should be measured by comparing months situated in the same position of two successive cycles. Within this framework he was able to reach the conclusion that Portugal was experiencing the highest rates of inflation ever.

In spite of the theoretical interest of this type of argument, the knowledge of the inflation rate in each year will always, for obvious reasons, be important. It will then be arguable whether we should compare averages of months or prices in specific months. There are pros and cons with both methods, but if we believe that inflation is accelerating, the best way may be to compare prices related to a specific month.

For the above reasons, a decision was taken to use the prices for January of each year, so that, for instance, the inflation rate in 1977 is given by P77 = (P in January 1978)/(P in January 1977). The choice of January is arbitrary (it is the first month of the year...).

Additional advantages are that by picking up a single month seasonal problems are avoided, and above all, the computational burden is alleviated. Furthermore we should always bear in mind that this type of study does not aim to get an exact measure for the inflation rate, but rather to ascertain whether different households face significantly different rates of inflation.

It is important to stress the different way the weights were computed in this study in comparison to the way they were obtained by the INE using the same survey.[1] The INE sums all the expenditure made by all households on item i, and then divides that figure by the expenditures made by all households on the total number of items. This means that the final weights will be more influenced by the higher expenditures, and so they will tend to represent the expenditure pattern of households situated in the upper half of the income distribution. The greater the discrepancy of expenditure patterns among households the greater will be the bias in favour of the more privileged households.

The only way to avoid this bias, and to get "democratic" weights is to calculate them for each household and then to average them out. This has been adopted in this study since the computation of prices indices for each household will already have been done. These computed price indices were the following: annual indices (P_{71}, P_{72},...,P_{78}) using the "old" prices; annual indices (P_{77},P_{78},...P_{81}) using the "new" prices for the Continent and for Oporto and Lisbon; indices for longer periods (P_{71-76} with the old prices, and P_{77-81} with the new ones). As the old prices include housing rents, for the period 1971-76, price indices without rents were also computed to allow a direct comparison with those for 1977-81.

No consideration was given to the period 1971-78(old indices) because it was found that the indices P_{77} and P_{78} provided conflicting results when compared with the new ones. As an example, for the North region, in 1977, the food inflation rate was around 31% when calculated with the old prices for Oporto and around 20% when the new prices for the Continent or Oporto were used. Some other discrepancies can be observed by direct comparison of the two sets of prices. One reason for

1. Apart from the fact that the INE considers only a subset of the total (see section 2.3), while in this study all the households are included. See also the final section of this chapter.

these discrepancies lies in the fact that the INE recorded for the old indices the price increases of the products[1] most often consumed by the reference households (1950/51 in the case of Oporto) while for the new indices the same happened but in relation to 1973/74. It is to be expected that there will be considerable differences between the products consumed in the two dates (and their corresponding price increases).

Another source of discrepancy is the change of weights that occurred between1950/51 and 1973/74 - important due to the higher degree of aggregation in the old indices. To show this consider the following illustration: In the old indices information is only available for the price increases of the aggregate "cereals", whereas for the new indices information exists for four of its components (rice, flour, pasta and bread). Even if the individual commodities are the same in both cases, the weights for rice, flour, pasta and bread are certainly different between 1948 and 1973/74. Consequently different price indices would occur for "cereals" with the old and the new indices. This source of error would not exist if the level of aggregation was the same.

3.3. <u>Income and the Differential Inflation</u>

As income is generally recognized as being an important factor for the existence of price differentials, the next step was to investigate their relationship in the case of Portugal. Households were therefore grouped into eight income groups[2], and the average price indices and standard deviations were computed for each of them.

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1. Very detailed and precisely defined (by their physical characteristics, brand, etc.).

2. The groups (adopted in the HES) were: < 18, 18-30, 30-48, 48-60, 60-90, 90-120, 120-180 and $\geqslant 180$. They refer to annual income and the units are thousands of escudos. As a reference it can be said that the average basic wage in industry in January 1974 corresponded to an annual income of around 45 thousand escudos.

As the household expenditure survey of 1973/74 includes no data on income (households were only asked to situate themselves in one of the eight proposed income brackets), total expenditure was used as a proxy for income.

The results for the four regions are given in tables 2 to 5 for the periods 1971-76 and 1977-81, and tables A8 to A15 (in appendix) for annual periods.[1] One of the most immediate and striking features arising from a simple inspection of these tables is the high degree of price dispersion. This is indicated by the high values for the standard deviations. To illustrate the extent of this dispersion consider the price distribution for 1975 in the North Region (Table 6). There is a marked tendency for the price index to decrease with income. The difference between the lowest and highest classes being 7.4 percentage points. However there is also a considerable dispersion within the classes as can be seen by their standard deviations (ranging from 4.1 to 7.5). Assuming the household EPI's to be normally distributed within the groups[2] a 10% interval around each group's mean and around the "all households" mean were calculated.

It is of interest to compare the percentage of households lying in the interval around each group mean with the percentage obtained for the same group but with the interval considered around the "all households" mean. These results are also given in table 6, and they are striking: firstly, the percentage of households lying in the considered intervals is small, and secondly the interval around the "all households" mean covers almost the same percentage of households in each group as a similar interval around its own mean (with the important exceptions of the lowest and

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1. Due to the great number of tables, the results for annual periods are presented in appendix.
2. A reasonable assumption given the size of the sample.

TABLE 2

NORTH

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 18	248.5 (19.4)	246.1 (22.5)	292.8 (19.8)	283.7 (32.2)	269
18-30	246.0 (17.8)	244.3 (19.3)	292.9 (20.0)	284.5 (31.0)	442
30-48	243.2 (15.9)	241.3 (17.3)	292.7 (18.3)	281.8 (30.5)	811
48-60	241.3 (13.5)	239.4 (14.5)	292.0 (17.7)	280.0 (33.4)	559
60-90	239.8 (13.3)	237.6 (14.3)	292.0 (15.8)	280.5 (30.9)	1058
90-120	240.3 (12.9)	238.0 (14.0)	289.7 (15.9)	277.0 (31.4)	515
120-180	241.1 (14.3)	238.6 (15.4)	289.2 (14.9)	279.6 (26.1)	411
≥ 180	243.7 (15.0)	239.7 (17.1)	287.7 (17.1)	277.6 (27.2)	273
All households	242.2 (15.1)	240.0 (16.5)	291.5 (17.3)	280.6 (30.7)	4338

Note: Standard deviations in parentheses

(1) "Old" prices for Oporto, including housing rents

(2) "Old" prices for Oporto, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Oporto

TABLE 3

CENTRE

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 18	265.6 (23.9)	254.8 (24.7)	287.4 (17.2)	388
18-30	256.9 (20.0)	248.0 (20.4)	285.8 (18.2)	588
30-48	250.9 (17.1)	242.1 (16.6)	285.5 (17.4)	893
48-60	247.8 (18.8)	238.8 (17.4)	285.6 (17.7)	514
60-90	243.9 (18.9)	234.1 (16.9)	283.6 (16.8)	903
90-120	237.7 (20.4)	226.0 (16.9)	282.2 (16.7)	447
120-180	229.0 (25.1)	216.5 (21.2)	285.1 (18.2)	305
≥ 180	225.9 (35.1)	206.5 (23.0)	287.0 (18.2)	179
All Households	247.2 (23.1)	236.9 (22.2)	285.0 (17.5)	4217

Note: Standard deviations in parentheses

(1) "Old" prices for Coimbra and Viseu, including housing rents

(2) "Old" prices for Coimbra and Viseu, excluding housing rents

(3) Prices for the Continent

TABLE 4

LISBON

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 18	275.5 (25.0)	268.5 (29.1)	289.5 (21.1)	292.4 (22.7)	225
18-30	275.6 (21.7)	270.3 (24.0)	289.0 (20.4)	292.3 (22.0)	430
30-48	276.3 (17.0)	271.6 (18.7)	288.4 (18.6)	292.2 (20.7)	937
48-60	278.7 (15.5)	274.2 (17.4)	289.5 (19.1)	293.5 (21.5)	623
60-90	279.9 (14.6)	274.4 (16.4)	288.4 (17.1)	292.1 (19.2)	1359
90-120	282.7 (14.5)	278.4 (16.5)	288.4 (16.1)	291.8 (18.1)	884
120-180	284.4 (14.8)	280.1 (17.1)	287.6 (17.3)	289.7 (19.4)	705
≥ 180	284.0 (16.4)	279.7 (18.8)	286.2 (16.6)	285.3 (18.5)	469
All Households	280.0 (16.7)	275.4 (18.8)	288.3 (17.9)	291.4 (20.0)	5632

Note: Standard deviations in parentheses

(1) "Old" prices for Lisbon, including housing rents

(2) "Old" prices for Lisbon, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Lisbon

TABLE 5

SOUTH

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 18	254.8 (20.3)	247.3 (21.7)	278.8 (16.7)	230
18-30	252.4 (19.0)	246.6 (19.3)	280.4 (16.3)	338
30-48	249.1 (16.5)	243.4 (16.0)	279.2 (16.7)	434
48-60	246.4 (16.3)	240.0 (15.4)	280.3 (16.0)	208
60-90	245.8 (15.6)	238.6 (14.2)	279.4 (16.1)	284
90-120	246.2 (14.3)	238.7 (14.0)	283.6 (17.1)	110
120-180	240.9 (15.0)	231.0 (12.4)	285.8 (21.2)	74
≥ 180	234.0 (36.9)	222.0 (33.0)	279.2 (41.7)	50
All Households	248.7 (18.6)	241.9 (18.4)	280.1 (17.9)	1728

Note: Standard deviations in parentheses

(1) "Old" prices for Evora and Faro, including housing rents

(2) "Old" prices for Evora and Faro, excluding housing rents

(3) Prices for the Continent

TABLE 6

NORTH - 1975

Percentage of Households in 10% intervals around the means

Income	Price distribution 1975	% Around the group mean	% Around the "All Households" mean	Difference	No. of Observ.
< 18	129.6 (7.5)	30.7	23.3	-10.4	269
18-30	127.9 (6.4)	33.7	29.2	- 4.5	442
30-48	126.5 (5.3)	38.3	36.6	- 1.7	811
48-60	125.9 (4.7)	41.8	41.3	- 0.5	559
60-90	125.1 (4.4)	43.1	43.7	0.6	1058
90-120	124.4 (4.1)	44.8	45.0	0.2	515
120-180	123.4 (4.1)	43.2	41.2	- 2.0	411
≥ 180	122.2 (4.4)	38.6	33.6	- 5.0	273
All Households	125.6 (5.3)		37.1		4338

highest income groups). This could lead to the conclusion that there is no substantial gain in considering group specific indices, as the "all households" and average price index is representative of approximately the same proportion of households. The problem is that this last proportion is biased, the bias increasing with the difference between the group specific mean and the "all households" mean.

If we now consider the class of income 18-30 then there are 33.7% of the group's households within the interval (125.1, 130.7) and 29% in the interval (123, 128). This latter interval lies almost entirely to the left of the group mean so that the sole consideration of the "all households" mean would be representative of 29.2% of those group households that suffer less inflation.

But, do all price distributions behave like the 1975 North distribution, where the price increases for the lowest income class is 29.6% but then decreases with income to reach "only" 22.2% for the highest class? By no means. For the same year in the Centre and in Lisbon the same pattern prevailed, with a staggering difference of 11.3 percentage points in the Centre (34.3% against 22%) and a more modest one of 2.5 percentage points (26.1% - 23.6%) in Lisbon. In the South practically all groups experienced a 20.5% inflation rate.

To test for differences among the means, an analysis of variance (ANOVA) was carried out for all price distributions. However with very big samples almost any difference will be significant (we are operating within the statistical infinity) so caution is necessary when making any conclusion. At the 0.001 level of significance the null hypothesis that all the means are equal would only be accepted for the South distributions in 1971, 1972 and 1975 and is only just rejected for the Centre 1972 distribution.

With the "old" indices, and for annual periods, the rule is for price indices to decrease with income. The exception being the year 1976 for all regions, where on average the highest income households experienced 4 percentage points more inflation than the lowest ones. This same pattern but for lower values happened in Lisbon in the years 1972 and 1973.[1]

The trend set in 1976 for higher income groups to face higher price increases continued in 1977 (around 6.5 more percentage points), in all regions (Continent "new" price indices - tables A12 to A15). But for the years 1978 and 1979 the pattern is more complex. In 1978 price indices increased first with income and then decreased, the opposite happening in 1979, but in both years the highest income groups experienced less inflation overall than the lowest ones. In 1980 this last conclusion is reversed, but in 1981 there is again a very strong and marked tendency for price indices to decrease with income. The difference between the lowest and highest income groups in on average 10 points (30% against 20%) and the results for 1981 are crucial for the overall conclusions drawn for the period 1977-81.

It should be stressed that although these trends can be detected, they do not necessarily imply that different income groups will have significantly different price indices. The analysis of variance only tests for differences among all the means, not between two specific means, and it may be very influenced by an extreme or "outlier" value.[2]

Having studied the annual price differentials, we would be inclined to say that for the period 1971-76, higher income households suffered less from inflation than low income ones, at least in the North, Centre

1. The years 1977 and 1978 are not studied with the "old" indices for the reasons mentioned in 3.2, although the results for them are included in the tables in the appendix.

2. However in these results there is generally a smooth behaviour and no evidence of any outliers.

and South. In Lisbon, the picture was not so clear, but the reverse might have happened. Tables 2 to 5 confirm these deductions. In the North, during that period, price indices decreased with income, until they reached a minimum with the middle income groups, and then rose again but in such a way that the bottom income group still had an index value 5 percentage points higher than the top one. In the Centre there was monotonicity, and the difference between the highest and lowest income group was an astonishing 39.7 points. In the South there is also an overall inverse relationship between price indices and income (there is only a "slight" problem with two contiguous classes), and the difference between the highest and powest price indices is 20.8 points.

For Lisbon there is a direct relationship between income and price indices, but the range is much smaller, only 8.5 percentage points.

If housing rents are excluded, the ranking of households by inflation rates stays the same, but those rates are now absolutely lower: for the whole period on average 2 percentage points lower in the North, 19.4 in the Centre, 5 in Lisbon and 7 in the South. But the range of price indices between the lowest and highest income groups is now higher: 6.7 percentage points in the North, 48.3 in the Centre, 11.2 in Lisbon and 25.3 in the South.[1]

The average annual rates of inflation in the four regions in the period 1971-76 (including housing rents) were the following: 15.9% in the North, 16.3% in the Centre, 18.7% in Lisbon and 16.4% in the South. Excluding rents these values were 15.7% in the North, 15.5% in the Centre, 18.4% in Lisbon and 15.9% in the South. They show that the average annual inflation rate in Lisbon was about 2.5 percentage points higher than in the other regions.

1. The INE recorded in "Housing rents" the rents to be paid for new houses with four rooms and with approximately the same area as those rented in the year of the original survey. As nowadays new houses have in general fewer rooms and less area, the index will not be very representative if the rises in rents for the bigger houses are much different from the smaller ones. It is difficult to reach any conclusion on an "a priori" basis.

It needs to be said that the usual tests for differences of means (in this case between regions) always tend to reject the null hypothesis that they are equal, due to the very large number of observations, and this is no exception.[1] However we can see that there are quite different values for each region, particularly if individual income groups are compared.

Differences in inflation rates among regions in the period 1971-76 may be due to the joint factors of different prices observed in each city and different expenditure weights in each region. In the period 1977-81, when the Continent prices are employed, only the differences in expenditure weights will be an explanation for any variations detected among the regions. (see tables 2 to 5). The average annual inflation rate was 23.9% in the North, 23.3% in the Centre, 23.6% in Lisbon and 22.9% in the South. Price indices decrease with income in the North, where the range is 5.1 percentage points, and in Lisbon where that value is 3.0. The same happens in the Centre with the exception of the two highest classes. In the South there are no very definite patterns, although the upper middle incomes tend to experience higher inflation rates than the lower middle ones.

When the new prices for Oporto and Lisbon are employed in the North and Lisbon regions respectively (instead of those for the Continent) two facts deserve mention: firstly that inflation would be on average 10 percentage points lower in the North and 2 points higher in Lisbon, and secondly that the monotonicity in the North is considerably disturbed. This serves to emphasize the importance of the construction of regional price indices.

One of the main criticisms for the division of Portugal into the four regions (North, Centre, Lisbon and South) is that they are not homogeneous (although this problem may not be so acute in Lisbon). In fact the main division in Portugal is between the more developed and industrial "litoral"[2]

1. For instance, this test would accept that the "All Households" mean for the North (247.2) is significantly different from the corresponding one for the South (248.7) at the 0.05 level of significance and only rejects it at the 0.01 level.

2. Coastal areas.

and the less developed and agricultural interior. The North, Centre and South regions will then encompass both types of area so that if any overall conclusions for them may be an average of quite different results. The importance of this for our study is that as the INE gets its regional price information in the cities (such as Oporto) it might be misleading to apply these prices to households in the North region who live far away from the city such as in the agricultural interior. To get more accurate results, we should isolate the households actually living in the city of Oporto and analyse the price distributions that result from applying the commodity prices collected for Oporto. This also holds for all the other cities where separate price information is obtained.

Since it is impossible to exactly relate those households living in the cities with the available price information, a separate analysis was conducted with those households living in the smallest administrative regions which lie within the cities - the "concelho".[1]

A drawback is that there is a considerable reduction in the number of households analysed so that the "confidence intervals" for our conclusions may have to be broadened. This problem may be particularly important in the South region where the number of households is considerably less than in the others.

Tables 7 to 10 show the price distribution by income class for the periods 1971-76 and 1977-81 for the "concelhos" of Oporto (North Region), Coimbra and Viseu (Centre), Lisbon (Lisbon) and Evora and Faro (South). The number of households as a percentage of the total in each region is: 18.7% in the North, 8.7% in the Centre, 37.7% in Lisbon and 12.5% in the South.

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1. As the administrative regions are quite different in Portugal and in England, it is impossible to translate precisely the word "concelho". Maybe the words "district council" or "municipality" are closest in meaning.

TABLE 7

NORTH - "CONCELHO" OF OPORTO

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 18	251.7 (13.3)	250.0 (15.3)	297.8 (16.2)	291.0 (19.1)	15
18-30	244.7 (15.3)	242.7 (16.7)	292.6 (23.8)	287.8 (24.9)	38
30-48	243.0 (12.4)	240.5 (14.0)	292.7 (17.7)	287.2 (20.9)	89
48-60	241.1 (11.6)	237.3 (14.1)	290.2 (13.7)	282.0 (26.7)	74
60-90	241.4 (12.0)	239.0 (13.5)	289.6 (15.2)	283.2 (31.0)	178
90-120	243.0 (11.6)	240.2 (12.9)	291.1 (14.8)	287.2 (17.9)	142
120-180	243.2 (12.8)	240.5 (14.1)	289.5 (14.5)	282.2 (24.9)	133
≥ 180	242.2 (13.8	238.7 (15.8)	286.2 (17.1	275.7 (28.5)	144
All Households	242.6 (12.7)	239.8 (14.3)	289.9 (16.2)	283.1 (25.9)	813

Note: Standard deviations in parentheses

(1) "Old" prices for Oporto, including housing rents

(2) "Old" prices for Oporto, excluding housing rents

(3) Prices for the Continent

(4) "New' prices for Oporto

TABLE 8

CENTRE - "CONCELHOS" OF COIMBRA AND VISEU

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 18	263.2 (25.9)	246.3 (23.3)	290.5 (15.4)	14
18-30	252.3 (19.1)	240.5 (21.6)	284:6 (14.1)	32
30-48	249.2 (12.3)	236.8 (14.3)	285.7 (14.1)	43
48-60	246.0 (16.1)	236.1 (14.6)	291.3 (16.4)	35
60-90	241.6 (17.9)	229.8 (16.9)	284.2 (14.3)	95
90-120	233.9 (18.2)	222.3 (16.0)	279.3 (16.8)	72
120-180	229.1 (21.4)	218.3 (20.1)	278.7 (15.9)	38
≥ 180	222.7 (34.2)	203.0 (23.2)	286.0 (18.1)	38
All Households	239.9 (22.5)	227.3 (21.1)	284.0 (15.9)	367

Note: Standard deviations in parentheses

(1) "Old" prices for Coimbra and Viseu, including housing rents

(2) "Old" prices for Coimbra and Viseu, excluding housing rents

(3) Prices for the Continent

TABLE 9

LISBON - "CONCELHO" OF LISBON

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 18	279.1 (23.5)	273.2 (27.2)	293.4 (22.9)	297.5 (24.4)	50
18-30	282.9 (21.1)	277.8 (23.9)	296.3 (21.0)	299.9 (23.0)	115
30-48	281.8 (17.6)	277.3 (19.8)	293.9 (19.0)	298.4 (21.6)	268
48-60	283.2 (14.7)	278.9 (16.4)	296.2 (19.0)	301.4 (21.3)	189
60-90	283.2 (13.4)	279.0 (15.4)	293.5 (17.9)	297.7 (20.1)	508
90-120	284.3 (16.0)	280.2 (18.3)	290.6 (16.5)	293.9 (18.8)	384
120-180	284.8 (14.5)	280.7 (16.7)	289.1 (18.0)	291.2 (20.1)	334
≥ 180	281.4 (16.1)	277.0 (17.9)	285.6 (16.2)	284.2 (18.5)	276
All Households	283.1 (15.9)	278.8 (18.0)	291.7 (18.3)	294.8 (20.8)	2124

Note: Standard deviations in parentheses

(1) "Old" prices for Lisbon, including housing rents

(2) "Old" prices for Lisbon, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Lisbon

TABLE 10

SOUTH - "CONCELHOS" OF EVORA AND FARO

Price Distributions by Income Class

Income \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 18	253.4 (24.6)	241.3 (26.2)	284.6 (20.2)	23
18-30	251.9 (26.8)	243.2 (26.4)	283.3 (17.3)	26
30-48	250.0 (20.0)	240.6 (19.1)	281.9 (18.6)	47
48-60	258.1 (14.3)	245.3 (15.3)	292.7 (13.8)	26
60-90	250.7 (13.4)	239.7 (11.4)	283.0 (15.4)	41
90-120	247.7 (13.6)	238.0 (12.9)	294.7 (17.5)	21
120-180	240.1 (12.0)	227.7 (10.9)	290.3 (21.1)	17
≥ 180	238.0 (19.7)	225.9 (14.2)	283.2 (18.3)	15
All Households	249.9 (19.2)	239.1 (18.7)	285.9 (17.9)	216

Note: Standard deviations in parentheses

(1) "Old" prices for Evora and Faro, including housing rents

(2) "Old" prices for Evora and Faro, excluding housing rents

(3) Prices for the Continent

Comparison of the results obtained in the "concelho" of Oporto with the results obtained for the North region lead us to conclude that: 1) The average rate of inflation for all households in both periods is practically the same; 2) The same pattern of differential inflation exists; 3) The difference in the inflation rates between the lowest and the highest income households is accentuated in the "concelho" case: around 4 more percentage points in 1971-76 and 6 points more in 1977-81 compared with the continent prices; 4) The price indices for the Continent and for Oporto are now closer.

For the Centre region, the results for "Coimbra and Viseu" provide similar patterns for the price distributions in both periods. The differential inflation between the lowest and highest income groups is practically as was found for the region in the period 1971-76 where housing rents are included but is lower by 5 percentage points where they are excluded. For the period 1977 to 1981 the difference is increased by 4 percentage points.

In Lisbon, inflation rates are about 0.2 percentage points higher per year for the households living in the"concelho"of Lisbon when compared with all the households in the region. The distributional pattern is only changed for the 1971-76 distribution when housing rents are included. However the highest income households have a price index in relation to the lowest income ones overstated by 6.5 points in the period 1971-76 but understated by 4.5 points in 1977-81. This means that our conclusions for the whole region may be overstating the equalising effect of differential inflation in 1971-76 but understanding the inequality for the period 1977-81.

In the South the consideration of households living in Evora and Faro leads to relative small numbers of them to be taken in each income class, so that our conclusions need to be properly qualified. However, it seems that in this case the inequality aspect of differential prices is exaggerated in 1971-76 when all households are considered and that the apparent equality effect in the period 1977-81 may not really exist.

The main conclusions of this section are the following:

1) There are significant differences in price indices experienced by households when they are grouped by income class.

2) These differences may be quite large (11 percentage points in one year).

3) In general the pattern in all the regions is for inflation to be higher for lower income groups, either with annual or longer periods.

4) The main exception to (3) above is for Lisbon during the period 1971-76 when inflation was greater for high income households.

5) The largest effect on inequality was felt in the Centre region in the same period (1971-76).

6) If we base our conclusions on those households who actually live in the cities ("concelhos") where the commodity price indices are available, then the results for the regions tend to underestimate the inequality bias in the North, Centre and South but to overestimate the equality bias in Lisbon for the period 1971-76.

3.4. Differential Inflation and the Age of the Household

The age of the household has also been regarded as a determinant factor for the existence of price differentials among households. The usual claim is that price indices will rise with age, so that senior citizens will be particularly affected. To verify this claim for Portugal, households were classified by the age of their head, and the means and standard deviations of the price indices experienced by each group was computed, using the available data on commodity price indices. Four age groups were studied - less than 25, 25 to 45, 45 to 65 and greater than 65 years of age - and they roughly correspond to the groups considered in the literature on the subject.

The annual results for the period 1971-76 and 1977-81, and for the four regions are shown in the appendix (tables A16 to A23). An analysis of variance, (ANOVA) was again carried out for all price distributions to test the null hypothesis of equal means among the groups. For the period 1971-76 that hypothesis was not rejected (at the 0.001 level of significance) in 1974 (North) and in 1976 (Lisbon and South). Excluding these cases, and the region of Lisbon, it can be said that in fact the general rule is for price indices to increase with age (although there are some cases where price indices are not monotonic). In Lisbon there was a direct relationship between age and price indices only in 1975 when the eldest households experienced 3.3 percentage points more inflation than the youngest ones. However, this difference might by itself compensate the opposite and smaller ones in the previous years. In fact, (and this is a conclusion valid for all the regions and all the years), the range in price indices in each year is relatively small: its maximum is around 3.7, but most of them are below 2 percentage points.

During the period 1977-81 (continent prices) there are many cases where significant statistical differences among the means cannot be detected: in 1977 (North), 1978 (South), 1979 (North, Lisbon and South) and in 1981 (Lisbon). Again the range in price indices is relatively low (if compared with the results obtained for income), with a maximum of 4 percentage points (1981-Centre), and with most values below 3 percentage points.

Apart from 1980 in all regions, when there was a negative relationship between age and prices, in all the other years older households suffered more inflation than the younger ones; 1981 is particularly noticeable in this respect, because the highest differences tend to exist in this year (4 percentage points in the Centre and South and around 2.7 in the North and Lisbon.)

Comparison of these results with those obtained for the distributions by income class suggest that income is a more important factor than age for the existence of differential price indices: with age as the control variable, not only are there more cases where the group means may be taken as equal, but also, the differences among them are much smaller. However, these annual results express the tendency for older households to experience higher inflation rates.

This is absolutely confirmed when the results for the longer periods 1971-76 and 1977-81 are examined (tables 11 to 14). In effect, it can be said that all these price distributions show price indices increasing with age. Some of the variations in price indices are however not statistically significant, as is the case with the 1971-76 distributions in Lisbon (with and without housing rents) and the 1977-81 distributions in the Centre and the South.

For the period 1971-76 all households in the Lisbon region can be thought of having experienced the same inflation rates when ranked by the

TABLE 11

NORTH

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 25	241.9 (14.0)	239.7 (14.9)	284.3 (21.4)	266.8 (38.1)	44
25-45	240.5 (14.6)	238.3 (15.7)	290.7 (15.8)	280.3 (29.4)	1461
45-65	242.2 (14.5)	240.1 (15.7)	291.4 (17.0)	280.2 (31.0)	1870
⩾ 65	244.8 (16.7)	242.3 (18.9)	293.1 (19.6)	282.5 (31.6)	963
All Households	242.2 (15.1)	240.0 (16.5)	291.5 (17.3)	280.6 (30.7)	4338

Note: Standard deviations in parentheses

(1) "Old" prices for Oporto, including housing rents

(2) "Old" prices for Oporto, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Oporto

TABLE 12

CENTRE

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 25	236.8 (20.8)	225.4 (22.2)	279.6 (17.1)	52
25-45	240.7 (22.5)	230.8 (21.8)	284.8 (16.7)	1210
45-65	246.8 (22.0)	236.6 (20.9)	285.1 (16.9)	1895
≥ 65	255.7 (23.1)	244.9 (22.2)	285.3 (19.3)	1060
All Households	247.2 (23.1)	236.9 (22.2)	285.0 (17.5)	4217

Note: Standard deviations in parentheses

(1) "Old" prices for Coimbra and Viseu, including housing rents

(2) "Old" prices for Coimbra and Viseu, excluding housing rents

(3) Prices for the Continent

TABLE 13

LISBON

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 25	280.2 (19.2)	275.0 (21.5)	280.4 (20.0)	282.8 (22.9)	86
25-45	279.7 (15.7)	275.3 (17.6)	287.5 (16.5)	290.5 (18.6)	2089
45-65	279.8 (16.0)	275.4 (18.0)	288.9 (17.7)	292.0 (19.7)	2398
≥ 65	281.0 (19.7)	275.6 (22.6)	289.5 (20.2)	292.2 (22.6)	1059
All Households	280.0 (16.7)	275.4 (18.8)	288.3 (17.9)	291.4 (20.0)	5632

Note: Standard deviations in parentheses

(1) "Old" prices for Lisbon, including housing rents

(2) "Old" prices for Lisbon, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Lisbon

TABLE 14

SOUTH

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 25	237.2 (16.9)	231.8 (17.8)	274.2 (16.1)	15
25-45	242.4 (18.4)	236.1 (18.3)	281.2 (20.0)	470
45-65	248.0 (17.9)	241.8 (17.8)	279.8 (16.7)	782
≥ 65	256.6 (17.0)	248.3 (17.4)	279.6 (17.7)	461
All Households	248.7 (18.6)	241.9 (18.4)	280.1 (17.9)	1728

Note: Standard deviations in parentheses

(1) "Old" prices for Evora and Faro, including housing rents

(2) "Old" prices for Evora and Faro, excluding housing rents

(3) Prices for the Continent

age of their head, while in the North only around 2.7 percentage points separate the eldest and youngest groups. Yet in the Centre and the South that difference reaches a substantial 19 percentage points.

In the period 1977-81 the opposite happens: the biggest differences occur in Lisbon and Oporto (around 9 points[1]) but there are no significant differences in other regions.[2]

The results for the "concelhos" (tables 15 to 18) are not in conflict with the previous conclusions. Again no significant differences among the means exist in Lisbon in the period 1971-76, although now for the same period we should also include Oporto (North). However the last result is mainly due to the value obtained for the youngest group (<25) and to its very small number of observations (7). For the other groups significant differences can be found (about 2.7 percentage points) in the distribution with housing rents, but not when housing rents are excluded.

The "concelhos" of the Centre and the South show again the biggest differences. If we believe the results for the "concelhos" to be a better guide, then the inequality effect of inflation with age was overstated in the Centre and understated in the South. But we must again bear in mind the influence of the small number of observations in the age class <25[3] on our conclusions.

In the period 1977-81 the most salient feature of the results for the "concelhos" in relation to their respective regions is the much more pronounced inequality effects of inflation in Lisbon and the Centre. Now in Lisbon the range in price indices is around 16 points (against 9 before) and around 21 percentage points in the Centre[4] (against no significant

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1. 15.7 when the new prices for Oporto are considered.
2. Although in both the Centre and the South the values for the youngest households were around 5.5 percentage points lower than the ones for the eldest households. However the other groups present more or less the same values.
3. Only 1 in the "concelhos" of the South and 6 in those of the Centre.
4. 7 if the group 25 is excluded.

TABLE 15

NORTH - "CONCELHO" OF OPORTO

Price Distributions by Age of Head of Household

Years / Age	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 25	245.2 (12.6)	239.5 (15.6)	282.6 (23.4)	275.8 (20.9)	7
25-45	241.8 (12.3)	238.9 (13.8)	287.7 (13.9)	279.4 (30.3)	265
45-65	242.4 (12.3)	240.0 (13.4)	290.8 (16.4)	285.1 (24.5)	374
≥ 65	244.2 (13.8)	240.7 (16.8)	291.7 (18.2)	284.8 (20.5)	167
All Households	242.6 (12.7)	239.8 (14.3)	289.9 (16.2)	283.1 (25.9)	813

Note: Standard deviations in parentheses

(1) "Old" prices for Oporto, including housing rents

(2) "Old" prices for Oporto, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Oporto

TABLE 16

CENTRE - "CONCELHOS" OF COIMBRA AND VISEU

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 25	234.8 (13.6)	219.5 (15.1)	267.3 (13.1)	6
25-45	234.7 (21.9)	222.5 (20.4)	281.1 (15.6)	114
45-65	241.3 (21.8)	228.9 (20.5)	284.5 (15.8)	173
≥ 65	245.0 (24.1)	231.7 (22.6)	288.6 (15.4)	74
All households	239.9 (22.5)	227.3 (21.1)	284.0 (15.9)	367

Note: Standard deviations in parentheses

(1) "Old" prices for Coimbra and Viseu, including housing rents

(2) "Old" prices for Coimbra and Viseu, excluding housing rents

(3) Prices for the Continent

TABLE 17

LISBON - "CONCELHO" OF LISBON

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)	No. Observ.
< 25	281.7 (23.5)	276.6 (25.8)	277.0 (18.7)	278.9 (22.1)	38
25-45	283.5 (15.0)	279.2 (17.2)	290.4 (16.7)	293.4 (19.3)	690
45-65	283.0 (15.2)	278.8 (17.3)	292.7 (18.1)	296.0 (20.6)	946
≥ 65	283.0 (17.6)	278.4 (20.4)	293.0 (20.3)	295.7 (22.9)	450
All Households	283.1 (15.9)	278.8 (18.0)	291.7 (18.3)	294.8 (20.8)	2124

Note: Standard deviations in parentheses

(1) "Old" prices for Lisbon, including housing rents

(2) "Old" prices for Lisbon, excluding housing rents

(3) Prices for the Continent

(4) "New" prices for Lisbon

TABLE 18

SOUTH - "CONCELHOS" OF EVORA AND FARO

Price Distributions by Age of Head of Household

Age \ Years	1971-76 (1)	1971-76 (2)	1977-81 (3)	No. Observ.
< 25	233.5 (0.0)	228.6 (0.0)	301.0 (0.0)	1
25-45	243.3 (13.6)	234.1 (14.3)	289.6 (18.7)	75
45-65	249.5 (19.6)	239.9 (19.2)	283.3 (15.0)	99
≥ 65	263.1 (20.7)	246.6 (21.9)	284.9 (21.8)	41
All Households	249.9 (19.2)	239.1 (18.7)	285.9 (17.9)	216

Note: Standard deviations in parentheses

(1) "Old" prices for Evora and Faro, including housing rents

(2) "Old" prices for Evora and Faro, excluding housing rents

(3) Prices for the Continent

differences before).

Once again, the results for the "concelhos" suggest that, if anything, the inequality effects of inflation may be underestimated when all the households for the region are included in the analysis.

3.5. A Regression Analysis

However important the income and the household's age may be for the existence of differential price indices, there is no assurance that some other factors such as the occupation of the head or the number of children may not have at least the same importance. Furthermore, in the previous sections we were not able to isolate and to measure the effects of the variables income and age, as the price indices were not broken down simultaneously by those two variables.

To obviate these difficulties a regression analysis was carried out where the price indices for each household were regressed on several socio-econonic variables. From the available information in the HES, four sets of dummy variables were selected: the first relates to the occupation of the head of the household, the second to his/her age, the third to the number of children resident in the household of less than eighteen years of age and the fourth relates to the location of the household (urban or rural area).

The only non-dummy variable to be included is income, and as before total expenditure is taken as its proxy.

The full list of the adopted dummy variables[1] is as follows:

D1 - Employers and self-employed in agriculture and industry.
D2 - Workers in agriculture and industry.
D3 - Members of professional bodies, executives, etc.
D4 - Non-actives, e.g. pensioners.
D5 - Others (such as employees in service industries).

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1. Initially there was a higher level of disaggregation, namely in the occupational class dummy set, but preliminary results suggested the above type of aggregation.

AH < 25	- Age of head less than 25 years
AH 25-45	- Age of head between 25 and 45 years
AH 45-65	- Age of head between 45 and 65 years
AH ≥ 65	- Age of head greater than or equal to 65 years
NC = 0	- Number of children equal to zero.
NC = 1	- Number of children equal to 1.
NC = 2	- Number of children equal to 2.
NC = 3	- Number of children equal to 3.
NC ≥ 4	- Number of children greater than or equal to 4.
Urban[1]	- If the household is situated in an urban area

As a constant is included in the regressions, the variables D5, AH< 25 and NC = 0 are not included to avoid the "dummy trap". A linear specification for the equation was chosen; some alternative specifications were tried, such as using the log of income instead of income, but only in a very few cases did any appreciable gains occur.[2]

In this type of study, where we want to make comparisons between different years and different regions, and where inevitably the estimated coefficients for some variables are not significant, we face a dilemma: either we drop those variables and try to get the best possible fit in each case and in so doing any comparison will be very difficult if not impossible; or we stick to the same specification, include all the variables, and the results will be directly comparable but we will not necessarily be using the best possible specification. In our case the second option was adopted, not only because it was essential to be able to make direct comparisons, but also because, as mentioned, no substantial gains would result in treating each case separately.

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1. In the HES 1973/74 an urban area is defined as one with more than 10,000 inhabitants.

2. Those gains were also virtually non-existent when "equivalent income" was substituted for income. Equivalent income was computed by dividing total income by the total consumption units in each household (using the ILO scale mentioned in the previous chapter).

Tables A24 to A31 in the appendix show the annual results for the four regions. The first striking feature is the low values exhibited by the R^2s. However, given the total number of observations in each regression, and the fact that we are dealing with cross-section data, a value of 0.20 for the R^2 may be indicating a "reasonable" fit and o.25 a "good" one.[1]

It is hardly a surprise to get these low values, if we remember the great dispersion in price indices existent in each income class, as indicated by the high values for the standard deviations. But here we are much more interested in regression than in correlation, and as such in the regression coefficients (and their statistical significance).

As far as the income variable is concerned only on three occasions is its coefficient insignificant: in 1972 (North and South) and in 1975 (South). With the exception of the year 1972 in the North, the analysis of variance for the price distributions by income also implies equal means for the income classes.

It is interesting to compare the results obtained with the price distributions by income class in the previous section and the coefficients for the income variable obtained with the regression analysis. In all cases the sign of the coefficients agrees with the price indices pattern observed in the income classes. The sign of the coefficients is almost always negative in the North, Centre and South, confirming the inverse relationship between income and price indices. The exceptions are 1972 (Centre), 1973 (North), 1976 (North, Centre and South). Only in Lisbon for the years 1974 and 1975 do the income coefficients show a negative sign.

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1. In chapter 5 this problem will be dealt with in more detail, namely the relationship between R^2, the estimated coefficients and the number and grouping of observations.

The highest absolute value for the income coefficient was in 1974 (South): the actual number -0.036 means that for each additional 100 thousand annual escudos inflation would be lower 3.6 percentage points. As the lowest and highest income classes are < 18 and ≥180 thousand escudos, if we assume the average income difference between them to be 180 thousand escudos, then the highest income class would experience lower inflation by 6.5 percentage points. That different is 11.6 if we use the data on the price distributions by income class. This means that other factors reinforce the effects of the variable income. The same happens in all the other years and regions with the exception of the year 1975 in Lisbon where the differentials due to the variable income are greater than those observed in the previous tabulations by income.

The results obtained in the same period for the set of dummy variables related with age are not so significant. It is rare to obtain significant coefficients and even more so for all of them at the same time. When any conclusions can be drawn, such as in 1971 in Lisbon, 1973 (North and Centre) and 1976 (North and South) they confirm the results obtained previously for those years with the price distributions by age.

If we consider the variables related to the number of children, some unexpected results were obtained. Not only were their coefficients in general significant, but also they indicate a trend for price indices to decrease with the number of children. This is particularly evident in Lisbon, and for all the regions the exceptions refer to the year 1974 (North, Centre and Lisbon). The maximum differences in the inflation experienced by childless households in relation to those with four or more children were 2.5 percentage points in 1971 (Lisbon) and 4 in 1974 (also in Lisbon).

The occupation of the head of the household seems not to be an important factor for the existence of price differentials. The coefficients of the dummy variables are not usually significant, and even when they are, the differences are very small. If anything can be concluded, it may be that the employers tend to experience inflation rates 0.2 percentage points higher than the workers.

The coefficients of the urban variable are almost always significant, but they change in sign from year to year, and only twice are they in absolute value greater than 1. Only in Lisbon can it be said that urban households frequently experience higher inflation than rural ones, but again by less than 1 percentage point per year.

The different explanatory power of the variables as evidenced in the period 1971-76 is confirmed for the yearly observations within the period 1977-81 using the Continent commodity prices (see tables A28 to A31 in appendix). The coefficients on income are always significant as are nearly always those of the urban variables. The number of children is again an important factor for the existence of price differentials, although in fewer cases than before. However, as their influence tends to pull into opposite directions in successive years, the end result for the whole period may be very small or even zero.

The variables related to the age of the household in the relatively few cases where all of them have significant coefficients confirm the conclusions already drawn. Again it is clear that occupational variables are not important and consequently nothing can be said about the inflation rates experienced by the employers and the workers.

The sign of the coefficient on the income variable is positive in 1977 and 1980 but negative in all other years, for all the regions. This coefficient is highest in absolute value in 1981, when it reached in the Centre region (-0.034) meaning a reduction in inflation of 3.4 points for each additional hundred thousand escudos. The annual response of inflation to income tends to be higher in the Centre region and to be lower in the South, North and Lisbon (in this order).

Tables 19 to 22 show the results obtained for the whole periods, 1971-76 and 1977-81. In the former period the income coefficients are all negative except for Lisbon. Exclusion of the housing rents accentuates the effects of the income variable. Once more we observe the high values reached by the income coefficients in the Centre region where for each additional thousand escudos there was 11.4 (16 if housing rents are excluded) percentage points lower inflation.

The regression results confirm the importance of the number of children in the North and in Lisbon, where price differentials actually decrease with the number of children. Households, with four or more children had price indices which were lower by 6 percentage points (9.6 in Lisbon) than for childless households.

Older households faced higher inflation rates in the Centre and the South, the difference between the eldest and youngest ones being 13 points in the Centre and 15 in the South. The importance of occupation is once more negligible. The sign of the urban variable is only significantly positive in Lisbon (around +3.3).

For the period 1977-81, the income coefficients are always negative, although they are not significant in the Centre and the South. The age of the household is important in the regions North, Centre and South, where older households face higher rates of inflation. An indication of that importance is given by the fact that the difference between the differential rates of the eldest and youngest households is more than double the difference between the highest and lowest income groups (calculated from the estimated income coefficients). Urban households tend to experience higher rates of inflation, particularly in Lisbon (+6.4 points).

TABLE 19

REGRESSIONS - NORTH

	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)
Constant	244.3* (105.7)	242,5* (95.7)	295.0* (106.6)	266.7* (56.2)
D1 (Employers, etc.)	0.308 (0.5)	0.504 (0.7)	0.287 (0.4)	2.175 (1.6)
D2 Workers	-1.037 (-1.7)	-0.713 (-1.1)	0.752 (1.0)	2.189 (1.7)
D3 (Profess., etc.)	2.329 (1.7)	2.384 (1.6)	1.652 (1.0)	2.497 (0.9)
D4 (Non-active)	-1.345 (-1.8)	-1.776* (-2.2)	-0.871 (-1.0)	-2.009 (-1.3)
AH 25-45	1.266 (0.6)	1.189 (0.5)	6.784* (2.5)	14.840* (3.1)
AH 45-65	0.718 (0.3)	1.033 (0.4)	7.634* (2.9)	13.730* (2.9)
AH ⩾ 65	1.703 (0.7)	1.655 (0.6)	9.106* (3.4)	15.820* (3.3)
NC = 1	-1.789* (-2.4)	-1.447 (-1.8)	-0.715 (-0.8)	-2.739 (-1.8)
NC = 2	-3.407* (-4.2)	-3.149* (-3.6)	0.130 (0.1)	-1.510 (-0.9)
NC = 3	-5.622* (-6.0)	-5.402* (-5.3)	0.106 (0.1)	-2.590 (-1.3)
NC ⩾ 4	-6.003* (-7.0)	-5.448* (-5.8)	0.761 (0.8)	-2.328 (-1.3)
URBAN	-0.689 (-1.4)	-1.170* (-2.1)	0.746 (1.3)	5.043* (4.9)
INCOME	-0.007* (-2.7)	-0.015* (-3.6)	-0.021* (-5.0)	-0.028* (-3.7)
R^2 adj	0.03	0.02	0.09	0.01
F	10.51	9.37	3.99	4.44
St. Dev.	14.9	16.3	17.2	30.6

Note: t - statistics in parenthesis
* - significant at 0.05%

(1) "Old" prices for Oporto, including housing rents
(2) "Old" prices for Oporto, excluding housing rents
(3) Prices for the Continent
(4) "New" prices for Oporto

TABLE 20

REGRESSIONS - CENTRE

	1971-76 (1)	1971-76 (2)	1977-81 (3)
Constant	246.0* (79.6)	237.9* (85.7)	278.8* (110.4)
D1 (Employers, etc.)	1.933* (2.1)	2.358* (2.9)	2.340* (3.1)
D2 Workers	-0.287 (-0.3)	0.336 (0.4)	3.286* (4.2)
D3 (Profess., etc.)	-0.485 (-0.2)	1.70 (0.7)	4.689* (2.2)
D4 (Non-active)	2.34* (2.0)	0.114 (0.1)	0.955 (1.0)
AH 25-45	4.94 (1.6)	5.026 (1.8)	4.564 (1.8)
AH 45-65	8.78* (2.9)	9.574* (3.5)	4.889* (2.0)
AH $\geqslant$ 65	13.0* (3.1)	13.0* (4.7)	5.401* (2.1)
NC = 1	-2.08* (-2.0)	-0.599 (-0.6)	-1.803* (-2.1)
NC = 2	-1.749 (-1.5)	0.880 (0.8)	0.218 (0.2)
NC = 3	-0.511 (-0.4)	2.597* (2.0)	-0.495 (-0.4)
NC $\geqslant$ 4	-0.324 (-0.2)	4.079* (3.1)	0.390 (0.3)
URBAN	0.427 (0.5)	-2.508* (-3.2)	0.177 (0.3)
INCOME	-0.114* (-18.9)	-0.160* (-29.3)	-0.005 (-1.0)
R^2 adj.	0.15	0.25	0.01
F	57.3	111.4	2.75
St. Dev.	21.3	19.2	17.4

Note: t - statistics in parenthesis
* - significant at the 0.05%

(1) "Old" prices for Coimbra and Viseu, including housing rents
(2) "Old" prices for Coimbra and Viseu, excluding housing rents
(3) Prices for the Continent

TABLE 21

REGRESSIONS - LISBON

	1971-76 (1)	1971-76 (2)	1977-81 (3)	1977-81 (4)
Constant	278.5* (152.8)	272.7* (131.6)	277.1* (139.8)	280.3* (126.8)
D1 (Employers etc.)	0.130 (0.2)	0.349 (0.4)	2.924* (3.5)	2.768* (3.0)
D2 Workers	-3.149* (-5.7)	-2.665* (-4.2)	0.351 (0.6)	0.962 (1.4)
D3 (Profess. etc.)	-2.768* (-2.7)	-3.486* (-3.0)	-0.369 (-0.3)	-0.977 (-0.8)
D4 (Non-active)	-0.731 (-1.0)	-1.372 (-1.7)	-1.165 (-1.5)	-1.612 (-1.9)
AH 25-45	1.842 (1.0)	2.291 (1.1)	7.616* (3.9)	8.766* (4.0)
AH 45-65	0.963 (-0.5)	-0.111 (-0.1)	8.665* (4.5)	9.748* (4.5)
AH ≥ 65	-0.646 (-0.3)	-0.195 (-0.1)	9.538* (4.7)	9.942* (4.4)
NC = 1	-3.189* (-5.1)	-2.851* (-4.0)	-0.183 (-0.3)	-0.299 (-0.4)
NC = 2	-5.284* (-7.0)	-4.971* (-5.8)	0.264 (0.3)	0.126 (0.1)
NC = 3	-9.576* (-8.4)	-9.530* (-7.3)	-1.341 (-1.1)	-1.506 (-1.1)
NC ≥ 4	-9.631* (-7.1)	-8.926* (-5.8)	0.013 (0.1)	-0.173 (-0.1)
URBAN	3.255* (6.9)	3.449* (6.5)	6.379* (12.5)	7.125* (12.5)
INCOME	0.032* (9.8)	0.035* (9.5)	-0.012* (-3.5)	-0.030* (-7.7)
R^2 adj	0.06	0.05	0.03	0.04
F	30.2	23.0	15.3	17.6
St. Dev.	16.2	18.4	17.6	19.6

Note: t - statistics in parenthesis. *significant at 0.05%

(1) "Old" prices for Lisbon, including housing rents
(2) "Old" prices for Lisbon, excluding housing rents
(3) Prices for the Continent
(4) "New" prices for Lisbon

TABLE 22

REGRESSIONS - SOUTH

	1971-76 (1)	1971-76 (2)	1977-81 (3)
Constant	241.7* (52.4)	237.5* (52.2)	274.1* (58.5)
D1 (Employers, etc.)	2.228 (1.7)	2.426 (1.8)	0.149 (0.1)
D2 Workers	-0.474 (-0.4)	1.00 (1.0)	-0.875 (-0.8)
D3 (Profess., etc.)	7.433 (1.7)	6.532 (1.5)	9.626* (2.2)
D4 (Non-active)	3.547* (2.4)	1.919 (1.3)	0.157 (0.1)
AH 25-45	6.699 (1.4)	4.702 (1.0)	8.896 (1.9)
AH 45-65	9.664* (2.1)	8.311 (1.8)	5.934 (1.3)
AH ≥ 65	15.45* (3.3)	12.75* (2.8)	5.057 (1.07)
NC = 1	-2.465 (-1.9)	-1.427 (-1.1)	-0.808 (-0.6)
NC = 2	-3.629* (-2.3)	-2.608 (-1.7)	-4.520* (-2.8)
NC = 3	-1.105 (-0.4)	0.616 (0.2)	0.181 (0.1)
NC ≥ 4	-0.790 (-0.3)	0.474 (0.1)	-1.795 (-0.6)
URBAN	-0.035 (-0.0)	-3.187* (3.1)	4.250* (4.1)
INCOME	-0.055* (-6.6)	-0.069* (-8.5)	-0.010 (-1.2)
R^2 adj.	0.11	0.12	0.01
F	17.6	18.6	2.95
St. Dev.	17.5	17.3	17.8

Note: t - statistics in parenthesis
* - significant at the 0.05%
(1) "Old" prices for Evora and Faro, including housing rents
(2) "Old" prices for Evora and Faro, excluding housing rents
(3) Prices for the Continent

The number of children is not an important variable in this period in any of the regions, as is not the occupation of the head of the household. Concerning this latter aspect, the only thing that can be concluded is that in the Centre region in this period, workers experienced more inflation than employers (+1.9 points) and the members of professions more than workers (+1.4). Finally, it should be said that the inequality effects of income and age are more pronounced in the North and Lisbon if instead of the Continent prices the commodity prices for the cities of Oporto and Lisbon are used instead.

In summary, the results obtained with the regression analysis lead to the following general conclusions:

1) They emphasise the importance of income to the existence of differential inflation - the more income the less inflation, with the only exception to this being the Lisbon region, from 1971 to 1976.

2) For periods when it is significant, the age of the head of the household is generally directly related to the inflation rate.

3) The same can be said for the number of children, but with an inverse relationship.

4) The occupation of the head of the household, by itself, is not an important factor in 'explaining' differential inflation.

5) The urban dummy variable was found to be significant but with opposite signs from year to year. There is more regularity in the Lisbon region where urban households tend to experience higher inflation rates than rural ones.

3.6. The Importance of the Expenditure Weights

With the adopted methodology, any difference in inflation rates among households belonging to the same region is due to their different expenditure weights; between regions those differences may also be the result of the non-identical commodity price rises observed in the cities. The non-uniformity of the results obtained for the four regions when the same set of prices was used (Continent prices) is an indication of some variation in expenditure weights.

Table 23 shows the expenditure weights by class of expenditure for all the regions, and for the ten following aggregates[1]: food, drink, clothing and footwear, tobacco, housing, household goods, fuel and light, transport, services and miscellaneous.[2] This table enables us to draw some interesting conclusions. As expected food is the most important item in any household budget. It represents around 60% of the lowest income household's total expenditure, but that percentage continually decreases with income until it reaches 30% for the top income ones. There are differences among the regions, perhaps the most important being that the "all households" average for food is about 6 points higher in the South than in the other regions. These higher values are the result of the higher weights exhibited by the lower income groups in the South.

In all the regions the percentage of total expenditure on drink increases first with income and then decreases, but the highest income households show a considerably lower percentage than those at the bottom of the scale. The "all households" average is much higher in the North

1. The expenditure weights using the maximum disaggregation (for the "old" and "new" prices) are shown in appendix - tables A32 to A39.

2. Housing includes housing rents and water; services comprise health, entertainment and communications (post-office, etc.); miscellaneous include personal and housing hygiene and domestic services.

TABLE 23
WEIGHTS BY CLASS OF EXPENDITURE

NORTH

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180	ALL
FOOD	59.66	57.68	54.55	52.92	49.40	45.95	41.54	30.93	49.98
DRINK	8.13	10.49	11.62	10.63	9.41	7.53	5.66	2.97	9.03
CLOTH.FOOTW	6.45	6.85	8.44	9.43	10.65	11.10	11.63	9.33	9.50
TOBACCO	1.04	1.40	1.34	1.34	1.41	1.28	1.07	0.75	1.27
HOUSING	12.35	9.57	8.81	8.59	9.23	10.24	10.92	17.99	10.12
HOUS.GOODS	0.61	1.38	2.35	3.15	4.23	4.60	5.90	7.34	3.62
FUEL,LIGHT	6.45	5.84	4.42	3.79	3.11	2.74	2.56	2.34	3.78
TRANSP.	0.85	0.93	1.58	2.34	3.84	5.91	8.83	13.13	4.04
SERVICES	3.05	4.33	4.85	5.77	6.36	7.97	8.31	10.27	6.21
MISCELLAN.	1.41	1.55	2.04	2.02	2.36	2.70	3.58	4.94	2.43

CENTRE

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180	ALL
FOOD	62.93	58.38	54.66	52.87	49.26	44.12	38.71	30.50	51.27
DRINK	5.86	8.45	9.13	8.70	7.62	5.81	4.75	2.59	7.41
CLOTH.FOOTW	6.93	8.47	10.21	10.71	11.90	13.29	12.68	11.46	10.64
TOBACCO	0.78	0.88	0.77	0.92	0.90	0.77	0.68	0.58	0.82
HOUSING	9.99	8.09	7.86	7.96	8.54	9.83	9.99	14.58	8.89
HOUS.GOODS	0.97	1.73	2.58	3.05	4.33	5.86	6.79	7.32	3.60
FUEL,LIGHT	7.13	5.17	4.41	4.15	3.14	2.89	2.79	2.03	4.08
TRANSP.	0.52	1.11	1.70	2.72	4.15	6.18	11.27	17.13	3.98
SERVICES	3.00	5.62	6.40	6.30	7.62	8.54	9.27	9.62	6.80
MISCELLAN.	1.88	2.10	2.30	2.61	2.52	2.70	3.07	4.20	2.50

LISBON

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180	ALL
FOOD	61.25	59.66	55.53	54.00	50.63	47.70	41.86	32.71	49.88
DRINK	2.83	4.81	5.68	5.15	4.68	3.74	2.78	1.82	4.21
CLOTH.FOOTW	5.72	6.84	7.69	8.66	8.67	8.85	8.34	7.20	8.11
TOBACCO	0.73	1.01	1.28	1.45	1.23	1.09	1.01	0.83	1.14
HOUSING	15.79	12.89	12.49	12.66	12.97	12.82	13.25	14.24	13.08
HOUS.GOODS	0.71	1.03	1.99	2.24	3.59	4.72	5.99	6.13	3.55
FUEL,LIGHT	5.36	4.68	3.86	3.23	2.98	2.66	2.49	2.07	3.19
TRANSP.	1.51	1.49	2.82	3.62	5.13	6.73	10.48	15.02	5.90
SERVICES	3.73	4.80	5.92	6.35	7.11	8.13	9.48	12.94	7.46
MISCELLAN.	2.36	2.79	2.73	2.62	3.00	3.56	4.35	7.03	3.46

SOUTH

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180	ALL
FOOD	66.26	62.70	59.52	54.47	51.21	47.74	37.33	30.46	56.73
DRINK	2.47	4.30	4.93	4.98	4.54	3.83	2.44	1.53	4.15
CLOTH.FOOTW	6.89	8.67	9.38	10.95	10.53	10.09	9.59	7.03	9.28
TOBACCO	1.22	2.21	2.09	2.07	2.04	1.96	1.17	0.99	1.91
HOUSING	11.07	8.82	8.53	9.19	10.17	10.28	12.94	15.22	9.76
HOUS.GOODS	0.84	1.19	2.47	3.53	4.11	5.48	5.38	6.13	2.82
FUEL,LIGHT	4.91	4.21	3.62	3.29	2.87	2.58	2.58	1.86	3.58
TRANSP.	0.61	0.63	1.94	2.91	4.41	8.15	16.99	20.76	3.61
SERVICES	3.02	3.46	4.81	5.81	7.10	6.31	7.92	10.62	5.20
MISCELLAN.	2.71	2.82	2.73	2.81	3.04	3.61	3.66	4.69	2.95

and Centre (9 and 7.4%) than in Lisbon and the South (around 4%).

Except for the top income class in the North and Centre, and the top two in Lisbon and the South, the clothing and footwear weights increase with income (from around 6.5% to 12% in the North and Centre, and to 9-10% in Lisbon and the South).

The same increases with income exist, and in a very marked way with the aggregates "household goods", "transport" and "services". The increased weighting for "transport" is particularly marked: from around 0.6% for the bottom households to 13% (North), 17% (Centre), 15% (Lisbon) and 20% (South) for the higher income ones.

The proportion of expenditures on "housing rents" first decreases with income and then increases, but within this pattern there are some significant differences among the regions. As anticipated, the average for Lisbon is higher than for the other regions - 13% against 10%.

The weights for "fuel and light" decrease monotonically with income, but while the final values are almost the same in all the regions (around 2%), the initial ones vary from the 6.5 - 7% in the North and Centre to 5% in Lisbon and the South.

This brief enumeration of the results and a simple inspection of table 23 show that there is a wide variation in expenditure weights and that they are closely related to the household's income. The differences between the regions are not as pronounced as those within them, but they exist nevertheless.

It is clear, for instance, that price increases in food, drink or fuel and light by themselves will have the effect of increasing inequality while those on clothing, household goods or transport will have the effect of reducing it. In chapter 6 we measure the effects on equality of 20% price increases in each of these aggregates.

Finally, it is worth mentioning that the weights used by the INE in the computation of the new price indices are the following.[1]

Food	45.64
Drink	3.98
Clothing and footwear	9.35
Tobacco	1.25
Housing	12.76
Household goods	5.24
Fuel, light	2.78
Transport	7.44
Services	7.98
Miscellaneous	3.58

These weights were also drawn from the HES 1973/74, but, among other things[2] the two bottom classes (<18, 18-30) and the top one (≥180) were excluded. The presumed intention is to arrive at values that are representative of the "average" household. Was this achieved? On an "a priori" basis we could cast some doubts on it, because as already emphasized, the way the weights are computed by the INE,[3] make them biased in favour of the higher income households.

If we compare the above weights with those in table 23, we can see that they roughly represent the expenditure weights of the following percentiles of income:[4]

1. Computed from the more disaggregated weights included in the annex of the book by Carvalho (1978).

2. See chapter 2, section 2.3 for a complete definition of the reference households adopted in the "new" indices.

3. And indeed, as far as I know, by almost all the similar bodies in the world.

4. The eight income classes represent approximately the following percentiles in each region: North - 1.6, 4.9, 17.8, 26.5, 50.9, 66.4, 82.9, 100; Centre - 2.6, 7.7, 23.6, 33, 57.8, 72.4, 86.5, 100; Lisbon - 1, 3.3, 13.5, 20.4, 43.5, 61.1, 80.4, 100; South - 4.9, 13.1, 36, 46.2, 67.4, 78.6, 89.7, 100.

	North	Centre	Lisbon	South
Food	65	70	65	79
Drink	86	87	50	70
Clothing and Footwear	26	20	-[1]	36
Household goods	70	65	70	75
Fuel, light	66	86	50	68
Transport	70	75	65	74
Services	66	63	60	89
Miscellaneous	83	90	61	78

No conclusions can be reached for "tobacco" due to the absence of any pattern, and for "housing" due to its U-shape behaviour. But if "clothing and footwear" is excluded, we can see that the expenditure weights used by the INE represent households above the 65th percentile in the North, the 63rd in the Centre, the 50th in Lisbon and the 68th in the South. Food, by far the most important aggregate of them all, is given a weight of 45.64 which corresponds to households always above the 65th percentile in all the regions. We can conclude, as expected, that the weights used by the INE reflect expenditure patterns of the higher income households.[2]

1. No class of income reaches the value 3.98.

2. We must bear in mind that this conclusion was arrived at by using very wide groupings of expenditure. Considerable variations exist within them, and they may be important with the disaggregation adopted in this study. The figures for the percentiles of income should then be regarded as rough approximations.

CHAPTER 4

The Cost-of-Living Index and the Linear Expenditure System

4.1 The (Static) Theory of Consumer Behaviour

Utility and preferences

A consumer is considered to be an individual, a household, or even a larger group with a common purpose. We assume the existence, for each (or for the representative) consumer a utility function of the form

$$u = f(q_1, \ldots\ldots, q_n) \tag{1}$$

where $q_1, q_2, \ldots\ldots, q_n$ are the quantities of the goods $Q_1, \ldots., Q_n$ that are consumed; u is the utility or satisfaction derived from the consumption of alternative consumption bundles. It is assumed that $f(q_1, \ldots, q_n)$ is continuous, has continuous first- and second-order partial derivatives and is a strictly quasi-concave function.[1] The partial derivatives of (1) are also assumed to be strictly positive, so that the consumer will always want more of all commodities. Some of these assumptions are made to allow the mathematical calculus to be employed.

The number of commodities n is a finite number and we suppose that each commodity can be characterized numerically. There exists a unit of measurement for each commodity (e.g. in litres, kilos or in quantity). This means that any bundle of the n commodities can be numerically expressed as a vector $(q_1, q_2, \ldots, q_n)$ where each component always refers to the same commodity.

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1 A scalar function $f(q_1, \ldots, q_n)$ is said to be quasi-concave if for q^1, q^o such that $f(q^1) \geq f(q^o)$ and for $0 \leq \lambda \leq 1$, $f(\lambda q^1 + (1-\lambda)q^o) \geq f(q^o)$.

The domain of the function (1) (commodity set) is a subset of the n-dimensional space R^n of real numbers, and can be defined by the following properties:

1 - The commodity vector $q = (q_1, \ldots, q_n)$ can have no negative components.

2 - If the bundle $q^o = (q_1^o, q_2^o, \ldots, q_n^o)$ is available to the consumer, then any bundle $\lambda q^o = (\lambda q_1^o, \lambda q_2^o, \ldots, \lambda q_n^o)$, with $0 \leq \lambda \leq 1$ will also be available to him. This is called the property of divisibility.

3 - The commodity set is unbounded from above, that is, if a bundle q^1 belongs to the set than any bundle q^2 where $q_i^2 \geq q_i^1$, $i = 1, \ldots, n$ also belongs to it.

The utility number u^i associated with a particular commodity bundle indicates that it is preferable or superior to any other bundle with lower numbers and inferior to those with higher numbers. In this sense the utility function is a numerical representation of a preference ordering.

Not all the preference orderings can be represented by a utility function with the properties mentioned above. To do so, preference orderings must satisfy some "axioms of choice', defined over some field of choice. Let us define a relation where there symbol R means 'at least as good as". Hence q^1Rq^2 means that the bundle of goods q^1 is at least as good as (is preferred to or indifferent to) the bundle q^2. Given the relation R, the axioms of choice needed for the existence of a utility function are as follows:

Axiom 1 - Reflexivity. For any bundle q, qRq.

It has a trivial meaning, although it is mathematically necessary. It says that each bundle is as good as itself.

Axiom 2 - Completeness. For any two bundles q^1, q^2, either

$$q^1 R q^2 \text{ or } q^2 R q^1.$$

This axiom is sometimes called the axiom of comparability or of connectedness. It says that given any two bundles the consumer is always able to judge between them. When q^1Rq^2 and q^2Rq^1 hold simultaneously we say that q^1 is indifferent to q^2.

Axiom 3 - Transitivity or consistency. If q^1Rq^2 and q^2Rq^3, then q^1Rq^3.

If, given any three bundles, the first is preferred (indifferent) to the second, and the second is preferred (indifferent) to the third, then the first must be preferred (indifferent) to the third.

This axiom is central to the theory of choice and it implies that the consumer's preferences must be consistent: the consumer never contradicts himself.

Axioms 1 to 3 define a preordering or weak ordering. Although some preorderings can be represented by utility functions, there are cases where they cannot. Perhaps the most cited one is the lexicographic ordering.[1] In the lexicographic ordering commodity bundles are ordered in the same way as words in the dictionary. An example can be a situation where a consumer always prefers a bundle which contains alcohol, so that between two bundles he will always choose that containing more alcohol. The other commodities will only be important for his choice if the bundles have the same quantity or no alcohol.

It is obvious that this preference relation satisfies axioms 1 to 3, but that it is impossible to number the bundles in such a way that bundles with higher numbers are preferred to those with lower numbers. Each bundle has no points (other than itself) to which it is indifferent. There are no indifference surfaces and so no utility function exists; the preferences are discontinuous.

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1 See Debreu (1959)

To rule out lexicographics orderings a new axiom is needed,

Axiom 4 - Continuity. For any bundle q^1, the set of bundles "at least as good as" q^1 and the set of bundles to which q^1 is "at least as good as" are closed, that is, contain their own boundaries.

Debreu (1959) proves that axioms 1 to 4 are sufficient conditions for the existence of a real-valued utility function which is a continuous function of the quantities consumed, such that $u(q^o) \leq u(q^1)$ when $q^o R q^1$. Debreu does not prove that the utility function is unique. Any monotonic transformation of a utility function is also a utility function - utility is ordinal rather than cardinal.

Axiom 5 - Convexity. If $q^1 R q^o$, then for $0 \leq \lambda \leq 1$, $\lambda q^1 + (1-\lambda) q^o R q^o$.

This is equivalent to saying that indifference curves are convex to the origin and it is a necessary and sufficient condition for the utility function to be quasi-concave.

Axiom 6 - Non-satiation. The consumer will never be satisfied, that is, he will always want to consume more of each and all the goods. This implies that the partial derivatives of the utility function will be positive.[1]

Utility and demand

When the consumer's preferences satisfy all the axioms of choice and his commodity (feasible) set has the properties mentioned above, the consumption analysis can be carried out within the framework of the utility function (1) with the advantage that very well known and powerful calculus techniques can be used.

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1 It also implies that the equilibrium point of the consumer will be on his budget line and will not be an interior point.

The consumer is supposed to purchase optimal quantities of the n consumer goods, given his budget constraint and his utility function. The budget constraint is given by his fixed income y. This income y will be allocated among all the items in such a way that total expenditures will be exactly equal to it. The problem of saving (dissaving) is not considered. y can be looked at as the amount the consumer dedicates to consumption after having decided the amount he is going to save.

The prices p_i $(i = 1,\dots, n)$ of the n commodities are also supposed to be fixed. From this it follows that the optimal quantities $q_i (i = 1,\dots, n)$ purchased by the consumer (his consumption plan) will be those that optimise his utility function subject to the budget constraint. That is, the consumer

$$\text{Maximizes } u = f(q_1,\dots, q_n)$$

$$\text{s.t.} \qquad y = \Sigma\, p_i\, q_i$$

This is a classical calculus problem. To solve it, we have to form the Lagrangean function

$$L = u(q_1,\dots, q_n) + \lambda(y - \Sigma\, p_i\, q_i)$$

where λ is the Lagrange multiplier. The first order conditions for a maximum are obtained by differentiating L with respect to q_i and λ,

$$\frac{\partial L}{\partial q_i} = \frac{\partial u}{\partial q_i} - \lambda p_i \qquad (i = 1, \dots, n)$$

$$\frac{\partial L}{\partial \lambda} = y - \Sigma\, p_i\, q_i$$

and by setting them to zero,

$$\frac{\partial u}{\partial q_i} = \lambda p_i \qquad (i = 1, \dots, n)$$

$$\Sigma\, p_i\, q_i = y \qquad (2)$$

The second order conditions for the existence of a global maximum are that the bordered Hessian determinants must alternate in sign i.e.

$$\begin{vmatrix} u_{11} & u_{12} & -p_1 \\ u_{21} & u_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{vmatrix} > 0, \qquad \begin{vmatrix} u_{11} & u_{12} & u_{13} & -p_1 \\ u_{21} & u_{22} & u_{23} & -p_2 \\ u_{31} & u_{32} & u_{33} & -p_3 \\ -p_1 & -p_2 & -p_3 & 0 \end{vmatrix} < 0,$$

$$\ldots, (-1)^n \begin{vmatrix} u_{11} & u_{12} & \ldots & u_{1n} & -p_1 \\ u_{21} & u_{22} & \ldots & u_{2n} & -p_2 \\ u_{n1} & u_{n2} & \ldots & u_{nn} & -p_n \\ -p_1 & -p_2 & \ldots & -p_n & 0 \end{vmatrix} > 0 \qquad (3)$$

where the u_{ij} are the second order partial derivatives. These second order conditions will be automatically satisfied if the utility function is strictly quasi-concave.

The solution of the (n + 1) first order conditions (2) provide the n optimal values of q_i and the equilibrium value of λ.

Each q_i will be a function of all prices and of y,[1] and these functions are called the demand functions.

The first n equations of (2) can be written as

$$\frac{\partial u/\partial q_1}{p_1} = \frac{\partial u/\partial q_2}{p_2} = \ldots = \frac{\partial u/\partial q_n}{p_n} = \lambda \qquad (4)$$

which express the fact that[2] in equilibrium the marginal utility of the last unit of value (e.g. pound) spent on each item must be equal for all of them.

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1 This result can be achieved by using the implicit function theorem. See Intriligator (1971).

2 In a cardinal utility interpretation.

The first order condition

$$\frac{\partial u}{\partial q_i} = \lambda\ p_i$$

can be written as

$$\frac{\partial u}{\partial (p_i\ q_i)} = \lambda$$

because p_i is a constant. At equilibrium, an additional pount spent on any good i must provide the same increase in utility, λ. The product $p_i\ q_i$ is the expenditure on good i. So λ can be said to be the marginal utility of the total expenditure,

i.e. $$\lambda = \frac{\partial u}{\partial y}$$

If the utility function is replaced by a monotonic transformation of it, the demand functions obtained in the two cases by constrained maximisation are the same. To see this, let us consider the transformation $v = F(u)$, with $F' > 0$. The first order conditions of the constrained maximization of v are,

$$\frac{\partial v}{\partial q_i} - \lambda'\ p_i = 0 \qquad (i = 1, \ldots, n)$$

$$y = \Sigma\ p_i\ q_i \qquad (5)$$

λ' is the new Lagrange multiplier associated with the maximization of v. The above first n conditions can be written as

$$F'\ \frac{\partial u}{\partial q_i} - \lambda'\ p_i = 0$$

or

$$\frac{\partial u}{\partial q_i} - \frac{\lambda'}{F'}\ p_i = 0$$

But λ'/F' is equal to λ, since $\lambda' = F'(\partial u/\partial y) = F'\lambda$. The first order conditions (5) are then equivalent to (2). As the second-order conditions will also be satisfied,[1] we have proved the ordinality of the utility function.

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1 See Lancaster (1968)

Up until now we have used "direct" utility, that is, utility functions whose arguments are the quantities of the commodities bought by the consumer (q_i's). But the constrained maximization of the direct utility function leads to a system of demand equations of the type

$$q_i^o = g_i(p_1, \ldots, p_n, y) \qquad (i = 1, \ldots, n)$$

where q_i^o is the optimal quantity of q_i. If the optimal q_i^o's are substituted for the q_i's in the direct utility function, we end up with what is called an indirect utility function:

$$u^* = f[g_1(p_1, \ldots, p_n, y), g_2(p_1, \ldots, p_n, y), \ldots, g_n(p_1, \ldots, p_n, y)]$$

$$= f^* (p_1, \ldots, p_n, y)$$

The arguments of the indirect utility function are now all the prices and income.

Demand functions

Given the hypotheses about the utility function, the implicit function theorem guarantees the existence of an optimal solution $(q_1^o, \ldots, q_n^o)$ for the system of the first order conditions (2). That optimal solution is a function of all the prices and income,

i,e,

$$q_1^o = g_1(p_1, \ldots, p_n, y)$$

$$q_n^o = g_n(p_1, \ldots, p_n, y) \qquad (6)$$

and

$$\lambda^o = \lambda^o(p_1, \ldots, p_n, y)$$

The system (6) of the functions $g_i (i = 1, \ldots, n)$ is called a demand system. Each demand function will have some properties which are a direct result of the demand functions being obtained by constrained maximization of a utility function. Only the functions possessing those properties can be called demand functions.

The properties are useful in the estimation of complete demand systems like (6), or even of a single demand function because they constitute restrictions on their specification and parameter values, with the result that less data is necessary to estimate them.

The properties of the demand functions are the following:

1 - Homogeneity of degree zero A demand function is homogeneous of degree zero in prices and income. This means that if all prices and income are multiplied by the same positive constant K, the quantity demanded will remain unaltered. This result can be achieved from the first order conditions (2) which can be written as

$$\frac{\partial u/\partial q_i}{\partial u/\partial q_j} = \frac{p_i}{p_j} \qquad i \neq j$$

$$y = \Sigma\, p_i\, q_i \tag{7}$$

If we multiply all the prices and income by K, the resultant system is

$$\frac{\partial u/\partial q_i}{\partial u/\partial q_j} = \frac{K\, p_i}{K\, p_j} = \frac{p_i}{p_j}$$

$$Ky = K\, \Sigma\, p_i\, q_i \quad \text{or} \quad y = \Sigma\, p_i\, q_i$$

which is equal to (7).

Since the demand functions are homogeneous of degree zero on income and prices, then by Euler's theorem we can write

$$p_i \frac{\partial q_i}{\partial p_i} + \dots + p_n \frac{\partial q_i}{\partial p_n} + y \frac{\partial q_i}{\partial y} = 0$$

or

$$\sum_j p_j \frac{\partial q_i}{\partial p_j} + y \frac{\partial q_i}{\partial y} = 0 \qquad (i,j = 1, \dots, n) \tag{8}$$

If we divide (8) by q_i we obtain

$$\sum_j \frac{p_i}{q_i} \frac{\partial q_i}{\partial p_j} + \frac{y}{q_i} \frac{\partial q_i}{\partial y} = 0 \qquad (9)$$

Expression (9) states that the sum of all direct and cross price elasticities $\left(\sum_j \frac{p_i}{q_i} \frac{\partial q_i}{\partial p_j}\right)$ and the income elasticity $\left(\frac{y}{q_i} \frac{\partial q_i}{\partial y}\right)$ must be equal to zero.

2 - Adding up The demand functions must be such that the sum of the estimated or predicted expenditure values on each commodity must always be equal to total income, that is, the budget constraint must always be satisfied.

This is called the adding up restriction and can be stated in a different way:

Since $$y = \sum_i p_i q_i$$

by differentiating with respect to y, we get

$$\sum_i p_i \frac{\partial q_i}{\partial y} = 1$$

or, since p_i is a constant

$$\sum \frac{\partial (p_i q_i)}{\partial y} = 1 \qquad (10)$$

$\partial(p_i q_i)/\partial y$ is the marginal budget share or the marginal propensity to consume good i. From (10) we conclude that the sum of the marginal propensities to consume must be equal to one.

3 - Symmetry and Negativity - The Slutsky equation The quantities purchased by a rational consumer will always satisfy the system (2) of the first order conditions. If his income is changed and/or if the prices of the commodities are different, he will modify his consumption pattern, so

that, the quantities (q_i) purchased will vary. In spite of this the new prices, quantities and income will also satisfy (2). To analyse the new situation, let us derive the total differential of the equations in (2), for the simplified case where there are only two commodities:[1]

$$\begin{aligned} u_{11}\,dq_1 + u_{12}dq_2 - p_1\,d\lambda &= \lambda\,dp_1 \\ u_{21}\,dq_1 + u_{22}dq_2 - p_2\,d\lambda &= \lambda\,dp_2 \\ -p_1\,dq_1 - p_2\,dq_2 &= -d_y + q_1\,dp_1 + q_2\,dp_2 \end{aligned} \tag{11}$$

This is a system of three equations for the three unknowns, dq_1, dq_2 and $d\lambda$, if the terms on the right are regarded as constants.

Let D be the determinant of the matrix A of the coefficients,

$$A = \begin{bmatrix} u_{11} & u_{12} & -p_1 \\ u_{21} & u_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{bmatrix}$$

and D_{ij} (i,j = 1,2) be the cofactor of the element in line i and column j. The solution of (11), by Cramer's rule is

$$dq_1 = \frac{\lambda D_{11}dp_1 + \lambda D_{21}dp_2 + D_{31}(-dy + q_1\,dp_1 + q_2\,dp_2)}{D} \tag{12}$$

$$dq_2 = \frac{\lambda D_{12}dp_1 + \lambda D_{22}dp_2 + D_{32}(-dy + q_1\,dp_1 + q_2\,dp_2)}{D} \tag{13}$$

Dividing both sides of (12) by dp_1 and assuming p_2 and y constant ($dp_2 = dy = 0$), we get

$$\frac{\partial q_1}{\partial p_1} = \frac{D_{11}\lambda}{D} + q_1\,\frac{D_{31}}{D} \tag{14}$$

which gives an expression for the variation in the purchases of Q_1 when its price increases, all other things remaining constant.

From (12) we can also arrive at

$$\frac{\partial q_1}{\partial y} = -\frac{D_{31}}{D} \tag{15}$$

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1 For the more general case see for instance Phlips (1974).

If we consider a price change which is compensated by an income change such that the consumer is left on his initial indifference curve[1]. In this case the change in total utility is zero, so that

$$du = u_1\, dq_1 + u_2\, dq_2 = 0$$

since $u_1/u_2 = p_1/p_2$, then $p_1\, dq_1 + p_2\, dq_2 = 0$. Hence, from the last equation in (9)

$$-dy + q_1\, dp_1 + q_2\, dp_2 = 0$$

and from (10)

$$\left(\frac{\partial q_1}{\partial p_1}\right)_{u=const} = \frac{D_{11}\,\lambda}{D} \qquad (16)$$

Therefore equation (14) can be written as

$$\frac{\partial q_1}{\partial p_1} = \left(\frac{\partial q_1}{\partial p_1}\right)_{u=const} - q_1 \left(\frac{\partial q_1}{\partial y}\right)_{prices=const} \qquad (17)$$

Equation (17) is known as the Slutsky equation. If we multiply (17) by p_1/q_1 and only the last term by y/y, we obtain an alternative expression in terms of elasticities,

$$\varepsilon_{11} = \xi_{11} - \alpha_1\, \eta_1 \qquad (18)$$

Expression (18) indicates that the price elasticity of the ordinary or uncompensated demand curve (ε_{11}) is equal to the price elasticity of the compensated demand curve (ξ_{11}) less the income elasticity (η_1) multiplied by the proportion of total expenditures spent on Q_1.

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1 This means that the consumer is on a "compensated demand curve". This curve is also called a Hicksian demand curve and can be obtained by minimizing the consumer's expenditures subject to the constraint that his utility is at a fixed level u^o. The system of demand equations (4) are termed "uncompensated" or ordinary demand curves.

The first term on the right of (17) is called the 'substitution effect' (the rate at which the consumer substitutes Q_1 for other commodities when p_1 changes and he moves along the same indifference curve). The second term is the 'income effect' and measures the changes in the purchases of Q_1 when income changes and the prices remain constant.

All of the above analysis can be carried out in relation to the commodity Q_2, with essentially the same results.

Of the substitution and income effects, the only definite conclusion is that the own substitution effect will always be negative. Let us consider the expression for the substitution effect given in (16): D will always be positive because (see expression (4)), and also by assumption the marginal utilities and the prices are positive; $D_{11}=-p_2^2$, that is $D_{11} < 0$, implying that the own substitution effect is negative.

$$\frac{\partial q_i}{\partial p_j} = \frac{D_{ji}\lambda}{D} + q_j \frac{D_{3i}}{D} = \left(\frac{\partial q_i}{\partial p_j}\right)_{u=const} - q_j \left(\frac{\partial q_i}{\partial y}\right)_{prices=const.} \qquad (19)$$

and in elasticity form

$$\varepsilon_{ij} = \xi_{ij} - \alpha_j \eta_i \qquad (20)$$

Let $S_{ij} = D_{ji}\lambda/D$. S_{ij} is the substitution effect when the quantity of the <u>ith</u> commodity is adjusted as a result of a variation in the <u>jth</u> price. We already know that S_{ii} is always negative, but nothing can be said in relation to the sign of S_{ij}. However, we can conclude that the matrix of the substitution effects S_{ij} is symmetric, that is $S_{ij}=S_{ji}$. The reason is that as D is a symmetric determinant,[1] $D_{ij}=D_{ji}$ so that

$$S_{ij} = \frac{D_{ji}\lambda}{D} = \frac{D_{ij}\lambda}{D} = S_{ji} \qquad (21)$$

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1 Its matrix is symmetric. The reason is that it is formed by a matrix of continuous second-order partial derivatives (Symmetric by Young's theorem), bordered by the prices p_i in such a way that it stays symmetric.

4.2 The Linear Expenditure System

Let us suppose that we want to estimate demand functions by using regression methods. The first and most likely specification would be linear, of the form,

$$q_i = \sum_j \alpha_{ij} p_j + \beta_i y \qquad (22)$$

However, although in the expression above, the quantity demanded for a good is a function of all the prices and of income, it will only represent a demand function if it satisfies the general properties (restrictions) of a demand function: homogeneity, adding-up, symmetry of the substitution effects and negativity of the direct substitution effects.

It is immediately clear that expression (22) does not satisfy the homogeneity restriction: in fact if we multiply all the prices and income by a positive constant the quantity demanded instead of remaining the same is also multiplied by that constant. Although in a less obvious way, we could also show that the other restrictions are not satisfied.

The problem now is to find out whether it is possible to modify expression (22) in such a way that it will satisfy the restrictions and at the same time keep its linear specification. It was in this way that Klein and Rubin (1947/48) first introduced the Linear Expenditure System when they were attempting to construct a true cost-of-living index.[1]

Homogeneity can be introduced in (22) simply by dividing the independent variables by the price of one good, say p_i:

$$q_i = \sum_j \alpha_{ij} \frac{p_j}{p_i} + \beta_i \frac{y}{p_i} \qquad (23)$$

The symmetry of the cross-substitution effects implies that (from (19) and (21)),

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1 See also Phlips (1974). For an exposition of the LES in matrix terms (and for its first empirical estimation for the UK) see Stone (1954).

$$\frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial y} = \frac{\partial q_j}{\partial p_i} + q_i \frac{\partial q_j}{\partial y} \qquad (i \neq j)$$

so that we must have, from (23),

$$\frac{\alpha_{ij}}{p_i} + q_j \frac{\beta_i}{p_i} = \frac{\alpha_{ji}}{p_j} + q_i \frac{\beta_j}{p_j} \qquad (i \neq j)$$

or, by multiplying through by $p_i\ p_j$,

$$\alpha_{ij}\ p_j + \beta_i\ p_j\ q_j = \alpha_{ji}\ p_i + \beta_j\ p_i\ q_i \tag{24}$$

Substituting (23) for q_i and q_j in (24), we obtain

$$\alpha_{ij}p_j+\beta_i\left[\sum_{K=1}^{n} \alpha_{jK}p_K+\beta_j y\right] = \alpha_{ji}p_i+\beta_j\left[\sum_{K=1}^{n} \alpha_{in}p_K+\beta_i y\right]$$

or

$$\alpha_{ij}p_j+\beta_i \sum_K \alpha_{jK}p_K = \alpha_{ji}p_i+\beta_j \sum_K \alpha_{iK}p_K \qquad (i \neq j)$$

which can be written as

$$(\alpha_{ij} + \beta_i\alpha_{jj})\ p_j + \beta_i\alpha_{ji}p_i + \beta_i \sum_K \alpha_{jK}p_K$$

$$= (\alpha_{ji} + \beta_j\ \alpha_{ii})\ p_i + \beta_j\ \alpha_{ij}\ p_j + \beta_j \sum_K \alpha_{iK}\ p_K$$

for $K \neq i \neq j$

The coefficients of the p_K $(K = 1, \ldots, n)$ on the two sides must be equal (the equality must hold for all price systems), so that

$$\alpha_{ij} = \beta_i\left[\alpha_{jj}/(\beta_j-1)\right]$$

$$\alpha_{ii} = \beta_i\ (\alpha_{ji}/\beta_j) - (\alpha_{ji}/\beta_j) \qquad (i \neq j \neq K)$$

$$\alpha_{iK} = \beta_i(\alpha_{jK}/\beta_j)$$

If we define

$$\gamma_K = \alpha_{jK}/\beta_j \qquad \text{for } K \neq j$$

$$= \alpha_{jK}/(\beta_j-1) \qquad \text{for } K=j$$

then

$$\alpha_{iK} = \beta_i \gamma_K - \delta_{iK} \gamma_i \qquad (i, K=1, \ldots, n) \qquad (25)$$

where δ_{iK} is the Kronecker delta,

i.e.,
$$\delta_{iK} = 0 \quad \text{for } i \neq K$$
$$= 1 \quad \text{for } i = K$$

Equation (25) implies the proportionality of the cross-price derivatives to the income derivatives.

Imposition of the symmetry restriction (25) on the equation (23), leads to

$$\begin{aligned} p_i q_i &= \sum_j [\beta_i \gamma_j - \delta_{ij} \gamma_i] p_j + \beta_i y \\ &= \beta_i \sum_j p_j \gamma_j - p_i \gamma_i + \beta_i y \end{aligned} \qquad (26)$$

The adding-up restriction requires that

$$\begin{aligned} y = \sum_i p_i q_i &= \sum_i \beta_i \sum_j p_j \gamma_j - \sum_i p_i \gamma_i + \sum_i \beta_i y \\ &= \sum_j p_j \gamma_j [\sum_i \beta_i - 1] + y \sum_i \beta_i \end{aligned}$$

So that we must have $\sum_i \beta_i = 1$. This means that imposition of all the restrictions except the negativity of the direct substitution effect leads to the following expression (from (26)),

$$q_i = -\gamma_i + \frac{\beta_i}{p_i} (y + \sum_j p_j \gamma_j) \text{ with } \sum_i \beta_i = 1$$

or, since γ_i is any constant

$$q_i = \gamma_i + \frac{\beta_i}{p_i} (y - \sum_j p_j \gamma_j) \text{ with } \sum_i \beta_i = 1$$

The negativity of the direct substitution effect implies that

$$S_{ii} = \frac{\partial q_i}{\partial p_i} + q_i \frac{\partial x_i}{\partial y} < 0$$

so that in the system (27),

$$S_{ii} = -\frac{\beta_i}{p_i^2}(y - \sum_j p_j\gamma_j) - \frac{\beta_i}{p_i}\gamma_i + x_i\frac{\beta_i}{p_i}$$

$$= -\frac{q_i-\gamma_i}{p_i} + \frac{\beta_i}{p_i}(q_i - \gamma_i)$$

$$= \frac{q_i-\gamma_i}{p_i}(\beta_i - 1) < 0$$

so that $q_i > \gamma_i$ and $0 < \beta_i < 1$.

We can conclude then, that the equations of the Linear Expenditure System (LES) can be expressed as follows:

$$q_i = \gamma_i + \frac{\beta_i}{p_i}(y - \sum_j p_j \gamma_j)$$

or

$$p_i q_i = p_i \gamma_i + \beta_i(y - \sum_j p_j \gamma_j) \qquad (28)$$

with $0 < \beta_i < 1$, $\sum_i \beta_i = 1$ and $q_i > \gamma_i$

Samuelson (1947/48) suggested an interpretation for (28). Consumers would first try to secure their minimum subsistence levels by acquiring the 'minimum required quantities' γ_i of the commodities. After that they allocate the remaining income by all the commodities following the marginal propensities to consume β_i. Thus, $p_i\gamma_i$ measures the 'subsistence income' and $y - \sum_j p_j\gamma_j$ is the 'supernumerary income'.

The trouble with this interpretation is that it is only meaningful when the γ_i's are positive and there is no theoretical reason why this should always be so.

Samuelson (1947/48) and Geary (1950/51) have worked out the utility function that by constrained maximization leads to the LES in (28). It can be done as follows:

From (2) we know that at equilibrium $\partial u/\partial q_i = \lambda p_i$. Also at equilibrium, the increase in utility from any variation in the quantities purchased of the commodities is zero,

i.e.

$$du = \sum_i \frac{\partial u}{\partial q_i} \, dq_i = 0$$

so that

$$du = \sum_i \lambda p_i \, dq_i = 0$$

$$= \sum_i p_i \, dq_i = 0$$

But from (28)

$$p_i = \frac{\beta_i}{q_i - \gamma_i} \left(y - \sum_j p_j \gamma_j\right)$$

leading to

$$du = \sum_i \frac{\beta_i}{q_i - \gamma_i} \, r \, dq_i = 0 \qquad (29)$$

where $r = y - \sum_j p_j \gamma_j$. Since r may be eliminated in (29),

$$du = \sum_i \frac{\beta_i}{q_i - \gamma_i} \, dq_i = 0$$

or, by integrating

$$u = \int \sum_i \frac{\beta_i}{q_i - \gamma_i} \, dq_i$$

and finally

$$u = \sum_i \beta_i \log (q_i - \gamma_i) \qquad (30)$$

with $0 < \beta_i < 1$, $\sum \beta_i = 1$. This function is called the Stone-Geary utility function. Since any monotonic transformation of a utility function is also a utility function, the Stone-Geary function is often expressed as

$$u = \prod_i \left(q_i - \gamma_i\right)^{\beta_i} \qquad (31)$$

4.3 The Cost-of-Living Index in the LES

We saw in Chapter 2[1] that the "true" cost-of-living index between two periods, 0 and t, is the ratio of the minimum costs necessary to reach a specific indifference curve (utility level or standard of living) when the prices are different in the two periods.

The reference utility level can be any but usually is either the utility level of the base period (0) or of the final period (t). In the former case we are dealing with a base-weighted and in the latter with a current-weighted true cost-of-living index. In this chapter we will consider only base-weighted indices.

We are now in possession of all the information necessary to construct cost-of-living indices for the LES. Namely, we know the expressions for the utility and demand functions,

$$u = \Pi\ (q_i - \gamma_i)^{\beta_i} \qquad 0 < \beta_i < 1,\ \sum_i \beta_i = 1 \tag{32}$$

$$p_i\ q_i = p_i\ \gamma_i + \beta_i\ (y - \sum_j p_j\ \gamma_j) \tag{33}$$

Substituting the demand functions (33) into the utility function(32), we obtain the indirect utility function,

$$u = \Pi_i \left[\frac{\beta_i}{p_i} (y - \sum_j p_j\ \gamma_j) \right]^{\beta_i}$$

$$= \left(y - \sum_j p_j\ \gamma_j\right)^{\Sigma \beta_i} \Pi \left(\frac{\beta_i}{p_i}\right)^{\beta_i}$$

or finally

$$u = (y - \sum_j p_j\ \gamma_j)\ \Pi_i \left(\frac{\beta_i}{p_i}\right)^{\beta_i} \tag{34}$$

Let $a_t = \Sigma \gamma_j\ p_{jt}$ and $b_t = \Pi \left(\frac{p_{it}}{\beta_i}\right)^{\beta_i}$. Then (34) can be written as

$$u = (y - a)\ b^{-1}$$

1 Section 2.2.

In the initial period, with prices p_o and income level y_o, the utility level will be

$$u_o = (y_o - q_o)\, b_o^{-1}$$

In period $\underline{t}$, with prices p_t, the minimum income level y_t necessary to get u_o must satisfy the equality

$$u_o = (y_o - a_o)\, b_o^{-1} = (y_t - a_t)\, b_t^{-1}$$

So that

$$y_t b_t^{-1} = (y_o - a_o)\, b_o^{-1} + a_t\, b_t^{-1}$$

and finally,

$$y_t = a_t + (y_o - a_o)\, \frac{b_t}{b_o}$$

The cost-of-living index is then given by the ratio y_t/y_o, that is

$$CLI = \frac{C(u_o, p_t)}{C(u_o, p_o)} = \frac{y_t}{y_o} = \frac{1}{y_o}\left[a_t + (y_o - a_o)\, \frac{b_t}{b_o}\right]$$

$$= \frac{a_t}{a_o} + \left(1 - \frac{a_o}{y_o}\right) \frac{b_t}{b_o}$$

$$= \left(\frac{a_o}{y_o}\right) \frac{a_t}{a_o} + \left(1 - \frac{a_o}{y_o}\right) \frac{b_t}{b_o} \qquad (35)$$

Following the interpretation given by Samuelson for the LES parameters γ_i and β_i, we can say that "necessary" goods are those with low expenditure elasticities, i.e. high γ_i and low β_i. "Luxuries" will have low γ_i and high β_i.

The cost-of-living index is a weighted average of a_t/a_o and b_t/b_o. But $\frac{a_t}{a_o} = \frac{\Sigma\gamma_i p_{it}}{\Sigma\gamma_i p_{io}}$ is an arithmetic price index using "commited purchases"[1] as weights, so giving "necessities" high weights. And $\frac{b_t}{b_o} = \Pi\left(\frac{p_{it}}{p_{io}}\right)^{\beta_i}$ is a geometric price index using marginal propensities to consume as weights,

1 Muellbauer (1974)

giving high weights to "luxuries". The conclusion is that from (35) b_t/b_o will tend to dominate for a rich person and a_t/a_o will tend to dominate for a poor person.

4.4 The Cost-of-Living Index in Portugal

Martins and Oliveira (1979) estimated the parameters of the LES for Portugal. They used time-series data on the consumption expenditures from 1953 to 1973 published by the DCP (Central Planning Department). The price indices utilized were those implicit in the consumption series.

The estimated Linear Expenditure System was a variant of (33) where the β_i's were dynamized, that is

$$p_i q_i = p_i \gamma_i + (\beta_{oi} + \theta \beta_{1i}) (y - \sum_j p_j \gamma_j) \qquad (36)$$

θ is the time variable ($\theta = 1$ in 1953). Of course $\Sigma\beta_i$ must still be equal to unity and that implies that $\Sigma\beta_o = 1$ and $\Sigma\beta_1 = 0$. The method of estimation was the weighted-mean maximum likelihood (WMML) developed of Deaton.[1]

The paremeter estimates are indicated in table 1. The R^2's (not shown) were always high for all the equations. For each product the trend was very well defined, although the oscillations were generally understated, but in general the adjustment quality was deemed to be "quite acceptable".

With the estimates in table 1, computation of the income elasticities allows us to conclude that in 1973 the "luxuries" were (elasticity greater than one): Health and hygiene, Sugar and confectionary, Other services, Clothing and footwear, Entertainment, Education, Light and fuel, Transport and communications, Vehicles, Durables, Other food and soft drinks. All the other items were "normal" goods.

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1 See Deaton (1975a).

The estimated parameters also enable us to construct cost-of-living indices by using expression (35). Since in the previous chapter price indices for different expenditure classes were constructed based on the 1973/74 HES and on the prices for the six cities and the Continent, the cost-of-living indices can be computed in such a way that some direct comparisons are made possible.

For that reason the year 1973 was taken as the reference year for the utility level, tastes, prices and income. That is, in the expression

$$CLI = \frac{C(u_o, p_t)}{C(u_o, p_t)}$$

for the cost-of-living index, u_o is the utility level reached by consumers at prices p_o in 1973, and p_t are the prices in the other years. As the β_i's were dynamized, different tastes are assumed throughout the years. In order to preserve the ordinality of the utility function we have to keep those tastes constant, at a certain level. As mentioned above the chosen level was the one in 1973, so that the β_i's were calculated with $\theta = 20$.[1]

The CLI can be computed for different values of the income variable y_o in (35). In this study those were: 13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000 escudos and they correspond to the mean of each of the eight HES' expenditure classes in the region of Lisbon.[2]

There is the problem of what price information to use in (35). In the previous chapter we concluded that the variations in regional prices were an important factor for the behaviour of differential inflation. In

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1 The results for the CLI's were almost identical when the tastes for 1976 or 1981 were considered.

2 The variations between regions are small in this respect and are irrelevant as concerns the computations of the CLI's.

TABLE 1

Parameters of the Linear Expenditure System

Expenditure Groups	γ	β_0	β_1
1 Cereals	618.82	.06615	-.00214
	(25.40261)	(.00932)	(.00039)
2 Vegetables, fruit	820.28	.13391	-.00437
	(53.07078)	(.01398)	(.00045)
3 Meat and poultry	550.72	.01825*	.00299
	(20.18643)	(.01114)	(.00044)
4 Fish	282.96	-.00974*	.00249
	(8.81119)	(.00817)	(.00029)
5 Milk, cheese, eggs	193.62	.04092	-.00109
	(17.71438)	(.00627)	(.00025)
6 Fats	239.22	.01816	-.00004*
	(11.17825)	(.00576)	(.00024)
7 Coffee, tea, etc.	103.78	.02091	-.00051
	(9.06939)	(.00372)	(.00016)
8 Sugar, confectionary	143.25	.02453	-.00030*
	(11.80183)	(.00498)	(.00021)
9 Other food	18.22	.00637	.00010*
	(4.21607)	(.00224)	(.00009)
10 Alcoholic drinks	627.78	-.06301	.00724
	(20.47831)	(.01961)	(.00065)
11 Soft drinks	40.18	-.00499*	.00129
	(4.66595)	(.00435)	(.00017)
12 Tobacco	129.18	.02518	-.00062
	(12.61236)	(.00507)	(.00021)
13 Clothing, footwear	753.60	.27966	-.00530
	(108.61348)	(.01503)	(.00056)
14 Rents, water	340.90	.06226	-.00137
	(27.66243)	(.00873)	(.00036)
15 Light and fuel	93.07	.04516	-.00125
	(16.23890)	(.00400)	(.00017)
16 Other housing expend.	443.66	-.03037	.00180
	(14.15092)	(.00906)	(.00036)
17 Health and Hygiene	279.32	.05614	.00018*
	(26.26950)	(.00643)	(.00026)
18 Transp. and Communications	447.86	.11130	.00021*
	(50.85735)	(.00863)	(.00037)
19 Entertainment	191.79	.03012	.00143
	(19.24676)	(.00658)	(.00026)
20 Education	64.27	.02522	-.00035
	(12.15430)	(.00439)	(.00017)
21 Other services	60.26	-.00008*	.00060
	(4.68632)	(.00329)	(.00013)
22 Vehicles	143.18	.05018	-.00005*
	(25.07811)	(.00501)	(.00020)
23 Durables	285.82	.09377	-.00044*
	(39.11094)	(.00657)	(.00029)

Note: Standard deviations in parentheses

* statistically not significant

Source: Martins and Oliveira (1979)

the period 1971-76 the two extreme cases occurred in the regions of Lisbon and in the Centre: in the former case price indices generally increased with income, so that for the whole period there was a gap of around 8.5 percentage points between the top and the bottom classes; in the latter one there was an inverse relationship between income and price indices and the corresponding gap was around 40 percentage points.

To evaluate the importance of the price information for the computation of the CLI's a separate analysis was carried out with the two sets of prices. The prices for the Continent were used in the period 1977-81.

An additional difficulty lies on the fact that the commodity aggregation of the LES is different from those of either the "old" or the "new" indices. The approximate correspondence between them was worked out (see table A40 in the appendix), with the exception of the items "Other services" and "Vehicles" where no such correspondence was possible. This enabled the construction of price indices for each of the remaining 21 LES expenditure items.[1]

The expenditure weights utilized were those for the "all households" class in each region in the case of the prices for the cities. The expenditure weights for the Continent prices were computed by a weighted average of the "all households" weights in each region.[2]

Since only 21 of the original 23 LES items are included in the analysis, and since the sum of the β_i's must be unity new betas - β' were computed by the expression

$$\beta'_i = \frac{\beta_i}{\sum_{j=1}^{21} \beta_j} \qquad i=1, \ldots, 21$$

1 Only 20 in the period 1977-81 since no price information exists for the aggregate "Rents".

2 Using the total number of households in each region as the weights.

TABLE 2

Cost-of-Living Indices (CLI) for Different Expenditure Levels

Old prices for Coimbra and Viseu (base Jan. 1974)

	$\frac{a_t}{a_o}$	$\frac{b_t}{b_o}$	CLI - 1973/74 Expenditure classes (thousands of escudos)							
			< 18	18-30	30-48	48-60	60-90	90-120	120-180	≥ 180
JAN 1972	0.845	0.865	0.855	0.860	0.862	0.863	0.863	0.864	0.864	0.865
JAN 1973	0.894	0.910	0.902	0.906.	0.907	0.908	0.909	0.909	0.909	0.910
JAN 1974	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
JAN 1975	1.271	1.219	1.246	1.233	1.228	1.225	1.223	1.222	1.221	1.220
JAN 1976	1.569	1.419	1.496	1.461	1.445	1.438	1.433	1.429	1.426	1.423
JAN 1977	1.751	1.628	1.691	1.662	1.649	1.643	1.639	1.636	1.634	1.631

Note - The expenditure levels considered in each expenditure class were: 13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000 escudos.

$$a_t/a_o = (\Sigma\gamma_i p_{it})/(\Sigma\gamma_i p_{io})$$

$$b_t/b_o = \Pi(p_{it}/p_{io})^{\beta_i}$$

TABLE 3

Cost-of-Living Indices (CLI) for Different Expenditure Levels

Old prices for Lisbon (base Jan. 1974)

	$\frac{a_t}{a_o}$	$\frac{b_t}{b_o}$	CLI - 1973/74 Expenditure classes (thousands of escudos)							
			< 18	18-30	30-48	48-60	60-90	90-120	120-180	≥ 180
JAN 1972	0.805	0.776	0.791	0.784	0.781	0.780	0.779	0.778	0.777	0.777
JAN 1973	0.866	0.839	0.853	0.847	0.844	0.843	0.842	0.841	0.840	0.840
JAN 1974	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
JAN 1975	1.333	1.302	1.318	1.311	1.308	1.306	1.305	1.304	1.304	1.303
JAN 1976	1.599	1.510	1.556	1.535	1.525	1.520	1.518	1.515	1.514	1.512
JAN 1977	1.887	1.805	1.847	1.828	1.819	1.815	1.812	1.810	1.809	1.807

Note - The expenditure levels considered in each expenditure class were:

13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000 escudos.

$a_t/a_o = (\Sigma\gamma_i p_{it})/(\Sigma\gamma_i p_{io})$

$b_t/b_o = \Pi(p_{it}/p_{io})^{\beta_i}$

TABLE 4

Cost-of-Living Indices (CLI) for Different Expenditure Levels

Continent Prices

(Change of base to Jan 1974 made with the old prices for Coimbra and Viseu)

	$\frac{a_t}{a_o}$	$\frac{b_t}{b_o}$	CLI - 1973/74 Expenditure classes (thousands of escudos)							
			< 18	18-30	30-48	48-60	60-90	90-120	120-180	≥ 180
JAN 1978	2.203	2.063	2.131	2.100	2.086	2.079	2.075	2.072	2.070	2.067
JAN 1979	2.702	2.529	2.613	2.574	2.557	2.549	2.543	2.539	2.536	2.533
JAN 1980	3.231	3.006	3.115	3.065	3.042	3.032	3.025	3.020	3.016	3.011
JAN 1981	3.678	3.508	3.590	3.552	3.535	3.527	3.522	3.518	3.515	3.511
JAN 1982	4.660	4.235	4.442	4.347	4.304	4.284	4.271	4.261	4.254	4.245

Note - The expenditure levels considered in each expenditure class were:

13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000 escudos.

$a_t/a_o = (\Sigma\gamma_i p_{it})/(\Sigma\gamma_i p_{io})$

$b_t/b_o = \Pi(p_{it}/p_{io})^{\beta_i}$

TABLE 5

Cost-of-Living Indices (CLI) for Different Expenditure Levels

Continent Prices

(Change of base to Jan. 1974 made with the old prices for Lisbon)

	$\frac{a_t}{a_o}$	$\frac{b_t}{b_o}$	CLI - 1973/74 Expenditure classes (thousands of escudos)							
			< 18	18-30	30-48	48-60	60-90	90-120	120-180	≥ 180
JAN 1978	2.442	2.327	2.383	2.357	2.346	2.340	2.337	2.334	2.332	2.330
JAN 1979	3.022	2.851	2.934	2.896	2.879	2.871	2.866	2.862	2.859	2.855
JAN 1980	3.576	3.388	3.480	3.438	3.419	3.410	3.404	3.400	3.397	3.393
JAN 1981	4.066	3.954	4.008	3.983	3.972	3.967	3.963	3.961	3.959	3.956
JAN 1982	5.140	4.773	4.951	4.869	4.832	4.815	4.803	4.795	4.789	4.781

Note - The expenditure levels considered in each expenditure class were:

13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000 escudos.

$a_t/a_o = (\Sigma\gamma_i p_{it})/(\Sigma\gamma_i p_{io})$

$b_t/b_o = \Pi(p_{it}/p_{io})^{\beta_i}$

TABLE 6

Cost-of-Living Indices (CLI) for Different Expenditure Levels

Continent Prices (base 1976)

	$\frac{a_t}{a_o}$	$\frac{b_t}{b_o}$	CLI - 1973/74 Expenditure classes (thousands of escudos)							
			< 18	18-30	30-48	48-60	60-90	90-120	120-180	≥ 180
JAN 1978	1.416	1.401	1.408	1.405	1.403	1.402	1.402	1.402	1.401	1.401
JAN 1979	1.772	1.717	1.744	1.732	1.726	1.724	1.722	1.721	1.720	1.719
JAN 1980	2.057	2.043	2.050	2.046	2.045	2.044	2.044	2.043	2.043	2.043
JAN 1981	2.353	2.385	2.369	2.376	2.380	2.381	2.382	2.383	2.383	2.384
JAN 1982	2.967	2.881	2.923	2.904	2.895	2.891	2.888	2.886	2.885	2.883

Note - The expenditure levels considered in each expenditure class were:

13000, 24000, 39000, 55000, 75000, 102000, 143000 and 277000.

$a_t/a_o = (\Sigma\gamma_i p_{it}/(\Sigma\gamma_i p_{io})$

$b_t/b_o = \Pi(p_{it}/p_{io})^{\beta_i}$

Of course, the γ_i's remain the same.

The base year for the price information of the "old indices" is 1963, and for the "new" ones is 1976. While there is no difficulty in changing the base to 1973[1] with the "old" price information, the same does not happen with the "new" one. In fact, as we know the price indices of the 20 items, for instance in January 1977 (base 1973), with both the Lisbon and the Centre commodity prices, and at the same time the price indices for the same items and month with the Continent commodity prices (base 1976). We are able to change the base from 1976 to 1973. However we will inevitably end up with two different results.

Furthermore, this procedure is far from being accurate because, as was discussed in the previous chapter, not only the level of aggregation but also the individual commodity prices are quite different in the "old" and "new" indices. This was the reason why cost-of-living indices for the period 1977-81 with base year 1976 were also computed.

The results are indicated in tables 2 to 6. The first conclusion to be drawn is that the cost-of-living indices either increase or decrease with income, and that the middle classes never show at the same time higher (lower) indices than the extreme classes. That is, the cost-of-living indices are monotonic in income.

Only twice do the CLI's increase with income - in January 1972 (Lisbon prices) and in January 1981 (Continent prices - base 1976) - so we can conclude that low income classes are more affected by price rises. We must not forget that the values for each year (in January) are being compared with those for January 1974, so that they will comprise the effects of the previous years.

In January 1977 the poorer households experienced an inflation rate

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1 As in this thesis the price information always refers to January of each year, the prices in January 1974 are taken as representing the prices in the year 1973.

6% above that suffered by the better off households in the Centre and 4% in Lisbon. In January 1982 that difference was 17% (19%) when the change of base (to 1973) of the Continent prices was made with the Lisbon (Centre) prices, but as mentioned above serious reservations must be put on these results. When the base is 1976 the gap in the same month if 4%.

In chapter 3 the rate of inflation in any one year was computed by dividing the price index in January of the following year by the price index in January of the same year. If we do the same with the cost-of-living indices (see tables 7 to 9) we can make direct comparisons of the results obtained with both approaches.

Comparison of table 7 with table A9 enables us to conclude that both of them agree on whom suffer most with inflation in each year in the Centre region: the bottom households in 1973, 1974 and 1975 and the top ones in 1976. In 1972 there are no significant differences. Where they disagree is in the magnitude of those differences. The cost-of-living indices always scale them down (e.g. from 9, 12 and 4 percentage points to 2.6, 3.5 and 1.6 in 1974, 1975 and 1976 respectively).

Exactly the same happens in the region of Lisbon (see tables 8 and A10): total agreement on those experiencing most inflation in each year but a substantial narrowing of differences by the CLI's (with a maximum of around 2 percentage points).

It is more difficult to compare the results in table 9 (Continent prices) with those on tables A12 to A15, because table 9 refers to the CLI's computed for the whole Continent while the other tables refer to the price indices for each region in isolation. But we can conclude that only in 1979 the CLI's "disagree" with the price indices. Again the range in the CLI's is considerably lower than the range in the price indices.

TABLE 7

COST-OF-LIVING INDICES

(OLD PRICES FOR COIMBRA AND VISEU)

Income \ Years	1972	1973	1974	1975	1976
< 18	105.5	110.9	124.6	120.1	113.0
18-30	105.3	110.4	123.3	118.5	113.8
30-48	105.2	110.3	122.8	117.7	114.1
48-60	105.2	110.1	122.5	117.4	114.3
60-90	105.2	110.0	122.3	117.1	114.4
90-120	105.2	110.0	122.2	116.9	114.5
120-180	105.2	110.0	122.1	116.8	114.6
≥ 180	105.2	109.9	122.0	116.6	114.6

Note: computed from table 2.

TABLE 8

COST-OF-LIVING INDICES

(OLD PRICES FOR LISBON)

Income \ Years	1972	1973	1974	1975	1976
< 18	107.8	117.2	131.8	118.1	118.7
18-30	108.0	118.1	131.1	117.1	119.1
30-48	108.1	118.5	130.8	116.6	119.2
48-60	108.1	118.8	130.6	116.4	119.4
60-90	108.1	118.8	130.5	116.3	119.4
90-120	108.1	118.9	130.4	116.2	119.4
120-180	108.1	119.0	130.4	116.1	119.5
≥ 180	108.1	119.0	130.3	116.0	120.0

Note: Computed from table 3.

TABLE 9

COST-OF-LIVING INDICES

(CONTINENT PRICES)

Income \ Years	1978	1979	1980	1981
< 18	123.9	117.5	115.6	123.4
18-30	123.3	118.1	116.1	122.2
30-48	123.0	118.5	116.4	121.6
48-60	123.0	118.6	116.5	121.4
60-90	122.8	118.7	116.5	121.2
90-120	122.8	118.7	116.6	121.1
120-180	122.8	118.8	116.6	121.1
≥ 180	122.7	118.8	116.7	120.9

Note: Computed from table 6

4.5 Conclusions

Perhaps the main (and remarkable) conclusion of this chapter is the fact that (with only one exception) the CLI's implicit in the LES parameters estimated from time-series data, ranked households in each year in exactly the same way as the price indices based on a Laspeyres formula with weights drawn from a cross-section data.

The narrower differences in inflation rates among income classes obtained with the cost-of-living indices may have a theoretical explanation: the CLI's allow the existence of substitution effects, that is, allow consumers to buy less of one good when its price increases, while the Laspeyres index assumes that the same quantity of the good is always bought. As we saw in chapter 2, the Laspeyres index is an upper limit to a base-weighted cost-of-living index.[1]

But how much faith can we put on the results given by the CLI's implicit in the LES parameters? There are two possible types of criticism: one relates to the formula (35) which enables us to calculate the CLI's, and the other to the properties of the Linear Expenditure System itself.

Let us begin with expression (35). If we differentiate it in order to income, we get

$$\frac{\partial CLI}{\partial y} = - \frac{1}{y^2} a_o \left[\frac{a_t}{a_o} - \frac{b_t}{b_o} \right]$$

As the a's and b's are constants, the above expression will be strictly positive or negative (unless $\gamma_i = 0$, leading to $a_o = 0$), depending on the size of a_t/a_o relative to b_t/b_o. We can thus conclude that the CLI is monotonic in income. For instance, it will rise in income if luxuries (b_t/b_o) rise in price more than necessities (a_t/a_o).[2] In our case

1 Also, rigorously we should not compare the cost-of-living indices (computed for households with the same composition) with the price indices in chapter 3 (which refer to households with a wide variety of compositions

2 Except where a_o is negative. This case only arises when the sum of "necessary" expenditures at base prices is negative, which requires that some of the price elasticities to be greater than one.

the opposite generally happens (see tables 2 to 6) leading the CLI's to fall in income.

This monotonicity property which was first demonstrated by McCarthy (1979) is very restrictive because it never allows the middle income households to experience the highest or lowest inflation rates. Since in our case the computed price indices were also generally monotonic in income this in-built property of the LES cost-of-living index was not very worrying.

The parameter estimates of the LES (and the implied CLI's) may also be particularly sensitive to the inclusion of new observations and to data revisions, as was demonstrated by Irvine and McCarthy (1980).[1]

But perhaps the main criticism of the LES (or for that matter of all the demand systems with underlying additive preferences) is that, among other things, it implies an almost exact proportionality of expenditure and price elasticities. Since there is no a priori reasoning for that to be true, the estimates may be severely distorted.

Deaton (1974, 1975a, 1975b, 1978) gives evidence of that distortion and concludes that "if the price to be paid for the theoretical consistency of demand models is the necessity of assuming additive preferences, then the price is too high".[2]

The inevitable conclusion is that we must have serious doubts on the accuracy of the computed CLI's.

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1 Who criticized the conclusion obtained by Muellbauer (1974) that in the UK (1964-72) there was a significant inequality effect induced by differential prices.

2 Deaton (1974).

Chapter 5

Equivalence Scales

5.1. Introduction

Income is a very crude measure for comparing the welfare of two households. The reason is that households differ in size, sex, age composition, education and other characteristics: a lower income household may be better off than a higher income one if it is, for instance of smaller size and/or has younger children.

Equivalence scales are numbers (index numbers) which enable us to express a household's income in the same reference unit, so that direct comparisons between households are made possible.

One of the simplest ways to make such comparisons is to calculate each household's per capita income. Although an improvement in relation to the sole consideration of total income, this procedure is still very deficient: a four adult household is quite different from a household with two adults and two young children. The equivalence scales provide us with a more sophisticated way of head counting where generally the effects of sex and age are taken account of.

The problem of constructing equivalence scales is formally identical to that of constructing cost-of-living indices. In this latter case we compare the minimum costs for a given household to reach a certain utility level when the prices of the commodities vary; in the former case we compare the minimum costs necessary for a household with a given composition to reach a certain utility level, with the minimum costs for a 'reference' household to reach the same utility level (assuming that both face the same prices). That is,

$$s = \text{equivalence scale} = \frac{c\ (u, p, a\ )}{c\ (u, p, a_o\ )} \quad (1)$$

Where u is the utility level, p the prices, a the composition of any household and a_o the composition of the reference household.

The composition of the reference household may vary depending on the particular study: a single adult household, a two-adult household, a two-adult with two young children, etc. The utility level is usually defined on current consumption and the household composition is treated as exogenous.

This way of looking at equivalence scales implies that it is possible to arrive at exactly the same demand functions for different types of households provided that their composition is adequately modelled. However some economists[1] have rejected this possibility.

The hypothesis of adequately modelling the effects of household composition in demand functions leads to considerable simplifications: instead of having to estimate them for each household type, a single demand function can be estimated for all of them. This is particularly useful because usually there is insufficient data to establish separate demand functions.

Although equivalence scales are useful in the estimation of demand functions, sometimes (as in the present study) we are directly interested in the computation of the equivalence scales themselves. A great deal of attention has recently been devoted to equivalence scales because they are important in relation to government welfare policies, e.g. in the introduction of family allowances and tax relief for larger families, as well as in defining the poverty line.

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1. e.g. Allen (1942) and Forsyth (1960).

There are three alternative ways of computing equivalence scales:

1 - By using nutritional or physiological studies

2 - By political decisions, voting or administrative conventions

3 - By investigating the expenditure patterns of households

Equivalence scales based mainly on nutritional needs have been widely used. Most of them compare the energy requirements (in calories) of different age and sex groups in normal health with the energy requirements of a healthy normal adult male (the reference unit). For instance, if the adult male needs 3000 calories per day and a 10 year old child only 2100 calories, the equivalence scale is 1 for the adult and 0.7 for the child.

The equivalence scales used in Portugal to choose the reference households for the CPI (suggested by the ILO[1]) is a nutritional scale. Another well-known nutritional scale is the Amsterdam scale, which was used for instance by Stone (1953):

Age group	Male	Female
under 14 years	0.52	0.52
14 - 17 years	0.98	0.90
18 years and over	1.00	0.90

The main objection to the use of nutritional scales is that they are based on normative judgements rather than on market behaviour. That is, it is one thing to say what an adult or child <u>ought</u> to consume of food in order to get the optimum quantities of calories, proteins, vitamins, etc.[2], and quite another thing as to what they <u>actually</u> consume.

There is also no reason why the equivalence scales should only be based on food intakes: the relative requirements on other items of

1. Also called the Atwater scale. See chapter 2, section 2.3.
2. It needs to be emphasized that the so-called nutritional scales often relate only to the optimum number of calories.

consumption (clothing, footwear, housing, etc.) are equally important.

Since the second method of arriving at equivalence scales (voting, administrative conventions, etc.) is clearly void of any scientific basis, we are left with the third method, i.e. the estimation of equivalence scales by an empirical investigation of the expenditure behaviour of households.

A considerable amount of literature exists related to this empirical estimation, and we can say that there are basically three approaches for solving this problem: Engel's, Prais-Houthakker's and Barten's approach, although the last two can be considered as something of a generalization of the Engel methodology.

5.2. Engel's Approach

The study of the relationship between food expenditure and income based on cross-section household surveys led Ernest Engel in 1857 to formulate what is now called Engel's first law: food expenditure increases with income but at a lesser rate, or in other words, food expenditure is inelastic with respect to income.

An immediate implication is that the proportion of total expenditure spent on food is higher for poorer households than for richer ones. Engel also observed that larger households also spent proportionately more on food than smaller ones. He then concluded that the proportion spent on food could be used as an indirect measure of welfare: those households with the same food share would have the same level of real income, irrespective of their composition.[1] If we then compare the money incomes of two households in that situation, we have a measure or index of the cost of reaching the same welfare level[2] when the family composition is

1. The same could be said in relation to other commodities other than food whose shares vary systematically with total expenditure.

2. Here identified with real income.

different, i.e. we are able to calculate an equivalence scale.

Let s^h be the equivalence scale of household h. From (1) we can see that s^h is a function of the household's characteristics a^h, i.e.

$$s^h = s (a^h) \qquad (2)$$

Where a^h is a vector of the household's characteristics, such as number of school age children, number of adolescents, number of adults, etc.

Expression (2) implies that the demand functions, the direct and indirect utility functions and the cost function will be the same across all households if they are expressed in the same equivalent unit, say in equivalent adult per capita terms.

In the case of the demand functions, they can be written as

$$\frac{q_i^h}{s (a^h)} = g_i \left(\frac{y^h}{s (a^h)} , p \right) \qquad (3)$$

i.e., the demand for good i per equivalent adult is a function of total expenditure per equivalent adult and of the prices of all the goods.

The proportion of total expenditure spent on good i, w_i, is given by

$$w_i^h = \frac{p_i \, q_i^h}{y^h} = \frac{p_i \, g_i \left(y^h / s (a^h), p \right)}{y^h / s (a^h)}$$

so that w_i is a function of $y / s (a)$, i.e.

$$w_i^h = F_i \left(\frac{y^h}{s (a^h)} \right)$$

For any two households h and r to spend the same budget proportion w_i on good i we must have

$$\frac{y^h}{s(a^h)} = \frac{y^r}{s(a^r)}$$

If the household r is the reference household $s(a^r)$ is equal to unity, so that

$$s(a^h) = \frac{y^h}{y^r}.$$

Engel's method is then justified provided the demand functions can be expressed as in (3). Of course, different levels of the budget share w generally lead to different equivalence scales.

5.3. The Prais-Houthakker and Barten Approaches

Let us write (3) in a simplified way, for household h,

$$\frac{q_i}{s} = g_i\left(\frac{y}{s}\right) \tag{4}$$

Since the prices p are taken as constant the above expression represents an Engel curve.[1] The equivalence scales s in Engel's model (4) are the same for all the commodities i. This implies, for instance, that the relative needs of children in relation to adults are the same, say in milk, alcohol, tobacco, clothing, etc. which is absurd.

Sydenstricker and King (1921) recognized the necessity of constructing different scales for different commodities[2], and their approach was later rediscovered and taken up by Prais and Houthakker (1955), who

1. An Engel curve shows the relationship between expenditure on good i and total income y, when the prices are constant.

2. In their case the division was only between food and non-food commodities. Their arguments were presented in a very discursive and somewhat confusing way, and not in a formalized manner as Prais and Houthakker did later on.

generalized the Engel curve (4) to

$$\frac{q_i}{s_i} = g_i\left(\frac{y}{s_o}\right) \tag{5}$$

The s_i's are called the commodity specific scales and s_o is the income scale (a weighted average of the commodity scales[1]). The interpretation of (5) is basically the same as that of (4): the quantity per equivalent adult demanded for each commodity is a function of the income per equivalent adult. But now we have different deflators for the expenditures on each commodity and for income. The quotient y / s_o can be considered as a measure of the household's welfare.

Within a given household, a certain type of person j is characterized by a set of weights w_{ij} for each commodity i[2]. If there are a_j persons of type j, the specific size of the household for commodity is is equal to

$$s_i = \sum_j w_{ij}\, a_j$$

The income or general scale s_o is given by

$$s_o = \sum_j w_j\, a_j$$

When the weights w_j are a weighted average of the commodity specific weights

i.e. $$w_j = \sum_i \frac{k_i / w_{ij}}{/k_i}$$

Prais and Houthakker (1955) have shown (using the adding-up restriction which (5) must satisfy) that the weights k_i are approximately proportional to the expenditure on commodity i.

1. See below
2. All the weights will be unity for the reference (e.g. the adult male).

Let $q_i / s_i = q_i^*$ (the needs corrected demand for good i). In the Prais-Houthakker model (5) q_i^* is only a function of y / s_o - the welfare level. Since the s_i's are different for each commodity, the model is only consistent with consumer theory if the utility function is such that the indifference curves are right angles (sometimes called Leontieff indifference curves)[1]. That is, the Prais-Houthakker model implies no substitutability between goods.

Barten (1964) assumed that the utility function of any household could be written as

$$u = f\left(\frac{q_i}{s_1}, \frac{q_2}{s_2}, \ldots, \frac{q_n}{s_n}\right) \qquad (6)$$

where again the s_i's are the commodity specific scales and are unity for the reference household. Equation (6) means that if any household consumes exactly s_i times of each commodity i, as the reference household, it will experience the same level of utility.

Let $p_i^* = p_i s_i$. In the Barten model the consumer will then

Maximise $u = f(q^*)$

Subject to $\sum p_i q_i = \sum p_i^* q_i^* = y$

implying that his demand functions will be given by[2]

$$\frac{q_i}{s_i} = q_i^* = g_i\left(\frac{y}{p_i^*}, \ldots, \frac{y}{p_n^*}\right)$$

and the cost function by

$$y = c(u, p_1^*, \ldots, p_n^*)$$

The conclusion is that variations in family composition are identical

1. See Deaton and Muellbauer (1980), Muellbauer (1977).
2. See Muellbauer (1977).

to variations in prices. Changes in family composition modify prices, both absolutely and relatively. An additional child will make milk and ice cream relatively more expensive and whisky and cigarettes relatively cheaper. Or 'a penny bun costs threepence when you've a wife and child.'[1]

The estimation of the Prais-Houthakker and Barten models

Let us suppose that we have cross-section data on the consumption expenditure and income of a wide variety of households which differ by their composition. Let us create two subsamples: one relating to the reference households and the other to households which differ from the reference households by the inclusion of an additional person of type t. There are n commodities and we estimate separately for each subsample their respective Engel curves.

The equivalence scale coefficients are unity for the reference households. The problem is to estimate the specific and income scales for the person t.[2]

$$q_i = g_i (y) \qquad i = 1, \ldots , n \qquad (7)$$

and for the second subsample

$$q_i = h_i (y) \qquad i = 1, \ldots , n \qquad (8)$$

We have to determine the (n + 1) coefficients w_{it} ($i = 1, \ldots , n$) (commodity specific coefficients) and w_t (income coefficient) in such a way that the Engel curves h (y) will coincide with g (y) when the equivalence coefficients are taken account of,

1. Gorman (1976).
2. See Cramer (1969).

i.e. $$q_i = \frac{h_i(y)}{(1 + w_{it})} = q_i\left(\frac{y}{1 + w_t}\right) \qquad \text{for all } i$$

Since w_t is a function of w_{it}, there are altogether n coefficients to be estimated and n paired comparisons of parameter estimates available, so that no apparent difficulty exists[1]. However, since both sets of Engel curves (7) and (8) must satisfy the adding-up or budget constraint restriction, in either set one function can be derived from the (n - 1) others. As a result there are only (n - 1) independent pairs of parameter estimates, which is insufficient to determine the n specific coefficients w_{it}. This is an example of the identification problem: we can only compute the relative values of the scales, not the absolute ones.[2]

Prais and Houthakker (1955) were unaware of this problem. They were able to estimate the specific coefficients because they assumed the income coefficients (w_t) to be unity for all t (which they regarded as a mere computational simplification).

Nicholson (1949) suggested that the income coefficients of children could be estimated if we consider the commodities for which the child's specific coefficients may be taken as zero. Such commodities are, for instance, tobacco, drink, and men's clothing and women's clothing. His results were disappointing, mainly due to the fact that these commodities are liable to important observational errors and usually have larger disturbances than others.

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1. Of course the specific form for the Engel curves will lead to different estimated coefficients. We will suppose that the best specification is adopted.

2. The formal proof of the lack of identification in Barten's model is in Muellbauer (1974b) and of the Prais-Houthakker's model in Muellbauer (1980).

In the Barten model, a way of removing the lack of identification is to consider some variation in the relative prices of the commodities, i.e. to use pooled time-series and cross-section data. The same cannot be done in relation to the Prais-Houthakker model due to its zero price substitution effects.

The Prais-Houthakker model has been the most used of the two models in the empirical estimation of equivalence scales,[1] but most authors do not pay the necessary attention to the identification problem. A common way of overcoming that problem has been to use the scales for food given by nutritional studies and then to calculate the remaining scales for the other commodities.[2] However it has also been found that these scales may be particularly sensitive to the initial values assumed for the nutritional scales,[3] so that they may not be very useful.

Muellbauer (1977) estimated the Barten model using pooled cross-section and time-series data (1968-73) for the U.K. He also estimated the Prais-Houthakker model on this same cross-section data conditional on the food scales estimated for the Barten model. He concluded that the two models provided quite different results, the Barten model typically giving much lower values for both the commodity and general scales. The main reason for this being the lack of substitution of the Prais-Houthakker model. The Barten model seemed to be a better way of

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1. See e.g. Singh (1972), Singh and Nagar (1973), Blokland (1976), McClements (1977, 1978).
2. A rather odd procedure since one of the main objectives of the empirical models is precisely to avoid the use of such nutritional scales. Also, given the importance of food in the consumer's budget the estimation of empirical food scales should be of primordial importance.
3. See McClements (1977, 1979) and Muellbauer (1979a, 1979b).

incorporating household composition effects, but of Muellbauer's preliminary results have suggested that it exaggerates the substitution effects. The overall conclusion seems to be that we must put serious reservations on the utilization of either the Barten or Prais-Houthakker models in the estimation of complete systems of equivalence scales.

In practice Engel's approach of estimating equivalence scales is still widely used, mainly due to its simplicity. When it is applied, the aim is not generally to obtain equivalence scales for different commodities, but usually only for food, or for some basket of goods sometimes called 'necessities' (including for instance, food, drink, clothing and footwear).[1]

Since the alternative to not using empirical scales is to use nutritional scales which are based on food requirements; and since food is by far the single most important expenditure item for most households, the use of Engel's method seems to be perfectly reasonable.[2]

5.4. The Engel Curves

We have seen that the Engel curves play a crucial role in the empirical estimation of equivalence scales, whatever the approach adopted. However, nothing has been said about the best specification of the Engel functions or about the econometric problems of their estimation. So, before attempting to estimate equivalence scales for Portugal we will dedicate this section to the discussion of those problems.

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1. See Friedman (1952), Seneca and Taussig (1971).
2. But there are other alternatives. In section 5.5 we will also use a method which was put forward by Brown (1954), based on the Prais-Houthakker model.

Forms of Engel Curves

Consumer theory can only help us on the choice of the Engel curve specification in a rather limited way: the Engel curves must be such that the sum of all the expenditures made by the households on each commodity must be equal to total income. This is the familiar adding-up or budget restriction, and it implies that the sum of the marginal propensities to consume must be equal to unity, or in elasticity terms, that the income elasticities for each commodity weighted by their budget shares must also sum to unity.

Oddly enough this restriction is rarely satisfied by the most common types of functions used in the empirical work. Fortunately, as Worswick and Champernowne (1954/55), Nicholson (1957) and Champernowne (1957) have shown, if the data satisfies the adding-up restriction the resulting parameter estimates will also approximately satisfy that restriction, whatever the function, at least within the observed income range.

If the theory is of not much help, some intuitive plausible considerations may be a better guide to the desirable properties of an Engel function. Let us consider a hypothetical commodity and an arbitrary consumer whose income is increasing.[1] We can think of a (positive) income threshold where the consumer adds that commodity to his purchases. Near that threshold income any income increase will lead to a more than proportional increase in the expenditures on that commodity, i.e., the income elasticity will be greater than unity and the commodity can be considered a 'luxury'.

1. Of course in an Engel curve framework we assume the constancy of all commodity prices.

Since in this interval the slope of the Engel curve is increasing, the slopes of all the other Engel curves must be declining in order that the budget restriction is satisfied.

The same will happen with our commodity when new ones are introduced: the income elasticity will be declining and when it is less than unity the commodity will be a 'necessity.' Eventually we will reach a saturation point where the slope of the Engel curve and the income elasticity will both be zero. We can even think that beyond that point the commodity will become an inferior good, i.e. additional income increases will lead to reductions in the expenditure in that commodity.

So, ideally an Engel curve should be able to represent luxuries, necessities and inferior goods. There is some empirical evidence for the validity of this proposition.[1] However much will depend upon the commodity classification and on the level of aggregation adopted. For a reasonable degree of aggregation we may find that, at least within the observed income range, the commodity is always either a luxury, a necessity or an inferior good. Even for fairly disaggregated income classifications it has been found that only the consideration of a very wide range of incomes would allow us to observe all three phases.

The ideal Engel curve is then sigmoid in shape and possesses a point of inflexion. Some mathematical candidates are for instance the lognormal distribution function, the logistic distribution function and the log reciprocal function. They have been used in empirical studies, but since the sigmoid character of the Engel curves is generally

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1. See Brown and Deaton (1972).

not observed the reasons mentioned above, some other functions have been used instead.[1] From these functions, perhaps the most important are the following:[2]

$$e_i = a_i + b_i\, y + u_i \quad (9)$$

$$e_i = a_i + b_i \log y + u_i \quad (10)$$

$$\log e_i = a_i + b_i \log y + u_i \quad (11)$$

$$w_i = a_i + b_i \log y + u_i \quad (12)$$

Where w_i is again the proportion of total expenditure spent on good i and u_i is the error term.

All these functions have the great advantage of allowing the use of linear regression methods. For all of them we assume that the stochastic vector u_i has the usual properties of zero expectation, constant variance and independence of y. Of course this cannot be true for all of them when they are estimated from the same data set, but usually this fact is ignored. One implication of this procedure is that the R^2's obtained for each of them cannot be directly compared.

Equation (9) is the linear Engel curve and it was used extensively by Allen and Bowley (1935). It is a good approximation in the region where the income elasticity is around unity. The expression for the

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1. Also, in the case of the lognormal function it is relatively more difficult to estimate.

2. We use $e_i = p_i\, q_i$ (expenditure on commodity i) in the Engel functions instead of q_i as both formulations are equivalent due to the price constancy. Also in cross-section studies the actual data generally refers to e_i and not to q_i.

   For other types of functions see Prais and Houthakker (1955), Leser (1963), Aitchison and Brown (1957) and Wold and Jureen (1953).

income elasticity

$$\frac{1}{\frac{a_i}{b_i\, y} + 1}$$

implies that when a_i is positive the elasticity is less than unity and increases with income (with unity as the upward limit). This property of the demand elasticities for necessities to increase with income is not generally observed and as a result this function generally provides a poor fit to the data.

The semi-logarithmic specification for the Engel curve (10) possesses income elasticities which are given by the following formula.

$$y_i = \frac{\partial \log e_i}{\partial \log y} = \frac{1}{e_i} \cdot \frac{\partial e_i}{\partial \log y} = \frac{b_i}{e_i}$$

With $b_i > o$, the elasticity is always less than unity and continually declines to zero. The semi-logarithmic function is then particularly suited for the first part of the range of necessities.

It can be immediately seen that the income elasticity of the double-logarithmic function (11) is a constant and is equal to b_i. This function is particularly useful when we observe the lower part of the sigmoid curve where the commodity is a luxury or at least the income elasticity has no tendency to decline.

However this function generally provides very good fits even for other parts of the sigmoid curve and given the immediate computation of the income elasticity it has been widely used. When using data for individual households and a disaggregated commodity classification the incidence of zero values may preclude the utilization of this function, but it rarely happens.

The Engel curve (12) was first estimated by Working (1943) and then successfully used by Leser (1963) and plays an important role for instance in Deaton and Muellbauer (1980).

One of its great advantages is that it satisfies the adding-up restriction. Adding-up requires that $\Sigma w_i = 1$ and this is satisfied when

$$\Sigma a_i = 1 \text{ and } \Sigma b_i = o \quad .$$

These conditions will be automatically satisfied if (12) is estimated by ordinary least-squares.

The income elasticity is given by

$$1 + \frac{b_i y}{e_i}$$

so that the model allows luxuries ($b_i > o$), necessitates ($b_i < o$) and inferior goods. It also implies a decline in the elasticities with rising income.

Estimation of Engel Curves

Engel curves are usually estimated from cross-section data derived from budget surveys. Unfortunately these surveys are primarily designed to aid the estimation of the population means of single variables rather than the estimation of the relationships between the variables. However nothing can be done to remedy such eventual imperfections of the data.

In addition there are also some problems related to the variable income in an Engel curve. Firstly there is the tendency for households to understate or conceal their income when asked for it in survey questionnaires. Furthermore, any insistence in obtaining detailed and precise data on the household's income is generally counterproductive and may reduce the response rate. This has led frequently to the elimination of questions on income or only to asking households to situate themselves in one of the proposed income brackets. This means that generally the information on total expenditure is much more reliable than the information on total income.

If an Engel curve is estimated by ordinary least-squares using income as the independent variable, the true income coefficient will then be systematically understated. The same will happen even if the recorded income differs from the 'true' income by a purely random error of observation.[1]

A second problem relates to what Friedman (1957) has called the permanent income hypothesis. He argued that only permanent income is the determinant variable for the expenditure decisions of the households, not observed income. Permanent income will always differ from observed income (the one recorded in the budget surveys), since the latter contains a 'transitory' and short-run component.[2] As a result, even if there are no observational errors, ordinary least-squares would also lead to biased estimates.

Given the above problems with the income variable it is often suggested that total expenditure is used in its place, because the observational errors would be considerably reduced, and OLS can be applied. Consequently, instead of estimating

$$e_i = a_{oi} + a_{1i}\,y + u_i \quad (i = 1,\ldots n) \qquad (13)$$

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1. This is the classical case of errors in variables.

2. This is one of the reasons why group means are sometimes preferred to individual observations in the Engel curve estimation. The average income of a group of households would be closer to the concept of permanent income than individual income.

we should estimate

$$e_i = b_{oi} + b_{1i} E + v_i \tag{14}$$

where

$$E_i = \Sigma e_i .$$

But in so doing we are running into another set of problems. In (13) we can reasonably assume that there is no 'feed-back' between the various expenditures and income, i.e., although the expenditures on each item depend on the household's income, income does not depend on the individual expenditures. We can think of situations when a household tries to increase his income to be able to make certain additional expenditures, but there is no evidence that this situation is generalized or at least successful. If so, the variables y and u are independent and OLS techniques can be directly applied to (13).

The same does not happen with equation (14). In here it is clear that not only the individual expenditures e_i depend on total expenditure E, but also that total expenditure will also depend on each individual expenditure. Therefore, if income is exogenous, we can represent the structure of Engel curves by the following linear system[1]:

$$\begin{aligned} e_i &= a_{oi} + a_{1i}\, y + u_i \quad (i = 1, \ldots, n) \\ E &= \Sigma e_i = a_o + a_1\, y + z \end{aligned} \tag{15}$$

where u_i and z are stochastic variables with the usual properties and are uncorrelated with y. The last equation is the sum of the structural

1. Linear Engel curves are used only for the sake of simplicity.

equations for the n individual categories of consumption.

If e_i' and E' are the systematic parts of e_i and E respectively, then,

$$e_i' = a_{oi} + a_{1i}\, y$$

$$E' = a_o + a_1\, y$$

the relationship between them being given by

$$e_i' = b_{oi} + b_{1i}\, E$$

Where, from (15)

$$b_{oi} = a_{oi} - a_{1i}\, a_o \,/\, a_1$$

$$b_{1i} = a_{1i} \,/\, a_1$$

When we estimate this relationship using the observed variables e_i and E (expression (14)),

$$e_i = b_{oi} + b_{1i}\, E + v_i$$

v_i is a linear combination of the stochastic variables u_i and z,

$$v_i = u_i - b_{1i}\, z$$

and from (15) we can conclude that v_i and E are correlated. If we apply OLS to (14), the estimate of the coefficient b_{1i}, which is given by[1]

$$\frac{\Sigma\, e_i\, E}{\Sigma\, E^2} = b_{1i} + \frac{\Sigma\, v_i\, E}{\Sigma\, E^2}$$

1. Where the variables are expressed as differences from their means.

is biased and inconsistent. This is after all, a classical problem of simultaneous-equation bias.

Summers (1959) was the first to emphasize this question when he criticised Prais and Houthakker (1955) for not giving the necessary importance to this problem. Prais and Houthakker did not have data on income so that they used total expenditure as its proxy. They were aware of the fact that the independent variable would be correlated with the stochastic term but they stated that 'so long as the item of expenditure is a small proportion of the budget it is not to be expected that serious biases will result...'.

In a comment on Summers' article, Prais (1959) also argued that if we think that a household first decides on its total expenditure and then allocates it to the various commodities[1] the simultaneous-equation bias would not exist. Furthermore, since the sum of the income coefficients must be unity some commodities would have upward and some others downward biases, but on the whole he thought them likely to be small.

However Liviatan (1961) found that those biases may be considerable even for individual commodities and successfully tested the hypothesis that income is exogenous and total expenditure endogenous. Given the problems with the inclusion of the income variables in an Engel curve he suggested the use of total expenditure and in order to avoid the correlation with the stochastic term the use of the 'instrumental variables' technique.

To employ the method of instrumental variables we must find a variable X that is correlated with the systematic part of E, but at the same time is not correlated with the stochastic variables u and z (and therefore with v).

1. After all this is the assumption generally put forward when describing the static consumer theory (see chapter 4).

If such variable can be found we can get consistent estimates of b_{1i} since the probability limit of its instrumental variables estimate is given by

$$\frac{\text{Cov}\ (\ e_i\ ,\ X\ )}{\text{Cov}\ (\ E,\ X)} = b_{1i} + \frac{\text{Cov}\ (v_i\ ,\ X\ )}{\text{Cov}\ (\ E\ ,\ X\ )} = b_{1i}$$

Liviatan suggests the use of recorded income y as an instrumental variable. The reasons are: 1) y has a relatively close correlation with y′, and thus with the systematic part of the total expenditure E ; 2) It has no correlation with the random variables in expenditure, i.e., income is not influenced by expenditures(it is exogenous to the consumers' expenditure decisions).

So, while y can not be used to obtain estimates of a_{1i} in (13) it can be used to obtain consistent estimates of b_{1i} in (14). When we only know at which income class the households belong (or say they belong[1]) the way to obtain those consistent estimates will be to compute for each income class the average expenditure on each item together with the average total expenditure and then to apply OLS to estimate the Engel functions.

But again the resolution of one problem is going to originate another of a different kind; namely, when we employ group means instead of individual observations, if the individual stochastic variables are homoskedastic then the group stochastic ones will be heteroskedastic:[2]

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1. As is the case with the 1973/74 HES in Portugal.
2. See Cramer (1969).

Let us assume that households are grouped in relation to a certain variable, and let h be the index which represents each group. Within each group, individual households are denoted by $j = 1, 2, \ldots, n_h$. We also assume that all individual data satisfies the relation

$$e_{hj} = a + b\,E_{hj} + u_{hj} \quad \text{for all h and j}$$

and the stochastic variables u_{hj} have zero mean, independence of one another and are homoskedastic, i.e.

$$\text{Var}\,(u_{hj}) = \sigma^2 \quad \text{for all h and j}$$

It follows immediately that if we create the variables $\bar{e}_h$, $\bar{E}_h$ and $\bar{u}_h$, defined as

$$\bar{e}_h = \frac{1}{n_h} \sum_j e_{hj}, \quad \bar{E}_h = \frac{1}{n_h} \sum_j E_{hj}, \quad \bar{u}_h = \frac{1}{n_h} \sum_j u_{hj}$$

they will satisfy the equation

$$\bar{e}_h = a + b\,\bar{E}_h + \bar{u}_h \tag{16}$$

The disturbances $\bar{u}_h$ will also have zero mean and be independent, but since they are the mean of n_h independent variates with the same variance σ^2, their variance will be given by

$$\text{Var}\,(\bar{u}_h) = \frac{\sigma^2}{n_h}$$

with the implication that (16) is heteroskedastic. The efficient estimation of a and b in (16) will then require that the group means are weighted by n_h, i.e., by the number of observations that they represent. Weighted least-squares regression will then lead to unbiased and efficient estimates of the coefficients and we will overcome the problem of using group means. Prais and Aitchison (1954) have shown that the usual classification of the individual households by income in Engel curve analysis also minimizes the increase in the variance of the estimate that is inevitable with grouping.

Cramer (1964) has confirmed these results and shown that regressions based on individual observations and on weighted income group means both yield unbiased estimates of the same parameter; furthermore the two estimates are generally highly correlated. .

However, when we use regression methods on grouped data, the values obtained for the correlation coefficients are highly unsatisfactory. As Cramer points out in the same article 'with the advent of large-scale computing facilities, several analyses of budget surveys based on individual observations have become available, and they all confirm that the correlations are much lower than the values to which we have become accustomed by the use of grouped data. For most commodities taken separately, income seldom accounts for more than 20 per cent of the individual variation of expenditure.'

He then derived an approximate relation between $\bar{R}^2$ and R^2 (the coefficients of determination based on grouped and non-grouped observations, respectively),

$$\frac{\bar{R}^2}{1 - \bar{R}^2} = \frac{N}{t} \cdot \frac{R^2}{1 - R^2}$$

where N is the total number of observations and t the number of groups. From it we can conclude that it is relatively easy to find, for instance, R^2 values of 0.90 with grouped observations when that value is about 0.05 with the individual ones.

A final problem in the estimation of Engel curves has to be mentioned. We have arrived at heteroskedasticity with the grouping of observations by assuming that the individual ones were homoskedastic. But it is well known that the variance usually increases with income, so that our initial assumption is not adequate.

However, this problem, if not totally solved, is at least minimized when we use double-logarithmic Engel functions[1] or Engel functions where the dependent variable is expressed in expenditure shares[2] rather than in levels of expenditure.

5.5. Empirical Equivalence Scales for Portugal

In a first attempt to estimate equivalence scales for Portugal we shall use Engel's approach. Since it is based on the assumption that the proportion of total expenditure spent on food is an indicator of a household's real income (and welfare), the most immediate course of action is to estimate Engel curves for different household types where the dependent variable is precisely that proportion. If we further assume that the coefficient of the income variable will be the same for each household type and that different compositions will only affect the constant term we are led to the estimation of a single Engel function but with several sets of dummy variables to take account of the household's characteristics.

This has been done for the four regions in Portugal using the data on each individual household. The proportion of expenditure on food was considered in a 'lato sensu', that is, including drink. The sets of dummy variables were the same as in chapter 3 and they refer to the occupation of the head of the household, his age, the number of children

1. See Cramer (1969).
2. See Leser (1963).

and the household's location. Of course we are particularly interested on the equivalence scales for children.

The only non-dummy variable-income (total expenditure) was taken either in levels or in logarithms, i.e., two specifications for the Engel curves were considered,

$$W = a + k_1 D_1 + \dots + k_n D_n + b y \tag{15}$$

and

$$W = a + k_1 D_1 + \dots + k_n D_n + b \log y \tag{16}$$

where the D_i's are the dummy variables and the k_i's their coefficients. Expression (16) is the Working-Leser Engel curve (12) 'expanded' by the dummy variables. The regression results of the estimation of (15) and (16) are shown on tables 1 and 2 respectively.

Let us first estimate the equivalence scales for children using the results on table 1 for the North and Lisbon regions (since in the other regions we did not get significant coefficients for the dummy variables that refer to the number of children).

It is clear that the equivalence scales will vary with the level of income (or the proportion spent on food), but it is less clear whether the household's characteristics other than the number of children will affect the equivalence scales for children. To obtain a clear picture of the properties of the equivalence scales that are inherent to the specification (15) we can reason as follows:

From the expression

$$W = a + k_1 D_1 + \dots + k_n D_n + b y$$

TABLE 1

REGRESSIONS

(Dependent variable -
% of total expenditure spent on food and drink)

	NORTH	CENTRE	LISBON	SOUTH
Constant	63.8* (33.1)	60.5* (31.9)	54.9* (37.6)	65.0* (19.3)
D1 (Employers)	4.03* (7.38)	3.28* (5.88)	1.97* (3.21)	0.60 (0.61)
D2 (Workers)	2.41* (4.68)	2.82* (4.80)	2.95* (6.59)	3.10* (4.03)
D3 (Profess. etc)	-3.46* (-2.98)	-0.15 (-0.1)	-1.74* (-2.15)	4.76 (1.51)
D4 (Non-active, etc)	-1.83* (-2.99)	-0.96 (-1.36)	-1.33* (-2.35)	-1.01 (-0.95)
AH 25-45	0.90 (0.47)	2.76 (1.48)	3.83* (2.66)	1.94 (0.57)
AH 45-65	5.16* (2.72)	6.19* (3.35)	8.03* (5.64)	4.34 (1.30)
AH ⩾ 65	6.40* (3.29)	7.63* (4.02)	9.06* (6.12)	5.21 (1.54)
NC = 1	1.28* (2.07)	0.14 (0.22)	2.00* (3.97)	0.68 (0.71)
NC = 2	2.35* (3.51)	2.52* (3.54)	3.57* (5.85)	1.93 (1.67)
NC = 3	3.70* (4.76)	4.12* (4.59)	5.86* (6.37)	5.46* (2.90)
NC ⩾ 4	6.76* (9.47)	5.96* (6.70)	10.36* (9.54)	2.76 (1.31)
URBAN	-5.99* (-14.3)	-5.23* (-9.86)	0.58 (1.54)	-5.10* (-6.76)
INCOME	-0.129* (-37.0)	-0.140* (-34.2)	-0.120* (-40.2)	-0.170* (-21.7)
R^2 adj	0.39	0.33	0.30	0.32
F	215.9	162.4	184.7	64.5
St. Dev.	12.4	13.1	13.0	12.8

TABLE 2

REGRESSIONS

(Dependent variable -
% of total expenditure spent on food and drink)

	NORTH	CENTRE	LISBON	SOUTH
Constant	171.5* (45.6)	170.2* (43.1)	171.5* (50.6)	162.0* (25.5)
D1 (Employers)	4.67* (8.43)	4.04* (7.22)	1.91* (3.11)	0.52 (0.52)
D2 (Workers)	3.18* (6.10)	3.73* (6.35)	3.52* (7.90)	3.81* (4.85)
D3 (Profess. etc)	-6.0* (-5.16)	-4.67* (-3.02)	-4.03* (-5.06)	0.31 (0.1)
D4 (Non-active, etc)	-1.93* (-3.10)	-0.65 (-0.92)	-1.86* (-3.29)	-0.78 (-0.72)
AH 25-45	0.53 (0.28)	2.38 (1.27)	4.48* (3.11)	2.19 (0.63)
AH 45-65	4.83* (2.51)	5.76* (3.11)	8.25* (5.79)	4.31 (1.27)
AH ⩾ 65	4.29* (2.18)	5.08* (2.66)	6.99* (4.72)	3.49 (1.01)
NC = 1	2.10* (3.34)	1.42* (2.22)	2.66* (5.27)	1.40 (1.44)
NC = 2	3.56* (5.20)	3.66* (5.09)	4.13* (6.77)	2.41* (2.04)
NC = 3	4.64* (5.86)	5.41* (5.97)	6.08* (6.61)	6.08* (3.16)
NC ⩾ 4	8.23* (11.3)	7.76* (8.62)	10.01* (9.23)	1.85 (0.87)
URBAN	-5.35* (-12.5)	-4.80* (-8.96)	1.80* (4.69)	-5.42* (-7.02)
LOG (INCOME)	-10.73* (-34.9)	-11.03* (-33.7)	-11.53* (-40.2)	-9.99* (-19.6)
R^2 adj	0.38	0.33	0.30	0.29
F	201.1	159.4	184.6	56.6
St. Dev.	12.6	13.1	13.0	13.0

let, without loss of generality, the first l ($l < n$) dummy variables refer to the number of children. Let also the reference household be characterized by the variables D_i , D_{l+1} , ... , D_{l+n} , with $i = 1$, ... , l and assume a certain level of income $\bar{y}_i$.

For that level of income, the reference household has the proportion of total expenditure spent on food given by

$$\bar{W}_i = a + k_i + k_{l+i} + \dots + k_{l+n} + b\,\bar{y}_i$$

or

$$\bar{W}_i = k_i + k + b\,\bar{y}_i$$

with $k = a + k_{l+1} + \dots + k_{l+n}$

For another household j whose differentiating characteristic is expressed by the dummy variable D_j ($j = 1$, ... , l) , the level of income which implies the same proportion $\bar{W}_i$ is calculated from

$$\bar{W}_i = k_j + k + b\,y_i$$

or

$$\bar{y}_j = \frac{\bar{W}_i - k_j - k}{b} = \frac{k_i + k + b\,\bar{y}_i - k_j - k}{b}$$

$$= \frac{k_i - k_j + b\,\bar{y}_i}{b}$$

so that the equivalence scale is given by

$$\frac{\bar{y}_j}{y_i} = \frac{k_i - k_j + b\,\bar{y}_i}{b\,\bar{y}_i} = \frac{k_i - k_j}{b\,\bar{y}_i} + 1 \qquad (17)$$

This is a decreasing function of the income level of reference $\bar{y}_i$ and an increasing function of the difference between the dummy coefficients.

It is also independent of the other characteristics of the reference household (given by k). This means that in our case the equivalence scales for children are independent of the occupation of the head of the household, his age or the household's location (urban or rural area).[1]

Furthermore, the difference between the incomes of equal welfare, $\bar{y}_j - \bar{y}_i$ is equal to

$$\bar{y}_j - \bar{y}_i = \frac{k_i - k_j + b\,\bar{y}_i}{b} - \bar{y}_i$$

$$= \frac{k_i - k_j}{b}$$

which is a constant and is independent of the income level y. It implies that the cost of an additional child is always the same, whatever the household's income level. It does not follow, of course, that the cost of the third child will be equal to the cost of the first, second or any other child.

From table 1 the equivalence scales for households with one, two, three and four or more children were calculated assuming that the reference is the childless household. The scales are presented below for four values of y (30, 50, 80 and 150 thousands of escudos) on the standardised hypothesis that the scale for the childless household is equal to two:

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1. This is a general hypothesis in the theory of equivalence scales.

North

Income level	Childless household	With 1 child	With 2 children	With 3 children	With ≥ 4 children
30	2	2.66	3.21	3.91	5.49
50	2	2.40	2.72	3.15	4.10
80	2	2.25	2.46	2.72	3.30
150	2	2.13	2.24	2.38	2.70

Lisbon

Income level	Childless household	With 1 child	With 2 children	With 3 children	With ≥ 4 children
30	2	3.11	3.98	5.26	7.76
50	2	2.67	3.19	3.95	5.45
80	2	2.42	2.74	3.22	4.16
150	2	2.22	2.40	2.65	3.15

A childless household in the North region with an income of 30 thousands of escudos would be as well off as a household with three children and with an income of 58.7 (30 x 3.91 / 2) thousands of escudos. In Lisbon that value would be 78.9.

As income increases the equivalence scales markedly decrease (a consequence of (17)). The equivalence scales are higher in Lisbon than in the North, implying that the cost of an additional child is higher in this region. In effect, the costs of additional children in both regions are the following:

	North	Lisbon
1st child	9.92	16.67
2nd child	8.30	13.08
3rd child	10.46	19.08
4th child	23.72	37.50

These results would suggest that there are economies of scale from the first to the second child and diseconomies from the second to the third. Not much can be said from the third to the fourth since in this latter case we are dealing with an open class (four or more children).

If we now use the Engel curves (16) and adopt the same procedure as before, we arrive at the following expression:

$$\frac{\bar{y}_j}{y_i} = e^{\frac{k_i - k_j}{b}}$$

meaning that in this case the equivalence scale of an additional child is a constant, independent of the level of income. It will vary as the child considered is the first, the second, etc.

The cost of an additional child is now an increasing function of income,

$$\bar{y}_j - \bar{y}_i = \left(e^{\frac{k_i - k_j}{b}} - 1 \right) \bar{y}_i$$

Using the data on table 2, the equivalence scales are as follows:

	Childless households	With 1 child	With 2 children	With 3 children	With $\geq$ 4 children
North	2	2.44	2.78	3.08	4.30
Centre	2	2.28	2.78	3.26	4.04
Lisbon	2	2.52	2.86	3.36	4.76

The scales for the South were not computed since there are only two significant coefficients for the children's summy variables.

The scales for Lisbon are again higher than for the other regions. If we compare these scales with those computed from table 1 we can conclude that they are close to the scales obtained for the income level 50 in the North region and between 50 and 80 in Lisbon.

This exercise and the results obtained serve to emphasize the importance of the specification of Engel curves to the final values for the equivalence scales. That specification will also depend a great deal on the use that we intend to give to the estimated scales. Sometimes we are interested in 'allowing' equivalence scales to vary with income for instance for tax purposes[1], on other occasions it is enough to have scales that are independent of income.

The methodology adopted can be criticized on several grounds. Firstly, there is the general criticism of obtaining equivalence scales with Engel's approach, but this problem has already been dealt with; secondly, since the Engel curves were estimated from individual observations and total expenditure was used as an independent variable, we might have obtained biased estimates for our coefficients; thirdly, we are ignoring the effects of age and sex on the equivalence scales for children; fourthly, our reference households (childless households) are not homogeneous - they may be for example either a couple without children or a six adult household; and finally we are assuming that the income coefficient is the same for all household types without testing that hypothesis.

To obviate some of these criticisms we shall use another methodology which is based on the one put forward by Brown (1954):

Let us assume that a double-logarithmic Engel function adequately represents the relationship between individual expenditure and income (total expenditure) in each of the (previously) well defined and homogeneous household types. This assumption can of course be empirically tested.

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1. See e.g. Seneca and Taussig (1971) who used quadratic Engel functions.

Let the differences in household composition be adequately modelled by the Prais-Houthakker Engel function (5). If so, the Engel functions for each household type can be written as

$$\frac{e_i}{\sum_j w_{ij}\, a_j} = c_i \left(\frac{y}{\sum_j w_j\, a_j} \right)^{d_i} \qquad (18)$$

Where w_{ij} and w_j are again the commodity specific and income scales, a_j is the number of persons of type j in each household and c_i and d_i are constants. w_{ij} and w_j depend only on the type of person, not on the household category, e.g. the equivalence scales for an adolescent are assumed to be the same whatever the age, sex and number of the other persons living with him in the household.

Since the equivalence scales are constant for each household type h, we can write $w_{ih} = \sum_j w_{ij}\, a_j$, $w_h = \sum_j w_j\, a_j$, and the Engel function (18) becomes

$$\frac{e_i}{w_{ih}} = c_i \left(\frac{y}{w_h} \right)^{d_i}$$

when this function is estimated for each household type by using

$$\log e_i = a_i + b_i \log y$$

the estimate of a_i will be an estimate of log $(w_{ih}\, e_i\, w_h^{-d_i})$ and the estimate of b_i an estimate of d_i. This means that the estimated income elasticities should be the same for each household type, within sampling error, and this can also be tested by the use of a covariance analysis.

If the income elasticities are the same for each household type, then we can rewrite the Engel function (18) as

$$z_i = e_i \left(\frac{\Sigma w_j \, a_j}{y} \right)^{d_i} = c_i \left(\Sigma w_{ij} \, a_j \right) \qquad (19)$$

where e_i, y, a_j and d_i are known values. If we assume some likely values for w_j , the left hand side of (19) is a known quantity, implying that we can obtain estimates for the w'_{ij} by ordinary least-squares. Obviously these estimates will depend on the postulated values for w_j, but within certain limits no great variation will exist.

In order to use the above methodology, households were classified into 11 household types:

1 - Single
2 - Couple
3 - CI (couple and infant)
4 - C(II) (couple and two or more infants)
5 - CS (couple and child)
6 - CS(I) (couple, child and one or more infants)
7 - C(SS) (couple and two or more children)
8 - C(SS)(I) (couple, two or more children and one or more infants)
9 - CA (couple and adolescent)
10 - CA(SI)(couple,adolescent and one or more children or infants)
11 - C(AA) ($\overline{SI}$) (couple, two or more adolescents, with or without children or infants)

These household types can be considered as 'pure' families since no other adults beyond the parents are included. The infants (I) are defined as being less than 5 years, the children (S) 5 to 13 years and the adolescents (A) 14 to 17 years. The aim is to compute equivalence scales for the single and couple households and for the infants, children and adolescents. The reference household is the couple.

A further discrimination by sex was not possible since it would lead to an insufficient number of observations within certain categories. The adopted eleven household types represent 2278 households in the North region (52.5%), 2499 in the Centre (59.3%), 3455 in Lisbon (61.3%) and 1112 in the South (64.4%).

The Engel curves for each household type were estimated for three aggregates: food, food + drink, food + drink + clothing + footwear, so that we will obtain three sets of equivalence scales corresponding to those aggregates.

For each household type the average expenditure on each aggregate and average total expenditure were computed for each of the eight income classes the households said they belonged to.[1] The Engel curves were estimated by weighted least-squares, taking into account the number of observations in each income class. As seen previously this procedure should lead to consistent estimates of the income coefficient.

The four types of Engel functions (9) to (12) were estimated for each case, the significance of the coefficients, the R^2's and the plots of the actual and fitted values were examined and the conclusion was that the double-logarithmic specification was on the whole at least as good as any of the others.

For each household type Engel curves were estimated when there were at least six observations. The results are shown on tables 3 to 5. An analysis of covariance was carried out to test for the equality of all the slopes (elasticities) to a common value. That common value is calculated from the individual regressions and it represents the average within class slope.[2] The corresponding F test always rejects that

1. Tables A41 to A44 in the appendix show all these values for the four regions in relation to the aggregate 'food + drink + clothing + footwear.'

2. See e.g. Blalock (1979).

TABLE 3

Income Elasticities per Household Type
(Double logarithmic Engel functions) FOOD

	NORTH		CENTRE		LISBON		SOUTH	
	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2
SINGLE	0.664 (.03)	0.99	0.686 (.07)	0.94	0.656 (.07)	0.94	0.651 (.06)	0.97
COUPLE	0.695 (.04)	0.98	0.614 (.04)	0.97	0.671 (.01)	0.98	0.674 (.04)	0.98
CI	0.535 (.05)	0.95	0.448 (.09)	0.85	0.630 (.06)	0.95	0.373 (.13)	0.66
CS	0.633 (.08)	0.92	0.526 (.06)	0.93	0.677 (.08)	0.93	---	--
CA	0.488 (.08)	0.86	0.506 (.10)	0.81	0.750 (.03)	0.99	0.513 (.06)	0.94
C(II)	0.570 (.06)	0.94	0.567 (.04)	0.97	0.540 (.07)	0.90	0.635 (.07)	0.95
C(SS)	0.538 (.02)	0.99	0.581 (.07)	0.93	0.526 (.07)	0.90	0.645 (.10)	0.91
C(SI)	0.676 (.10)	0.89	0.691 (.04)	0.98	0.519 (.08)	0.90	0.536 (.22)	0.54
C(SS)(I)	0.553 (.07)	0.92	0.593 (.04)	0.97	0.643 (.05)	0.97	0.638 (.08)	0.93
CA(SI)	0.496 (.13)	0.78	---	--	---	--	---	--
C(AA)($\overline{SI}$)	0.624 (.08)	0.92	0.703 (.08)	0.93	0.697 (.18)	0.75	0.575 (.23)	0.62
Average within class	0.643		0.611		0.654		0.632	

TABLE 4

Income Elasticities per Household Type
(Double logarithmic Engel functions) FOOD AND DRINK

	NORTH		CENTRE		LISBON		SOUTH	
	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2
SINGLE	0.638 (.05)	0.97	0.698 (.02)	0.99	0.653 (.06)	0.95	0.684 (.07)	0.96
COUPLE	0.637 (.04)	0.97	0.575 (.05)	0.95	0.656 (.02)	0.99	0.687 (.05)	0.97
CI	0.441 (.04)	0.94	0.378 (.09)	0.78	0.598 (.06)	0.94	0.393 (.12)	0.73
CS	0.435 (.08)	0.85	0.488 (.09)	0.83	0.722 (.04)	0.98	0.535 (.04)	0.97
CA	0.495 (.05)	0.92	0.547 (.05)	0.96	0.601 (.05)	0.96	0.603 (.09)	0.90
C(II)	0.558 (.08)	0.89	0.526 (.07)	0.89	0.646 (.07)	0.98	---	--
C(SS)	0.468 (.02)	0.99	0.528 (.06)	0.93	0.494 (.07)	0.92	0.671 (.09)	0.93
C(SI)	0.536 (.06)	0.93	0.507 (.06)	0.92	0.482 (.06)	0.89	0.626 (.08)	0.93
C(SS)(I)	0.621 (.11)	0.84	0.602 (.02)	0.99	0.475 (.08)	0.91	0.492 (.22)	0.50
CA(SI)	0.285 (.16)	0.45	---	--	---	--	---	--
C(AA)($\overline{SI}$)	0.512 (.07)	0.92	0.661 (.07)	0.95	0.703 (.18)	0.76	0.571 (.25)	0.57
Average within class	0.578		0.591		0.630		0.644	

TABLE 5

Income Elasticities per Household Type (Double logarithmic Engel functions)   FOOD + DRINK + CLOTHING + FOOTWEAR

	NORTH		CENTRE		LISBON		SOUTH	
	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2	Elasticity	R^2
SINGLE	0.718 (.04)	0.98	0.742 (.05)	0.98	0.738 (.05)	0.97	0.706 (.07)	0.96
COUPLE	0.720 (.04)	0.98	0.667 (.05)	0.97	0.718 (.02)	0.99	0.741 (.05)	0.98
CI	0.540 (.04)	0.97	0.486 (.08)	0.86	0.629 (.05)	0.96	0.502 (.09)	0.88
CS	0.655 (.08)	0.92	0.592 (.07)	0.92	0.667 (.06)	0.95	---	--
CA	0.542 (.07)	0.92	0.581 (.10)	0.86	0.739 (.03)	0.99	0.593 (.05)	0.97
C(II)	0.607 (.04)	0.97	0.583 (.06)	0.94	0.546 (.05)	0.95	0.654 (.07)	0.95
C(SS)	0.575 (.03)	0.99	0.629 (.05)	0.97	0.553 (.06)	0.93	0.765 (.08)	0.96
C(SI)	0.682 (.08)	0.92	0.702 (.04)	0.99	0.506 (.05)	0.95	0.691 (.14)	0.82
C(SS)(I)	0.553 (.06)	0.94	0.689 (.04)	0.98	0.612 (.05)	0.96	0.654 (.09)	0.91
CA(SI)	0.467 (.15)	0.69	---	--	---	--	---	--
C(AA)($\overline{SI}$)	0.694 (.06)	0.97	0.696 (.05)	0.97	0.695 (.14)	0.84	0.636 (.21)	0.70
Average within class	0.674		0.663		0.692		0.700	

possibility, so that the elasticities were found to be significantly different between the eleven household types.

A similar finding played an important role in Forsyth's (1960) conclusion that empirical equivalence scales cannot be computed. He used only six observations also related to group means. However, as Kemsley pointed out in the discussion of Forsyth's paper, the computation of variances and covariances from group means does not necessarily provide identical values to those obtained from individual observations. Our conclusion about the existence of different elasticities must then be properly qualified. In any case the average within class slopes constitute 'best guesses' for common elasticity values, and as such they were used in the subsequent calculations.[1]

Entering those estimates for d_i in (19) and knowing the average number of each type of person in each household class (see table 6), the w_{ij} were computed from (19) using OLS under two hypotheses: in the first (H1) the income scales for infants, children and adolescents were set equal to unity; in the second (H2) those scales were assumed to be 0.6, 0.7 and 0.8 respectively. The best estimates were obtained with no constant term, and the coefficient estimates were always statistically significant.

The resulting equivalence scales are shown on tables 7 and 8, again under the normalizing assumption that the value for a couple is 2. The equivalence scales thus estimated may be deemed as 'plausible' or 'sensible', i.e., there is no 'a priori' basis for rejecting them. The exception is the South region where there is the tendency for adolescents (A) to have lower values than children (S). The explanation might lie on

1. Although the confidence on the resulting estimated scales will always be undermined by the failure, or at least inconclusiveness of the test.

TABLE 6

Average Number of Children per Family Type

	NORTH			CENTRE			LISBON			SOUTH		
	I	S	A	I	S	A	I	S	A	I	S	A
Single	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Couple	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CI	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0
C(II)	2.2	0.0	0.0	2.1	0.0	0.0	2.1	0.0	0.0	2.0	0.0	0.0
CS	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0
CS(I)	1.5	1.0	0.0	1.2	1.0	0.0	1.1	1.0	0.0	1.1	1.0	0.0
C(SS)	0.0	2.6	0.0	0.0	2.4	0.0	0.0	2.2	0.0	0.0	2.2	0.0
C(SS)(I)	1.3	2.7	0.0	1.3	2.6	0.0	1.2	2.4	0.0	1.1	2.4	0.0
CA	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	1.0
CA(SI)	1.1	2.1	1.0	1.1	1.9	1.0	1.2	1.6	1.0	1.2	1.7	1.0
C(AA)($\overline{SI}$)	0.1	0.7	2.0	0.0	0.7	2.1	0.1	0.6	2.1	0.1	0.6	2.0
All households	0.3	0.8	0.3	0.2	0.5	0.2	0.2	0.4	0.2	0.1	0.3	0.2

Note: I - Infants (0-4 years)
S - Children (5-13 years)
A - Adolescents (14-17 years)

TABLE 7

Equivalence Scales using
Brown's Methodology (H1)*

FOOD

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.09	1.09	1.03	1.09	1.07
Couple	2.00	2.00	2.00	2.00	2.00
I	0.65	0.64	0.68	0.67	0.66
S	0.66	0.73	0.74	0.75	0.72
A	0.79	0.75	0.81	0.75	0.78

FOOD + DRINK

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.04	1.05	0.99	1.06	1.03
Couple	2.00	2.00	2.00	2.00	2.00
I	0.60	0.62	0.68	0.67	0.64
S	0.61	0.70	0.72	0.78	0.69
A	0.81	0.75	0.81	0.75	0.79

FOOD + DRINK + CLOTHING + FOOTWEAR

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.00	1.01	0.98	1.02	1.00
Couple	2.00	2.00	2.00	2.00	2.00
I	0.72	0.73	0.74	0.75	0.73
S	0.74	0.81	0.80	0.88	0.80
A	0.96	0.89	0.92	0.85	0.91

(*) In this hypothesis the income scales for the infants (I), children (S) and adolescents (A) are assumed to be unity

TABLE 8

Equivalence Scales using
Brown's Methodology (H2)*

FOOD

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.16	1.16	1.09	1.13	1.13
Couple	2.00	2.00	2.00	2.00	2.00
I	0.44	0.43	0.46	0.37	0.44
S	0.50	0.57	0.57	0.66	0.56
A	0.68	0.64	0.69	0.59	0.66

FOOD + DRINK

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.10	1.11	1.10	1.13	1.11
Couple	2.00	2.00	2.00	2.00	2.00
I	0.35	0.43	0.46	0.44	0.42
S	0.47	0.56	0.55	0.61	0.54
A	0.72	0.65	0.70	0.63	0.68

FOOD + DRINK + CLOTHING + FOOTWEAR

	NORTH	CENTRE	LISBON	SOUTH	WEIGHTED AVERAGE
Single	1.07	1.07	1.05	1.09	1.06
Couple	2.00	2.00	2.00	2.00	2.00
I	0.49	0.49	0.49	0.50	0.49
S	0.57	0.63	0.61	0.68	0.61
A	0.82	0.77	0.79	0.72	0.78

(*) In this hypothesis the income scales are assumed to be: 0.6 for the infants (I), 0.7 for the children (S) and 0.8 for the adolescents (A).

the lack of accuracy which results from the reduced number of observations in many household types in this region (see table A44 in the appendix).

Another conclusion is that there is no substantial variation between regions so that we may be legitimately tempted to compute equivalence scales for the Continent as a whole. That was done by a weighted average of the scales in each region, the weights being the number of 'pure' households in each region. These averages are also shown on the tables.

The hypothesis H1 is clearly an extreme one, while H2 is much more plausible. In the next chapter we will use the equivalence scales for the Continent obtained under H2 for the aggregate 'food + drink + clothing + footwear', which is,

Single	-	1.06
Couple	-	2
I	-	0.49
S	-	0.61
A	-	0.78

We call this scale the HES (household expenditure survey) scale.

Chapter 6

Inequality and Differential Inflation in Portugal

6.1 Inequality Measures

Inequality measures are usually employed in economics for comparisons of income distributions which refer to one or more countries or regions. More often we want to be able to rank these distributions by the degree of inequality and the inequality measures are the main tool in reaching this objective.

Most of the measures intend to capture in a single real number the degree of inequality in a given distribution. Since there is a complete ordering within the real numbers it follows that we can always unequivocally rank any two distinctions. We shall see later that this fact may lead to very misleading conclusions. Furthermore, and as Sen (1973) points out, maybe our notion of inequality is inherently incomplete, i.e., maybe there are frequent situations when we cannot compare two distributions, in which case the inequality measures would be inadequate to form a ranking.

We can divide the measures presented in the literature into two categories: the 'objective' measures, usually employing some statistical indicator and with no reference to social welfare, and the 'normative' ones which are built up in such a way that a higher degree of inequality corresponds to a lower level of social welfare. However, it is very difficult to separate out completely the objective and normative aspects so that in the 'objective' measures we can consider 'normative' aspects and vice versa.

'Objective' measures of inequality

One of the simplest measures is the range (R). It can be defined as the difference between the highest and the lowest income levels as a ratio of mean income. Thus R is expressed by,

$$R = (\text{Max } y_i - \text{Min } y_i) / \mu$$

Where y_i is the income of person i (i = 1,..., n) and μ is the mean income.

The trouble with the range is that it ignores the distribution in between the extremes and so it is not a very useful indicator.

The relative mean deviation (RMD) considers all the distribution and can be described as the sum of the absolute values of all the differences between each income level and the mean income, as a proportion of total income. That is,

$$RMD = \sum_i | y_i - \mu | / \sum_i y_i$$

With perfect equality RMD = 0 and with all income going to one person, RMD = 2 (n - 1)/n. The main problem with this measure is that it is not sensitive to transfers of income on the same side of the mean income. If we accept the so-called Pigou (1912) - Dalton (1920) condition, which states that any transfer from a poorer person to a richer one must always increase the degree of inequality the RMD violates this condition.

The variance, defined as

$$V = \sum_i (y_i - \mu)^2 / n$$

clearly satisfies the Pigou-Dalton conditions, but it depends on the mean income level. Two distributions with different mean income levels cannot be compared by their variance. However, this can be corrected by using the coefficient of variation (CV), which is the standard deviation of the distribution (the positive square root of the variance), divided by the mean income level,

$$CV = V^{\frac{1}{2}}/\mu$$

The coefficient of variation attaches equal weights to transfers of income at different income levels. That is, a transfer of income say, from a person with income level 100 to one with 90 is considered equal to a similar transfer from a person with income level 1 000 100 to one with 1 000 090. We are tempted to say that a greater importance should be given to the former transfer. However, this is a normative statement and would not be universally accepted.

If we want to emphasize the lower end of the distribution, we can use the variance of the logarithms (VL) - or its square root, the standard deviation of the logarithms - this can be expressed as

$$VL = \sum (\log y_i - \log \mu)^2/n$$

Since the logarithms reduce the differences at high income levels, then VL as a measure of inequality will cause problems if we think the social welfare function should be a concave function of individual incomes (a normative aspect). The variance of logarithms may violate the Pigou-Dalton condition because it becomes very insensitive to transfers among the rich. There can be a negative impact on the variance of logarithms indicating a decrease in equality arising from a transfer of income from a relatively rich household to an even richer one. Finally, it needs to be said that this measure should be adopted if we believe the incomes are lognormally distributed.[1]

Theil (1967) has proposed a measure that is derived from the notion of entropy in information theory. Theil's measure is defined as

$$T = \sum_i s_i \log s_i/p_i$$

1. See Aitchison and Brown (1957). For Portugal I have concluded that the lognormal distribution is a reasonable approximation to the distributions of the basic wages and salaries in industry (see Santos (1980) and (1983)).

where s_i is the income share of group i and p_i is the population share of the same group. When per capita income is the same in all classes, s_i / p_i is unity for each group and Theil's measure is zero. When all income belongs to one individual or group, the index value is log N, where N is the number of individuals or groups. Perhaps the main attraction of Theil's index is its aggregation properties which enable the easy decomposition of the total income inequality into its components.[1]

Some other measures could be stated and discussed, as for example Kuznets's measure, Champernowne's measure and some other statistical measures of dispersion like the interdecile or interquantile measures[2]; but perhaps one of the best ways of throwing some light onto the problems of inequality measurement is to discuss, in length, one of the most used indicators of inequality in economic theory: the Gini (1912) coefficient and the related Lorenz curves.

Before that we note that, with the exception of the variance, all the measures studied (including the Gini coefficient), have one property in common: they are invariant if everyone's income is raised in the same proportion. Is this desirable? Should we give greater importance to inequality when a society is rich (since it 'can afford to be inequality-conscious') as to when it is poor? Again this is a normative question about which there is unlikely to be a universal answer.

Let us return to the Lorenz curves and the Gini coefficient. If we plot on a graph the proportion of the population (arranged from the poorest to the richest) on the abcissa, and the percentages of income received by the bottom % of the population on the ordinate we obtain a Lorenz curve.

2. See e.g. Fishlow (1972).

3. The interdecile or interquantile measures are often utilized and they constitute a privileged way of observing directly what happens to the different income groups. They are closely linked to the Gini coefficient as they are usually obtained from the intermediate calculations needed to compute it.

As 0% of the population receive 0% income and 100% of the population receive 100% of income, the Lorenz curve has these two points as its limits. The length of the axis is 100% or 1. If there is absolute equality it happens that x% of the population will always receive x% of the income.

So, the diagonal of the square (see fig.1) represents the line of absolute equality.

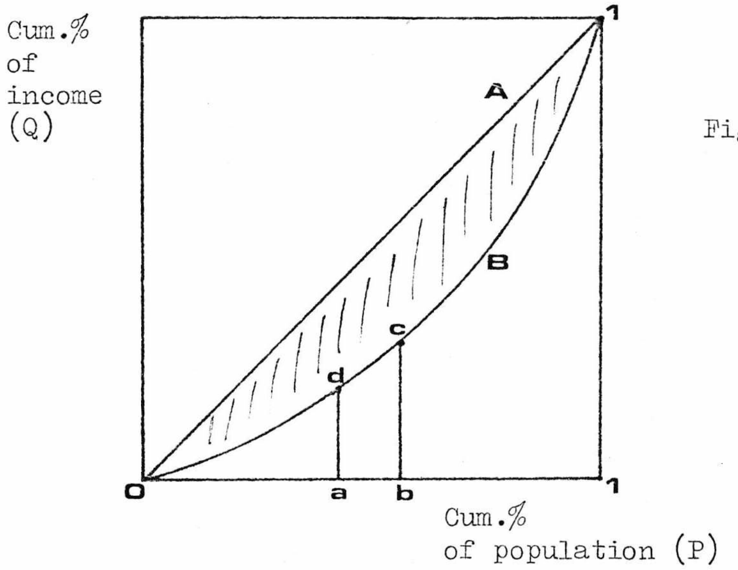

Fig.1. Lorenz curve

A - Line of absolute equality

B -- Lorenz curve

Since in all empirical income distributions the bottom income groups have received proportionately less income, the Lorenz curve will lie below the diagonal and its slope will not fall as we move to increased shares of the population.[1]

The Gini coefficient of concentration can be given an interpretation in terms of fig.1: it is the ratio of the shaded area (between the Lorenz curve and the line of absolute equality) to the area of the triangle below (or above) the line of absolute equality. Perhaps this possibility of a suggestive visualization of the Gini coefficient is one of the causes for its popularity.

The Gini coefficient of concentration is closely related to the coefficient of mean difference (also due to Gini). The coefficient of

1. See Kendall (1963) for a mathematical proof of these statements.

mean difference is the average of the differences of all the possible pairs of (in this case) income values, taken regardless of sign. The coefficient of concentration is equal to the coefficient of mean difference divided by twice the mean income (see Kendall 1963). From this close relationship we can draw some interesting conclusions:

- The Gini coefficient takes differences over all pairs of incomes while the other measures already studied only consider differences to the mean
- The differences considered are absolute differences and not proportionate differences
- The sensitivity of the Gini coefficient depends not on the size of the income levels but on the numbers of people in between them. The rate of substitution between the person with the i-th highest income and the one with j-th highest income is j/i (see Sawyer (1976). This means that, given a transfer between any two persons, the influence on the Gini coefficient will be greatest, the more people exist between these income levels. It also implies that the Gini coefficient has more sensitivty to transfers in the middle income levels.

There are several ways of calculating the Gini coefficient. Usually the data on income distribution is presented in grouped form, so that we can only plot a finite number of points of the Lorenz curve, such as poinds d and c in fig.1. The Lorenz curve is then obtained by interpolation between these points. If the interpolation is made by straight lines, the formula for the Gini coefficient (using the trapezoidal rule) is given by (see fig.1)

$$G = 1 - \sum (ab) \quad (ad + bc)$$

or

$$G = 1 - \sum_{i} p_i \, (Q_i + Q_{i-1}) \qquad (1)$$

Where p_i is the percentage of population in class i and Q_i the cumulative proportion of income up to that class. Formula (1) provides of course a lower bound for the Gini coefficient, but is quite satisfactory if the number of population groups is eight or more.[1]

A more sophisticated method requires the fitting of a continuous function to the plotted points (e.g. polynomial, cubic-spline, etc) and then by integration the computation of the area of inequality. This is rarely necessary, but as we shall see in the next section sometimes may be indispensable.

The Gini coefficient will be zero when there is perfect equality (the Lorenz curve is on the diagonal) and unity when the income belongs to one person or income group.

The Gini coefficient has been subject to several criticisms. For instance, Schutz (1951) argues that the shapes of the areas between the Lorenz curve and the line of absolute equality may be infinitely varied without any change in the value of the ratio of concentration. From this we can see the 'crudity' and 'ambiguity' of that measure. He proposes instead the use of an alternative indicator: the slopes of the points on a Lorenze curve. The argument is that this measure is more directly related to inequality as we are comparing 'the amounts of income that individuals or groups get with their proportionate-equal shares.'

While it is true that there is some relative insensitiveness of the Gini coefficient, the measure proposed by Schutz is nothing else than the relative mean deviation and it is thus insensitive to transfers between people on the same side of the mean. The improvement may not exist at all!

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1. See Gastwirth (1972).

Eltëto and Frigyes (1968) emphasize again the relative insensitivity of the concentration ratio as well as the difficulties in computing it from empirical data (due to the several approximations, and therefore errors that are committed). The measures they propose to obviate these deficiencies are:

$$u = \frac{m}{m_1} \quad , \quad v = \frac{m_2}{m_1} \quad , \quad w = \frac{m_2}{m} .$$

where

$$m = E\ (\ y\ ),\ m_1 = E\ (\ y\ /\ y \leq m\ ),\ m_2 = E\ (\ y\ /\ y \geq m\ )$$

and y is the income of an income unit selected at random.

That is, m is the mean income, m_1 (m_2) is the mean income of those with an income smaller (equal to or greater) than m. v is a measure of inequality for the entire distribution and u and w for the lower and upper parts, respectively. The range of the measure is $[1, +\infty]$ and so their sensitivity is, they say, greater than in the Gini coefficient.[1] The advantages of these indicators are, following their authors; plausible economic interpretation; easy to compute even from grouped data; suited not only to express the degree of the income inequality, but are decomposable in such a way that the components so obtained show to what extent certain factors contribute to the inequality; have geometric interpretation related to the Lorenz curve and can be simply expressed with the parameters of the distributions usually used in income distribution, as for instance the two-parameter lognormal distribution.

There is no room here for describing in full the reasons for these desirable attributes, some of which are evident. However, as Atkinson (1970)

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1. Which is not true, as there are the same number of points between 0 and 1 as between 1 and ∞.

has observed, the three measures are simple transforms of the relative mean deviation and as such they suffer from the same criticism as Schultz's measure does.

An interesting point was raised by Paglin (1975) in relation to the use of the Lorenz curves and the Gini coefficients in comparisons of income distributions over time.[1] The problem is that we are measuring inequality in relation to the line of perfect equality, and this line implies that everybody should have exactly the same income. He argues that no single person or society would agree with such a notion of equality since it is inevitable, for instance, that young and old adults (or families) will have considerably less income than middle age ones. The line of equality would 'overspecify' the conditions of equality.

He argues that perfect equality should be defined as a situation in which all families have equal lifetime income, not the same income during their lifetime. However there are considerable and insurmountable difficulties in computing lifetime incomes for a growing economy. Paglin proposes that for each year the reference line be the Lorenz curve that is obtained for the age distribution of incomes (the P- reference line) instead of the perfect equality line. His measure of inequality, which he calls the Paglin-Gini would be equal to the difference between the (normal) Gini coefficient and the Gini coefficient obtained for the age distribution of incomes. The lifetime income is so assumed to be equal to the age distribution of incomes in each year.

Within this framework Paglin was able to conclude that estimates of inequality in the U.S. had been overstated by 50 per cent and that inequality from 1947 to 1972 had declined by 23 per cent (contrary to the general opinion of its constancy).

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1. Although his remarks will hold good for other inequality measures too.

Not surprisingly Paglin's article originated a considerable controversy[1], but from it we may conclude that the point raised by Paglin is an important one and that although his proposed measure is not a perfect way of solving the problem it goes a long way in the right direction and makes us question our inequality notions.

If we go back to the 'traditional' Lorenz curves and suppose that we are comparing two income distributions, e.g. either in the same country but in different dates or between different countries but at the same date, let the Lorenz curves be as depicted in fig.2, where they intersect once.

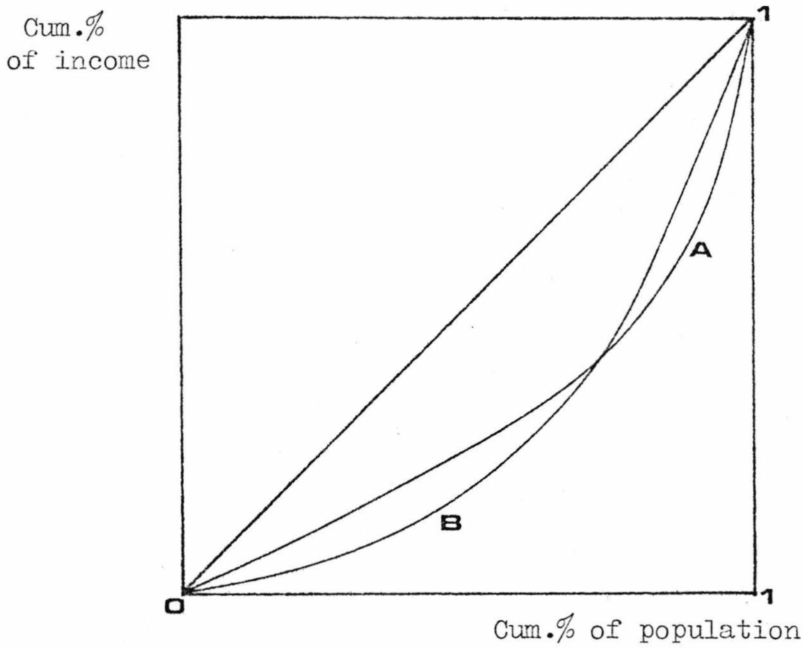

Fig.2 Interesecting Lorenz curves

It is easily seen that distribution A is more equal at the bottom and distribution B at the top. If we use the measures already studied to rank both distributions we will obtain contradictory results:

1. See Danziger, Haveman and Smolensky (1977), Nelson (1977), Johnson (1977), Minarik (1977), Kurien (1977), Paglin (1977, 1979) and Wertz (1979).

sometimes distribution A will be considered more equal, sometimes distribution B. The reason is that, as we have seen, some measures give more weight to the bottom of the distributions, while others give it to the middle or to the top.

In such a case what is our final decision? It depends on our personal judgements (either to favour the equality at the bottom or at the top) or sometimes on the purpose of the study being carried out. However we can say that one of the first things to do when studying income inequality is to plot the Lorenz curves. If they intersect it is advisable to calculate always more than one indicator. If not it will depend on the kind of study we are doing, but surely the inequality measures will not rank distributions consistently. But we will return to this matter when discussing the 'normative' measures.

Normative measures

As we have seen, underlying the usual statistical measures of income inequality there is some concept of social welfare. Dalton (1920) argued that we should build up such measures starting precisely from the form of the social welfare function. His main assumptions are that the level of social welfare is given by the sum of the individual utilities (an 'utilitarian' assumption) and that there is a strictly concave utility function for all the individuals.

Dalton's measure relates the actual levels of aggregate utility and the level of total utility that would be obtained if income were equally divided. It follows from his assumptions that the maximum social welfare is obtained when income is equally divided, so that his measure is the ratio of the actual social welfare to the maximum social welfare attainable with the present levels of income.

Atkinson (1970) further developed Dalton's approach. He criticized the Dalton measure on the grounds that it is not invariant with respect to positive linear transformations of the utility function. He also assumed a social welfare function whichis an additively separable and symmetric function of individual incomes. Thus, the distributions should be ranked according to:

$$w = \int_{o}^{y'} U(y)\ f(y)\ dy \qquad (2)$$

where $o \leq y \leq y'$

He further assumes that the utility functions U (y) are increasing and concave (both traditional assumptions). From this point on, he relies heavily on the results already acquired in the field of decision-making under uncertainty,[1] as the formal problems are identical: ranking distributions according to (2) is formally identical to ranking probability distributions f (y) according to expected utility; saying that U (y) is concave is equivalent to the proposition that a person is risk averse.

He then proves an important result: when the Lorenz curves of two distributions do not intersect there is no ambiguity in their ranking. Even using different utility functions our ranking will always be the same whatever the inequality measure used, with the only proviso that the utility functions be increasing and concave: the distribution closer to the diagonal is more equal than the other. It should however be borne in mind that these results were obtained for distributions with equal means. Sen (1973) proves that Atkinson's result can be extended to the case when

1. See Rothschild and Stiglitz(1969), Hadar and Russel (1969), Hanoch and Levy (1969).

the social welfare function is non-additive, but emphasizes that the case of different means is much more serious.

To form a complete ranking of the distributions, U (y) needs to be specified up to a monotonic linear transformation. To fulfill this requirement, Atkinson introduces the concept of an 'equally distributed equivalent level of income' (y_{EDE}) which is defined as the level of income per head that if equally distributed would give the same level of social welfare as the present distribution.

$$U(y_{EDE}) \int_{0}^{y'} f(y)\, dy = \int_{0}^{y'} U(y)\, f(y)\, dy$$

The measure of inequality he proposes is then,

$$A = 1 - \frac{y_{EDE}}{\mu}$$

The lower (A), the lower is inequality. (A) can also have a suggestive interpretation. If for example A = 0.4 then we can conclude that if income were equally distributed only 60% of the present income would be necessary to achieve the same level of social welfare. If we require (A) to be independent of the mean level of incomes (which is the case with most other inequality measures) - that is, when distributions C = f (y) and D = f (ky), (k = constant) have the same degree of inequality - then U (y) has the form[1]:

$$U(y) = A' + B \frac{y^{1-e}}{1-e}, \quad e \neq 1$$

$$\text{or} \quad U(y) = \ln(y), \quad e = 1$$

and $e \geq 0$ for concavity.

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1. See Arrow (1965), Pratt (1964) on constant relative inequality - aversion.

In the case of discrete and grouped distributions (those found in practically all empirical studies), it implies that Atkinson's measure be defined as

$$A = 1 - \left[\sum_i \left(\frac{y_i}{\bar{y}} \right)^{1-e} f_i \right]^{1/1-e}$$

Where y_i is the income of those in the ith income range, f_i is the proportion of the population with incomes in the ith range and $\bar{y}$ is the mean income. We must choose the value for e which is a measure of the degree of 'inequality aversion'. As e increases more weight is given to transfers at the bottom of the distribution. $e \longrightarrow \infty$ implies that only transfers to the very lowest income group are taken into account; $e = 0$ indicates distributions are ranked according to total income. As an empirical finding, it has been observed that with $e = 1$, A will rank distributions in approximately the same way as the Gini coefficient, and with $e = 2$ approximately as the standard deviation of the logarithms.

The case for using the Atkinson measure against the Gini coefficient was emphasized by Newbury (1970) who has shown that there exists no additive utility function which ranks income distributions in the same way as the Gini coefficient. However Sheshinski (1972) gives an example of a non-additive group welfare function which does reflect the Gini ranking.

Atkinson's work was important in several respects: firstly it emphasized the degree of arbitrariness that can be introduced when ranking income distributions only by the use of one statistical measure, particularly when the Lorenz curves intersect; secondly it definitely ruled out any pretending neutrality of the statistical inequality measures relating to social welfare attitudes; and thirdly it led to a new inequality measure, constructed no doubt from a criticizable (utilitarian) framework, but whose best characteristic is the existence of a parameter (e) directly related to the importance that we give to the transfers of income along the income scale.

As a practical matter it remains the fact that in any empirical work we should always compute more than one inequality measure because each of them will reveal some aspect of inequality that the others will not.

6.2. Inequality and the Differential Inflation - The Empirical Evidence.

When analysing the behaviour of the price indices by income class (chapter 3) we observed the general tendency for price indices to decrease with income.[1] However, significant differences were found to exist between the four regions.

The purpose of this section is to characterize the inequality in the regions and to analyse the effects on it of differential inflation, by using some of the inequality measures studied in the previous section. At first we will deal with the historical experience in Portugal by considering the two (longer) periods 1971-76 and 1977-81, and next we will simulate the effects on inequality of 20% price increases in each of ten expenditure aggregates.

As indicated in chapter 3 the choice of those two periods is due to the marked differences in price information between them. Again, using the same notation the index P71-76 measures the inflation between January 1971 and January 1977 and it relates to the price information obtained in the six cities ('old' price indices). For the period January 1977 to January 1982, the price index P77-81 refers to the Continent prices (equal in all the regions).

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1. Income was also the single most important variable in the 'explanation' of price differentials.

The methodology used for the determination of the impact of inflation on inequality is as follows:

1) The income distribution is characterized in one year (1973/74 - the year in which the HES was carried out), using several definitions of the income y.

2) That distribution is assumed to be the same in all the years of the analysed inflationary period (1971-1981).[1]

3) For each individual household a new variable is computed, y/P, where P is the price index observed in one of the two sub-periods in question.

4) Finally, comparison of the distributions of y and of y/P, by comparing their respective inequality measures, indicates the effects of the differential inflation on income distribution.

This is a straightforward method of isolating the differential effects of inflation. However it does have deficiencies. Perhaps the most important one is the assumption of the constancy of the income distribution over time with the concomitant implication of the constancy of the expenditure patterns.[2]

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1. It should be noted that the 'same' income distribution does not imply the same incomes for all the households over all the years. It allows increases or decreases in them, provided they are in the same proportion for all the households. The reason is that the inequality measures employed are invariant to those proportional variations.

2. We could also have used the cost-of-living indices computed in chapter 4 to measure the effects of the differential inflation on inequality. However, the criticisms made to those CLI's (see chapter 4) and also the opportunity that the price indices provide to use and analyse all the data available, strongly favour their sole consideration.

The inequality measures utilized are the Gini coefficient and the Atkinson index (for three values of e - 1, 1.5 and 2). They are common measures of inequality and their simultaneous consideration should provide enough information for adequately describing inequality. They are computed from grouped data and in order to improve their accuracy, thirteen income groups were identified,[1] instead of the eight so far employed in the previous chapters. The household's total consumption expenditure was again taken as a proxy to income.

The distribution of the households' incomes is sometimes the only distribution studied in a HES. As the previous chapter on equivalence scales made clear, that procedure is incorrect since it does not take into account the households' composition. In our case that composition is taken into consideration by computing for each household the income per consumption unit (using the ILO scale) and the income per adult equivalent (using the HES scale estimated in the previous chapter) and by analysing the resulting distributions.

To study the impact of the differential inflation, two new variables were created from each of those three, by dividing them either by P71-76 or by P77-81. That is, in addition to the variables income, income per consumption unit and income per adult equivalent, we create the variables:

income/P71-76
income/P77-81
income per consumption unit/P71-76
income per consumption unit/P77-81
income per adult equivalent P/71-76

and finally income per adult equivalent/P77-81

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1. Those groups were: under 10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70, 70-80, 80-90, 90-100, 100-120, 120-180 and $\geq$180 thousands of escudos.

An additional complication lies in the fact that since the HES scale was computed only for 'pure' households (see chapter 5), the variables based on that scale could rigorously only be considered within that subsample. As a consequence, the distributions of all the other variables were examined either for the 'all households' and the 'pure households' samples. The values of the inequality measures for all the distributions are shown in tables 1 to 6 (all households) and 7 to 15 ('pure' households).

The first conclusion is that the households' income distribution is more equal in Lisbon, followed by the North (see table 1). As concerns the Centre and the South, the different rankings obtained for two values of the inequality aversion parameter e, in the Atkinson measure is a sign that their Lorenz curves intersect. The Centre is more equal for the top incomes and more unequal for the bottom ones.[1]

It is interesting to compare these results with the Paglin-Gini measure. To calculate the Paglin-Gini, the age distribution of incomes was studied in all the regions by considering six age classes: under 25, 25-35, 35-45, 45-55, 55-65 and $\geq$ 65 years of age. Since the Paglin-Gini is the value obtained by the difference between the Gini coefficient and the Age-Gini, some significant errors may occur if we calculate those measures using formula (1) and a different number of points (13 for the income distribution and 6 for the age distribution). For that reason a cubic-spline was fitted to all the distributions and the Gini coefficients calculated by integration.

1. We can see from table 7 that the corresponding income distributions for the 'pure'families have some distinct features, such as the substantial reduction in inequality in the South, leading to the crossing of its Lorenz curve with that for Lisbon. Although these are interesting aspects, perhaps revealing the importance of the existence of other adults in the household (eventual additional wage earners) they are not essential in our study. We will base our conclusions on the 'all households' sample and will use the results for the 'pure' households only to draw conclusions in relation to the variables that use the HES scale.

Table 1
Inequality Measures (All households)
Income

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.375	0.384	0.362	0.391
A_1	0.219	0.228	0.206	0.231
$A_{1.5}$	0.315	0.323	0.300	0.323
A_2	0.403	0.408	0.388	0.403

Table 2
Inequality Measures (All households)
Income per consumption unit

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.224	0.247	0.255	0.249
A_1	0.082	0.098	0.100	0.097
$A_{1.5}$	0.114	0.134	0.142	0.136
A_2	0.141	0.164	0.181	0.171

Table 3
Inequality Measures (All households)
Income/P_{71-76}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.375	0.407	0.356	0.429
A_1	0.221	0.252	0.200	0.275
$A_{1.5}$	0.318	0.355	0.291	0.368
A_2	0.409	0.443	0.378	0.446

Table 4
Inequality Measures (All households)
Income/P_{77-81}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.377	0.385	0.363	0.411
A_1	0.222	0.228	0.207	0.254
$A_{1.5}$	0.318	0.325	0.301	0.346
A_2	0.406	0.410	0.389	0.423

Table 5
Inequality Measures (All households)
Income per consumption unit/P_{71-76}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.226	0.276	0.249	0.273
A_1	0.083	0.120	0.095	0.117
$A_{1.5}$	0.115	0.162	0.135	0.161
A_2	0.144	0.197	0.173	0.198

Table 6
Inequality Measures (All households)
Income per consumption unit/P_{77-81}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.227	0.247	0.257	0.255
A_1	0.084	0.098	0.100	0.103
$A_{1.5}$	0.117	0.134	0.143	0.143
A_2	0.144	0.164	0.182	0.178

Table 7
Inequality Measures ("Pure" households)
Income

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.376	0.397	0.360	0.365
A_1	0.221	0.237	0.204	0.205
$A_{1.5}$	0.316	0.329	0.298	0.290
A_2	0.401	0.407	0.386	0.365

Table 8
Inequality Measures ("Pure" households)
Income per consumption unit

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.245	0.284	0.274	0.251
A_1	0.098	0.127	0.115	0.100
$A_{1.5}$	0.138	0.171	0.164	0.140
A_2	0.174	0.209	0.209	0.176

Table 9
Inequality Measures ("Pure" households)
Income per adult equivalent

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.284	0.308	0.299	0.281
A_1	0.127	0.145	0.135	0.122
$A_{1.5}$	0.177	0.198	0.193	0.172
A_2	0.221	0.243	0.245	0.217

Table 10
Inequality Measures ("Pure" households)
Income/P_{71-76}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.377	0.420	0.353	0.375
A_1	0.223	0.262	0.197	0.215
$A_{1.5}$	0.320	0.361	0.288	0.302
A_2	0.407	0.442	0.374	0.379

Table 11
Inequality Measures ("Pure" households)
Income/P_{77-81}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.379	0.399	0.362	0.364
A_1	0.224	0.239	0.206	0.203
$A_{1.5}$	0.320	0.332	0.300	0.288
A_2	0.406	0.411	0.388	0.363

Table 12
Inequality Measures ("Pure" households)
Income per consumption unit/P_{71-76}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.250	0.313	0.267	0.262
A_1	0.101	0.151	0.109	0.108
$A_{1.5}$	0.142	0.202	0.156	0.151
A_2	0.179	0.243	0.200	0.189

Table 13
Inequality Measures ("Pure" households)
Income per consumption unit/P_{77-81}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.249	0.286	0.276	0.249
A_1	0.101	0.128	0.116	0.098
$A_{1.5}$	0.141	0.173	0.166	0.138
A_2	0.178	0.211	0.211	0.174

Table 14
Inequality Measures ("Pure" households)
Income per adult equivalent/P_{71-76}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.287	0.332	0.292	0.292
A_1	0.128	0.166	0.130	0.131
$A_{1.5}$	0.180	0.225	0.185	0.184
A_2	0.226	0.275	0.235	0.230

Table 15
Inequality Measures ("Pure" households)
Income per adult equivalent/P_{77-81}

	NORTH	CENTRE	LISBON	SOUTH
GINI	0.288	0.310	0.301	0.279
A_1	0.129	0.146	0.137	0.121
$A_{1.5}$	0.180	0.200	0.195	0.170
A_2	0.225	0.246	0.247	0.214

The values obtained were:

	Gini	Age-Gini	Paglin-Gini
North	0.378	0.112	0.266
Centre	0.388	0.140	0.248
Lisbon	0.366	0.108	0.258
South	0.394	0.140	0.254

The value for the Gini coefficients are of course higher than those shown in table 1, but they are close to them (on average + 0.004). Lisbon has also the most equal age distribution of incomes and the Centre and the South the most unequal ones.[1] But although Lisbon has both the most equal income and age distribution it is not ranked as the most equal region by the Paglin-Gini measure.

Since the Age-Gini coefficient is an indication of the expected lifetime incomes, the conclusion is that households in Lisbon can expect a more even lifetime income profile than for instance those living in the Centre. Since in the Paglin index inequality has to be measured in relation to that expected income, the position of Lisbon relative to the other regions is not so surprising. However we can not forget that we are dealing with only one year and that, as Paglin himself stressed, his measure should be used with time-series and not with cross-section data.

An example of the eventual misleading character of the conclusions, drawn from the sole examination of the total income distribution by household, is reflected in table 2. This table shows the value of the inequality measures

1. Relying only on the rankings given by the Gini coefficient. Nevertheless from the drawn Lorenz curves we could observe that the only problem refers to the South Lorenz curve which crosses all the others at relatively low incomes.

for the income per consumption unit distributions. The first conclusion is that when the family composition is taken account of, the income distributions are considerably more equal (e.g. the Gini coefficient is about 35% lower on average in all the regions). The ranking of the regions (using the Gini coefficient) by increasing order of inequality is now North, Centre, South and Lisbon, in contrast to Lisbon, North, Centre and South before. Lisbon moves from the most equal to the most unequal region (all the others keep their relative positions). With the values obtained for A_e (giving more weight to the lower range of incomes) the corresponding rankings were Lisbon, North and South, Centre in contrast to North, Centre, South and Lisbon. The same conclusion is obtained for Lisbon, although the relative position of the Centre is now altered.

The Lorenz curves for the income per consumption unit distributions are depicted in fig.3,[1] and the main feature is the insensitivity of the inequality measures to the crossing of the Lisbon Lorenz curve at the very top incomes. Perhaps a lower value of e in the Atkinson index would have captured that detail.

From tables 8 and 9 we can infer that the use of the ILO scale may be underestimating inequality. In effect, if we have more faith on the equivalence scales empirically estimated (the HES scale), we would conclude that the values for the Gini coefficients may be underestimated by on average about 11% and those for A_2 by about 21%.

In order to quantify the influence of the price increases in 1971-76 and in 1977-81 and to obtain a concise and clear picture of this whole process, the percentage variation of the Gini and the A_2 coefficients, due

1. In figs. 3 to 5 the Lorenz curves were drawn by using cubic-spline interpolations.

to those prices and in relation to the variables income, income per consumption unit and income per adult equivalent, were computed. In other words, the percentage variation of the Gini and A_2 coefficients were computed for the pairs of distributions income and income/P71-76, income per consumption unit and income per consumption unit/P71-76, etc. The reasons for choosing only these two measures is fundamentally for the sake of simplicity and also because the Gini coefficient and A_2 will be particularly sensitive to transfers of income in the middle and in the bottom of the distributions respectively, i.e., they will be particularly attentive to the worse-off.

The price increases in 1971-76 led to the following variations in the Gini coefficients (in percentage terms).

	North	Centre	Lisbon	South
Income	+ 0.0	+ 6.0	- 1.7	+ 9.7
Income per consumption unit	+ 0.9	+11.7	- 2.4	+ 9.6
Income per adult equivalent	+ 1.0	+ 7.8	- 2.3	+ 3.9

and to A_2:

	North	Centre	Lisbon	South
Income	+ 1.5	+ 8.6	- 2.6	+ 10.7
Income per consumption unit	+ 2.1	+20.1	- 4.4	+ 15.6
Income per adult equivalent	+ 2.3	+13.2	- 4.1	+ 6.0

These results corroborate the conclusions obtained in chapter 3, but also provide some additional information. We note that in general the effect of inflation is more pronounced when the household's income is adjusted by its composition than when it is not. The only exception is the South, particularly when we analyse only the 'pure' households (income per adult equivalent). If we take an average of the three percentage variations, we conclude that the differential inflation in the period 1971-76 increased the Gini coefficient by 0.6% in the North, 8.5% in the Centre, 7.7% in the South, and reduced it by 2.1% in Lisbon. The corresponding values for A_2 are: + 2% in the North, + 14% in the Centre, + 10.8% in the South and - 3.7% in Lisbon.

To put these numbers into perspective let us consider the Gini coefficient for the basic wages and salaries in industry. This was 0.260 in January 1972[1] and 0.196 in January 1977 - a reduction of 27.6% (see Santos (1980)). If we assume that there was the same reduction in the Gini coefficients for the (money) income distributions in all the regions in that period we would conclude that the differential inflation would have wiped out about 2.4% of that reduction in the North, 34.6% in the Centre, 31.3% in the South and emphasized the reduction in Lisbon by about 8.5%. These numbers indicate the importance of the differential inflation effects on inequality and the variations that exist among the regions. Fig.4 depicts the Lorenz curves for the variable income per consumption unit/P71-76, and we can observe the outward movement of the Centre and South curves in opposition to the inward movement of the Lisbon curve if we compare this figure with fig.3.

Concerning the period 1977-81, the percentage variations of the Gini coefficients were:

	North	Centre	Lisbon	South
Income	+ 0.5	+ 0.3	+ 0.3	+ 5.1
Income per consumption unit	+ 1.3	+ 0.0	+ 0.8	+ 2.4
Income per adult equivalent	+ 1.4	+ 0.6	+ 0.7	- 0.7

1. There is no data for January 1971.

FIG.3

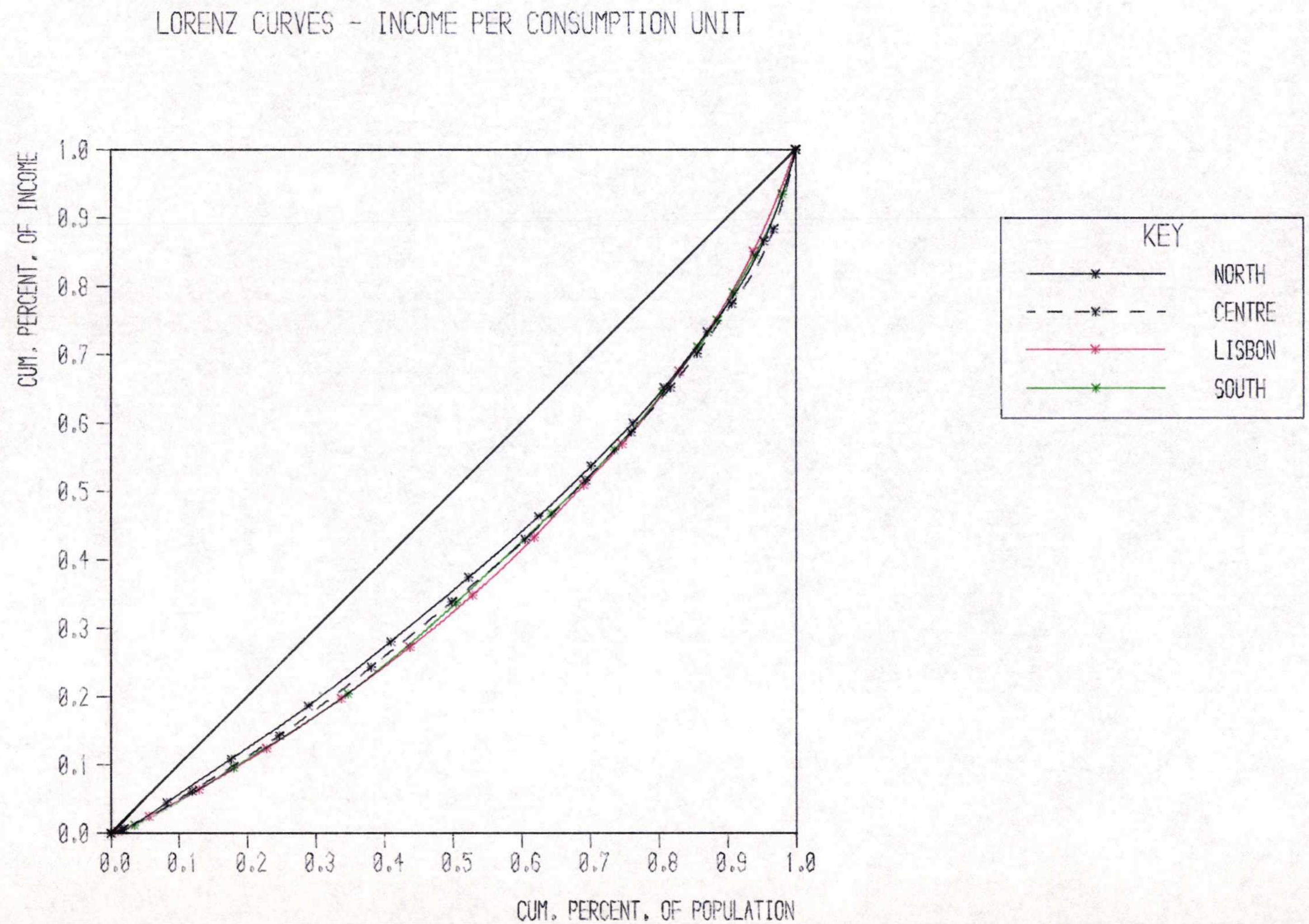

LORENZ CURVES - INCOME PER CONSUMPTION UNIT
CUM. PERCENT. OF INCOME
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
CUM. PERCENT. OF POPULATION
KEY
NORTH
CENTRE
LISBON
SOUTH

FIG.4

LORENZ CURVES - INCOME PER CONSUMPTION UNIT/P71-76

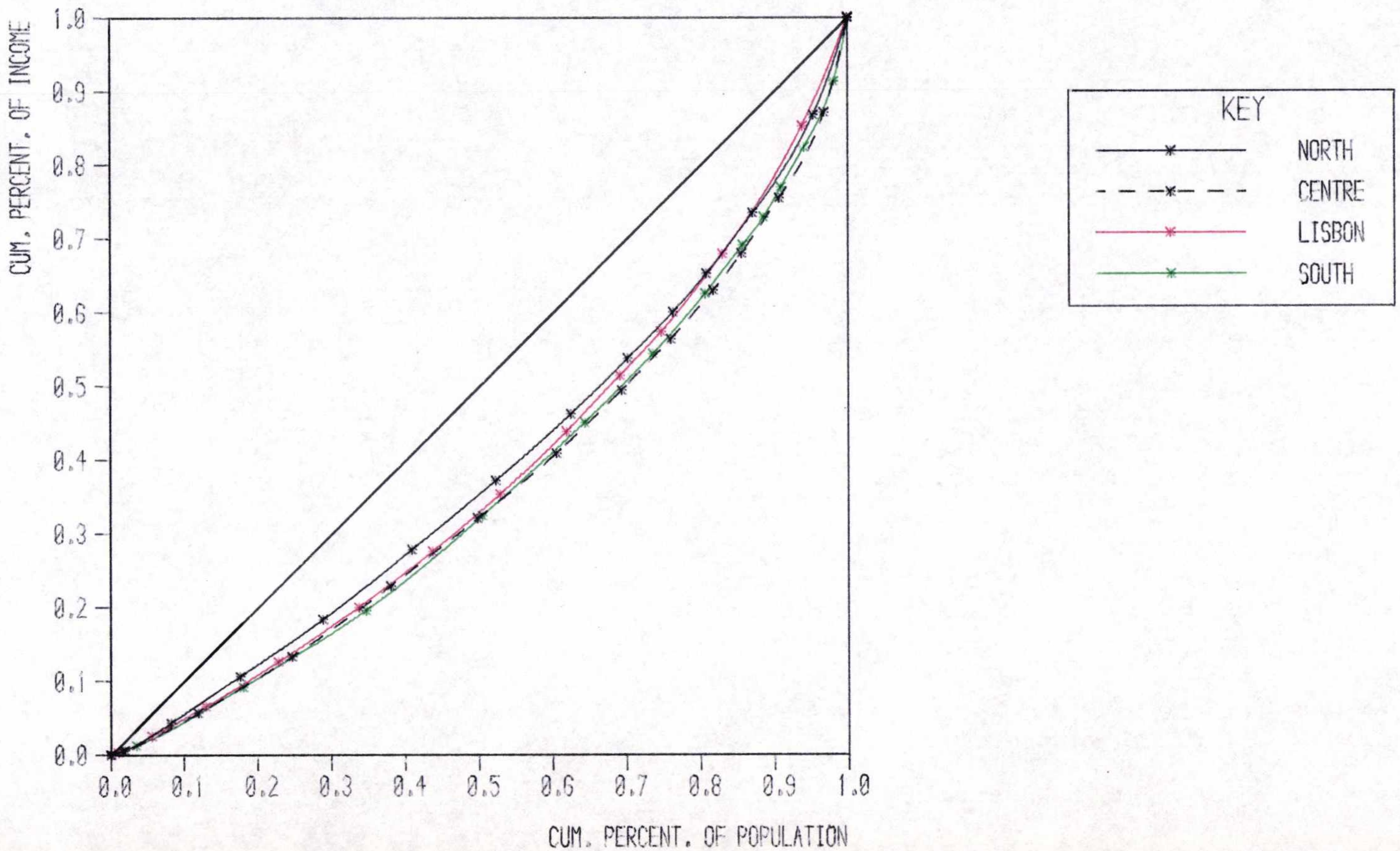
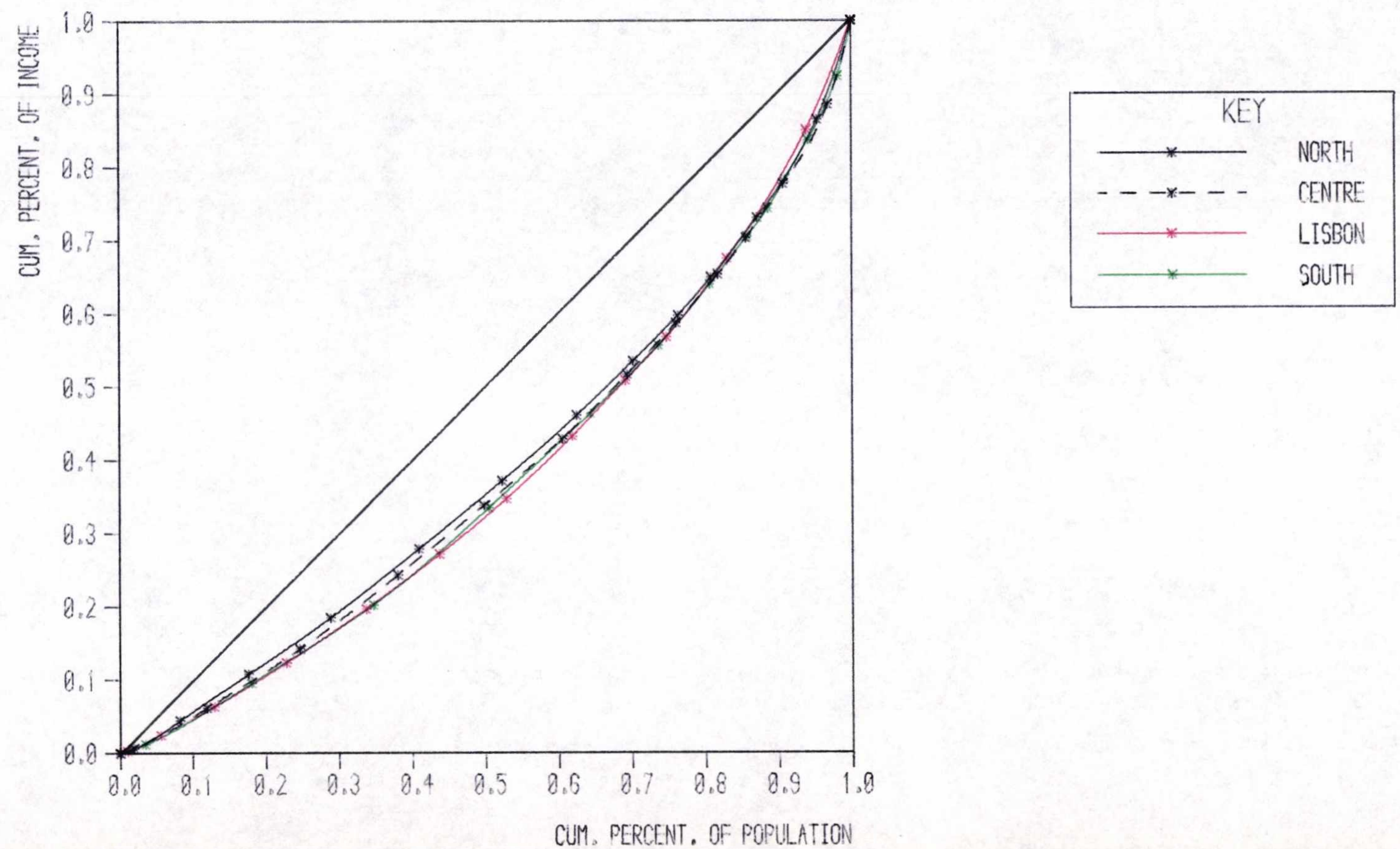


FIG.5

LORENZ CURVES - INCOME PER CONSUMPTION UNIT/P7781



There is an odd result for the South related to the income per adult equivalent. This variable is only computed for the 'pure' households and within them we would obtain similar negative figures for the two other variables. It may once more indicate that the inclusion of additional adults may be an important factor for differential inflation, but it may also be related to the relatively low number of households (concentrated in mainly low income classes) that exist in the South region where we consider only the 'pure' families.[1]

In the other regions, the averages of 1.1% (North), 0.3% (Centre) and 0.6% (Lisbon) indicate that while for all of them differential inflation induced inequality, it was to a smaller extent than in the previous period. But we should not forget that for this period we have assumed the existence for all the regions of the same average Continent price increases. Fig.5 represents the Lorenz curves for the variable income per consumption unit/P77-81 and if we compare it with fig.3 we can hardly detect any modifications given the small variations that happened.

However, considering that from January 1977 to January 1980 the Gini coefficients of the distributions of the basic wages and salaries in industry remained practically the same, and making the same assumptions as before, we can conclude that differential inflation might have been the only important force tending to increase inequality in that period.

As for A_2, the percentages were:

	North	Centre	Lisbon	South
Income	+ 0.7	+ 0.5	+ 0.3	+ 5.0
Income per consumption unit	+ 2.1	+ 0.0	+ 0.6	+ 4.1
Income per adult equivalent	+ 1.8	+ 1.2	+ 0.8	+ 1.4

1. But see below the results for A_2.

No odd results exist now in the South, and the averages are 1.5% (North), 0.9% (Centre), 0.6% (Lisbon) and 3.5% (South), again comparatively smaller than the corresponding ones for the previous period.

A Simulation study

The results obtained for the period 1977-81 in which the same price information is employed suggest that there are some differences between the regions as concerns the effects of price increases on inequality. In order to estimate those differences and to measure the consequences of price increases on different expenditure aggregates we have simulated a 20% price increase in each of the ten expenditure aggregates considered in chapter 3 (see table 23 in Section 3.6).

Since the data in that table refers only to eight income classes, the Gini and Atkinson measures were recomputed using the information on the mean income and number of people in each class. The next step was to assume a 20% increase in the price of each aggregate (e.g. food), and to calculate the corresponding overall rate of inflation (P) for each income class, using the expenditure weights shown in table 23. This enabled us to obtain the distribution y/P and of its related Gini and Atkinson indices. The comparison of the initial and final values of those measures allow us to quantify the effects on inequality of the simulated price rises.

The results are shown in table 16, but before analysing them some comments are in order. Firstly since we are only dealing with eight observations, the precision of the inequality measures is reduced, and of course their values will be lower than those obtained with thirteen observations.[1] Secondly since 20% price rises in one aggregate correspond

1. Compare the values for the Gini and Atkinson coefficients in tables 16 and 1.

Table 16
Effects on the Gini and Atkinson (e=2) Coefficients
of a 20% Increase in the Price of the Expenditure Aggregates

	NORTH		CENTRE		LISBON		SOUTH	
	GINI	A2	GINI	A2	GINI	A2	GINI	A2
Income	0.3667	0.3879	0.3757	0.3936	0.3534	0.3704	0.3833	0.3848
1-Food	+0.0011	+0.0045	+0.0011	+0.0052	+0.0012	+0.0051	+0.0012	+0.0047
2-Drink	+0.0007	+0.0007	+0.0006	+0.0002	+0.0004	+0.0003	+0.0003	-0.0004
3-Cloth. and footw.	0	-0.0013	0	-0.0016	+0.0002	-0.0006	+0.0001	-0.0009
4-Tobacco	0	0	0	0	0	0	0	0
5-Housing	-0.0007	-0.0002	-0.0004	-0.0002	-0.0002	+0.0003	-0.0005	0
6-Hous. goods	-0.0003	-0.0015	-0.0003	-0.0014	-0.0003	-0.0014	-0.0003	-0.0011
7-Fuel, light	0	+0.0011	+0.0001	+0.0011	0	+0.0008	+0.0001	+0.0006
8-Transport	-0.0008	-0.0021	-0.0009	-0.0023	-0.0009	-0.0026	-0.0012	-0.0023
9-Services	-0.0003	-0.0015	-0.0001	-0.0015	-0.0004	-0.0016	-0.0003	-0.0012
10-Miscellaneous	-0.0002	-0.0006	0	-0.0003	-0.0003	-0.0006	-0.0001	-0.0002

to a much lower overall rate of inflation, the inequality measures had to be computed with four decimal places for some differences to emerge. Even so the Atkinson index with e = 1 was remarkably insensitive to these small variations. Given that no significant additional information was gained with e = 1.5, table 16 only presents the results for the Gini coefficient and for A_2.

In spite of the problems mentioned above, the results obtained should be reasonably accurate and comparisons between the regions made possible since the same procedure was adopted for all of them.

The first overall conclusion to be drawn from table 16 is that price increases in food, drink and fuel and light increase inequality in all the regions with the opposite happening for the other expenditure aggregates.[1] This statement has however to be qualified because there are some exceptions. If we emphasize the lower end of the distribution, a price increase in drink in the South reduces inequality.[2] With the same welfare judgement price rises in clothing and footwear reduce inequality, but if the middle of the distribution is emphasized the price rises are neutral in the North and Centre and increase inequality in Lisbon and the South.

The higher effects are associated with food and with transport. A 20% increase in food leads to increases in the Gini coefficient of 0.34% in Lisbon, 0.31% in the South, 0.30% in the North and 0.29% in the Centre. The corresponding effects on A_2 are 1.38% in Lisbon, 1.32% in the Centre, 1.22% in the South and 1.12% in the North, implying that both measures only agree that the effect of price increases in food is most felt in Lisbon.

1. Excluding tobacco, which due to its low expenditure weights make the inequality measures insensitive to the tobacco price increases.

2. This is clear from direct inspection of table 23.

In relation to transport, the percentage decreases in the Gini coefficient are: -0.31% in the South, -0.26% in Lisbon, -0.24% in the Centre and -0.22% in the North. A_2 only disagrees in the ranking of the two first regions, considering that Lisbon should take the first place.

It would be tedious and uninteresting to enumerate the differences between the regions in relation to the other expenditure aggregates. Suffices it to say that the results in table 16 stress once more that while there are differences between the regions in expenditure weights, they are relatively small.[1] The contrasting results obtained for instance between the Centre and Lisbon regions in the period 1971-76 where fundamentally due to the distinct price series utilized in each region, not the outcome of substantial different expenditure weights. But perhaps the main virtue of table 16 is the possibility it offers of roughly assessing the likely effect on equality of a given 'menu' of price rises.

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1. Not forgetting that we are ignoring the variations within the expenditure aggregates.

Conclusions

Since almost all the chapters in this thesis include their own conclusions, it is necessary in this section to mention only the most important results obtained, to discuss eventual shortcomings of the methodology and to propose courses of action and new lines of research.

The main results can be briefly summarized as:

1) There is overwhelming evidence, in the period of study adopted, that a single and average index such as the CPI cannot adequately represent the rates of inflation experienced by different households.

2) The price indices are related in a systematic way to the household's characteristics, although not always in the same direction. Of all the characteristics investigated (income, age, number of children, occupation and location), income is by far the most important.

3) As a rule, the higher the income the less inflation. But there are exceptions. The most important one concerns the region of Lisbon in the period 1971-76 when higher income households experienced higher inflation rates.

4) This particular result for Lisbon is in complete contrast to those found in the other regions for the same period, especially in the Centre and South. For instance in the Centre, and for the whole period, the highest income class experienced a rate of inflation lower by about 40 percentage points than the lowest income

class. In the regression analysis (Centre region) the coefficient of the income variable suggests that for each additional annual hundred thousand escudos the rate of inflation would be lower by 11.4 percentage points. As a consequence, the Gini coefficient of the income distribution is higher by about 9% when the differential inflation is taken account of.

5) During the period 1977-81, the year 1981 is particularly significant to the overall conclusion that inflation decreases with income. In that year the range in the inflation rates by income class is about 10 percentage points.

6) The results for the 'concelhos' suggest that the results may be conservative in relation to the inequality effects of inflation.

7) The greater uniformity between the regions in the period 1977-81 in relation to 1971-76 is due to the application of the same price information for all of them in the 1977-81 period.
The analysis of expenditure weights by income class in all regions leads us to the conclusion that although there are some variations, they could not by themselves explain the contrasting results obtained in 1971-76. This conclusion is important since it stresses the shortcomings of employing a unique CPI to describe the price changes for the whole country.

8) The Cost-of-Living indices computed from the parameters of the Linear Expenditure System agree remarkably well with the results obtained with the constant weight price

indices, although they tend to smooth out some of the differences. We might think that this is a natural consequence of allowing for the substitution effects, but the serious deficiencies of either the LES and its cost-of-living formula cast some doubts on that interpretation.

9) Concerning the income distribution in the regions, we could conclude that inequality is substantially reduced when we take into account the household composition. But with the equivalence scales estimated in this thesis we obtain higher values for the inequality measures than with the use of the ILO nutritional scale.

10) The effects of differential inflation seem to be emphasized when total household income is deflated by the equivalence scales.

Concerning the criticisms of the methodology: Firstly we did not allow the expenditure weights to vary over time in the computation of the price indices (due to there being only one survey available), and secondly we had to assume the same commodity price changes for all income classes. As far as the first criticism goes, it could be argued that the variation in the expenditure weights is allowed for in the Linear Expenditure Cost-of-Living indices (and that basically the same conclusions are obtained), but it is obvious that a better answer to this can only be given by making use of the 1980 Expenditure Survey.

In relation to the second criticism, although this thesis constitutes an improvement by allowing commodity price increases to differ from region to region, it still assumes that they are the same for all income classes. Ideally we should characterize for each income class the commodities more frequently consumed and then observe their price increases over the years. For the same type of commodities there is some evidence that the 'poor pay more' in absolute terms, but there is no evidence that the percentage price increases for these goods is any higher. The assumption therefore in this thesis is that although some classes pay more than others for the same or equivalent commodities, the rates of price increases are the same in all of them. There is no empirical evidence to contradict or confirm this assumption.

This leads me to propose one further line of research: To investigate in detail the type of commodities bought by the lower and higher income classes and to measure their associated, separate price increases. Of course there is also the advantage that the information given by the 1980 HES can be utilized and that the results obtained compared with those for the 1973/74 HES.

The conclusions of this thesis are sufficiently impressive, I think, for the National Statistical Institute (INE) to start publishing a separate price index for at least a low income class (perhaps pensioners?) and also for computing CPI's for some other cities in Portugal other than Lisbon and Oporto.

Appendix

TABLE A1

"OLD" PRICE INDICES - OPORTO

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1971	P1972	P1973	P1974	P1975	P1976	P1977	P1978
1-CEREALS	101.4	103.2	104.5	149.0	108.5	97.0	129.6	128.1
2-VEGETABLES	103.7	100.0	117.6	158.1	139.9	163.0	109.3	88.8
3-POTAT,GREENS	132.4	125.7	101.2	107.8	196.3	100.3	102.3	97.5
4-FRUIT	112.6	98.8	108.0	129.0	214.9	91.3	119.9	134.6
5-CONFECTIONARY	103.5	108.6	101.4	115.8	110.8	130.0	121.6	138.2
6-FATS	109.2	84.4	147.2	136.4	100.7	107.7	111.7	118.5
7-CONDIMENTS	-	-	-	-	-	-	-	-
8-SUGAR,COFFEE	100.0	100.0	100.0	165.5	161.6	107.3	227.1	104.5
9-MEAT	118.0	101.5	123.6	129.2	115.4	119.9	144.8	111.7
10-POULTRY	107.0	100.0	105.1	127.6	121.8	116.8	111.2	116.7
11-FISH	133.5	103.4	104.3	112.6	162.0	140.1	133.6	120.2
12-MILK,EGGS	106.1	97.8	106.3	127.5	131.5	93.6	127.5	115.9
13-MEALS OUT	115.3	103.1	125.2	138.9	144.5	100.0	145.4	115.3
14-DRINKS	114.6	112.6	116.8	124.0	104.5	101.8	155.5	172.8
15-CLOTHING	108.9	103.9	106.8	111.6	100.0	102.7	125.4	150.8
16-FOOTWEAR	108.5	117.4	111.2	113.6	127.8	119.0	120.2	132.2
17-HOUSING	117.6	113.2	122.6	100.0	128.2	121.5	98.7	100.0
18-FUEL,LIGHT	100.0	100.0	100.0	107.7	105.5	154.1	201.4	126.3
19-WATER	100.0	100.0	100.0	100.0	156.3	120.0	100.0	100.0
20-PERSON.HYG.	102.2	103.1	132.9	203.7	121.3	96.1	126.4	117.1
21-HOUSING HYG.	167.6	100.4	100.0	108.3	104.0	123.9	113.5	159.6
22-FURNITURE	106.8	104.7	111.9	126.0	111.8	110.6	113.1	137.4
23-DOMEST.SERV.	116.9	122.1	108.8	110.3	100.0	100.0	100.0	100.0
24-TRANSPORT	100.0	100.0	149.1	122.1	107.5	150.4	100.0	120.8
25-COMUNICATIONS	100.0	100.0	100.0	185.6	163.6	100.0	111.8	169.9
26-HEALTH	120.4	100.0	98.9	101.2	136.9	102.7	147.4	100.0
27-ENTERTAINS	100.2	89.7	105.9	149.3	101.2	130.0	129.6	103.5
28-TOBACCO	100.0	100.0	100.0	131.5	143.3	100.0	100.0	161.6

TABLE A2

"OLD" PRICE INDICES - VISEU AND COIMBRA

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1971	P1972	P1973	P1974	P1975	P1976	P1977	P1978
1-CEREALS	101.1	101.0	100.1	164.3	101.1	103.3	137.4	117.8
2-VEGETABLES	113.9	108.5	118.3	134.1	135.4	160.0	124.2	77.3
3-POTAT,GREENS	132.0	109.2	119.3	124.5	223.2	89.3	92.3	95.9
4-FRUIT	120.9	93.6	114.0	141.3	158.0	106.3	128.8	137.5
5-CONFECTIONARY	104.1	106.0	99.4	116.8	115.4	140.6	139.7	123.7
6-FATS	107.7	103.4	142.5	128.5	101.6	115.0	116.4	100.2
7-CONDIMENTS	148.3	105.4	100.0	111.4	140.3	119.7	132.6	114.0
8-SUGAR,COFFEE	100.1	100.0	102.2	163.1	166.6	105.2	154.6	107.9
9-MEAT	115.8	110.7	122.4	123.6	123.1	124.2	116.2	118.2
10-POULTRY	117.1	101.9	114.5	130.5	121.2	137.3	101.3	97.0
11-FISH	117.5	101.9	126.2	152.5	121.2	114.7	153.3	134.6
12-MILK,EGGS	103.3	99.7	110.5	132.8	121.2	102.7	120.8	124.8
13-MEALS OUT	116.1	101.1	102.2	102.9	170.0	101.4	149.4	130.0
14-DRINKS	100.5	109.9	116.3	118.5	101.6	103.3	215.2	164.4
15-CLOTHING	110.3	104.8	103.3	112.2	101.2	114.8	101.2	101.3
16-FOOTWEAR	102.8	113.8	106.7	107.8	136.7	124.6	127.5	135.3
17-HOUSING	124.8	116.3	121.6	124.2	124.3	135.6	117.2	112.9
18-FUEL,LIGHT	104.5	100.0	99.8	110.8	150.6	112.2	116.0	171.1
19-WATER	130.5	100.0	100.0	100.0	100.0	106.1	100.0	100.0
20-PERSON.HYG.	108.1	103.2	109.4	147.9	101.2	112.2	126.2	123.2
21-HOUSING HYG.	113.3	130.4	124.4	111.5	102.0	100.1	192.6	106.9
22-FURNITURE	104.7	103.6	112.1	121.3	125.3	109.0	108.3	134.2
23-DOMEST.SERV.	126.9	112.4	125.6	114.1	117.1	100.0	122.0	100.0
24-TRANSPORT	100.0	100.0	100.0	101.5	110.4	114.5	100.0	113.6
25-COMUNICATIONS	100.0	100.0	100.0	134.3	167.3	100.0	112.2	148.5
26-HEALTH	102.1	107.5	110.5	106.8	127.5	120.3	120.6	105.2
27-ENTERTAINS	103.0	102.3	103.2	147.4	100.2	127.4	130.5	108.0
28-TOBACCO	100.0	100.0	100.0	134.9	141.1	100.0	100.0	129.5

TABLE A3

"OLD" PRICE INDICES - LISBON

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1971	P1972	P1973	P1974	P1975	P1976	P1977	P1978
1-CEREALS	100.0	100.7	100.0	166.5	100.1	101.0	137.4	115.0
2-VEGETABLES	107.9	101.2	119.4	160.5	125.9	179.3	101.6	94.4
3-POTAT,GREENS	117.4	112.4	103.0	114.3	208.4	106.1	98.8	98.0
4-FRUIT	105.1	89.0	117.0	133.7	130.0	118.4	118.8	154.0
5-CONFECTIONARY	112.2	101.7	100.9	121.3	133.1	123.5	142.4	113.1
6-FATS	101.7	109.7	122.7	146.1	101.8	106.1	116.7	104.6
7-CONDIMENTS	-	-	-	-	-	-	-	-
8-SUGAR,COFFEE	100.5	100.0	104.6	152.9	173.1	100.8	117.5	109.7
9-MEAT	105.1	113.3	111.1	136.4	133.9	132.7	97.7	132.3
10-POULTRY	126.7	100.0	124.8	106.9	142.4	121.3	117.2	120.4
11-FISH	126.8	104.7	144.2	128.9	119.8	139.5	148.7	124.7
12-MILK,EGGS	103.7	100.1	107.8	131.3	131.7	90.9	124.6	127.4
13-MEALS OUT	136.7	115.9	135.1	143.1	104.3	113.3	106.0	135.3
14-DRINKS	101.7	115.0	122.7	144.4	97.7	113.1	151.2	173.8
15-CLOTHING	111.6	113.5	137.6	120.3	100.7	103.2	104.0	109.3
16-FOOTWEAR	107.6	122.8	112.5	117.2	107.8	104.2	144.8	151.2
17-HOUSING	135.0	114.6	125.5	80.3	144.1	122.5	87.8	116.6
18-FUEL,LIGHT	100.0	100.0	107.3	124.3	112.9	122.3	147.5	142.8
19-WATER	150.0	100.0	100.0	100.0	100.0	100.0	133.3	125.0
20-PERSON.HYG.	106.1	104.8	118.1	123.0	100.5	131.6	105.3	132.2
21-HOUSING HYG.	129.1	138.4	107.4	144.8	101.8	115.2	125.9	150.4
22-FURNITURE	109.2	106.4	118.5	132.4	107.7	121.9	124.8	137.0
23-DOMEST.SERV.	117.1	119.4	112.3	126.4	105.1	98.8	125.0	100.0
24-TRANSPORT	100.0	122.8	123.4	117.0	142.9	128.2	100.0	138.2
25-COMUNICATIONS	100.0	100.0	100.0	184.0	165.2	100.0	113.1	167.0
26-HEALTH	115.3	100.0	110.0	145.7	100.0	173.8	89.5	126.9
27-ENTERTAINS	100.3	85.5	110.1	158.5	102.3	130.1	116.0	123.7
28-TOBACCO	100.0	100.0	100.0	132.9	147.2	100.0	100.0	161.9

TABLE A4

"OLD" PRICE INDICES - EVORA AND FARO

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1971	P1972	P1973	P1974	P1975	P1976	P1977	P1978
1-CEREALS	100.0	101.1	100.1	159.0	101.8	101.1	168.2	122.8
2-VEGETABLES	130.4	113.4	115.1	161.2	112.2	181.0	83.1	97.2
3-POTAT,GREENS	126.5	102.7	135.6	118.2	167.3	93.6	119.2	108.9
4-FRUIT	113.6	88.2	112.2	139.6	153.9	120.8	115.8	135.6
5-CONFECTIONARY	106.1	112.6	99.5	115.3	141.0	134.0	110.9	117.1
6-FATS	100.2	111.5	137.2	130.6	101.3	121.2	110.6	102.3
7-CONDIMENTS	101.3	113.0	105.2	118.6	124.4	102.8	154.1	112.3
8-SUGAR,COFFEE	100.0	100.0	101.0	159.5	170.5	152.8	150.7	105.0
9-MEAT	106.3	124.2	117.8	130.6	114.6	117.4	132.2	121.1
10-POULTRY	113.1	102.1	110.5	114.0	126.5	124.6	110.4	122.7
11-FISH	118.6	113.9	121.3	114.8	153.8	132.5	131.5	141.8
12-MILK,EGGS	101.3	103.9	106.1	125.8	113.4	104.7	134.8	115.8
13-MEALS OUT	112.3	113.5	108.8	140.3	119.9	108.6	131.9	127.5
14-DRINKS	107.3	107.8	129.9	110.8	102.0	101.4	157.6	215.8
15-CLOTHING	109.4	109.1	127.1	105.4	102.5	106.0	100.9	113.6
16-FOOTWEAR	106.2	114.9	119.8	117.6	120.8	133.6	127.1	132.0
17-HOUSING	123.9	136.6	128.0	113.3	113.5	111.2	141.6	112.6
18-FUEL,LIGHT	103.0	100.9	103.6	121.5	116.4	114.6	149.3	142.9
19-WATER	100.0	100.0	100.0	114.6	129.5	100.0	100.0	100.0
20-PERSON.HYG.	101.6	100.7	123.7	118.8	120.0	100.8	122.9	129.9
21-HOUSING HYG.	102.0	97.2	112.6	134.2	121.3	109.7	112.7	103.2
22-FURNITURE	109.1	111.7	111.6	116.5	119.8	104.1	112.4	151.0
23-DOMEST.SERV.	118.0	136.0	117.1	101.9	122.0	100.0	100.0	138.4
24-TRANSPORT	100.6	100.8	100.7	111.0	122.5	151.2	105.2	126.4
25-COMUNICATIONS	100.0	100.0	100.0	137.0	172.0	100.0	118.3	143.2
26-HEALTH	105.0	103.3	119.6	101.4	119.9	136.3	129.6	118.4
27-ENTERTAINS	104.6	97.2	106.4	118.7	109.6	101.8	107.1	154.0
28-TOBACCO	100.0	100.0	100.0	144.2	146.8	100.0	100.0	165.8

TABLE A5

"NEW" PRICE INDICES - CONTINENT

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1977	P1978	P1979	P1980	P1981
1-RICE	100.0	143.6	130.3	120.4	148.7
2-FLOUR	114.2	129.5	122.1	113.3	144.1
3-PASTA	119.7	110.9	118.1	119.9	164.6
4-BREAD	135.4	114.7	123.0	118.8	131.3
5-STARCHES	69.2	104.8	239.4	74.1	136.7
6-PULSES	99.4	98.5	124.5	139.6	154.9
7-CHICKPEA	161.3	103.2	107.5	106.5	151.3
8-VEGETABLES	120.6	104.4	119.3	124.1	131.1
9-FRUIT	160.4	121.0	98.7	122.9	170.9
10-GOAT	102.0	125.7	131.4	110.4	113.5
11-LAMB	106.8	120.8	128.7	115.9	115.4
12-PORK	89.9	124.4	148.4	87.9	107.4
13-BEEF	162.6	126.2	163.8	103.2	107.9
14-OFFAL	100.6	122.3	141.5	98.6	97.8
15-SAUSAGES	118.9	116.8	143.6	95.4	100.4
16-POULTRY	111.2	127.5	111.5	110.4	117.2
17-FRESH FISH	135.2	132.4	114.8	129.9	98.8
18-FROZEN FISH	84.5	117.8	187.0	107.6	162.8
19-SHELLFISH	119.4	138.9	135.8	108.5	97.2
20-CANNED FISH	150.7	112.4	128.4	128.0	124.4
21-DRIED FISH	134.9	122.6	103.2	111.4	185.1
22-EGGS	118.8	109.8	138.4	94.3	162.6
23-MILK	130.1	116.4	116.3	110.3	150.8
24-DRIED MILK	119.8	129.9	119.0	117.3	122.7
25-MILK PRODUCTS	155.1	125.7	117.7	113.4	119.3
26-OILS	116.0	105.9	130.0	105.5	159.1
27-BUTTER	174.6	123.1	105.2	100.2	175.3
28-MARGARINE	125.5	130.7	116.3	116.9	117.4
29-SUGAR	100.0	115.4	106.7	124.9	143.0
30-JELLY	194.1	99.8	98.7	102.1	128.2
31-CONFECTIONARY	115.7	118.1	118.6	117.3	124.3
32-COCOA	253.6	110.1	108.4	105.2	108.8
33-COFFEE	209.6	100.0	122.4	108.1	103.0
34-TEA	178.4	105.9	124.2	101.7	121.7
35-CONDIMENTS	140.8	116.8	146.8	109.3	106.2
36-PREPARED FOOD	116.5	124.8	117.0	109.9	123.6
37-MEALS OUT	129.6	125.2	120.0	117.2	119.7
38-TABLE WINE	172.8	181.7	66.9	99.3	134.1
39-OTHER WINES	132.6	141.4	137.2	141.7	125.9
40-BEER	106.6	126.4	126.8	108.3	113.2
41-MINERAL WATER	108.3	130.8	152.0	91.2	124.4
42-OTHER DRINKS	118.1	119.8	112.8	118.0	131.1

TABLE A5 (CONT.)

	P1977	P1978	P1979	P1980	P1981
43-CLOTHING(MEN)	119.7	117.8	126.3	137.6	120.3
44-TAILOR(MEN)	118.6	111.0	127.4	127.7	117.9
45-U/WEAR(MEN)	126.5	113.1	131.8	132.1	112.9
46-CLOTH.(WOMEN)	117.1	120.7	129.4	137.3	118.8
47-TAILOR(WOMEN)	123.1	119.5	130.0	132.8	116.9
48-U/WEAR(WOMEN)	112.0	113.3	132.0	126.2	108.1
49-CLOTH.(CHILD)	120.3	113.9	127.5	132.6	113.6
50-LAUNDRY	122.9	121.4	141.9	128.1	120.7
51-FOOTW.(MEN)	118.8	115.6	151.1	112.9	96.0
52-FOOTW.(WOMEN)	127.0	114.9	157.9	109.8	103.3
53-FOOTW.(CHILD)	126.1	119.2	152.8	122.4	106.1
54-FOOTW.REPAIR	127.8	119.2	137.1	114.6	117.6
55-WATER	124.8	116.3	125.0	100.0	133.6
56-GAS	113.0	162.7	122.6	124.9	112.7
57-ELECTRICITY	142.8	142.2	127.4	119.5	124.8
58-ELECTR.APPL.	146.5	104.1	102.3	114.4	109.0
59-FURNITURE	120.9	109.6	123.3	126.8	119.0
60-HOUS.TEXTILES	114.3	107.5	116.8	117.3	110.3
61-OTHERS	143.9	103.6	115.5	117.7	121.0
62-CUTLERY,ETC.	143.5	114.6	117.7	131.9	114.8
63-REPAIR,MAINT.	124.1	113.0	110.6	117.4	122.1
64-HYG.PRODS.	117.3	119.3	125.2	114.0	118.2
65-SERVICES(HYG)	159.1	114.4	126.8	117.7	116.9
66-SURGIC.GOODS	237.2	101.7	117.5	101.8	164.6
67-MEDICINES	129.3	103.5	105.4	106.0	107.1
68-DOCTORS	129.0	119.8	125.5	131.1	117.7
69-TOBACCO	100.0	142.0	139.2	116.8	118.7
70-PRIV.TRANSP.	148.6	119.2	125.9	128.2	117.6
71-URB.TRANSP.	100.0	126.0	129.2	122.7	155.2
72-SUBURB.TRANSP	100.0	144.7	123.2	128.9	155.4
73-LONG DIST.TRA	100.0	116.4	122.3	125.7	142.1
74-TAXIS	100.0	131.0	126.9	123.8	147.1
75- POST-OFFICE	121.1	129.5	129.5	126.3	100.0
76-TELEPHONE	140.5	157.1	118.4	130.7	100.0
77-EDUCATION	118.8	112.6	108.1	113.8	118.7
78-ENTERTAINS	116.0	113.4	122.2	118.5	113.6

TABLE A6

"NEW" PRICE INDICES - OPORTO

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1977	P1978	P1979	P1980	P1981
1-RICE	100.0	142.0	129.6	120.5	147.0
2-FLOUR	112.9	131.8	120.4	114.0	144.7
3-PASTA	119.7	109.2	117.3	120.5	166.5
4-BREAD	133.6	114.1	122.3	119.2	130.9
5-STARCHES	69.8	110.0	236.4	68.6	149.5
6-PULSES	77.7	109.0	114.0	136.5	156.2
7-CHICKPEA	162.6	106.0	102.2	98.4	148.2
8-VEGETABLES	110.8	111.5	119.8	123.1	130.0
9-FRUIT	150.8	126.3	95.4	120.0	195.3
10-GOAT	99.3	121.4	134.3	115.6	110.7
11-LAMB	102.5	115.1	134.7	118.7	112.7
12-PORK	90.6	128.1	139.4	92.9	103.2
13-BEEF	178.3	129.1	152.1	106.9	107.1
14-OFFAL	93.0	124.3	127.4	92.5	89.8
15-SAUSAGES	129.2	112.9	140.4	95.1	104.4
16-POULTRY	110.8	126.8	113.7	112.1	123.3
17-FRESH FISH	142.1	111.7	108.2	118.7	104.0
18-FROZEN FISH	116.3	128.6	154.7	97.7	163.4
19-SHELLFISH	113.7	134.2	152.7	88.4	112.7
20-CANNED FISH	152.0	107.5	127.7	126.9	127.3
21-DRIED FISH	171.8	124.5	98.5	115.2	178.5
22-EGGS	124.7	110.2	138.5	92.1	167.0
23-MILK	120.0	121.7	118.6	110.2	154.9
24-MILK PRODUCTS	178.4	120.5	119.5	118.1	115.5
25-OILS	117.3	105.6	128.4	106.5	150.1
26-FATS	139.0	129.1	113.4	111.8	127.5
27-SUGAR	100.0	115.4	106.7	124.9	137.7
28-JELLY	191.4	99.8	99.1	100.0	127.0
29-CONFECTIONARY	118.5	120.8	120.4	119.8	127.6
30-COCOA	262.9	110.6	110.3	101.9	109.7
31-COFFEE	255.1	97.5	128.6	97.5	104.0
32-TEA	172.6	107.4	130.3	109.1	101.8
33-CONDIMENTS	136.6	113.5	148.4	113.4	104.0
34-PREPARED FOOD	121.4	126.8	118.8	109.9	115.4
35-MEALS OUT	140.7	115.4	128.9	111.2	122.0
36-ALCHOOL.DRINK	164.7	175.1	62.3	106.0	139.8
37-SOFT DRINKS	110.7	127.3	129.9	108.2	123.4

TABLE A6 (CONT.)

	P1977	P1978	P1979	P1980	P1981
38-CLOTHING(MEN)	114.7	118.0	125.0	143.2	114.2
39-TAILOR(MEN)	112.6	118.3	123.5	125.4	116.2
40-U/WEAR(MEN)	124.9	120.0	129.8	122.3	121.3
41-CLOTH.(WOMEN)	105.5	116.0	129.7	140.7	123.0
42-TAILOR(WOMEN)	127.0	111.6	131.6	135.6	110.6
43-U/WEAR(WOMEN)	103.2	122.0	125.2	123.9	97.8
44-CLOTH.(CHILD)	96.2	126.3	123.3	127.7	117.5
45-LAUNDRY	116.2	119.8	122.1	121.3	129.4
46-FOOTW.(MEN)	134.0	120.8	170.5	101.1	103.3
47-FOOTW.(WOMEN)	132.3	115.3	155.6	107.8	105.1
48-FOOTW.(CHILD)	116.7	124.5	150.9	101.9	122.8
49-FOOTW.REPAIR	130.8	115.8	131.9	111.9	117.6
50-WATER	128.2	100.0	100.0	100.0	169.4
51-GAS	100.0	164.4	123.4	127.9	109.1
52-ELECTRICITY	174.2	106.8	100.0	100.0	100.0
53-ELECTR.APPL.	119.1	109.3	97.4	116.2	114.8
54-FURNITURE	113.4	103.3	127.0	129.9	121.6
55-HOUS.TEXTILES	96.3	110.4	128.2	106.4	126.8
56-OTHERS	151.5	97.4	111.0	138.6	110.4
57-CUTLERY,ETC.	118.1	127.6	127.6	124.4	117.4
58-REPAIR,MAINT.	134.2	114.8	126.7	107.3	124.5
59-HYG.PRODS.	117.5	111.3	130.4	106.9	127.5
60-SERVICES(HYG)	125.5	122.9	141.3	117.8	121.7
61-SURGIC.GOODS	215.2	102.8	114.7	105.6	155.0
62-MEDICINES	129.3	103.5	105.5	106.0	107.1
63-DOCTORS	113.5	121.9	131.9	111.5	118.5
64-TOBACCO	100.0	142.0	139.1	115.7	120.1
65-PRIV.TRANSP.	148.6	119.2	125.9	128.2	117.6
66-URB.TRANSP.	100.0	158.5	110.2	145.5	147.8
67-SUBURB.TRANSP	100.0	170.7	122.0	128.9	157.9
68-LONG DIST.TRA	-	-	-	-	-
69-TAXIS	100.0	134.6	127.0	124.4	146.0
70- POST-OFFICE	123.0	128.1	129.7	127.6	100.0
71-TELEPHONE	146.3	156.0	120.3	129.9	100.0
72-EDUCATION	124.4	105.2	101.9	122.6	112.3
73-ENTERTAINS	111.1	119.4	116.9	124.5	110.6

TABLE A7

"NEW" PRICE INDICES - LISBON

(P YEAR X: PRICES IN JAN. X+1/PRICES IN JAN. X)

	P1977	P1978	P1979	P1980	P1981
1-RICE	100.0	147.1	132.0	120.2	148.7
2-FLOUR	114.1	129.2	122.0	113.2	143.9
3-PASTA	108.1	112.2	118.4	118.1	164.5
4-BREAD	133.7	113.1	122.7	119.0	131.4
5-STARCHES	64.3	106.5	244.7	75.5	133.3
6-PULSES	104.3	95.8	124.6	143.2	153.7
7-CHICKPEA	161.0	104.4	109.2	108.7	152.0
8-VEGETABLES	127.2	97.4	121.5	114.5	154.8
9-FRUIT	145.3	140.7	93.8	128.2	166.9
10-GOAT	105.0	132.4	122.7	106.5	120.6
11-LAMB	112.1	125.1	124.4	118.1	117.7
12-PORK	88.6	125.3	147.3	83.5	111.7
13-BEEF	174.4	126.1	167.7	102.0	108.9
14-OFFAL	95.3	129.2	141.7	92.9	98.1
15-SAUSAGES	122.3	118.6	140.4	96.0	101.1
16-POULTRY	111.5	124.4	111.9	110.2	115.9
17-FRESH FISH	133.6	137.3	113.4	132.5	89.8
18-FROZEN FISH	82.3	121.8	174.2	97.3	189.1
19-SHELLFISH	110.8	139.8	146.6	100.4	91.5
20-CANNED FISH	149.4	115.4	129.5	129.7	123.2
21-DRIED FISH	131.3	131.9	100.6	105.3	189.2
22-EGGS	118.1	107.3	142.2	94.4	156.1
23-MILK	113.1	121.5	117.4	110.6	154.0
24-MILK PRODUCTS	156.7	125.7	116.6	114.5	118.5
25-OILS	114.4	107.0	130.1	107.4	162.3
26-FATS	146.8	128.3	112.3	107.3	138.4
27-SUGAR	100.0	115.4	106.7	124.9	146.0
28-JELLY	194.4	100.4	100.0	105.9	129.4
29-CONFECTIONARY	113.8	120.3	119.5	118.9	125.0
30-COCOA	245.1	108.5	111.5	107.5	108.2
31-COFFEE	208.0	100.1	124.0	107.3	102.7
32-TEA	184.5	101.9	123.9	99.7	127.1
33-CONDIMENTS	145.2	115.9	146.5	109.4	109.0
34-PREPARED FOOD	115.5	132.8	119.0	113.0	124.1
35-MEALS OUT	130.7	120.8	120.0	122.2	116.9
36-ALCHOOL.DRINK	173.6	177.3	67.1	100.5	133.0
37-SOFT DRINKS	112.2	122.9	118.1	113.0	133.3

TABLE A7 (CONT.)

	P1977	P1978	P1979	P1980	P1981
38-CLOTHING(MEN)	117.7	120.2	130.3	137.4	118.6
39-TAILOR(MEN)	113.0	112.7	127.8	126.2	111.3
40-U/WEAR(MEN)	128.1	112.8	138.8	129.5	109.4
41-CLOTH.(WOMEN)	117.8	125.2	128.5	137.9	116.7
42-TAILOR(WOMEN)	114.9	121.5	126.8	137.9	116.6
43-U/WEAR(WOMEN)	109.1	112.1	128.5	135.1	102.4
44-CLOTH.(CHILD)	123.8	114.7	125.3	134.0	110.6
45-LAUNDRY	125.5	123.2	150.2	129.0	117.9
46-FOOTW.(MEN)	116.3	117.7	140.7	114.3	93.4
47-FOOTW.(WOMEN)	129.5	117.0	152.3	109.9	102.4
48-FOOTW.(CHILD)	131.9	119.3	144.1	126.6	106.3
49-FOOTW.REPAIR	127.0	120.2	140.5	115.7	119.1
50-WATER	132.8	135.5	141.1	100.0	131.0
51-GAS	100.0	160.0	121.0	120.4	118.1
52-ELECTRICITY	141.1	149.3	130.7	122.2	127.6
53-ELECTR.APPL.	130.7	109.9	98.0	110.7	112.8
54-FURNITURE	115.9	112.6	122.3	148.4	109.9
55-HOUS.TEXTILES	107.4	96.4	137.7	117.8	102.0
56-OTHERS	147.6	103.2	130.4	124.2	112.0
57-CUTLERY,ETC.	160.7	99.1	118.3	138.0	119.9
58-REPAIR,MAINT.	119.8	118.5	114.0	121.0	131.4
59-HYG.PRODS.	115.2	114.1	126.7	121.6	108.6
60-SERVICES(HYG)	108.9	97.3	158.0	110.2	114.7
61-SURGIC.GOODS	251.6	100.5	117.8	102.5	163.9
62-MEDICINES	129.3	106.4	102.5	106.0	107.1
63-DOCTORS	104.4	127.7	116.5	124.0	116.6
64-TOBACCO	100.0	141.1	138.9	116.1	120.0
65-PRIV.TRANSP.	148.6	119.2	125.9	128.2	117.6
66-URB.TRANSP.	100.0	146.5	128.5	123.8	146.8
67-SUBURB.TRANSP	100.0	140.2	121.1	127.4	153.3
68-LONG DIST.TRA	-	-	-	-	-
69-TAXIS	100.0	134.0	126.9	123.7	147.4
70- POST-OFFICE	121.1	129.5	129.5	126.3	100.0
71-TELEPHONE	123.3	157.2	118.0	130.9	100.0
72-EDUCATION	111.2	104.3	106.1	125.3	107.3
73-ENTERTAINS	114.5	112.5	129.6	119.4	110.5

TABLE A8

NORTH

Price Distributions by Income Class

Income \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 18	113.0 (2.5)	104.7 (2.7)	114.6 (4.5)	125.9 (5.2)	129.6 (7.5)	112.5 (4.8)	132.8 (7.3)	121.3 (7.3)	269
18-30	113.2 (2.3)	104.5 (2.4)	114.4 (3.8)	125.7 (4.4)	127.9 (6.4)	113.2 (4.4)	132.6 (6.1)	123.3 (6.6)	442
30-48	113.2 (2.1)	104.4 (1.9)	114.3 (3.0)	125.6 (4.2)	126.5 (5.3)	113.3 (3.5)	131.4 (5.2)	124.9 (5.9)	811
48-60	113.1 (2.0)	104.1 (1.8)	114.3 (2.7)	125.3 (3.7)	125.9 (4.7)	113.7 (3.0)	130.5 (4.8)	124.2 (5.4)	559
60-90	112.7 (1.9)	103.9 (1.6)	114.8 (3.0)	125.0 (3.5)	125.1 (4.4)	114.1 (3.3)	129.2 (4.7)	124.9 (4.8)	1058
90-120	112.4 (2.0)	103.7 (1.6)	115.6 (3.5)	124.8 (3.9)	124.4 (4.1)	115.0 (3.7)	127.9 (5.1)	123.9 (5.1)	515
120-180	111.8 (2.1)	103.7 (1.5)	116.7 (4.5)	124.4 (3.3)	123.4 (4.1)	116.2 (4.8)	125.9 (5.0)	123.1 (4.5)	411
⩾ 180	111.5 (2.9)	104.3 (2.4)	118.8 (5.6)	122.1 (5.7)	122.2 (4.4)	118.6 (5.6)	121.8 (6.9)	119.6 (5.5)	273
All Households	112.7 (2.2)	104.9 (1.9)	115.1 (3.8)	125.0 (4.2)	125.6 (5.3)	114.3 (4.2)	129.4 (6.0)	123.9 (5.7)	4338

Note: Prices for OPORTO

Standard deviations in parentheses

TABLE A9

CENTRE

Price Distributions by Income Class

Income \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 18	112.2 (2.3)	105.4 (1.4)	114.4 (4.0)	130.0 (6.5)	134.3 (10.1)	112.5 (4.3)	128.5 (9.4)	118.8 (7.1)	388
18-30	111.4 (2.1)	105.6 (1.3)	114.7 (3.1)	129.4 (5.3)	130.0 (7.5)	113.2 (3.6)	129.5 (8.8)	119.1 (5.6)	588
30-48	111.1 (2.0)	105.7 (1.3)	114.6 (2.6)	129.0 (4.2)	126.9 (6.0)	113.9 (3.1)	129.5 (7.3)	119.7 (4.8)	893
48-60	111.0 (2.1)	105.7 (1.4)	114.5 (2.7)	128.1 (4.2)	125.6 (5.5)	114.5 (3.1)	128.7 (7.1)	119.9 (5.0)	514
60-90	110.9 (2.0)	105.7 (1.3)	113.9 (2.6)	127.1 (4.1)	124.7 (5.0)	115.1 (2.8)	127.1 (6.3)	119.1 (3.8)	903
90-120	110.7 (2.3)	105.7 (1.4)	113.0 (2.5)	125.3 (4.0)	123.8 (4.7)	115.7 (2.8)	124.7 (5.9)	118.9 (3.5)	447
120-180	110.2 (2.6)	105.3 (1.7)	111.9 (3.2)	123.5 (5.1)	122.9 (4.8)	115.8 (2.8)	123.1 (6.5)	118.9 (4.0)	305
≥ 180	110.4 (3.7)	105.6 (2.4)	111.3 (3.4)	121.2 (4.9)	122.0 (4.3)	116.8 (3.9)	119.3 (5.3)	117.8 (3.1)	179
All Households	111.0 (2.3)	105.6 (1.4)	113.9 (3.1)	127.5 (5.2)	126.5 (7.0)	114.4 (3.4)	127.4 (7.7)	119.2 (4.8)	4217

Note: Prices for Coimbra and Viseu

Standard deviations in parentheses

TABLE A10

LISBON

Price Distributions by Income Class

Years / Income	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 18	113.7 (5.6)	107.6 (2.5)	117.8 (4.6)	129.6 (7.5)	126.1 (7.2)	117.3 (5.3)	116.3 (6.4)	122.7 (5.1)	225
18-30	113.1 (5.6)	107.9 (2.5)	118.5 (4.0)	130.4 (6.9)	124.2 (6.2)	118.0 (5.2)	116.4 (5.9)	124.7 (5.1)	430
30-48	112.7 (4.4)	108.0 (2.2)	118.7 (3.0)	129.6 (6.5)	124.5 (5.5)	118.9 (4.7)	116.1 (5.2)	126.1 (4.1)	937
48-60	112.9 (3.9)	108.2 (2.1)	119.2 (2.7)	128.8 (6.4)	124.7 (5.1)	119.5 (4.5)	115.1 (5.1)	126.5 (3.6)	623
60-90	113.1 (3.6)	108.5 (2.2)	119.5 (2.4)	128.1 (5.9)	124.5 (4.8)	119.9 (4.2)	114.2 (4.5)	127.0 (3.1)	1359
90-120	113.5 (3.5)	108.8 (2.3)	120.0 (2.4)	127.8 (5.6)	124.1 (4.5)	120.4 (4.3)	113.3 (4.3)	127.4 (3.0)	884
120-180	113.7 (3.8)	109.4 (2.6)	120.4 (2.5)	127.1 (5.7)	123.9 (5.1)	120.9 (4.7)	111.9 (4.4)	127.9 (3.1)	705
⩾ 180	113.5 (4.2)	109.9 (3.2)	120.3 (2.4)	126.2 (7.4)	123.6 (5.7)	121.8 (4.5)	110.2 (4.7)	127.5 (3.7)	469
All Households	113.2 (4.1)	108.6 (2.5)	119.5 (2.9)	128.3 (6.4)	124.3 (5.3)	119.8 (4.7)	114.1 (5.2)	126.7 (3.8)	5632

Note: Prices for Lisbon

Standard deviations in parentheses

TABLE A11

SOUTH

Price Distributions by Income Class

Income \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 18	109.1 (2.5)	110.2 (3.1)	117.4 (3.5)	129.4 (4.7)	120.5 (5.2)	115.7 (3.9)	131.0 (9.2)	121.7 (6.9)	230
18-30	109.0 (2.1)	109.7 (3.1)	117.6 (2.7)	128.4 (4.0)	120.5 (4.1)	115.9 (4.1)	128.7 (9.1)	124.4 (7.8)	338
30-48	109.0 (2.1)	109.8 (3.1)	117.7 (2.5)	126.2 (3.6)	120.5 (3.2)	116.2 (3.5)	127.6 (9.6)	126.2 (5.8)	434
48-60	109.1 (2.0)	110.2 (3.1)	117.7 (2.5)	124.6 (3.2)	120.4 (2.8)	115.9 (3.7)	126.6 (10.0)	127.3 (5.5)	208
60-90	109.2 (2.0)	110.6 (3.4)	117.4 (2.3)	123.4 (3.3)	120.5 (2.6)	116.6 (3.9)	127.7 (12.4)	127.2 (5.2)	284
90-120	109.2 (2.0)	111.0 (3.3)	116.4 (2.8)	122.9 (3.7)	121.1 (2.9)	117.4 (5.0)	124.9 (8.9)	127.8 (4.4)	110
120-180	108.9 (2.7)	110.9 (4.2)	115.1 (3.9)	120.1 (3.1)	120.6 (2.4)	120.0 (7.7)	124.5 (12.2)	127.1 (3.9)	74
≥ 180	108.2 (6.6)	110.7 (8.3)	113.3 (7.1)	117.8 (6.8)	119.6 (6.4)	119.6 (10.9)	112.5 (10.5)	126.9 (8.2)	50
All Households	109.0 (2.4)	110.2 (3.5)	117.3 (3.1)	125.7 (4.8)	120.5 (3.7)	116.4 (4.5)	127.7 (10.3)	125.7 (6.5)	1728

Note: Prices for Evora and Faro

Standard deviations in parentheses

TABLE A12

NORTH

Price Distributions by Income Class
(Continent Prices)

Income \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 18	124.9 (7.6)	124.1 (7.5)	127.3 (10.4)	113.0 (4.1)	132.3 (7.4)	269
18-30	126.7 (6.7)	125.4 (6.4)	125.5 (9.1)	112.2 (3.5)	131.5 (6.1)	442
30-48	126.9 (5.4)	126.0 (5.7)	124.8 (6.8)	112.3 (3.4)	131.1 (5.5)	811
48-60	127.3 (5.4)	125.4 (5.3)	124.8 (6.9)	112.7 (3.3)	130.4 (5.3)	559
60-90	128.0 (4.8)	124.7 (4.5)	124.8 (5.4)	113.8 (3.1)	129.1 (4.7)	1058
90-120	128.5 (4.8)	123.7 (4.3)	125.1 (4.8)	114.8 (3.1)	127.1 (4.3)	515
120-180	129.2 (4.5)	122.6 (3.3)	125.8 (4.0)	116.2 (3.0)	125.1 (4.0)	411
≥ 180	131.1 (15.5)	121.3 (2.9)	125.4 (3.3)	117.9 (3.3)	122.4 (3.7)	273
All Households	127.7 (5.6)	124.5 (5.3)	125.2 (6.5)	113.7 (3.7)	129.0 (5.8)	4338

Note: Standard deviations in parentheses

TABLE A13

CENTRE

Price Distributions by Income Class
(Continent Prices)

Income \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 18	122.9 (7.0)	120.9 (6.6)	129.4 (9.0)	113.2 (4.0)	132.8 (6.6)	388
18-30	124.3 (5.9)	122.5 (6.1)	127.2 (7.4)	112.8 (3.6)	131.3 (5.7)	588
30-48	125.6 (5.2)	123.2 (5.1)	125.8 (6.0)	112.9 (3.3)	130.1 (5.0)	893
48-60	126.3 (5.0)	123.2 (4.7)	125.5 (5.5)	113.6 (3.3)	128.9 (4.7)	514
60-90	126.6 (4.9)	122.5 (3.9)	125.6 (4.6)	114.3 (3.1)	127.6 (4.5)	903
90-120	127.1 (4.6)	121.6 (3.3)	125.7 (3.9)	115.8 (3.0)	125.7 (4.1)	447
120-180	129.2 (5.1)	121.0 (2.9)	125.3 (3.3)	117.3 (3.3)	124.1 (4.4)	305
≥ 180	131.0 (5.8)	120.3 (2.5)	125.4 (2.7)	119.1 (3.2)	121.9 (3.2)	179
All Households	126.1 (5.7)	122.3 (4.8)	126.2 (5.9)	114.2 (3.7)	128.6 (5.7)	4217

Note: Standard deviations in parentheses

TABLE A14

LISBON

Price Distributions by Income Class
(Continent Prices)

Income \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 18	125.5 (6.9)	121.3 (5.1)	127.1 (8.0)	114.6 (3.4)	131.0 (7.7)	225
18-30	127.0 (6.0)	122.7 (5.2)	125.6 (6.1)	114.2 (2.9)	129.6 (6.0)	430
30-48	127.5 (5.2)	123.3 (4.6)	125.4 (5.6)	114.5 (3.0)	127.9 (5.2)	937
48-60	128.2 (5.3)	123.2 (3.8)	125.8 (5.1)	115.0 (2.7)	126.9 (4.8)	623
60-90	128.7 (4.7)	122.7 (3.2)	125.8 (4.5)	115.4 (2.7)	125.9 (4.4)	1359
90-120	129.6 (4.4)	122.3 (2.8)	125.7 (3.7)	116.2 (2.6)	124.7 (3.9)	884
120-180	130.4 (4.9)	121.7 (2.8)	125.3 (3.9)	117.4 (2.8)	123.3 (3.7)	705
≥ 180	131.8 (4.6)	121.0 (2.2)	125.0 (2.7)	118.7 (2.6)	121.0 (3.0)	469
All households	128.8 (5.2)	122.5 (3.7)	125.6 (4.8)	115.7 (3.1)	125.9 (5.3)	5632

Note: Standard deviations in parentheses

TABLE A15

SOUTH

Price Distributions by Income Class
(Continent Prices)

Income \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 18	123.0 (6.3)	119.7 (5.0)	127.8 (7.8)	114.3 (4.0)	130.1 (5.9)	230
18-30	124.2 (5.3)	121.1 (5.0)	126.3 (6.3)	114.5 (3.2)	129.2 (5.1)	338
30-48	125.3 (4.6)	121.4 (4.0)	125.0 (4.6)	114.7 (2.8)	128.2 (4.5)	434
48-60	126.4 (5.0)	121.6 (3.6)	125.4 (4.5)	115.3 (2.9)	126.3 (4.2)	208
60-90	126.7 (5.0)	121.3 (3.4)	125.2 (3.9)	115.7 (2.6)	125.7 (3.5)	284
90-120	128.5 (4.9)	121.3 (2.7)	126.1 (3.9)	116.7 (3.0)	123.7 (3.7)	110
120-180	130.7 (5.9)	120.6 (2.9)	124.9 (3.2)	118.4 (3.3)	122.5 (3.0)	74
≥ 180	130.6 (9.5)	119.0 (7.0)	124.3 (7.3)	118.1 (7.6)	119.8 (6.9)	50
All Households	125.7 (5.7)	121.0 (4.3)	125.6 (5.5)	115.2 (3.5)	127.2 (5.2)	1728

Note: Standard deviations in parentheses

TABLE A16

NORTH

Price Distribution by Age of Head of Household

Age \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 25	112.3 (1.8)	103.9 (1.9)	116.5 (3.7)	125.4 (3.8)	126.1 (5.0)	112.7 (4.6)	128.8 (5.5)	124.2 (5.5)	44
25-45	112.3 (2.1)	103.8 (1.7)	115.5 (4.0)	125.1 (3.7)	124.7 (4.8)	114.6 (4.3)	128.2 (5.5)	124.3 (4.9)	1461
45-65	112.8 (2.1)	104.1 (1.9)	114.9 (3.4)	125.2 (3.9)	125.5 (5.0)	114.3 (3.9)	129.6 (5.6)	124.3 (5.5)	1870
⩾ 65	113.3 (2.4)	104.5 (2.3)	114.9 (3.9)	124.7 (5.1)	127.0 (6.2)	113.9 (4.5)	131.0 (7.2)	122.4 (7.0)	963
All Households	112.7 (2.2)	104.1 (1.9)	115.1 (3.8)	125.0 (4.2)	125.6 (5.3)	114.3 (4.2)	129.4 (6.0)	123.9 (5.7)	4338

Note: Prices for OPORTO

Standard deviations in parentheses

TABLE A17

CENTRE

Price Distribution by Age of Head of Household

Age \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 25	110.8 (1.9)	105.2 (1.1)	112.3 (3.3)	125.8 (5.7)	125.6 (7.0)	114.4 (3.4)	125.2 (7.2)	118.6 (4.0)	52
25-45	110.6 (2.1)	105.5 (1.4)	113.3 (2.9)	126.8 (5.2)	125.0 (5.8)	114.7 (3.0)	126.2 (7.2)	119.1 (3.8)	1210
45-65	111.0 (2.2)	105.7 (1.4)	114.0 (2.9)	127.5 (4.7)	126.0 (6.3)	114.6 (3.4)	127.8 (7.7)	119.4 (4.8)	1895
≥ 65	111.6 (2.4)	105.7 (1.6)	114.5 (3.3)	128.4 (5.8)	129.3 (8.5)	114.0 (3.8)	128.2 (8.1)	119.1 (5.7)	1060
All Households	111.0 (2.3)	105.6 (1.4)	113.9 (3.1)	127.5 (5.2)	126.5 (7.0)	114.5 (3.4)	127.4 (7.7)	119.2 (4.8)	4217

Note: Combined prices for Coimbra and Viseu

Standard deviations in parentheses

TABLE A18

LISBON

Price Distributions by Age of Head of Household

Age \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 25	115.0 (5.1)	108.7 (2.8)	120.9 (3.6)	128.4 (6.5)	121.6 (6.3)	119.0 (4.0)	112.1 (5.6)	127.7 (3.2)	86
25-45	113.0 (3.8)	109.0 (2.5)	119.8 (2.6)	127.8 (5.8)	124.3 (4.9)	119.7 (3.9)	113.6 (4.8)	127.4 (3.4)	2089
45-65	112.9 (3.9)	108.4 (2.4)	119.4 (2.7)	128.8 (6.2)	124.2 (5.2)	119.8 (4.5)	114.5 (5.1)	126.8 (3.7)	2398
≥ 65	114.2 (4.9)	108.2 (2.4)	118.7 (3.6)	128.3 (7.6)	124.9 (6.0)	120.0 (6.3)	114.3 (6.0)	124.9 (4.4)	1059
All Households	113.2 (4.1)	108.6 (2.5)	119.5 (2.9)	128.3 (6.4)	124.3 (5.3)	119.8 (4.7)	114.1 (5.2)	126.7 (3.8)	5632

Note: Prices for Lisbon

Standard deviations in parentheses

TABLE A19

SOUTH

Price Distributions by Age of Head of Household

Age \ Years	1971	1972	1973	1974	1975	1976	1977	1978	No. of Observ.
< 25	109.0 (1.3)	110.5 (2.0)	116.9 (3.5)	124.1 (4.1)	119.3 (3.6)	113.9 (5.8)	124.6 (8.2)	131.3 (7.4)	15
25-45	109.0 (2.6)	109.7 (3.5)	116.8 (3.4)	124.5 (4.6)	119.8 (3.5)	116.6 (5.3)	125.4 (8.3)	126.7 (5.9)	470
45-65	109.0 (2.1)	110.0 (3.2)	117.3 (2.9)	125.6 (4.4)	120.6 (3.7)	116.4 (4.4)	127.2 (10.2)	126.5 (6.5)	782
≥ 65	109.7 (2.5)	111.0 (3.8)	117.8 (2.9)	127.1 (5.2)	121.0 (3.8)	116.4 (4.1)	131.0 (11.4)	123.1 (6.2)	461
All Households	109.0 (2.4)	110.2 (3.5)	117.3 (3.1)	125.7 (4.8)	120.5 (3.7)	116.4 (4.6)	127.7 (10.3)	125.7 (6.5)	1728

Note: Prices for Evora and Faro

Standard deviations in parentheses

TABLE A20

NORTH

Price Distributions by Age of Head of Household
(Continent Prices)

Age \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 25	127.4 (5.0)	123.3 (5.8)	122.6 (4.7)	116.0 (3.4)	127.4 (5.5)	44
25-45	127.9 (5.1)	123.6 (4.5)	125.2 (5.4)	114.6 (3.5)	128.3 (5.4)	1461
45-65	127.5 (5.5)	124.9 (5.2)	125.3 (6.4)	113.5 (3.5)	129.1 (5.6)	1870
⩾ 65	128.0 (6.4)	125.3 (6.2)	124.9 (8.1)	112.8 (3.8)	130.2 (6.4)	963
All Households	127.7 (5.8)	124.5 (5.3)	125.2 (6.5)	113.7 (3.7)	129.0 (5.8)	4338

Note: Standard deviations in parentheses

TABLE A21

CENTRE

Price Distributions by Age of Head of Household
(Continent Prices)

Age \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 25	125.2 (4.4)	120.4 (3.8)	126.1 (5.7)	116.7 (3.7)	126.2 (5.9)	52
25-45	126.9 (5.3)	121.8 (4.1)	125.9 (4.8)	115.1 (3.6)	127.4 (5.3)	1210
45-65	126.0 (5.6)	122.8 (4.9)	126.0 (5.7)	114.0 (3.7)	128.5 (5.4)	1895
⩾ 65	125.5 (6.3)	122.0 (5.5)	126.9 (7.3)	113.2 (3.7)	130.2 (6.1)	1060
All Households	126.1 (5.7)	122.3 (4.8)	126.2 (5.9)	114.2 (3.7)	128.6 (5.7)	4217

Note: Standard deviations in parentheses

TABLE A22

LISBON

Price Distributions by Age of Head of Household
(Continent Prices)

Age \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 25	125.9 (5.3)	121.6 (3.3)	124.9 (3.9)	117.5 (3.4)	124.7 (5.2)	86
25-45	129.0 (4.9)	122.1 (3.2)	125.6 (4.2)	116.2 (3.0)	125.1 (4.6)	2089
45-65	128.8 (5.2)	122.7 (3.9)	125.6 (4.8)	115.6 (3.0)	126.0 (5.1)	2398
⩾ 65	128.7 (5.8)	122.6 (4.2)	125.7 (5.8)	114.8 (3.0)	127.4 (6.2)	1059
All Households	128.8 (5.2)	122.5 (3.7)	125.6 (4.8)	115.7 (3.1)	125.9 (5.3)	5632

Note: Standard deviations in parentheses

TABLE A23

SOUTH

Price Distributions by Age of Head of Household
(Continent Prices)

Age \ Years	1977	1978	1979	1980	1981	No. of Observ.
< 25	126.6 (3.9)	120.6 (4.8)	124.3 (3.5)	116.2 (3.5)	124.6 (4.8)	15
25-45	126.8 (5.9)	121.0 (4.1)	125.4 (4.5)	115.8 (3.6)	126.1 (5.0)	470
45-65	125.6 (5.5)	121.3 (4.2)	125.7 (5.5)	115.2 (3.4)	127.1 (5.1)	782
⩾ 65	124.8 (5.8)	120.6 (4.6)	126.2 (6.2)	114.6 (3.4)	128.6 (5.5)	461
All Households	125.7 (5.7)	121.0 (4.3)	125.8 (5.5)	115.2 (3.5)	127.2 (5.2)	1728

Note: Standard deviations in parentheses

TABLE A24

REGRESSIONS - NORTH

	1971	1972	1973	1974	1975	1976	1977	1978
Constant	112.7* (340.5)	104.0* (354.5)	115.9* (212.0)	126.1* (203.9)	127.6* (162.6)	111.7* (186.5)	131.5* (162.6)	124.3* (149.1)
D1 (Employers, etc)	0.274* (2.9)	0.651* (7.8)	-0.245 (-1.6)	-0.288 (-1.6)	-0.859* (-3.9)	0.459* (2.7)	1.497* (6.5)	1.285* (5.4)
D2 Workers	0.104 (1.2)	0.118 (1.5)	-0.27 (-1.8)	0.105 (0.6)	-0.042 (-0.2)	-0.517* (-3.2)	0.360 (1.7)	1.517* (6.8)
D3 (Profess., etc)	-0.529* (-2.6)	0.186 (1.0)	0.755* (2.3)	0.617 (1.6)	-0.66 (-1.4)	0.749* (2.1)	-0.646 (-1.3)	-1.375* (-2.7)
D4 Non-active	0.067 (0.6)	-0.07 (-0.7)	-0.098 (-0.6)	-0.591* (-3.9)	-0.098 (-0.4)	0.138 (0.7)	-0.665* (-2.6)	-0.572* (-2.2)
AH 25-45	0.231 (0.7)	-0.088 (-0.3)	-0.674 (-1.2)	-0.29 (-0.5)	-0.531 (-0.7)	1.825* (3.1)	-0.083 (-0.1)	-0.55 (-0.7)
AH 45-65	0.5 (1.5)	0.118 (0.4)	-1.778* (-3.3)	0.266 (0.4)	-0.025 (-0.0)	1.159* (2.0)	1.174 (1.5)	0.274 (0.3)
AH ≥ 65	0.722* (2.2)	0.418 (1.4)	-1.767* (-3.2)	-0.188 (-0.3)	0.694 (0.9)	0.908 (1.5)	1.703* (2.1)	-1.128 (-1.3)
NC = 1	-0.2 (-1.9)	-0.253* (-2.7)	-0.413* (-2.4)	0.562* (2.8)	-0.456 (-1.8)	-0.065 (-0.3)	-0.298 (-1.2)	0.859* (3.2)
NC = 2	-0.152 (-1.3)	-0.129 (-1.3)	-1.036* (-5.4)	0.595* (2.8)	-0.457 (-1.7)	-0.479* (-2.3)	0.056 (0.2)	1.345 (4.6)
NC = 3	-0.264* (-2.0)	-0.115 (-1.0)	-1.558* (-7.1)	0.804 (3.2)	-0.511 (-1.6)	-1.055* (-4.4)	0.551 (1.7)	1.672* (5.0)
NC ≥ 4	-0.363* (-3.0)	0.02 (0.2)	-2.147* (-10.6)	1.537* (6.7)	-0.386 (-1.3)	-1.474* (-6.6)	1.024* (3.4)	2.331* (7.5)
URBAN	0.066 (0.9)	-0.418* (-6.6)	0.298* (2.5)	-0.828* (-6.2)	-0.027 (-0.2)	0.508* (3.9)	-1.678* (-9.6)	-1.281* (7.1)
INCOME	-0.006* (-10.6)	0.000 (0.6)	0.015* (17.7)	-0.014* (-13.7)	-0.019* (-15.4)	0.019* (19.4)	-0.034* (-26.6)	-0.012* (-9.3)
R^2 adj.	0.07	0.05	0.13	0.08	0.09	0.14	0.25	0.12
F	23.8	18.8	48.7	29.6	34.3	54.1	114.6	45.1
St.Dev.	2.13	1.89	3.52	3.99	5.06	3.86	5.21	5.37

Obs.: Prices for Oporto

* - significant at 0.05%

t - statistics in parentheses

TABLE A25

REGRESSIONS - CENTRE

	1971	1972	1973	1974	1975	1976	1977	1978
Constant	111.0* (346.9)	105.0* (515.2)	113.0* (267.3)	127.7* (188.2)	129.6* (137.3)	113.0* (242.8)	128.0* (124.1)	119.0* (172.0)
D1 (Employers, etc)	0.061 (0.7)	0.433* (7.2)	0.811* (6.5)	0.643* (3.2)	-0.823* (-3.0)	-0.313* (-2.3)	2.345* (7.7)	0.917* (4.5)
D2 Workers	-0.081 (-0.8)	0.131* (2.1)	0.603* (4.6)	0.624* (3.0)	-1.221* (-4.2)	-0.268 (-1.9)	1.688* (5.3)	0.535* (2.5)
D3 (Profess., etc)	-0.059 (-0.2)	-0.057 (-0.3)	0.111 (0.3)	1.109* (2.0)	-0.458 (-0.6)	-0.759* (-2.0)	0.032 (0.0)	0.014 (0.0)
D4 Non-active	0.427* (3.6)	0.332* (4.4)	0.426* (2.7)	0.012 (0.1)	-0.605 (-1.7)	0.367* (2.1)	0.102 (0.3)	-0.029 (-0.1)
AH 25-45	0.086 (0.3)	0.308 (1.5)	1.002* (2.4)	0.317 (0.5)	-0.128 (-0.1)	0.608 (1.3)	0.187 (0.2)	0.620 (0.9)
AH 45-65	0.253 (0.8)	0.448* (2.3)	1.482* (3.6)	1.444* (2.2)	0.037 (0.0)	0.503 (1.1)	1.383 (1.4)	0.454 (0.7)
AH ⩾ 65	0.510 (1.6)	0.395 (1.9)	1.625* (3.8)	1.854* (2.7)	1.667 (1.8)	0.262 (0.6)	0.971 (0.9)	-0.039 (0.1)
NC = 1	-0.245* (-2.3)	-0.033 (-0.5)	-0.170 (-1.2)	0.591* (2.6)	-1.449* (-4.6)	0.266 (1.7)	-0.559 (-1.6)	-0.618* (-2.7)
NC = 2	-0.267* (-2.2)	-0.238* (-3.9)	-0.056 (-0.4)	1.359* (5.3)	-1.303* (-3.6)	-0.233 (-1.3)	0.058 (0.1)	-0.484 (-1.9)
NC = 3	-0.266 (-1.8)	-0.096 (-1.0)	-0.024 (-0.1)	1.707* (5.3)	-0.784 (-1.8)	-0.570* (-2.6)	0.667 (1.4)	-0.529 (-1.6)
NC ⩾ 4	-0.5* (-3.3)	-0.141 (-1.5)	0.015 (0.0)	2.685* (8.4)	-0.89* (-2.0)	-1.009* (-4.6)	1.2* (2.5)	-1.015* (-3.1)
URBAN	0.534* (6.0)	0.111* (2.0)	-0.297* (-2.5)	-0.718* (-3.7)	-0.362 (-1.4)	0.912* (7.0)	-2.754* (-9.5)	-0.206 (-1.1)
INCOME	-0.004* (-6.7)	0.001* (2.1)	-0.012* (-13.9)	-0.033* (-25.1)	-0.031* (-16.5)	0.015* (16.3)	-0.033* (-16.2)	-0.004* (-3.1)
R^2 adj.	0.05	0.03	0.09	0.19	0.13	0.11	0.15	0.01
F	17.1	8.9	35.0	75.4	51.0	41.2	57.6	5.08
St. Dev.	2.21	1.41	2.92	4.68	6.51	3.21	7.12	4.77

Obs.: Prices for Coimbra and Viseu

* - significant at 0.05%

t - statistics in parentheses

TABLE A26

REGRESSIONS - LISBON

	1971	1972	1973	1974	1975	1976	1977	1978
Constant	115.0* (259.3)	108.4* (409.5)	120.6* (384.0)	129.6* (186.9)	121.4* (205.7)	117.9* (229.6)	113.0* (211.2)	127.0* (307.0)
D1 (Employers, etc)	-0.673* (-3.6)	0.211 (1.9)	-0.236 (-1.8)	0.285 (1.0)	0.322 (1.3)	0.191 (0.9)	0.723* (3.2)	0.061 (0.4)
D2 Workers	-1.15* (-8.5)	-0.06 (-0.7)	-0.315* (-3.3)	0.752* (3.6)	-0,232 (-1.3)	-0.249 (-1.6)	1.358* (8.3)	0.340* (2.7)
D3 (Profess., etc)	0.492* (2.0)	-0.109 (-0.7)	-0.353* (-2.0)	0.187 (0.5)	-0.61 (-1.9)	-0.805* (-2.8)	-0.273 (-0.9)	-0.696* (-3.0)
D4 Non-active	0.425* (2.5)	-0.225* (-2.2)	-0.397* (-3.3)	-1.13* (-4.2)	0.655* (2.9)	0.464* (2.3)	-0.401 (-1.9)	-0.692* (-4.3)
AH 25-45	-1.10* (-2.5)	0.329 (1.3)	-0.782* (-2.5)	-1.33 (-1.9)	2.894* (5.0)	0.724 (1.4)	1.127* (2.1)	-0.268 (-0.7)
AH 45-65	-2.089* (-4.8)	-0.466 (-1.8)	-1.54* (-5.0)	0.560 (0.8)	2.657* (4.6)	0.649 (1.3)	2.721* (5.2)	-0.969* (-2.4)
AH $\geq$ 65	-1.551* (-3.4)	-0.597* (-2.2)	-2.27* (-7.1)	0.642 (0.9)	2.90* (4.8)	0.868 (1.7)	2.660* (4.6)	-2.413* (-5.7)
NC = 1	-0.993* (-6.5)	-0.442* (-4.9)	-0.516* (-4.8)	0.713* (3.0)	0.10* (10.5)	-0.147 (-0.8)	0.795* (4.3)	-0.310* (-2.2)
NC = 2	-1.48* (-8.0)	-0.827* (-7.5)	-0.984* (-7.6)	1.562* (5.4)	-0.005 (-0.0)	-0.367 (-1.7)	1.610* (7.3)	-0.506* (-3.0)
NC = 3	-2.156* (-7.8)	-1.231* (-7.4)	-1.665* (-8.5)	2.623* (6.0)	-0.134 (-0.4)	-1.259* (-3.9)	2.605* (7.8)	-1.214* (-4.7)
NC $\geq$ 4	-2.563* (-7.8)	-1.323* (-6.7)	-2.19* (-9.4)	4.314* (8.4)	0.181 (0.4)	-2.07* (-5.5)	4.151* (10.4)	-2.166* (-7.1)
URBAN	0.886* (7.8)	-0.085 (-1.3)	0.432* (5.3)	-0.993* (-5.6)	0.669* (4.5)	0.305* (2.3)	0.03 (0.2)	0.599* (5.6)
INCOME	0.002* (2.2)	0.008* (17.6)	0.006* (10.6)	-0.014* (-11.2)	-0.03* (-3.2)	0.014* (15.1)	-0.023* (-24.4)	0.007* (9.4)
R^2 adj.	0.09	0.09	0.08	0.07	0.01	0.05	0.17	0.09
F	43.0	41.5	38.0	33.1	6.95	25.6	87.5	41.4
St. Dev.	3.94	2.35	2.79	6.15	5.24	4.56	4.75	3.67

Obs.: Prices for Lisbon

* - significant at 0.05%

t - statistics in parentheses

TABLE A27

REGRESSIONS - SOUTH

	1971	1972	1973	1974	1975	1976	1977	1978
Constant	109.1* (178.0)	110.3* (125.7)	117.6* (150.6)	126.9* (119.3)	119.5* (124.6)	113.3* (96.0)	126.9* (48.2)	130.0* (79.7)
D1 (Employers, etc)	0.460* (2.6)	0.048 (0.2)	0.683* (3.0)	-0.233 (-0.7)	0.424 (1.5)	-0.339 (-1.0)	-0.954 (-1.2)	1.290* (2.7)
D2 Workers	-0.329* (-2.3)	-0.568* (-2.8)	0.487* (2.7)	0.431 (1.8)	-0.330 (-1.5)	-0.175 (-0.6)	-1.363* (-2.3)	1.644* (4.4)
D3 (Profess., etc)	0.431 (0.8)	1.595* (2.0)	-0.314 (-0.4)	0.766 (0.8)	1.518 (1.7)	0.443 (0.4)	-2.174 (-0.9)	-1.024 (-o.7)
D4 Non-active	0.516* (2.7)	0.808* (2.9)	0.621* (2.5)	-0.673* (-2.0)	0.277 (0.9)	-0.05 (-0.1)	0.349 (0.4)	-0.805 (-1.6)
AH 25-45	-0.215 (-0.3)	-0.067 (-0.8)	-0.043 (-0.1)	-0.072 (-0.1)	0.471 (0.5)	3.007* (2.5)	2.098 (0.8)	-4.334* (-2.6)
AH 45-65	0.016 (0.0)	-0.381 (-0.4)	0.158 (0.2)	1.003 (1.0)	1.257 (1.3)	2.578* (2.2)	2.697 (1.0)	-4.839* (-3.0)
AH ⩾ 65	0.361 (0.6)	0.165 (0.2)	0.308 (0.4)	2.072 (1.9)	1.481 (1.5)	2.793* (2.4)	5.005 (1.9)	-7.589* (-4.6)
NC = 1	-0.263 (-1.5)	-0.736* (-3.0)	-0.250 (-1.1)	0.244 (0.8)	0.197 (0.7)	-0.133 (-0.4)	-1.317 (-1.8)	-0.927* (-2.0)
NC = 2	-0.339 (-1.6)	-0.969* (-3.2)	-0.514 (-1.9)	0.656 (1.8)	0.076 (0.2)	-0.990* (-2.4)	-1.784* (-2.0)	1.084* (-2.0)
NC = 3	-0.175 (-0.5)	-0.946 (-1.9)	0.260 (0.6)	1.745* (2.9)	0.364 (0.7)	-1.490* (-2.3)	-1.950 (-1.3)	-0.952 (-1.0)
NC ⩾ 4	-0.144 (-0.4)	-0.871 (-1.6)	0.248 (0.5)	1.370* (2.1)	-0.239 (-0.4)	-0.216 (-0.3)	-3.227* (-2.0)	-2.032* (-2.0)
URBAN	0.754* (5.5)	1.347* (6.9)	0.269 (1.5)	-1.764* (-7.4)	0.032 (0.1)	-0.953* (-3.6)	-0.606 (-1.0)	0.364 (1.0)
INCOME	-0.004* (-3.2)	0.002 (1.0)	-0.014* (-9.8)	-0.036* (-18.8)	-0.002 (-1.1)	0.018* (8.4)	-0.016* (-3.4)	0.012* (4.0)
R^2 adj.	0.06	0.08	0.08	0.29	0.02	0.04	0.05	0.08
F	9.97	12.2	12.4	54.7	3.18	6.99	8.54	13.3
St. Dev.	2.33	3.33	2.97	4.04	3.64	4.48	10.0	6.19

Obs.: Prices for Evora and Faro

* - significant at 0.05%

t - statistics in parentheses

TABLE A28

REGRESSIONS - NORTH

	1977	1978	1979	1980	1981
Constant	126.8* (151.2)	124.2* (161.2)	122.1* (123.0)	114.9* (235.7)	129.3* (165.7)
D1 (Employers, etc.)	0.683* (2.9)	2.281* (10.4)	-0.898* (-3.2)	-1.728* (-12.5)	0.095 (0.4)
D2 Workers	-0.310 (-1.4)	0.767* (3.7)	-0.324 (-1.2)	-0.465* (-3.6)	0.752* (3.6)
D3 (Profess., etc.)	0.934 (1.8)	-0.342 (-0.7)	1.002 (1.7)	0.508 (1.7)	-1.261* (-2.7)
D4 Non-active	-0.109 (-0.4)	-0.282 (-1.2)	-0.210 (-0.7)	0.120 (0.8)	0.066 (0.3)
AH 25-45	0.676 (0.8)	0.423 (0.6)	2.849* (2.9)	-1.435* (-3.0)	0.727 (0.9)
AH 45-65	-0.401 (-0.5)	1.388 (1.8)	3.077* (3.1)	-2.543* (-5.3)	2.259* (2.9)
AH ≥ 65	0.186 (0.2)	1.577* (2.0)	2.796* (2.8)	-2.920* (-5.9)	2.986* (3.8)
NC = 1	-0.868* (-3.2)	-0.297 (-1.2)	0.459 (1.4)	-0.171 (-1.1)	0.568* (2.3)
NC = 2	-1.182* (-4.0)	-0.042 (-0.2)	0.335 (1.0)	-0.352* (-2.1)	1.253* (4.6)
NC = 3	-1.516* (-4.5)	-0.060 (-0.2)	0.223 (0.6)	-0.672* (-3.4)	2.159* (6.8)
NC ≥ 4	-1.805* (-5.8)	0.209 (0.7)	-0.123 (-0.3)	-0.953* (-5.3)	3.206* (11.1)
URBAN	0.248* (1.4)	-1.203* (-7.2)	2.030* (9.4)	0.886* (8.4)	-1.932* (-11.4)
INCOME	0.017* (13.0)	-0.012* (-9.8)	-0.005* (-3.4)	0.019* (24.2)	-0.031* (-24.8)
R^2 adj.	0.06	0.11	0.03	0.26	0.25
F	23.3	41.5	11.3	119.2	110.6
St. Dev.	5.40	4.96	6.40	3.14	5.03

Obs.: Prices for the Continent

* - significant at 0.05%

t - statistics in parentheses

TABLE A29

REGRESSIONS - CENTRE

	1977	1978	1979	1980	1981
Constant	123.3* (156.7)	120.5* (176.3)	126.5* (148.7)	115.1* (242.3)	129.3* (180.3)
D1 (Employers, etc.)	0.224 (1.0)	2.092* (10.4)	-0.175 (-0.7)	-1.446* (-10.3)	0.466* (2.2)
D2 Workers	0.352 (1.4)	1.240* (5.9)	-0.343 (-1.3)	-0.559* (-3.8)	0.778* (3.5)
D3 (Profess., etc.)	0.042 (0.1)	0.679 (1.2)	1.198 (1.7)	-0.174 (-0.4)	0.441 (0.7)
D4 Non-active	0.273 (0.9)	0.440 (1.7)	-0.042 (-0.1)	-0.099 (-0.6)	-0.204 (-0.8)
AH 25-45	1.939* (2.5)	1.069 (1.6)	-0.159 (-0.2)	-1.235* (-2.6)	0.470 (0.7)
AH 45-65	0.862 (1.1)	1.808* (2.7)	0.126 (0.2)	-2.079* (-4.5)	1.633* (2.3)
AH ≥ 65	0.878 (1.1)	0.897 (1.3)	0.795 (0.9)	-2.350* (-4.9)	2.510* (3.5)
NC = 1	-0.818* (-3.1)	-0.358 (-1.6)	0.497 (1.8)	0.036 (0.2)	-0.202 (-0.8)
NC = 2	-0.592* (-2.0)	-0.090 (-0.4)	0.132 (0.4)	-0.093 (-0.5)	0.724* (2.7)
NC = 3	-1.361* (-3.7)	0.329 (1.0)	0.733 (1.8)	-0.667* (-3.0)	0.816* (2.4)
NC ≥4	-1.759* (-4.8)	-0.114 (-0.4)	1.056* (2.6)	-0.689* (-3.1)	1.751* (5.2)
URBAN	0.836* (3.8)	-0.498* (-2.6)	0.923* (3.9)	0.902* (6.8)	-2.285* (-11.4)
INCOME	0.025* (16.4)	-0.007* (-5.5)	-0.012* (-7.0)	0.022* (23.4)	-0.034* (-23.9)
R^2 adj.	0.09	0.05	0.02	0.23	0.23
F	33.1	19.6	6.16	97.1	100.2
St. Dev.	5.43	4.71	5.87	3.28	4.94

Obs.: Prices for the Continent

* - significant at 0.05%

t - statistics in parentheses

TABLE A30

REGRESSIONS - LISBON

	1977	1978	1979	1980	1981
Constant	123.8* (221.1)	121.5* (297.6)	124.7* (231.2)	116.7* (372.7)	126.7* (238.9)
D1 (Employers, etc.)	0.882* (3.7)	0.593* (3.5)	-0.127 (-0.6)	-0.365* (-2.8)	0.340 (1.5)
D2 Workers	-0.202 (-1.2)	0.368* (3.0)	-0.234 (-1.4)	-0.424* (-4.4)	0.656* (4.0)
D3 (Profess., etc.)	0.240 (0.8)	-0.003 (-0.0)	0.211 (0.7)	-0.020 (-0.1)	-0.558 (-1.9)
D4 Non-active	0.189 (0.9)	-0.181 (-1.2)	-0.222 (-1.1)	-0.180 (-1.5)	-0.159 (-0.7)
AH 25-45	2.994* (5.4)	0.950* (2.4)	0.483 (0.9)	-1.304* (-4.2)	0.422 (0.8)
AH 45-65	2.571* (4.7)	1.111* (2.8)	0.845 (1.6)	-2.080* (-6.8)	1.629* (3.1)
AH ⩾ 65	2.754* (4.8)	0.813* (2.0)	1.011 (1.8)	-2.627* (-8.3)	2.635* (4.9)
NC = 1	-0.308 (-1.6)	-0.488* (-3.5)	0.637* (3.4)	-0.434* (-4.0)	0.504* (2.8)
NC = 2	-0.232 (-1.0)	-0.530* (-3.1)	0.681* (3.0)	-0.706* (-5.4)	0.936* (4.2)
NC = 3	-1.247* (-3.6)	-0.828* (-3.2)	1.170* (3.5)	-1.211* (-6.2)	1.599* (4.8)
NC ⩾ 4	-2.266* (-5.5)	-0.546 (-1.8)	1.513* (3.8)	-1.722* (-7.4)	3.226* (8.2)
URBAN	1.520* (10.6)	1.218* (11.6)	0.656* (4.7)	-0.022 (-0.3)	-0.716* (-5.2)
INCOME	0.017* (16.6)	-0.008* (-10.8)	-0.005* (-4.9)	0.015* (27.6)	-0.026* (-27.4)
R^2 adj.	0.10	0.05	0.01	0.18	0.20
F	49.5	23.4	4.9	97.6	106.8
St. Dev.	4.97	3.62	4.8	2.8	4.7

Obs.: Prices for the Continent

* - significant at 0.05%

t - statistics in parentheses

TABLE A31

REGRESSIONS - SOUTH

	1977	1978	1979	1980	1981
Constant	124.9* (87.7)	120.7* (107.9)	125.0* (87.3)	115.5* (131.5)	126.8* (101.5)
D1 (Employers, etc.)	-0.379 (-0.9)	0.417 (1.3)	0.242 (0.6)	-0.070 (-0.3)	0.045 (0.1)
D2 Workers	-0.433 (-1.3)	0.638* (2.5)	-0.621 (-1.9)	-0.731* (-3.6)	0.624* (2.2)
D3 (Profess., etc.)	1.970 (1.5)	1.248 (1.2)	2.525 (1.9)	-0.417 (-0.5)	-0.154 (-0.1)
D4 Non-active	0.212 (0.5)	-0.178 (-0.5)	-0.276 (-0.6)	-0.094 (-0.3)	0.464 (1.2)
AH 25-45	0.989 (0.7)	0.941 (0.8)	0.784 (0.5)	-0.203 (-0.2)	1.316 (1.0)
AH 45-65	-0.546 (-0.4)	0.647 (-0.6)	1.295 (0.9)	-0.766 (-0.9)	2.064 (1.7)
AH $\geq$ 65	-1.083 (-0.8)	-0.182 (-0.2)	1.748 (1.2)	-1.231 (-1.4)	2.952* (2.3)
NC = 1	-0.232 (-0.6)	-0.957* (-3.0)	0.733 (1.8)	0.159 (0.6)	0.019 (0.1)
NC = 2	-1.617* (-3.3)	-1.157* (-3.0)	0.596 (1.2)	-0.258 (-0.9)	0.044 (0.1)
NC = 3	-1.205 (-1.5)	-0.916 (-1.5)	0.950 (1.2)	-0.352 (-0.7)	1.747* (2.5)
NC $\geq$ 4	-1.110 (-1.2)	-1.812* (-2.6)	1.247 (1.4)	0.194 (0.4)	1.150 (1.5)
URBAN	1.545* (4.9)	0.769* (3.1)	0.152 (0.5)	0.375 (1.9)	-1.083* (-3.9)
INCOME	0.022* (8.8)	-0.006* (-2.9)	-0.011* (-4.2)	0.012* (7.8)	-0.032* (-14.4)
R^2 adj.	0.10	0.02	0.01	0.07	0.18
F	15.7	3.65	2.59	11.4	30.7
St. Dev.	5.41	4.25	5.44	3.34	4.74

Obs.: Prices for the Continent

* - significant at 0.05%

t - statistics in parentheses

TABLE A32

WEIGHTS BY CLASS OF EXPENDITURE ("OLD" INDICES) - NORTH

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-CEREALS	11.72	9.76	9.02	8.08	7.04	5.80	4.45	2.21
2-VEGETABLES	1.53	1.92	1.86	1.58	1.19	1.02	0.78	0.37
3-POTAT,GREENS	9.01	8.10	7.21	6.26	5.05	3.95	3.28	1.89
4-FRUIT	1.57	1.36	1.77	2.08	2.39	2.31	2.24	1.56
5-CONFECTIONARY	0.32	0.33	0.35	0.35	0.45	0.46	0.55	0.43
6-FATS	9.22	8.64	7.86	7.27	6.23	5.15	4.17	2.47
7-CONDIMENTS	0.22	0.22	0.21	0.19	0.19	0.18	0.14	0.09
8-SUGAR,COFFEE	3.85	2.75	2.47	2.19	2.00	1.65	1.29	0.84
9-MEAT	4.24	6.44	7.75	8.58	9.26	9.42	9.21	7.54
10-POULTRY	1.58	2.55	2.71	3.22	2.95	2.70	2.30	1.23
11-FISH	5.34	6.35	6.82	7.03	6.71	6.08	5.33	4.08
12-MILK,EGGS	1.94	2.08	2.46	2.67	2.85	2.62	2.57	1.98
13-MEALS OUT	9.12	7.18	4.06	3.42	3.09	4.61	5.23	6.24
14-DRINKS	8.13	10.49	11.62	10.63	9.41	7.53	5.66	2.97
15-CLOTHING	5.26	5.55	6.97	7.78	8.90	9.42	9.88	8.08
16-FOOTWEAR	1.19	1.30	1.47	1.65	1.75	1.68	1.75	1.25
17-HOUSING	12.15	9.43	8.64	8.42	9.03	10.01	10.68	17.73
18-FUEL,LIGHT	6.45	5.84	4.42	3.79	3.11	2.74	2.56	2.34
19-WATER	0.20	0.14	0.17	0.17	0.20	0.23	0.24	0.26
20-PERSON.HYG.	0.87	0.72	1.05	1.03	1.19	1.30	1.48	1.46
21-HOUSING HYG.	0.51	0.59	0.80	0.75	0.81	0.76	0.72	0.58
22-FURNITURE	0.61	1.38	2.35	3.15	4.23	4.60	5.90	7.34
23-DOMEST.SERV.	0.03	0.24	0.19	0.24	0.36	0.64	1.38	2.90
24-TRANSPORT	0.85	0.93	1.58	2.34	3.84	5.91	8.83	13.13
25-COMUNICATIONS	0.15	0.21	0.24	0.25	0.36	0.62	0.75	0.97
26-HEALTH	1.36	1.76	1.74	2.03	2.17	2.82	2.51	3.57
27-ENTERTAINS	1.54	2.36	2.87	3.49	3.83	4.53	5.05	5.73
28-TOBACCO	1.04	1.40	1.34	1.34	1.41	1.28	1.07	0.75

TABLE A33

WEIGHTS BY CLASS OF EXPENDITURE ("OLD" INDICES) - CENTRE

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-CEREALS	11.58	9.63	8.28	7.00	6.15	4.69	3.65	2.24
2-VEGETABLES	3.59	2.99	2.40	2.18	1.95	1.37	0.94	0.52
3-POTAT,GREENS	10.97	9.40	7.78	6.39	5.60	4.11	3.12	1.99
4-FRUIT	1.62	1.83	2.08	2.40	2.28	2.24	1.98	1.94
5-CONFECTIONARY	0.17	0.27	0.36	0.38	0.53	0.62	0.65	0.62
6-FATS	10.54	9.45	8.36	7.90	6.60	5.44	4.15	2.69
7-CONDIMENTS	0.33	0.29	0.30	0.24	0.23	0.20	0.14	0.10
8-SUGAR,COFFEE	4.30	3.36	2.70	2.30	2.04	1.60	1.39	0.99
9-MEAT	3.62	5.77	7.53	8.79	8.98	9.12	7.82	7.26
10-POULTRY	1.46	2.64	2.73	3.31	3.07	2.68	2.40	1.61
11-FISH	5.55	5.76	6.32	6.02	5.94	5.06	5.09	3.73
12-MILK,EGGS	2.47	3.11	3.54	3.50	3.26	3.18	2.74	2.44
13-MEALS OUT	6.73	3.88	2.28	2.46	2.63	3.81	4.64	4.37
14-DRINKS	5.86	8.45	9.13	8.70	7.62	5.81	4.75	2.59
15-CLOTHING	5.53	6.91	8.44	8.87	10.00	11.29	10.92	10.01
16-FOOTWEAR	1.40	1.56	1.77	1.84	1.90	2.00	1.76	1.45
17-HOUSING	9.81	7.94	7.71	7.77	8.36	9.59	9.75	14.34
18-FUEL,LIGHT	7.13	5.17	4.41	4.15	3.14	2.89	2.79	2.03
19-WATER	0.18	0.15	0.15	0.19	0.18	0.24	0.24	0.24
20-PERSON.HYG.	0.73	0.90	1.04	1.19	1.23	1.43	1.44	1.62
21-HOUSING HYG.	1.03	0.95	0.99	1.17	1.00	0.90	0.94	0.75
22-FURNITURE	0.97	1.73	2.58	3.05	4.33	5.86	6.79	7.32
23-DOMEST.SERV.	0.12	0.25	0.27	0.25	0.29	0.37	0.69	1.83
24-TRANSPORT	0.52	1.11	1.70	2.72	4.15	6.18	11.27	17.13
25-COMUNICATIONS	0.12	0.20	0.20	0.38	0.30	0.49	0.49	0.97
26-HEALTH	1.08	2.35	2.36	1.89	2.86	3.02	3.21	3.27
27-ENTERTAINS	1.80	3.07	3.84	4.03	4.46	5.03	5.57	5.38
28-TOBACCO	0.78	0.88	0.77	0.92	0.90	0.77	0.68	0.58

TABLE A34

WEIGHTS BY CLASS OF EXPENDITURE ("OLD" INDICES) - LISBON

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-CEREALS	12.11	8.76	7.25	6.11	4.81	3.61	2.57	1.62
2-VEGETABLES	2.35	2.13	1.83	1.68	1.36	1.13	0.86	0.49
3-POTAT,GREENS	6.23	5.13	4.71	4.29	3.77	3.26	2.60	1.75
4-FRUIT	2.98	3.19	3.20	3.43	3.27	3.14	2.76	1.99
5-CONFECTIONARY	0.39	0.44	0.51	0.55	0.56	0.68	0.70	0.70
6-FATS	8.09	8.95	7.09	6.12	5.39	4.42	3.39	2.11
7-CONDIMENTS	0.21	0.34	0.28	0.23	0.18	0.15	0.11	0.08
8-SUGAR,COFFEE	3.62	2.87	2.22	1.82	1.56	1.39	1.09	0.81
9-MEAT	6.02	8.17	9.53	10.47	10.90	10.46	9.10	7.32
10-POULTRY	2.15	2.68	3.28	3.29	3.00	2.46	1.82	1.16
11-FISH	7.64	7.25	7.65	7.90	7.20	7.03	5.89	4.57
12-MILK,EGGS	4.14	4.28	4.37	4.27	4.18	3.90	3.34	2.65
13-MEALS OUT	5.32	5.47	3.61	3.84	4.45	6.07	7.63	7.46
14-DRINKS	2.83	4.81	5.68	5.15	4.68	3.74	2.78	1.82
15-CLOTHING	4.47	5.54	6.15	7.08	7.08	7.35	6.99	6.22
16-FOOTWEAR	1.25	1.30	1.54	1.58	1.59	1.50	1.35	0.98
17-HOUSING	15.20	12.33	11.96	12.20	12.47	12.33	12.78	13.79
18-FUEL,LIGHT	5.36	4.68	3.86	3.23	2.98	2.66	2.49	2.07
19-WATER	0.59	0.56	0.53	0.46	0.50	0.49	0.47	0.45
20-PERSON.HYG.	0.99	1.08	1.16	1.18	1.25	1.36	1.46	1.57
21-HOUSING HYG.	1.12	1.33	1.26	1.14	1.13	1.08	1.01	0.76
22-FURNITURE	0.71	1.03	1.99	2.24	3.59	4.72	5.99	6.13
23-DOMEST.SERV.	0.25	0.38	0.31	0.30	0.62	1.12	1.88	4.70
24-TRANSPORT	1.51	1.49	2.82	3.62	5.13	6.73	10.48	15.02
25-COMUNICATIONS	0.39	0.47	0.65	0.70	0.82	1.08	1.18	1.31
26-HEALTH	1.60	2.25	2.36	2.49	2.58	2.77	2.91	3.87
27-ENTERTAINS	1.74	2.08	2.91	3.16	3.71	4.28	5.39	7.76
28-TOBACCO	0.73	1.01	1.28	1.45	1.23	1.09	1.01	0.83

TABLE A35

WEIGHTS BY CLASS OF EXPENDITURE ("OLD" INDICES - SOUTH)

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-CEREALS	13.42	10.91	7.87	6.75	5.22	4.40	2.82	1.79
2-VEGETABLES	3.02	3.52	2.83	2.27	2.01	1.89	1.00	0.53
3-POTAT,GREENS	6.36	5.50	4.62	4.20	3.59	3.06	2.11	1.67
4-FRUIT	2.66	3.16	3.99	3.72	3.49	3.13	2.83	2.04
5-CONFECTIONARY	0.43	0.43	0.55	0.67	0.67	0.83	0.50	0.72
6-FATS	11.54	10.64	9.54	7.98	6.78	5.42	4.02	2.52
7-CONDIMENTS	0.42	0.39	0.37	0.34	0.25	0.24	0.12	0.06
8-SUGAR,COFFEE	4.07	3.58	2.69	2.34	2.04	1.69	1.28	0.96
9-MEAT	7.93	8.86	10.26	10.76	10.88	10.70	8.05	7.72
10-POULTRY	2.93	2.32	3.91	3.20	3.68	2.79	2.41	1.98
11-FISH	5.16	5.53	5.62	6.10	5.61	5.88	4.66	3.83
12-MILK,EGGS	4.80	4.97	4.88	4.58	4.60	3.76	3.33	2.53
13-MEALS OUT	3.47	2.89	2.39	1.56	2.39	3.95	4.20	4.11
14-DRINKS	2.47	4.30	4.93	4.98	4.54	3.83	2.44	1.53
15-CLOTHING	5.29	6.93	7.62	8.98	8.71	8.27	8.15	6.07
16-FOOTWEAR	1.60	1.74	1.76	1.97	1.82	1.82	1.44	0.96
17-HOUSING	10.59	8.43	8.13	8.85	9.79	9.92	12.51	14.86
18-FUEL,LIGHT	4.91	4.21	3.62	3.29	2.87	2.58	2.58	1.86
19-WATER	0.48	0.39	0.40	0.34	0.38	0.36	0.43	0.36
20-PERSON.HYG.	1.17	1.30	1.20	1.23	1.28	1.31	1.28	1.29
21-HOUSING HYG.	1.34	1.47	1.44	1.33	1.30	1.16	0.75	0.66
22-FURNITURE	0.84	1.19	2.47	3.53	4.11	5.48	5.38	6.13
23-DOMEST.SERV.	0.20	0.05	0.09	0.25	0.46	1.14	1.63	2.74
24-TRANSPORT	0.61	0.63	1.94	2.91	4.41	8.15	16.99	20.76
25-COMUNICATIONS	0.05	0.10	0.21	0.29	0.42	1.03	0.92	1.07
26-HEALTH	1.60	1.47	2.08	2.11	3.46	2.22	3.17	2.85
27-ENTERTAINS	1.37	1.89	2.52	3.41	3.22	3.06	3.83	6.70
28-TOBACCO	1.22	2.21	2.09	2.07	2.04	1.96	1.17	0.99

TABLE A36

WEIGHTS BY CLASS OF EXPENDITURE ("NEW" INDICES) - NORTH

	≤18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-RICE	4.10	3.03	2.80	2.35	2.19	1.76	1.47	0.93
2-FLOUR	0.83	1.24	1.37	1.33	0.86	0.70	0.37	0.32
3-PASTA	0.45	0.46	0.44	0.38	0.36	0.30	0.28	0.15
4-BREAD	8.13	6.03	5.22	4.76	4.31	3.67	2.87	1.64
5-STARCHES	6.93	6.03	5.42	4.68	3.70	2.88	2.38	1.46
6-PULSES	1.17	1.27	1.35	1.12	0.89	0.67	0.51	0.27
7-CHICKPEA	0.01	0.05	0.05	0.06	0.04	0.04	0.03	0.02
8-VEGETABLES	3.70	3.57	3.00	2.61	2.20	1.95	1.64	1.30
9-FRUIT	1.80	1.54	1.97	2.29	2.64	2.60	2.54	1.99
10-GOAT	0.00	0.08	0.05	0.10	0.09	0.18	0.27	0.22
11-LAMB	0.10	0.17	0.23	0.16	0.33	0.15	0.17	0.21
12-PORK	0.90	1.30	1.83	1.92	2.04	2.18	2.08	1.85
13-BEEF	2.59	3.68	3.72	4.66	5.63	5.86	6.13	5.82
14-OFFAL	0.22	0.17	0.34	0.45	0.47	0.56	0.54	0.48
15-SAUSAGES	1.17	2.01	2.50	2.32	1.79	1.75	1.37	1.13
16-POULTRY	1.64	2.66	2.78	3.32	3.13	2.93	2.54	1.59
17-FRESH FISH	1.91	1.84	1.99	1.96	2.04	2.11	2.14	2.29
18-FROZEN FISH	0.62	0.52	0.65	0.64	0.59	0.56	0.51	0.25
19-SHELLFISH	0.02	0.08	0.18	0.15	0.16	0.16	0.24	0.32
20-CANNED FISH	0.06	0.07	0.09	0.15	0.13	0.13	0.11	0.10
21-DRIED FISH	3.48	4.54	4.58	4.79	4.48	3.84	3.06	2.35
22-EGGS	0.59	0.76	0.86	0.92	0.91	0.83	0.89	0.69
23-MILK	1.32	1.19	1.35	1.43	1.48	1.44	1.35	1.14
24-DRIED MILK	0.09	0.06	0.18	0.17	0.23	0.18	0.09	0.06
25-MILK PRODUCTS	0.25	0.34	0.32	0.43	0.54	0.51	0.61	0.67
26-OILS	10.24	9.32	8.24	7.52	6.40	5.38	4.30	2.89
27-BUTTER	0.03	0.04	0.05	0.07	0.08	0.08	0.14	0.21
28-MARGARINE	0.19	0.23	0.30	0.33	0.36	0.28	0.28	0.19
29-SUGAR	3.44	2.25	1.98	1.69	1.52	1.19	0.94	0.62
30-JELLY	0.05	0.05	0.07	0.08	0.10	0.13	0.09	0.10
31-CONFECTIONARY	0.35	0.37	0.39	0.39	0.50	0.52	0.62	0.56
32-COCOA	0.00	0.02	0.02	0.01	0.05	0.05	0.04	0.04
33-COFFEE	0.97	0.70	0.64	0.59	0.52	0.44	0.39	0.34
34-TEA	0.04	0.04	0.02	0.03	0.02	0.02	0.02	0.03
35-CONDIMENTS	0.25	0.24	0.23	0.21	0.21	0.20	0.16	0.11
36-PREPARED FOOD	0.05	0.14	0.07	0.05	0.09	0.11	0.12	0.32
37-MEALS OUT	10.28	8.06	4.55	3.84	3.44	5.13	5.99	7.63
38-TABLE WINE	8.64	10.86	11.74	10.60	9.13	7.08	5.09	3.00
39-OTHER WINES	0.16	0.13	0.16	0.10	0.16	0.25	0.31	0.30
40-BEER	0.05	0.19	0.36	0.47	0.57	0.56	0.48	0.21
41-MINERAL WATER	0.03	0.12	0.08	0.12	0.11	0.11	0.16	0.17
42-OTHER DRINKS	0.17	0.18	0.23	0.23	0.29	0.28	0.24	0.17

TABLE A36 (CONT.)

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
43-CLOTHING(MEN)	0.98	1.44	1.81	2.28	2.98	3.10	3.34	3.22
44-TAILOR(MEN)	0.03	0.05	0.08	0.09	0.16	0.17	0.10	0.10
45-U/WEAR(MEN)	0.79	1.07	1.31	1.58	1.63	1.76	1.84	1.41
46-CLOTH.(WOMEN)	2.89	1.68	2.02	1.90	2.01	2.55	3.06	3.18
47-TAILOR(WOMEN)	0.14	0.16	0.14	0.14	0.17	0.14	0.10	0.05
48-U/WEAR(WOMEN)	0.92	0.91	0.92	0.97	1.14	1.28	1.27	0.93
49-CLOTH.(CHILD)	0.23	0.65	1.21	1.39	1.54	1.37	1.29	0.97
50-LAUNDRY	0.14	0.08	0.09	0.10	0.12	0.07	0.13	0.09
51-FOOTW.(MEN)	0.39	0.50	0.51	0.60	0.68	0.64	0.68	0.52
52-FOOTW.(WOMEN)	0.78	0.61	0.63	0.70	0.67	0.68	0.77	0.68
53-FOOTW.(CHILD)	0.06	0.20	0.35	0.37	0.44	0.36	0.33	0.22
54-FOOTW.REPAIR	0.14	0.10	0.12	0.13	0.15	0.19	0.19	0.15
55-WATER	0.25	0.17	0.20	0.21	0.23	0.27	0.28	0.34
56-GAS	2.72	2.16	1.86	1.78	1.43	1.13	1.01	0.63
57-ELECTRICITY	0.78	0.93	1.02	1.14	1.23	1.49	1.66	2.19
58-ELECTR.APPL.	0.06	0.27	0.51	0.61	0.95	1.25	1.81	2.00
59-FURNITURE	0.15	0.59	1.14	1.66	2.52	2.54	3.10	5.78
60-HOUS.TEXTILES	0.43	0.52	0.64	0.73	0.71	0.87	1.03	0.99
61-OTHERS	0.04	0.12	0.25	0.40	0.42	0.42	0.55	0.23
62-CUTLERY,ETC.	4.39	4.03	2.92	2.10	1.72	1.37	1.14	0.98
63-REPAIR,MAINT.	0.01	0.00	0.01	0.01	0.03	0.03	0.11	0.11
64-HYG.PRODS.	0.99	0.81	1.16	1.14	1.32	1.46	1.72	1.84
65-SERVICES(HYG)	0.00	0.14	0.07	0.17	0.20	0.53	0.50	1.45
66-SURGIC.GOODS	0.02	0.04	0.03	0.04	0.03	0.08	0.02	0.04
67-MEDICINES	1.47	1.65	1.51	1.54	1.69	1.95	1.65	2.19
68-DOCTORS	0.14	0.11	0.36	0.48	0.46	0.57	0.65	0.70
69-TOBACCO	1.14	1.50	1.45	1.45	1.56	1.40	1.20	0.91
70-PRIV.TRANSP.	0.00	0.03	0.38	0.85	2.24	4.33	8.24	14.21
71-URB.TRANSP.	0.22	0.32	0.51	0.62	0.84	1.05	0.86	0.51
72-SUBURB.TRANSP	0.00	0.00	0.04	0.10	0.14	0.10	0.11	0.05
73-LONG DIST.TRA	0.53	0.55	0.68	0.86	0.81	0.89	0.56	0.45
74-TAXIS	0.21	0.10	0.11	0.11	0.15	0.13	0.11	0.09
75- POST-OFFICE	0.14	0.10	0.07	0.05	0.06	0.04	0.03	0.04
76-TELEPHONE	0.03	0.14	0.22	0.26	0.37	0.69	0.87	1.21
77-EDUCATION	0.05	0.15	0.33	0.67	1.03	1.17	1.53	2.44
78-ENTERTAINS	1.70	2.51	2.95	3.29	3.38	4.17	4.60	4.95

TABLE A37

WEIGHTS BY EXPENDITURE CLASS ("NEW" INDICES) - CENTRE

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-RICE	2.91	2.20	2.03	1.58	1.30	1.05	0.81	0.63
2-FLOUR	1.13	1.57	1.36	1.27	1.07	0.70	0.60	0.32
3-PASTA	0.63	0.51	0.42	0.40	0.32	0.23	0.24	0.12
4-BREAD	8.27	6.19	5.14	4.34	4.00	3.20	2.44	1.68
5-STARCHES	8.33	7.03	5.71	4.69	4.07	2.96	2.25	1.36
6-PULSES	2.47	2.03	1.64	1.50	1.33	1.00	0.57	0.28
7-CHICKPEA	0.06	0.12	0.08	0.17	0.11	0.08	0.07	0.05
8-VEGETABLES	5.20	4.19	3.52	2.92	2.71	1.99	1.65	1.38
9-FRUIT	1.83	1.98	2.26	2.60	2.51	2.48	2.20	2.34
10-GOAT	0.14	0.11	0.18	0.52	0.37	0.92	0.49	0.63
11-LAMB	0.02	0.48	0.40	0.64	0.58	0.44	0.40	0.35
12-PORK	0.77	1.02	1.78	1.62	2.17	2.13	1.50	1.69
13-BEEF	0.66	1.20	1.98	2.96	3.43	3.85	4.31	4.22
14-OFFAL	0.07	0.22	0.26	0.40	0.44	0.53	0.37	0.58
15-SAUSAGES	2.32	3.24	3.59	3.48	2.87	2.25	1.69	1.20
16-POULTRY	1.62	2.91	2.96	3.58	3.34	2.95	2.62	2.05
17-FRESH FISH	2.08	2.06	2.26	2.21	2.12	1.76	1.99	1.93
18-FROZEN FISH	0.29	0.54	0.50	0.52	0.53	0.57	0.42	0.29
19-SHELLFISH	0.04	0.01	0.03	0.08	0.11	0.13	0.20	0.24
20-CANNED FISH	0.11	0.13	0.17	0.17	0.20	0.25	0.18	0.10
21-DRIED FISH	3.64	3.57	3.88	3.55	3.54	2.95	2.90	1.89
22-EGGS	0.72	0.95	0.94	0.94	0.82	0.83	0.67	0.59
23-MILK	0.86	1.09	1.38	1.35	1.39	1.35	1.21	1.12
24-DRIED MILK	0.18	0.20	0.26	0.30	0.25	0.12	0.13	0.20
25-MILK PRODUCTS	1.02	1.15	1.28	1.19	1.11	1.21	1.04	1.02
26-OILS	11.21	9.65	8.39	7.82	6.44	5.31	4.02	2.71
27-BUTTER	0.05	0.08	0.11	0.16	0.16	0.17	0.17	0.24
28-MARGARINE	0.42	0.56	0.57	0.59	0.63	0.52	0.42	0.32
29-SUGAR	3.68	2.69	2.04	1.71	1.40	1.08	0.88	0.64
30-JELLY	0.14	0.10	0.12	0.13	0.15	0.12	0.14	0.13
31-CONFECTIONARY	0.19	0.32	0.40	0.43	0.60	0.68	0.72	0.72
32-COCOA	0.10	0.11	0.15	0.09	0.15	0.14	0.13	0.11
33-COFFEE	0.86	0.73	0.57	0.52	0.49	0.41	0.38	0.31
34-TEA	0.06	0.05	0.03	0.03	0.02	0.03	0.03	0.02
35-CONDIMENTS	0.37	0.32	0.32	0.26	0.25	0.22	0.16	0.12
36-PREPARED FOOD	0.14	0.06	0.09	0.10	0.10	0.21	0.27	0.40
37-MEALS OUT	7.28	4.33	2.53	2.72	2.84	4.25	5.20	5.03
38-TABLE WINE	6.11	8.44	9.07	8.30	7.11	5.22	3.96	2.04
39-OTHER WINES	0.07	0.18	0.12	0.20	0.21	0.43	0.48	0.49
40-BEER	0.10	0.23	0.31	0.55	0.50	0.39	0.40	0.26
41-MINERAL WATER	0.03	0.05	0.04	0.05	0.07	0.07	0.13	0.12
42-OTHER DRINKS	0.13	0.20	0.27	0.27	0.34	0.26	0.33	0.29

TABLE A37 (CONT.)

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
43-CLOTHING(MEN)	1.17	1.96	2.50	2.67	2.97	3.24	3.15	3.29
44-TAILOR(MEN)	0.05	0.07	0.13	0.15	0.18	0.20	0.27	0.27
45-U/WEAR(MEN)	1.13	1.81	1.87	1.98	2.01	2.20	2.13	2.05
46-CLOTH.(WOMEN)	2.06	1.76	2.11	1.97	2.43	3.14	3.19	3.21
47-TAILOR(WOMEN)	0.15	0.12	0.15	0.17	0.15	0.18	0.19	0.24
48-U/WEAR(WOMEN)	1.17	0.99	1.21	1.21	1.45	1.72	1.59	1.44
49-CLOTH.(CHILD)	0.16	0.49	0.95	1.20	1.47	1.53	1.38	1.04
50-LAUNDRY	0.18	0.21	0.16	0.20	0.16	0.15	0.12	0.19
51-FOOTW.(MEN)	0.47	0.66	0.72	0.79	0.76	0.77	0.73	0.61
52-FOOTW.(WOMEN)	0.82	0.65	0.72	0.65	0.73	0.86	0.74	0.71
53-FOOTW.(CHILD)	0.07	0.19	0.26	0.33	0.37	0.37	0.29	0.22
54-FOOTW.REPAIR	0.20	0.22	0.21	0.23	0.22	0.19	0.18	0.18
55-WATER	0.22	0.18	0.18	0.21	0.20	0.29	0.28	0.30
56-GAS	2.65	2.08	1.96	1.75	1.59	1.35	1.26	1.00
57-ELECTRICITY	0.67	0.63	0.79	0.85	0.92	1.01	1.02	1.31
58-ELECTR.APPL.	0.11	0.24	0.53	0.67	0.87	1.29	1.59	1.86
59-FURNITURE	0.17	0.51	0.86	1.14	1.85	2.68	3.39	4.93
60-HOUS.TEXTILES	0.58	0.79	0.93	1.02	1.27	1.61	1.82	1.50
61-OTHERS	0.24	0.34	0.49	0.46	0.70	0.78	0.83	0.72
62-CUTLERY,ETC.	5.84	3.99	3.15	3.24	2.07	1.85	1.90	1.08
63-REPAIR,MAINT.	0.00	0.01	0.01	0.01	0.02	0.02	0.05	0.06
64-HYG.PRODS.	0.82	0.99	1.14	1.31	1.35	1.60	1.62	1.90
65-SERVICES(HYG)	0.14	0.37	0.37	0.08	0.55	0.54	0.88	1.25
66-SURGIC.GOODS	0.06	0.06	0.04	0.04	0.06	0.02	0.04	0.02
67-MEDICINES	0.86	1.65	1.85	1.44	2.03	1.99	1.75	1.68
68-DOCTORS	0.10	0.47	0.31	0.47	0.46	0.79	0.84	0.78
69-TOBACCO	0.86	0.94	0.84	1.01	0.98	0.84	0.75	0.67
70-PRIV.TRANSP.	0.02	0.21	0.72	1.82	2.95	5.25	10.87	18.58
71-URB.TRANSP.	0.02	0.04	0.10	0.09	0.18	0.25	0.31	0.14
72-SUBURB.TRANSP	0.01	0.00	0.05	0.04	0.08	0.11	0.06	0.09
73-LONG DIST.TRA	0.48	0.79	0.78	0.72	1.02	0.78	0.73	0.59
74-TAXIS	0.03	0.11	0.18	0.13	0.18	0.27	0.07	0.13
75- POST-OFFICE	0.06	0.11	0.11	0.12	0.10	0.12	0.07	0.06
76-TELEPHONE	0.07	0.12	0.11	0.31	0.25	0.45	0.49	1.09
77-EDUCATION	0.06	0.15	0.42	0.92	1.13	1.67	2.23	2.59
78-ENTERTAINS	2.01	3.30	3.98	3.74	4.10	4.45	4.45	4.02

TABLE A38

WEIGHTS BY CLASS OF EXPENDITURE ("NEW" INDICES) - LISBON

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-RICE	2.17	1.43	1.18	1.01	0.87	0.68	0.53	0.34
2-FLOUR	0.63	0.83	0.48	0.41	0.32	0.24	0.18	0.21
3-PASTA	1.41	1.03	0.80	0.63	0.48	0.36	0.25	0.14
4-BREAD	10.30	6.75	5.76	4.90	3.85	2.86	2.07	1.35
5-STARCHES	4.50	3.17	2.84	2.61	2.18	1.82	1.43	0.94
6-PULSES	1.44	1.15	0.96	0.86	0.63	0.48	0.33	0.21
7-CHICKPEA	0.19	0.11	0.18	0.13	0.12	0.10	0.08	0.04
8-VEGETABLES	3.90	3.85	3.42	3.18	2.94	2.63	2.23	1.61
9-FRUIT	3.58	3.66	3.67	3.93	3.76	3.63	3.22	2.46
10-GOAT	0.00	0.04	0.05	0.08	0.18	0.16	0.22	0.18
11-LAMB	0.95	0.79	0.99	0.88	1.03	0.94	0.82	0.62
12-PORK	1.18	1.53	1.80	1.75	1.84	1.55	1.29	1.29
13-BEEF	2.83	4.16	5.32	6.85	7.12	7.43	6.69	5.67
14-OFFAL	0.28	0.56	0.73	0.74	0.84	0.68	0.54	0.46
15-SAUSAGES	1.88	2.26	1.90	1.66	1.50	1.24	1.08	0.88
16-POULTRY	2.59	3.15	3.75	3.74	3.43	2.80	2.09	1.43
17-FRESH FISH	4.92	4.59	5.06	5.53	4.98	4.82	4.05	3.40
18-FROZEN FISH	0.64	0.72	0.67	0.53	0.44	0.42	0.30	0.17
19-SHELLFISH	0.01	0.13	0.27	0.32	0.32	0.42	0.42	0.59
20-CANNED FISH	0.20	0.32	0.29	0.24	0.24	0.21	0.21	0.16
21-DRIED FISH	3.31	2.54	2.48	2.40	2.32	2.22	1.86	1.37
22-EGGS	1.16	1.13	1.06	0.92	1.05	0.94	0.82	0.74
23-MILK	2.24	2.45	2.59	2.64	2.53	2.23	1.95	1.48
24-DRIED MILK	0.36	0.44	0.33	0.26	0.15	0.17	0.12	0.13
25-MILK PRODUCTS	1.18	0.94	1.02	1.08	1.11	1.16	1.05	0.98
26-OILS	8.01	8.79	6.72	5.79	5.12	4.08	3.13	1.98
27-BUTTER	0.22	0.35	0.35	0.32	0.35	0.35	0.35	0.30
28-MARGARINE	1.16	1.01	0.92	0.86	0.70	0.63	0.49	0.36
29-SUGAR	2.79	2.05	1.49	1.22	0.97	0.77	0.58	0.38
30-JELLY	0.12	0.09	0.13	0.15	0.11	0.13	0.13	0.12
31-CONFECTIONARY	0.48	0.50	0.58	0.63	0.65	0.79	0.82	0.87
32-COCOA	0.10	0.30	0.17	0.12	0.13	0.16	0.13	0.09
33-COFFEE	1.20	0.77	0.72	0.56	0.53	0.51	0.42	0.38
34-TEA	0.06	0.07	0.04	0.03	0.03	0.03	0.02	0.03
35-CONDIMENTS	0.25	0.39	0.31	0.26	0.21	0.17	0.13	0.09
36-PREPARED FOOD	0.23	0.26	0.28	0.37	0.36	0.50	0.43	0.44
37-MEALS OUT	6.20	6.32	4.29	4.41	5.14	6.98	8.88	9.06
38-TABLE WINE	2.61	4.50	5.12	4.57	3.95	3.05	2.17	1.30
39-OTHER WINES	0.05	0.07	0.13	0.20	0.28	0.27	0.30	0.31
40-BEER	0.21	0.33	0.63	0.52	0.53	0.46	0.34	0.25
41-MINERAL WATER	0.09	0.18	0.14	0.13	0.14	0.12	0.14	0.16
42-OTHER DRINKS	0.25	0.30	0.33	0.36	0.41	0.33	0.26	0.20

TABLE A38 (CONT.)

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
43-CLOTHING(MEN)	0.99	1.53	1.85	2.36	2.41	2.67	2.64	2.58
44-TAILOR(MEN)	0.12	0.07	0.06	0.04	0.12	0.08	0.07	0.05
45-U/WEAR(MEN)	0.99	1.01	1.58	1.59	1.69	1.61	1.36	1.21
46-CLOTH.(WOMEN)	1.40	1.66	1.67	1.90	1.87	2.12	2.28	2.07
47-TAILOR(WOMEN)	0.14	0.17	0.07	0.07	0.09	0.12	0.11	0.10
48-U/WEAR(WOMEN)	0.80	0.96	0.78	0.89	0.80	0.78	0.68	0.63
49-CLOTH.(CHILD)	0.08	0.26	0.57	0.77	0.80	0.83	0.73	0.54
50-LAUNDRY	0.31	0.33	0.22	0.23	0.18	0.18	0.19	0.16
51-FOOTW.(MEN)	0.50	0.50	0.64	0.64	0.65	0.64	0.57	0.46
52-FOOTW.(WOMEN)	0.67	0.68	0.68	0.71	0.70	0.64	0.63	0.45
53-FOOTW.(CHILD)	0.05	0.11	0.21	0.25	0.27	0.26	0.22	0.14
54-FOOTW.REPAIR	0.23	0.20	0.21	0.20	0.18	0.18	0.15	0.14
55-WATER	0.76	0.69	0.65	0.57	0.60	0.59	0.56	0.60
56-GAS	4.20	3.54	2.86	2.36	2.06	1.75	1.67	1.34
57-ELECTRICITY	1.52	1.52	1.28	1.31	1.30	1.31	1.23	1.31
58-ELECTR.APPL.	0.13	0.20	0.46	0.59	1.14	1.51	1.92	1.58
59-FURNITURE	0.08	0.20	1.07	1.11	1.91	2.67	3.48	3.87
60-HOUS.TEXTILES	0.38	0.45	0.43	0.44	0.56	0.68	0.89	1.22
61-OTHERS	0.23	0.26	0.22	0.35	0.43	0.49	0.52	0.44
62-CUTLERY,ETC.	2.16	2.00	1.80	1.43	1.44	1.32	1.29	0.96
63-REPAIR,MAINT.	0.01	0.03	0.02	0.02	0.03	0.06	0.07	0.13
64-HYG.PRODS.	1.17	1.24	1.33	1.36	1.45	1.59	1.72	1.94
65-SERVICES(HYG)	0.08	0.17	0.12	0.16	0.29	0.49	0.54	1.51
66-SURGIC.GOODS	0.20	0.12	0.07	0.06	0.06	0.07	0.04	0.09
67-MEDICINES	1.69	2.06	1.98	2.15	2.12	2.22	2.10	1.93
68-DOCTORS	0.01	0.17	0.47	0.43	0.47	0.43	0.70	1.25
69-TOBACCO	0.82	1.11	1.43	1.62	1.40	1.24	1.19	1.02
70-PRIV.TRANSP.	0.24	0.25	0.78	1.44	2.91	4.63	9.35	16.44
71-URB.TRANSP.	0.39	0.64	0.81	0.83	1.17	1.11	1.12	0.50
72-SUBURB.TRANSP	0.01	0.08	0.19	0.28	0.22	0.30	0.37	0.16
73-LONG DIST.TRA	0.95	0.59	1.11	1.32	1.18	1.15	0.85	0.38
74-TAXIS	0.18	0.15	0.21	0.13	0.25	0.34	0.22	0.29
75- POST-OFFICE	0.12	0.13	0.09	0.04	0.06	0.06	0.06	0.06
76-TELEPHONE	0.41	0.48	0.72	0.81	0.95	1.25	1.37	1.65
77-EDUCATION	0.01	0.19	0.31	0.58	0.87	1.29	2.17	3.96
78-ENTERTAINS	2.07	2.29	3.14	3.16	3.52	3.82	4.34	5.73

TABLE A39

wWEIGHTS BY CLASS OF EXPENDITURE ("NEW" INDICES) - SOUTH

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
1-RICE	1.93	1.73	1.23	1.15	0.88	0.79	0.56	0.40
2-FLOUR	1.38	1.19	0.81	0.58	0.56	0.25	0.29	0.34
3-PASTA	1.43	1.05	0.79	0.65	0.54	0.52	0.25	0.20
4-BREAD	10.27	7.99	5.74	4.98	3.81	3.29	2.18	1.47
5-STARCHES	4.30	3.42	2.73	2.39	2.08	1.94	1.27	1.10
6-PULSES	1.92	1.66	1.52	0.92	0.87	0.69	0.43	0.33
7-CHICKPEA	0.47	0.55	0.49	0.43	0.32	0.36	0.14	0.09
8-VEGETABLES	3.73	4.20	3.35	3.28	2.96	2.49	1.80	1.35
9-FRUIT	3.02	3.49	4.41	4.09	3.88	3.51	3.33	2.78
10-GOAT	0.04	0.23	0.24	0.18	0.13	0.34	0.26	0.22
11-LAMB	2.33	2.49	3.10	2.72	3.37	2.33	2.16	1.46
12-PORK	1.16	1.64	1.63	1.87	1.71	1.77	1.07	1.33
13-BEEF	0.56	0.82	1.23	2.38	3.03	4.36	3.80	4.12
14-OFFAL	0.17	0.24	0.34	0.52	0.57	0.42	0.54	0.45
15-SAUSAGES	4.75	4.72	4.83	4.18	3.32	2.82	1.80	1.70
16-POULTRY	3.22	3.05	4.03	3.44	4.00	3.08	2.89	2.71
17-FRESH FISH	3.92	4.16	3.75	4.74	3.97	4.49	3.66	3.06
18-FROZEN FISH	0.55	0.48	0.36	0.46	0.44	0.39	0.39	0.26
19-SHELLFISH	0.06	0.07	0.16	0.24	0.29	0.51	0.43	0.44
20-CANNED FISH	0.19	0.23	0.27	0.31	0.23	0.25	0.16	0.18
21-DRIED FISH	1.10	1.15	1.64	1.07	1.37	1.03	0.93	0.72
22-EGGS	0.77	0.96	1.12	1.19	1.20	1.03	0.87	0.90
23-MILK	2.72	2.65	2.22	2.19	2.42	2.23	1.91	1.35
24-DRIED MILK	0.26	0.27	0.22	0.13	0.14	0.06	0.06	0.05
25-MILK PRODUCTS	1.63	1.56	1.73	1.50	1.36	0.96	1.05	0.89
26-OILS	11.14	10.07	8.82	7.25	6.29	4.82	3.69	2.48
27-BUTTER	0.05	0.13	0.18	0.30	0.27	0.26	0.33	0.28
28-MARGARINE	1.65	1.39	1.30	1.18	0.96	0.89	0.64	0.55
29-SUGAR	3.27	2.51	1.77	1.46	1.16	0.91	0.68	0.61
30-JELLY	0.11	0.14	0.09	0.12	0.15	0.11	0.07	0.08
31-CONFECTIONARY	0.48	0.46	0.61	0.75	0.75	0.96	0.55	0.87
32-COCOA	0.28	0.29	0.25	0.28	0.24	0.35	0.33	0.16
33-COFFEE	0.87	0.95	0.78	0.69	0.71	0.49	0.39	0.30
34-TEA	0.03	0.04	0.02	0.01	0.02	0.01	0.03	0.06
35-CONDIMENTS	0.47	0.42	0.40	0.36	0.28	0.27	0.14	0.07
36-PREPARED FOOD	0.26	0.14	0.17	0.42	0.22	0.11	0.20	0.27
37-MEALS OUT	3.87	3.15	2.61	1.70	2.70	4.43	4.78	4.70
38-TABLE WINE	2.05	3.73	4.05	4.09	3.52	2.77	1.88	0.75
39-OTHER WINES	0.20	0.20	0.28	0.18	0.34	0.49	0.19	0.60
40-BEER	0.27	0.44	0.72	0.72	0.66	0.69	0.50	0.31
41-MINERAL WATER	0.10	0.06	0.07	0.09	0.12	0.10	0.11	0.13
42-OTHER DRINKS	0.11	0.20	0.20	0.30	0.29	0.15	0.10	0.10

TABLE A39 (CONT.)

	<18	18-30	30-48	48-60	60-90	90-120	120-180	≥180
43-CLOTHING(MEN)	1.33	2.20	2.20	2.77	2.60	2.39	2.90	1.86
44-TAILOR(MEN)	0.24	0.22	0.31	0.31	0.34	0.30	0.28	0.26
45-U/WEAR(MEN)	1.51	2.00	2.15	2.40	2.21	2.03	2.05	1.16
46-CLOTH.(WOMEN)	1.37	1.48	1.52	1.85	1.86	2.18	1.86	2.28
47-TAILOR(WOMEN)	0.22	0.18	0.24	0.24	0.25	0.25	0.48	0.09
48-U/WEAR(WOMEN)	0.79	0.70	0.91	1.00	1.07	0.92	1.23	0.67
49-CLOTH.(CHILD)	0.14	0.33	0.64	0.95	1.05	0.99	0.65	0.67
50-LAUNDRY	0.26	0.33	0.18	0.13	0.16	0.14	0.08	0.10
51-FOOTW.(MEN)	0.71	0.76	0.76	0.84	0.72	0.76	0.68	0.43
52-FOOTW.(WOMEN)	0.88	0.68	0.74	0.77	0.69	0.79	0.65	0.51
53-FOOTW.(CHILD)	0.02	0.21	0.21	0.29	0.33	0.34	0.17	0.13
54-FOOTW.REPAIR	0.22	0.25	0.20	0.24	0.28	0.15	0.17	0.09
55-WATER	0.56	0.45	0.45	0.40	0.44	0.43	0.54	0.48
56-GAS	3.90	3.40	2.73	2.38	1.99	1.61	1.46	1.01
57-ELECTRICITY	0.54	0.70	0.85	1.01	1.03	1.19	1.57	1.25
58-ELECTR.APPL.	0.13	0.22	0.47	0.69	0.97	1.51	1.64	2.55
59-FURNITURE	0.19	0.45	1.00	1.65	2.06	3.03	2.74	3.05
60-HOUS.TEXTILES	0.36	0.39	0.68	0.94	0.80	0.95	1.06	1.15
61-OTHERS	0.23	0.20	0.43	0.61	0.61	0.58	0.69	0.38
62-CUTLERY,ETC.	2.64	2.18	2.00	1.79	1.74	1.51	1.07	0.87
63-REPAIR,MAINT.	0.00	0.01	0.05	0.03	0.09	0.00	0.00	0.35
64-HYG.PRODS.	1.30	1.42	1.31	1.35	1.45	1.49	1.55	1.51
65-SERVICES(HYG)	0.24	0.28	0.15	0.38	0.57	0.16	0.13	0.08
66-SURGIC.GOODS	0.08	0.02	0.04	0.18	0.06	0.10	0.03	0.01
67-MEDICINES	1.43	1.26	1.85	1.29	2.28	1.58	2.94	2.79
68-DOCTORS	0.02	0.05	0.17	0.50	0.94	0.65	0.48	0.77
69-TOBACCO	1.33	2.39	2.25	2.24	2.23	2.15	1.35	1.19
70-PRIV.TRANSP.	0.17	0.25	1.28	2.09	3.26	8.02	17.24	22.86
71-URB.TRANSP.	0.01	0.02	0.06	0.04	0.09	0.04	0.05	0.02
72-SUBURB.TRANSP	0.00	0.00	0.06	0.07	0.16	0.05	0.03	0.02
73-LONG DIST.TRA	0.34	0.39	0.60	0.67	1.04	0.88	1.37	0.45
74-TAXIS	0.09	0.02	0.07	0.04	0.06	0.05	0.09	0.18
75- POST-OFFICE	0.05	0.07	0.10	0.07	0.09	0.40	0.24	0.01
76-TELEPHONE	0.00	0.05	0.16	0.30	0.39	0.84	0.89	1.36
77-EDUCATION	0.00	0.22	0.46	0.63	0.62	0.81	1.80	2.30
78-ENTERTAINS	1.60	1.89	2.42	3.35	3.33	2.95	2.99	6.14

TABLE A40

Correspondence between the items of the LES

and those of the old and new indices

LES	Old	New
1 Cereals	1	1+...+4
2 Vegetables, fruit	2+3+4	5+...+9
3 Meat and poultry	9+10	10+...+16
4 Fish	11	17+...+21
5 Milk, cheese, eggs	12	22+...+25
6 Fats	6	26+27+28
7 Coffee, tea, etc.	8	32+33+34
8 Sugar, confectionary	5	29+30+31
9 Other food	7	35
10 Alcoholic drinks	14	38+39+40
11 Soft drinks	14	41+42
12 Tobacco	28	69
13 Clothing, footwear	15+16	43+...+54
14 Rents and water	17+19	55 (water)
15 Light and fuel	18	56+57
16 Other housing expend.	21+23	63
17 Health and hygiene	20+26	64+...+68
18 Transp. and commun.	24+25	70+...+76
19 Entertainment	27	78
20 Education	27	77
21 Other services	-	-
22 Vehicles	-	-
23 Durables	22	58+...+62

TABLE A41

Data for Engel curves - NORTH

(y - declared income group, e - expenditure on food + drink + clothing + footwear, E - total expenditure)

	SINGLE			COUPLE			CI			C(II)			CS			CS(I)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	11.63	16.28	249	17.09	22.96	169	31.03	35.08	2	20.18	22.55	2	16.71	19.99	4	21.17	25.02	2
18-30	17.47	26.28	94	25.08	34.51	248	30.18	39.68	6	27.70	48.15	6	38.20	46.90	13	33.32	46.32	12
30-48	21.35	38.60	46	30.27	44.30	193	35.16	53.35	32	36.70	51.54	21	38.07	48.97	27	37.20	50.52	35
48-60	27.66	52.80	18	37.22	57.53	94	38.53	57.85	21	42.76	60.60	22	40.04	63.86	22	41.73	61.28	24
60-90	29.46	67.28	10	41.51	70.45	82	46.50	82.04	32	43.14	69.74	24	43.84	74.00	38	44.65	74.12	27
90-120	51.50	99.64	3	51.78	108.16	41	56.27	109.73	11	55.61	76.53	1	54.13	102.71	8	60.20	102.46	6
120-180	37.01	74.69	2	54.33	114.70	34	58.48	156.57	10	86.33	159.25	4	66.14	145.00	10	82.50	207.28	3
≥ 180	28.59	81.11	3	71.14	179.60	16	80.00	217.13	8	89.87	256.75	4	70.96	233.52	3	72.42	187.97	1

	C(SS)			C(SS)(I)			CA			CA(SI)			C(AA)($\overline{SI}$)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	27.19	32.33	2	17.66	23.98	4	22.03	26.15	5	-	-	-	-	-	-
18-30	33.03	42.88	20	40.94	51.50	11	32.16	40.43	22	42.96	50.42	4	39.63	51.00	5
30-48	37.81	51.36	48	40.08	54.61	46	40.62	55.62	15	41.64	51.98	5	37.31	49.53	10
48-60	39.40	57.30	33	43.63	61.70	37	42.48	61.84	26	54.86	68.00	9	44.25	60.88	12
60-90	47.42	73.59	40	48.04	78.07	44	48.77	76.15	19	51.08	81.34	8	54.35	76.24	17
90-120	52.78	85.03	19	75.16	102.37	4	55.17	88.13	6	52.09	77.83	3	71.91	111.73	10
120-180	61.59	134.77	12	81.33	157.59	6	60.78	160.48	9	65.30	117.45	1	82.77	136.69	4
≥ 180	87.74	228.85	10	80.72	193.19	4	77.83	185.07	3	-	-	-	105.26	279.47	2

TABLE A42

Data for Engel curves - CENTRE

(y - declared income group, e - expenditure on food + drink + clothing + footwear, E - total expenditure)

	SINGLE			COUPLE			CI			C(II)			CS			CS(I)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	11.56	16.56	310	18.29	25.15	243	26.00	29.14	2	22.49	30.96	1	21.66	27.00	9	17.21	19.69	1
18-30	17.33	27.08	88	22.80	32.32	399	31.99	44.62	11	28.48	38.44	11	32.53	48.92	20	27.27	32.52	11
30-48	21.37	36.08	27	29.82	43.77	270	35.43	59.34	35	33.92	52.41	17	39.32	57.96	52	43.24	60.20	39
48-60	27.33	54.10	15	35.27	58.17	92	43.30	70.63	28	48.31	72.55	6	43.56	68.06	37	39.95	61.06	11
60-90	30.11	63.07	12	39.97	78.15	83	45.59	77.84	30	46.52	71.72	10	50.77	94.98	26	51.14	93.91	18
90-120	38.98	82.58	2	46.48	97.06	30	54.88	121.70	8	56.36	103.27	4	46.53	87.90	9	56.19	125.15	8
120-180	12.88	39.49	1	57.37	174.45	24	56.14	176.36	7	71.36	204.33	1	72.19	130.06	5	72.51	139.94	8
≥ 180	14.03	47.30	1	66.27	158.47	16	-	-	-	83.56	312.67	2	52.53	272.28	2	82.30	238.55	1

	C(SS)			C(SS)(I)			CA			CA(SI)			C(AA)($\overline{SI}$)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	40.38	45.70	1	-	-	-	45.67	51.85	2	-	-	-	-	-	-
18-30	29.60	36.25	28	31.60	39.48	12	27.44	34.26	34	47.64	61.35	8	33.80	44.17	5
30-48	39.94	56.92	39	39.12	56.42	42	38.46	58.17	42	40.55	65.63	10	45.31	59.88	13
48-60	47.51	69.16	36	44.71	64.21	28	39.04	62.37	24	46.67	62.92	8	43.73	69.38	14
60-90	45.70	77.98	23	52.43	79.63	9	53.23	88.75	17	49.15	68.50	5	53.54	87.17	12
90-120	59.35	97.70	8	59.40	106.50	4	48.85	78.08	2	86.26	144.60	3	86.69	155.65	3
120-180	63.89	119.76	6	92.74	185.89	4	52.03	88.85	5	-	-	-	125.62	305.52	2
≥ 180	84.84	211.15	3	-	-	-	113.91	255.37	3	-	-	-	165.44	373.10	1

TABLE A43

Data for Engel curves - LISBON
(y - declared income group, e - expenditure on food + drink + clothing + footwear, E - total expenditure)

	SINGLE			COUPLE			CI			C(II)			CS			CS(I)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	11.78	18.06	230	18.08	24.98	148	21.93	28.74	3	18.43	24.42	2	21.79	32.69	8	25.74	33.77	1
18-30	18.94	30.71	111	24.02	35.77	254	26.32	38.00	10	27.18	34.07	2	22.27	31.87	12	38.12	48.70	8
30-48	19.42	36.99	59	30.23	46.44	396	36.13	56.68	62	33.63	50.86	19	35.50	52.83	73	38.62	53.60	34
48-60	33.48	66.09	34	35.19	60.18	221	40.03	67.83	57	41.91	65.50	25	42.64	65.64	86	41.20	63.18	37
60-90	33.52	72.41	33	43.04	77.52	240	48.05	85.06	61	52.14	92.99	20	50.47	87.03	95	52.06	90.89	63
90-120	24.42	63.40	17	49.68	101.46	101	60.51	125.00	32	51.70	86.44	4	58.37	108.54	39	57.39	99.51	21
120-180	54.62	122.71	8	57.73	128.37	78	72.04	147.39	18	51.02	125.48	9	64.77	123.45	38	65.12	120.41	15
≥ 180	33.24	109.95	5	76.13	172.25	50	64.23	185.31	15	95.96	218.86	9	86.11	183.78	12	71.69	207.52	9

	C(SS)			C(SS)(I)			CA			CA(SI)			C(AA)($\overline{SI}$)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	45.18	48.25	1	-	-	-	26.62	37.80	2	-	-	-	-	-	-
18-30	29.72	39.23	13	31.98	41.73	4	38.71	52.44	13	-	-	-	49.38	65.58	2
30-48	37.44	55.25	47	40.67	56.60	25	34.28	49.00	49	39.15	53.46	10	53.29	68.16	3
48-60	15.81	71.56	48	47.87	65.43	17	44.03	64.18	50	44.20	74.53	3	37.72	56.35	11
60-90	51.63	88.54	48	54.60	84.28	16	47.34	84.74	56	50.82	86.07	5	68.60	102.96	14
90-120	64.44	113.45	16	59.30	119.06	3	64.79	124.71	19	75.41	124.61	5	83.13	128.02	8
120-180	67.33	137.33	28	74.32	165.67	6	62.21	137.50	11	-	-	-	53.88	124.96	5
≥ 180	72.08	223.28	16	81.38	239.06	7	93.42	224.21	6	-	-	-	111.35	266.20	4

TABLE A44

Data for Engel curves - SOUTH

(y - declared income group, e - expenditure on food + drink + clothing + footwear, E - total expenditure)

	SINGLE			COUPLE			CI			C(II)			CS			CS(I)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	10.51	14.06	119	16.18	21.48	159	-		-	18.36	22.56	1	23.03	27.88	4	15.21	21.93	1
18-30	14.60	22.46	45	21.71	29.03	144	25.85	33.28	16	27.59	39.25	2	26.36	32.82	16	30.96	43.96	4
30-48	22.10	38.00	19	25.98	39.06	132	33.28	50.04	18	34.89	59.10	6	29.54	40.84	33	34.37	45.41	20
48-60	25.88	41.86	5	32.04	47.20	62	39.22	60.78	7	39.18	60.54	11	37.41	64.73	19	31.66	43.99	4
60-90	21.16	47.14	4	37.93	62.01	33	42.74	92.73	9	47.86	84.17	4	44.02	71.86	15	47.25	81.63	15
90-120	13.93	45.82	1	47.14	113.24	13	49.88	75.35	4	-	-	-	45.85	92.15	7	49.33	90.19	2
120-180	-	-	-	54.81	114.05	6	46.88	133.20	3	-	-	-	53.45	90.54	1	62.05	118.12	2
≥ 180	-	-	-	47.82	96.31	2	-	-	-	-	-	-	-	-	-	-	-	-

	C(SS)			C(SS)(I)			CA			CA(SI)			C(AA)($\overline{SI}$)		
y	e	E	N	e	E	N	e	E	N	e	E	N	e	E	N
< 18	19.15	23.17	3	15.78	32.92	1	23.54	29.06	2	-	-	-	-	-	-
18-30	31.46	42.12	12	24.83	29.46	2	26.90	42.41	16	30.48	37.67	1	19.17	35.37	1
30-48	35.58	47.01	12	36.63	45.17	7	28.11	36.70	23	-	-	-	43.06	53.07	4
48-60	50.69	69.82	10	33.24	60.82	1	39.73	62.07	10	50.97	64.50	1	37.00	45.98	1
60-90	39.23	53.31	7	38.20	51.30	2	48.16	69.70	5	50.21	66.48	3	62.61	99.83	3
90-120	67.52	143.01	2	51.12	98.80	2	53.87	119.02	5	66.58	85.59	1	45.16	100.92	3
120-180	-	-	-	-	-	-	43.60	73.56	2	-	-	-	83.39	179.38	1
≥ 180	-	-	-	103.45	226.81	1	-	-	-	-	-	-	-	-	-

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