Reliability Assessment for Consecutive-k-out-of-n: F Retrial Systems under Poisson Shocks

Mingjia Li[a], Linmin Hu[b,*], Shaomin Wu[c], Bing Zhao[b], Yan Wang[b]

[a]*School of Economics and Management, Yanshan University, 438 West Hebei Street, Qinhuangdao, Hebei 066004, China*
[b]*School of Science, Yanshan University, 438 West Hebei Street, Qinhuangdao, Hebei 066004, China*
[c]*Kent Business School, University of Kent, Canterbury, Kent CT2 7FS, United Kingdom*

Abstract

This paper derives reliability indices of a linear consecutive-k-out-of-n: F system with retrial under Poisson shocks. The system fails if and only if at least k consecutive components from n components fail and is maintained by one repairman. When a component fails during the repairman's unavailability, it will be waiting until the repairmen becomes available. The failure of a component may be caused by its intrinsic characteristics such as ageing and deterioration or extrinsic factors such as shocks. It is assumed that a component will fail once the magnitude of a shock is greater than a threshold. At any time, a component is at one of the three states: working, waiting for repair, and under repair. For some systems, we need to obtain reliability indices for practical use. Hence, this paper uses the Markov chain to model the transition between states and obtains several reliability indices. The parameter sensitivity of the system reliability indices is analysed with numerical experiments.

Keywords: Consecutive-k-out-of-n: F system; Poisson shocks; Retrial; Availability; Cost-benefit

---

1. Introduction

A k-out-of-n system is composed of n components, and it fails only if the amount of failed components is k or more. It is a fairly commonly used redundant system and was analysed by many authors, see [1–3] for examples. The consecutive-k-out-of-n system as a particular k-out-of-n system, which fails only if there are k or more consecutive components fails [4–6]. In the real life, there are many examples that are consecutive-k-out-of-n: F systems. For example, if there are 3 or more failed street lights which are consecutive, the street light system needs repair. It is therefore of great significance to study the reliability of such a system to ensure system availability.

---

*Corresponding author

Email addresses: ming_jia_l@163.com (Mingjia Li), linminhu@ysu.edu.cn (Linmin Hu), s.m.wu@kent.ac.uk (Shaomin Wu), zhaobing@ysu.edu.cn (Bing Zhao), yanwang_w@163.com (Yan Wang)

Suggested citation: Mingjia Li, Linmin Hu, Shaomin Wu, Bing Zhao, Yan Wang, Reliability assessment for consecutive-k-out-of-n: F retrial systems under Poisson shocks, Applied Mathematics and Computation, Volume 448, 2023, 127913.

In the following, we use C(k, n: F) to represent this type of system, which can be abbreviated as C(k, n) systems. The C(k, n: F) system was first proposed by Kontoleon [7]. Zhang and Wang [8] studied the reliability of a linear C(2, n: F) system and obtained the transition probability for consecutive systems. Cheng and Zhang [9] derived the transition probabilities between C(k, n) system states under the assumption that key components are preferentially repaired. Yuan and Cui [10] proposed the C(k, n: F) system where multiple repairmen take multiple vacations and provided the analytical solution of several reliability indices. On the basis of Birnbaum importance, Si et al. [11] generalized many importance measures for binary, multistate, and consecutive systems. Wang et al. [12] analyzed the optimization problem of linear C(k, n: F) systems using a genetic algorithm. For other related work on the reliability of C(k, n) systems, refer to [13–15].

In reliability analysis, a central problem is dealing with situations of the availability of repairmen. If we assume that there is only one repairman available, then a failed component will be repaired directly if the repairman is available, or it will be waiting in the cache for the time when the repairman is available. In this paper, we refer to *the cache* as a retrial. Retrial is a term that was first introduced and then widely used in queuing theory. Retrial queues are characterized by the feature that a customer who finds the server busy upon his arrival is obliged to leave the service area and repeats his demand after some time called "retrial time" [16].

Some authors apply the concept of retrial to research on system reliability and maintenance. Krishnamoorthy and Ushakumari [17] established a reliability model of a retrial system in view of failed units having no waiting room (as all positions are occupied). Kuo et al. [18] analyzed the cost-benefit ratios of a k-out-of-n: G repairable system with retrial feature and mixed standby components. Wang et al. [19] introduced a cost model to determine the optimal values of some parameters in a retrial system. Chen [20] extended the work of Wang et al. [19] to a scenario where the server is assumed able to provide partial service after a breakdown. Subsequently, Yang and Tsao [21] derived certain reliability indices of a repairable k-out-of-n: G system with retrial feature and working vacations on the basic of the matrix-analytic method. Recently, Wang et al. [22] compared four unreliable systems with retrial feature and preventive maintenance, and performed cost-benefit analysis.

In practice, the failure of a system can be affected by intrinsic factors (ageing or deterioration of a component) as well as extrinsic factors (shocks). Barlow and Proschan [23] elaborated on the life distribution problem of a single-component system under Poisson shocks. Zhang et al. [24] studied the reliability and maintenance costs of a deteriorating system which may fail due to its degradation or random shocks. Janani [25] analyzed a queueing system with a server and two types of failure. Wu and Wu [26] studied a two-component system under Poisson shocks, where the repair time of the failed components and the repairman's vacation time are assumed to obey the general distributions. Segovia and Labeau [27] studied a multi-state system under shocks by using the matrix-analytic method. Zhao et al. [28] proposed a general multi-state balanced system under shocks, and obtained several reliability indices through finite Markov chain imbedding technique.

Some technical systems are repairable linear C(k, n: F) retrial systems and they are subject to shocks of various types in the real world. The system presented in this study can be used for modeling the telecommunication system, which is widely used in earthquakes, forestry, meteorology, transportation and other departments as an important emergency communication system. On the basis of the development trend of intelligent automation of products in the future, we can develop an intelligent automatic telecommunication system. The system composes of n relay stations, each of which can transmit signals to at least k consecutive relay stations. This system fails if and only if k or more of n relay stations in succession fail. Failure of a relay station can be caused by its ageing or external electromagnetic interference. The repair requests of failed relay stations can be sent to the repair station by the intelligent patrol robot with on-line monitoring function. The failed relay station will be performed maintenance directly if the state of the intelligent repair equipment is idle. Otherwise, the repair requests are saved in the information storage generator installed in the intelligent patrol robot and then look for the opportunity for repair after a period of time. This paper aims to serve this need.

The main contribution of this study is reflected in the following three aspects:

- The paper performs reliability assessment for C(k, n: F) systems with retrial under Poisson shocks.
- This paper derives several propositions and obtains the state transition rate matrix.
- The paper obtains three main reliability indices of the system: availability, reliability and mean time to first failure, the effect of several parameters on system reliability indices is analyzed, and cost/benefit analysis are performed.

As aforementioned, some linear C(k, n: F) retrial systems are subject to shocks of various types in the real world. Hence, reliability analysis of such systems provides engineers with methods to aid in their decision making. As such, this paper has managerial implications.

The rest of the paper is arranged as follows. Section 2 makes assumptions for a repairable linear C(k, n: F) retrial system under Poisson shocks, and obtains the transition rates between states of the system. Section 3 derives system reliability indices. Section 4 provides numerical experiments to illustrate the effect of various parameters on several system reliability indices, and analyzes the cost/benefit of the system. Finally, Section 5 wraps up the work of this text and suggests future research.

2. Model development

In this section, we first propose the system model to be studied. Then the transition rates between system states are given in the second subsection based on several assumptions in the first subsection.

The system model we consider here consists of one repairman and n components sorted in linear order, and it fails if and only if the amount of failed components which are consecutive is not less than k, namely linear C(k, n: F) repairable system. The failure of the system component may be caused by its intrinsic characteristics or extrinsic shocks. The failed component will enter a retrial space when the repairman is busy. The model is analyzed based on some assumptions as follows.

2.1. Model assumptions

A1 The failure of a component may be caused by an intrinsic factor or an extrinsic factor. Each component has two possible states, working or failed. The working time X of each component obeys the exponential distribution with rate λ_a. The arrival of the shocks as a homogeneous Poisson process with intensity λ_b, and the magnitude $\hat{Y}$ of each shock that the system suffers obeys a distribution function Φ. The repair time of a failed component in the system obeys the exponential distribution with rate μ.

A2 All components in the system are affected by shocks. The threshold of magnitude of a shock that causes a component to fail is a random variable υ with the cumulative distribution function H, and the component fails once the magnitude $\hat{Y}$ of a shock is greater than υ.

A3 If the system state restored to a working state after the repairman has repaired a component, it should be referred to as a critical component. Otherwise, if repairing a component does not make the system state change from failure to working, then the component should be referred to as a non-critical component.

A4 Failed components can be changed into new components after repair. When the state of the system is in failure, it is possible that components that have not yet failed enter the failure state.

A5 When a component is in a failure state, if the repairman's state is idle, the component will be repaired directly. Otherwise, the failed component goes into the retrial space to wait for repair and looks for the opportunity for repair after a period of time. The retrial time of component in the retrial space obeys an exponential distribution with rate γ.

A6 Critical components should be repaired first when the system fails. A critical component will be repaired directly if the component fails when the repairman is on idle. The repairman must immediately stop repairing the non-critical component and start repairing the critical component if a critical one fails while the repairman is repairing a non-critical one. A newly arrived component in the system will enter the retrial space while the repairman is working on a critical component. The retrial rule for components in the retrial space is FIFO (first in first out).

A7 At time $t = 0$, the state of the repairman is idle and all components are new. The random variables in this model are all independent of each other.

A8 A shock may cause any number of components to fail simultaneously.

According to these assumptions, the system states are defined in the next subsection. The general form of the transition rates between states of the system are expressed based on the derived propositions.

2.2. *System state analysis*

Let $U(t)$ denote the repairman is idle or busy at time t, and let $Z(t)$ denote the amount of components in the orbit when the system is in working or failure state at time t, as defined below.

$$U(t) = \begin{cases} 0, & \text{if the repairman is idle at time } t, \\ 1, & \text{if the repairman is busy at time } t. \end{cases}$$

$$Z(t) = \begin{cases} z_-, & \text{if there are } -z_- \text{ failed components in the retrial orbit and the system} \\ & \text{is in working state at time } t, z_- = 0, -1, \cdots, -k, \cdots, -\eta, \\ z_+, & \text{if there are } z_+ \text{ failed components in the retrial orbit and the system is} \\ & \text{in failure state at time } t, z_+ = k-1, k, \cdots, n-1, \end{cases}$$

where $\eta = n - \lfloor n/k \rfloor$. The symbol $\lfloor\ \rfloor$ means round down; that is, $\lfloor n/k \rfloor$ is the largest of the integers less than or equal to n/k.

According to the assumptions of the model, $\{U(t), Z(t), t \geq 0\}$ is a continuous-time Markov process. The state space can be represented as $\Psi = \{(u, z), u = 0, 1, z = 0, -1, \cdots, -(\eta-1)\} \cup \{(0, -\eta)\} \cup \{(1, z), z = k-1, k, \cdots, n-1\} \cup \{(0, z), z = 2k-1, 2k, \cdots, n-1, 2k \leq n\}$.

The system working state set and the failure state set are $B = \{(u, z), u = 0, 1, z = 0, -1, \cdots, -(\eta-1)\} \cup \{(0, -\eta)\}$ and $E = \{(0, z), z = 2k-1, 2k, \cdots, n-1, 2k \leq n\} \cup \{(1, z), z =$

$k-1, k, \cdots, n-1\}$, respectively. The system state probabilities at time t are defined as follows:

$P_{0,z}(t) = P\{U(t) = 0, Z(t) = z\},\ z = 0, -1, -2, \cdots, -\eta, 2k-1, \cdots, n-1$, and

$P_{1,z}(t) = P\{U(t) = 1, Z(t) = z\},\ z = 0, -1, \cdots, -(\eta-1), k-1, k, \cdots, n-1$,

respectively. Then the Laplace transform of state probability $P_{u,z}(t)$ is $P^*_{u,z}(s) = \int_0^\infty \mathrm{e}^{-st} P_{u,z}(t)$ $\mathrm{d}t$. Based on Assumption A2, the probability of a single component failure is $P_{\hat{y}} = P(\upsilon \le \hat{y}) = H(\hat{y})$ when the magnitude of the shock is $\hat{y}$. Therefore, the probability of a single component failure caused by the shock with magnitude $\hat{Y}$ is $H(\hat{Y})$. Let $Y = H(\hat{Y})$, then

$$P(Y \le y) = P\left(H(\hat{Y}) \le y\right) = P\left(\hat{Y} \le H^{-1}(y)\right) = \varPhi\left(H^{-1}(y)\right) = \varPhi H^{-1}(y).$$

Let $G = \varPhi H^{-1}$, then the distribution function of Y is $G(y)$. $Y = H(\hat{Y})$ is a function that increases as $\hat{Y}$ increases, representing the probability that a single component fails due to a shock.

Let A_{il} be the event that a shock causes $l-i$ components to fail, and there are $n-l$ components that are still working in the system. It is noted that i and $l(l \ge i)$ represent the numbers of failed components before and after shock, respectively. Let ω_{il} be the probability of occurrence of A_{il}, then

$$\omega_{il} = \int_0^1 P(A_{il}\,|Y = y)\mathrm{d}G(y) = \int_0^1 (1-y)^{n-l} y^{l-i} \mathrm{d}G(y).$$

In order to obtain the transition rates between the states of the system, we give the following propositions.

Proposition 1. *Let $M_{-(u-z)}$ denote the amount of likely situations that a repairable linear C(k, n: F) system with retrial feature is in state (u, z), where $1 \le u - z \le \eta$, we have*

$$M_{-(u-z)} = \sum_{\sigma=0}^{\alpha} (-1)^\sigma \begin{pmatrix} n+z-u+1 \\ \sigma \end{pmatrix} \begin{pmatrix} n-k\sigma \\ u-z-k\sigma \end{pmatrix}, \tag{1}$$

where $\alpha = \min(n+z-u+1, \lfloor (u-z)/k \rfloor)$, and $\sum\limits_{r=\tau}^{\kappa} \Lambda(r) \equiv 0$, $\kappa < \tau$. $M_{-(u-z)} = 1$ when $(u, z) = (0, 0)$.

Proof. The state (u, z) represents that the system is in the working state and $u - z$ components fail when $1 \le u - z \le \eta$. According to Theorem 1 in literature [9], we can obtain the formula for the calculation of $M_{-(u-z)}$. □

Proposition 2. *Let M_j^- be the number of likely situations of state $(1, j-1)$ which can be transferred to state $(0, -(j-1))$, where $2k \le j \le \eta + 1$, then*

$$\begin{aligned} M_j^- =& (n-j+1) \times N(j-k, n-j+1) \\ =& (n-j+1) \times \sum_{\sigma=0}^{\beta} (-1)^\sigma \begin{pmatrix} n-j+1 \\ \sigma \end{pmatrix} \begin{pmatrix} n-k-k\sigma \\ j-k-k\sigma \end{pmatrix}, \end{aligned} \tag{2}$$

where $\beta = \min(n-j+1, \lfloor (j-k)/k \rfloor)$, *and* $\sum_{r=\tau}^{\kappa} \Lambda(r) \equiv 0$, $\kappa < \tau$.

Proof. When j components fail in the system, $n-j$ components in the working state. The $n-j$ working components divide j failed components into $n-j+1$ sections in the linear consecutive system. The number of consecutive failed components must be at least k since state $(1, j-1)$ is a failed state. Furthermore, the amount of failed components which are consecutive is not more than $k-1$ except the k failed components since the state of the system changes to $(0, -(j-1))$ after the repairman has repaired a component. Therefore, the number of different situations that the state of the system changes from failed state $(1, j-1)$ to working state $(0, -(j-1))$ is equivalent to the number of situations where $j-k$ identical balls are placed in $n-j+1$ different urns where the maximum amount of balls is $k-1$. In addition, the k consecutive balls can be deposited in any one of $n-j+1$ urns. Thus the number of different situations M_j^- is $(n-j+1) \times N(j-k, n-j+1)$, where the calculation of $N(j-k, n-j+1)$ can be obtained according to Lemma 1 in literature [9] . □

According to Proposition 2, the probabilities of the state of the system changes from failed state $(1, j-1)$ to state $(0, -(j-1))$ and state $(0, j-1)$ are $M_j^- \Big/ \Big(C_n^j - M_{-j}\Big)$ and $\Big(C_n^j - M_{-j} - M_j^-\Big) \Big/ \Big(C_n^j - M_{-j}\Big)$ after the repairman has repaired a component, respectively, where $2k \leq j \leq \eta + 1$.

Proposition 3. *Let λ denote the component's failure rate caused by intrinsic factors or extrinsic shocks. The probability of each situation of working state (u, z_1) is the same, that is $p_{(u,z_1)h_{(u,z_1)}} = 1 \big/ M_{-(u-z_1)}$. Then the transition rates from state (u, z_1) to state $(1, -z_2)$ and state $(1, z_2)$ are $C_{1+z_2}^{u-z_1} \lambda M_{-(1+z_2)} \big/ M_{-(u-z_1)}$ and $\Big[C_{n-(u-z_1)}^{1+z_2-(u-z_1)} - C_{1+z_2}^{u-z_1} M_{-(1+z_2)} \big/ M_{-(u-z_1)}\Big] \lambda$, respectively, where $1 + z_2 > u - z_1, z_2 = k-1,\ k, \cdots, \eta - 1$.*

Proof. There are $M_{-(1+z_2)}$ different situations of state $(1, -z_2)$ based on Proposition 1. In addition, there are $C_{1+z_2}^{u-z_1}$ possible ways of the system state changes from state (u, z_1) to any case of state $(1, -z_2)$. Therefore, the amount of likely situations in which the system state changes from state (u, z_1) to state $(1, -z_2)$ is equal to $M_{-(1+z_2)} C_{1+z_2}^{u-z_1}$. According to the transition probability for consecutive systems proposed in literature [29], we have

$$
\begin{aligned}
p_{(u,z_1),(1,-z_2)}(\Delta t) = & \sum_{h_{(u,z_1)}=1}^{M_{(u,z_1)}} p_{(u,z_1),h_{(u,z_1)}} p_{h_{(u,z_1)}(1,-z_2)}(\Delta t) \\
= & \frac{\{\text{Total number of ways of transferring from } (u,z_1) \text{ to } (1,-z_2)\}}{M_{-(u-z_1)}} \lambda \Delta t \\
& + o(\Delta t) \\
= & \frac{C_{1+z_2}^{u-z_1} M_{-(1+z_2)}}{M_{-(u-z_1)}} \lambda \Delta t + o(\Delta t),
\end{aligned}
$$

Furthermore, there are $C_{n-(u-z_1)}^{1+z_2-(u-z_1)}$ likely situations where the number of failed components increases from $u - z_1$ to $1 + z_2$. Then the transition rate from state (u, z_1) to state $(1, z_2)$ is $\left[C_{n-(u-z_1)}^{1+z_2-(u-z_1)} - C_{1+z_2}^{u-z_1} M_{-(1+z_2)} / M_{-(u-z_1)}\right] \lambda$. □

Based on Assumptions A1, A4, A5 and A8 and the above analyses, the transition rates between states of the system can be obtained as follows:

- $(0, z-1) \to (1, z), (1, z) \in B \,|(0, z) \to (1, z-1), (0, z) \in E$: The transition rate is the retrial rate γ if a failed component in the retrial orbit will begin to be repaired.

- $(1, z) \to (0, z), (1, z) \in B \cup \{(1, z), z = \eta + 1, \cdots, n-1, 2k \leq n\} \,|(1, z) \to (0, -z)$, $z = k-1, k, \cdots, \min(2k-2, \eta)$: The transition rate is μ when the state of the system does not change after repairing a failed component. Based on Assumptions A3 and A6, the transition rate is also μ when the state of the system changes from failure to working after repairing a failed component.

- $(1, z) \to (0, -z) \,|(0, z)$ $z = 2k-1, 2k, \cdots, \eta, 2k-1 \leq \eta$: If the system state is failure, it may be restored to a working state or remain in the failed state after the repairman has repaired a component. According to Proposition 2, the two transition rates are $\left[M_{z+1}^{-} / \left(C_n^{z+1} - M_{-(z+1)}\right)\right] \mu$ and $\left[1 - M_{z+1}^{-} / \left(C_n^{z+1} - M_{-(z+1)}\right)\right] \mu$, respectively.

- $(0, z) \,|(1, z+1) \to (1, z), z = -(\eta-1), -(\eta-2), \cdots, -(k-1), k \leq \eta$: The transition occurs if the system state remains in working after a component fails. Then the transition rate is $(1-z)\left(\lambda_a + \omega_{-z,1-z}\lambda_b\right) M_{z-1}/M_z$ according to Proposition 3.

- $(0, z) \,|(1, z+1) \to (1, -z), z = -(\eta-1), -(\eta-2), \cdots, -(k-1), k \leq \eta$: The transition occurs if the state of the system changes from working to failure after a component fails. Then the transition rate is $\left[n + z - (1-z) M_{z-1}/M_z\right]\left(\lambda_a + \omega_{-z,1-z}\lambda_b\right)$ according to Proposition 3.

- $(1,-(z-1)) \to (1,-z), z=1,2,\cdots,k-2, k\geq 3\,|(0,-z) \to (1,-z), z=0,1,\cdots,k-2, k\geq 2$: The transition occurs if the system state remains in working after a component fails. The transition rate of this process is $(n-z)\left(\lambda_a+\omega_{z,z+1}\lambda_b\right)$.

- $(1,-(\eta-1))\,|(0,-\eta) \to (1,\eta)$: The transition occurs if the state of the system changes from working to failure after a component fails. The transition rate of this process is $(n-\eta)\left(\lambda_a+\omega_{\eta,\eta+1}\lambda_b\right)$.

- $(1,z-1) \to (1,z), z=k,k+1,\cdots,n-1, k\leq n-1\,|(0,z) \to (1,z), z=2k-1,2k,\cdots,n-1, 2k\leq n$: The transition occurs if the system state remains in failure after a component fails. The transition rate of this process is $(n-z)\left(\lambda_a+\omega_{z,z+1}\lambda_b\right)$.

- $(u,z_1) \to (1,-z_2)\,|(1,z_2), u-z_2<z_1\leq 0, z_2=k-1,k,\cdots,\eta-1, k\leq\eta$: The system may remain in a working state or transfer from a working state to a failed state after two or more components fail due to shock. The two transition rates are $C_{1+z_2}^{u-z_1}\omega_{u-z_1,1+z_2}\lambda_b M_{-(1+z_2)}/M_{-(u-z_1)}$ and $\left[C_{n-(u-z_1)}^{1+z_2-(u-z_1)}-M_{-(1+z_2)}C_{1+z_2}^{u-z_1}/M_{-(u-z_1)}\right]\omega_{u-z_1,1+z_2}\lambda_b$ according to Proposition 3.

- $(u,z_1) \to (1,-z_2), u-z_2<z_1\leq 0, z_2=1,2,\cdots,k-2, k\geq 3$: The transition occurs if the system remains in a working state after two or more components fail. The transition rate of this process is $C_{n-(u-z_1)}^{1+z_2-(u-z_1)}\omega_{u-z_1,1+z_2}\lambda_b$.

- $(u,z_1) \to (1,z_2), u-z_2<z_1\leq 0, z_2=\eta,\eta+1,\cdots,n-1$: The transition occurs if the state of the system changes from working to failure after two or more components fail. The transition rate of this process is $C_{n-(u-z_1)}^{1+z_2-(u-z_1)}\omega_{u-z_1,1+z_2}\lambda_b$.

- $(1,z_1) \to (1,z_2), k-1\leq z_1<z_2-1, z_2=k+1,k+2,\cdots,n-1, k+1\leq n-1$: The transition occurs if the system remains in a failed state after two or more components fail. The transition rate of this process is $C_{n-(1+z_1)}^{z_2-z_1}\omega_{1+z_1,1+z_2}\lambda_b$ when the repairman is busy.

- $(0,z_1) \to (1,z_2), 2k-1\leq z_1<z_2, z_2=2k,2k+1,\cdots,n-1, 2k\leq n-1$: The transition occurs if the system remains in a failed state after two or more components fail. The transition rate of this process is $C_{n-z_1}^{1+z_2-z_1}\omega_{z_1,1+z_2}\lambda_b$ when the repairman is idle.

3. Reliability indices analysis

In this section, we analyze some reliability indices of the proposed system model. The transition rate matrix of the system is expressed by the state transition rate obtained in Section 2. Then, the calculation steps of system availability, reliability function and mean time to first failure are given.

In the following analysis, let $\lambda_{(u_1,z_1),(u_2,z_2)}$ and $\mu_{(u_1,z_1),(u_2,z_2)}$ denote the transition rates of the system state from (u_1, z_1) to (u_2, z_2) due to component failure and repair, respectively. In order to derive the reliability indices, let the vector $\boldsymbol{P}(t)$ of the transient-state probabilities be

$$\begin{aligned}\boldsymbol{P}(t) = \big(&P_{00}(t), P_{0,-1}(t), \cdots, P_{0,-\eta}(t), P_{10}(t), \cdots, P_{1,-(\eta-1)}(t), P_{0,2k-1}(t), \cdots,\\ &P_{0,n-1}(t), P_{1,k-1}(t), \cdots, P_{1,n-1}(t)\big).\end{aligned}$$

Based on the transition rates, the Kolmogorov-Feller matrix equation is expressed as follows:

$$\boldsymbol{P}'(t) = \boldsymbol{P}(t)\boldsymbol{L}_1, \tag{3}$$

where $\boldsymbol{L}_1$ denote the transition rate matrix. According to the repairman's state and whether the system is in failure state, $\boldsymbol{L}_1$ is divided into 4×4 block matrix as follows:

$$\boldsymbol{L}_1 = \begin{pmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{12} & \boldsymbol{L}_{13} & \boldsymbol{L}_{14} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} & \boldsymbol{L}_{23} & \boldsymbol{L}_{24} \\ \boldsymbol{L}_{31} & \boldsymbol{L}_{32} & \boldsymbol{L}_{33} & \boldsymbol{L}_{34} \\ \boldsymbol{L}_{41} & \boldsymbol{L}_{42} & \boldsymbol{L}_{43} & \boldsymbol{L}_{44} \end{pmatrix},$$

each block of $\boldsymbol{L}_1$ is expressed as follows.

$$\boldsymbol{L}_{11} = \text{diag}\left(-W_0, -\left(W_1+\gamma\right), \cdots, -\left(W_{k-1}+\gamma\right), -\left(F_k+\gamma\right), \cdots, -\left(F_\eta+\gamma\right)\right),$$

where $\boldsymbol{L}_{11}$ is a matrix of order $(\eta+1)\times(\eta+1)$,

$$W_i = \sum_{z_1=i}^{\eta-1} \lambda_{(0,-i),(1,-z_1)} + \sum_{z_2=k-1}^{n-1} \lambda_{(0,-i),(1,z_2)}, i = 0, 1, \cdots, k-1,$$

$$F_v = \sum_{z_1=v}^{\eta-1} \lambda_{(0,-v),(1,-z_1)} + \sum_{z_2=v}^{n-1} \lambda_{(0,-v),(1,z_2)}, v = k, k+1, \cdots, \eta-1,$$

$$F_d = \sum_{z_2=\eta}^{n-1} \lambda_{(0,-\eta),(1,z_2)},$$

$$\boldsymbol{L}_{12} = \begin{pmatrix} \lambda_{(0,0),(1,0)} & \lambda_{(0,0),(1,-1)} & \cdots & \lambda_{(0,0),(1,-(\eta-1))} \\ \gamma & \lambda_{(0,-1),(1,-1)} & \cdots & \lambda_{(0,-1),(1,-(\eta-1))} \\ 0 & \gamma & \cdots & \lambda_{(0,-2),(1,-(\eta-1))} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma \end{pmatrix}_{(\eta+1)\times\eta},$$

$$\boldsymbol{L}_{13} = \mathbf{0}_{(\eta+1)\times(n-2k+1)},$$

$$\boldsymbol{L}_{14}=\begin{pmatrix}\lambda_{(0,0),(1,k-1)} & \lambda_{(0,0),(1,k)} & \cdots & \lambda_{(0,0),(1,\eta)} & \cdots & \lambda_{(0,0),(1,n-1)}\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ \lambda_{(0,-(k-1)),(1,k-1)} & \lambda_{(0,-(k-1)),(1,k)} & \cdots & \lambda_{(0,-(k-1)),(1,\eta)} & \cdots & \lambda_{(0,-(k-1)),(1,n-1)}\\ 0 & \lambda_{(0,-k),(1,k)} & \cdots & \lambda_{(0,-k),(1,\eta)} & \cdots & \lambda_{(0,-k),(1,n-1)}\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_{(0,-\eta),(1,\eta)} & \cdots & \lambda_{(0,-\eta),(1,n-1)}\end{pmatrix}_{(\eta+1)\times(n-k+1)},$$

$$\boldsymbol{L}_{21}=\begin{pmatrix}\mu & & & & \\ & \mu & & & \\ & & \ddots & & \\ & & & \mu & 0\end{pmatrix}_{\eta\times(\eta+1)},$$

$$\boldsymbol{L}_{22}=\left(\begin{matrix}-I_1 & \lambda_{(1,0),(1,-1)} & \cdots & \lambda_{(1,0),(1,-(k-2))} & \lambda_{(1,0),(1,-(k-1))} & \cdots\\ 0 & -I_2 & \cdots & \lambda_{(1,-1),(1,-(k-2))} & \lambda_{(1,-1),(1,-(k-1))} & \cdots\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots\\ 0 & 0 & \cdots & -I_{k-1} & \lambda_{(1,-(k-2)),(1,-(k-1))} & \cdots\\ 0 & 0 & \cdots & 0 & -Q_k & \cdots\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots\\ 0 & 0 & \cdots & 0 & 0 & \cdots\\ 0 & 0 & \cdots & 0 & 0 & \cdots\end{matrix}\right.$$

$$\left.\begin{matrix}\lambda_{(1,0),(1,-(\eta-2))} & \lambda_{(1,0),(1,-(\eta-1))}\\ \lambda_{(1,-1),(1,-(\eta-2))} & \lambda_{(1,-1),(1,-(\eta-1))}\\ \vdots & \vdots\\ \lambda_{(1,-(k-2)),(1,-(\eta-2))} & \lambda_{(1,-(k-2)),(1,-(\eta-1))}\\ \lambda_{(1,-(k-1)),(1,-(\eta-2))} & \lambda_{(1,-(k-1)),(1,-(\eta-1))}\\ \vdots & \vdots\\ -Q_{\eta-1} & \lambda_{(1,-(\eta-2)),(1,-(\eta-1))}\\ 0 & -\left(\sum_{z_2=\eta}^{n-1}\lambda_{(1,-(\eta-1)),(1,z_2)}+\mu\right)\end{matrix}\right)_{\eta\times\eta},$$

$$I_x = \sum_{z_1=x}^{\eta-1} \lambda_{(0,-x),(1,-z_1)} + \sum_{z_2=k-1}^{n-1} \lambda_{(0,-x),(1,z_2)} + \mu, x = 1, 2, \cdots, k-1,$$

$$Q_m = \sum_{z_1=m}^{\eta-1} \lambda_{(0,-m),(1,-z_1)} + \sum_{z_2=m}^{n-1} \lambda_{(0,-m),(1,z_2)} + \mu, m = k, k+1, \cdots, \eta-1,$$

$$\boldsymbol{L}_{23} = \mathbf{0}_{\eta\times(n-2k+1)},$$

$$\boldsymbol{L}_{24} = \begin{pmatrix} \lambda_{(1,0),(1,k-1)} & \lambda_{(1,0),(1,k)} & \cdots & \lambda_{(1,0),(1,\eta)} & \cdots & \lambda_{(1,0),(1,n-1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \lambda_{(1,-(k-2)),(1,k-1)} & \lambda_{(1,-(k-2)),(1,k)} & \cdots & \lambda_{(1,-(k-2)),(1,\eta)} & \cdots & \lambda_{(1,-(k-2)),(1,n-1)} \\ 0 & \lambda_{(1,-(k-1)),(1,k)} & \cdots & \lambda_{(1,-(k-1)),(1,\eta)} & \cdots & \lambda_{(1,-(k-1)),(1,n-1)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{(1,-(\eta-1)),(1,\eta)} & \cdots & \lambda_{(1,-(\eta-1)),(1,n-1)} \end{pmatrix}_{\eta\times(n-k+1)},$$

$$\boldsymbol{L}_{31} = \mathbf{0}_{(n-2k+1)\times(\eta+1)}, \qquad \boldsymbol{L}_{32} = \mathbf{0}_{(n-2k+1)\times\eta},$$

$$\boldsymbol{L}_{33} = \text{diag}\left(-\left(\sum_{z=2k-1}^{n-1} \lambda_{(0,2k-1),(1,z)} + \gamma\right), -\left(\sum_{z=2k}^{n-1} \lambda_{(0,2k),(1,z)} + \gamma\right), \cdots, -\left(\lambda_{(0,n-1),(1,n-1)} + \gamma\right)\right),$$

where $\boldsymbol{L}_{33}$ is a matrix of order $(n-2k+1)\times(n-2k+1)$,

$$\boldsymbol{L}_{34} = \begin{pmatrix} \mathbf{0} & \boldsymbol{D}_{34} \end{pmatrix}_{(n-2k+1)\times(n-k+1)},$$

where $$\boldsymbol{D}_{34} = \begin{pmatrix} \gamma & \lambda_{(0,2k-1),(1,2k-1)} & \cdots & \lambda_{(0,2k-1),(1,n-2)} & \lambda_{(0,2k-1),(1,n-1)} \\ 0 & \gamma & \cdots & \lambda_{(0,2k),(1,n-2)} & \lambda_{(0,2k),(1,n-1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \gamma & \lambda_{(0,n-1),(1,n-1)} \end{pmatrix}_{(n-2k+1)\times(n-2k+2)},$$

$$\boldsymbol{L}_{41} = \begin{pmatrix} \mathbf{0} & \boldsymbol{D}_{41} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}_{(n-k+1)\times(\eta+1)},$$

where $\boldsymbol{D}_{41} = \text{diag}\left(\mu, \cdots, \mu, \mu_{(1,2k-1),(0,-(2k-1))}, \cdots, \mu_{(1,\eta),(0,-\eta)}\right)$ is a matrix of order $(\eta-k+2)\times(\eta-k+2)$,

$$L_{42} = \mathbf{0}_{(n-k+1)\times\eta}, \qquad L_{43} = \begin{pmatrix} \mathbf{0} \\ D_{43} \end{pmatrix}_{(n-k+1)\times(n-2k+1)},$$

where $D_{43} = \mathrm{diag}\left(\mu_{(1,2k-1),(0,2k-1)}, \cdots, \mu_{(1,\eta),(0,\eta)}, \mu, \cdots, \mu\right)$ is a matrix of order $(n-2k+1) \times (n-2k+1)$,

$$L_{44} = \begin{pmatrix} -\left(\sum_{z=k}^{n-1} \lambda_{(1,k-1),(1,z)} + \mu\right) & \lambda_{(1,k-1),(1,k)} & \cdots & \lambda_{(1,k-1),(1,n-1)} \\ 0 & -\left(\sum_{z=k+1}^{n-1} \lambda_{(1,k),(1,z)} + \mu\right) & \cdots & \lambda_{(1,k),(1,n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\mu \end{pmatrix}_{(n-k+1)\times(n-k+1)}.$$

According to the Kolmogorov-Feller matrix equation, the state probabilities of the system can be solved. The calculation method of system availability is given below.

3.1. Availability of the system

Solve the following equations by using the Laplace transform. And then we can use the inverse Laplace transform to figure out the state probability.

$$\begin{cases} \boldsymbol{P}'(t) = \boldsymbol{P}(t)\boldsymbol{L}_1, \\ \boldsymbol{P}(0) = (1, 0, \cdots, 0). \end{cases} \tag{4}$$

The transient-state availability of the system is as follows:

$$A(t) = \sum_{z_1=0}^{\eta} P_{0,-z_1}(t) + \sum_{z_2=0}^{\eta-1} P_{1,-z_2}(t). \tag{5}$$

Next, the steady-state availability can be calculated by using the final value theorem and the Laplace transform.

$$A(\infty) = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA^*(s) = \lim_{s\to 0} \left[\sum_{z_1=0}^{\eta} sP^*_{0,-z_1}(s) + \sum_{z_2=0}^{\eta-1} sP^*_{1,-z_2}(s) \right]. \tag{6}$$

Detailed algorithm steps of system availability are shown in Algorithm 1.

Algorithm 1 Calculate system availability.

Step 1. Define the repairman's state and the number of components in the retrial orbit as $U(t)$ and $Z(t)$, respectively.
Step 2. Establish the continuous-time Markov process $\{U(t), Z(t), t \geq 0\}$ with state space Ψ.
Step 3. Give the transition rates between the system states (u_1, z_1) and (u_2, z_2), $(u_1, z_1), (u_2, z_2) \in \Psi$.
Step 4. Divide the transition rate matrix $\boldsymbol{L}_1$ into 4×4 block matrices.
Step 5. Solve Eq. (4) by using the Laplace transform.
Step 6. Get the state probabilities $P_{u,z}(t)$, $(u, z) \in \Psi$ by using the inverse Laplace transform.
Step 7. Calculate the transient-state availability $A(t)$ based on Eq. (5).
Step 8. Let $t \to \infty$, obtain the steady-state availability $A(\infty)$.

The system reliability is the probability that the system will work before time t. Therefore, the system reliability function can be obtained by changing all the failure states in the system into absorbing states. Then the mean time to first failure of the system can be obtained.

3.2. *Reliability function and mean time to first failure*

Make all the failure states into absorbing states to obtain the reliability function $R(t)$, then another Markov process $\{\tilde{U}(t), \tilde{Z}(t), t \geq 0\}$ is obtained. Next, we provide the equations about the working states $\{\tilde{P}_{u,z}(t), (u, z) \in B\}$, according to the Kolmogorov-Feller forward equation.

$$\begin{cases} \tilde{\boldsymbol{P}}'(t) = \tilde{\boldsymbol{P}}(t)\boldsymbol{L}_2, \\ \tilde{\boldsymbol{P}}(0) = (1, 0, \cdots, 0), \end{cases} \tag{7}$$

where $\boldsymbol{L}_2 = \begin{pmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{12} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{pmatrix}$ and $\tilde{\boldsymbol{P}}(t) = \left(\tilde{P}_{00}(t), \tilde{P}_{0,-1}(t), \cdots, \tilde{P}_{0,-\eta}(t), \tilde{P}_{10}(t), \cdots, \tilde{P}_{1,-(\eta-1)}(t)\right)$.
The reliability function can be calculated through the Laplace transform method as follows.

$$R(t) = \tilde{\boldsymbol{P}}(t)\mathbf{e}_w, \tag{8}$$

where $\mathbf{e}_w$ is a $(2\eta + 1)$-order column vector where all the elements are equal to 1.

The mean time to first failure which is represented by MTTFF, is

$$\text{MTTFF} = -\tilde{\boldsymbol{P}}(0)\boldsymbol{L}_2^{-1}\mathbf{e}_w. \tag{9}$$

Detailed algorithm steps of system reliability and MTTFF are shown in Algorithm 2.

Algorithm 2 Calculate system reliability and MTTFF.

Step 1. Change all failed states to absorbing states in the system to establish a new Markov process $\{\tilde{U}(t), \tilde{Z}(t), t \geq 0\}$.
Step 2. Delete the transition rates related to the failed states in matrix $\boldsymbol{L}_1$ to obtain matrix $\boldsymbol{L}_2$.
Step 3. Solve Eq. (7) by using the Laplace transform.
Step 4. Get the working state probabilities $\{\tilde{P}_{u,z}(t), (u, z) \in B\}$ by using the inverse Laplace transform.
Step 5. Calculate the reliability function $R(t)$ and MTTFF based on Eqs. (8) and (9).

4. An application and numerical results

To illustrate the proposed reliability model for a linear C(k, n: F) retrial system under Poisson shocks, this section uses a case in which intelligent automation is designed in a real communication system. There are four relay stations in the communication system between two places. Both stations 2 and 3 can receive signals from station 1, and both stations 3 and 4 can receive signals from station 2. Therefore, the communication between source and destination will not be affected if station 2 or 3 fails. But the communication between source and destination will be disconnected if stations 2 and 3 fail at the same time. This is a typical linear C(2, 4: F) system. Maintenance on the system is completed by an intelligent patrol robot and an intelligent repair equipment. It is assumed that the intelligent patrol robot and intelligent repair equipment are completely reliable. The system will be subjected to external electromagnetic interference under a random uncertainty environment, and the arrival of electromagnetic interference is a Poisson process. When the relay station fails, the repair request is sent to the intelligent repair equipment by the intelligent patrol robot. The relay station will be repaired directly if the state of the intelligent repair equipment is idle. The repair request will be saved in the information storage generator installed in the intelligent patrol robot when the intelligent repair equipment is busy, and the repair request will be continuously attempted after a period of time.

The specific configuration of the system is as follows. The numbers of relay stations, intelligent patrol robots and intelligent repair equipment are 4, 1 and 1, respectively. Each relay station can transmit signal to the next two relay stations at most. The distribution Φ of electromagnetic interference magnitude $\hat{Y}$ is the same as the distribution H of threshold υ, then the distribution $G = \Phi H^{-1}$ of Y is a uniform distribution in the interval (0, 1). Based on Eqs. (4)-(9) in Section 3, some system reliability indices can be obtained. Set some parameters as base values: the relay station failure rate λ_a due to aging is 0.04, the relay station failure rate λ_b due to external electromagnetic interference is 0.04, the repair rate μ of the intelligent repair equipment is 1, and retrial rate γ of the repair request in the information storage generator is 0.5. Then, we change the value of each parameter in turn to perform a numerical analysis on the system reliability indices. In addition, the cost/benefit of the systems with and without retrial is compared.

4.1. Numerical analysis of reliability indices

We show the effect of several parameters on the system reliability (see Figs. 1-4), instantaneous availability (see Figs. 5-8), steady-state availability (see Figs. 9-12) and MTTFF (see Tables 1-4). From these figures and tables, the following conclusions can be drawn.

- The system reliability function $R(t)$ decreases with time t increasing, and shock magnitude λ_b has a significant influence on $R(t)$. Moreover, $R(t)$ decreases as λ_a and λ_b increase, and increases as μ and γ increase.
- The system instantaneous availability $A(t)$ tends to be steady when $t = 18$ at the base case. The shock magnitude λ_b has a greater impact on $A(t)$ than relay station failure rate λ_a.
- The repair rate μ and the retrial rate γ have little influence on system reliability $R(t)$, but they have an obvious influence on system instantaneous availability $A(t)$. The change is particularly significant when $\mu < 1$ and $\gamma < 0.3$.

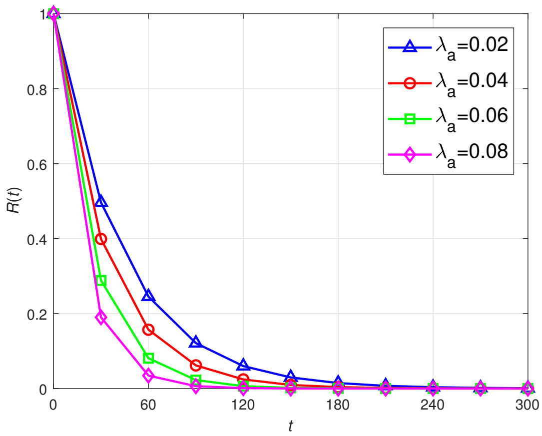

Fig. 1. $R(t)$ under different values of λ_a.

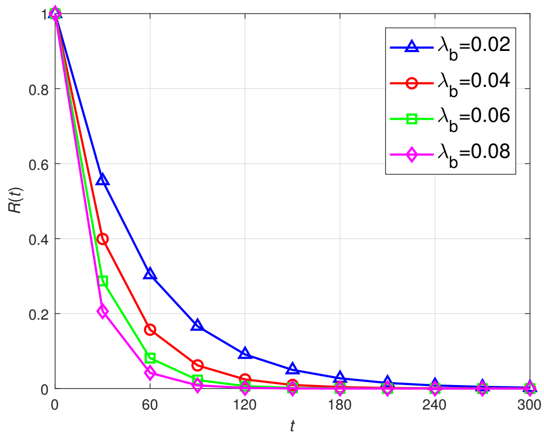

Fig. 2. $R(t)$ under different values of λ_b.

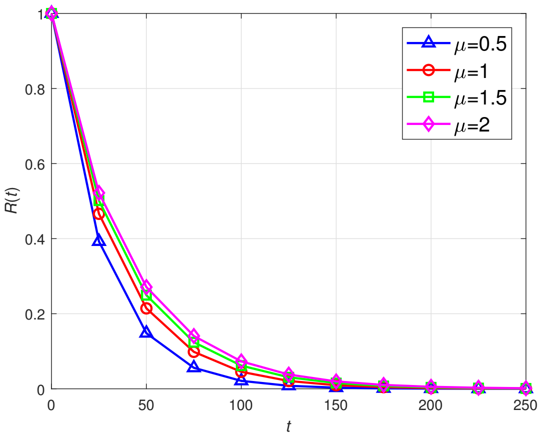

Fig. 3. $R(t)$ under different values of μ.

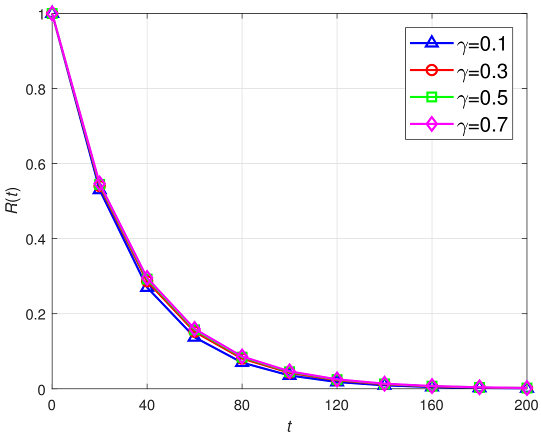

Fig. 4. $R(t)$ under different values of γ.

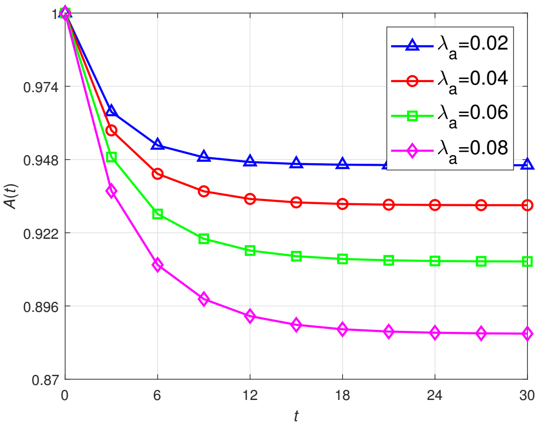

Fig. 5. $A(t)$ under different values of λ_a.

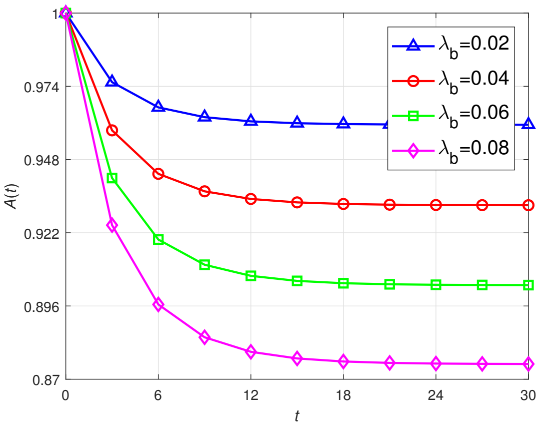

Fig. 6. $A(t)$ under different values of λ_b.

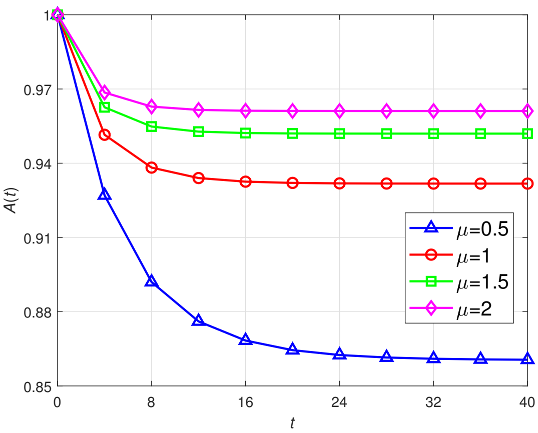

Fig. 7. $A(t)$ under different values of μ.

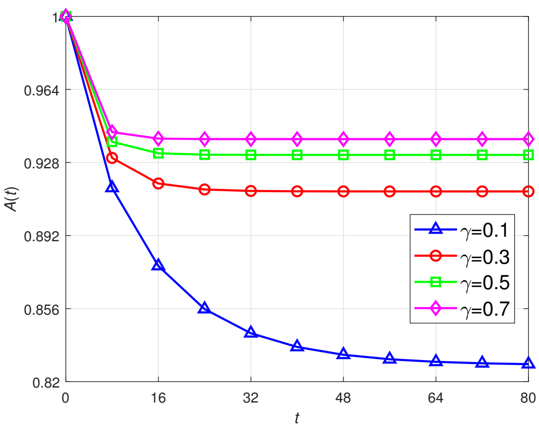

Fig. 8. $A(t)$ under different values of γ.

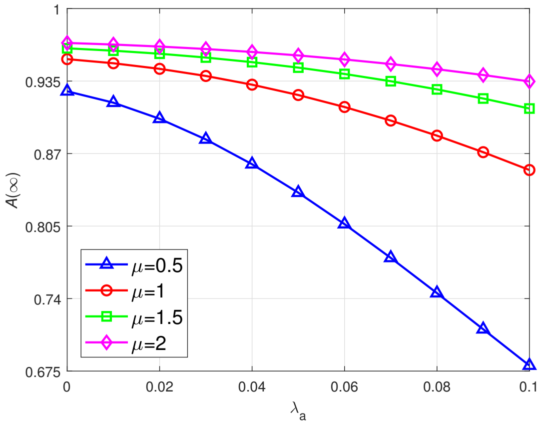

Fig. 9. $A(\infty)$ versus λ_a for various values of μ.

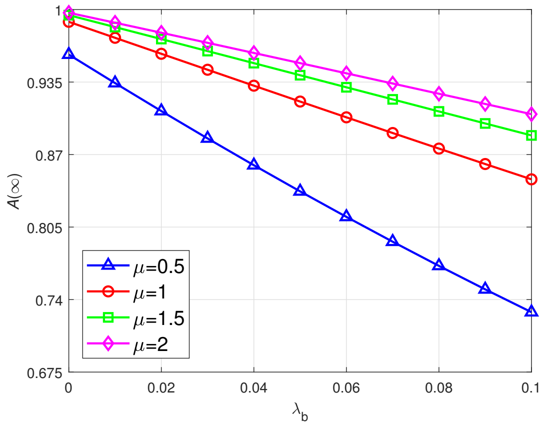

Fig. 10. $A(\infty)$ versus λ_b for various values of μ.

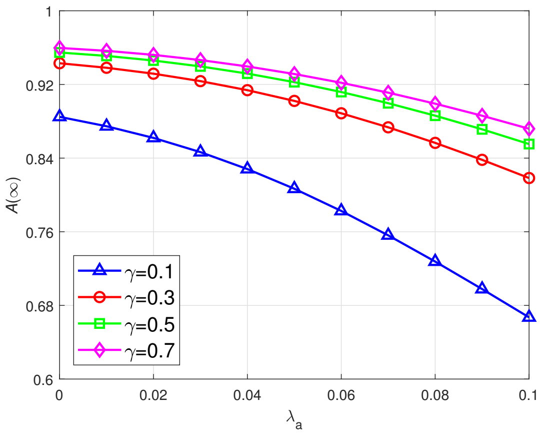

Fig. 11. $A(\infty)$ versus λ_a for various values of γ.

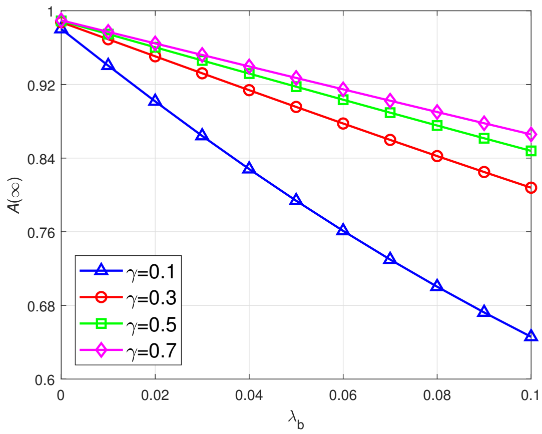

Fig. 12. $A(\infty)$ versus λ_b for various values of γ.

Table 1. MTTFF for various values of λ_a and μ.

λ_a	$\mu = 0.5$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$
0.01	45.1663	47.1263	47.8918	48.2994	48.5526
0.02	38.6158	42.7407	44.5840	45.6270	46.2977
0.03	32.1062	37.6242	40.4524	42.1676	43.3180
0.04	26.5892	32.5944	36.0726	38.3330	39.9182
0.05	22.2000	28.0802	31.8519	34.4637	36.3769
0.06	18.7761	24.2205	28.0128	30.7893	32.9063
0.07	16.1042	20.9987	24.6397	27.4347	29.6434
0.08	13.9994	18.3351	21.7337	24.4482	26.6613
0.09	12.3198	16.1363	19.2552	21.8300	23.9862
0.10	10.9608	14.3151	17.1495	19.5545	21.6152

Table 2. MTTFF for various values of λ_a and γ.

λ_a	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 0.7$	$\gamma = 0.9$
0.01	46.2041	46.9286	47.1263	47.2186	47.2720
0.02	41.2966	42.4057	42.7407	42.9024	42.9976
0.03	35.8369	37.1785	37.6242	37.8467	37.9802
0.04	30.6756	32.0842	32.5944	32.8578	33.0187
0.05	26.1990	27.5515	28.0802	28.3621	28.5372
0.06	22.4780	23.7069	24.2205	24.5026	24.6808
0.07	19.4392	20.5199	20.9987	21.2689	21.4424
0.08	16.9674	17.9002	18.3351	18.5868	18.7509
0.09	14.9501	15.7475	16.1363	16.3666	16.5188
0.10	13.2920	13.9707	14.3151	14.5234	14.6629

- The system is more prone to failure as relay station failure rate λ_a and shock magnitude λ_b increase, hence the steady-state availability $A(\infty)$ gradually decreases. In addition, steady-state availability $A(\infty)$ gradually increases as the repair rate μ and the retrial rate γ increase, respectively. It can be found that $A(\infty)$ is more sensitive to parameters λ_a and λ_b when $\mu < 1$ and $\gamma < 0.3$.
- MTTFF decreases with the increase of λ_a and λ_b, and increases with the increase of μ and γ. Furthermore, λ_a and λ_b have a more significant impact on MTTFF than μ and γ.

Table 3. MTTFF for various values of λ_b and μ.

λ_b	$\mu = 0.5$	$\mu = 1$	$\mu = 1.5$	$\mu = 2$	$\mu = 2.5$
0.01	47.4881	69.3029	85.6126	98.2180	108.2424
0.02	37.6265	50.3906	58.7362	64.5965	68.9335
0.03	31.1584	39.5853	44.6979	48.1170	50.5621
0.04	26.5892	32.5944	36.0726	38.3330	39.9182
0.05	23.1896	27.7011	30.2357	31.8530	32.9735
0.06	20.5616	24.0846	26.0232	27.2453	28.0852
0.07	18.4692	21.3029	22.8399	23.8008	24.4577
0.08	16.7637	19.0968	20.3497	21.1286	21.6590
0.09	15.3470	17.3045	18.3485	18.9950	19.4342
0.10	14.1514	15.8195	16.7052	17.2522	17.6232

Table 4. MTTFF for various values of λ_b and γ.

λ_b	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 0.7$	$\gamma = 0.9$
0.01	62.2998	67.4907	69.3029	70.2251	70.7838
0.02	46.3443	49.3387	50.3906	50.9273	51.2528
0.03	36.9097	38.8827	39.5853	39.9458	40.1650
0.04	30.6756	32.0842	32.5944	32.8578	33.0187
0.05	26.2487	27.3099	27.7011	27.9047	28.0294
0.06	22.9422	23.7728	24.0846	24.2481	24.3487
0.07	20.3783	21.0472	21.3029	21.4380	21.5214
0.08	18.3317	18.8825	19.0968	19.2109	19.2817
0.09	16.6601	17.1217	17.3045	17.4025	17.4636
0.10	15.2689	15.6614	15.8195	15.9049	15.9584

4.2. Cost/benefit analysis

The cost/benefit of the systems with and without retrial under Poisson shocks is compared in this section. For the intelligent automatic communication system, the repair request of failed relay stations can be sent to the intelligent repair equipment by the information storage generator installed in the intelligent patrol robot. The failed relay station will be performed maintenance directly if the state of the intelligent repair equipment is idle. Otherwise, the repair requests stored in the information storage generator will be repeatedly sent to the intelligent repair equipment after a period of time, so that the failed relay station can be restored to the working state. We assume that the annual cost of the information storage generator, i.e., the annual amortization and operating expenses (AE), is \$620. The failure information of the relay station will be directly sent to the intelligent repair equipment by the intelligent patrol robot in the general non-retrial system. If the intelligent repair equipment is busy, the information of the failed relay station will be stored in the intelligent repair equipment's repository and be queued in the repository for repair in the order of arrival. Therefore, there is a storage fee in this case, that is, the annual occupancy cost (OC) of the repository. In the following cost/benefit analysis, the costs of the retrial and non-retrial systems are the AE of the information storage generator and the OC of the repository, respectively, and the benefits refer to mean time to first failure and steady-state availability. In Figs. 13-16, costs/MTTFF and costs/$A(\infty)$ of the two systems are compared and analyzed.

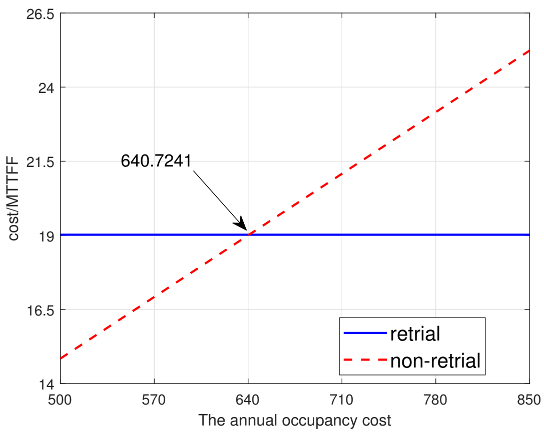

Fig. 13. cost/MTTFF varies with occupancy cost.

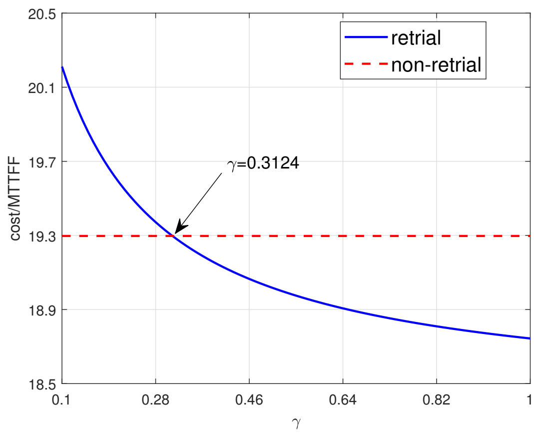

Fig. 14. cost/MTTFF varies with retrial rate.

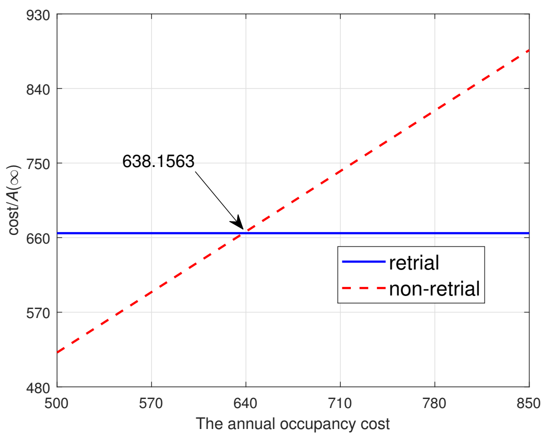

Fig. 15. cost/$A(\infty)$ varies with occupancy cost.

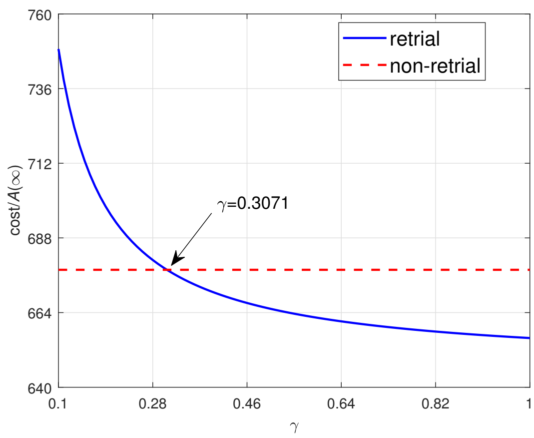

Fig. 16. cost/$A(\infty)$ varies with retrial rate.

Fig. 13 shows the change of cost/MTTFF with OC. We can see that the cost/MTTFF of the system with retrial feature is smaller than that of the system without retrial feature when OC > \$640.7241. We assume that the annual occupancy cost of the repository is \$650 in Fig. 14. We observe that the cost/MTTFF of the system with retrial feature is smaller than that of the system without retrial feature when $\gamma > 0.3124$. Fig. 15 shows the change of cost/$A(\infty)$ with OC. We can see that the cost/$A(\infty)$ of the system with retrial feature is smaller than that of the system without retrial feature when OC > \$638.1563. We still assume that the annual occupancy cost of the repository is \$650 in Fig. 16. It can be seen that the cost/$A(\infty)$ of the system with retrial feature is smaller than that of the system without retrial feature when $\gamma > 0.3071$. Based on the above analysis, we can choose the system with a lower cost/benefit according to the OC and the retrial rate in practical problems when designing the system.

5. Conclusions

This text proposed a linear C(k,n: F) system with retrial feature under Poisson shocks. The system can be used to simulate the telecommunication system in practical engineering. The retrial feature and Poisson shock make the model more practical. It is assumed that the working time, repair time and retrial time of the components in the system follow exponential distributions. The failure of components in the system can be induced by intrinsic factors or extrinsic shocks. Based on the definitions and assumptions, the availability and reliability indices were given by using Laplace transform and Markov process theory. In the numerical experiments, the effect of several parameters on reliability function, instantaneous availability, steady-state availability and MTTFF were analyzed. In addition, we compared the cost/benefit of the systems with and without retrial, including cost/MTTFF and cost/$A(\infty)$. The system developers and managers can design the system according to the estimated costs and parameter settings. When the OC of the non-retrial system exceeds a certain threshold, the retrial system with lower cost/benefit should be considered. In addition, in the case of the selected retrial system, the retrial rate should exceed a certain threshold to ensure that the retrial is of value to the system design.

This work merely considered that the cost is constant in the cost-benefit analysis. In the future, we will extend the cost/benefit to the function of several controllable parameters of the system, and minimize them by determining the optimal values of controllable parameters. When a large number of components lead to complex system states, it is difficult to acquire the accurate system reliability indices according to the current modeling methods and solving algorithms. Therefore, the state aggregation method and numerical algorithm can be considered for approximate processing. Moreover, Assumption A4 in Section 2.1 in this paper assumes that a component may fail while the system is being repaired and the paper did not distinguish the difference between the failure rates of the component during its operating state and during its idle state (i.e., at the state while the system is being repaired). The loading intensity on the component during the two states may differ, and this assumption may therefore be too restrictive. Our future work will also aim to relax this assumption to distinguish the difference.

Acknowledgments

This work was supported by the National Natural Science Foundation of China [grant number 72071175], and the Project of Hebei Key Laboratory of Software Engineering [grant number 22567637H].

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] L. Yuan, Reliability analysis for a k-out-of-n: G system with redundant dependency and repairmen having multiple vacations, Appl. Math. Comput. 218 (24) (2012) 11959–11969. https://doi.org/10.1016/j.amc.2012.06.006.

[2] D. Lyu, S. Si, Dynamic importance measure for the k-out-of-n: G system under repeated random load, Reliab. Eng. Syst. Saf. 195 (2020) 106720. https://doi.org/10.1016/j.ress.2019.106720.

[3] J. Wang, X. Zhu, Joint optimization of condition-based maintenance and inventory control for a k-out-of-n: F system of multi-state degrading components, Eur. J. Oper. Res. 290 (2) (2021) 514–529. https://doi.org/10.1016/j.ejor.2020.08.016.

[4] D. T. Chiang, S.-C. Niu, Reliability of consecutive-k-out-of-n: F system, IEEE T. Reliab. 30 (1) (1981) 87–89. https://doi.org/10.1109/TR.1981.5220981.

[5] F. K. Hwang, Simplified reliabilities for consecutive-k-out-of-n systems, SIAM J. Algebr. Discrete Methods 7 (2) (1986) 258–264. https://doi.org/10.1137/0607029.

[6] D. T. Chiang, R.-F. Chiang, Relayed communication via consecutive-k-out-of-n: F system, IEEE T. Reliab. 35 (1) (1986) 65–67. https://doi.org/10.1109/TR.1986.4335348.

[7] J. M. Kontoleon, Reliability determination of a r-successive-out-of-n: F system, IEEE T. Reliab. 29 (5) (1980) 437–437. https://doi.org/10.1109/TR.1980.5220921.

[8] Y. Zhang, T. Wang, Repairable consecutive-2-out-of-n: F system, Microelectron. Reliab. 36 (5) (1996) 605–608. https://doi.org/10.1016/0026-2714(95)00183-2.

[9] K. Cheng, Y. L. Zhang, Analysis for a consecutive-k-out-of-n: F repairable system with priority in repair, Int. J. Syst. Sci. 32 (5) (2001) 591–598. https://doi.org/10.1080/00207720119474.

[10] L. Yuan, Z.-D. Cui, Reliability analysis for the consecutive-k-out-of-n: F system with repairmen taking multiple vacations, Appl. Math. Model. 37 (7) (2013) 4685–4697. https://doi.org/10.1016/j.apm.2012.09.008.

[11] S. Si, J. Zhao, Z. Cai, H. Dui, Recent advances in system reliability optimization driven by importance measures, Front. Eng. Manag. 7 (3) (2020) 335–358. https://doi.org/10.1007/s42524-020-0112-6.

[12] D. Wang, S. Si, Z. Cai, J. Zhao, Reliability optimization of linear consecutive-k-out-of-n: F systems driven by reconfigurable importance, Reliab. Eng. Syst. Saf. 216 (2021) 107994. https://doi.org/10.1016/j.ress.2021.107994.

[13] S. Eryilmaz, Parallel and consecutive-k-out-of-n: F systems under stochastic deterioration, Appl. Math. Comput. 227 (2014) 19–26. https://doi.org/10.1016/j.amc.2013.10.081.

[14] S. D. Dafnis, F. S. Makri, A. N. Philippou, The reliability of a generalized consecutive system, Appl. Math. Comput. 359 (2019) 186–193. https://doi.org/10.1016/j.amc.2019.04.056.

[15] I. S. Triantafyllou, m-consecutive-k-out-of-n: F structures with a single change point, Mathematics 8 (12) (2020) 2203. https://doi.org/10.3390/math8122203.

[16] G. Choudhury, J.-C. Ke, A batch arrival retrial queue with general retrial times under Bernoulli vacation schedule for unreliable server and delaying repair, Appl. Math. Model. 36 (1) (2012) 255–269. https://doi.org/10.1016/j.apm.2011.05.047.

[17] A. Krishnamoorthy, P. V. Ushakumari, Reliability of a k-out-of-n system with repair and retrial of failed units, Top 7 (2) (1999) 293–304. https://doi.org/10.1007/BF02564728.

[18] C.-C. Kuo, S.-H. Sheu, J.-C. Ke, Z. G. Zhang, Reliability-based measures for a retrial system with mixed standby components, Appl. Math. Model. 38 (19) (2014) 4640–4651. http://doi.org/ 10.1016/j.apm.2014.03.005.

[19] K.-H. Wang, T.-C. Yen, J.-Y. Chen, Optimization analysis of retrial machine repair problem with server breakdown and threshold recovery policy, J. Test. Eval. 46 (6) (2018) 2630–2640. https://doi.org/10.1520/JTE20160149.

[20] W.-L. Chen, System reliability analysis of retrial machine repair systems with warm standbys and a single server of working breakdown and recovery policy, Systems Eng. 21 (1) (2018) 59–69. https://doi.org/10.1002/sys.21420.

[21] D.-Y. Yang, C.-L. Tsao, Reliability and availability analysis of standby systems with working vacations and retrial of failed components, Reliab. Eng. Syst. Saf. 182 (2019) 46–55. https://doi.org/10.1016/j.ress.2018.09.020.

[22] K.-H. Wang, C.-H. Wu, T.-C. Yen, Comparative cost-benefit analysis of four retrial systems with preventive maintenance and unreliable service station, Reliab. Eng. Syst. Saf. 221 (2022) 108342. https://doi.org/10.1016/j.ress.2022.108342.

[23] R. E. Barlow, F. Proschan, Statistical theory of reliability and life testing: probability models, Tech. rep., Florida State Univ Tallahassee (1975).

[24] N. Zhang, M. Fouladirad, A. Barros, J. Zhang, Reliability and maintenance analysis of a degradation-threshold-shock model for a system in a dynamic environment, Appl. Math. Model. 91 (2021) 549–562. https://doi.org/10.1016/j.apm.2020.09.047.

[25] B. Janani, Transient analysis of differentiated breakdown model, Appl. Math. Comput. 417 (2022) 126779. https://doi.org/10.1016/j.amc.2021.126779.

[26] Q. Wu, S. Wu, Reliability analysis of two-unit cold standby repairable systems under Poisson shocks, Appl. Math. Comput. 218 (1) (2011) 171–182. https://doi.org/10.1016/j.amc.2011.05.089.

[27] M. C. Segovia, P.-E. Labeau, Reliability of a multi-state system subject to shocks using phase-type distributions, Appl. Math. Model. 37 (7) (2013) 4883–4904. https://doi.org/10.1016/j.apm.2012.09.055.

[28] X. Zhao, S. Wang, X. Wang, Y. Fan, Multi-state balanced systems in a shock environment, Reliab. Eng. Syst. Saf. 193 (2020) 106592. https://doi.org/10.1016/j.ress.2019.106592.

[29] M. Li, L. Hu, R. Peng, Z. Bai, Reliability modeling for repairable circular consecutive-k-out-of-n: F systems with retrial feature, Reliab. Eng. Syst. Saf. 216 (2021) 107957. https://doi.org/10.1016/j.ress.2021.107957.