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System structure based decentralized sliding mode output tracking control for nonlinear interconnected systems

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Abstract
In this article, a decentralized tracking control scheme is proposed for a class of nonlinear interconnected systems with uncertainties using sliding mode technique. Both matched nonlinear uncertainty and mismatched known nonlinear interconnections are considered. Under the condition that the nominal isolated subsystems have relative degrees, a geometric transformation is applied to transfer the interconnected system into a new nonlinear interconnected system with a special structure to facilitate the system analysis and design. Then, a composite sliding surface is designed in terms of tracking errors, and decentralized controllers are proposed to drive the system states to the designed sliding surface in finite time and maintain a sliding motion on it thereafter. A set of conditions are developed to guarantee that the output tracking errors converge to zero asymptotically while all system state variables are bounded. The considered interconnected systems are nonlinear and it is not required that either the isolated subsystems or the isolated nominal subsystems are linearizable. The desired output signals are allowed to be time-varying. Finally, the developed results are applied to an inverted coupled-pendulum system. Simulation demonstrates that the proposed control scheme is effective.

KEYWORDS
decentralized control, nonlinear interconnected systems, output tracking, robust control, sliding mode control

1 | INTRODUCTION

With the advancement of modern technology, there comes a need to deal with more complex systems, which may be large-scale, meeting practical engineering requirements. Large-scale interconnected systems are usually composed of a set of dynamical subsystems which might be distributed over large space.[1,2] The communications between those different subsystems may become difficult or expensive due to the data transfer over large distances. In particular, when the data-transformation paths connecting various subsystems are broken or blocked, some data may be lost, or in the worst case, no data from the other subsystems may be available at all. Centralized control will not work in this case. Conversely,

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decentralized control needs local information only, and it does not require any of the other subsystems' state information. Thus it provides high reliability for the control of large-scale interconnected systems in reality.

During the past few decades, many results have been obtained for interconnected systems.[3] A fuzzy controller based on a reduced-order observer is designed for interconnected systems using integral sliding mode technique in Reference 4. Mahmoud proposed a decentralized control strategy for interconnected time-delay systems in Reference 5 where the considered system is linear. A finite-time control strategy is presented for nonlinear interconnected systems with dead-zone input in Reference 6, and robust controllers are designed for an interconnected multimachine power system using output feedback sliding mode techniques in Reference 7. Also, a decentralized control scheme is proposed for fully nonlinear interconnected system with time delay in Reference 8. Recently, a decentralized predictor method based controller is designed for large-scale systems in Reference 9 where the considered interconnected systems are linear and strong limitation on the interconnections between subsystems is required. It should be noted that most of the existing results are focused on stabilization instead of tracking control using either state feedback or output feedback control. Compared with the stabilization, tracking problem is more difficult, and thus the results obtained for tracking control are much limited, particularly for the large-scale nonlinear interconnected systems using decentralized control.

It is well known that tracking problem is a very interesting topic in engineering control. Most of existing work related to tracking control is focused on centralized control (see, e.g., References 10-12). Although decentralized tracking control for interconnected systems is studied in References 13 and 14, and distributed tracking control for interconnected systems with communication constraints is considered in Reference 15, it is required that the isolated subsystems of the considered interconnected systems are linear in References 13-15. Narendra and Zhang study a class of linear interconnected systems in Reference 16 where model reference tracking control is focused. Tracking control for interconnected systems is considered in Reference 17 using integral reinforcement learning. However, it is required that the interconnected terms are matched. More recently, Han and Yan propose an observer-based adaptive tracking control of large-scale stochastic nonlinear systems in Reference 18 which increases the dimension of the closed-loop system and thus it will increase the computational load required for implementation. It should be pointed out that most of the existing results about tracking control for interconnected systems are not decentralized, which implies the communication between each controller of one subsystem and all the other subsystems is essential with unobstructed channel for data transfer. This is not convenient for practical implementation. Li, Tong, and Yang proposed a decentralized event-triggered control scheme in Reference 19 using observer based feedback control, which guarantees that both the tracking performance and the stability of the closed-loop interconnected system but may increase the computation load greatly. Decentralized event-triggered tracking control is also designed for nonlinear interconnected systems with unknown interconnections in Reference 20. However, it is required, in References 19 and 20, that all of the isolated subsystems have a triangular structure. It should be mentioned that sliding mode control, as a popular method due to its high robustness,[21,22] has been widely applied to deal with tracking problems (see, e.g., References 23-26). However, the results on decentralized tracking control using sliding mode techniques for nonlinear interconnected systems are very few specifically when the desired signal is time-varying, and the tracking errors are convergent to zero asymptotically. An adaptive fuzzy control based on dynamic surface sliding mode technique is designed for prescribed output tracking in Reference 27 which can only be applied to the specific multi-machine power systems and unfortunately the designed controller is not decentralized.

In this article, a class of nonlinear interconnected systems is considered where both the matched uncertainty in the isolated subsystems and the mismatched interconnections are considered. A nonlinear coordinate transformation is introduced to explore the nominal isolated subsystems' structure, which transfers the interconnected systems to the required form, facilitating the system analysis and control design by using the structure of interconnections. The sliding surface is designed based on the tracking errors, and the sliding mode stability is achieved as well. A decentralized sliding mode control scheme is proposed to drive the nonlinear interconnected systems to the designed sliding surface in finite time. Compared with adaptive control approaches, there is a less restriction on the uncertainty bound and the structure when using sliding mode control, which means the uncertainty is allowed to have a more general form. The main contributions in this article can be summarized as follows:

- The designed controller is decentralized and the desired output signals are time-varying. This is in comparison with the existing work for interconnected system which needs either the other subsystems information available for design or the desired signals are constant.
- The developed result guarantees that the system outputs can track the desired outputs asymptotically while the system states are bounded.

- The considered interconnected systems are nonlinear with nonlinear disturbances which are bounded by nonlinear functions. It is not required that the nominal isolated subsystems are linearizable.
- The interconnection terms are mismatched, and the developed results has a high robustness.

Finally, the obtained results are applied to a coupled-pendulums system, and the simulation demonstrates that the method proposed in this article is effective.

2 | SYSTEM DESCRIPTION AND BASIC ASSUMPTIONS

Consider a nonlinear large-scale interconnected system formed by N subsystems as follows

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)\big(u_i + \varphi_i(x_i)\big) + p_i(x_i)\psi_i(x), \\ y_i &= h_i(x_i) \qquad i = 1, 2, \dots, N, \end{aligned} \tag{1}$$

where $x_i \in \Pi_i \subset R^{n_i}$, $u_i \in R$, and $y_i \in R$ are the states, input and output of the ith subsystem respectively, Π_i are neighborhoods of the origin, $x = col(x_1, x_2, \dots, x_N) \in \Pi$, and $\Pi := \Pi_1 \times \cdots \times \Pi_N \in R^{\sum_{i=1}^{N} n_i}$. The terms $\varphi_i(x_i) \in R$ are matched uncertainties. The terms $p_i(x_i)\psi_i(x) \in R^{n_i}$ represent the interconnection of the ith subsystem where $p_i(x_i) \in R^{n_i}$ are known used to describe the structure of the interconnections, and the terms $\psi_i(x) \in R$ are known which is used to describe the interconnections for $i = 1, 2, \dots, N$. All of the nonlinear terms are assumed to be smooth enough in their arguments to guarantee the existence and uniqueness of the system solutions.

In this article, the local case will be considered, and the considered domain may not be specified in the subsequence unless it is necessary. It should be noted that each subsystem in system (1) is assumed to be single-input and single-output for simplifying the analysis. The following definitions are introduced firstly for readers' convenience.

Definition 1. Consider system (1). The system

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)\big(u_i + \varphi_i(x_i)\big) \\ y_i &= h_i(x_i) \qquad i = 1, 2, \dots, N \end{aligned} \tag{2}$$

is called the ith isolated subsystem of the system (1), and the system

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \qquad i = 1, 2, \dots, N \end{aligned} \tag{3}$$

is called the ith nominal isolated subsystem of the system (1).

Definition 2. Consider system (1) with desired output signals $y_{id}(t)$ for $i = 1, 2, \dots, N$. If the controller u_i of the ith subsystem depends on the time t, the state x_i and the desired output signal $y_{id}(t)$ of the ith subsystem only, that is,

$$u_i = u_i(t, x_i, y_{id}) \qquad i = 1, 2, \dots, N, \tag{4}$$

then (4) is called decentralized static state feedback tracking control.

The objective of this article is, for a given desired output signal $y_{id}(t)$, to design a decentralized control such that the output $y_i(t)$ can track the desired signal $y_{id}(t)$ asymptotically, that is,

$$\lim_{t\to\infty} |y_i(t) - y_{id}(t)| = 0, \tag{5}$$

for $i = 1, 2, \dots, N$, while all the state variables of the interconnected system (1) are bounded.

To deal with the tracking problem stated above, some assumptions on the considered system (1) are introduced at first.

Assumption 1. There exist known continuous functions $\rho_i(x_i)$ in domain Π_i such that for $x_i \in \Pi_i$ with $i = 1, 2, \dots, N$.

$$|\varphi_i(x_i)| \leq \rho_i(x_i).$$

Remark 1. Assumption 1 implies that the uncertainties $\varphi_i(x_i)$ in the system (1) are required to be bounded for $i = 1, 2, \dots, N$, and the bounds are known. The bounds on the uncertainties will be used to design a decentralized controller later to cancel the effects of the corresponding uncertainties to enhance the robustness.

Assumption 2. For system (1), the triple (f_i, g_i, h_i) has an uniform relative degree r_i^a in the domain Π_i, the triple (f_i, p_i, h_i) has an uniform relative degree r_i^b in the domain Π_i, and $r_i^a = r_i^b$ for $i = 1, 2, \dots, N$. Furthermore, both distributions generated by the column vectors of function matrices $g_i(x_i)$ and $p_i(x_i)$ respectively, are involutive in the domain Π_i for $i = 1, 2, \dots, N$.

Remark 2. The definition of the relative degree for a nonlinear control systems is available in Reference 28. The uniform relative degree in Assumption 2 implies that, for any point $x_i \in \Pi_i$, the system has relative degree, and the relative degree is independent of $x_i \in \Pi_i$. For further discussion about the relative degree, see Reference 28.

Remark 3. Assumption 2 is the limitation to both the structure of the nominal isolated subsystems (3) and the structure of the distribution of the interconnections of system (1). It should be pointed out that the methodology developed in this article can be directly extended to the case $r_i^a < r_i^b$. Here, the condition $r_i^a = r_i^b$ is imposed on system (1) in Assumption 2 just for simplification of the later analysis and description. Similar limitation has been employed in Reference 29.

Assumption 3. The desired output signals $y_{id}(t)$ and their time derivatives up to the r_i^ath order are smooth, known and bounded for all $t \in [0, \infty)$.

Remark 4. Assumption 3 is the limitation to the desired signals. It requires that the ideal output signals $y_{id}(t)$ are differentiable for sufficient times. This assumption is quite standard in tracking control and usually is satisfied in most cases in reality. However, if the desired signal $y_{id}(t)$ is not continuous in reality due to some engineering limitation, this work may not be applied.

3 | SYSTEM STRUCTURE ANALYSIS

Consider the nonlinear interconnected system in (1). Under Assumption 2, it follows from Reference 28 that there exist diffeomorphisms $z_i = T_i(x_i)$ defined in Π_i, described by

$$\begin{bmatrix} x_{i,1} \\ x_{i,2} \\ \vdots \\ x_{i,(r_i^a-1)} \\ x_{i,r_i^a} \\ x_{i,(r_i^a+1)} \\ \vdots \\ x_{i,n_i} \end{bmatrix} \xrightarrow{z_i=T_i(x_i)} \begin{bmatrix} z_{i,1} \\ z_{i,2} \\ \vdots \\ z_{i,(r_i^a-1)} \\ z_{i,r_i^a} \\ z_{i,(r_i^a+1)} \\ \vdots \\ z_{i,n_i} \end{bmatrix} =: \begin{bmatrix} \xi_{i,1} \\ \xi_{i,2} \\ \vdots \\ \xi_{i,(r_i^a-1)} \\ \xi_{i,r_i^a} \\ \eta_{i,(r_i^a+1)} \\ \vdots \\ \eta_{i,n_i} \end{bmatrix} \tag{6}$$

and the feedback transformation

$$u_i = \varpi_i^{-1}(x_i)(-\varsigma_i(x_i) + v_i), \tag{7}$$

where $\varsigma_i(x_i)$ and $\varpi_i(x_i)$ are defined by

$$\varsigma_i(x_i) = L_{f_i}^{r_i^a} h_i(x_i), \tag{8}$$

$$\varpi_i(x_i) = L_{g_i} L_{f_i}^{r_i^a-1} h_i(x_i), \tag{9}$$

where v_i is the new controller to be designed later, and the notation $L_{g_i}L_{f_i}^{r_i^a-1}h_i(x_i)$ denotes Lie derivative (see e.g., Reference 28).

In the new coordinates z_i, the terms $\varsigma_i(x_i)$ and $\varpi_i(x_i)$ in (8) and (9) are, respectively, denoted by

$$\alpha_i(z_i) = \varsigma_i(x_i)|_{x_i=T_i^{-1}(z_i)},$$

$$\beta_i(z_i) = \varpi_i(x_i)|_{x_i=T_i^{-1}(z_i)}.$$

Then, under the diffeomorphism (6) and the feedback transformation (7), it follows from Reference 28 that in the new coordinates z_i, the system (1) can be described by

$$\begin{aligned}
\dot{\xi}_{i,1} &= \xi_{i,2},\\
\dot{\xi}_{i,2} &= \xi_{i,3},\\
&\dots\\
\dot{\xi}_{i,(r_i^a-1)} &= \xi_{i,r_i^a},\\
\dot{\xi}_{i,r_i^a} &= v_i(t) + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z),\\
\dot{\eta}_{i,(r_i^a+1)} &= q_{i,(r_i^a+1)}(z_i) + \Gamma_{i,(r_i^a+1)}\delta_i(z),\\
&\dots\\
\dot{\eta}_{i,n_i} &= q_{i,n_i}(z_i) + \Gamma_{i,n_i}\delta_i(z),\\
y_i &= \xi_{i,1},
\end{aligned} \tag{10}$$

where $z_i := col(\xi_i, \eta_i)$ with $\xi_i := col(\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i^a})$ and $\eta_i := col(\eta_{i,(r_i^a+1)}, \dots, \eta_{i,n_i})$, $z = col(z_1, z_2, \dots, z_N)$, and

$$\tau_i(z_i) = \varphi_i(x_i)|_{x_i=T_i^{-1}(z_i)}, \tag{11}$$

$$\gamma_i(z_i) = L_{p_i}L_{f_i}^{r_i^b-1}h_i(x_i)|_{x_i=T_i^{-1}(z_i)}, \tag{12}$$

$$\delta_i(z) = \psi_i(x)|_{x=T^{-1}(z)}. \tag{13}$$

The system (10) can be expressed in a compact form as

$$\dot{\xi}_i = A_i\xi_i + B_i[v_i + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z)], \tag{14}$$

$$\dot{\eta}_i = q_i(\xi_i, \eta_i) + \Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \dots, \xi_N, \eta_N), \tag{15}$$

$$y_i = C_i\xi_i \qquad i = 1, 2, \dots, N, \tag{16}$$

where $z_i = col(\xi_i, \eta_i)$ with $\xi_i \in R^{r_i^a}$ and $\eta_i \in R^{(n_i - r_i^a)}$. The triple (A_i, B_i, C_i) with appropriate dimensions has a standard *Brunovsky* form as follows

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & 1\\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0\\ 0\\ \vdots\\ 0\\ 1 \end{bmatrix}, \tag{17}$$

$$C_i = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}, \tag{18}$$

$q_i(\xi_i, \eta_i)$ and $\Gamma_i(\xi_i, \eta_i)$ are the last $n_i - r_i^a$ rows of the vectors

$$\left[\frac{\partial T_i}{\partial x_i} f_i(x_i)\right]_{x_i=T_i^{-1}(z_i)} \quad \text{and} \quad \left[\frac{\partial T_i}{\partial x_i} p_i(x_i)\right]_{x_i=T_i^{-1}(z_i)},$$

respectively.

Remark 5. It should be pointed out that the diffeomorphism $z_i = T_i(x_i) = col(\xi_i, \eta_i) = col(\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i^a}, \eta_{i,(r_i^a+1)}, \dots, \eta_{i,n_i})$ given in (6) is not unique. From Reference 28, a way to choose the diffeomorphism can be given as follows: $\xi_i = col(h_i(x_i), L_{f_i} h_i(x_i), \dots, L_{f_i}^{r_i^a} h_i(x_i))$, for $i = 1, 2, \dots, N$. $\eta_i = col(\eta_{i,(r_i^a+1)}, \dots, \eta_{i,n_i})$ where η_{ij} can be obtained by solving the equations $L_{g_i}\eta_{ij} = 0$ for $i = 1, 2, \dots, N$ and $j = r_i^a + 1, \dots, n_i$.

Remark 6. However, from (14)–(16), it is clear to see that in this article, it is not required that the nominal subsystems of system (1) are feedback linearizable. If the relative degree $r_i^a = n_i$, then the system (10) will have the following form

$$\begin{aligned}
\dot{\xi}_{i,1} &= \xi_{i,2}, \\
\dot{\xi}_{i,2} &= \xi_{i,3}, \\
&\dots \\
\dot{\xi}_{i,(n_i-1)} &= \xi_{i,n_i}, \\
\dot{\xi}_{i,n_i} &= v_i(t) + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z), \\
y_i &= \xi_{i,1}.
\end{aligned} \tag{19}$$

In this case the nominal isolated subsystem of interconnected system (14) is completely feedback linearizable and thus the nonlinear part relating to the dynamics of variables η_i in system (15) disappears.

4 | DECENTRALIZED OUTPUT TRACKING CONTROL

In the subsequence, the nonlinear interconnected systems (14)–(16) are to be focused. The main results will be presented in this section. Firstly, a sliding surface in terms of tracking errors will be proposed. Then, a decentralized controller based on sliding mode technique will be designed to implement the output tracking, and the boundedness of the considered interconnected system will be discussed.

4.1 | Sliding surface design

It is assumed that the desired output signals $y_{id}(t)$ satisfy Assumption 3. For (16), the output tracking errors e_i are defined by

$$e_i = y_i(t) - y_{id}(t) \qquad i = 1, 2, \dots, N. \tag{20}$$

The following sliding functions are introduced

$$S_i(\cdot) = e_i^{(r_i^a-1)} + a_{i,1} e_i^{(r_i^a-2)} + \cdots + a_{i,(r_i^a-2)} e_i^{(1)} + a_{i,(r_i^a-1)} e_i^{(0)}, \tag{21}$$

where $e_i^{(r_i^a-1)}, e_i^{(r_i^a-2)}, \dots$, and $e_i^{(1)}$ denote the $(r_i^a - 1)$th order, $(r_i^a - 2)$th order, $\dots$, and the 1st order derivatives of $e_i(t)$ respectively, $e_i^{(0)} := e_i(t)$, and $a_{i,1}, a_{i,2}, \dots, a_{i,(r_i^a-1)}$ are a set of design parameters, which are chosen such that the following polynomials

$$\lambda^{r_i^a-1} + a_{i,1}\lambda^{r_i^a-2} + \cdots + a_{i,(r_i^a-2)}\lambda + a_{i,(r_i^a-1)} \tag{22}$$

are *Hurwitz* stable for $i = 1, 2, \dots, N$. Then, the composite sliding surface for interconnected system (14)–(16) can be described by

$$\{S = col(S_1, S_2, \dots, S_N) |\ S_i = 0,\ i = 1, 2, \dots, N\}, \tag{23}$$

where S_i are defined in (21). From the design above, it is clear to see that when $S_i = 0$,

$$\lim_{t\to\infty} |e_i(t)| = 0.$$

This implies that when sliding motion occurs,

$$\lim_{t\to\infty} |y_i(t) - y_{id}(t)| = \lim_{t\to\infty} |e_i(t)| = 0, \tag{24}$$

that is, the outputs $y_i(t)$ of system (1) can track the ideal signal $y_{id}(t)$ asymptotically for $i = 1, 2, \dots, N$. The following result is now ready to be presented:

Theorem 1. *Consider the interconnected system (14)–(16). Under Assumption 3, when the system (14)–(16) is limited to moving on the sliding surface (23), the following results hold:*

(i) $\lim_{t\to\infty} |y_i(t) - y_{id}(t)| = \lim_{t\to\infty} |e_i(t)| = 0$ *for* $i = 1, 2, \dots, N$
(ii) The state variables ξ_i *in (14) are bounded for* $i = 1, 2, \dots, N$.

Proof. The result in (i) has been shown above (see (24)). The remains are to prove that the result in (ii) holds.

When system (14)–(16) is constrained to the sliding surface (21), it follows that

$$S_i = e_i^{(r_i^a - 1)} + a_{i,1} e_i^{(r_i^a - 2)} + \dots + a_{i,(r_i^a - 2)} e_i^{(1)} + a_{i,(r_i^a - 1)} e_i^{(0)} = 0.$$

Then,

$$e_i^{(r_i^a - 1)} = -a_{i,1} e_i^{(r_i^a - 2)} - \dots - a_{i,(r_i^a - 2)} e_i^{(1)} - a_{i,(r_i^a - 1)} e_i^{(0)}.$$

Let

$$e_{i,1} \triangleq e_i^{(0)} = e_i.$$

Then, the following error dynamics are obtained

$$\begin{aligned}
\dot{e}_{i,1} &= e_i^{(1)} \triangleq e_{i,2}, \\
\dot{e}_{i,2} &= e_i^{(2)} \triangleq e_{i,3}, \\
&\dots \\
\dot{e}_{i,(r_i^a - 2)} &= e_i^{(r_i^a - 2)} \triangleq e_{i,(r_i^a - 1)}, \\
\dot{e}_{i,(r_i^a - 1)} &= -a_{i,1} e_{i,(r_i^a - 1)} - \cdots - a_{i,(r_i^a - 2)} e_{i,2} - a_{i,(r_i^a - 1)} e_{i,1}.
\end{aligned}$$

Therefore the sliding mode dynamics of system (14)–(15) are given by the following equation by rewriting the system above in a compact form

$$\dot{\epsilon}_i = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 1 \\ -a_{i,(r_i^a - 1)} & -a_{i,(r_i^a - 2)} & -a_{i,(r_i^a - 3)} & \dots & -a_{i,1} \end{bmatrix}}_{E_i} \epsilon_i, \tag{25}$$

where $\epsilon_i = col(e_{i,1}, e_{i,2}, \dots, e_{i,(r_i^a-1)})$. It should be noted that the entries of the last row of matrix E_i: $a_{i,1}, a_{i,2}, \dots, a_{i,(r_i^a-1)}$ forms the *Hurwitz* polynomial (22). Therefore, system (25) is *Hurwitz* stable which implies that

$$\lim_{t\to\infty} |\epsilon_i(t)| = 0. \tag{26}$$

Further, from (24) and (26)

$$\lim_{t\to\infty} \left\| \begin{matrix} \xi_{i,1} - y_{id}^{(0)} \\ \xi_{i,2} - y_{id}^{(1)} \\ \vdots \\ \xi_{i,(r_i^a-1)} - y_{id}^{(r_i^a-2)} \\ \xi_{i,r_i^a} - y_{id}^{(r_i^a-1)} \end{matrix} \right\| = 0.$$

From Assumption 3, the desired output signal $y_{id}(t)$ and its derivatives: $y_{id}^{(1)}, y_{id}^{(2)}, \dots, y_{id}^{(r_i^a-1)}$ are bounded in $t \in [0, \infty]$. It follows that the state variables $\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i^a}$ are bounded and thus the states ξ_i in (14) are bounded when the states of the system are limited to the sliding surface (23).

Hence, the result follows. ■

Remark 7. It is well known that the sliding mode is a reduced-order system. Section 4.1 shows that for system (14)–(16) with the sliding surface given in (23), the corresponding sliding mode dynamics are system (25). Theorem 1 shows that the sliding mode is asymptotically stable and the partial of state variable ξ_i is bounded. Next, the decentralized controllers are to be designed to guarantee the reachability, and the boundedness of the partial states η_i will be discussed as well.

4.2 | Decentralized sliding mode controller design

Now, the objective is to design a decentralized state-feedback controller based on sliding mode technique such that the states of the controlled system (14)–(16) can be driven to the designed sliding surface (21) in finite time.

Since $z_i = T_i(x_i)$ is a diffeomorphism, from Assumption 1 and definitions of $\tau_i(z_i)$ and $\delta_i(z)$ in (11) and (13) respectively, it follows that there are continuous functions $\rho_i'(z_i)$ such that in the considered neighborhood of the origin

$$|\tau_i(z_i)| \le \rho_i'(z_i), \tag{27}$$

where $\rho_i'(\cdot)$ depends on the transformation $z_i = T_i(x_i)$ and $\rho_i(\cdot)$ in Assumption 1. Since $\rho_i(\cdot)$ are known, the bound $\rho_i'(\cdot)$ can be obtained from $z_i = T_i(x_i)$.

For system (14)–(16), the following control law is proposed

$$v_i = -\dot{S}_i + y_i^{(r_i^a)} - \left(K_i(z_i) + |\beta_i(z_i)|\rho_i'(z_i) + \frac{1}{2}|\gamma_i(z_i)|^2\right) sgn(S_i) \qquad i = 1, 2, \dots, N, \tag{28}$$

where the function $K_i(z_i)$ is the feedback gain to be designed later. $S_i(\cdot)$ is given in (21) and $sgn(\cdot)$ is the *sign* function. It is clear that the controllers v_i in (28) are decentralized.

Remark 8. From the structure of the control (28), it follows that only the variables $z_i, y_i, y_i^{(r_i^a)}$ and $y_{id}(t)$ are used in the ith control v_i, which are available locally. Specially from (10), $y_i^{(r_i^a)}$ is actually the first order derivative of the state x_{i,r_i^a}, which totally depends on the local state x_{i,r_i^a}. Therefore, from the coordinate transformation $z_i = T_i(x_i)$ and the relationship between u_i and v_i in (7), it is straightforward to see that the designed controllers are decentralized.

Theorem 2. *Under Assumptions 1 and 3, the nonlinear interconnected system (14)–(16) is driven to the sliding surface (21) in finite time by the controller (28) if the control gain $K_i(z_i)$ satisfies*

$$\sum_{i=1}^{N} K_i(z_i) > \frac{1}{2}\sum_{i=1}^{N} |\delta_i(z)|^2 + \sigma_i, \tag{29}$$

where σ_i is a positive constant.

Proof. The closed-loop system obtained by applying control law (28) into system (14)–(16) can be described by

$$\dot{\xi}_i = A_i\xi_i + B_i[-\dot{S}_i + y_i^{(r_i^a)} - \left(K_i(z_i) + |\beta_i(z_i)|\rho_i'(z_i) + \frac{1}{2}|\gamma_i(z_i)|^2\right) sgn(S_i) + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z)], \tag{30}$$

$$\dot{\eta}_i = q_i(\xi_i, \eta_i) + \Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \ldots, \xi_N, \eta_N), \tag{31}$$

$$y_i = C_i\xi_i. \qquad i = 1, 2, \ldots, N. \tag{32}$$

With the special structure of the triple (A_i, B_i, C_i) in (14)–(16), it follows

$$\begin{aligned}
y_i &= \xi_{i,1}, \\
y_i^{(1)} &= \xi_{i,2}, \\
&\cdots \\
y_i^{(r_i^a-1)} &= \xi_{i,r_i^a}, \\
y_i^{(r_i^a)} &= \dot{\xi}_{i,r_i^a} = -\dot{S}_i + y_i^{(r_i^a)} - \left(K_i(z_i) + |\beta_i(z_i)|\rho_i'(z_i) + \frac{1}{2}|\gamma_i(z_i)|^2\right) sgn(S_i) + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z).
\end{aligned} \tag{33}$$

From the last equation in (33),

$$\dot{S}_i = -\left(K_i(z_i) + |\beta_i(z_i)|\rho_i'(z_i) + \frac{1}{2}|\gamma_i(z_i)|^2\right) sgn(S_i) + \beta_i(z_i)\tau_i(z_i) + \gamma_i(z_i)\delta_i(z). \tag{34}$$

Then, from (27)–(34), and according to the basic inequality $ab \le \frac{1}{2}(a^2 + b^2)$,

$$\begin{aligned}
S^{\top}\dot{S} &= \sum_{i=1}^{N} S_i\dot{S}_i \\
&= \sum_{i=1}^{N}\left(-\left(K_i(z_i) + |\beta_i(z_i)|\rho_i'(z_i) + \frac{1}{2}|\gamma_i(z_i)|^2\right)|S_i| + \beta_i(z_i)\tau_i(z_i)S_i + \gamma_i(z_i)\delta_i(z)S_i\right) \\
&\le \sum_{i=1}^{N}\left(-K_i(z_i)|S_i| - \frac{1}{2}|\gamma_i(z_i)|^2|S_i| + \frac{1}{2}(|\gamma_i(z_i)|^2 + |\delta_i(z)|^2)|S_i|\right) \\
&= \sum_{i=1}^{N}\left(-K_i(z_i) + \frac{1}{2}|\delta_i(z)|^2\right)|S_i|.
\end{aligned} \tag{35}$$

It follows from (35), (29) and the basic inequality $\left(\sum_{i=1}^{N}|S_i|\right)^2 \ge \sum_{i=1}^{N}|S_i|^2$ that

$$S^T\dot{S} < -\sigma\sum_{i=1}^{N}|S_i| \le -\sigma\|S\|, \tag{36}$$

where $\sigma := min_i\{\sigma_i\} > 0$ due to $\sigma_i > 0$ for $i = 1, 2, \ldots, N$, meaning that the reachability condition holds for the closed-loop interconnected system (30)–(31). Hence, the result follows. ∎

Remark 9. Based on the analysis above and from the feedback transformation (28), it follows that the decentralized controller

$$u_i = \varpi_i^{-1}(x_i)\left[-\varsigma_i(x_i) - \dot{S}_i + y_i^{(r_i^a)} - \left(K_i(T_i(x_i)) + |\varpi_i(x_i)|\rho_i(x_i) + \frac{1}{2}|L_{p_i}L_{f_i}^{r_i^b-1}h_i(x_i)|^2\right) sgn(S_i)\right] \tag{37}$$

can drive the system (1) to the corresponding sliding surface in finite time, where $p_i = p_i(x_i), f_i = f_i(x_i)$ and S_i is defined in (21).

4.3 | The boundedness of system states

In this subsection, the boundedness of the closed-loop system (30)–(31) is analyzed. The following assumptions are needed.

Assumption 4. The functions $q_i(\xi_i, \eta_i)$ in system (30)-(31) satisfy the *Lipschitz* condition with the *Lipschitz* constants L_{q_i} uniformly for η_i in the considered domain. Moreover, there exists a *Lyapunov* function $V_{i0}(\eta_i)$ such that

$$\begin{aligned} \chi_{i1}\|\eta_i\|^2 \le V_{i0}(\eta_i) \le \chi_{i2}\|\eta_i\|^2, \\ \frac{\partial V_{i0}}{\partial \eta_i} q_i(0, \eta_i) \le -\chi_{i3}\|\eta_i\|^2, \\ \left\|\frac{\partial V_{i0}}{\partial \eta_i}\right\| \le \chi_{i4}\|\eta_i\|, \end{aligned} \tag{38}$$

where χ_{i1}, χ_{i2}, χ_{i3}, and χ_{i4} are positive constants for $i = 1, 2, \dots, N$.

Remark 10. The Assumption 4 implies that

$$\|q_i(\xi_i, \eta_i) - q_i(0, \eta_i)\| \le L_{q_i}\|\xi_i - 0\|. \tag{39}$$

Assumption 4 is the limitation to the nonlinear term $q_i(\xi_i, \eta_i)$ in (30)–(31). It also implies that the zero dynamics $\dot{\eta}_i = q_i(0, \eta_i)$ of the nominal system of system (30)–(31) is asymptotically stable.

Assumption 5. There exist positive constants κ_{1j} and κ_{2j} such that

$$\|\Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \dots, \xi_N, \eta_N)\| \le \sum_{j=1}^{N}(\kappa_{1j}\|\xi_j\| + \kappa_{2j}\|\eta_j\|), \tag{40}$$

for $i = 1, 2, \dots, N$.

Remark 11. Assumption 5 will hold if the inequalities $\|\Gamma_i(\xi_i, \eta_i)\| \le \kappa_{1i}\|\xi_i\| + \kappa_{2i}\|\eta_i\|$ hold for $i = 1, 2, \dots, N$.

Theorem 3. *Under Assumptions 3–5, the states of the closed-loop system (30)–(31) are bounded if the matrix $W^T + W$ is positive definite where the matrix W is defined as*

$$W := \begin{bmatrix} \chi_{13} - \chi_{14}\kappa_{21} & -\chi_{14}\kappa_{22} & \cdots & -\chi_{14}\kappa_{2N} \\ -\chi_{24}\kappa_{21} & \chi_{23} - \chi_{24}\kappa_{22} & \cdots & -\chi_{24}\kappa_{2N} \\ \vdots & \vdots & \ddots & \\ -\chi_{N4}\kappa_{21} & -\chi_{N4}\kappa_{22} & \cdots & \chi_{N3} - \chi_{N4}\kappa_{2N} \end{bmatrix}, \tag{41}$$

where χ_{ij} and κ_{lj} satisfy the Assumptions 4 and 5 for $i = 1, 2, \dots, N, j = 1, 2, 3, 4$ and $l = 1, 2$.

Proof. From Theorem 1, it follows that the variables $\xi_i = col(\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i^a})$ with $i = 1, 2, \dots, N$ are bounded when the sliding motion occurs if Assumption 3 holds. Theorem 2 shows that the interconnected system can be driven to the sliding surface in finite time. From Theorems 1 and 2, it follows that the variables $\xi_i = col(\xi_{i,1}, \xi_{i,2}, \dots, \xi_{i,r_i^a})$ with $i = 1, 2, \dots, N$

are bounded. Therefore, there exist constants $C_i > 0$ such that in the considered domain,

$$\|\xi_i\| \leq C_i \qquad i = 1, 2, \ldots, N. \tag{42}$$

The remain is to prove that the variables η_i in the closed-loop system (30)–(31) are bounded for $i = 1, 2, \ldots, N$.

It should be noted that from (42), the variables ξ_i in the system (31) are bounded and can be considered as parameters defined in a compact set. For this system, consider the following *Lyapunov* candidate function

$$V(\eta_1, \eta_2, \ldots, \eta_N) = \sum_{i=1}^{N} V_{i0}(\eta_i),$$

where $V_{i0}(\eta_i)$ is defined in Assumption 4. Then, the time derivative of the *Lyapunov* function $V(\cdot)$ along the trajectories of system (30)–(31) is given by

$$\begin{aligned}
&\dot{V}(\eta_1, \eta_2, \ldots, \eta_N) \\
&= \sum_{i=1}^{N} \frac{\partial V_{i0}(\eta_i)}{\partial \eta_i} \left[q_i(\xi_i, \eta_i) + \Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \ldots, \xi_N, \eta_N) \right] \\
&= \sum_{i=1}^{N} \left[\frac{\partial V_{i0}(\eta_i)}{\partial \eta_i} q_i(0, \eta_i) + \frac{\partial V_{i0}(\eta_i)}{\partial \eta_i} (q_i(\xi_i, \eta_i) - q_i(0, \eta_i)) \right] + \sum_{i=1}^{N} \frac{\partial V_{i0}(\eta_i)}{\partial \eta_i} \left[\Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \ldots, \xi_N, \eta_N) \right].
\end{aligned} \tag{43}$$

Further, from (44) and Assumptions 4 and 5, it follows

$$\begin{aligned}
&\dot{V}(\eta_1, \eta_2, \ldots, \eta_N) \\
&\leq \sum_{i=1}^{N} \left(-\chi_{i3}\|\eta_i\|^2 + \chi_{i4} L_{qi} C_i \|\eta_i\| + \left\| \frac{\partial V_{i0}(\eta_i)}{\partial \eta_i} \right\| \|\Gamma_i(\xi_i, \eta_i)\delta_i(\xi_1, \eta_1, \ldots, \xi_N, \eta_N)\| \right) \\
&\leq \sum_{i=1}^{N} \left(-\chi_{i3}\|\eta_i\|^2 + \chi_{i4} L_{qi} C_i \|\eta_i\| + \chi_{i4}\|\eta_i\| \sum_{j=1}^{N} (\kappa_{1j}\|\xi_j\| + \kappa_{2j}\|\eta_j\|) \right) \\
&\leq \sum_{i=1}^{N} \left(-\chi_{i3}\|\eta_i\|^2 + \chi_{i4} L_{qi} C_i \|\eta_i\| + \sum_{j=1}^{N} \chi_{i4}\kappa_{1j} C_i \|\eta_i\| + \sum_{j=1}^{N} \chi_{i4}\kappa_{2j} \|\eta_i\|\|\eta_j\| \right) \\
&= -\left(\sum_{i=1}^{N} \chi_{i3}\|\eta_i\|^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} \chi_{i4}\kappa_{2j}\|\eta_i\|\|\eta_j\| - \sum_{i=1}^{N}\sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j})\|\eta_i\| \right) \\
&= -\frac{1}{2} \left(\|\eta_1\|, \ldots, \|\eta_N\| \right) (W + W^T) \begin{pmatrix} \|\eta_1\| \\ \|\eta_2\| \\ \vdots \\ \|\eta_N\| \end{pmatrix} + \sum_{i=1}^{N}\sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j})\|\eta_i\| \\
&\leq -\frac{1}{2} \lambda_{min}(W + W^T)\|\eta\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j})\|\eta_i\| \\
&= -\frac{1}{2} \lambda_{min}(W + W^T) \sum_{i=1}^{N} \|\eta_i\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j}) \|\eta_i\| \\
&= -\frac{1}{2} \sum_{i=1}^{N} \left\{ \lambda_{min}(W + W^T)\|\eta_i\| - \sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j}) \right\} \|\eta_i\| \\
&\leq 0,
\end{aligned} \tag{44}$$

where $\|\eta\| := \|(\|\eta_1\|, \|\eta_2\|, \ldots, \|\eta_N\|)^T\|$, if

$$\|\eta_i\| \geq \frac{\sum_{j=1}^{N} \chi_{i4} C_i (L_{qi} + \kappa_{1j})}{\lambda_{min}(W)}.$$

Then, from theorem 4.18 in Reference 30, the variables η_i are bounded for $i = 1, 2, \ldots, N$.

Hence, the result follows. ■

Remark 12. From Remark 6, if $r_i^a = n_i$, the considered system can be fully linearized and thus the dynamical equation (31) disappears. In this case, Assumptions 4–5 are unnecessary, and the interconnection terms are completely matched. This can be regarded as a special case of the results developed in this article.

5 | SIMULATION EXAMPLE

Consider two inverted pendulums connected by a spring as shown in Figure 1. Each pendulum is controlled by a servomotor which provides a torque input u_i at the pivot. It is assumed that θ_i and $\dot{\theta}_i$ represent the angular position and velocity of the pendulums respectively for $i = 1, 2$. The model which describes the motion of the pendulums is given by (see, Reference 31)

$$\begin{aligned} \dot{x}_{1,1} &= x_{1,2}, \\ \dot{x}_{1,2} &= \frac{u_1}{J_1} + \beta_1(x_1)\tau_1(x_1) + \gamma_1(x_1)\delta_1(x) + \frac{kr}{2J_1}(l - b), \\ y_1 &= x_{1,1}, \end{aligned} \tag{45}$$

and

$$\begin{aligned} \dot{x}_{2,1} &= x_{2,2}, \\ \dot{x}_{2,2} &= \frac{u_2}{J_2} + \beta_2(x_2)\tau_2(x_2) + \gamma_2(x_2)\delta_2(x) - \frac{kr}{2J_2}(l - b), \\ y_2 &= x_{2,1}, \end{aligned} \tag{46}$$

where $x_{1,1} = \theta_1$, $x_{2,1} = \theta_2$, $x_{1,2} = \dot{\theta}_1$, and $x_{2,2} = \dot{\theta}_2$ are system states. It is assumed that $x_{1,1}$ and $x_{2,1}$ are measurable, which are system outputs.

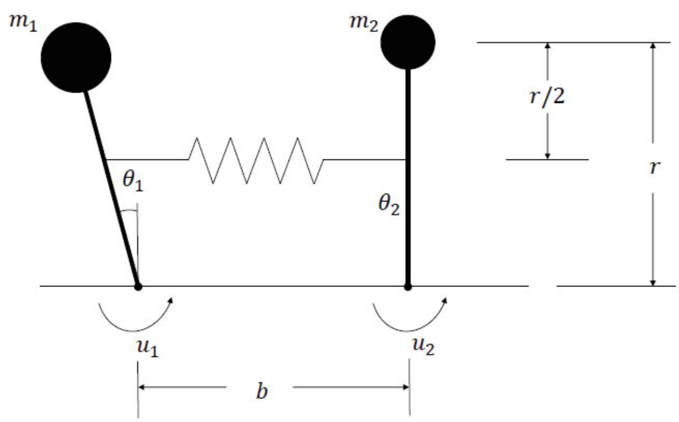

FIGURE 1 Two inverted pendulums connected by a spring

It should be pointed out that system (45)–(46) above has already been in the form of system (14) where

$$\beta_1 = \frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1}, \qquad \beta_2 = \frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2},$$
$$\tau_1(x_1) = sin(x_{1,1}), \qquad \tau_2(x_2) = sin(x_{2,1}),$$
$$\gamma_1(x_1) = \frac{kr^2}{4J_1}, \qquad \gamma_2(x_2) = \frac{kr^2}{4J_2},$$
$$\delta_1(x) = sin(x_{2,1}), \qquad \delta_2(x) = sin(x_{1,1}).$$

From Reference 31, the parameters are chosen as m_1 =2 kg and $m_2 = 2.5$ kg represent the end masses of the pendulum. $J_1 = 0.5$ kg m^2 and $J_2 = 0.625$ kg m^2 are the moments of inertia. $g = 9.81$ m/s^2 is the gravitational acceleration. $k = 100$ N/m is the spring constant of the connecting spring. $r = 0.5$ m is the pendulum height and $l = 0.5$ m is the natural length of the spring. The distance between the pendulum hinges is $b = 0.5$ m, where $b = l$.

By direct calculation

$$|\tau_1(x_1)| = |sin(x_{1,1})| \le 1 = \rho_1(x_1),$$

$$|\tau_2(x_2)| = |sin(x_{2,1})| \le 1 = \rho_2(x_2).$$

Here, both the value of σ_i for $i = 1, 2$ are designed as 0.1. It can be verified that the relative degree $r_i^a = r_i^b = 2$ for $i = 1, 2$. The nominal subsystems can be feedback linearized. For simulation purposes, the initial states are chosen as $x_{1,1}(0) = 1$ and $x_{2,1}(0) = -0.8$. And the desired output signals $y_{id}(t)$ are chosen as

$$y_{1d} = 0.5\sin(t), \qquad y_{2d} = 5e^{-t}. \tag{47}$$

It is clear that Assumption 3 is satisfied. Let

$$e_1 = y_1 - y_{1d}, \qquad e_2 = y_2 - y_{2d},$$
$$\dot{e}_1 = \dot{y}_1 - \dot{y}_{1d}, \qquad \dot{e}_2 = \dot{y}_2 - \dot{y}_{2d},$$
$$S_1 = \dot{e}_1 + a_1 \cdot e_1, \qquad S_2 = \dot{e}_2 + a_2 \cdot e_2, \tag{48}$$

where the sliding function parameters are chosen as $a_1 = 2$ and $a_2 = 3$. Then from (28), the control laws can be described by

$$u_1 = J_1 \left(-\dot{S}_1 + y_1^{(2)} - K_1(x_1)\mathrm{sgn}(S_1) \right) \tag{49}$$

and

$$u_2 = J_2 \left(-\dot{S}_2 + y_2^{(2)} - K_2(x_2)\mathrm{sgn}(S_2) \right), \tag{50}$$

where, based on (29), the value of the control gain $K_i(\cdot)$ is chosen as 19.72 for $i = 1, 2$. By direct calculation, Assumptions 4–5 as well as the conditions of Theorems 1–3 are satisfied. Therefore, the outputs of the closed-loop system formed by applying controllers (49)–(50) to the system (45)–(46) can track the desired signals in (47) asymptotically.

The tracking results are shown in Figure 2 with a good tracking performance as expected. Each angular position y_i of the subsystem can track the ideal reference y_{id} for $i = 1, 2$, at around 2 seconds despite the interactions between the subsystems. The time responses of the states of the system (45)–(46) are presented in Figure 3 where the system states are bounded. The simulation demonstrates that the results developed in this article are effective and in consistence with the theoretical results.

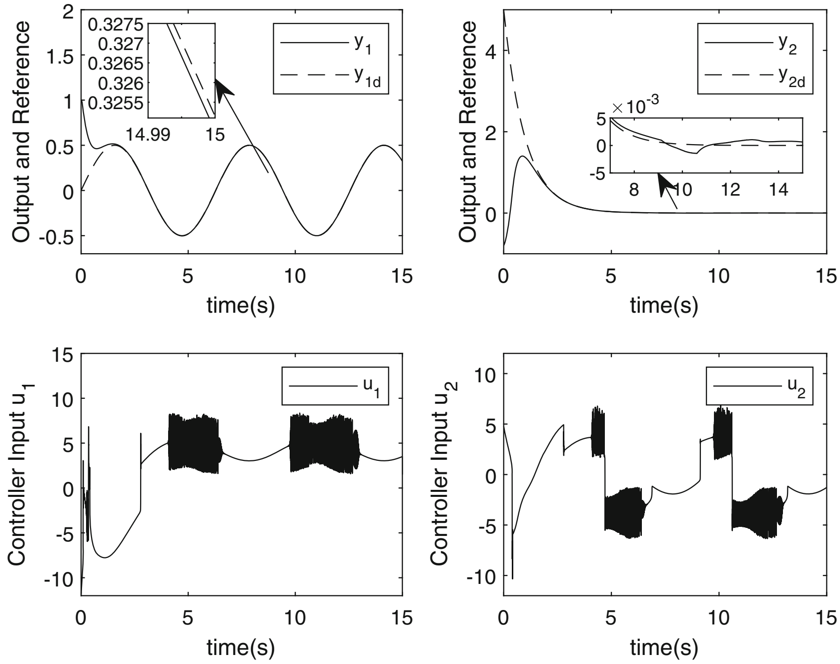

FIGURE 2 Time responses of system's output, the desired output (upper), and controller inputs of system (45)–(46)

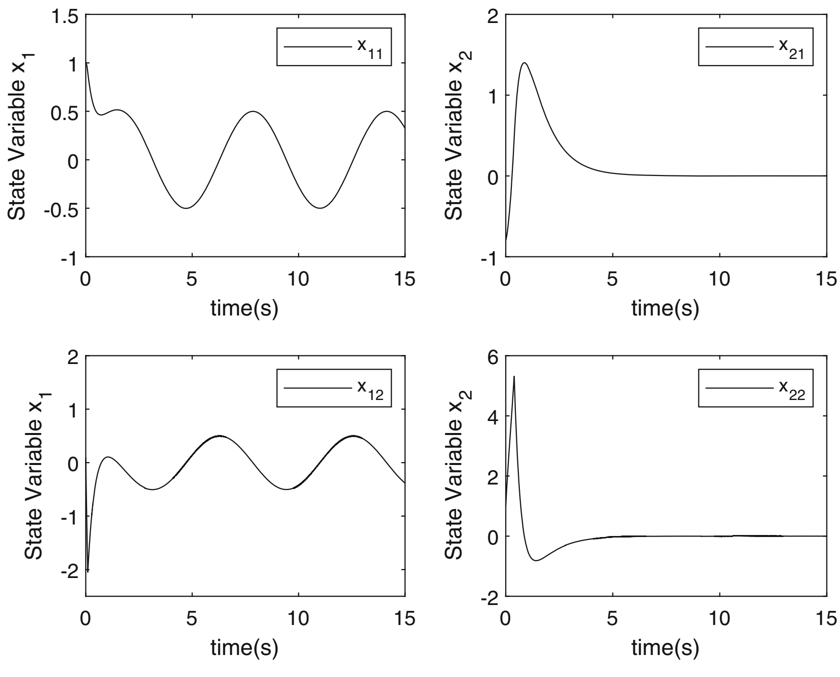

FIGURE 3 Evolution of state variables of system (45)–(46)

6 | CONCLUSIONS

A decentralized sliding mode control scheme for output tracking of a class of nonlinear interconnected systems has been proposed in this article. The developed results can guarantee asymptotic output tracking performance while the bounded state variables are maintaining across the closed-loop systems. The designed controllers are decentralized and the desired reference signals are time-varying. It is not required that either the interconnected system or the isolated subsystems of the interconnected systems are linearizable. Also, the developed results can be extended to the case when the isolated subsystems have multiple-input and multiple-output. Thus, the method developed in this article is suitable for a wide class of large-scale nonlinear interconnected systems.

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CONFLICT OF INTEREST

There are no conflict of interests for this article.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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