A cost-informed component maintenance index and its applications*

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Abstract: All systems and components are unreliable in the sense that they will fail. While a failed component in a system is being repaired, preventive maintenance (PM) may be conducted on the other components to improve the reliability of the system. The selection of different components for PM may result in a variety of maintenance policies with different cost implications. It is therefore necessary to develop appropriate tools such as importance measures to guide the selection of components for PM in order to minimize relevant cost. There is little research, nevertheless, that jointly minimizes the total expected cost of maintenance and meanwhile maximizes the number of components for PM. This paper proposes an importance index, Cost-Informed Component Maintenance Index (CICMI) to aid in such a joint optimization. It also derives some properties of the proposed index and different maintenance policies, respectively. Subject to cost constraints, a method is proposed to optimize the number of components for PM. A case study on a reactor coolant system is performed to illustrate the applicability of the proposed methods.

Keywords: Maintenance; Cost; Importance measure; Opportunistic maintenance; Optimization

1. Introduction

A large-scale complex system is normally composed of many different components. Due to components ageing, deterioration, and/or other reasons, the chance for a system to fail increases over time. Timely conducting preventive maintenance (PM) on those components is therefore needed to retain the system at a specified level of availability.

System performance and reliability can be improved by effectively planning maintenance interventions. With limited maintenance capacity in practice, it is often not possible to perform PM on every component in a system (Wu et al., 2016). Therefore, making the best use of available resources to maximize the reliability of

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a system is needed (Wu et al., 2016; Truong Ba et al., 2017; Tian et al., 2014). Recent developments have revealed that importance indexes in reliability engineering can provide valuable information to constantly optimize a specific objective function. For example, Peng et al. (2012) proposed an importance measure discussing how certain degraded components affect the reliability of the system. Some importance indexes have been proposed to guide PM, which may be used to maximize system performance (Dui et al., 2019), to aid maintenance selection (Wu and Coolen, 2013; Dui et al., 2017), or to perform maintenance for joint importance (Dui et al., 2020).

Predecessors have conducted research in the use of importance measure to select components for PM (Vaurio, 2011). Borgonovo et al. (2016) proposed an importance to guide PM considering mean failure time. Zhang et al. (2020) used Griffith importance and integrated importance to determine the maintenance sequence for the components of the heave compensation system. Dui et al. (2020) used joint integrated importance to guide PM so as to improve system performance as much as possible. Meshkat and Mahmoudi (2017) proposed a joint reliability importance to study maintenance problems. Chen and Feng (2020) proposed a new group importance measure for maintenance, which combines the system structure and the cost of PM. Du and Si (2019) proposed an importance to rank the contribution of component maintenance. At the same time, group maintenance can improve maintenance efficiency. Liu et al. (2014) proposed the importance measure of revenue cost to rank important components and gave the best maintenance level. Vu et al. (2016) proposed a preventive group maintenance policy considering the average remaining life of components. Ahmadi (2020) distinguished whether to perform preventive maintenance or corrective maintenance through component states, and then formulated a maintenance decision model based on the length of the life cycle.

While a component fails, other components can be selected for PM. This is an opportunistic maintenance policy, in which recent publications include (Li et al., 2021; Chien et al., 2019; Dui et al., 2017). Erguido et al. (2017) proposed a dynamic opportunistic maintenance policy in consideration of dynamic factors such as weather. Zhang et al. (2017) studied the opportunistic maintenance when the system was shut down for maintenance. Levitin et al. (2019) studied the opportunistic maintenance policy for the cost-effective scheduling to minimize the total expected loss. Jafari et al. (2018) developed models and algorithms for collecting corrective maintenance information. Then the opportunity preventive maintenance policy under corrective maintenance for component are given. Wu and Coolen **Error! Reference source not found.**

considered two types of costs for the opportunistic maintenance, one is the cost caused by system failure, and the other is the maintenance cost. However, it does not consider the joint influence between failure components and components for PM. The failure of certain components will cause the failure of the system, which depends on the specific system structure. Zhao et al., (2007) gave a opportunistic maintenance model considering the system structure and applied it to the sleeper system. The cost-based IIM proposed by Dui (2017) can also be used for opportunistic preventive maintenance, but it does not consider the influence of the structural position of the component in the system on the maintenance decision.

While a component failed and is being repaired, PM can be performed on some components. If the maintenance resource such as maintenance budget or the number of available repairmen is limited, it is necessary to maximize the performance of the system at the lowest cost. This raises a question: which components and how many components should be chosen for PM, considering the expected cost of maintenance? When different components fail, the selection of different components for PM may result in different maintenance policies. It is therefore intriguing to develop appropriate tools such as importance indexes to guide the selection of PM components in order to minimize the cost impact during the selection process.

This paper proposed a new importance index to guide the development of maintenance policies for the scenario when a system or a group of components fails, considering the cost of maintenance such as cost of repairing components, cost of repairing the system and cost of PM on components. The proposed method selects components for PM based not only on the maintenance cost of the components, but also on the extent to which components can improve the performance of the system at the lowest cost. Meanwhile, on the mathematical level, some properties are derived to gain a more in-depth understanding. The impact of critical and non-critical components on PM policies are considered. The joint impact of different components on system performance and maintenance costs is investigated. The paper also illustrates the application of this proposed index on series-parallel and parallel-series systems, and discusses the index combined with maintenance policies. Finally, a method to optimize the number of components for PM is proposed.

The remainder of this paper is structured as follows. Section 2 analyzes the total expected maintenance cost of the system. Section 3 proposes a Cost-Informed Component Maintenance Index (CICMI), derives some properties, and discusses the CICMI for series-parallel and parallel-series systems. Section 4 discusses several issues regarding the PM policies, and the optimization model of the CICMI for multiple components PM. Section 5 demonstrates the validity of the measure using a case of a reactor coolant system. Section 6 concludes the paper and proposes future research suggestions.

Notations

c_i	Cost per failure of component i
$c_{s,i}$	Expected cost per system failure caused by the failure to component i
c_{p_j}	PM cost of component j
$p_i(t)$	Reliability of component i
$\boldsymbol{p}(t)$	$(p_1(t), \ldots, p_n(t))$
x_i	Indicator: $x_i = 1$ if component i is working at time t, $x_i = 0$ otherwise
$(x_i, p_i(t))$	$(p_1(t), p_2(t), \ldots, p_{i-1}(t), x_i(t), p_1(t), p_{i+1}(t), \ldots, p_n(t))$
$\phi(X(t))$	System structure function with domain $\{0,1\}$ at time t, and range $\{0,1\}$, $\phi\big(X(t)\big) = \phi(x_1(t), x_2(t), \ldots, x_n(t))$
$\phi(x_i, p_i(t))$	$\phi(p_1(t), p_2(t), \ldots, p_{i-1}(t), x_i, p_1(t), p_{i+1}(t), \ldots, p_n(t))$
$Pr[x_i(t) = 1]$	The probability that component i works at time t
$C(t)$	Expected total cost of maintaining the system within time $(0, t)$
$C(0_i, t)$	Expected total cost of maintaining the system within time $(0, t)$ when component i fails; meanwhile, as a function of the reliability of other components, it can also be expressed as $C(0_i, p(t))$
$I_{j\|i}^{C}$	Cost-Informed Component Maintenance Index
$\boldsymbol{O}_i^{\boldsymbol{C}}$	The priority ranking of components selected for PM when component i fails

Assumptions

a) Suppose a system is composed of *n* components.

b) When maintenance (either PM or CM) is conducted on a component, the component must stop operating.

c) When the system fails due to the failure of a component, PM can be performed on other components.

d) Component states in this system are statistically independent of each other.

e) The system and components have only two states: working (i.e., state 1) and failed (i.e., state 0).

f) Components in the system can be critical or non-critical. The failure of a critical component will cause the system to fail while the failure of a non-critical one will not.

2 The total expected maintenance cost of the system

Suppose that the failure of a component can be announced immediately, i.e, it is a self-announcing component. Only the failure of a critical component can cause the system to fail. Denote $c_{s,i}$ as the cost of system failure and repair due to component i. If the failed component is not a critical one, its failure will not cause the system to fail and it will not incur the cost of system failure, but only incur cost of repairing this failed component or cost of PM on other components. Hence, in addition to the cost of PM, the total expected maintenance cost of the system within time interval $(0, t)$ is given by

$$C(t) = \sum_{i=1}^{n} \left\{ \{ c_{s,i} \Pr[\phi(0_i, \mathbf{1}) = 0] + c_i + C_i^P(t) \} \Pr[x_i(t) = 0] \right\}, \quad (1)$$

where c_i is the cost of repairing the failed component i. $\Pr[\phi(0_i, X(t)) = 0]$ is the probability that the system is at state 0 when component i fails. $\Pr[x_i(t) = 0]$ is the probability that component i fails and is a function of time. $C_i^P(t)$ represents the expected cost of PM on other components when component i fails.

PM can be performed on components that have failed while CM (repair) is performed on failed components. If PM is performed on a component, j, say, then this implies that the component is at working state and its probability is $Pr[x_j(t) = 1]$. We now discuss the expressions of $C_i^P(t)$ for two situations of critical and non-critical components.

Assuming component i is critical, if it fails, the expected cost of PM on other components is obtained by

$$C_i^P(t) = \sum_{j=1, j \neq i}^{n} c_{p_j} Pr[x_j(t) = 1] \Pr[\phi(0_i, 0_j, \mathbf{1}_{ij}) = 0] = \sum_{j=1, j \neq i}^{n} c_{p_j} Pr[x_j(t) = 1], \quad (2)$$

where c_{p_j} is the cost of PM on component j. $Pr[\phi(0_i, 0_j, \mathbf{1}_{ij}) = 0]$ is the probability that the system is at state 0 when both components i and j fail, but the others are working. Because component i is critical, $Pr[\phi(0_i, 0_j, \mathbf{1}_{ij}) = 0] = 1$.

However, if a non-critical component, i, say, fails, the number of other components for PM is limited. Then, PM can be performed on a component, j, say, which should not lead to the system to stop operating or result in unnecessary cost of system failure. Thus, component j should meet two conditions: it is not a critical component and components i and j do not form a cut set.

Assume component i is non-critical. When it fails, the cost of PM on other components is obtained by

$$C_i^P(t) = \sum_{j=1,j\neq i}^{n} c_{p_j}\, Pr\left[x_j(t) = 1\right] \Pr\left[\phi\left(0_i, 0_j, \mathbf{1}_{ij}\right)=1\right], \tag{3}$$

where $(0_i, 0_j, \mathbf{1}_{ij})$ represents both components i and j stop working while the other components are working. The system is still working if a non-critical component fails. So $\Pr\left[\phi(0_i, 0_j, \mathbf{1}_{ij})=1\right]$ is the probability that the system does not fail when component j is selected for PM, which implies that component j should meet two aforementioned conditions.

We can derive some properties of the total expected maintenance cost of the system as follows.

Property 1

(1) Assume that the maintenance cost and properties of components i and l are the same. If $Pr[x_i(t) = 1] > Pr[x_l(t) = 1]$, then $C(0_i, t) > C(0_l, t)$.

(2) Assume that the reliabilities of components i and l are the same. If $c_{s,i}\, Pr[\phi(0_i, \mathbf{1}) = 0] + c_i > c_{s,l}\, Pr[\phi(0_l, \mathbf{1}) = 0] + c_l$ and, $c_{p_i} < c_{p_l}$, then $C(0_i, t) > C(0_l, t)$.

The proof of this property and those of the other properties are given in Appendix.

The first part in Property 1 shows that the expected cost function caused by a component failure relates to the reliability of the failed components. When all other conditions are kept constant, the higher the reliability of the failed components, the greater the expected total cost of the system due to the failure of the components.

The second part in Property 1 shows that when component reliability is not considered, the cost of repairing a component and the system due to the failure of a component will positively affect the expected total cost of the system. However, the PM cost of the failed components will negatively affect the total cost, which can be interpreted as follows: If the cost of PM on a component is low, it is not suitable for CM. In other words, when a component fails, the total expected maintenance cost is higher than the PM cost on other components.

3 Cost-informed component maintenance index

3.1 Definitions and some properties

This section studies the impact of the cost of maintaining non-failed components on the cost of system maintenance while a failed component is being repaired. We first investigate the situation of the failed components: whether the failed component is a critical component or not.

Definition 1 Assuming component i is critical, when it fails, the impact of component j on the maintenance cost of the system can be defined by

$$I_{j|i}^{C} = -\frac{\partial C(0_i, p(t))}{\partial p_j(t)}. \tag{4}$$

Definition 2 Assuming component i is non-critical, when it fails, the impact of component j on the maintenance cost of the system can be defined by

$$I_{j|i}^{C} = -\,\phi(0_i, 0_j, \mathbf{1}_{ij}) \frac{\partial C(0_i, p(t))}{\partial p_j(t)}, \tag{5}$$

where $(0_i, 0_j, \mathbf{1}_{ij})$ represents components i and j stop working and all the other components are working; $\phi(0_i, 0_j, \mathbf{1}_{ij})$ is able to restrict critical components so that critical components cannot be selected for PM.

We refer the $I_{j|i}^{C}$ of component i to as the Cost-Informed Component Maintenance Index (CICMI). If a component i fails, CICMI suggests the magnitude of the impact of the cost due to maintaining component j on the expected total cost when component j is selected for PM. One may then rank the values of $I_{j|i}^{C}$ in ascending order, which prioritizes components for PM to reduce the total cost of maintaining the system.

When component i fails, other components can be ranked in terms of the priority for PM according to $I_{j|i}^{C}$. The best CICMI matrix for PM based on cost can be given by

$$\boldsymbol{J_i} = \left[I_{j_{(1)}|i}^{C}, I_{j_{(2)}|i}^{C}, \dots, I_{j_{(n-1)}|i}^{C}\right], \tag{6}$$

where $I_{j_{(k)}|i}^{C}$ decreases in k. $\boldsymbol{J_i}$ suggests which component j may be selected for PM so that the total cost of maintaining the system can be minimized, given that component i has failed.

The priority ranking of components selected for PM can be given by

$$\boldsymbol{O_i^C} = \{j_{(1)}, j_{(2)}, \dots, j_{(n-1)}\}, \tag{7}$$

where $j_{(n-1)}$ represents component j corresponding to $\boldsymbol{J_i}$ at the $n-1$ position. This set can help maintenance analysts make judgments. They need to consider not only the cost of maintaining each individual component but also the cost of maintaining the entire system. PM decisions can be made more specifically for the failure of different components.

Similar to Property 1, we can derive some properties of CICMI as follows.

Property 2

(1) Assume that components j and h have the same reliability and maintenance cost. When component i fails, if $p_j(t) < p_h(t)$, then $I^C_{j|i} \geq I^C_{h|i}$.

(2) Assume that the reliability values of components j and h are the same. When a component i fails, if $c_{p_h} > c_{p_j}$, $c_{s,j}\, Pr\left[\phi\left(0_j, \mathbf{1}\right) = 0\right] + c_j > c_{s,h}\, Pr[\phi(0_h, \mathbf{1}) = 0] + c_h$, then $I^C_{j|i} \geq I^C_{h|i}$.

The first part in Property 2 means that when we do not consider the cost, it is a better decision for us to maintain the current components with lower reliability. The second part in Property 2 means that when reliability is not considered, it is more appropriate to perform PM on components with lower PM cost but higher in the sum of the cost of system failure and the cost of repairing failed components.

Property 3 For the failure situations of two different components, assume that (1) the states of the remaining components except for the two components remain the same, and (2) only one of components i and l may fail. If $p_i(t) < p_l(t)$, then $I^C_{j|i}(x_l(t) = 1) > I^C_{j|l}(x_i(t) = 1)$.

The meaning of Property 3 in reality can be expressed as repairing components with lower reliability is more valuable.

3.2 Discussions on series-parallel and parallel-series systems

This subsection uses two series-parallel systems and a parallel-series system to illustrate the proposed method, as shown in Figs. 1, 2, and 3, respectively.

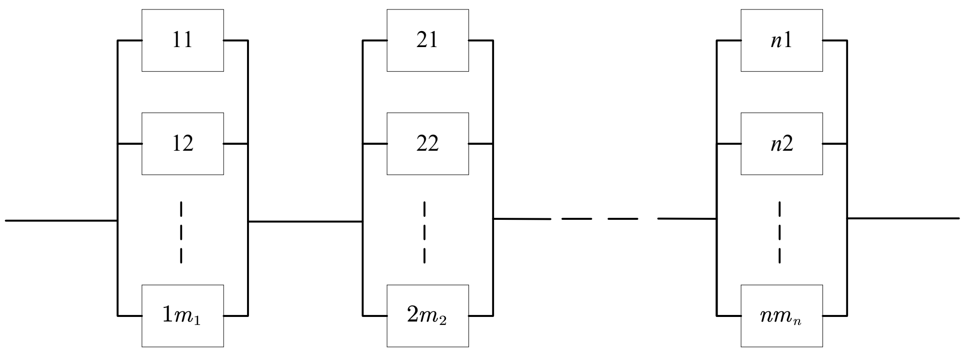


Fig. 1 A series-parallel system

In Fig.1, we can see that there is no critical component in the series-parallel system and $c_{s,i}$ will not be incurred. The expected cost of the system is

$$C(t) = \sum_{i=1}^{n} \left\{ \{c_i + C_i^P(t)\}\, Pr[x_i(t) = 0] \right\}. \quad (8)$$

The expected cost is derived as follows

$$C(t) = \{c_{11} + C_{11}^P(t)\}(1 - p(x_{11})) + \{c_{12} + C_{12}^P(t)\}(1 - p(x_{12})) + \cdots + \{c_{nm_n} + C_{nm_n}^P(t)\}\left(1 - p\left(x_{nm_n}\right)\right).$$

The total number of components is $m_1 + m_2 + \cdots + m_n = \sum_{a=1}^{n} m_a = M$. We denote $x_i = x_j = x_k = \{x_{11}, x_{12}, \ldots, x_{1m_1}, x_{21}, x_{22}, \ldots, x_{2m_2}, \ldots, x_{n1}, \ldots, x_{nm_n}\}$, where $i = j = k = 1,2,\ldots,M$. That is, the components are numbered according to the order of the parallel group set, from top to bottom, and from left to right as 1, 2, 3,..., M.

If $m_a > 2$, for $a = 1,2,\ldots,n,$, or if there is no second-order cut set in the system, then for any component i and component j, $\phi(0_i, 0_j, \mathbf{1}_{ij}) = 1$ holds. Plugging it into Equation (3) and Equation (5), we have

$$C_i^P(t) = \sum_{j=1, j\neq i}^{n} c_{p_j} Pr[x_j(t) = 1], \tag{9}$$

and

$$I_{j|i}^C = -\frac{\partial C(0_i, p(t))}{\partial p_j(t)} = -\frac{\partial \left\{ \begin{matrix} \{c_1 + C_1^P(t)\}(1 - p(x_1)) + \{c_2 + C_2^P(t)\}(1 - p(x_2)) + \cdots \\ + \{c_{i-1} + C_{i-1}^P(t)\}(1 - p(x_{i-1})) + \{c_{i+1} + C_{i+1}^P(t)\}(1 - p(x_{i+1})) \end{matrix} \right\}}{\partial p_j(t)}$$

$$= c_j + C_j^P(t) - c_{p_j}\{\sum_{k=1,k\neq j,i}^{M}(1 - p_k(t)) + 1\} = c_j + \sum_{k=1,k\neq i,j}^{M} c_{p_k} p_k(t) - c_{p_j}\{\sum_{k=1,k\neq j,i}^{M}(1 - p_k(t)) + 1\}. \tag{10}$$

If $m_a = 2$, $a = 1,2,\ldots,n$, i.e., two components are connected in parallel, as shown in Fig.2.

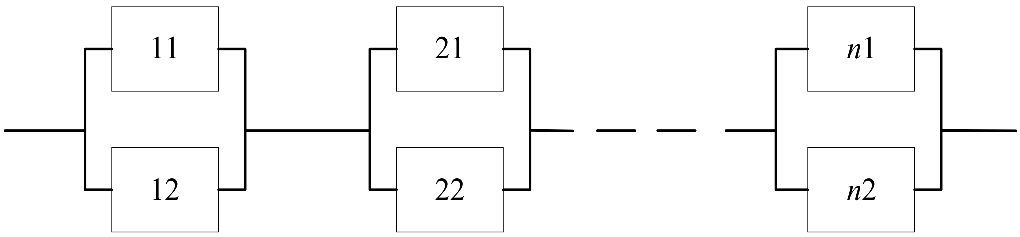


Fig. 2 series-parallel system when $m_a = 2$

If one of the two components in parallel fails, PM cannot be carried out. The component in parallel with component j is component o. Then we obtain

$$I_{j|i}^C = -\frac{\partial C(0_i, p(t))}{\partial p_j(t)}$$

$$= -\frac{\partial \left\{ \{c_{i1} + C_{i1}^P(t)\}(1 - p_{i1}(t)) + \{c_{i2} + C_{i2}^P(t)\}(1 - p_{i2}(t)) + \cdots + \{c_{iM} + C_{iM}^P(t)\}(1 - p_{iM}(t)) \right\}}{\partial p_j(t)}$$

$$= c_j + \sum_{k=1,k\neq i,j,o}^{2n} c_{p_k} p_k(t) - c_{p_j}\left\{ \sum_{k=1,k\neq j,i,o}^{2n} (1 - p_k(t)) + 1 \right\}. \tag{11}$$

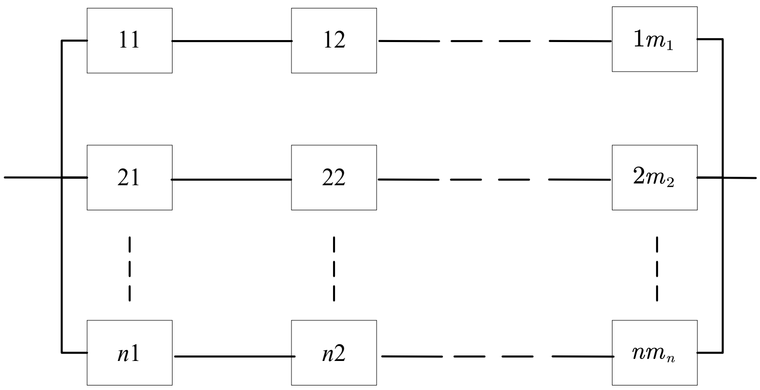


Fig. 3 parallel-series system

In Fig. 3, we give the structure of the parallel system. Apparently, if a component has failed and PM is performed on another component, the system will not fail. This case is similar to the series -parallel system for $m_a > 2$. i.e., $I^C_{j|i} = c_j + \sum_{k=1,k\neq i,j}^{M} c_{p_k} p_k(t) - c_{p_j}\left\{\sum_{k=1,k\neq j,i}^{M}\left(1 - p_k(t)\right) + 1\right\}$, where $i = j = k = \left\{x_{11}, x_{12}, \dots, x_{1m_1}, x_{21}, x_{22}, \dots, x_{2m_2}, \dots, x_{n1}, \dots, x_{nm_n}\right\}$.

4 Optimization model of CICMI for PM policies

4.1 Analysis of PM policies

If a component is non-critical, the failure will not cause the system to fail. This leads to Policy 1 for system failure. Whatever a component is non-critical or critical, the failure of a component can be immediately located. This leads to Policy 2 for component failure. In the following, we give detailed discussions.

Policy 1. The failure of a component cannot be immediately identified. But if the system fails, repairmen will check the system to locate the cause of the failure. The reason for the failure of the system may be due to the failed components containing critical components, or it may be due to the failed components being all non-critical components but constituting a cut set. Assume that there are n_0 minimal cut sets in the system. The set of components $i_1, i_2, \dots, i_{m_i}$ is the ith minimum cut set. The number of components in the cut set is m_i. This means that the failure of all components of this set will cause the system to fail. Therefore, when components $i_1, i_2, \dots, i_{m_i}$ fail, they are maintained and the system has to stop working. In this situation, PM can be performed on other components.

Generally, under Policy 1, the system fails because at least one minimal cut set failed, and the PM can be performed on all other components. The cost incurred in time interval $(0, t)$ is composed of three components: the cost of repairing the system due to the failures of the n_0 minimal cut sets, the cost of repairing each component in the failed minimal cut sets, and the cost of all components in the system except the component in the failed minimal cut sets. Thus, we have

$$C(t) = \sum_{i=1}^{n_0} \left\{ c_{s,i} + \sum_{j=1}^{m_i} c_{i_j} + \sum_{z=1}^{n-m_i} c_{p_z} Pr[x_z = 1, t] \right\} Pr\left[0_{i_1}, 0_{i_2}, \dots, 0_{i_{m_i}}\right], \quad (12)$$

where $c_{s,i}$ is the expected cost per system failed, due to the failure of the ith minimum cut set, and c_{i_j} is the expected cost per failure of component i_j. $\sum_{z=1}^{n-m_i} c_{p_z}$ represents the sum of cost on PM for components that are not included in the ith cut set. $\Pr\left[(0_{i_1}, 0_{i_2}, \dots, 0_{i_{m_i}})\right]$ represents the probability that the cut set $\{i_1, i_2, \dots, i_{m_i}\}$ failed. When a set of components composed of a cut set fails, the impact of a component j on the maintenance cost of the system can be defined by

$$I^C_{j|i_1,i_2,\dots,i_{m_i}}(t) = \frac{-\partial C(0_{i_1}, 0_{i_2}, \dots, 0_{i_{m_i}}, p(t))}{\partial p_j(t)}. \quad (13)$$

For specific application scenarios, if components $\{i_1, i_2, \dots, i_{m_i}\}$ fail, we can choose the component with the largest $I^C_{j|i_1,i_2,\dots,i_{m_i}}(t)$ as the first component to perform PM because this component can minimize the total cost. Then, according to the ranking of $I^C_{j|i_1,i_2,\dots,i_{m_i}}(t)$, we can select components for PM.

Similar to Property 3, we give the property of CICMI under Policy 1 as follows.

Property 4 Suppose a system failure is caused by the failures of two different cut sets. Assume that (1) the states of the remaining components except for the components included in cut sets k and l remain the same, (2) component j does not participate in forming the minimum cut sets k and l, and (3) only one of cut sets k and l can cause the system to fail. If $Pr\left[0_{k_1}, 0_{k_2}, \dots, 0_{k_{m_k}}\right] > Pr\left[0_{l_1}, 0_{l_2}, \dots, 0_{l_{m_l}}\right]$, then $I^C_{j|k_1,k_2,\dots,k_{m_k}}(t) > I^C_{j|l_1,l_2,\dots,l_{m_l}}(t)$.

Policy 2. The failure of a component can be immediately identified. While CM on the failed component is being performed, it can perform PM on other components. If the failed component is critical, PM can be performed on all other components. However, if the failed components are non-critical components, it requires that the components with PM cannot form a cut set.

If components with PM performed constitute cut sets, then the system will fail and results in increased maintenance cost. The total expected system cost function in which multiple components can be performed on maintenance is given by

$$C^S(t) = \sum_{i=1}^{n} \left\{ \left\{ c_{s,i}\, Pr[\phi(0_i, \mathbf{1}) = 0] + c_i + C_i^{SP}(t) \right\} Pr[x_i(t) = 0] \right\}, \tag{14}$$

$$C_i^{SP}(t) = \sum_{z=1}^{m} c_{p_{j_z}}\, Pr\left[x_{j_z}(t) = 1\right] Pr\left[\phi\left(0_i, 0_{j_1}, \dots, 0_{j_{z-1}}, \mathbf{1}_{i,j_1,j_2,\dots,j_{z-1}}\right) = 1\right], \tag{15}$$

where $C_i^{SP}(t)$ represents the PM cost of the system when component i fails. The maximum number of components to be performed on PM is m. $(0_i, 0_{j_1}, \dots, 0_{j_{z-1}}, 1_{i,j_1,j_2,\dots,j_{z-1}})$ represents those components $i, j_1, j_2, \dots, j_{z-1}$ stop working while all the other components are working. Therefore, Equation (14) takes into account the situation of the multiple components' PM at the same time, which is related to the component structure in the system and can affect the PM plan.

Similarly, we can obtain the CICMI under the condition of the multiple components' maintenance at the same time. Assuming component i is critical or non-critical, when it fails, the impact of component j on the maintenance cost of the system can be defined by

$$I_{j|i}^{SC} = -\frac{\partial C^S\left(0_i, p(t)\right)}{\partial p_j(t)}, \tag{16}$$

and

$$I_{j|i}^{SC} = -\,\phi\left(0_i, 0_j, \mathbf{1}_{ij}\right) \frac{\partial C^S\left(0_i, p(t)\right)}{\partial p_j(t)}, \tag{17}$$

respectively.

4.2 Optimization model for multiple components' PM

We assume PM is performed on only two components. When the component i, which can be critical or non-critical, fails, the impact of components j and k on the maintenance costs of the system can be defined by

$$I_{j,k|i}^{C} = -\frac{\partial^2 C\left(0_i, p(t)\right)}{\partial p_j(t) \partial p_k(t)}, \tag{18}$$

and

$$I_{j,k|i}^{C} = -\phi\left(0_i, 0_j, \mathbf{1}_{ij}\right) \phi(0_i, 0_k, \mathbf{1}_{ik})\, \frac{\partial^2 C\left(0_i, p(t)\right)}{\partial p_j(t) \partial p_k(t)}, \tag{19}$$

respectively, where components j and k are any two indexes of the $n-1$ components, then there are C_{n-1}^2 combinations. Different sets of components j, k and l will get the corresponding $I_{j,k|i}^{C}$. The combinations are ranked according to the descending order of the value of $I^C_{j,k|i}$ to form a set $\boldsymbol{O}^C_{2|i}$. $\boldsymbol{O}^C_{2|i} = \left\{\{j,k\}_{(1)}, \{j,k\}_{(2)}, \dots, \{j,k\}_{(C^2_{n-1})}\right\}$, where $\{j,k\}_{(1)}$ indicates that the combination of components j and k maximizes the value of $I^C_{j,k|i}$**.** Selecting the first combination of PM can reduce the system maintenance cost to the greatest extent.

PM is performed on only three components. Similarly, we can obtain

$$I^C_{j,k,l|i} = -\frac{\partial^3 C\left(0_i, p(t)\right)}{\partial p_j(t)\partial p_k(t)\partial p_l(t)}, \tag{20}$$

and

$$I^C_{j,k,l|i} = -\phi\left(0_i, 0_j, \mathbf{1}_{ij}\right)\phi\left(0_i, 0_k, \mathbf{1}_{ik}\right)\phi\left(0_i, 0_l, \mathbf{1}_{il}\right)\frac{\partial^3 C\left(0_i, p(t)\right)}{\partial p_j(t)\partial p_k(t)\partial p_l(t)} \tag{21}$$

where components j, k,and l are any three of the $n-1$ components, then there are C^3_{n-1} combinations. The combinations are obtained according to the descending order of the value of $I^C_{j,k,l|i}$ to form a set $\boldsymbol{O}^C_{3|i} = \left\{\{j,k,l\}_{(1)}, \{j,k,l\}_{(2)}, \dots, \{j,k,l\}_{(C^3_{n-1})}\right\}$, where $\{j,k,l\}_{(1)}$ indicates that the combination of components j, k and l that maximise $I^C_{j,k,l|i}$, $\{j,k,l\}_{(2)}$ and indicates that the combination of components j, k and l that make $I^C_{j,k,l|i}$ the second largest, and so on.

When performing PM, we often not only perform PM on one component, but also on multiple components to improve system reliability. As such, we investigate the impact on the total cost of maintaining a group of components. Generalizing to k components, we can also obtain

$$I^C_{j_1,j_2,\dots,j_k|i} = -\frac{\partial^k C\left(0_i, p(t)\right)}{\partial p_{j_1}(t)\partial p_{j_2}(t)\dots\partial p_{j_k}(t)}, \tag{22}$$

and

$$I^C_{j_1,j_2,\dots,j_k|i} = -\phi\left(0_i, 0_{j_1}, \mathbf{1}_{ij_1}\right)\phi\left(0_i, 0_{j_2}, \mathbf{1}_{ij_2}\right)\dots\phi\left(0_i, 0_{j_k}, \mathbf{1}_{ij_k}\right)\frac{\partial^k C\left(0_i, p(t)\right)}{\partial p_{j_1}(t)\partial p_{j_2}(t)\dots\partial p_{j_k}(t)}, \tag{23}$$

respectively, where $j_1, j_2, \dots, j_k (k < n-1)$ are any k of the $n-1$ components. There are C^k_{n-1} combinations. The combinations are ranked according to the descending order of the value of $I^C_{j_1,j_2,\dots,j_k|i}$ to form a set $\boldsymbol{O}^C_{k|i} = \left\{\{j_1,j_2,\dots,j_k\}_{(1)}, \{j_1,j_2,\dots,j_k\}_{(2)}, \dots, \{j_1,j_2,\dots,j_k\}_{(C^k_{n-1})}\right\}$, where $\{j_1,j_2,\dots,j_k\}_{(1)}$ indicates that the combination of components $j_1, j_2, \dots, j_k$ that make $I^C_{j_1,j_2,\dots,j_k|i}$ the largest**.**

It is still considered that maintenance should be carried out one by one. From the perspective of the system, what is the optimal number of components for PM? Consequently, we may as well compare the combinations in sets $\boldsymbol{O}^C_{1|i}$, $\boldsymbol{O}^C_{2|i}$,…, $\boldsymbol{O}^C_{k|i}$ according to the corresponding CICMI to select the maximum value.

In applications, it may be more common to consider specific cost constraints in maintenance policies. Therefore, this article gives the arrangement of maintaining components according to cost constraints.

When one component fails, to perform PM on other components one by one, we need to solve the following integer programming.

$$max \sum_{j=1, j\neq i}^{n} I^C_{j|i} z_j, \tag{24}$$

subject to $\sum_{j=1,j\neq i}^{n} c_j z_j \leq C$,

where c_j is the cost of the PM component j, C is the total cost constraint of PM, z_j is the decision variable representing whether component j should be chosen for PM and it can only be 0 or 1. Subsequently, $\sum_{j=1,j\neq i}^{n} z_j$ is expressed as the total number of components that are performed on PM. For policy 1 above, the integer programming can be changed to:

$$max \sum_{j=1, j\neq i_1, i_2, \ldots, i_{m_i}}^{n} I^C_{j|i_1, i_2, \ldots, i_{m_i}} z_j, \tag{25}$$

subject to $\sum_{j=1,j\neq i_1,i_2,\ldots,i_{m_i}}^{n} c_j z_j \leq C$.

This integer programming can solve the problem of PM plan when cut sets cause the system to fail.

The above integer programming problem cannot solve the optimization of the number of components for PM. For policy 2, when considering multiple components for PM, the difference between the selection of maintenance components here and the selection of the above maintenance plan lies in the need to investigate whether the failed components are critical components. If a critical component, i, say, fails, PM can be performed on the other components. When component i is a non-critical component, the other non-critical components for PM cannot form a cut set and cause system failure. Subsequently, we construct the corresponding integer programming model.

$$max\ \phi\left(0_i, x_{j_1}, \ldots, x_{j_n}, \mathbf{1}_{i,j_1,j_2,\ldots,j_n}\right) \sum_{j=1, j\neq i}^{n} I^{SC}_{j|i} z_j, \tag{26}$$

subject to $\sum_{j=1,j\neq i}^{n} c_j z_j \leq C$,

where z_j indicates that the PM component j is selected, then it is 1, otherwise it is

0. $\phi(0_i, x_{j_1}, \dots, x_{j_n}, \mathbf{1}_{i,j_1,j_2,\dots,j_n})$ ensures that the selected PM components will not cause system to fail.

5 Application to reactor coolant system

In this section, we use a reactor coolant system (Fig. 4) to illustrate the proposed method. As the most important system of nuclear power plants, the main coolant system may cause huge economic losses and serious social impact if a critical component fails. PM on the main coolant system can reduce the probability of system failure, reduce maintenance cost, and therefore avoid the negative impact on society. This section uses the proposed CICMI to analyze the reactor coolant system and further illustrates the application and effectiveness of CICMI.

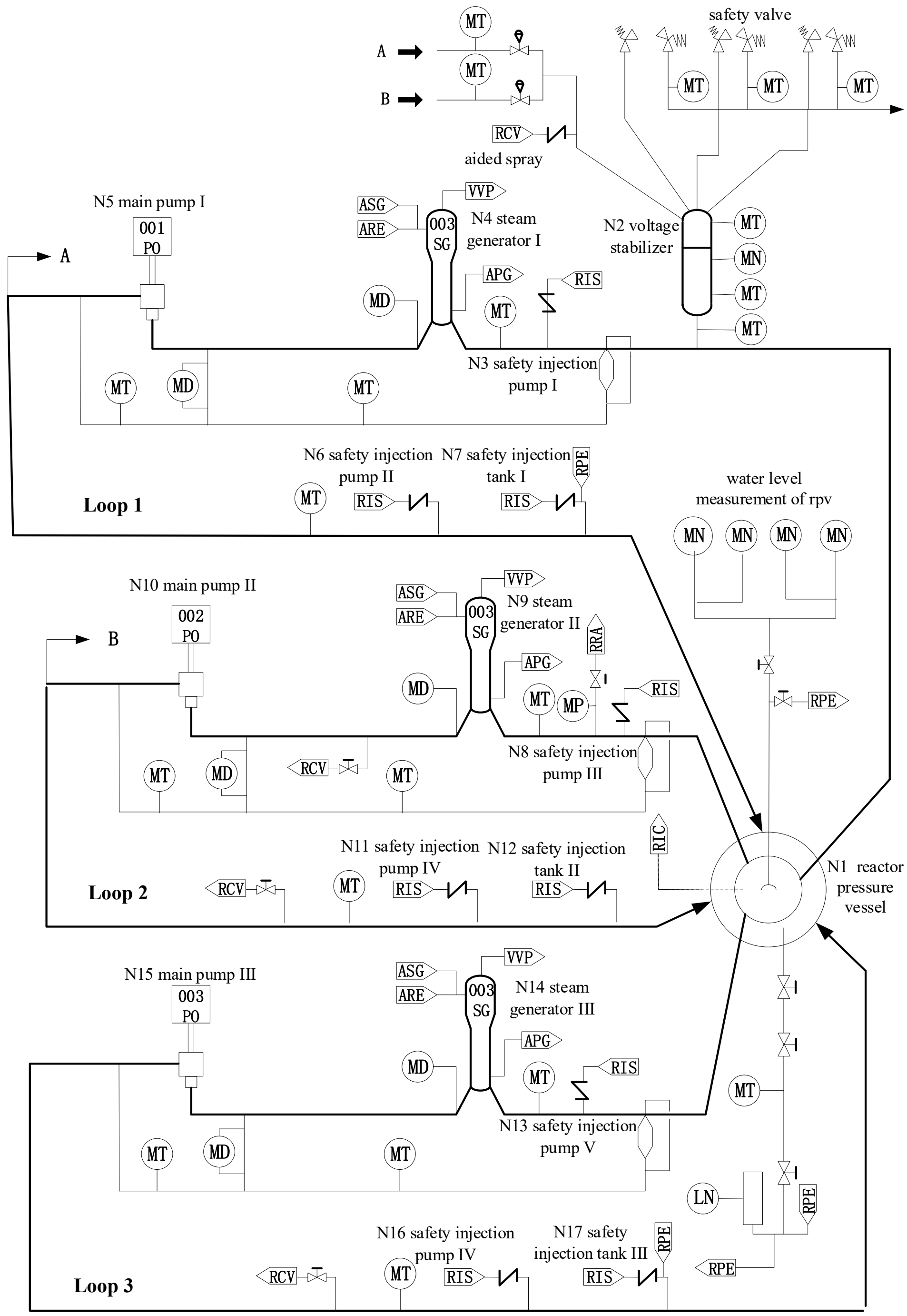


Fig. 4 Schematic diagram of main coolant system

Fig. 4 shows the system components of the reactor coolant system. The structure of the main coolant system is quite complex. It can be seen from Fig. 4 that there are three loops in the system. While the whole system is still running and if a component in one loop fails, the two other loops are operating and will not be affected by the failure of the component.

Table 1 lists the 17 main components of the system. Among the main components, loop 1 consists of components N2, N3, N4, N5, N6, and N7. Loop 2 consists of components N8, N9, N10, N11, and N12. Loop

3 consists of components N13, N14, N15, N16 and N17. The components of a loop are connected in series, and the loops are in parallel. Component N1 is a critical component, so its failure causes the entire system to fail. The repair cost, system failure cost, and PM cost for each component are shown in Table 2.

Table 1 The major components

Code	Name	Code	Name
N1	pressure reactor reaction vessels	N10	main pump II
N2	voltage stabilizer	N11	safety injection pump IV
N3	safety injection pump I	N12	safety injection tank II
N4	steam generator I	N13	safety injection pump V
N5	main pump I	N14	steam generator III
N6	safety injection pump II	N15	main pump III
N7	safety injection tank I	N16	safety injection pump IV
N8	safety injection pump III	N17	safety injection tank III
N9	steam generator II		

Table 2 Reactor coolant system related costs

NO.	Code	$c_{s,i}$	c_i	c_{pi}	NO.	Code	$c_{s,i}$	c_i	c_{pi}
1	N1	27896	33157	21364	10	N10	32562	29875	15533
2	N2	23562	25752	17654	11	N11	13245	12864	8873
3	N3	13245	12864	8873	12	N12	29345	13453	10743
4	N4	35623	22245	11863	13	N13	13245	12864	8873
5	N5	32562	29875	15533	14	N14	35623	22245	11863
6	N6	13245	12864	8873	15	N15	32562	29875	15533
7	N7	29345	13453	10743	16	N16	13245	12864	8873
8	N8	13245	12864	8873	17	N17	29345	13453	10743
9	N9	35623	22245	11863					

The Weibull distribution is widely used in reactor systems (Prabhakar et al., 2004). We assume that the failure time of component i follows the two-parameter Weibull distribution $W\ (t;\ \theta_{1i}, \gamma_{1i})$, and repair time follows the Weibull distribution $W\ (t;\ \theta_{2i}, \gamma_{2i})$. Table 3 lists the scale and shape parameters of each component's time to failure and repair time.

Table 3 The scale and shape parameters of each component's failure time and repair time.

NO.	Code	θ_{1i}	γ_{1i}	θ_{2i}	γ_{2i}	NO.	Code	θ_{1i}	γ_{1i}	θ_{2i}	γ_{2i}
1	N1	2150	3.46	12	2.78	10	N10	880	2.14	20	2.13
2	N2	250	3.92	7	2.12	11	N11	2600	2.03	8	2.37
3	N3	2600	2.03	8	2.37	12	N12	180	2.43	10	2.23
4	N4	100	2.36	5	3.12	13	N13	2600	2.03	8	2.37
5	N5	880	2.14	20	2.13	14	N14	100	2.36	5	3.12
6	N6	2600	2.03	8	2.37	15	N15	880	2.14	20	2.13
7	N7	180	2.43	10	2.23	16	N16	2600	2.03	8	2.37
8	N8	2600	2.03	8	2.37	17	N17	180	2.43	10	2.23
9	N9	100	2.36	5	3.12						

The CICMI of a component relates to its own PM costs and repair cost when a component fails. As maintenance cost increase, CICMI increases. This implies that if a component fails and incurs higher maintenance cost, it is more valuable to perform PM. When a component fails, the CICMI relates to the component location in the system. The impact of the component location for PM on the value of CICMI depends only on whether the component is a critical component or not. In the following, we will conduct a more specific analysis on the CICMI under the failure of critical components and non-critical components, as shown in Fig. 5 and Fig.6.

Fig. 5 is for the critical component N1 (pressure reactor reaction vessels). Because the CICMI of the components of the same type are the same, we only select one of the same types for drawing. Fig. 6 is for the non-critical components N2 (voltage stabilizer), N3 (safety injection pump), N4 (steam generator), N5 (main pump), and N7 (safety injection tank).

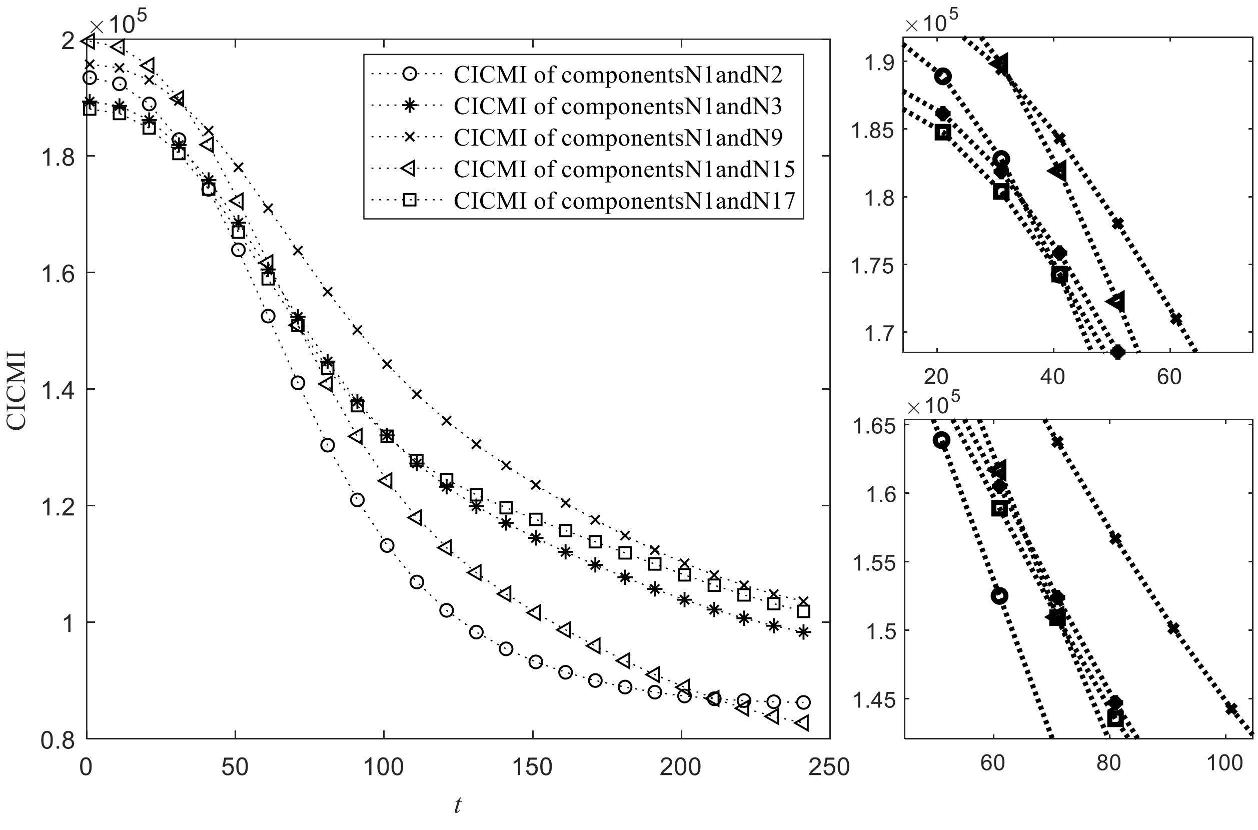


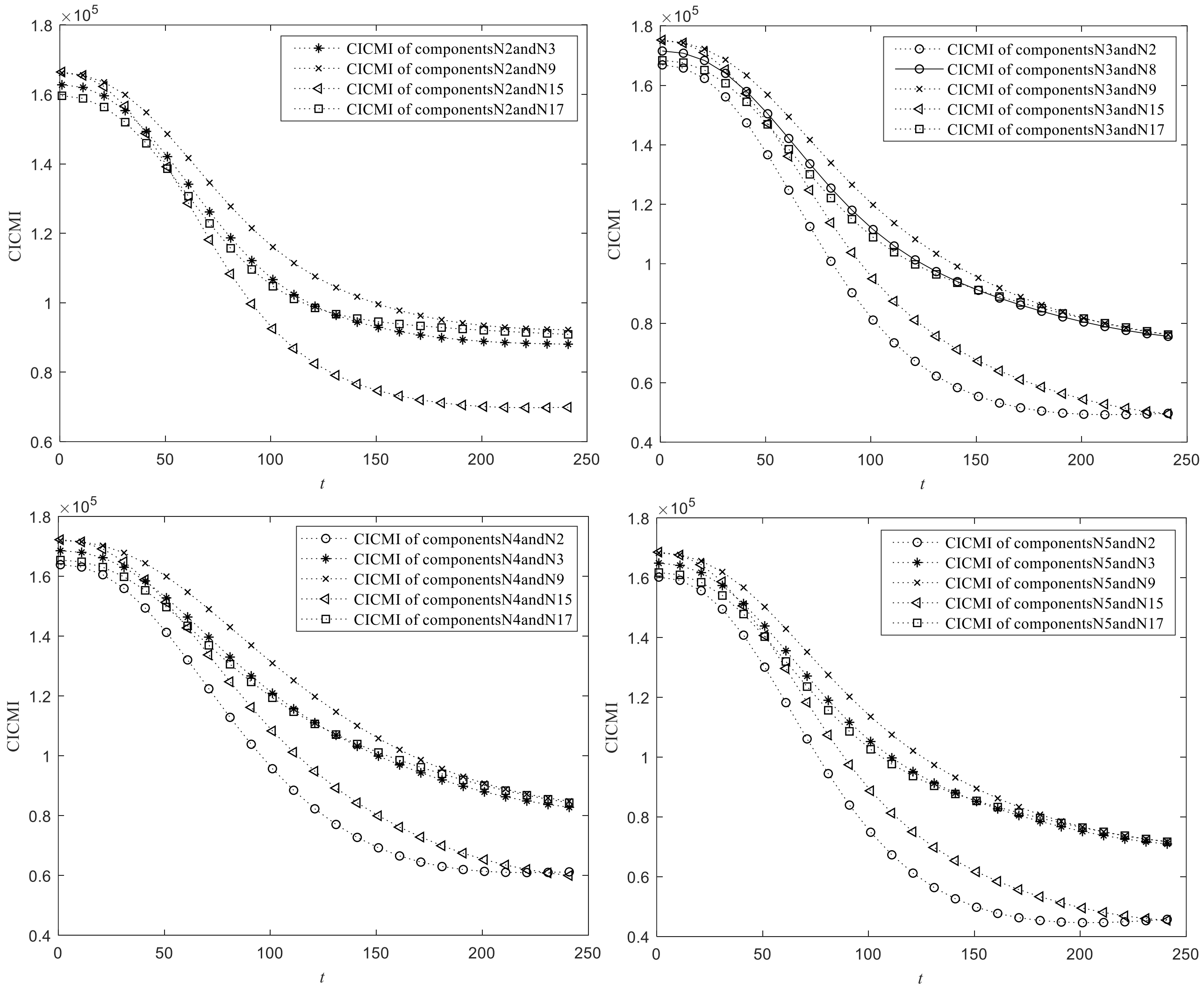
Fig. 5 CICMI when critical component N1 fails

Fig. 5 shows the CICMI over time when the critical component N1 fails. CICMI is affected by not only the cost associated with its own PM components but also the reliability of other component selected for PM. Obviously, the CICMI curves of different components are interleaved with each other. Consequently, we can find that the priority of PM components sorted according to CICMI has changed over time as in Table 4. This further shows that the model is very useful, and PM can be more reasonably arranged according to the priority of specific time, to reduce cost as much as possible.

Table 4 PM priority O_{N1}^{C} at different times under Weibull distribution (critical component N1 fails)

t	O_{N1}^{C}
10	N15, N9, N2, N3, N17
35	N9, N15, N2, N3, N17
50	N9, N15, N3, N17, N2
65	N9, N3, N15, N17, N2
80	N9, N3, N17, N15, N2
200	N9, N17, N3, N15, N2
250	N9, N17, N3, N2, N15

When the non-critical redundant components N2, N3, N4, N5 and N7 fail, the CICMI for the corresponding components of the PM is shown in Fig. 6.



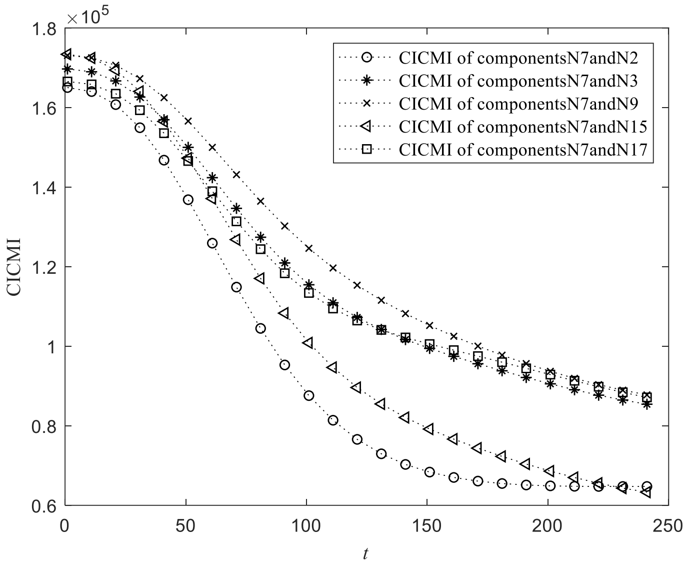
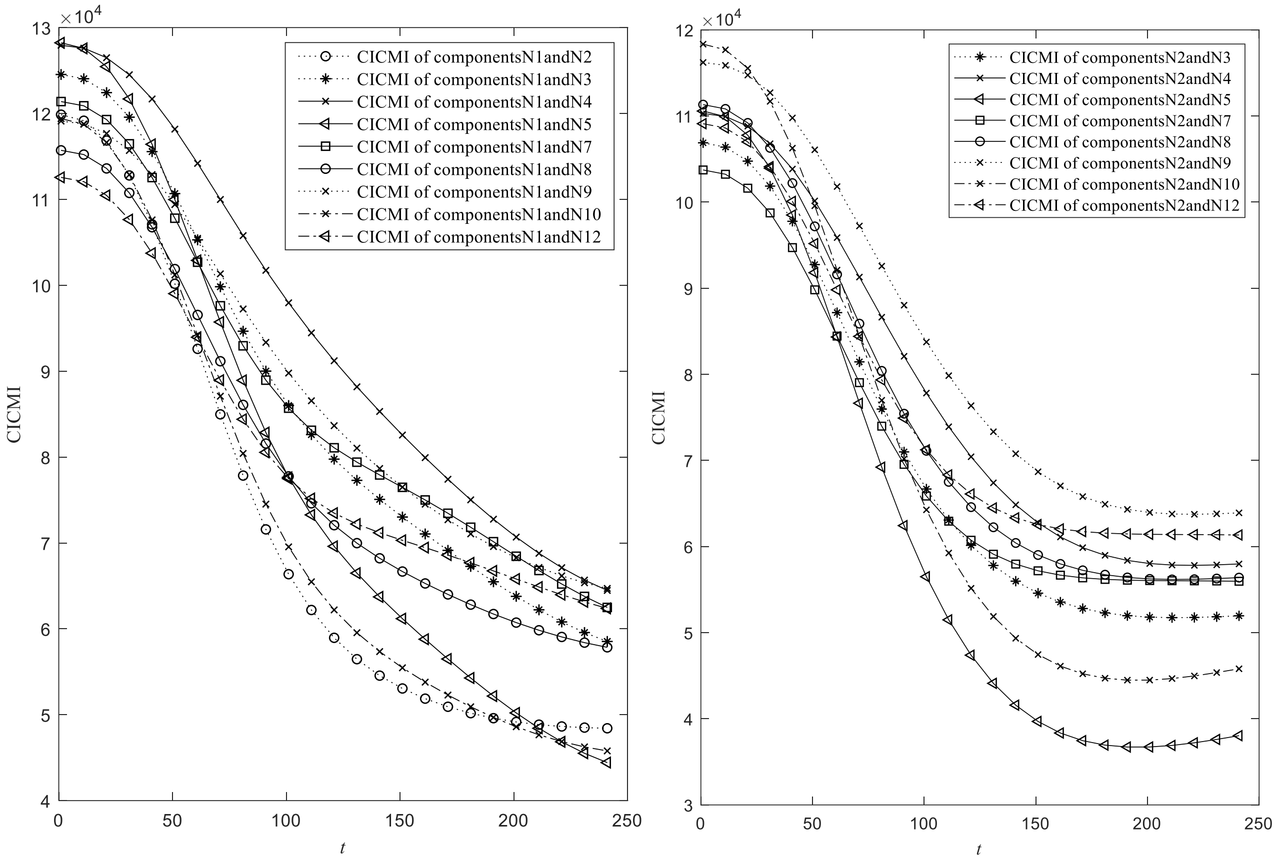
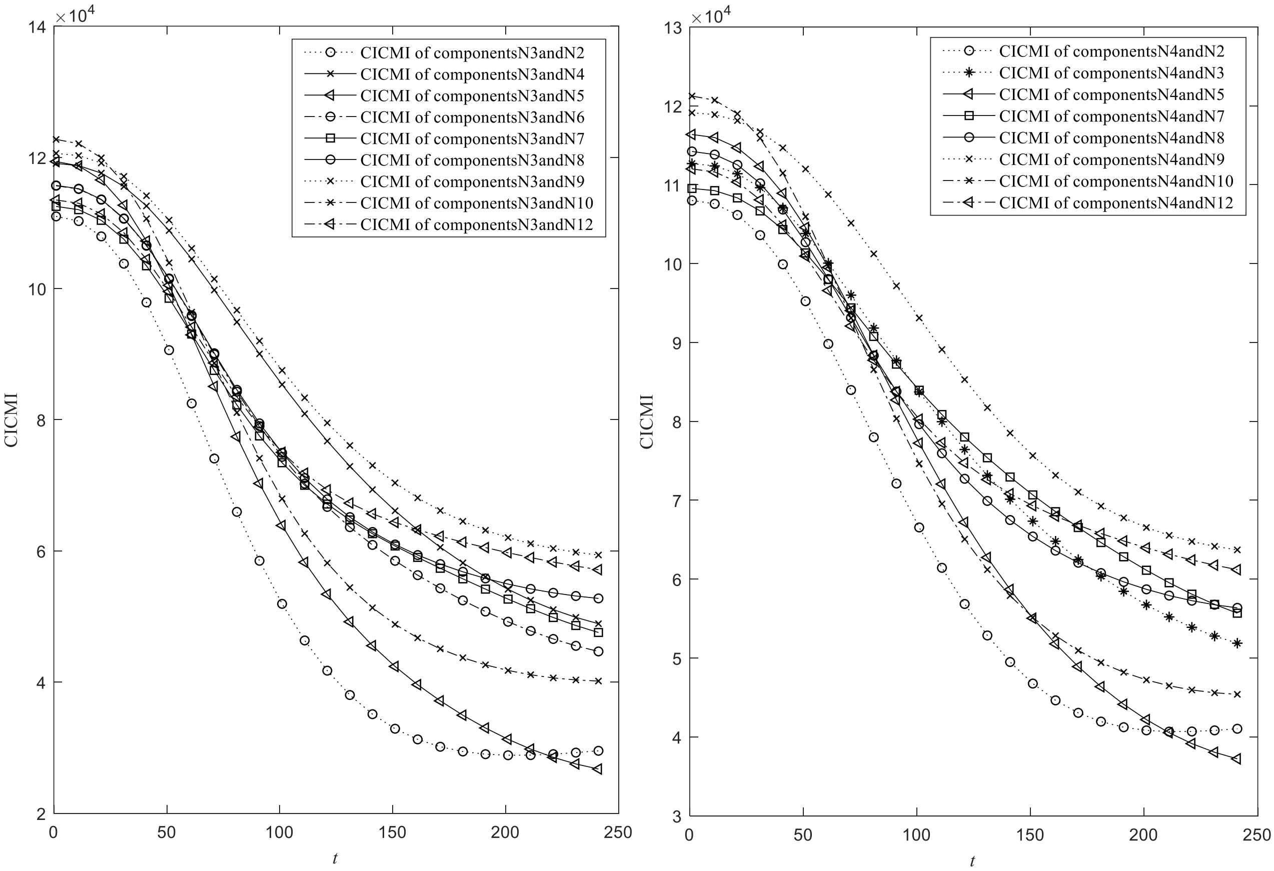


Fig. 6 CICMI when non-critical components fail

From Fig. 6, if $t = 100$ and component N2 fails, components selected for PM have the priority rank $\boldsymbol{O}_{\boldsymbol{N2}}^{\boldsymbol{C}} = (\mathrm{N9}, \mathrm{N3}, \mathrm{N17}, \mathrm{N15})$. If $t = 230$, $\boldsymbol{O}_{\boldsymbol{N2}}^{\boldsymbol{C}} = (\mathrm{N9}, \mathrm{N17}, \mathrm{N3}, \mathrm{N15})$. Similarly, if $t = 100$, and component N3 fails, $\boldsymbol{O}_{\boldsymbol{N2}}^{\boldsymbol{C}} = (\mathrm{N9}, \mathrm{N8}, \mathrm{N17}, \mathrm{N15}, \mathrm{N2})$. If $t = 230$, $\boldsymbol{O}_{\boldsymbol{N2}}^{\boldsymbol{C}} = (\mathrm{N17}, \mathrm{N9}, \mathrm{N8}, \mathrm{N15}, \mathrm{N2})$. We can conclude that the priority of components sorted by CICMI evolves over time. At different time points, components selected for PM may be different. In addition, if different components fail, components selected for PM may be different. This also shows the flexibility and usefulness of the proposed method, which can provide repairmen with the optimized total maintenance cost at the time when a component fails.

For this reactor, the components in loop 2 and loop 3 are of the same type, which implies that the same type of components in the two loops are the same in structure. Such two components include components N8 and N13, components N9 and N14, components N10 and N15, and components N12 and N17. Components N3 and N6, components N8 and N11, and components N13 and N16 not only have the same type of components but also are the same in structure. Hence, the CICMI of these two components are the same. Fig. 7 depicts the CICMI under multiple components maintenance at the same time when a component fails. If PM is performed on the remaining components at the same time, and the system has not failed, we cannot perform PM on all the remaining components. Then the location of the component in the system becomes more important, which also leads to the difference between Fig. 7 and Fig. 6.


×10^4
CICMI
t
CICMI of componentsN1andN2
CICMI of componentsN1andN3
CICMI of componentsN1andN4
CICMI of componentsN1andN5
CICMI of componentsN1andN7
CICMI of componentsN1andN8
CICMI of componentsN1andN9
CICMI of componentsN1andN10
CICMI of componentsN1andN12
CICMI of componentsN2andN3
CICMI of componentsN2andN4
CICMI of componentsN2andN5
CICMI of componentsN2andN7
CICMI of componentsN2andN8
CICMI of componentsN2andN9
CICMI of componentsN2andN10
CICMI of componentsN2andN12


×10^4
CICMI
t
CICMI of componentsN3andN2
CICMI of componentsN3andN4
CICMI of componentsN3andN5
CICMI of componentsN3andN6
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CICMI of componentsN3andN8
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CICMI of componentsN3andN12
CICMI of componentsN4andN2
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CICMI of componentsN4andN12

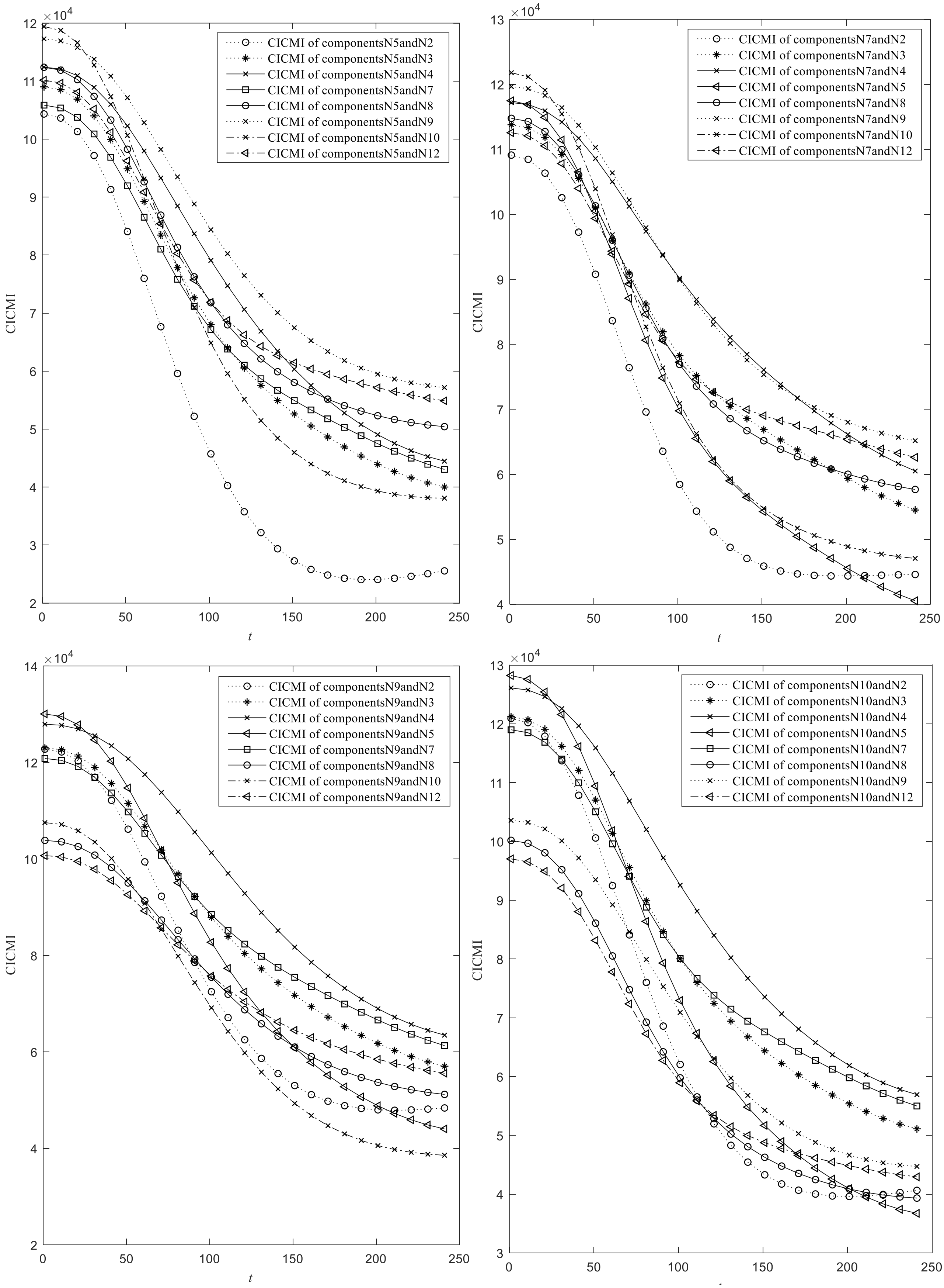

Fig. 7 CICMI under multiple components maintenance at the same time

From Fig. 7, if $t = 150$ and component N1 fails, the priority ranking of components for PM is $\boldsymbol{O_{N1}^{SC}} = (\mathrm{N4}, \mathrm{N7}, \mathrm{N9}, \mathrm{N3}, \mathrm{N12}, \mathrm{N8}, \mathrm{N5}, \mathrm{N10}, \mathrm{N2})$.

If $t = 230$, then $\boldsymbol{O}_{\boldsymbol{N1}}^{\boldsymbol{SC}} = (\text{N4}, \text{N10}, \text{N7}, \text{N12}, \text{N3}, \text{N8}, \text{N2}, \text{N10}, \text{N5})$. Similarly, if $t = 150$ and component N3 fails, then $\boldsymbol{O}_{\boldsymbol{N1}}^{\boldsymbol{SC}} = (\text{N9}, \text{N4}, \text{N12}, \text{N8}, \text{N7}, \text{N6}, \text{N10}, \text{N5}, \text{N2})$. If $t = 230$, then $\boldsymbol{O}_{\boldsymbol{N1}}^{\boldsymbol{SC}} = (\text{N9}, \text{N23}, \text{N8}, \text{N4}, \text{N7}, \text{N6}, \text{N10}, \text{N2}, \text{N5})$. We can see that the priority of the components evolves over time. In addition, when component N1 fails, for component N5 and component N10 at different times, the corresponding CICMI is also different. It is also reflected in the failure of other components. Moreover, for the same type of components (such as components N4 and N9, components N5 and N10), the corresponding CICMI is different. Consequently, the PM will be more complicated, and it can be accurate to the specific location of the component in the system, rather than the type of components.

Based on Equation (24), we now analyze the relationship between the number of components based on the component priority rank and cost constraints in Fig 8.

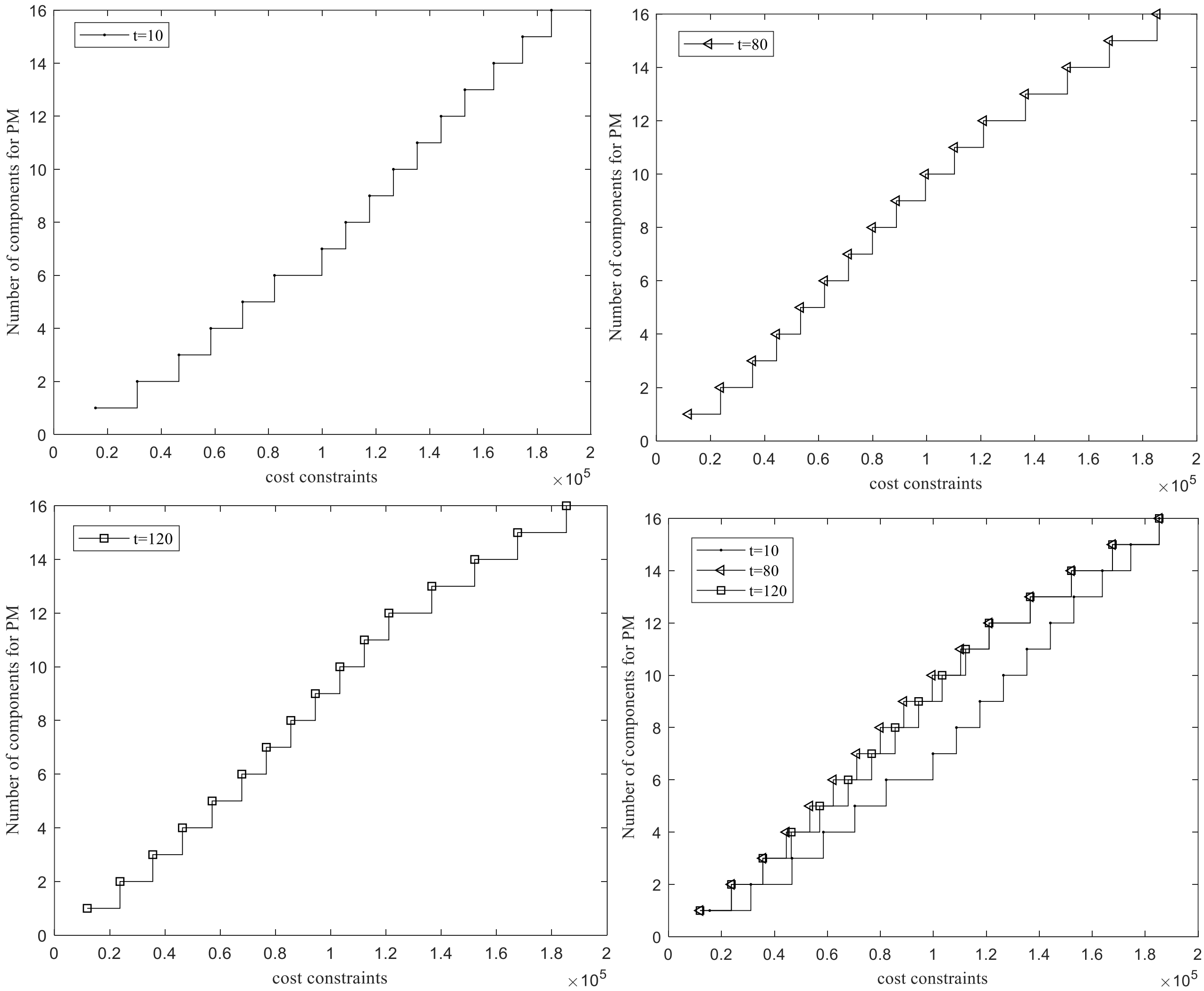

Fig. 8 Number of components for PM under the cost constraints

Fig. 8 shows different repair capacity with the corresponding cost when component N1 fails. The number of components for PM can be determined, subject to cost constraints. For example, when the cost constraint is

80000 at $t = 10$, the optimal number of components for PM is 5. However, when $t = 80$, the optimal number of components for PM is 8, and when $t = 120$, the optimal number of components for PM is 7. The occurrence of this situation is also due to the change in the priority of components for PM. This shows that the discussion of the number of components for PM is very necessary.

Meanwhile, considering multiple components maintenance at the same time, we discuss the components for PM under the constraints of the cost of the maintenance based on Equation (26). When component N1 fails and the cost constrain is 90000, we can obtain the PM policy for different periods, as shown in Table 5, where 1 represents the corresponding component for PM, and 0 means that no PM is performed on this component.

Table 5 PM policy in different periods when component N1 fails

time	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12	N13	N14	N15	N16	N17
10	0	1	1	1	1	1	1	0	0	0	0	0	1	0	1	0	0
20	0	0	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0
30	0	0	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0
40	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
50	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
60	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	1
70	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
80	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	1	1
90	0	0	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0
100	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	1

From Table 5, we can see that for different periods, the components needing PM have also changed, which is reflected in the system structure. At $t = 10$, we need to perform PM on components N2, N3, N4, N5, N6, N7, N13, and N15. The number of components obtained from the best solution is 8. However, at $t = 20$, we need to perform PM on components N3, N4, N5, N6, N7, N8, N9, N11 and N12. The number of components given by the best solution is 9 when the cost constraint is met. This can indicate that as time changes, the number of components for PM may also change.

6 Conclusions and future work

This paper proposed an importance index, Cost-Informed Component Maintenance Index (CICMI) for optimizing preventive maintenance policies. It then applied the CICMI to analyze series-parallel and parallel-serial systems. For different policies such as group failure and simultaneous maintenance, some properties of the CICMI were given. Considering the cost constraints, this paper optimized the number of components for PM and maintenance policies. Finally, a case study on a reactor was given to illustrate the applicability of the proposed measures.

Numerical examples show that CICMI is not only affected by the costs associated with its own PM components, but also by the reliability and cost of other components selected for PM. When a component fails, the priority of the component performing PM is not fixed, but changes over time. In other words, the components for PM at different times under cost constraints are different. Over time, the number of PM components under cost constraints may also change.

Our future work will be focusing on extending the proposed index to multistate systems. Cost in this paper is assumed fixed, which can be extended to a time-dependent variable. We may comprehensively investigate cost and system reliability to discuss which state is most suitable for components to be performed on PM.

Acknowledgements

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Appendix

Proof of Property 1

(1) Considering the total expected system cost function under different component failures,

$$C(0_i,t) = \sum_{k=1,k\neq i,l}^{n} \Big\{\{c_{s,k}\, Pr[\phi(0_k,\mathbf{1}) = 0] + c_k + C_k^P(t)\}\, Pr[x_k(t) = 0]\Big\}$$

$$+c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + C_i^P(t) + \{c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + C_l^P(t)\}\, Pr[x_l(t) = 0],$$

$$C(0_l,t) = \sum_{k=1,k\neq i,l}^{n} \Big\{\{c_{s,k}\, Pr[\phi(0_k,\mathbf{1}) = 0] + c_k + C_k^P(t)\}\, Pr[x_k(t) = 0]\Big\}$$

$$+\{c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + C_i^P(t)\}\, Pr[x_i(t) = 0] + c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + C_l^P(t),$$

$$C(0_i,t) - C(0_l,t) = \{c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + C_i^P(t)\}(1 - Pr[x_i(t) = 0])$$
$$+\{c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + C_l^P(t)\}\,(Pr[x_l(t) = 0] - 1)$$

$$= \left\{c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] + c_{p_l} Pr[x_l(t) = 1]\right\}(1 - Pr[x_i(t) = 0])$$

$$-\left\{c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] + c_{p_i} Pr[x_i(t) = 1]\right\}(1 - Pr[x_l(t) = 0])$$

$$= \left\{c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] + c_{p_l} Pr[x_l(t) = 1]\right\} Pr[x_i(t) = 1]$$

$$-\left\{c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] + c_{p_i} Pr[x_i(t) = 1]\right\} Pr[x_l(t) = 1]$$

$$= \left\{c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1]\right\} Pr[x_i(t) = 1]$$

$$-\left\{c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1]\right\} Pr[x_{l(t)} = 1]$$

$$+\left(c_{p_l} - c_{p_i}\right) Pr[x_l(t) = 1]\, Pr[x_i(t) = 1].$$

Let $c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] = a_1$, $c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] + c_l + \sum_{z=1,z\neq i,l}^{n} c_{p_z}\, Pr[x_z(t) = 1] = a_2$, $c_{p_l} - c_{p_i} = b_1$. Subsequently, $a_1 - a_2 = c_{s,i}\, Pr[\phi(0_i,\mathbf{1}) = 0] + c_i - c_{s,l}\, Pr[\phi(0_l,\mathbf{1}) = 0] - c_l$. Then we can obtain, $C(0_i,t) - C(0_l,t) = a_1\, Pr[x_i(t) = 1] - a_2\, Pr[x_l(t) = 1] + b_1\, Pr[x_l(t) = 1]\, Pr[x_i(t) = 1]$.

When the maintenance cost and properties of component i and component l is the same, we have $Pr[\phi(0_i,\mathbf{1}) = 0] = Pr[\phi(0_l,\mathbf{1}) = 0]$, then $a_1 = a_2$. Then we have $C(0_i,t) - C(0_l,t) = a_1(Pr[x_i(t) =$

$1] - Pr[x_l(t) = 1])$. Consequently, $a_1 > 0$, if $Pr[x_i(t) = 1] > Pr[x_l(t) = 1]$, then $C(0_i, t) > C(0_l, t)$.

(2) When the reliability values of component i and component l are the same, we have $C(0_i, t) - C(0_l, t) = a_1 \, Pr[x_i(t) = 1] - a_2 \, Pr[x_l(t) = 1] + b_1 \, Pr[x_l(t) = 1] \, Pr[x_i(t) = 1] = (a_1 - a_2)R + b_1 \, R^2$.

Duo to $R \geq 0$, when $c_{s,i} \, Pr[\phi(0_i, \mathbf{1}) = 0] + c_i > c_{s,l} \, Pr[\phi(0_l, \mathbf{1}) = 0] + c_l, c_{p_i} < c_{p_l}$, that is, $a_1 - a_2 > 0$ and $b_1 > 0$. Hence, we can obtain $C(0_i, t) > C(0_l, t)$. ■

Proof of Property 2

(1) When failed component i is a non-critical component and components j and h are also non-critical components, $\phi(0_i, 0_j, \mathbf{1}_{ij})$and $\phi(0_i, 0_h, \mathbf{1}_{ih})$are both 1. At this time, the expression of $I_{j|i}^C$ is the same as when failed component i is critical component. When failed component i is a non-critical component and components j and h are both critical components, $\phi(0_i, 0_j, \mathbf{1}_{ij})$ and $\phi(0_i, 0_h, \mathbf{1}_{ih})$ are both 0, then $I_{j|i}^C = I_{h|i}^C$. When failed component i is the critical component, we have

$$I_{j|i}^C = -\frac{\partial C(0_i, t)}{\partial p_j(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,j}^{n} \left\{\left\{c_{s,k} \, Pr[\phi(0_k, \mathbf{1}) = 0] + c_k + C_k^P(t)\right\} Pr[x_k(t) = 0]\right\} + c_{s,i} \, Pr[\phi(0_i, \mathbf{1}) = 0] + c_i + C_i^P(t) + \left\{c_{s,j} \, Pr\left[\phi\left(0_j, \mathbf{1}\right) = 0\right] + c_j + C_j^P(t)\right\} Pr\left[x_j(t) = 0\right]}{\partial p_j(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,j}^{n} \left\{\left\{c_{s,k} \, Pr[\phi(0_k, \mathbf{1}) = 0] + c_k + C_k^P(t)\right\} Pr[x_k(t) = 0]\right\}}{\partial p_j(t)} - c_{p_j} + c_{s,j} \, Pr\left[\phi\left(0_j, \mathbf{1}\right) = 0\right] + c_j + C_j^P(t)$$

$$= -c_{p_j} \sum_{k=1,k\neq i,j}^{n} Pr[x_k(t) = 0] - c_{p_j} + c_{s,j} \, Pr\left[\phi\left(0_j, \mathbf{1}\right) = 0\right] + c_j + \sum_{z=1,z\neq j,l}^{n} c_{p_z} \, Pr[x_z(t) = 1] + c_{p_l} Pr[x_l(t) = 1]$$

$$= -c_{p_j} \sum_{k=1,k\neq i,j,l}^{n} Pr[x_k(t) = 0] - c_{p_j} \, Pr[x_l(t) = 0] - c_{p_j} + c_{s,j} \, Pr\left[\phi\left(0_j, \mathbf{1}\right) = 0\right] + c_j + \sum_{z=1,z\neq j,l}^{n} c_{p_z} \, Pr[x_z(t) = 1]$$

$$+ c_{p_l} Pr[x_l(t) = 1].$$

$$I_{h|i}^C = -\frac{\partial C(0_i, t)}{\partial p_l(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,h}^{n} \left\{\left\{c_{s,k} \, Pr[\phi(0_k, \mathbf{1}) = 0] + c_k + C_k^P(t)\right\} Pr[x_k(t) = 0]\right\} + c_{s,i} \, Pr[\phi(0_i, \mathbf{1}) = 0] + c_i + C_i^P(t)}{\partial p_h(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,h}^{n} \left\{\left\{c_{s,k} \, Pr[\phi(0_k, \mathbf{1}) = 0] + c_k + C_k^P(t)\right\} Pr[x_k(t) = 0]\right\}}{\partial p_h(t)} - c_{p_h} + c_{s,h} \, Pr[\phi(0_h, \mathbf{1}) = 0] + c_h + C_h^P(t)$$

$$= -c_{p_h} \sum_{k=1,k\neq i,h}^{n} Pr[x_k(t)=0] - c_{p_h} + c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] + c_h + \sum_{z=1,z\neq j,h}^{n} c_{p_z}\, Pr[x_z(t)=1] + c_{p_j} Pr\big[x_j(t)=1\big]$$

$$= -c_{p_h} \sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] - c_{p_h}\, Pr\big[x_j(t)=0\big] - c_{p_h} + c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] + c_h$$

$$+ \sum_{z=1,z\neq j,h}^{n} c_{p_z}\, Pr[x_z(t)=1] + c_{p_j} Pr\big[x_j(t)=1\big].$$

$$I_{j|i}^{C} - I_{h|i}^{C} = -c_{p_j} \sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] - c_{p_j}\, Pr[x_h(t)=0] - c_{p_j} + c_{s,j}\, Pr\big[\phi\big(0_j,\mathbf{1}\big)=0\big] + c_j + c_{p_h} Pr[x_h(t)=1]$$

$$-\left\{-c_{p_h} \sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] - c_{p_h}\, Pr\big[x_j(t)=0\big] - c_{p_h} + c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] + c_h + c_{p_j} Pr\big[x_j(t)=1\big]\right\}$$

$$= -c_{p_j}\left(\sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] + Pr[x_h(t)=0] + 1 + Pr\big[x_j(t)=1\big]\right) + c_{s,j}\, Pr\big[\phi\big(0_j,\mathbf{1}\big)=0\big] + c_j$$

$$+c_{p_h}\left(\sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] + Pr\big[x_j(t)=0\big] + Pr[x_h(t)=1] + 1\right) - c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] - c_h$$

$$= -c_{p_j}\left(\sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] + 2 - p_h(t) + p_j(t)\right) + c_{s,j}\, Pr\big[\phi\big(0_j,\mathbf{1}\big)=0\big] + c_j$$

$$+c_{p_h}\left(\sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] + 2 - p_j(t) + p_h(t)\right) - c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] - c_h.$$

When the maintenance cost and properties of component j and component h is the same, we can obtain $I_{j|i}^{C} - I_{h|i}^{C} = 2c_{p_h}(p_h(t) - p_j(t))$. Hence, when component i fails, if $p_j(t) < p_h(t)$, then $I_{j|i}^{C} \geq I_{h|i}^{C}$.

(2) When the reliability values of component j and component h are the same,

$I_{j|i}^{C} - I_{h|i}^{C} = \left(\sum_{k=1,k\neq i,j,h}^{n} Pr[x_k(t)=0] + 2 - p_j(t) + p_h(t)\right)\left(c_{p_h} - c_{p_j}\right) + c_{s,j}\, Pr\big[\phi\big(0_j,\mathbf{1}\big)=0\big] + c_j - c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] - c_h$.

It is not difficult to draw that if $c_{p_h} > c_{p_j}$, $c_{s,j}\, Pr\big[\phi\big(0_j,\mathbf{1}\big)=0\big] + c_j > c_{s,h}\, Pr[\phi(0_h,\mathbf{1})=0] + c_h$, then $I_{j|i}^{C} \geq I_{h|i}^{C}$.

Specially, in the serial-parallel system,

$$I_{j|i}^{C} = c_j + \sum_{z=1,z\neq i,j}^{n} c_{p_z}\, p_z(t) - c_{p_j}\left\{\sum_{k=1,k\neq i,j}^{M} \big(1 - p_k(t)\big) + 1\right\}$$

$$= c_j + \sum_{z=1,z\neq i,j,h}^{n} c_{p_z} p_z(t) - c_{p_j}\left\{\sum_{k=1,k\neq i,j,h}^{M}\big(1-p_k(t)\big)+1\right\} + c_{p_h}p_h(t) - c_{p_h}\big(2-p_h(t)\big)$$

$$= c_j + \sum_{z=1,z\neq i,j,h}^{n} c_{p_z} p_z(t) - c_{p_j}\left\{\sum_{k=1,k\neq i,j,h}^{M}\big(1-p_k(t)\big)+1\right\} - 2c_{p_h}\big(1-p_h(t)\big).$$

$$I_{h|i}^{C} = c_h + \sum_{z=1,z\neq i,j,h}^{n} c_{p_z} p_z(t) - c_{p_h}\left\{\sum_{k=1,k\neq i,j,h}^{M}\big(1-p_k(t)\big)+1\right\} - 2c_{p_j}\left(1-p_j(t)\right).$$

Then we have $I_{j|i}^{C} - I_{h|i}^{C} = c_j - c_h + 2c_{p_j}(p_l(t) - p_j(t))$. More specifically in the serial-parallel system, when $c_j = c_h$, if $p_h(t) < p_j(t)$, then $I_{j|i}^{C} > I_{h|i}^{C}$. Considering component i fails, when $p_h(t) = p_j(t)$, if $c_j > c_h$, then $I_{j|i}^{C} > I_{h|i}^{C}$. ■

Proof of Property 3

$$I_{j|i}^{C} = -\frac{\partial C(0_i,t)}{\partial p_j(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,l}^{n}\left\{\left\{c_{s,k}\, Pr[\phi(0_k,\mathbf{1})=0] + c_k + C_k^P(t)\right\} Pr[x_k(t)=0]\right\} + c_{s,i}\, Pr[\phi(0_i,\mathbf{1})=0] + c_i + C_i^P(t) + \left\{c_{s,l}\, Pr[\phi(0_l,\mathbf{1})=0] + c_l + C_l^P(t)\right\} Pr[x_l(t)=0]}{\partial p_j(t)}$$

$$= -\frac{\partial \sum_{k=1,k\neq i,l}^{n}\left\{\left\{c_{s,k}\, Pr[\phi(0_k,\mathbf{1})=0] + c_k + C_k^P(t)\right\} Pr[x_k(t)=0]\right\}}{\partial p_j(t)} - c_{p_j} - c_{p_j} Pr[x_l(t)=0].$$

$$I_{j|l}^{C} = \frac{\partial C(0_l,t)}{\partial p_j(t)} = -\frac{\partial \sum_{k=1,k\neq i,l}^{n}\left\{\left\{c_{s,k}\, Pr[\phi(0_k,\mathbf{1})=0] + c_k + C_k^P(t)\right\} Pr[x_k(t)=0]\right\}}{\partial p_j(t)} - c_{p_j} - c_{p_j} Pr[x_i(t)=0].$$

For the failure of two different components, the states of the remaining components except for the two components remains the same. When only one of component i and component l fail, we have $I_{j|i}^{C}(x_l(t)=1) - I_{j|l}^{C}(x_i(t)=1) = c_{p_j} Pr[x_i(t)=0] - c_{p_j} Pr[x_l(t)=0] = c_{p_j}\big(p_l(t) - p_i(t)\big)$. Duo to $c_{p_j} > 0$, if $p_i(t) < p_l(t)$, then $I_{j|i}^{C} \geq I_{j|l}^{C}$.

Specially, in the serial-parallel system,

$I_{j|i}^{C} = c_j + C_j^P(t) - c_{p_j}\sum_{k=1,k\neq j,i,l}^{M}\big(1-p_k(t)\big) - c_{p_j}\big(2-p_l(t)\big)$,

$I_{j|l}^{C} = c_j + C_j^P(t) - c_{p_j}\sum_{k=1,k\neq j,i,l}^{M}\big(1-p_k(t)\big) - c_{p_j}\big(2-p_i(t)\big)$,

$I_{j|i}^{C}(x_l(t)=1) - I_{j|l}^{C}(x_i(t)=1) = c_{p_j}\big(p_l(t) - p_i(t)\big)$. ■

Proof of Property 4

$$C(t)=\sum_{i=1}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]$$

$$=\sum_{i=1,i\neq k,l}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]$$

$$+\left\{c_{s,k}+\sum_{j=1}^{m_k}c_{k_j}+\sum_{z=1}^{n-m_k}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{k_1},0_{k_2},\ldots,0_{k_{m_k}}\right]$$

$$+\left\{c_{s,l}+\sum_{j=1}^{m_l}c_{k_j}+\sum_{z=1}^{n-m_l}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{l_1},0_{l_2},\ldots,0_{l_{m_l}}\right],$$

$$C\left(0_{k_1},0_{k_2},\ldots,0_{k_{m_k}},X(t)\right)=\sum_{i=1,i\neq k,l}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]$$

$$+\left\{c_{s,k}+\sum_{j=1}^{m_k}c_{k_j}+\sum_{z=1}^{n-m_k}c_{p_z}Pr[x_z(t)=1]\right\}$$

$$+\left\{c_{s,l}+\sum_{j=1}^{m_l}c_{k_j}+\sum_{z=1}^{n-m_l}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{l_1},0_{l_2},\ldots,0_{l_{m_l}}\right],$$

$$C\left(0_{l_1},0_{l_2},\ldots,0_{l_{m_l}},X(t)\right)=\sum_{i=1,i\neq k,l}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]$$

$$+\left\{c_{s,k}+\sum_{j=1}^{m_k}c_{k_j}+\sum_{z=1}^{n-m_k}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{k_1},0_{k_2},\ldots,0_{k_{m_k}}\right]$$

$$+\left\{c_{s,l}+\sum_{j=1}^{m_l}c_{k_j}+\sum_{z=1}^{n-m_l}c_{p_z}Pr[x_z(t)=1]\right\}.$$

When component j does not participate in forming the minimum cut set k and l,

$$I^C_{j|k_1,k_2,\ldots,k_{m_k}}(t)=-\frac{\partial C\left(0_{k_1},0_{k_2},\ldots,0_{k_{m_k}},X(t)\right)}{\partial p_j(t)}$$

$$=-\frac{\partial\sum_{i=1,i\neq k,l}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]}{\partial p_j(t)}-c_{p_j}$$

$$-c_{p_j}Pr\left[0_{l_1},0_{l_2},\ldots,0_{l_{m_l}}\right],$$

$$I^C_{j|l_1,l_2,\ldots,l_{m_l}}(t)=-\frac{\partial\sum_{i=1,i\neq k,l}^{n_0}\left\{c_{s,i}+\sum_{j=1}^{m_i}c_{i_j}+\sum_{z=1}^{n-m_i}c_{p_z}Pr[x_z(t)=1]\right\}Pr\left[0_{i_1},0_{i_2},\ldots,0_{i_{m_i}}\right]}{\partial p_j(t)}-c_{p_j}$$

$$-c_{p_j}Pr\left[0_{k_1},0_{k_2},\ldots,0_{k_{m_k}}\right].$$

For the failure of the system caused by two different cut sets, except for the components included in the cut sets k and l, the states of the remaining components remain the same. When only one of cut sets k and l can lead to system failure, we have $I^C_{j|k_1,k_2,\ldots,k_{m_k}}(t)-I^C_{j|l_1,l_2,\ldots,l_{m_l}}(t)=c_{p_j}\left(Pr\left[0_{k_1},0_{k_2},\ldots,0_{k_{m_k}}\right]-Pr\left[0_{l_1},0_{l_2},\ldots,0_{l_{m_l}}\right]\right)$. Considering $c_{p_j}>0$, if $Pr\left[0_{k_1},0_{k_2},\ldots,0_{k_{m_k}}\right]>Pr\left[0_{l_1},0_{l_2},\ldots,0_{l_{m_l}}\right]$, then $I^C_{j|k_1,k_2,\ldots,k_{m_k}}(t)>I^C_{j|l_1,l_2,\ldots,l_{m_l}}(t)$. ■