Applications of Game Theory and Epistemic Logic to Fact-Checking

Can Başkent[1], Sanjay Bhattacherjee[2], and Shujun Li[2]

[1] Department of Computer Science, Middlesex University, London, UK
c.baskent@mdx.ac.uk
[2] School of Computing, University of Kent, Canterbury, UK
{s.bhattacherjee, s.j.li}@kent.ac.uk

Abstract. In this paper, we present a game theoretical and an epistemic logical approach to fact-checking. Game theory is the study of strategic reasoning whereas epistemic logic is the formal study of reasoning about and with knowledge. Fact-checking is strategic because the process involves more than one agent. It is epistemic as the process is all about verifying the correctness of one or more pieces of information – hence checking whether it is indeed "knowledge". We discuss a variety of typical fact-checking scenarios and present game theoretical and epistemic logical analysis for them.

Keywords: Fact-checking · Game theory · Epistemic logic · False information · Fake news · Modelling

1 Introduction

The attack on the US Capitol on January 6, 2021 can be criticised from a plethora of angles. One important aspect of this event was the role of fake news or alt-truth related to it. Combined with mass hysteria and certain other elements of behavioural economics, people's approach to "truth" as well as to "information" has shaped the event. A substantial amount of information that provoked the masses were fake. The information was often not vetted or checked by credible sources, yet put in mass circulation with the help of social media and bulletin boards. Most analysts agree on one thing about the attack: people who participated in the attack should have checked those "facts" before taking any action. Fact-checking is increasingly important in today's world. It shapes our relationship with information, knowledge, beliefs, rationality and strategic thinking. In this work, we study fact-checking from two perspectives: game theory and epistemic logic.

Game theory is the study of strategic behaviour. It allows us to analyse the role of strategic reasoning in fact-checking, and more importantly, helps us identify states of equilibrium. If fact-checking is seen as a game between agents like fact-checkers, journalists and the general public, game theory provides us with the required methodology and toolkit to describe a solution concept, an equilibrium (or its non-existence) for a variety of situations.

Epistemic logic, on the other hand, is the study of knowledge-based reasoning using logical tools. Epistemic logic has proved very useful in solving epistemic puzzles (such as *muddy children*), describing belief revision and knowledge updates [10]. Fact-checking almost always includes an element of knowledge: fact-checkers are either verifying some piece of information or refuting it in a multi-agent setting. The role of epistemic logic in fact-checking may hence sound familiar. In fact, what fact-checking accomplishes can be seen as an instance of the problem of whether *knowledge is "justified true belief"*. Fact-checkers ensure that the piece of information in question is true and *justified*. Is it then knowledge? This is very relevant for Gettier's approach who challenged the orthodox "justified true belief" idea for knowledge [15]. This paper presents a relatable approach as well: fact-checking does more than "justified true belief" approach to establish knowledge.

There are two main reasons to justify the choice of game theory and epistemic logic to approach the phenomenon of fact-checking. First, these formalisms allow us to study *game theoretic rationality* behind fact-checking of news items by the agents. Second, epistemic logic explains the knowledge element of fact-checking, as it studies how a piece of information becomes an agent's knowledge. Strategic reasoning and formal epistemology are popular areas, yet they have been scantly applied to the study of fact-checking to the best of our knowledge; hence this paper.

Our contribution to the discipline through this paper is twofold. First, we identify some non-cooperative and cooperative games that occur in a range of fact-checking scenarios. These are games of pure conflict, pure cooperation, the prisoners' dilemma, the good Samaritan game, voting games, etc. Second, using epistemic logical syntax, we represent the knowledge situation of agents involved in such fact-checking scenarios. Arguably, our contributions establish the first steps of studying fact-checking from the aforementioned perspectives. In this paper we focus on fact-checking of fake news (i.e., newsworthy information), but our approaches can be extended to any *check-worthy* information.

The rest of the paper is organised as follows. In Section 2, we provide a review of the related literature. In Section 3, we briefly describe some fact-checking scenarios from the real world that are suitable for game theoretical or epistemic logical analysis. Section 4 shows how some of the scenarios give rise to commonly studied games. After introducing the fundamentals of epistemic logic, we present a formal analysis of some of the scenarios from an epistemic logical perspective in Section 5. Finally, we conclude with some ideas for future work.

2 A Review of the Literature

Research on misinformation, fake news and deception has seen a recent impetus from humanities, economic theory and computational sciences. A recent survey by Zhou and Zafarani presents a forward-looking view of the area, highlighting "knowledge-based, style-based, propagation-based, and source-based" approaches with an extensive bibliography [39].

A majority of work on the spread of misinformation uses graph theoretical and algorithmic *diffusion models*. Diffusion models are relatively easy to work with and make it more natural to discuss and produce results in computational complexity. A recent work by Budak, Agrawal and El Abbadi uses diffusion models to formalise the spread of misinformation [8]. This allows them to formalise the "eventual influence limitation" problem and show that it is NP-hard. A similar problem was discussed by Nguyen, Yan and Thai [27] using diffusion models as well. They proposed methods to find a small set of highly influential "good" nodes that may be used to prevent the spread of misinformation. Chen *et al.*, on the other hand, distinguished centralised and decentralised problems in blocking misinformation, and presented a discussion from both algorithmic and game theoretical perspectives. Tong *et al.* described a similar problem where they allowed an arbitrary number of cascades [34]. A detailed overview of the area combining game and decision theories for deception models can be found in [18].

Fact-checking has produced a plethora of work in social and political sciences for foreseeable reasons. The topic has been featured in mass-media [16] and large online platforms have produced fact-checking tools [12], thus becoming a part of common lexicon. Walter *et al.* presented a meta-analysis of the effectiveness of fact-checking in correcting political misinformation, where they argued that the impact of "fact-checking weakens when refuting only parts of a claim" [36]. The *epistemology* of fake news is presented from a variety of different perspectives in [5]. Particularly, the epistemology of deceit in the current climate is analysed in [24]. Such work presents the philosophical and epistemic dimensions of fake news and other types of misinformation.

The usefulness of the fact-checking industry in providing resolution over facts to the recipients of the news was critiqued in [25] through comparative epistemology. Fact-checking and misinformation studies within the context of epidemiology and medicine have become a central debate due to the Covid-19 pandemic. Krause *et al.* argued that the Covid-19 "misinfodemic" must be approached from a risk communication perspective [19]. However, the influence of fact-checking is not unilateral. In [37], the authors pointed out that the pre-existing beliefs and political inclinations and knowledge of the recipients determine the ability to correct political misinformation through fact-checking.

Game theory has been a popular methodology for security from many angles [33]. It has been used to study cyber deception in general [28]. A game between fact-checkers and regulators has been studied in [38]. It has been a fruitful tool to analyse the spread and mitigation of fake news and misinformation, particularly using evolutionary game theory [35], game-theoretic opinion models [13], and spatial games for distributed fact-checking [17].

To the best of our knowledge, fake news and fact-checking of newsworthy information have not been studied using epistemic logic. A broader approach using dynamic logic explains computationally how one's belief can be changed by external information [10,14]. Fact-checking can be considered a case study for dynamic logic with a lot of potential for extensive formal modelling and analysis.

As emphasised by Lazer *et al.*, the authors of the current paper believe that we must "promote interdisciplinary research to reduce the spread of fake news and to address the underlying pathologies it has revealed" [21]. This paper attempts at presenting some novel approaches in that direction.

3 Fact-Checking Scenarios

The nature of fact-checking indicates that there is always a game theoretical element since any fact-checking scenario would involve at least two parties with conflicting or cooperative interests: a fact-checker and the author of the information that is being checked. There are however more complicated scenarios where more parties are involved. In this section, we give some typical examples of fact-checking scenarios, which are by no means exhaustive, but can interest game theorists and logicians in different ways. Most such scenarios involve more than two entities, so graph-based games are of relevance. Some of the scenarios can be combined to form a complicated ecosystem with multiple parallel and sequential games.

The most basic scenario is between *fact-checkers and information creators*, where the author creates a piece with a fact for which they *know* the truth value (real or fake) while the fact-checker *asserts* its truth value. A number of fact-checkers may form one or more legal entities called *association(s) of fact-checkers* to cooperate for collective benefits, but some of them from the same association may be competing with each other for better reputation.

Information creators may want to conduct fact-checking in *traditional media outlets*, as *independent journalists* and as *"We Media"* (i.e., grass-root media outlets often run by a single or a small group of individuals who are often not trained/certified journalists). Each such group of information creators can engage in different types of cooperative fact-checking games, but also with some level of cooperation between them to maintain the overall health of the (sub-)sector. If we extend the concept of "We Media" to every Internet user, i.e., we consider Internet users is an independent information creators, we have the scenario of *user-generated content (UGC)*, where most Internet users are so-called "prosumers" because they both produce and consume information online. Some of the UGC creators are also active fact-checkers, and comment on other UGC creators' posts to conduct their own individual checks and maybe point out factual errors. Different UGC creators compete with each other for attention, but also cooperate with each other to facilitate spreading of information and fact-checking. Between *professional and grass-root level fact-checkers*, there could be cooperation and conflict while fact-checking each other.

The scenarios can get more complicated by the presence or intervention of *regulatory public bodies* in the fact-checking process. News outlets, fact-checkers and the regulatory public bodies have their own respective strategies while sharing knowledge with each other. These scenarios can get complicated further when *government-operated/controlled/funded* bodies complete with *private media outlets* for fact-checking. The former may have advantages over the latter because of their closer relationships with the regulators. If we further broaden the scope to fact-checking between nation states, there is competition and cooperation while establishing narratives on matters of common interest, and also scope of mediation by the United Nations and other global bodies.

Going back to groups of individuals and organisations, we can have some challenging fact-checking scenarios. For groups of *politicians, government officials, religious bodies and organisations headed by spiritual leaders*, their boundaries between regulators, information creators and fact-checkers can be blurry. A very niche fact-checking scenario occurs between *criminals* and *victims*, where the former can deliberately or unconsciously cause harm to the latter, via creating and spreading mis- and disinformation. Note that an individual person can have multiple attributes, and hence they can belong to any of the above-mentioned groups.

One special type of scenario involves *online (social media) platforms* and their *account holders*. While the former has the power to apply restrictions on the latter, and both parties can be creators and propagators of false information, and also fact-check each other. Note that one person (including an owner of an online platform) can hold one or more online accounts, and one online account can be controlled/used by more than one person or organisation. So these types of scenarios can be quite interesting to analyse.

The last example is something we call *fact-checking fact-checkers*, where a professional or grass-root level fact-checker can fact-check other fact-checker's fact-checking reports. This can lead to complicated fact-checking graphs describing which fact-checkers fact-checked what fact-checking reports of what other fact-checkers. It is possible that a fact-checker fact-checks its own fact-checking report, i.e., self-correct a fact-checking report. Such interactions can be very dynamic and involve multiple iterations.

4 Game Theoretical Analysis of Some Scenarios

A comprehensive analysis of the scenarios discussed in Section 3 falls beyond the scope of the current paper. Instead, in what follows, we show the theoretical richness of fact-checking by identifying the games occurring in some of the scenarios. We start with two-player games for which the preliminaries are described as part of the first game in Section 4.1. All other preliminaries are introduced subsequently wherever relevant.

4.1 Game of Competing Fact-Checkers

We consider the *game of competing fact-checkers* where two fact-checkers are independently scrutinising a news item published by a third party and competing to *gain better reputation by being more accurate*.

The fact-checkers form a set of *players* $\mathbf{P} = \{P_1, P_2\}$. The set of *actions* is the same for each player: $A = A_1 = A_2 = \{\text{asserts fake}, \text{asserts real}\} = \{\mathtt{f}, \mathtt{r}\}$. The players take their actions *simultaneously*, without the knowledge of the action of the other player. The outcome of a game is called its *action profile* and is denoted as a tuple (x, y) where $x, y \in A$ are simultaneous actions of P_1 and P_2 respectively. The set of all possible action profiles for the game of competing fact-checkers is

$$\mathbf{A} = \{(\mathtt{r}, \mathtt{f}), (\mathtt{r}, \mathtt{r}), (\mathtt{f}, \mathtt{f}), (\mathtt{f}, \mathtt{r})\}.$$

We assume that the truth value of the news item will eventually be established. Hence, we associate an independent truth value with the news item as *actually fake/false* or *actually real/true.*

Each player $P_i \in \mathbf{P}$ has a preference $\pi_i(\mathbf{A})$ of the outcomes in $\mathbf{A}$. A preference $\pi(\mathbf{A})$ is an ordered set starting from the most preferred outcome to the least preferred one.

> *The foremost preference of both players will be to assert correctly. Therein, they will prefer the other player's assertion to be incorrect.*

The preference of each player is determined accordingly in the following two scenarios.

When the news is fake. P_1's preference is: $\pi_1(\mathbf{A}) = ((\mathtt{f}, \mathtt{r}), (\mathtt{f}, \mathtt{f}), (\mathtt{r}, \mathtt{r}), (\mathtt{r}, \mathtt{f}))$. Similarly, the preference of P_2 is: $\pi_2(\mathbf{A}) = ((\mathtt{r}, \mathtt{f}), (\mathtt{f}, \mathtt{f}), (\mathtt{r}, \mathtt{r}), (\mathtt{f}, \mathtt{r}))$. The pay-off matrix for this scenario is given in Table 1a. We see that P_1 receives a better payoff by asserting the news to be fake (its actual truth value), irrespective of the strategy of P_2. Similarly, P_2 receives a better payoff by asserting the news to be fake, irrespective of the strategy of P_1. This is same as in the prisoner's dilemma game [30].

Hence, the correct assertion of "fake" by both players is the only state of *pure strategy equilibrium.*

	P_2 f	P_2 r
P_1 f	(2, 2)	(3, 0)
P_1 r	(0, 3)	(1, 1)

(a) When the news is *actually fake*

	P_2 r	P_2 f
P_1 r	(2, 2)	(3, 0)
P_1 f	(0, 3)	(1, 1)

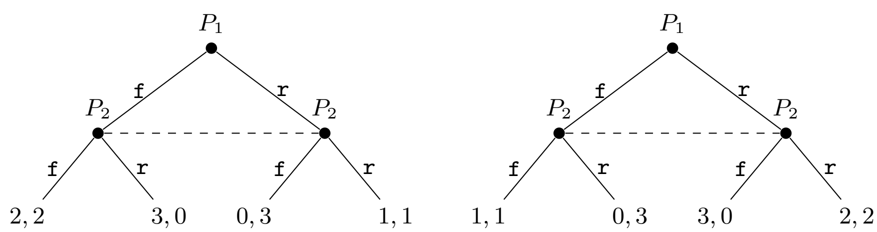
(b) When the news is *actually real*

Fig. 1: All action profiles and their payoffs for the game of competing fact-checkers.

When the news is real. P_1's preference is: $\pi_1(\mathbf{A}) = ((\mathtt{r}, \mathtt{f}), (\mathtt{r}, \mathtt{r}), (\mathtt{f}, \mathtt{f}), (\mathtt{f}, \mathtt{r}))$. Similarly, P_2's preference is: $\pi_2(\mathbf{A}) = ((\mathtt{f}, \mathtt{r}), (\mathtt{r}, \mathtt{r}), (\mathtt{f}, \mathtt{f}), (\mathtt{r}, \mathtt{f}))$. The pay-off matrix for this scenario has been portrayed in Table 1b. As before, we see that P_1 receives a better payoff by asserting the news to be real (its actual truth value), irrespective of the strategy of P_2. Similarly, P_2 receives a better payoff by asserting the news to be real, irrespective of the strategy of P_1.

Hence, the correct assertion of "real" by both players is the only state of *pure strategy equilibrium* and the game is the same as the prisoner's dilemma.

Extensive Form Epistemics of the Fact-Checking Game. The game of competing fact-checkers has an epistemic element which can best be described in an extensive form. Let's start with the case when the news is *fake*, where the dashed line indicates the epistemic indistinguishability. That is in Figure 2a, from P_1's perspective, P_2's action of either `r` or `f` is indistinguishable as P_1 does not know what P_2 played. Similarly for P_2. Figure 2b represents the case when the news is *real.*



(a) When the news is *fake.* (b) When the news is *real.*

Fig. 2: Extensive form representation of the game of competing fact-checkers.

Prisoners' Dilemma. Irrespective of whether the news item is actually real or fake, the players always get a better pay-off by asserting the truth value correctly. This shows that the game of competing fact-checkers is the same as the prisoner's dilemma game. The extensive form representations also illustrate the similarities.

"We Media". The non-cooperative game between two "We Media" agents where they fact-check each other's content may involve several kinds of utilities. One of the most important driving forces of such individuals is *reputation* as a function of accuracy of facts. The game of competing fact-checkers models the game between two such agents.

Independent Journalism. A fact-checking game between an independent journalist and one influenced by a funding body is a game of competing fact-checkers.

User-Generated Content (UGC). Two users that have generated content on the same topic and are also fact-checking each other are also playing the game of competing fact-checkers.

4.2 The Author vs. Fact-Checker Game

Consider the fundamental fact-checking scenario involving two parties: the author A of a piece of information that is being checked and a fact-checker C. The author knows if a fact F stated in their piece is *actually* fake (`f`) or real (`r`). However, in both cases they want the fact-checker to be convinced by the article and believe the fact to be real ($\mathtt{r}$). The fact-checker, on the other hand, would want to be accurate – assert fake (denoted by $\mathtt{f}$) when the author has provided a false fact (also denoted with $\mathtt{f}$) and assert real (denoted by $\mathtt{r}$) when the when the author's fact is true (also denoted by $\mathtt{r}$). Table 1 provides the action profiles and their corresponding payoffs for the players in this *unique game*.

The only state of equilibrium is when the author has stated a true fact and the fact-checker ratifies it.

Table 1: Author vs. Fact-Checker Game: All possible action profiles (A, C) such that $A, C \in \{\mathtt{r}, \mathtt{f}\}$ and the corresponding payoffs.

	$C = \mathtt{r}$	$C = \mathtt{f}$
$A = \mathtt{r}$	$(+1, +1)$	$(-1, -1)$
$A = \mathtt{f}$	$(+1, -1)$	$(-1, +1)$

4.3 Fact-Checking Game of Pure Conflict

A game of pure conflict can occur in several fact-checking scenarios. Consider a political scenario with only two parties – P_1 in power and P_2 in opposition. A disruptive social incident has occurred that has the potential to cause a huge anti-government public uproar. Each of these parties employ their own fact-checking companies to weave their respective narratives. A fact F that is central to the event, is in contention. The narratives of parties P_1 ans P_2 declare $F_1, F_2 \in \{\mathtt{r}, \mathtt{f}\}$ as the truth value of F respectively. If $F_1 = F_2$ (F is considered to be either true or false by both narratives), then it works to the advantage of the ruling party. If F is considered to be true by one narrative and false by another ($F_1 = \neg F_2$), then it works to the advantage of the opposition party. The action profiles and payoffs are denoted in Table 2.

This game is analogous to the *game of matching pennies*. There is no clear preference of strategy for the players, and as a result there is no *equilibrium*. Games of pure conflict are always zero-sum although the converse is not true.

Table 2: Game of Pure Conflict: All possible action profiles (F_1, F_2) such that $F_1, F_2 \in \{\mathtt{r}, \mathtt{f}\}$ and the corresponding payoffs.

	$F_2 = \mathtt{r}$	$F_2 = \mathtt{f}$
$F_1 = \mathtt{r}$	$(+1, -1)$	$(-1, +1)$
$F_1 = \mathtt{f}$	$(-1, +1)$	$(+1, -1)$

4.4 Fact-Checking Game of Pure Cooperation

A game of pure cooperation (no conflict) can occur in several fact-checking scenarios as well. Consider a two-party political system as before. A social media outlet has claimed that the perks available to the elected members of parliament (MPs) are used for unofficial or personal use by all MPs. Each party employs their own fact-checking companies to weave their respective narratives. A central fact F in the final narrative is in contention and is key to maintaining the trust of common people on their MPs. The narratives of parties P_1 and P_2 declare $F_1, F_2 \in \{\mathtt{r}, \mathtt{f}\}$ as the truth value of F, respectively. If $F_1 = F_2$, it works to the advantage of both parties. If $F_1 = \neg F_2$, it would lead to further distrust and confusion among the common people and would disadvantage both parties. The action profiles and payoffs are denoted in Table 3.

The coordination game is a common example of pure cooperation where players P_1 and P_2 both have a clear preference of strategy that results in *no conflict*. There are *multiple states of pure strategy equilibria*, whenever $F_1 = F_2$.

Table 3: Game of Pure Cooperation: All possible action profiles (F_1, F_2) such that $F_1, F_2 \in \{\mathtt{r}, \mathtt{f}\}$ and the corresponding payoffs.

	$F_2 = \mathtt{r}$	$F_2 = \mathtt{f}$
$F_1 = \mathtt{r}$	$(+1, +1)$	$(-1, -1)$
$F_1 = \mathtt{f}$	$(-1, -1)$	$(+1, +1)$

4.5 Fact-Checking Game of Narratives

Let us consider a scenario within a fact-checking organisation where a fact F is handled by a team of two fact-checkers P_1 and P_2 who are supposed to jointly produce a verdict on F. The fact-checking happens in two stages. First, they will each conduct independent investigations and provide a verdict (*true* or *false*) with supporting narratives on F to each other. P_1 and P_2 have conflicting belief-systems and hence inevitably produce opposite verdicts with contradicting narratives. They try to convince each other about their own narrative after which they get a second chance to revise their verdicts. They finally produce a joint verdict and a supporting narrative that gets published by the organisation. Their actions in this game are $\{$*stick*, *deviate*$\}$ – they either stick to their initial narrative or deviate from it. The fact-checker behind the published narrative wins credibility for doing a good job in producing the initial narrative and then convincing the other fact-checker about its truth value. The following are the different action profiles and their corresponding payoffs as represented in Table 4.

– (*deviate*, *stick*): P_1 deviates and agrees with P_2's initial narrative. Hence, the payoffs are $(0, +2)$.

- (*stick*, *deviate*): P_2 deviates and agrees with P_1's initial narrative. Hence, the payoffs are $(+2, 0)$.
- (*deviate*, *deviate*): P_1 and P_2 both deviate from their respective narratives and arrive at a middle ground. Hence, their individual payoffs $(+1, +1)$ are less than what they would have had if they could have stuck to their initial narratives.
- (*stick*, *stick*): P_1 and P_2 are both adamant about not changing their initial narratives and reach an impasse. In this case, they are both suffering a loss for not being able to produce a verdict as a team.

This game is analogous to the *game of chicken*, and hence a deviating fact-checker may be said to have "chickened out".

Table 4: Game of Narratives: All possible action profiles (A_1, A_2) such that $A_1, A_2 \in \{\text{Deviate}, \text{Adamant}\}$ and the corresponding payoffs.

	P_2 Deviates	P_2 Sticks
P_1 Deviates	$(+1, +1)$	$(0, +2)$
P_1 Sticks	$(+2, 0)$	$(-1, -1)$

4.6 Fact-Checking Game of Verdicts

Now consider a similar scenario where a fact F has to be checked by a team of two fact-checkers P_1 and P_2 on contract. Say their respective verdicts are $F_1 = \mathtt{r}$ and $F_2 = \mathtt{f}$ such that $F_2 = \neg F_1$. Although they differ in their initial verdicts, they are required to reach an agreement on their final joint verdict to get paid for their work. The joint verdict has to be one of $\{\mathtt{r}, \mathtt{f}\}$. The following are the different action profiles and their corresponding payoffs as represented in Table 5.

- $(\mathtt{f}, \mathtt{r})$ and $(\mathtt{r}, \mathtt{f})$: Unless they agree on a verdict, they do not get paid for their works. So, in both these states their payoffs are $(0, 0)$.
- $(\mathtt{r}, \mathtt{r})$: P_1 had given an initial verdict $F_1 = \mathtt{r}$ and could convince P_2 to agree on it. This is the best outcome for P_1 and hence results in the highest individual payoff for P_1. This is not the best outcome for P_2, but is certainly better than reaching an impasse. Hence, the payoffs are $(2, 1)$.
- $(\mathtt{f}, \mathtt{f})$: P_2 had given an initial verdict $F_2 = \mathtt{f}$ and could convince P_1 to agree on it. For similar reasons as above, the payoffs are $(1, 2)$.

4.7 Voluntary Fact-Checkers' Group Game

Consider a large social media group of voluntary fact-checkers $\{P_1, \ldots, P_n\}$. When a news item F is posted by one of the members P_c of the group to fact-check, any one of the n members (including P_c) may fact-check and provide

Table 5: Game of Verdicts: All possible action profiles (F_1, F_2) such that $F_1, F_2 \in \{\mathtt{r}, \mathtt{f}\}$ and the corresponding payoffs.

	$F_2 = \mathtt{r}$	$F_2 = \mathtt{f}$
$F_1 = \mathtt{r}$	$(2,1)$	$(0,0)$
$F_1 = \mathtt{f}$	$(0,0)$	$(1,2)$

their report to the group. If nobody fact-checks F, the payoff is 0 for everyone in the group. If a member P_s of the group does the fact-checking and shares their report with the group, every player other than P_c gets a payoff x. The payoff of P_s who actually did the fact-checking is $x - \delta$ because fact-checking requires spending resources (time, money, etc.) that reduces their payoff by δ. This game is analogous to the *good Samaritan game* where equilibrium is attained only when all members of the group adopt a *mixed strategy* and does the fact-checking probabilistically once for every N posts in the group (here, N determines how active the group is).

4.8 Association of Fact-Checkers Game

An association of n fact-checkers may follow a democratic decision-making process while producing a verdict on a fact F. Such a mechanism may be modelled as a voting game with n players $P = \{P_1, \dots, P_n\}$. This is a cooperative game where the players in a set $S \subseteq P$ may individually arrive at the same conclusion $F_S \in \{\mathtt{r}, \mathtt{f}\}$ after checking a fact F while the conclusion of $P \setminus S$ is $F_{P \setminus S} = \neg F_S$. However, only some of these sets have the ability to *pass* a verdict. This ability is captured by the characteristic function

$$T(S) = \begin{cases} 1 & \text{if S can pass a verdict,} \\ 0 & \text{otherwise.} \end{cases}$$

We may define the *core* as a subset of fact-checkers who procure maximum individual utility by providing the same joint verdict.

In general, a common voting game assigns individual weights to each of its players. Such games are called *weighted majority voting games* [9, 11] that occur in the decision-making procedures of various public bodies like the International Monetary Fund (IMF) and the European Union (EU) [22, 23]. The weights assigned to each of the players could be based on some socio-economic parameter like population or economic contribution while some players may be given special powers to ensure that no verdict is passed without their consent (blockers) [7].

An association of fact-checkers may function similarly by assigning the weight $w_i, i \in \{1, \dots, n\}$ to its player P_i. A subset $S \subseteq P$ can pass a verdict if its weight $w(S) = \sum_{P_i \in S} w_i$ is greater than (or equal to) a fraction q of the total weight $\omega = \sum_{i=1}^{n} w_i$ of all players. A player $P_i \in P$ may enjoy a certain amount of *voting power* in the system that may be measured using one of several metrics [26,29,32] depending upon their applicability. The assignment of weights to players may consider the fact that the resultant voting powers may demonstrate a level of inequality in their ability to influence a verdict [6, 20].

Regulators of a News Outlet. The fact-checking game between a regulator and a publisher was studied in [38] and a mixed strategy Nash equilibrium was shown to exist. However, a regulator in a fact-checking scenario may work with multiple subordinate fact-checkers to vote on the verdict on a fact F. A verdict can only be passed with the consent of the regulator, while the subordinate fact-checkers also have a say in the matter. Such a scenario may also be modelled using weighted majority voting games with (unique) blockers as studied in [7].

5 An Epistemic Approach to Fact-Checking

A comprehensive epistemic analysis of the scenarios discussed in Section 3 falls beyond the scope of the current work. Instead, in what follows, in order to show the theoretical richness of fact-checking and its relevance for multi-agent systems, we discuss some of those scenarios from an epistemic logical angle.

5.1 Preliminaries

Epistemic logic is a family of formal systems which use logical methods to analyse knowledge situations in multi-agent systems and artificial intelligence. The topics of fake news and fact-checking are ideal domains of inquiry where formal methods of epistemic logic can be very useful. In particular, we focus on multi-agent epistemic logic, doxastic logic and dynamic epistemic logic for fact-checking purposes.

Let us start with setting up the notations. Let P be a set of countably-many propositional variables, and A be a set of agents. The syntax of epistemic logic is given using a Backus-Naur form as follows where $p \in P$ and $a \in A$.

$$varphi := p \ \mid \ \neg\varphi \ \mid \ \varphi \wedge \varphi \ \mid \ K_a\varphi$$

The formula $K_a\varphi$ reads "the agent a knows that φ". Disjunction and implication operators, $\vee$ and $\longrightarrow$, respectively, are taken as abbreviations in the usual sense. The dual of the K_a operator is denoted by $\hat{K}_a$ and defined as $K_a\varphi = \neg\hat{K}_a\neg\varphi$.

Epistemic logical syntax can express complex statements succinctly. For example, the syntax allows us to express formulas like $K_a(K_b\varphi \wedge K_c\varphi)$ which reads as "Agent a knows that both agents b and c know φ".

Epistemic Logic A model of epistemic logic is a tuple $M = (W, \{R_a\}_{a \in A}, V)$, where W is a non-empty set of possible worlds, R_a is a binary equivalence relation on W, and V is a valuation. The equivalence relation R_a varies over set of agents a in A, and is taken as reflexive, symmetric and transitive. The valuation function V specifies which propositional variables are true in each possible world.

The semantics of epistemic logic is given as follows for model M. We will drop the subscript M if it is clear from the context. The formula $w \models_M p$ will read that the proposition p is satisfied at state $w \in W$ in model M.

$w \models_M p$	iff	$w \in V(p)$
$w \models_M \neg\varphi$	iff	not $w \models_M \varphi$
$w \models_M \varphi \wedge \psi$	iff	$w \models_M \varphi$ and $w, \models_M \psi$
$w \models_M K_a\varphi$	iff	for all $v \in W$ such that wR_av, we have $v \models_M \varphi$

The axioms of epistemic logic are given as follows.

- All axioms of propositional logic
- $K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$
- $K_a\varphi \to \varphi$
- $K_a\varphi \to K_aK_a\varphi$
- $\neg K_a\varphi \to K_a\neg K_a\varphi$

A logic is S5 if it is axiomatised by the above axioms. In this work, we take our system of epistemic logic as $S5_n$ for n-many agents.

Doxastic Logic: The Logic of Beliefs Often, agents may not acquire knowledge – which is a strong propositional attitude. Instead, they may *believe* in a proposition. In this case, we resort to doxastic logic – the logic of beliefs.

Doxastic logic uses the same syntax with one difference: the modal operator is $B_a\varphi$, instead of $K_a\varphi$. Dual of B_a is defined as before and denoted by $\hat{B}_a$.

The axioms of doxastic logic are given as follows.

- All axioms of propositional logic
- $B_a(\varphi \to \psi) \to (B_a\varphi \to B_a\psi)$
- $\neg B_a\bot$
- $B_a\varphi \to B_aB_a\varphi$
- $\hat{B}_a\varphi \to B_a\hat{B}_a\varphi$

A logic is KD45 if it is axiomatised by the above axioms. In this work, we take our system of doxastic logic as $KD45_n$ for n-many agents.

Common Knowledge If every agent a in a group A knows φ, we say "φ is mutual knowledge amongst A", denoted $E_A\varphi$. In short, $E_A(\varphi) = \varphi \wedge \bigwedge_{a \in A} K_a\varphi$.

We define common knowledge of φ amongst a set of agents A, denoted $C_A\varphi$ as follows.

$$C_A(\varphi) = \varphi \wedge E_A\varphi \wedge E_AE_A\varphi \wedge \cdots \wedge E_A^k\varphi \wedge \ldots$$

Notice that the above formula for common knowledge is an infinitary one.

Common knowledge is an interesting and a fruitful concept in epistemic logic and game theory [2,4]. Perhaps the most intuitive description of common knowledge is given as follows, underlying how central the concept is in game theory. Imagine a classroom full of students. Assume that suddenly rain breaks off and all the students realise it. Not only they individually realise that the rain has started, but also they observe that all the other students have realised it too. As they collectively observed the rain, and moreover observed that the others have observed the rain as well, the fact that rain has started becomes common knowledge amongst the students.

Dynamic Epistemic Logic Dynamic Epistemic Logic is the umbrella term for the systems that are developed to "study modal logics of model change" [3]. Public Announcement Logic (PAL), which we will focus here, is a dynamic epistemic logic. The language of PAL extends the basic modal language by the formulas of type $[\varphi]\psi$. This formula reads as "after a public announcement of φ, ψ is true". In this context, external announcements are considered truthful. After receiving an announcement, all agents update their knowledge, thus the model is updated.

The semantics of $[\varphi]\psi$ is given as follows for the model $M = (W, \{R_a\}_{a\in A}, V)$:

$$w \models_M [\varphi]\psi \text{ iff } w \models_M \varphi \text{ implies } w \models_{M!\varphi} \psi,$$

where the model $M!\varphi$ is the updated model after the announcement of φ. The model $M!\varphi = (W_\varphi, \{R_{a_\varphi}\}_{a\in A}, V_\varphi)$ is defined as follows: $W_\varphi = \{w \in W : w \models_M \varphi\}$, $R_{a_\varphi} = R_a \cap (W_\varphi \times W_\varphi)$, and $V_\varphi = V \cap W_\varphi$.

Mathematically, PAL is equi-expressible as the modal epistemic logic as each formula of PAL can be effectively reduced to a formula of modal epistemic logic.

5.2 Multiple Iterations of Fact-Checking, Epistemic Logically

Let us start by considering the example of "Multiple Iterations" discussed in Section 3 where fact-checkers fact-check each other.

Let us start by assuming that fact-checker Players 1 and 2 report different information about the same news φ, that is φ_1 and φ_2, respectively. This is an important epistemic assumption as it allows fact-checkers to have some room for error. At this moment, what we know is either φ or its negation is true, that is $\varphi \vee \neg\varphi$. Moreover, we have $B_1\varphi_1$ and $B_2\varphi_2$. Notice that we do not assume that either of the fact-checkers are accurate. Thus, we do not have $\varphi_1 \to \varphi$ or $\varphi_2 \to \varphi$.

Now, let us assume Players 1 and 2 fact-check each other. This is an important supposition as it assumes that fact-checkers can be mistaken or partisan. This is indeed one of common motivations behind fact-checking other fact-checkers.

When Player 1 fact-checks Player 2, he forms a belief about φ_2. It is either $B_1\varphi_2$ or $B_1\neg\varphi_2$. Briefly, $B_1\varphi_2 \vee B_1\neg\varphi_2$. Similarly for Player 2: $B_2\varphi_1 \vee B_2\neg\varphi_1$.

This process can be iterated. Player 1 can fact-check Player 2:

$$B_1(B_2\varphi_1 \vee B_2\neg\varphi_1) \vee B_1\neg(B_2\varphi_1 \vee B_2\neg\varphi_1).$$

Player 2 can check Player 1, too:

$$B_2(B_1\varphi_2 \vee B_1\neg\varphi_2) \vee B_2\neg(B_1\varphi_2 \vee B_1\neg\varphi_2).$$

This procedure can be iterated up to ω. Thus, multiple iterations of fact-checking φ between Players 1 and 2 produce the following formula $\text{IFC}_{1,2}(\varphi)$.

$$\begin{aligned}\text{IFC}_{1,2}(\varphi) :=& (\varphi \vee \neg\varphi) \wedge (B_1\varphi_1 \wedge B_2\varphi_2) \wedge \\ & [B_1(B_2\varphi_1 \vee B_2\neg\varphi_1) \vee B_1\neg(B_2\varphi_1 \vee B_2\neg\varphi_1)] \wedge \\ & [B_2(B_1\varphi_2 \vee B_1\neg\varphi_2) \vee B_2\neg(B_1\varphi_2 \vee B_1\neg\varphi_2)] \wedge \dots\end{aligned}$$

Iterated fact-checking can only happen if we assume that fact-checkers may not be accurate; otherwise, there would be no need to fact-check other fact-checkers. This explains why we used the belief operators B_i rather than the knowledge operator K_i. This is because, if we initially assumed that $K_1\varphi_1$ and $K_2\varphi_2$, then by the axioms of epistemic logic, we would have φ_1 and φ_2 true, which would make no sense for iterated fact-checking.

5.3 Associations of Fact-Checkers, Epistemic Logically

Next, we consider the scenario of "Associations of Fact-Checkers", presented in Section 3 where fact-checkers form groups.

Let us assume that we have $n > 1$ fact-checkers forming $m \leq n$ associations. We start with some notations. If Players $i, j, k \in \{1, \ldots, n\}$ form a coalition, and jointly believe in a proposition φ, we denote their joint belief with $B_{i,j,k}\varphi$. In this case, $B_{i,j,k}\varphi := B_i\varphi \wedge B_j\varphi \wedge B_k\varphi$. Similarly, we denote the coalitions as $c_1, \ldots, c_m$. Each coalition c_i will fact-check φ and declare their outcome as φ_i. For simplicity, we do not count in the possibility of multiple iterations, as the previous formula IFC can easily be extended to multiple agents, where each coalition acts as an individual agent.

The epistemic case of fact-checker associations for n fact-checkers forming m associations for the piece of news φ for a one-shot game can be expressed as follows.

$$
\begin{aligned}
\mathrm{AFC}_{n|m}(\varphi) := &(\varphi \vee \neg\varphi) \ \wedge (B_{c_1}\varphi_1 \wedge \cdots \wedge B_{c_m}\varphi_m) \wedge \\
&[B_{c_1}(B_{c_2}\varphi_2 \vee B_{c_2}\neg\varphi_2) \vee \\
&B_{c_1}\neg(B_{c_2}\varphi_2 \vee B_{c_2}\neg\varphi_2) \vee \cdots \vee \\
&B_{c_1}(B_{c_m}\varphi_m \vee B_{c_m}\neg\varphi_m) \vee \\
&B_{c_1}\neg(B_{c_m}\varphi_m \vee B_{c_m}\neg\varphi_m)] \wedge \\
&\cdots \\
&[B_{c_m}(B_{c_1}\varphi_1 \vee B_{c_1}\neg\varphi_1) \vee \\
&B_{c_m}\neg(B_{c_1}\varphi_1 \vee B_{c_1}\neg\varphi_1) \vee \cdots \vee \\
&B_{c_m}(B_{cm-1}\varphi_{m-1} \vee B_{cm-1}\neg\varphi_{m-1}) \vee \\
&B_{c_m}\neg(B_{cm-1}\varphi_{m-1} \vee B_{cm-1}\neg\varphi_{m-1})] \wedge \\
&\ldots,
\end{aligned}
$$

where $\Sigma_{i=1}^{m}|c_i| = n$.

This allows us to compare the epistemic strength of n fact-checkers forming different associations and coalitions. For example, we can consider coalitions $\{c_i\}_{1 \leq m \leq n}$ and $\{c'_i\}_{1 \leq l \leq n}$ with $\Sigma_{i=1}^{m}|c_i| = n$ and $\Sigma_{i=1}^{l}|c'_i| = n$. In this case, we can compare the coalitions by contrasting the following two formulas: $\mathrm{AFC}_{n|m}(\varphi)$ with respect to association $\{c_i\}$ and $\mathrm{AFC}_{n|l}(\varphi)$ with respect to association $\{c'_i\}$.

A major advantage of this approach is that it allows us to understand how different associations create different epistemic situations. Moreover, this formula given above makes it easier to check from a SAT-solver's perspective, comparing the logical satisfiability of the formula with respect to different associations $\{c_i\}$ and $\{c'_i\}$.

5.4 "We Media", Epistemic Logically

Given a finite set of agents A in our epistemic model, it is possible to create two partitions of A: A_r and A_j where $A_r \cup A_j = A$, and A_r denotes the set of readers and A_j denotes the set of journalists who belong to news outlets. This partitioning makes it easier to trace the epistemic interaction between readers and journalists. For example, the formula $K_j\varphi \rightarrow K_r\varphi$ for $\varphi \in \Phi$ denotes *honest journalism* where a a journalist j writes about all he knows about the subject (that is denoted by the epistemic closure of Φ) so that the reader r knows about them, too.

In "We Media", however, there cannot be any partitioning as the distinction between journalists and readers is blurry. In an epistemic system of "We Media", we cannot be specific about formulas in the form of $K_j\varphi \rightarrow K_r\varphi$. Therefore, there will be no set A_j where for each $j \in A_j$, we have $K_j\varphi \rightarrow K_r\varphi$ for $\varphi \in \Phi$. Traditional media separates A_j and A_r, making it easier to analyse the sets A_j and A_r. "We Media" costs us that epistemic certainty.

5.5 Independent Journalism, Epistemic Logically

A difference between "We Media" and independent journalism can be expressed by epistemic logic. In independent journalism, we allow individuals that are not in A_j, that is those people who do not belong to a news outlet, to produce news. In this case, the epistemic model will still have sets A_r and A_j where A_r and A_j do not necessarily form a partition. There can be some agents $i \in A$ such that $i \notin A_r \cup A_j$.

Consequently, when independent journalists write, everyone knows about it: $K_i\varphi \rightarrow E_{A_r \cup A_j}(\varphi)$ for $\varphi \in \Phi$ for some set of formulas Φ. The syntax of epistemic logic thus allows us to distinguish groups of agents: independent journalists, mainstream journalists and readers.

5.6 User-Generated Content (UGC), Epistemic Logically

Epistemically, UGC blurs the distinction between writers (content generators) and readers. The approach is similar to what we have presented for "We Media" and "Independent Journalism".

Let A_w and A_r be the set of writers and readers, respectively. UGC then is generated by the agents $i \in A_w \cap A_r$, where $A_w, A_r \subseteq A$. If φ is a proposition generated by users, then $K_i\varphi$ for *some* $i \in A_w \cap A_r$.

Proposition 1. *Let φ be a piece of UGC. If a reader r and a writer w knows about it, then there exists a user $u \in A_w \cap A_r$ such that $K_r\varphi \rightarrow K_u\varphi$ and $K_u\varphi \rightarrow K_w\varphi$.*

Proof. If the reader r knows that he himself created the content φ, then $u := r$, and the first part of the result follows. If r knows that it is not himself who created the content, he knows that some other user u generated the content, and r knows that u knows about φ. Thus, $K_r\varphi \to K_u\varphi$. Similarly, if u himself generated the content, then $u := w$, if not w knows that someone else wrote the content.

Hence, $K_r\varphi \to K_u\varphi \to K_w\varphi$.

The above treatment discusses UGC in isolation. It can also be combined with "We Media" and "Independent Journalists" to create a finer partition of agents A. This would generate similar results as presented in Proposition 1.

5.7 Regulators of News Outlets, Epistemic Logically

Determining the dominant fact-checker or the regulator, where multiple fact-checkers are present, requires a voting system. For example, where the multiple fact-checkers are present, some cases can be forwarded to the legal system. In this case, courts may need to act as the dominant fact-checkers whose decisions overrule those of fact-checkers.

Let $1, 2, \dots, n$ be fact-checkers, and R be a regulator fact-checker. In this case, R decides on what the fact-checkers believe. So, we have:

$$(B_1\varphi_1 \vee \dots \vee B_n\varphi_n) \to B_R\varphi_i$$

for some $i \in \{1, \dots, n\}$.

This formulation makes it clear that the regulator chooses out of what the fact-checkers have come up with, rather than fact-checking themselves. If we drop the condition on the formula within the scope of the modality B_R, then we obtain a formula describing an unrestricted regulator – a regulator who comes up with its own fact-checking. In this case, we have the following:

$$(B_1\varphi_1 \vee \dots \vee B_n\varphi_n) \to B_R\varphi.$$

5.8 Fact-Checking Fact-Checkers, Dynamic Epistemic Logically

Public announcement logic offers an alternative formalism to the scenario of "Fact-Checking Fact-Checkers".

Let agents $1, 2 \in A$ be fact-checkers for φ_1, φ_2, respectively. Let $B_1\varphi_1$ and assume that it is publicly announced: imagine a fact-checker publishing their findings. Now, after this update, 2 publishes their finding about φ_2 in the updated model. We then have the following:

$$w \models_M [B_1\varphi_1]B_2\varphi_2,$$

which reduces to $w \models_{M!B_1\varphi_1} B_2\varphi_2$. This pattern can be extended to n-agents easily: $w \models_M [B_1\varphi_1] \dots [B_{n-1}\varphi_{n-1}]B_n\varphi_n$.

Since in PAL $[\varphi][\psi]\chi \iff [\psi][\varphi]\chi$, the order of fact-checking the fact-checkers does not matter. Along the same lines, it is possible to express self fact-checking: $w \models_M [B_1\varphi]B_1\varphi_1$.

But, this stabilises just after one step:

$$w \models_M [B_1\varphi]B_1\varphi_1 \leftrightarrow [B_1\varphi][B_1\varphi]\varphi_1.$$

Therefore, a fact-checker can fact-check itself dynamically only once. The consecutive fact-checking activities will not lead to any model updates.

PAL for fact-checking is a fruitful area. It allows us to combine a variety of epistemic attitudes with the scenarios we consider in Section 3. We leave it for future work.

6 Conclusion and Future Work

Game theory is a broad field. Game theoretical approach to fact-checking underlines the *homo economicus* element of fact-checking. A long-term goal of the game theoretical approach is to offer a *behavioural economical* angle for fact-checking to explain the systematically irrational behaviour of *homo economics* [1].

Epistemic logic, on the other hand, helps us relate fact-checking to multi-agent systems and knowledge engineering. A long-term goal of the epistemic approach is to provide an in-depth analysis of the *dynamic* elements of knowledge and belief in fact-checking. The epistemic approach clarifies the distinction between facts, knowledge and belief formally and clearly. We can thus talk about "facts we do know", "facts we can know" and "facts we can check". The prior two have been analysed extensively in the epistemic literature [31] whereas the latter one not quite so.

Tools and techniques from game theory and epistemic logic allow us to discuss the nuances and previously unnoticed theoretical subtleties of the subject. Eventually, this helps us understand the depth and the breadth of fact-checking, which can hopefully inform political and sociological approaches to the topic. This paper aims at this ambitious goal.

An immediate next step of our work is to focus on the *dynamic epistemic game theory* of fact-checking. Epistemic game theory studies the role of knowledge in game theory. How knowledge impacts agents' strategising around fake news is a question of interest for epistemic game theory. By doing so, we can formalise how agents' knowledge does or does not change by fake news and how the knowledge relies on fact-checking.

Among scenarios this work does not cover, an interesting one is *blockchain for fact-checking*, i.e., a distributed ledger facilitating cooperation between fact-checkers. This also allows us to discuss the knowledge situation of fact-checkers using a public database with immutable history as well as the strategising between the agents and the environment. For instance, we can discuss how such a blockchain presents a definitive solution for the iterated fact-checking by effectively providing dynamic epistemic updates.

This work presents initial analysis of fact-checking using game theory and epistemic logic. We leave more in-depth analyses for future work.

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