Different costs-informed component preventive maintenance with system lifetime changes*

Hongyan Dui[a, b†], Chi Zhang[c], Tianzi Tian[d], Shaomin Wu[e]

[a]*School of Management, Zhengzhou University, Zhengzhou 450001, China*

[b]*School of Business, Luoyang Vocational & Technical College, Luoyang, Henan, 471000, China*

[c]*Laboratory of Science and Technology on Integrated Logistics Support, College of Intelligence Science and Technology, National University of Defense Technology, Changsha 410073, China*

[d]*School of Reliability and System Engineering, Beihang University, Beijing 100191, China*

[e]*Kent Business School, University of Kent, Canterbury, Kent CT2 7FS, UK*

Abstract: System safety assessment is a technique aiming at identifying hazards for the system under analysis and showing the compliance with the safety requirements. In order to increase the safety of a system, one may select components in the system for preventive maintenance, under the constraints of maintenance cost, maintenance time and the availability of maintenance staff. In different maintenance policies, maintenance cost can differ. This paper proposes some measures for component preventive maintenance considering maintenance effectiveness, based on which the expected costs due to a component and the system are investigated, respectively. Three different maintenance cost scenarios are analyzed for different maintenance policies. Considering both cost and maintenance constraints, components are optimally chosen for preventive maintenance. An application of a hydraulic system for an aircraft is then used to illustrate the proposed method.

Keywords: Maintenance policy; Different cost; System Lifetime; Importance measure

1. Introduction

Reducing the economic loss caused by a component failure and increasing the system availability through effective maintenance activities are important in reliability engineering and they can be achieved through proper preventive maintenance, which can extend the system lifetime. Optimally scheduling preventive maintenance can provide a scientific basis for the engineers in their making managerial decisions.

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† Corresponding author. duihongyan@zzu.edu.cn. Tel: (86) 371- 67781582, Fax: (86) 371- 67781582.

However, improperly improving system reliability may increase failure losses. The current literature lacks a discussion of maintenance policy on the relationship between system reliability modifications and relevant different costs from the perspective of the system lifetime, which will be addressed in this paper.

The continuous development of technical systems and the increasing reliance on equipment have led to the increasing importance of ensuring component lifetime and maintenance economy. In order to better maintain system and manage system health (Lu et al., 2012; Haddad et al., 2012), many researchers have conducted related studies on these. Iscioglu (2021) analysed the remaining life function of a polymorphic system and evaluated the situation when the life cycles are independent and dependent on each other. Bohlooli-Zefreh et al. (2021) further applied a cost function to obtain the best replacement strategy for the system based on the proposed failure model. In terms of application, Shafiee et al. (2019) proposed a combined network analysis process and topic analysis model to select scenarios and low-risk maintenance strategies for different component sets of the system. Levitin et al. (2018) studied a method to evaluate the expected unfinished part of the task and the survivability of the system experiencing internal failures and external shocks. Tan et al. (2011) proposed an index for the remaining life that considered the degree of damage in the time period. It's used to evaluate maintenance policies to improve system lifetime.

Importance measures can be used to identify weak parts of a system (Levitin et al., 2003; Si et al., 2020; Dui et al., 2022; Wu et al., 2016; Gravette et al., 2015; Xu et al., 2020; Huang et al., 2020). Some authors have therefore considered the use of importance measure for maintenance, for example, Nguyen et al. (2017) proposed a joint predictive maintenance for a system with a complex structure and multiple different components. The predictive state index and the structural importance measure of the component are combined to construct preventive maintenance. Dui et al. (2019) proposed an extended joint integrated importance measure to guide the corrective maintenance of faulty components and preventive maintenance of operational components, aiming to improve the system lifetime. Further, this importance measure is used in more practical researches (Zhang et al., 2020; Dui et al., 2021). According to the structure function, Marichal and Mathonet (2013) gave an explicit expression of the Barlow–Proschan importance measure in general. It measured the probability that the failure of a given component will cause the system to fail when component life is independent. How to balance system reliability and expected loss is a key issue. Levitin et al. (2021a) gave the optimization of cyclic preventive replacement with reusable elements exposed to shocks. From the perspective of system lifetime, Navarro and Durante (2017) proposed the representation of the reliability function of the system lifetime in time based on the coherent system under different backgrounds. Zhang et al. (2020) proposed an iterative algorithm to calculate the reliability and extend the lifetime of the backup system in the case of backup redundancy. Zhao et al. (2021) studied the reliability of critical systems under different failure modes, and derived system reliability indicators based on the constructed model, including system life and remaining life distribution functions.

However, in a complex system, high reliability does not necessarily mean low failure loss. Integrating the failure rate and the cost of failure can better minimize the system failure loss during a system's life cycle. Xu et al. (2012) gave the optimal replacement strategy to minimize the expected system unit time cost while meeting its reliability requirements. Levitin et al. (2019) analyzed the maintenance cost effective of imperfect inspections. Vu et al. (2016) developed a cost model that considers the economic and structural dependencies between components. When it comes to losses, related literature has different discussions on the cost classification of maintenance. Dui et al. (2017) considered the impact of maintenance costs on system reliability. Wu and Coolen (2013) considered two types of costs, in which one is the cost due to system failure, and the other is the cost of maintenance. In fact, the maintenance cost of components should also change with the reliability of components and cannot be simply regarded constant.

Resource limitation is also a problem for current engineering companies. Issues such as optimizing resource allocation and maintenance optimization have been discussed (Levitin and Lisnianski, 2000; Si et al., 2019; Liu et al., 2020). Yuan et al. (2015) gave a potential cost index, which was used to exclude measures that exceed budget. At the same time, they constructed a knapsack problem that is to solve the corresponding measures to maximize the system lifetime under the limited budget, and give examples. Jafary et al. (2017) proposed a simple method of explicit correlation parameters to characterize related component failures, and then gave the best maintenance policy to minimize system failures. Levitin et al. (2021b) studied the optimal multiple replacement and maintenance scheduling considering the resource limitation. One needs to conduct an in-depth investigation on planning issues on the system performance optimization from the perspective of selection of maintenance components under resource constraints.

If one can further know the magnitude of the loss caused due to system failures in the life cycle, one can optimally choose maintenance policies. The current issues still lack some thinking:

- While improving the reliability of components, will it also increase the possible losses caused by system failure? How to weigh the system reliability and expected loss?

- When it comes to losses, how to consider the maintenance cost? It is not comprehensive to only consider maintenance cost when a component fails, so how to consider other factors?

- Will the reliability and maintenance costs of other components affect the preventive maintenance policy? How to determine the components of preventive maintenance and improve the system reliability to a greater extent?

The main contributions of this paper are three aspects:

(1) This paper gives the expected losses from failures of the system by considering the cost and failure rate, and gradually construct the change of system lifetime and maintenance cost.

(2) For three different cost scenarios, the paper proposes the corresponding expected cost functions for component failures and working conditions. Among them, the cost setting can help maintenance personnel to identify the components that need improving.

(3) We consider three limitations: total maintenance cost, maintenance time and the ability of maintenance personnel. After specifying specific restrictions, a preventive maintenance (PM) plan can be scheduled to maximize the performance. In addition, it also analyzes the cost changes affected by the maintenance policy.

This paper proposes a new maintenance index that takes into account different maintenance cost scenarios, which is dedicated to solving the maintenance optimization problem. The rest of this paper is organized as follows. Section 2 gives different cost analysis with maintenance effectiveness being considered. Section 3 studies component preventive maintenance on the expected losses. Section 4 verifies the correctness and effectiveness of the model through the case analysis of a hydraulic system. The conclusions and future work are provided in Section 5.

2. Different cost analysis on system lifetime change

Denote the system lifetime distribution by $F = F(\boldsymbol{q}(\boldsymbol{t})) = F(q_1(t), q_2(t), \ldots, q_n(t))$, where $q_i(t)$ Denote the system lifetime distribution by $F = F(\boldsymbol{q}(\boldsymbol{t})) = F(q_1(t), q_2(t), \ldots, q_n(t))$, where $q_i(t)$ is the lifetime distribution of component *i*. Corrective maintenance is performed upon component failure. The change of the system lifetime distribution due to restoring component *i* from the failed state to the functioning state is $F\left(0_i, \boldsymbol{X}(t)\right) - F(1_i, \boldsymbol{X}(t))$. Denote the failure rate of component k as $\lambda_k(t)$, which equals $\lambda_k(t) = \dfrac{dF_k(t)/dt}{R_k(t)}$.

The well-known joint failure importance given by $JFI(i,k) = F(0_i, 0_k, \boldsymbol{X}(t)) + F(1_i, 1_k, \boldsymbol{X}(t)) - F(1_i, 0_k, \boldsymbol{X}(t)) - F(0_i, 1_k, \boldsymbol{X}(t))$.

Once component i fails, the system lifetime distribution $F(q)$ becomes $F(0_i, \boldsymbol{X}(t))$. The probability density function of the system, given that component i has failed, is given by

$$\frac{dF(0_i, \boldsymbol{X}(t))}{dt} = \frac{dF_k(t)}{dt}\frac{dF(0_i, \boldsymbol{X}(t))}{dF_k(t)} = \frac{dF_k(t)}{dt}\Big(F(0_i, 0_k, \boldsymbol{X}(t)) - F(0_i, 1_k, \boldsymbol{X}(t))\Big)$$

$$= \lambda_k(t)R_k(t)\Big(F(0_i, 0_k, \boldsymbol{X}(t)) - F(0_i, 1_k, \boldsymbol{X}(t))\Big)$$

Let $I_k(t)_{X_i(t)=0} = \lambda_k(t)R_k(t)\ F(0_i,\mathbf{0}_k,\boldsymbol{X}(t)) - F(0_i,\mathbf{1}_k,\boldsymbol{X}(t))$. Then $I_k(t)_{X_i(t)=0}$ is the effect of component k on system lifetime for failed component i.

Similarly, the probability density function of the system, given that component i is working, is given by

$$\frac{dF(1_i, \boldsymbol{X}(t))}{dt} = \frac{dF_k(t)}{dt}\frac{dF(1_i, \boldsymbol{X}(t))}{dF_k(t)} = \frac{dF_k(t)}{dt}\Big(F(1_i, 0_k, \boldsymbol{X}(t)) - F(1_i, 0_k, \boldsymbol{X}(t))\Big)$$

$$= \lambda_k(t)R_k(t)\Big(F(1_i, 0_k, \boldsymbol{X}(t)) - F(1_i, 1_k, \boldsymbol{X}(t))\Big)$$

Let $I_k(t)_{X_i(t)=1} = \lambda_k(t)R_k(t)\ F(1_i,\mathbf{0}_k,\boldsymbol{X}(t)) - F(1_i,\mathbf{1}_k,\boldsymbol{X}(t))$. Then $I_k(t)_{X_i(t)=1}$ is the effect of component k on the system lifetime for working component i.

When component i is repaired from state 0 to state 1, the change of system lifetime is $F(0_i, \boldsymbol{X}(t)) - F(1_i, \boldsymbol{X}(t))$. Thus, we can obtain

$$\frac{d[F(0_i,\boldsymbol{X}(t))\text{-}F(1_i,\boldsymbol{X}(t))]}{dt} = \lambda_k(t)R_k(t)[F(0_i,0_k,\boldsymbol{X}(t)) - F(0_i,1_k,\boldsymbol{X}(t))] - \lambda_k(t)R_k(t)[F(1_i,0_k,\boldsymbol{X}(t)) - F(1_i,1_k,\boldsymbol{X}(t))]$$
$$= \lambda_k(t)R_k(t)[F(0_i,0_k,\boldsymbol{X}(t)) + F(1_i,1_k,\boldsymbol{X}(t)) - F(0_i,1_k,\boldsymbol{X}(t)) - F(1_i,0_k,\boldsymbol{X}(t))] = \lambda_k(t)R_k(t)JFI(i,k)$$

Let $I_k(t)_{X_i(t)} = I_k(t)_{X_i(t)=0} - I_k(t)_{X_i(t)=1}$. Then $I_k(t)_{X_i(t)}$ is the effect of component k on the system lifetime when component i is restored from state 0 to state 1.

In a real system, higher reliability does not necessarily mean smaller losses from failure. Improperly increasing the system reliability may increase the losses from failure. Consequently, to solve the problem of minimizing system failure loss during the lifecycle of the production system, it is necessary to consider the cost in combination with the different component failures.

Let $JFI_c(i,k)_{X_i(t)=0} = c_{k|X_i(t)=0}\ F\ 0_i,0_k,\boldsymbol{X}\ t\ - F\ 0_i,1_k,\boldsymbol{X}\ t$ represent the losses from failures per unit time caused by component k when component i is failed. The $c_{k|X_i(t)=0}$ is the maintenance cost of improving component k when component i is at state 0.

Then, let $JFI_c(i,k)_{X_i(t)=1} = c_{k|X_i(t)=1}\left(F\left(1_i,0_k,\boldsymbol{X}(t)\right) - F\left(1_i,1_k,\boldsymbol{X}(t)\right)\right)$ represent the losses from failures per unit time caused by component k when component i is working. The $c_{k|X_i(t)=1}$ is the maintenance cost of improving component k when component i is at state 1.

Consequently, the joint loss importance can be defined as

$$I_1(i,k)_{X_i(t)} = JFI_c(i,k)_{X_i(t)=0} - JFI_c(i,k)_{X_i(t)=1}, \tag{1}$$

and

$$I_1(i,k)_{X_i(t)} = c_{k|X_i(t)=0}\left(F\left(0_i,\mathbf{0}_k,X(t)\right) - F\left(0_i,\mathbf{1}_k,X(t)\right)\right) - c_{k|X_i(t)=1}\left(F\left(1_i,\mathbf{0}_k,X(t)\right) - F\left(1_i,\mathbf{1}_k,X(t)\right)\right). \tag{2}$$

Equation (2) describes the contribution of component k to the change of system loss when repairing component i or performing preventive maintenance on component k.

Scenario 1: considering the cost of maintaining the system and components.

We consider two types of costs: the cost of maintaining the system and the cost of maintaining components in the following.

If component i is a critical component, then when it is at state 0, the system will fail after component i fails. Without considering the repair time, if component k is repaired or preventively maintained, when the system has failed, regardless of whether the component k is a critical component, it will not incur system cost. As a result, we only need to consider the maintenance cost.

When component i is a non-critical component, if component k is a critical component, the failure of component k will incur costs. Subsequently, if component k is not a critical component, then only the maintenance cost can be calculated. Consequently, when component i fails, we have

$$c_{k|X_i(t)=0} = c_k \cdot \Pr\left(\phi\left(0_i,\mathbf{1}_i\right) < K\right) + \Pr\left(\phi\left(0_i,\mathbf{1}_i\right) \geq K\right) \cdot \left[c_{s,k} \cdot \Pr\left(\phi\left(0_i,0_k,\mathbf{1}_{i,k}\right) < K\right) + c_k\right], \tag{3}$$

where c_k is the cost incurred due to maintaining component k and $c_{s,k}$ is the system cost of maintaining component k. $\left(0_i,\mathbf{1}_i\right)$ represents that component i stop working and all the other components are working and $\phi\left(0_i,\mathbf{1}_i\right) < K$ means that when the component i fails, the state of the system will be lower than the failure threshold K, i.e., the system fails. $\Pr\left(\phi\left(0_i,\mathbf{1}_i\right) < K\right)$ ensures that the system will not fail when the component i is selected for maintenance. In addition to being a critical component, component i can only be a non-critical component, then $\Pr\left(\phi\left(0_i,\mathbf{1}_i\right) < K\right) + \Pr\left(\phi\left(0_i,\mathbf{1}_i\right) \geq K\right) = 1$.

In addition, if component i does not fail, then we only need to consider whether the system fails during the process of maintaining component k, which leads to system cost. In other words, whether component k is a critical component is the key in this case. So, we have

$$c_{k|X_i(t)=1} = c_{s,k} \cdot \Pr\left(\phi\left(0_k, \mathbf{1}_k\right) < K\right) + c_k. \tag{4}$$

Scenario 2: dividing the cost of maintaining components specifically based on scenario 1.

In fact, under actual application conditions, the cost of preventive maintenance and corrective maintenance of component k are different. We can distinguish whether component k is undergoing preventive maintenance or corrective maintenance based on the reliability of component k. Define c_k^{pf} as the cost of preventive maintenance for component k, and c_k^f as the maintenance cost for component k failure. Then equations (3) and (4) can be changed to

$$c_{k|X_i(t)=0} = \left(\theta_k c_k^f + (1-\theta_k)c_k^{pf}\right)\Pr\left(\phi\left(0_i, \mathbf{1}_i\right) < K\right) + \Pr\left(\phi\left(0_i, \mathbf{1}_i\right) \geq K\right)\begin{bmatrix} c_{s,k} \cdot \Pr\left[\phi\left(0_i, 0_k, \mathbf{1}_{i,k}\right) < K\right] \\ +\theta_k c_k^f + (1-\theta_k)c_k^{pf} \end{bmatrix}, \tag{5}$$

$$c_{k|X_i(t)=1} = c_{s,k} \cdot \Pr\left(\phi\left(0_k, \mathbf{1}_k\right) < K\right) + \theta_k c_k^f + (1-\theta_k)c_k^{pf}, \tag{6}$$

where θ_k is a 0-1 variable, it can be used to choose whether to perform preventive maintenance or corrective maintenance based on the reliability of component k. We can choose a reliability threshold to determine whether to perform corrective maintenance or preventive maintenance.

Scenario 3: related to the component reliability.

The relationship between cost and reliability of each component can be obtained based on past experience or data of similar components. However, in many cases, such data is not available. To be more specific, cost is a monotonically increasing function of component reliability. According to Si et al., 2019, the cost distribution function can be given as

$$C_k(t) = a_k e^{\left[(1-f_k)\frac{R_k(t)-R_{k,\min}}{R_{k,\max}-R_k(t)}\right]} \tag{7}$$

where f_k is the feasibility of increasing the reliability of component k, and it assumes values between 0 and 1. $R_{k,\min}$ represents minimum reliability of component i. Then $R_{k,\max}$ means the maximum achievable reliability of the component i. a_k is the cost coefficient corresponding to each component.

The feasibility parameter i.e., f_k is a constant, which indicates the difficulty of improving the reliability of the components relative to the rest of the components in the system. Many authors have proposed weighting factors for assigning reliability, which can be used to quantify feasibility. These weights depend on certain influencing factors. Other literatures also summarize some complex methods, such as component complexity, technical level, operating status, criticality, etc.

According to the proposed cost distribution function, we can express the maintenance cost as $c_k(t) = a_k e^{\left[(1-f_k)\frac{R_k^{am}(t)-R_{k,\min}}{R_{k,\max}-R_k^{am}(t)}\right]} - a_k e^{\left[(1-f_k)\frac{R_k^{bm}(t)-R_{k,\min}}{R_{k,\max}-R_k^{bm}(t)}\right]}$, where $R_k^{am}(t)$ represents the reliability of the component after repair, and $R_k^{bm}(t)$ represents the reliability of the component before repair.

Depend on the above definition, whether component i fails or not will affect the feasibility factor in the cost function for the maintenance of component k. In order to better illustrate the effect of combining the above cost functions, we give the following analysis

$$c_k(t)_{X_i(t)=0} = a_k e^{\left[(1-f_{k|X_i(t)=0})\frac{R_k^{am}(t)-R_{k,\min}}{R_{k,\max}-R_k^{am}(t)}\right]} - a_k e^{\left[(1-f_{k|X_i(t)=0})\frac{R_k^{bm}(t)-R_{k,\min}}{R_{k,\max}-R_k^{bm}(t)}\right]}, \tag{8}$$

and

$$c_k(t)_{X_i(t)=1} = a_k e^{\left[(1-f_{k|X_i(t)=1})\frac{R_k^{am}(t)-R_{k,\min}}{R_{k,\max}-R_k^{am}(t)}\right]} - a_k e^{\left[(1-f_{k|X_i(t)=1})\frac{R_k^{bm}(t)-R_{k,\min}}{R_{k,\max}-R_k^{bm}(t)}\right]}, \tag{9}$$

where $f_{k|X_i(t)=0}$ and $f_{k|X_i(t)=1}$ are the feasibility of repairing component k when component i fails and works, respectively.

Proposition 1. For the different kinds of components i and k,

- If components i and k are critical components, then $f_{k|X_i(t)=0} > f_{k|X_i(t)=1}$.
- If components i and k are non-critical components, then $f_{k|X_i(t)=0} \le f_{k|X_i(t)=1}$.
- If component i is a critical component and component k is a non-critical component, then $f_{k|X_i(t)=0} = f_{k|X_i(t)=1}$.
- If component i is a non-critical component and component k is a critical component, then $f_{k|X_i(t)=0} = f_{k|X_i(t)=1}$.

Proof. If components i and k are critical components, then when component i fails, the system has stopped running. Currently, repairing component k does not need to bear the system cost. While repairing component k when component i is operating normally, additional system costs need to be borne. So we have $c_k(t)_{X_i(t)=0} < c_k(t)_{X_i(t)=1}$.

For the formula $c_k(t) = a_k e^{\left[(1-f_k)\frac{R_k^{am}(t)-R_{k,\min}}{R_{k,\max}-R_k^{am}(t)}\right]} - a_k e^{\left[(1-f_k)\frac{R_k^{bm}(t)-R_{k,\min}}{R_{k,\max}-R_k^{bm}(t)}\right]}$, we can construct a new function as $c(x) = e^{\left[(1-x)\mathbf{R}^a\right]} - e^{\left[(1-x)\mathbf{R}^b\right]}$, where $\mathbf{R}^a = \frac{R_k^{am}(t)-R_{k,\min}}{R_{k,\max}-R_k^{am}(t)}$ and $\mathbf{R}^b = \frac{R_k^{bm}(t)-R_{k,\min}}{R_{k,\max}-R_k^{bm}(t)}$. Because $R_k^{am}(t) > R_k^{bm}(t)$, we can get $\mathbf{R}^a > \mathbf{R}^b$.

Because $\frac{dc(x)}{dx} = \mathbf{R}^b e^{\left[(1-x)\mathbf{R}^b\right]} - \mathbf{R}^a e^{\left[(1-x)\mathbf{R}^a\right]} < 0$, $c(x)$ is a monotonically decreasing function. According to function $c(x)$ and $c_k(t)_{X_i(t)=0} < c_k(t)_{X_i(t)=1}$, we can obtain $f_{k|X_i(t)=0} > f_{k|X_i(t)=1}$.

When components i and k are non-critical components, if components i and k fail at the same time and cause the system to stop running $\phi\ 0_i,0_k,\boldsymbol{X}(t)\ < K$, then $c_k(t)_{X_i(t)=0} > c_k(t)_{X_i(t)=1}$. If components i and k fail at the same time and cause the system is still operating normally $\phi\ 0_i,0_k,\boldsymbol{X}(t)\ \geq K$, then $c_k(t)_{X_i(t)=0} = c_k(t)_{X_i(t)=1}$. Thus, we can obtain $f_{k|X_i(t)=0} \leq f_{k|X_i(t)=1}$.

If component i is a critical component and component k is a non-critical component, then we can obtain $c_k(t)_{X_i(t)=0} = c_k(t)_{X_i(t)=1}$. On the contrary, we get $c_k(t)_{X_i(t)=0} = c_k(t)_{X_i(t)=1}$. So $f_{k|X_i(t)=0} = f_{k|X_i(t)=1}$.

□

3 Component preventive maintenance on the expected losses

According to section 2, considering the second-order joint effect, the cost contribution of maintaining component k on the system lifetime when component i is at state 0 can be given

$$I_2(i,k)_{X_i(t)=0} = \lambda_k(t) R_k(t)_{X_i(t)=0} JFI_c(i,k)_{X_i(t)=0} \tag{10}$$

It can be used to illustrate the contribution of component k to the extension of system lifetime due to the avoidance of maintenance costs and system losses due to the absence of failures. Similarly, the contribution of maintaining component k to system when component i is at state 1 can be given

$$I_2(i,k)_{X_i(t)=1} = \lambda_k(t) R_k(t)_{X_i(t)=1} JFI_c(i,k)_{X_i(t)=1}. \tag{11}$$

In order to better reflect the combined influence of the reliability of the two components on the system, the reliability of the other component when a component is in a working or failing state needs to be given. Jafary et al. (2020) proposed the reliability impact of positive and negative correlations between two components.

$$R_k\ t\ _{X_i\ t\ =1} = E\left[R_k\ t\ \mid R_i\ t\ =1\right] = \frac{p_i\ t\ p_k\ t\ + \rho_{i,k}\sigma_i\ t\ \sigma_k\ t}{p_i\ t}. \tag{12}$$

where $\sigma_i(t)=\sqrt{Var\ R_i(t)}=\sqrt{p_i(t)q_i(t)}$, $p_i(t)+q_i(t)=1$. $R_i(t)=1$ if component i is reliable and $R_i(t)=0$ otherwise. For simplicity, success and failure are alternatively denoted with $p_i(t)$ and $q_i(t)$. The correlation between a pair of components *i* and *k* is denoted $\rho_{i,k}$.

Then multiplying the numerator and denominator by $\sigma_i(t)$ in equation (12), we can get

$$R_k(t)_{X_i(t)=1}=p_k(t)+\frac{\rho_{i,k}\sigma_i^2(t)\sigma_k(t)}{\sigma_i(t)p_i(t)}=p_k(t)+\frac{\rho_{i,k}\sigma_k(t)p_i(t)q_i(t)}{\sigma_i(t)p_i(t)}=p_k(t)+\frac{\rho_{i,k}\sigma_k(t)q_i(t)}{\sigma_i(t)}. \quad (13)$$

Similarly, we have

$$\begin{aligned}
R_k(t)_{X_i(t)=0}&=E\left[R_k(t)\,|\,R_i(t)=0\right]=\frac{p_i(t)p_k(t)+\rho_{i,k}\sigma_i(t)\sigma_k(t)}{q_i(t)}\\
&=\frac{\left(1-q_i(t)\right)p_k(t)+\rho_{i,k}\sigma_i(t)\sigma_k(t)}{q_i(t)}=\frac{p_k(t)}{q_i(t)}-p_k(t)+\frac{\rho_{i,k}\sigma_i(t)\sigma_k(t)}{q_i(t)}\\
&=\frac{p_k(t)}{q_i(t)}-p_k(t)+\frac{\rho_{i,k}\sigma_i^2(t)\sigma_k(t)}{q_i(t)\sigma_i(t)}=\frac{p_k(t)}{q_i(t)}-p_k(t)+\frac{\rho_{i,k}\sigma_k(t)p_i(t)q_i(t)}{q_i(t)\sigma_i(t)}\\
&=\frac{p_k(t)}{q_i(t)}-p_k(t)+\frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}.
\end{aligned} \quad (14)$$

When component *i* is difficult or impossible to observe the real-time state, equations (13) and (14) are mainly used to obtain the state information of component *i* through component *k*. Components *k* and *i* are associated and components *k* is easily observed.

Consequently, the joint lifetime importance can be defined as

$$\begin{aligned}
&I_2(i,k)_{X_i(t)}=I_2(i,k)_{X_i(t)=0}-I_2(i,k)_{X_i(t)=1}\\
&=\lambda_k(t)R_k(t)_{X_i(t)=0}JFI_c(i,k)_{X_i(t)=0}-\lambda_k(t)R_k(t)_{X_i(t)=1}JFI_c(i,k)_{X_i(t)=1}\\
&=\lambda_k(t)R_k(t)_{X_i(t)=0}c_{k|X_i(t)=0}\left(F(0_i,\mathbf{0}_k,\boldsymbol{X}(t))-F(0_i,\mathbf{1}_k,\boldsymbol{X}(t))\right)\\
&-\lambda_k(t)R_k(t)_{X_i(t)=1}c_{k|X_i(t)=1}\left(F(1_i,\mathbf{0}_k,\boldsymbol{X}(t))-F(1_i,\mathbf{1}_k,\boldsymbol{X}(t))\right)
\end{aligned} \quad (13)$$

It describes the contribution of component *k* to the system when repairing component *i* or performing preventive maintenance on it. The contribution of maintaining component *i* to extend the life of the system as much as possible and avoid failure losses can be expressed as $I_3(i)_{X_i(t)}$ .Then we have expected lifetime importance,

$$\begin{aligned}
I_3(i)_{X_i(t)}&=\sum_{k=1,k\neq i}^{n}\left[\lambda_k(t)R_k(t)_{X_i(t)=0}JFI_c(i,k)_{X_i(t)=0}-\lambda_k(t)R_k(t)_{X_i(t)=1}JFI_c(i,k)_{X_i(t)=1}\right]\\
&=\sum_{k=1,k\neq i}^{n}\begin{bmatrix}\lambda_k(t)R_k(t)_{X_i(t)=0}c_{k|X_i(t)=0}\left(F(0_i,\mathbf{0}_k,\boldsymbol{X}(t))-F(0_i,\mathbf{1}_k,\boldsymbol{X}(t))\right)\\-\lambda_k(t)R_k(t)_{X_i(t)=1}c_{k|X_i(t)=1}\left(F(1_i,\mathbf{0}_k,\boldsymbol{X}(t))-F(1_i,\mathbf{1}_k,\boldsymbol{X}(t))\right)\end{bmatrix}=\sum_{k=1,k\neq i}^{n}I_2(i,k)_{X_i(t)}.
\end{aligned} \quad (14)$$

It represents the expected contribution of repair component *i* to the prolonged system lifetime. Based on the three different cost scenarios, we need to further analyse these three scenarios separately.

Scenario 1

According to the reliability effect of positive and negative correlations between two components and Equations (13), (14), we can derive Equations (3) and (4) as

$$
\begin{aligned}
& c_{k|X_i(t)=0} = c_k \cdot \Pr\left\{R_U\left(\frac{p_1(t)}{q_i(t)} - p_1(t) + \frac{p_i(t)\rho_{i,1}\sigma_1(t)}{\sigma_i(t)}, \frac{p_2(t)}{q_i(t)} - p_2(t) + \frac{p_i(t)\rho_{i,2}\sigma_2(t)}{\sigma_i(t)}, \ldots, \frac{p_j(t)}{q_i(t)} - p_j(t) + \frac{p_i(t)\rho_{i,j}\sigma_j(t)}{\sigma_i(t)}\right) < \tau\right\} + \\
& \Pr\left\{R_U\left(\frac{p_1(t)}{q_i(t)} - p_1(t) + \frac{p_i(t)\rho_{i,1}\sigma_1(t)}{\sigma_i}, \frac{p_2(t)}{q_i(t)} - p_2(t) + \frac{p_i(t)\rho_{i,2}\sigma_2(t)}{\sigma_i}, \ldots, \frac{p_j(t)}{q_i(t)} - p_j(t) + \frac{p_i(t)\rho_{i,j}\sigma_j(t)}{\sigma_i}\right) \geq \tau\right\} \cdot \\
& \left[c_{s,k} \cdot \Pr\left\{R_U\left(\frac{p_1(t)}{q_k(t)} - p_k(t) + \frac{p_k(t)\rho_{k,1}\sigma_1(t)}{\sigma_k(t)}, \frac{p_2(t)}{q_k(t)} - p_2(t) + \frac{p_k(t)\rho_{k,2}\sigma_2(t)}{\sigma_k(t)}, \ldots, \frac{p_j(t)}{q_k(t)} - p_j(t) + \frac{p_k(t)\rho_{k,j}\sigma_j(t)}{\sigma_k(t)}\right) < \tau\right\} + c_k\right] \\
& , j \in \{1,2,\ldots,n\}, j \neq i \,\&\, j \neq k,
\end{aligned}
$$

$$
\begin{aligned}
& c_{k|X_i(t)=1} = c_{s,k} \cdot \Pr\left\{R_U\left(p_1(t) + \frac{q_k(t)\rho_{k,1}\sigma_1(t)}{\sigma_k(t)}, p_2(t) + \frac{q_k(t)\rho_{k,2}\sigma_2(t)}{\sigma_k(t)}, \ldots, p_j(t) + \frac{q_k(t)\rho_{k,j}\sigma_j(t)}{\sigma_k(t)}\right) < \tau\right\} + c_k, \\
& j \in \{1,2,\ldots,n\} \,\&\, j \neq k,
\end{aligned}
$$

where $R_U(t)$ represents the reliability of the system at time *t*, and τ is the system reliability threshold. When the reliability of the system is lower than the threshold τ, the system fails. We can calculate the reliability of component *k* by observing the reliability of component *i*. Similarly, we can calculate the reliability of other components related to component *i,* based on which one can obtain the overall performance reliability of the system at time *t*.

Scenario 2

Similar to scenario 1, we can derive Equations (5) and (6) as

$$
\begin{aligned}
& c_{k|X_i(t)=0} = c_k \Pr\left\{R_U\left(\frac{p_1(t)}{q_i(t)} - p_1(t) + \frac{p_i(t)\rho_{i,1}\sigma_1(t)}{\sigma_i(t)}, \frac{p_2(t)}{q_i(t)} - p_2(t) + \frac{p_i(t)\rho_{i,2}\sigma_2(t)}{\sigma_i(t)}, \ldots, \frac{p_j(t)}{q_i(t)} - p_j(t) + \frac{p_i(t)\rho_{i,j}\sigma_j(t)}{\sigma_i(t)}\right) < \tau\right\} + \\
& \Pr\left\{R_U\left(\frac{p_1(t)}{q_i(t)} - p_1(t) + \frac{p_i(t)\rho_{i,1}\sigma_1(t)}{\sigma_i(t)}, \frac{p_2(t)}{q_i(t)} - p_2(t) + \frac{p_i(t)\rho_{i,2}\sigma_2(t)}{\sigma_i(t)}, \ldots, \frac{p_j(t)}{q_i(t)} - p_j(t) + \frac{p_i(t)\rho_{i,j}\sigma_j(t)}{\sigma_i(t)}\right) \geq \tau\right\} \cdot \\
& \left[\begin{aligned} & c_{s,k} \cdot \Pr\left\{R_U\left(t|\frac{p_1(t)}{q_k(t)} - p_k(t) + \frac{p_k(t)\rho_{k,1}\sigma_1(t)}{\sigma_k(t)}, \frac{p_2(t)}{q_k(t)} - p_2(t) + \frac{p_k(t)\rho_{k,2}\sigma_2(t)}{\sigma_k(t)}, \ldots, \frac{p_j(t)}{q_k(t)} - p_j(t) + \frac{p_k(t)\rho_{k,j}\sigma_j(t)}{\sigma_k(t)}\right) < \tau\right\} \\ & + \theta_k c_k^f + (1-\theta_k) c_k^{pf} \end{aligned}\right] \\
& , j \in \{1,2,\ldots,n\}, j \neq i \,\&\, j \neq k,
\end{aligned}
$$

$$
\begin{aligned}
& c_{k|X_i(t)=1} = c_{s,k} \Pr\left\{R_U\left(p_1(t) + \frac{q_k(t)\rho_{k,1}\sigma_1(t)}{\sigma_k(t)}, p_2(t) + \frac{q_k(t)\rho_{k,2}\sigma_2(t)}{\sigma_k(t)}, \ldots, p_j(t) + \frac{q_k(t)\rho_{k,j}\sigma_j(t)}{\sigma_k(t)}\right) < \tau\right\} + \\
& \theta_k c_k^f + (1-\theta_k) c_k^{pf}, j \in \{1,2,\ldots,n\} \,\&\, j \neq k,
\end{aligned}
$$

where θ_k is a 0-1 variable, it can be used to choose whether to perform preventive $R_{k,\min} = \min R_k(t)_{X_i(t)=1}$ maintenance or failure maintenance based on the state of component k.

Scenario 3

According to Equation (7), $R_{i,\min}$ represents the minimum reliability of component i. $R_{i,\max}$ represents the maximum achievable reliability of the component i. For Equations (8) and (9), we have $R_{k,\min} = \min\{R_k(t)_{X_i(t)=0}\}$ and $R_{k,\max} = \max\{R_k(t)_{X_i(t)=0}\}$ when component i failed. Similarly, $R_{k,\max} = \max\{R_k(t)_{X_i(t)=1}\}$ when component i is reliable. Then we have

$$c_k(t)_{X_i(t)=0} = a_k e^{\left[(1-f_{k|X_i(t)=0})\frac{\left(\frac{p_k^{am}(t)}{q_i(t)} - p_k^{am}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right) - \min\{R_k(t)_{X_i(t)=0}\}}{\max\{R_k(t)_{X_i(t)=0}\} - \left(\frac{p_k^{am}(t)}{q_i(t)} - p_k^{am}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right)}\right]} - a_k e^{\left[(1-f_{k|X_i(t)=0})\frac{\left(\frac{p_k^{bm}(t)}{q_i(t)} - p_k^{bm}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right) - \min\{R_k(t)_{X_i(t)=0}\}}{\max\{R_k(t)_{X_i(t)=0}\} - \left(\frac{p_k^{bm}(t)}{q_i(t)} - p_k^{bm}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right)}\right]},$$

$$c_k(t)_{X_i(t)=1} = a_k e^{\left[(1-f_{k|X_i(t)=1})\frac{\left(\frac{p_k^{am}(t)}{q_i(t)} - p_k^{am}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right) - \min\{R_k(t)_{X_i(t)=0}\}}{\max\{R_k(t)_{X_i(t)=0}\} - \left(\frac{p_k^{am}(t)}{q_i(t)} - p_k^{am}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right)}\right]} - a_k e^{\left[(1-f_{k|X_i(t)=1})\frac{\left(\frac{p_k^{bm}(t)}{q_i(t)} - p_k^{bm}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right) - \min\{R_k(t)_{X_i(t)=0}\}}{\max\{R_k(t)_{X_i(t)=0}\} - \left(\frac{p_k^{bm}(t)}{q_i(t)} - p_k^{bm}(t) + \frac{p_i(t)\rho_{i,k}\sigma_k(t)}{\sigma_i(t)}\right)}\right]}.$$

When component i undergoes repair, given the fixed maintenance cost C and fixed maintenance time t, we should determine the components for PM to maximize the avoidable losses. On the other hand, we need to consider the impact of changes in the number of maintenance personnel on preventive maintenance decisions. In other words, we need to solve the following integer programming. According to the classification of different cost scenarios, similarly, integer programming should also be discussed in these three scenarios.

Scenario 1

$$\max \sum_{k=1,k\neq i}^{n} I_2(i,k)_{X_i(t)} \cdot z_k, \tag{15}$$

$$s.t.\begin{cases} z_k \in \{0,1\} \\ c_i + \sum\limits_{k=1,k\neq i}^{n} \begin{bmatrix} c_k \cdot \Pr\{\phi(0_i,\mathbf{1}_i) < K\} \\ +\Pr\{\phi(0_i,\mathbf{1}_i) \geq K\} \cdot \left[c_{s,k} \cdot \Pr\{\phi(0_i,0_k,\mathbf{1}_{i,k}) < K\} + c_k\right] \end{bmatrix} \cdot z_k \leq C, \\ \sum\limits_{k=1,k\neq i}^{n} \left(t_{k0}^{pf} + \varepsilon \cdot t_k^{pf}\right) \cdot z_k \leq t_{i0}^{f} + \delta \cdot t_i^{f}, \varepsilon \in \{0,1\}, \delta \in \{0,1\} \end{cases}$$

Scenario 2

$$\max \sum_{k=1,k\neq i}^{n} I_2(i,k)_{X_i(t)} \cdot z_k, \tag{16}$$

$$
s.t.\begin{cases}
z_k \in \{0,1\} \\
c_i + \sum_{k=1,k\neq i}^{n} \begin{bmatrix} c_k \cdot \Pr\left(\phi\left(0_i, \mathbf{1}_i\right) < K\right) \\ + \Pr\left(\phi\left(0_i, \mathbf{1}_i\right) \geq K\right) \cdot \left[c_{s,k} \cdot \Pr\left(\phi\left(0_i, 0_k, \mathbf{1}_{i,k}\right) < K\right) + \theta_k c_k^f + (1-\theta_k) c_k^{pf}\right] \end{bmatrix} \cdot z_k \leq C, \\
\sum_{k=1,k\neq i}^{n} \left(t_{k0}^{pf} + \varepsilon \cdot t_k^{pf}\right) \cdot z_k \leq t_{i0}^f + \delta \cdot t_i^f, \varepsilon \in (0,1), \delta \in (0,1)
\end{cases}
$$

Scenario 3

$$
\max \sum_{k=1,k\neq i}^{n} I_2(i,k)_{X_i(t)} \cdot z_k, \tag{17}
$$

$$
s.t.\begin{cases}
z_k \in \{0,1\} \\
c_i + \sum_{k=1,k\neq i}^{n} \left[a_k e^{\left[(1-f_k)\frac{R_k^{am}(t) - R_{k,\min}}{R_{k,\max} - R_k^{am}(t)}\right]} - a_k e^{\left[(1-f_k)\frac{R_k^{bm}(t) - R_{k,\min}}{R_{k,\max} - R_k^{bm}(t)}\right]} \right] \cdot z_k \leq C, \\
\sum_{k=1,k\neq i}^{n} \left(t_{k0}^{pf} + \varepsilon \cdot t_k^{pf}\right) \cdot z_k \leq t_{i0}^f + \delta \cdot t_i^f, \varepsilon \in (0,1), \delta \in (0,1)
\end{cases}
$$

in which c_i represents the maintenance cost for component *i*, z_k is the decision variable represents whether component *k* should be maintained or not, T represents the total maintenance time, t_i^f and t_k^{pf} represent the maintenance time of component *i* and the preventive maintenance time of component *k,* respectively, and t_{i0}^f, t_{k0}^{pf} represent the necessary time required for corrective maintenance and preventive maintenance of the corresponding component, respectively. δ and ε are the influence factors of corrective maintenance personnel and preventive maintenance personnel, respectively.

In the three different cost scenarios, different cost constraint functions are reflected in the different $c_{k|X_i(t)}$ formula. It is necessary to maximize the contribution of the remaining components to system when component *i* fails. It also satisfies that all repairable maintenance costs and preventive maintenance costs are less than the total maintenance cost. Meanwhile the total preventive maintenance time is less than the corrective maintenance time.

Note that z_k can only take values from 0 and 1. After failed component being removed, there are $n-1$ components left. Therefore, it suffices to test the 2^{n-1} combinations of z_k for deriving the optimal maintenance policy. Under the optimal maintenance policy $\{z_k^*, k \neq i\}$, the number of maintained components is $\sum_{k\neq i} z_k^*$.

4. Application to hydraulic system

A hydraulic system of 2H/2E structure is as in Fig. 1. Two of the systems (2H) use traditional hydraulic-powered actuation systems, and the other two (2E) are powered by electricity. In Fig. 1, the green hydraulic system and the yellow hydraulic system are located on both sides of the wings symmetrically. A total of 8 Engineer Driven Pumps (EDP) and 4 Electric Motor pumps (EMP) forms the pump source of the two main hydraulic systems. And it provides hydraulic power for the aircraft's main flight control, landing gear, front wheel turning and other related systems. All EDPs are connected to the engine through a clutch, and closing any EDP alone will not affect other EDP work and system-level performance.

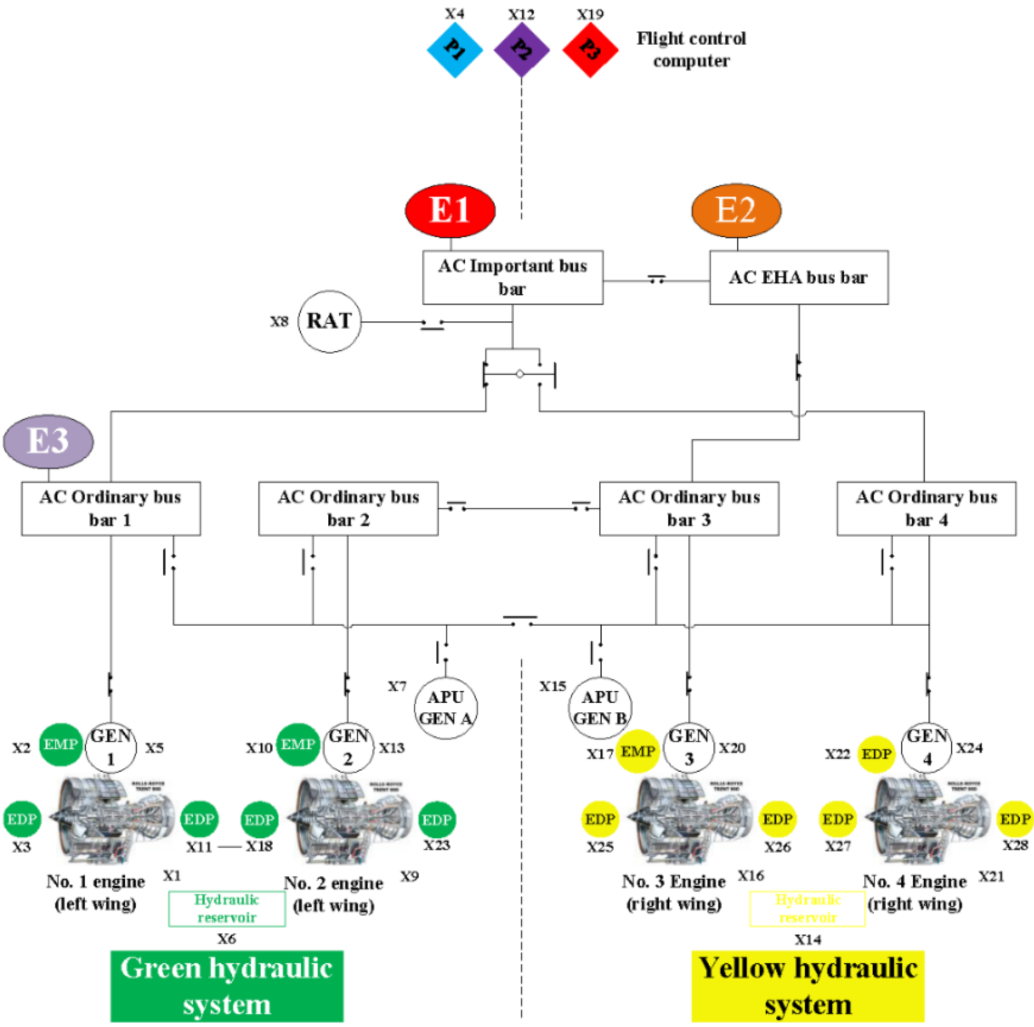

Fig. 1 hydraulic system

There are many important components in the hydraulic system, some of which will greatly affect the reliability of the entire aircraft if they fail. As such, keeping them in operating is very crucial and preventive maintenance on them is therefore needed to maximize system performance. Some important components in the hydraulic system are shown in Table 1.

Table 1 Important components in the hydraulic system

Code	Name	Code	Name
X1	Engine No.1	X15	APU Generator No.2
X2	Electric Motor pump No.1	X16	Engine No.3
X3	Engineer Driven Pump No.1	X17	Electric Motor pump No.3
X4	Flight control computer No.1	X18	Engineer Driven Pump No.3
X5	Generator No.1	X19	Flight control computer No.3
X6	Hydraulic reservoir No.1	X20	Generator No.3
X7	APU Generator No.1	X21	Engine No.4
X8	Ram Air Turbine	X22	Electric Motor pump No.4
X9	Engine No.2	X23	Engineer Driven Pump No.4
X10	Electric Motor pump No.2	X24	Generator No.4
X11	Engineer Driven Pump No.2	X25	Engineer Driven Pump No.5
X12	Flight control computer No.2	X26	Engineer Driven Pump No.6
X13	Generator No.2	X27	Engineer Driven Pump No.7
X14	Hydraulic reservoir No.2	X28	Engineer Driven Pump No.8

Assuming that each component obeys the Weibull distribution $W(t,\theta,\gamma)$, the reliability expression of each component is $R(t)=\exp[-(\frac{t}{\theta})^{\gamma}]$. The scale and shape parameters of the failure time of each component are shown in Table 2. Table 3 shows the corrective maintenance and preventive maintenance costs for each type of component.

Table 2 The scale and shape parameters of the failure time of each component

No.	Component	Code	θ	γ
1	Engine	X1, X9, X16, X21	4385	0.95
2	Electric Motor pump	X2, X10, X17, X22	1643	1.13
3	Engineer Driven Pump	X3, X11, X18, X23, X25, X26, X27, X28	2045	1.43
4	Flight control computer	X4, X12, X19	3015	1.24
5	Generator	X5, X13, X20, X24	3963	0.68
6	Hydraulic reservoir	X6, X14	3364	0.21
7	APU Generator	X7, X15	3648	0.79
8	Ram Air Turbine	X8	4031	0.46

Table 3 Repair and preventive maintenance cost of each type of component

No.	Component	Code	Maintenance cost	PM cost
1	Engine	X1, X9, X16, X21	7000	3200
2	Electric Motor pump	X2, X10, X17, X22	4000	1900
3	Engineer Driven Pump	X3, X11, X18, X23, X25, X26, X27, X28	3900	1700
4	Flight control computer	X4, X12, X19	2600	1000
5	Generator	X5, X13, X20, X24	3500	1500
6	Hydraulic reservoir	X6, X14	3000	1400
7	APU Generator	X7, X15	3600	1500
8	Ram Air Turbine	X8	4200	1800

Next, we will analyse the behaviour of joint loss importance when components X5, and X6 fail. Since the joint loss importance of the same type of components is exactly same, we only illustrate one of the 8 different types of component groups. Fig. 2 shows the behaviour of the joint loss importance (simply expressed as I_1) with 8 different types of components under cost scenario 1 when components X5, and X6 fail over time.

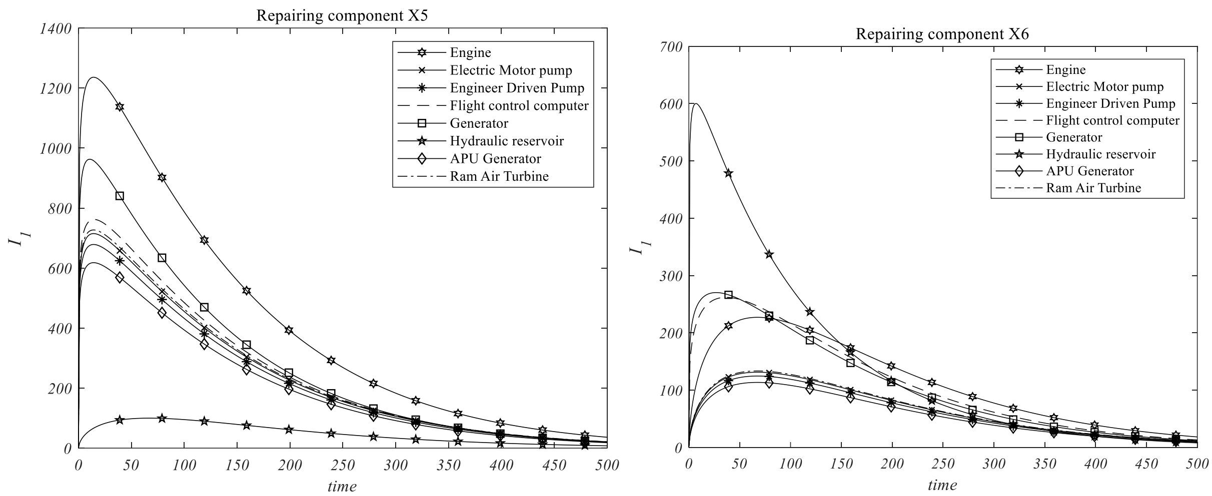

Fig. 2 Joint loss importance in cost scenario 1

From Fig. 2, when components X5 fail, the joint loss importance of the engine changes significantly with the changes in reliability. By comparison, the joint loss importance of hydraulic reservoir fluctuates smoothly with reliability changes. Moreover, the engine's joint loss importance has always been higher than other components. This shows that when component X5 fails, the failure causes the greatest losses. Consequently, it is wise to choose the engine for preventive maintenance when component X5 fails. On the contrary, if the hydraulic reservoir fails under the same circumstances, its preventive maintenance is not important. When repairing component X6, the engine does not always stay in the top position in terms of the joint loss importance value. Similarly, it is not the value of the joint loss importance of the hydraulic reservoir, which is ranked last but the APU Generator. Meanwhile, we can find the joint loss importance when different types of components fail, and there are intersections of the joint loss importance changes over time when component X6 fails.

Fig. 3 shows the joint loss importance under cost scenario 2 when repairing componentsX5, or X6.

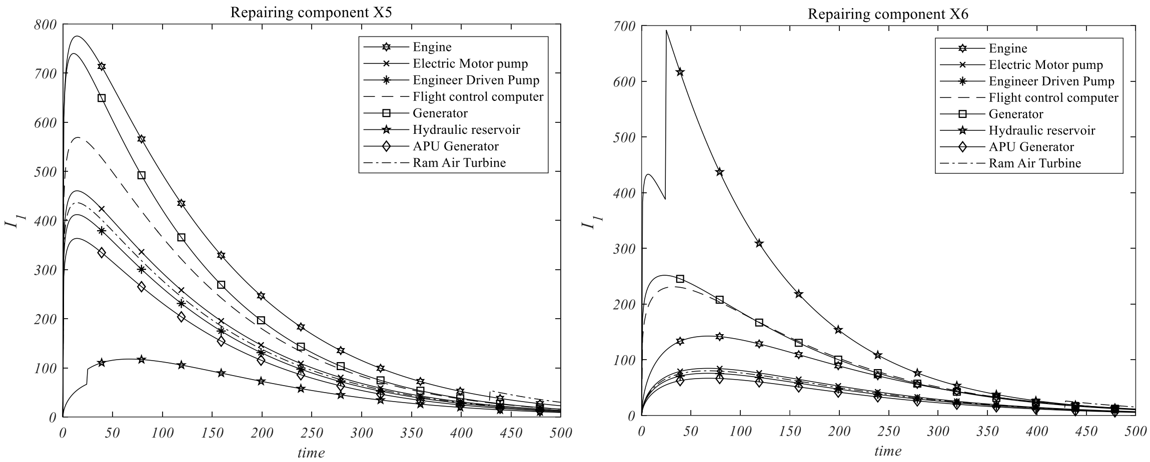

Fig. 3 Joint loss importance in cost scenario 2

From Fig. 3, the joint loss importance's behaviour is similar to that in cost scenario 1, but the jumping part will cause significant differences between the two. In the practical application process, considering the cost constraints, if a component fails, the selection scheme of preventive maintenance components based on this importance will be more variable. As time goes on, the joint failure importance of the failed component and the remaining components will gradually approach 0 due to the decrease in component reliability, causing the joint loss importance of the two components to approach 0. In Fig. 3, there is also a crossover phenomenon. For example, when component X5 fails, at t=400, an engine failure has the greatest impact on system losses, but at t=450, Ram Air Turbine ranks first. This also indicates that as the reliability of components changes, the joint effects of different components and failed components contribute differently to system losses.

Fig. 4 shows the joint loss importance in cost scenario 3 when components X5, and X6 fail respectively.

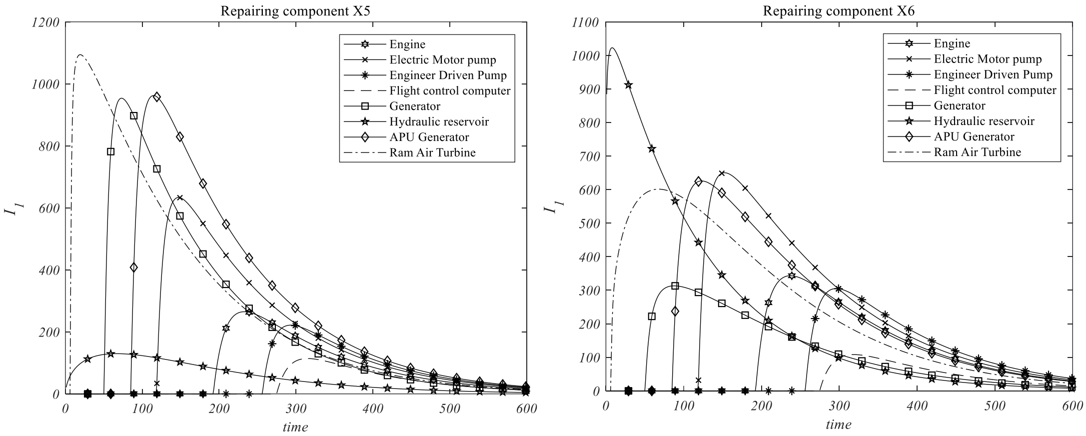

Fig. 4 Joint loss importance in cost scenario 3

Form Fig. 4, when the component X6 is repaired, the component with the highest joint loss importance always changes, that is, the component that has the greatest impact on system losses always changes. From t=10 to t=400, the component changes with the highest joint loss importance value are: Hydraulic reservoir, Ram Air Turbine, APU generator, Electric Motor Pump, and Engineer Driven Pump. Similarly, if there are cost constraints, the preventive maintenance programs given at different times should also be different.

Next, we analyze the joint lifetime importance (simply expressed as I_2) under three different cost scenarios according to Figs. 5, 6, and 7, respectively.

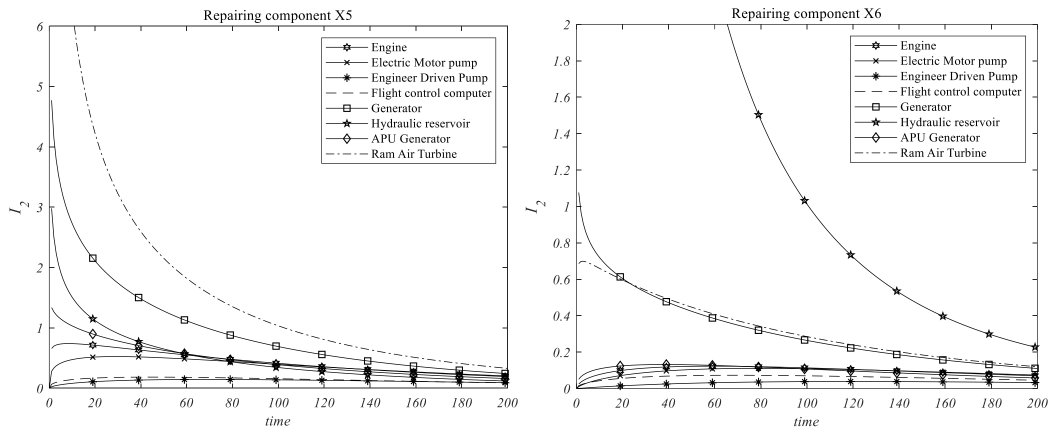

Fig. 5 Joint Lifetime importance in cost scenario 1

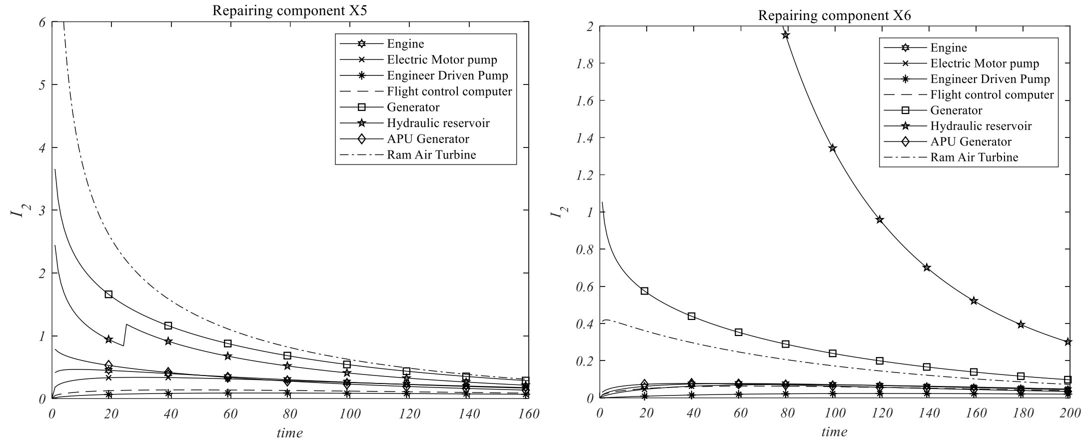

Fig. 6 Joint Lifetime importance in cost scenario 2

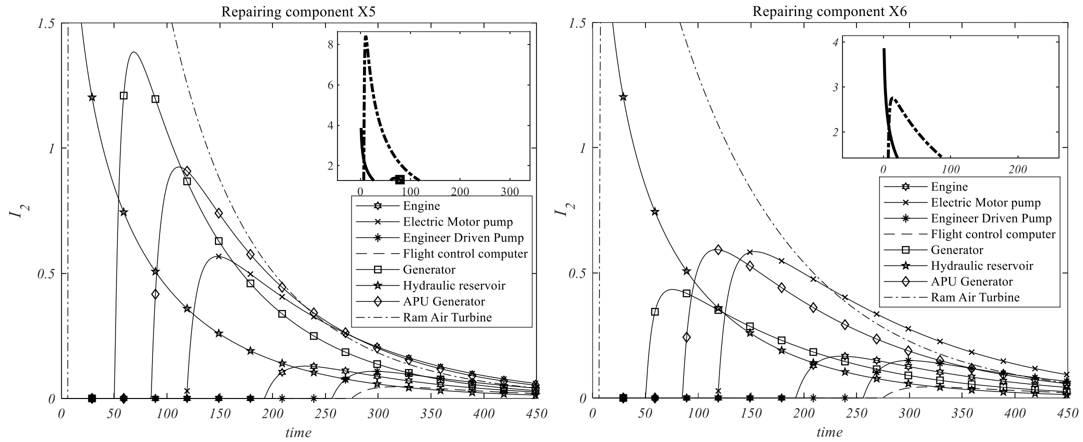

Fig. 7 Joint Lifetime importance in cost scenario 3

Fig. 5 specifically describes the joint lifetime importance behavior due to the failures of components X5, and X6 and different types of components in cost scenario 1. From Fig. 5, if the impact on systemlifetime is considered, when repairing component X5, if Ram Air Turbine fails, the impact on system losses always ranks first. This is different from the above consideration of the impact on system losses. Further, when the repairing component X6, the Hydraulic reservoir has the highest joint lifetime importance, and the Ram Air Turbine ranks the second most important. In other words, the results of different component failures are also different. In this way, the joint effect of system components is particularly important in preventive maintenance strategies. This phenomenon also appears in Fig. 6, i.e. in cost scenario 2. Compared with cost scenarios 1 and 2, Fig. 7 reflects that the changing of components that have the greatest impact on system are more complex as costs change over time.

We give the expected lifetime importance (simply expressed as I_3) for four different components in different cost scenarios in Fig. 8. Subsequently, in cost scenario 1, before t=15, if component X6 does not fail, then the system will gain greater benefits. But after t=15, the importance of component X6 gradually decreases, and after t=22, it ranks last among the four components. At the same time, component X1 has the highest importance. Consequcetly, before t=15, if we want to gain the higher importance, then we should focus on component X6. After t=15, we should change our focus to component X1. Also in cost scenarios 2 and 3, at different times, the curves of the Fig. 8 can be produced by different components cross.

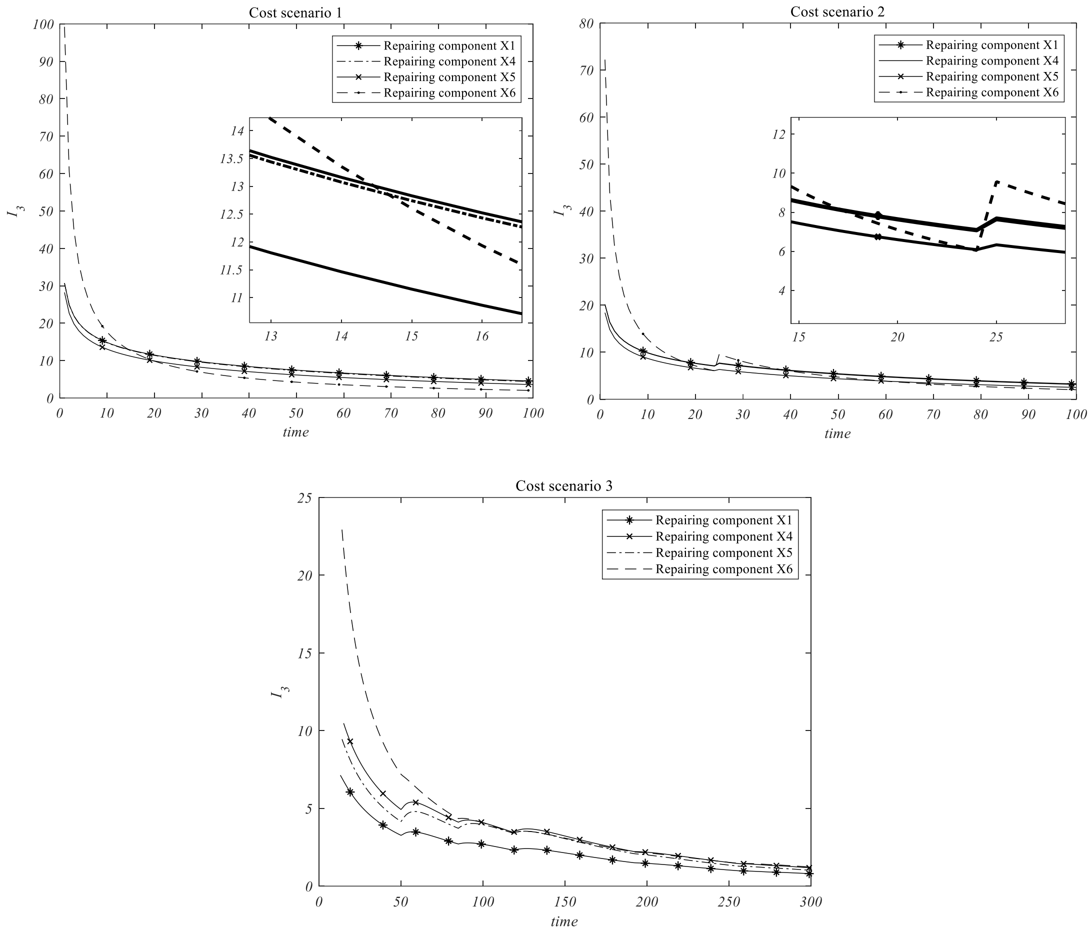

Fig. 8 Lifetime importance in three different cost scenarios for different components

In the following, we give the best maintenance scheme of the eight categories of components under the condition of maximizing the avoidable losses. In order to simplify the calculation, we stipulate that only one component of each type can be repaired. Corrective maintenance and preventive maintenance time of each component are given in Table 4. Table 5 and Table 6 respectively show the maintenance policy when components X5 and X6 fail at t=100 under cost scenario 1. The numbers in the first column on the left represent different categories of components. The second line represents the cost constraint ($\times 10^4$). Among them, 3:1/3:3/3:5 represents the ratio of corrective maintenance staff (CMS) to preventive maintenance staff (PMS). The 0 in the table means that the component is not subject to preventive maintenance, and 1 means that preventive maintenance is required. For example, 0/1/0 means that when the ratio of CMS to PMS is 3:1, 3:3 or 3:5, the component should be not repaired, repaired or not repaired. The ratio is used to reflect the comparison of the maintenance capabilities of preventive maintenance and corrective maintenance. In the table, we can see the changes in the scheme caused by different cost constraints and different ratios of CMS to PMS. In order to see the changes of the scheme more intuitively, Fig. 9 is given to analyze the influence of constraint conditions on expected lifetime importance.

Table 4 Maintenance and preventive maintenance time of each component

No.	Component	Code	Maintenance time	PM time
1	Engine	X1, X9, X16, X21	30	10
2	Electric Motor pump	X2, X10, X17, X22	15	3.5
3	Engine Driven Pump	X3, X11, X18, X23, X25, X26, X27, X28	14.5	3
4	Flight control computer	X4, X12, X19	8	2
5	Generator	X5, X13, X20, X24	12.5	2.5
6	Hydraulic reservoir	X6, X14	10	2
7	APU Generator	X7, X15	13	3
8	Ram Air Turbine	X8	16	4

Table 5 maintenance scheme when components X5 failed at t=100 under cost scenario 1

X5	CMS/PMS = 3:1/3:3/3:5												
T=100	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**	0/0/0	0/0/0	0/0/**1**
2	0/**1/1**	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	0/0/0	**1/1/1**	**1**/0/0	**1**/0/0	**1/1/1**	**1/1/1**	**1**/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
6	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/0	0/0/**1**	0/0/**1**	0/0/0
7	**1**/0/0	0/0/0	0/**1/1**	0/**1/1**	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1**/0	0/**1/1**	0/**1/1**	0/**1/1**
8	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**

Table 6 maintenance scheme when components X6 failed at t=100 under cost scenario 1

X6	CMS/PMS = 3:1/3:3/3:5												
T=100	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
2	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**

3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
6	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
7	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**	0/0/**1**	0/0/**1**
8	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**

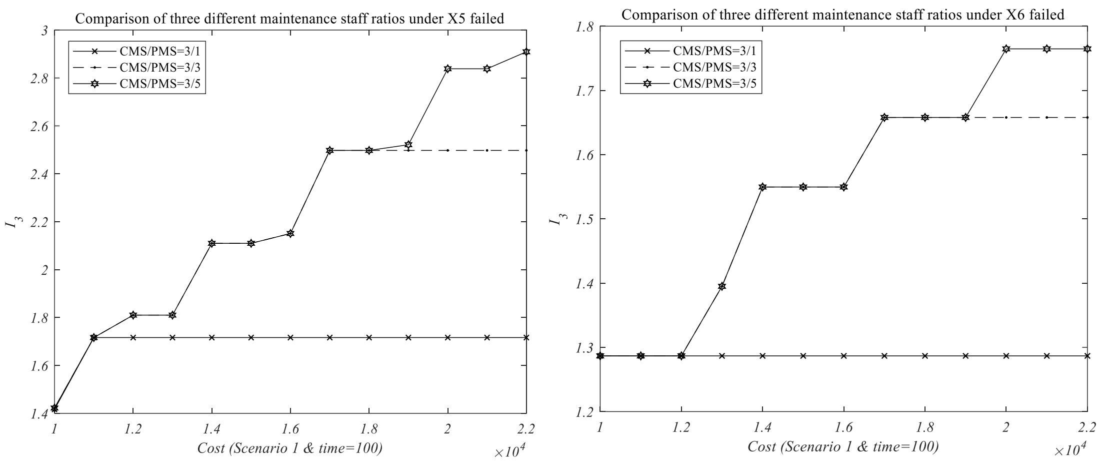

Fig. 9 Comparison of the different maintenance staff at under cost scenario 1

Fig. 9 shows a comparison of different maintenance staff at t=100 under cost scenario 1 when components X5 and X6 respectively fail. The expected lifetime importance with the change of cost constraints formed by the ratios of three CMS to PMS is initially an upward trend with the increase of cost constraints. When component X5 fails, after the cost constraint is greater than 1.1×10^4, the curve representing CMS/PMS=3/1 no longer rises. This is because under the condition of CMS/PMS=3/1, the corrective maintenance will be over quickly, there is no extra time for preventive maintenance, and it will not help to increase the cost. However, the curve representing CMS/PMS=3/3 still rises. This shows that the cost constraint is between 1.2×10^4 and1.9×10^4. When the cost constraint exceeds 1.9×10^4, the limiting factor is the ratio of CMS to PMS.

Table 7 and Table 8 give the maintenance scheme at t=100 in cost scenario 2, respectively, when component X5 and X6 are being repaired. Compared with cost scenario 1, it can be seen that in the case of cost scenario 2, due to the existence of preventive maintenance costs, the total cost can be allocated more reasonably to save the total cost. Combining Table 5 and Table 7, it is faster to reach the same number of repaired components in cost scenario 2, so that the expected lifetime importance is improved faster. This phenomenon can also be found in the comparison between Table 6 and Table 8. Moreover, in Fig. 10, the curve representing CMS/PMS=3/5 has experienced a sharp rise, which can also explain the changes brought about by segmentation costs.

Table 7 maintenance scheme when components X5 failed at t=100 under cost scenario 2

X5	CMS/PMS = 3:1/3:3/3:5												
T=100	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**	0/**1**/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
2	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/0/**1**	0/0/0	0/0/**1**	0/0/**1**	0/0/**1**	0/0/**1**
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
6	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1**/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
7	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/0/0	0/0/**1**	0/0/**1**	0/0/**1**	0/0/0	0/0/0	0/0/0	0/0/0
8	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**

Table 8 maintenance scheme when components X6 failed at t=100 under cost scenario 2

X6	CMS/PMS = 3:1/3:3/3:5												
T=100	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
2	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	0/0/0	**1/1/1**	**1**/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
6	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
7	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**	0/0/**1**	0/0/**1**	0/0/**1**	0/0/**1**	0/0/**1**	0/0/**1**
8	**1/1/1**	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**

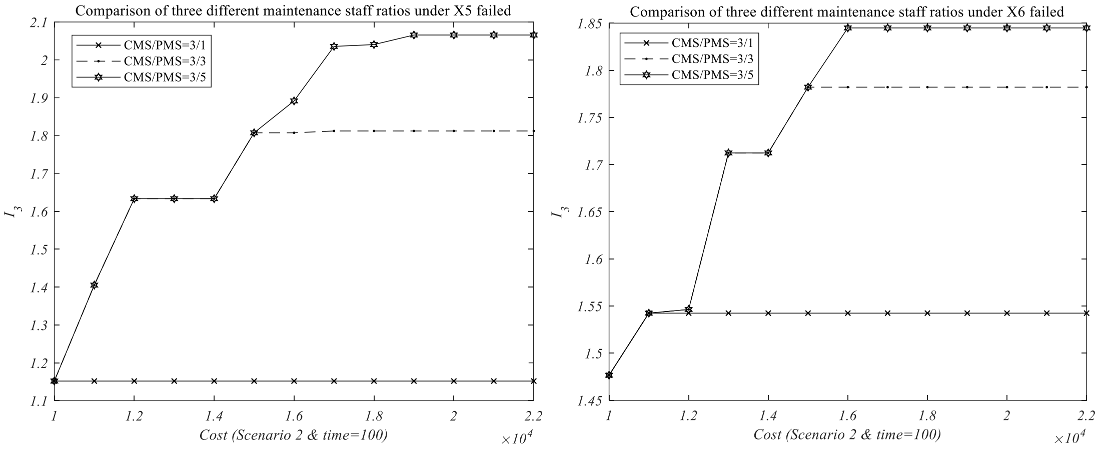

Fig. 10 Comparison of the different maintenance staff under cost scenario 2

Table 9 and Table 10 give the maintenance scheme at t=100 and t=300 in cost scenario 3, respectively, when component X5 is being repaired. Table 11 and Table 12 give the maintenance scheme at t=100 and t=300 in cost scenario 3, respectively, when component X6 is repaired. Due to the limitation of the maintenance threshold in the third cost scenario, different components should be repaired at different time points, which leads to different preventive maintenance strategies at different time points. Two time points, t=100 and t=300, are selected here. When t=100, only some components can be repaired, and when t=300 all components can be repaired because all of them are below the repair threshold.

Table 9 maintenance scheme when components X5 failed at t=100 under cost scenario 3

X5	CMS/PMS = 3:1/3:3/3:5										
T=100	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
2	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	0/0/0	0/0/0	0/0/0	0/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1**/0/0	**1/1/1**	**1/1/1**
6	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0
7	0/0/0	0/0/0	**1/1/1**	**1/1/1**	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**
8	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**

Table 10 maintenance scheme when components X5 failed at t=300 under cost scenario 3

X5	CMS/PMS = 3:1/3:3/3:5												
T=300	1.3	1.5	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**	0/0/0

2	0/0/0	0/**1/1**	0/**1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
3	0/0/0	**1**/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/0	0/0/**1**
4	**1/1/1**	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/0/0	0/0/**1**	0/0/0	0/0/0
5	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**
6	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
7	0/0/0	0/0/0	**1**/0/0	**1**/0/0	**1**/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
8	**1/1/1**	**1/1/1**	**1/1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**

Table 11 maintenance scheme when components X6 failed at t=100 under cost scenario 3

X6	CMS/PMS = 3:1/3:3/3:5									
T=100	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
2	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
5	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
6	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
7	0/0/0	**1/1/1**	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**
8	0/0/0	0/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**

Table 12 maintenance scheme when components X6 failed at t=300 under cost scenario 3

X6	CMS/PMS = 3:1/3:3/3:5												
T=300	2.1	2.3	2.5	2.7	2.9	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.6
1	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
2	0/0/0	0/0/0	0/0/0	0/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1**/0/0	**1/1/1**	**1**/0/0	**1**/0/0	**1/1/1**	**1/1/1**
3	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
4	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/**1**
5	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
6	0/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	**1**/0/0	**1**/0/0	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**
7	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0	0/**1/1**	0/0/0	0/**1/1**	0/**1/1**	0/0/0	0/0/0
8	**1/1/1**	**1/1/1**	**1/1/1**	**1/1/1**	0/0/0	0/0/0	0/0/0	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**	0/**1/1**

Figs. 11 and 12 give the comparison of the different maintenance staff under cost scenario 3 when components X5 and X6 fail, respectively.

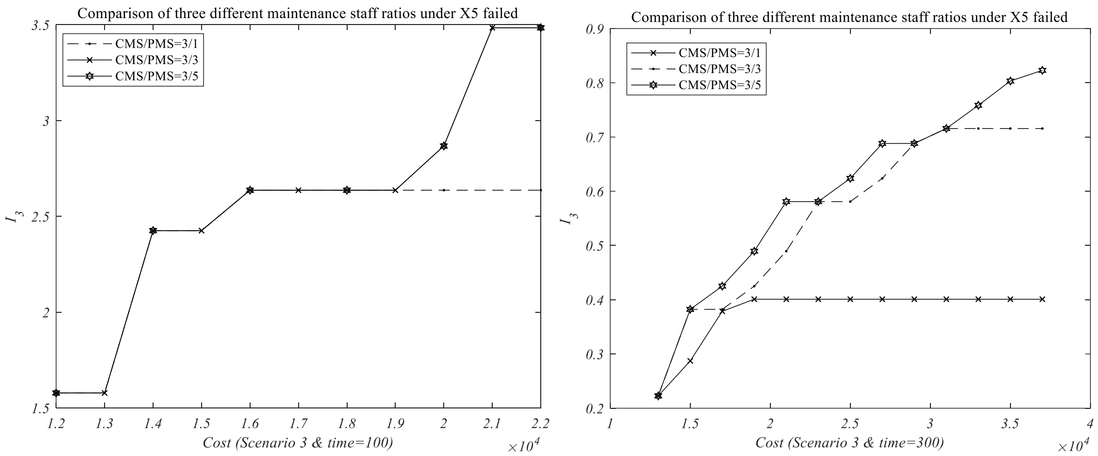

Fig. 11 Comparison of the different maintenance staff under cost scenario 3 when component X5 fails

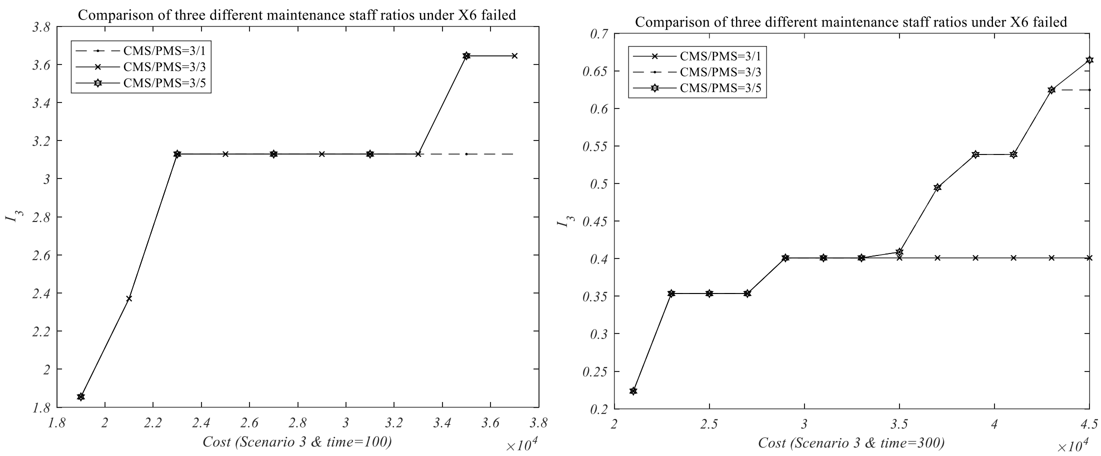

Fig. 12 Comparison of the different maintenance staff under cost scenario 3 when component X6 fails

From Figs. 11 and 12, in the case of t=100, the early stage is mainly constrained by cost constraints. Whether it is X5 failure or X6 failure, the maintenance improvement of the three different maintenance personnel ratios is the same. As the total maintenance cost continues to rise, when the proportion of preventive maintenance staff is high, more components can be selected for preventive maintenance to increase the value of contribution to system lifetime. However, due to the limitation of the number of repairable parts, even if the ratio of preventive maintenance personnel is increased, the expected lifetime importance will not be improved. When t=300 is selected, all components are under the maintenance threshold and can be repaired. When component X5 failed, as the total cost increases, the higher the ratio of preventive maintenance staff, the more preventive maintenance components will be, and the greater the contribution to the performance of the system. When X6 failed, the curve is similar to the previous cost scenarios. In the case of $c = 3.3 \times 10^4$ and $c = 4.3 \times 10^4$, it represents the change of preventive maintenance strategy from cost restriction to maintenance personnel ratio restriction.

5. Conclusions and future work

This paper proposed some maintenance indexes that consider different maintenance cost scenarios for optimising maintenance policies. It considered three maintenance cost scenarios. The proposed methods are applied to maintain the hydraulic system of the aircraft. Furthermore, the failure rate of the system components was used to determine the failure conditions of the components in the system, and the joint lifetime importance of the entire system when different components fail was given. In this paper, three maintenance cost scenarios were considered. When the component contributed more to the lifetime of the system, more costs were required to restore its performance.

For future study, we plan to consider the multi-stage cost of the life cycle of the system, such as the cost of the production stage and the cost of the operation stage. Then we plan to analyse the maintenance measures based on the life cycle and the ability of different components to influence the losses of the system.

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