Information in Corporate Bond Yield for Optimal Policy Rules

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Dedication

This thesis is dedicated to the Almighty God- in whom I live, and move, and have my being, and also in memory of my late biological parents- Mr Olaniyi Emmanuel Kolawole and Ms Kehinde Adesola Soetan whom I sadly have little memories of due to various issues of life. Lastly, this thesis is dedicated in loving memory of my grandmother Mrs Christianah Olubunmi Adekanmbi who took care of me from birth, showered me with unwavering love, and never stopped believing in me.

Declaration

I humbly certify that this thesis as I have presented for examination for the award of PhD degree in Economics at the University of Kent is solely my own work.

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Abstract

This thesis examines the information in corporate bond yields relevant to conducting simple optimal policies. It is a compendium of three self-contained essays, each addressing emerging questions that pertain to the impact of financial outcomes on the macroeconomy. The 2007-2009 financial crisis demonstrated the importance for policy authorities of considering developments in the financial sector in devising their techniques for managing business cycle fluctuations.

The first essay entitled ***"Corporate Bond Spreads in a DSGE Model"*** presents a medium-scale dynamic stochastic general equilibrium (DSGE) model as in Smets and Wouters (2007), with endogenously determined government and corporate bond yields. This approach is taken to assess the effect of financial conditions on macroeconomic outcomes. In particular, it explores the information that corporate bond yields convey in relation to the macroeconomy. The model incorporates an Epstein-Zin preference, as in Epstein and Zin (1989), and financial friction in the form of a costly verification problem (Bernanke et al., 1999a). The corporate bond spread is investigated as a proxy for the external finance premium in DeGraeve (2008) and Gilchrist et al. (2009). Based on the contribution of these studies, the model in this essay is embedded with bond pricing equations and a stochastic discount factor, which are consistent with a recursive Epstein-Zin preference and financial friction, as in Bernanke et al. (1999a) and following the formulation in Christensen and Dib (2008). The standard Bayesian technique is employed to estimate model parameters, using macroeconomic and bond yield data as observables. The 10-year BAA grade corporate bond yield is preferred because it explains the business cycle fluctuation better than other bond yields. In addition, it clearly contains information that is not included in the 10-year government bond yield. Furthermore, the implied corporate bond spread is sizeable, and the magnitude of its volatility aligns with the data. Unsurprisingly, investment is most sharply affected by an external financial premium shock. The fluctuation in investment is well captured in the estimated model, and the estimated investment adjustment parameter is thus not excessively large. Also evident from the analysis is the structural break in the corporate bond spread before and after Paul Volcker's term as chair of the Federal Reserve Board of Governors.

The second essay entitled ***"Optimal Policy Rules and Corporate Bond Premium"*** extends the model developed in the first essay with interest in a macroprudential-like monetary policy. In the model, policy makers take into account financial conditions implied by corporate bond yields, in addition to the traditional macro-economic indicators such as inflation and the output gap. More specifically, the essay investigates the information contained in financial indicators for optimal monetary policy. The essay extends the standard Taylor rule to allow the central bank to respond to the corporate bond premium and other related financial indicators, such as net worth and the asset ratio. The optimal rule-based policy that maximises households' conditional welfare is computed. Adding this information improves welfare and also reduces the volatility of inflation by almost half.

Lastly, the third essay entitled ***"Government Investment and External Finance Premium"*** examines how monetary and fiscal policies are affected by the financial friction within one model. It assesses how the interventions of the government and the central bank jointly affect entrepreneurs' balance sheets, particularly focusing on the impact that government spending has on the cost of sourcing external finance. Furthermore, it offers an understanding of the multiplier effects of fiscal instruments over a short and long horizon by evaluating its present value multiplier (as in Mountford and Uhlig (2009)) on output, consumption, and investment. The results show the advantage of government investment over other fiscal instruments; it reduces the cost of external finance while also generating the highest short-run and long-run output and investment multipliers.

Key words: DSGE Model; Corporate Bond Premium; Financial Friction; Epstein-Zin Preference; Optimal Policy.

Contents

Abstract iv

1 Corporate Bond Spread in a DSGE Model 1

1.1 Introduction . . . 1
1.2 Model Set-Up . . . 9
1.2.1 Households . . . 9
1.2.2 Labour Market . . . 11
1.2.3 Goods Market . . . 13
1.2.4 Entrepreneurs . . . 16
1.2.5 Financial Intermediary and Asset Pricing . . . 19
1.2.6 Aggregate Resource Constraint and Government . . . 20
1.3 Estimation . . . 21
1.3.1 Data . . . 22
1.3.2 Estimation Procedure . . . 22
1.3.3 Calibration and Prior Distributions . . . 24
1.4 Results and Discussions . . . 25
1.4.1 Implications of Financial Friction and Bond Yield Data in Estimation . . . 25
1.4.2 Coefficient of Relative Risk Aversion and Model Moment . . . 36
1.4.3 Impulse Responses . . . 40
1.4.4 Historical Shock Decomposition . . . 43
1.5 Conclusion . . . 46

2 Optimal Policy Rules and Corporate Bond Premium 60

2.1 Introduction . . . 60
2.2 Literature Review . . . 65
2.2.1 Financial Shocks and Business Cycle . . . 65
2.2.2 Monetary Policy and Financial Indicators . . . 66
2.3 The Model . . . 67
2.3.1 Household . . . 68
2.3.2 Labour Market . . . 70
2.3.3 Capital Goods Producers . . . 72
2.3.4 Consumer Goods Market . . . 72
2.3.5 Financial Intermediary . . . 77
2.3.6 Aggregate Resource Constraint and Government . . . 79
2.4 Methodology . . . 80
2.4.1 Simple and Implementable Rules . . . 80

2.4.2 Consumption Equivalent Welfare Cost 84
2.5 Calibration and Estimation 85
2.6 Results 86
2.6.1 Posterior Estimates 86
2.6.2 Optimal Policy and Welfare Analysis 88
2.6.3 Impulse Response 97
2.7 Conclusion 100

3 Government Investment and External Finance Premium 110
3.1 Introduction 110
3.2 Related Literatures on Quantifying the Effect of Fiscal Policies 113
3.3 The Model 114
3.3.1 Household 114
3.3.2 Labour Market 117
3.3.3 Capital Goods Producers 119
3.3.4 Consumer Goods Market 119
3.3.5 Financial Intermediary 122
3.3.6 Aggregate Resource Constraint and Government 126
3.4 Estimation and Calibration 129
3.4.1 Calibration 129
3.4.2 Estimation 130
3.5 Fiscal Multipliers 134
3.5.1 Impulse Response 139
3.6 Conclusion 143

4 Conclusion 181

Bibliography 193

List of Figures

1.1 Movement of Treasury and Corporate Bond Yields of different Maturities 3
1.2 Historical Treasury and Corporate Bond Yields 3
1.3 Corporate Bond Spreads 1984-2018 4
1.4 40-Quarter Corporate Bond Spreads in Data and Model 30
1.5 Government and Corporate Bond Yields in Data and Model 30
1.6 Response to Monetary Shock for Great Inflation and Moderation Era 34
1.7 Response to Productivity Shock for Great Inflation and Moderation Era 35
1.8 Mean Term Premium with Changing Risk Aversion Coefficient σ_E 36
1.9 Cross- and Auto-correlation of Output, Consumption, Investment, Hours, and Wages . . 37
1.10 Cross-correlation of Output, Consumption, Investment, Hours, and Wages 38
1.11 Cross-correlation of Inflation, Interest Rate, Treasury Yield, Corporate Yield and Spread . 39
1.12 Cross- and Auto-correlation of Inflation, Interest Rate, Treasury and Corporate Yield, Spread 39
1.13 Response to Positive Productivity Shock 41
1.14 Response to Tightening Monetary Policy Shock 42
1.15 Response to Adverse Financial Shock 43
1.16 Historical Shock Decomposition of Changes in Investment 44
1.17 Historical Shock Decomposition of Inflation 44
1.18 Historical Shock Decomposition of Nominal Interest Rate 45
1.19 Historical Shock Decomposition of Change in Output 45
1.20 Historical Shock Decomposition of Change in Consumption 46
1.21 Identification Strength of Prior Mean 55
1.22 Multivariate Convergence Diagnostics of 1,000,000 Draws 55
1.23 Smoothed Variables of Observable- Model 5 56
1.24 The Fit of Model 3 and Model 4 56
1.25 Cross- and Auto-correlation of Output, Consumption, Investment, Hours, and Wages . . 58
1.26 Cross-correlation of Output, Consumption, Investment, Hours, and Wages 58
1.27 Cross-correlation of Inflation, Interest Rate, Treasury Yield, Corporate Yield and Spread . 59
1.28 Cross- and Auto-correlation of Inflation, Interest Rate, Treasury and Corporate Yield, Spread 59

2.1 Conditional Welfare Varying ψ_π and ψ_{cb} 93
2.2 Conditional Welfare Varying ψ_{cb} 94
2.3 Welfare of Alternative Monetary Policy with Varying ψ_{cb} 95
2.4 Response to Tightening Monetary Policy Shock for Policy 1b 98
2.5 Response to Adverse External Finance Shock for Policy 1b 99

2.6 Response to Positive Productivity Shock for Policy 1b 100
2.7 Multivariate Convergence Diagnostics for 100,000 Draws 108
2.8 Response to Adverse External Finance Shock for Policy 1b 108
2.9 Response to tightening monetary Policy Shock for Policy 1b 109
2.10 Response to Positive Productivity Shock for Policy 1b 109

3.1 Cumulative Present-value Multipliers 134
3.2 Fiscal Multipliers on Output 137
3.3 Fiscal Multipliers on Consumption 138
3.4 Fiscal Multipliers on Investment 138
3.5 Impulse Response to One Standard Deviation Increase in Government Consumption Shock 140
3.6 Impulse Response toOne Standard Deviation Increase in Government Investment shock . 140
3.7 Impulse Response toOne Standard Deviation Increase in Transfer Shock 141
3.8 Impulse Response toOne Standard Deviation Increase in Capital Tax Shock 141
3.9 Impulse Response toOne Standard Deviation Increase in Labour Tax Shock 142
3.10 Impulse Response to One Standard Deviation Increase in Consumption Tax Shock 142
3.11 Multivariate Convergence Diagnostics for 2,000,000 Draws 180

List of Tables

1.1 Calibrated Parameters and Sources . . . 24
1.2 Estimation Results with and without Financial Friction and Data . . . 26
1.3 Prior and Posterior Distribution for Model 4 Estimation . . . 27
1.4 Great Inflation and Moderation Estimates . . . 33
1.5 First and Second Moment in Data and Model . . . 40
1.6 Prior and Posterior Distribution of Model 5 . . . 57

2.1 Calibrated Parameters and Sources . . . 86
2.2 Prior and posterior distribution of estimated parameters . . . 87
2.3 Bounded Optimal Policy Parameters and Consumption Equivalent Welfare Cost . . . 89
2.4 Unbounded Optimal Policy Parameters and Consumption Equivalent Welfare Cost . . . 90
2.5 Leverage as Policy Instrument as Decomposed by Specific Shocks . . . 92
2.6 Standard Deviation of Bounded Optimal Policy Parameters and Data . . . 96
2.7 Standard Deviation of Unbounded Optimal Policiy Parameters and Data . . . 97

3.1 Calibrated Parameters and Sources . . . 130
3.2 Prior and Posterior Distribution of Fiscal Parameters . . . 132
3.3 Prior and Posterior Distribution of Structural and Exogenous Parameters (non-fiscal) . . . 133
3.4 Present-value Fiscal Multipliers and Productive Capital . . . 135

1. Corporate Bond Spread in a DSGE Model

Abstract

This chapter assesses the information contained in the corporate bond yield that is relevant to the macroeconomy. Building on the workhorse medium-scale dynamic stochastic general equilibrium (DSGE) model of Smets and Wouters (2007), a stochastic discounting factor consistent with Epstein and Zin (1989), and financial friction Bernanke et al. (1999a) in the form of a costly verification problem are embedded in the model. The modification allows for a one-to-one comparison of bond yields in the model with data. Model parameters are estimated by the standard Bayesian method using macroeconomic and bond-yield (government and corporate) data. More specifically, the estimation methodology incorporates the information content of a 10-year BAA-grade corporate bond yield. Results show that estimates made using corporate bond yield data fit the model better; the implied corporate bond spread is sizeable, and the magnitude of its volatility aligns with the data. The fluctuation in investment is well captured in the model such that the estimated investment adjustment cost is not too high. Not surprisingly, investment is most sharply affected by external financial premium shocks. Also noticeable is the structural break in the corporate bond spread before and after Paul Volcker's term as chair of the Federal Reserve Board of Governors. Finally, the result reveals the relationship between the balance sheet condition of firms and the risk premium on the bonds they issue.

1.1 Introduction

Against the backdrop of the financial crises experienced by some industrialised countries and the world at large, there has been an awakening to the relevance of financial market conditions to economic fluctuations (Gerali et al., 2010; Rossana, 2015; Del Negro et al., 2016). The direst financial crisis since the Great Depression was the 2007-2009 global financial crisis (GFC). The crisis began in the United States (US) with the bursting of the housing bubble after an unexpected drop in house prices (Gerali et al., 2010). This resulted in the distress of banks that, at the time, had large portfolios of subprime loans backed by mortgages. The subprime mortgage crisis negatively impacted other asset classes, leading to liquidity problems in the interbank lending market and massive yield spread, causing a strain on businesses and household-credit conditions. This credit crunch rippled outward, culminating in a global financial crisis, with a sharp decline in economic activities globally. This account suggests that economic crises are attributable not only to a decline in economic activities but also to underlying failures in the financial industry (Gerali et al., 2010). Furthermore, the contractionary phase of the business cycle is a consequence of the excesses generated during the expansionary phase, making it necessary to understand the boom in order to make sense of the burst (Mian and Sufi, 2018). This then suggests the need for macroeconomic models to more fully reflect financial and credit conditions.

The underlying assumptions in macroeconomic models that do not reflect financial intermediation imply that a borrower's balance sheet has no effect on their optimal spending choices. Meanwhile, inasmuch as the movement of financial asset prices affects household wealth, it has a direct effect on their spending. Therefore, integrating financial conditions into a macroeconomic model provides a theoretical link between a household's wealth (an indicator of their financial capabilities) and the economic activities in which they can engage. According to Cochrane (2005), asset prices are key in explaining the allocation of consumption and investment over time and states. Therefore, the viability of a macroeconomic model should be ascertained by determining its ability to match asset prices.

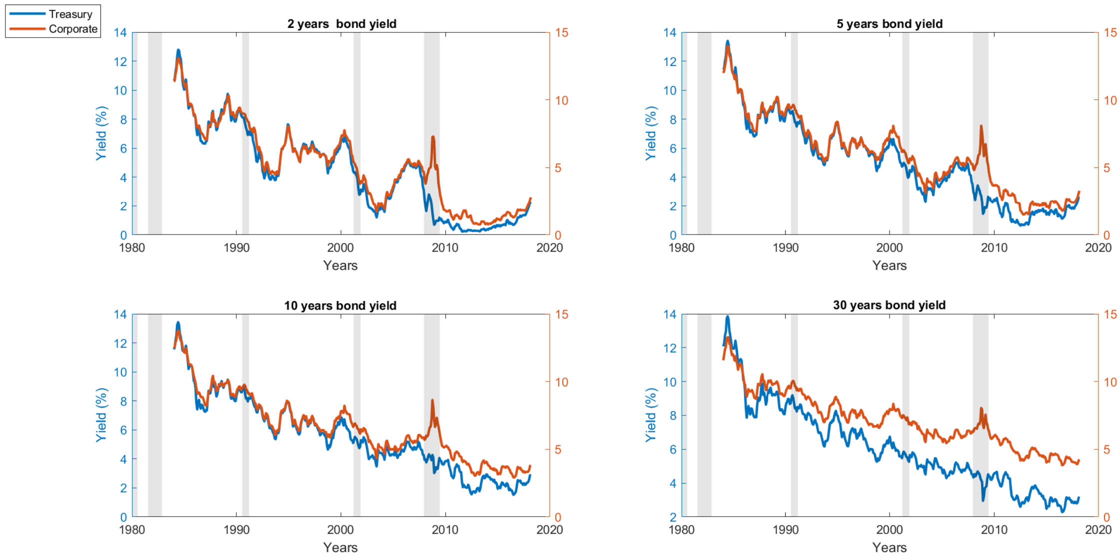
To this end, this chapter presents a medium-scale DSGE model with government and corporate bond yields to assess the effect of financial conditions on macroeconomic outcomes with a focus on the US business cycle. The corporate bond spread[1] is a key default-risk indicator that is observed and analysed here. Such an indicator shows the ability of firms to obtain credit because it reflects the condition of their balance sheets. Although corporate bond spread is treated as a credit spread in some existing studies, this chapter takes this understanding further by arguing that credit risk is one of the factors influencing the spread. [2] One key finding in respect of all the factors identified as influencing the spread is that increased corporate bond spread could reflect a distortion in the supply of credit. A number of studies emphasise the importance of corporate bond spread in predicting economic outcomes (see Gilchrist and Zakrajsek (2011) for a list of these). In addition, Kuehn and Schmid (2014) show empirically that corporate defaults tend to cluster in recession.

Historically, as illustrated in Figure 1.1, both government and corporate bond yields co-move (i.e. follow similar trends). However, there is an exception to this during the GFC, with increasing corporate bond yields coinciding with a decrease in government bond yields. The widening corporate bond spread observed during the financial crisis reflects both the disruption of credit supply as a result of aggravated corporate balance sheets and the crunch felt in the system of financial intermediation. In the aftermath of the crisis, the spread has remained wider than its pre-crisis extent. Generally, the yield on long-term government and corporate bonds tends to be relatively stable. This is because the short rate is mostly affected by monetary policy (Figure 1.2). The countercyclical attribute of corporate bond spread is evident and tends to be greater for longer maturing bonds, as seen in Figure 1.3. However, during the GFC, the 2-year bond had the highest spread, providing another exception to the short-term corporate bond spread being the highest. This is partly a reflection of market expectations and investors' belief that the crisis was a short-term concern. Structural breaks are evident in the 10-year corporate spread before and after the GFC.

[1] The difference between corporate and Treasury bond yields of the same maturity.

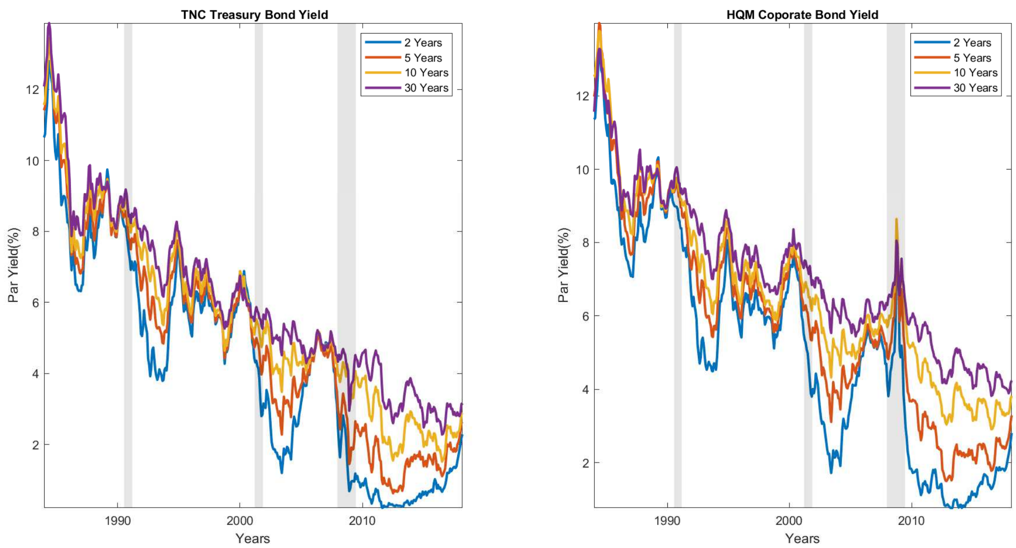
[2] Swanson (2009) considers a risk-neutral approach in quantifying the contribution of default, taxes and systematic risk in corporate bond spread. Collin-Dufresne et al. (2001) investigates the determinant of changes in corporate bond spread against changes in some proxies (e.g. leverage). These two studies concluded that default risk only explains some portion of corporate bond spread. Other factors include liquidity premium, call and conversion features, and the asymmetric tax treatment of corporate and treasury bonds (Huang and Huang, 2012; Kuehn and Schmid, 2014).

Figure 1.1: Movement of Treasury and Corporate Bond Yields of different Maturities



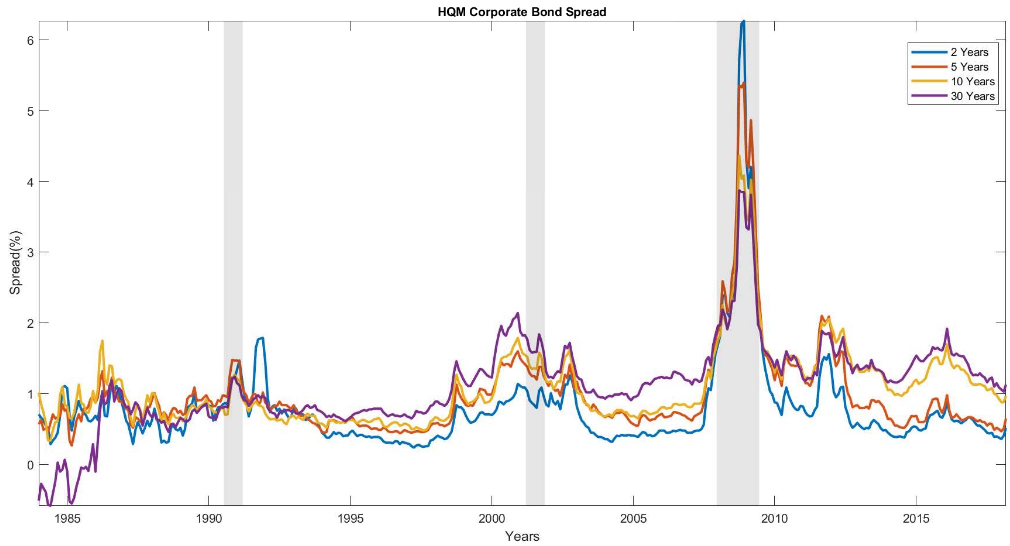
TNC Treasury Bond Yields and HQM Corporate Bond Yields Curve - Monthly average par yields (Department of Treasury, Accessed in July 2019)

Figure 1.2: Historical Treasury and Corporate Bond Yields



TNC Treasury Bond Yields and HQM Corporate Bond Yields Curve - Monthly average par yields (Department of Treasury, Accessed in July 2019)

Figure 1.3: Corporate Bond Spreads 1984-2018



TNC Treasury Bond Yields and HQM Corporate Bond Yields Curve - Monthly average par yields (Department of Treasury, Accessed in July 2019)

The debate around the forecasting ability of financial variables has persisted for decades, with some being of the view that financial asset prices are forward looking and financial variables are thus valuable in forecasting. Financial indicators such as stock market indices and term spread[3], amongst others, have received attention for their usefulness in the forecast of economic activities. Nevertheless, the decision in this chapter to choose a debt market indicator stems from the fact that outstanding long-term bonds in the US and other countries are worth more than equities. Hence, the influence of the debt market on the macroeconomy cannot be underestimated as it concerns the largest class of security. Furthermore, looking at the modelling options in respect of returns on equities and bonds, it is easier to model the latter since the former relies solely on the performance of the issuing corporation.

Amongst all possible debt market indicators, the term spread (referred to interchangeably as the term premium), which contains information on expected inflation and monetary policy, receives the most attention in the existing literature. According to Campbell and Shiller (1991), the term spread is the compensation for investing in long-term bonds. However, it does not capture credit risk, and the GFC is an example of the impact credit risk has on the economy. According to King et al. (2007b), corporate bond spread can signal a deterioration in macroeconomic conditions. As such, the contribution of the corporate debt market to business cycle fluctuations is enormous; it is responsible for a much larger share of that fluctuation than the government debt market. Corporate bond investors do not, as in the case of the government bond, only assume interest rate risk. Amongst other things, they are exposed

[3]The difference between long-term government bond yield and short-term Treasury yield, often regarded as term premium.

to credit risk (Huang and Huang, 2012; Chun et al., 2014). As such, models that are successful in explaining term premiums (i.e. capturing interest rate risk) can be modified to account for credit risk.

Evidence from Stock and Waston (2003) suggests that the ability of term premiums to forecast economic activities has declined since 1985. Several empirical studies (see Zhang (2002); Gilchrist et al. (2009); for a list) suggest the forecasting power of corporate bond spread. This potential lies within securities with low to medium default probability with a long time to maturity (Gilchrist et al., 2009). An example is Zhang (2002), where output forecast from high-yield corporate debt is observed to be more accurate than using investment-grade yields. According to Chun et al. (2014), a shift in corporate bond spread precedes the actual economic cycle, implying its ability to predict upcoming recessions. King et al. (2007a) shows that indices of corporate bond spread contain vital information about the near-term possibility of recession. Elton et al. (2001); Campbell and Taksler (2003); Cremers et al. (2008) all consider varying aspects of corporate bond spread and the bond yield. For example, Campbell and Taksler (2003) examine how the volatility of equity affects yields on corporate bonds, with their result indicating that the volatility of equity has a direct relationship with the cost of borrowing for corporate bond issuers.

It is worth noting that it has been difficult to capture some asset price features in economic models. A prominent example of this is the equity premium puzzle, which receives more attention in earlier studies (Hordahl et al., 2008). According to Mehra and Prescott (1985), a general equilibrium model with friction is most likely to successfully account for the magnitude of the average equity premium. Some features of the bond yield data have also been considered puzzling (e.g., theoretical models have been unable to generate a sufficiently large and variable bond risk premium) as Chun et al. (2014); Kuehn and Schmid (2014) submit. For example, the credit spread puzzle refers to the inability of structural models to explain yield spreads and default rates (Huang and Huang, 2012). This chapter contributes to the literature by investigating this puzzle and its implications for macroeconomic outcomes. This is achieved by theoretically modelling the debt market for government and corporate bonds within a DSGE model that fits macroeconomics variables as in Smets and Wouters (2007).

Earlier non-consumption-based asset pricing models such as Duffie and Kan (1996); Dai and Singleton (2000); Huang and Huang (2012) consider various aspects of bond yield in affine models. Duffee (2002) also documents the fitting of the term premium in certain affine models, some of which fail to replicate the key relationship between expected returns and the slope of the yield curve. This is due to the no-arbitrage condition on which the model is based. However, this condition introduces some restrictions on the model's term premium variability. Affine models are appealing because of the flexibility they offer in terms of modelling the conditional mean, volatilities, and jumps of desired variables. Furthermore, the model is analytically tractable. Although some of the highlighted finance studies are successful in generating large and varying bond premiums, there is less clarity in the economic justification for their results.

Backus et al. (1989) reconcile varying bond features with macroeconomic conditions by providing an economic justification for the observed large and varying term premium in a consumption-based asset pricing model of an endowment economy. The assumptions underlying the economic justification are that investors have Epstein-Zin recursive utility and that consumption and inflation are related in such a way that positive inflation surprises lead to reduced growth in consumption. These two assumptions suggest that investors deserve a premium for holding nominal bonds because positive inflation is followed by a decline in bond value and consumption. Bansal and Yaron (2004) corroborate this, taking on these assumptions for an endowment economy and matching the magnitude and variability of risk premium in bond and foreign exchange markets. Campbell and Cochrane (1999) are also able to explain varying asset pricing dynamics in a consumption-based model with external habits in an endowment economy, with emphasis on equity premium puzzles.

Chen et al. (2009) and Bhamra et al. (2010) also attempt to capture credit risk in an endowment economy by jointly modelling equity and corporate bond prices. They do so with the aim of structurally analysing corporate financing decisions. However, in an endowment economy, bond prices only depend on the joint stochastic process for consumption and inflation, neglecting the production sector. This is a disadvantage since it limits the assessment of the impact of policy changes on the consumption and inflation conditions. A typical example of this limitation is the difficulty in assessing the effects of a change in monetary policy; it would require several years' worth of data to estimate. However, this limitation can be addressed in a more structured DSGE model.

The findings of Donaldson et al. (1990), den Haan (1995), and Wu (2006) reveal that the bond premium puzzle is present in standard real business cycle models.[4] New macroeconomics models such as in Christiano et al. (2005) and Smets and Wouters (2003, 2007) feature habit formation in consumption and recent studies of the term structure adopt nominal rigidities. Models with these features have an advantage over those of earlier generations in the sense that they can better match impulse responses of the economy to nominal and technology shocks (Campbell, 2003). The results of Rudebusch and Swanson (2008) also show that integrating habit formation in consumption and nominal rigidities could help in fitting the term premium, but with the disadvantage of distorting the model's ability to fit other macroeconomic variables. As a result, the bond premium puzzle remains.

Extending the study further, Rudebusch and Swanson (2012) make the assumption that investors have recursive Epstein-Zin preferences and are faced with long-run economic risk. Like Backus et al. (1989), Rudebusch and Swanson (2012) work with a canonical DSGE model, and three key findings emerge. First, they suggest a straightforward modification to existing DSGE models that will be helpful in bringing the model close to bond pricing facts. Second, the model provides a better explanation than existing studies by providing an economic interpretation of the results. Third, the study provides an intuitive response to the ever-puzzling issue of why the yield curve slopes upward. However, the model in Rudebusch and Swanson (2012) requires a high level of risk aversion in matching asset prices. Therefore,

[4]This puzzle refers to the inability of the model to replicate the magnitude in data.

there is the possibility of the model understating the true quantity of the risk faced by bondholders. In furthering an accurate fit, the authors note that modelling the linkage between long-run inflation risk and term premiums will allow better insight into long-term bond yields. Having established how term premium analysis is attainable in a structured DSGE model with Epstein-Zin preferences, it is worth extending such models to study defaultable debt (i.e. corporate bonds); this chapter seeks to do so.

Building on Smets and Wouters (2007), the medium-scale DSGE model is modified by embedding a simpler version of Bernanke et al. (1999a) financial friction, similar to Christensen and Dib (2008). The friction is aimed at providing a theoretical linkage between agents' financial health and the economic activities with which they can engage. This financial-friction specification is selected for its ability to allow asset price movements to impose credit market imperfections (DeGraeve, 2008). Since Epstein-Zin preferences are essential for the successful calibration of financial indicators, the modified model also incorporates the preferences, as in Rudebusch and Swanson (2012). These adjustments enabled us to derive a proper Stochastic Discounting Factor (SDF) used in specifying the bond pricing equations within the model. First, it is worth highlighting the financial accelerator mechanism and how it relates to bond prices.

The inability of entrepreneurs to fully fund capital purchases drives them to source funds externally at an agreed loan rate higher than the presiding nominal risk-free rate (i.e., $R_{t,t+1}$). The loan rate is contractual (i.e., determined in period t but to be repaid in period $t+1$), with its nominal value given by

$$R^N_{t,t+1} = S_{t,t+1} R_{t,t+1},$$

where the external finance premium $S_{t,t+1}$ is

$$S_{t,t+1} = \varepsilon^s_t S\left(\frac{Q_{t-1}K_{t-1}}{N_{t-1}}\right), \quad \text{where} \quad S'() < 0 \quad \text{and} \quad S(1) = 1.$$

The nominal SDF ($\Lambda^N_{t+1} = \frac{\Lambda_{t,t+1}}{\pi_{t,t+1}}$) is used in evaluating the bond price during period t. The period-t price and yield to maturity of a government bond maturing in period n is expressed as

$$B^g_{t,n} = \mathbb{E}_t\left[\Lambda^N_{t,t+1} B^g_{t+1,n-1}\right], \qquad R^g_{t,n} = \left(\frac{1}{B^g_{t,n}}\right)^{\frac{1}{n}}.$$

In the same vein, the price and yield to maturity of a nominal corporate bond are modelled as

$$B^c_{t,n} = \mathbb{E}_t\Big[\Lambda^N_{t+1} B^c_{t+1,n-1}\Big]\Big(\frac{1}{S_{t+1}}\Big), \qquad R^c_{t,n} = \Big(\frac{1}{B^c_{t,n}}\Big)^{\frac{1}{n}}$$

Consistent with asset pricing theory, as shown above, the bond price in this chapter is the discounted expected future bond price calculated recursively. [5]

The model is solved and evaluated using perturbation methods. The standard Bayesian estimation technique is adopted to estimate model parameters. The estimation procedure incorporates macroeconomic information and government and corporate bond yield data.

DeGraeve (2008) and Gilchrist et al. (2009) are the two studies identified as closely related to the work of this chapter. However, neither of the two theoretically link macroeconomic variables to explicit bond pricing equations. Although Gilchrist et al. (2009) estimate the model using the long-maturing corporate bond spread as a proxy for external finance premium, DeGraeve (2008) indicates that no straightforward representation of financial data is possible through model variables; this limits their study to comparing a model-simulated external finance premium (i.e., after estimating with macroeconomic data) with various proxies thereof (e.g. bank loans, credit standards, debt to GDP ratio).

As is the case with the model here, Gilchrist et al. (2009) indicates that the external finance premium depends on an exogenous financial disturbance (ε^s_t). This is perceived as a shock to the credit supply as a result of changes in the efficiency of financial intermediaries. An additional feature in the financial-friction specification is the shock to entrepreneurial net worth. These two financial shocks are excluded in DeGraeve (2008). As one of the contributions to knowledge, this chapter sets out a link between model variables and the debt market (i.e., government and corporate bonds), allowing a one-to-one comparison of model bond yields with data. This linkage might be a result of the incorporation of Epstein-Zin preferences in the model presented here; their inclusion allows me to define a proper SDF, which DeGraeve (2008) and Gilchrist et al. (2009) do not consider. This enhances the study of how corporate bond spread affects the macroeconomy.

The results of this chapter indicate the modified model's ability to match macroeconomic and financial

[5] The price of n-period nominal bond is evaluated as follows

$$\begin{aligned}
B^g_{t,1} &= \mathbb{E}_t\Big[\Lambda^N_{t,t+1}\Big] \\
B^g_{t,2} &= \mathbb{E}_t\Big[\Lambda^N_{t,t+2}\Big] = \mathbb{E}_t\Big[\Lambda^N_{t,t+1} B^g_{t+1,1}\Big] \\
&\vdots \\
B^g_{t,n} &= \mathbb{E}_t\Big[\Lambda^N_{t,t+n}\Big] = \mathbb{E}_t\Big[\Lambda^N_{t,t+1} B^g_{t+1,n-1}\Big]
\end{aligned}$$

data without compromise. The estimated model also incorporates financial information (which, in this instance, comprises financial friction and corporate bond yield data), thereby providing a better description of macroeconomic dynamics than when this information is excluded. This improved descriptive ability indicates that financial disruption has a significant economic impact in the propagation of the US business cycle. In addition, the model implies corporate bond spread is sizeable, and the magnitude of its volatility aligns with the data. This addresses the limitations in existing studies that have been unable to generate variable and sizeable corporate bond spread, as observed in Chun et al. (2014). The result shows further that historical shock decomposition aligns with historical narratives of the effect on the economy of various shocks. Noticeable also is a structural break in the corporate bond spread before and after Paul Volcker's term as chair of the Federal Reserve Board of Governors.

The remainder of this chapter is structured as follows: Section 1.2 presents the structure and attributes of the modified medium-scale DSGE model and the nonlinear equations derived for each sector of the economy. Section 1.3 captures the methodology used in solving and estimating the model, while Section 1.4 presents and discusses the results. Section 1.5 delivers the conclusion and recommendations for future extensions of the research.

1.2 Model Set-Up

The model builds on Smets and Wouters (2003, 2007), in which the authors introduce a DSGE model for the US economy that incorporates many types of real and nominal frictions. The following are added to the model: (i) Epstein-Zin preferences as in Rudebusch and Swanson (2012) to help differentiate the coefficient of risk aversion from the elasticity of substitution; (ii) Financial friction as in Bernanke et al. (1999a) in the form of a cost-verification problem to endogenously describe the behaviour of the financial industry. The financial friction takes on a simple form, as in Christensen and Dib (2008). Furthermore, the following adjustments are made to the benchmark model: (i) Fixed cost in production is removed because of the introduction of financial friction; (ii) Capital utilisation is redefined. However, unlike Smets and Wouters (2007), where wage- and price-markup shocks are used alongside the Kimball aggregator, this chapter uses a Dixit-Stiglitz aggregator to aggregate prices in the goods and labour market, similar to Smets and Wouters (2003). In addition, the wage mark-up shock is replaced by a labour supply shock, while the price mark-up shock is retained and follows an AR(1) process. The following sub-sections describe the equilibrium conditions of each sector of the economy.

1.2.1 Households

There exists a continuum of households that are individually represented by $j \in [0,1]$. Each of these households has a non-separable period utility that is a function of consumption goods $C_t(j)$ and labour $L_t(j)$:

$$U(C_t(j), L_t(j)) = \varepsilon_t^b \Big(\frac{1}{1-\sigma_C}\Big)\Big(C_t(j) - \eta_c C_{t-1}(j)\Big)^{1-\sigma_C} \exp\Big(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w (L_t(j))^{1+\sigma_L}\Big), \tag{1.2.1}$$

where ε_t^b is the household preference shock; η_c is the external habit formation parameter; ε_t^w is the labour supply shock; σ_C is the elasticity of substitution in goods, and σ_L is the elasticity of labour disutility. At each point in time, the individual household derives satisfaction from consumption goods relative to a proportion of past consumption. In addition, they have disutility from supplying their labour services to firms.

To differentiate elasticity of substitution from the coefficient of risk aversion, a household is said to have a recursive Epstein-Zin preference given by:

$$V_t(j) = U(C_t(j), L_t(j)) + \beta v_t(j) \tag{1.2.2}$$

and

$$v_t(j) = \mathbb{E}_t(V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}},$$

where β is the subjective discount factor, and σ_E is the Epstein-Zin preference parameter that controls the coefficient of relative risk aversion. According to Rudebusch and Swanson (2012), a higher magnitude of σ_E in absolute terms is a reflection of how much the household dislikes risk. Household j in period t consumes $C_t(j)$ and invests in nominal government bonds issued through financial intermediary $B_t(j)$. This bond is discounted at the risk-free rate, R_t. They decide how many hours to work $L_t(j)$, receiving a nominal wage of $W_t^h(j)$. In addition, they receive dividend Div_t from their labour union. T_t is a lump-sum government transfer. Their intertemporal budget constraint is therefore expressed as:

$$C_t(j) + \frac{B_t(j)}{R_{t+1}P_t} - T_t \leq \frac{W_t^h(j)L_t(j)}{P_t} + \frac{B_{t-1}(j)}{P_t} + \frac{Div_t}{P_t}, \tag{1.2.3}$$

where P_t is the price level at time t. Household j's optimisation problem is to maximise V_0 subject to (1.2.2) and their intertemporal budget constraint (1.2.3). The following are the equilibrium equations from the household's optimisation problem:

$$\frac{W_t^h}{P_t} = -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} \tag{1.2.4}$$

$$\Xi_t = U_C(C_t, L_t), \tag{1.2.5}$$

with $U_C(C_t, L_t)$ and $U_L(C_t, L_t)$ representing marginal consumption and marginal disutility, respectively. Also, the real SDF at time t for a payoff in time $t+1$ is given as

$$\Lambda_{t,t+1} = \beta \left[\frac{V_{t+1}}{\left(\mathbb{E}_t V_{t+1}^{1-\sigma_E}\right)^{\frac{1}{1-\sigma_E}}} \right]^{-\sigma_E} \frac{\Xi_{t+1}}{\Xi_t} \tag{1.2.6}$$

$$\frac{1}{R_{t+1}} = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\pi_{t+1}} \right] \tag{1.2.7}$$

1.2.2 Labour Market

A household directly supplies their homogeneous labour to an intermediate labour union which, in turn, differentiates labour services and sets wages subject to Calvo pricing. This differentiated labour is packaged by individuals called labour packers. There are two resulting sub-sectors in the labour market, as discussed in the following subsection.

1.2.2.1 Labour Packers

The labour $H_t(l)$, as differentiated by the labour union, is bought and packaged by labour packers. Labour packers aggregate the labour supplied into an index H_t; this step employs a Dixit-Stiglitz aggregator. They thereafter sell the indexed labour H_t at W_t to intermediate goods firms owned by entrepreneurs. Labour packers thus maximise their profit as follows:

$$\begin{aligned} \max_{H_t(l)} \quad & W_t H_t - \int_0^1 W_t(l) H_t(l) \, \mathrm{d}l \\ \text{s.t} \quad & H_t = \left(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, \mathrm{d}l \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \end{aligned}$$

This optimisation gives the labour demand schedule as

$$H_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\epsilon_w} H_t. \tag{1.2.8}$$

The wage received by labour packers, which is also the cost of wages faced by entrepreneurs, is

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w}\, \mathrm{d}l\right)^{\frac{1}{1-\epsilon_w}} \tag{1.2.9}$$

1.2.2.2 Labour Unions

Labour unions hire the raw labour force L_t from households, which is followed by training to differentiate that labour based on skills. They take the marginal rate of substitution as the cost of labour services in their negotiations with labour packers. The mark-up above this marginal disutility is distributed among the households in the form of dividend Div_t, as seen in the household's budget constraint. The union is subjected to nominal rigidities and can only adjust wages in each period with a probability of $1-\zeta_w$, therefore optimising the wage over the period during which they cannot change the price. In the period when they are unable to re-optimise wages, they partially index previously optimised wages to reflect lagged inflation.

$$\begin{aligned}
\max_{\tilde{W}_t(l)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s}\Big(W_{t+s}(l)H_{t+s}(l) - W_{t+s}^h L_{t+s}(l)\Big) \\
\text{s.t} \quad & H_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}}\right)^{-\epsilon_w} H_{t+s} \\
& H_{t+s}(l) = L_{t+s}(l) \\
& W_{t+s}(l) = \tilde{W}_t(l)\Big(\prod_{l=1}^{s} \pi_{t+l-1}^{l_w} \pi_{ss}^{1-l_w}\Big) \quad \text{for} \quad s = 1, \ldots, \infty
\end{aligned}$$

Solving the above optimisation problem gives

$$\frac{\tilde{W}_t}{W_t} = \left(\frac{\epsilon_w}{1-\epsilon_w}\right)\left(\frac{G_t^{w2}}{G_t^{w1}}\right), \tag{1.2.10}$$

where

$$G_t^{w1} = W_t H_t + \mathbb{E}_t\left[\zeta_w \Lambda_{t,t+1}\left(\frac{W_t}{W_{t+1}}\right)^{1-\epsilon_w}\left(\pi_t^{l_w}\pi_{ss}^{1-l_w}\right)^{1-\epsilon_w} G_{t+1}^{w1}\right] \quad (1.2.11)$$

and

$$G_t^{w2} = W_t^h H_t + \mathbb{E}_t\left[\zeta_w \Lambda_{t,t+1}\left(\frac{W_t}{W_{t+1}}\right)^{-\epsilon_w}\left(\pi_t^{l_w}\pi_{ss}^{1-l_w}\right)^{-\epsilon_w} G_{t+1}^{w2}\right]. \quad (1.2.12)$$

The aggregate wage expression is

$$W_t = \left[(1-\zeta_w)\tilde{W}_t^{1-\epsilon_w} + \zeta_w\left(\gamma\pi_{t-1}^{l_w}\pi_{ss}^{1-l_w}W_{t-1}\right)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}. \quad (1.2.13)$$

There is wage dispersion cost as a result of the discrepancy between labour demanded and labour supplied. This implies that

$$H_t \neq L_t, \quad \text{and} \quad W_t \neq W_t^h \implies L_t = \Delta_t^w H_t$$

The explicit expression from wage dispersion cost considering the sticky wage is

$$\Delta_t^w = (1-\zeta_w)\int_0^1\left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w}\mathrm{d}l + \zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w}\Delta_{t-1}^w. \quad (1.2.14)$$

1.2.3 Goods Market

Capital and consumer goods are produced in the goods market. The consumer goods section is comprised of retailers, wholesalers and entrepreneurs.

1.2.3.1 Capital-Goods Producers

Capital-goods producers use the final good I_t to produce investment goods $\tilde{I}_t$. They work in a perfectly competitive environment and are faced with the cost of changing the flow of investments. As a result, they choose the quantity of investment I_t to maximise their profit,

$$\Pi_t^I = Q_t \tilde{I}_t - I_t$$

$$\max_{I_t \tilde{I}_t} \quad \mathbb{E}_0 \Big[\sum_{t=0}^{\infty} \Lambda_{0,t} \Pi_t^I \Big]$$
$$\text{s.t} \quad \tilde{I}_t = \varepsilon_t^i \Bigg[I_t - \frac{\Psi}{2} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big)^2 I_t \Bigg]$$

$$1 = Q_t \varepsilon_t^i \Bigg[1 - \Psi \frac{I_t}{I_{t-1}} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big) - \frac{\psi}{2} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big)^2 \Bigg] + \mathbb{E}_t \Bigg[\Lambda_{t,t+1} Q_{t+1} \varepsilon_{t+1}^i \Psi \Big(\frac{I_{t+1}}{I_t} \Big)^2 \Big(\frac{I_{t+1}}{I_t} - \gamma \Big) \Bigg],$$

where Ψ is the investment adjustment shock and ε_t^i is the shock to investment.

1.2.3.2 Retailers

The retailers are the producers of the final good Y_t. The final good is composed of differentiated goods from wholesalers $Y_t(i)$. It is allocated to consumption, investment and government expenditure. The technology used in transforming these differentiated goods is given in the form of a Dixit-Stiglitz aggregator. Their optimisation problem gives the following demand-curve expression:

$$Y_t(i) = \Big(\frac{P_t(i)}{P_t} \Big)^{-\epsilon_p} Y_t \tag{1.2.15}$$

The Law of Motion (LOM) for price is

$$P_t = \Big(\int_0^1 P_t(i)^{1-\epsilon_p} \, \mathrm{d}i \Big)^{\frac{1}{1-\epsilon_p}} \tag{1.2.16}$$

1.2.3.3 Wholesalers

Wholesalers buy intermediate goods M_t from the entrepreneur who owns firms. These intermediate goods are differentiated without cost, taking the demand function earlier derived from retailers' First Order Condition. Wholesalers' prices are subject to Calvo pricing, which introduces nominal rigidities into the model. The optimal price is set by wholesalers and is allowed to re-optimise, giving rise to the optimisation problem below

$$\max_{\tilde{P}_t(i)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Lambda_{t,t+s} \Big[\tilde{P}_t(i) X_{s,t}^p Y_{t+s}(i) - MC_{t+s} M_{t+s}(i) \Big]$$

$$\text{s.t} \quad Y_{t+s}(i) = Y_{t+s} \bigg(\frac{\tilde{P}_t(i) X_{s,t}^p}{P_{t+s}} \bigg)^{-\epsilon_p}$$

$$Y_{t+s}(i) = \frac{M_{t+s}(i)}{\varepsilon_t^p}$$

where

$$X_{s,t}^p = \begin{cases} 1 & \text{for} \quad s = 0 \\ \Big(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} \Big) & \text{for} \quad s = 1 \ldots \infty \end{cases}$$

The first constraint is the demand schedule (1.2.15), which is obtained from the retailer optimisation problem. However, the second constraint represents a linear production function used for differentiating the intermediate good. Where ε_t^p is the production function for transforming the intermediate good, and $\tilde{P}_t(i)$ is the newly optimised price. The inability to change this price for some period allows wholesalers to only partially index the current price to lagged inflation represented by $X_{t,s}^p$. The probability of being able to re-optimise the price is ζ_p, while ι_p measures the degree of price indexation. Solving the optimisation problem above gives the optimal newly set price for those allowed to reset the price as

$$\frac{\tilde{P}_t(i)}{Pt} = \bigg(\frac{\epsilon_p}{\epsilon_p - 1} \bigg) \bigg(\frac{G_t^{p2}}{G_t^{p1}} \bigg)$$

where

$$G_t^{p1} = P_t Y_t + \mathbb{E}_t \bigg[\zeta_p \Lambda_{t,t+1} \bigg(\frac{\pi_t^{l_p} \pi_{ss}^{1-l_p}}{\pi_{t+1}} \bigg)^{1-\epsilon_p} G_{t+1}^{p1} \bigg]$$

and

$$G_t^{p2} = \varepsilon_t^p MC_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{-\varepsilon_t^p} G_{t+1}^{p2}\right]$$

Aggregate price (1.2.16) is the sum of the newly reset price and the partially indexed price, which is expanded as

$$P_t = \left((1-\zeta_p)\tilde{P}_t(i)^{1-\epsilon_p} + \zeta_p\left(P_{t-1}\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}\right)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}$$

Simplifying the above gives

$$1 = (1-\zeta_p)\left(\frac{\tilde{P}_t(i)}{P_t}\right)^{1-\epsilon_p} + \zeta_p\left(\frac{\pi_{t-1}^{l_p}\pi_{ss}^{1-l_p}}{\pi_t}\right)^{1-\epsilon_p} \tag{1.2.17}$$

The price dispersion expression is derived as follows,

$$\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i$$

$$\Delta_t^p = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p \tag{1.2.18}$$

1.2.4 Entrepreneurs

Each entrepreneur owns a firm i that uses the following technology

$$M_t(i) = \varepsilon_t^a\left(K_{t-1}(i)U_t^k(i)\right)^{\alpha}\left(\gamma^t H_t(i)\right)^{1-\alpha}. \tag{1.2.19}$$

They hire labour H_t and pay a wage of W_t to the labour union (which, in turn, pays households). They also buy investment goods $\tilde{I}_t$ from capital good producers. This is added to their existing stock of capital K_{t-1} to make up for the next period's capital K_t, where K_t denotes the capital choice at the end of period $t-1$ for use in period t production. It is believed that an entrepreneur cannot fully finance the purchase of the capital they need with their net worth. Therefore, they borrow in the form of loans from banks to finance desired capital stock in excess of their net worth. Entrepreneurs are also impacted by idiosyncratic shocks that affect their capital holdings, which is why not all survive the system. The entrepreneur's problem is given below.

$$\begin{aligned}
&\max_{H_t,K_t,U_t^k} \mathbb{E}_t\Big[\sum_{s=0}^{\infty}(1-\varphi)^{s-1}\Lambda_{t,t+s}N_{t+s}\Big] \\
&\text{s.t} \quad D_{t-1} = Q_{t-1}K_{t-1} - N_{t-1} \\
&\quad N_t = MC_tM_t - W_tH_t + (1-\delta_t^k)Q_tK_{t-1} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}D_{t-1} \\
&\quad M_t \leq \varepsilon_t^a\Big(K_tU_t^k\Big)^{\alpha}\Big(\gamma^tH_t\Big)^{1-\alpha} \\
&\quad K_t \leq (1-\delta_t^k)K_{t-1} + \tilde{I}_t \\
&\quad \delta_t^k = \delta + \psi_u\frac{MPK_{ss}}{Q_{ss}}\Big(U_t^{k\,\frac{1}{\psi_u}} - 1\Big)
\end{aligned}$$

At each period, entrepreneurs survive with the probability φ. The individual entrepreneur's concern is mostly for their lifetime net worth, suggesting their need to maximise that amount. The first constraint identifies how much more an entrepreneur needs to borrow to make up for the purchase of capital for the next period's production. This fund is obtainable from households through the financial intermediaries that set the terms for entrepreneurs. The second constraint is the LOM of net worth and shows that the entrepreneur retains the capital share after depreciation less debt repayments. The third constraint is the production function, followed by the LOM of capital, which is the fourth constraint. Finally, the fifth constraint is depreciation. The marginal product of capital is given below:

$$\text{MPK}_t = \alpha\frac{\text{MC}_tM_t}{K_{t-1}} \tag{1.2.20}$$

Solving the entrepreneur's optimisation problem gives the optimum capital utilisation as

$$U_t^k = \left(\frac{\frac{\text{MPK}_t}{Q_t}}{\frac{\text{MPK}_{ss}}{Q_{ss}}}\right)^{\psi_u} \tag{1.2.21}$$

Equation (1.2.21) determines the relationship between the utilisation rate and marginal product of capital. The elasticity of utilisation cost with respect to input from capital is represented by ψ_u and $R^k_{t,t+1}$ is the realised return on capital, which is different from the return on loan $(R^N_{t,t+1})$ requested by the financial intermediary and agreed on at the loan's inception.

$$\mathbb{E}_t\Big[\Lambda_{t,t+1}R^K_{t,t+1}\Big] = \mathbb{E}_t\Big[\Lambda_{t,t+1}\frac{1}{\pi_{t,t+1}}\Big]R^N_{t,t+1} \tag{1.2.22}$$

The partial derivative of K_t, implies

$$\mathbb{E}_t\Big[R^K_{t,t+1}\Big] = \mathbb{E}_t\Big[\frac{\text{MPK}_{t+1}}{Q_t} + (1-\delta^k_{t+1})\frac{Q_{t+1}}{Q_t}\Big] \tag{1.2.23}$$

The expression (1.2.23) is the ex-post real return on capital. It is the sum of income gain (i.e. marginal real revenue on capital that is evaluated by real capital) and capital gain (i.e. the real price change of remaining capital). However, it is worth noting that $\frac{R^N_{t,t+1}}{\pi_{t,t+1}} \neq R^K_{t,t+1}$ because the marginal product of capital MPK_{t+1} and $\pi_{t,t+1}$ depends on the realisation of the shocks at $t+1$.

The LOM representation of the real net worth of all entrepreneurs is expressed as follows.

$$N_t = \varphi E_t + (1-\varphi)\varepsilon^N_t, \tag{1.2.24}$$

where E_t represents surviving entrepreneur's net worth and is given by

$$E_t = \Big(R^K_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}\Big)Q_{t-1}K_{t-1} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}}N_{t-1} \tag{1.2.25}$$

Since the amount borrowed by entrepreneur is $D_{t-1} = Q_{t-1}K_{t-1} - N_{t-1}$, the nominal debt repayment as agreed at time $t-1$ is $R^N_{t-1,t}(Q_{t-1}K_{t-1} - N_{t-1})$. With $\frac{R^N_{t-1,t}}{\pi_{t-1,t}}$ being the ex-ante nominal contract rate deflated by the ex-post realised inflation, the net worth of existing entrepreneur E_t is the product of realised gross return and capital less the product of the contracted borrowing rate and the amount of borrowing.

1.2.5 Financial Intermediary and Asset Pricing

Financial intermediaries act as go-betweens for households and entrepreneurs in respect of financial transactions by lending the money deposited by households (in the form of bonds, loans etc.) to entrepreneurs at a rate higher than the risk-free rate ($R_{t,t+1}$). While it is cheap for entrepreneurs to obtain internal finance, it is costly to source funds externally. External funds can be obtained either by loans, bonds or equity or a myriad of other forms. The existence of market imperfection as a result of asymmetric information between market participants is probably a good explanation for the high cost of obtaining external funding. Entrepreneurs cannot obtain loans at the risk-free rate because financial intermediaries cannot easily observe their output. It is costly for financial intermediaries to observe the realised return of entrepreneurs since they have to pay a state verification cost, suggesting that the cost of sourcing external finance is different from the economy's risk-free rate. At time t, the rate of return on the loans to entrepreneurs from time t to $t+1$ is agreed on and is given by

$$R^N_{t,t+1} = S_{t,t+1} R_{t,t+1} \tag{1.2.26}$$

The nominal external finance premium is

$$S_{t,t+1} = S\left(\varepsilon^s_t \frac{Q_{t-1}K_{t-1}}{N_{t-1}}\right), \quad \text{where} \quad S'() < 0 \quad \text{and} \quad S(1) = 1 \tag{1.2.27}$$

The external finance premium, as seen in (1.2.27), increases the amount entrepreneurs borrow relative to their net worth. Like every contract between borrowers and lenders, there is a need for the borrower to post collateral, in this case, the net worth of the entrepreneur observed by the financial intermediary before the loan issuance. Therefore, the spread $S_{t,t+1}$ is known at time t and has a functional form given by

$$S_{t,t+1} = \varepsilon^s_t S_{ss} \left(\frac{Q_{t-1}K_{t-1}}{N_{t-1}} \frac{N_{ss}}{K_{ss}}\right)^{\psi_S}, \tag{1.2.28}$$

where ψ_S is the elasticity of the external finance premium to the entrepreneur's leverage and S_{ss} is a constant. It is assumed that lenders only know the aggregate $\frac{Q_{t-1}K_{t-1}}{N_{t-1}}$ and not individual entrepreneurs' level. The exogenous disturbance to the external finance premium ε^s_t can be interpreted as a shock to the credit supply that influences the functionality of the financial intermediary.

1.2.5.1 Asset Pricing

In theory, the current price of any asset should equal the expected future value that is discounted stochastically. Nominal government and corporate bond prices at time t of bonds maturing in n period are defined as follows.

$$B^g_{t,n} = \mathbb{E}_t\left[\Lambda^N_{t+1} B^g_{t+1,n-1}\right] \tag{1.2.29}$$

$$B^c_{t,n} = \mathbb{E}_t\left[\Lambda^N_{t+1} B^c_{t+1,n-1}\right]\left(\frac{1}{S_{t+1}}\right) \tag{1.2.30}$$

Their returns (yield to maturity) are given as

$$R^g_{t,n} = \left(\frac{1}{B^g_{t,n}}\right)^{\frac{1}{n}} \tag{1.2.31}$$

$$R^c_{t,n} = \left(\frac{1}{B^c_{t,n}}\right)^{\frac{1}{n}} \tag{1.2.32}$$

With reference to the expressions for government and corporate bonds above, the corporate bond spread can easily be deduced in the DSGE model as:

$$C^b_{t,n} = R^c_{t,n} - R^g_{t,n} \tag{1.2.33}$$

1.2.6 Aggregate Resource Constraint and Government

To aggregate the model variables, it is standard practice in macroeconomics to assume that a representative household chooses consumption and labour. This assumption was therefore adopted in aggregating household choices of consumption and labour. However, to aggregate firms (i.e. the labour and consumer goods markets), price and wage dispersions are defined and given as (1.2.18) and (1.2.14), respectively. The central bank follows a nominal interest rate rule, adjusting its instrument in response to deviations of inflation and output from their respective target levels as follows.

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y}\right]^{1-\rho_R} \left[\frac{\left(\frac{Y_t}{Y_{t-1}}\right)}{\left(\frac{Y_t^f}{Y_{t-1}^f}\right)}\right]^{\psi_{\mathrm{dy}}} \varepsilon_t^r, \tag{1.2.34}$$

where R_{ss} is the steady-state nominal rate, π_{ss} is the steady-state inflation, and the natural output Y_t^f is the output in the flexible price and wage economy. The interest-rate smoothing parameter is denoted by ρ_R and the shock to the monetary policy ε_t^r is

$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r, \quad \eta_t^r \sim \mathcal{N}(0, \sigma_r). \tag{1.2.35}$$

Market clearing implies that the following holds:

$$Y_t = C_t + G_t + I_t \tag{1.2.36}$$

The implication of (1.2.36) is that the final good produced in the economy is allocated to consumption, government spending, and investment. Government spending is expressed relative to the steady-state output path as

$$\varepsilon_t^g = \frac{G_t}{y_{ss}\gamma^t}, \tag{1.2.37}$$

where ε_t^g follows an exogenous AR(1) process:

$$\ln \varepsilon_t^g = (1-\rho_g)\ln g + \rho_g \ln \varepsilon_{t-1}^g + \rho_{ga} \ln \varepsilon_{t-1}^a + \eta_t^g, \quad \eta_t^g \sim \mathcal{N}(0, \sigma_g) \tag{1.2.38}$$

1.3 Estimation

Bayesian estimation is used to estimate the DSGE model presented in this chapter. This technique combines the likelihood function with the prior distributions in approximating the posterior mode, which is then used as the starting value for a random walk Metropolis algorithm. Before estimation, the optimisation problem for each sector is solved using the perturbation method. In addition, the equilibrium equations derived from solving the aforementioned optimisation problems were detrended using the deterministic trend γ, while nominal variables were replaced by their real counterparts. The detrended equilibrium equations were entered into Dynare in a non-linear fashion to retain the coefficient of risk aversion. Dynare generated a level linearisation around the deterministic steady-state of each model variable. The solution of the rational expectation system after solving the optimisation problem of the model is of the form:

$$s_t = As_{t-1} + B\eta_t \tag{1.3.1}$$

$$o_t = Cs_{t-1} + D\nu_t \tag{1.3.2}$$

$$\eta_t \sim \mathcal{N}(0, \Omega) \quad \text{and} \quad \nu_t \sim \mathcal{N}(0, \Theta),$$

where s_t is a vector comprised of the endogenous and exogenous processes of the model.

These variables are expressed as level deviations from their steady-state values. Vector η_t comprises the noise in respect of the shock processes. Matrices A and B are both functions of the structural parameters of the model. The vector representing our data (observables) is o_t while ν_t is the measurement error shock. In the same manner, matrices C and D are functions of the parameters governing the observable variables.

1.3.1 Data

In reconstructing the seven quarterly US data in Smets and Wouters (2007), changes were noted in the base year of some of the data. The real GDP is now chained to 2012, which is different from the 1996 base year in Smets and Wouters (2007). While corporate bond data is only available for a short period, the BAA corporate bond yield is inferred using Moody's Seasoned BAA corporate bond yield relative to the yield on the 10-year (40 quarters) Treasury constant maturity and 10-Year Treasury constant maturity rates FRED (Accessed in September 2019b).

1.3.2 Estimation Procedure

Estimating the model presented in this chapter takes place in stages and with different observables. The estimation procedure is divided into five model specifications, namely models 1 to 5. Only macroeconomic data is used in estimating models 1 and 2. For the estimation of the remaining models, data in addition to the macroeconomic data are used: model 3 is estimated using the 40-quarter government bond yield; model 4 uses the 40-quarter BAA corporate yield; and model 5 uses the 40-quarter BAA corporate yield and BAA spread. There is no structural difference between models 2, 3, 4 and 5; with respect to model 1, the financial transmission mechanism is turned off. Therefore, model 1 strictly omits financial friction.

These models are estimated using the same seven macroeconomic data as in Smets and Wouters (2007), namely: (i) the log difference of real GDP; (ii) the log difference of real consumption; (iii) the log difference of real investment; (iv) the log difference of real wage; (v) log hours; (vi) log inflation (GDP deflator); and (vii) the federal funds rate. The log-transformed 40-quarter government and BAA corporate bond yields relating the yields in the model with data are expressed as:

$$\log(R^g_{t,40}) = \frac{-\log(B^g_{t,40})}{10}$$

$$\log(R^c_{t,40}) = \frac{-\log(B^c_{t,40})}{10}$$

Due to the complexity of estimating the study's non-linear model, measurement errors are added to all observable equations. The measurement equation (1.3.2) is explicitly represented as

$$o_t = \begin{pmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \\ \bar{R}^g_{t,40} \\ \bar{R}^c_{t,40} \\ C^{bs}_{t,40} \end{pmatrix} = \begin{pmatrix} \log\left(\frac{\gamma y_t}{y_{t-1}}\right) \times 100 \\ \log\left(\frac{\gamma c_t}{c_{t-1}}\right) \times 100 \\ \log\left(\frac{\gamma i_t}{i_{t-1}}\right) \times 100 \\ \log\left(\frac{\gamma w_t}{w_{t-1}}\right) \times 100 \\ \log\left(\frac{l_t}{l_{ss}}\right) \times 100 \\ \log(\pi_t) \times 100 \\ \log(r_t) \times 100 \\ \log(R^g_{t,40}) \times 100 \\ \log(R^c_{t,40}) \times 100 \\ C^b_{t,40} \end{pmatrix} + \begin{pmatrix} \varepsilon_{to,y} \\ \varepsilon_{to,c} \\ \varepsilon_{to,i} \\ \varepsilon_{to,w} \\ \varepsilon_{to,l} \\ \varepsilon_{to,p} \\ \varepsilon_{to,r} \\ \varepsilon_{to,gb} \\ \varepsilon_{to,cb} \\ \varepsilon_{to,cs} \end{pmatrix} \quad (1.3.3)$$

The expression in (1.3.3) relates model variables to data, where l and dl are 100 times log and the log difference of the data variables, respectively. Note that dlP_t stands for the log difference of the GDP deflator (i.e P_t represents the GDP price deflator). For the model variables, γ is the common gross quarterly trend growth rate to real GDP, consumption, investment and wages, where $\bar{R}^g_{t,t+40}$ and $\bar{R}^c_{t,t+40}$ are the percentage yields on 40-quarter government and BAA corporate bonds, respectively. Lastly, $C^{bs}_{t,40}$ represents the BAA corporate bond spread in the data.

Dynare offers two ways of specifying measurement errors in estimation. One option is to treat these errors as a special case of structural shocks. The second option is to use Dynare's inbuilt capacity to specify measurement error. The latter approach is adopted because it treats measurement errors as additive serially uncorrelated shocks to the observation equation. The resulting smoothed variables from estimation do not include contributions from measurement errors.

1.3.3 Calibration and Prior Distributions

Due to the constrained solution method offered in Dynare for non-linear models, some parameters cannot be identified, justifying the need for calibration of the following parameters as presented in Table 1.1. All parameters in Table 1.1 were fixed throughout the estimation procedure over the sample period of 1966-2006. However, these parameters change when estimating the model over sub-periods of great inflation and moderation. The two parameters recalibrated for consistency with the sample average in periods of great inflation and moderation are S_{ss} and π_{ss}.

The parameters governing Epstein-Zin preferences are important for a model like the one here, which is intended to price risky assets. This preference parameter differentiates the coefficient of risk aversion from the elasticity of substitution. However, risk aversion in an Epstein-Zin preference is not easily computed. Meanwhile, Rudebusch and Swanson (2012), use the closed-form expression for risk aversion as derived from Swanson (2009). However, the utility function in this chapter differs from the one in Rudebusch and Swanson (2012) – in that it includes external habit formation - and a completely different utility function.

Prior to estimating the model, the parameters (σ_E, σ_C, and σ_H) that this chapter has in common with Rudebusch and Swanson (2012) relating to risk aversion, are fixed to the same value. After estimation, all parameters are fixed to their estimated and calibrated values except the Epstein-Zin preference parameter (σ_E), which is calibrated to the value that gives an average term premium of 100 basis points. This approach is further discussed in subsection 1.4.2.

Table 1.1: Calibrated Parameters and Sources

	Baseline	Description	Source
σ_E	-150	Epstein-Zin parameter	Rudebusch and Swanson (2012)
σ_C	2.00	IES in consumption	Rudebusch and Swanson (2012)
σ_H	3.00	Elasticity of labour	Rudebusch and Swanson (2012)
β	0.995	Subjective Discounting factor	Smets and Wouters (2007)
η_g	0.18	Government spending	Smets and Wouters (2007)
δ	0.025	Capital Depreciation rate	Smets and Wouters (2007)
θ_p	5	Price elasticity of substitution	Christensen and Dib (2008)
θ_w	5	Wage elasticity of substitution	Christensen and Dib (2008)
S_{ss}	1.0064	Steady-state external finance premium	Calculated from prime loan spread
π_{ss}	1.009	Steady-state gross inflation	Sample average
φ	0.9728	Entrepreneur survival rate	Christensen and Dib (2008)

The steady-state external finance premium is set to 1.0064, corresponding to an annual risk spread of 254 basis points, equal to the average spread in the sample between the business prime lending rate and the three-month Treasury bill rate between 1965 and 2006. The choice of the initial values prior to the parameters being estimated follows Smets and Wouters (2007), where the stochastic processes, including the external finance premium shock, net worth and measurement errors, are assumed to follow an inverse-gamma distribution. Furthermore, the persistence in the AR(1) processes follows a beta distribution. However, unlike Smets and Wouters (2007), the mean and degrees of freedom of both the structural parameters and the stochastic processes increased. In addition, no boundary is set for the estimated parameters and the elasticity of the external finance premium is assumed to have a beta distribution similar to Gilchrist et al. (2009)[6] with a mean of 0.10 and a standard deviation of 0.02.

1.4 Results and Discussions

This section presents and discusses the results of the analysis in this chapter. The convergence and identification diagnostics from the estimation procedure, along with other results, are reported in Appendix 1.5.

1.4.1 Implications of Financial Friction and Bond Yield Data in Estimation

The estimation results of the baseline model with and without financial friction using macroeconomic data as in Smets and Wouters (2007) are reported. In addition to using macroeconomic data, the baseline model is estimated using 40-quarter government and BAA corporate bond yield data. In another instance, BAA corporate bond spread is used with 40-quarter BAA corporate bond yield data.

The results of the first analysis, as presented in Table 1.2, represent an investigation into the implications of financial friction and the different yield data in the baseline model. The model 1 estimates, as shown in Table 1.2, correspond to the modified model in which the financial transmission channel is shut down by restricting $\psi_S = 0$, and $S_{ss} = 1$ such that entrepreneurs are able to obtain external financing at the risk-free rate. This implies that the steady-state return on capital is $R^k_{ss} = \frac{1}{\beta}$. In addition, in the absence of financial friction, the shock to the external finance premium is irrelevant. Therefore, the model with no financial friction contains no external finance premium shock. The columns corresponding to models 2-5 contain the estimation of the full modified model with the financial transmission channel but with different observation data.

Model 2 is estimated with only the seven macroeconomic data. Model 3 is estimated using the seven macroeconomic data and 40-quarter government bond yields, and Model 4 uses macroeconomic and 40-quarter BAA corporate yields. The Model 5 estimation uses the seven macroeconomic data, 40-quarter BAA and corporate spread. Overall, the parameters are quite similar across the models. Most of the similarity is associated with rigidity in both price and wage. Consistent with DeGraeve (2008), preference shock is less volatile when the credit condition is incorporated (as in model 2), yet it is more

[6]A higher mean is set for the elasticity of external finance premium. Gilchrist et al. (2009) had a mean prior of 0.07 while that here is 0.10

persistent.

Table 1.2: Estimation Results with and without Financial Friction and Data

Log data density	Model 1 -972.62	Model 2 -982.58	Model 3 -1128.34	Model 4 -1088.78	Model 5 -1128.47	Description
σ_a	0.47	0.45	0.50	0.45	0.50	Productivity shock
σ_b	2.26	1.15	4.26	1.32	2.99	Preference shock
σ_g	2.09	1.12	1.79	2.18	1.73	Government shock
σ_i	6.52	6.68	0.48	3.40	1.94	Investment shock
σ_r	0.27	0.22	0.19	0.23	0.19	Monetary shock
σ_p	0.47	0.49	0.71	0.48	0.76	Price mark-up shock
σ_w	2.90	2.57	2.67	2.31	2.45	Labour supply shock
σ_n	0.46	0.46	0.46	0.46	0.46	Net worth shock
σ_s	n.a.	0.41	0.92	0.39	0.40	External finance shock
ρ_a	0.98	0.97	0.998	0.98	0.996	Productivity persistence
ρ_b	0.27	0.77	0.99	0.43	0.98	Preference persistence
ρ_g	0.98	0.75	0.80	0.99	0.77	Government persistence
ρ_i	0.59	0.66	0.94	0.97	0.97	Investment persistence
ρ_r	0.78	0.16	0.81	0.17	0.79	Monetary persistence
ρ_p	0.23	0.86	0.66	0.83	0.63	Price mark-up persistence
ρ_w	0.998	0.96	0.99	0.98	0.99	Labour supply persistence
ρ_n	0.50	0.50	0.49	0.50	0.50	Net worth persistence
ρ_s	n.a.	0.16	0.51	0.70	0.79	External premium persistence
ρ_{ga}	0.47	0.62	0.91	0.52	0.85	Government & output
ρ_R	0.35	0.73	0.38	0.71	0.38	Rate smoothing
ψ_π	2.00	1.70	1.37	1.47	1.34	P-inflation
ψ_y	0.17	0.003	0.05	0.01	0.05	P-output gap
ψ_{dy}	0.20	0.30	0.29	0.33	0.29	P-output growth
$\bar{\pi}_{ss}$	0.61	0.61	0.61	0.61	0.61	Steady-state inflation
$\bar{\gamma}$	0.46	0.46	0.28	0.50	0.32	Trend growth rate
Ψ	5.65	5.67	3.53	1.84	3.26	Adjustment cost
η_C	0.57	0.39	0.48	0.41	0.49	Consumption habit
ζ_w	0.80	0.92	0.92	0.90	0.91	Wage stickiness
ζ_p	0.74	0.72	0.77	0.71	0.77	Price stickiness
ι_w	0.64	0.69	0.70	0.62	0.76	Wage indexation
ι_p	0.43	0.34	0.51	0.35	0.58	Price indexation
ψ_u	0.64	0.20	0.24	0.19	0.38	Elasticity of utilisation
α	0.28	0.16	0.12	0.12	0.10	Capital share
ψ_S	0.00	0.04	0.097	0.06	0.10	Elasticity of external finance

Model 1- No financial friction with macro data in estimation. Model 2- Full model with macro data in estimation. Model 3- Full model with macro and government yield in estimation. Model 4- Full model with macro and BAA yield in estimation. Model 5- Full model with macro, BAA yield and BAA spread in estimation. P in the description column implies monetary policy response, and estimation is over the period 1966-2006.

Table 1.3: Prior and Posterior Distribution for Model 4 Estimation

		Prior distribution			Posterior distribution			
Description		Distr	Mean	St.Dev	Mode	Mean	5%	95%
Productivity shock	σ_a	$\mathcal{I}$	1.00	10.00	0.45	0.45	0.38	0.52
Preference shock	σ_b	$\mathcal{I}$	1.00	10.00	1.32	1.33	1.01	1.67
Government shock	σ_g	$\mathcal{I}$	1.00	10.00	2.18	1.78	1.23	2.40
Investment shock	σ_i	$\mathcal{I}$	1.00	10.00	3.40	3.97	3.09	5.11
Monetary shock	σ_r	$\mathcal{I}$	1.00	10.00	0.23	0.22	0.18	0.26
Price mark-up shock	σ_p	$\mathcal{I}$	1.00	10.00	0.48	0.59	0.31	0.83
Labour supply shock	σ_w	$\mathcal{I}$	1.00	10.00	2.31	2.36	1.81	2.87
Net worth shock	σ_n	$\mathcal{I}$	1.00	10.00	0.46	0.71	0.24	1.26
External finance shock	σ_s	$\mathcal{I}$	1.00	10.00	0.39	0.40	0.27	0.52
Productivity persistence	ρ_a	$\mathcal{B}$	0.50	0.25	0.98	0.98	0.97	0.996
Preference persistence	ρ_b	$\mathcal{B}$	0.50	0.25	0.44	0.45	0.25	0.67
Government persistence	ρ_g	$\mathcal{B}$	0.50	0.25	0.99	0.94	0.89	0.996
Investment persistence	ρ_i	$\mathcal{B}$	0.50	0.25	0.97	0.95	0.93	0.98
Monetary persistence	ρ_r	$\mathcal{B}$	0.50	0.25	0.17	0.32	0.04	0.83
Price mark-up persistence	ρ_p	$\mathcal{B}$	0.50	0.25	0.83	0.64	0.30	0.97
Labour supply persistence	ρ_w	$\mathcal{B}$	0.50	0.25	0.98	0.98	0.97	0.99
Net worth persistence	ρ_n	$\mathcal{B}$	0.50	0.25	0.50	0.48	0.09	0.86
External premium persistence	ρ_s	$\mathcal{B}$	0.50	0.25	0.70	0.74	0.63	0.85
Government & output	ρ_{ga}	$\mathcal{N}$	0.50	0.25	0.52	0.63	0.22	1.03
Adjustment cost	Ψ	$\mathcal{N}$	4.00	1.50	1.84	2.56	1.56	3.71
Consumption habit	η_C	B	0.70	0.10	0.41	0.45	0.37	0.53
Wage stickiness	ζ_w	$\mathcal{B}$	0.50	0.10	0.90	0.90	0. 86	0.93
Price stickiness	ζ_p	$\mathcal{B}$	0.50	0.10	0.71	0.74	0.67	0.81
Wage indexation	ι_w	$\mathcal{B}$	0.50	0.15	0.62	0.56	0.32	0.80
Price indexation	ι_p	$\mathcal{B}$	0.50	0.15	0.35	0.42	0.21	0.64
Elasticity of utilisation	ψ_u	$\mathcal{B}$	0.50	0.15	0.19	0.17	0.07	0.27
Capital share	α	$\mathcal{N}$	0.30	0.05	0.10	0.12	0.11	0.13
Rate smoothing	ρ_R	$\mathcal{B}$	0.75	0.10	0.71	0.66	0.40	0.82
P-inflation	ψ_π	$\mathcal{N}$	1.50	0.25	1.47	1.40	1.07	1.67
P-output gap	ψ_y	$\mathcal{N}$	0.125	0.05	0.01	0.07	0.001	0.15
P-output growth	ψ_{dy}	$\mathcal{N}$	0.125	0.05	0.33	0.31	0.25	0.37
Trend growth rate	$\bar{\gamma}$	$\mathcal{N}$	0.40	0.10	0.50	0.45	0.38	0.52
Elasticity of external finance	ψ_S	$\mathcal{N}$	0.10	0.02	0.06	0.08	0.05	0.10

This table gives the prior alongside the mode, 5 and 95 percentile of the posterior distribution of estimated parameters using corporate bond yield alongside with the seven macroeconomic data as observables. Note that posterior distribution is obtained using Metropolis-Hastings algorithm using 1,000,000 draws. Also, $\mathcal{I}, \mathcal{B}, \mathcal{N}, \mathcal{G}$, and $\mathcal{U}$ all denotes Inverse-gamma, Beta, Normal, and Gamma.

The estimates in models 1 and 2 are the closest to DeGraeve (2008) because they are both estimated using only macroeconomic data with financial data excluded. The parameters exhibiting the most profound changes are those relating to preference shock and the utility function. Specifically, a lower external habit formation is observed in the model with financial friction. This parameter is known to make consumption processes less persistent. Since there is financial friction in the model, this generates sufficient internal business cycle propagation - the reason for a lower external habit in such a model (DeGraeve, 2008). Similarly, preference shock is less volatile in the presence of financial friction but more persistent. A notable exception is that the inclusion of government yields as observables (see models 3 and 5) distorts the comparability of the result obtained across the board.

The estimates of model 1, particularly the structural parameters, are, for the most part, comparable with Smets and Wouters (2007). However, there are some notable differences; for example, interest-rate smoothing is much more limited in the model, and it is lower than the set prior mean. Also, the estimated response of monetary policy to the output gap is double that in Smets and Wouters (2007). The share of capital in production is also higher. The most profound difference comes from the standard error of the stochastic processes; this is notable as there are differences in the stochastic structure of these models. For example, the model includes labour supply shock and not wage mark-up shock. It also includes preference shock and not the risk premium shock of Smets and Wouters (2007); the risk premium shock is comparable to the external finance premium shock as both create a wedge between the risk-free and loan rates.

There is no doubt that the inclusion of financial variables and data in the estimations burdens the model, causing a deterioration in the marginal likelihood. The difference between the results in columns two and four of Table 1.2 is due to the inclusion of BAA corporate yield bond data in estimating the baseline model. The marginal likelihood when only macroeconomic data (i.e. models 1-2) are included has the range $(-983, -973)$, which changes to $(-1129, -1089)$ when bond yields are included as observables(models 3-5). However, when the observables are the BAA corporate bond yields rather than the government bond yields, the estimation result performs better with a marginal likelihood difference of 39.56.

The estimation using only macroeconomic data can be used to establish that the estimates with BAA corporate yield are not far-fetched. However, there is a significant difference between these estimations in respect of the parameters associated with the external finance premium and investment. Furthermore, using bond yield information leads to a significant decrease in the standard error of investment shock coupled with a marginal decrease in the external finance premium shock. Broadly speaking, the standard deviation of investment shock is significantly reduced when bond yield data are included as observables (models 3-5). However, the persistence of this shock increases, ranging from $0.94 - 0.97$ compared to the range of $0.59 - 0.66$ obtained when only macroeconomic data were used in the estimation. Price indexation and rigidity in the presence of BAA yield are reduced relative to the non-financial friction model. Similarly, wage indexation reduces marginally, but wage stickiness is higher. In the various models, estimates of the elasticity of external premium are significantly non-zero, higher than the 5% calibration in Bernanke et al. (1999a) and the estimate of Christensen and Dib (2008), taking on the highest 10% value when estimating with government bond yield; this is the same as in DeGraeve (2008).

The DSGE model that incorporates credit market imperfection seems to generate more internal persistence, especially when the model is estimated using bond yield data because this requires somewhat less volatile exogenous processes than model 1 (the model without financial friction). The external finance premium shock generates a wedge between the risk-free rate and the loan rate. This is tantamount to the risk premium shock in Smets and Wouters (2007). The standard deviation of the shock is lowest in Model 4, which incorporates corporate bond yield information, has. However, it is mostly persistent in this instance. The share of capital in production α is lower than the estimates in Smets and Wouters (2007) with financial friction models; this is particularly the case when the model estimation incorporates bond yield data.

Overall, the majority of the parameters are fairly stable when estimating with different bond data types and in models with or without financial friction. However, the estimation when the only financial variable is the corporate bond yield performs better than that with the government bond yield taking into account the estimated parameters and model's fit of the observables, in particular, the fit of corporate bond spread when the government or corporate bond yields are used in the estimation.

A comparative analysis of the performance of government and corporate yield in estimating the baseline model is depicted in Figure 1.4. This shows corporate bond spread both in the data and model using either government or corporate bond yields as observables. Similarly, Figure 1.5 shows the bond yields (government and corporate) in the data and the model.

An interesting feature of the work in this chapter concerns the estimated corporate bond spread in Figure 1.4. First, government yield as an observable (model 3, top-right panel) is less informative for corporate bond spread. While the spread increases during recessions (as in data- blue line), this is contradicted by the estimated spread using the government bond yield. However, estimates with the corporate bond yield capture the increase in the spread observed during recessions. In respect of model 5, where corporate bond yield and corporate bond spread are the observables, the result indicates overfitting (lower-right panel). Figure 1.5 confirms that, as observables, corporate bond yields are preferable to government bond yields. The top-left panel shows government and corporate bond yields in the data; the top-right panel shows the same yield for model 3 (estimates with government yield); the lower-left panel for model 4 (estimates with corporate yield); and the lower-right panel for model 5.

The overfitting of model 5 is apparent as the estimated bond yield is almost indistinguishable from the data. While model 3 only uses the government yield, the fit of simulated corporate yield is not appropriately captured. Meanwhile, for model 4, the simulated government yield closely aligns with data even though the model is only estimated using corporate bond spread.

Figure 1.4: 40-Quarter Corporate Bond Spreads in Data and Model

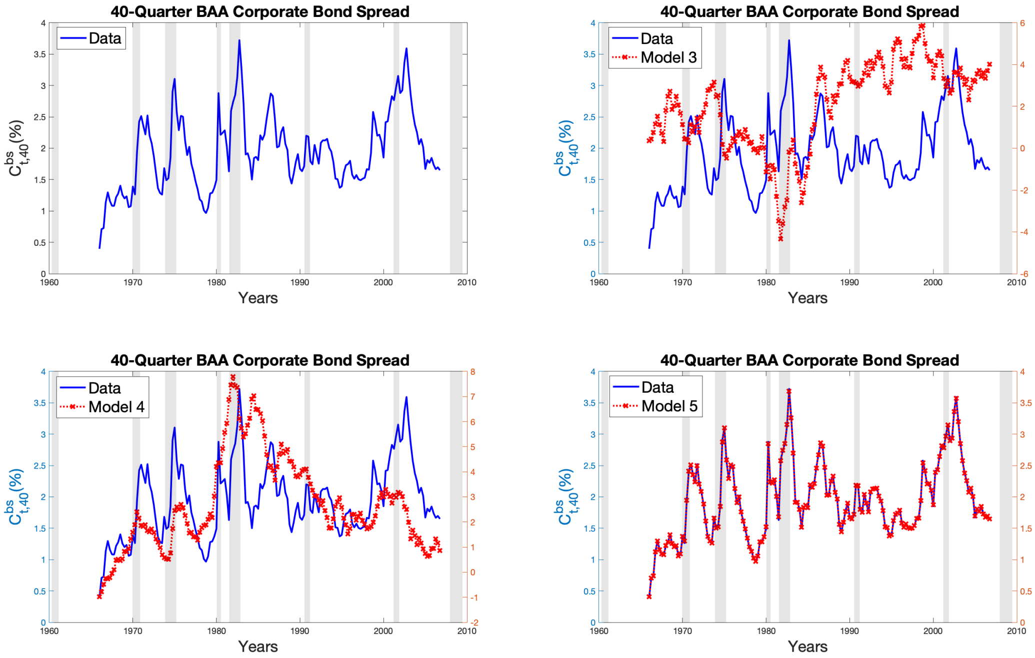
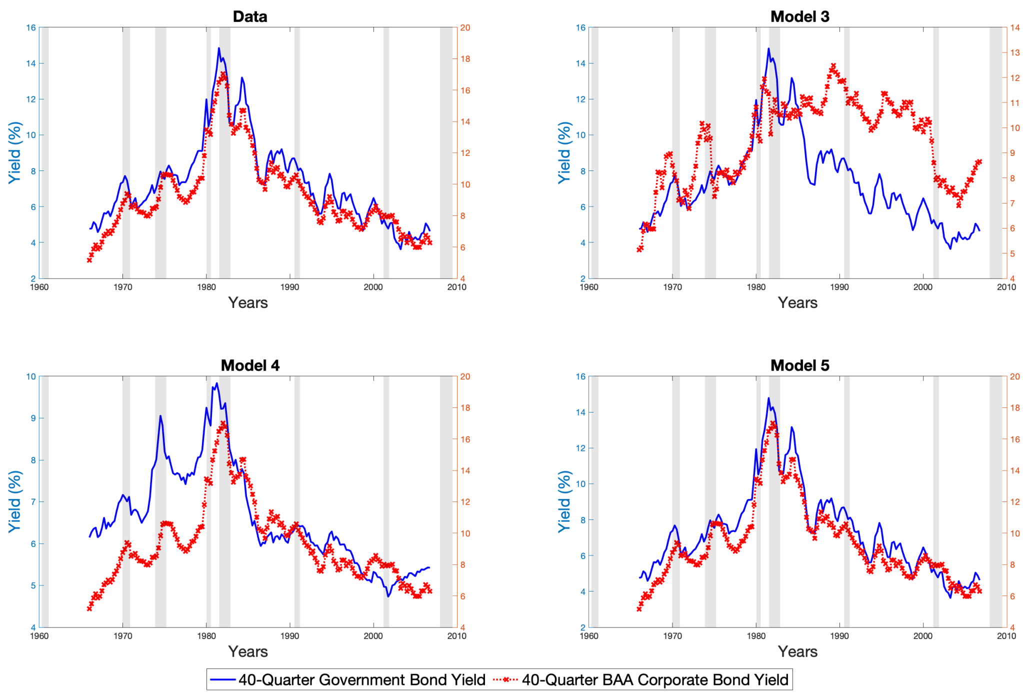


Figure 1.5: Government and Corporate Bond Yields in Data and Model



There are two distinctive regimes in the estimated corporate bond spreads for models 3 and 4 (Figures 1.4 and 1.5). First, the gap between the estimated corporate bond spread and data widens over the period 1966-1984 (which is within the Great Inflation era). Second, the gap between model estimates and data is close during the period 1984-2006 (which falls in the period of the Great Moderation). The only exception is the corporate bond spread in the aftermath of the dot-com bubble burst and the 9/11 attacks.

The observed gap in levels in the first regime is possibly linked to failed monetary policy and credit market conditions at the time. Policies in operation in this era include the wage and price control (1971-1974) introduced by Nixon's administration and the Whip Inflation Now (WIN) programme. In the second era, some level of sustained moderation was experienced. This suggests that the observed structural break in the corporate bond spread might be linked to different monetary policy regimes. However, economists have not yet reached a consensus on the exact period of transition between the eras of Great Inflation and the Great Moderation.

The perceived structural change in the corporate bond spread described in this chapter is partly consistent with the result in Chun et al. (2014), an empirical study on detecting regime shifts in credit spread over the 1987-2009 period. Chun et al. (2014) use the same methodology in identifying regime shift from the credit spread for federal funds rate and index of tightening loan standards. The study overlaps observed monetary policy and credit supply regimes on credit spread. Their result shows the influence of monetary policy and credit supply on the dynamics of credit spreads.

Unlike Chun et al. (2014), the estimation in this chapter begins in 1966 and covers the eras of Great Inflation and Great Moderation. Powell (2019) presents a detailed discourse on the two distinctive regimes observed in corporate yield spread (as shown in Figure 1.4) and the defining monetary policies. During the Great Inflation, interest rates kept rising and even spiked sharply towards the end of the 1970s. At this point, investment slowed, productivity was hindered, and the balance sheet worsened. The rising inflation is thought of as the factor influencing the economic uneasiness (History, Accessed in July 2020a).

Powell (2019) also confirms that these two distinctive periods in respect of the corporate bond spread fall into two different monetary-policy eras. The policy era relevant to the estimated corporate bond spread in Figures 1.4 and 1.5 is described as one in which "policy breeds macroeconomic instability and great inflation". Specifically, the period (1966-1982) in which the estimated corporate bond spread fell was known for "stop and go" policy that could not suppress the rising inflation pressure. This resulted in inflation and inflation expectation rising through four expansions before Paul Volcker took over the administration of the Federal Reserve. Policies in the Great Inflation era made allowance for excessive growth in the supply of money (History, Accessed in July 2020a).

The second era covered here (i.e., 1983-2006) is known as the Great Moderation. This era, which was successful in achieving price stability and expansions, was more stable than the first. Upon the appointment of Paul Volcker in August of 1979, the administration took firm control of reserve and money growth, which gradually slowed the increasing inflation. In addition to the tight reserve management, credit control measures were introduced in early 1980 alongside the Monetary Control Act. The tightening credit conditions led to a spike in interest rates, causing a drop in lending activities and an increase in unemployment. However, with continued efforts, the administration's commitment yielded positive outcomes, and they were able to bring a definitive stop to the rising inflation of the early 1980s (Powell, 2019). The economy during this era recorded reduced unemployment and the longest period of sustained growth and stability since World War II (History, Accessed in July 2020b). Aligning the estimated corporate spread with these narratives suggests that corporate bond spread responds to, and is structurally affected by, different monetary policy regimes.

The model is estimated and compared over two sub-samples similar to Smets and Wouters (2007) to further explore the structural change seen in Figure 1.4. The mode of the posterior distribution of the model parameters for the sub-sample estimation is reported in Table 1.4. One issue that must be considered is whether the estimated model parameters behave in a way that supports the classification by Powell (2019). For example, the reason volatility of inflation and output rose in the first era and fell in the second era must be explored.

Before conducting the sub-sample estimation, the steady-state of the external finance premium and inflation (S_{ss} and π_{ss}) is recalibrated to the sample average of each sub-period. During the Great Inflation, the gross loan prime spread (equivalent to the steady-state of the external finance premium) is $S_{ss} = 1.0021$ (0.21%) and the corresponding gross inflation is $\pi_{ss} = 1.014$ (1.04%). During the Great Moderation, the prime rate is higher, $S_{ss} = 1.0074$ (0.74%), while inflation notably reduces $\pi_{ss} = 1.0062$ (0.62%). Calibrating these parameters to their sample average is instrumental to being able to successfully estimate the model[7]

The sub-sample estimation shows that there are significant changes to the parameters governing the exogenous processes. In particular, the standard errors of the shocks, excluding investment and labour supply, fall in the second era (Great Moderation). While it appears the standard error for the majority of the shocks is higher in the first era, the variability of some of these shocks is higher when compared to the second sub-sample[8]. Meanwhile, the estimated standard deviation of all parameters relating to exogenous process is higher during the Great Inflation (excluding investment see column SD in Table 1.4). Similar to Smets and Wouters (2007), the persistence of external finance premium shock has fallen in the second sub-sample [9]. Some economists have related the cause of the Great Moderation to three things, namely: structural change, good economy, and good luck (History, Accessed in July 2020b).

[7]If these parameters are set otherwise, the sub-sample estimation process fails without finding a model.

[8]Variance (i.e. $\frac{\sigma^2}{1-\rho^2}$) of the AR(1) process for preference, monetary policy, price mark-up, investment and wage supply shocks are somewhat higher during the Great Moderation

[9]External finance shock in this model is similar to the risk premium shock in Smets and Wouters (2007).

Table 1.4: Great Inflation and Moderation Estimates

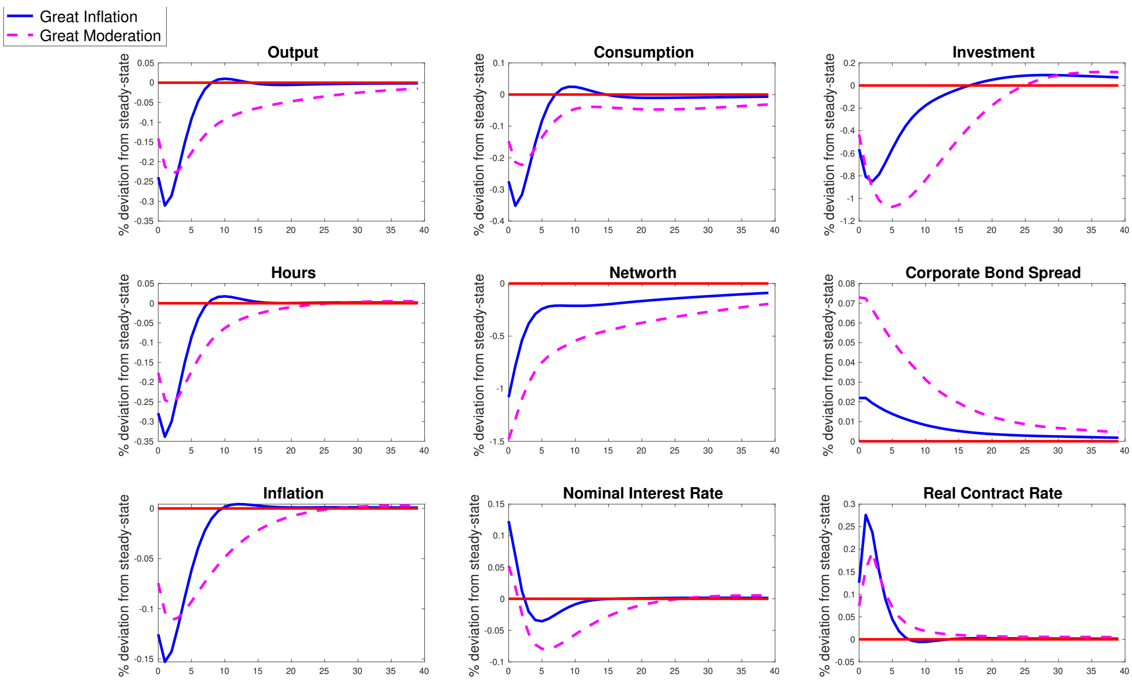
	Structural Parameters					Shock Processes			
	1966:1-1979:2		1984:1-2006:4			1966:1-1979:2		1984:1-2006:4	
	Mode	SD	Mode	SD		Mode	SD	Mode	SD
Ψ	2.47	0.80	6.33	1.23	σ_a	0.56	0.07	0.39	0.04
η_C	0.47	0.05	0.54	0.06	σ_b	1.64	0.26	1.37	0.23
ζ_w	0.86	0.03	0.85	0.02	σ_g	2.35	0.37	0.69	0.24
ζ_p	0.58	0.06	0.85	0.02	σ_i	2.44	0.42	6.19	1.03
ι_w	0.67	0.14	0.53	0.17	σ_r	0.24	0.03	0.15	0.02
ι_p	0.34	0.15	0.43	0.12	σ_p	0.53	0.17	0.38	0.11
ψ_u	0.55	0.15	0.20	0.07	σ_w	2.01	0.38	2.43	0.38
α	0.08	0.01	0.13	0.01	σ_n	0.46	0.19	0.46	0.19
ρ_R	0.77	0.05	0.46	0.07	σ_s	0.39	0.09	0.39	0.07
ψ_π	1.57	0.16	2.13	0.22	ρ_a	0.98	0.01	0.98	0.01
ψ_y	0.12	0.05	0.13	0.05	ρ_b	0.18	0.14	0.81	0.06
$\psi_{\mathrm{d}y}$	0.25	0.05	0.03	0.01	ρ_g	0.75	0.09	0.59	0.26
$\bar{\gamma}$	0.52	0.07	0.47	0.05	ρ_i	0.92	0.05	0.79	0.05
ψ_s	0.09	0.02	0.10	0.02	ρ_r	0.24	0.05	0.86	0.06
					ρ_p	0.54	0.26	0.84	0.04
					ρ_w	0.96	0.01	0.98	0.01
					ρ_n	0.50	0.50	0.51	0.50
					ρ_s	0.80	0.07	0.69	0.06
					ρ_{ga}	0.51	0.24	0.66	0.24
						Measurement Error			
					$\sigma_{o,y}$	0.49	0.07	0.29	0.04
					$\sigma_{o,c}$	0.05	0.02	0.37	0.04
					$\sigma_{o,i}$	0.05	0.02	0.05	0.02
					$\sigma_{o,l}$	0.05	0.02	0.05	0.02
					$\sigma_{o,r}$	0.04	0.01	0.03	0.01
					$\sigma_{o,cb}$	0.04	0.01	0.04	0.01
					$\sigma_{o,w}$	0.29	0.04	0.64	0.05
					$\sigma_{o,\pi}$	0.20	0.04	0.10	0.01

The calibrated steady-state for the external finance premium 1966:1-1979:2 is $1.0021(0.21\%)$, while for the second sub-period it is $1.0074(0.74\%)$. Also, the steady-state gross inflation is $1.014(1.04\%)$ for the first sub-period and $1.0062(0.62\%)$ for the second sub-period.

The shift from manufacturing to services during the Great Moderation (i.e., 1984-2006) is one of the things thought to have reduced the volatility of economic activities. Also, policies that influenced the deregulation in many industries were believed to increase economic flexibility giving an enabling environment for the economy to adjust more smoothly to shocks, alongside the openness for international trade constituted the "good economy" reason for reduced economic volatilities. The most controversial of all is the possibility that there was good luck at the time. This idea is wrapped in the notion that shocks hitting the US economy during the Great Moderation were smaller compared to the large adverse shocks experienced during the Great Inflation. The chairman of the Bank of England pointed out, however, that "shocks are not measured directly, only their consequences" (Bean, 25 August 2009). Meanwhile, there were many "large shocks" that hit the economy during the Great Moderation, such as the Latin American debt crisis of the 1980s, the failure of Continental Illinois Bank in 1984, the stock market crash of 1987, the Asian financial crisis in 1997, the dot-com crash in 2000. Evidence of this can be seen in the increase in the estimated standard errors of investment and labour supply shock during the Great Moderation (i.e. Table 1.4). These shocks are not exactly "small shocks". Therefore, the reduced volatility during the Great Moderation could be investigated as a time of structural change and a good economy (good policy).

The central bank's response to the economy's output gap marginally increases in the second sub-sample, whereas there is a significant decline in their response to the growth in output. The central bank has become flexible in the second sub-sample with a decrease in interest-rate smoothing and a strong response to inflation. This era captures the period during which Paul Volcker's administration extended a definite measure to halt the rising inflation.

Figure 1.6: Response to Monetary Shock for Great Inflation and Moderation Era

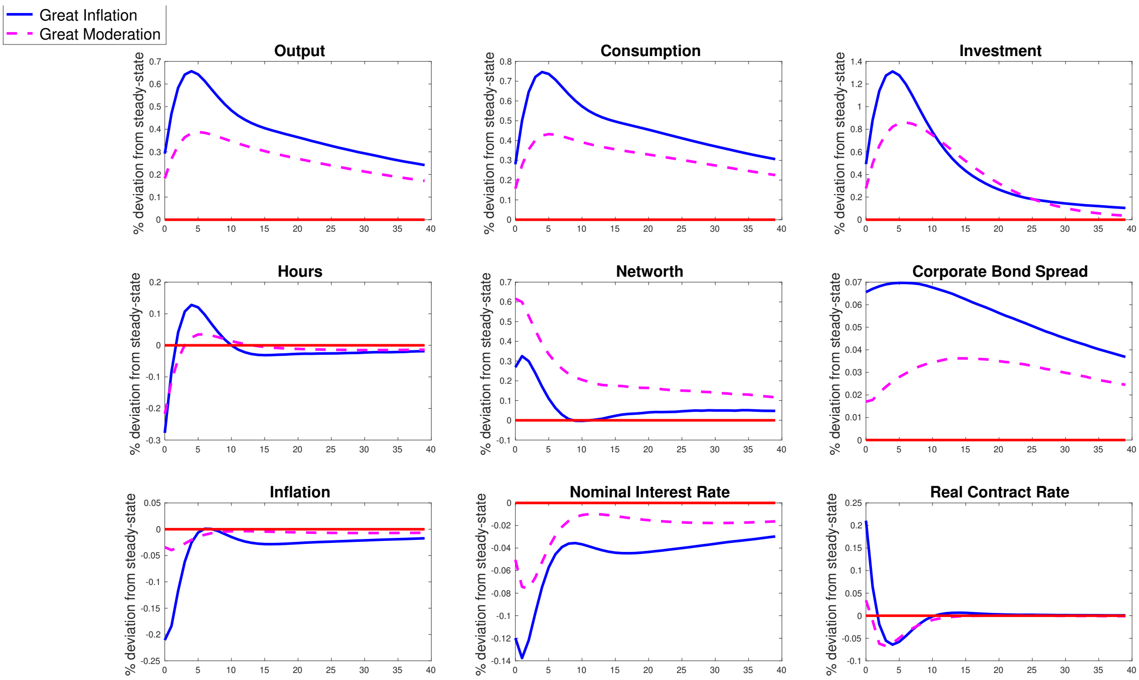


Note: The impulse response represents percentage deviations of variables from their steady-state and it is in response to a one standard deviation shock. It shows response to one standard deviation contractionary monetary policy shock as estimated for the specific era.

Also worth noting is the increase in the estimated degree of price stickiness and price indexation during the second era. While wage stickiness remains relatively stable, wage indexation falls. The latter is consistent with the notion that low and stable inflation may reduce the cost of not adjusting prices. This lengthens the average price duration leading to a flatter Phillips curve (Smets and Wouters, 2007). Evidence also indicates an increase in real rigidities in the second era due to the increase in investment adjustment costs. Noticeable also is the contribution of measurement errors in the two eras. The most profound contributors in the first era are the measurement error in respect of output growth and inflation. This implies that, as compared to the case of the Great Moderation, the model could not sufficiently account for changes in output and inflation during the Great Inflation.

Due to the limitations of the analysis and identification of structural breaks in non-linear DSGE models, the impulse response function shown uses the estimates from the two sub-periods mentioned above. Figure 1.6 shows the response of some macroeconomic and financial variables to a one standard deviation contradictory shock. Similarly, Figure 1.7 shows the response to a positive productive shock. In both cases, the response of the corporate bond spread in the two sub-periods (Great Inflation and Great Moderation) is mostly distinguishable from other responses.

Figure 1.7: Response to Productivity Shock for Great Inflation and Moderation Era



Note: The impulse response represents percentage deviations of variables from their steady-state and it is in response to one standard deviation shock. It shows response to one standard deviation positive productivity shock as estimated for the specific era.

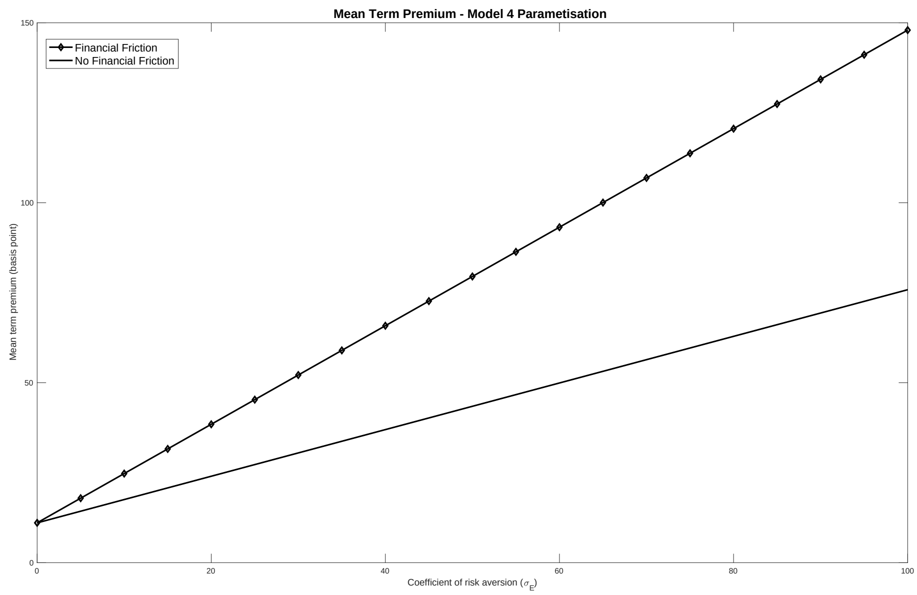
1.4.2 Coefficient of Relative Risk Aversion and Model Moment

This subsection presents the effects of increasing the coefficient of risk aversion and the model implied moment.

1.4.2.1 Coefficient of Relative Risk Aversion

As in Rudebusch and Swanson (2012), Figure 1.8 illustrates how the mean of the term premium changes with a varying risk aversion coefficient in models with and without financial friction. All parameters excluding the coefficient of risk aversion (σ_E) are fixed either to their estimated (using model 4 in Table 1.3) or calibrated values before calculating the corresponding term premium for varying σ_E. Term premium is defined as the difference between the yield to maturity on a 40-quarter bond (discounted using SDF) and the risk-neutral yield to maturity.[10] It increases as the risk aversion coefficient increases. This implies that any specified level of mean term premium can be achieved solely by making the household more risk averse.

Figure 1.8: Mean Term Premium with Changing Risk Aversion Coefficient σ_E



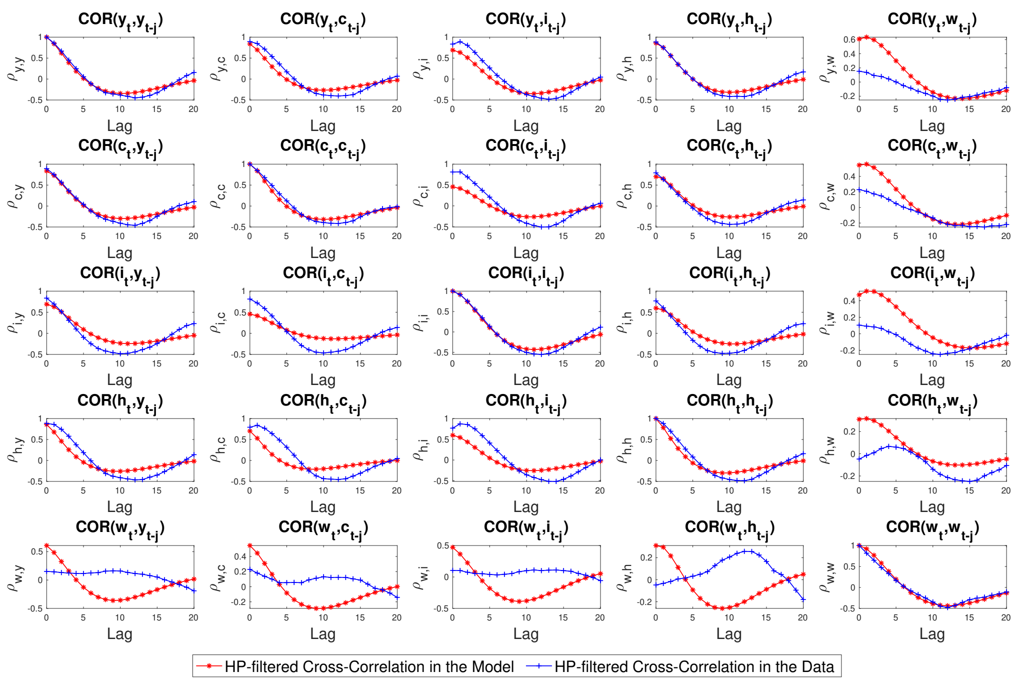
Interestingly, the model in this chapter requires a lesser risk aversion coefficient $\sigma_E = 65$ that equivalently gives a mean term premium of 100 basis point compared to $\sigma_E = 150$ in Rudebusch and Swanson (2012). The level of risk aversion needed for the same mean term premium in a model without financial friction is higher; hence, having financial friction is plausible because there are internal processes to indicate households are risk averse.

[10]See equations 30-33 in Rudebusch and Swanson (2012) for the expression regarding the term premium.

1.4.2.2 Implied Model Cross-Correlation

The implied model moments are calculated using the estimates of model 4 (full model with 40-quarter BAA used in estimation) and are compared with the empirical moments (detrended using a Hodrick-Prescott filter.[11]) as in Smets and Wouters (2007) In Figures1.9 to 1.12, the blue line represents the empirical cross-correlation and the red line is the model-simulated cross-correlation, where output is y_t, consumption c_t, investment i_t, hours h_t, wage w_t, inflation π_t, interest rate r_t, government bond yield r_t^{gb}, corporate yield r_t^{cb}, corporate bond spread c_t^{bs}. The Figures show the ability of the presented DSGE model in replicating both positive and negative cross-correlation of the macroeconomic and financial data. Model simulated cross-correlation is generated by running a stochastic simulation (with the HP filter option) after the model parameters are set to the estimated values in Table 1.3 alongside risk aversion coefficient set to $\sigma_E = 65$footnote The value resulting in a term premium of 100 basis point as earlier noted.

Figure 1.9: Cross- and Auto-correlation of Output, Consumption, Investment, Hours, and Wages



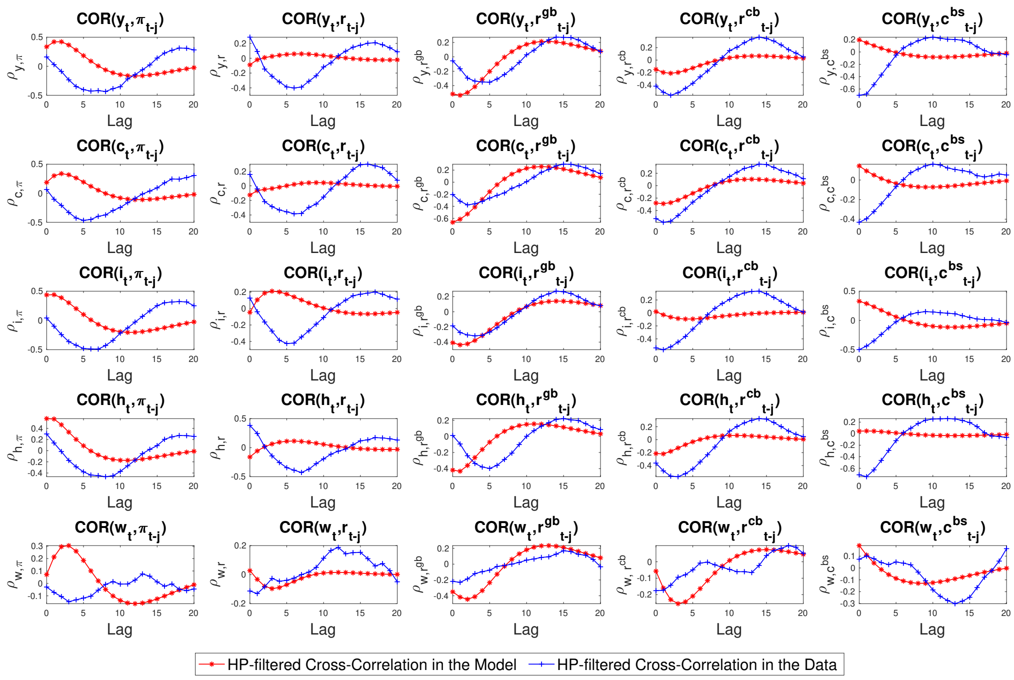
The horizontal axis represents the time lag, and the vertical axis the corresponding correlation. For example, Figure 1.9 shows the cross- and auto-correlations of output (y_t), consumption (c_t), investment (i_t), hours (h_t), and wage (w_t). More precisely, in the first row COR(y_t, y_{t-1}) is the autocorrelation of output at different lags, while COR(y_t, c_{t-1}) is the cross-correlation of output with consumption where

[11]This is because first difference magnifies the noise of the cross-correlation and distorts model-data comparison (see Figures1.25 to 1.28 in Appendix 1.5)

output is leading and consumption is lagging by $t-1$. Note that the contemporaneous correlation is when the horizontal axis is zero (i.e., lag $=0$). The cross-correlation when output (row 1), consumption (row 2), investment (row 3) and hours (row 4) are leading (in Figure 1.9) replicates the empirical moment to the substantial level. However, when wage is leading (row 5), the fit of the model and empirical cross-correlation is reduced. Meanwhile, the model is able to replicate the autocorrelation of wage even though it is unable to fully replicate its cross-correlation with other macroeconomic variables.

In Figures 1.9 and 1.10, macroeconomic variables are leading. The leading variables are: row 1- output; row 2- consumption; row 3- investment; row 4- hours, and row 5- wage. Meanwhile in Figures 1.11 and 1.12, financial variables are leading and the variables are: row 1-inflation; row 2- interest rate; row 3- government yield; row 4- corporate yield, and row 5- corporate spread.

Figure 1.10: Cross-correlation of Output, Consumption, Investment, Hours, and Wages



Considering that the model is only estimated using the growth of output, consumption, investment, and wage, the simulated cross-correlation (in levels) of these macroeconomic variables significantly replicates the observed cross-correlation in data. Also, government yield and corporate bond spread were not used in estimating the model. Yet, the simulated cross-correlation using HP-filter still corresponds to data to a reasonable extent.

Figure 1.11: Cross-correlation of Inflation, Interest Rate, Treasury Yield, Corporate Yield and Spread

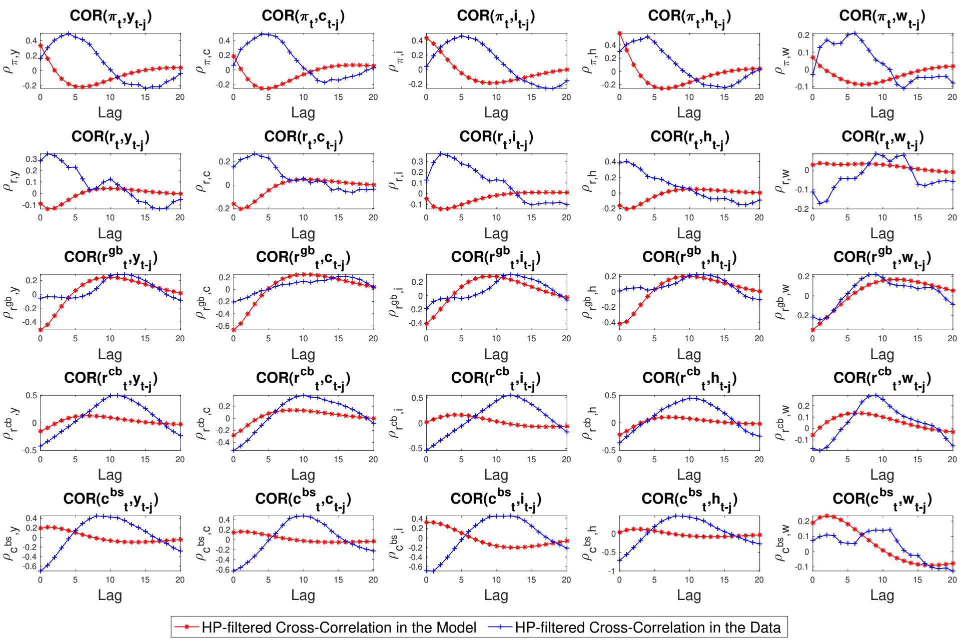


Figure 1.12: Cross- and Auto-correlation of Inflation, Interest Rate, Treasury and Corporate Yield, Spread

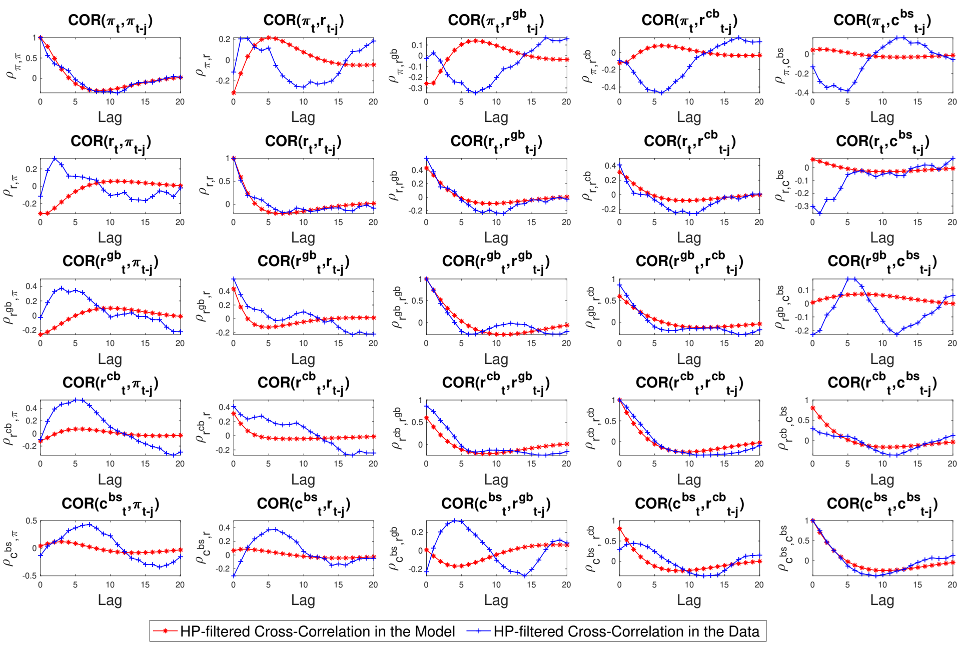


Table 1.5: First and Second Moment in Data and Model

	std($\mathrm{d}y_t$)	std($\mathrm{d}c_t$)	std($\mathrm{d}i_t$)	std($\mathrm{d}w_t$)	std($C^b_{t,n}$)	std(π_t)	std(r_t)	std($R^c_{t,n}$)	std($R^g_{t,n}$)
A. Empirical Standard Deviation									
Data	0.85	0.69	2.03	0.60	0.63	0.59	0.82	2.62	2.45
B. Theoretical Model Standard Deviation									
Model 3	0.71	0.62	3.26	0.24	1.38	1.05	1.33	3.86	2.91
Model 4	0.70	0.73	2.43	0.25	1.18	0.53	0.68	1.43	1.59
Model 5	0.81	1.14	4.43	0.37	2.17	2.49	3.44	4.29	5.86
C. Model Estimated Standard Deviation									
Model 3	0.65	0.55	2.02	0.20	1.81	0.57	0.81	1.81	2.46
Model 4	0.63	0.57	2.02	0.20	1.91	0.56	0.81	2.62	1.20
Model 5	0.66	0.49	1.85	0.21	0.63	0.57	0.81	2.62	2.45

Table 1.5 presents the empirical and model-simulated standard deviation of some model variables. Panel A reports the empirical standard deviation in the data for the 1960-2006 period, while Panel B shows the model simulated moments after running a stochastic simulation when parameters are fixed to their estimated or calibrated values. The reported moment in Panel C corresponds to the calculated standard deviation of the smoothed variables (i.e., the time series for the 1960-2006 period) produced by the estimated model. The difference between the data moment (Panel A) and the model estimated moment (Panel C) is captured by the measurement errors on the observed variables.

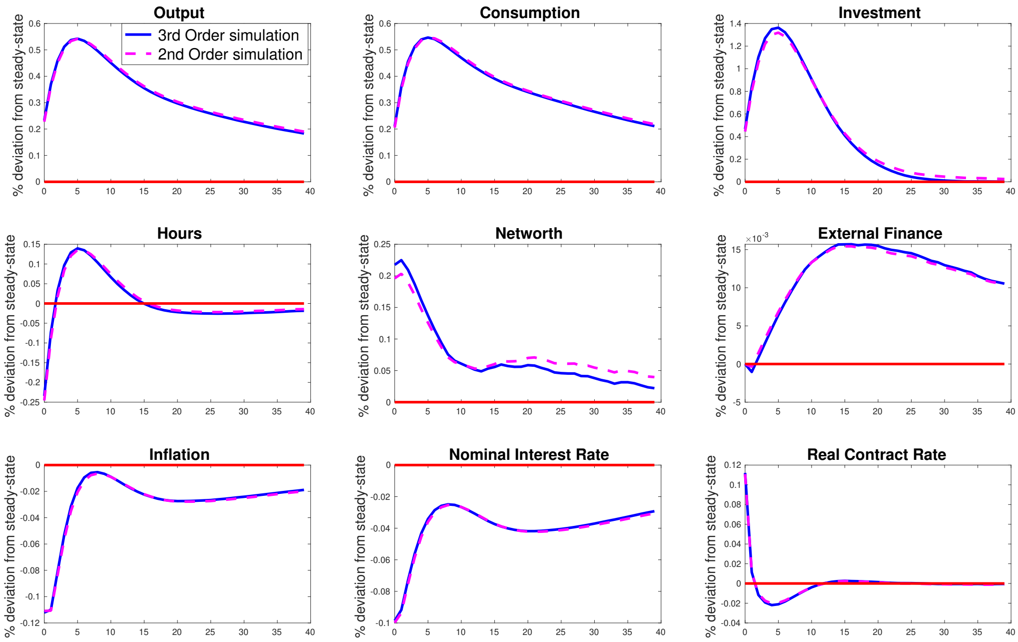
On average, the model simulated moment is comparable with the data, with Model 4 mostly preferred. Although the model fit to change in wage is less substantial and lower than it is in the data, if consideration is given to the Kimball aggregator, as in Smets and Wouters (2007), it will be possible to generate a time-varying wedge in wage. However, the limitations of solving a non-linear model with time-varying wage restricted the choice of goods aggregator in this study.

1.4.3 Impulse Responses

There is a need to study the transmission of shocks in the model, which was achieved by plotting the impulse responses of key macroeconomic variables to three structural shocks, namely: (i) productivity; (ii) monetary policy; and (iii) financial (external finance premium) shocks.

Model parameters are fixed to their calibrated or estimated values (i.e. model 4 estimates in Table 1.3) to illustrate these dynamics. Second and third-order model simulated impulse responses are presented for comparison.

Figure 1.13: Response to Positive Productivity Shock

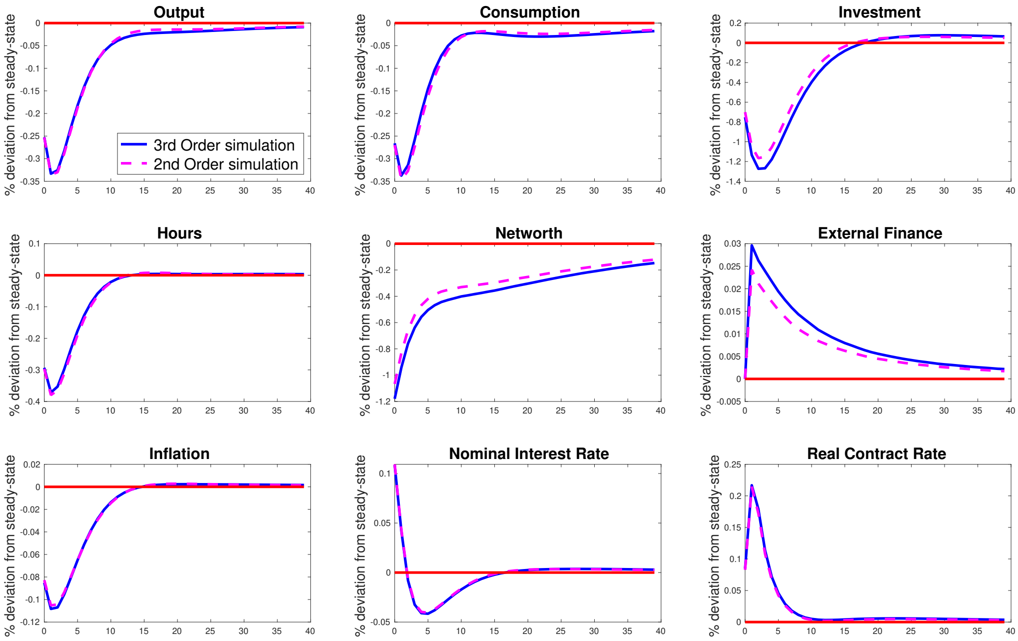


Note: The impulse response represents percentage deviations of variables from their steady-state and it is in response to one standard deviation shock.

In Figures 1.13 to 1.15, the real contract rate is the deflated contractual nominal loan rate at which entrepreneurs obtain loans. On impact, a positive productivity shock expands aggregate demand, and, in the same way, output and net worth increase. Furthermore, hours significantly drop on impact to about 25% but increase around the second quarter after impact. Improved productivity potentially boosts investor confidence, improving the risk-on sentiment, which then increases the value and quantity of investment. The cost of obtaining external funds is, however, mildly affected. The decline in inflation increases the real cost of repaying existing debt. Although the second and third-order responses are indistinguishable for all the macroeconomic variables shown in Figure 1.13; this changes in response to net worth, with the magnitude of impact differing for both (second and third-order) responses in the short and long run.

A contractionary monetary policy shock, as in Figure 1.14, leads to an instantaneous increase in the nominal interest rate. This immediate impact causes output, investment and inflation to fall. Due to a low return on capital with a high real interest rate to be paid on existing debt, net worth falls significantly by about 120 basis points, making external finance more expensive. In other words, the increased external finance premium is an effect of entrepreneurs' depreciated balance sheet (due to increased leverage). The increased cost of external finance and decreased return on capital, in turn, discourage entrepreneurs, thereby decreasing the demand for capital (explaining the decline in investment). With the contractionary monetary policy shock, the difference between the second and third-order responses is apparent in net worth, the external finance premium, and, to a lesser extent, investment.

Figure 1.14: Response to Tightening Monetary Policy Shock

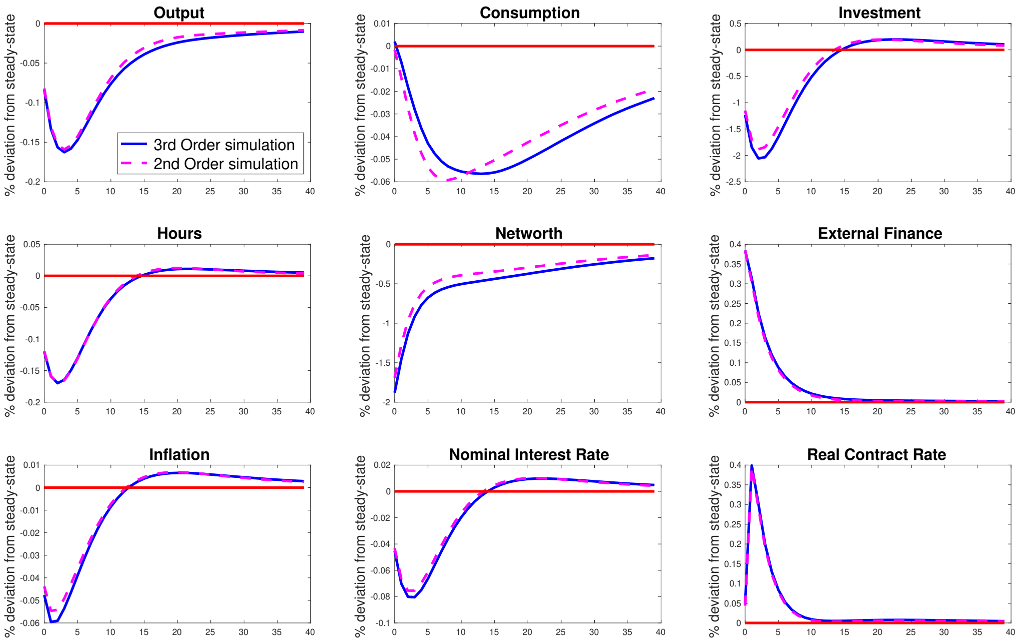


Note: The impulse response represents percentage deviations of variables from their steady-state and it is in response to one standard deviation shock.

The resulting response of a one standard deviation adverse financial shock (external finance premium shock) as in Figure 1.15 is an important mechanism to note, especially in assessing the importance of financial friction in the model. First, on impact, the shock causes an increase in external finance premium to about 40 basis points making external finance costly. This unanticipated adverse financial shock tightened financial conditions, leading to a slowdown in economic activities. Output declines by about 10 basis points, and throughout the 40 quarters, output remains below its initial pre-shock level. This effect is, however, mild in respect of consumption. However, investment is significantly affected with a decline of about 150 basis points on impact. The economic downturn is magnified partly by the substantial decline of about 200 basis points in net worth.

All these adverse effects and disinflation culminated in the easing of monetary policy, which results in a decline in the nominal interest rate. Notably, in this instance, the third-order response is reflected in the consumption, output (about $10 - 30$ quarters after impact), and investment ($1 - 15$ quarters after impact). Inflation and the nominal interest rate are also mildly affected two quarters after impact. Again, the second and third-order simulated responses of net worth are distinguishable.

Figure 1.15: Response to Adverse Financial Shock



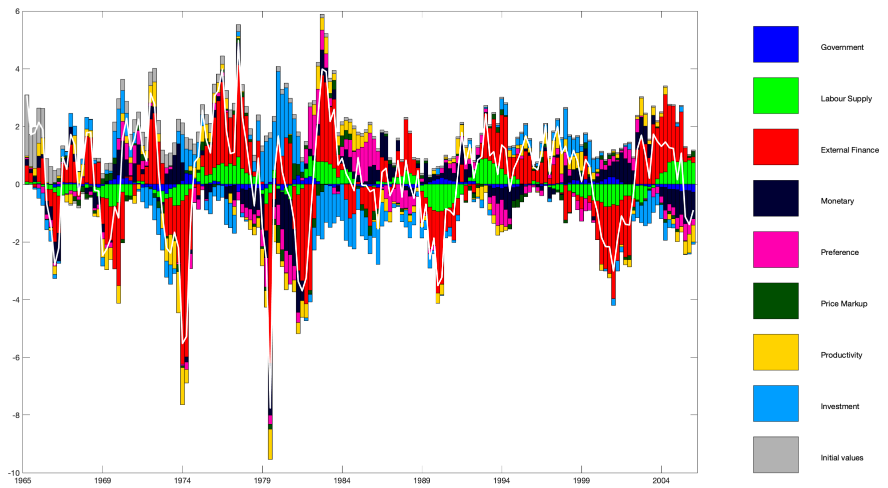
Note: The impulse response represents percentage deviations of variables from their steady-state and it is in response to one standard deviation shock.

1.4.4 Historical Shock Decomposition

The historical shock decomposition implied by the model is reported to assess how the shock emanating from the financial market affects the economy as compared to other shocks. Figures 1.16 to 1.20 portrays the historical shock decomposition of changes in investment, inflation, nominal interest rate, output and consumption, expressed as a deviation from their steady state alongside estimated model shock contributions.

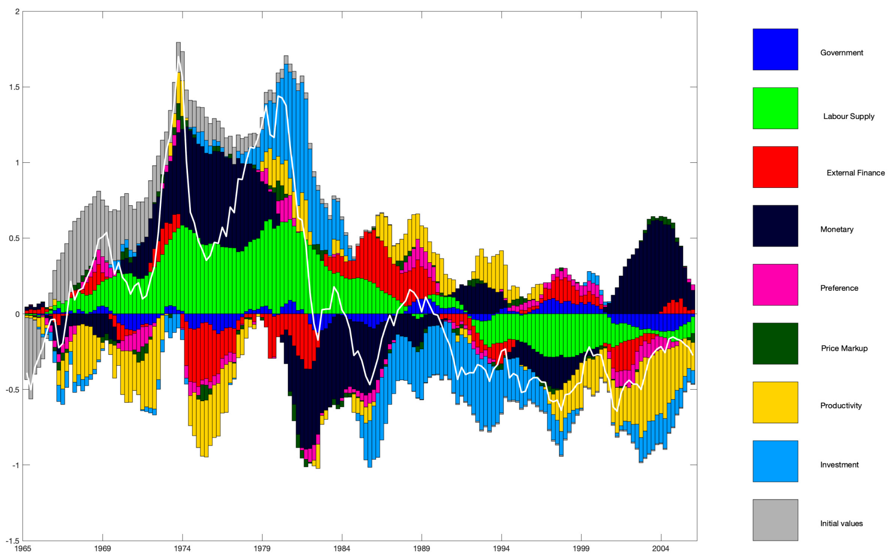
The financial shock substantially accounts for the historical dynamics of changes in investment, particularly the declines during recession (see Figure 1.16). In addition, financial shock accounts for the dynamics of inflation, nominal interest rates, and output to a reasonable extent. The role of monetary policy, financial conditions, and unemployment (denoted here by the labour supply shock) are, according to the decomposition, consistent with historical narratives. For example, the rise in inflation in Figure 1.17 from the mid-1960s could not be controlled by the "stop and go" monetary policy in place at the time; rather, inflation kept rising. The estimated contribution of monetary policy shock supports the rise in inflation until Paul Volcker's administration put in place a definitive policy to counter the trend in the early 1980s. Paul Volcker's impressive monetary policy regime led to a decline in inflation. Labour supply, investment, external finance, and productivity shocks are all instrumental to the inflation dynamics observed.

Figure 1.16: Historical Shock Decomposition of Changes in Investment



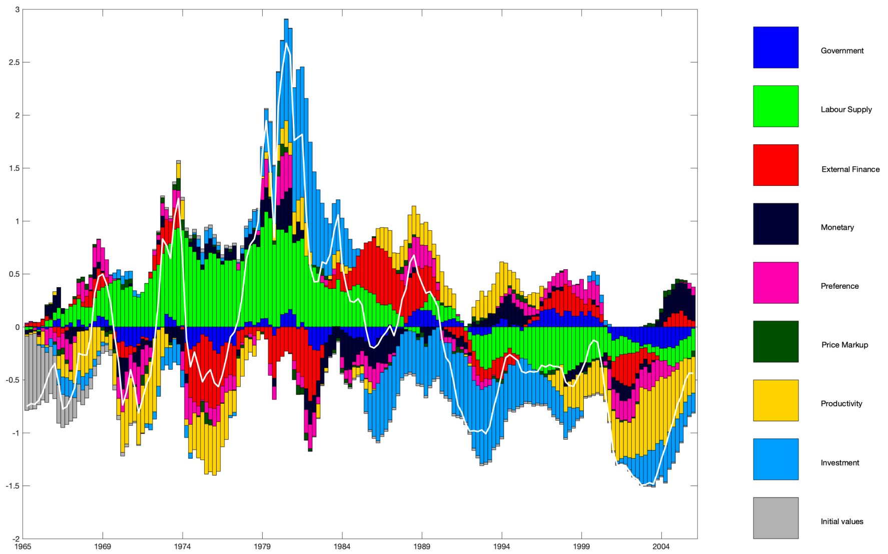
The white line plot represents the actual variable as a percentage point deviation from the steady state. And the coloured bars indicate each shocks contribution.

Figure 1.17: Historical Shock Decomposition of Inflation



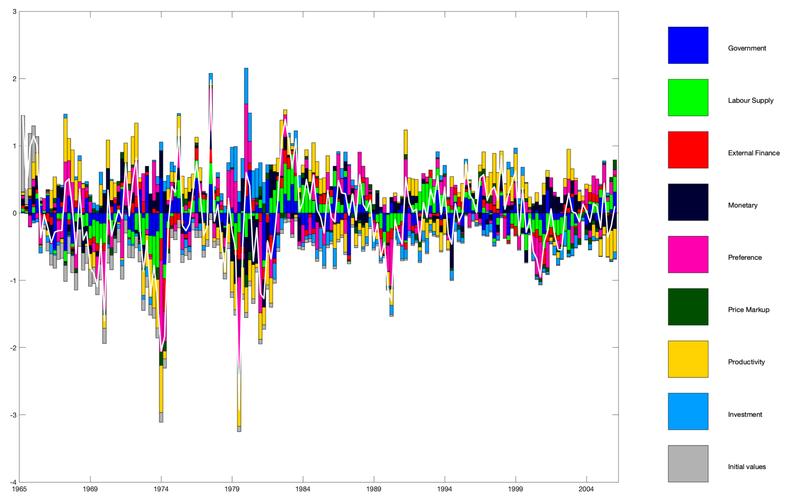
The white line plot represents the actual variable as a percentage point deviation from the steady state. And the coloured bars indicate each shocks contribution

Figure 1.18: Historical Shock Decomposition of Nominal Interest Rate



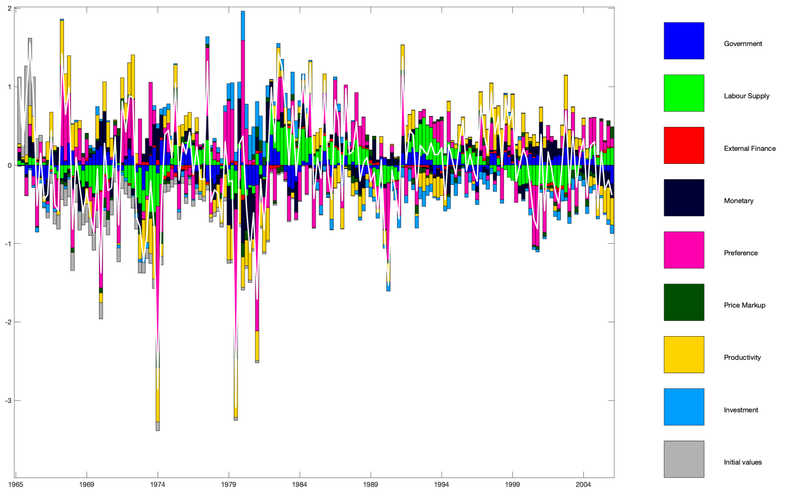
The white line plot represents the actual variable as a percentage point deviation from the steady state. And the coloured bars indicate each shocks contribution

Figure 1.19: Historical Shock Decomposition of Change in Output



The white line plot represents the actual variable as a percentage point deviation from the steady state. And the coloured bars indicate each shocks contribution

Figure 1.20: Historical Shock Decomposition of Change in Consumption



The white line plot represents the actual variable as a percentage point deviation from the steady state. And the coloured bars indicate each shocks contribution

An interesting observation can be made regarding the contribution of external finance premium shock to the historical decomposition of the nominal interest rate. As shown in Figure 1.18, this shock contributes to nearly all the reduction in the nominal rate, which implies that the central bank has had to react to developments in the financial industry. Preference shock mostly explains the dynamics of growth in consumption; it is also seen as a significant contributor to the decline in output growth in the 1970s.

1.5 Conclusion

This chapter contributes to the pool of research on the effect of financial conditions on the dynamics of macroeconomic outcomes, focusing on the corporate bond yield. It does so by theoretically linking the debt market (government and corporate bonds) with the macroeconomy in such a way that the model is able to account for the magnitude and volatility of the corporate bond spread that has otherwise been difficult to capture in existing studies. The answers provided by this chapter are made plausible by incorporating a preference that differentiates risk aversion from the elasticity of substitution in a medium-scale DSGE model in which financial friction is embedded.

A particular novelty of this chapter is its straightforward link between bond (government and corporate) prices and yields of the model with data. This is also the case with the link between the corporate bond spread model and data; hence, a comparative analysis of the model performance is possible. Having solved the model using the perturbation method and estimated model parameters using the Bayesian method, including macroeconomic and bond yield data, the best estimation outcome is from the model using 10-year BAA grade corporate bond yield alongside macroeconomics data. The government yield performs poorly in this regard, and including information from corporate and government yields led to overfitting. The results indicate the ability of the model presented to match both macroeconomic and financial moments without compromise. Most importantly, the magnitude and volatility of the estimated model's corporate bond spread are comparable with the data observations. This points to the model's advantage over those in existing studies that have been unable to generate variable and sizeable corporate bond spread. Unlike that in Rudebusch and Swanson (2012), the model here is able to attain a term premium of 100 basis points without making households excessively risk averse. That is, the structure of the model and its incorporation of the corporate bond yield substantially explains the risk attitude observed in the economy.

Further results from estimating the model indicate the importance of the financial accelerator mechanism. For example, the estimates of parameters relating to the transmission of financial friction are non-negligible and significant. The implication of financial friction on the amplification of the business cycle is also tangible. In addition, the historical decomposition of shocks on observable data shows that disturbances emanating from the financial industry potentially affect macroeconomic outcomes, most especially investment. Hence, it can be concluded that financial disruption has an economically significant impact on the propagation of US business cycles. Furthermore, the analysis in this chapter suggests the possibility of a structural break in the corporate bond spread. While this stance has not been thoroughly justified in a theoretical sense, the estimated parameters over the two sub-periods of the observed break suggest the possibility.

Further research extending the work of this chapter could further develop the model to investigate optimal monetary policy rules that incorporate information from financial variables. That is, the traditional Taylor-type rule of inflation and output targets could be extended to include responses to indicators such as corporate bond spread and could be used to assess welfare improvements. This recommendation reflects the finding concerning the historical shock decomposition of nominal interest rates, which shows that central banks have, at some point, had to respond to developments in the financial industry in setting their rate.

Appendix 1

I: Detrended Model Equations

The model has been detrended using the deterministic growth rate γ, and nominal variables now transformed to real. Detrended model variables are therefore denoted by small letters. For example

$k_t = \frac{K_t}{\gamma^t}$, $w_t = \frac{W_t}{\gamma^t P_t}$, $q_t = \frac{Q_t}{P_t}$, $\bar{\beta} = \beta\gamma^{-\sigma_c t}$

$mc_t = \frac{MC_t}{P_t}$, $\bar{V}_t = \frac{V_t}{\gamma^{t(1-\sigma_C)}}$

Euler Equation

$$1 = \mathbb{E}_t\left[\lambda_{t,t+1}\frac{1}{\pi_{t+1}}\right]R_{t,t+1} \tag{1.5.1}$$

Stochastic Discounting Factor

$$\lambda_{t,t+1} = \bar{\beta}\left(\frac{\gamma^{1-\sigma_C}\bar{V}_{t+1}}{\bar{v}_t}\right)^{-\sigma_E}\frac{\xi_{t+1}}{\xi_t} \tag{1.5.2}$$

Marginal Utility of Consumption

$$\xi_t = \varepsilon_t^{\beta}\left(c - \frac{\eta_c}{\gamma}c_{t-1}\right)^{-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right) \tag{1.5.3}$$

Marginal Utility of Labour

$$w_t^h = \varepsilon_t^w\left(c_t - \frac{\eta_c}{\gamma}c_{t-1}\right)L_t^{\sigma_L} \tag{1.5.4}$$

Epstein-Zin Preference

$$\bar{V}_t = \varepsilon_t^{\beta}\left[\frac{1}{1-\sigma_C}\left(c - \frac{\eta_c}{\gamma}c_{t-1}\right)^{1-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right)\right] + \bar{\beta}\bar{v}_t \tag{1.5.5}$$

Value Function

$$\bar{v}_t = \mathbb{E}_t\left[\gamma^{(1-\sigma_C)(1-\sigma_E)}\bar{V}_{t+1}^{1-\sigma_E}\right]^{\frac{1}{1-\sigma_E}} \tag{1.5.6}$$

Production Function

$$m_t = \varepsilon_t^a \left(U_t^k \frac{k_{t-1}}{\gamma} \right)^{\alpha} \left(L_t \right)^{1-\alpha} \tag{1.5.7}$$

Real wage

$$w_t = (1-\alpha) mc_t \frac{m_t}{H_t} \tag{1.5.8}$$

Marginal Product of Capital

$$mpk_t = \alpha mc_t \frac{\gamma m_t}{k_{t-1}} \tag{1.5.9}$$

Capital Utilisation

$$U_t^k = \left(\frac{\frac{mpk_t}{q_t}}{\frac{mpk_{ss}}{q_{ss}}} \right)^{\psi_u} \tag{1.5.10}$$

Capital Depreciation

$$\delta_t^k = \delta + \psi_u \left(\frac{mpk_t}{q_t} - \frac{mpk_{ss}}{q_{ss}} \right) \tag{1.5.11}$$

Law of Motion of Capital

$$k_t = (1-\delta_t^k) \frac{k_{t-1}}{\gamma} + \varepsilon_t^i i_t - \frac{\psi \gamma}{2} \left(\frac{i_t}{i_{t-1}} - i_t \right)^2 i_t \tag{1.5.12}$$

Tobin's Q

$$1 = q_t \left[\varepsilon_t^i - \psi \gamma \frac{i_t}{i_{t-1}} \left(\frac{\gamma i_t}{i_{t-1}} - \gamma \right) - \frac{\gamma}{2} \left(\frac{\gamma i_t}{i_{t-1}} - \gamma \right)^2 \right] + \mathbb{E}_t \left[\lambda_{t,t+1} q_{t+1} \psi \left(\frac{\gamma i_{t+1}}{i_t} \right)^2 \left(\frac{\gamma i_{t+1}}{i_t} - \gamma \right) \right] \tag{1.5.13}$$

Realised Return and Loan Rate

$$\mathbb{E}_t [\lambda_{t,t+1} R_{t,t+1}^R] = \mathbb{E}_t \left[\lambda_{t,t+1} \frac{1}{\pi_{t,t+1}} \right] R_{t,t+1}^N \tag{1.5.14}$$

Realised Capital Return

$$R^R_{t-1,t} = \frac{mpk_t}{q_{t-1}} + (1-\delta^k_t)\frac{q_t}{q_{t-1}} \tag{1.5.15}$$

Nominal Contract Rate

$$R^N_{t,t+1} = S_{t,t+1} R_{t,t+1} \tag{1.5.16}$$

External Finance Premium

$$S_{t,t+1} = \varepsilon^s_t S_{ss}\left(\frac{q_{t-1}k_{t-1}}{n_{t-1}}\frac{n_{ss}}{k_{ss}}\right)^{\psi_S} \tag{1.5.17}$$

Law of Motion of Net worth

$$n_t = \varphi \bar{E}^s_t + (1-\varphi)\exp(\varepsilon^N_t) \tag{1.5.18}$$

Net worth of Surviving Entrepreneurs

$$\bar{E}^s_t = \left(R^R_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}\right) q_{t-1}\frac{k_{t-1}}{\gamma} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}} n_{t-1} \tag{1.5.19}$$

Stock Price Equations

$$\tilde{p}_t = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\left(\frac{g^{p2}_t}{g^{p1}_t}\right) \tag{1.5.20}$$

$$g^{p1}_t = y_t + \mathbb{E}_t\left[\zeta_p \lambda_{t,t+1}\gamma\left(\frac{\pi_t^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_{t+1}}\right)^{1-\epsilon_p} g^p_{t+1}1\right] \tag{1.5.21}$$

$$g^{p2}_t = y_t mc_t \varepsilon^p_t + \mathbb{E}_t\left[\zeta_p \lambda_{t,t+1}\gamma\left(\frac{\pi_t^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_{t+1}}\right)^{-\epsilon_p} g^{p2}_{t+1}\right] \tag{1.5.22}$$

$$\tilde{p}_t = \left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{\frac{1}{1-\epsilon_p}} \tag{1.5.23}$$

$$\Delta^p_t = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta^p_{t-1} \tag{1.5.24}$$

Stick Wage Equations

$$\frac{\tilde{w}_t}{w_t} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\left(\frac{g_t^{w2}}{g_t^{w1}}\right) \tag{1.5.25}$$

$$g_t^{w1} = w_t H_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\gamma\left(\frac{w_t}{w_{t+1}}\frac{\pi_t^{l_w}\pi_s^{1-l_w}}{\pi_{t+1}}\right)^{1-\epsilon_w} g_{t+1}^{w1}\right] \tag{1.5.26}$$

$$g_t^{w2} = w_t^h H_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\gamma\left(\frac{w_t}{w_{t+1}}\frac{\pi_t^{l_w}\pi_{ss}^{1-l_w}}{\pi_{t+1}}\right)^{1-\epsilon_w} g_{t+1}^{w2}\right] \tag{1.5.27}$$

$$\frac{\tilde{w}_t}{w_t} = \left[\frac{1 - \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{1.5.28}$$

$$\Delta_t^w = (1-\zeta_w)\left[\frac{1 - \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{-\frac{\epsilon_w}{1-\epsilon_w}} + \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{\pi_t}\right)^{-\epsilon_w}\Delta_{t-1}^w \tag{1.5.29}$$

Labour Market Equilibrium

$$L_t = \Delta_t^w H_t \tag{1.5.30}$$

Goods Market Equilibrium

$$m_t = \Delta_t^p y_t \tag{1.5.31}$$

Market Clearing

$$y_t = c_t + i_t + \varepsilon_t^g y_{ss} \tag{1.5.32}$$

Monetary Policy

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_1}\left(\frac{y_t}{y_{ss}}\right)^{\psi_2}\right]^{1-\rho_R}\left(\frac{\frac{y_t}{y_{t-1}}}{\frac{y_t^f}{y_{t-1}^f}}\right)^{\psi_3}\varepsilon_t^r \tag{1.5.33}$$

Nominal Stochastic Discounting Factor

$$\Lambda_{t+1}^N = \frac{\Lambda_{t+1}}{\pi_{t+1}} \tag{1.5.34}$$

Nominal Government Bond Price

$$B^g_{t,n} = \mathbb{E}_t\Big[\Lambda^N_{t+1} B^g_{t+1,n-1}\Big] \tag{1.5.35}$$

Government Bond Yield

$$R^g_{t,n} = \left(\frac{1}{B^g_{t,n}}\right)^{\frac{1}{n}} \tag{1.5.36}$$

Nominal Corporate Bond Price

$$B^c_{t,n} = \mathbb{E}_t\Big[\Lambda^N_{t+1} B^c_{t+1,n-1}\Big]\left(\frac{1}{S_{t+1}}\right) \tag{1.5.37}$$

Corporate Bond Yield

$$R^c_{t,n} = \left(\frac{1}{B^c_{t,n}}\right)^{\frac{1}{n}} \tag{1.5.38}$$

Corporate Bond Spread

$$C^b_{t,n} = R^c_{t,n} - R^g_{t,n} \tag{1.5.39}$$

Exogenous Processes

$$\ln \varepsilon^\beta_t = \rho_\beta \ln \varepsilon^\beta_{t-1} + \eta^\beta_t \tag{1.5.40}$$
$$\ln \varepsilon^p_t = \rho_p \ln \varepsilon^p_{t-1} + \eta^p_t \tag{1.5.41}$$
$$\ln \varepsilon^a_t = \rho_a \ln \varepsilon^a + \eta^a_t \tag{1.5.42}$$
$$\ln \varepsilon^w_t = \rho_w \varepsilon^w_t + \varepsilon^w_t \tag{1.5.43}$$
$$\ln \varepsilon^i_t = \rho_i \ln \varepsilon^i_{t-1} + \eta^i_t \tag{1.5.44}$$
$$\ln \varepsilon^r_t = \rho_r \ln \varepsilon^r_{t-1} + \eta^r_t \tag{1.5.45}$$
$$\ln \varepsilon^g_t = \rho_g \ln \varepsilon^g_{t-1} + \rho_{ga} \ln \varepsilon^a_{t-1} + \eta^g_t \tag{1.5.46}$$
$$\ln \varepsilon^S_t = \rho_S \ln \varepsilon^S_{t-1} + \eta^S_t \tag{1.5.47}$$
$$\ln \varepsilon^N_t = \rho_N \ln \varepsilon^N + \eta^N_t \tag{1.5.48}$$

where $\quad \eta^k_t \sim \mathcal{N}(0, \sigma_k) \qquad k = [a, \beta, w, p, i, r, g, N, S]$

II: Steady State

$$
\begin{aligned}
U_{ss}^k &= 1 \\
\Delta_s s^p &= 1 \\
\Delta_s s^w &= 1 \\
\tilde{p}_{ss} &= 1 \\
\lambda_{ss} &= \bar{\beta} \\
\delta_{ss}^k &= \delta \\
S_{ss} &= S_{ss} \\
R_{ss} &= \frac{\pi_{ss}}{\bar{\beta}} \\
R_{ss}^N &= S_{ss} R_{ss} \\
R_{ss}^N &= S_{ss} R_{ss} \\
R_{ss}^R &= \frac{R_{ss}^N}{\pi_{ss}} \\
\frac{i_{ss}}{k_{ss}} &= \frac{\gamma - 1 - \delta}{\gamma} \\
mc_{ss} &= \frac{\epsilon_p - 1}{\epsilon_p} \\
mpk_{ss} &= R_{ss}^R - (1 - \delta_{ss}^k) \\
\frac{m_{ss}}{k_{ss}} &= \frac{mpk_{ss}}{\gamma \alpha mc_{ss}} \\
\frac{l_{ss}}{k_{ss}} &= \left(\gamma^\alpha \frac{m_{ss}}{k_{ss}} \right)^{\frac{1}{1-\alpha}} \\
w_{ss} &= (1 - \alpha) mc_{ss} \frac{m_{ss}}{k_{ss}} \div \left(\frac{l_{ss}}{k_{ss}} \right) \\
\tilde{w}_{ss} &= w_{ss} \\
w_{ss}^h &= \left(\frac{\epsilon_w - 1}{\epsilon_w} \right) w_{ss} \\
\frac{c_{ss}}{k_{ss}} &= \frac{m_{ss}}{k_{ss}} (1 - g_{ss}) - \frac{i_{ss}}{k_{ss}} \\
l_{ss} &= \left[\left(\frac{\epsilon_w - 1}{\epsilon_w} \right) w_{ss} \left(\frac{l_{ss}}{k_{ss}} \div \frac{c_{ss}}{k_{ss}} \right) \left(\frac{\gamma}{\gamma - \eta_c} \right) \right]^{\frac{1}{1+\sigma_L}}
\end{aligned}
$$

$$
\begin{aligned}
h_{ss} &= l_{ss} \\
k_{ss} &= l_{ss}\left(\frac{k_{ss}}{l_{ss}}\right) \\
i_{ss} &= k_{ss}\left(\frac{i_{ss}}{k_{ss}}\right) \\
y_{ss} &= k_{ss}\left(\frac{m_{ss}}{k_{ss}}\right) \\
m_{ss} &= y_{ss} \\
c_{ss} &= k_{ss}\left(\frac{c_{ss}}{k_{ss}}\right) \\
g_{ss}^{p1} &= \frac{y_{ss}}{1-\zeta_p\bar{\beta}\gamma} \\
g_{ss}^{p2} &= \frac{y_{ss}mc_{ss}}{1-\zeta_p\bar{\beta}\gamma} \\
g_{ss}^{w1} &= \frac{h_{ss}w_{ss}}{1-\zeta_w\bar{\beta}\gamma} \\
g_{ss}^{w2} &= \frac{h_{ss}w_{ss}^{h}}{1-\zeta_w\bar{\beta}\gamma} \\
\bar{V}_{ss} &= \left(\frac{1}{1-\sigma_C}\right)\left(c_{ss}-\frac{\eta_c}{\gamma}c_{ss}\right)^{1-\sigma_C}\exp\left(\frac{\sigma_C-1}{1+\sigma_L}l_{ss}^{1+\sigma_L}\right)\left(\frac{1}{1-\bar{\beta}\gamma^{1-\sigma_C}}\right) \\
\bar{v}_{ss} &= \gamma^{1-\sigma_C}\bar{V}_{ss} \\
\xi_{ss} &= \left(c_{ss}-\frac{\eta_c}{\gamma}c_{ss}\right)^{-\sigma_C}\exp\left(\frac{\sigma_C-1}{1+\sigma_L}l_{ss}^{1+\sigma_L}\right)
\end{aligned}
$$

III: Identification and Estimation Output

Figure 1.21: Identification Strength of Prior Mean

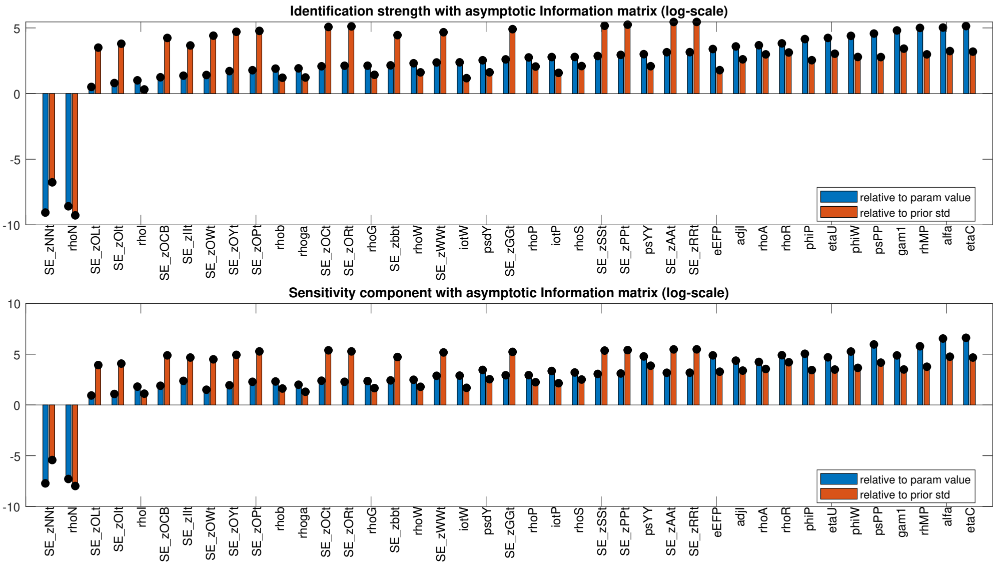


Figure 1.22: Multivariate Convergence Diagnostics of 1,000,000 Draws

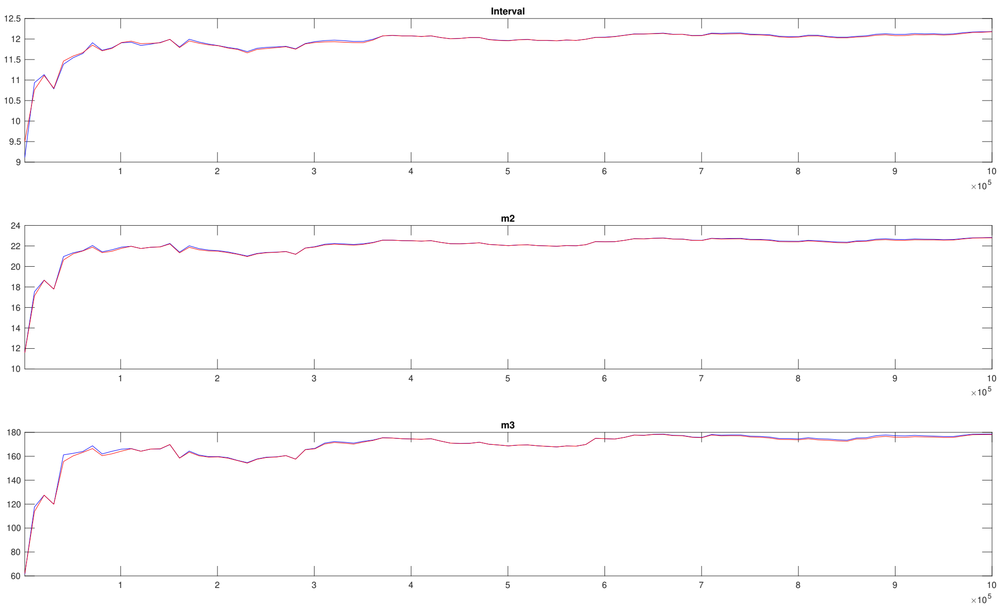


Figure 1.23: Smoothed Variables of Observable- Model 5

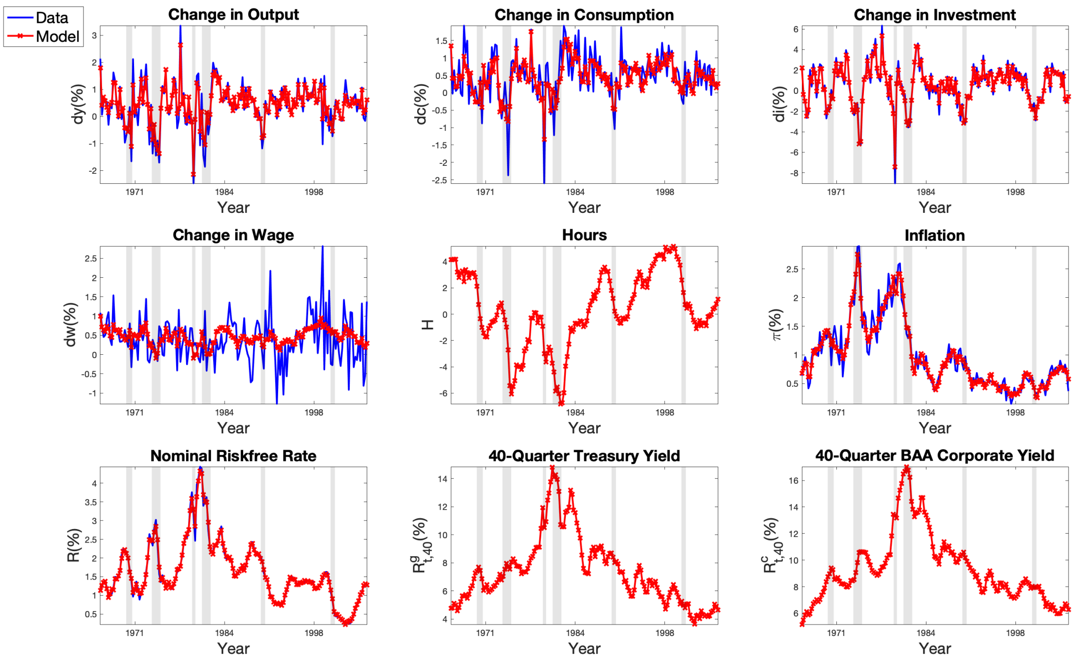


Figure 1.24: The Fit of Model 3 and Model 4

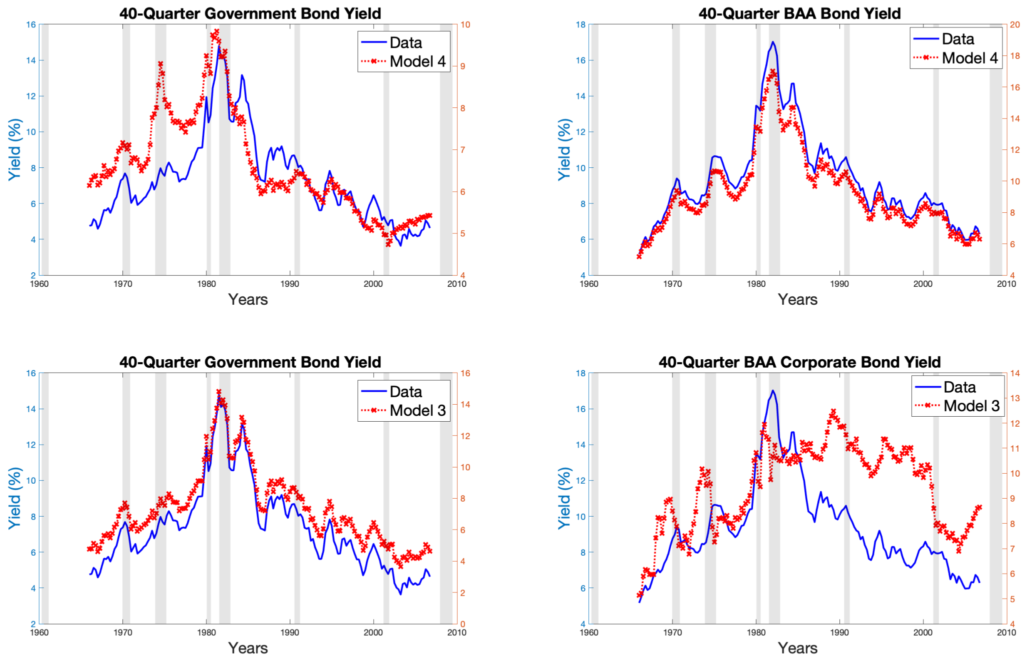


Table 1.6: Prior and Posterior Distribution of Model 5

		Prior distribution			Posterior distribution			
Description		Distr	Mean	St.Dev	Mode	Mean	5%	95%
Productivity shock	σ_a	$\mathcal{I}$	1.00	10.00	0.50	0.50	0.42	0.59
Preference shock	σ_b	$\mathcal{I}$	1.00	10.00	2.99	3.87	2.05	6.03
Government shock	σ_g	$\mathcal{I}$	1.00	10.00	1.73	1.80	1.51	2.10
Investment shock	σ_i	$\mathcal{I}$	1.00	10.00	1.94	2.04	1.70	2.37
Monetary shock	σ_r	$\mathcal{I}$	1.00	10.00	0.19	0.20	0.16	0.23
Price mark-up shock	σ_p	$\mathcal{I}$	1.00	10.00	0.76	0.81	0.48	1.12
Labour supply shock	σ_w	$\mathcal{I}$	1.00	10.00	2.45	2.50	1.96	3.04
Net worth shock	σ_n	$\mathcal{I}$	1.00	10.00	0.46	0.89	0.23	1.71
External finance shock	σ_s	$\mathcal{I}$	1.00	10.00	0.40	0.43	0.30	0.55
Productivity persistence	ρ_a	$\mathcal{B}$	0.50	0.25	0.996	0.995	0.99	0.998
Preference persistence	ρ_b	$\mathcal{B}$	0.50	0.25	0.98	0.99	0.98	0.99
Government persistence	ρ_g	$\mathcal{B}$	0.50	0.25	0.77	0.77	0.67	0.86
Investment persistence	ρ_i	$\mathcal{B}$	0.50	0.25	0.97	0.97	0.95	0.99
Monetary persistence	ρ_r	$\mathcal{B}$	0.50	0.25	0.79	0.78	0.72	0.84
Price mark-up persistence	ρ_p	$\mathcal{B}$	0.50	0.25	0.63	0.62	0.44	0.80
Labour supply persistence	ρ_w	$\mathcal{B}$	0.50	0.25	0.99	0.99	0.98	0.997
Net worth persistence	ρ_n	$\mathcal{B}$	0.50	0.25	0.50	0.49	0.09	0.88
External premium persistence	ρ_s	$\mathcal{B}$	0.50	0.25	0.79	0.78	0.71	0.86
Government & output	ρ_{ga}	$\mathcal{N}$	0.50	0.25	0.85	0.85	0.46	1.24
Adjustment cost	Ψ	$\mathcal{N}$	4.00	1.50	3.26	3.22	2.07	4.28
Consumption habit	η_C	B	0.70	0.10	0.49	0.49	0.43	0.54
Wage stickiness	ζ_w	$\mathcal{B}$	0.50	0.10	0.91	0.90	0. 87	0.93
Price stickiness	ζ_p	$\mathcal{B}$	0.50	0.10	0.77	0.77	0.71	0.82
Wage indexation	ι_w	$\mathcal{B}$	0.50	0.15	0.76	0.73	0.57	0.89
Price indexation	ι_p	$\mathcal{B}$	0.50	0.15	0.58	0.58	0.38	0.78
Elasticity of utilisation	ψ_u	$\mathcal{B}$	0.50	0.15	0.38	0.39	0.21	0.56
Capital share	α	$\mathcal{N}$	0.30	0.05	0.10	0.17	0.14	0.20
Rate smoothing	ρ_R	$\mathcal{B}$	0.75	0.10	0.38	0.40	0.27	0.52
P-inflation	ψ_π	$\mathcal{N}$	1.50	0.25	1.34	1.36	1.24	1.48
P-output gap	ψ_y	$\mathcal{N}$	0.125	0.05	0.05	0.07	0.001	0.12
P-output growth	ψ_{dy}	$\mathcal{N}$	0.125	0.05	0.29	0.28	0.21	0.35
Steady-state inflation	$\bar{\pi}_{ss}$	$\mathcal{G}$	0.625	0.10	0.61	0.63	0.46	0.79
Trend growth rate	$\bar{\gamma}$	$\mathcal{N}$	0.40	0.10	0.32	0.33	0.23	0.43
Elasticity of external premium	ψ_S	$\mathcal{U}$	0.10	0.02	0.10	0.10	0.07	0.13

Note that posterior distribution is obtained using Metropolis-Hastings algorithm using 1,000,000 draws. Also, $\mathcal{I},\mathcal{B},\mathcal{N}$, and $\mathcal{G}$, all denotes Inverse-gamma, Beta, Normal, and Gamma distributions respectively.

Cross-Correlation with First Difference Data

Figure 1.25: Cross- and Auto-correlation of Output, Consumption, Investment, Hours, and Wages

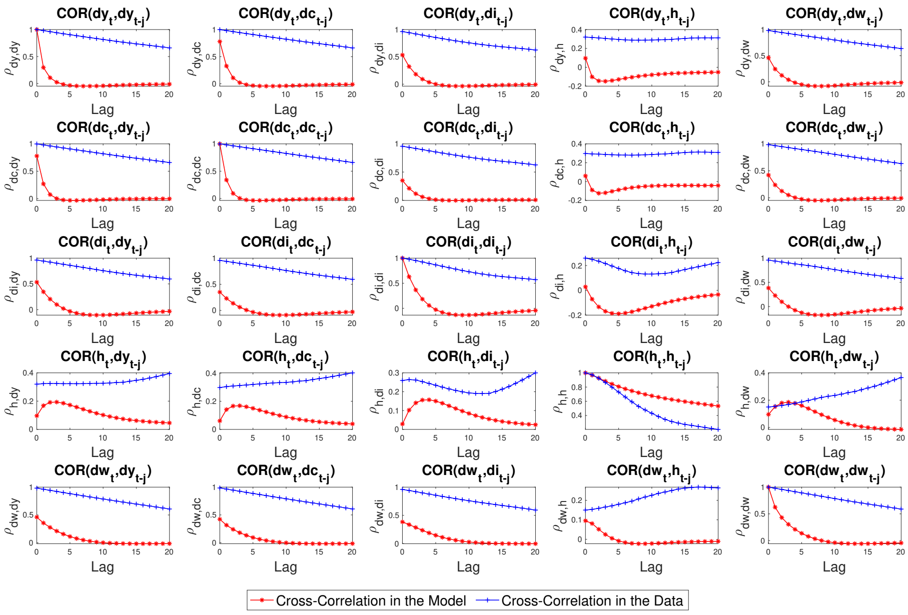


Figure 1.26: Cross-correlation of Output, Consumption, Investment, Hours, and Wages

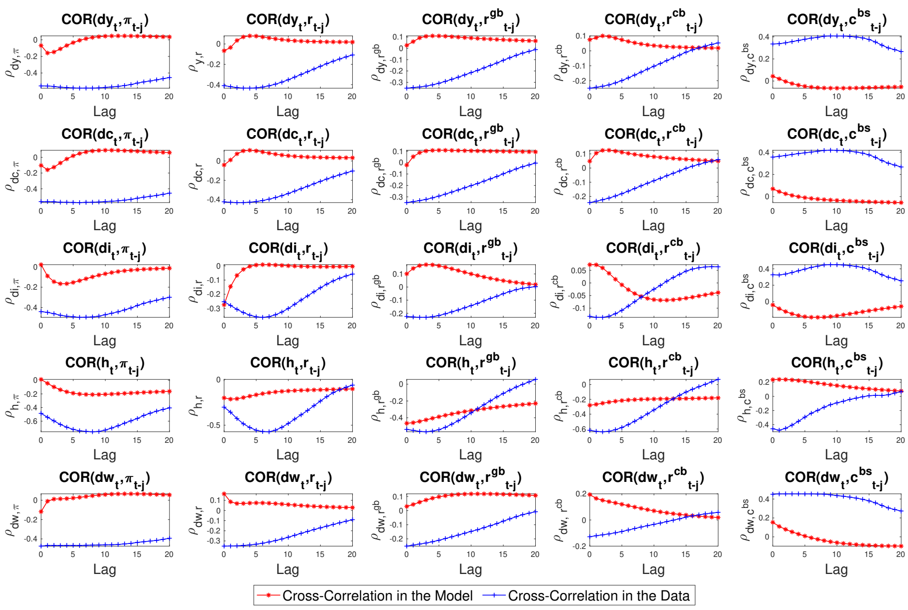


Figure 1.27: Cross-correlation of Inflation, Interest Rate, Treasury Yield, Corporate Yield and Spread

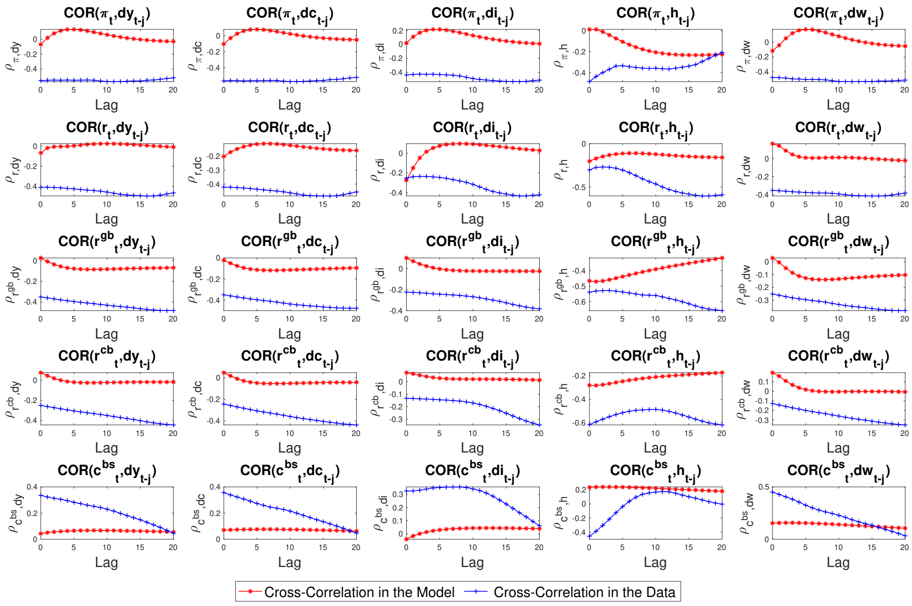
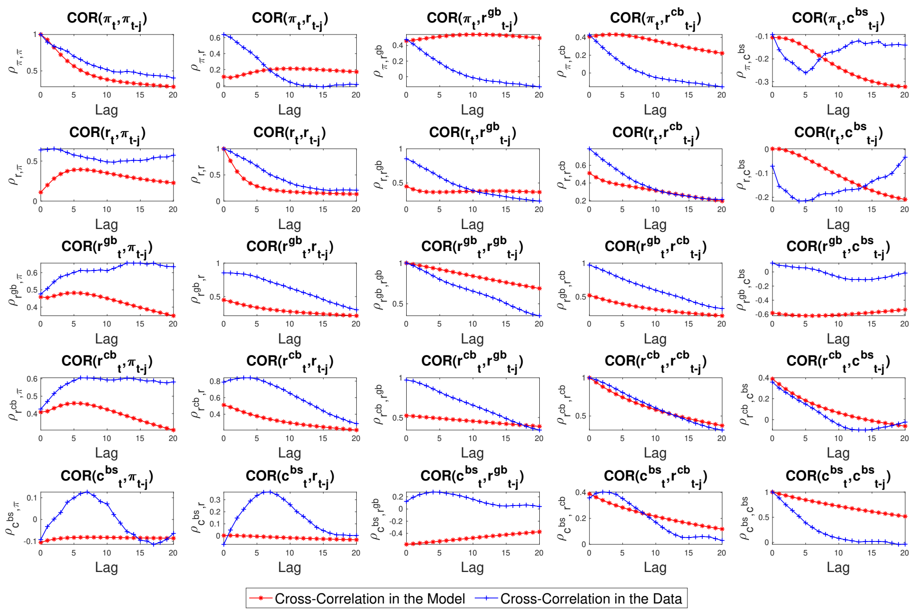


Figure 1.28: Cross- and Auto-correlation of Inflation, Interest Rate, Treasury and Corporate Yield, Spread



2. Optimal Policy Rules and Corporate Bond Premium

Abstract

This chapter explores what it means for the real economy if the central bank considers corporate bond premiums and other financial indicators such as net worth and the asset ratio in its design of monetary policy. This is done in the context of a dynamic stochastic general equilibrium (DSGE) model that accounts for the feedback between financial conditions and the real economy. The benchmark Taylor rule is extended to include the first difference of these financial indicators as an additional policy instrument. Similar to Schmitt-Grohe and Uribe (2007), the chapter analyses the quantitative implications of these adjustments in an optimal, simple and implementable monetary policy that maximises household welfare relative to a flexible price economy. The results show that the introduction of financial indicators appears to alter the optimal monetary policy in a significant way. In particular, the easing of monetary policy is found to be the optimal response to an increasing corporate bond premium, while the tightening of interest rates is optimal for increasing entrepreneur net worth; in this instance, welfare is improved, and inflation is less volatile (having been almost halved).

2.1 Introduction

Traditionally, monetary policy has targeted inflation and output stability, an approach well summarised by the Taylor rule. According to Taylor and Williams (2008), a monetary policy mechanism where the federal funds rate directly affects inflation and real output in macroeconomic models is a simplification. In reality, the interest rates on longer-term or higher-risk loans are the rates with a direct impact on the real economy and which alter spending decisions. Therefore, the ongoing adoption of the simplistic monetary mechanism is due to the fact that the spread in interest rates has been negligible and somewhat constant over time.

The tightening credit conditions (e.g., rising credit spread and deteriorating balance sheets) and falling real economic activities during the 2007-2009 financial crisis[1] revealed, amongst other things, that price and output stability do not entirely ensure macroeconomic or financial stability. Rectifying the aftermath of this crisis has been costly –central bankers have had to adopt unconventional policies deviating from a systematic monetary policy mechanism that is traceable. Since then, many have suggested that the central bank should consider financial stability indicators alongside its usual inflation and output targets in designing effective monetary policy (Chen and Columba, 2016; Nair and Anand, 2020).

[1]During this period, the spread on interest rates became significant and volatile, causing huge disruption to the real economy.

The aforementioned led to a re-evaluation of the importance of financial conditions in the real economy (Fiore and Tristani, 2012). As a result, the debate on the inclusion of measures of financial vulnerability in the design of monetary-policy rules has also resurfaced in the literature[2] (Blanchard et al., 2013). For example, the Bank of England has established a Financial Policy Committee. Nonetheless, there is an active discourse regarding the setting of the key policy rate and whether or not the central bank should respond to financial stability. While Borio (2014) argues the relevance of including financial-vulnerability measures into monetary policy rules, Bernanke (2013) insists on monetary policy focusing on inflation and output targeting. In an earlier study, Cecchetti et al. (2000) claims that the central bank should consider responding to asset-price misalignments in its usual policy decisions. This claim is based on the possibility that such policy mechanisms could reduce the formation of asset price bubbles and, therefore, reduce the risk of the adverse effects of a bust in investment cycles that can jeopardise macroeconomic stability.

The slope of the term structure of interest rates has been a consideration in setting the policy rate. This was done in reference to the increasing flatness of the US yield curve at the time that the federal funds rate was increasing. However, there has been a shift since the 2007-2009 financial crisis. Credit risk premiums have now become a prominent feature of financial conditions influencing monetary policy (McCulley and Toloui, 2008). Hence, the question of what measures of financial vulnerability are worthwhile policy instruments continues to linger in the literature. Scholars such as Woodford (2012); Gilchrist and Zakrajsek (2012) and Borio (2014) suggest indicators like leverage, credit or bond spreads, and house prices. Some of these indicators are adopted in defining the objectives of macroprudential policies in restricting banks' credit growth by adjusting banks' lending over the business cycle and preventing risk-taking (Lazopoulos and Gabriel, 2019).

Evidence shows that central banks have had to react to financial shocks and distress emanating from the credit market. A typical example is seen in the policy reaction in the US in the late 1980s when the interest rate was reduced by a magnitude greater than that suggested by the Taylor rule due to the effect of the increased capital requirement. In its reaction to the 2007-2009 financial crisis, the Federal Reserve drastically reduced the federal fund rate. This reaction was implemented despite official statistics not yet having indicated a decline in real GDP and no inflation having been otherwise observed. Obviously, the indicators spurring such reactions must reflect developments in the financial sector (Mishkin and Westelius, 2008). Quantitative easing (QE), amongst other non-conventional methods, has been adopted by central banks to curb the effect of financial crises.

The implication of such a non-conventional monetary policy reaction is that central banks react to indicators other than GDP and inflation in an unsystematic way. This establishes the premise for the questions this chapter addresses. One of the focus areas here is assessing the implication of extending monetary policy mechanisms of allowing for varying financial indicators. Furthermore, it investigates the

[2] This idea is often regarded as the lean-against-the-wind policies, whereby the usual Taylor-type monetary policy rule is extended to include financial variables (Taylor and Williams, 2008).

extent to which the modification will improve the economy's response to disturbances. These questions are crucial as it is important that monetary policy takes into account the reactions from the financial market. This is because the private sector makes its own assessment of the central bank's action in designing its strategy. Knowing that the distress emanating from credit markets ripples and reflects on the broader economy, it is essential for central banks to continuously observe events in the financial markets (Taylor and Williams, 2008).

This chapter contributes to the literature by presenting a model that reflects the intermediation process of the financial sector and also extends the monetary transmission mechanism to reflect financial indicators. Specifically, it explores the information that corporate bond spreads provide for optimal monetary policy and their influence on household welfare.[3] According to Lazopoulos and Gabriel (2019), financial instability is captured by the spread between lending and policy rates; hence, inferring information from such an indicator for the purpose of designing a monetary policy mechanism could potentially curtail any financial imbalance that could jeopardise macroeconomic and financial stability. Consideration is also given to the information relevant to an optimal monetary policy contained within other financial indicators such as leverage and external finance premium.

The model is estimated using macroeconomic and 40-quarter BAA corporate yield data, allowing the corporate bond premium to be accounted for.[4] Similar to Schmitt-Grohe and Uribe (2007), the chapter investigates the quantitative implications of financial variables in an optimal, simple and implementable monetary policy relative to a flexible price economy. More precisely, corporate bond spread (that already reflects the corporate bond premium), leverage, external finance premium, and net worth are individually considered as policy instruments additional to the usual inflation and output targeting. Thereafter, the chapter compares the performance of these alternative policies with that of the standard Taylor-type policies by looking at their consumption-equivalent welfare losses to a flexible price economy. It then looks at the implications of these policies for macroeconomic outcomes in the face of shocks, specifically policy and financial shocks.

The novelty of this work is that it investigates a macroprudential-type policy in a model where corporate and government bonds are endogenously determined. This theoretical linkage enhances the model's ability to replicate the magnitude and volatility of corporate bond spread observed in data without compromising its fit for macroeconomic variables.[5] The spread is influenced by the endogenous external finance premium, which is also the source of distortion in capital and investment dynamics. In this instance, the increasing corporate bond spread is a reflection of financial distortion as a result of the distorted allocation of capital and investment, which affects aggregate demand.

[3] The difference between corporate and government bond yield of the same maturity.

[4] Corporate bond premium is regarded as the aspect of the corporate bond yield in the data that the presented model could not account for. Hence, it is an additive shock to the corporate bond yield in the model, and it is orthogonal to the state of the economy (i.e. financial inclusive).

[5] Being able to define a proper stochastic discounting factor contributes to the success of the model's ability to match corporate bond spread in the data. In addition, the structure of the financial intermediation process such that a shock that affects its efficiency is incorporated.

Another key idea is that the analysis of the corporate bond spread incorporates the information from the data that is otherwise not captured in the model. According to Gilchrist and Zakrajsek (2011) the information not otherwise captured is the financial bond premium reflecting the cyclical movements in the relationship between default risk and credit spreads.[6] citeMoneyGil argue that the premium is a representation of the switch in the risk attitudes of corporate-debt investors and financial intermediaries. This premise thus makes it possible to analyse the benefit of augmenting monetary policy with a first-difference rule using financial variables as instruments. That is, the chapter considers the benefit of the nominal interest rate (amongst other financial indicators) in responding to fluctuations in the corporate bond spread.

There are five key questions this chapter seeks to address: First, what is the relevance of financial indicators in monetary policy rule? Second, how does the shock emanating from the financial sector affect monetary policy rules with financial indicators? Third, which financial indicator, when included in the policy rule, stabilises the economy better? Fourth, what should be the monetary policy response to increased corporate bond spread? Fifth, how is welfare improved in the presence of alternative monetary policies in a flexible price economy?

These questions are answered through the adoption of an abridged version of Smets and Wouters (2003, 2007) as the baseline model underlying the analysis in this chapter. Although the model does not include a structural representation of financial intermediation, it is modified to include financial friction and a recursive Epstein-Zin preference. This allows the definition of a proper stochastic discounting factor (SDF) by differentiating the elasticity of substitution from risk aversion. In doing so, the modification allows us to address the limitations of standard New-Keynesian models that are without financial friction.

The modified model provides a time-varying wedge between the interest rate earned on savings and the interest rate at which loans can be obtained. This allows a theoretical analysis of the spread between government and BAA corporate bond yields. The benefit of the financial accelerator mechanism for the analysis in this model is not limited to being able to fit financial variables. According to Christiano et al. (2010), financial factors are prime determinants of economic fluctuations, suggesting that a robust approach to monetary policy must include attention to these factors. In an early study, Bernanke and Gertler (1995) note that the effect of monetary policy on interest rates is magnified by the endogenous changes to firms through the cost they incur sourcing external finance. [7]

Most of the theoretical models used for the evaluation of alternative monetary policies do not consider the existence of financial friction intermediation. In such models, it is assumed that there is a single interest rate at which households and firms earn returns on savings and can borrow against future income. This implies that a breakdown in the financial market will have no allocative effects on the economy (Curdia and Woodford, 2016). Therefore, it is impossible to analyse the consequence that financial indicators would have on monetary policy or to align how the cyclical variations in the financial sector

[6] In this chapter, we regard this premium as the corporate bond premium.

[7] The difference between the funds raised internally by the firm and the one sourced externally through debt or equity.

affect the real economy. Spread between interest rates available to different classes of borrowers was one of the indicators of stress in the financial sector during the 2007-2009 financial crisis; the unusual increase and the volatility of these spreads were enormous (Taylor and Williams, 2008). Taylor (2008) suggests that the intercept term in a Taylor rule for monetary policy should be adjusted downward in proportion to the observed increase in spreads.

Upon deriving the model's optimal intra- and intertemporal decisions, the model is estimated using the same seven macroeconomic observables in Smets and Wouters (2007) and the 40-quarter BAA corporate yield. [8] However, some parameters are calibrated to values in existing studies (see Table 2.1) or the sample average. The benchmark monetary policy with which the model is estimated responds to inflation, output gap (defined relative to a flexible price economy) and the growth in output. In the flexible price economy, the financial friction mechanism is not fully operational. It only allows for the steady-state uniformity of real capital returns in both economies (sticky and flexible). Thereafter, model parameters are set to their estimated or calibrated values before the optimal policy and welfare analysis is conducted. The optimal policy and welfare analysis is conducted under the full set of model disturbances, excluding monetary policy shock. In all, this study considered instances where optimal policy parameters are restrictive (i.e. parameters are bounded) as in Schmitt-Grohe and Uribe (2007) and those where they are non-restrictive such that the optimal policy parameters are unbounded (see Table 2.4).

The analysis in this chapter shows that targeting financial stability improves the effectiveness of monetary policy in stabilising the aggregate economy and, at the same time, improves household welfare relative to the traditional output and inflation targeting. Optimal monetary policy suggestions for an alternative economy responding to corporate bond spread, leverage, and external finance premium is that monetary policy should be reduced when these indicators are increasing.[9] The magnitude by which the nominal interest rate should be reduced is dependent on the type of macroeconomic and financial indicators used in the policy rule. For example, extending the traditional Taylor rule (inflation and output gap) to include growth in corporate bond spread suggests that the nominal interest rate should be reduced by two units for every unit increase in corporate bond spread (when policy parameters are bounded). If policy parameters are unbounded, the suggested reduction in the nominal interest rate is approximately three units. However, there is consistency in the implied monetary policy response to increasing leverage, regardless of the composition of the policy rule. If leverage is the financial indicator used, the policy suggestion implies an approximately one-unit reduction in the nominal interest rate for a one unit increase in leverage.

The monetary policies augmented by financial indicators mostly reduce the volatility of inflation and investment relative to the Taylor-rule (estimated and optimal) policy. In addition, there is a fall in the volatility of financial variables (government bond yield, corporate bond yield, and corporate bond

[8] Data series is reconstructed and updated to cover the period 1966-2006.

[9] The traditional Taylor rule of inflation and output target is regarded to here as the benchmark economy.

spread) compared to the estimated benchmark policy. Monetary policy plays a clear role when the economy is subject to a shock emanating from the financial industry. Conclusively, if central banks were to maximise welfare rather than pursue the mandate to stabilise inflation and output, their policy would respond more aggressively to financial variables and less to the output gap, which is detrimental to welfare. An important aspect of our findings is that monetary authorities should be sufficiently flexible and actively respond to increasing corporate bond spread by lowering the nominal interest rate.

The remainder of this chapter is structured as follows. Section 2.2 outlines and briefly addresses the related literature. Section 2.3 presents the modified model and derived equilibrium equations. Section 2.4 sets out the methodology adopted in finding the optimal policy parameter and in conducting the welfare analysis, while Section 2.6 presents and discusses the results. Section 2.7 delivers the conclusions and recommendations for the future progress of the research.

2.2 Literature Review

The ideas in this chapter can be traced to three strands of research in existing literatures. The first is the literature on the importance of financial shocks to the business cycle, which warranted the study of DSGE models featuring financial friction. The second is the literature focusing on how monetary policy should optimally respond to financial variables. The third is the literature on the methodology for welfare analysis.

2.2.1 Financial Shocks and Business Cycle

There is increasing awareness of the relevance of financial market conditions to economic fluctuations (Gerali et al., 2010; Rossana, 2015; Del Negro et al., 2016). The impact of the 2007-2009 global financial crisis, which started in the US following the housing-bubble burst caused by the unexpected fall in house prices, was enormous (Gerali et al., 2010). The emergence of the crisis resulted in distress for banks that, at the time, had large portfolios of subprime loans backed by mortgages. The initial collapse impacted other asset classes negatively, leading to problems in interbank lending market liquidity and massive yield spread, causing a strain on businesses and household credit conditions. This credit crunch culminated in a global financial crisis, with a sharp decline in economic activities globally. Therefore, economic crises are not only traceable to a decline in economic activities but also to underlying failures in the financial industry (Gerali et al., 2010). This event made it clear to macroeconomists that financial shocks have a lasting effect. Furthermore, the recovery from this shock has been slow compared to past recessions (Cerra and Saxena, 2008; Claessens et al., 2009; Reinhart and Rogoff, 2009; Claessens et al., 2012). This suggests the need for macroeconomic models to reflect financial and credit conditions.

The usual canonical and New-Keynesian models do not reflect how financial and credit markets conditions affect the real economy, as Christiano et al. (2005) observe. The assumption underlying such models is that a borrower's balance sheet has no effect on their optimal spending choices. However, inasmuch as financial-asset price movement affects a household's wealth, it will have a direct effect on their spending. Christiano et al. (2014) suggests that risk shocks specific to leveraged entrepreneurs account for a large share of business cycle fluctuations. A class of financial models such as that in Huang and Huang (2012); Chun et al. (2014) are left to fit asset prices by failing to consider the real economy. This only considers the reduced-form processes of economic variables influencing SDF, which is an integral component of asset pricing. According to Cochrane (2005), asset prices are essential in explaining the allocation of consumption and investment over time and state. As a result, the viability of a macroeconomic model should be ascertained by its ability to match asset prices. Key stakeholders, investors and policy makers are also interested in knowing how macroeconomic variables interact with asset prices. Therefore, integrating financial conditions into a macroeconomic model will be helpful to better explain business cycle fluctuations.

2.2.2 Monetary Policy and Financial Indicators

Before the 2007-2009 financial crisis, financial stability was usually examined from the perspective of asset price deviations from their fundamental value. In the course of the dot-com bubble and the ensuing crash, discussions emerged in the literature about considering asset prices as a monetary policy instrument. Bernanke et al. (1999b) and Lowe and Borio (2002) suggest that monetary policy need not react to asset prices but rather clean the effects of bubbles once they burst. Gilchrist and Leahy (2002) notes that including asset prices in monetary policy rules marginally improves output and inflation stability. The drawback of these studies is their non-consideration of welfare analysis. Hence, their evaluations of the implications of financial indicators in monetary policy are based solely on the output-inflation stabilisation framework. This makes it difficult to measure the benefit of alternative monetary policy specifications that responds to financial indicators. However, Cecchetti et al. (2002) believes the possible gain of including asset price information in designing monetary policy depends on the source of the shock affecting the underlying asset that is being considered as a policy tool. The study also mentions the possibility of the central bank identifying large misalignments in asset prices ahead of time, allowing it to be well-positioned to respond to any adverse effects.

However, the 2007-2009 financial crisis reopened this debate; monetary policy has been ineffective in curtailing the enormous accumulated loss in output. As a result, it is important for monetary policy to account for financial booms to avoid being overburdened during busts. While it is true that the crisis saw massive asset price misalignment, there was also excessive growth in credit and leverage. This is consistent with the finding in Cecchetti et al. (2002) that varying aspects of asset prices have different implications for the conduct of monetary policy in enhancing economic outcomes. Oscar Jorda and Taylor (2012) shows that the gravity of recessions is systematically linked to a build-up of excessive leverage during preceding expansions. This suggests that the reason for considering credit-driven bubbles as an important indicator is that they are easy to monitor and predict than asset price bubbles (Adrian and Shin, 2010). This discourse leads to a new prospect of defining financial stability in terms of indicators such as spreads, leverage, and credit growth. Overall, the events following the 2007-2009 crisis, as well as the accumulated knowledge about financial crises, moved the argument that monetary policy should react to financial booms and not only to busts; both are at the centre of discussions in academic and policy-making circles.

A considerable number of studies examine the implications of augmenting the monetary policy mechanism to include a response to financial variables. For example, Faia and Monacelli (2007) in a close economy set-up, find that responding to asset prices with a Taylor-rule policy improves welfare if the response to inflation is mild. Gilchrist and Zakrajsek (2011) show that allowing monetary policy to respond to credit spread dampens the adverse effect of financial disruption on the real economy. In Curdia and Woodford (2016) heterogeneity in household spending is considered as the source of credit friction, showing that it is optimal to respond to credit spread when the economy is affected by financial shocks. Angeloni and Faia (2013) indicate that a monetary policy that aggressively responds to inflation and asset prices or bank leverage improves welfare when coupled with capital requirements, compared to the usual inflation and output targeted Taylor-type monetary policy. Lacoviell (2005) considers the information provided by house prices and debt indexation relevant to monetary policy.

These studies do not investigate optimal, simple and implementable policies, as in the study by Schmitt-Grohe and Uribe (2007) of an estimated DSGE model. Their results are specific to the characteristics of the model, choice of financial friction, and type of financial variable included in the policy rule. This chapter thus analyses the effect of augmenting the traditional Taylor-type monetary policy rule to account for corporate bond spread (amongst other indicators) in a credit channel model. Corporate bond spread is endogenously determined in the model, and it reflects the corporate bond premium.[10] According to Gilchrist and Zakrajsek (2011), bond premium is a timely indicator identifying stress in the financial industry.

There are two traditional approaches in the literature to the implementation of optimal policy. The first is reflected in Schmitt-Grohe and Uribe (2007) on the monetary policy coefficient that maximises a second-order approximation of the household utility function. The second approach focuses on achieving the central bank's objective of minimising loss. The loss function is a weighted average of variances of inflation, output gap and changes to the nominal interest rate. This chapter adopts the first approach of welfare maximising in its exploration of an optimal policy rule.

2.3 The Model

The model presented is similar to the one analysed in Chapter 1. The financial friction mechanism for the flexible price economy is modified by only making it count for the steady-state values of return on capital; this distinguishes the effect of financial friction, particularly its welfare implications, on the flexible price economy. This modification then allows for the definition of the output gap for the sticky-

[10] The excess bond premium is the difference between corporate bond yield in data and what model predict it to be.

price economy relative to output in the flexible price economy. In addition, the model in Chapter 1 paved the way for a fairly efficient comparison of welfare in a sticky economy with optimal policies relative to a flexible price and frictionless economy. Another minor deviation from the model set-up presented in Chapter 1 is in the specification of a shock to the external finance premium. In this chapter, the shock directly affects the leverage of entrepreneurs and, hence, it is also impacted by the elasticities of changes in leverage. Regardless of the specification, it is still a shock to credit supply and represents the functionality of financial intermediaries.

Again, the model builds on Smets and Wouters (2003, 2007) which introduces a DSGE model for the US economy that incorporates many types of real and nominal frictions. The following are added to the model: (i) an Epstein-Zin preference as in Rudebusch and Swanson (2012) to help differentiate the coefficient of risk aversion from the elasticity of substitution; and (ii) Financial friction, as in Bernanke et al. (1999a), in the form of a cost-verification problem, to endogenously describe financial industry behaviour. The financial friction added takes on a simple form, as in Christensen and Dib (2008). Furthermore, the following adjustments are made to the benchmark Smets and Wouters (2003, 2007) model: (i) Fixed cost in production is removed because of the introduction of financial friction; and (ii) Capital utilisation is redefined. However, unlike Smets and Wouters (2007), where wage and price mark-up shocks are included alongside the Kimball aggregator, this chapter follows an approach similar to Smets and Wouters (2003) and uses the Dixit-Stiglitz aggregator for aggregating prices in goods and labour markets. In addition, a wage mark-up shock is replaced with labour supply, and the price mark-up shock is assumed to follow an AR(1) process.

2.3.1 Household

Household j maximise non-separable instantaneous utility, which is a function of goods $C_t(j)$ and labour $L_t(j)$

$$U(C_t, L_t) = \varepsilon_t^{\beta}\left(\frac{1}{1-\sigma_C}\right)\left(C_t(j) - \eta_c C_{t-1}(j)\right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^{w}\left(L_t(j)\right)^{1+\sigma_L}\right) \tag{2.3.1}$$

where ε_t^{β} is the household's preference shock; η_c is the external habit formation parameter; ε_t^{w} is the labour supply shock; σ_C is the elasticity of substitution in goods, and σ_L is the elasticity of labour disutility. The elasticity of substitution is differentiated from the coefficient of risk aversion by households having a recursive Epstein-Zin preference given by

$$V_t(j) = U(C_t(j), L_t(j)) + \beta v_t(j) \tag{2.3.2}$$

where

$$v_t(j) = \mathbb{E}_t(V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}}$$

Each household j in period t consumes $C_t(j)$ and invests in government bonds issued through financial intermediary $B_t(j)$. This bond is discounted at the risk-free rate, R_t. They decide how many hours to work $L_t(j)$, pay tax T_t to the government and receive dividends Div_t from the labour union. The household budget constraint is given by:

$$C_t(j) + \frac{B_t(j)}{R_{t+1}P_t} \leq \frac{W_t^h(j)L_t(j)}{P_t} + \frac{B_{t-1}(j)}{P_t} + \frac{Div_t}{P_t} + T_t \tag{2.3.3}$$

Therefore, household j's optimisation problem is to maximise V_0 subject to (2.3.2) and the budget constraint (2.3.3), where P_t is the price level at time t. The following are the equilibrium equations from the household's optimisation problem:

$$\frac{W_t^h}{P_t} = -\frac{U_L(C_t, L_t)}{U_C(C_t, L_t)} \tag{2.3.4}$$

$$\Xi_t = U_C(C_t, L_t), \tag{2.3.5}$$

with $U_C(C_t, L_t)$ and $U_L(C_t, L_t)$ representing marginal consumption and marginal disutility, respectively. The SDF at time t for a pay off in time $t+1$ is given as

$$\Lambda_{t,t+1} = \beta \left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}} \right]^{-\sigma_E} \frac{\Xi_{t+1}}{\Xi_t} \tag{2.3.6}$$

$$\frac{1}{R_{t+1}} = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\pi_{t+1}} \right] \tag{2.3.7}$$

2.3.2 Labour Market

Households directly supply their homogeneous labour to an intermediate labour union which, in turn, differentiates their labour services and sets wages subject to Calvo pricing. This differentiated labour is packaged by individuals called labour packers. As a result, there are two sub-sectors in the labour market, as discussed in the following.

2.3.2.1 Labour Packers

The labour-union differentiated labour $H_t(l)$ is bought and packaged by labour packers who sell H_t for W_t, thus maximising their profit as

$$\max_{H_t(l)} \quad W_t H_t - \int_0^1 W_t(l) H_t(l)\, \mathrm{d}l$$
$$\text{s.t} \quad H_t = \left(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}}\, \mathrm{d}l \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

Their optimisation problem above gives the labour demand schedule as

$$H_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} H_t. \tag{2.3.8}$$

The wage received by labour packers, which is also the cost of wage to entrepreneurs, is

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w}\, \mathrm{d}l \right)^{\frac{1}{1-\epsilon_w}} \tag{2.3.9}$$

2.3.2.2 Labour Unions

Labour unions hire a raw labour force L_t from households with training conducted to differentiate them based on their skills. They take the marginal rate of substitution as the cost of labour services in their negotiations with labour packers. The mark-up above this marginal disutility is distributed to the households in the form of dividend Div_t, as seen in the household's budget constraint. The union is subjected to nominal rigidities and can only readjust wages with probability $1 - \zeta_w$ in each period; therefore, they optimise wages over the period in which they cannot change the price. In the period when they are unable to re-optimise wages, they partially index the previously optimised wage to reflect lagged inflation.

$$\max_{\tilde{W}_t(l)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \Big(W_{t+s}(l) H_{t+s}(l) - W_{t+s}^h L_{t+s}(l) \Big)$$

$$\text{s.t} \quad H_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}$$

$$H_{t+s}(l) = L_{t+s}(l)$$

$$W_{t+s}(l) = \tilde{W}_t(l) \Big(\prod_{l=1}^{s} \pi_{t+l-1}^{l_w} \pi_{ss}^{1-l_w} \Big) \quad \text{for} \quad s = 1, \ldots, \infty$$

Solving the above optimisation problem gives

$$\frac{\tilde{W}_t}{W_t} = \left(\frac{\epsilon_w}{1-\epsilon_w} \right) \left(\frac{G_t^{w2}}{G_t^{w1}} \right) \tag{2.3.10}$$

where

$$G_t^{w1} = W_t H_t + \mathbb{E}_t \left[\zeta_w \Lambda_{t,t+1} \left(\frac{W_t}{W_{t+1}} \right)^{1-\epsilon_w} \left(\pi_t^{l_w} \pi_{ss}^{1-l_w} \right)^{1-\epsilon_w} G_{t+1}^{w1} \right] \tag{2.3.11}$$

and

$$G_t^{w2} = W_t^h H_t + \mathbb{E}_t \left[\zeta_w \Lambda_{t,t+1} \left(\frac{W_t}{W_{t+1}} \right)^{-\epsilon_w} \left(\pi_t^{l_w} \pi_{ss}^{1-l_w} \right)^{-\epsilon_w} G_{t+1}^{w2} \right] \tag{2.3.12}$$

The aggregate wage expression is

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{1-\epsilon_w} + \zeta_w \Big(\gamma \pi_{t-1}^{l_w} \pi_{ss}^{1-l_w} W_{t-1} \Big)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \tag{2.3.13}$$

There is wage dispersion cost as a result of the discrepancy between labour demanded and labour supplied. This implies that

$$H_t \neq L_t, \quad \text{and} \quad W_t \neq W_t^h \implies L_t = \Delta_t^w H_t$$

The explicit expression of the wage dispersion cost considering the sticky wage is

$$\Delta_t^w = (1-\zeta_w)\int_0^1 \left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w} \mathrm{d}l + \zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w}\Delta_{t-1}^w. \tag{2.3.14}$$

2.3.3 Capital Goods Producers

Capital goods producers take the final good I_t to produce investment goods $\tilde{I}_t$. They work in a perfectly competitive environment and are faced with the cost of changing the flow of investments. As a result, they choose the quantity of investment I_t to maximise their profit

$$\Pi_t^I = Q_t\tilde{I}_t - I_t$$

$$\max_{I_t\tilde{I}_t} \quad \mathbb{E}_0\left[\sum_{t=0}^{\infty}\Lambda_{0,t}\Pi_t^I\right]$$
$$\text{s.t} \quad \tilde{I}_t = \varepsilon_t^i\left[I_t - \frac{\Psi}{2}\left(\frac{I_t}{I_{t-1}}-\gamma\right)^2 I_t\right]$$

$$1 = Q_t\varepsilon_t^i\left[1-\Psi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-\gamma\right)-\frac{\psi}{2}\left(\frac{I_t}{I_{t-1}}-\gamma\right)^2\right]+\mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}\varepsilon_{t+1}^i\Psi\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t}-\gamma\right)\right]$$

where Ψ is the investment adjustment shock and ε_t^i is the shock to investment.

2.3.4 Consumer Goods Market

This comprises retailers, wholesalers, and entrepreneurs.

2.3.4.1 Retailers (Final Good Producer)

The retailers are the final good (Y_t) producers. The final good is a composition of differentiated goods from wholesalers $Y_t(i)$. It is allocated to consumption, investment and government expenditure. The technology used in transforming these differentiated goods is given in the form of a Dixit-Stiglitz aggregator. Their optimisation problem gives the demand curve expression as:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t \tag{2.3.15}$$

The Law of Motion (LOM) for price is

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_p}\,\mathrm{d}i\right)^{\frac{1}{1-\epsilon_p}} \tag{2.3.16}$$

2.3.4.2 Wholesalers

Wholesalers buy intermediate goods M_t from the entrepreneur who owns firms. These intermediate goods are differentiated without cost, taking the demand function previously derived from retailers' first-order conditions. Wholesale prices are subject to Calvo pricing, which introduces nominal rigidities into the model. Similar to Smets and Wouters (2007), the optimal price set by wholesalers allowed to re-optimise gives rise to the following optimisation problem.

$$\begin{aligned}
\max_{\tilde{P}_t(i)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Lambda_{t,t+s}\left[\tilde{P}_t(i) X^p_{s,t} Y_{t+s}(i) - MC_{t+s} M_{t+s}(i)\right] \\
\text{s.t} \quad & Y_{t+s}(i) = Y_{t+s}\left(\frac{\tilde{P}_t(i) X^p_{s,t}}{P_{t+s}}\right)^{-\epsilon_p} \\
& Y_{t+s}(i) = \varepsilon^p_t M_{t+s}(i),
\end{aligned}$$

where

$$X^p_{s,t} = \begin{cases} 1 & \text{for} \quad s = 0 \\ \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) & \text{for} \quad s = 1 \dots \infty. \end{cases}$$

The first constraint is the demand schedule (2.3.15), which is obtained from the retailer optimisation problem. The second constraint represents a linear production function used to differentiate the intermediate good, where ε^p_t is the production function for transforming the intermediate good, and $\tilde{P}_t(i)$ is the newly optimised price. The inability to change this price for some period allows wholesalers to only partially index the current price to lagged inflation represented by $X^p_{t,s}$. The probability of being able to re-optimise price is ζ_p and ι_p measures the degree of price indexation. Solving the optimisation problem above gives the optimal newly set price for those allowed to reset price as

$$\frac{\tilde{P}_t(i)}{Pt} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\left(\frac{G_t^{p2}}{G_t^{p1}}\right),$$

where

$$G_t^{p1} = P_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{1-\epsilon_p} G_{t+1}^{p1}\right]$$

and

$$G_t^{p2} = \varepsilon_t^p MC_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{-\varepsilon_t^p} G_{t+1}^{p2}\right].$$

Aggregate price (2.3.16) is the sum of the newly reset price and the partially indexed price, which is expanded as

$$P_t = \left((1-\zeta_p)\tilde{P}_t(i)^{1-\epsilon_p} + \zeta_p\left(P_{t-1}\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}\right)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}.$$

Simplifying the above gives

$$1 = (1-\zeta_p)\left(\frac{\tilde{P}_t(i)}{P_t}\right)^{1-\epsilon_p} + \zeta_p\left(\frac{\pi_{t-1}^{l_p}\pi_{ss}^{1-l_p}}{\pi_t}\right)^{1-\epsilon_p} \tag{2.3.17}$$

The price dispersion expression is derived as follows.

$$\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i$$

$$\Delta_t^p = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p. \tag{2.3.18}$$

2.3.4.3 Entrepreneurs

Each entrepreneur owns a firm i that uses the following technology

$$M_t(i) = \varepsilon_t^a\Big(K_{t-1}(i)U_t^k(i)\Big)^{\alpha}\Big(\gamma^t H_t(i)\Big)^{1-\alpha} \tag{2.3.19}$$

Each entrepreneur hires labour H_t paying W_t as a wage to the labour union (which, in turn, pays households). They also buy investment goods $\tilde{I}_t$ from capital goods producers. This portion is added to their existing stock of capital K_{t-1} to make up for the next period's capital K_t, where K_t denotes the capital choice at the end of period $t-1$ that is to be used for period t production. It is believed that entrepreneurs cannot fully finance the purchase of capital needed with their net worth. Therefore, they borrow in the form of loans from banks to finance the capital stock desired in excess of their net worth. Entrepreneurs also experience idiosyncratic shocks that affect their capital holdings; for this reason, not all entrepreneurs survive the system. The entrepreneur's problem is given below

$$\begin{aligned}
\max_{H_t,K_t,U_t^k} \mathbb{E}_t&\Big[\sum_{s=0}^{\infty}(1-\varphi)^{s-1}\Lambda_{t,t+s}N_{t+s}\Big] \\
\text{s.t}\quad & D_{t-1} = Q_{t-1}K_{t-1} - N_{t-1} \\
& N_t = MC_tM_t - W_tH_t + (1-\delta_t^k)Q_tK_{t-1} - \frac{R_{t-1,t}^N}{\pi_{t-1,t}}D_{t-1} \\
& M_t \le \varepsilon_t^a\Big(K_tU_t^k\Big)^{\alpha}\Big(\gamma^tH_t\Big)^{1-\alpha} \\
& K_t \le (1-\delta_t^k)K_{t-1} + \tilde{I}_t \\
& \delta_t^k = \delta + \psi_u\frac{MPK_{ss}}{Q_{ss}}\Big(U_t^{k\,\frac{1}{\psi_u}} - 1\Big)
\end{aligned}$$

At each period, entrepreneurs survive with the probability φ. The individual entrepreneur's concern is mostly for their lifetime net worth, suggesting they wish to maximise this. Each of the above constraints in the entrepreneur's problem is explained as follows: The first constraint indicates how much more an entrepreneur needs to borrow to make up for the purchase of capital for the next period's production. This fund is obtainable from households through the financial intermediaries that set the terms for entrepreneurs. The second constraint is the LOM of net worth. This indicates that the entrepreneur retains the capital share after depreciation, less debt repayments. The third constraint is the production function, followed by the LOM of capital, which is the fourth constraint. Lastly, the fifth constraint is depreciation.

The marginal product of capital is given below:

$$MPK_t = \alpha \frac{MC_t M_t}{K_{t-1}} \tag{2.3.20}$$

Solving the entrepreneur's optimisation problem gives the optimum capital utilisation as

$$U_t^k = \left(\frac{\frac{MPK_t}{Q_t}}{\frac{MPK_{ss}}{Q_{ss}}} \right)^{\psi_u} \tag{2.3.21}$$

Equation (2.3.21) determines the relationship between the utilisation rate and the marginal product of capital. The elasticity of utilisation cost with respect to input from capital is ψ_u and $R_{t,t+1}^k$ is the realised return on capital; the latter is different from the return requested by the financial intermediary on loan $(R_{t,t+1}^N)$ that was agreed at the loan's inception.

$$\mathbb{E}_t \left[\Lambda_{t,t+1} R_{t,t+1}^K \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{1}{\pi_{t,t+1}} \right] R_{t,t+1}^N \tag{2.3.22}$$

The partial derivative of K_t, implies that the return on capital is

$$\mathbb{E}_t \left[R_{t,t+1}^K \right] = \mathbb{E}_t \left[\frac{MPK_{t+1}}{Q_t} + (1 - \delta_{t+1}^k) \frac{Q_{t+1}}{Q_t} \right] \tag{2.3.23}$$

The expression (2.3.23) is the ex-post real return on capital. It is the sum of income gain (i.e., the marginal real revenue on capital, which is evaluated in terms of real capital) and capital gain (i.e., the real price change of remaining capital). However, it is important to note that $\frac{R_{t,t+1}^N}{\pi_{t,t+1}} \neq R_{t,t+1}^K$ because the marginal product of capital MPK_{t+1} and $\pi_{t,t+1}$ depend on the realisation of the shocks at $t+1$.

The LOM representation of the real net worth of all entrepreneurs is as follows.

$$N_t = \varphi E_t + (1-\varphi)\varepsilon_t^N, \tag{2.3.24}$$

where E_t represents surviving entrepreneurs' net worth and is given by

$$E_t = \left(R^K_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}\right)Q_{t-1}K_{t-1} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}}N_{t-1}. \tag{2.3.25}$$

Since the amount borrowed by entrepreneurs is $D_{t-1} = Q_{t-1}K_{t-1} - N_{t-1}$, the nominal debt repayment as agreed at time $t-1$ is $R^N_{t-1,t}(Q_{t-1}K_{t-1} - N_{t-1})$. The ex-ante nominal contract rate deflated by ex-post realised inflation is $\frac{R^N_{t-1,t}}{\pi_{t-1,t}}$ and the net worth of existing entrepreneurs E_t is the product of realised gross returns and capital less the product of the contracted borrowing rate and the amount of borrowings.

2.3.5 Financial Intermediary

Financial intermediaries act as a go-between for households and entrepreneurs in financial transactions. They lend money deposited by households (in the form of bonds, loans etc.) to entrepreneurs at a rate higher than the risk-free rate ($R_{t,t+1}$). Although it is cheap for entrepreneurs to obtain internal finance, it is costly to source funds externally. External funds can be obtained through loans, bonds, equity and other sources. The existence of market imperfections in the form of asymmetric information between market participants is probably a good explanation for the high cost of external funds. Entrepreneurs cannot obtain loans at the risk-free rate because financial intermediaries cannot easily observe their output. It is costly for financial intermediaries to observe the realised returns of entrepreneurs as they (the banks) have to pay a state-verification cost; this suggests that the cost of sourcing external finance is different from the economy's risk-free rate. At time t, the rate of return on the loans to entrepreneurs from time t to $t+1$ is agreed on and is given by

$$R^N_{t,t+1} = S_{t,t+1}R_{t,t+1}. \tag{2.3.26}$$

The nominal external finance premium is

$$S_{t,t+1} = S\left(\varepsilon_t^s \frac{Q_{t-1}K_{t-1}}{N_{t-1}}\right), \quad \text{where} \quad S'() < 0 \quad \text{and} \quad S(1) = 1 \tag{2.3.27}$$

The external finance premium, seen in equation (2.3.27), is increasing in the amount entrepreneurs borrow relative to their net worth. Like every contract between borrowers and lenders, there is a need for the borrower to post collateral, which, in this case, is the entrepreneur's net worth as observed by the financial intermediary before the loan issuance. Therefore, the spread $S_{t,t+1}$ is known at time t, and it has a functional form given by

$$S_{t,t+1} = S_{ss}\left(\varepsilon_t^s \frac{Q_{t-1}K_{t-1}}{N_{t-1}} \frac{N_{ss}}{K_{ss}}\right)^{\psi_S}, \tag{2.3.28}$$

where ψ_S is the elasticity of external finance premium to entrepreneur's leverage. The constant S_{ss} is assumed based on the reasoning that lenders only know the aggregate $\frac{Q_{t-1}K_{t-1}}{N_{t-1}}$ and not the level for individual entrepreneurs. ε_t^s is an exogenous disturbance to the entrepreneur's leverage at time t.

2.3.5.1 Asset Pricing

In theory, the current price of any asset should equal the expected future value that is discounted stochastically. The nominal government and corporate bond prices at time t of bonds maturing in n period are defined as follows

$$B_{t,n}^g = \mathbb{E}_t\Big[\Lambda_{t+1}^N B_{t+1,n-1}^g\Big] \tag{2.3.29}$$

$$B_{t,n}^c = \mathbb{E}_t\Big[\Lambda_{t+1}^N B_{t+1,n-1}^c\Big]\left(\frac{1}{S_{t+1}}\right), \tag{2.3.30}$$

where the nominal SDF is $\Lambda_{t+1}^N = \frac{\Lambda_{t+1}}{\pi_{t+1}}$ and the respective return (yield to maturity) in the model is given as

$$R_{t,n}^g = \left(\frac{1}{B_{t,n}^g}\right)^{\frac{1}{n}} \tag{2.3.31}$$

$$R_{t,n}^c = \left(\frac{1}{B_{t,n}^c}\right)^{\frac{1}{n}} \tag{2.3.32}$$

The corporate bond premium in the data is expressed as the sum of the corporate yield in the model and a white noise factor capturing the aspect of the yield not captured in the presented model. The shock is specified such that it is orthogonal to the structural disturbances in the model. The corporate spread in the data is expressed as

$$R_{t,n}^{c\text{data}} = R_{t,n}^{c} + \eta_t^c, \tag{2.3.33}$$

where $R_{t,n}^{c\text{data}}$ represents the corporate bond yield in the data, and $R_{t,n}^{c}$, model's corporate yield. The corporate bond premium captures the aspect of the corporate bond yield in data that is not captured strictly by the model; this is expressed as

$$C_{t,n}^{\text{premium}} = R_{t,n}^{c\text{data}} - R_{t,n}^{c} \tag{2.3.34}$$

Therefore, 2.3.34 is regarded as the corporate bond premium. Corporate bond spread (reflecting the bond premium) can easily be deduced in the DSGE model, using the expressions above for government and corporate bonds, as:

$$C_{t,n}^{b} = R_{t,n}^{c\text{data}} - R_{t,n}^{g} \tag{2.3.35}$$

2.3.6 Aggregate Resource Constraint and Government

To aggregate the model variables, it is standard practice in macroeconomics to assume that a representative household chooses consumption and labour. This assumption is adopted in aggregating household choices of consumption and labour. However, to aggregate firms (labour and consumer goods markets), price and wage dispersions, obtained in (2.3.18) and (2.3.14), respectively, are defined. The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations in inflation and output from their respective target levels;

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y}\right]^{1-\rho_R} \left[\frac{\left(\frac{Y_t}{Y_{t-1}}\right)}{\left(\frac{Y_t^f}{Y_{t-1}^f}\right)}\right]^{\psi_{\mathrm{d}y}} \varepsilon_t^r, \tag{2.3.36}$$

where R_{ss} is the steady-state nominal rate, π_{ss} is the steady-state inflation, and Y_t^f the output in a flexible price economy. The interest-rate smoothing parameter is denoted by ρ_R, and the shock to the monetary policy ε_t^r is

$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r, \quad \eta_t^r \sim \mathcal{N}(0, \sigma_r). \tag{2.3.37}$$

Market clearing implies that the following holds.

$$Y_t = C_t + G_t + I_t \tag{2.3.38}$$

The implication of (2.3.38) is that the final good produced in the economy is allocated to consumption, government and investment. Government spending is expressed relative to the steady-state output path as

$$\varepsilon_t^g = \frac{G_t}{y_{ss}\gamma^t}, \tag{2.3.39}$$

where ε_t^g follows an exogenous AR(1) process:

$$\ln \varepsilon_t^g = (1-\rho_g)\ln g + \rho_g \ln \varepsilon_{t-1}^g + \rho_{ga} \ln \varepsilon_{t-1}^a + \eta_t^g, \quad \eta_t^g \sim \mathcal{N}(0, \sigma_g) \tag{2.3.40}$$

2.4 Methodology

This section discusses the methodology for setting up the welfare criterion to evaluate the optimal policy rule. First, the welfare objective criterion is set up and thereafter, the alternative Taylor rules, the optimal parameters of which are to be calculated, are specified.

2.4.1 Simple and Implementable Rules

Adopting the methodology of Schmitt-Grohe and Uribe (2007), optimal monetary policies are evaluated by computing the second-order approximation of the policy functions that maximises households' lifetime utility using the perturbation method. The economy with a simple Taylor rule responding solely to inflation and output (i.e. output growth) is regarded as the benchmark economy. In addition to responding to inflation and output, policies in alternative economies respond to corporate bond spread, leverage, external finance premium, and net worth. Overall, two main policy versions are considered for both the benchmark and alternative economies.

The monetary policy rule in (2.3.36) with the exclusion of the shock is the first version for the benchmark economy. In addition to the instrument in the benchmark economy, the corresponding policy for alternative economies responds to financial indicators.[11] The second policy version is a stylised policy rule where, in the benchmark, the target is a combination of inflation and output gap, or inflation and output growth. In both cases, there is interest-rate smoothing. The corresponding version two policy for alternative economies, however, replaces the response to output with the financial indicator peculiar to each of the alternative economies. It is important to note that these policy rules are without monetary policy shock.

Conducting optimal policy analysis is synonymous with finding the monetary rule parameter combinations (i.e. ρ_R, ψ_π, ψ_y, $\psi_{\mathrm{d}y}$, and ψ_{cb}) that maximise household lifetime welfare. According to Schmitt-Grohe and Uribe (2007), for a policy to be implementable, three requirements must be met. First, the policy rule must ensure local uniqueness of the rational expectations of equilibrium. Second, the rule must induce a non-negative equilibrium for the nominal interest rate, which is achieved by imposing $2\sigma_R < R_{ss}$, where σ_R is the unconditional standard deviation of the nominal interest rate. Third, the policy coefficients must be limited to the arbitrary interval $[0, 3]$. This chapter considers a somewhat relaxed interval bound for the policy parameters, especially for ψ_{cb}, the magnitude and dimensions of which are not specifically known but must be established. The first step in implementing optimal policy and welfare analysis is to find policy parameters that maximise household lifetime conditional utility. These parameters are sought using the household's recursive Epstein-Zin preference V_t (see (2.3.2)) as the objective function.

The aggregate conditional welfare using Epstein-Zin preference is

$$\mathbb{E}_0[V_t] = \mathbb{E}_0\bigg[U(C_t L_t) + \beta v_t\bigg], \tag{2.4.1}$$

where $\mathbb{E}_0$ represents the conditional expectation, and $v_t = \mathbb{E}_t(V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}$. According to Schmitt-Grohe and Uribe (2004a), the appropriate welfare measure is conditional welfare. Different monetary policy regimes can be associated with different stochastic steady states in which unconditional welfare will give varying starting points for each policy regime. The transitional dynamics from the stochastic steady state of unconditional welfare make it inappropriate for our analysis since monetary policy parameters (i.e., ρ_R, ψ_π, ψ_y, and $\psi_{\mathrm{d}y}$) are not reflected. Hence, welfare analysis conditioned on being in the non-stochastic steady state will enhance the comparability of welfare across the different economies. This is due to the fact that all economies start at the same initial point if conditioned on the deterministic steady state before transitioning. Based on the above, the conditional lifetime utility of households in the benchmark economy has the following expression.

[11]The alternative economies are labelled using the name of the financial indicator they respond to. For example, the row labelled corporate spread in Panel B of Table 2.3 implies that corporate bond spread is the financial indicator in the policy rule- this is in addition to inflation and output target in the benchmark economy.

$$\mathbb{E}_0[V_t^b] = \mathbb{E}_0\Bigg[U(C_t^b L_t^b) + \beta v_t^b\Bigg], \tag{2.4.2}$$

where the instantaneous utility function in the benchmark economy is

$$U(C_t^b, L_t^b) = \varepsilon_t^{\beta}\frac{1}{1-\sigma_C}\Big(C_t^b - \eta_c C_{t-1}^b\Big)^{1-\sigma_C}\exp\Big(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{b^{1+\sigma_L}}\Big). \tag{2.4.3}$$

The corresponding conditional lifetime utility for alternative economies has a superscript of a and is given as

$$\mathbb{E}_0[V_t^a] = \mathbb{E}_0\Bigg[U(C_t^a L_t^a) + \beta v_t^a\Bigg] \tag{2.4.4}$$

with

$$U(C_t^a, L_t^a) = \varepsilon_t^{\beta}\frac{1}{1-\sigma_C}\Big(C_t^a - \eta_c C_{t-1}^a\Big)^{1-\sigma_C}\exp\Big(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{a^{1+\sigma_L}}\Big). \tag{2.4.5}$$

It is assumed that in the benchmark economy, monetary policy only targets macroeconomic indicators by responding to inflation and output. There are two such policies that will be compared within alternative economies (i.e., responding to financial indicators). Hence, the monetary policy rules adopted in the benchmark economy are

- Benchmark version 1

$$\frac{R_t}{R_{ss}} = \Big(\frac{R_{t-1}}{R_{ss}}\Big)^{\rho_R}\Bigg[\Big(\frac{\pi_t}{\pi_{ss}}\Big)^{\psi_\pi}\Big(\frac{Y_t}{Y_t^f}\Big)^{\psi_y}\Bigg]^{1-\rho_R}\Bigg[\frac{(\frac{Y_t}{Y_{t-1}})}{(\frac{Y_t^f}{Y_{t-1}^f})}\Bigg]^{\psi_{\mathrm{d}y}}$$

- Benchmark version 2

Policy 2a

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y}\right]^{1-\rho_R}$$

Policy 2b

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left[\frac{\left(\frac{Y_t}{Y_{t-1}}\right)}{\left(\frac{Y_t^f}{Y_{t-1}^f}\right)}\right]^{\psi_{\mathrm{d}y}}\right]^{1-\rho_R}$$

The first policy in the benchmark economy responds to inflation, output gap and change in output. The second policy regime has two versions – one responds to inflation and output gap, while the other responds to inflation and change in output. All these policies consider interest-rate smoothing. Similarly, the alternative economy has two main policies that correspond to the benchmark economy. In the alternative economy, there are two sub-policy versions comparable to benchmark version 1. There is just one policy rule for the second version of the alternative economies. The second policy version is a simple rule responding to inflation and various financial indicators. Generally speaking, in the alternative economy, the central bank is assumed to be monitoring the growth in some financial indicators. These policies are expressed as

- Alternative version 1

Policy 1a

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y} \left(\frac{X_t}{X_{t-1}}\right)^{\psi_{cb}}\right]^{1-\rho_R} \left[\frac{\left(\frac{Y_t}{Y_{t-1}}\right)}{\left(\frac{Y_t^f}{Y_{t-1}^f}\right)}\right]^{\psi_{\mathrm{d}y}}$$

Policy 1b

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y} \left(\frac{X_t}{X_{t-1}}\right)^{\psi_{cb}}\right]^{1-\rho_R}$$

- Alternative version 2

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{X_t}{X_{t-1}}\right)^{\psi_{cb}}\right]^{1-\rho_R}$$

In the alternative policies above, $X_t = \{C_{t,40}^b, \frac{Q_t K_t}{N_t}, S_{t,t+1}, N_t\}$ are the financial indicators (i.e. corporate bond spread, leverage, external finance premium, and net worth) incorporated in the policy rule. Additionally, ψ_{cb} is the parameter associating the policy response to these indicators. In this instance, leverage is a measure of the loan to value ratio. In simple terms, it is the ratio of capital required by entrepreneurs to remain in business relative to their net worth. Welfare improvement is analysed for the benchmark and alternative economies relative to the flexible price economy[12]. This is expressed in consumption units using the certainty conditional and unconditional consumption equivalent.

2.4.2 Consumption Equivalent Welfare Cost

It would be uninformative to only compare the welfare levels of economies. A more promising comparison is to find the welfare cost in consumption units. To evaluate the consumption-equivalent welfare cost, let λ be the portion of consumption in the flexible price economy that households are willing to forgo to be well off either in the benchmark or alternative economies. The expressions (2.4.6) and (2.4.7) relate the welfare loss in the flexible price economy to the benchmark and alternative economies, respectively.

$$\mathbb{E}_0[V_t^b] = \mathbb{E}_0\Big[U((1-\lambda^c)C_t^f L_t^f) + \beta v_t^f\Big] \tag{2.4.6}$$

$$\mathbb{E}_0[V_t^a] = \mathbb{E}_0\Big[U((1-\lambda^c)C_t^f L_t^f) + \beta v_t^f\Big], \tag{2.4.7}$$

where λ^c, as in equations (2.4.6) and (2.4.7), represents the conditional welfare loss to the flexible price economy if households decide to be in economies with benchmark and alternative policies, respectively.

$$\mathbb{E}_0[V_t^f] = \mathbb{E}_0\Big[U(C_t^f L_t^f) + \beta v_t^f\Big] \tag{2.4.8}$$

That is, λ^c measures the proportion of the reference (flexible price) economy's consumption process lost to sticky price economies under different monetary policy regimes. As a robustness check, the unconditional consumption equivalent loss λ^u is calculated. A positive value for λ^c and λ^u indicates that welfare is higher in the flexible price economy. The positive unit is the proportion of households' consumption they are willing to forfeit in a flexible price economy to be in either the benchmark or alternative economies. Hence we evaluate the consumption equivalent welfare cost of the benchmark and alternative monetary policy relative to the time-invariant stochastic equilibrium allocation in a flexible price economy conditioned on the steady state using Pfeifer and Born (Accessed in June 2020a) and the unconditional welfare cost using the code Pfeifer and Born (Accessed in June 2020b).

[12] Prices are fully flexible, and household's also have recursive Epstein-Zin preference.

The full set of model disturbances is included, other than the monetary policy shock that is excluded in the enactment of the optimal policy rule and consumption-equivalent welfare analysis. This inclusion is essential because the exogenous processes that are included in the make-up of the model presented are an integral source of business cycle fluctuations. This is also due to the fact that the computation of the optimal policy rule and welfare is dependent on the structure of shocks to the economy.

2.5 Calibration and Estimation

The majority of the model parameters are estimated. Several others are calibrated before proceeding to establish the optimal policy rule and welfare analysis. The DSGE model is estimated using a Bayesian estimation technique with the simple Taylor rule as the operational monetary policy. The model is estimated using the same seven macroeconomic data used in Smets and Wouters (2007) alongside the 40-quarter nominal BAA corporate yield over the period 1966-2006. The preference parameters (σ_C and σ_L) are calibrated to the same value as in Rudebusch and Swanson (2012). The Epstein-Zin preference parameter σ_E is calibrated to -65 (see Chapter 1), resulting in a mean term premium of 100 basis points. Price and wage elasticities are calibrated to 5, implying a 25% mark-up. The welfare and optimal policy analysis establishes the gross steady-state inflation calibration to 1.005, which is equivalent to the annual 2% central bank inflation target (Smets and Wouters, 2007). The other preference parameters (σ_C and σ_L) are calibrated to the same value as in Rudebusch and Swanson (2012). The steady-state external finance premium is set to 1.0064, corresponding to an annual risk spread of 256 basis points, equal to the sample average spread between the business prime lending rate and the three-month Treasury bill rate.

The prior of all parameters follows the same distribution as in Smets and Wouters (2007). However, unlike Smets and Wouters (2007), the mean and degrees of freedom are relaxed (i.e. increased) for all the stochastic processes. In addition, no boundary is set for all estimated parameters. Parameters governing the stochastic process of the financial intermediation process are also assumed to follow an inverse-gamma distribution with a beta-distributed persistence. The elasticity of the external finance premium assumes a normal distribution similar to Gilchrist et al. (2009) with a mean of 0.10 and standard deviation of 0.02.

The model is estimated using macroeconomic and 40-quarter BAA corporate yield data. An additive measurement error shock to the model's corporate bond yield that is orthogonal to the state of the economy helps in capturing the corporate bond premium. Hence, the corporate bond spread adopted as an additional instrument for monetary policy is defined as the difference between the 40-quarter corporate bond yield in the data (i.e. the corporate bond yield in the model plus the corporate bond premium) and the 40-quarter government bond yield.

Table 2.1: Calibrated Parameters and Sources

	Baseline	Description	Source
σ_E	-65	Epstein-Zin parameter	Calibrated in Chapter 1
σ_C	2.00	IES in consumption	Rudebusch and Swanson (2012)
σ_H	3.00	Elasticity of labour	Rudebusch and Swanson (2012)
β	0.995	Subjective discounting factor	Smets and Wouters (2007)
η_g	0.18	Government spending	Smets and Wouters (2007)
δ	0.025	Capital depreciation rate	Smets and Wouters (2007)
θ_p	5	Price elasticity of substitution	Christensen and Dib (2008)
θ_w	5	Wage elasticity of substitution	Christensen and Dib (2008)
S_{ss}	1.0064	Steady state external finance premium	Calculated from prime loan spread
π_{ss}	1.005	Steady state gross inflation	Smets and Wouters (2007)
φ	0.9728	Entrepreneur survival rate	Christensen and Dib (2008)

2.6 Results

In this section, the consequences of adopting alternative specifications of the monetary policy rule are presented and discussed from the perspective of the effects of including a direct response to some financial indicators. In addition, it includes an analysis of the performance of the alternative monetary policies relative to the benchmark policy when the economy is subject to a financial shock; this is done by inspecting the impulse response.

2.6.1 Posterior Estimates

Table 2.1 shows the calibrated parameters and their sources. The result from the Bayesian estimation of the model is presented in Table 2.2, showing the mode, mean and the 5th and 95th percentile of the posterior distribution. Productivity, government spending, labour supply shock, and investment shocks are highly persistent. The shock to the external finance premium is similarly persistent, with an AR(1) coefficient of 89. The posterior mean of external-consumption habit formation is smaller than its prior mean. This is expected because the financial accelerator mechanism coupled with the informational content of the BAA yield enhances the model's ability to generate sufficient internal propagation.

The investment adjustment cost is less than the estimates in DeGraeve (2008) because the dynamics of investment are better captured in this model. The mean posterior estimate of the elasticity of external finance is 5%. This is frequently the calibrated value (see Bernanke et al. (1999a) for example) and is close to the estimate in Christensen and Dib (2008), but somewhat lower than the estimated value in DeGraeve (2008). The importance of the financial accelerator mechanism is supported.

Subsequent to the estimation procedure, model parameters are set to their estimated (Table 2.2) and calibrated values. The optimal policy rule is then identified, and the consumption-equivalent welfare benefit is calculated.

Table 2.2: Prior and posterior distribution of estimated parameters

Description		Prior distribution			Posterior distribution			
		Distr	Mean	St.Dev	Mode	Mean	5%	95%
Productivity shock	σ_a	$\mathcal{I}$	1.00	10.00	0.47	0.47	0.40	0.54
Preference shock	σ_b	$\mathcal{I}$	1.00	10.00	1.22	1.35	1.07	1.63
Government shock	σ_g	$\mathcal{I}$	1.00	10.00	2.07	1.99	1.57	2.41
Investment shock	σ_i	$\mathcal{I}$	1.00	10.00	2.98	2.64	2.19	3.09
Monetary shock	σ_r	$\mathcal{I}$	1.00	10.00	0.30	0.30	0.26	0.33
Price mark-up shock	σ_p	$\mathcal{I}$	1.00	10.00	1.08	1.09	0.82	1.36
Labour supply shock	σ_w	$\mathcal{I}$	1.00	10.00	3.44	3.49	3.02	3.94
Net worth shock	σ_n	$\mathcal{I}$	1.00	10.00	0.46	1.11	0.22	2.14
External finance shock	σ_s	$\mathcal{I}$	1.00	10.00	2.64	4.29	1.72	6.92
Corporate premium shock	η_t^c	$\mathcal{I}$	0.10	2.00	0.04	0.05	0.02	0.07
Productivity persistence	ρ_a	$\mathcal{B}$	0.50	0.25	0.98	0.98	0.97	0.99
Preference persistence	ρ_b	$\mathcal{B}$	0.50	0.25	0.21	0.24	0.07	0.40
Government persistence	ρ_g	$\mathcal{B}$	0.50	0.25	0.99	0.99	0.98	0.997
Investment persistence	ρ_i	$\mathcal{B}$	0.50	0.25	0.98	0.98	0.97	0.99
Monetary persistence	ρ_r	$\mathcal{B}$	0.50	0.25	0.22	0.28	0.16	0.40
Price mark-up persistence	ρ_p	$\mathcal{B}$	0.50	0.25	0.82	0.80	0.68	0.93
Labour supply persistence	ρ_w	$\mathcal{B}$	0.50	0.25	0.99	0.995	0.99	0.999
Net worth persistence	ρ_n	$\mathcal{B}$	0.50	0.25	0.49	0.49	0.10	0.88
External premium persistence	ρ_s	$\mathcal{B}$	0.50	0.25	0.89	0.87	0.79	0.96
Government & output	ρ_{ga}	$\mathcal{N}$	0.50	0.25	0.49	0.49	0.08	0.86
Adjustment cost	Ψ	$\mathcal{N}$	4.00	1.50	3.74	4.33	2.68	5.94
Consumption habit	η_C	B	0.70	0.10	0.43	0.46	0.39	0.54
Wage stickiness	ζ_w	$\mathcal{B}$	0.50	0.10	0.61	0.61	0.57	0.65
Price stickiness	ζ_p	$\mathcal{B}$	0.50	0.10	0.69	0.67	0.61	0.73
Wage indexation	ι_w	$\mathcal{B}$	0.50	0.15	0.76	0.72	0.55	0.89
Price indexation	ι_p	$\mathcal{B}$	0.50	0.15	0.20	0.24	0.10	0.38
Elasticity of utilization	ψ_u	$\mathcal{B}$	0.50	0.15	0.28	0.24	0.12	0.36
Capital share	α	$\mathcal{N}$	0.30	0.05	0.12	0.13	0.11	0.15
Rate smoothing	ρ_R	$\mathcal{B}$	0.75	0.10	0.63	0.63	0.56	0.71
P-inflation	ψ_π	$\mathcal{N}$	1.5	0.25	1.41	1.55	1.33	1.77
P-output gap	ψ_y	$\mathcal{N}$	0.125	0.05	0.05	0.11	0.06	0.17
P-output growth	ψ_{dy}	$\mathcal{N}$	0.125	0.05	0.20	0.14	0.07	0.20
Trend growth rate	$\bar{\gamma}$	$\mathcal{N}$	0.40	0.10	0.42	0.44	0.38	0.51
Elasticity of external finance	ψ_S	$\mathcal{B}$	0.10	0.02	0.07	0.05	0.03	0.07

Note that posterior distribution is obtained using Metropolis-Hastings algorithm using 100,000 draws. Also, $\mathcal{I}, \mathcal{B}, \mathcal{N}$, and $\mathcal{G}$, all denotes Inverse-gamma, Beta, Normal, and Gamma distributions respectively.

The benchmark economy requires policy parameters in its two policy versions that maximise household welfare (2.4.2). Likewise, the alternative economies require policy parameters that maximise household welfare (2.4.4), which is the conditional lifetime utility of households in an economy operating under the alternative policy. After using the perturbation method to compute a second-order approximation to the policy function and lifetime utility for each of the economies under consideration, the optimal policy parameters and welfare analysis are calculated using numerical methods.

2.6.2 Optimal Policy and Welfare Analysis

The results after conducting the analysis in respect of the optimal policy rule and welfare are reported in Tables 2.3 and 2.4. In Table 2.3, policy parameters are constrained to the interval $\psi_\pi \in [1, 3]$, $\psi_y, \psi_{\text{dy}} \in [0, 3]$ and $\psi_{cb} \in [-3, 3]$ before evaluating the optimal parameters that maximise welfare. In Table 2.4 the parameters are not bound (i.e. $\psi_\pi \in [1, \infty]$, $\psi_y, \psi_{\text{dy}} \in [0, \infty]$ and $\psi_{cb} \in [-\infty, \infty]$). In both the bounded and unbounded optimal-parameter cases, Panel A corresponds to the first and second version of the benchmark policy, while Panels B, C and D are for the alternative economies 1a, 1b and 2, respectively. In sum, the policy parameters (i.e. $\rho_R, \psi_\pi \psi_y, \psi_{\text{dy}}$, and ψ_{cb}) that maximise welfare under the specified policies are reported as is the corresponding consumption-equivalent welfare cost. The unconditional and conditional welfare loss relative to the flexible economy are represented by λ^u and λ^c, respectively.

The parameter bounds in Table 2.3 are similar to those arbitrarily chosen in Schmitt-Grohe and Uribe (2007). The result obtained here by setting this bound is partly consistent with the study. In particular, the optimal response to inflation in all the policies analysed is close to the bound, as is the response to the output gap the least. Schmitt-Grohe and Uribe (2007) do not consider a policy rule with output growth or financial indicators. The analysis here indicates that the central bank should react more to economic (inflation and output) and financial conditions.

Broadly speaking (when parameters are bound or unbounded), in the alternative economies where financial indicators are introduced into the Taylor rule, the optimised policy parameters suggest that the central bank should be flexible and consider responding more aggressively in the short-run to inflation, output (to a lesser extent) and financial indicators, rather than leaving it for the long-run. This appears to be the case based on the interest rate inertial (interest-rate smoothing) being less substantial (Schmitt-Grohe and Uribe, 2007). In addition, the results in this chapter show a small response to the output gap but a strong response to output growth.

Results from the bounded optimal policy indicate strict stabilisation of inflation for all economies, particularly the benchmark economies responding only to macroeconomic indicators (see Panel A of Table 2.3). The inflation coefficient of the optimised rule takes the largest value allowed in the search, of 3. In the alternative economies (with bounded and unbounded optimal policy parameters) where the benchmark Taylor rule is extended to include financial indicators, the optimal policy parameter that maximises welfare suggests almost no interest rate smoothing (an exception is the policy with an external finance premium, indicating mild interest rate smoothing).

Table 2.3: Bounded Optimal Policy Parameters and Consumption Equivalent Welfare Cost

	ρ_R	ψ_π	ψ_y	ψ_{dy}	ψ_{cb}	$\lambda^u(\%)$	$\lambda^c(\%)$
A. Benchmark versions							
Estimated	0.6323	1.4116	0.0503	0.1991	——	0.2264	0.2200
Optimal rule version 1	0.7127	2.9986	0.1430	1.7138	——	0.1326	0.1560
Optimal rule version 2a	0.1711	3.0000	0.1922	——	——	0.1476	0.1639
Optimal rule version 2b	0.0661	3.0000	——	0.2787	——	0.1469	0.1639
B. Alternative version 1a							
Corporate spread	0.3136	2.9997	0.2367	0.3187	-3.0000	0.0889	0.1168
Leverage	0.0000	2.9060	0.7060	0.3875	-1.6574	0.0669	0.0996
External finance	0.5114	3.0000	0.1323	0.9339	-2.9913	0.1268	0.1488
Networth	0.0051	3.0000	0.4562	0.2047	0.3584	0.0769	0.1072
C. Alternative version 1b							
Corporate spread	0.0000	2.9631	0.4552	——	-2.1674	0.0924	0.1219
Leverage	0.0000	2.8367	0.5685	——	-1.3805	0.0675	0.0986
External finance	0.0020	3.0000	0.3002	——	-0.1164	0.1433	0.1631
Networth	0.0000	2.9996	0.7681	——	0.5030	0.0753	0.1072
D. Alternative version 2							
Corporate spread	0.0000	2.9996	——	——	-1.2151	0.1123	0.1342
Leverage	0.0027	3.0000	——	——	-1.3007	0.0735	0.1020
External finance	0.0015	2.9992	——	——	-3.0000	0.1233	0.1438
Networth	0.0580	3.0000	——	——	0.3226	0.0887	0.1141

The optimal policy parameters are calculated setting the following parameter bounds $\rho_R \in [0,1]$, $\psi_\pi \in [1,3]$, $\psi_y, \psi_{\text{dy}} \in [0,3]$ and $\psi_{cb} \in [-3,3]$. We initialise all the parameter to their estimated values before finding the optimal values that maximises welfare. Meanwhile, initial value of ψ_{cb} was set to 0 as this parameter was not originally in the model, hence has not been initially estimated.

The output gap takes on the least response in the benchmark versions (1 and 2) and alternative version 1a. However, the response to the output gap in the unbounded alternative optimal policy 1a with leverage ranks higher than the response to output growth. The optimised rule is quite effective as it delivers welfare levels remarkably close to those achieved under the flexible price economy. At the same time, the optimal rules induce a stable rate of inflation; the standard deviation of inflation is relatively low compared to the data (see Tables 2.6 2.7), a feature that also characterises the Ramsey policy.

Consistent with Schmitt-Grohe and Uribe (2007) and Faia and Monacelli (2007), the strong output-gap stabilisation is not welfare improving (this is discussed in subsection 2.6.2.1).

Table 2.4: Unbounded Optimal Policy Parameters and Consumption Equivalent Welfare Cost

	ρ_R	ψ_π	ψ_y	ψ_{dy}	ψ_{cb}	$\lambda^u(\%)$	$\lambda^c(\%)$
A. Benchmark versions							
Estimated	0.6323	1.4116	0.0503	0.1991	——	0.2264	0.2200
Optimal rule version 1	0.8548	4.5114	0.1018	1.6279	——	0.1270	0.1512
Optimal rule version 2a	0.0000	3.2779	0.1540	——	——	0.1439	0.1632
Optimal rule version 2b	0.8789	7.2998	——	18.7760	——	0.1170	0.1480
B. Alternative version 1a							
Corporate spread	0.8316	4.5166	0.1089	1.6583	-0.0168	0.1260	0.1509
Leverage	0.0002	9.9017	2.1918	1.5572	-3.2148	0.0485	0.0899
External finance	0.8296	4.6310	0.3213	1.8379	-0.1378	0.1230	0.1502
Networth	0.0000	16.6544	3.7491	4.1088	1.5675	0.0562	0.1016
C. Alternative version 1b							
Corporate spread	0.0005	4.4978	0.0000	——	-3.3305	0.0743	0.1078
Leverage	0.0079	4.3865	0.3010	——	-1.6352	0.0616	0.0945
External finance	0.1908	4.5329	0.2118	——	-0.0679	0.1421	0.1643
Networth	0.0000	7.2532	1.6639	——	0.7018	0.0637	0.1020
D. Alternative version 2							
Corporate spread	0.0000	5.7612	——	——	-11.8807	0.0394	0.0842
Leverage	0.0000	6.1004	——	——	-1.7850	0.0624	0.0961
External finance	0.1035	4.8685	——	——	-5.4526	0.1170	0.1441
Networth	0.0000	4.7163	——	——	0.3892	0.0792	0.1092

Unconstrained optimal policy and welfare analysis. Optimal policy parameters are calculated by not limiting their bound $\psi_\pi \in [1, \infty]$, $\psi_y, \psi_{dy} \in [0, \infty]$ and $\psi_{cb} \in [-\infty, \infty]$. The only limiting parameter is the interest smoothing $\rho_R \in [0, 1]$. In finding the optimal benchmark policy, parameters are initialise to the estimated value. Meanwhile, the optimal parameter for the benchmark version 1 is used as the initial conditions for the alternative economies. In addition, the initial value of ψ_{cb} is set to 0 as this parameter was not originally in the model, hence has not been estimated.

A stronger policy response to corporate spread, leverage, and external finance premium is recommended and implies a reduction in nominal interest rate. As these indicators increase, the optimal policy rule is that the central bank should be flexible in setting the policy rate. This flexibility implies that preference should be given to forward-looking instruments, hence less interest rate smoothing. Increasing corporate bond spread, leverage, and external finance premiums are possible financial stress indicators. Therefore, a policy rule that suggests a strong response to these indicators is reasonable; it is important to urgently respond to financial stress indicators to prevent consequential and economy-wide spillover. Increasing net worth suggests that the policy rate should be increased as a way of curbing excessive growth that could promote a boom-burst pattern.

The negative policy response to corporate bond spread, leverage, and external finance premium suggests that the central bank should ease the nominal interest rate when there is an increase in these financial indicators. This is because an increase in these indicators is most likely a reflection of a distortion in the supply of credit and the ability to obtain credit, making it difficult and expensive for entrepreneurs to get funds. The ripple effects on the economy cause reduced productivity and increased unemployment, amongst other impacts. A reduction in the interest rate as suggested by the optimal policy rule (i.e. negative ψ_{cb}) will therefore ease the pressure on the economy. Taylor (2008) indicates that the continual use of unconventional measures in easing monetary policy could result in periods of high inflation as in the past, and this could come along with frequent and severe recessions. However, Taylor (2008) also indicates the benefit of systematically adjusting monetary policy for financial sector stress by subtracting a smoothed version of the spread from the interest rate that would otherwise be suggested when targeting inflation and real GDP.[13]

It is essential to note that the policy response appropriate during a period of financial turmoil will be different from the response when the financial market is functioning normally. Times of financial disruption require that policy makers are flexible in their approach by responding appropriately and in a timely manner to the macroeconomic implications of financial market factors.[14] Such adjustments to the policy rule will reduce the likelihood of adverse feedback effects on macroeconomic outcomes and, in the long run, will improve the predictability of any financial stress. This will allow investors and other stakeholders to factor these possibilities into their strategies.

The suggested monetary policy easing in response to increasing leverage seems puzzling (see Tables 2.3 and 2.4); one would expect that as entrepreneurs' leverage increases, so too should the interest rate. In this instance, the optimal policy is evaluated by including all structural shocks except a monetary policy shock. There are two qualitative responses to increasing leverage even though, quantitatively, monetary policy easing has been the optimal response to increasing leverage.

Table 2.5 presents the optimal policy response to an increase in leverage when different shocks are driving the economy to analyse the two qualitative outcomes. The coefficient of the monetary policy response to increasing leverage is mixed, and the net response depends solely on the relative effect of

[13] The spread between Libor at 3-month maturity and an index of overnight federal funds rates expected for the same period (Taylor, 2008).

[14] With less interest rate inertia when required.

the shock. For example, when firms anticipate future possibilities, they want to expand their balance sheet to capture business opportunities; hence, an increase in leverage is positive. Therefore the optimal monetary policy response to increasing leverage depends on the force driving the economy.

Table 2.5: Leverage as Policy Instrument as Decomposed by Specific Shocks

	ρ_R	ψ_π	ψ_y	ψ_{dy}	ψ_{cb}
Productivity Shock	0.0916	2.9999	0.0000	1.1812	0.0088
Investment Shock	0.0000	2.9996	0.3298	0.1695	-2.4908
Financial Shock	0.4815	2.5876	0.0838	0.9312	-0.0734
Government Shock	0.0001	3.0000	0.1215	2.9656	0.0225
Preference Shock	0.3075	3.0000	0.4892	0.4137	0.1209
Price Mark-up Shock	0.2699	1.0713	0.0077	0.4151	-0.1022
Labour Supply Shock	0.5400	2.9931	0.2251	0.0156	0.7262

The alternative version 1a is the specification used in this analysis. By excluding every other shocks, the contributions of individual shocks in the conduct of optimal policy are evaluated. Policy parameter are bounded by 3.

If the central bank strongly targets financial stability through monitoring corporate bond spread, leverage, external finance premiums and net worth, it is possible to attain a considerable welfare level and still achieve inflation and output stabilisation. Interestingly, as Panel D of Tables 2.3 and 2.4 reports, a simple policy rule reacting only to inflation and the respective financial indicators can ensure higher welfare levels than the simple Taylor rule of inflation and output-gap stabilisation.

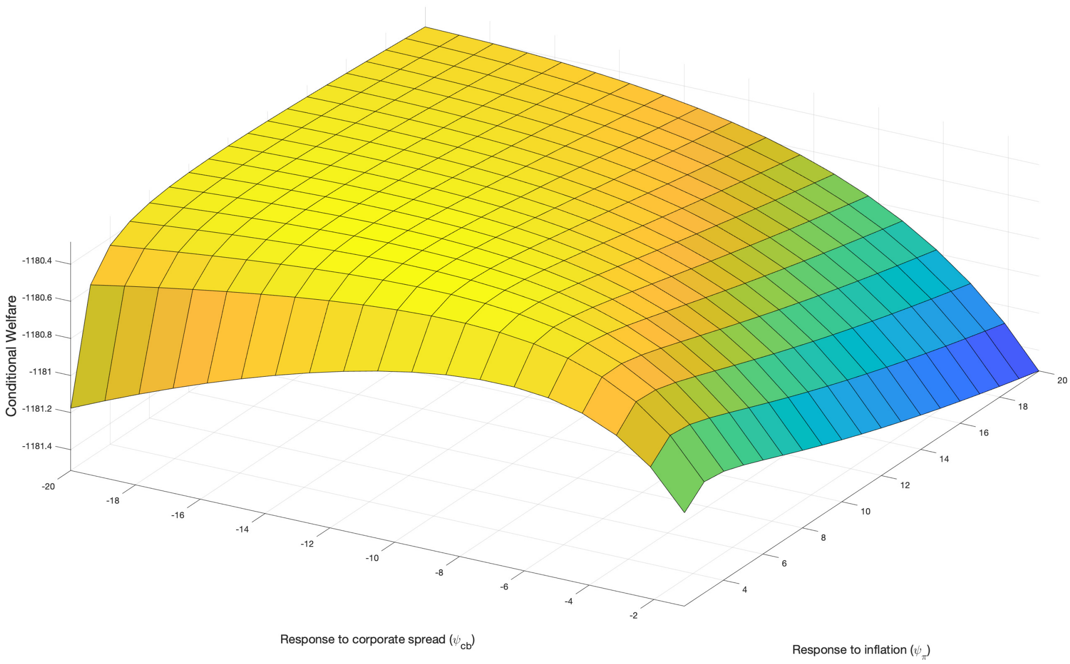
As regards the consumption-equivalent welfare loss of the benchmark and alternative economies relative to the flexible price economy, the estimated policy (i.e., setting the policy parameters to their estimated value without establishing the optimal policy rule) has the highest conditional and unconditional welfare loss (equivalently the lowest level of welfare). However, when consideration is given to financial indicators in the design of monetary policy, welfare is higher, and the consumption-equivalent welfare loss to a flexible price economy is reduced. For example, in Table 2.3, the estimated and optimal benchmark policies (version 1) lose 0.2200% and 0.1560% per unit of their consumption process, respectively, relative to the flexible price economy. The corresponding loss 0.1168% if, in addition to the inflation and output target in the benchmark economy, there is a response to corporate bond spread. In other words, $\lambda^c = 0.2200$ implies that households would be giving up that $(\%)$ unit of their consumption stream under the flexible economy to be well off in a sticky-price economy with an estimated Taylor rule policy.

In the same vein, a value of $\lambda^c = 0.1168$ in Panel B for corporate spread implies that households would be giving up that $(\%)$ unit of their consumption stream under the flexible economy to be well off in an economy that considers the growth of corporate spread alongside the usual inflation and output targets of the benchmark economy. Higher welfare in this analysis translates to a lower consumption loss relative to the flexible price economy; that is, a policy regime with higher welfare is synonymous with a regime that generates a higher consumption-equivalent gain.

2.6.2.1 Conditional Welfare Surface

In this next section, the effect of the policy response to inflation and corporate bond spread on the surface of conditional welfare is explored. This is done by varying the coefficient of inflation ψ_π and corporate bond spread ψ_{cb} when monetary policy only responds to inflation and corporate bond spread without interest rate smoothing.

Figure 2.1: Conditional Welfare Varying ψ_π and ψ_{cb}

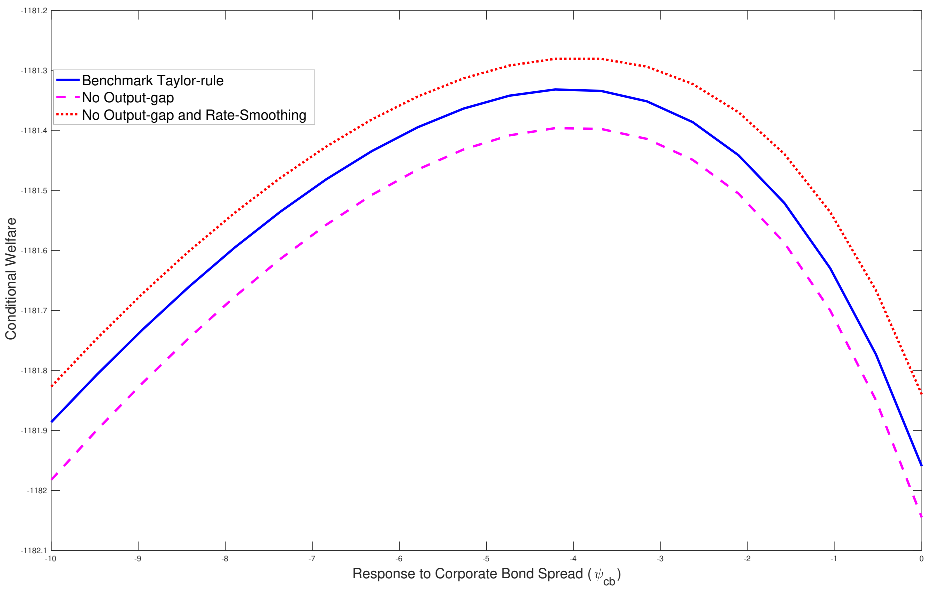


Monetary policy reacting only to inflation and corporate bond spread with no interest rate smoothing. Response to corporate spread is on x-axis and response to inflation on y-axis,

Figure 2.1 shows the conditional welfare surface of the aforementioned policy specifications. This image shows that welfare is improved for not so large values of inflation, but a high-level response to corporate bond spread improves welfare more. This confirms the result in Panel D of Table 2.4- a policy version that only responds to inflation and corporate bond spread. In addition, it shows that the welfare surface is concave in corporate bond spread coefficient.

The concavity of the welfare surface in response to corporate bond spread is further analysed by fixing the coefficient of inflation to a set value, and the equivalent welfare with a varying coefficient of corporate bond spread is calculated. In addition, this exercise is extended to investigate the impact that output and interest rate smoothing has on welfare. In doing so, three scenarios are considered.

Figure 2.2: Conditional Welfare Varying ψ_{cb}



These policies are extended to respond to corporate bond spread in addition to the instruments marked by the legend. Other policy parameters are fixed at $\psi_\pi = 1.4116$, $\psi_y = 0.0503$ and $\rho_R = 0.6323$.

These scenarios are as follows: i) extending the typical Taylor rule [15] to include the response to corporate spread; ii), considering only inflation and corporate bond spread with interest rate smoothing; and iii) having no interest rate smoothing but including inflation and corporate bond spread targeting only.

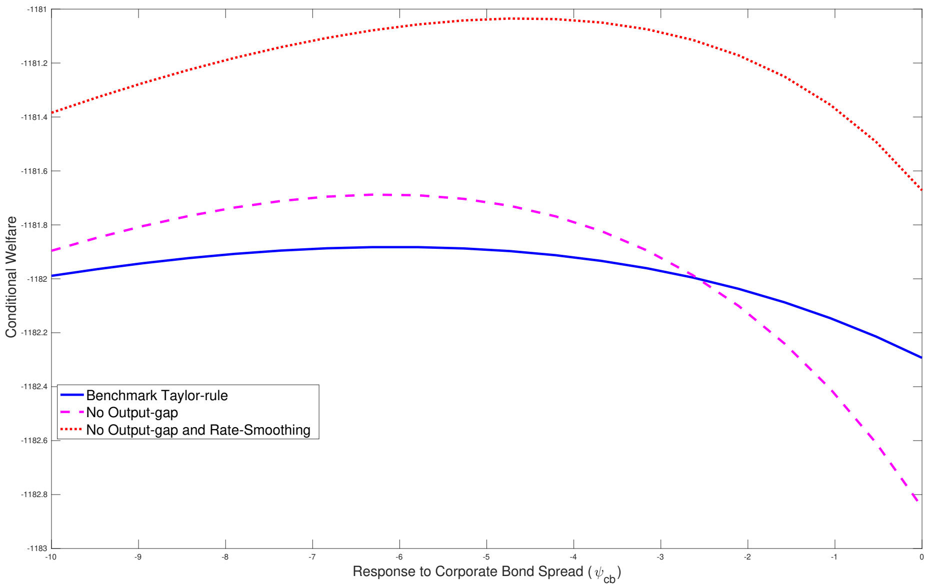
In scenario 1, the policy parameters are ρ_R, ψ_π, ψ_y and ψ_{cb}, in scenario 2 the parameters are ρ_R, ψ_π, and ψ_{cb}, and in scenario 3 there are only two parameters ψ_π, and ψ_{cb}.[16] The difference between Figures 2.2 and 2.3 comes from the parametrisation of the policy parameters. In Figure 2.2, all these parameters excluding ψ_{cb} are fixed to their estimated values ($\rho_R = 0.6323, \psi_\pi = 1.4116$ and $\psi_y = 0.0503$). In Figure 2.3 the parameters are calibrated to the common values in the literature (i.e., $\rho_R = 0.9, \psi_\pi = 1.5$ and $\psi_y = 0.5$). Each of the figures shows the corresponding consumption welfare when ψ_{cb} is varied in the interval $\in [-10, 0]$. The solid blue line correspond to scenario 1, the dashed magenta line to scenario 2, and the dotted red line to scenario 3.

[15]

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\psi_y}\right]^{1-\rho_R}$$

[16]Where ρ_R is interest rate smoothing, ψ_π is for inflation, ψ_y is for output, and ψ_{cb} is for corporate bond spread.

Figure 2.3: Welfare of Alternative Monetary Policy with Varying ψ_{cb}



These policies are extended to respond to corporate bond spread in addition to the instruments marked by the legend. Other policy parameters are fixed at $\psi_\pi = 1.5$, $\psi_y = 0.5$ and $\rho_R = 0.9$

From Figures 2.2 and 2.3, there is no doubt that including the growth of the corporate bond spread in designing an optimal policy is welfare improving. In particular, a policy rule responding only to inflation and corporate bond spread outperforms the benchmark policy (that is extended to include corporate bond spread alongside output gap). Also noticeable is that responding to output is not beneficial to welfare. In Figure 2.2 when the policy response to the output gap is somehow muted $\psi_y = 0.0503$, welfare is better than in the case of the policy without a response to the output gap (the dashed magenta plot). However, this changes when the response to the output gap is increased $\psi_y = 0.5$, and the policy without reference to the output gap is more welfare improving (see Figure 2.3). Responding to the output gap then becomes welfare inferior to following a policy rule that excludes it for the corporate bond spread response $\psi_{cb} \in [-10, -3)$.[17]

2.6.2.2 Empirical and Model Simulated Standard Deviation of Different Policies

The actions of monetary policy authorities are often intended to stabilise the economy by making decisions that ensure the best macroeconomic outcomes. The standard deviation of some macroeconomic and financial variables is examined to identify the implications of the analysed policy regimes. Table 2.6 corresponds to the model simulated standard deviation when optimal policy parameters are bounded, while Table 2.7 references the unbounded policy parameters.

[17] Note the interval $[-10, -3)$ is partly open and close. The conclusion made for welfare does not include the point where corporate bond spread response (ψ_{cb}) is -3 (Figure 2.3).

Table 2.6: Standard Deviation of Bounded Optimal Policy Parameters and Data

	std($\mathrm{d}y_t$)	std($\mathrm{d}c_t$)	std($\mathrm{d}i_t$)	std($\mathrm{d}w_t$)	std($C^b_{t,n}$)	std(π_t)	std(r_t)	std($R^c_{t,n}$)	std($R^g_{t,n}$)
Empirical Moment									
Data (1966-2006)	0.85	0.69	2.03	0.60	0.63	0.59	0.82	2.62	2.45
A. Benchmark Taylor Rule									
Estimated	0.71	0.82	2.00	0.69	1.53	0.67	0.74	1.88	1.77
Optimal rule version 1	0.69	0.78	2.00	0.60	1.45	0.40	0.82	1.35	0.79
B. Alternative version 1a									
Corporate spread	0.75	0.87	1.83	0.67	1.29	0.28	0.83	1.33	0.90
Leverage	1.07	1.27	3.82	0.80	2.07	0.83	3.01	3.73	2.14
External finance	0.70	0.81	1.80	0.65	1.40	0.33	0.77	1.32	0.82
Networth	0.80	0.94	1.74	0.71	1.18	0.29	1.08	1.26	0.93
C. Alternative version 1b									
Corporate spread	0.73	0.83	1.97	0.66	1.42	0.26	0.78	1.30	0.82
Leverage	1.01	1.19	3.59	0.76	2.00	0.69	2.75	3.34	1.85
External finance	0.72	0.83	2.09	0.65	1.47	0.29	0.83	1.74	0.97
Networth	0.84	0.99	1.74	0.73	1.23	0.33	1.23	1.38	1.06

Moments obtained under all model shocks. The corresponding policy parameters are set to the values in Table 2.3.

Note that only a few policy suggestions are considered. Panel A shows the second moment for the estimated and optimal benchmark policy. Panel B shows the extension of the benchmark policy that includes financial variables, while Panel C is for the alternative policy version 1b, in which monetary policy responds to inflation, output gap, and financial indicators with interest rate smoothing. Interestingly, the alternative economies are able to reduce the fluctuation observed in the macroeconomic and financial variables considered. More precisely, these policies outperform the traditional Taylor rule of inflation and output targeting in stabilising inflation. While it appears that leverage is welfare improving, it does not ensure macroeconomic or financial stability.

The standard deviation of the selected macroeconomic and financial variables is highest when monetary policy includes leverage as its instrument. Notably, amongst all the financial indicators considered, corporate bond spread stabilises inflation the most, and it consistently stabilises other macroeconomic and financial variables.

Table 2.7: Standard Deviation of Unbounded Optimal Policiy Parameters and Data

	std(dy_t)	std(dc_t)	std(di_t)	std(dw_t)	std($C^b_{t,n}$)	std(π_t)	std(r_t)	std($R^c_{t,n}$)	std($R^g_{t,n}$)
Empirical Moment									
Data (1966-2006)	0.85	0.69	2.03	0.60	0.63	0.59	0.82	2.62	2.45
A. Benchmark Taylor Rule									
Estimated	0.71	0.82	2.00	0.69	1.53	0.67	0.74	1.88	1.77
Optimal rule version 1	0.69	0.79	2.02	0.60	1.45	0.40	0.73	1.22	0.67
B. Alternative version 1a									
Corporate spread	0.69	0.79	2.03	0.60	1.45	0.37	0.74	1.25	0.67
Leverage	1.26	1.51	4.42	0.91	2.21	0.55	3.62	3.28	1.50
External finance	0.69	0.79	2.05	0.60	1.45	0.37	0.78	1.38	0.69
Networth	0.83	0.97	1.95	0.73	1.13	0.24	1.15	1.32	0.79
C. Alternative version 1b									
Corporate spread	0.80	0.93	1.93	0.70	1.25	0.21	1.15	1.09	0.72
Leverage	1.11	1.33	3.91	0.82	2.10	0.32	3.07	2.47	1.02
External finance	0.75	0.85	2.19	0.68	1.48	0.19	0.80	1.51	0.75
Networth	0.86	1.01	1.86	0.75	1.13	0.24	1.32	1.34	0.86

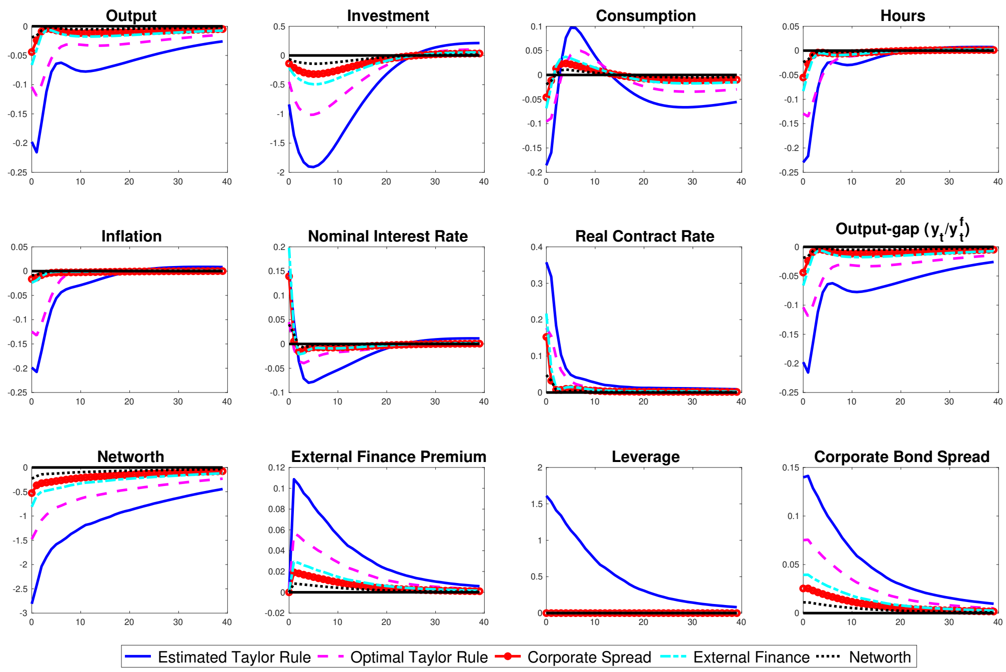
Moments obtained under all model shocks. The corresponding policy parameters are set to the values in Table 2.4.

2.6.3 Impulse Response

How the optimised policies alter the responses of macroeconomic and financial outcomes to model distortion is assessed by computing the response of model variables to various shocks using the welfare criterion mentioned earlier. Model parameters, other than those in the monetary policy rule, are fixed to their calibrated or estimated values in Tables 2.1 and 2.2. For each of the different economies (i.e., different policy regimes), the policy parameters are set to their unbounded optimal values in Table 2.4. Figures 2.4 to 2.6 show the impulse response for selected macroeconomic and financial variables under different monetary policy regimes to tightening monetary policy shock, adverse financial shock and positive productivity shock. All are related to policy 1 of the benchmark economy and policy 1b for the alternative economies, excluding the policy version with leverage as the instrument. This exclusion is due to the observation that using leverage as a policy instrument leads to increased macroeconomic and financial fluctuations (check Table 2.6 and 2.7). A full set of impulse responses, including when leverage is used as the policy instrument, are included in Appendix 2.7. The impulse response function is simulated at order 3.

A one-standard-deviation contractionary monetary policy shock, as in Figure 2.4, leads to an instantaneous increase in the nominal interest rate for all policies. This immediate impact causes output, investment, and inflation to fall. Due to a low return on capital with a high real interest rate to be paid on existing debt, the net worth of entrepreneurs falls significantly – about 300 and 150 basis points for the estimated and optimal Taylor rule policies. Entrepreneurs' net worth is mildly impacted when monetary policy considers financial indicators, as are other variables in response to the shock. The huge decline in entrepreneurs' net worth for the benchmark policy increases the cost of external finance. An increased external finance premium is an effect of entrepreneurs' depreciated balance sheet (which is due to increased leverage of about 150 basis points). The increased cost of external finance and the increase in the real loan-contract rate cause a decrease in the demand for capital (because of the decline in investment). In sum, if financial indicators are used as policy instruments, the distortion of macroeconomic and financial variables is better stabilised, relative to the case where monetary policy only targets inflation and output.

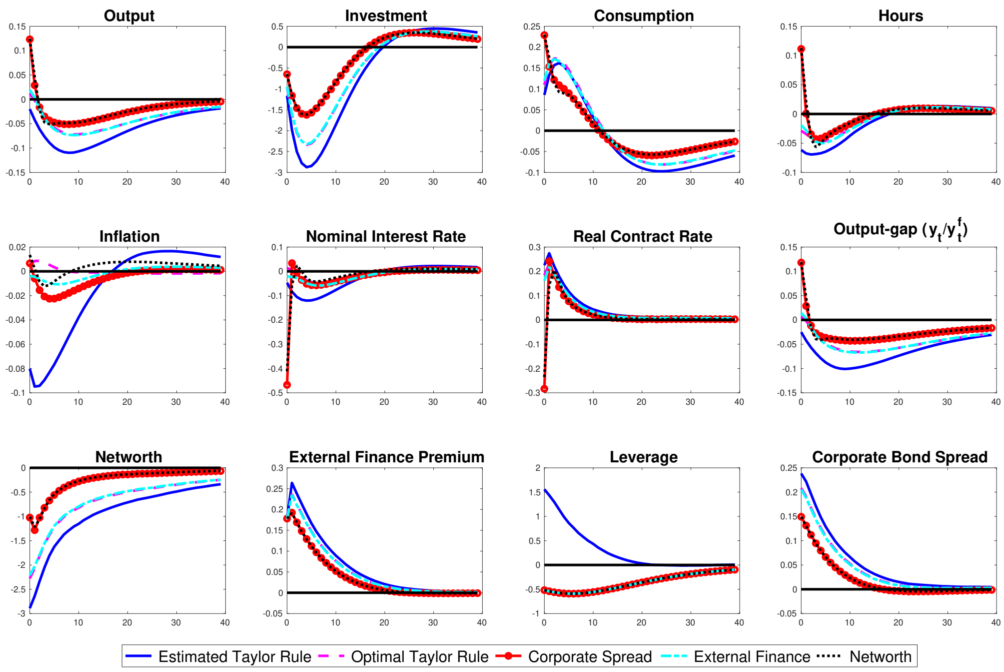
Figure 2.4: Response to Tightening Monetary Policy Shock for Policy 1b



The figure depict the impulse response to one-standard deviation tightening monetary shock. All variables are expressed as percentage deviation from their respective steady-state.

A shock to the external finance premium causes a significant decrease in economic activities. The resulting response of a one-standard-deviation external finance premium shock (Figure 2.5) requires aggressive easing of monetary policy by a reduction of close to 50 basis points when the corporate bond spread is used as a policy instrument. Similarly, when net worth is used as a policy instrument, this calls for an easing of nominal interest rates by 40 basis points. The policy considering external finance and the estimated benchmark policy only decline marginally. Net worth is most affected, with a decline of around 300 basis points upon impact in the benchmark economy with the estimated policy parameters. The optimal Taylor rule and external finance policy saw a decline of 200 basis points. By contrast, the alternative economies with corporate bond spread and net worth only decline by 100 basis points upon impact. Corporate bond spread in itself responds almost indifferently in the alternative economies. Ideally, an adverse financial shock that increases the cost of sourcing external finance and makes asset prices decline increases leverage. However, if consideration is given to financial indicators as monetary policy instruments, leverage is not adversely affected when a financial shock hits. This effect is presented in Figure 2.5 where leverage increases on impact for the estimated monetary policy economy. The amplification of this effect leads to a reduction in output and investment. However, the negative effect of adverse financial shock is dampened when monetary policy accounts for financial indicators and, at the same time, implies inflation and output gap stabilisation.

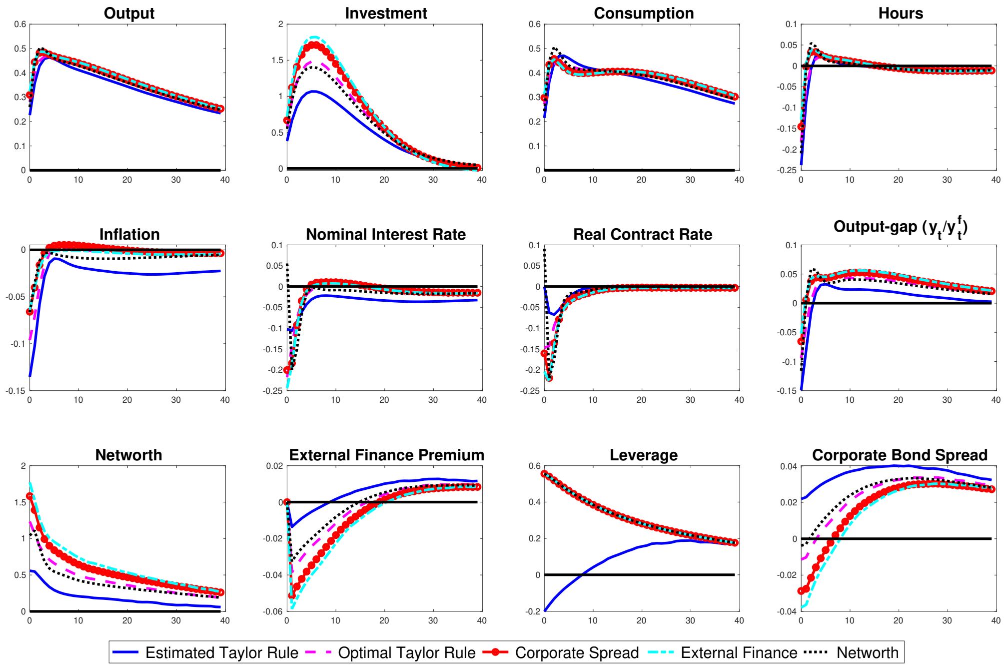
Figure 2.5: Response to Adverse External Finance Shock for Policy 1b



The figure depict the impulse response to one standard deviation adverse financial shock. All variables are expressed as percentage deviation from their respective steady-state.

Aggregate demand, output, and consumption increase with a positive productivity shock (Figure 2.6). The growth certainty implied from this shock improves the value of capital, which, in turn, improves net worth. As a result, entrepreneurs are encouraged to take out more loans, hence the increase in leverage in the alternative economies. However, the benchmark economy that does not account for financial indicators sees a decline in leverage. While investment improves greatly for alternative economies, the cost of obtaining external funds is lower. The decline in inflation increases the real cost of repaying existing debt.

Figure 2.6: Response to Positive Productivity Shock for Policy 1b



The figure depict the impulse response to one standard deviation positive productivity shock. All variables are expressed as percentage deviation from their respective steady-state.

2.7 Conclusion

Financial instability is reflected in the risk it poses to macroeconomic and financial outcomes. Hence monetary policy must be responsive, and necessary measures must be put in place to reduce the macroeconomic risk; the financial market will then be able to return to its normal functioning state. This chapter contributes to the body of research on the inclusion of financial variables in monetary policy rules. It is motivated by the view that such policies can better stabilise the economy in the presence of financial shocks, such as the 2007-2009 financial crisis.

The analysis in this chapter is conducted using an estimated model with financial friction and bond pricing equations. This allows a structural account of the evolution of financial stress indicators, such as the corporate bond spread. First, the growth of four financial indicators (corporate bond spread, leverage, external finance premium, and net worth) are considered monetary policy instruments. Numerical methods are then adopted to calculate the policy coefficients maximising household lifetime welfare alongside the corresponding consumption-equivalent welfare benefits. In addition, the transmission of structural shocks is examined in relation to suggested alternative monetary policies (i.e. with financial indicators). The main conclusions are highlighted below.

First, in a bid to offset the negative effects of the turmoil on macroeconomic outcomes, interest rates should systematically be adjusted downwards - a result consistent with Taylor (2008). The optimal policy response suggests that interest rates should be reduced when there is an increase in corporate bond spread, leverage, and external premiums. An increase in these indicators reflects possible distortion in the supply of credit; hence, the optimal policy implication of an interest rate cut is plausible. Putting this measure in place can curtail the adverse feedback that times of financial turmoil have on macroeconomic outcomes. It also allows the financial market from which the turmoil emerged to recover to its normal functioning state. A financial boom does not occur in isolation, and a subsequent burst is possible. Thus, an indicator such as increasing net worth according to the Taylor rule suggests that interest rates should be increased to curtail the looming bubble burst. Achieving the aforementioned results relies solely on the extent to which the monetary policy is effective, timely, decisive and flexible (McCulley and Toloui, 2008).

Second, the inclusion of financial indicators in monetary policy indicates that central banks' must be willing to be flexible in attempting to address the effect of financial conditions on macroeconomic outcomes. In particular, they must be willing to respond less to interest rate inertia than they otherwise would; if periods of financial severity are left to deteriorate further as a result of a lack of decisive and appropriate policy reaction, the resulting effects on macroeconomic outcomes may be heightened to such an extent that there is less room for easy correction. However, such a policy response can be eased once the economy returns to normal with the focus of policy on the usual macroeconomic instruments. However, from time to time, policy makers need to monitor the information emanating from the financial market. Consistent with Gilchrist and Zakrajsek (2011), the results from this chapter suggest that if the central bank aggressively responds to financial indicators in a timely manner, it may be able to reduce the chances of the interest rate being constrained by the zero lower bound.

Third, in the model, following a financial shock, it is optimal when the central bank reacts more strongly to financial variables and less intensively to inflation. Such policies have been shown to better stabilise the real economy than a typical Taylor policy responding to inflation output. At the same time, welfare is improved for the alternative economies- with the external finance premium being the only exception. Amongst the alternative policies considered, responding to increasing corporate bond spread is mostly welfare improving (in the unconstrained and constrained case). In the constrained policy case, responding to leverage seemingly generates the greatest welfare improvement. Lastly, responding to the output gap is welfare inferior, which is consistent with the observation in Schmitt-Grohe and Uribe (2007).

Appendix 2

I: Detrended Model Equations

The model has been detrended using the deterministic growth rate γ, and nominal variables now transformed to real. Detrended model variables are therefore denoted by small letters. For example

$k_t = \frac{K_t}{\gamma^t}$, $w_t = \frac{W_t}{\gamma^t P_t}$, $q_t = \frac{Q_t}{P_t}$, $\bar{\beta} = \beta\gamma^{-\sigma_c t}$ $mc_t = \frac{MC_t}{P_t}$, $\bar{V}_t = \frac{V_t}{\gamma^{t(1-\sigma_C)}}$

Euler Equation

$$1 = \mathbb{E}_t\Big[\lambda_{t,t+1}\frac{1}{\pi_{t+1}}\Big]R_{t,t+1} \tag{2.7.1}$$

Stochastic Discounting Factor

$$\lambda_{t,t+1} = \bar{\beta}\Big(\frac{\gamma^{1-\sigma_C}\bar{V}_{t+1}}{\bar{v}_t}\Big)^{-\sigma_E}\frac{\xi_{t+1}}{\xi_t} \tag{2.7.2}$$

Marginal Utility of Consumption

$$\xi_t = \varepsilon_t^{\beta}\Big(c - \frac{\eta_c}{\gamma}c_{t-1}\Big)^{-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right) \tag{2.7.3}$$

Marginal Utility of Labour

$$w_t^h = \varepsilon_t^w\Big(c_t - \frac{\eta_c}{\gamma}c_{t-1}\Big)L_t^{\sigma_L} \tag{2.7.4}$$

Epstein-Zin Preference

$$\bar{V}_t = \varepsilon_t^{\beta}\left[\frac{1}{1-\sigma_C}\Big(c - \frac{\eta_c}{\gamma}c_{t-1}\Big)^{1-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right)\right] + \bar{\beta}\bar{v}_t \tag{2.7.5}$$

Value Function

$$\bar{v}_t = \mathbb{E}_t\Big[\gamma^{(1-\sigma_C)(1-\sigma_E)}\bar{V}_{t+1}^{1-\sigma_E}\Big]^{\frac{1}{1-\sigma_E}} \tag{2.7.6}$$

Production Function

$$m_t = \varepsilon_t^a\Big(U_t^k\frac{k_{t-1}}{\gamma}\Big)^{\alpha}\Big(L_t\Big)^{1-\alpha} \tag{2.7.7}$$

Real wage

$$w_t = (1-\alpha) mc_t \frac{m_t}{H_t} \tag{2.7.8}$$

Marginal Product of Capital

$$mpk_t = \alpha mc_t \frac{\gamma m_t}{k_{t-1}} \tag{2.7.9}$$

Capital Utilisation

$$U_t^k = \left(\frac{\frac{mpk_t}{q_t}}{\frac{mpk_{ss}}{q_{ss}}} \right)^{\psi_u} \tag{2.7.10}$$

Capital Depreciation

$$\delta_t^k = \delta + \psi_u \left(\frac{mpk_t}{q_t} - \frac{mpk_{ss}}{q_{ss}} \right) \tag{2.7.11}$$

LOM of Capital

$$k_t = (1-\delta_t^k) \frac{k_{t-1}}{\gamma} + \varepsilon_t^i i_t - \frac{\psi\gamma}{2} \left(\frac{i_t}{i_{t-1}} - i_t \right)^2 i_t \tag{2.7.12}$$

Tobin's Q

$$1 = q_t \left[\varepsilon_t^i - \psi\gamma \frac{i_t}{i_{t-1}} \left(\frac{\gamma i_t}{i_{t-1}} - \gamma \right) - \frac{\gamma}{2} \left(\frac{\gamma i_t}{i_{t-1}} - \gamma \right)^2 \right] + \mathbb{E}_t \left[\lambda_{t,t+1} q_{t+1} \psi \left(\frac{\gamma i_{t+1}}{i_t} \right)^2 \left(\frac{\gamma i_{t+1}}{i_t} - \gamma \right) \right] \tag{2.7.13}$$

Realised Return and Loan Rate

$$\mathbb{E}_t [\lambda_{t,t+1} R_{t,t+1}^R] = \mathbb{E}_t \left[\lambda_{t,t+1} \frac{1}{\pi_{t,t+1}} \right] R_{t,t+1}^N \tag{2.7.14}$$

Realised Capital Return

$$R_{t-1,t}^R = \frac{mpk_t}{q_{t-1}} + (1-\delta_t^k) \frac{q_t}{q_{t-1}} \tag{2.7.15}$$

Nominal Contract Rate

$$R^N_{t,t+1} = S_{t,t+1} R_{t,t+1} \tag{2.7.16}$$

External Finance Premium

$$S_{t,t+1} = \varepsilon^s_t S_{ss} \left(\frac{q_{t-1} k_{t-1}}{n_{t-1}} \frac{n_{ss}}{k_{ss}} \right)^{\psi_S} \tag{2.7.17}$$

LOM of Net worth

$$n_t = \varphi \bar{E}^s_t + (1-\varphi)\exp(\varepsilon^N_t) \tag{2.7.18}$$

Net worth of Survived Entrepreneurs

$$\bar{E}^s_t = \left(R^R_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}} \right) q_{t-1} \frac{k_{t-1}}{\gamma} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}} n_{t-1} \tag{2.7.19}$$

Stick Price Equations

$$\tilde{p}_t = \left(\frac{\epsilon_p}{\epsilon_p - 1} \right) \left(\frac{g^{p2}_t}{g^{p1}_t} \right) \tag{2.7.20}$$

$$g^{p1}_t = y_t + \mathbb{E}_t \left[\zeta_p \lambda_{t,t+1} \gamma \left(\frac{\pi^{\iota_p}_t \pi^{1-\iota_p}_{ss}}{\pi_{t+1}} \right)^{1-\epsilon_p} g^p_{t+1} 1 \right] \tag{2.7.21}$$

$$g^{p2}_t = y_t mc_t \varepsilon^p_t + \mathbb{E}_t \left[\zeta_p \lambda_{t,t+1} \gamma \left(\frac{\pi^{\iota_p}_t \pi^{1-\iota_p}_{ss}}{\pi_{t+1}} \right)^{-\epsilon_p} g^{p2}_{t+1} \right] \tag{2.7.22}$$

$$\tilde{p}_t = \left[\frac{1 - \zeta_p \left(\frac{\pi^{\iota_p}_{t-1} \pi^{1-\iota_p}_{ss}}{\pi_t} \right)^{1-\epsilon_p}}{1-\zeta_p} \right]^{\frac{1}{1-\epsilon_p}} \tag{2.7.23}$$

$$\Delta^p_t = (1-\zeta_p) \left[\frac{1 - \zeta_p \left(\frac{\pi^{\iota_p}_{t-1} \pi^{1-\iota_p}_{ss}}{\pi_t} \right)^{1-\epsilon_p}}{1-\zeta_p} \right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p \left(\frac{\pi^{\iota_p}_{t-1} \pi^{1-\iota_p}_{ss}}{\pi_t} \right)^{-\epsilon_p} \Delta^p_{t-1} \tag{2.7.24}$$

Stick Wage Equations

$$\frac{\tilde{w}_t}{w_t} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\left(\frac{g_t^{w2}}{g_t^{w1}}\right) \tag{2.7.25}$$

$$g_t^{w1} = w_t H_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\gamma\left(\frac{w_t}{w_{t+1}}\frac{\pi_t^{l_w}\pi_s^{1-l_w}}{\pi_{t+1}}\right)^{1-\epsilon_w} g_{t+1}^{w1}\right] \tag{2.7.26}$$

$$g_t^{w2} = w_t^h H_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\gamma\left(\frac{w_t}{w_{t+1}}\frac{\pi_t^{l_w}\pi_{ss}^{1-l_w}}{\pi_{t+1}}\right)^{1-\epsilon_w} g_{t+1}^{w2}\right] \tag{2.7.27}$$

$$\frac{\tilde{w}_t}{w_t} = \left[\frac{1 - \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota w}\pi_{ss}^{1-\iota w}}{\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{2.7.28}$$

$$\Delta_t^w = (1-\zeta_w)\left[\frac{1 - \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{\iota w}\pi_{ss}^{1-\iota w}}{\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{-\frac{\epsilon_w}{1-\epsilon_w}} + \zeta_w\left(\frac{w_{t-1}}{w_t}\frac{\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{\pi_t}\right)^{-\epsilon_w}\Delta_{t-1}^w \tag{2.7.29}$$

Labour Market Equilibrium

$$L_t = \Delta_t^w H_t \tag{2.7.30}$$

Goods Market Equilibrium

$$m_t = \Delta_t^p y_t \tag{2.7.31}$$

Market Clearing

$$y_t = c_t + i_t + \varepsilon_t^g y_{ss} \tag{2.7.32}$$

Monetary Policy

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_1}\left(\frac{y_t}{y_{ss}}\right)^{\psi_2}\right]^{1-\rho_R}\left(\frac{\frac{y_t}{y_{t-1}}}{\frac{y_t^f}{y_{t-1}^f}}\right)^{\psi_3}\varepsilon_t^r \tag{2.7.33}$$

Exogenous Processes

$$\ln \varepsilon_t^\beta = \rho_\beta \ln \varepsilon_{t-1}^\beta + \eta_t^\beta \tag{2.7.34}$$
$$\ln \varepsilon_t^p = \rho_p \ln \varepsilon_{t-1}^p + \eta_t^p \tag{2.7.35}$$
$$\ln \varepsilon_t^a = \rho_a \ln \varepsilon^a + \eta_t^a \tag{2.7.36}$$
$$\ln \varepsilon_t^w = \rho_w \varepsilon_t^w + \varepsilon_t^w \tag{2.7.37}$$
$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i \tag{2.7.38}$$
$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r \tag{2.7.39}$$
$$\ln \varepsilon_t^g = \rho_g \ln \varepsilon_{t-1}^g + \rho_{ga} \ln \varepsilon_{t-1}^a + \eta_t^g \tag{2.7.40}$$
$$\ln \varepsilon_t^S = \rho_S \ln \varepsilon_{t-1}^S + \eta_t^S \tag{2.7.41}$$
$$\ln \varepsilon_t^N = \rho_N \ln \varepsilon^N + \eta_t^N, \tag{2.7.42}$$

where $\eta_t^k \sim \mathcal{N}(0, \sigma_k) \qquad k = [a, \beta, w, p, i, r, g, N, S]$

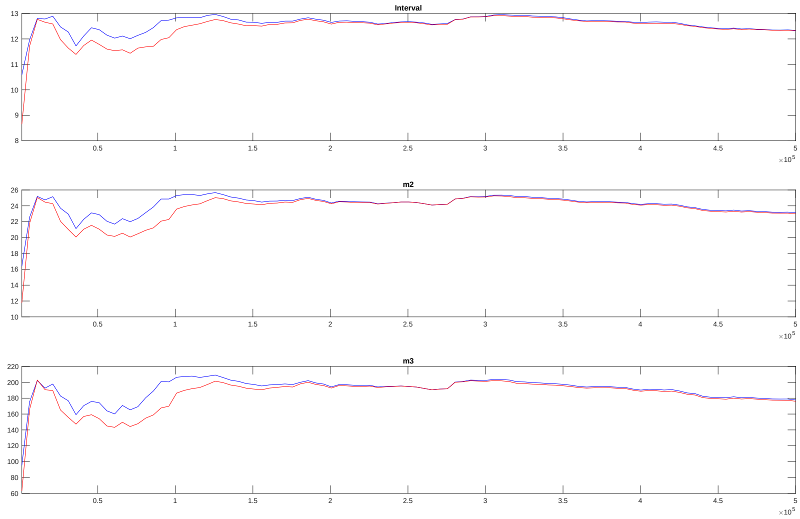
II: Steady State

$$U_{ss}^k = 1$$
$$\Delta_s s^p = 1$$
$$\Delta_s s^w = 1$$
$$\tilde{p}_{ss} = 1$$
$$\lambda_{ss} = \bar{\beta}$$
$$\delta_{ss}^k = \delta$$
$$S_{ss} = S_{ss}$$
$$R_{ss} = \frac{\pi_{ss}}{\bar{\beta}}$$
$$R_{ss}^N = S_{ss} R_{ss}$$
$$R_{ss}^R = \frac{R_{ss}^N}{\pi_{ss}}$$
$$\frac{i_{ss}}{k_{ss}} = \frac{\gamma - 1 - \delta}{\gamma}$$
$$mc_{ss} = \frac{\epsilon_p - 1}{\epsilon_p}$$
$$mpk_{ss} = R_{ss}^R - (1 - \delta_{ss}^k)$$

$$
\begin{aligned}
\frac{m_{ss}}{k_{ss}} &= \frac{mpk_{ss}}{\gamma\alpha mc_{ss}} \\
\frac{l_{ss}}{k_{ss}} &= \left(\gamma^{\alpha}\frac{m_{ss}}{k_{ss}}\right)^{\frac{1}{1-\alpha}} \\
w_{ss} &= (1-\alpha)mc_{ss}\frac{m_{ss}}{k_{ss}} \div \left(\frac{l_{ss}}{k_{ss}}\right) \\
\tilde{w}_{ss} &= w_{ss} \\
w^{h}_{ss} &= \left(\frac{\epsilon_w - 1}{\epsilon_w}\right)w_{ss} \\
\frac{c_{ss}}{k_{ss}} &= \frac{m_{ss}}{k_{ss}}(1-g_{ss}) - \frac{i_{ss}}{k_{ss}} \\
l_{ss} &= \left[\left(\frac{\epsilon_w - 1}{\epsilon_w}\right)w_{ss}\left(\frac{l_{ss}}{k_{ss}} \div \frac{c_{ss}}{k_{ss}}\right)\left(\frac{\gamma}{\gamma-\eta_c}\right)\right]^{\frac{1}{1+\sigma_L}} \\
h_{ss} &= l_{ss} \\
k_{ss} &= l_{ss}\left(\frac{k_{ss}}{l_{ss}}\right) \\
i_{ss} &= k_{ss}\left(\frac{i_{ss}}{k_{ss}}\right) \\
y_{ss} &= k_{ss}\left(\frac{m_{ss}}{k_{ss}}\right) \\
m_{ss} &= y_{ss} \\
c_{ss} &= k_{ss}\left(\frac{c_{ss}}{k_{ss}}\right) \\
g^{p1}_{ss} &= \frac{y_{ss}}{1-\zeta_p\bar{\beta}\gamma} \\
g^{p2}_{ss} &= \frac{y_{ss}mc_{ss}}{1-\zeta_p\bar{\beta}\gamma} \\
g^{w1}_{ss} &= \frac{h_{ss}w_{ss}}{1-\zeta_w\bar{\beta}\gamma} \\
g^{w2}_{ss} &= \frac{h_{ss}w^{h}_{ss}}{1-\zeta_w\bar{\beta}\gamma} \\
\bar{V}_{ss} &= \left(\frac{1}{1-\sigma_C}\right)\left(c_{ss} - \frac{\eta_c}{\gamma}c_{ss}\right)^{1-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}l_{ss}^{1+\sigma_L}\right)\left(\frac{1}{1-\bar{\beta}\gamma^{1-\sigma_C}}\right) \\
\bar{v}_{ss} &= \gamma^{1-\sigma_C}\bar{V}_{ss} \\
\xi_{ss} &= \left(c_{ss} - \frac{\eta_c}{\gamma}c_{ss}\right)^{-\sigma_C}\exp\left(\frac{\sigma_C - 1}{1+\sigma_L}l_{ss}^{1+\sigma_L}\right)
\end{aligned}
$$

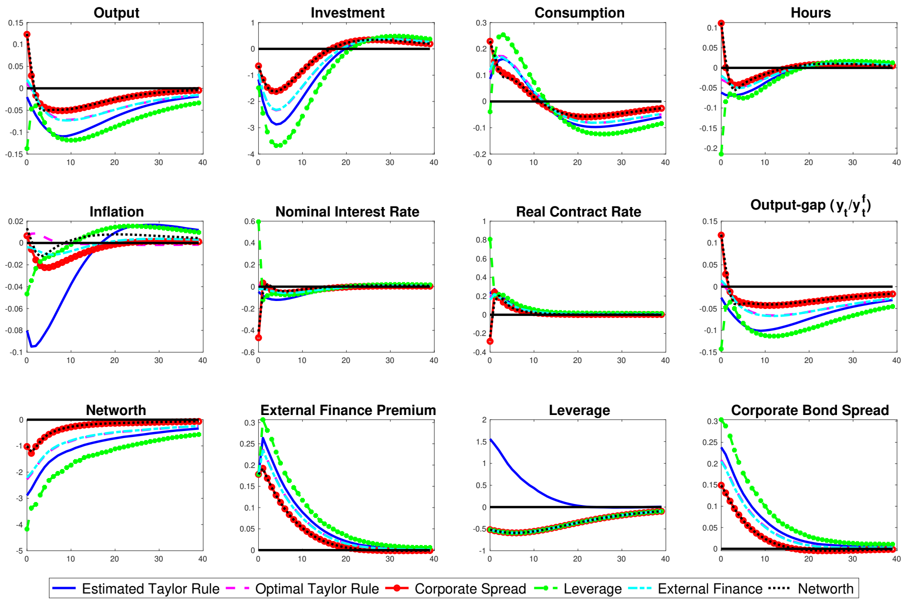
III: Convergence

Figure 2.7: Multivariate Convergence Diagnostics for 100,000 Draws



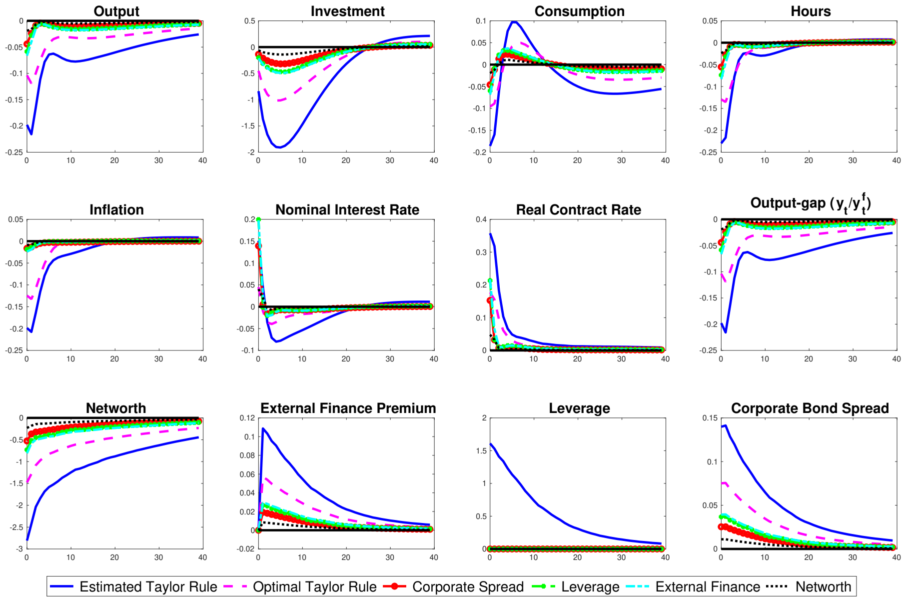
IV: Impulse Response with Leverage

Figure 2.8: Response to Adverse External Finance Shock for Policy 1b



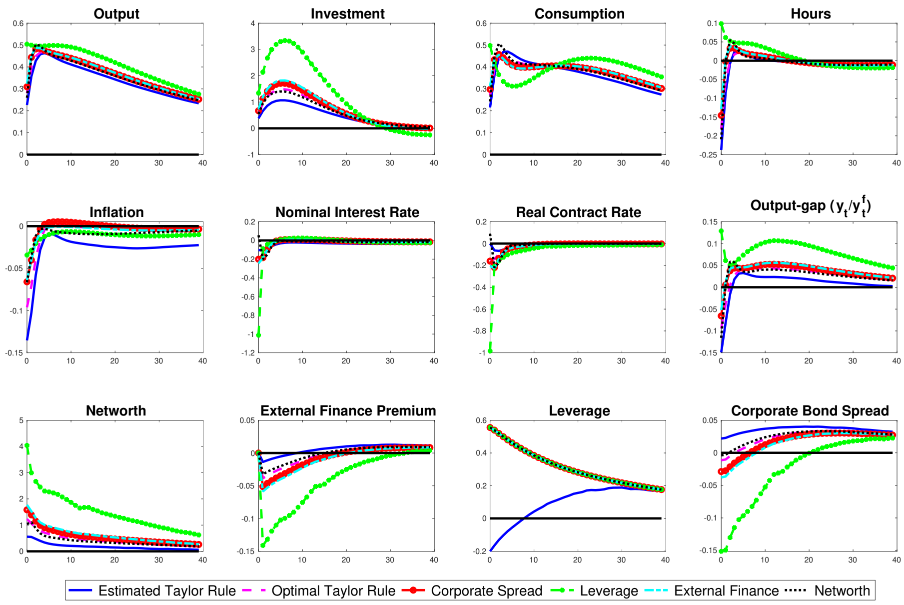
Impulse response to one standard deviation adverse financial shock. Variables are expressed as percentage deviation from their respective steady-state.

Figure 2.9: Response to tightening monetary Policy Shock for Policy 1b



Impulse response to one standard deviation tightening monetary shock. Variables are expressed as percentage deviation from their respective steady-state.

Figure 2.10: Response to Positive Productivity Shock for Policy 1b



Impulse response to one standard deviation positive productivity shock. Variables are expressed as percentage deviation from their respective steady-state.

3. Government Investment and External Finance Premium

Abstract

This chapter examines the combined impact of monetary and fiscal policies on macroeconomic and financial outcomes. In particular, it explores the dynamics of fiscal transmission in the presence of financial friction. It assesses how the joint intervention of monetary and fiscal authorities alters entrepreneurs' balance sheets, especially in respect of the cost of sourcing external finance. Furthermore, it offers an understanding of the multiplier effect of fiscal instruments over a short- and long-term horizon by evaluating its present-value multiplier (as in Mountford and Uhlig (2009)) on output, consumption, and investment. Decomposing government spending to investment and consumption components, this chapter shows that government investment has a negative effect on credit spread and also has higher short-run and long-run multipliers than government consumption. The result also suggests that the presence of the financial friction mechanism leads to a more substantial multiplier than is otherwise observed in a model without financial friction.

3.1 Introduction

The 2007-2009 global financial crisis (GFC) and the 2019 COVID-19 pandemic are instances where economies around the world witnessed a flurry of monetary and fiscal policy interventions intended to bring stability. For instance, the US federal government, in responding to the current COVID-19 crisis, adopted a number of policies to deliver a fiscal stimulus to the economy and thereby relief to individuals and businesses affected by the pandemic. Similarly, the central bank introduced significant monetary policy measures to complement the aforementioned fiscal stimulus. Such measures are not new; the US has in the past engaged in a wide range of similar policy interventions. For example, its response to the 'war on terrorism' is reflected in the two major tax cuts of 2001 and 2003. The 2007–2009 GFC is another significant example of a crisis requiring policy intervention in recent US history.

A key component of this chapter is identifying which fiscal tool can best ensure economic stability, taking into account the financial accelerator mechanism. The chapter also aims to investigate how monetary and fiscal policy interventions jointly affect the degree of financial friction. In particular, it assesses the impact of productive public capital on the economy in the presence of financial friction.

Undoubtedly, financial markets play a decisive role in business cycle transmission in the context of unfavourable economic conditions. The severity of the GFC prompted the 2009 US congress to pass a number of fiscal stimulus bills alongside other financial rescue programmes. Such policies have been identified as countercyclical instruments for stabilising the economy. One of the fiscal bills passed was the $787 billion American Recovery and Reinvestment Act (ARRA)(Leeper et al., 2010). Capital was also injected into some private sectors to prevent bankruptcy. The peculiarity of ARRA, as compared to previous policies, is that it relies more on government spending than tax cuts – almost two-thirds of the funds allocated were for government spending and transfers. Interestingly, prior to the enactment of ARRA, infrastructure expenditure was rarely used as a countercyclical fiscal tool. Policies containing core government investment packages are commendable as they directly impact the stock of public capital, which can play a key role in increasing long-run output and improving standards of living (Ramey, 2020).

Fiscal policy is important because it can impact supply through instruments such as infrastructural expenditure and spending that enhances human capital. Government investment spending is perceived by many policy makers to be preferable to government consumption in providing economic stimulation during a recession; in the aftermath of a recession, investment spending can stimulate the economy and, in the short-run, through standard income and multiplier effects, can return output to its potential. In addition, government investment spending can also change the path of potential output (Ramey, 2020).

Similar packages have been received with some scepticism, one of the arguments being that the policy will further increase the deficit. According toRamey (2020), government spending provides two benefits: Keynesian demand stimulus in the short-run and neo-classical supply stimulus in the long run. Hence, if output remains higher because of the long-run effect of increased public capital, the tax base will expand, and the tax rate required to foot the deficit will be less.

As is well known, policy makers use the instruments available to react to economic conditions (Philippopoulos et al., 2015), with monetary policy often targeting inflation and the output gap, while fiscal policy reacts to the state of public finances. However, it is essential to study the joint impact of these intertwined policy strands. For example, the central bank's ability to control inflation and influence real activity relies on fiscal behaviour and, in addition, people's expectations of it. Furthermore, according to Leeper and Leith (2016), the joint coordination of monetary and fiscal policy could potentially provide room for nominal government debt expansion. Alternatively, an increase in the policy interest rate could lead to an increase in nominal net worth, aggregate demand, and the price level. Hence, an uncooperative act of one policy authority may jeopardise the efforts of the other in achieving its objectives (Leeper, 2010). Therefore, this paper aims to provide a theoretical framework for assessing the effectiveness of joint monetary and fiscal policy as a tool for economic stability, particularly in the presence of financial friction.

The theoretical work of Baxter and King (1993) and Ludger and Andreas (2003) shows that an increase in government demand leads to a strong negative wealth effect, which results in an increase in employment, but a decrease in private consumption and wages. However, this result is due to the authors' assumption in the model that government spending is a wasted resource. Barro (1990), who was the first to introduce public capital stock in the production function of an endogenous growth model, indicates that even though the government does not engage directly in production nor own capital, it purchases a flow of output (including services of roads, sewers etc.) from the private sector. These purchased services are made available as input for private-sector production. Turnovsky (2000) and Ghosh and Mourmouras (2004), amongst others, go on to examine the influence on production that the presence of public capital has in different capacities.

The aforementioned studies only examine fiscal policy in models that are without financial friction, thereby underestimating the information relevant to the business cycle that is embedded in financial markets. However, some fiscal policy interventions are enacted because of the impact of financial disruption on macroeconomic outcomes. Moreover, theory and empirical analysis support the significant role of financial friction in macroeconomic dynamics. Therefore, it is important to give consideration to the presence of financial friction in fiscal policy analysis. The essence of financial market dynamics is further corroborated by Leeper (2010), who clearly state that dynamics, expectations and asset prices all play a fundamental part in the transmission of the impact of monetary and fiscal policies.

This chapter is of significance because it integrates the theoretical findings by highlighting the joint dynamics of monetary and fiscal policies; it does so using a dynamic stochastic general equilibrium (DSGE) model that accounts for credit market imperfections. The financial friction and capital tax dynamics in the chapter closely relate to the findings in Fernandez-Villaverde (2010) where the financial friction dynamics are a costly verification problem, as in Bernanke et al. (1999b). Furthermore, capital tax is applied on net returns on deposits. A simplification of the financial friction mechanism as in Christensen and Dib (2008) is adopted here.

This chapter differs from Fernandez-Villaverde (2010) in the following ways: First, unlike Fernandez-Villaverde (2010), it is assumed here that government spending (i.e., consumption and investment) is productive. Second, here all fiscal instruments respond to debt levels and, similarly, both types of policies (i.e., fiscal and monetary) react to the output gap. By contrast, in Fernandez-Villaverde (2010), taxes are solely exogenous with no reference to real variables or debt. Third, several of the model parameters presented in this chapter are estimated; those remaining are calibrated. In addition, this chapter allows for a capital tax to be levied on entrepreneurs' dividends, alongside the levy on net returns on deposits.

There are three key questions this chapter seeks to address: i) what is the effect of fiscal policy (accounting for the impact of credit market imperfections on credit spread)?; ii) which aspect of government spending has the greatest effect on credit spread?; and iii) what are the implications of the financial friction mechanism for the transmission of fiscal policies?

A medium-scale DSGE model is presented to answer these questions. The baseline is the New Keynesian model in Smets and Wouters (2007) with some structural augmentations. These include the introduction of an Epstein-Zin preference, financial friction, and the fiscal sector. The fiscal instruments include government consumption, government investment, transfer, labour tax, consumption tax, and the capital tax applied to net returns on deposits and entrepreneurs' dividends. A substantial subset of the parameters is estimated using US macroeconomic and federal government fiscal data, while the rest are calibrated either to their averages in the data or taken from existing studies. Amongst the parameters that are estimated are the fiscal and monetary policy coefficients. Thereafter, the implications of the financial friction mechanism for fiscal transmission are evaluated using present-value fiscal multipliers.

The chapter's contribution to the area of study is the evidence it provides of the interaction of monetary and fiscal policies and their effects on credit-constrained firms. Specifically, the model presented shows that government spending is not wasteful. In particular, government consumption directly affects private consumption decisions, and government investment influences public capital, which, in turn, affects marginal productivity. The analysis shows that productive capital leads to higher marginal productivity. Furthermore, government investment can lower the cost of the external finance premium. At the same time, it is the most effective fiscal multiplier for output and investment.

The remainder of this chapter is structured as follows. Section 3.2 highlights several related studies on quantifying fiscal multipliers. Section 3.3 discusses the theoretical set-up of the model. Section 3.4 presents the estimation procedure and discussion of the results. Section 3.5 sets out the model dynamics – the fiscal multipliers and impulse response. Section 3.6 contains the conclusion and recommendations.

3.2 Related Literatures on Quantifying the Effect of Fiscal Policies

A number of empirical studies try to quantify the effects of fiscal multipliers over the business cycle (see Ramey (2011a) for a comprehensive list of studies analysing fiscal multipliers for the US). However, these studies are mostly based on a linear structural vector autoregression (SVAR) or linearised DSGE model, hence limiting the state-dependent multipliers. Candelon and Lieb (2013) use a regime-switching approach for a non-linear structural vector autoregressive (VAR) model and observe that the effect of fiscal policies is differentiated across the business cycle. Furthermore, the government spending multiplier is stronger on impact than it is for tax multipliers, and this effect is more pronounced in a recession.

Similarly, Auerbach and Gorodnichenko (2012) suggest that fiscal multipliers vary over the business cycle, with spending multipliers more sizeable in recessions than in expansions. Bachmann and Sims (2012) also find evidence of the varying effect of fiscal policy over the business cycle by estimating a non-linear VAR that allows for differentiating the effect of government spending in 'normal' times versus recessions. In particular, they identify government investment as more effective relative to consumption during economic downturns, as it predicts long-term productivity.

Ramey and Zubairy (2018) seek to empirically identify the reason the government spending multiplier in the US varies. They investigate whether this variation is because of the measure of slackness (i.e., the unemployment rate) in the economy or the near-zero lower bound interest rate. The results support the idea that fiscal multipliers vary over the business cycle.

While the studies just mentioned do not consider the impact of monetary policy on the fiscal multipliers, Cogan et al. (2010); Christiano et al. (2011a); Coenen et al. (2012); Carrillo and Céline (2013), for example, find that in New Keynesian DSGE models, government spending multipliers are more substantial when the zero lower bound is binding for monetary policy. Leeper et al. (2017) also show that long-run government spending multipliers vary for the different regime combinations of fiscal and monetary policies in operation; the multiplier is larger under a passive monetary/active fiscal policy regime than it is under an active monetary/passive fiscal regime. The short-run multiplier is comparable in the two regimes.

How fiscal policies are communicated and anticipated by private agents is also a possible factor influencing fiscal multipliers. For example, the empirical work of Ricco et al. (2016) shows how the communication of fiscal policy affects its impact. Some studies, such as Galí et al. (2007); Eggertssonn and Krugman (2012), find that allowing for heterogeneity in households (such that some are financially constrained while others are not) has important implications for the transmission of fiscal policy. Studies like Ramey (2011b); Leeper et al. (2012, 2013) also suggest that the effects of fiscal policies are often anticipated ahead of their impact.

3.3 The Model

The model builds on Smets and Wouters (2003, 2007), which introduces a DSGE model for the US economy that incorporates several real and nominal frictions. The following are added to the model: (i) Epstein-Zin preference as in Rudebusch and Swanson (2012) to help differentiate the coefficient of risk aversion from the elasticity of substitution; (ii) Financial friction as in Bernanke et al. (1999b), in the form of a cost verification problem, to endogenously describe the behaviour of the financial industry. Financial friction takes a simple form, as in Christensen and Dib (2008); and (iii) A fiscal policy sector with a wide range of instruments that are endogenous and modelled following Leeper et al. (2009), except for consumption tax that is entirely exogenous. Furthermore, the following adjustments are made to the benchmark Smets and Wouters (2003, 2007) model: (i) Fixed cost in production is removed because of the introduction of financial friction; (ii) Capital utilisation is redefined. (iii) The output gap is defined as the deviation of output from its steady state. Unlike Smets and Wouters (2007), where there are wage and price mark-up shocks alongside a Kimball aggregator, this chapter uses a Dixit-Stiglitz aggregator for aggregating prices in the goods and labour markets. In addition, the two mark-up shocks are replaced by labour supply and marginal cost shocks, respectively. The subsections that follow describe the equilibrium conditions of each sector of the economy.

3.3.1 Household

There is a continuum of households indexed with $j \in [0, 1]$, with effective consumption $(\tilde{C}_t(j))$, which is a composite of private consumption $(C_t(j))$ and government consumptions services (G_t^C). Effective consumption is expressed as follows.

$$\tilde{C}_t(j) = \left[(1-\eta_g)^{\frac{1}{\psi_c}} C_t(j)^{\frac{\psi_c - 1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}} G_t^{C\,\frac{\psi_c - 1}{\psi_c}} \right]^{\frac{\psi_c}{\psi_c - 1}}$$

The functional form of effective consumption takes on a non-separable constant elasticity of substitution (CES) aggregator following Bouakez and Rebei (2007); Pappa (2009); Coenen et al. (2013); Ercolani and e Azevedo (2014), amongst others. Allowing private and government consumption to be non-separable, leads to co-movement in the two goods, and it is economically traceable. For example, government spending on education and healthcare has a direct impact on households' private consumption decisions. The parameter $\eta_g \in (0, 1)$ is the share of government consumption expenditure in effective consumption, whileψ_c is the elasticity between private and government consumption. As $\psi_c \to 0$, private and public consumption become perfect complements, and as $\psi_c \to \infty$, they tend to become perfect substitutes.

Households derive utility from their effective consumption $\tilde{C}_t(j)$ and disutility from their labour effort $L_t(j)$, with instantaneous utility given below,

$$U(\tilde{C}_t, L_t) = \varepsilon_t^{\beta} \frac{1}{1-\sigma_C} \left(\tilde{C}_t(j) - \eta_c \tilde{C}_{t-1}(j) \right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L} \varepsilon_t^{w} L_t^{1+\sigma_L}(j) \right), \tag{3.3.1}$$

where η_c is the external consumption habit formation parameter,ε_t^{β} is preference shock,and ε_t^{w} is labour supply shock. The elasticity of substitution is differentiated from the coefficient of risk aversion by households having a recursive Epstein-Zin preference given by

$$V_t(j) = U(\tilde{C}_t(j), L_t(j)) + \beta v_t(j), \tag{3.3.2}$$

where

$$v_t(j) = \mathbb{E}_t (V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}}$$

The household budget constraint is given by

$$(1+\tau_t^c) C_t(j) + \tfrac{D_t(j)}{P_t} + \tfrac{B_t(j)}{P_t} \leq (1-\tau_t^l) \tfrac{W_t^h(j) L_t(j)}{P_t} + R_{t-1,t}^{D} \tfrac{D_{t-1}(j)}{P_t} + \left(1 + (1-\tau_t^k)(R_{t-1,t} - 1) \right) \tfrac{B_{t-1}(j)}{P_t} + \tfrac{Div_t}{P_t} + Z_t$$

Each household j in period t spend on private consumption $C_t(j)$, invest by making a nominal deposit $B_t(j)$ at the financial intermediary which pays uncontingent gross nominal rate R_t. Household also have investment in government (public) debt $D_t(j)$ yielding an uncontigent gross nominal rate R_t^D.They decide how many hours to work $L_t(j)$, receives lump sum transfer Z_t from government, also receives dividend Div_t from labour union. Meanwhile, they pay distortionary taxes $\tau_t^c, \tau_t^l, \tau_t^k$ which are levied on consumption, labour earnings and net returns on deposit.

Thus household j's optimisation problem is to maximise V_0bychoosing private consumption $C_t(j)$, hours $L_t(t)$,deposit $B_t(j)$, public debt $D_t(j)$ and preference V_t. Appendix A provides the detail of the optimisation problem and solutions. However, the equilibrium equations arrived at after solving household problem are given below.

Households' stochastic discounting factor (SDF) at time t for a payoff in time $t+1$ is given as

$$\Lambda_{t,t+1} = \beta \left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}} \right]^{-\sigma_E} \frac{\Xi_{t+1}}{\Xi_t}, \tag{3.3.3}$$

where $\beta \in (0,1)$ is the subjective discount factor and

$$\Xi_t = \frac{\varepsilon_t^\beta (\tilde{C}_t - \eta_c \tilde{C}_{t-1})^{-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L} \varepsilon_t^w L_t^{1+\sigma_L}\right) \left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}}}{(1+\tau_t^c)}$$

The household's decision on government debt and deposits with financial intermediaries gives the following expressions

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}} \right] R_{t,t+1}^D \tag{3.3.4}$$

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}} \Big(1 + (1-\tau_{t+1}^k)(R_{t,t+1} - 1) \Big) \right] \tag{3.3.5}$$

The real wage is given as

$$\frac{W_t^h}{P_t} = \frac{(\tilde{C}_t - \eta_c \tilde{C}_{t-1})\varepsilon_t^w L_t^{\sigma_L}(1+\tau_t^c)}{\left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}}(1-\tau_t^l)} \tag{3.3.6}$$

3.3.2 Labour Market

Households directly supply their homogeneous labour to an intermediate labour union, which, in turn, differentiates the labour services and sets wages subject to Calvo pricing. This differentiated labour is packaged by individuals called labour packers. As a result, there are two sub-sectors in the labour market, as discussed in the following subsection.

3.3.2.1 Labour Packers

The labour-union differentiated labour $H_t(l)$ is bought and packaged by labour packers. Labour packers sell H_t for W_t, thus maximising their profit as

$$\max_{H_t(l)} \quad W_t H_t - \int_0^1 W_t(l) H_t(l)\, \mathrm{d}l$$
$$\text{s.t} \quad H_t = \left(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}}\, \mathrm{d}l\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

Their optimisation problem above gives the labour demand schedule as

$$H_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\epsilon_w} H_t. \tag{3.3.7}$$

The wage received by labour packers, which is also the cost of wages to entrepreneurs, is

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w}\, \mathrm{d}l\right)^{\frac{1}{1-\epsilon_w}} \tag{3.3.8}$$

3.3.2.2 Labour Unions

Labour unions hire raw labour force L_t from households with subsequent training conducted for their differentiation based on skills. They take the marginal rate of substitution as the cost of labour services in their negotiations with labour packers. The mark-up above this marginal disutility is distributed to the households in the form of dividend Div_t, as seen in the household budget constraint. The union is subjected to nominal rigidities and can only readjust wages with probability $1-\zeta_w$ in each period, therefore optimising wages over the period in which they cannot change the price. In the period when they are unable to re-optimise wages, they partially index the previously optimised wage to reflect lagged inflation.

$$\max_{\tilde{W}_t(l)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \Big(W_{t+s}(l) H_{t+s}(l) - W_{t+s}^h L_{t+s}(l) \Big)$$

$$\text{s.t} \quad H_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}$$

$$H_{t+s}(l) = L_{t+s}(l)$$

$$W_{t+s}(l) = \tilde{W}_t(l) \bigg(\prod_{l=1}^{s} \pi_{t+l-1}^{l_w} \pi_{ss}^{1-l_w} \bigg) \quad \text{for} \quad s = 1, \ldots, \infty$$

Solving the above optimisation problem gives

$$\frac{\tilde{W}_t}{W_t} = \left(\frac{\epsilon_w}{1-\epsilon_w} \right) \left(\frac{G_t^{w2}}{G_t^{w1}} \right) \tag{3.3.9}$$

where

$$G_t^{w1} = W_t H_t + \mathbb{E}_t \left[\zeta_w \Lambda_{t,t+1} \left(\frac{W_t}{W_{t+1}} \right)^{1-\epsilon_w} \left(\pi_t^{l_w} \pi_{ss}^{1-l_w} \right)^{1-\epsilon_w} G_{t+1}^{w1} \right] \tag{3.3.10}$$

and

$$G_t^{w2} = W_t^h H_t + \mathbb{E}_t \left[\zeta_w \Lambda_{t,t+1} \left(\frac{W_t}{W_{t+1}} \right)^{-\epsilon_w} \left(\pi_t^{l_w} \pi_{ss}^{1-l_w} \right)^{-\epsilon_w} G_{t+1}^{w2} \right] \tag{3.3.11}$$

The aggregate wage expression is

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{1-\epsilon_w} + \zeta_w \left(\gamma \pi_{t-1}^{l_w} \pi_{ss}^{1-l_w} W_{t-1} \right)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}} \tag{3.3.12}$$

There is a wage dispersion cost because of the discrepancy between labour demanded and labour supplied. This implies that

$$H_t \neq L_t, \quad \text{and} \quad W_t \neq W_t^h \implies L_t = \Delta_t^w H_t$$

The explicit expression of the wage dispersion cost considering the sticky wage is

$$\Delta_t^w = (1-\zeta_w)\int_0^1 \left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w} \mathrm{d}l + \zeta_w \left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w} \Delta_{t-1}^w. \tag{3.3.13}$$

3.3.3 Capital Goods Producers

Capital goods producers take the final good I_t to produce investment goods $\tilde{I}_t$. They work in a perfectly competitive environment and are faced with the cost of changing the flow of investments. As a result, they choose the quantity of investment I_t to maximise their profit

$$\Pi_t^I = Q_t \tilde{I}_t - I_t$$

$$\max_{I_t \tilde{I}_t} \quad \mathbb{E}_0\Big[\sum_{t=0}^{\infty} \Lambda_{0,t}\Pi_t^I\Big]$$
$$\text{s.t} \quad \tilde{I}_t = \varepsilon_t^i \left[I_t - \frac{\Psi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2 I_t\right]$$

$$1 = Q_t\varepsilon_t^i\left[1 - \Psi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \gamma\right) - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}\varepsilon_{t+1}^i\Psi\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - \gamma\right)\right],$$

where Ψ is the investment adjustment shock and ε_t^i is the shock to investment.

3.3.4 Consumer Goods Market

This market comprises retailers, wholesalers, and entrepreneurs.

3.3.4.1 Retailers(Final Good Producer)

The retailers are the producers of the final good Y_t. The final good is composed of differentiated goods from wholesalers $Y_t(i)$. It is allocated to consumption, investment and government expenditure. The technology used in transforming these differentiated goods is given in the form of a Dixit-Stiglitz aggregator. Retailers' optimisation problem gives the demand curve expression as:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t \tag{3.3.14}$$

And the Law of Motion (LOM) for price is

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_p}\,\mathrm{d}i\right)^{\frac{1}{1-\epsilon_p}} \tag{3.3.15}$$

3.3.4.2 Wholesalers

Wholesalers buy intermediate goods M_t from the entrepreneur who owns firms. These intermediate goods are differentiated without cost, taking the demand function earlier derived from retailers' first-order condition (FOC). Wholesaler's prices are subject to Calvo pricing, which introduces nominal rigidities into the model. The optimal price set by wholesalers that are allowed to re-optimise gives rise to the optimisation problem below.

$$\begin{aligned}
\max_{\tilde{P}_t(i)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Lambda_{t,t+s}\left[\tilde{P}_t(i) X_{s,t}^p Y_{t+s}(i) - MC_{t+s} M_{t+s}(i)\right] \\
\text{s.t} \quad & Y_{t+s}(i) = Y_{t+s}\left(\frac{\tilde{P}_t(i) X_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p} \\
& Y_{t+s}(i) = \varepsilon_t^p M_{t+s}(i),
\end{aligned}$$

where

$$X_{s,t}^p = \begin{cases} 1 & \text{for} \quad s = 0 \\ \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) & \text{for} \quad s = 1\ldots\infty \end{cases}$$

The first constraint is the demand schedule (3.6.19), which is obtained from the retailer optimisation problem. However, the second constraint represents a linear production function used for differentiating the intermediate good, whereε_t^p is the production function for transforming the intermediate good, and $\tilde{P}_t(i)$ is the newly optimised price. The inability to change this price for some period means that wholesalers are only able to partially index the current price to lagged inflation, represented by $X_{t,s}^p$. The probability of being able to re-optimise price is ζ_p, while ι_p measures the degree of price indexation. Solving the optimisation problem above gives the optimal newly set price for those allowed to reset the price as

$$\frac{\tilde{P}_t(i)}{Pt} = \Big(\frac{\epsilon_p}{\epsilon_p - 1}\Big)\Big(\frac{G_t^{p2}}{G_t^{p1}}\Big),$$

where

$$G_t^{p1} = P_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{1-\epsilon_p} G_{t+1}^{p1}\right]$$

and

$$G_t^{p2} = \varepsilon_t^p MC_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{-\varepsilon_t^p} G_{t+1}^{p2}\right]$$

Aggregate price (3.3.15) is the sum of the newly reset price and the partially indexed price, which is expanded as

$$P_t = \left((1-\zeta_p)\tilde{P}_t(i)^{1-\epsilon_p} + \zeta_p\Big(P_{t-1}\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}\Big)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}$$

Simplifying the above gives

$$1 = (1-\zeta_p)\Big(\frac{\tilde{P}_t(i)}{P_t}\Big)^{1-\epsilon_p} + \zeta_p\Big(\frac{\pi_{t-1}^{l_p}\pi_{ss}^{1-l_p}}{\pi_t}\Big)^{1-\epsilon_p} \tag{3.3.16}$$

The price dispersion expression is derived as follows.

$$\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i$$

$$\Delta_t^p = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p \tag{3.3.17}$$

3.3.5 Financial Intermediary

Financial intermediaries act as a go-between for households and entrepreneurs for financial transactions, lending household's deposited money (in the form of bond B_t) to entrepreneurs as a loan A_t at a rate R_t^N higher than the risk-free rate R_t. While it is inexpensive for entrepreneurs to obtain internal finance, it is costly to source funds externally. External funds can be obtained through loans, bonds, equity or from various other sources. The existence of market imperfections in the form of asymmetric information between market participants is probably a good explanation for the high cost of obtaining external funds. Entrepreneurs cannot obtain loans at the risk-free rate because financial intermediaries cannot easily observe their output. It is costly for financial intermediaries to observe the realised returns of entrepreneurs; they have to pay a state-verification cost, suggesting that the cost of sourcing external finance is different from the economy's risk-free rate. This chapter assume that the nominal rate of return $R_{t,t+1}^N$ on an entrepreneur's loan from period t to $t+1$ is fixed at time t expressed as

$$R_{t,t+1}^N = S_{t,t+1}R_{t,t+1}, \tag{3.3.18}$$

where $S_{t,t+1}$ is the spread between risk-free and loan rates, and it increases by the amount entrepreneurs borrow relative to their net worth. Like every contract between borrowers and lenders, there is a need for the borrower to post collateral, which, in this case, is the net worth of the entrepreneur observed by the financial intermediary before the loan issuance.

$$\begin{aligned} S_{t,t+1} &= S\left(\varepsilon_t^s \frac{Q_t K_t}{N_t}\right) \quad \text{where} \quad S'() < 0 \quad \text{and} \quad S(1) = 1 \\ &= S_{ss}\left(\varepsilon_t^s \frac{Q_t K_t}{N_t}\frac{N_{ss}}{K_{ss}}\right)^{\psi_S} \end{aligned}$$

The external finance premium $S_{t,t+1}$is assumed based on the reasoning that lenders only know the aggregate $\frac{Q_t K_t}{N_t}$ and not that of the individual entrepreneurs. The constant S_{ss} is calibrated to data, and ε_t^s is an exogenous disturbance to the external finance premium. This can be interpreted as shock to the credit supply, which represents the functionality of financial intermediaries. The elasticity of external finance premium to entrepreneur's leverage is ψ_S. Hence, if entrepreneurs borrow $A_{t,t+1}$ at t they must repay $R^N_{t,t+1} A_{t,t+1}$ to the financial intermediary. Although financial intermediaries only recover the uncontingent rate $R_{t,t+1}$, which is rebated to households as the earnings on their deposits. The difference between the loan rate $R^N_{t,t+1}$ and $R_{t,t+1}$ is the loss to default and intermediation. By market clearing, loans must be equal to deposits $A_{t,t+1} = B_{t,t+1}$[1].

3.3.5.1 Asset Pricing

In theory, the current price of any asset should equal the expected future value that is discounted stochastically. Government and corporate bond prices at time t of a bond maturing in n period are defined as follows.

$$B^g_{t,n} = \mathbb{E}_t \left[\Lambda^N_{t+1} B^g_{t+1,n-1} \right] \tag{3.3.19}$$

$$B^c_{t,n} = \mathbb{E}_t \left[\Lambda^N_{t+1} B^c_{t+1,n-1} \right] \left(\frac{1}{S_t} \right), \tag{3.3.20}$$

where $\Lambda^N_{t+1} = \frac{\Lambda_{t+1}}{\pi_{t+1}}$ is the nominal discounting factor and the respective return (yield to maturity) in the model is given as

$$R^g_{t,n} = \left(\frac{1}{B^g_{t,n}} \right)^{\frac{1}{n}} \tag{3.3.21}$$

$$R^c_{t,n} = \left(\frac{1}{B^c_{t,n}} \right)^{\frac{1}{n}} \tag{3.3.22}$$

From the expressions for government and corporate bonds above, the corporate bond spread can easily be deduced in the DSGE model as:

[1] Note that $B_{t,t+1}$ is household's deposit with financial intermediary and $A_{t,t+1}$ is the loan entrepreneurs took from the financial intermediary

$$C_{t,n}^{b} = R_{t,n}^{c} - R_{t,n}^{g} \tag{3.3.23}$$

3.3.5.2 Entrepreneurs

Each entrepreneur owns a firm i that uses the following technology

$$M_t(i) = \varepsilon_t^a \left[\left(K_{t-1}(i)U_t^k(i)\right)^{\alpha} \left(\gamma^t H_t(i)\right)^{1-\alpha} \right]^{1-\theta_G} {K_{t-1}^G}^{\theta_G} \tag{3.3.24}$$

the technology employed by entrepreneurs comprises privately-owned capital K_{t-1}, labour H_t and public capital K_{t-1}^G. In a similar vein to Smets and Wouters (2007), γ^t is the labour augmenting deterministic growth rate in the economy and ε_t^a is the total factor productivity (TFP) shock.

Each entrepreneur hires labour H_t paying W_t as a wage to the labour union (which, in turn, pays households). They also buy a portion of private investment goods $\tilde{I}_t$ from investment good producers. The purchased investment good is added to their existing stock of private capital K_{t-1} to make up for the next period's private capital K_t, which is the private capital choice at the end of the period $t-1$ to be used for the period t production. Entrepreneurs also take advantage of the public capital provided by the government and denoted by K_{t-1}^G, where θ_G represents the elasticity of output with respect to public capital.

At each period, entrepreneurs survive with the probability φ; hence, the concern of entrepreneurs is mostly for their lifetime net worth and its maximisation. The optimisation problem is set out below.

$$\underset{H_t, K_t, U_t^k}{\text{maximise}} \quad \mathbb{E}_t \left[\sum_{s=0}^{\infty} (1-\varphi)^{s-1} \Lambda_{t,t+s} N_{t+s} \right] \tag{3.3.25a}$$

$$\text{subject to} \quad A_{t-1} = Q_{t-1}K_{t-1} - N_{t-1} \tag{3.3.25b}$$

$$N_t = MC_t M_t - W_t H_t + (1-\delta_t^k) Q_t K_{t-1} - \frac{R_{t-1,t}^N}{\pi_{t-1,t}} A_{t-1} \tag{3.3.25c}$$

$$M_t \leq \varepsilon_t^a \left[\left(K_{t-1}(i)U_t^k(i)\right)^{\alpha} \left(\gamma^t H_t(i)\right)^{1-\alpha} \right]^{1-\theta_G} {K_{t-1}^G}^{\theta_G} \tag{3.3.25d}$$

$$K_t \leq (1-\delta_t^k) K_{t-1} + \tilde{I}_t \tag{3.3.25e}$$

$$K_t^G = (1-\delta_G) K_{t-1}^G + G_t^I \tag{3.3.25f}$$

$$\delta_t^k = \delta + \psi_u \frac{\alpha(1-\theta_G) MC_{ss} M_{ss}}{Q_{ss} K_{ss}} \left({U_t^k}^{\frac{1}{\psi_u}} - 1 \right) \tag{3.3.25g}$$

Each of the above constraints in the entrepreneur's problem can be explained. The first constraint represents entrepreneurs' balance sheet condition, captured in 3.3.25b. This constraint indicates how much an entrepreneur has to borrow at the end of period $t-1$ for the purchase of the capital K_{t-1} needed for production in period t. In addition to their end-of-period real net worth (N_{t-1}), they obtain external funding in the form of a loan A_{t-1} to be able to obtain the required capital. This external funding is obtainable from household deposits with financial intermediaries. The financial intermediaries thus set the terms for entrepreneurs.

The second constraint in3.3.25c is the LOM of net worth. This shows that the entrepreneur retains the capital share after depreciation less loan repayments. The third constraint in 3.3.25d is the Cobb-Douglas production function, which allows for variable capital utilisation U_t^k and public capital also used in production. The constraint in 3.3.25e is more or less redundant since entrepreneurs directly choose K_t, given K_{t-1}. Hence the purpose of 3.3.25e is in defining the capital good $\tilde{I}_t$. The dynamics of public capital and how government investment translates into productive public capital are represented in 3.3.25f and δ_G is the rate at which public capital depreciates. The last constraint 3.3.25g gives the expression for depreciation as a function of the marginal product of capital.

Given the production function, the marginal product of capital is given as

$$\mathsf{MPK}_t = \alpha(1-\theta_G)\frac{\mathsf{MC}_t M_t}{K_{t-1}} \tag{3.3.26}$$

Solving the entrepreneur's optimisation problem gives the optimum capital utilisation as

$$U_t^k = \left(\frac{\frac{\mathsf{MPK}_t}{Q_t}}{\frac{\mathsf{MPK}_{ss}}{Q_{ss}}}\right)^{\psi_u} \tag{3.3.27}$$

Equation (3.6.30) determines the relationship between the utilisation rate and the marginal product of capital, where ψ_u is the elasticity of utilisation cost with respect to input from capital.

The partial derivative of K_t implies the following

$$\mathbb{E}_t\left[R^K_{t,t+1}\right] = \mathbb{E}_t\left[\frac{\mathsf{MPK}_{t+1}}{Q_t} + (1-\delta^k_{t+1})\frac{Q_{t+1}}{Q_t}\right] \tag{3.3.28}$$

Where $R^k_{t,t+1}$ is the ex-post real return on capital. It is the sum of income gain (i.e., the marginal real revenue on capital, which is evaluated by real capital) and capital gain (i.e. the real price change of the remaining capital).However, it should be noted that $\frac{R^N_{t,t+1}}{\pi_{t,t+1}} \neq R^K_{t,t+1}$ because the marginal product of capital $R^k_{t,t+1}$and $\pi_{t,t+1}$ depends on the realisation of the shocks at $t+1$. Furthermore, $R^N_{t,t+1} \neq \mathbb{E}_t\left[\pi_{t,t+1} R^K_{t,t+1}\right]$ because $\pi_{t,t+1}$ and $R^K_{t,t+1}$ are not independent.

$$\mathbb{E}_t\Big[\Lambda_{t+1} R^K_{t,t+1}\Big] = \mathbb{E}_t\Big[\Lambda_{t,t+1}\frac{1}{\pi_{t,t+1}}\Big] R^N_{t,t+1} \tag{3.3.29}$$

The real net worth of all entrepreneurs has its LOM representation as

$$N_t = \varphi E_t + (1-\varphi)\varepsilon^N_t \tag{3.3.30}$$

where ε^N_t represents the seed transferred from exiting to incoming entrepreneurs, and E_t is the net worth of surviving entrepreneurs given by

$$E_t = \left(R^K_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}\right) Q_{t-1} K_{t-1} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}} N_{t-1} \tag{3.3.31}$$

Since the amount borrowed by entrepreneurs is $A_{t-1} = Q_{t-1}K_{t-1} - N_{t-1}$, thus the nominal debt repayment as agreed at time $t-1$ is $R^N_{t-1,t}(Q_{t-1}K_{t-1} - N_{t-1})$, with $\frac{R^N_{t-1,t}}{\pi_{t-1,t}}$ being the ex-ante nominal contract rate deflated by the ex-post realised inflation. The net worth of existing entrepreneur E_t is the product of realised gross returns and capital less the product of the contracted borrowing rate and the amount of borrowing.

3.3.6 Aggregate Resource Constraint and Government

In aggregating the model variables, it is important to first note that it is a standard practice in macroeconomics to assume that a representative household chooses consumption and labour.

3.3.6.1 Monetary Policy

The central bank follows a nominal interest rate rule by adjusting its instruments in response to deviations of inflation and output from their respective target levels.

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi}\left(\frac{Y_t}{Y_{ss}}\right)^{\psi_y}\right]^{1-\rho_R}\varepsilon^r_t, \tag{3.3.32}$$

where R_{ss} is the steady-state nominal rate, π_{ss} and Y_{ss} are the steady state of inflation and output, respectively. The interest-rate smoothing parameter is denoted by ρ_R and the shock to the monetary policy ε_t^r is

$$\ln \varepsilon_t^r = \rho_r \ln \varepsilon_{t-1}^r + \eta_t^r, \quad \eta_t^r \sim \mathcal{N}(0, \sigma_r). \tag{3.3.33}$$

3.3.6.2 Fiscal Policy

The government decides on the fiscal instruments it will use to satisfy its budget constraint given by

$$G_t^C + G_t^I + Z_t + \frac{R_{t-1}^D D_{t-1}}{\pi_t} = \tau_t^c C_t + \tau_t^l W_t L_t + \tau_t^k \left((R_{t-1} - 1) \frac{B_{t-1}}{\pi_t} + \varphi N_{t-1} \right) + D_t \tag{3.3.34}$$

In the government budget above, capital tax τ_t^k is levied on net interest earnings on bond $(R_{t-1} - 1)\frac{B_{t-1}}{\pi_t}$ and entrepreneur's dividend φN_{t-1}.

All fiscal instruments except consumption tax respond to the deviations of lagged government debt D_{t-1} and output Y_t from their respective steady states. All are affected by AR(1) exogenous processes.

$$\frac{\tau_t^k}{\tau_{ss}^k} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_K} \left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_K} \varepsilon_t^K \tag{3.3.35}$$

$$\frac{\tau_t^l}{\tau_{ss}^l} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_L} \left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_L} \varepsilon_t^L \tag{3.3.36}$$

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_Z} \left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_Z} \varepsilon_t^T \tag{3.3.37}$$

$$\frac{G_t^C}{G_{ss}^C} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GC}} \left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GC}} \varepsilon_t^{GC} \tag{3.3.38}$$

$$\frac{G_t^I}{G_{ss}^I} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GI}} \left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GI}} \varepsilon_t^{GI} \tag{3.3.39}$$

$$\frac{\tau_t^c}{\tau_{ss}^c} = \varepsilon_t^C, \tag{3.3.40}$$

where the ε_t^J for $J = K, L, C, GC, GI$ follows AR(1) processes as:

$$\ln \varepsilon_t^J = \rho_J \ln \varepsilon_{t-1}^J + \eta_t^J, \quad \eta_t^J \sim \mathcal{N}(0, \sigma_J) \tag{3.3.41}$$

In particular, the shocks to taxes are believed to co-move; hence the exogenous processes are expressed as

$$\begin{aligned}
\ln \varepsilon_t^K &= \rho_K \ln \varepsilon_{t-1}^K + \eta_t^K + \rho_{l,k}\eta_t^L + \rho_{k,c}\eta_t^C \\
\ln \varepsilon_t^L &= \rho_L \ln \varepsilon_{t-1}^L + \eta_t^L + \rho_{l,k}\eta_t^K + \rho_{c,l}\eta_t^C \\
\ln \varepsilon_t^C &= \rho_C \ln \varepsilon_{t-1}^C + \eta_t^C + \rho_{k,c}\eta_t^K + \rho_{c,l}\eta_t^L
\end{aligned}$$

3.3.6.3 Intermediate Goods Market Clearing

The aggregate of the goods market

$$\begin{aligned}
M_t &= \int_0^1 M_t(i)\,\mathrm{d}i \\
M_t &= Y_t \int_0^1 P_t(i)^{-\epsilon_p}\,\mathrm{d}i = Y_t \Delta_t^p
\end{aligned}$$

Also, for the labour market

$$\begin{aligned}
L_t &= \int_0^1 L_t(i)\,\mathrm{d}i \\
L_t &= H_t \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_w} \mathrm{d}i = H_t \Delta_t^w,
\end{aligned}$$

where price and wage dispersions are (3.3.17) and (3.3.13), respectively.

3.3.6.4 Final Goods Market Clearing

Market clearing implies that the following holds

$$Y_t = C_t + I_t + G_t^C + G_t^I \tag{3.3.42}$$

The implication of (3.6.39) is that the final good produced in the economy is allocated to private consumption (C_t), private investment (I_t), government consumption (G_t^C) and government investment (G_t^I).

3.4 Estimation and Calibration

The model parameters are partly calibrated and partly estimated. The estimation procedure is carried on using US quarterly macroeconomic and federal fiscal data from 1983Q1 to 2007Q4. The estimation period is chosen to ensure stability and equilibrium determinacy.

3.4.1 Calibration

The difficulty in identifying some of the parameters suggests the need for calibration. Some of these parameters are calibrated to values in existing studies, while others are calibrated to the average of historical data. Table 3.1 shows the calibrated parameters and their sources.

The parameters governing the Epstein-Zin preference differentiate the coefficient of risk aversion from the elasticity of substitution and are calibrated to the value in Rudebusch and Swanson (2012).

Other preference parameters (σ_C and σ_L) are calibrated to the same value as in Rudebusch and Swanson (2012). The steady-state external finance premium is set to 1.0075, corresponding to an annual risk spread of 300 basis points, equal to the sample average spread between the business prime lending rate and the three-month treasury bill rate. The estimation period is from 1983 to 2007.

The steady state of all fiscal variables is calibrated to their sample mean (i.e., 1983Q1-2007Q4). The only exception is government transfers and the productivity of public capital. Steady-state government transfers are calculated based on the assumed values of the other relevant pre-determined variables that are consistent with the model equations. The productivity of public capital (θ_G), which is an integral component in determining the effectiveness of government investment, has been investigated in some studies with no consensus on the value (see, Leeper et al. (2010) for a list of studies). This chapter takes on the value in Baxter and King (1993); that is, the setting $\theta_G = 0.05$. The parameter η_g denoting the share of government consumption in effective consumption is calculated to be 7%. This is because the fiscal dynamics captured in the estimation are only for the federal government. However, if the state government fiscal dynamics are included, the value of η_g over the same estimation period is 18%.

Table 3.1: Calibrated Parameters and Sources

	Baseline	Description	Source
σ_E	-148.3	Epstein-Zin parameter	Rudebusch and Swanson (2012)
σ_C	2.00	IES in consumption	Rudebusch and Swanson (2012)
σ_L	3.00	Elasticity of labour	Rudebusch and Swanson (2012)
β	0.995	Subjective Discounting factor	Smets and Wouters (2007)
δ	0.025	Capital Depreciation rate	Smets and Wouters (2007)
θ_p	5	Price elasticity of substitution	Christensen and Dib (2008)
θ_w	5	Wage elasticity of substitution	Christensen and Dib (2008)
S_{ss}	1.0075	Steady-state external finance premium	Sample average
π_{ss}	1.005	Steady-state gross inflation	Smets and Wouters (2007)
φ	0.9728	Entrepreneur survival rate	Christensen and Dib (2008)
α	0.36	Share of capital	Leeper et al. (2010)
θ_G	0.05	Productivity of public capital	Leeper et al. (2010)
τ_k	0.19	Capital tax rate	Sample average
τ_l	0.21	Labour tax rate	Sample average
τ_c	0.02	Consumption tax rate	Sample average
g^c_{ss}/y_{ss}	0.05	Government consumption share in output	Sample average
g^i_{ss}/y_{ss}	0.02	Government investment share in output	Sample average
b_{ss}/y_{ss}	0.56	Debt ratio	Calculated from data
η_g	0.07	Share of government consumption in effective consumption	Sample average

3.4.2 Estimation

The estimation period (1983Q1 to 2007Q4) spans the regimes of active monetary policy and passive fiscal policy(Bhattarai et al., 2012) to ensure the determinacy of equilibrium conditions in the New Keynesian model presented in this chapter. Active monetary policy is characterised as the case where interest rates respond strongly to inflation. Passive fiscal policy is where taxes respond strongly to outstanding debt, which is consistent with the definition in Leeper (1991). The estimation period is selected as it excludes the period of high inflation in the 1970s. Monetary policy over this period is active as it was governed by the Taylor rule and the fiscal policy is thus passive.

There are 12 observables used in estimating the model, namely, consumption, investment, wage, government consumption, government investment, labour tax revenue, consumption tax revenue, capital tax revenue,[2] government transfer, inflation, hours, and nominal interest rate. Fiscal data are limited to that of the federal government because of the availability of comprehensive government debt data. A detailed description of data sources and transformation is captured in Leeper et al. (2009). All observables are detrended using first differences except for inflation, hours and the nominal interest rate. Similar to Smets and Wouters (2007), the model assumes a balanced growth path and that variables grow at the

[2]This is the sum of net returns and dividend revenue.

trend rate γ. A Bayesian estimation technique is adopted in estimating the model. This is implemented by first using Sim's optimisation routine to maximise the log posterior function and thereafter using the random walk Metropolis-Hastings algorithm to sample from the posterior distribution. A total of $2,000,000$ draws were obtained from the posterior samples using a random walk Metropolis-Hasting algorithm.

The choice of prior distribution follows studies such as Smets and Wouters (2007), Leeper et al. (2010),DeGraeve (2008) and Kliem and Kriwoluzky (2014). More precisely, fiscal policy coefficients representing output stabiliser and debt follow the specification of Leeper et al. (2010). The co-movement between capital and the labour tax rate $\rho_{l,k}$ is assumed to follow a normal distribution with a mean of 0.25 and a standard deviation of 0.1. The remaining co-movement parameters $\rho_{k,c}$ for capital and consumption, and $\rho_{c,l}$ for consumption and the labour tax rate also follow a normal distribution with a mean of 0.05 and a standard deviation of 0.1, while interest-rate smoothing ρ_R and the persistence of all exogenous processes follow Smets and Wouters (2007).

The monetary policy response to inflation and output takes a gamma distribution, but the mean and standard deviation of the prior is set to the values in Smets and Wouters (2007). Investment adjustment cost follows a gamma distribution as in Smets and Wouters (2007). However, the prior distribution pertaining to the standard error of the exogenous processes is relaxed to have a mean of 1 and a standard deviation of 10.The elasticity of the external finance premium follows DeGraeve (2008), and the elasticity between private and government consumption ψ_c is assumed to follow a gamma distribution with a mean of 0.30 and a standard deviation of 0.10.

3.4.2.1 Posterior Estimates

Tables 3.2 (fiscal parameters) and 3.3(structural parameters) report the mode, mean, 5% and 95% of the posterior distribution of the estimated model. The estimated mean persistence of fiscal instruments is higher than the set prior mean. Similar to Leeper et al. (2009), the 95% posterior intervals for all parameters do not contain zero except for parameters relating to the co-movement between consumption and labour/capital. The persistence to all shocks except for monetary policy shocks, which are estimated to be highly persistent and higher than the set prior mean of 0.5.

Table 3.2: Prior and Posterior Distribution of Fiscal Parameters

		Prior distribution			Posterior distribution			
Description		Distr	Mean	St.Dev	Mode	Mean	5%	95%
Output response								
Capital tax	ψ_K	$\mathcal{G}$	1.00	0.30	1.50	1.48	0.77	2.18
Labour tax	ψ_L	$\mathcal{G}$	0.50	0.25	0.32	0.47	0.12	0.80
Transfer	ψ_Z	$\mathcal{G}$	0.20	0.10	0.16	0.22	0.05	0.38
Govt consumption	ψ_{GC}	$\mathcal{G}$	0.07	0.05	0.04	0.07	0.003	0.15
Govt investment	ψ_{GI}	$\mathcal{G}$	0.07	0.05	0.03	0.07	0.003	0.14
Debt response								
Capital tax	ω_K	$\mathcal{G}$	0.40	0.20	0.23	0.28	0.19	0.35
Labour tax	ω_L	$\mathcal{G}$	0.40	0.20	0.08	0.08	0.02	0.12
Transfer	ω_Z	$\mathcal{G}$	0.40	0.20	0.09	0.08	0.02	0.14
Govt consumption	ω_{GC}	$\mathcal{G}$	0.40	0.20	0.10	0.09	0.03	0.14
Govt investment	ω_{GI}	$\mathcal{G}$	0.40	0.20	0.08	0.07	0.003	0.14
ARcoefficient								
Capital tax	ρ_K	$\mathcal{B}$	0.50	0.25	0.68	0.71	0.52	0.90
Labour tax	ρ_l	$\mathcal{B}$	0.50	0.25	0.93	0.93	0.87	0.98
Consumption tax	ρ_C	$\mathcal{B}$	0.50	0.25	0.97	0.97	0.94	0.998
Govt transfer	ρ_Z	$\mathcal{B}$	0.50	0.25	0.996	0.98	0.95	0.999
Govt consumption	ρ_{GC}	$\mathcal{B}$	0.50	0.25	0.96	0.96	0.93	0.99
Govt investment	ρ_{GI}	$\mathcal{B}$	0.50	0.25	0.997	0.99	0.97	1.00
Standard error								
Capital tax	σ_K	$\mathcal{I}$	1.00	10.00	3.85	3.83	3.27	4.39
Labour tax	σ_L	$\mathcal{I}$	1.00	10.00	1.82	1.89	1.63	2.09
Consumption tax	σ_C	$\mathcal{I}$	1.00	10.00	2.97	3.05	2.69	3.41
Govt transfer	σ_Z	$\mathcal{I}$	1.00	10.00	2.62	2.67	2.35	2.99
Govt consumption	σ_{GC}	$\mathcal{I}$	1.00	10.00	3.20	3.28	2.89	3.66
Govt investment	σ_{GI}	$\mathcal{I}$	1.00	10.00	2.50	2.57	2.26	2.88
Shock co-movement								
Labour & capital	$\rho_{l,k}$	$\mathcal{N}$	0.25	0.10	0.24	0.25	0.19	0.32
Capital & consumption	$\rho_{k,c}$	$\mathcal{N}$	0.05	0.10	0.07	0.06	-0.01	0.14
Consumption & labour	$\rho_{c,l}$	$\mathcal{N}$	0.05	0.10	0.001	0.01	-0.06	0.07

Note that posterior distribution is obtained using Metropolis-Hastings algorithm using $2,000,000$ draws.Also, $\mathcal{I}, \mathcal{B}, \mathcal{N}$, and $\mathcal{G}$all denotes Inverse-gamma, Beta, Normal, Gamma anddistributions respectively.

In order of magnitude, the mean estimate of the fiscal policy parameter to debt is highest for capital tax, followed by government consumption, labour tax and transfers, with government investment the least responsive. However, the only significant estimate for the debt dynamics comes from capital tax and is 0.28. Capital tax has the highest response to aggregate output, whereas government spending is less responsive. The estimated co-movement between capital and labour tax suggests that any tax legislation on one tends to simultaneously affect the other. By contrast, the changes in consumption tax do not affect labour or capital tax.

Table 3.3: Prior and Posterior Distribution of Structural and Exogenous Parameters (non-fiscal)

		Prior distribution			Posterior distribution			
Description		Distr	Mean	St.Dev	Mode	Mean	5%	95%
Standard error								
Productivity shock	σ_a	$\mathcal{I}$	1.00	10.00	0.41	0.40	0.33	0.46
Preference shock	σ_b	$\mathcal{I}$	1.00	10.00	0.48	0.89	0.28	1.49
Investment shock	σ_i	$\mathcal{I}$	1.00	10.00	0.62	0.53	0.26	0.81
Monetary shock	σ_r	$\mathcal{I}$	1.00	10.00	0.13	0.14	0.12	0.16
Marginal cost shock	σ_p	$\mathcal{I}$	1.00	10.00	2.89	4.19	1.09	7.70
Labour supply shock	σ_w	$\mathcal{I}$	1.00	10.00	2.52	2.58	2.10	3.06
Net worth shock	σ_n	$\mathcal{I}$	1.00	10.00	0.46	0.92	0.23	1.71
External finance shock	σ_s	$\mathcal{I}$	1.00	10.00	0.46	1.44	0.78	2.11
ARcoefficient								
Productivity persistence	ρ_a	$\mathcal{B}$	0.50	0.25	0.98	0.98	0.95	0.9999
Preference persistence	ρ_b	$\mathcal{B}$	0.50	0.25	0.63	0.58	0.23	0.92
Investment persistence	ρ_i	$\mathcal{B}$	0.50	0.25	0.997	0.82	0.69	0.98
Monetary persistence	ρ_r	$\mathcal{B}$	0.50	0.25	0.40	0.42	0.27	0.58
Marginal cost persistence	ρ_p	$\mathcal{B}$	0.50	0.25	0.64	0.41	0.02	0.78
Labour supply persistence	ρ_w	$\mathcal{B}$	0.50	0.25	0.997	0.98	0.96	0.9999
Net worth persistence	ρ_n	$\mathcal{B}$	0.50	0.25	0.54	0.49	0.10	0.88
Externalpremium persistence	ρ_s	$\mathcal{B}$	0.50	0.25	0.995	0.76	0.65	0.90
Structural parameters								
Consumption habit	η_C	B	0.70	0.10	0.86	0.78	0.63	0.92
Wage stickiness	ζ_w	$\mathcal{B}$	0.50	0.10	0.50	0.50	0.42	0.58
Price stickiness	ζ_p	$\mathcal{B}$	0.50	0.10	0.73	0.76	0.66	0.85
Wage indexation	ι_w	$\mathcal{B}$	0.50	0.15	0.47	0.48	0.24	0.72
Price indexation	ι_p	$\mathcal{B}$	0.50	0.15	0.29	0.36	0.14	0.59
Elasticity of utilisation	ψ_u	$\mathcal{B}$	0.50	0.15	0.65	0.60	0.42	0.79
Rate smoothing	ρ_R	$\mathcal{B}$	0.75	0.10	0.87	0.85	0.81	0.89
P-inflation	ψ_π	$\mathcal{G}$	1.50	0.25	1.97	2.08	1.71	2.38
P-output gap	ψ_y	$\mathcal{G}$	0.125	0.05	0.03	0.05	0.02	0.08
Trend growth rate	$\bar{\gamma}$	$\mathcal{N}$	0.40	0.10	0.42	0.41	0.33	0.49
Adjustment cost	Ψ	$\mathcal{N}$	4.00	1.50	3.01	3.54	2.41	4.66
External finance elasticity	ψ_S	$\mathcal{U}$	0.25	0.14	0.21	0.39	0.26	0.50
Consumption goods elasticity	ψ_c	$\mathcal{G}$	0.30	0.10	0.54	0.54	0.37	0.72

Note that posterior distribution is obtained using Metropolis-Hastings algorithm using $2,000,000$ draws. Also, $\mathcal{I}$, $\mathcal{B}$, $\mathcal{N}$, $\mathcal{G}$, and $\mathcal{U}$ all denotes Inverse-gamma, Beta, Normal, Gamma and Uniform distributions respectively.

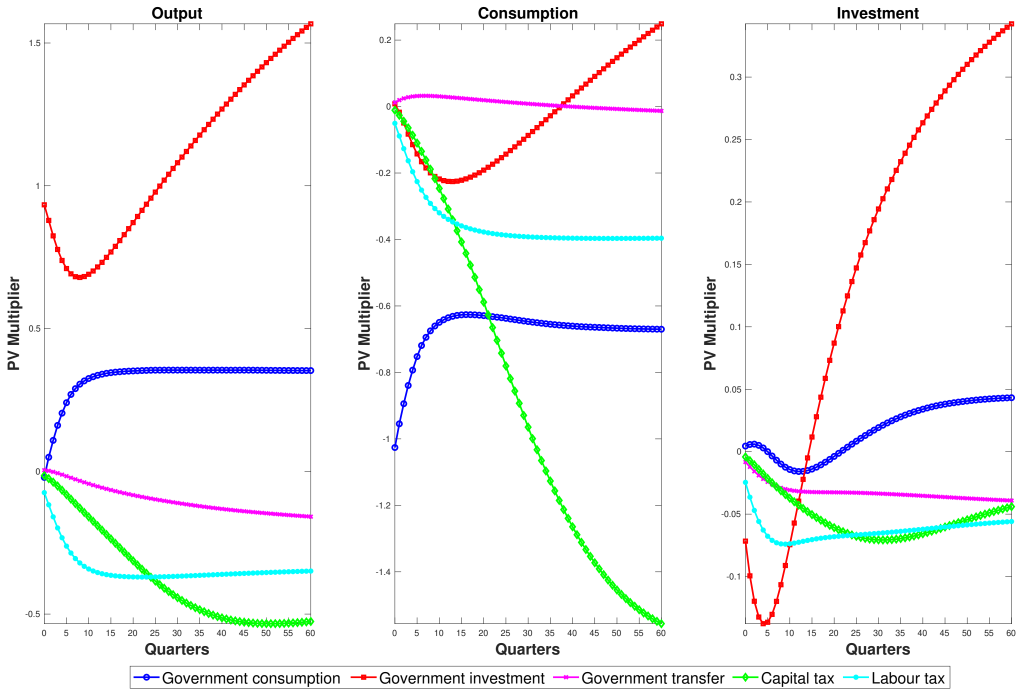
3.5 Fiscal Multipliers

The effect of fiscal shocks on output, consumption and investment are usually expressed in terms of fiscal multipliers. Following Mountford and Uhlig (2009) and as in Leeper et al. (2009), this chapter reports the present-value fiscal multipliers (see 3.4). There is no doubt that these multipliers are preferred over multipliers on impact solely because the present-value multipliers tend to show the full dynamics of the impact of fiscal shocks and also discount future macroeconomic effects. The present-value multiplier over a k horizon is calculated using the expression below

$$\frac{\sum_{i=0}^{k}\left(\prod_{j=0}^{i} R_{t+j}^{-1}\right)\Delta Y_{t+i}}{\sum_{i=0}^{k}\left(\prod_{j=0}^{i} R_{t+j}^{-1}\right)\Delta G_{t+i}}, \tag{3.5.1}$$

where G_t represents the fiscal instrument, and the impact multiplier is evaluated at $i = 0$. Figure 3.1 shows the present-value multiplier dynamics for 60 quarters. The long-term government investment multiplier is higher for output and investment, but the capital tax long-term multiplier for consumption is higher than others. The impact multiplier of government investment is close to one.

Figure 3.1: Cumulative Present-value Multipliers



The figure depict the cumulative present-value multipliers of one unit increase in each fiscal tools on output,private-consumption and investment.

Table 3.4: Present-value Fiscal Multipliers and Productive Capital

	Productivity of public capital ($\theta_G = 0.05$)						$\theta_G = 0.10$		
	Benchmark fiscal rule			Alternative fiscal rule			Benchmark fiscal rule		
	Y	C	I	Y	C	I	Y	C	I
	Government consumption present-value multipliers								
Impact	-0.021	-1.026	0.005	-0.022	-1.026	0.005	-0.047	-1.051	0.005
5 quarters	0.240	-0.752	0.000	0.243	-0.750	0.001	0.217	-0.775	-0.001
10 quarters	0.325	-0.649	-0.014	0.329	-0.645	-0.014	0.299	-0.673	-0.017
25 quarters	0.354	-0.637	0.008	0.356	-0.634	0.008	0.318	-0.666	0.001
∞	0.348	-0.673	0.042	0.345	-0.673	0.041	0.280	-0.725	0.026
	Government investment present-value multipliers								
Impact	0.933	0.008	-0.072	0.936	0.007	-0.071	1.030	0.133	-0.100
5 quarters	0.710	-0.143	-0.137	0.718	-0.135	-0.135	0.919	0.119	-0.190
10 quarters	0.690	-0.218	-0.075	0.699	-0.209	-0.073	1.014	0.123	-0.095
25 quarters	0.978	-0.143	0.147	0.988	-0.129	0.149	1.673	0.421	0.268
∞	2.154	0.731	0.437	2.153	0.760	0.437	4.034	2.220	0.789
	Capital tax present-value multipliers								
Impact	-0.015	-0.011	-0.004	-0.015	-0.011	-0.004	-0.013	-0.001	-0.004
5 quarters	-0.082	-0.110	-0.021	-0.082	-0.111	-0.021	-0.070	-0.100	-0.020
10 quarters	-0.160	-0.246	-0.037	-0.159	-0.249	-0.037	-0.131	-0.222	-0.033
25 quarters	-0.384	-0.781	-0.068	-0.380	-0.785	-0.067	-0.234	-0.648	-0.032
∞	-0.431	-1.607	-0.021	-0.422	-1.597	-0.054	0.288	-0.941	0.118
	Labour taxpresent-value multipliers								
Impact	-0.075	-0.050	-0.025	-0.075	-0.051	-0.025	-0.074	-0.050	-0.025
5 quarters	-0.262	-0.226	-0.068	-0.262	-0. 227	-0.067	-0.260	-0.226	-0.067
10 quarters	-0.342	-0.320	-0.074	-0.342	-0.323	-0.073	-0.336	-0.318	-0.071
25 quarters	-0.370	-0.387	-0.067	-0.366	-0.391	-0.066	-0.350	-0.375	-0.059
∞	-0.336	-0.390	-0.051	-0.329	-0.391	-0.049	-0.276	-0.343	-0.036
	Government transfer present-value multipliers								
Impact	0.004	0.012	-0.008	0.004	0.013	-0.009	0.003	0.010	-0.007
5 quarters	-0.017	0.032	-0.024	-0.018	0.033	-0.025	-0.017	0.028	-0.020
10 quarters	-0.044	0.031	-0.031	-0.045	0.031	-0.032	-0.044	0.026	-0.026
25 quarters	-0.098	0.014	-0.033	-0.100	0.012	-0.034	-0.106	0.005	-0.031
∞	-0.197	-0.037	-0.043	-0.194	-0.039	-0.043	-0.251	-0.083	-0.052

Present-value multipliers of output (Y),private consumption (C),and private investment (I).The first six column reportsmultipliers when productivity of public capital is 0.05,where the benchmark rule is when debt and output are jointly reflected in fiscal policy rule.While the alternative rule is solely when debt is the only variable fiscal rule respond to. The last three column however reports the multiplier when public capital productivity is increased to 0.10 and the fiscal policy rule responds to output.

The present-value multipliers in Figure3.4 are impacted by changes in public capital productivity θ_G. The figure also shows that there are no significant changes in fiscal multipliers whether the underlying fiscal instrument responds to the output gap or not.

Under the baseline parametrisation of public capital productivity $\theta_G = 0.05$, government investment has the highest output multiplier of 0.933 on impact, which declines for a few quarters but increases after 25 quarters. After infinity quarters, the multiplier peaks with a value of 2.154. The government investment output multiplier becomes more significant when public capital productivity is increased to 0.10, where the impact multiplier is 1.030. After infinity quarters, the multiplier is 4.034, close to the value in Baxter and King (1993) for the same level of public capital productivity. The investment multiplier builds up over time upon a government investment shock, peaking at the infinity horizon. The government spending (i.e., consumption and investment spending) multipliers for consumption are nearly all negative, except for government investment on impact and at the infinity horizon.

As for the evaluation of tax multipliers, an increase in the specific tax policy will result in a negative value for the macroeconomic variables, and the result implies a reversal or a tax cut. This is so because the tax multipliers, as reported in 3.4 are evaluated by increasing the tax shock. Tax-cut policies (i.e., capital and labour) that reduce tax revenue by 1 unit translate over time into an increase in output, consumption and investment. However, the magnitude of the effect of tax multipliers on output is not comparable with that of the government investment multiplier. In the long-run (i.e., at the infinity horizon and under the baseline calibration of $\theta_G = 0.05$), the government investment multiplier's effect on output is 2.154 (for a 1 unit increase in government investment), capital and labour taxes with the same unit increment in their revenue have a multiplier of 0.431 and 0.336, respectively.

The capital tax multiplier's effects on consumption are seemingly the highest of all the fiscal instruments considered. At around 25 quarters from impact, the capital tax multiplier reduces private consumption by more than half the corresponding increase in tax revenue, with a multiplier value of -0.781. However, the multiplier peaks at the infinity quarter with a multiplier value of -1.607. Interpreting this result for a corresponding tax reduction policy implies that a capital tax cut policy that reduces capital tax revenue by 1 unit translates to an increase in private consumption. This increment peaks at the infinity horizon with 1.607 consumption units.

Most studies of fiscal multipliers in New Keynesian models that find a spending multiplier of less than unity reach this conclusion based on their abstraction of productive government spending. However, the significant government spending multiplier (particularly the long-term government investment multiplier) in this chapter is due to the adoption of productive government spending both in consumption (directly in relation to household utility) and investment (through productive capital affecting the marginal product of capital); this is consistent with some of the conclusions in Ramey (2011a).

3.5.0.1 Fiscal Multipliers with Financial Friction Mechanism-off

The transmission of fiscal policy is greatly affected by the presence of a financial friction mechanism. This is evident in Figures 3.2 to 3.4 which compare the cumulative present value of fiscal multipliers in a model with and without a financial friction mechanism. Capital tax is redundant in a model with the financial mechanism turned off because entrepreneurs can go to the market to raise funds without issuing bonds. Hence, the caveat is that some fundamental issues remain since bonds and other assets are perfect substitutes. If capital tax is imposed on bonds alone, there will be a violation of the no-arbitrage rule. Therefore, the implication of including the financial friction mechanism is investigated using government investment, government consumption, government transfers and labour tax as fiscal instruments.

The thick red plot in Figures 3.2 to 3.4 represents the cumulative present-value multipliers in a model with the financial friction mechanism fully operational. The blue dotted line depicts the fiscal multiplier when the financial friction mechanism is turned off. The models in both cases abstract from having capital tax in order to analyse the difference in the multipliers. It appears that the impact and short-term multiplier in these two scenarios are close. However, it is worth examining if there is a wide difference in their long-term multiplier.

Figure 3.2: Fiscal Multipliers on Output

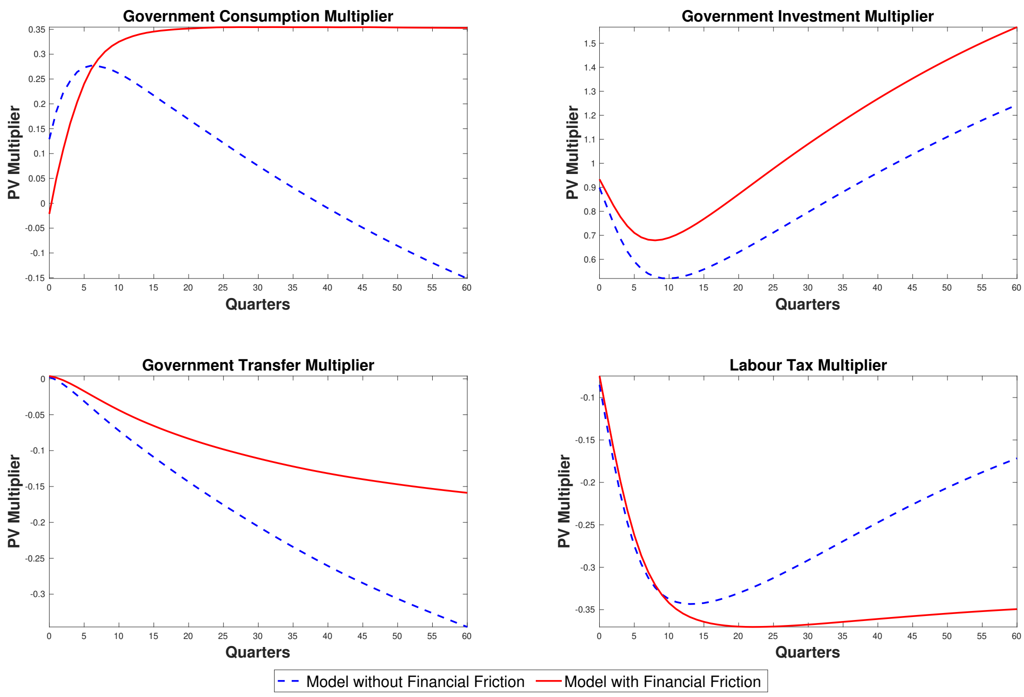


Figure 3.3: Fiscal Multipliers on Consumption

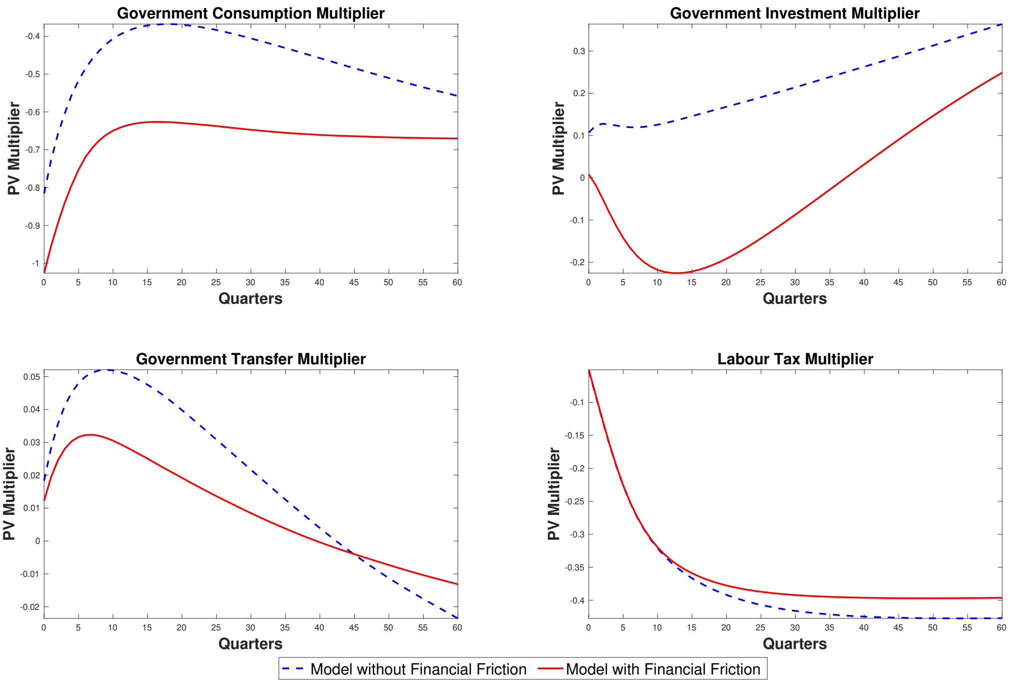
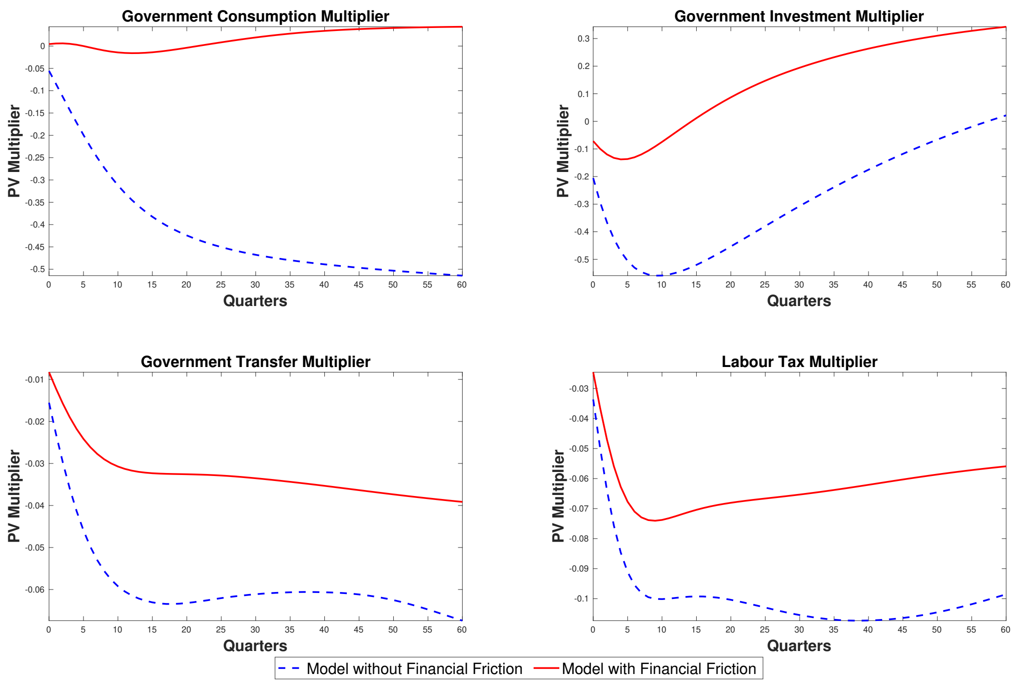


Figure 3.4: Fiscal Multipliers on Investment



3.5.1 Impulse Response

This subsection presents and discusses the transmission of fiscal shocks to some macroeconomic and financial variables. Figure 3.5 to 3.10 plots the impulse response of selected model variables to a one-standard-deviation fiscal shock with two different values of public capital productivity (i.e.$\theta_G = 0.05$ and $\theta_G = 0.1$). The impulse response unit on the $y-$axis represents the percentage log deviation of the variables of interest from their steady states. Following a one-standard-deviation shock to government consumption, wealth reduces, leading to an increase in labour effort. Hence, output gradually increases and peaked at about 5 quarters after impact before it gradually decreases but has stayed above its steady state, up until the $40th$ quarter shown in the dynamics. However, increasing the productivity of public capital ($\theta_G = 0.1$) has little or no effect on the dynamics of macroeconomic and financial variables after the transmission of government consumption shock. Meanwhile, the dynamics of the government investment shock (see Figure 3.6) are more pronounced, and the change in the value of public capital is evident. Output and consumption increase with increased public capital productivity, and the long-run effect gives more incentive to invest.

Generally, the transmission of government spending shocks is the most expansionary of all the fiscal shocks considered. This is because a government spending shock increases aggregate demand, and output in the model is somewhat demand determined. Another important feature to consider is that the shock to government investment raises inflation. This is reflected in entrepreneurs' wealth through the Fisher effect. The increased inflation reduces the real cost of the external finance premium the entrepreneur will pay on their loan. Therefore, the crowding out of private investment by government investment is minimised. Furthermore, the government debt burden is also minimised compared to the moment of impact of the government consumption shock. While Fernandez-Villaverde (2010), with a non-separable government spending shock, shows the negative effect on external finance, the result from this chapter (having separated government investment from its consumption counterpart) reveals that it is the former that actually reduces external finance.

A tax reduction brings about a smaller initial expansion in output. Standard theory suggests that an increase in capital tax leads to an instantaneous decrease in investment, labour, and output, while consumption increases due to agents sacrificing investment for consumption (Baxter and King, 1993; Braun, 1994; Leeper and Yang, 2008). However, in this estimated model (see Figure 3.8), the correlation between capital tax and labour tax translate into a decrease in consumption once capital tax increases. This is because labour tax is expected to increase due to its correlation, which then makes households work more and consume less.

Figure 3.5: Impulse Response to One Standard Deviation Increase in Government Consumption Shock

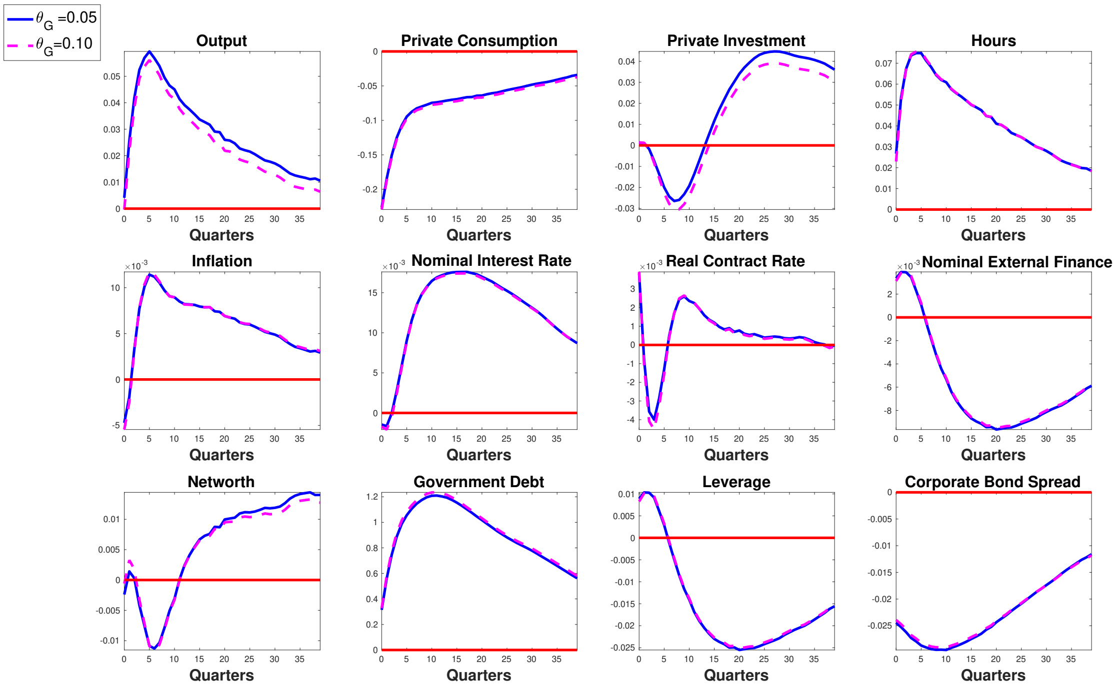


Figure 3.6: Impulse Response toOne Standard Deviation Increase in Government Investment shock

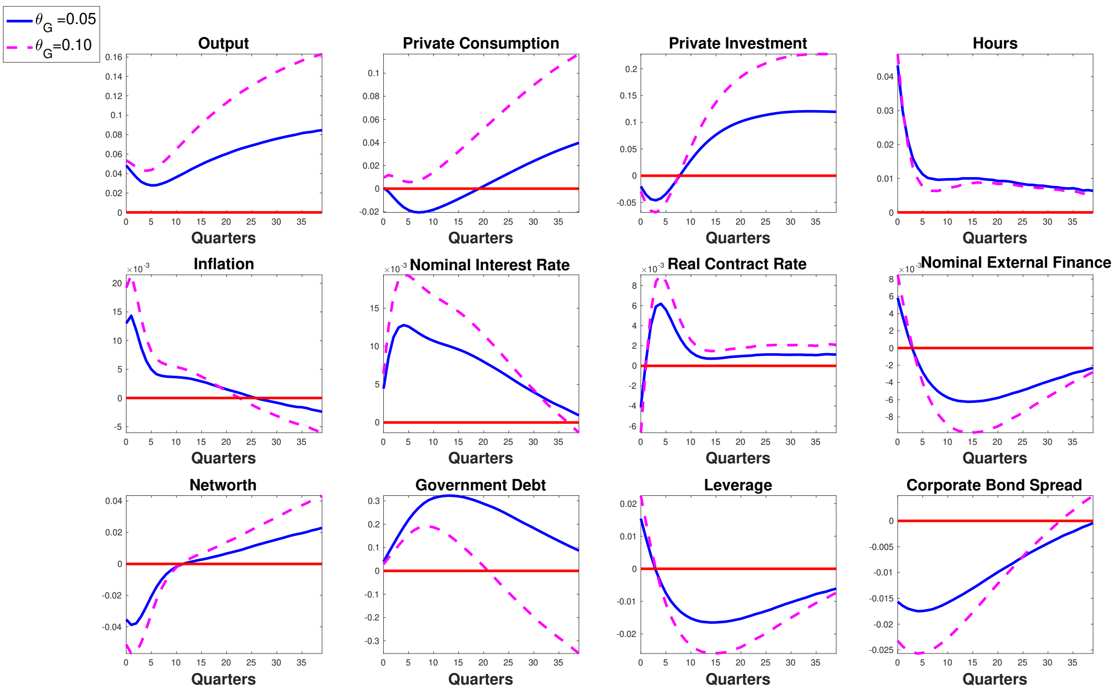


Figure 3.7: Impulse Response toOne Standard Deviation Increase in Transfer Shock

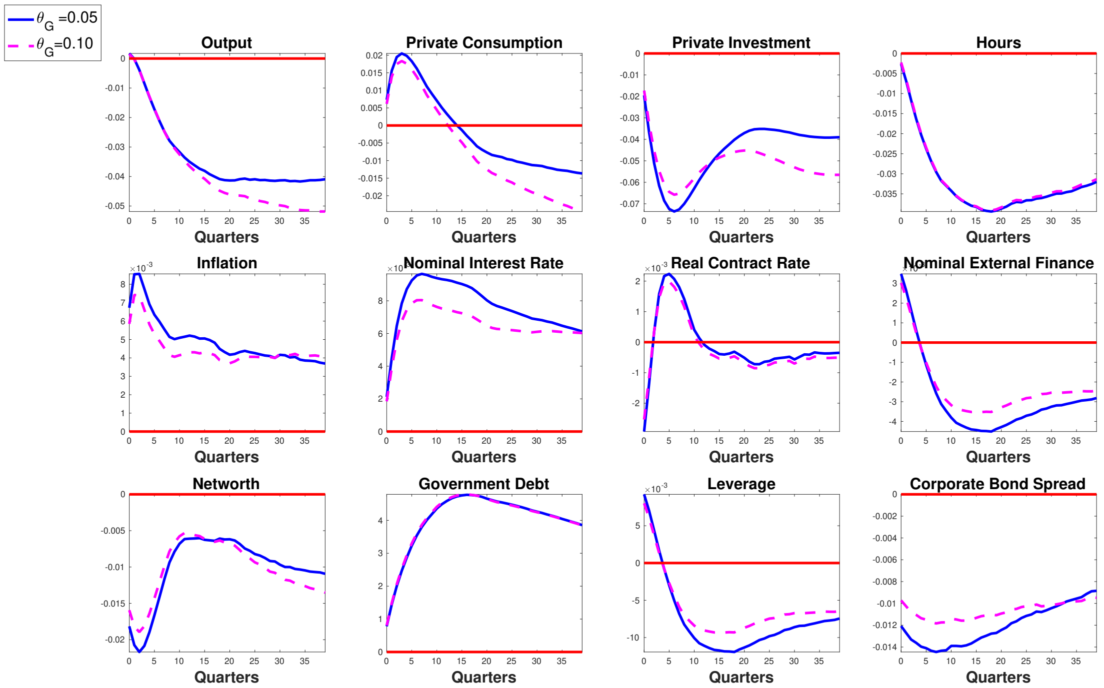


Figure 3.8: Impulse Response toOne Standard Deviation Increase in Capital Tax Shock

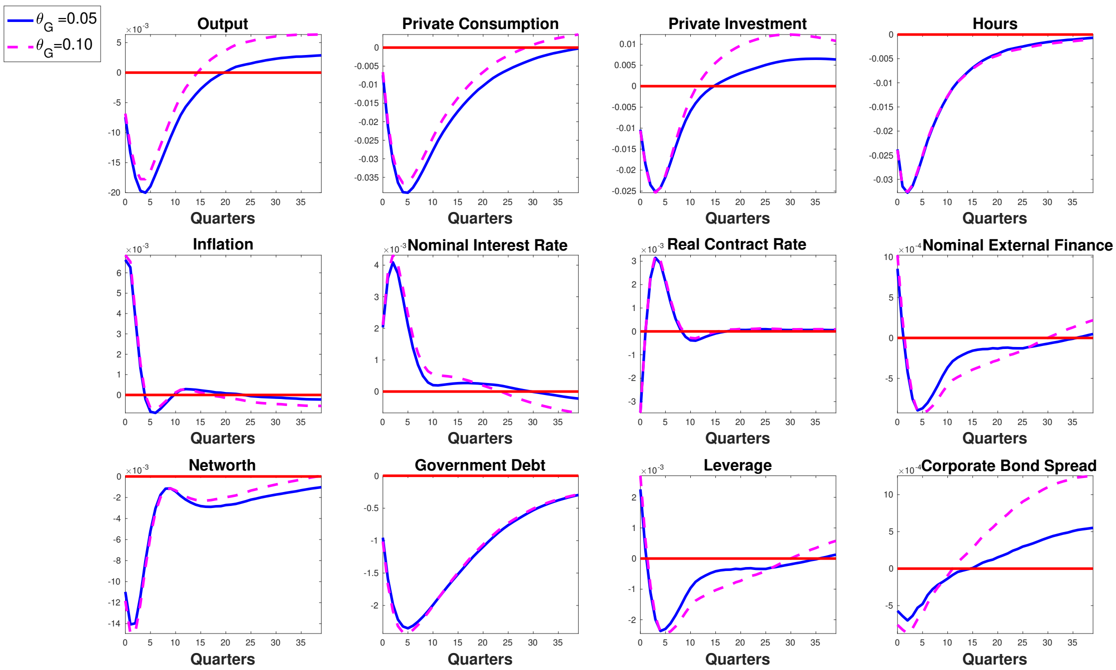


Figure 3.9: Impulse Response toOne Standard Deviation Increase in Labour Tax Shock

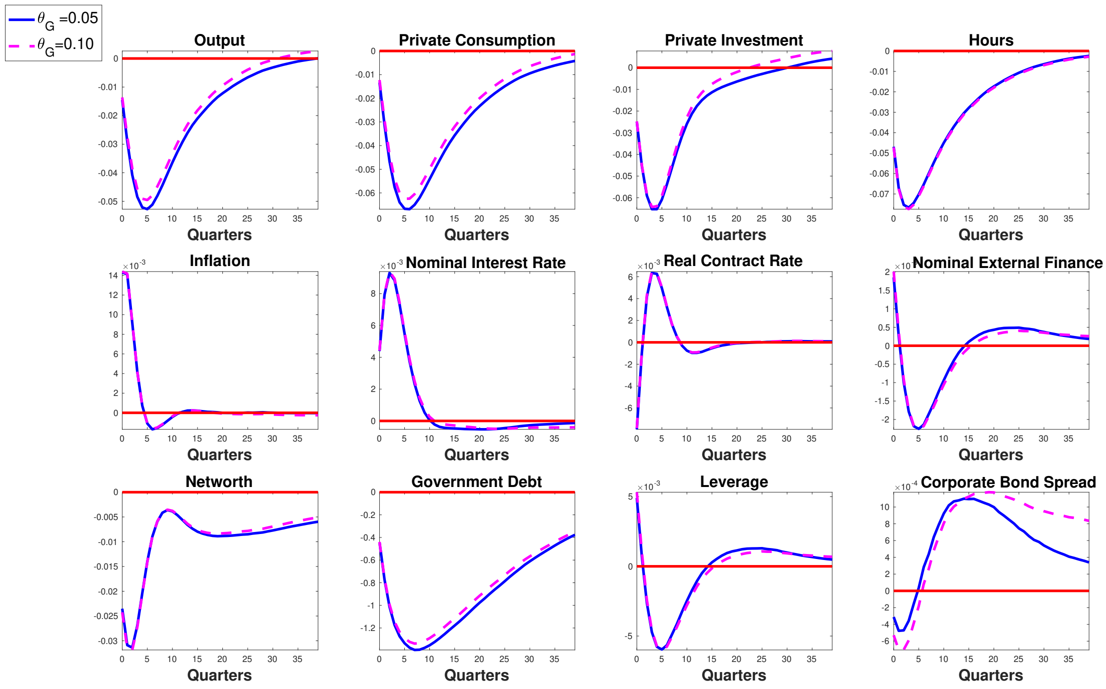
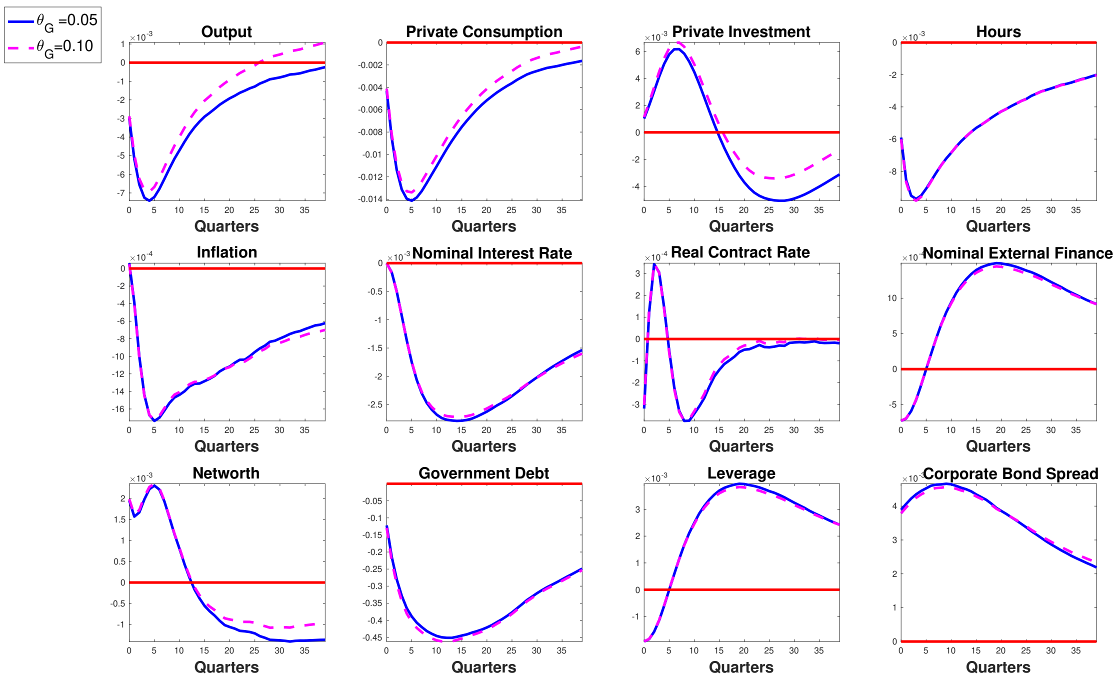


Figure 3.10: Impulse Response to One Standard Deviation Increase in Consumption Tax Shock



3.6 Conclusion

This chapter has explored the joint implications of fiscal and monetary policy on macroeconomic and financial outcomes in the context of a New Keynesian model. It provides the theoretical basis and mechanism for a joint interaction of fiscal and monetary policy with financial friction. The analyses carried out in the chapter were aimed at varying several objectives. The first was to identify the multiplier effects of the different fiscal tools. The second was to observe the interplay of fiscal shocks and their transmission's effects on macroeconomic and financial variables, particularly the external finance premium. In furtherance of these objectives, the chapter evaluated the influence of financial friction on fiscal multipliers were evaluated.

Using the medium-scale DSGE modelSmets and Wouters (2003, 2007) as the benchmark, this chapter has shown the important mechanism (i.e., that of productive public capital) through which government investment reduces the cost of the external finance premium. In addition, it revealed that government investment has substantial short and long-term multiplier effects on output and private investment. Having identified this channel through which government investment decisions can influence financial friction, the chapter gives room to assess the extent of the potential effects that different policy interventions could bring about in the economy.

Similar to Leeper and Leith (2016), the results from the analyses in this chapter confirm that it is not optimal to dissociate inflation stabilisation from debt stabilisation by delegating the former to monetary policy and the latter to fiscal policy. Rather, an optimal approach to obtaining the best macroeconomic and financial outcomes is the coordination of monetary and fiscal policies; for the most part, the response of fiscal policy to the output gap is higher than the response of monetary policy.

The key point of this chapter is premised on the view that increasing productive public capital increases aggregate output significantly and also potentially reduces the crowding-out effect of private consumption and investment. Consistent with Ramey (2020), this chapter has shown that long-run multipliers can be sizeable once government capital is productive. In addition, an increase in government investment also lowers the cost of external finance premium more. Therefore, public capital plays an important role in the dynamics of aggregate output and, therefore, should not be regarded as wasteful. Further research could explore the implication of a binding zero lower bound with unconventional policies.

Appendix 3

I: Optimisation Problems

Household Sector

To start with,we define individual household's effective consumption$(\tilde{C}_t(j))$ which is a composite of private consumption $(C_t(j))$ and government consumptions services (G_t^C).The functional form of the effective consumption takes on a non-separable CES aggregator in the likes of Bouakez and Rebei (2007), Pappa (2009), Coenen et al. (21013), Ercolani and Azevedo (2014) and Pappa et al. (2015).

The aggregator hence is given as

$$\tilde{C}_t(j) = \left[(1-\eta_g)^{\frac{1}{\psi_c}} C_t(j)^{\frac{\psi_c - 1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}} G_t^{C\,\frac{\psi_c-1}{\psi_c}}\right]^{\frac{\psi_c}{\psi_c - 1}}$$

the parameter $\eta_g \in (0,1)$ is the share of government consumption expenditure in the effective consumption.While ψ_c is the elasticity between private and government consumption.As $\psi_c \to 0$,private and public consumption becomes perfect complements and as $\psi_c \to \infty$ they are tend to become perfect substitutes. Allowing private and government consumption to be non-separable implying co-movement in the two good iseconomically trackable.For example,government spendings such as education and healthcare is able to have direct impact on household's private consumption decision.

Household j maximise nonseperable utility which is a function of goods $\tilde{C}_t(j)$ and labour $L_t(j)$

$$U(\tilde{C}_t, L_t) = \varepsilon_t^{\beta} \frac{1}{1-\sigma_C}\left(\tilde{C}_t(j) - \eta_c \tilde{C}_{t-1}(j)\right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L} \varepsilon_t^w L_t^{1+\sigma_L}(j)\right), \tag{3.6.1}$$

where η_c is the consumption habit formation parameter.

To differentiate elasticity of substitution from the coefficient of risk aversion,household is said to have recursive Epstein-Zin preference given by

$$V_t(j) = U(\tilde{C}_t(j), L_t(j)) + \beta v_t(j), \tag{3.6.2}$$

where

$$v_t(j) = \mathbb{E}_t(V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}}$$

The household budget constrain is given by

$$(1+\tau_t^c)C_t(j) + \frac{D_t(j)}{P_t} + \frac{B_t(j)}{P_t} \leq (1-\tau_t^l)\frac{W_t^h(j)L_t(j)}{P_t} + R_{t-1,t}^D\frac{D_{t-1}(j)}{P_t} + \Big(1+(1-\tau_t^k)(R_{t-1,t}-1)\Big)\frac{B_{t-1}(j)}{P_t} + \frac{Div_t}{P_t} + Z_t \tag{3.6.3}$$

Each household$j$ in period t spend on private consumption $C_t(j)$,invest by making a nominal deposit $B_t(j)$ at the financial intermediary which pays uncontingentgross nominalrate R_t.Household also have investment in government (public) debt $D_t(j)$ yielding an uncontigent gross nominal rate R_t^D.They decide how many hours to work $L_t(j)$,receives lump sum transfer Z_t from government,also receives dividend Div_t from labour union.Meanwhile,they pay distortionary taxes $\tau_t^c, \tau_t^l, \tau_t^k$ which are leviedon consumption,labour earnings and net returns on deposit.

Thus household j's optimisation problem is to maximise V_0 where its choices are private consumption $C_t(j)$, hours $L_t(t)$,government bond $B_t(j)$ and V_t. This choices are made subject to (3.6.2) and its budget constraint (3.6.3)

Setting up a Lagrangian for the problem, we have

$$\mathcal{L} = \begin{bmatrix} V_0(j) + \sum_{t=0}^{\infty}\mu_t\Big(U(\tilde{C}_t(j), L_t(j)) + \beta(\theta_{t+1|t}V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}} - V_t(j)\Big) \\ +\sum_{t=0}^{\infty}\lambda_t\Big((1-\tau_t^l)\frac{W_t^h(j)L_t(j)}{P_t} + R_{t-1,t}^D\frac{D_{t-1}(j)}{P_t} + \Big(1+(1-\tau_t^k)(R_{t-1,t}-1)\Big)\frac{B_{t-1}(j)}{P_t} \\ +\frac{Div_t}{P_t} + Z_t - (1+\tau_t^c)C_t(j) - \frac{D_t(j)}{P_t} - \frac{B_t(j)}{P_t}\Big) \end{bmatrix}$$

In the Lagrangian above, θ is just a probabilityparametrisation used in expressing expectation. Since household choose $C_t(j)$,$L_t(j)$,$D_t(j)$ and $B_t(j)$ the partial derivative of the Lagrangian with respect to the choices and in addition the partial derivative with respect to $V_t(j)$ are sought after. Writing down the FOC while dropping out the j index gives the following expressions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t} &= \mu_t U(\tilde{C}_t(j), L_t(j)) - \lambda_t(1+\tau_t^c) \\ &\implies \mu_t U(\tilde{C}_t(j), L_t(j)) = \lambda_t(1+\tau_t^c) \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = \mu_t U(\tilde{C}_t(j), L_t(j)) + \lambda_t \frac{W_t^h}{P_t}(1 - \tau_t^l)$$
$$\implies \mu_t U(\tilde{C}_t(j), L_t(j)) = -\lambda_t \frac{W_t^h}{P_t}(1 - \tau_t^l)$$

$$\frac{\partial \mathcal{L}}{\partial D_t} = -\frac{\lambda_t}{P_t} + \frac{R_{t+1}^D \lambda_{t+1}}{P_{t+1}}$$
$$\implies \frac{\lambda_t}{R_{t+1}^D P_t} = \mathbb{E}_t\Big[\frac{\lambda_{t+1}}{P_{t+1}}\Big]$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\frac{\lambda_t}{P_t} + \Big(1 + (1 - \tau_{t+1}^k)(R_{t,t+1} - 1)\Big)\frac{\lambda_{t+1}}{P_{t+1}}$$
$$\implies \frac{\lambda_t}{P_t} = \mathbb{E}_t\Big[\Big(1 + (1 - \tau_{t+1}^k)(R_{t,t+1} - 1)\Big)\frac{\lambda_{t+1}}{P_{t+1}}\Big]$$

$$\frac{\partial \mathcal{L}}{\partial V_t} = -\mu_t + \varepsilon_{t-1}^{\beta}\beta\mu_{t-1}\theta_{t|t-1}V_t^{-\sigma_E}\Big(\theta_{t|t-1}V_t^{1-\sigma_E}\Big)^{\frac{\sigma_E}{1-\sigma_E}}$$
$$\implies \mu_t = \varepsilon_{t-1}^{\beta}\beta\mu_{t-1}\mathbb{E}_{t-1}V_t^{-\sigma_E}\Big(\mathbb{E}_{t-1}V_t^{1-\sigma_E}\Big)^{\frac{\sigma_E}{1-\sigma_E}}$$

the conditional probabilities have been transformed to conditional expectation. However, we make another transformation mainly for the Lagrangian multipliers whereby we define discounted Lagrangian multipliers $\tilde{\lambda}_t = \beta^{-1}{\theta_{t|0}}^{-1}\lambda_t$ and $\tilde{\mu}_t = \beta^{-1}{\theta_{t|0}}^{-1}\mu_t$

Substituting the transformed multiplier into equilibrium equations we have

$$\tilde{\lambda}_t = \tilde{\mu}_t \frac{U(\tilde{C}_t(j), L_t(j))}{(1 + \tau_t^c)} \tag{3.6.4}$$

$$\tilde{\mu}_t U(\tilde{C}_t(j), L_t(j)) = -\tilde{\lambda}_t \frac{W_t^h}{P_t}(1 - \tau_t^l) \tag{3.6.5}$$

$$\frac{\tilde{\lambda}_t}{R_{t+1}P_t} = \mathbb{E}_t\left[\frac{\tilde{\lambda}_{t+1}}{P_{t+1}}\right] \tag{3.6.6}$$

$$\frac{\tilde{\lambda}_t}{P_t} = \mathbb{E}_t\left[\left(1 + (1-\tau^k_{t+1})(R_{t,t+1}-1)\right)\frac{\tilde{\lambda}_{t+1}}{P_{t+1}}\right] \tag{3.6.7}$$

$$\tilde{\mu}_t = \tilde{\mu}_{t-1}\varepsilon^{\beta}_{t-1}\beta\left(\mathbb{E}_{t-1}V_t^{1-\sigma_E}\right)^{\frac{\sigma_E}{1-\sigma_E}}V_t^{-\sigma_E} \tag{3.6.8}$$

the above transformation helps to translate utility at time t to utility at time 0. From (3.6.4) and (3.6.5) we have

$$-\frac{W^h_t}{P_t} = \frac{U_L(\tilde{C}_t(j), L_t(j))}{U_C(\tilde{C}_t(j), L_t(j))}\frac{(1+\tau^c_t)}{(1-\tau^l_t)} \tag{3.6.9}$$

Rewriting (3.6.6) as

$$\tilde{\lambda}_t = R^D_{t+1}\mathbb{E}_t\left[\tilde{\lambda}_{t+1}\frac{P_t}{P_{t+1}}\right]$$

replacing the $\tilde{\lambda}_t$ in the above with (3.6.4) gives

$$\tilde{\mu}_t\frac{U_C(\tilde{C}_t(j), L_t(j))}{(1+\tau^c_t)} = R^D_{t+1}\mathbb{E}_t\left[\tilde{\mu}_{t+1}\frac{U_C(\tilde{C}_{t+1}(j), L_{t+1}(j))}{(1+\tau^c_{t+1})}\frac{P_t}{P_{t+1}}\right] \tag{3.6.10}$$

and from (3.6.8)

$$\frac{\tilde{\mu}_t}{\tilde{\mu}_{t-1}} = \varepsilon^{\beta}_{t-1}\beta\left(\mathbb{E}_{t-1}V_t^{1-\sigma_E}\right)^{\frac{\sigma_E}{1-\sigma_E}}V_t^{-\sigma_E}$$

it must be that $\frac{\tilde{\mu}_t}{\tilde{\mu}_{t-1}} = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}$ this equivalence is given by

$$\frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} = \varepsilon_t^\beta \beta \left(\mathbb{E}_t V_{t+1}^{1-\sigma_E} \right)^{\frac{\sigma_E}{1-\sigma_E}} V_{t+1}^{-\sigma_E} \tag{3.6.11}$$

Also from (3.6.10)

$$\frac{\tilde{\mu}_t}{\tilde{\mu}_{t+1}} = R_{t+1}^D \mathbb{E}_t \left[\left(\frac{U_C(\tilde{C}_{t+1}, L_{t+1})}{U_C(\tilde{C}_t, L_t)} \right) \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \frac{P_t}{P_{t+1}} \right] \tag{3.6.12}$$

Transforming (3.6.12) to $\frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}$ so we can equate to (3.6.11)

$$\beta \left(\mathbb{E}_t V_{t+1}^{1-\sigma_E} \right)^{\frac{\sigma_E}{1-\sigma_E}} V_{t+1}^{-\sigma_E} = \frac{1}{R_{t+1}^D} \mathbb{E}_t \left[\left(\frac{U_C(\tilde{C}_t, L_t)}{U_C(\tilde{C}_{t+1}, L_{t+1})} \right) \left(\frac{1+\tau_{t+1}^c}{1+\tau_t^c} \right) \frac{P_{t+1}}{P_t} \right] \tag{3.6.13}$$

Thus we have

$$U_C(\tilde{C}_t, L_t) = \varepsilon_t^\beta \beta R_{t+1}^D \mathbb{E}_t \left[\left(\mathbb{E}_t V_{t+1}^{1-\sigma_E} \right)^{\frac{\sigma_E}{1-\sigma_E}} V_{t+1}^{-\sigma_E} U_C(\tilde{C}_{t+1}, L_{t+1}) \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \frac{P_t}{P_{t+1}} \right] \tag{3.6.14}$$

Let P_τ be the price of a state contingent bond that pays $\$1$ at time τ, $t \leq \tau$ and pays 0 otherwise. Inserting this state contingent into household's optimisation problem $t < \tau$ gives

$$P_{\tau|t} = \varepsilon_t^\beta \beta \mathbb{E}_t \left[\left(\mathbb{E}_t V_{t+1}^{1-\sigma_E} \right)^{\frac{\sigma_E}{1-\sigma_E}} V_{t+1}^{-\sigma_E} \frac{U_C(\tilde{C}_{t+1}, L_{t+1})}{U_C(\tilde{C}_t, L_t)} \left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \frac{P_t}{P_{t+1}} P_{\tau|t+1} \right] \tag{3.6.15}$$

where $P_{\tau|t}$ is the price of the state contingent bond as given in time t which matures in time τ, the same explanation goes for $P_{\tau|t+1}$. Thus leading us to household's SDF.

Let

$$\Xi_t = \frac{U_C(\tilde{C}_t, L_t)}{(1+\tau_t^c)}$$

the discounting factor at time t for a payoff in time $t+1$ is given as

$$\Lambda_{t,t+1} = \varepsilon_t^{\beta}\beta\left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}}\right]^{-\sigma_E}\frac{\Xi_{t+1}}{\Xi_t} \tag{3.6.16}$$

substituting the SDF into (3.6.14)

$$\frac{1}{R_{t+1}^D} = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\pi_{t+1}}\right] \tag{3.6.17}$$

In all,the household optimazation problem gives the following equations

$$\begin{aligned}
\tilde{C}_t(j) &= \left[(1-\eta_g)^{\frac{1}{\psi_c}}C_t(j)^{\frac{\psi_c-1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}}G_t^{C\,\frac{\psi_c-1}{\psi_c}}\right]^{\frac{\psi_c}{\psi_c-1}} \\
U(\tilde{C}_t, L_t) &= \varepsilon_t^{\beta}\frac{1}{1-\sigma_C}\left(\tilde{C}_t(j) - \eta_c\tilde{C}_{t-1}(j)\right)^{1-\sigma_C}\exp\left(\frac{\sigma_C-1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}(j)\right) \\
V_t(j) &= U(\tilde{C}_t(j), L_t(j)) + \beta v_t(j) \\
v_t(j) &= \mathbb{E}_t(V_{t+1}^{1-\sigma_E}(j))^{\frac{1}{1-\sigma_E}} \\
U_C(\tilde{C}_t, L_t) &= (\tilde{C}_t - \eta_c\tilde{C}_{t-1})^{-\sigma_C}\exp\left(\frac{\sigma_C-1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right)\left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}} \\
\frac{W_t^h}{P_t} &= \frac{(\tilde{C}_t - \eta_c\tilde{C}_{t-1})\varepsilon_t^w L_t^{\sigma_L}(1+\tau_t^c)}{\left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}}(1-\tau_t^l)}
\end{aligned}$$

$$\begin{aligned}
\Xi_t &= \frac{U_C(\tilde{C}_t, L_t)}{(1+\tau_t^c)} \\
\Lambda_{t,t+1} &= \varepsilon_t^{\beta}\beta\left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}}\right]^{-\sigma_E}\frac{\Xi_{t+1}}{\Xi_t} \\
1 &= \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}}\right]R_{t,t+1}^D \\
1 &= \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}}\left(1 + (1-\tau_{t+1}^k)(R_{t,t+1}-1)\right)\right]
\end{aligned}$$

Retailers(Final Good Producer)

They are the final good Y_t producers, the final good is a composition of differentiated goods from wholesalers $Y_t(i)$. This final good is allocated to private consumption C_t, private investment I_t,government consumption G_t^C and government investment G_t^I.

The technology used in transforming this differentiated goods is given in form of a Dixit-Stiglitz aggregator.

They maximise their profit

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, \mathrm{d}i\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.6.18}$$

$$\begin{aligned} \max_{Y_t(i), Y_t} \quad & P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, \mathrm{d}i \\ \text{s.t} \quad & Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, \mathrm{d}i\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \end{aligned}$$

$P_t$is the price of the final good,$P_t(i)$ is the price for intermediary good i, and ϵ_p is the elasticity of substitution among varieties

Setting up a Lagrangian

$$\mathcal{L} = P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, \mathrm{d}i + \lambda_t^{MC} \left[\left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, \mathrm{d}i\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} - Y_t \right]$$

Finding the partial derivatives

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Y_t} &= P_t - \lambda_t^{MC} \\ \frac{\partial \mathcal{L}}{\partial Y_t(i)} &= -P_t(i) + \lambda_t^{MC} \left[\left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} \, \mathrm{d}i\right)^{\frac{1}{\epsilon_p - 1}} Y_t(i)^{\frac{-1}{\epsilon_p}} \right] \end{aligned}$$

Applying FOC[3] we have

$$
\begin{aligned}
P_t &= \lambda_t^{MC} \\
P_t(i) &= \lambda_t^{MC} Y_t^{\frac{1}{\epsilon_p}} Y_t(i)^{\frac{-1}{\epsilon_p}} \\
&= P_t Y_t^{\frac{1}{\epsilon_p}} Y_t(i)^{\frac{-1}{\epsilon_p}}
\end{aligned}
$$

Simplifying the algebra we have the demand curve expression as

$$
Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t \tag{3.6.19}
$$

And now, going by

$$
P_t Y_t = \int_0^1 P_t(i) Y_t(i)\,\mathrm{d}i
$$

substituting (3.6.19)

$$
\begin{aligned}
P_t Y_t &= Y_t \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i \\
P_t &= \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i \\
P_t^{1-\epsilon_p} &= \int_0^1 P_t(i)^{1-\epsilon_p}\,\mathrm{d}i
\end{aligned}
$$

Thus we have LOM[4] for price

$$
P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_p}\,\mathrm{d}i\right)^{\frac{1}{1-\epsilon_p}} \tag{3.6.20}
$$

[3] First Order Condition
[4] Law of Motion

Wholesalers This individuals buy intermediate goods M_t from the entrepreneur who owns firms. This

intermediate goods are differentiated without cost,taking the demand function earlier derived from retailers FOC. Wholesalers prices are subject to Calvo pricing(1996) introducing nominal rigidities into the model.

Under Calvo pricing with partial indexation,the optimal price set by wholesalers that is allowed to re-optimise gives rise to the optimisation problem below

$$\max_{\tilde{P}_t(i)} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Lambda_{t,t+s} \Big[\tilde{P}_t(i) X_{s,t}^p Y_{t+s}(i) - MC_{t+s} M_{t+s}(i) \Big]$$
$$\text{s.t} \quad Y_{t+s}(i) = Y_{t+s} \Big(\frac{P_t(i) X_{s,t}^p}{P_{t+s}} \Big)^{-\epsilon_p}$$
$$Y_{t+s}(i) = \frac{M_{t+s}(i)}{\varepsilon_t^p}$$

The second constraint as seen above represent a linear production used for differentiating the intermediate good. $\tilde{P}_t(i)$ is the newly set price that each firm will be stuck with for some period only indexed to past inflation represented by$X_{t,s}^p$,and ζ_p is the probability of not being able to re-optimise price and the parameter l_p is for indexation

$$X_{s,t}^p = \begin{cases} 1 & \text{for} \quad s = 0 \\ \Big(\prod_{l=1}^{s} \pi_{t+l-1}^{l_p} \pi_*^{1-l_p} \Big) & \text{for} \quad s = 1 \dots \infty \end{cases}$$

Setting up a Lagrangian and finding the partial derivative with respect to $\tilde{P}_t(i)$

$$\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \Lambda_{t,t+s} \Big[\tilde{P}_t(i) X_{s,t}^p - \varepsilon_t^p MC_{t+s} \Big] Y_{t+s} \Big(\frac{P_t(i) X_{s,t}^p}{P_{t+s}} \Big)^{-\epsilon_p}$$
$$\frac{\partial \mathcal{L}}{\partial \tilde{P}_t(i)} = \begin{bmatrix} X_{s,t}^p Y_{t+s}(i) - \epsilon_p \tilde{P}_t(i) X_{s,t}^p Y_{t+s} \Big(\frac{\tilde{P}_t(i) X_{s,t}^p}{P_{t+s}} \Big)^{-\epsilon_p - 1} \frac{X_{s,t}^p}{P_{t+s}} \\ + \epsilon_p \varepsilon_t^p MC_{t+s} Y_{t+s} \Big(\frac{\tilde{P}_t(i) X_{s,t}^p}{P_{t+s}} \Big)^{-\epsilon_p - 1} \frac{X_{s,t}^p}{P_{t+s}} \end{bmatrix}$$

multiplying through by $\tilde{P}_t(i)$ and applying FOC to the partial derivative

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}\left[\tilde{P}_t(i)X_{s,t}^pY_{t+s}(i)-\epsilon_p\tilde{P}_t(i)X_{s,t}^pY_{t+s}\left(\frac{\tilde{P}_t(i)X_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}+\epsilon_p\varepsilon_t^pMC_{t+s}Y_{t+s}\left(\frac{\tilde{P}_t(i)X_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\right]=0$$

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}\left[\tilde{P}_t(i)X_{s,t}^pY_{t+s}(i)-\epsilon_p\tilde{P}_t(i)X_{s,t}^pY_{t+s}(i)+\epsilon_p\varepsilon_t^pMC_{t+s}Y_{t+s}(i)\right]=0$$

We move on to factorise the above expression

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\left[\tilde{P}_t(i)X_{s,t}^p-\epsilon_p\tilde{P}_t(i)X_{s,t}^p+\epsilon_p\varepsilon_t^pMC_{t+s}\right]=0$$

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\left[(1-\epsilon_p)\tilde{P}_t(i)X_{s,t}^p+\epsilon_p\varepsilon_t^pMC_{t+s}\right]=0$$

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\left[\epsilon_p\varepsilon_t^pMC_{t+s}-(\epsilon_p-1)\tilde{P}_t(i)X_{s,t}^p\right]=0$$

Rewriting the expression derived above into two parts

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)(\epsilon_p-1)\tilde{P}_t(i)X_{s,t}^p=\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\epsilon_p\varepsilon_t^pMC_{t+s}$$

$$\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\tilde{P}_t(i)X_{s,t}^p=\left(\frac{\epsilon_p}{\epsilon_p-1}\right)\mathbb{E}_t\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}(i)\varepsilon_t^pMC_{t+s}$$

substituting $Y_{t+s}(i)$

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{\tilde{P}_t(i)X_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\tilde{P}_t(i)X_{s,t}^p\right]=\left(\frac{\epsilon_p}{\varepsilon_t^p-1}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{\tilde{P}_t(i)X_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\varepsilon_t^pMC_{t+s}\right]$$

and to simplifying the sum, we first introduce $\frac{P_t}{P_t}$ to both the LHS[5] and RHS[6]

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_t}{P_{t+s}}\right)^{1-\epsilon_p}P_{t+s}X_{s,t}^{p^{1-\epsilon_p}}\right]\left(\frac{\tilde{P}_t(i)}{P_t}\right)^{1-\epsilon_p}=\left(\frac{\epsilon_p}{\epsilon_p-1}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_tX_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\varepsilon_t^pMC_{t+s}\right]$$

[5] Left Hand Side
[6] Right Hand Side

factoring out $\left(\frac{\tilde{P}_t(i)}{P_t}\right)^{-\epsilon_p}$

$$\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_t}{P_{t+s}}\right)^{1-\epsilon_p}P_{t+s}X_{s,t}^{p^{1-\epsilon_p}}\right]\frac{\tilde{P}_t(i)}{P_t}=\left(\frac{\varepsilon_t^p}{\epsilon_p-1}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_tX_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\varepsilon_t^pMC_{t+s}\right]$$

Thus

$$\frac{\tilde{P}_t(i)}{Pt}=\left(\frac{\epsilon_p}{\epsilon_p-1}\right)\left(\frac{G_t^{p2}}{G_t^{p1}}\right)$$

where

$$G_t^{p2}=\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_tX_{s,t}^p}{P_{t+s}}\right)^{-\epsilon_p}\varepsilon_t^pMC_{t+s}\right]$$

and

$$G_t^{p1}=\mathbb{E}_t\left[\sum_{s=0}^{\infty}\zeta_p^s\Lambda_{t,t+s}Y_{t+s}\left(\frac{P_tX_{s,t}}{P_{t+s}}\right)^{1-\epsilon_p}\right]$$

Expanding the sums recursively,

$$G_t^{p1}=\left(\zeta_p^0\Lambda_{t,t}Y_t\left(\frac{P_tX_{0,t}^p}{P_t}\right)^{1-\epsilon_p}\quad+\zeta_p^1\Lambda_{t,t+1}Y_{t+1}\left(\frac{P_tX_{1,t}^p}{P_{t+1}}\right)^{1-\epsilon_p}\quad+\zeta_p^2\Lambda_{t,t+2}Y_{t+2}\left(\frac{P_tX_{2,t}^p}{P_{t+2}}\right)^{1-\epsilon_p}\quad+\dots\right)$$

Recalling $X_{s,t}$ and using the following ratios

$$\frac{P_{t+2}}{P_t}=\pi_{t+1}\pi_{t+2}$$
$$\Lambda_{t,t+2}=\Lambda_{t,t+1}\Lambda_{t+1,t+2}$$

truncating the sum recursively gives the following expression.

$$G_t^{p1} = Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_{ss}^{1-l_p}}{\pi_{t+1}}\right)^{1-\epsilon_p} G_{t+1}^{p1}\right]$$

For LHS

$$G_t^{p2} = \begin{pmatrix} \zeta_p^0 \Lambda_{t,t} Y_t \left(\frac{P_t X_{0,t}^p}{P_t}\right)^{-\epsilon_p} \varepsilon_t^p MC_t \\ +\zeta_p^1 \Lambda_{t,t+1} Y_{t+1} \left(\frac{P_t X_{1,t}^p}{P_{t+1}}\right)^{-\varepsilon_t} \epsilon_{t+1}^p MC_{t+1} \\ +\zeta_p^2 \Lambda_{t,t+2} Y_{t+2} \left(\frac{P_t X_{2,t}^p}{P_{t+2}}\right)^{-\epsilon_p} \epsilon_{t+2}^p MC_{t+2} \\ +\ldots \end{pmatrix}$$

Recalling $X_{s,t}$ and using the following ratios

$$\frac{P_{t+2}}{P_t} = \pi_{t+1}\pi_{t+2}$$
$$\Lambda_{t,t+2} = \Lambda_{t,t+1}\Lambda_{t+1,t+2}$$

Also considering that $\frac{P_t}{P_{t+1}} = {\pi_{t+1}}^{-1}$, the sum in its recursive form is

$$G_t^{p2} = \varepsilon_t^p MC_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{l_p}\pi_*^{1-l_p}}{\pi_{t+1}}\right)^{-\varepsilon_t^p} G_{t+1}^{p2}\right]$$

Aggregate price is thus given by

$$1 = (1-\zeta_p)\left(\frac{\tilde{P}_t(i)}{P_t}\right)^{1-\epsilon_p} + \zeta_p\left(\pi_{t-1}^{l_p}\pi_{ss}^{1-l_p}\pi_t^{-1}\right)^{1-\epsilon_p} \tag{3.6.21}$$

from (3.6.21) we have

$$\frac{\tilde{P}_t(i)}{P_t} = \left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{\frac{1}{1-\epsilon_p}} \tag{3.6.22}$$

Price Dispersion The Price dispersion expression is derived as follows;

$$\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i$$

$$\Delta_t^p = (1-\zeta_p)\int_0^1 \left(\frac{\tilde{P}_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i + \zeta_p\left(\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}\frac{P_{t-1}(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i$$
$$\Delta_t^p = (1-\zeta_p)\int_0^1 \left(\frac{\tilde{P}_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\int_0^1 \left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\epsilon_p} \mathrm{d}i$$
$$\Delta_t^p = (1-\zeta_p)\int_0^1 \left(\frac{\tilde{P}_t(i)}{P_t}\right)^{-\epsilon_p} \mathrm{d}i + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p$$

$$\Delta_t^p = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p \tag{3.6.23}$$

Entrepreneurs

Each entrepreneur owns a firm i that uses the following technology

$$M_t(i) = \varepsilon_t^a\left(K_{t-1}(i)U_t^k(i)\right)^{\alpha}\left(\gamma^t H_t(i)\right)^{1-\alpha}{K_{t-1}^G}^{\theta_G} \tag{3.6.24}$$

[7]

[7] γ^t is labour augmenting deterministic growth rate in the economy and ε_t^a is the Total Factor Productivity (TFP) shock.

the technology employed by entrepreneurs comprises of privately owned capital (K_{t-1}), labour (H_t), and public capital K^G_{t-1}

Each entrepreneurhire labour H_t paying W_t as wage tolabour union (which in turn pay households), they also buy a portion of private investment goods $\tilde{I}_t$ from investment good producers. This portion is added to their existing stock of private capital K_{t-1} to make up for next period private capital K_t. Where K_t denote the private capital choice at the end of period $t-1$ which is to be used for period t production. They also take advantage of the installed public capital by the government denoted by K^G_t, where θ_G represents the elasticity of output with respect to public capital.

Entrepreneur have expected lifetime with probability of survival given as φ implying expected lifetimeof $\frac{1}{1-\varphi}$ this is to ensure that entrepreneur'snet worth is not enough to finance new capital acquisition for next period production.

So they are faced with external finance premium that is in addition to risk-free rate.

At time t, the rate of return $R^N_{t,t+1}$ on their loan from time t to $t+1$ is agreed on at time t. Thus leading to costly state verification. The nominal external finance premium is $\text{PREM}_{t,t+1}$

$$R^N_{t,t+1} = \text{PREM}_{t,t+1} R_{t,t+1} \tag{3.6.25}$$

note that $R_{t,t+1}$ is representing the risk-free rate. The spread $\text{PREM}_{t,t+1}$ is also known at time t and is given as

$$\text{PREM}(.) = S\left(\frac{N_t}{Q_t K_t}\right)$$

$$\text{PREM}_{t,t+1} = \text{PREM}_{ss}\left(\varepsilon^S_t \frac{Q_t K_t}{N_t}\frac{N_{ss}}{K_{ss}}\right)^{\psi_S} \tag{3.6.26}$$

PREM_{ss} is a constant and it is assume based on the reasoning that lenders only know the aggregate $\frac{Q_t K_t}{N_t}$ not individual entrepreneurs level.

In all, each entrepreneur maximises their wealth which in this case is represented as their networth N_t. The probability for an entrepreneur to survive and continue production is φ, thus the expected lifetime is$\frac{1}{1-\varphi}$

$$\begin{aligned}
&\max_{H_t,K_t,U_t^k} \mathbb{E}_t\Big[\sum_{s=0}^{\infty}(1-\varphi)^{s-1}\Lambda_{t,t+s}N_{t+s}\Big] \\
&\text{s.t} \quad N_{t-1} = Q_{t-1}K_{t-1} - D_{t-1} \\
&\quad N_t = MC_tM_t - W_tH_t + (1-\delta_t^k)Q_tK_{t-1} - \frac{R_{t-1,t}^N}{\pi_{t-1,t}}D_{t-1} \\
&\quad M_t \leq \varepsilon_t^a\big(K_{t-1}(i)U_t^k(i)\big)^{\alpha}\big(\gamma^tH_t(i)\big)^{1-\alpha}{K_{t-1}^G}^{\theta_G} \\
&\quad K_t \leq (1-\delta_t^k)K_{t-1} + \tilde{I}_t \\
&\quad \delta_t^k = \delta + \psi_u\frac{\alpha MC_{ss}M_{ss}}{Q_{ss}K_{ss}}\Big({U_t^k}^{\frac{1}{\psi_u}} - 1\Big)
\end{aligned}$$

The interpretation of each of the constraints as seen above is explained below

First constraint is regarded as the balance sheet constraint, it tells the difference of what an entrepreneur has left after paying the debt D_{t-1}.

The second constraint is the Law of Motion (LOM) of net worth. This constrain indicate that entrepreneur retains capital share and capital after depreciation minus debt repayments.

Third constraint is the production function, followed by the LOM of capital. And lastly the depreciation.

From the balance sheet constrain we have

$$D_{t-1} = Q_{t-1}K_{t-1} - N_{t-1}$$

and substituting into LOM of net worth

$$N_t = MC_tM_t - W_tH_t + \left(1 - \delta_t^k - \frac{R_{t-1,t}^N}{\pi_{t-1,t}}\frac{Q_{t-1}}{Q_t}\right)Q_tK_{t-1} - \frac{R_{t-1,t}^N}{\pi_{t-1,t}}N_{t-1} \tag{3.6.27}$$

Setting up a Lagrangian

$$\mathcal{L} = \mathbb{E}_t\Big[\sum_{s=0}^{\infty}(1-\varphi)^{s-1}\Lambda_{t,t+s}N_{t+s}\Big]$$

where N_{t+s} in the Lagrangian is the expression in (3.6.27)

Finding the partial derivatives

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial H_t} &= \Lambda_{t-1,t}\Bigg[-W_t + (1-\alpha)MC_t\Big(\frac{M_t}{H_t}\Big)\Bigg] \\
\frac{\partial \mathcal{L}}{\partial K_t} &= \mathbb{E}_t\Bigg[\Lambda_{t,t+1}(1-\varphi)^{s-1}\Big(\alpha MC_{t+1}\Big(\frac{M_{t+1}}{K_t}\Big) + (1-\delta^k_{t+1})Q_{t+1} - \frac{R^N_{t,t+1}}{\pi_{t,t+1}}Q_t\Big)\Bigg] \\
\frac{\partial \mathcal{L}}{\partial U^k_t} &= \Lambda_{t-1,t}\Bigg[\alpha MC_t\Big(\frac{M_t}{U^k_t}\Big) - \frac{\alpha(1-\theta_G)MC_{ss}M_{ss}}{K_{ss}}{U^k_t}^{\frac{1}{\psi_u}-1}Q_t K_{t-1}\Bigg]
\end{aligned}
$$

Applying FOC to the above set of partial derivatives and considering that SDF $\Lambda_{t-1,t} \neq 0$ we have the following equations:

From ∂H_t we have

$$
W_t = (1-\alpha)MC_t\frac{M_t}{H_t} \tag{3.6.28}
$$

From ∂U^k_t we have

$$
\alpha MC_t\frac{M_t}{U^k_t} = \frac{\alpha MC_{ss}M_{ss}}{K_{ss}Q_{ss}}{U^k_t}^{\frac{1}{\psi_u}-1}Q_t K_{t-1}
$$

Divide through by K_{t-1} and multiply through by U^k_t

$$
\alpha MC_t\frac{M_t}{K_{t-1}} = \frac{\alpha MC_{ss}M_{ss}}{K_{ss}Q_{ss}}{U^k_t}^{\frac{1}{\psi_u}}Q_t
$$

Defining the marginal product of capital as follows

$$
MPK_t = \alpha MC_t\frac{M_t}{K_{t-1}} \tag{3.6.29}
$$

divide through by Q_t

$$\frac{MPK_t}{Q_t} = \frac{MPK_{ss}}{Q_{ss}} U_t^{k^{\frac{1}{\psi_u}}}$$

thus we have an explicit expression for utilisation

$$U_t^k = \left(\frac{\frac{MPK_t}{Q_t}}{\frac{MPK_{ss}}{Q_{ss}}} \right)^{\psi_u} \tag{3.6.30}$$

Before we simplified the partial derivative with respect to K_t, let us define another variable $R_{t,t+1}^k$ which is the realised return on capital, this is different from the requested return on loan $R_{t,t+1}^N$ which is known at the inception of the loan.

$$\mathbb{E}_t \Big[\Lambda_{t,t+1} R_{t,t+1}^K \Big] = \mathbb{E}_t \Big[\Lambda_{t,t+1} \frac{1}{\pi_{t,t+1}} \Big] R_{t,t+1}^N \tag{3.6.31}$$

Now directly from the partial derivative of K_t, FOC[8] implies

$$\mathbb{E}_t \Big[R_{t,t+1}^K \Big] = \mathbb{E}_t \left[\frac{MPK_{t+1}}{Q_t} + (1 - \delta_{t+1}^k) \frac{Q_{t+1}}{Q_t} \right] \tag{3.6.32}$$

Noting that $\frac{R_{t,t+1}^N}{\pi_{t,t+1}} \neq R_{t,t+1}^K$ because the marginal product of capital MPK_{t+1} and $\pi_{t,t+1}$depends on the realisation of the shocksat $t + 1$

Labour Sector

Household directly supply their homogeneous labour to an intermediate labour union which inturn differentiates the labour services and set wages subject to a Calvo schemeiates.

Labour Packers The hired differentiated labour is made into a single labour quantity index by labour packers

[8] First Order Condition

$$H_t = \left[\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, \mathrm{d}l \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{3.6.33}$$

The differentiated labour $H_t(l)$ by labour union is bought and packaged by labour packers. Labour packers sells H_t for W_t thus maximising

$$\begin{aligned} \max_{H_t(l)} \quad & W_t H_t - \int_0^1 W_t(l) H_t(l) \, \mathrm{d}l \\ \text{s.t} \quad & H_t = \left(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, \mathrm{d}l \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \end{aligned}$$

substituting the labour demand index into the optimisation problem for an unconstrained problem. We setting up a Lagrangian

$$\mathcal{L} = W_t \bigg(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, \mathrm{d}l \bigg)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t(l) H_t(l) \, \mathrm{d}l$$

differentiating with respect to $H_t(l)$ and applying FOC

$$\begin{aligned} W_t(l) &= W_t \bigg(\int_0^1 H_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} \, \mathrm{d}l \bigg)^{\frac{1}{\epsilon_w - 1}} H_t(l)^{\frac{-1}{\epsilon_w}} \\ W_t(l) &= W_t H_t^{\frac{1}{\epsilon_w}} H_t(l)^{\frac{-1}{\epsilon_w}} \end{aligned}$$

Thus we have

$$H_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} H_t \tag{3.6.34}$$

And thus the wage received by labour packers can be derive as follows

$$W_t H_t = \int_0^1 W_t(l) H_t(l) \, \mathrm{d}l \tag{3.6.35}$$

substituting the optimum labour $H_t(l)$ (3.6.34) into (3.6.35) and simplifying gives the wage index

$$W_t = \left(\int_0^1 W_t(l)^{1-\epsilon_w} \, \mathrm{d}l \right)^{\frac{1}{1-\epsilon_w}} \tag{3.6.36}$$

Labour Unions

They hire raw labour force from households and give them some trainings in a way to differentiate them based on their skills.

Labour union takes this marginal rate of substitution as the cost of labour servicesin their negotiations with labour packers. The mark-up above this marginal disutility is distributed to the households.

The union is subjected to nominal rigidities, they can readjust wages with probability $1 - \zeta_w$ in each period. And thus they optimise wage over the period which they are stuck, and for those time where they do not readjust wage they increase by the deterministic growth rate γ and the weighted average of steady-state inflation π_{ss}

$$\begin{aligned}
\max_{\tilde{W}_t(l)} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \Big(W_{t+s}(l) H_{t+s}(l) - W_{t+s}^h L_{t+s}(l) \Big) \\
\text{s.t} \quad & H_{t+s}(l) = \left(\frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} \\
& H_{t+s}(l) = L_{t+s}(l) \\
& W_{t+s}(l) = \tilde{W}_t(l) \Big(\prod_{l=1}^{s} \pi_{t+l-1}^{l_w} \pi_{ss}^{1-l_w} \Big) \quad \text{for} \quad s = 1, \dots, \infty
\end{aligned}$$

Setting up Lagrangian, by first substituting the constraints

$$\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \Big(X_{t,s} \tilde{W}_t(l) - W_t^h \Big) \left(\frac{W_{t+s}(l)}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s}$$

finding the partial derivative with respect to $\tilde{W}_t(l)$ and factorising

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left[X_{s,t} H_{t+s}(l) + \Big(X_{s,t} \tilde{W}_t(l) - W_t^h \Big) \left(-\epsilon_w \left(\frac{X_{t+s} \tilde{W}_t(l)}{W_{t+s}} \right)^{-\epsilon_w - 1} \left(\frac{X_{s,t}}{W_{t+s}} \right) H_{t+s} \right) \right] = 0$$

$$X_{t+s} = \begin{cases} 1 & \text{for} \quad s = 0 \\ \left(\prod_{l=1}^{s} \pi_{t+l-1}^{l_w} \pi_{ss}^{1-l_w} \right) & \text{for} \quad s = 1, \dots \infty \end{cases}$$

multiplying through by $\tilde{W}_t(l)$ and simplifying

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left[X_{s,t} \tilde{W}_t(l) H_{t+s}(l) - \epsilon_w \left(X_{s,t} \tilde{W}_t(l) - W_t^h \right) H_{t+s}(l) \right] = 0$$
$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} H_{t+s}(l) \left[X_{s,t} \tilde{W}_t(l) - \epsilon_w \left(X_{s,t} \tilde{W}_t(l) - W_t^h \right) \right] = 0$$
$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} H_{t+s}(l) \left[(1 - \epsilon_w) X_{s,t} \tilde{W}_t(l) + \epsilon_w W_t^h \right] = 0$$
$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} H_{t+s}(l) \left[\epsilon_w W_t^h - (\epsilon_w - 1) X_{s,t} \tilde{W}_t(l) \right] = 0$$

Substituting $H_{t+s}(l)$

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{\tilde{W}_t X_{s,t}^w}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} \left(\epsilon_w W_t^h - (\epsilon_w - 1) X_{s,t} \tilde{W}_t(l) \right) = 0$$

$$\epsilon_w \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{\tilde{W}_t X_{s,t}^w}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} W_t^h = (\epsilon_w - 1) \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{\tilde{W}_t X_{s,t}^w}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} X_{s,t} \tilde{W}_t(l)$$

The sums above still has the optimum wage $\tilde{W}_t(l)$ as a variable, we make some modifications. The modification for LHS is to introduce $\left(\frac{W_t}{W_t} \right)^{-\epsilon_w}$

$$\epsilon_w \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t \tilde{W}_t X_{s,t}^w}{W_t W_{t+s}} \right)^{-\epsilon_w} H_{t+s} W_t^h$$

factoring out the non-stochastic component (without subscript s)

$$\epsilon_w \left[\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t X_{s,t}^w}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} W_t^h \right] \left(\frac{\tilde{W}_t}{W_t} \right)^{-\epsilon_w}$$

Also, for the RHS weintroduce $\left(\frac{W_t}{W_t}\right)^{1-\epsilon_w}$

$$(\epsilon_w - 1)\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t \tilde{W}_t X_{s,t}^w}{W_t W_{t+s}} \right)^{1-\epsilon_w} H_{t+s} W_{t+s}$$

also factorising out the non-stochastic component we have

$$(\epsilon_w - 1)\left[\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t X_{s,t}^w}{W_{t+s}} \right)^{1-\epsilon_w} H_{t+s} W_{t+s} \right] \left(\frac{\tilde{W}_t}{W_t} \right)^{1-\epsilon_w}$$

equating LHS to RHS and simplify

factoring out the non-stochastic component (without subscript s)

$$\epsilon_w \left[\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t X_{s,t}^w}{W_{t+s}} \right)^{-\epsilon_w} H_{t+s} W_t^h \right] = (\epsilon_w - 1)\left[\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t X_{s,t}^w}{W_{t+s}} \right)^{1-\epsilon_w} H_{t+s} W_{t+s} \right] \left(\frac{\tilde{W}_t}{W_t} \right)$$

Thus we can rewrite the expression above as

$$\frac{\tilde{W}_t}{W_t} = \left(\frac{\epsilon_w}{\epsilon_w - 1} \right) \left(\frac{G_t^{w2}}{G_t^{w1}} \right)$$

where

$$G_t^{w1} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s} \left(\frac{W_t X_{s,t}^w}{W_{t+s}} \right)^{1-\epsilon_w} H_{t+s} W_{t+s} \right]$$

and

$$G_t^{w2} = \mathbb{E}_t\Bigg[\sum_{s=0}^{\infty} \zeta_w^s \Lambda_{t,t+s}\bigg(\frac{W_t X_{s,t}^w}{W_{t+s}}\bigg)^{-\epsilon_w} H_{t+s} W_t^h\Bigg]$$

Expanding the sum

$$G_t^{w1} = W_t H_t + \zeta_w \Lambda_{t,t+1}\bigg(\frac{W_t X_{1,t}^w}{W_{t+1}}\bigg)^{1-\epsilon_w} H_{t+1} W_{t+1} + \zeta_w^2 \Lambda_{t,t+2}\bigg(\frac{W_t X_{2,t}^w}{W_{t+2}}\bigg)^{1-\epsilon_w} H_{t+2} W_{t+2} + \dots$$

with further simplification, the recursive expression for G_t^{w1} is

$$G_t^{w1} = W_t H_t + \mathbb{E}_t\Bigg[\zeta_w \Lambda_{t,t+1}\bigg(\frac{W_t}{W_{t+1}}\bigg)^{1-\epsilon_w}\Big(\pi_t^{l_w}\pi_{ss}^{1-l_w}\Big)^{1-\epsilon_w} G_{t+1}^{w1}\Bigg]$$

also

$$G_t^{w2} = W_t^h H_t + \zeta_w \Lambda_{t,t+1}\bigg(\frac{W_t X_{1,t}^w}{W_{t+1}}\bigg)^{-\epsilon_w} H_{t+1} W_{t+1}^h + \zeta_w^2 \Lambda_{t,t+2}\bigg(\frac{W_t X_{2,t}^w}{W_{t+2}}\bigg)^{-\epsilon_w} H_{t+2} W_{t+2}^h + \dots$$

and here is the recursive form

$$G_t^{w2} = W_t^h H_t + \mathbb{E}_t\Bigg[\zeta_w \Lambda_{t,t+1}\bigg(\frac{W_t}{W_{t+1}}\bigg)^{-\epsilon_w}\Big(\pi_t^{l_w}\pi_{ss}^{1-l_w}\Big)^{-\epsilon_w} G_{t+1}^{w2}\Bigg]$$

The aggregate wage expression is

$$W_t = \bigg[(1-\zeta_w)\tilde{W}_t^{1-\epsilon_w} + \zeta_w\Big(\gamma\pi_{t-1}^{l_w}\pi_{ss}^{1-l_w} W_{t-1}\Big)^{1-\epsilon_w}\bigg]^{\frac{1}{1-\epsilon_w}} \tag{3.6.37}$$

from which we have

$$\frac{\tilde{W}_t(i)}{W_t} = \left[\frac{1 - \zeta_w \left(\frac{W_{t-1} \pi_{t-1}^{\iota_w} \pi_{ss}^{1-\iota_w}}{W_t} \right)^{1-\epsilon_w}}{1 - \zeta_w} \right]^{\frac{1}{1-\epsilon_w}}$$

Wage Dispersion

There exist wage dispersion cost, this implies that

$$H_t \neq L_t$$
$$W_t \neq W_t^h$$

$$L_t = \int_0^1 H_t(l)\, \mathrm{d}l$$

recall that

$$H_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} H_t$$

$$L_t(l) = \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} H_t\, \mathrm{d}l$$

where the wage dispersion cost

$$\Delta_w = \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} \mathrm{d}l$$
$$L_t = \Delta_t^w H_t$$

The LOM of wage dispersion cost is

$$\Delta_t^w = (1-\zeta_w)\int_0^1 \left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w} \mathrm{d}l + \zeta_w \int_0^1 \left(\frac{W_{t-1}(l)\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w} \mathrm{d}l$$
$$\Delta_t^w = (1-\zeta_w)\int_0^1 \left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w} \mathrm{d}l + \zeta_w \left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w} \int_0^1 \left(\frac{W_{t-1}(l)}{W_{t-1}}\right)^{-\epsilon_w} \mathrm{d}l$$

which is

$$\Delta_t^w = (1-\zeta_w)\int_0^1 \left(\frac{\tilde{W}_t(l)}{W_t}\right)^{-\epsilon_w} \mathrm{d}l + \zeta_w \left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w} \Delta_{t-1}^w$$

$$\Delta_t^w = (1-\zeta_w)\left[\frac{1-\zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{W_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}} + \zeta_w \left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w} \Delta_{t-1}^w$$

Investment Good Producers

Investment takes the final goods I_t to produce investment goods $\tilde{I}_t$

They choose the quantity of investment I_t to maximise their profits given as

$$\Pi_t^I = Q_t\tilde{I}_t - I_t$$

$$\max_{I_t\tilde{I}_t} \quad \mathbb{E}_0\left[\sum_{t=0}^{\infty}\Lambda_{0,t}\Pi_t^I\right]$$
$$\text{s.t} \quad \tilde{I}_t = \varepsilon_t^i I_t - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2 I_t$$

Setting up a Lagrangian

$$\mathcal{L} = \mathbb{E}_t \Lambda_{0,t} \left[\sum_{t=0}^{\infty} Q_t \tilde{I}_t - I_t + \lambda_t^i \Big(\varepsilon_t^i I_t - \frac{\psi}{2} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big)^2 I_t - \tilde{I}_t \Big) \right]$$

and finding the partial derivatives

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tilde{I}_t} &= \Lambda_{0,t}(Q_t - \lambda_t^i) \\
\frac{\partial \mathcal{L}}{\partial I_t} &= \Lambda_{0,t} \left[-1 + \lambda_t^i \Big(\varepsilon_t^i - \psi \frac{I_t}{I_{t-1}} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big) - \frac{\psi}{2} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big)^2 \Big) \right] \\
&\quad + \mathbb{E}_t \left[\Lambda_{0,t+1} \lambda_{t+1}^i \psi \Big(\frac{I_{t+1}}{I_t} - \gamma \Big) \Big(\frac{I_{t+1}}{I_t} \Big)^2 \right]
\end{aligned}$$

applying FOC we have

$$\begin{aligned}
\lambda_t^i &= Q_t \\
1 &= Q_t \left[\varepsilon_t^i - \psi \frac{I_t}{I_{t-1}} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big) - \frac{\psi}{2} \Big(\frac{I_t}{I_{t-1}} - \gamma \Big)^2 \right] + \mathbb{E}_t \left[\frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} Q_{t+1} \psi \Big(\frac{I_{t+1}}{I_t} \Big)^2 \Big(\frac{I_{t+1}}{I_t} - \gamma \Big) \right]
\end{aligned}$$

Government Policies

The central bank follows a nominal interest rate rule by adjusting its instrument in response to deviations of inflation and output from their respective target levels;

$$\frac{R_t}{R_{ss}} = \Big(\frac{R_{t-1}}{R_{ss}} \Big)^{\rho_R} \left[\Big(\frac{\pi_t}{\pi_{ss}} \Big)^{\psi_\pi} \Big(\frac{Y_t}{Y_{ss}} \Big)^{\psi_y} \right]^{1-\rho_R} \varepsilon_t^r \tag{3.6.38}$$

where R_{ss},π_{ss},and Y_{ss} are the deviations of nominal interest rate,inflation,and output from their respective steady-states.

Market clearing implies that the following holds

$$Y_t = C_t + I_t + G_t^C + G_t^I \tag{3.6.39}$$

where G_t^C and G_t^I are government consumption and investment respectively. The available public capital for the beginning of time t production is expressed as

$$K_t^G = (1-\delta_G)K_{t-1}^G + \frac{G_t^I}{Y_t} \tag{3.6.40}$$

where δ_G is the rate at which public capital depreciates.

Government decides the fiscal instruments to satisfy its budget constraint given by

$$G_t^C + G_t^I + Z_t + \frac{R_{t-1}^D D_{t-1}}{\pi_t} = \tau_t^c C_t + \tau_t^l W_t L_t + \tau_t^k\left((R_{t-1}-1)\frac{B_{t-1}}{\pi_t} + \varphi N_{t-1}\right) + D_t \tag{3.6.41}$$

In the government budget above,capital tax τ_t^k is levied onnet interest earning on bond $(R_{t-1}-1)\frac{B_{t-1}}{\pi_t}$ and entrepreneur's dividend φN_{t-1}. All fiscal instrument except consumption tax responds tothe deviations of lagged government debt $D_{t-1}$and output $Y_t$from there respective steady-states. Consumption tax is exogenous, since government debt is an observable in estimation and debt is constructed through the accumulation of government net borrowing consistent with the NIPA concept, consumption taxes are necessary for model receipts to equal actual receipt Leeper et al. (2009).Meanwhile, all are affected by AR(1) exogenous processes.

$$\frac{\tau_t^k}{\tau_{ss}^k} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_K}\left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_K}\varepsilon_t^K \tag{3.6.42}$$

$$\frac{\tau_t^l}{\tau_{ss}^l} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_L}\left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_L}\varepsilon_t^L \tag{3.6.43}$$

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_Z}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_Z}\varepsilon_t^T \tag{3.6.44}$$

$$\frac{G_t^C}{G_{ss}^C} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GC}}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GC}}\varepsilon_t^{GC} \tag{3.6.45}$$

$$\frac{G_t^I}{G_{ss}^I} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GI}}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GI}}\varepsilon_t^{GI} \tag{3.6.46}$$

$$\frac{\tau_t^c}{\tau_{ss}^c} = \varepsilon_t^C \tag{3.6.47}$$

$$\tag{3.6.48}$$

where the ε_t^J for $J = K, L, C, GC, GI$follows AR(1) processes as:

$$\ln \varepsilon_t^J = \rho_J \ln \varepsilon_{t-1}^J + \eta_t^J, \quad \eta_t^J \sim \mathcal{N}(0, \sigma_J) \tag{3.6.49}$$

In particular,the shocks to taxes are believed to co-move hence the exogenous processes are expressed as

$$\begin{aligned}
\ln \varepsilon_t^K &= \rho_K \ln \varepsilon_{t-1}^K + \eta_t^K + \rho_{l,k}\eta_t^L + \rho_{k,c}\eta_t^C \\
\ln \varepsilon_t^L &= \rho_L \ln \varepsilon_{t-1}^L + \eta_t^L + \rho_{l,k}\eta_t^K + \rho_{c,l}\eta_t^C \\
\ln \varepsilon_t^C &= \rho_C \ln \varepsilon_{t-1}^C + \eta_t^C + \rho_{k,c}\eta_t^K + \rho_{c,l}\eta_t^L
\end{aligned}$$

II:Equilibrium Equations

$$\tilde{C}_t = \left[(1-\eta_g)^{\frac{1}{\psi_c}} C_t^{\frac{\psi_c-1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}} {G_t^C}^{\frac{\psi_c-1}{\psi_c}}\right]^{\frac{\psi_c}{\psi_c-1}}$$

$$U(\tilde{C}_t, L_t) = \varepsilon_t^\beta \frac{1}{1-\sigma_C}\left(\tilde{C}_t - \eta_c \tilde{C}_{t-1}\right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right)$$

$$V_t = U(\tilde{C}_t, L_t) + \beta v_t$$

$$v_t = \mathbb{E}_t(V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}$$

$$U_C(\tilde{C}_t, L_t) = (\tilde{C}_t - \eta_c \tilde{C}_{t-1})^{-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w L_t^{1+\sigma_L}\right)\left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}}$$

$$\frac{W_t^h}{P_t} = \frac{(\tilde{C}_t - \eta_c \tilde{C}_{t-1})\varepsilon_t^w L_t^{\sigma_L}(1+\tau_t^c)}{\left[(1-\eta_g)\frac{\tilde{C}_t}{C_t}\right]^{\frac{1}{\psi_c}}(1-\tau_t^l)}$$

$$\Xi_t = \varepsilon_t^\beta \frac{U_C(\tilde{C}_t, L_t)}{(1+\tau_t^c)}$$

$$\Lambda_{t,t+1} = \varepsilon_t^\beta \beta\left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}}\right]^{-\sigma_E}\frac{\Xi_{t+1}}{\Xi_t}$$

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}}\right] R_{t,t+1}^D$$

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{\pi_{t,t+1}}\left(1 + (1-\tau_{t+1}^k)(R_{t,t+1} - 1)\right)\right]$$

$$M_t = \varepsilon_t^a\left[(K_{t-1}U_t^k)^\alpha(\gamma^t H_t)^{1-\alpha}\right]^{1-\theta_G} {K_{t-1}^G}^{\theta_G}$$

$$W_t = (1-\alpha)\mathsf{MC}_t \frac{M_t}{H_t}$$

$$\mathsf{MPK}_t = \alpha \mathsf{MC}_t \frac{M_t}{K_{t-1}}$$

$$U_t^k = \left(\frac{\frac{\mathsf{MPK}_t}{Q_t}}{\frac{\mathsf{MPK}_{ss}}{Q_{ss}}}\right)^{\psi_u}$$

$$\delta_t^k = \delta + \psi_u\left(\frac{\mathsf{MPK}_t}{Q_t} - \frac{\mathsf{MPK}_{ss}}{Q_{ss}}\right)$$

$$K_t = (1-\delta_t^k)K_{t-1} + \varepsilon_t^i I_t - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2 I_t$$

$$1 = Q_t\left[\varepsilon_t^i - \psi\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - \gamma\right) - \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}\psi\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - \gamma\right)\right]$$

$$\mathbb{E}_t[\Lambda_{t,t+1}R_{t,t+1}^R] = \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{1}{\pi_{t,t+1}}\right]R_{t,t+1}^N$$

$$R^R_{t-1,t} = \frac{\mathsf{MPK}_t}{Q_{t-1}} + (1-\delta^k_t)\frac{Q_t}{Q_{t-1}}$$

$$R^N_{t,t+1} = S_{t,t+1} R_{t,t+1}$$

$$S_{t,t+1} = S_{ss}\left(\varepsilon^s_t \frac{Q_t K_t}{N_t}\frac{N_{ss}}{K_{ss}}\right)^{\psi_S}$$

$$N_t = \varphi E^s_t + (1-\varphi)(\varepsilon^N_t)$$

$$E^s_t = \left(R^R_{t-1,t} - \frac{R^N_{t-1,t}}{\pi_{t-1,t}}\right) Q_{t-1}K_{t-1} + \frac{R^N_{t-1,t}}{\pi_{t-1,t}} N_{t-1}$$

$$\frac{\tilde{P}_t}{P_t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\left(\frac{G^{p2}_t}{G^{p1}_t}\right)$$

$$G^{p1}_t = Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_{t+1}}\right)^{1-\epsilon_p} G^{p1}_{t+1}\right]$$

$$G^{p2}_t = \varepsilon^p_t \mathsf{MC}_t Y_t + \mathbb{E}_t\left[\zeta_p \Lambda_{t,t+1}\left(\frac{\pi_t^{\iota_p}\pi_{s}^{1-\iota_p}}{\pi_{t+1}}\right)^{-\epsilon_p} G^{p2}_{t+1}\right]$$

$$\frac{\tilde{P}_t(i)}{P_t} = \left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{s}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{\frac{1}{1-\epsilon_p}}$$

$$\Delta^p_t = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta^p_{t-1}$$

$$\frac{\tilde{W}_t}{W_t} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\left(\frac{G^{w2}_t}{G^{w1}_t}\right)$$

$$G^{w1}_t = W_t H_t + \mathbb{E}_t\left[\zeta_w \Lambda_{t,t+1}\left(\frac{W_t}{W_{t+1}}\right)^{1-\epsilon_w}\left(\pi_t^{l_w}\pi_{ss}^{1-l_w}\right)^{1-\epsilon_w} G^{w1}_{t+1}\right]$$

$$G^{w2}_t = W^h_t H_t + \mathbb{E}_t\left[\zeta_w \Lambda_{t,t+1}\left(\frac{W_t}{W_{t+1}}\right)^{-\epsilon_w}\left(\pi_t^{l_w}\pi_{ss}^{1-l_w}\right)^{-\epsilon_w} G^{w2}_{t+1}\right]$$

$$\frac{\tilde{W}_t(i)}{W_t} = \left[\frac{1-\zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{W_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}}$$

$$\Delta_t^w = (1-\zeta_w)\left[\frac{1-\zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{W_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}} + \zeta_w\left(\frac{W_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{W_t}\right)^{-\epsilon_w}\Delta_{t-1}^w$$

$$M_t = \Delta_t^p Y_t$$

$$L_t = \Delta_t^w H_t$$

$$Y_t = C_t + I_t + G_t^C + G_t^I$$

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi}\left(\frac{Y_t}{Y_t^f}\right)^{\psi_y}\right]^{1-\rho_R}\varepsilon_t^r$$

$$K_t^G = (1-\delta_G)K_{t-1}^G + \frac{G_t^I}{Y_t}$$

$$\frac{\tau_t^k}{\tau_{ss}^k} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_K}\left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_K}\varepsilon_t^K$$

$$\frac{\tau_t^l}{\tau_{ss}^l} = \left(\frac{Y_t}{Y_{ss}}\right)^{\psi_L}\left(\frac{D_{t-1}}{D_{ss}}\right)^{\omega_L}\varepsilon_t^L$$

$$\frac{Z_t}{Z_{ss}} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_Z}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_Z}\varepsilon_t^T$$

$$\frac{G_t^C}{G_{ss}^C} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GC}}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GC}}\varepsilon_t^{GC}$$

$$\frac{G_t^I}{G_{ss}^I} = \left(\frac{Y_t}{Y_{ss}}\right)^{-\psi_{GI}}\left(\frac{D_{t-1}}{D_{ss}}\right)^{-\omega_{GI}}\varepsilon_t^{GI}$$

$$\frac{\tau_t^c}{\tau_{ss}^C} = \varepsilon_t^C$$

$$D_t = G_t^C + G_t^I + Z_t + \frac{R_{t-1}^D D_{t-1}}{\pi_t} - \tau_t^c C_t - \tau_t^l W_t L_t - \tau_t^k\left((R_{t-1}-1)\frac{B_{t-1}}{\pi_t} - \varphi N_{t-1}\right)$$

III:Detrending

The variables that are originally in capital letter, the detrended version will be denoted by the small letter. This process is carried out using the deterministic growth rate γ

$k_t = \frac{K_t}{\gamma^t}$, $w_t = \frac{W_t}{\gamma^t P_t}$, $q_t = \frac{Q_t}{P_t}$, $\bar{\beta} = \beta\gamma^{-\sigma_c t}$

$mc_t = \frac{MC_t}{P_t}$, $\bar{V}_t = \frac{V_t}{\gamma^{t(1-\sigma_C)}}$

Equations are detrended in accordance to how they appeared in the listing of equilibrium equations

$$\tilde{c}_t = \left[(1-\eta_g)^{\frac{1}{\psi_c}} {c_t}^{\frac{\psi_c-1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}} {g_t^C}^{\frac{\psi_c-1}{\psi_c}}\right]^{\frac{\psi_c}{\psi_c-1}}$$

$$U(\tilde{c}_t, l_t) = \varepsilon_t^\beta \frac{1}{1-\sigma_C}\left(\tilde{c}_t - \eta_c \frac{\tilde{c}_{t-1}}{\gamma}\right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w l_t^{1+\sigma_L}\right)$$

$$\tilde{V}_t = U(\tilde{c}_t, l_t) + \beta \tilde{v}_t$$

$$\tilde{v}_t = \mathbb{E}_t(\tilde{V}_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}$$

$$U_c(\tilde{c}_t, l_t) = (\tilde{c}_t - \eta_c \frac{\tilde{c}_{t-1}}{\gamma})^{-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L}\varepsilon_t^w l_t^{1+\sigma_L}\right)\left[(1-\eta_g)\frac{\tilde{c}_t}{c_t}\right]^{\frac{1}{\psi_c}}$$

$$w_t^h = \frac{(\tilde{c}_t - \eta_c \frac{\tilde{c}_{t-1}}{\gamma})\varepsilon_t^w l_t^{\sigma_L}(1+\tau_t^c)}{\left[(1-\eta_g)\frac{\tilde{c}_t}{c_t}\right]^{\frac{1}{\psi_c}}(1-\tau_t^l)}$$

$$\xi_t = \varepsilon_t^\beta \frac{U_c(\tilde{c}_t, l_t)}{(1+\tau_t^c)}$$

$$\lambda_{t,t+1} = \varepsilon_t^\beta \beta \left[\frac{\tilde{V}_{t+1}}{(\mathbb{E}_t \tilde{V}_{t+1}^{1-\sigma_E})^{\frac{1}{1-\sigma_E}}}\right]^{-\sigma_E} \frac{\xi_{t+1}}{\xi_t}$$

$$\frac{1}{R_{t+1}} = \mathbb{E}_t\left[\frac{\lambda_{t,t+1}}{\pi_{t+1}}\right]$$

$$m_t = \varepsilon_t^a \left[\left(\frac{k_{t-1}}{\gamma}u_t^k\right)^\alpha {h_t}^{1-\alpha}\right]^{1-\theta_G}\left(\frac{k_{t-1}^G}{\gamma}\right)^{\theta_G}$$

$$w_t = (1-\alpha)\mathsf{mc}_t \frac{m_t}{h_t}$$

$$\mathsf{mpk}_t = \alpha \mathsf{mc}_t \frac{\gamma m_t}{k_{t-1}}$$

$$u_t^k = \left(\frac{\frac{\mathsf{mpk}_t}{q_t}}{\frac{\mathsf{mpk}_{ss}}{q_{ss}}}\right)^{\psi_u}$$

$$\delta_t^k = \delta + \psi_u\left(\frac{\mathsf{mpk}_t}{q_t} - \frac{\mathsf{mpk}_{ss}}{q_{ss}}\right)$$

$$k_t = (1-\delta_t^k)\frac{k_{t-1}}{\gamma} + \varepsilon_t^i i_t - \frac{\psi}{2}\left(\frac{\gamma i_t}{i_{t-1}} - \gamma\right)^2 i_t$$

$$1 = q_t\left[\varepsilon_t^i - \psi\frac{\gamma i_t}{i_{t-1}}\left(\frac{\gamma i_t}{i_{t-1}} - \gamma\right) - \frac{\psi}{2}\left(\frac{\gamma i_t}{i_{t-1}} - \gamma\right)^2\right] + \mathbb{E}_t\left[\lambda_{t,t+1} q_{t+1}\psi\left(\frac{\gamma i_{t+1}}{i_t}\right)^2\left(\frac{\gamma i_{t+1}}{i_t} - \gamma\right)\right]$$

$$\mathbb{E}_t[\lambda_{t,t+1}R_{t,t+1}^R] = \mathbb{E}_t\left[\lambda_{t,t+1}\frac{1}{\pi_{t,t+1}}\right]R_{t,t+1}^N$$

$$R_{t-1,t}^R = \frac{\mathsf{mpk}_t}{q_{t-1}} + (1-\delta_t^k)\frac{q_t}{q_{t-1}}$$

$$R_{t,t+1}^N = s_{t,t+1}R_{t,t+1}$$

$$s_{t,t+1} = s_{ss}\left(\varepsilon_t^s \frac{q_t k_t}{n_t}\frac{n_{ss}}{k_{ss}}\right)^{\psi_S}$$

$$n_t = \varphi E_t^s + (1-\varphi)(\varepsilon_t^N)$$

$$E_t^s = \left(R_{t-1,t}^R - \frac{R_{t-1,t}^N}{\pi_{t-1,t}}\right) q_{t-1}\frac{k_{t-1}}{\gamma} + \frac{R_{t-1,t}^N}{\pi_{t-1,t}}\frac{n_{t-1}}{\gamma}$$

$$\tilde{P}_t^{opt} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\left(\frac{g_t^{p2}}{g_t^{p1}}\right)$$

$$g_t^{p1} = y_t + \mathbb{E}_t\left[\zeta_p \lambda_{t,t+1}\left(\frac{\pi_t^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_{t+1}}\right)^{1-\epsilon_p}\gamma g_{t+1}^{p1}\right]$$

$$g_t^{p2} = \varepsilon_t^p \mathsf{mc}_t y_t + \mathbb{E}_t\left[\zeta_p \lambda_{t,t+1}\left(\frac{\pi_t^{\iota_p}\pi_s^{1-\iota_p}}{\pi_{t+1}}\right)^{-\epsilon_p} g_{t+1}^{p2}\right]$$

$$\tilde{P}_t^{opt} = \left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_s^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{\frac{1}{1-\epsilon_p}}$$

$$\Delta_t^p = (1-\zeta_p)\left[\frac{1-\zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{1-\epsilon_p}}{1-\zeta_p}\right]^{-\frac{\epsilon_p}{1-\epsilon_p}} + \zeta_p\left(\frac{\pi_{t-1}^{\iota_p}\pi_{ss}^{1-\iota_p}}{\pi_t}\right)^{-\epsilon_p}\Delta_{t-1}^p$$

$$\frac{\tilde{w}_t}{w_t} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\left(\frac{g_t^{w2}}{g_t^{w1}}\right)$$

$$g_t^{w1} = w_t h_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\left(\frac{w_t}{w_{t+1}}\right)^{1-\epsilon_w}\left(\frac{\pi_t^{l_w}\pi_{ss}^{1-l_w}}{\pi_{t+1}}\right)^{1-\epsilon_w}\gamma g_{t+1}^{w1}\right]$$

$$g_t^{w2} = w_t^h h_t + \mathbb{E}_t\left[\zeta_w \lambda_{t,t+1}\left(\frac{w_t}{w_{t+1}}\right)^{-\epsilon_w}\left(\frac{\pi_t^{l_w}\pi_{ss}^{1-l_w}}{\pi_{t+1}}\right)^{-\epsilon_w}\gamma g_{t+1}^{w2}\right]$$

$$\frac{\tilde{w}_t(i)}{w_t} = \left[\frac{1-\zeta_w\left(\frac{w_{t-1}\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{w_t\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}}$$

$$\Delta_t^w = (1-\zeta_w)\left[\frac{1-\zeta_w\left(\frac{w_{t-1}\pi_{t-1}^{\iota_w}\pi_{ss}^{1-\iota_w}}{w_t\pi_t}\right)^{1-\epsilon_w}}{1-\zeta_w}\right]^{\frac{1}{1-\epsilon_w}} + \zeta_w\left(\frac{w_{t-1}\pi_{t-1}^{l_w}\pi_{ss}^{1-1_w}}{w_t\pi_t}\right)^{-\epsilon_w}\Delta_{t-1}^w$$

$$m_t = \Delta_t^p y_t$$

$$l_t = \Delta_t^w h_t$$

$$y_t = c_t + i_t + g_t^C + g_t^I$$

$$\frac{R_t}{R_{ss}} = \left(\frac{R_{t-1}}{R_{ss}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi_{ss}}\right)^{\psi_\pi}\left(\frac{y_t}{y_t^f}\right)^{\psi_y}\right]^{1-\rho_R}\varepsilon_t^r$$

$$k_t^G = (1-\delta_G)\left(\frac{k_{t-1}^G}{\gamma}\right) + \frac{g_t^I}{y_t}$$
$$\frac{\tau_t^k}{\tau_{ss}^k} = \left(\frac{y_t}{y_{ss}}\right)^{\psi_K}\left(\frac{d_{t-1}}{d_{ss}}\right)^{\omega_K}\varepsilon_t^K$$
$$\frac{\tau_t^l}{\tau_{ss}^l} = \left(\frac{y_t}{y_{ss}}\right)^{\psi_L}\left(\frac{d_{t-1}}{d_{ss}}\right)^{\omega_L}\varepsilon_t^L$$
$$\frac{z_t}{z_{ss}} = \left(\frac{y_t}{y_{ss}}\right)^{-\psi_Z}\left(\frac{d_{t-1}}{d_{ss}}\right)^{-\omega_Z}\varepsilon_t^T$$
$$\frac{g_t^C}{g_{ss}^C} = \left(\frac{y_t}{y_{ss}}\right)^{-\psi_{GC}}\left(\frac{d_{t-1}}{d_{ss}}\right)^{-\omega_{GC}}\varepsilon_t^{GC}$$
$$\frac{g_t^I}{g_{ss}^I} = \left(\frac{y_t}{y_{ss}}\right)^{-\psi_{GI}}\left(\frac{d_{t-1}}{d_{ss}}\right)^{-\omega_{GI}}\varepsilon_t^{GI}$$
$$\frac{\tau_t^c}{\tau_{ss}^C} = \varepsilon_t^C$$
$$d_t = g_t^C + g_t^I + z_t + \frac{R_{t-1}^D d_{t-1}}{\pi_t} - \tau_t^c C_t - \tau_t^l w_t L_t - \tau_t^k\left((R_{t-1}-1)\frac{b_{t-1}}{\pi_t} - \varphi n_{t-1}\right)$$

Exogenous Processes

The following are the exogenous process

$$\ln\varepsilon_t^\beta = \rho_\beta \ln\varepsilon_{t-1}^\beta + \eta_t^\beta, \quad \eta_t^\beta \sim \mathcal{N}(0,\sigma_\beta) \tag{3.6.50}$$
$$\ln\varepsilon_t^p = \rho_p \ln\epsilon_{t-1}^p + \eta_t^p, \quad \eta_t^p \sim \mathcal{N}(0,\sigma_p) \tag{3.6.51}$$
$$\ln\varepsilon_t^a = \rho_z \ln\epsilon_{t-1}^a + \eta_t^a, \quad \eta_t^a \sim \mathcal{N}(0,\sigma_a) \tag{3.6.52}$$
$$\ln\varepsilon_t^w = \rho_w \ln\varepsilon_{t-1}^w + \eta_t^w \quad \eta_t^w \sim \mathcal{N}(0,\sigma_w) \tag{3.6.53}$$
$$\ln\varepsilon_t^i = \rho_i \ln\varepsilon_{t-1}^i + \eta_t^i, \quad \eta_t^i \sim \mathcal{N}(0,\sigma_i) \tag{3.6.54}$$
$$\ln\varepsilon_t^r = \rho_r \ln\varepsilon_{t-1}^r + \eta_t^r, \quad \eta_t^r \sim \mathcal{N}(0,\sigma_r) \tag{3.6.55}$$
$$\ln\varepsilon_t^s = \rho_s \ln\varepsilon_{t-1}^s + \eta_t^s, \quad \eta_t^s \sim \mathcal{N}(0,\sigma_s) \tag{3.6.56}$$
$$\ln\varepsilon_t^n = \rho_n \ln\varepsilon_{t-1}^n + \eta_t^n, \quad \eta_t^n \sim \mathcal{N}(0,\sigma_n) \tag{3.6.57}$$
$$\tag{3.6.58}$$

IV:Steady State

$$\pi_{ss} = \text{given} \qquad \text{from data}$$
$$\frac{g^C_{ss}}{y_{ss}} = \text{given} \qquad \text{from data}$$
$$\frac{g^I_{ss}}{y_{ss}} = \text{given} \qquad \text{from data}$$
$$s_{ss} = \frac{b_{ss}}{y_{ss}} \qquad \text{from data}$$
$$\tau^c_{ss} = \text{given} \qquad \text{from data}$$
$$\tau^l_{ss} = \text{given} \qquad \text{from data}$$
$$\tau^k_{ss} = \text{given} \qquad \text{from data}$$
$$s_{ss} = \text{given} \qquad \text{from data}$$
$$\lambda_{ss} = \beta \qquad \text{from EQ2}$$
$$R_{ss} = \frac{\pi_{ss}}{\beta} \qquad \text{from EQ1}$$
$$q_{ss} = 1 \qquad \text{from EQ12}$$
$$u^k_{ss} = 1 \qquad \text{from EQ9}$$
$$\Delta^p_{ss} = 1 \qquad \text{from EQ23}$$
$$\Delta^w_{ss} = 1 \qquad \text{from EQ28}$$
$$\delta^k_{ss} = \delta \qquad \text{from EQ10}$$
$$R^N_{ss} = s_{ss} R_{ss} \qquad \text{from EQ15}$$
$$R^R_{ss} = \frac{R^N_{ss}}{\pi_{ss}} \qquad \text{from EQ13}$$
$$\text{mc}_{ss} = \frac{\epsilon_p - 1}{\epsilon_p} \qquad \text{from EQ19}$$
$$\text{mpk}_{ss} = R^R_{ss} - (1-\delta) \qquad \text{from EQ14}$$
$$n_{ss} = \frac{1-\varphi}{1-\varphi\frac{R^R_{ss}}{\gamma}} \qquad \text{from EQ17}$$
$$E_{ss} = R^R_{ss}\frac{n_{ss}}{\gamma} \qquad \text{from EQ18}$$
$$k^G_{ss} = \frac{g^I_{ss}}{\delta_G y_{ss}} \qquad \text{from EQ34}$$

Auxiliary steady-state variables

$$\frac{i_{ss}}{k_{ss}} = \frac{\gamma - 1 + \delta}{\gamma} \qquad \text{from EQ11}$$

$$\frac{m_{ss}}{k_{ss}} = \frac{\mathsf{mpk}_{ss}}{\alpha\gamma\mathsf{mc}_{ss}} \qquad \text{from EQ8}$$

$$\frac{l_{ss}}{k_{ss}} = \left(\gamma^{\alpha}\frac{m_{ss}}{k_{ss}}\left(\frac{\gamma}{k_{ss}^{G}}\right)^{\theta_G}\right)^{\frac{1}{1-\alpha}} \qquad \text{from EQ6}$$

$$\frac{c_{ss}}{k_{ss}} = \frac{m_{ss}}{k_{ss}}\left(1 - \frac{g_{ss}^{I}}{y_{ss}} - \frac{g_{ss}^{C}}{y_{ss}}\right) - \frac{i_{ss}}{k_{ss}} \qquad \text{from EQ24}$$

$$\frac{g_{ss}^{C}}{k_{ss}} = \left(\frac{g_{ss}^{C}}{y_{ss}}\right)\left(\frac{m_{ss}}{k_{ss}}\right)$$

$$\frac{\tilde{c}_{ss}}{k_{ss}} = \left[(1-\eta_g)^{\frac{1}{\psi_c}}\left(\frac{c_{ss}}{k_{ss}}\right)^{\frac{\psi_c - 1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}}\left(\frac{g_{ss}^{C}}{k_{ss}}\right)\right]^{\frac{\psi_c}{\psi_c - 1}}$$

$$w_{ss} = (1-\alpha)\mathsf{mc}_{ss}\left(\frac{m_{ss}}{k_{ss}}\right)\left(\frac{k_{ss}}{l_{ss}}\right) \qquad \text{from EQ7}$$

$$\tilde{w}_{ss} = w_{ss} \qquad \text{from EQ27}$$

$$w_{ss}^{h} = \left(\frac{\epsilon_w - 1}{\epsilon_w}\right)\tilde{w}_{ss} \qquad \text{from EQ24}$$

$$l_{ss} = \left[\left(\frac{\epsilon_w - 1}{\epsilon_w}\right)\left(\frac{1-\tau_{ss}^{l}}{1+\tau_{ss}^{c}}\right)\left(\frac{\gamma}{\gamma - \eta_c}\right)\left(\frac{l_{ss}}{k_{ss}} \times \frac{k_{ss}}{\tilde{c}_{ss}}\right)\left(\frac{\tilde{c}_{ss}}{k_{ss}} \times \frac{c_{ss}}{k_{ss}}\right)^{\frac{1}{\psi_c}}(1-\eta_g)^{\frac{1}{\psi_c}} w_{ss}\right]^{\frac{1}{\sigma_L + 1}}$$

$$h_{ss} = l_{ss}$$

$$k_{ss} = l_{ss}\left(\frac{k_{ss}}{l_{ss}}\right)$$

$$i_{ss} = k_{ss}\left(\frac{i_{ss}}{k_{ss}}\right)$$

$$m_{ss} = k_{ss}\left(\frac{m_{ss}}{k_{ss}}\right)$$

$$y_{ss} = m_{ss}$$

$$c_{ss} = k_{ss}\left(\frac{c_{ss}}{k_{ss}}\right)$$

$$g_{ss}^{C} = y_{ss}\left(\frac{g_{ss}^{C}}{y_{ss}}\right)$$

$$g_{ss}^{I} = y_{ss}\left(\frac{g_{ss}^{I}}{y_{ss}}\right)$$

$$b_{ss} = y_{ss} s_{ss}$$

$$g_{ss}^{p1} = \frac{y_{ss}}{1 - \zeta_p \gamma} \qquad \text{from EQ20}$$

$$g_{ss}^{p2} = \frac{\mathsf{mc}_{ss} y_{ss}}{1 - \zeta_p \gamma} \qquad \text{from EQ21}$$

$$g_{ss}^{w1} = \frac{w_{ss} h_{ss}}{1 - \zeta_p \gamma} \qquad \text{from EQ25}$$

$$g_{ss}^{w2} = \frac{w_{ss}^{h} h_{ss}}{1 - \zeta_p \gamma} \qquad \text{from EQ26}$$

$$\xi_{ss} = \left(c_{ss} - \frac{\eta_c}{\gamma} c_{ss} \right)^{-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1 + \sigma_L} L_{ss}^{1+\sigma_L} \right) \left(\frac{1}{(1 + \tau_{ss}^{c})} \right) \left[(1 - \eta_g) \frac{\tilde{c}_t}{c_t} \right]^{\frac{1}{\psi_c}} \qquad \text{from EQ2}$$

$$\bar{V}_{ss} = \frac{\frac{1}{1-\sigma_C} \left(c_{ss} - \eta_c c_{ss} \right)^{1-\sigma_C} \exp\left(\frac{\sigma_C - 1}{1+\sigma_L} l_{ss}^{1+\sigma_L} \right)}{1 - \beta \gamma} \qquad \text{from EQ5a}$$

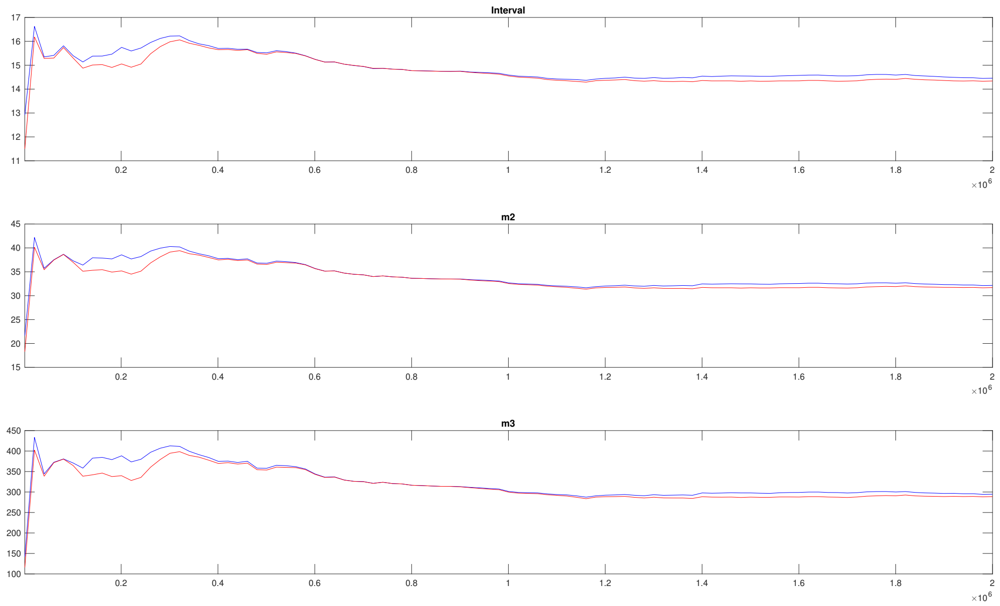
$$\bar{v}_{ss} = \gamma \bar{V}_{ss} \qquad \text{from EQ5b}$$

$$\tilde{c}_{ss} = \left[(1 - \eta_g)^{\frac{1}{\psi_c}} c_{ss}^{\frac{\psi_c - 1}{\psi_c}} + \eta_g^{\frac{1}{\psi_c}} g_{ss}^{C \, \frac{\psi_c - 1}{\psi_c}} \right]^{\frac{\psi_c}{\psi_c - 1}}$$

$$z_{ss} = \tau_{ss}^{c} c_{ss} + \tau_{ss}^{l} w_{ss} l_{ss} + \tau_{ss}^{k} \varphi \frac{n_{ss}}{\gamma} + b_{ss} - g_{ss}^{C} - g_{ss}^{I} - \frac{R_{ss} b_{ss}}{\pi_{ss}} \qquad \text{from EQ33}$$

V: Estimation

Figure 3.11: Multivariate Convergence Diagnostics for 2,000,000 Draws



4. Conclusion

This thesis examined the information contained in corporate bond yields relevant to enacting simple optimal monetary and fiscal policies in a three-part analysis. A medium-scale DSGE model with financial friction was constructed to examine the role of the information contained in corporate bond yields in explaining macroeconomic and financial outcomes. An important feature of the model is the straightforward link between bond (government and corporate) prices and yields in the model with data. The corporate bond spread model-data linkage is similarly straightforward, hence giving room for a comparative analysis of the model performance. The model is solved using the perturbation method, and it is estimated using the Bayesian method. In estimating the model, macroeconomic and bond yield data were used in the first and second chapters, while macroeconomic and fiscal (federal government) data were used in the third chapter.

Interestingly, the simulated corporate bond spread is sizeable, and the magnitude of its volatility is substantially consistent with the observations in the data, even though corporate bond yields are the only additional observations in the estimation beyond the seven macroeconomic data of Smets and Wouters (2007). These results suggest that to design an optimal policy response to enhance macroeconomic and financial stability, policy makers should utilise the information from corporate bond yields and other financial indicators, such as leverage, the external finance premium, and net worth.

Based on the results obtained in the first chapter, the second chapter further investigates the usefulness of the information content of financial indicators for the conduct of optimal monetary policy. The analysis extended the traditional Taylor rule of inflation and output targeting to include financial indicators as additional policy instruments. In this context, an optimal, simple and implementable monetary policy that maximises household welfare relative to a flexible price and frictionless economy is solved numerically (Schmitt-Grohe and Uribe, 2007). The results generated by this analysis show that the inclusion of financial stress indicators, such as corporate bond spread, alters monetary policy in a significant way. This suggests the easing of monetary policy is the optimum response to increasing corporate bond spread because the indicator is a reflection of distortion in the supply of credit which potentially can lead to reduced productivity and increased unemployment. Notably, the easing of monetary policy, as suggested by the inclusion of corporate bond spread, improves welfare as well as reduces the volatility of inflation by almost half. Also, the result indicates that the central bank should be flexible enough to consider adjusting the policy rate to reflect current financial conditions in addition to its inflation-targeting objective. Similar to the result obtained in Schmitt-Grohe and Uribe (2007), the result also shows that targeting output-gap is welfare detrimental as it makes inflation more volatile.

The third part of this study demonstrated the implication of joint fiscal and monetary policies on macroeconomic and financial outcomes. To achieve this, it offers an understanding of the multiplier effects of fiscal instruments over a short and long horizon by evaluating its present value multiplier (as in Mountford and Uhlig (2009)) on output, consumption, and investment. In addition, it assesses how the financial friction mechanism alters the transmission of fiscal policy. The results obtained support the joint coordination of monetary and fiscal policies in stabilising the real economy and managing public debt.

Government spending is decomposed into consumption and investment component, both of which are assumed to be productive, to examine the extent productive government spending impacts optimal allocation and alters the financial friction mechanism. For example, government consumption is reflected in household utility, and government investment improves the marginal productivity of capital through the stock of productive public capital. This study showed that government investment generates substantial short and long-run output and investment multipliers that are not otherwise captured by most New Keynesian models. Government investment not only generates significant multipliers it also reduces the cost of sourcing external finance. Consistent with Ramey (2020), this study confirms that long-run multipliers can be sizeable once government capital is productive.

Fundamentally, this thesis supports the view that corporate bond yields contain vital business cycle information, which, when given consideration, can facilitate proactive policy making. The analysis establishes the position that corporate bond spread is a key indicator, signalling financial imbalances that could potentially propagate a recession. It, therefore, serves as a useful tool for better stabilising the macroeconomy. In contrast to the idea that the gain from targeting asset prices vanishes when monetary policy becomes more anti-inflationary, as in Faia and Monacelli (2007), this thesis shows that strong inflation targeting does not attenuate the benefit from targeting financial stress indicators such as the corporate bond spread. Overall, it confirms that financial indicators such as corporate bond spread are useful monetary policy tools that can enhance macroeconomic and financial stability. The importance of coordination between monetary and fiscal policies has also been identified.

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