VALIDATING INTRA-DAY RISK PREMIUM IN CROSS-SECTIONAL RETURN CURVES

Abstract. This paper investigates the cross-sectional asset pricing for intra-day return curves. By introducing a functional Fama-MacBeth regression approach, the validation of the intra-day risk premium associated with the Fama-French Carhart factors is examined. The empirical evidence reveals that these common risk factors show weak explainability to the entire cross-sectional intra-day returns, despite significant risk premiums that are discovered in specific half-hour time-spans in bullish sentiment.

JEL Classification: C12, C52, C58, G12, G14

Keywords: Cross-sectional asset pricing, Intra-day return curves, Fama-MacBeth regression, Factor model, Risk premium.

1. Introduction

Due to the availability of large datasets, the systemic comovement in individual stocks has been studied in a finer resolution. Consequently, finding an appropriate multi-factor model has become one of the most controversial topics in empirical asset pricing. Related works are recently discussed from two aspects: dealing with the explosion of risk factors (Harvey et al., 2016) and tackling the challenges of presenting high-frequency information (Pelger, 2020). In this paper, we focus on the latter topic and verify systemic risk factors in cross-sectional intra-day returns.

The exposures derived from factor models can change during the observation period. This is well known for studying the risk premium of cross-sectional returns at a daily or lower frequency, while it is also shown to be the case at an intra-day high-frequency level. High-frequency information has been considered in this scope of literature, for example, incorporating intra-day systemic trading and institutional fund flows to better understand the predictability of daily cross-sectional stock returns (Heston et al., 2010), or calibrating more efficient daily estimators by aggregating risk exposures obtained at intra-day intervals (Li et al., 2017). However, studying the time-varying feature of intra-day factor exposures itself receives less attention. Andersen et al. (2020) empirically evidenced significant intra-day variation in market risk exposures, i.e., betas. This systematic variation can be explained under their conjecture by different responses of intra-day stock returns on market shocks. Thus, estimating time-varying intra-day risk exposures is crucial to identify the source of shocks. Traditionally, we could use rolling-window and filter-type approaches to obtain time-varying betas, e.g., Adrian and Franzoni (2009). Extending these methods to develop time-varying exposures at a high-frequency level is challenging, though, so the demand of practitioners who mainly work with such data cannot meet.

This difficulty is easily overcome by analysing curve data, as modelling intra-day return curves automatically produces time-varying coefficients at an intra-day level. Different from discrete observations, the intra-day return curves well preserve the intra-day movement patterns. Studying the cross-sectional risk premium of these curves provides the implications of the systemic risk factors at an intra-day level. Moreover, the methodology of curve data modelling is firmly grounded in the theory of functional data analysis (FDA) (Ramsay and Silverman, 2005; Horváth and Kokoszka 2012). Proliferations can be seen to use this statistical tool in several recent studies in finance for finding curve-type risk factors and deriving time-varying risk exposures, e.g., Kokoszka et al. (2018), Cao et al. (2020), Horváth et al. (2020), and Nadler and Sancetta (2020).

Such a method allow us to obtain time-vary factor exposures in a high-frequency context, but the controversial question remains and is now transplanted to finding "right" curve-type risk factors. We, therefore, try to fulfil these gaps and study the validation of the risk premium for curve-type common risk factors, providing a way to identify the source of systemic variation of cross-sectional intra-day stock returns.

This paper considers cross-sectional asset pricing in an FDA-preferred high-frequency environment. Our model framework provides time-varying (or curve-type) factor exposures and risk premiums over the intra-day interval. A functional Fama-MacBeth regression is introduced to investigate validities of curve-type common risk factors. Based on a high-dimensional intra-day return data set ranged from 2004 to 2016, we test the validity of intra-day risk premiums associated with the Fama-French and Carhart factors (Fama and French, 1993; Carhart 1997). Estimating the model in annual sub-samples, we find weak evidence that these factors explain cross-sectional intra-day return curves over the entire intra-day window, despite some explanations indicated in specific trading hours of intra-day intervals in the bull market. Our contributions can be summarised two-fold: 1) we propose a novel method to test the significance of risk premium at an intra-day level, and the results indicate the effectiveness or refusal of the corresponding risk factor; 2) we identify the weak explanatory ability of Fama-French Carhart factors to cross-sectional intra-day return curves, which bridges works to search robust risk factors in both bullish and bearish sentiment.

The remaining part is structured as follows. Section 2 describes the data. The method and empirical analysis are presented in Section3. Section 4 concludes remarks and discusses possible future work.

2. high-dimensional Intra-day return curves

Our paper uses a high-frequency return dataset at a 5-minute frequency, including the S&P 500 composites from January 2004 to December 2016, collected from the WRDS TAQ Millisecond trades database by Pelger (2020). The dataset forms a balanced panel with $N = 332$ assets over $3,273$ trading days. On each trading day, the return starts at 9:35 am and delivers 77 discrete observations. For a notational convenience, we denote the discrete high-frequency return data observed at a regular spaced grid as u_j at day t by $r_t(u_j)$, $j \in [1, J = 77]$.

To treat these dense observed data as curves, the discrete high-frequency returns $r_t(u_j)$ are smoothed into a continuous intra-day return (IDR) curve $r_t(u)$, $u \in [0, 1]$ using suitable smoothing techniques (Ramsay and Silverman, 2005). In order to account for a changing market status, we separate the entire sample into annual sub-samples, and similar treatment is also adopted in Pelger (2020) and Andersen et al. (2020). Figure 1 shows an example of stacked plots of IDR curves in 2016 for four representative assets from the energy, financial, information technology and consumer staples sectors, respectively. The figure exhibits the intra-day movements of the assets: Chevron and Bank of America experience larger intra-day variations, followed by IBM; the IDR curves of Coca-Cola display relatively less intra-day variations.

Insert Figure [1] about here.

Table 1 shows the summary statistics of the IDR curves of sample years from 2004 to 2016. We observe sensible results such as that the standard derivations of intra-day return curves are high during the financial crisis period of 2008-2009. The definition of related statistics can be referred to Appendix A. Moreover, we test the property for each of IDR curves by using a series of recently developed hypothesis tests (Horváth et al., 2014; Kokoszka et al., 2017; Górecki et al., 2018; Rice et al., 2020a). The results indicate that most IDR curves exhibit stationary, serially uncorrelated, conditional heteroscedastic, and non-Gaussian distributed sequences, which roughly satisfy the condition of market efficient hypothesis. We skip the P-values for each of the tests on each asset to save some space.

Insert Table [1] about here.

In addition, the dataset extends market excess return, size (smb), value (hml), momentum factors (Fama and French, 1993; Carhart, 1997), and the risk-free rate into a high-frequency context, with a time window matching the S&P 500 composites. Although these factors have been extensively discussed in the literature, we are the first to consider them as intra-day curve-type risk factors. The correlations among these risk factors are analysed by calculating their sample correlation operators (Ramsay and Silverman, 2005). Figure 2 exhibits the correlation operators between the intra-day market beta and momentum, and the size and value factors. The plots show that both pairs are barely or low correlated across the intra-day trading intervals, and this pattern remains the same for other pairs of risk factors that have not been reported. Hence, the intra-day risk factors are suitable to fit a linear regression model without concern of multicollinearity.

Insert Figure [2] about here.

3. Functional Fama-MacBeth regression and empirical findings

The Fama-MacBeth (F-M) regression (Fama and MacBeth, 1973) has become a cornerstone in testing and verifying the risk premium. Their two-step regression approach provides an empirical framework to test the validation of the implication of the Capital Asset Pricing Model, and it has been extensively used in investigating other multi-factor models. Considering a scalar return panel r_t^i, $i \in [1, N]$, $t \in [1, T]$, the first step is to regress cross-sectional returns on M common risk factors $\{F_{1,t}, \dots, F_{M,t}\}$, and the time series regression is estimated for each asset i,

$$r_t^i = \beta_0^i + \beta_1^i F_{1,t} + \cdots + \beta_M^i F_{M,t} + \varepsilon_t^i.$$

This results in a sequence of factor loadings or exposures $\hat{\beta}_m^i$ for corresponding risk factors $F_{m,t}$, $m \in [1, M]$. In the second step, the F-M regression gets the risk premium coefficient γ_m^t for common factor $F_{m,t}$ by regressing the cross-section of asset returns r_i^t on the factor exposures at each time point t,

$$r_i^t = \gamma_0^t + \gamma_1^t \hat{\beta}_1^i + \cdots + \gamma_M^t \hat{\beta}_M^i + \epsilon_i^t.$$

The risk premium coefficients should statistically deviate from zero using the Newey-West corrected t statistic if the corresponding common risk factor has adequate explanatory capability.

We now adapt F-M regression into a functional data context so that the risk premium can be derived and assessed at an intra-day level. The functional F-M regression retains a two-step approach that is carried out with functional linear regression models. Although the response is the IDR curves, the risk factors as explanatory variables, do not necessarily have to be the same type of curve data; they can also be scalar observations at a daily frequency. This generalisation is useful because the information on risk factors is rarely observed at a finer intra-day level, resulting in an intractable issue for many empirical studies. As a result, we consider two versions of functional linear regression models in the first step: a concurrent function-to-function and a function-to-scalar regression. In the former case, we regress the IDR curves of ith asset on intra-day curve-type common factors $F_{m,t}(u)$, $m \in [1, M]$:

$$r_t^i(u) = \beta_0^i(u) + \beta_1^i(u) F_{1,t}(u) + \cdots + \beta_M^i(u) F_{M,t}(u) + \varepsilon_t^i(u). \tag{1}$$

Alternatively, we run the regression on daily common factors $F_{m,t}$ in the first step,

$$r_t^i(u) = \beta_0^i(u) + \beta_1^i(u) F_{1,t} + \cdots + \beta_M^i(u) F_{M,t} + \varepsilon_t^i(u), \tag{2}$$

where $\beta_m^i(u)$ describe how the IDR curves are exposed to the corresponding risk factor over the intra-day trading hours. Thus, for each risk factor, we obtain N factor exposures as the regression is estimated with each asset from $i = 1$ to N.

The second step is to regress the cross-sectional IDR curves on the factor exposures at each time step. Because either model from step one produces curve-type factor exposures $\beta_m^i(u)$, here we use a function-to-function linear regression model:

$$r_i^t(u) = \gamma_0^t(u) + \gamma_1^t(u)\beta_1^i(u) + \cdots + \gamma_1^t(u)\beta_M^i(u) + \epsilon_i^t(u). \tag{3}$$

where the curve loading $\gamma_m^t(u)$ represents the risk premium on the intra-day interval at time point t. Thus, for each risk factor, we obtain a sequence of $\{\gamma_m^1(u), \ldots, \gamma_m^T(u)\}$ by estimating the regression across the testing period from $t = 1$ to T.

The averaged risk premium curve $\gamma_{m,T}(u) = 1/T \sum_{t=1}^T \gamma_m^t(u)$, that is generated from a valid common risk factor $F_{m,t}$ or $F_{m,t}(u)$, should be significant, or statistically deviated from zero. In the hypothesis testing, we therefore aim to test the null hypothesis $\mathcal{H}_0$: $\mathbb{E}[\gamma_m^t(u)] = 0$. This can be detected by using the norm-based test $\Lambda_T = T||\gamma_{m,T}(u)||$ proposed by Rice et al. (2020b). Under $\mathcal{H}_0$, the statistic converges to a limit distribution of $\sum_{\ell=1}^{\infty} \xi_\ell \mathcal{N}_\ell^2$, where $\mathcal{N}_\ell$, $\ell > 1$ are independent and identically distributed Gaussian random variables, and ξ_ℓ, $\ell > 1$ are the eigenvalues of the covariance operator with kernel $C(u, v) = cov(\gamma_0(u), \gamma_0(v))$. The null hypothesis is rejected if the statistic Λ_T is larger than the computed critical values.

An immediate application with our dataset investigates the explanatory ability of Fama-French Carhart risk factors on cross-sectional IDR curves. Considering that the risk factors can perform variously under different market sentiments, we use the monthly S&P 500 index and detect the market status through the method proposed by Pagan and Sossounov (2003). The results indicate that the years include 2004, 2005, 2006, 2010, 2012, 2013, 2014, 2016 fall into bull markets, and only the year of 2008 fall into the bear market. This result is not surprising as Pagan and Sossounov (2003) showed that their test could detect the bearish duration about two times less than the bullish duration. The remaining annual sub-samples, i.e., 2007, 2009, 2011, and 2015 are composed of months mixed with bearish and bullish sentiments. We thereby do not consider these sub-samples in analysing the effect of the market status.

Figure 3 shows an example of the curve-type factor exposure and risk premium for the market excess return factor estimated by Model (1) and (3) with a sub-sample of 2013. This annual sub-sample is chosen as a representative because it is relatively recent and in bullish sentiment. We thus expect to observe estimated risk exposures under such a market status. From the left-hand subplot, we observe time-varying factor exposures during the intra-day interval, and their values are generally above one for all cross-sectional stocks. Also, we find from the right-hand subplot that this factor generates positive risk premiums during midday trading hours. This effect becomes rather manifest when the market is booming, i.e., in May and November 2013.

Insert Figure [3] about here.

Table 2 exhibits the P-values of the test Λ_T for factor exposures and risk premiums estimated by using intra-day and daily risk factors. The daily Fama-French Carhart factors are collected for the same sample period from the Kenneth French data library. Although most of the risk exposures in step one are significant, the results indicate that none of the considered common risk factors generate a valid risk premium to account for the cross-sectional IDR curves over the entire intra-day interval. The finding implies that unlike the cross-sections of scalar returns that can be explained by Fama-French Carhart factors, the systemic comovement of cross-sectional IDR curves is more difficult to explain or predict given the involvement of rich intra-day variations. The economic rationale behind this variation can be potentially attributed to high-frequency traders, whose activities accelerate the intra-day information reflection speed, generating a more efficient and less-predictable market.

Insert Table [2] about here.

The above test assesses the validity of risk premium over the entire intra-day interval, while some market participants working with high-frequency information may pay more attention to the systemic comovement in the intra-day time-spans. To tackle with this, we project the curve loadings $\gamma_m^t(u)$ onto a finite number of B-spline basis functions $\phi_j(u)$, $1 \leq j \leq K$ that $\gamma_m^t(u) \approx \sum_{k=1}^{K} \zeta_{m,k}^t \phi_k(u)$. The variation of the loading curve

$\gamma_m^t(u)$ is then approximated by K number of score vectors. Since the B-spline bases are linearly independent and uniform across the grids on the interval $[0, 1]$, the scores $\zeta_{m,k}^t$ implicate the risk premium for factor $F_{m,t}$ or $F_{m,t}(u)$ at the kth time-span of the intra-day interval on date t. Suppose we project an intra-day risk premium curve $\gamma_m^t(u)$ onto 13 B-spline bases and obtain 13 scalar scores. Each of these scores represents the risk premium over a half-hour time-span across the intra-day interval from 9:30am to 4:00pm, as shown in Figure 4. For instance, the score by projecting onto the first B-spline basis stands for the risk premium in the time-span of 9:30am–10:00am. Thus, the significance of risk premium at the kth intra-day time-span can be tested through the Newey-West corrected t statistic $\frac{\zeta_{m,T,k}}{\sigma_{\zeta_{m,T,k}}/\sqrt{T}}$, $k \in [1, 13]$, where $\zeta_{m,T,k}$ and $\sigma_{\zeta_{m,T,j}}$ are the sample average and heteroskedasticity and autocorrelation consistent standard deviation of the sequence $\{\zeta_{m,k}^1, \ldots, \zeta_{m,k}^T\}$, respectively.

Insert Figure [4] about here.

In the application, we test the validation of Fama-French Carhart risk factors over half-hour time-spans in the annual samples from 2004 to 2016. Figure 5 shows the proportions of the 13 annual samples that the risk factors generate valid risk premiums over intra-day time-spans. By fitting models (1) and (3), the upper subplot shows some validities of the intra-day risk factors in specific half-hour intervals. Comparatively, the lower subplot shows the results by regressing models (2) and (3) with daily risk factors, indicating that, except for the market excess return showing slight explanatory power in afternoon trading hours, the remaining factors seldom add interpretation to cross-sectional IDR curves. The benefit of using more informative curve-type factors are clear: 1) in concordance with Figure 3, the market excess return explains the cross-sections of IDR curves during the midday trading hours; 2) the hml value risk factor adds explanatory power to cross-sectional IDR curves in late-day trading hours; 3) though smb size and momentum are still weak in interpretability, the risk premiums derived from these risk factors account for the cross-sectional variation of IDR curves in one or two samples. To enhance our understanding, Figure 6 exhibits the proportions of valid risk premiums associated with intra-day risk factors in bull and bear annual sub-samples. We find these risk premiums are more inclined to be valid in a bull market, particularly during mid-day trading hours, while they become invalid in bearish sentiment. This result is consistent with conventional findings that the risk premiums generated by the market anomalies of cross-sectional stock returns often vanish during the market turmoil.

Insert Figure [5] and [6] about here.

4. Conclusions

This paper considers a classic problem of cross-sectional asset pricing but on a high-frequency data type – intra-day return curves. The intra-day risk premiums associated with Fama-French Carhart risk factors are derived and assessed under a functional Fama-MacBeth regression approach. The results indicate that these common risk factors show weak explainability to the entire cross-sectional intra-day returns, despite significant risk premiums that are discovered in half-hour time-spans in bull markets. Future work avenues include testing the common risk factors according to the industry or firm characteristics. Also, since the observed risk factors may potentially be miss-specified, one can decompose latent common risk factors through high-dimensional functional principal component analysis. Meanwhile, the function-to-function regression models can be replaced by the fully functional linear regression model described in Chapter 8 of Horváth and Kokoszka (2012), which allows us to derive an intra-day risk premium surface showing a cross intra-day effect. However, testing the significance of a surface sequence still remains an open question.

Appendix A. Definition of basic statistics for intra-day return curves

Following the standard assumptions in functional data analysis, we let the IDR curves $r_t(u)$ to be squared integrable functions drawn from a separable $L^2[0,1]$ Hilbert space. The $L^2[0,1]$ space is equipped by an inner product $\langle r_1, r_2 \rangle = \int r_1(u) r_2(u) du$ for $r_1, r_2 \in L^2[0,1]$, which leads to a norm of $||r(u)|| = [\int r^2(u)du]^{1/2}$, for $\int = \int_0^1$. We now can define the mean and standard deviation of IDR curves on the ith asset as follows:

$$\mu_{i,T}(u) = \frac{1}{T} \sum_{t=1}^{T} r_{i,t}(u), \quad 1 \le t \le T, 1 \le i \le N;$$

$$\sigma_{i,T}(u) = [\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}(u) - \mu_{i,T}(u))^2]^{1/2}.$$

Let $r_t^f(u)$ be the intra-day risk-free rate. The Sharpe ratio of IDR curves from the asset i can be derived over the intra-day interval:

$$SR_{i,T}(u) = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}(u) - r_t^f(u))}{[\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}(u) - r_t^f(u))^2]^{1/2}}. \tag{4}$$

As a preliminary analysis of the samples, we take the norm of averaged mean, standard deviation and Sharpe ratios of cross-sectional IDR curves:

$$\mu_{N,T} = ||\frac{1}{N}\sum_{i=1}^{N}\mu_{i,T}(u)||, \quad \sigma_{N,T} = ||\frac{1}{N}\sum_{i=1}^{N}\sigma_{i,T}(u)||, \quad SR_{N,T} = ||\frac{1}{N}\sum_{i=1}^{N}SR_{i,T}(u)||.$$

References

Adrian, T., Franzoni, F. (2009). Learning about beta: time-varying factor loadings, expected returns, and the conditional CAPM. *Journal of Empirical Finance*, *16*, 537–556. https://doi.org/10.1016/j.jempfin.2009.02.003.

Andersen, T. G., Thyrsgaard, M., Todorov, V. (2020). Recalcitrant Betas: Intraday Variation in the Cross-Sectional Dispersion of Systematic Risk. *Quantitative Economics, Forthcoming.*

BarndorffNielsen, O. E., Shephard, N. (2004). Econometric analysis of realized covariation: High frequency based covariance, regression, and correlation in financial economics. *Econometrica*, *72*, 885–925.

Cao, R., Horváth, L., Liu, Z., Zhao, Y. (2020). A study of data-driven momentum and disposition effects in the Chinese stock market by functional data analysis. *Review of Quantitative Finance and Accounting*, *54*, 335–358. https://doi.org/10.1007/s11156-019-00791-x.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*, 57–82. https://doi.org/10.2307/2329556.

Fama, E. F., French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*, 3–56. https://doi.org/10.1016/0304-405X(93)90023-5.

Fama, E. F., MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of political economy*, *81*, 607–636. https://www.jstor.org/stable/1831028.

Górecki, T., Hörmann, S., Horváth, L., Kokoszka, P. (2018). Testing normality of functional time series. *Journal of time series analysis*, *39*, 471–487. https://doi.org/10.1111/jtsa.12281.

Heston, S. L., Korajczyk, R. A., Sadka, R. (2010). Intraday patterns in the crosssection of stock returns. *The Journal of Finance*, *65*, 1369–1407. https://doi.org/10.1111/j.1540-6261.2010.01573.x

Nadler, P., Sancetta, A. (2020). Empirical asset pricing with functional factors. *Available at SSRN 3518236.*

Harvey, C. R., Liu, Y., Zhu, H. (2016). ...and the cross-section of expected returns. *The Review of Financial Studies*, *29*, 5–68.

Horváth, L., Kokoszka, P. (2012). *Inference for functional data with applications*. Springer.

Horváth, L., Kokoszka, P., Rice, G. (2014). Testing stationarity of functional time series. *Journal of Econometrics*, *179*, 66–82. https://doi.org/10.1016/j.jeconom.2013.11.002.

Horváth, L., Li, B., Li, H., Liu, Z. (2020). Time-varying beta in functional factor models: Evidence from China. *The North American Journal of Economics and Finance*, *54*, 101283. https://doi.org/10.1016/j.najef.2020.101283.

Kokoszka, P., Miao, H., Reimherr, M., Taoufik, B. (2018). Dynamic functional regression with application to the cross-section of returns. *Journal of Financial Econometrics*, *16*, 461–485. https://doi.org/10.1093/jjfinec/nbx027.

Kokoszka, P., Rice, R., Shang, H. L. (2017). Inference for the autocovariance of a functional time series under conditional heteroscedasticity. *Journal of Multivariate Analysis*, *162*, 32–50. https://doi.org/10.1016/j.jmva.2017.08.004.

Li, J., Todorov, V., Tauchen, G. (2017). Jump regressions. *Econometrica*, *85*, 173–195. https://doi.org/10.3982/ECTA12962

Pagan, A. R., Sossounov, K. A. (2003). A simple framework for analysing bull and bear markets. *Journal of applied econometrics*, *18*, 23–46. https://doi.org/10.1002/jae.664.

Pelger, M. (2020). Understanding systematic risk – a highfrequency approach. *The Journal of Finance*. https://doi.org/10.1111/jofi.12898.

Ramsay, J. O., Silverman, B. W. (2005). *Functional Data Analysis.* Wiley Online Library.

Rice, G., Wirjanto, T., Zhao, Y. (2020a) Tests for conditional heteroscedasticity of functional data. *Journal of Time Series Analysis*, , *41*, 733–758. https://doi.org/10.1111/jtsa.12532

Rice, G., Wirjanto, T., Zhao, Y. (2020b) Forecasting value at risk with intra-day return curves. *International Journal of Forecasting*, *36*, 1023–1038. https://doi.org/10.1016/j.ijforecast.2019.10.006.

Tables

Table 1. Summary statistics of the cross-sectional IDR curves, with μ_{max} and μ_{min} presenting the maximum and minimum expected return across N assets.

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
$\mu_{N,T}$	1.16e-04	1.10e-04	1.08e-04	1.22e-04	2.55e-04	2.15e-04	1.22e-04	1.32e-04	1.04e-04	9.70e-05	9.29e-05	1.06e-04	1.12e-04
μ_{max}	3.46e-04	3.06e-04	2.51e-04	5.14e-04	8.23e-04	8.17e-04	3.21e-04	3.86e-04	3.89e-04	2.84e-04	2.46e-04	1.21e-03	4.89e-04
μ_{min}	4.90e-05	5.19e-05	4.60e-05	5.94e-05	1.14e-04	7.53e-05	5.62e-05	5.78e-05	4.68e-05	4.76e-05	4.45e-05	5.16e-05	5.00e-05
$\sigma_{N,T}$	1.71e-03	1.61e-03	1.60e-03	1.87e-03	3.91e-03	3.13e-03	1.90e-03	2.02e-03	1.56e-03	1.37e-03	1.36e-03	1.66e-03	1.71e-03
$SR_{N,T}$	2.92e-02	3.24e-02	3.38e-02	3.35e-02	4.25e-02	4.52e-02	3.99e-02	4.52e-02	3.81e-02	3.88e-02	4.00e-02	3.50e-02	3.74e-02

Table 2. P-values of Λ_T test of factor exposures and risk premiums estimated by using models (1), (2) and (3) with daily and intra-day risk factors, respectively.

Panel A: Intra-day Risk Factor $F_{m,t}(u)$													
Step 1: $\beta^i_m(u)$ in Model (1)													
	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Market return	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMB	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HML	0.00	0.00	0.18	0.48	0.00	0.00	0.05	0.31	0.00	0.00	0.00	0.00	0.00
Momentum	0.00	0.05	0.47	0.00	0.00	0.43	0.00	0.21	0.15	0.00	0.00	0.49	0.00
Step 2: $\gamma^t_m(u)$ in Model (3)													
Market return	0.39	0.45	0.47	0.45	0.50	0.51	0.51	0.47	0.49	0.49	0.47	0.49	0.50
SMB	0.50	0.52	0.52	0.50	0.42	0.43	0.49	0.50	0.48	0.49	0.50	0.47	0.50
HML	0.47	0.46	0.50	0.47	0.50	0.49	0.49	0.42	0.49	0.59	0.59	0.59	0.60
Momentum	0.48	0.47	0.29	0.23	0.26	0.47	0.51	0.49	0.61	0.46	0.50	0.49	0.43
Panel B: Daily Risk Factor $F_{m,t}$													
Step 1: $\beta^i_m(u)$ in Model (2)													
Market return	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMB	0.39	0.09	0.00	0.00	0.00	0.00	0.11	0.00	0.05	0.00	0.00	0.02	0.00
HML	0.48	0.50	0.29	0.45	0.09	0.00	0.00	0.48	0.00	0.40	0.47	0.03	0.47
Momentum	0.15	0.45	0.42	0.07	0.26	0.28	0.00	0.49	0.10	0.28	0.50	0.00	0.33
Step 2: $\gamma^t_m(u)$ in Model (3)													
Market return	0.43	0.48	0.50	0.50	0.50	0.51	0.50	0.49	0.47	0.50	0.41	0.39	0.50
SMB	0.44	0.49	0.47	0.43	0.51	0.50	0.50	0.50	0.47	0.42	0.49	0.46	0.49
HML	0.51	0.47	0.49	0.50	0.49	0.49	0.52	0.52	0.49	0.47	0.49	0.41	0.46
Momentum	0.27	0.48	0.47	0.36	0.47	0.40	0.49	0.50	0.50	0.49	0.49	0.45	0.45

Figures

Figure 1. Plots of IDR curves of four representative assets in 2016.

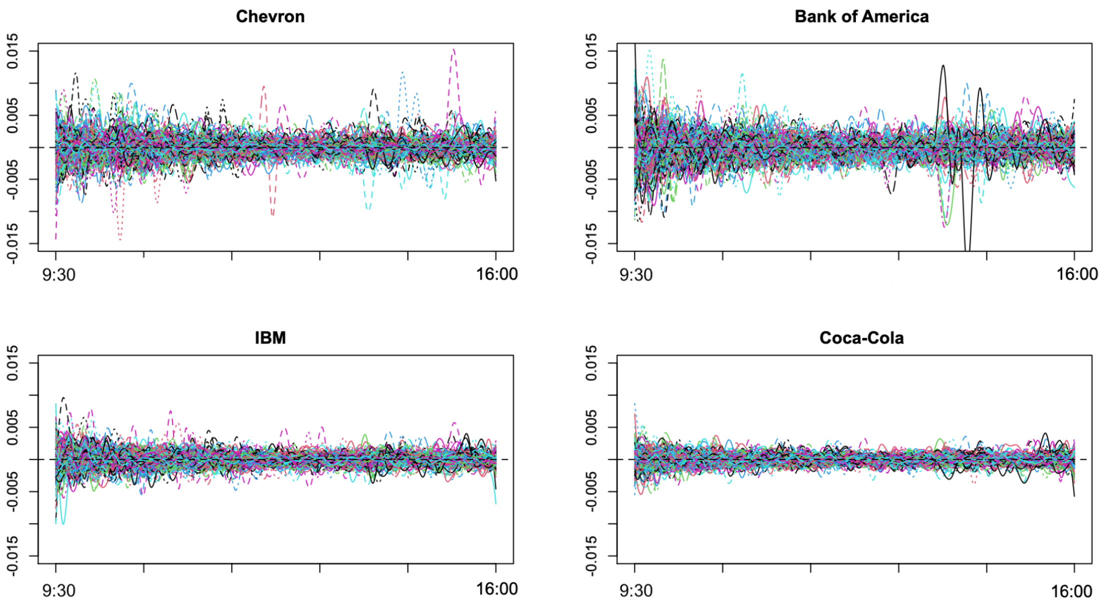


Figure 2. Correlation operators between market excess return and momentum, and between SMB and HML intra-day factors, with x/y-axis representing intra-day interval from 9:30 am to 4:00 pm.

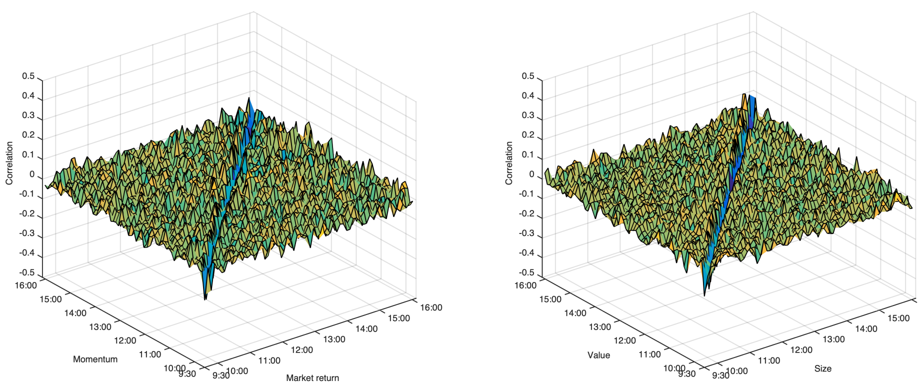


Figure 3. Intra-day factor exposure and risk premium of Market excess return estimated from models (1) and (3) with a sample of 2013.

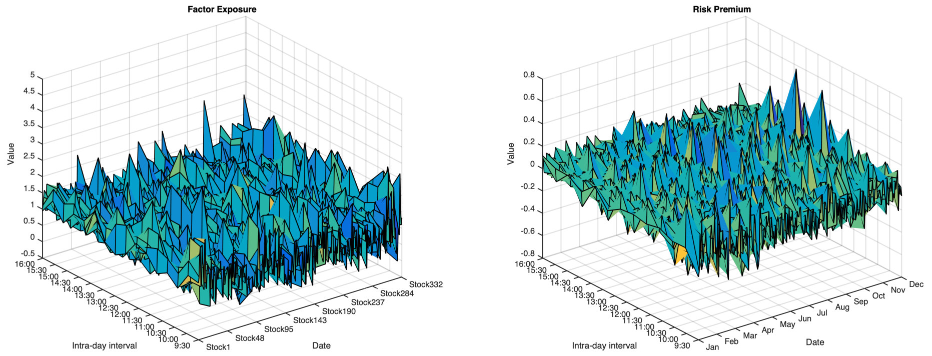


Figure 4. B-spline bases functions for projecting intra-day risk premium onto intervals.

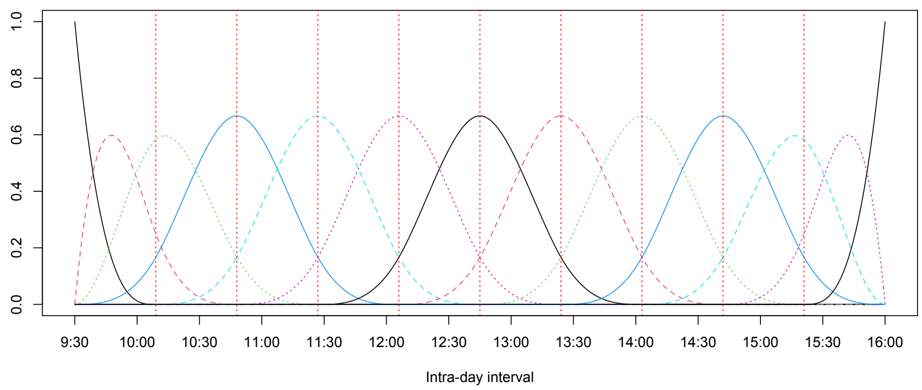
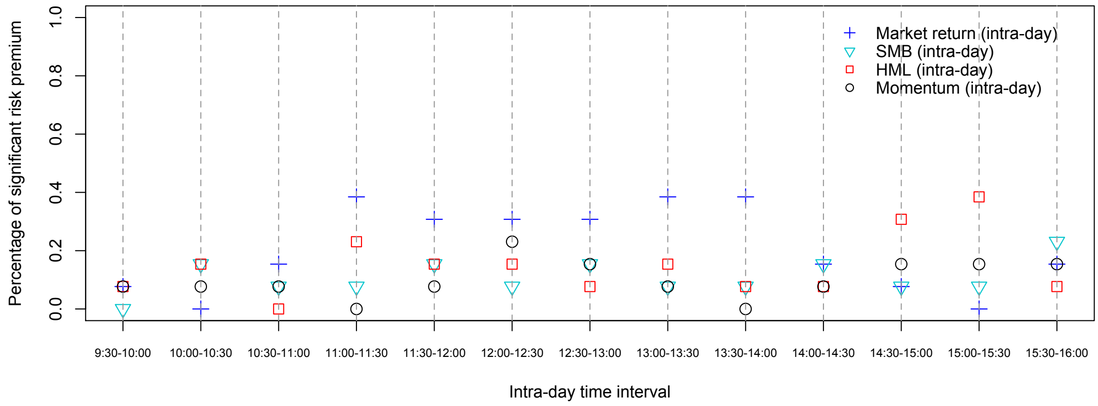


FIGURE 5. Percentage of the valid risk premium over intra-day trading hours in 13 annual samples that are tested using the Newey-West corrected t statistics at a 95% significance level, with the upper subplot showing the results using intra-day risk factors (models (1) and (3)) and the lower subplot showing the results using daily risk factors (models (2) and (3)).



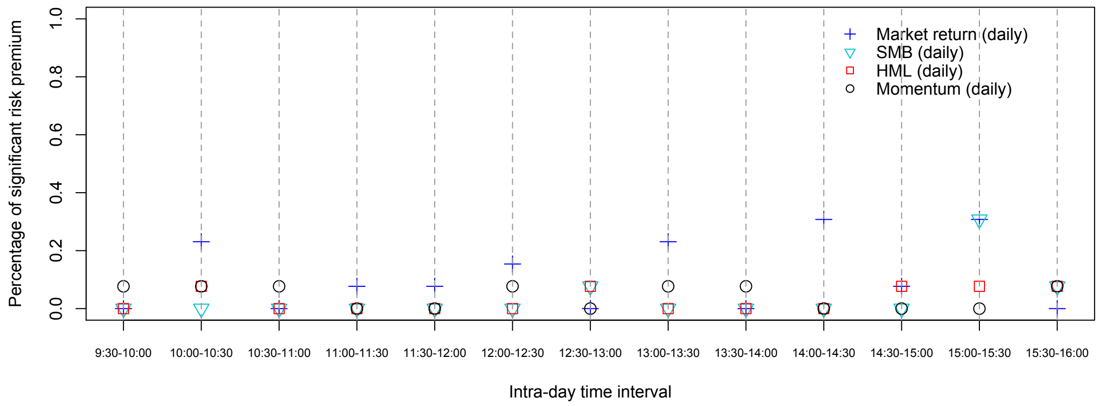


FIGURE 6. Percentage of the valid risk premium over intra-day trading hours in bull and bear markets that are tested through the Newey-West corrected t statistics at a 95% significance level, with both subplots showing the results using intra-day risk factors (models (1) and (3)). The bull and bear markets are detected via the algorithm proposed by Pagan and Sossounov (2003). Since the bull and bear may across our sample period, we eliminate the sample years mixed with bull and bear market sentiments.