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Cooperative Fault Estimation for A Class of Heterogeneous Multi-agents with Stochastic Nonlinearities Based on Finite Impulse Response Filter

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Summary

This paper investigates the cooperative fault estimation problem for a class of heterogeneous multi-agent systems, in which the agent dynamics are governed by linear discrete time-varying systems with nonidentical dimensions subject to stochastic nonlinearities. A finite impulse response (FIR) filter based fault estimation scheme is developed via relative outputs to estimate the possible faults of the local and neighboring agents simultaneously. An analytical redundancy expressed in terms of all the states in the previous time window is originally established for deriving the fault estimation signal. The prior variance information coupled with fault estimation error in nonlinear form, is fully considered to design performance index through analysis of random matrix inequality. The optimal FIR filter gain is analytically obtained with computational efficiency by searching the minimum point of the relevant matrix trace function. Illustrative examples are finally provided to demonstrate the effectiveness and advantages of the developed results.

KEYWORDS:
Heterogeneous multi-agent systems, stochastic nonlinearity, cooperative fault estimation, FIR filter, computational efficiency.

1 | INTRODUCTION

Thanks to the unique ability of real-time information sharing over a network, autonomous multi-agent systems (MASs) usually complete more challenging tasks than an individual agent. Consequently, related results on MASs have naturally attached more attention due to the broad applications such as formation of satellites, flocking of mobile vehicles, smart grids, intelligent transportation systems, scheduling of automated highway systems, and so forth [1,2,3,4]. As the size and complexity of the considered systems rapidly increase and the long-time uninterrupted operation of the equipment or the existence of interactions over the communication network, MASs are inevitably susceptible to various faults. On the one hand, an undetected fault in any individual agent may induce interruptions, and even result in synchronization degradation or instability as time evolves and/or environment changes. What is worse, it may be transmitted and amplified in the whole system through complex communication networks, further resulting in a disaster. In adiition, for the network of multiagent systems, it is unrealistic and infeasible to solve the FDI problem using a centralized architecture, given the stringent constraints on the computational resources and communication bandwidth in practice. Therefore, detecting and estimating possible faults in each agent at an early stage are of significant importance to maintain safe and reliable operations for such a class of large scale systems[5,6,7,8].

In virtue of the easy expansibility and high efficiency, there has been growing attention in designing distributed fault diagnosis schemes associated with the practice demand. Up to now, several results for fault diagnosis of MASs have been achieved based on adaptive observers[9,10]; the unknown input observers[11,12]; the distributed Kalman filter[13] and the interval observer technology[14]. Nevertheless, all the aforementioned previous works on MASs fault diagnosis only identify the presence of the possible fault, while further information with regard to its shape and amplitude is not available. As for the pioneer works on fault estimation for MASs, a sliding mode observer and a robust unknown input observer are established for liner MASs[15] and[16], as well as the nonlinear MASs case[17], based on the relative state information or relative output. Unfortunately, the fault diagnosis especially fault estimation schemes of the heterogeneous MASs, which allow the agents dynamics and even their state dimensions to be nonidentical, have not received much attention. On the one hand, it is usually impossible to access relative state information or absolute output for heterogeneous MASs. On the other hand, compared with state information transmission, not only process faults, but also measurement noises and sensor faults corresponding to the local agent are transmitted to neighboring agents in the transmission process of output information. Accordingly, it will be more challenging to achieve fault estimation from the available information affected by multi-source disturbance or noise information for a class of heterogeneous MASs.

It is quite common in engineering practice that system parameters may suffer from nonlinearity and stochasticity, which may cause undesirable dynamic behaviors such as oscillation or even instability. Therefore, the relevant analysis and synthesis problems have also been one of the main reseearch streams in the area, see e.g.[18,19,20,21]. Different from the nonlinearities bounded by the linear-like form (e.g. Lipschitz and sector conditions) or state dependent noise, stochastic nonlinearities described by statistical means have received particular attention probably due to the high manoeuvrability of the tracked target, intermittent network congestion, random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, and modification of the operating point of a linearized model of nonlinear systems. Unfortunately, the available research on stochastic nonlinearity mainly focuses on the filtering[22,23] and control[24,25] for the signal system, while that on the fault estimation of heterogeneous MASs is still far from mature[26].

Recall that, no matter whether the homogeneous or heterogeneous agents, almost all of the existing results utilize the infinite impulse response (IIR) structure based observer to realize fault diagnosis via all the past inputs and outputs in a recursive manner. Generally speaking, all the possible errors induced by modeling uncertainty, computational error, initial state, noise covariance and nonlinearity may be accumulated inevitably over time. In such cases, IIR structure based observer may provide poor performance[27,28,29]. Besides, if it is intended to estimate all the possible faults of the whole MASs, each individual agent should be costomarily equipped with a fault estimator from the sporadic applications for MASs. Usually this is cost efficiency or unnecessary[15,16,17]. To overcome these shortcomings, a nonlinear observer is established in[30] to estimate the possible fault existing in the local agent and its neighbors via the relative outputs. However, it is applied to a class of heterogeneous MASs disturbed by the Lipschitz nonlinearity, which is usually more or less conservative among the applications involved randomly fluctuated network conditions and/or communication constraints.

In view of the robustness advantage brought by the finite impluse response (FIR) based structure, fault diagnosis strategy with time window constraint (such as parity space method, FIR-type filter and the finite memory observer) has been widely used in fault detection[31,32,33] and fault estimation[34,35,36] for a single system. However, it should be pointed out that almost all analytic redundancies in the work mentioned above are in the form of either current[37] or history state[34] in the present time window. The former requires the singularity of the system matrix of the applied systems, while the latter brings a little heavy computation for updating the filter gain in each time instant, especially with the increasing of the time window length for time-varying systems. To the best of our knowledge, FIR filter based fault estimtion for heterogeneous MASs with stochastic nonlinearity has not been paid adequate attention despite its clear engineering significance.

Summarizing the above discussions, the cooperative fault estimation of heterogeneous MASs with stochastic nonlinearity is considered via the FIR filter in this paper. The main contributions of this paper are summarized as follows:

- **(i)** A more general heterogeneous MASs, whose dynamics are represented by the linear discrete time-varying systems with unknown disturbances and stochastic nonlinearity, is considered.

- **(ii)** By using the relative output information transmitted by the communication topology, which is also affected by disturbances and faults of the nearby agents, a FIR filter is established to implement the fault estimation of local and neighboring agents simultaneously.

(iii) A novel analytic redundancy expressed in terms of all the states in the previous time window is originally constructed to derive fault estimation signal, providing significant computational efficiency in calculating the optimal filter gain.

The remainder of this paper is organized as follows. In Section II, preliminaries are presented and the problem under consideration is briefly introduced. In Section III, through analysis of random matrix inequality, the known matrix information coupled with the corresponding state in nonlinear form induced from the stochastic uncertainty is extracted to construct a novel performance index to make the fault estimation error as small as possible in stochastic sense. Then, comparison analysis with the existing similar work is provided regarding the amount of calculation. In Section IV, we present the illustrative examples to show the effectiveness and the superiority of the proposed algorithm comparing with the existing similar work. Section V concludes this paper.

2 | PRELIMINARIES AND PROBLEM FORMULATION

2.1 | Graph theory and notations

Let the logical interdependence between N individuals be described by an wighted graph $\mathscr{G} = \{\mathscr{V}, \mathscr{E}, \mathscr{A}\}$, where $\mathscr{V} = \{\mathscr{V}_1, \dots, \mathscr{V}_N\}$ is a nonempty finite set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of edges links, in which an edge is represented by an ordered pair of distinct nodes. $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix capturing the edge weights with entries $a_{ij} > 0$ if node $\mathscr{V}_i$ can receive the information from node $\mathscr{V}_j$, i.e. $(\mathscr{V}_j, \mathscr{V}_i) \in \mathscr{E}$. Otherwise $a_{ij} = 0$. Self-loops are not considered here, i.e. $(\mathscr{V}_i, \mathscr{V}_i) \notin \mathscr{E}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$; $l_{ij} = -a_{ij}, \forall i \neq j$. $M_i = \{\mathscr{V}_j \in \mathscr{V} : (\mathscr{V}_p, \mathscr{V}_j) \in \mathscr{E}, p \neq j\}$ represents the nearest neighboring set of the agent i. Operator $\otimes$ denotes the Kronecker product of matrices. I_n denotes the n-dimensional identify matrix.

2.2 | System description

Consider a group of N heterogeneous agents interacted over a known graph to achieve the desired objective. The dynamic of the i-th individual node is described by the following stochastic discrete time-varying system,

$$\begin{cases} x_i(q+1) = A_i(q)x_i(q) + B_{d,i}(q)d_i(q) + B_{f,i}(q)f_i(q) + g_i(q, x_i(q), \delta_i(q)), \\ \quad y_i(q) = C_i(q)x_i(q) + D_{d,i}(q)d_i(q) + D_{f,i}(q)f_i(q) + q_i(q, x_i(q), \zeta_i(q)), \\ \quad x_i(0) = x_{0,i}, \end{cases} \tag{1}$$

where $i = 1, 2, \dots, N$. $x_i(q) \in \mathbb{R}^{n_i}$ is the state vector of the agent i, $y_i(q) \in \mathbb{R}^m$ is its output vector, $d_i(q) \in \mathbb{R}^{n_{d_i}}$ denotes the unknown disturbance and $f_i(q) \in \mathbb{R}^{n_{f_i}}$ is the system fault. $A_i(q)$, $B_{d,i}(q)$, $B_{f,i}(q)$, $C_i(q)$, $D_{d,i}(q)$, and $D_{f,i}(q)$ are all of known real time-varying matrices with compatible dimensions. The stochastic nonlinear functions $g_i(q, x_i(q), \delta_i(q))$ and $q_i(q, x_i(q), \zeta_i(q))$ stand for the stochastic parameter fluctuations. $\delta_i(q)$ and $\zeta_i(q)$ are mutually independent zero mean Gaussian noise sequences with unit variances, which are mutually uncorrelated to the initial state $x_{0,i}$ described by a random vector. The statistical characteristics are assumed to satisfy

$$\mathrm{E}\{g_i(q, x_i(q), \delta_i(q))|x_i(q)\} = \mathbf{0},\ i = 1, \dots, N, \mathrm{E}\{q_i(q, x_i(q), \zeta_i(q))|x_i(q)\} = \mathbf{0},\ i = 1, \dots, N, \tag{2}$$

$$\mathrm{E}\left\{ \begin{bmatrix} g_i(q, x_i(q), \delta_i(q)) \\ q_i(q, x_i(q), \zeta_i(q)) \end{bmatrix} \begin{bmatrix} g_j(q, x_i(q), \delta_j(q)) \\ q_j(q, x_i(q), \zeta_j(q)) \end{bmatrix}^T \middle| x_i(q) \right\} = \mathbf{0},\ i \neq j, \tag{3}$$

$$\mathrm{E}\left\{ \begin{bmatrix} g_i(q, x_i(q), \delta_i(q)) \\ q_i(q, x_i(q), \zeta_i(q)) \end{bmatrix} \begin{bmatrix} g_i(t, x_i(t), \delta_i(t)) \\ q_i(t, x_i(t), \zeta_i(t)) \end{bmatrix}^T \middle| x_i(q) \right\} = \begin{cases} \mathbf{0}, & q \neq t; \\ \sum_{p=1}^{m_1} \Pi_p^i(q) x_i^T(q) \Gamma_{p,i}(q) x_i(q), & q = t. \end{cases}, \tag{4}$$

where $\mathbf{0}$ is the zero vector/matrix with appropriate dimension and m_1 is a known nonnegative integer. $\Pi_p^i(q) = \mathrm{diag}\{\Pi_{p,1}^i(q), \Pi_{p,2}^i(q)\}$, $\Gamma_{p,i}(q)$, $\Pi_{p,1}^i(q)$ and $\Pi_{p,2}^i(q)$ are positive-definite matrices with compatible dimensions.

Remark 1. Associated with the first- and second-order statistics defined in (2)-(4), the stochastic nonlinearities $g_i(q, x_i(q), \delta_i(q))$ and $q_i(q, x_i(q), \zeta_i(q))$ can characterize several well-studied nonlinear stochastic phenomenons, such as systems with stochastic vectors whose powers depend on the sector-bounded nonlinear function or signum function of the states, and the corresponding deterministic coefficient matrix (state-dependent multiplicative noises)[38]. As such, the considered heterogeneous MASs

(1) involving unpredictable sensor and actuation stochastic uncertainties in this paper are more general than the existing works[10,14,16].

2.3 | Problem formulation

Considering that relative states and absolute outputs are not easy or even impossible to obtain in many practice situations, especially for heterogeneous multi-agents[39,40], the relative outputs are employed to achieve the fault estimation for system (1).

Furthermore, through the known communication topology, the relative output information obtained from the i-th agent is expressed as

$$\bar{y}_i(q) = \sum_{j\in M_i} a_{ij}(y_i(q) - y_j(q)). \tag{5}$$

Remark 2. It is assumed that each agent can measure the relative output $\bar{y}_i(q)$, which implies that the output dimension of each agent should be the same despite the heterogeneity of the considered agents[15,41]. These non-introspective agents operating in such cooperative and distributed environments, are practically relevant and widespread in many areas. For example, two vehicles in close proximity may be able to measure their relative distance without either of them having knowledge of their absolute positions and the localization problem of the unmanned aerial vehicle-unmanned aerial vehicle joint formation.

From (1) and (5), not only the possible faults and unknown uncertainties of the monitored agent, but also that of the neighboring agents, are injected into the current output $\bar{y}_i(q)$. Based on this, the auxiliary augmented model under fault propagation is ready to be established.

For the selected i-th agent, define

$$\bar{x}_i(q) = \begin{bmatrix} x_i(q) \\ x_{i_1}(q) \\ \vdots \\ x_{i_{|M_i|}}(q) \end{bmatrix},\ \bar{d}_i(q) = \begin{bmatrix} d_i(q) \\ d_{i_1}(q) \\ \vdots \\ d_{i_{|M_i|}}(q) \end{bmatrix},\ \bar{f}_i(q) = \begin{bmatrix} f_i(q) \\ f_{i_1}(q) \\ \vdots \\ f_{i_{|M_i|}}(q) \end{bmatrix},\ \bar{x}_i(0) = \begin{bmatrix} x_{0,i} \\ x_{0,i_1} \\ \vdots \\ x_{0,i_{|M_i|}} \end{bmatrix}, \tag{6}$$

$$\bar{g}_i(q, x_i(q), \delta_i(q)) = \begin{bmatrix} g_i(q, x_i(q), \delta_i(q)) \\ g_{i_1}(q, x_i(q), \delta_i(q)) \\ \vdots \\ g_{i_{|M_i|}}(q, x_i(q), \delta_i(q)) \end{bmatrix}, \bar{q}_i(q, x_i(q),\ \zeta_i(q)) = \begin{bmatrix} q_i(q, x_i(q), \zeta_i(q)) \\ q_{i_1}(q, x_i(q), \zeta_i(q)) \\ \vdots \\ q_{i_{|M_i|}}(q, x_i(q), \zeta_i(q)) \end{bmatrix}, \tag{7}$$

where the neighbor agents of the i-th agent are labelled as nodes $i_1, \cdots, i_{|M_i|}$. $|M_i|$ represents the cardinality of the nearest neighboring set for the agent i.

With (6) and (7), an auxiliary augmented system interfered by all the possible faults and unknown disturbance of the i-th agent itself and its neighbors is described as

$$\begin{cases} \bar{x}_i(q+1) = \bar{A}_i(q)\bar{x}_i(q) + \bar{B}_{d,i}(q)\bar{d}_i(q) + \bar{B}_{f,i}(q)\bar{f}_i(q) + \bar{g}_i(q, x_i(q), \delta_i(q)), \\ \bar{y}_i(q) = C_{L,i}(\bar{C}_i(q)\bar{x}_i(q) + \bar{D}_{d,i}(q)\bar{d}_i(q) + \bar{D}_{f,i}(q)\bar{f}_i(q) + \bar{q}_i(q, x_i(q), \zeta_i(q)), \\ \bar{x}_i(0) = \bar{x}_{0,i}, \end{cases} \tag{8}$$

where

$$\bar{A}_i(q) = \mathrm{diag}\{A_i(q), A_{i_1}(q), \cdots, A_{i_{|M_i|}}(q)\},\ \bar{C}_i(q) = \mathrm{diag}\{C_i(q), C_{i_1}(q), \cdots, C_{i_{|M_i|}}(q)\}, \tag{9}$$

$$\bar{B}_{d,i}(q) = \mathrm{diag}\{B_{d,i}(q), B_{d,i1}(q), \cdots, B_{d,i_{|M_i|}}(q)\},\ \bar{D}_{d,i}(q) = \mathrm{diag}\{D_{d,i}(q), D_{d,i1}(q), \cdots, D_{d,i_{|M_i|}}(q)\}, \tag{10}$$

$$\bar{D}_{f,i}(q) = \mathrm{diag}\{D_{f,i}(q), D_{f,i1}(q), \cdots, D_{f,i_{|M_i|}}(q)\},\ C_{L,i} = \begin{bmatrix} L_{ii} & L_{i_1} & \cdots & L_{i_{|M_i|}} \end{bmatrix} \otimes I_m, \tag{11}$$

$$\bar{B}_{f,i}(q) = \mathrm{diag}\{B_{f,i}(q), B_{f,i1}(q), \cdots, B_{f,i_{|M_i|}}(q)\}. \tag{12}$$

Remark 3. Notice that, the change of the considered topology will cause the output matrix $C_{L,i}\bar{C}_i(q)$ to change accordingly, but will not affect the fault estimator design.

In practice, sometimes it is impossible or cost efficiency to construct fault estimators for some specific agents. In such cases, it is necessary to design fault estimation scheme for some selected agents with higher intelligence to achieve the fault estimation of the agent itself and its neighbor agents simultaneously. From the augmented system (8), it can be seen that all the possible faults from the agent itself and its neighbor agents, are contained. Thus, this provides potential for realizing fault estimation of the entire MASs while reducing unnecessary hardware cost.

To establish the FIR filter based fault estimator, the corresponding relative outputs should be collected, firstly.

Considering that the analytical redundancy commonly used in the past may induce the calculation burden in obtaining filter gain[34,36], we propose a novel analytical form here.

For a given integer $p > 0$, combining (8) from time instant $q-p$ to q together yields

$$\begin{aligned} Y_{i,p}(q) =& \tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{A}_i(q-1)X_i(q-1) + \tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{d,i}(q-1)D_i(q-1) + \tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{f,i}(q-1)F_i(q-1) \\ &+ \tilde{C}_{L,i}(q)\tilde{C}_i(q)G_i(q-1) + \tilde{C}_{L,i}(q)\tilde{D}_{d,i}(q)D_i(q) + \tilde{C}_{L,i}(q)\tilde{D}_{f,i}(q)F_i(q) + \tilde{C}_{L,i}(q)Q_i(q), \end{aligned} \tag{13}$$

where

$$Y_{i,p}(q) = \begin{bmatrix} \bar{y}_i(q-p) \\ \bar{y}_i(q-p+1) \\ \vdots \\ \bar{y}_i(q) \end{bmatrix},\ D_i(q) = \begin{bmatrix} \bar{d}_i(q-p) \\ \bar{d}_i(q-p+1) \\ \vdots \\ \bar{d}_i(q) \end{bmatrix},\ F_i(q) = \begin{bmatrix} \bar{f}_i(q-p) \\ \bar{f}_i(q-p+1) \\ \vdots \\ \bar{f}_i(q) \end{bmatrix}, X_i(q-1) = \begin{bmatrix} \bar{x}_i(q-p-1) \\ \bar{x}_i(q-p) \\ \vdots \\ \bar{x}_i(q-1) \end{bmatrix}, \tag{14}$$

$$G_i(q-1) = \begin{bmatrix} \bar{g}_i(q-p-1, x_i(q-p-1), \delta_i(q-p-1)) \\ \bar{g}_{i_1}(q-p, x_i(q-p), \delta_i(q-p)) \\ \vdots \\ \bar{g}_{i_{|M_i|}}(q-1, x_i(q-1), \delta_i(q-1)) \end{bmatrix}, Q_i(q) = \begin{bmatrix} \bar{q}_i(q-p, x_i(q-p), \zeta_i(q-p)) \\ \bar{q}_{i_1}(q-p+1, x_i(q-p+1), \zeta_i(q-p+1)) \\ \vdots \\ \bar{q}_{i_{|M_i|}}(q, x_i(q), \zeta_i(q)) \end{bmatrix}, \tag{15}$$

and the diagonal coefficient matrices are given by

$$\tilde{C}_i(q) = \mathrm{diag}(\bar{C}_i(q-p), \bar{C}_i(q-p+1), \ldots, \bar{C}_i(q)), \tilde{C}_{L,i} = \mathrm{diag}(C_{L,i}, C_{L,i}, \ldots, C_{L,i}), \tag{16}$$

$$\tilde{A}_i(q-1) = \mathrm{diag}(\bar{A}_i(q-p-1), \bar{A}_i(q-p), \ldots, \bar{A}_i(q-1)), \tilde{B}_{d,i}(q-1) = \mathrm{diag}(\bar{B}_{d,i}(q-p-1), \bar{B}_{d,i}(q-p), \ldots, \bar{B}_{d,i}(q-1)), \tag{17}$$

$$\tilde{B}_{f,i}(q-1) = \mathrm{diag}(\bar{B}_{f,i}(q-p-1), \bar{B}_{f,i}(q-p), \ldots, \bar{B}_{f,i}(q-1)), \tilde{D}_{d,i}(q) = \mathrm{diag}(\bar{D}_{d,i}(q-p), \bar{D}_{d,i}(q-p+1), \ldots, \bar{D}_{d,i}(q)), \tag{18}$$

$$\tilde{D}_{f,i}(q) = \mathrm{diag}(\bar{D}_{f,i}(q-p), \bar{D}_{f,i}(q-p+1), \ldots, \bar{D}_{f,i}(q)). \tag{19}$$

Then, the designed FIR based fault estimator is described by

$$\hat{\bar{f}}_i(q) = \begin{bmatrix} \hat{f}_i(q) \\ \hat{f}_{i_1}(q) \\ \vdots \\ \hat{f}_{i_{|M_i|}}(q) \end{bmatrix} = p_i(q)Y_{i,p}(q),\ i = 1, 2, \ldots, N, \tag{20}$$

where $p_i(q)$ denotes the FIR filter gain for the i-th agent to be designed later. $\hat{\bar{f}}_i(q)$ is the corresponding fault estimation based on its finite relative output information $Y_{i,p}(q)$.

The fault estimation error $r_i(q)$ is defined as

$$r_i(q) = \hat{\bar{f}}_i(q) - \bar{f}_i(q), \tag{21}$$

where the fault $\bar{f}_i(q)$ defined in (6) can be rewritten by $\bar{f}_i(q) = W_f F_i(q)$ with $F_i(q)$ defined in (14), and $W_f = \overbrace{\begin{bmatrix} 0 & \cdots & 0 & I \end{bmatrix}}^{p+1} \in \mathrm{R}^{(n_{f_i}+n_{f_{i_1}}+\cdots+n_{f_{i_{|M_i|}}})\times(p+1)(n_{f_i}+n_{f_{i_1}}+\cdots+n_{f_{i_{|M_i|}}})}$.

Now, it is clear to see that the fault estimation problem for system (1) has been transferred to finding the optimal filter gain $p_i^*(q)$ for the FIR in (20) such that the estimator error $r_i(q)$ converges to 0.

3 | MAIN RESULT

In this section, a cooperative fault estimation algorithm is to be presented based on the FIR filters for heterogeneous MASs with stochastic nonlinearities.

3.1 | Problem analysis and reformulation

Associated with (13), the fault estimation error dynamics are given as

$$\begin{aligned}
r_i(q) =& p_i(q)Y_{i,p}(q) - W_f F_i(q) \\
=& p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{A}_i(q-1)X_i(q-1) + p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{d,i}(q-1)D_i(q-1) \\
&+ p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{f,i}(q-1)F_i(q-1) + p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)G_i(q-1) \\
&+ p_i(q)\tilde{C}_{L,i}(q)\tilde{D}_{d,i}(q)D_i(q) + p_i(q)\tilde{C}_{L,i}(q)\tilde{D}_{f,i}(q)F_i(q) + p_i(q)\tilde{C}_{L,i}(q)Q_i(q) - W_f F_i(q) \\
\triangleq& T_i(q)W_i(q),
\end{aligned} \tag{22}$$

where

$$\bar{X}_i(q) = \begin{bmatrix} X_i(q-1) \\ G_i(q-1) \\ Q_i(q) \end{bmatrix}, \bar{D}_i(q) = \begin{bmatrix} D_i(q-1) \\ D_i(q) \end{bmatrix}, \quad \bar{F}_i(q) = \begin{bmatrix} F_i(q-1) \\ F_i(q) \end{bmatrix}, \quad T_i(q) = \begin{bmatrix} T_i^a(q) & T_i^b(q) & T_i^c(q) \end{bmatrix}, \quad W_i(q) = \begin{bmatrix} \bar{X}_i(q) \\ \bar{D}_i(q) \\ \bar{F}_i(q) \end{bmatrix}, \tag{23}$$

$$T_i^a(q) = \begin{bmatrix} p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{A}_i(q-1) & p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q) & p_i(q)\tilde{C}_{L,i}(q) \end{bmatrix}, \tag{24}$$

$$T_i^b(q) = \begin{bmatrix} p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{d,i}(q-1) & p_i(q)\tilde{C}_{L,i}(q)\tilde{D}_{d,i}(q) \end{bmatrix}, \tag{25}$$

$$T_i^c(q) = \begin{bmatrix} p_i(q)\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{f,i}(q-1) & p_i(q)\tilde{C}_i(q)\tilde{D}_{f,i}(q) - W_f \end{bmatrix}. \tag{26}$$

Considering both the target system (1) and the fault estimator (20) contain the stochastic information, the performance index is selected as:

$$\mathrm{E}(r_i^T(q)r_i(q)) = \mathrm{E}(W_i^T(q)T_i^T(q)T_i(q)W_i(q)) = \mathrm{tr}(\mathrm{E}(W_i^T(q)T_i^T(q)T_i(q)W_i(q))) = \mathrm{tr}(T_i^T(q)T_i(q)\mathrm{E}(W_i(q)W_i^T(q))). \tag{27}$$

Different from the deterministic system with Lipschitz nonlinearity [14] or sector bounded nonlinearity [42], and the stochastic system with additive noise [26], multiplicative noise [43] or stochastic nonlinear system where stochastic property is independent of the state [44], the variance information of the stochastic nonlinear considered in this paper is coupled with the state information via a non-linear form in $\mathrm{E}(W_i(q)W_i^T(q))$, which cannot be ignored in $\mathrm{E}(r_i^T(q)r_i(q))$ for obtaining satisfactory fault estimation in (27). Consequently, how to extract the prior known matrices $\Pi_p(q)$ and $\Gamma_{p,i}(q)$ from the variance information of stochastic nonlinearity, is of intractable challenge.

According to [36], any quadratic matrices $T_i^T(q)T_i(q)$ and $\mathrm{E}(W_i(q)W_i^T(q))$ satisfy

$$\mathrm{E}(r_i^T(q)r_i(q)) \leq \mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(\mathrm{E}(W_i(q)W_i^T(q))). \tag{28}$$

Associated with (28) and the stochastic property of the augmented vector $\bar{X}_i(q)$, we proceed with extracting the stochastic property from random nonlinearity in $\mathrm{E}(W_i(q)W_i^T(q))$ and further deriving a novel performance index, which is completely decoupled from unknown variables $X_i(q)$, $\bar{D}_i(q)$, and $\bar{F}_i(q)$ to constrain fault estimation error. To this end, the following propositions are provided firstly.

Proposition 1. The variances of the stochastic nonlinear functions $G_i(q-1)$ and $Q_i(q)$ in (15) are bounded by

$$\mathrm{E}(G_i(q-1)G_i^T(q-1)) \leq M_{gi}(q-1)N_{gi}(q-1), \tag{29}$$

$$\mathrm{E}(Q_i(q)Q_i^T(q)) \leq M_{qi}(q)N_{qi}(q), \tag{30}$$

where

$$M_{gi}(q-1) = \mathrm{diag}\{m_{gi}^{11}, \ldots, m_{gi}^{1|M_i|}, m_{gi}^{21}, \ldots, m_{gi}^{2|M_i|}, \ldots, m_{gi}^{(p+1)1}, \ldots, m_{gi}^{(p+1)|M_i|}\}, \tag{31}$$

$$N_{gi}(q-1) = \mathrm{diag}\{n_{gi}^{11}, \ldots, n_{gi}^{1|M_i|}, n_{gi}^{21}, \ldots, n_{gi}^{2|M_i|}, \ldots, n_{gi}^{(p+1)1}, \ldots, n_{gi}^{(p+1)|M_i|}\}, \tag{32}$$

$$M_{qi}(q) = \mathrm{diag}\{m_{qi}^{11}, \ldots, m_{qi}^{1|M_i|}, m_{qi}^{21}, \ldots, m_{qi}^{2|M_i|}, \ldots, m_{qi}^{(p+1)1}, \ldots, m_{qi}^{(p+1)|M_i|}\}, \tag{33}$$

$$N_{qi}(q) = \mathrm{diag}\{n_{qi}^{11}, \ldots, n_{qi}^{1|M_i|}, n_{qi}^{21}, \ldots, n_{qi}^{2|M_i|}, \ldots, n_{qi}^{(p+1)1}, \ldots, n_{qi}^{(p+1)|M_i|}\}, \tag{34}$$

with

$$m_{gi}^{11}=\sum_{p=1}^{m}\Pi_{p,1}^{i}(q-p-1)\mathrm{tr}(\Gamma_{p,i}(q-p-1)),\ \ m_{gi}^{1|M_i|}=\sum_{p=1}^{m}\Pi_{p,1}^{i_{|M_i|}}(q-p-1)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-p-1)),\tag{35}$$

$$m_{gi}^{21}=\sum_{p=1}^{m}\Pi_{p,1}^{i}(q-p)\mathrm{tr}(\Gamma_{p,i}(q-p)),\ \ m_{gi}^{2|M_i|}=\sum_{p=1}^{m}\Pi_{p,1}^{i_{|M_i|}}(q-p)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-p)),\tag{36}$$

$$m_{gi}^{(p+1)1}=\sum_{p=1}^{m}\Pi_{p,1}^{i}(q-1)\mathrm{tr}(\Gamma_{p,i}(q-1)),\ \ m_{gi}^{(p+1)|M_i|}=\sum_{p=1}^{m}\Pi_{p,1}^{i_{|M_i|}}(q-1)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-1)),\tag{37}$$

$$n_{gi}^{11}=\mathrm{tr}(x_i(q-p-1)x_i^T(q-p-1))\otimes I_{n_i},\ \ n_{gi}^{1|M_i|}=\mathrm{tr}(x_{i_{|M_i|}}(q-p-1)x_{i_{|M_i|}}^T(q-p-1))\otimes I_{n_{i_{|M_i|}}},\tag{38}$$

$$n_{gi}^{21}=\mathrm{tr}(x_i(q-p-1)x_i^T(q-p-1))\otimes I_{n_i},\ \ n_{gi}^{2|M_i|}=\mathrm{tr}(x_{i_{|M_i|}}(q-p-1)x_{i_{|M_i|}}^T(q-p-1))\otimes I_{n_{i_{|M_i|}}},\tag{39}$$

$$n_{gi}^{(p+1)1}=\mathrm{tr}(x_i(q-1)x_i^T(q-1))\otimes I_{n_i},\ \ n_{gi}^{(p+1)|M_i|}=\mathrm{tr}(x_{i_{|M_i|}}(q-1)x_{i_{|M_i|}}^T(q-1))\otimes I_{n_{i_{|M_i|}}}.\tag{40}$$

The term $M_{qi}(q)$ in (33) can be obtained directly by replacing $\Pi_{p,1}^{i}(q-p-1)$ and $\Gamma_{p,i}(q-p-1)$ with $\Pi_{p,2}^{i}(q-p)$ and $\Gamma_{p,i}(q-p)$ respectively, in $M_{gi}(q-1)$, and $N_{qi}(q-1)$ can be obtained directly by replacing $x_i(q-p-1)$ and I_{n_i} with $x_i(q-p)$ and I_m respectively, in $N_{gi}(q)$, which are omitted here.

Proof. See Appendix A. □

From Proposition 1, it follows that

$$\mathrm{tr}(\mathrm{E}(\bar{X}_i(q)\bar{X}_i^T(q)))\le\mathrm{tr}(M_i^a(q)N_i^a(q)),\tag{41}$$

where $\bar{X}_i(q)$ defined in (23), and

$$M_i^a(q)=\begin{bmatrix}I_{X_i}&&0\\&M_{gi}(q-1)&\\0&&M_{qi}(q)\end{bmatrix},\ N_i^a(q)=\begin{bmatrix}\mathrm{E}(X_i(q-1)X_i^T(q-1))&&0\\&N_{gi}(q-1)&\\0&&N_{qi}(q)\end{bmatrix},\tag{42}$$

with the $(n_i+n_{i_1}+\cdots+n_{i_{|M_i|}})$ dimensional identify matrix denoted by I_{X_i}.

Based on Proposition 1 and (41), we can extract the prior variance information (4) in the nonlinear functions $g_i(q,x_i(q),\delta_i(q))$ and $q_i(q,x_i(q),\zeta_i(q))$ from $\mathrm{E}(W_i(q)W_i^T(q))$ and directly have

$$\mathrm{E}(r_i^T(q)r_i(q))\le\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q)N_i^b(q))\le\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))\mathrm{tr}(N_i^b(q)),\tag{43}$$

where

$$M_i^b(q)=\begin{bmatrix}M_i^1(q)&0\\0&I_{Ni}\end{bmatrix},\ N_i^b(q)=\begin{bmatrix}N_i^1(q)&0&0\\0&\bar{D}_i(q)\bar{D}_i^T(q)&\bar{D}_i(q)\bar{F}_i^T(q)\\0&\bar{F}_i(q)\bar{D}_i^T(q)&\bar{F}_i(q)\bar{F}_i^T(q)\end{bmatrix},\tag{44}$$

with the $2(n_{d_i}+n_{d_{i_1}}+\cdots+n_{d_{i_{|M_i|}}}+n_{f_i}+n_{f_{i_1}}+\cdots+n_{f_{i_{|M_i|}}})(p+1)$ dimensional identify matrix denoted by I_{N_i}.

Thus, the coupling problem caused by the complex analytical expressions of stochastic nonlinearity has been solved. Similar to[36], $\mathrm{tr}(N_i^b(q))$ is a determined term, which can be ignored in $\mathrm{E}(r_i^T(q)r_i(q))$. Then, the minimum point of $\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))$ can be found to realize the fault estimation under the satisfactory fault estimation level.

Furthermore, to analysis the term $\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))$ for the performance index selection process, the following proposition in[45] is needed.

Proposition 2 ([45]). Assume that the Hessian matrix $\nabla_{p(q)}^2 J(p^*(q))$ of $J(p_q^*)$ is continuous, matrix $p^*(q)$ is the only minimum point of convex function $J(p(q))$ if and only if the Gradient and Hessian matrices satisfy

$$\nabla_{p(q)}J(p^*(q))=0,\tag{45}$$

$$\nabla_{p(q)}^2 J(p^*(q))>0.\tag{46}$$

Following Proposition 2, no matter whether $\mathrm{tr}(T_i^T(q)T_i(q))$ or $\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))$ is chosen as the performance index, the prior variance information of the stochastic nonlinearity cannot affect the selection of the optimal FIR filter due to derivative operation. Thus, it is somewhat conservative to select the upper bound on $\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))$ to constrain the fault estimation error.

In view of the semi-definite property of matrices $T_i^T(q)T_i(q)$ and $M_i^b(q)$, we have

$$\mathrm{tr}(T_i^T(q)T_i(q)M_i^b(q)) \leq \mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q)). \tag{47}$$

Then, the prior variance information about $\Pi_p^i(q)$ and $\Gamma_{p,i}(q)$ of stochastic nonlinearities $q_i(q, x_i(q)$ and $\zeta_i(q))$, $g_i(q, x_i(q), \delta_i(q))$ is separated from $W_i(q)$. Based on this, we are now in a position to reformulate the problem in the following manner.

For the heterogeneous stochastic nonlinear system (1), find FIR filter gain $p_i(q)$ satisfying

$$\mathrm{E}(r_i^T(q)r_i(q)) < \mathrm{tr}(T_i(q)M_i^b(q)T_i^T(q))_{min} \tag{48}$$

such that the transient characteristics of the fault estimation error dynamics are bounded within a minimum range in the stochastic sense.

Remark 4. Considering the randomness of the new augmented disturbance vector $T_i(q)$, it is impossible to achieve complete decoupling from the fault of interest. Therefore, an upper bound on the error estimation error $\mathrm{tr}(T_i(q)M_i^b(q)T_i^T(q))_{min}$, where includes the prior variance information (4), is developed as the performance index to achieve the optimal robust fault estimation for the heterogeneous MASs (1) with the unknown disturbances and stochastic nonlinearities.

3.2 | FIR based cooperative fault estimation algorithm

For the fault estimator (20), the main result for system (1) is ready to be presented based on the Proposition 2.

Theorem 1. For the heterogeneous stochastic nonlinear system (1), the faults of itself and its neighbor agents can be estimated by the the optimal FIR filter (20) under the performance index (48), if the gain $p_i(q)$ is chosen as

$$p_i(q) = W_f \tilde{D}_{f,i}^T(q)\tilde{C}_{L,i}^T(q)R_i^{-T}(q), \tag{49}$$

and $R_i(q)$ defined by

$$\begin{aligned} R_i(q) =& \tilde{C}_{L,i}(q)(\tilde{C}_i(q)\tilde{A}_i(q-1)\tilde{A}_i^T(q-1)\tilde{C}_i^T(q) + \tilde{C}_i(q)\tilde{B}_{d,i}(q-1)\tilde{B}_{d,i}^T(q-1)\tilde{C}_i^T(q) \\ &+ \tilde{C}_i(q)\tilde{B}_{f,i}(q-1)\tilde{B}_{f,i}^T(q-1)\tilde{C}_i^T(q) + \tilde{C}_i(q)M_{gi}(q-1)\tilde{C}_i^T(q) \\ &+ \tilde{D}_{d,i}(q)\tilde{D}_{d,i}^T(q) + \tilde{D}_{f,i}(q)\tilde{D}_{f,i}^T(q) + M_{qi}(q))\tilde{C}_{L,i}^T(q), \end{aligned} \tag{50}$$

is positive definite.

Proof. From (23)-(26) and (44), it follows that

$$\mathrm{tr}(T_i(q)M_i^b(q)T_i^T(q))) = p_i(q)R_i(q)p_i^T(q) - p_i(q)\tilde{C}_{L,i}(q)\tilde{D}_{f,i}(q)W_f^T - W_f\tilde{D}_{f,i}^T(q)\tilde{C}_{L,i}^T(q)p_i^T(q) + W_fW_f^T.$$

If the existence condition in Proposition 2 holds, the optimal solution $p^*(q)$ for the minimum point of $\mathrm{tr}(T_i(q)M_i^b(q)T_i^T(q)))$ can be derived by solving the first order differential equation

$$\bigtriangledown_{p_i(q)}J(p_i^*(q)) = 2p_i^*(q)R_i(q) - 2W_f\tilde{D}_{f,i}^T(q)\tilde{C}_{L,i}^T(q) = 0. \tag{51}$$

Furthermore, it can easily get

$$\bigtriangledown_{p_i(q)}^2 J(p_i^*(q)) = \frac{\partial}{\partial^T(\mathrm{vec}(p_i^*(q)))}\left(\frac{\partial J(p_i^*(q)}{\partial^T(\mathrm{vec}(p_i^*(q)))}\right)^T = 2I_{n_{f_i}\times n_{f_i}} \otimes R_i(q). \tag{52}$$

Suppose that $R_i(q)$ is positive definite, it follows that the corresponding minimum solution $p_i^*(q)$ for $p_i(q)$ exists and the optimal FIR filter with gain $p_i^*(q)$ is expressed as in (49). This completes the proof. □

Remark 5. $R_i(q)$, defined in (52), is the key matrix that needs to be calculated to obtain the optimal filter gain. Associated with (16)-(19), (31) and (33), $R_i(q)$ can be calculated from the known system parameter matrices $A_i(q)$, $B_{d,i}(q)$, $B_{f,i}(q)$, $C_i(q)$, $D_{d,i}(q)$, $D_{f,i}(q)$, and the statistical characteristics $\Pi_p^i(q), \Gamma_{p,i}(q)$. According to the structure reflected in (50), $R_i(q)$ is a semi-positive definite symmetric matrix. Furthermore, as long as any one of $\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{A}_i(q-1)\tilde{A}_i^T(q-1)\tilde{C}_i^T(q)\tilde{C}_{L,i}^T(q)$, $\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{d,i}(q-1)\tilde{B}_{d,i}^T(q-1)\tilde{C}_i^T(q)\tilde{C}_{L,i}^T(q)$, $\tilde{C}_{L,i}(q)\tilde{C}_i(q)\tilde{B}_{f,i}(q-1)\tilde{B}_{f,i}^T(q-1)\tilde{C}_i^T(q)\tilde{C}_{L,i}^T(q)$, $\tilde{C}_{L,i}(q)\tilde{D}_{f,i}(q)\tilde{D}_{f,i}^T(q)\tilde{C}_{L,i}^T(q)$, $\tilde{C}_{L,i}(q)\tilde{C}_i(q)M_{gi}(q-1)\tilde{C}_i^T(q)\tilde{C}_{L,i}^T(q)$, $\tilde{C}_{L,i}(q)\tilde{D}_{d,i}(q)\tilde{D}_{d,i}^T(q)\tilde{C}_{L,i}^T(q)$, and $\tilde{C}_{L,i}(q)M_{qi}(q))\tilde{C}_{L,i}^T(q)$, is positive definite, then $R_i(q)$ satisfies the positive definite condition.

As it is assumed that the stochastic nonlinearity is independent of each other in section 2.2, then $D_{d,i}(q)$ has full row rank for all $q > 0$[46]. Consequently, with the definitions of (10) and (18), $\tilde{D}_{d,i}(q)\tilde{D}^T_{d,i}(q)$ satisfies positive definite property. From (9) and (11), $\tilde{C}_{L,i}$ is full row rank, which implies $R_i(q)$ positive definite. If the positive definite condition does not hold, an alternative way is to replace $R_i(q)$ with $R^*_i(q) = R_i(q) + \gamma_i I_i$ to obtain an alternative suboptimal solution, where γ_i is a positive real number as small as possible and I_i is an identity matrix with compatible dimensions.

The whole fault estimation framework can be described by a block diagram in Figure. 1

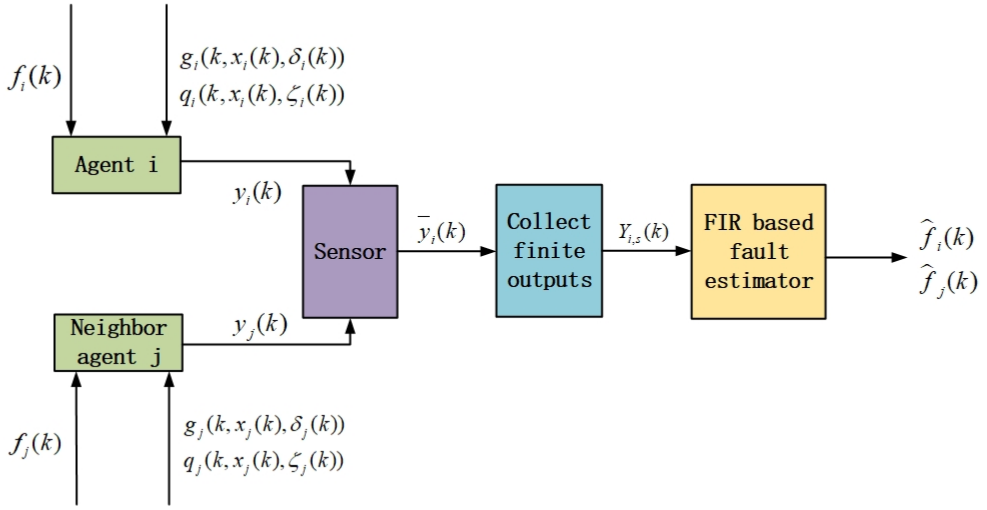

FIGURE 1 Block diagram of cooperative fault estimation for the selected key agent *i*

.

The specific fault estimators design steps for the whole MASs can be summarized as follows:

Algorithm 1 FIR filter based fault estimation for the whole MASs with stochastic nonlinearities

S1: Give appropriate time window length p for the selected i-th agent and initialize $q = p + 1$;

S2: Collect the measurement $Y_{i,p}(q)$ within the time window $[q - p, q]$;

S3: Calculate $\tilde{C}_{L,i}(q)$, $\tilde{D}_{f,i}(q)$, $R_i(q)$ according to (16),(19) and (50) and select the corresponding W_f according to different purposes;

S4: Obtain the optimal FIR filter gain $p^*_i(q)$ based on Theorem 1;

S5: Construct the FIR filter based estimator as (20) and derive the corresponding fault estimation results for each selected agent;

S6: Increase q by one and return to S2.

Remark 6. It should be noted that many practical systems can be described by the discrete time-varying stochastic nonlinear state space model (1), such as mobile robots, cooperating UAV team operations (surveillance and reconnaissance), formation flying of UAV's and satellites, vehicle platoons. For these systems, we can calculate the optimal filter gain $p^*_i(q)$ and construct the FIR filter based estimator as (20) to derive the corresponding fault estimation following the Algorithm 1 above. Considering the difference between practical application and theoretical research, if necessary, the optimal filter gain may need to be slightly adjusted to obtain better performance. The practical verification of the theoretical results developed in this paper will be further considered in the future.

4 | COMPUTATIONAL COST ANALYSIS

Generally speaking, the computational cost of updating the filter gain at each time instant can inevitably be increased substantially over the time window p[47]. Compared with the analytic redundancy derived in[37], which is expressed in terms of $x(q-p)$, all the coefficient matrices involved in the newly established one in (13) in the paper possess easy-to-obtain diagonal structures, as it is originally formulated in terms of all the state information $(x(q-p-1), \dots, x(q-1))$ in the previous time window $[q-p-1, q-1]$. Usually in the multi-agents system, the computing power of a sub-agent is often very limited. It is thus very necessary to develop a computationally efficient fault estimation algorithm. In the following, we will demonstrate that the proposed method indeed possesses computational advantages over the scenario considered in[37] owing to the new proposed redundancy in (13).

Introducing the notations

$$H_{i,dp}(q) = \begin{bmatrix} \bar{D}_{d,i}(q-p) & 0 & \cdots & \cdots & 0 \\ C_{L,i}\bar{C}_i(q-p+1)\bar{B}_{d,i}(q-p) & \bar{D}_{d,i}(q-p+1) & 0 & \cdots & 0 \\ C_{L,i}\bar{C}_i(q-p+2)\bar{A}_i(q-p+1)\bar{B}_{d,i}(q-p) & C_{L,i}\bar{C}_i(q-p+2)\bar{B}_{d,i}(q-p+1) & \bar{D}_{d,i}(q-p+2) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ C_{L,i}\bar{C}_i(q)\bar{A}_i(q-1)\cdots A_i(q-p+1)\bar{B}_{d,i}(q-p) & \cdots & C_{L,i}\bar{C}_i(q)\bar{A}_i(q-1)\bar{B}_{d,i}(q-2) & C_{L,i}\bar{C}_i(q)\bar{B}_{d,i}(q-1) & \bar{D}_{d,i}(q) \end{bmatrix}, \tag{53}$$

$$H_{i,gp}(q) = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ C_{L,i}\bar{C}_i(q-p+1) & 0 & 0 & \cdots & 0 \\ C_{L,i}\bar{C}_i(q-p+2)A_i(q-p+1) & C_{L,i}\bar{C}_i(q-p+2) & 0 & & \\ \cdots & \ddots & \ddots & \ddots & \vdots \\ C_{L,i}\bar{C}_i(q)\bar{A}_i(q-1)\cdots\bar{A}_i(q-p+1) & \cdots & C_{L,i}\bar{C}_i(q)\bar{A}_i(q-1) & C_{L,i}\bar{A}_i(q) & 0 \end{bmatrix}, \tag{54}$$

$$H_{i,op}(q) = \begin{bmatrix} C_{L,i}\bar{C}_i(q-p) \\ C_{L,i}\bar{C}_i(q-p+1)\bar{A}_i(q-p) \\ C_{L,i}\bar{C}_i(q-p+2)\bar{A}_i(q-p+1)\bar{A}_i(q-p) \\ \vdots \\ C_{L,i}\bar{C}_i(q)\bar{A}_i(q-1)\cdots\bar{A}_i(q-p) \end{bmatrix}, X_{i1}(q-p) = \begin{bmatrix} x_i(q-p) \\ x_{i1}(q-p) \\ \vdots \\ x_{i_{|M_i|}}(q-p) \end{bmatrix}, \tag{55}$$

which allow to express the analytic redundancy based on the form proposed in[37] in the following compact form

$$Y_{i,p}(q) = H_{i,op}(q)X_{i1}(q-p) + H_{i,dp}(q)D_i(q) + H_{i,fp}(q)F_i(q) + H_{i,gp}(q)G_i(q) + Q_i(q). \tag{56}$$

Following the similar way with the definition of $H_{i,dp}(q)$, $H_{i,fp}(q)$ can be constructed by replacing $\bar{B}_{d,i}(q-p)$ and $\bar{D}_{d,i}(q-p)$ with the corresponding $\bar{B}_{f,i}(q-p)$ and $\bar{D}_{f,i}(q-p)$, respectively.

In order to realize the fault estimation of system (1), the following optimal filter gain is employed based on (56)

$$p_{i1}(q) = W_f H_{i,fp}^T(q)R_{i,1}^{-T}(q), \tag{57}$$

where

$$R_{i,1}(q) = H_{i,op}(q)H_{i,op}^T(q) + H_{i,dp}(q)H_{i,dp}^T(q) + H_{i,fp}(q)H_{i,fp}^T(q) + H_{i,gp}(q)M_{gi}(q)H_{i,gp}^T(q) + M_{qi}(q) \tag{58}$$

In comparison with the existing fault estimation framework in[37], the computation complexity of $R_i(q)$ is much reduced in this work. As addition is much easier than multiplication in practice, let MD denote the number of multiplication and division involved in calculating the optimal FIR filter gain in[48].

Considering that the quadratic matrix needed to calculate the optimal filter gain is composed of the coefficient matrices in analytical redundancy, it is reasonable to evaluate the computational complexity by comparing the calculation cost differences between these two algorithms in obtaining the coefficient matrices involved in corresponding analytical redundancy. Please refer to the Table 1 for more details, where the argument 'k' is omitted for the convenience of analysis.

TABLE 1 Calculation cost differences for obtaining the coefficient matrices involved in corresponding analytical redundancy between our proposed algorithm and that in[37]

	Our proposed algorithm	Algorithm presented in[37]
The frequency of calculating $\bar{C}_i\bar{A}_i$	$p+1$	$\frac{3p^2-p}{2}$
The frequency of calculating $C_{L,i}\bar{C}_i$	$4(p+1)$	$\frac{3p^2+5p+2}{2}$
The frequency of calculating $\bar{C}_i\bar{B}_{d,i}$	$p+1$	p
The frequency of calculating $\bar{C}_i\bar{B}_{f,i}$	$p+1$	p
The frequency of calculating $\bar{A}_i\bar{A}_i$	0	$\frac{p^3+6p^2-p}{6}$
The frequency of calculating $\bar{A}_i\bar{B}_{d,i}$	0	p
The frequency of calculating $\bar{A}_i\bar{B}_{f,i}$	0	p

Obviously, our proposed analytical redundancy usually brings significant computational reduction in the application of single multi-agent fault estimation, especially with the increase of p.

In summary, different from the existing results in[15,16,17], the derived established filter can estimate the possible faults in the individual agent and its neighbors simultaneously utilizing the available relative output information only. This implies that some key nodes could be selected to be equipped with the desired FIR filter so as to estimate the faults in the whole MASs, which potentially reduces hardware cost. Moreover, due to the originality derived analytical redundancy, it also provides computational efficiency for calculating the filter gain in each time instant.

5 | ILLUSTRATIVE EXAMPLE

In this section, illustrative examples are introduced to demonstrate the flexibility and effectiveness of the developed FDI algorithm for a class of heterogeneous MASs subject to stochastic nonlinearity.

Consider a group of five heterogeneous agents whose dynamics are represented by (1), and the corresponding matrix coefficients are described as

$$A_1(q) = A_3(q) = A_5(q) = \begin{bmatrix} -0.8100 & -0.15\sin(0.39\pi q) & -0.0420 \\ -0.45e^{(\frac{q}{q+1})} & 0.1500 & -0.2250 \\ 0.1650 & -0.3000 & -0.15\frac{q}{q+1} \end{bmatrix}, A_2(q) = A_4(q) = \begin{bmatrix} 0.4050 & 0.297(\frac{q}{q+1}) \\ 0.4698 & 0.405e^{(0.2\pi\frac{q}{q+1})} \end{bmatrix}, \tag{59}$$

$$C_1(q) = C_3(q) = C_5(q) = \begin{bmatrix} -1.3440 & 0.56\sin(0.49\pi q) & 1.9040 \\ -1.3440 & 0.56\sin(0.49\pi q) & 1.9040 \end{bmatrix}, C_2(q) = C_4(q) = \begin{bmatrix} 1.0790 & 1.2450 \\ 1.1620 & 1.2450 \end{bmatrix}, \tag{60}$$

$$B_{d,1}(q) = B_{d,3}(q) = B_{d,5}(q) = \begin{bmatrix} 1.3 \\ 1.82 \\ 1.69 \end{bmatrix}, B_{d,2}(q) = B_{d,4}(q) = \begin{bmatrix} 1.17 \\ 1.69 \end{bmatrix}, B_{f,1}(q) = B_{f,3}(q) = B_{f,5}(q) = \begin{bmatrix} 0.72 \\ 0.6 \\ 0.44 \end{bmatrix}, \tag{61}$$

$$B_{f,2}(q) = B_{f,4}(q) = \begin{bmatrix} 0.66 \\ 1.14 \end{bmatrix}, D_{d,1}(q) = D_{d,3}(q) = D_{d,5}(q) = \begin{bmatrix} 1.17 \\ 1.69 \end{bmatrix}, D_{d,2}(q) = D_{d,4}(q) = \begin{bmatrix} 1.26 \\ 1.82 \end{bmatrix}, \tag{62}$$

$$D_{f,1}(q) = D_{f,3}(q) = D_{f,5}(q) = \begin{bmatrix} 2.52 \\ 2.1 \end{bmatrix}, D_{f,2}(q) = D_{f,4}(q) = \begin{bmatrix} 0.88 \\ 1.12 \end{bmatrix}. \tag{63}$$

The initial states $x_1(0), \dots, x_5(0)$ for each of agents are selected randomly with compatible dimensions. The stochastic nonlinearities $g_i(q, x_i(q), \delta_i(q))$ and $q_i(q, x_i(q), \zeta_i(q))$ are selected as in[26], which are independent of the random initial states.

$$g_i(q,x_i(q),\delta_i(q)) = \begin{bmatrix} 0.2 \\ 0.12 \\ 0.14 \end{bmatrix} (0.3\mathrm{sign}(x_i^{(1)}(q))x_i^{(1)}(q)\delta_i^{(1)}(q) + 0.2\mathrm{sign}(x_i^{(2)}(q))x_i^{(2)}(q)\delta_i^{(2)}(q)$$
$$+ 0.3\mathrm{sign}(x_i^{(3)}(q))x_i^{(3)}(q)\delta_i^{(3)}(q)), i \in \{1,3,5\}, \tag{64}$$
$$g_i(q,x_i(q),\delta_i(q)) = \begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix} (0.1\mathrm{sign}(x_i^{(1)}(q))x_i^{(1)}(q)\delta_i^{(1)}(q) + 0.2\mathrm{sign}(x_i^{(2)}(q))x_i^{(2)}(q)\delta_1^{(2)}(q)), i \in \{2,4\}, \tag{65}$$
$$q_i(q,x_i(q),\zeta_i(q)) = \begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix} (0.3\mathrm{sign}(x_i^{(1)}(q))x_i^{(1)}(q)\zeta_i^{(1)}(q) + 0.2\mathrm{sign}(x_i^{(2)}(q))x_i^{(2)}(q)\zeta_i^{(2)}(q)$$
$$+ 0.3\mathrm{sign}(x_i^{(3)}(q))x_i^{(3)}(q)\zeta_i^{(3)}(q)), i \in \{1,3,5\}, \tag{66}$$
$$q_i(q,x_i(q),\zeta_i(q)) = \begin{bmatrix} 0.3 \\ 0.14 \end{bmatrix} (0.1\mathrm{sign}(x_i^{(1)}(q))x_i^{(1)}(q)\zeta_i^{(1)}(q) + 0.2\mathrm{sign}(x_i^{(2)}(q))x_i^{(2)}(q)\zeta_i^{(2)}(q), i \in \{2,4\}. \tag{67}$$

The terms $x_i^{(j)}(q)$, $\delta_i^{(j)}(q)$ and $\zeta_i^{(j)}(q)$ denote the j_{th} elements of the system state $x_i(q)$, the stochastic variables $\delta_i(q)$ and $\zeta_i(q)$, respectively. $\delta_i(q)$ and $\zeta_i(q)$ are independent of the zero mean Gaussian noise sequences with unit variances. Obviously, the expectations and the covariances of the above stochastic nonlinearities meet the form defined in (2)-(4) with the integer $m = 1$. The parameter matrices involved are naturally arranged as

$$\Pi_{p,1}^j(q) = \begin{bmatrix} 0.2 \\ 0.12 \\ 0.14 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.12 \\ 0.14 \end{bmatrix}^T, \ j = 1,3,5, \ \Pi_{p,1}^j(q) = \begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix}^T, \ j = 2,4, \tag{68}$$
$$\Pi_{p,2}^j(q) = \begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix}^T, \ j = 1,3,5, \ \Pi_{p,2}^j(q) = \begin{bmatrix} 0.3 \\ 0.14 \end{bmatrix}\begin{bmatrix} 0.3 \\ 0.14 \end{bmatrix}^T, \ j = 2,4, \tag{69}$$
$$\Gamma_{p,i}(q) = \mathrm{diag}\{0.09, 0.04, 0.09\}, \ i = 1,3,5, \ \Gamma_{p,i}(q) = \mathrm{diag}\{0.01, 0.04\}, \ i = 2,4. \tag{70}$$

Suppose the topology utilized to exchange information is shown by the undirected graph in Figure. 2, where all the edge weights between the corresponding two agents are 1.

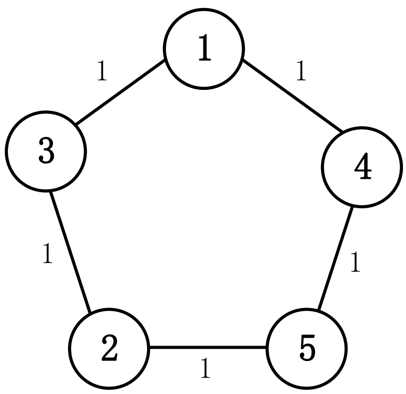

FIGURE 2 Communication topology of the considered MASs

Set the finite time window length $p = 3$. It it supposed that the sine-wave signal, acted in the following manner, affects the agents 1 and 3.

$$f_1(q) = \begin{cases} 1.6\sin(0.24\pi q), & 35 \le q \le 70, \\ 0, & \text{others}, \end{cases}, \ f_3(q) = \begin{cases} 0.8\cos(0.25\pi q), & 35 \le q \le 70, \\ 0, & \text{others}. \end{cases} \tag{71}$$

And the disturbance $d_i(q)$ $(i = 1, 2, \dots, 5)$ corresponding to each agent is illustrated by Figure. 3.

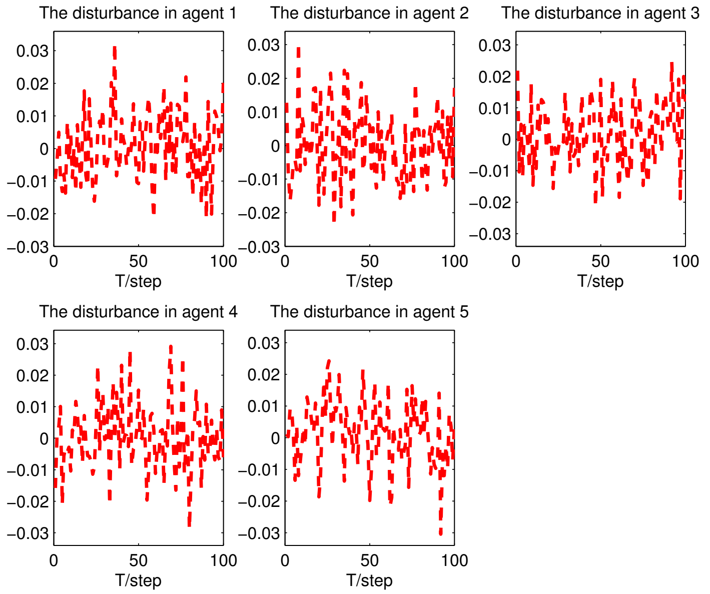

FIGURE 3 The distuabance involued in each agent

Select the agent 1 as the key agent and equip it with FIR filter based fault estimator (20), where $W_f = \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}, W_f \in R^{3\times 12}$. Apply the proposed method, it can be clearly seen from Figure. 4 and Figure. 5 that the fault characteristics corresponding to agents 1 and 3 are tracked timely and accurately even in the presence of stochastic nonlinearity.

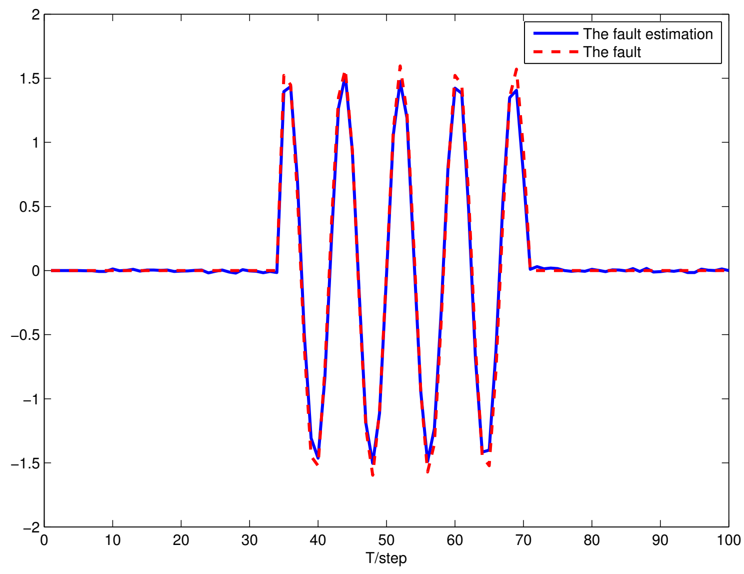

FIGURE 4 The time response of the fault signal and its estimation for agent 1

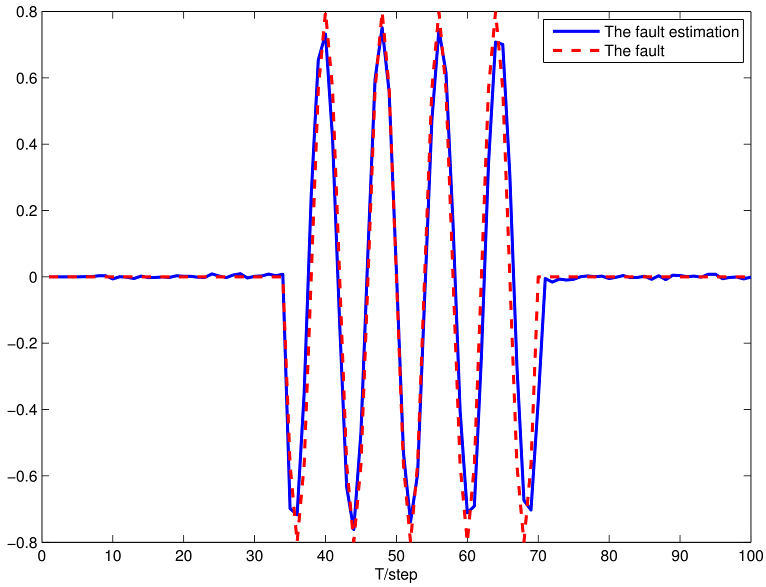

FIGURE 5 The time response of the fault signal and its estimation for agent 3

As described in [38], multiplicative noise acts as a special case of stochastic nonlinearity. Following the inequality (43), obtaining the optimal filter by directly solving the minimum points of $\mathrm{tr}(T_i^T(q)T_i(q))$ or $\mathrm{tr}(T_i^T(q)T_i(q))\mathrm{tr}(M_i^b(q))$ is the way that adopted in [36] for fault estimation of linear discrete time-varying systems with multiplicative noise. Now, let's compare our work with [36] in this regard to further illustrate the advantage of the proposed fault estimation algorithm.

In this part, define mean square error (MSE) as

$$\mathrm{MSE} = \frac{\sum_{j=1}^{M}(\bar{f}_i^j(q) - \hat{\bar{f}}_i^j(q))^2}{M}, \tag{72}$$

to quantify how much improvement our novel performance index has been made, where $\bar{f}_i^j(q)$ is the real fault of the j_{th} simulation for the i_{th} agent, $\hat{\bar{f}}_i^j$ is the corresponding fault estimation, and M is the total number of simulations.

Set $M = 200$. The MSE at each time instant in [36] and this paper is calculated respectively following (72) under the same conditions within the time interval [0, 100]. Obviously, the novel performance index can provide more accurate fault estimation results according to Figure. 6.

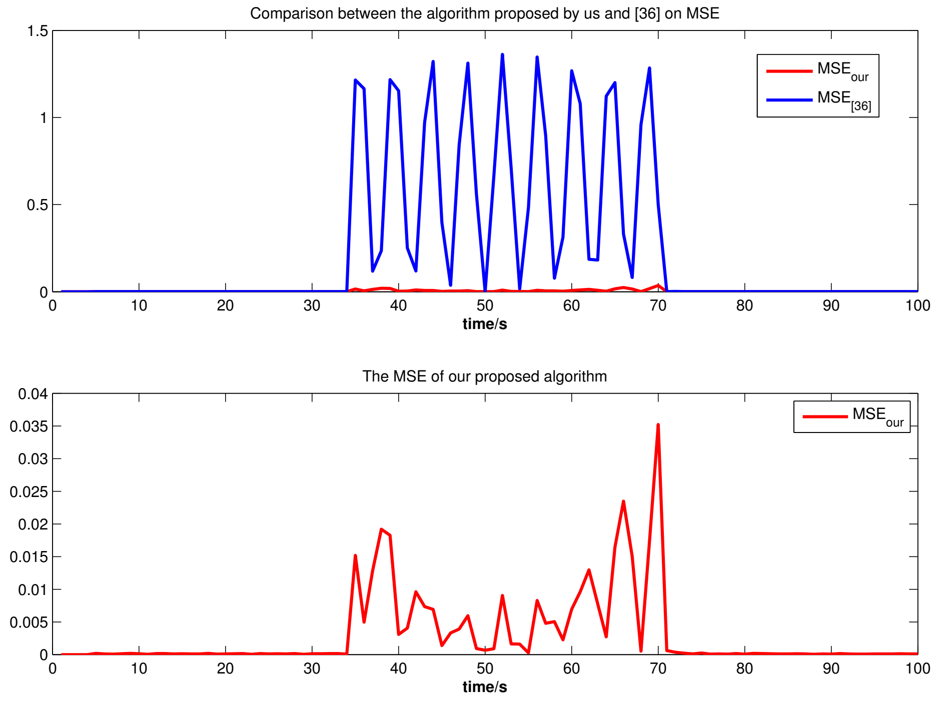

FIGURE 6 Comparision between the algorithm proposed in this paper and [36]

Furthermore, the actual calculation amount between the two algorithms can be obtained by following Table 1, and specifically,

$$\begin{aligned}
\mathrm{MD}_{ours} =& 3m(n_1+n_3+n_4)^2(p+1)+36m^3(p+1)+3m(n_1+n_3+n_4)(n_{d1}+n_{d3}+n_{d4}+n_{f1}+n_{f3}+n_{f4})(p+1) \\
=& 744, \qquad (73) \\
\mathrm{MD}_{[32]} =& 3m(n_1+n_3+n_4)^2\frac{3p^2-p}{2}+9m^3\frac{3p^2+5s+2}{2}+3m(n_1+n_3+n_4)(n_{d1}+n_{d3}+n_{d4}+n_{f1}+n_{f3}+n_{f4})p \\
&+(n_1+n_3+n_4)^3\frac{p^3+6p^2-p}{6}+(n_1+n_3+n_4)^2(n_{d1}+n_{d3}+n_{d4}+n_{f1}+n_{f3}+n_{f4})p \\
=& 14000. \qquad (74)
\end{aligned}$$

It is clear to see that our proposed algorithm based on the novel analytical redundancy (13) enjoys obvious computational advantages. To summarize, all the simulation results lead to the conclusion that the proposed algorithm provides clear evidences for the effectiveness and flexibility of the developed FIR based fault estimator for a class of heterogeneous multi-agents with stochastic nonlinearities.

6 | CONCLUSION

In this paper, FIR filter has been utilized to deal with cooperative fault estimation problem for a class of heterogeneous MASs with stochastic nonlinearities. To this end, an auxiliary augmentation system has been presented via integrating possible faults from each agent and its neighbors induced from the interconnections. This allows the filter to be equipped on a selected key agent to realize the fault estimation of itself and its neighbors simultaneously. All the system states in the previous time window have been originally utilized to construct analytical redundancy. The challenge of extracting prior variance information coupled with fault estimation error in nonlinear form to design performance index is solved. By solving the minimum point problem of the matrix trace function, the optimal FIR filter gain has been obtained with obvious computational advantages, which has been shown by the specific calculation cost analysis when compared with the previous work. At last, the numerical simulations have been provided to verify the feasibility and superiority of the proposed fault estimation approach.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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APPENDIX A: PROOF OF PROPOSITION 1

Associated with (2)-(4), it can be easily obtained that

$$\mathrm{E}(G_i(q-1)G_i^T(q-1)) = \mathrm{diag}\{g_i^{11}, \dots, g_i^{1(p+1)}, g_i^{21}, \dots, g_i^{2(p+1)}, \dots, g_i^{N1}, \dots, g_i^{N(p+1)}\}, \tag{75}$$

where

$$\begin{aligned} g_i^{11} =& g_i(q-p-1)g_i^T(q-p-1) = \sum_{p=1}^{m} \Pi_{p,1}^i(q-p-1)\mathrm{tr}(\Gamma_{p,i}(q-p-1)x_i(q-p-1)x_i^T(q-p-1)), \\ g_i^{1(p+1)} =& g_{i_{|M_i|}}(q-p-1)g_{i_{|M_i|}}^T(q-p-1) \qquad (76) \\ =& \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-p-1)\mathrm{tr}(\Gamma_{p,i}(q-p-1)x_{i_{|M_i|}}(q-p-1)x_{i_{|M_i|}}^T(q-p-1)), \qquad (77) \end{aligned}$$

$$g_i^{21} = g_i(q-p)g_i^T(q-p) = \sum_{p=1}^{m} \Pi_{p,1}^i(q-p)\mathrm{tr}(\Gamma_{p,i}(q-p)x_i(q-p)x_i^T(q-p)), \tag{78}$$

$$g_i^{2(p+1)} = g_i^{i_{|M_i|}}(q-p)g_{i_{|M_i|}}^T(q-p) = \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-p)\mathrm{tr}(\Gamma_{p,i}(q-p)x_{i_{|M_i|}}(q-p)x_{i_{|M_i|}}^T(q-p)), \tag{79}$$

$$g_i^{N1} = g_i(q-1)g_i^T(q-1) = \sum_{p=1}^{m} \Pi_{p,1}^i(q-1)\mathrm{tr}(\Gamma_{p,i}(q-1)x_i(q-1)x_i^T(q-1)), \tag{80}$$

$$g_i^{N(p+1)} = g_i^{i_{|M_i|}}(q-1)g_{i_{|M_i|}}^T(q-1) = \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-1)\mathrm{tr}(\Gamma_{p,i}(q-1)x_{i_{|M_i|}}(q-1)x_{i_{|M_i|}}^T(q-1)). \tag{81}$$

In light of the semi-positive nature of known matrix $\Gamma_{p,i}(q)$ and the quadratic matrix $x_i(q)x_i^T(q)$, the following inequalities hold:

$$g_i^{11} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i}(q-p-1)\mathrm{tr}(\Gamma_{p,i}(q-p-1))\mathrm{tr}(x_i(q-p-1)x_i^T(q-p-1)), \tag{82}$$

$$g_i^{1(p+1)} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-p-1)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-p-1))\mathrm{tr}(x_{i_{|M_i|}}(q-p-1)x_{i_{|M_i|}}^T(q-p-1)), \tag{83}$$

$$g_i^{21} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i}(q-p)\mathrm{tr}(\Gamma_{p,i}(q-p))\mathrm{tr}(x_i(q-p)x_i^T(q-p)), \tag{84}$$

$$g_i^{2(p+1)} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-p)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-p))\mathrm{tr}(x_{i_{|M_i|}}(q-p)x_{i_{|M_i|}}^T(q-p)), \tag{85}$$

$$g_i^{N1} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i}(q-1)\mathrm{tr}(\Gamma_{p,i}(q-1))\mathrm{tr}(x_i(q-1)x_i^T(q-1)), \tag{86}$$

$$g_i^{N(p+1)} \leq \sum_{p=1}^{m} \Pi_{p,1}^{i_{|M_i|}}(q-1)\mathrm{tr}(\Gamma_{p,i_{|M_i|}}(q-1))\mathrm{tr}(x_{i_{|M_i|}}(q-1)x_{i_{|M_i|}}^T(q-1)). \tag{87}$$

Recalling the definition of $M_{gi}(q-1)$ and $N_{gi}(q-1)$ in (31)-(32), one obtains

$$\mathrm{E}(G_i(q-1)G_i^T(q-1)) \leq M_{gi}(q-1)N_{gi}(q-1). \tag{88}$$

It can be obtained that $\mathrm{E}(Q_i(q)Q_i^T(q)) \leq M_{qi}(q)N_{qi}(q)$ following the same reasoning as for (88). This completes the proof.

$\square$