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Decentralised state feedback stabilisation for nonlinear interconnected systems using sliding mode control*

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ABSTRACT

In this paper, the stabilisation problem is considered for a class of nonlinear interconnected systems with matched uncertainties and mismatched unknown interconnections. A composite sliding surface is designed firstly, and a set of conditions is developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. Then, decentralised state feedback sliding mode control is proposed to drive the interconnected systems to the designed sliding surface in finite time, and a sliding motion is maintained thereafter. The bounds on the uncertainties and interconnections have more general nonlinear forms, which are employed in the control design to reject the effects of uncertainties and unknown interconnections to enhance the robustness. It is not required either the isolated nominal subsystems linearisable or the interconnections linearisable. Finally, a numerical simulation example is presented to demonstrate the effectiveness of the proposed control strategy.

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Nonlinear interconnected systems; decentralised control; state feedback; sliding mode control; stabilisation

1. Introduction

With the advancement of scientific technology, many industrial and commercial systems have become more complex, which can be modelled by large-scale interconnected systems. These systems usually consist of a set of composite objects through interactions, which are possibly different sorts of physical, natural, and artificial dynamics. Such a class of systems widely exists in the real world, for instance, modern power systems, transportation systems, aircraft systems and robot systems (Dang et al., 2020; Xiang et al., 2021; Yan et al., 2017). In reality, the existence of nonlinearities, uncertainties and interconnections makes the analysis and design for interconnected systems very difficult. Moreover, practical systems are prone to be affected by internal and external disturbances including modelling errors, parameter variations, temperature changes, pressure and mechanical loss, etc. Therefore, the study of complex interconnected systems with uncertainties and disturbances is full of challenges.

It should be noted that centralised control and decentralised control are two different approaches. Centralised control allows each controller to use the whole system states/outputs information and thus developed results usually have low conservatism. In decentralised control, each controller only adopts local states/outputs information of its own subsystem and cannot use the other subsystems' information. Therefore, decentralised control does not need information transfer between subsystems which can avoid the data transfer cost when compared with a centralised scheme. Moreover, if information transfer channels are blocked, centralised control usually does not work but decentralised control is not affected by it. Therefore, decentralised control is convenient for practical implementation, particularly when the interconnected systems are distributed in a large space. To be specific, decentralised control law consists of several local controllers, and each of these local controllers only uses its local state information of the corresponding subsystem. So the structure of decentralised control is usually more effective than centralised control from the implementation point of view. In the last few decades, decentralised control has received much attention and

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many results have been achieved (Misgeld et al., 2015; Sombría et al., 2012; Zhang, 2020; Zhou et al., 2020).

Sliding mode control (SMC) due to its high robustness has been recognised as an effective method for control systems with uncertainties. Niu et al. (2008) considered the sliding-mode control for nonlinear stochastic systems modelled by Ito stochastic differential equations. Yan et al. (2005) proposed a modified SMC which was able to deal with mismatched uncertainties, where dynamic feedback was employed. R. Xu and Özgüner (2008) presented an innovative SMC to stabilise a kind of under-actuated system in the cascade case. Khan et al. (2011) proposed a novel dynamic integral sliding mode controller for state-dependent matched and mismatched uncertainties. Kang and Fridman (2016) imposed an SMC combined with a backstepping method to control a cascade of equation-ODE systems with matched and mismatched disturbances. A neural network fuzzy SMC presented by Chiang and Chen (2017) was applied to pneumatic muscle actuators, where an adaptive training used neural network was able to establish a fuzzy SMC controller, and an integrator could minimise the tracking error. Feng et al. (2018) adopted the terminal SMC and the full-order terminal SMC to improve the performance of multiple-input multiple-output systems with mismatched uncertainties, respectively. It should be noted that all of the results mentioned above only considered centralised systems.

Recently, many researchers have focused on the decentralised SMC of the interconnected systems. Azizi et al. (2010) applied the decentralised SMC to the distributed simulation of differential-algebraic equation systems. But the proposed method was not suitable for general nonlinear systems. An adaptive decentralised SMC for a class of non-affine stochastic nonlinear interconnected systems was presented in Ning et al. (2016), which just estimated one adaptive parameter of each subsystem. However, uncertainties were not considered in Ning et al. (2016). Furthermore, a kind of novel decentralised state-feedback adaptive SMC was imposed by Mirkin et al. (2011) to large-scale interconnected systems with nonlinear interconnections and time-delay. The global decentralised discrete SMC for interconnected systems based on output feedback was employed by Mahmoud and Qureshi (2012). Although these two strategies achieved good results for specific interconnected systems, it is required that all the isolated subsystems are linear. A decentralised integral SMC combined with PID was proposed in Thien and Kim (2018) for unmanned aerial vehicles, where the control sensitivity with respect to the network topology was analysed, but the mismatched uncertainties were not considered. Ark et al. (2020) proposed an SMC scheme for load frequency problems in two area interconnected power systems, where mismatched uncertainties were not considered. A model-free decentralised sliding mode control (SMC) scheme for each subsystem is designed in Song et al. (2020) where it is required that the interconnection terms are matched and the considered interconnected systems are linear. Although many researchers have obtained the remarkable achievements of decentralised SMC, few people concentrated on the nonlinear interconnected systems with mismatched uncertainties and unknown interconnections at the same time. Due to the complexity of nonlinear systems, the technology of SMC combined with decentralised control for nonlinear interconnected systems with unknown interconnections is challenging and significant.

In this paper, a state feedback decentralised SMC scheme is proposed to stabilise a class of nonlinear interconnected systems. The considered interconnected systems possess both nonlinear interconnections and nonlinear isolated subsystems. A coordinate transformation is applied to transform all the isolated subsystems into the regular form to facilitate the controller design as well as the interconnected system analysis. Then, for the transformed system, a composite sliding surface is designed, and a set of conditions are developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable based on the Lyapunov theory. A state feedback SMC law is established to drive the system to the sliding surface in finite time and keep the sliding motion after that. The bounds on all uncertainties and interconnections have general nonlinear forms, which are employed in the decentralised control design to reduce the effects of uncertainties. It is shown that under certain conditions, the effect of the unknown interconnections can be completely cancelled by appropriate designed decentralised controllers with regard to the reachability analysis. At last, a numerical simulation example is provided to demonstrate the effectiveness of the proposed control strategy.

2. System description and problem formulation

For simplification of statement as well as readers' convenience, a few concepts are introduced at first.

Definition 2.1 (Khalil, 2002): A continuous function $\alpha : [0, a) \longrightarrow [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Definition 2.2 (Yan et al., 2014): A class $\mathcal{K}$ function is said to belong to class $\mathcal{KC}^1$ if it is continuously differentiable.

Consider nonlinear time-varying interconnected systems with matched disturbances and unknown interconnections consisted of n interconnected subsystems,

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)(u_i + \varphi_i(t, x_i)) + h_i(t, x), \quad i = 1, 2, \ldots, n, \tag{1}$$

where $x_i \in \Omega_i \subset \mathcal{R}^{n_i}$ (Ω_i denotes a neighbourhood of the origin), and $u_i \in \mathcal{R}^{m_i}$ are, respectively, state variables and inputs of the ith subsystem with $m_i < n_i$, $x := \text{col}(x_1, x_2, \ldots, x_n) \in \Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n$. It is assumed that the matrix function $g_i(\cdot) \in \mathcal{R}^{n_i \times m_i}$ is known and has full column rank; the nonlinear vector $f_i(\cdot) \in \mathcal{R}^{n_i}$ is known. The term $\varphi_i(\cdot)$ denotes matched disturbance, and $h_i(\cdot)$ represents the unknown interconnection. All nonlinear functions are assumed to be continuous in their arguments in the considered domain to guarantee the existence of system solutions.

Some definitions for system (1) are to be introduced as follows.

Definition 2.3: Consider system (1). The following system

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)(u_i + \varphi_i(t, x_i)), \quad i = 1, 2, \ldots, n, \tag{2}$$

is called the ith isolated subsystem of system (1). The system

$$\dot{x}_i = f_i(t, x_i) + g_i(t, x_i)u_i, \quad i = 1, 2, \ldots, n, \tag{3}$$

is called the ith nominal isolated subsystem of system (1).

It is well known that one of the major issues for interconnected systems is to design a controller such that interconnected system (1) has the desired performance if all nominal isolated subsystems (3) exhibit the required performance. Compared with centralised control, one of the important challenges for interconnected systems is to deal with interconnections, because for the decentralised case each controller is only allowed to use its own/local state information. The definition of the decentralised control is given as follows.

Definition 2.4 (Yan et al., 2017): Consider system (1). If the controller u_i for the ith subsystem only depends on the time t and states x_i, that is,

$$u_i = u_i(t, x_i), \quad i = 1, 2, \ldots, n. \tag{4}$$

Then, control (4) is called the decentralised state feedback controller for system (1).

Now, consider a nonlinear transformation,

$$z_i = T_i(x_i), \quad i = 1, 2, \ldots, n, \tag{5}$$

which is a diffeomorphism, i.e. the Jacobian matrices $\partial T_i / \partial x_i$ is nonsingular in the considered domain for $i = 1, 2, \ldots, n$. Then, transformation (5) defines a new coordinate $z = col(z_1, z_2, \ldots, z_n)$. In the new coordinate z, system (1) can be described by

$$\begin{aligned} \dot{z}_i &= \left[\frac{\partial T_i}{\partial x_i} \dot{x}_i \right]_{x_i = T_i^{-1}(z_i)} \\ &= \left[\frac{\partial T_i}{\partial x_i} \big(f_i(t, x_i) + g_i(t, x_i) \right. \\ &\quad \left. \cdot \, (u_i + \varphi_i(t, x_i)) + h_i(t, x) \big) \right]_{x_i = T_i^{-1}(z_i)}, \\ &\quad i = 1, 2, \ldots, n. \end{aligned} \tag{6}$$

It is assumed that system (1) in the new coordinates z can be described by

$$\dot{z}_{i1} = F_{i1}(t, z_{i1}, z_{i2}) + H_{i1}(t, z), \tag{7}$$

$$\begin{aligned} \dot{z}_{i2} &= F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2}) \\ &\quad \cdot (u_i + \Phi_i(t, z_{i1}, z_{i2})) + H_{i2}(t, z), \end{aligned} \tag{8}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$, $z_{i2} \in \Omega_{z_{i2}} \subset \mathcal{R}^{m_i}$, $z = \text{col}(z_1, z_2, \ldots, z_n)$, $z_i = \text{col}(z_{i1}, z_{i2}) \in \Omega_{T_i} \subset \mathcal{R}^{n_i}$,

$$\begin{aligned} \Omega_{T_i} := \Omega_{z_{i1}} \times \Omega_{z_{i2}} &:= \{(z_{i1}, z_{i2}) \mid (z_{i1}, z_{i2}) \\ &= T_i(x_i), x_i \in \Omega_i\}, \end{aligned}$$

and

$$\begin{bmatrix} F_{i1}(\cdot) \\ F_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} f_i(t, x_i)\right]_{x_i = T_i^{-1}(z_i)}, \tag{9}$$

$$H_i(\cdot) := \begin{bmatrix} H_{i1}(\cdot) \\ H_{i2}(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} h_i(t, x)\right]_{x_i = T_i^{-1}(z_i)}, \tag{10}$$

$$\begin{bmatrix} 0 \\ G_i(\cdot) \end{bmatrix} := \left[\frac{\partial T_i}{\partial x_i} g_i(t, x_i)\right]_{x_i = T_i^{-1}(z_i)}, \tag{11}$$

$$\Phi_i(\cdot) := [\varphi_i(t, x_i)]_{x_i = T_i^{-1}(z_i)}, \tag{12}$$

where $G_i(\cdot) \in \mathcal{R}^{m_i \times m_i}$ is nonsingular in the considered domain Ω_{T_i} for $i = 1, 2, \ldots, n$.

It should be noted that systems (7)–(8) are in the traditional regular form, which is very useful for the constructive application of the sliding mode paradigm.

Remark 2.1: It should be pointed out that there is no systematic method to find a coordinate transformation (5) to transfer system (1) to regular form (7)–(8). But the work in Marino and Tomei (1995) and Yan et al. (2014) can be referred to construct the corresponding transformation in certain cases.

In the following, the nonlinear interconnected systems (7)–(8) will be focused. The objective of this paper is to develop a state feedback decentralised SMC scheme, such that controlled systems (7)–(8) are uniformly asymptotically stable irrespective of disturbances and unknown interconnections. It should be emphasised that the results developed in this paper can be easily extended to all interconnected systems (1) which can be transformed to systems (7)–(8) by a known nonsingular transformation.

3. Sliding motion analysis and control synthesis

In this section, the sliding surface will be designed and the corresponding sliding motion is to be analysed. Then, a novel decentralised SMC strategy is to be proposed under the assumption that all system states are accessible.

3.1. Stability of sliding motion

Based on the specific structure of systems (7)–(8), the switching function for the ith subsystem can be selected as

$$s_i(z_i) = z_{i2}, \quad i = 1, 2, \ldots, n. \tag{13}$$

Then, the composite sliding function for interconnected systems (7)–(8) is given as

$$\begin{aligned} S(z) &= \operatorname{col}(s_1(z_1), s_2(z_2), \ldots, s_n(z_n)) \\ &= \operatorname{col}(z_{12}, z_{22}, \ldots, z_{n2}). \end{aligned} \tag{14}$$

So, the composite sliding surface is written by

$$\begin{aligned} \{&\operatorname{col}(z_1, z_2, \ldots, z_n) \mid s_i(z_i) \\ &= z_{i2} = 0 \text{ for } i = 1, 2, \ldots, n\}. \end{aligned} \tag{15}$$

When the interconnected system is limited to moving on the sliding surface (15), $z_{i2} = 0$ for $i = 1, 2, \ldots, n$. It follows from the structure of systems (7)–(8) that the corresponding sliding mode dynamics can be described by

$$\begin{aligned} \dot{z}_{i1} &= F_{i1s}(t, z_{i1}) + H_{i1s}(t, z_{11}, z_{21}, \ldots, z_{n1}), \\ & i = 1, 2, \ldots, n, \end{aligned} \tag{16}$$

where $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$ denotes the state of the sliding mode dynamics, and

$$F_{i1s}(\cdot) := F_{i1}(t, z_{i1}, z_{i2})|_{z_{i2}=0}, \tag{17}$$

$$H_{i1s}(\cdot) := H_{i1}(t, z)|_{z_{12}=0, \ldots, z_{n2}=0}, \tag{18}$$

where $F_{i1}(\cdot)$ and $H_{i1}(\cdot)$ are defined in (9) and (10, respectively. From (10), it is clear to see that the term $H_{i1s}(\cdot)$ comes from $h_i(t, x)$, which represents the unknown interconnection of the ith subsystems in (16) for $i = 1, 2, \ldots, n$.

In order to analyse the sliding motion governed by interconnected system (16) and related to the composite sliding surface (15), the following assumptions are needed.

Assumption 3.1: There exists the continuously differentiable functions $V_i(t, z_{i1}) : \mathcal{R}^+ \times \mathcal{R}^{n_i - m_i} \mapsto \mathcal{R}^+$ for $i = 1, 2, \ldots, n$, such that for any $z_{i1} \in \Omega_{z_{i1}}$ the following inequalities hold:

(i) $p_{i1}^2(\|z_{i1}\|) \le V_i(t, z_{i1}) \le p_{i2}^2(\|z_{i1}\|)$;

(ii) $\frac{\partial V_i(\cdot)}{\partial t} + (\frac{\partial V_i(\cdot)}{\partial z_{i1}})^T F_{i1s}(t, z_{i1}) \le -p_{i3}^2(\|z_{i1}\|)$;

(iii) $\|(\frac{\partial V_i(\cdot)}{\partial z_{i1}})^T\| \le p_{i4}(\|z_{i1}\|)$,

where the functions $p_{il}(\cdot)$ for $l = 1, 2, 3, 4$ are class $\mathcal{KC}^1$ functions.

From Definition 2.2, there are continuous functions $\varsigma_{il}(\cdot)$ such that for any $z_{i1} \in \Omega_{z_{i1}}$, $p_{il}(\cdot)$ can be decomposed as

$$p_{il}(\|z_{i1}\|) = \varsigma_{il}(\|z_{i1}\|)\|z_{i1}\|, \quad l = 1, 2, 3, 4, \tag{19}$$

where $\varsigma_{il}(\cdot)$ are continuous functions in $\mathcal{R}^+$ for $i = 1, 2, \ldots, n$ and $l = 1, 2, 3, 4$.

Remark 3.1: Assumption 3.1 implies that all the nominal isolated subsystems of interconnected system (16) are uniformly asymptotically stable. It is worth clarifying that Assumption 3.1 is usually required when the nominal sliding mode dynamics are fully nonlinear (see e.g. Yan et al., 2013, 2014, 2017). Moreover, if the nominal system is exponentially stable, then Assumption 3.1 will be satisfied. It should be mentioned that the fact that $\dot{z}_{i1} = F_{i1s}(t, z_{i1})$ is uniformly asymptotically stable does not mean that nominal system (7) is uniformly asymptotically stable. It should be pointed out that in Mahmoud and Qureshi (2012) and Yan et al. (2013), the whole interconnected systems need to satisfy the constraint conditions to ensure all the nominal isolated blue subsystems of (7)–(8) are asymptotically stable, while only (16) needs to satisfy the related conditions to guarantee that nominal isolated subsystems of the reduced-order subsystems (16) are asymptotically stable in this paper. Therefore, the approach proposed in this paper has more advantages than the work mentioned above in this regard.

Assumption 3.2: The interconnection term $H_{i1s}(\cdot)$ in system (16) satisfies

$$\begin{aligned}&\|H_{i1s}(t, z_{11}, z_{21}, \ldots, z_{n1})\| \\ &\quad \le \beta_i(t, z_{11}, z_{21}, \ldots, z_{n1}) \sum_{j=1}^{n} \|z_{j1}\|,\end{aligned} \tag{20}$$

where $\beta_i(\cdot)$ are known continuous functions for $i = 1, 2, \ldots, n$.

Remark 3.2: Assumption 3.2 ensures that the interconnections in (16) are bounded by known functions. However, the method developed in this paper can be applied to a wider class of interconnections, for example, (20) can be replace by

$$\begin{aligned}\|H_{i1s}(\cdot)\| \le\; &\beta_{1i}(\cdot)\|z_{11}\| + \beta_{2i}(\cdot)\|z_{21}\| \\ &+ \cdots + \beta_{ni}(\cdot)\|z_{n1}\|.\end{aligned}$$

It is required that β_{ji} are the constants for $i, j = 1, 2, \ldots, n$ in Mahmoud and Qureshi (2012). In this paper, $\beta_{ji}(\cdot)$ are known continuous functions which include the interconnections considered in Mahmoud and Qureshi (2012) as a special case in this regard. In reality, the bounds on uncertainties for a specific practical system usually can be obtained/estimated based on the prior knowledge and engineering experiences as well as statistical/historical data collected for the considered system. It should be noted that under certain conditions, the method proposed in Y. Yang Niu (2020) can be applied if the bounds on uncertainties are unknown.

The following result is ready to be presented.

Theorem 3.1: *Under Assumptions* 3.1 *and* 3.2*, the sliding motion associated with the sliding surface* (15) *of system* (7)–(8) *is uniformly asymptotically stable if the function matrix* $M^T(\cdot) + M(\cdot) > 0$ *in the considered domain* $z_{i1} \in \Omega_{z_{i1}} \subset \mathcal{R}^{n_i - m_i}$*, where* $M = (m_{ij}(\cdot))_{n\times n}$ *is a* $n \times n$ *function matrix with its entries defined by*

$$m_{ij} = \begin{cases} \varsigma_{i3}^2(\cdot) - \varsigma_{i4}(\cdot)\beta_i(\cdot), & i = j, \\ -\varsigma_{i4}(\cdot)\beta_i(\cdot), & i \ne j, \end{cases} \tag{21}$$

where $\varsigma_{i3}(\cdot)$ *and* $\varsigma_{i4}(\cdot)$ *are given in (19) and* $\beta_i(\cdot)$ *is defined in (20) for* $i, j = 1, 2, \ldots, n$.

Proof: From the analysis above, it is clear to see that system (16) is the sliding mode dynamics related to the composite sliding surface (15). The remaining is to show that system (16) is uniformly asymptotically stable.

Under the condition that $p_{il}(\cdot)$ is class $\mathcal{KC}^1$ function, the equations in (19) hold. For system (16), consider the candidate Lyapunov function

$$V(t, z_{11}, z_{21}, \ldots, z_{n1}) = \sum_{i=1}^{n} V_i(t, z_{i1}), \tag{22}$$

where $V_i(\cdot)$ is defined in Assumption 3.1. The time derivative of $V(\cdot)$ along the trajectory of system (16) is described as

$$\begin{aligned}&\dot{V}(t, z_{11}, z_{21}, \ldots, z_{n1}) \\ &\quad = \sum_{i=1}^{n} \dot{V}_i(t, z_{i1}) \\ &\quad = \sum_{i=1}^{n} \left(\frac{\partial V_i(\cdot)}{\partial t} + \left(\frac{\partial V_i(\cdot)}{\partial z_{i1}} \right)^T (F_{i1s}(\cdot) + H_{i1s}(\cdot)) \right)\end{aligned}$$

$$\leq \sum_{i=1}^{n}\left(\frac{\partial V_i(\cdot)}{\partial t}+\left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T F_{i1s}(\cdot)\right.$$
$$\left.+\left\|\left(\frac{\partial V_i(\cdot)}{\partial z_{i1}}\right)^T\right\|\cdot\|H_{i1s}(\cdot)\|\right). \tag{23}$$

From Assumptions 3.1 and 3.2, Equation (23) can be written as follows:

$$\dot V(t,z_{11},z_{21},\dots,z_{n1})$$
$$\leq \sum_{i=1}^{n}\left(-p_{i3}^2(\|z_{i1}\|)+p_{i4}(\|z_{i1}\|)\cdot\beta_i(\cdot)\sum_{j=1}^{n}\|z_{j1}\|\right). \tag{24}$$

According to Equation (19), it follows that

$$\dot V(t,z_{11},z_{21},\dots,z_{n1})$$
$$\leq \sum_{i=1}^{n}\left(-\varsigma_{i3}^2(\|z_{i1}\|)\|z_{i1}\|^2\right.$$
$$\left.+\ \varsigma_{i4}(\|z_{i1}\|)\|z_{i1}\|\beta_i(\cdot)\sum_{j=1}^{n}\|z_{j1}\|\right)$$
$$=-\sum_{i=1}^{n}\left(\varsigma_{i3}^2(\|z_{i1}\|)-\varsigma_{i4}(\|z_{i1}\|)\beta_i(\cdot)\right)\|z_{i1}\|^2$$
$$-\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\varsigma_{i4}(\|z_{i1}\|)\cdot\beta_i(\cdot)\|z_{i1}\|\cdot\|z_{j1}\|$$
$$-\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\varsigma_{i4}(\|z_{i1}\|)\cdot\beta_i(\cdot)\|z_{i1}\|\cdot\|z_{j1}\|$$
$$=-\frac{1}{2}Z^T\left(M^T+M\right)Z, \tag{25}$$

where $Z := \text{col}(\|z_{11}\|,\|z_{21}\|,\dots,\|z_{n1}\|)$ and M is the $n\times n$ matrix with entries defined in (21). Hence, the result follows from $M^T+M>0$. ■

Remark 3.3: Theorem 3.1 provides a set of sufficient conditions under which the sliding mode is uniformly asymptotically stable. The function matrix M in Theorem 3.1 only depends on $\varsigma_{i3}(\cdot)$, $\varsigma_{i4}(\cdot)$ and $\beta_i(\cdot)$, which are determined by the given system. The condition that $M^T+M>0$ with M defined in (21) implies the limitation to the mismatched interconnections.

3.2. Reachability analysis

A set of conditions have been developed in Theorem 3.1 to guarantee the sliding motion stability of the considered interconnected systems (7)–(8). The objective now is to design a decentralised state feedback SMC such that the interconnected system is driven to the sliding surface (15) in finite time.

For interconnected system (7)–(8), the corresponding reachability condition based on the composite sliding surface is described by

$$S^T(z)\dot S(z)\leq -\eta\|S(z)\|, \tag{26}$$

where $S(z)$ is defined by (14) and η is a positive constant.

Consider system (7)–(8). The following assumption is introduced for further analysis and control design.

Assumption 3.3: The uncertainties $\Phi_i(t,z_{i1},z_{i2})$ and $H_{i2}(t,z)$ in (8) satisfy

$$\|\Phi_i(t,z_{i1},z_{i2})\|\leq \xi_{i1}(t,z_{i1},z_{i2}), \tag{27}$$
$$\|H_{i2}(t,z)\|\leq \sum_{j=1}^{n}\epsilon_{ij}(t,z_j), \tag{28}$$

where $\xi_{i1}(t,z_{i1},z_{i2})$ and $\epsilon_{ij}(t,z_j)$ are known continuous functions.

It should be noted that Assumption 3.3 is the limitation to system uncertainties as well as interconnections. It is clear to see that the bounds on the uncertainties and interconnections are fully nonlinear, which are to be employed in the control design to reject the effects of them on the system performance. Construct the control law

$$u_i=-G_i^{-1}(t,z_{i1},z_{i2})F_{i2}(t,z_{i1},z_{i2})-G_i^{-1}(t,z_{i1},z_{i2})k_i\cdot\text{sgn}(z_{i2})$$
$$-G_i^{-1}(t,z_{i1},z_{i2})\left(\|G_i(t,z_{i1},z_{i2})\|\xi_{i1}(t,z_{i1},z_{i2})\text{sgn}(z_{i2})\right.$$
$$\left.+\frac{n}{2}z_{i2}+\frac{1}{2}\frac{z_{i2}}{\|z_{i2}\|^2}\sum_{j=1}^{n}\epsilon_{ji}^2(t,z_i)\right),\quad i=1,2,\dots,n, \tag{29}$$

where $F_{i2}(\cdot)$ is given in (9), $\xi_{i1}(\cdot)$ and $\sum_{j=1}^{n}\epsilon_{ij}(t,z_j)$ are given in (27) and (28), respectively, sgn$(\cdot)$ is the usual signum function, and k_i is the control gain which is a positive constant.

Remark 3.4: From the control structure in (29), it follows that the functions $\epsilon_{ij}(t,z_j)$ need to be vanished at $z_j = 0$ for $i, j = 1, 2, \dots, n$. This implies that the unknown interconnections $H_{i2}(t, z)$ must be vanished at the origin $z_{i2} = 0$ for $i = 1, 2, \dots, n$. Otherwise, it may result in infinite control due to the term $\frac{z_{i2}}{\|z_{i2}\|^2}$.

Theorem 3.2: *Under Assumption 3.3, nonlinear interconnected system (7)–(8) can be driven to the sliding surface (15) in finite time by the designed controller in (29) and maintains a sliding motion on it thereafter.*

Proof: From the definition of $s_i(z_i)$ in (13) and system (8),

$$\begin{aligned} s_i^T(z)\dot{s}_i(z_i) &= z_{i2}^T \dot{z}_{i2} \\ &= z_{i2}^T \left(F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2})\right. \\ &\quad \left. \times\ (u_i + \Phi_i(t, z_{i1}, z_{i2})) + H_{i2}(t, z)\right). \end{aligned} \tag{30}$$

Note, in Equation (30), the state variable $z = col(z_1, z_2, \dots, z_N)$ is involved through the interconnection terms $H_{i2}(t, z)$. However, for a decentralised scheme, the control u_i can only use the local states z_i. In order to cancel the effects caused by interconnections using decentralised control, it is necessary to consider the composite sliding surface in (14).

Then, from (14) and (30), it follows that

$$\begin{aligned} S^T(z)\dot{S}(z) &= \sum_{i=1}^{n} s_i^T(z_i)\dot{s}_i(z_i) \\ &= \sum_{i=1}^{n} z_{i2}^T \left(F_{i2}(t, z_{i1}, z_{i2}) + G_i(t, z_{i1}, z_{i2})\right. \\ &\quad \left. \times\ (u_i + \Phi_i(t, z_{i1}, z_{i2})) + H_{i2}(t, z)\right). \end{aligned} \tag{31}$$

Substituting the control u_i in (29) into Equation (31),

$$\begin{aligned} S^T(z)\dot{S}(z) = \sum_{i=1}^{n} z_{i2}^T \Bigg(& F_{i2}(\cdot) + G_i(\cdot)\Bigg(- G_i^{-1}(\cdot)F_{i2}(\cdot) \\ & - G_i^{-1}(\cdot)\left(\|G_i(\cdot)\|\xi_{i1}(\cdot)\mathrm{sgn}(z_{i2}) + \frac{n}{2}z_{i2}\right. \\ & \left. + \frac{1}{2}\frac{z_{i2}}{\|z_{i2}\|^2}\sum_{j=1}^{n}\epsilon_{ji}^2(\cdot)\right) \cdot - G_i^{-1}(\cdot)k_i \\ & \cdot \mathrm{sgn}(z_{i2}) + \Phi_i(\cdot)\Bigg) + H_{i2}(\cdot)\Bigg). \end{aligned} \tag{32}$$

Rearranging the associated terms in Equation (32), it follows that

$$\begin{aligned} & S^T(z)\dot{S}(z) \\ &= \sum_{i=1}^{n} \Bigg(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\|\xi_{i1}(\cdot) z_{i2}^T \mathrm{sgn}(z_{i2}) \\ &\quad + z_{i2}^T H_{i2}(\cdot) - \left(\frac{n}{2}z_{i2}^T z_{i2} + \frac{1}{2}\frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2}\sum_{j=1}^{n}\epsilon_{ji}^2(\cdot)\right) \\ &\quad - k_i z_{i2}^T \mathrm{sgn}(z_{i2})\Bigg) \\ &= \sum_{i=1}^{n}\left(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\|\xi_{i1}(\cdot) z_{i2}^T \mathrm{sgn}(z_{i2})\right) \\ &\quad + \left(\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n}\frac{n}{2}z_{i2}^T z_{i2} - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2}\epsilon_{ji}^2(\cdot)\right) \\ &\quad - \sum_{i=1}^{n} k_i z_{i2}^T \mathrm{sgn}(z_{i2}). \end{aligned} \tag{33}$$

Based on (27), (28) and the fact that $s^T\mathrm{sgn}(s) \geq \| s \|$ for any vectors s (see Lemma 1 in Yan & Edwards, 2008),

$$\begin{aligned} & \sum_{i=1}^{n}\left(z_{i2}^T G_i(\cdot)\Phi_i(\cdot) - \|G_i(\cdot)\|\xi_{i1}(\cdot) z_{i2}^T \mathrm{sgn}(z_{i2})\right) \\ &\quad \leq \sum_{i=1}^{n}\Big(\|z_{i2}\| \cdot \|G_i(\cdot)\| \cdot \|\Phi_i(\cdot)\| - \|z_{i2}\| \cdot \|G_i(\cdot)\| \\ &\quad\quad \cdot \xi_{i1}(\cdot)\Big) \leq 0. \end{aligned} \tag{34}$$

Then, by similar reasoning as in (34), and from (28)

$$\begin{aligned} & \sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n}\frac{n}{2}z_{i2}^T z_{i2} - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2}\epsilon_{ji}^2(\cdot) \\ &\quad \leq \sum_{i=1}^{n}\|z_{i2}\| \cdot \|H_{i2}(\cdot)\| - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\|z_{i2}\|^2 \\ &\quad\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\epsilon_{ji}^2(t, z_i) \\ &\quad = \sum_{i=1}^{n}\|z_{i2}\| \cdot \|H_{i2}(\cdot)\| - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\|z_{i2}\|^2 \\ &\quad\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\epsilon_{ij}^2(t, z_j). \end{aligned} \tag{35}$$

From the fact that $\frac{a^2+b^2}{2} \geq |a|\,|b|$, it follows that

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\|z_{i2}\|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\epsilon_{ij}^2(t,z_j) &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\left(\|z_{i2}\|^2 + \epsilon_{ij}^2(t,z_j)\right)\\ &\geq \sum_{i=1}^{n}\sum_{j=1}^{n}\|z_{i2}\|\epsilon_{ij}(t,z_j)\\ &= \sum_{i=1}^{n}\|z_{i2}\|\sum_{j=1}^{n}\epsilon_{ij}(t,z_j)\\ &\geq \sum_{i=1}^{n}\|z_{i2}\|\cdot\|H_{i2}(\cdot)\|. \end{aligned} \tag{36}$$

From (36) and (35),

$$\sum_{i=1}^{n} z_{i2}^T H_{i2}(\cdot) - \sum_{i=1}^{n}\frac{n}{2}z_{i2}^T z_{i2} - \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\frac{z_{i2}^T z_{i2}}{\|z_{i2}\|^2}\epsilon_{ji}^2(\cdot) \leq 0. \tag{37}$$

Substituting (34) and (37) into (33) yields

$$S^T(z)\dot{S}(z) \leq -\sum_{i=1}^{n} k_i z_{i2}^T \mathrm{sgn}(z_{i2}) \leq -\eta\sum_{i=1}^{n} z_{i2}^T\mathrm{sgn}(z_{i2}) \leq -\eta\|S\|, \tag{38}$$

where η is a positive constant which is chosen such that $\eta \leq \min\{k_1, k_2, \ldots, k_n\}$.

Inequality (38) shows that reachability condition (26) is satisfied, and thus interconnected systems (7)–(8) can be driven to the sliding surface (15) in finite time and maintain a sliding motion on it thereafter. Hence, the result follows. ■

According to SMC theory, Theorems 3.1 and 3.2 together show that the closed-loop system formed by applying control law (29) to interconnected systems (7)–(8) is uniformly asymptotically stable.

Remark 3.5: From the proof of Theorem 3.2, it is clear to see that both the matched uncertainties and the mismatched interconnection terms can be cancelled by the designed decentralised controllers in the reachability analysis, which is one of the main contributions in this paper. Such controllers can enhance the robustness against unknown interconnections even in the framework of a decentralised scheme. Moreover, the developed decentralised controllers can guarantee that the interconnected systems are driven to the composite sliding surfaces in finite time. As for how to estimate the finite reaching time, refer to the recent work in Li et al. (2020). It should be noted that the sliding motion is not robust to the mismatched interconnections in the sliding phase. Actually, the limitation to the mismatched interconnection is necessary for the sliding phase, which can be seen from the comments in Remark 3.3.

Remark 3.6: It should be emphasised that in this paper, the considered systems are fully nonlinear with nonlinear disturbances and nonlinear interconnections. It is not required that the nominal subsystems are linear, or the nominal subsystems are linearisable or partial linearisable. This is in comparison with most of the existing work (Z. Xu et al., 2020). Therefore, the methodology developed in this paper can be applied to a wide class of interconnected systems.

4. Simulation results

Consider the nonlinear interconnected system which is composed of two third-order subsystems

$$\dot{x_1} = \begin{bmatrix} -6x_{12}^2x_{13}^2 - 4x_{12}^2 - 2x_{11} \\ -3x_{12}x_{13}^2 - 3x_{12} + \frac{1}{16}(x_{12}^2 - x_{11})^2 \\ 3x_{12}^2x_{13} - 3x_{13} - \frac{1}{4}(x_{12}^2 - x_{11})\exp\{-t\}\cos(x_{13}t) \end{bmatrix} + \begin{bmatrix} -4(x_{13}^2\sin^2 t + 1) \\ 0 \\ 0 \end{bmatrix}(u_1 + \varphi_1(t,x_1)) + h_1(t,x), \tag{39}$$

$$\dot{x_2} = \begin{bmatrix} -8x_{21} + x_{23} \\ -7x_{22} + x_{23} \\ x_{21} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}(u_2 + \varphi_2(t,x_2)) + h_2(t,x), \tag{40}$$

where $x_i = \mathrm{col}(x_{i1}, x_{i2}, x_{i3}) \in \mathcal{R}^3$ and $u_i \in \mathcal{R}$ are, respectively, the state variables and inputs of the i-th subsystem for $i = 1, 2$. The terms $\varphi_i(\cdot)$ and $h_i(\cdot)$ for $i = 1, 2$ are matched disturbances and unknown interconnections, respectively.

Consider the transformation T_1 and T_2 defined by

$$T_1 : \begin{cases} z_{11}^a = x_{12}, \\ z_{11}^b = x_{13}, \\ z_{12} = \frac{1}{4}(x_{12}^2 - x_{11}), \end{cases} \quad \text{and}$$

$$T_2 : \begin{cases} z_{21}^a = x_{21}, \\ z_{21}^b = x_{21} + x_{22}, \\ z_{22} = x_{23}. \end{cases}$$

It is easy to find that the Jacobian matrices of T_1 and T_2 are given by

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(1/4) & (1/2)x_{12} & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which are nonsingular in the whole state space. By direct calculation, system (39)–(40) in the new coordinates is given by

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a \left(z_{11}^b\right)^2 - 3z_{11}^a + z_{12}^2 \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b - z_{12}\exp\{-t\}\cos\left(z_{11}^b t\right) \end{bmatrix} + H_{11}(\cdot), \tag{41}$$

$$\dot{z}_{12} = -2z_{12} + \frac{1}{2}z_{11}^a z_{12}^2 + (1 + \left(z_{11}^b\right)^2 \sin^2 t) \times (u_1 + \Phi_1(\cdot)) + H_{12}(\cdot), \tag{42}$$

$$\dot{z}_{21} = \left[-8z_{21}^a + z_{22} - 7z_{21}^b - z_{21}^a + 2z_{22}\right] + H_{21}(\cdot), \tag{43}$$

$$\dot{z}_{22} = z_{21}^a + (u_2 + \Phi_2(\cdot)) + H_{22}(\cdot), \tag{44}$$

where $H_{11}(\cdot) \in \mathcal{R}^2$ and $H_{i2}(\cdot) \in \mathcal{R}^1$ for $i = 1, 2$.

In order to demonstrate the theoretical results obtained in this paper, it is assumed that the uncertainties in (41)–(44) satisfy

$$|\Phi_1(\cdot)| \le (\|z_{11}^b\| + 1)\exp\{-t\}, \tag{45}$$

$$\|H_{11}\| \le \|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|, \tag{46}$$

$$\|H_{12}\| \le \sum_{j=1}^{2} \epsilon_{1j}(t, z_j) \le 0.25(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|), \tag{47}$$

$$\sum_{j=1}^{2} \epsilon_{1j}^2(t, z_j) \le 0.06(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|)^2, \tag{48}$$

$$|\Phi_2(\cdot)| \le \|z_{21}^b\| \sin^2 z_{22}, \tag{49}$$

$$\|H_{21}\| \le 1.618(z_{11}^a + \left(z_{11}^a\right)^2 - 4z_{12})^2 \sin^2 z_{22}, \tag{50}$$

$$\|H_{22}\| \le \sum_{j=1}^{2} \epsilon_{2j}(t, z_j) \le 0.40(z_{11}^a + \left(z_{11}^a\right)^2 - 4z_{12})^2 \sin^2 z_{22}, \tag{51}$$

$$\sum_{j=1}^{2} \epsilon_{2j}^2(t, z_j) \le 0.32(z_{11}^a + \left(z_{11}^a\right)^2 - 4z_{12})^4 \sin^4 z_{22}. \tag{52}$$

For (41)–(44), select the switching function $S(z) := \text{col}(z_{12}, z_{22})$. When the sliding motion occurs, $z_{12} = z_{22} = 0$. It can be obtained by direct calculation that the sliding mode dynamics are written as follows:

$$\dot{z}_{11} = \begin{bmatrix} -3z_{11}^a \left(z_{11}^b\right)^2 - 3z_{11}^a \\ 3\left(z_{11}^a\right)^2 z_{11}^b - 3z_{11}^b \end{bmatrix} + H_{11s}(\cdot), \tag{53}$$

$$\dot{z}_{21} = \begin{bmatrix} -8z_{21}^a \\ -7z_{21}^b - z_{21}^a \end{bmatrix} + H_{21s}(\cdot). \tag{54}$$

It is clear to see from (46) and (50) that

$$\|H_{11s}(\cdot)\| \le \|z_{11}^a\| \sin^2 t \le \|z\|, \tag{55}$$

$$\|H_{21s}(\cdot)\| = 0 \tag{56}$$

and thus $\beta_1 = 1$ and $\beta_2 = 0$.

For system (41)–(44), consider the candidate Lyapunov function as

$$V(\cdot) = V_1(\cdot) + V_2(\cdot),$$

where $V_1 = (z_{11}^a)^2 + (z_{11}^b)^2$ and $V_2 = (z_{21}^a)^2 + (z_{21}^b)^2$. By direct calculation,

$$p_{il}(\|z_{i1}\|) = \varsigma_{il}\|z_{i1}\|, \quad i = 1, 2, \quad l = 1, 2, 3, 4, \tag{57}$$

where ς_{il} for $i = 1, 2, l = 1, 2, 3, 4$ are the positive constants. It is easy to find that Assumption 3.1 holds and the $p_{il}(\cdot)$ satisfy (57) with

$$\varsigma_{11} = \varsigma_{12} = 1, \quad \varsigma_{13} = \sqrt{6}, \quad \varsigma_{14} = 2,$$
$$\varsigma_{21} = \varsigma_{22} = 1, \quad \varsigma_{23} = \sqrt{13}, \quad \varsigma_{24} = 2.$$

Then from (21), it follows by direct calculation that

$$M^T + M > 0. \tag{58}$$

According to Theorem 3.1, the designed sliding mode is asymptotically stable.

Based on (29), the designed control is given by

$$u_1(\cdot) = \frac{-2z_{12} + 0.5z_{11}^a z_{12}^2}{1 + \left(z_{11}^b\right)^2 \sin^2 t} - \frac{k_1 \mathrm{sgn}(z_{12})}{1 + \left(z_{11}^b\right)^2 \sin^2 t} - z_{12} - \left((\|z_{11}^b\| + 1)\exp\{-t\}\mathrm{sgn}(z_{12}) + \frac{0.03z_{12}(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|)^2}{\|z_{12}\|^2 \left(1 + \left(z_{11}^b\right)^2 \sin^2 t\right)} \right), \tag{59}$$

$$u_2(\cdot) = z_{21}^a - k_2 \mathrm{sgn}(z_{22}) - z_{22} - \left(\|z_{21}^b\| \sin^2 z_{22} \mathrm{sgn}(z_{22}) + \frac{0.16z_{22}}{\|z_{22}\|^2} \times \left(z_{11}^a + \left(z_{11}^a\right)^2 - 4z_{12} \right)^4 \cdot \sin^4 z_{22} \right), \tag{60}$$

where constants k_1 and k_2 are chosen as

$$k_1 = 0.2 \quad \text{and} \quad k_2 = 1.5.$$

From Theorems 3.1 and 3.2, it follows that controller (59)–(60) can stabilise interconnected system (41)–(44) uniformly asymptotically.

For simulation purposes, the initial states are chosen as $x_{10} = (-2, 7.5, 5)$ and $x_{20} = (6, 2, 3.5)$, and the uncertainties and interconnections are chosen as

$$H_{11} = \begin{bmatrix} -0.5\left(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|\right) \\ 0.7\left(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|\right) \end{bmatrix},$$

$$H_{12} = 0.05(\|z_{11}^a\| \sin^2 t + \|z_{12}\| + \|z_{22}\|),$$

$$\Phi_1(\cdot) = 0.9 \cdot (\|z_{11}^b\| + 1)\exp\{-t\},$$

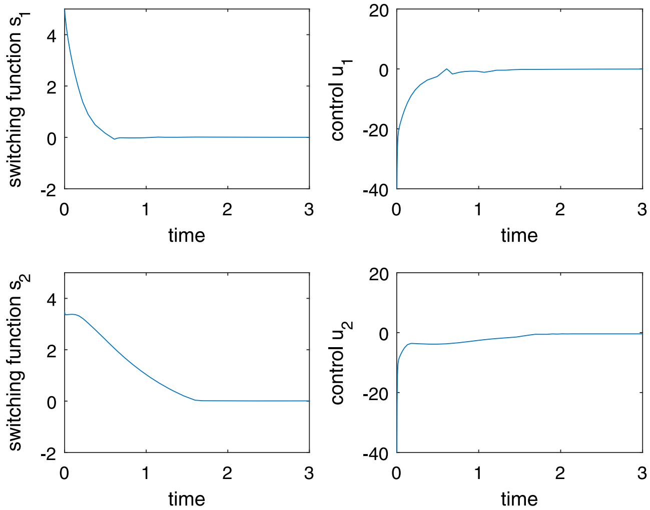

Figure 1. The time responses of the switching function s_1 and control signal u_1 (upper), and the time responses of the switching function s_2 and control signal u_2 (bottom) for $k_1 = 0.2$ and $k_2 = 1.5$.

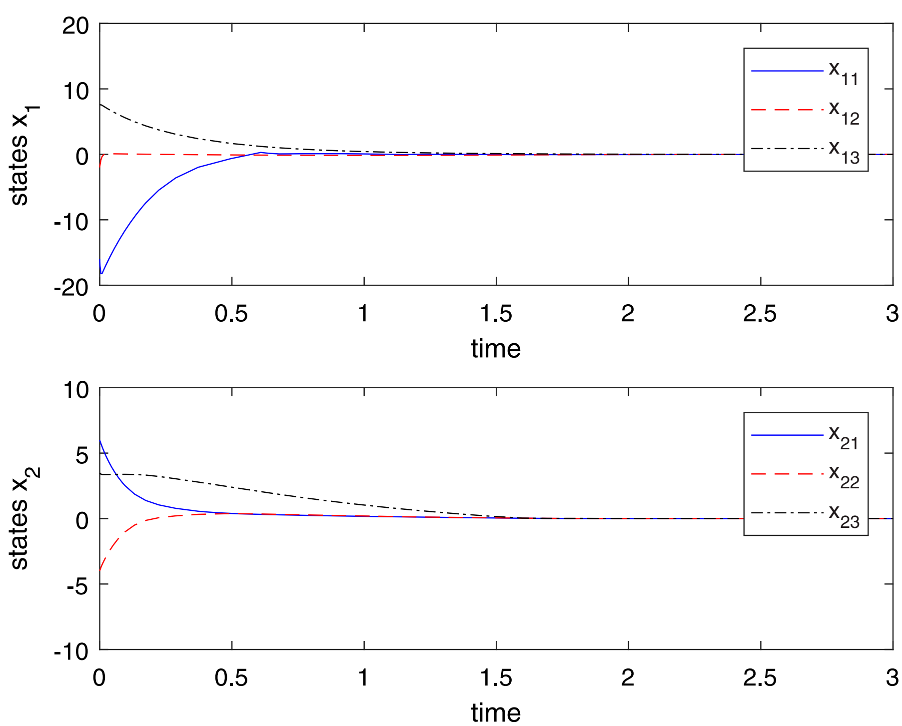

Figure 2. The time responses of the state variables of subsystem (39) (upper), and the time responses of the state variables of subsystem (40) (bottom) for $k_1 = 0.2$ and $k_2 = 1.5$.

$$H_{21} = \begin{bmatrix} -0.647(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22} \\ 0.323(z_{11}^a + (z_{11}^a)^2 - 4z_{12})^2 \sin^2 z_{22} \end{bmatrix}.$$

Figure 1 shows the control signals and the sliding functions with respect to time. The simulation results in Figure 2 show that the closed-loop system formed by applying control (59)–(60) to interconnected system (41)–(44) is uniformly asymptotically stable which is consistent with the obtained theoretical results.

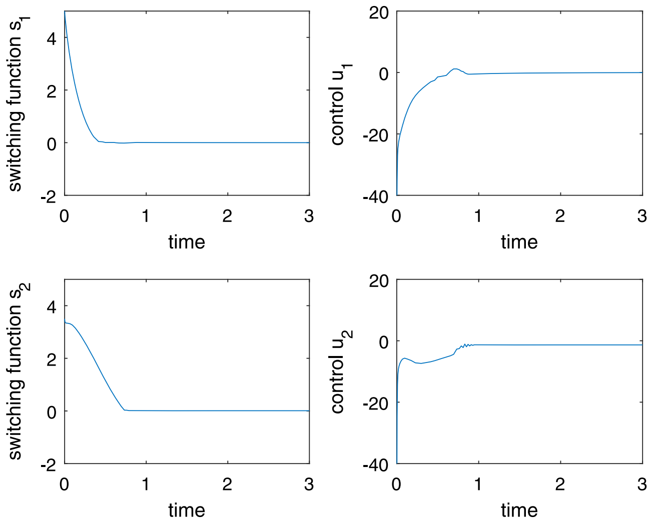

Figure 3. The time responses of the switching function s_1 and control signal u_1 (upper), and the time responses of the switching function s_2 and control signal u_2 (bottom) for $k_1 = 2.5$ and $k_2 = 5$.

It should be noted that the reachability constant depends on the parameters k_1 and k_2 which affect the convergent rates of sliding functions as well as system state variables. In order to demonstrate this, keep all the other parameters the same but increase k_1 and k_2 to $k_1 = 2.5$ and $k_2 = 5$. The simulation results are presented in Figures 3 and 4. It is clear to see, by comparing Figures 1 and 2 with Figures 3 and 4, that the bigger the values of k_1 and k_2 are, the faster the sliding functions and system state variables converge.

Remark 4.1: It should be noted that interconnected systems (39)–(40) are fully nonlinear where both matched uncertainties and unmatched interconnections are involved. Therefore, the methods proposed in the recent work in Ma and Xu (2021) and X. Yang and He (2021) cannot be applied to system (39)–(40). Although the considered interconnected systems are nonlinear in Ma and Xu (2021) and X. Yang and He (2021), it is required that the nominal subsystems have a triangle structure and the uncertainties/interconnections have a linear growth rate in Ma Xu (2021). Moreover, it is required that the interconnections are matched in X. Yang and He (2021).

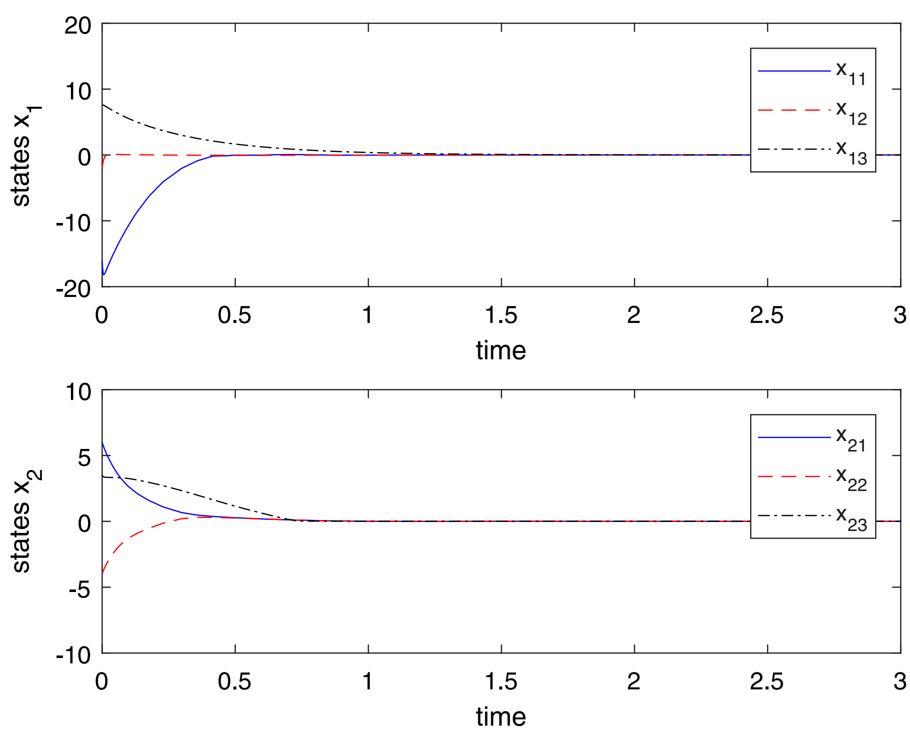

Figure 4. The time responses of the state variables of subsystem (39) (upper), and the time responses of the state variables of subsystem (40) (bottom) for $k_1 = 2.5$ and $k_2 = 5$.

5. Conclusion

A class of fully nonlinear interconnected systems with unknown nonlinear interconnections has been considered in this paper. A composite sliding surface has been designed, and a set of conditions has been developed to guarantee that the corresponding sliding motion is uniformly asymptotically stable. A novel decentralised state feedback control law is designed for the nonlinear interconnected systems to ensure that the interconnected system is driven to the designed sliding surface in finite time. The proposed strategy supplies an approach to improve the robustness for nonlinear interconnected systems in that effects of all matched uncertainties and mismatched interconnections can be rejected by the designed decentralised control regarding the reaching phase. Finally, numerical simulation results have been presented to show the effectiveness of the proposed methods. In the future, it is expected to extend the results developed in this paper to time delay nonlinear interconnected systems and use some strategies to reduce possible chattering towards practical applications.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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