ESSAYS ON AGGREGATE PRODUCTIVITY AND THE COLLATERAL CONSTRAINT

By

ANUP MULAY

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DR ALFRED DUNCAN

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DECLARATION

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, contains no material previously published by another person, nor material that has been accepted for the award of any other degree or diploma of any university or other institute of higher education, except where due acknowledgement has been made.

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STATEMENT OF CONJOINT WORK

All chapters were jointly co-authored with Dr. Alfred Duncan (University of Kent). In particular, the development of theoretical models and derivation of theoretical results were the outcome of collaborative working practices.

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ESSAYS ON AGGREGATE PRODUCTIVITY AND THE COLLATERAL CONSTRAINT

Abstract

We attempt to draw a straight line connecting financial shocks, portfolio reallocation by firms, and drop in aggregate productivity using a dynamic general equilibrium model. Our objective is to study whether the presence of financial frictions can produce business cycle-like response of variables to a financial shocks, as well as lower aggregate productivity. We propose a model that shows how an adverse shock to credit access for firms causes them to change the balance sheet portfolio composition of productive assets, which in turn causes loss of efficiency, and manifests as an efficiency wedge. We find some evidence for these dynamics in the data in the form of loss of measured TFP in countries after a financial shock, and a change in portfolio composition of assets for the U.S. data. Specifically, we find that post the 2008 crisis firms changed their allocation between assets of varying depreciation rates. We calibrate the model such that the impulse response functions are qualitatively similar to data facts.

We take the canonical real business cycle model as base and modify it to incorporate our hypothesis stated above. We develop the simple model without frictions first and discuss its unique features, then add the friction and discuss how the full model responds to shocks and how the response of variables is very close to a business cycle. Then we address some technical issues in the model to make it comparable to a standard model with financial frictions. We also conduct welfare analysis and compute welfare costs by comparing a model of constrained firms with one of unconstrained firms.

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INTRODUCTION

We will draw a straight line connecting financial frictions in the credit markets, changes in firm's balance sheet capital asset portfolio, and the lowering of productivity. We show that the changes in credit conditions will lead to firms' having lower access to credit, which leads to a lowering of aggregate productivity through the mechanism of balance sheet portfolio reallocation. Because the firms have several different types of assets on their balance sheet with different durations (which is loosely used here to imply the inverse of each assets' rate or depreciation), they can reshuffle their portfolio depending on what is optimum given the credit constraint. This reshuffling causes firms to choose sub optimal portfolios compared to the first best unconstrained portfolio, and this efficiency wedge looks qualitatively like a negative shock to aggregate productivity. We find some evidence for this in the data in the form of a fall in productivity after a financial shock (2008 crisis) across the globe, and some evidence of reshuffling of the balance sheet portfolio after a financial shock for U.S. data. We attempt to model this mechanism in a dynamic general equilibrium setting where there is a continuum of capital goods and firms face a collateral constraint which limits their access to finance. The main themes of our work are, firstly, that if the firms are credit constrained, a financial shock can lead to reallocation of funds which takes the economy further from its first best allocation and, secondly, that misallocation of resources in, as well as out of, equilibrium manifests itself as an efficiency wedge and might explain the loss of productivity after a financial shock.

The point that credit constrained firms might be the reason for amplification of shocks as well as for prolonged recessionary conditions has been studied extensively in the literature, most intuitively by Kiyotaki and Moore (1997). In their seminal paper, Kiyotaki and Moore (1997) show how the existence of leverage in firms can amplify productivity shocks as well as prolong a recession which otherwise would have not been as deep or as long lasting. Leveraged firms whose collateral value falls suddenly succumb to the multiplier mechanism where lower value of collateral leads to lower investment which leads to lower output and investment in the next period, and so forth. The fall in value of collateral, which is also productive capital, reduces demand from constrained firms and thus capital is reallocated to the unconstrained firms. Because the unconstrained firms have lower marginal productivity, and given reallocation of capital, output takes a while to recover from the shock. The mechanism of transmitting the productivity shock into prices and in turn into reallocation of capital is made possible by the existence of collateral constraints. If there were no credit constraints, marginal products of capital would be equal for the two groups and price of capital would be invariant to shocks and would stay at the level of discounted returns from using capital. There is no reallocation of capital on a productivity shock; the only impact would be on current period output. The presence of collateral constraints leads to an efficiency wedge which is a result of misallocation of productive capital. Although these are not the focus of their study - the focus being collateral constraints amplifying and prolonging business cycles - it is still interesting that the efficiency wedge in this case will likely move in the same direction as the productivity shock on impact, and whereas the shock dissipates relatively quickly, the movement of the wedge is more hump shaped and takes longer to fade away and return to its steady state value (which is always positive as long as credit constraints exist). In this piece of work, we develop a model which has collateral constraints similar to Kiyotaki and Moore (1997) model, but we do not rely on shocks to technology to generate business cycles. Our model attempts to capture events like the 2008 financial crisis where liquidity dried up and the value of collateral fell sharply due to a spate of repossessions by banks. Similar to Kiyotaki and Moore (1997), our model also relies on some form of misallocation to generate persistence but unlike their model our form of misallocation happens due to some assets having better collateralisability than other in recessionary conditions whereas their misallocation is along the extensive margin for the two groups in the model. Chapters one and two develop these ideas in great detail. We also talk in detail about the efficiency wedge and the efficient steady state in Chapter three.

While Kiyotaki and Moore (1997) assume separate technologies for the two agent groups and abstract from the more realistic situation where agents have access to the same technology and face idiosyncratic shocks and have different levels of accumulated wealth, Bernanke, Gertler, and Gilchrist (1999) (BGG) model this situation in another seminal paper. They model debt contracts using the costly state verification method (rather than the costly enforcement used by Kiyotaki and Moore (1997)) wherein they introduce idiosyncratic shocks to individual firms, in addition to an aggregate productivity shock to the economy. The aggregate shock materialises at the beginning of the period, which sets the price of capital and the rate of return. Individual firms then choose optimal capital and borrowings based on the returns and price. Finally the firm specific shock materialises which sets the firms idiosyncratic return on capital as well as its borrowing rate of interest, and the firm finds out whether it will default based on these things or not. The interest rate on borrowings as well as the probability of default are higher when economic conditions are recessionary. They show how individual firm's capital demand depends on the expected discounted return on capital. If the expected discounted return on capital is higher, the demand for capital is commensurately higher. This mechanism does not exist in the model by Kiyotaki and Moore (1997), because in their model capital expenditures are determined solely by net worth and not by discounted expected returns to capital. Firms in the Kiyotaki-Moore model are constrained for financing and rely on what they can borrow against their net worth solely to make the investment decision. In the BGG model, the capital expenditure is further affected in the aggregate by how net worth evolves. In case the default probabilities are very high in any given period, apart from the negative impact of discounted expected return on capital on individual firms, the fact that many firms will go bankrupt has a negative impact on the net worth which will be taken into the next period by the aggregate sector. So there is a dual impact; individual firms accumulate lower capital due to fall in expected returns, and even after that several firms go bankrupt due to higher default probability. This prolongs recessions in their model, especially when aided by nominal rigidities like sticky prices. However, the presence of sticky prices and their role in the New Keynesian models of bringing about a price dispersion in equilibrium which shows up as an efficiency wedge, the misallocation among firms as a result of access to finance is not clearly available.

The emphasis on efficiency wedge in our discussion comes from the work by Chari, Kehoe, and McGrattan (2007) (CKM) where they use U.S. business cycle data and a real business cycle model with time-varying wedges for efficiency, labour, investment, and government consumption, and find that efficiency wedges and labour wedges explain the long deep downturn of the 1930s and also are largely responsible for fluctuations thereafter over the business cycle. They show how, in a model with differential access to input financing for two agent groups, the friction can manifest as an efficiency wedge, a labour wedge and an investment wedge in the benchmark RBC model. This is fairly close to the model by Kiyotaki and Moore (1997), except that Kiyotaki and Moore (1997) have one sector being unconstrained. In this case the wedge will not just be an efficiency wedge, but will also have effects similar to an investment wedge and a labour wedge. The only way such a wedge can appear solely as an efficiency wedge is if, according to CKM, the average of the wedges faced by the two constrained sectors remains the same. In that situation, the financing friction will appear solely as a negative technology shock. For the model by BGG where risk averse consumers end up bearing all the risk from an adverse idiosyncratic shock, which leads to bankruptcies and a lowering of aggregate net wealth and thus capital accumulation, CKM show that this can be modelled in an RBC model as an investment wedge as well as a wedge where the rate of depreciation seems to be time varying[1]. The model we develop also has a wedge which causes depreciation to be time varying, though the channels are very different. The CKM benchmark RBC wedges are ad hoc, whereas our time varying depreciation is a result of optimising behaviour by constrained firms, as explained later in greater detail. The main message of CKM is that the four kinds of wedges measured in aggregate in data can be a result of several different frictions including financial friction of various kinds. However, CKM do not directly model how financial frictions might show up as a negative productivity shock, for instance. They rely on the mapping between the detailed model with financial frictions and a prototype benchmark model with time varying wedges, assuming the same allocations between the two models in equilibrium. We, however, show in our model exactly how the financial frictions that exist in credit markets might map into a lower productivity and mimic a negative technology shock qualitatively. We also have a wedge between the marginal product of capital and the (implied) rental rate of capital, which is a result of having several productive assets on the balance sheet but deploying only select few depending by constructing a production bundle of capital, and this wedge, together with the wedge that causes depreciation to be time varying, can be interpreted as the investment wedge mentioned by CKM. The investment wedge and the aggregate capital demand wedge (which affects the balance sheet portfolio composition) are determined by optimal choices in period t and these wedges then pin down the efficiency wedge for period $t+1$, making it a predetermined variable in each period.

Adding to the results of CKM (and changing them slightly), Buera and Moll (2015) argue that adding heterogeneity of some form (beyond the two agent setting) to a representative agent model changes how frictions at the individual agent level manifest at the

[1] CKM show that the mapping of

$$c_t + k_t = w_t l_t + (1-\tau_t^k) r_t k_{t-1} + (1 - \delta(1-\tau^k))k_{t-1} + T_t$$

mimics the effect of the suboptimal contracts of BGG.

aggregate economy level. Specifically, they show how heterogeneity of productivity among entrepreneurs leads to the result that a ‘credit crunch’ is isomorphic to a drop in aggregate productivity. However, in their model, this shock to the collateral constraint (i.e. the ‘credit crunch’) does not show up in the entrepreneurs’ aggregate Euler equation, which is an important result considering that most models of financial frictions treat the collateral constraint shock as something that changes investment incentives by distorting the Euler[2]. This result is obtained because bonds are in zero net supply on aggregate, and hence interest rate must adjust so that aggregate returns to capital equal the aggregate returns to wealth for entrepreneurs[3]. They also go on to show that if investment costs were heterogeneous, this friction would show up as an aggregate investment wedge, whereas if recruitment costs were heterogeneous, the friction would show up as a labour wedge. They find that the models which generate efficiency and labour wedges on aggregate from underlying heterogeneity are the ones where the response of macro variables appears qualitatively very similar to that seen in data post a financial crisis. This result is on the likes of CKM. The main takeaway from the work of Buera and Moll (2015) is that heterogeneity of some sort changes how wedges manifest in the aggregate model and abstracting from all forms of heterogeneity in modelling frictions might cause the understanding of the underlying mechanism generating wedges to be incomplete. We take this message on board and model heterogeneity of a different kind; the heterogeneity of productive capital assets. We model a continuum of capital assets, each with its own constant depreciation rate, from which firms choose optimally and construct their balance sheet portfolio. How changes in credit market conditions change the optimal choices and result in wedges which drive macro variables to produce business cycle-like responses is the issue we study in great detail in this piece of work. Like Buera and Moll (2015),

[2]However, there is an investment wedge on aggregate for the entire economy including the workers, the reason being workers’ inability to save by assumption. This is not the same investment wedge we see in CKM or BGG.

[3]Facilitated by a Cobb-Douglas production function.

our efficiency wedge results from aggregation over the underlying heterogeneity, but whereas Buera and Moll (2015) manage to map the idiosyncratic fluctuations into a deterioration of credit conditions at the aggregate level, we can actually map it in the other direction: a deterioration of credit conditions leading to a decline in aggregate productivity. This subtle point is fairly important because it shows that in a way, Buera and Moll (2015) still rely on productivity shocks to generate a link between financial frictions, the business cycle, and drop in aggregate productivity, just that they rely on idiosyncratic shocks to heterogeneous firms. We, however, show how a shock that originates in the financial sector can appear as, or be perceived as, a drop in productivity. This, we feel, is crucial in explaining fluctuations observed in macro variables post a financial shock, and is something that is missing from current models of financial frictions.

We introduce a different kind of misallocation of factors compared to the existing literature. Most existing literature introduces heterogeneity of firms and idiosyncratic shocks or firm specific taxes on factors to generate misallocation. There is also literature which links financial friction to misallocation through the lack of access to funds forcing firms to have lower factors of production. The way these firms then overcome the constraint is by relying on retaining earnings and relying on savings. However, the ability to save and the rate of accumulating these funds depends on idiosyncratic shocks to productivity. So, a change in individual productivity affects access to finance, which causes misallocation of factors, and hence aggregate productivity is affected. The changes in individual productivity are exacerbated by the financial friction, which reduces aggregate productivity. We rely on a very different set up and mechanism in our model. We introduce heterogeneity in asset types and have a representative aggregate firm. Also, our shocks actually originate in the financial sector and show up as a reduction in productivity because of the transmission mechanism of the model.

We find that our dynamic model can replicate some of the facts observed in data, and allows us to have a theoretical explanation reconciling a shock in the financial sector causing changes in the capital portfolio held by the aggregate firm which, in turn, is reflected as a lowering of productivity. We check the effects of alternative calibrations and preference specifications, and find that, except for minor differences, the results still hold.

We build an unconstrained version of the model in chapter one, add the collateral constraint to the model in chapter two, and conduct comparative welfare analysis in chapter three.

Chapter One

CAPITAL PORTFOLIO AND TIME VARYING DEPRECIATION

1.1 Introduction

The canonical real business cycle (RBC) model treats aggregate rate of depreciation of capital assets as constant. Even as the quantity of capital changes over the business cycle, the effective depreciation rate does not change, which seems counter intuitive. It would seem that as firms increase asset holding over the upswing, their depreciation charge would rise either from increased utilisation, or simply from having newer assets with a higher value on the balance sheet, and thus a higher depreciation charge even at existing rates[1].

As figure 1.1 shows that the effective rate of depreciation, calculated by dividing the current cost depreciation charge in billions of dollars by the current cost net stock of fixed assets in billions of dollars, yields a time varying effective rate of depreciation.

The rate of depreciation does appear to display some cyclical tendencies, but the extent will become clearer if we run a VAR with financial shocks and TFP shocks. It might well be that unconditionally it is almost acyclical, but conditional on a shock it appears cyclical. For example, for the great depression during the 1930s and the financial crisis in 2010s, the rate appears countercyclical. Similarly for the recession in the early 1990s. But for the recession

[1]Assuming that firms use the written down value method of depreciation where a percentage of assets remaining book value is written off each year.

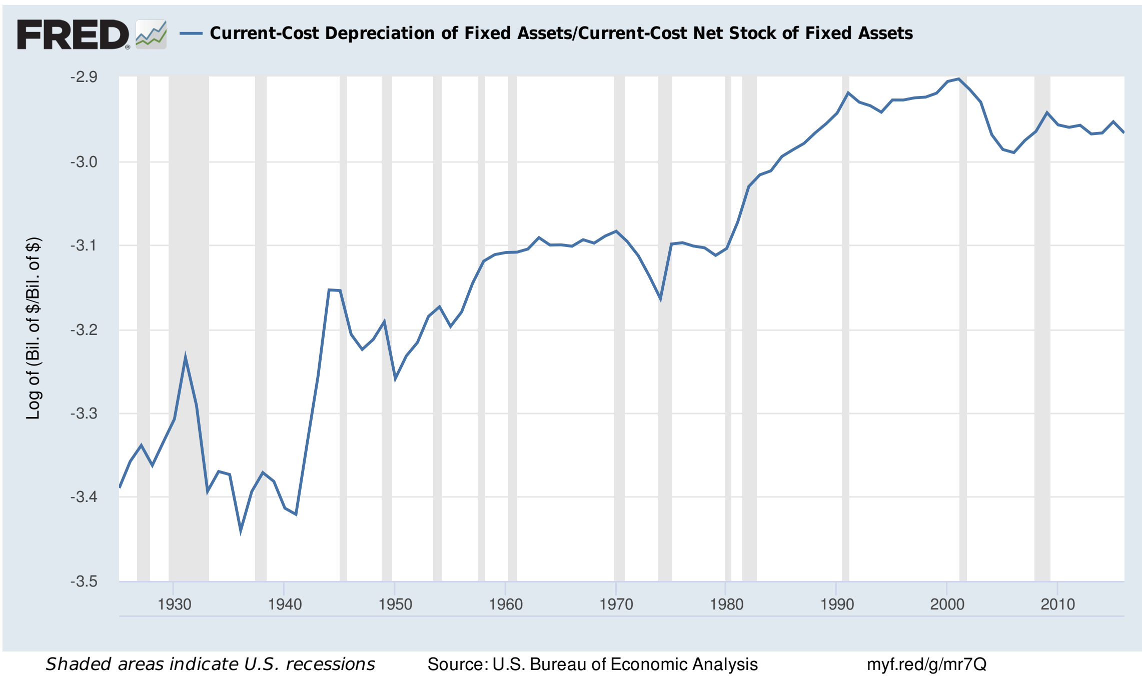


Figure 1.1 Variable rate of depreciation

in the early 1950s and late 1970s, it appears procyclical.

Greenwood et al. (1988) model a procyclical rate of depreciation by linking it to utilisation of the capital assets over the business cycle, while Collard and Kollintzas (2000) implicitly have a procyclical rate of depreciation in their model. Most models, like Greenwood et al. (1998), model the depreciation rate as being procyclical. Ambler and Paquet (1994) simply make the rate of depreciation an $AR(1)$ process, but their objective is to match some other observed facts from data.

We explain the stylised fact in figure 1.1 using the standard RBC model, with an extension wherein there will be a continuum of capital assets with individual constant rates of depreciation. The households will still own the capital assets and lease them to the firms. The firms will maximise profits by choosing a portfolio of capital to be used in production from all the assets leased from the household. This will allow us to distinguish between ‘balance sheet capital’, the sum of all individual types of capital bought by the households, and ‘portfolio’ capital, the productive capital actually deployed by the firms for production, which will be constructed using a standard Dixit-Stiglitz CES aggregator function. We show that the effective rate of depreciation depends on the assumption of the elasticity of substitution between the asset types, and if we assume they are complementary to each other, we can generate procyclical effective rate of depreciation. We also show how our model will lead to an ‘efficiency wedge’ kidn of expression which appears as a part of the aggregate TFP, and is present even in equilibrium.

We discuss related literature next, then set up the model and explain in detail how our mechanism works. We then present and discuss the results of the model, and then conclude.

1.2 Literature

Greenwood et al. (1988) first introduced a time varying rate of depreciation to the neoclassical model, in the form of variable capital utilisation. They introduce utilisation as a choice variable in a setting where there is a shock to the marginal efficiency of investments and find that this generates business cycle-like responses from all variables. In their specification,

$$Y = F(UK, ZH)$$

where utilisation U determines the rate of depreciation as $\delta = \delta(U_t)$ such that depreciation is a convex function of utilisation. An increase in the marginal efficiency of investment increases utilisation, which in turn increases labour demand, and because lower amount of investment is required to achieve the same amount of capital stock tomorrow, it also increases consumption, output, investment, and capital accumulation. Clearly, in this setting, utilisation is procyclical, and so is depreciation. The authors are more focussed on having a shock to marginal efficiency of investment translate into a business cycle, and in the process address the Short Run Increasing Returns to Labour (SRIRL) puzzle, (as well as generate a mechanism to make labour demand flatter and more elastic by reducing the impact of diminishing marginal product of labour through variable utilisation) and the procyclical depreciation is an incidental outcome of the model. In our model, the aggregate rate of depreciation turns out to be time varying because the portfolio of assets is reshuffled over the business cycle. Like Greenwood et al. (1988), the additional term shows up in the production function, and forms part of the Solow residual.

Bernstein and Nadiri (1988) take a different approach to introducing time varying rate of depreciation by treating the flow of undepreciated capital as an output of current production, to be used in future production. Specifically,

$$Y(t) = F(K_N(t), L(t), K_O(t))$$

where K_N is capital input and K_O is capital output, and the depreciation rate is defined as $\delta(t) = \frac{(K_N(t)-K_O(t))}{K_N(t)}$. The choice of K_O is also the choice of how much capital is to be utilised in the current period, and hence a choice of depreciation rate as well. An important feature of this setting is that the marginal product of capital ($F_{K_N} > 0$) is not necessarily equal to the marginal product of utilised capital ($-F_{K_O} > 0$), and if they were to be equal, it would give us the Greenwood et al. (1988) specification. The result of having this mechanism of capital input and output results in utilisation, or depreciation, being an intertemporal, forward looking decision which evaluates the trade off between current output and future output. These two features, namely the difference in marginal products of capital and utilised capital, as well as forward looking nature of depreciation rate are a feature of our model as well. Difference in marginal products of capital versus utilised capital arises in our model from the distinction between the balance sheet portfolio that the firm has on hand versus the production capital deployed for use, which consists of a combination of assets on the balance sheet. As for the forward looking depreciation rate, we show in the later sections how depreciation rate today depends on which assets are bought by the firm based on the phase of the business cycle.

Ambler and Paquet (1994) address the issue of high correlation generated by an RBC model between hours and average labour productivity, which is not reflected in the data, by introducing shocks to the rate of depreciation directly, so that $\delta = \delta_t$. A sudden 'destruction' of the capital shock in the form of a rise in δ_t raises the marginal productivity of capital and hence labour supply jumps to try and rebuild the capital stock. At the same time the lower capital stock reduces output and the average labour productivity as well. This addresses the correlation problem for hours and productivity and takes it closer to observed data. However, apart from this, there is no further insight into the impact or working of a time varying depreciation, or how it would vary over the business cycle.

Collard and Kollintzas (2000) do not explicitly introduce a variable for rate of depreciation, but account for it by modelling utilisation, maintenance, improvement, and scrapping activities of the firm. They distinguish between labour allocated to production activities and that allocated to maintenance activities. Higher allocation to production implies more utilisation and more depreciation with end of period capital being lower, whereas a higher allocation to maintenance implies lower depreciation and a higher stock of capital. The capital evolution equation is of the form

$$k_{t+1} = i_t + \tilde{k}_t$$

where the capital services $\tilde{k}_t$ are generated by some function $g(\cdot)$ as $\tilde{k}_t = B_t g(k_t, \tilde{h}_t, \hat{h}_t)$ where $\tilde{h}$ and $\hat{h}$ are the two labour allocations. The authors find that maintenance could potentially be an important determinant of other variables, and also lends a richer propagation mechanism to the standard RBC model. We aim to demonstrate with our model how a time varying depreciation might also account for some of the variation in the Solow residual. Boucekkine and Ruiz-Tamarit (2002) continue on the lines of Collard and Kollintzas (2000) in modelling a choice of maintenance services and investment, with investment entailing convex costs. The capital evolution equation is

$$K_{t+1} = I_t + (1 - \delta(m_t, u_t))K_t$$

where depreciation rate is explicitly a function of maintenance choice $m = \frac{M}{K_{-1}}$ and utilisation u. Similar to Collard and Kollintzas (2000), depreciation increases with utilisation and falls with maintenance. The additional modelling choice to be made is of the sign of the cross derivative of maintenance and utilisation which has an impact on the dynamics of the model. The authors also find that treating maintenance and investment as complements yields the best results from the model.

Boucekkine el al. (2008) model fixed as well as variable, endogenous maintenance costs which depend on the capital utilisation, and this allows them to differentiate between a depreciation rate and a scrapping rate. They find that in response to an investment specific positive shock, both depreciation and scrapping rate go up, which they take to be the change in composition of capital asset types due to improvement in technology leading to a shorter lifespan for any existing asset, as well a fall in utilisation. The change in composition is necessarily towards better more productive assets in their model, but in our model, the firm chooses different assets with corresponding accounting rates of depreciation, and a change in the overall composition of the portfolio is what yields the time varying rate in our case.

Albonico and Kalyvitis (2013) use the formulation of Collard and Kollintzas (2000) and add capital adjustment costs to the model and find that depreciation responds positively to positive shock to technology as well as to government spending, and negative shock to price of investment, and negatively to a negative labour supply shock and a positive preference shock. The positive response to technology shock is most intuitive, while the negative shock to investment price makes maintenance dearer which causes a rise in the rate of depreciation. A negative government shock increases labour supply, and thus utilisation, which results in a higher rate of depreciation. The fall in depreciation resulting from a negative shock to labour supply is also self explanatory, whereas a positive preference shock crowd out investment and reduces utilisation and thus depreciation rate. The authors also conduct a Bayesian estimation of the model and find a volatile and highly procyclical depreciation rate for Canada and the U.S. We will only deal with a shock to technology, and we also find a procyclical depreciation rate resulting from an increase in the capital stock, as well as change in the portfolio of assets held.

Boucekkine el al. (2008) model fixed as well as variable, endogenous maintenance costs which depend on the capital utilisation, and this allows them to differentiate between a depreciation rate and a scrapping rate. They find that in response to an investment specific positive shock, both depreciation and scrapping rate go up, which they take to be the change in composition of capital asset types due to improvement in technology leading to a shorter lifespan for any existing asset, as well a fall in utilisation. The change in composition is necessarily towards better more productive assets in their model, but in our model, the firm chooses different assets with corresponding accounting rates of depreciation, and a change in the overall composition of the portfolio is what yields the time varying rate in our case.

1.3 Model

Our model is based on the canonical real business cycle model (RBC) with two sectors; households and firms. We describe each sector in detail, and the solve for the equilibrium of the model. Our model will generate the following predictions:

Model Prediction 1.1 *Aggregate rate of depreciation is time varying and depends on the composition of the balance sheet portfolio.*

Model Prediction 1.2 *Composition of portfolio in steady state consists of more long term assets than short term assets as the marginal product of long term assets, though lower, is generated for an extended period of time.*

Model Prediction 1.3 *Response is greater for high depreciation assets as compared to low depreciation assets on impact of technology shock, except in case of very low depreciation assets and a negative technology shock. In that case, response is highest from very low depreciation assets.*

We will go through the each sector of the model in detail below.

1.3.1 The Household

The household is the same as in an RBC model with the following maximisation problem;

$$V(k(i)|A) = \max_{C,N,k(i)'} \{u(C,N) + \beta \mathbb{E}[V(k(i)'|A')]\} \tag{1.1}$$

subject to the following budget constraint[2];

$$C + \int_i k(i)' \text{ di} = \int_i (1-\delta(i))k(i) \text{ di} + WN + \int_i R^k(i)k(i) \text{ di} + \pi - T \qquad (\lambda) \tag{1.2}$$

[2] The integral is over asset types (i)

The household chooses consumption, labour supply, and end of period capital stock, with the only difference being that now households choose capital assets from a continuum with differing rates of depreciation, instead of just one type of capital. This choice leads to a change in the construction of the portfolio over the business cycle, which is discussed in detail further. Also, the rental income for the households consists of lending the entire portfolio of capital assets it holds, where each asset has its own rental rate.

The first order conditions are the following;

$$C: \qquad u_c(C_t, N_t) = \lambda_t \tag{1.3}$$

$$N: \qquad u_n(C_t, N_t) = \lambda_t W_t \tag{1.4}$$

$$k(i): \qquad \beta V'(k(i)'|A') = \lambda_t \tag{1.5}$$

The envelope condition is the following;

$$V'(k(i)|A') = \lambda_t[(1-\delta(i)) + R_t^k(i)] \tag{1.6}$$

Combining (1.5) and (1.6), we have the following capital Euler condition;

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1}[(1-\delta(i)) + R_{t+1}^k(i)] \tag{1.7}$$

(1.7) is almost the standard capital Euler equation, except that there will be one for each type of capital with its own depreciation rate $\delta(i)$ as well as its own rental rate $R^k(i)$. When combined with the firm optimality conditions, it will yield the demand function for each type of capital $k(i)$.

We use the CRRA utility function, so;

$$u(C, N) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \nu \frac{N_t^{1+\psi}}{1+\psi}$$

1.3.2 The Firm

The firm maximisation problem is altered slightly by the presence of a continuum of capital assets. Now the firm rents the entire holding of capital from the household and then constructs a bundle of production capital using a CES aggregator function, which is deployed to generate output. So now the firm needs to choose not just each asset type $k(i)$, but also the deployed production capital k, apart from the labour n. The optimisation problem now looks as under;

$$\max_{k,k(i),N} A_t k_t^{\alpha} N_t^{1-\alpha} - W_t N_t - \int R_t^k(i) k_t(i) \text{ di} \tag{1.8}$$

subject to;

$$k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \text{di} \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_{k,t}) \tag{1.9}$$

The first order conditions are as under;

$$k: \qquad F_{k,t} = \alpha A_t k_t^{\alpha-1} N_t^{1-\alpha} \tag{1.10}$$

$$k(i): \qquad R_t^k(i) = F_{k,t} k_t^{\frac{1}{\varepsilon}} k_t(i)^{-\frac{1}{\varepsilon}} \tag{1.11}$$

$$N: \qquad W_t = (1-\alpha) A_t k_t^{\alpha} N_t^{-\alpha} \tag{1.12}$$

The marginal product of the production capital is F_k, and not R^k as in the standard model. Intuitively it makes sense that the shadow value of the CES aggregator constraint is the marginal product of production capital. The rental rate for each type of capital is now not equal to the marginal product of the production capital, but is adjusted by the amount of that asset used as a proportion of the production portfolio given the elasticity of demand ε. An increase in the rental rate reduces the amount of asset $k(i)$ used in production, and the magnitude depends on elasticity of demand. For $\varepsilon \to \infty$, the capital assets become perfect substitutes, and for $\varepsilon = 0$, they become perfect complements. For $\varepsilon = 1$, the function is Cobb-Douglas. If assets are perfect substitutes, it is equivalent to having just one type of capital asset, and hence the rental rate is the same as the marginal product of production capital. Perfect complements will lead to something like a Leontief set of curves in the capital assets space wherein each asset is required in a certain quantity to form the production capital.

1.3.3 Demand function

Combining (1.11) and (1.7) yields the following demand function for each type of capital asset;

$$\begin{aligned} k_{t+1}(i) &= k_{t+1}\left(\frac{\beta\mathbb{E}_t[\lambda_{t+1}F_{k,t+1}]}{\lambda_t - \beta\mathbb{E}_t[\lambda_{t+1}](1-\delta(i))}\right)^{\varepsilon} + \underbrace{\text{Cov}[m, F_k]}_{\text{assumed}\approx 0} \\ &= k_{t+1}\left(\frac{\mathbb{E}_t[m_{t+1}F_{k,t+1}]}{1-\mathbb{E}_t[m_{t+1}](1-\delta(i))}\right)^{\varepsilon} \end{aligned} \tag{1.13}$$

where $m_{t+1} := \beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ is the stochastic discount factor. The shadow value of the household budget constraint λ, which is also the measure of lifetime marginal utility of wealth, plays an important role in determining the demand for any type of capital asset in our setting. In the canonical RBC setting, a similar expression would be obtained from rearranging the capital Euler equation;

$$\tilde{k}_{t+1} = \mathbb{E}_t\left[n_{t+1}\left(\frac{\alpha m_{t+1}}{1-m_{t+1}(1-\delta)}\right)^{\frac{1}{1-\alpha}}\right]$$

where $\tilde{k}$ is the capital stock in the RBC setting. The above does not, however, have the same 'demand function' interpretation as (1.13) because of the way a technology shock works its way through an RBC model. A positive shock to technology would increase productivity of capital and labour immediately, and with capital already fixed for production, would cause a rise in the demand for labour, and hence wages. Marginal product of capital would rise as well indicating that more capital is needed now because of increased labour input. A rise in output and wages raises consumption as well as investment, which generates an increase in next period's capital, as required. Given this, the above is merely stating a relationship between variables and not assigning any demand function like dynamic.

For our setting, the mechanism still holds as regards the size of production capital (which is chosen from balance sheet portfolio) required next period being larger given a positive shock to technology, but there is still the effect on what actually happens to the balance sheet portfolio of the household, and how its composition changes. Both questions can be answered if we can figure out what happens to the demand of each individual type of asset, which is what (1.13) tells us. It is not merely an expression showing relationship among variables, but actually a demand function which pins down change in demand for each type of asset.

We can see that conditional on the individual rate of depreciation and the elasticity of substitution, the demand for each type of asset rises when current marginal utility of wealth λ falls, that is, when there is an increase in consumption and an increase in the perception of lifetime wealth. If the assets are perfect complements, then each one has to be in the same proportion in the production capital. The higher is the degree of substitutability, the more is the reshuffling of assets upon changes in economic conditions.

To see how the balance sheet portfolio looks like in the steady state, we examine the below;

$$k(i) = k\left(\frac{\beta F_k}{1-\beta(1-\delta(i))}\right)^{\varepsilon}$$

Considering the two extremes of $\delta(i) = 0$ and $\delta(i) = 1$, we see that demand for longer term assets is higher than short term assets in the steady state, that is;

$$k\left(\frac{\beta F_k}{1-\beta}\right)^{\varepsilon}\Bigg|_{\delta(i)=0} > k\left(\frac{\beta F_k}{1}\right)^{\varepsilon}\Bigg|_{\delta(i)=1} \Rightarrow \frac{1}{1-\beta} > 1$$

To answer how reshuffling of assets happens within the balance sheet portfolio on being hit by a positive or a negative technology shock, we conduct a small numerical exercise to generate the following plot;

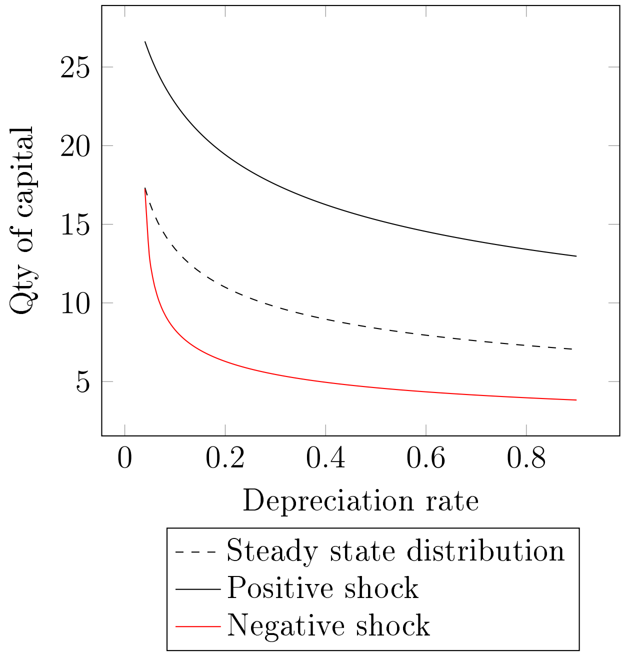


Figure 1.2 Steady state portfolio and reshuffling on shocks

Figure 1.2 shows the steady state distribution of assets by the dotted line. As discussed earlier, long term assets are more in quantity than short term ones. This is not, however, the 'value' of the assets, but just the quantity. Thus far, we have not distinguished between prices of different assets for the sake of simplicity. The black solid line shows how the portfolio looks like in the period the economy is hit with a positive technology shock. Although the portfolio is still heavy in long term assets, the percentage increase in short term assets is higher. This change in portfolio composition towards more short term assets causes depreciation to be procyclical.

If $\delta(i) = 0$, $k_{t+1}(i) = k_{t+1}\left(\frac{m_{t+1}F_{k,t+1}}{1-m_{t+1}}\right)^{\varepsilon}$, whereas if $\delta(i) = 1$, $k_{t+1}(i) = k_{t+1}(m_{t+1}F_{k,t+1})^{\varepsilon}$. The only difference in response comes from the denominator in the demand function when $\delta(i) = 0$. As the stochastic discount factor falls, the demand for very long term assets responds to a lesser magnitude than the demand for very short term assets. This explains the portfolio reallocation. The intuition is that with the technology shock, marginal productivity of all assets increases and agents move to the highly productive short term assets[3] to meet consumption demand and increase production.

The red solid line shows the response of assets to a negative technology shock. The peculiar thing about this response is that investment in long term assets barely falls for the very long term assets, but falls for the rest of the assets, making the curve steeper than before. The overall effect is to shrink the balance sheet portfolio, but the weights are now more towards long term assets and hence the average rate of depreciation will be lower than the steady state, generating, again, a procyclical movement.

Again, with a negative shock, the stochastic discount factor rises and for some very low depreciation assets the response of the demand is in the opposite direction $\left(\frac{m_{t+1}F_{k,t+1}}{1-m_{t+1}}\right) < 0$, implying that demand for these assets rises when all other asset demand is falling. Intuitively, as the economic environment deteriorates, agents invest in very long term assets which yield marginal product for a very long duration and these assets would not require frequent investment. Assets that need frequent reinvestment due to a shorter life span face a fall in demand. Although very long term asset demand goes up, the demand for intermediate term assets falls more than the demand for short term assets. This is due to the high marginal productivity of short term assets compared to medium term assets.

This behaviour by optimising firms to look for safe assets during recessions is explored further in chapters two and three. For the current specification, we discuss how the degree of substitutability also matters for asset reallocation after a shock using a calibration exercise in Appendix 1.C.

[3] We discuss marginal productivity of different duration assets in detail in chapter three. For now, because it is not central to our purpose in this chapter, we merely state that the marginal product of a long term asset is spread over several time periods and is thus lower each period than, say, an asset with a one period life span. Of course, the implicit assumption is that, provided all things are constant always, the marginal products of all assets is the same after adjusting for duration. We will come back to this point later in chapter three.

1.3.4 Second order approximations

We approximate the aggregate demand function $\int k(i)\text{di}$ and other expressions (which are mentioned below) using second order approximations around the average rate of depreciation $\int \delta(i) = \bar{\delta}$ which allows us to pin down the aggregate demand for all types of capital, $\bar{k} := \int k(i)\text{di}$, as a function of, among other variables, the variance of depreciation rates σ^2_δ. Following the approximation procedure detailed in Appendix 1.A, and the derivations in Appendix 1.B, the expression for balance sheet portfolio of capital is;

$$\begin{aligned} \bar{k}_{t+1} &= k_{t+1}\left(\frac{1-m_{t+1}(1-\bar{\delta})}{m_{t+1}\mathbb{E}_t[F_{k,t+1}]}\right)^{-\varepsilon}\left\{1+\frac{1}{2}\varepsilon(\varepsilon+1)\sigma^2_\delta\left(\frac{m_{t+1}}{1-m_{t+1}(1-\bar{\delta})}\right)^2\right\} \\ \Rightarrow \bar{k}_{t+1} &= k_{t+1}\left(\frac{1}{1-\tau^k_t}\right) \end{aligned} \tag{1.14}$$

where $\tau^k_t := 1 - \frac{\left(1+\mathbb{E}_tX_{t+1}\frac{\varepsilon-1}{\varepsilon+1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{1+\mathbb{E}_tX_{t+1}}$ and $\mathbb{E}_tX_{t+1} := \left(\frac{\varepsilon(\varepsilon+1)\sigma^2_\delta}{2(\mathbb{E}_t\bar{R}^k_{t+1})^2}\right)$.

$\mathbb{E}_t\bar{R}^k_{t+1} := \left(\frac{1-m_{t+1}(1-\bar{\delta})}{m_{t+1}}\right)$ is the average over all individual rental rates. Although not economically meaningful, it is used to simplify expressions.

As explained in Appendix 1.B, if there is no variance in depreciation rates, $\sigma^2_\delta = 0$, which implies there is only one type of asset, or if the assets are perfect complements, $\varepsilon = 0$, which implies that effectively there is only one combination of assets possible, then $\tau^k = 0$ and $\bar{k} = k$ as in the canonical RBC model.

We also approximate the production capital construction constraint in (1.9). Following the algebra detailed in Appendix 1.B, we get the following aggregate version of the capital Euler equation;

$$\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}\left(\overbrace{F_{k,t+1}\tau^\gamma_t}^{\bar{R}^k_{t+1}} + (1-\bar{\delta})\right) \tag{1.15}$$

where

$$\tau_t^{\gamma} := \left\{ 1 + \frac{1}{2}\varepsilon(\varepsilon-1)\sigma_{\delta}^2 \left(\frac{m_{t+1}}{1-m_{t+1}(1-\bar{\delta})} \right)^2 \right\}^{\frac{1}{\varepsilon-1}}$$

(1.15) is similar to the capital Euler in the canonical RBC model, except for the wedge τ^{γ}. If, as before, either variance of depreciation rates, or substitutability of capital assets is zero, we go back to the canonical RBC representation of the Euler.

The final approximation is for the non-depreciated capital each period, $\int(1-\delta(i))k_t(i)\text{di}$. We arrive at an expression for the time varying rate of depreciation, D, by approximating $(1-D_t)\bar{k}_t = \int(1-\delta(i))k_t(i)\text{di}$ and rearranging, as explained in Appendix 1.B;

$$D_t \equiv \bar{\delta} + \tau_{t-1}^D = 1 - (1-\bar{\delta}) \left\{ \frac{1 + \mathbb{E}_{t-1}X_t + \frac{\varepsilon\sigma_{\delta}^2}{\mathbb{E}_{t-1}\bar{R}_t^k(1-\bar{\delta})}}{1+\mathbb{E}_{t-1}X_t} \right\} \tag{1.16}$$

where

$$\tau_{t-1}^D = 1 - \left\{ \frac{1 + X_t + \frac{\varepsilon\sigma_{\delta}^2}{\mathbb{E}_{t-1}\bar{R}_t^k(1-\bar{\delta})}}{1+X_t} \right\} + \bar{\delta} \left[\left\{ \frac{1 + X_t + \frac{\varepsilon\sigma_{\delta}^2}{\mathbb{E}_{t-1}\bar{R}_t^k(1-\bar{\delta})}}{1+X_t} \right\} - 1 \right]$$

When $\sigma_{\delta}^2 = 0$ or $\varepsilon = 0$, $X_t = 0$ and $\tau^D = 0$, which implies $D_t = \bar{\delta} = D$.

1.3.5 Depreciation rates

There are three different measures of depreciation we have come across so far. The first, $\delta(i)$, is the individual rate of depreciation for asset (i). This rate is considered constant and set by taxation schedules. The theoretical minimum value is 0 for near permanent assets like land, and theoretical maximum is 1 for assets which can be fully depreciated in about a year.

The second measure of depreciation is the average of all of the above constant rates, $\bar{\delta} := \int_0^1 \delta(i)\text{di}$. This is some value between 0 and 1, depending on what we choose as the minimum and maximum rates of depreciation for assets. This is also the rate of depreciation around which our second order approximations are made, and the mean rate around which the variance σ_δ^2 is calculated.

The final measure, the time varying rate of depreciation D, is our contribution from this exercise, and it depends on how the portfolio composition changes on impact of a technology shock. We have already discussed the cyclical properties of D and have found it to be procyclical as a result of how the asset reallocation takes place.

1.3.6 The 'wedges'

We can have a representation of something that seems like an efficiency wedge in the production function as a result of being able to distinguish between the production capital and the balance sheet portfolio. By replacing the production capital in the production function by balance sheet portfolio, we obtain the following;

$$Y_t = A_t(1 - \tilde{\tau}_t)\bar{K}_t^{\alpha} N_t^{1-\alpha} \tag{1.17}$$

where

$$1 - \tilde{\tau}_t := \left(\frac{K_t}{\bar{K}_t}\right)^{\alpha}$$

In the above, $\tilde{\tau}$ is not the efficiency wedge, although it might seem that way at first glance. It is in fact a result of there being a continuum of capital goods being chosen in varying quantities. The choices are still optimal, and the reason for the differential is further explained in Chapter three. For now, we will merely state without details that the wedge-like variable in this model where financial frictions are absent is a representation of the difference between choosing a single type of capital asset versus choosing a continuum of capital assets with different rates of depreciation but the same marginal cost.

Similarly, the other two 'wedges', τ^{γ} and τ^{k} are not actual wedges in the conventional sense which distort optimal choices. The Euler 'wedge' τ^{γ} and the capital 'wedge' τ^{k} represent how, because of there being different types of capital assets with the same marginal cost but differing marginal products, the optimal investment choices result in differential holdings of capital assets and that the firm can choose the production bundle in addition to the balance sheet portfolio. If we were to set $\sigma_{\delta}^{2} = 0$ to indicate that there is only a single type of capital asset, τ^{γ} and τ^{k} would disappear and we would return to the standard RBC model.

The same applies for our most important wedge in this setting, the depreciation 'wedge'. τ^{d} is merely a representation of how the depreciation rate fluctuates on aggregate because of a change in the underlying composition of balance sheet portfolio. It does not indicate any deviation from the optimal: the underlying portfolio chosen is always optimal given the environment.

1.3.7 System equations

We first define the competitive equilibrium. The states $\boldsymbol{s}$ are portfolio capital $\bar{K}$, production capital K, effective depreciation rate D, and the shock process A.

Definition 1 *Recursive Equilibrium: A recursive competitive equilibrium is defined as a set of functions for (1) households' policies $C^{h}(\boldsymbol{s})$, $N^{h}(\boldsymbol{s})$, and $\bar{K}^{l}(\boldsymbol{s})$; (2) production firms' policies $K(\boldsymbol{s})$, $\bar{K}(\boldsymbol{s})$, and $N(\boldsymbol{s})$; (3) aggregate prices $W(\boldsymbol{s}$ and $R(\boldsymbol{s})$; (4) law of motion for aggregate states $\boldsymbol{s}' = \boldsymbol{\Phi}(\boldsymbol{s})$; such that (i) households policies satisfy its first order conditions; (ii) firms' policies are optimal and $V(\boldsymbol{s})$ satisfies the Bellman equation; (iii) wage and interest rates clear the labour and capital markets, and $m(\boldsymbol{s}) = \beta^{U_c(C',N')}/_{U_c(C,N)}$; (iv) the law of motion for $\boldsymbol{\Phi}(\boldsymbol{s})$ is consistent with individual decisions and the stochastic process for A.*

Following is the full system of equations that describes the entire model. The system is linearised and solved using perturbation methods around a deterministic steady state.

$$C_t^{-\sigma} = \lambda_t \tag{1.18}$$

Marginal Utlity of Consumption

$$\nu N_t^{\psi} = \lambda_t W_t \tag{1.19}$$

Labour Supply

$$W_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \tag{1.20}$$

Wages

$$F_{k,t} = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} \tag{1.21}$$

Marginal Product of Capital

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{1.22}$$

Production Function

$$Y_t = C_t + \underbrace{\bar{K}_{t+1} - (1 - D_t)\bar{K}_t}_{I_t} \tag{1.23}$$

National Income Accounting Identity

$$m_t = \beta \mathbb{E}_{t-1} \left(\frac{\lambda_t}{\lambda_{t-1}} \right) \tag{1.24}$$

SDF

$$\lambda_t = \mathbb{E}_t \lambda_{t+1} \left[F_{k,t+1} \tau_t^{\gamma} + (1 - \bar{\delta}) \right] \tag{1.25}$$

Capital Euler

$$\tau_t^{\gamma} = \left\{ 1 + \mathbb{E}_t X_{t+1} \frac{\varepsilon - 1}{\varepsilon + 1} \right\}^{\frac{1}{\varepsilon - 1}} \tag{1.26}$$

Euler wedge

$$\mathbb{E}_t X_{t+1} = \left(\frac{\varepsilon(\varepsilon + 1)\sigma_{\delta}^2}{2(\mathbb{E}_t \bar{R}_{t+1}^k)^2} \right) \tag{1.27}$$

Intermediate variable

$$\bar{R}_t^k = \left(\frac{1 - m_t(1 - \bar{\delta})}{m_t} \right) \tag{1.28}$$

Average rental rate

$$K_{t+1} = \bar{K}_{t+1}(1 - \tau_t^k) \tag{1.29}$$

Portfolio & B/S Capital

$$\tau_t^k := 1 - \frac{\left(\mathbb{E}_t X_{t+1} \frac{\varepsilon - 1}{\varepsilon + 1} \right)^{\frac{\varepsilon}{\varepsilon - 1}}}{1 + \mathbb{E}_t X_{t+1}} \tag{1.30}$$

Capital wedge

$$D_{t+1} = \bar{\delta} + \tau_t^D \tag{1.31}$$

Time varying depreciation rate

$$1 - \tilde{\tau}_t = \left(\frac{K_t}{\bar{K}_t} \right)^{\alpha} \tag{1.32}$$

New TFP Measure

$$A_t = A_{t-1}^{\rho} e^{u_t} \tag{1.33}$$

Technology shock

$$\tau_t^D := 1 - \left\{ \frac{1 + \mathbb{E}_t X_{t+1} + \frac{\varepsilon \sigma_\delta^2}{\mathbb{E}_t \bar{R}_{t+1}^k (1-\bar{\delta})}}{1 + \mathbb{E}_t X_{t+1}} \right\} + \bar{\delta} \left[\left\{ \frac{1 + \mathbb{E}_t X_{t+1} + \frac{\varepsilon \sigma_\delta^2}{\mathbb{E}_t \bar{R}_{t+1}^k (1-\bar{\delta})}}{1 + \mathbb{E}_t X_{t+1}} \right\} - 1 \right] \tag{1.34}$$

Depreciation wedge

In the above, $\bar{K} := \int_0^1 K(i)\mathrm{di}$ is aggregate capital that appears in the balance sheet, whereas K is the portfolio constructed according to (1.9). $\bar{R}^k := \int_0^1 R^k(i)\mathrm{di}$, where $R^k(i)$ is obtained from (1.11). $\bar{\delta} := \int_0^1 \delta(i)\mathrm{di}$.

If $\sigma_\delta^2 = 0$, then $X = 0 \Rightarrow \tau^\gamma = 1$; $\tau^K = 0$; $\tau^D = 0$. Putting these in the system above causes the equations to collapse into those of the canonical RBC Model.

There are 16 equations from (1.18) to (1.34) for the following 16 variables;

$$[C \;\; K \;\; N \;\; Y \;\; \lambda \;\; \Gamma \;\; \bar{K} \;\; D \;\; \bar{R}^k \;\; A \;\; \tau \;\; \tau^\gamma \;\; \tau^k \;\; \tau^D \;\; A \;\; X]$$

1.3.8 Model calibration

The parameters values used are as under;

Parameter Values

Parameter	Value	Description
β	0.995	Discount factor
α	$1/3$	Capital share
σ	1.01	Household intertemporal substitutability
ψ	1	Frisch elasticity
ν	1	Disutility from labour
ε	0.3	Substitutability of capital assets
$\bar{\delta}$	$0.057^{\dagger}$	Average rate of depreciation
σ_δ^2	0.01	Variance of depreciation rates
ρ	0.9	Shock persistence
σ_A^2	0.01	Variance of technology shock

†Calibrated to obtain an aggregate rate of depreciation $D = 0.025$ in steady state

Table 1.1 Parameters

The most important parameter, the degree of elasticity of substitution between capital assets, ε, is set to 0.3 implying that the assets are very close to being complements. We use this value of the parameter for our solution because we find that we get results which are closest to the canonical RBC model. Results from values which make assets more substitutable are presented in Appendix 1.C.4. The average depreciation rate $\bar{\delta}$ is set at a value of 0.065 which is slightly higher than the canonical RBC, and the reason is that we aim to get a steady state value for the effective rate of depreciation D of 0.035. As discussed, given that the balance sheet portfolio is heavier in long term assets, the effective depreciation rate turns out to be lower than the average rate used. The variance of depreciation rates σ_δ^2 is set at 0.01, although it might be argued that it can be anything up to 0.08 or thereabout. However, the role of this parameter is only to change the magnitude of response in the dynamics, and to some extent, change slightly the values in steady state, so it does not seem to be critical to the extent of the value of ε.

We use the discount factor β to imply an annual risk free rate of 2% ($\beta^4 = 0.98$). The share of capital α is set to the widely used value of 1/3. The intertemporal elasticity of substitution is set as close as possible to the log specification with a value of 1.01. The Frisch elasticity parameter and disutility of labour parameter are set to 1 to limit their impact on dynamics. The $AR(1)$ parameter ρ is set to 0.9, which implies the shock is fairly persistent.

1.3.9 Steady state analysis

In the steady state, $\bar{R}^k$ is pinned down by (1.28). In turn, X is also pinned down, and so are all the wedges τ^γ, τ^k, τ^D, as well as Γ. $A = 1$ in steady state. So the system reduces to 8 equations and 8 variables, as under;

$$C^{-\sigma} = \lambda \tag{1.35}$$

$$\nu N^{\psi+\alpha} = \lambda W \tag{1.36}$$

$$W = (1-\alpha)K^\alpha N^{1-\alpha} \tag{1.37}$$

$$Y = K^\alpha N^{1-\alpha} \tag{1.38}$$

$$Y = C + (\bar{\delta} + \tau^D)\bar{K} \tag{1.39}$$

$$F_k = \alpha K^{\alpha-1} N^{1-\alpha} \tag{1.40}$$

$$K = \bar{K}(1-\tau^K) \tag{1.41}$$

$$\tau = \left(\frac{K}{\bar{K}}\right)^\alpha \tag{1.42}$$

The variables are;

$$[C \;\; K \;\; N \;\; Y \;\; W \;\; \lambda \;\; \bar{K} \;\; \tau]$$

We put the above system through a nonlinear solver in Julia™ and obtain the following solution;

Steady State Values

Variable	Description	Model	RBC
C/Y	Consumption-Output ratio	0.72	0.72
$\bar{K}/Y$	Portfolio Capital-Output ratio	11.14	11.11
K/Y	Production Capital-Output ratio	8.47	11.11†
N	Labour	0.96	0.96
D	Time-varying aggregate depreciation	0.025	–

†Canonical RBC has only one measure of capital

Table 1.2 Steady State Values

The resulting measure of productivity in the steady state, τ, has value ~ 0.93 as compared to 1 in the canonical RBC model. The canonical RBC model does not consider any efficiency wedge arising from there being different types of capital assets to choose from, which is what we have here. This is similar to the steady state of a New Keynesian model with Calvo pricing where price dispersion exists even in equilibrium causing an efficiency wedge which lowers output. However, in the NK setting, policy implications are generated in the form of keeping prices constant to remove dispersion, whereas in our case there is no policy implication as yet. We will however revisit this in the later chapters. For now, the implication is that the output is not at the efficient level due to the fact that we have assumed assets to be complements. If, however, we change calibration of ε, we can change the value of τ by making assets more substitutable. (More details and results in Appendix 1.C.4.) We have that $\bar{K} > K$, and not all capital present on the balance sheet is deployed for production. This nuance is lost in the standard RBC model where there is no distinction drawn between balance sheet portfolio capital and production capital.

1.4 Results

We present the impulse response functions from a positive technology shock in this section, and then discuss those results.

1.4.1 Impulse response functions

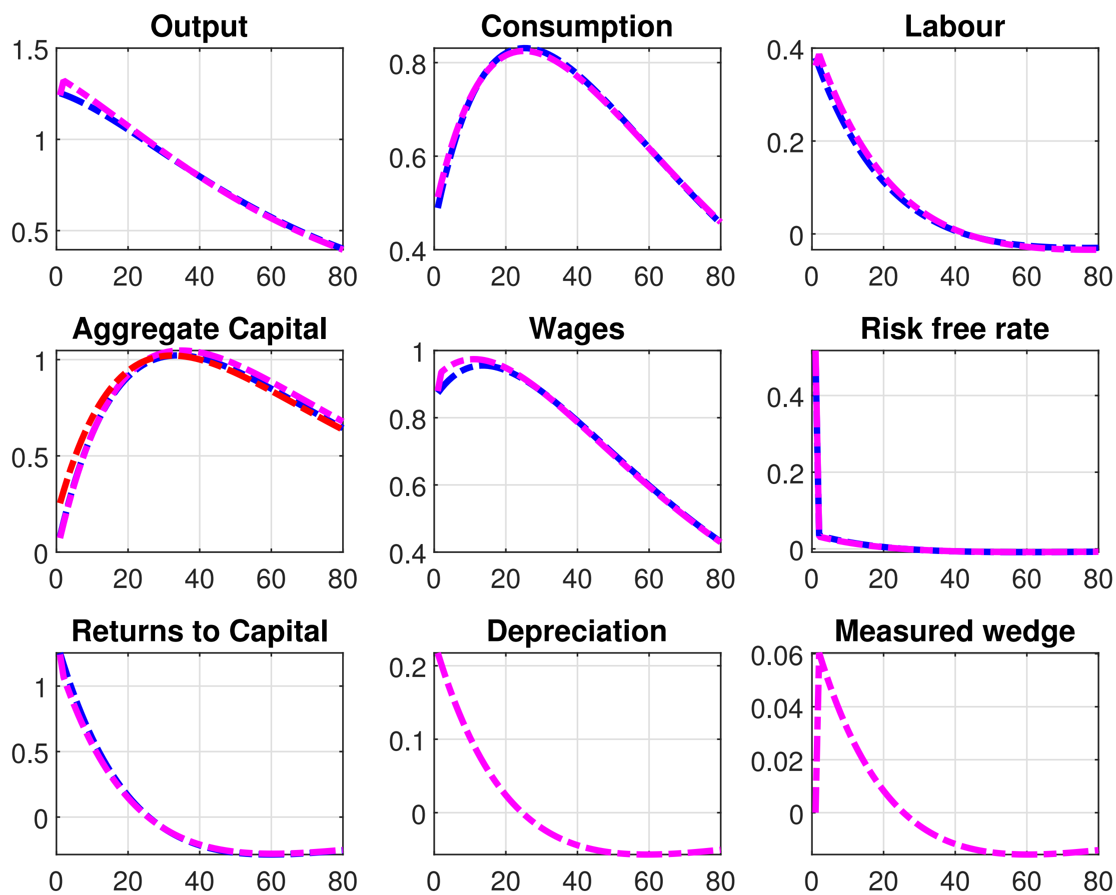


Figure 1.3 Complementary assets and a positive technology shock

Assets are complements. Portfolio Model, Canonical RBC Model, Production capital in Portfolio model

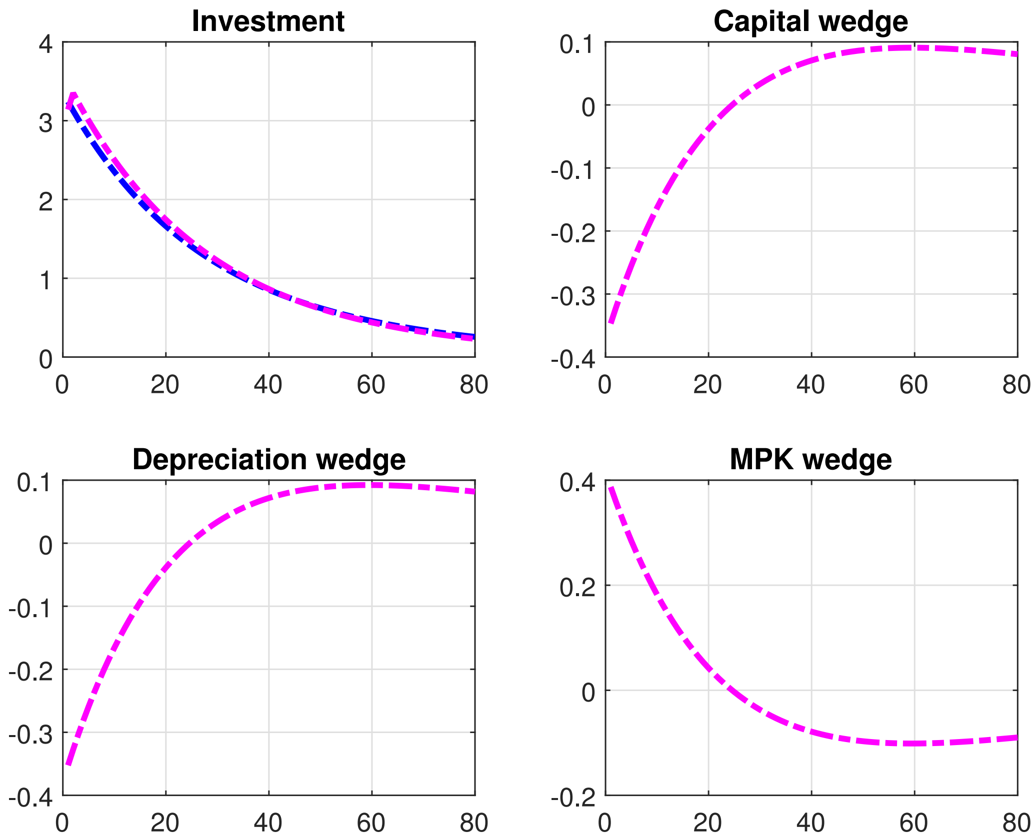


Figure 1.4 Complementary assets and a positive technology shock (contd.)

Assets are complements. Portfolio Model, Canonical RBC Model

1.4.2 Discussion

As can be seen in figures 1.3 and 1.4, the response to a positive technology shock is almost identical for both models. As the marginal products of labour and capital rise on impact, labour demand is higher which raises output and consumption also rises from a rise in wages and in output. Consumption smoothing causes households to save more and take advantage of the fact that capital is now below optimal. The risk free rate also rises inducing savings and thus investment. Higher demand for production capital also raises the rental rate of assets. The canonical RBC model has a higher impact on the risk free rate on impact because it uses a lower rate of depreciation, whereas the portfolio model uses $\bar{\delta}$, the average rate of depreciation, which is much higher than the effective rate of depreciation D. This explains the response of the first seven panels in figure 1.3 and the first panel of figure 1.4.

The rest of the variables are highly dependent on how we calibrate ε. Since we have assumed that the assets are more complementary in this case, we find that when the incentives are such that households want to accumulate more capital (as in the case being discussed), the increase needs to be relatively more spread out than in the case when assets are substitutes. Increasing substitutability causes the accumulation to be focussed on only a limited types of assets, because it is possible to use the same assets to replace all other types of assets. The case of substitutes is discussed in detail in Appendix 1.C.4.

As the rental rate R^k rises, the intermediate variable X falls, causing the capital wedge τ^k to fall as well. Since τ^k is negative in the steady state, a further fall increases the denominator of $\frac{1}{1-\tau^k}$ in (1.29). This explains the response of balance sheet portfolio $\bar{K}$ being lower than that of production capital K. Similarly, the depreciation wedge τ^D is also negative in the steady state, and a further fall causes the effective rate D to rise from (1.31). The MPK wedge τ^γ is positive in the steady state, and a fall in its value along with a simultaneous rise in the expected marginal product of capital tempers the response of the shock through the capital Euler in (1.25).

The measure of the efficiency wedge τ in (1.32) depends on what happens to the balance sheet portfolio as well as the production capital, which, in turn, depends on how we calibrate ε. In the present case of complementary assets, a positive technology shock increases production capital by more than the balance sheet portfolio, increasing the measure of TFP a period after impact. This creates a procyclical movement as observed in 1.3. If we were to carry out a production function decomposition based on the balance sheet capital, where we have not made a distinction between production capital and balance sheet capital, we might overestimate the TFP and the impact of the technology shock. The distinction between capital types could act as an omitted variable when decomposing the production function to obtain TFP.

An alternative calibration of ε where we consider assets to have a higher degree of substitutability changes many of the results above. Specifically, we find that effective depreciation is counter cyclical in that case, because the portfolio reshuffling favours long durations assets on a positive shock and short duration assets on a negative shock. This also causes the efficiency wedge to be counter cyclical. Detailed results on the steady state, as well as the dynamics of the model along with discussion are presented in Appendix 1.C.4.

1.4.3 Second moments for model with time varying depreciation

We also compute the second moments for variables in our model and compare them with moments from the data and from a standard RBC model. All data is from the BEA and is expressed as real per capita quarterly figures. Output (Y) is the GDP series starting from 1947 Q1 till 2020 Q3, Investment (I) is the sum of private fixed investment and durables consumption starting 2002 Q1 till 2020 Q3, Consumption (C) is the nondurables and services consumption starting 1947 Q1 till 2020 Q3, Labour hours (N) are the recorded hours in the nonfarm sector starting 1948 Q1 till 2020 Q3 and Wages (W) is the nonfarm compensation starting 1947 Q1 till 2020 Q3. All data are in logs and filtered using the HP filter. This methodology is the same as adopted by [19].

As for the RBC model and our model, we solve both models using parameter values stated previously. We then simulate each variable series for 300 periods and discard the first 100 periods. We then HP filter the data and compute moments based on the cyclical deviations.

Moments from data, RBC and modified model

	σ_X			σ_X/σ_Y			$ACF(1)$			$corr(X,Y)$		
Variable	Data	RBC	Model	Data	RBC	Model	Data	RBC	Model	Data	RBC	Model
Y	0.02	0.02	0.02	1.0	1.0	1.0	0.78	0.67	0.70	1.0	1.0	1.0
I	0.04	0.04	0.04	2.4	2.4	2.4	0.88	0.65	0.70	0.75	0.99	0.99
C	0.01	0.01	0.007	0.6	0.5	0.4	0.63	0.71	0.70	0.79	0.97	0.98
N	0.02	0.01	0.01	1.1	0.6	0.6	0.81	0.65	0.70	0.89	0.99	0.99
W	0.01	0.01	0.01	0.6	0.5	0.5	0.65	0.67	0.70	0.02	0.99	0.99

Table 1.3 Moments for model with time varying depreciation rate

We see in table 1.3 that we can match the data moments fairly well, apart from the correlation of wages with output for both RBC and our model. The correlations for other variables with output are also not as high in the data as compared to the model. However, we do almost as well as the RBC model in matching the standard deviations of variables. The autocovariances in the data are of a smaller magnitude than those generated by both the RBC and our models.

1.5 Conclusion

We modified the canonical RBC model to include a continuum of capital assets, each with its own depreciation rate, and changed the firm's optimisation problem slightly which also required the firms to choose a bundle of productive capital from this continuum of capital assets. The capital assets are owned by households and are chosen optimally. This distinction between production capital and the balance sheet portfolio capital produces a time varying efficiency wedge in the model. We also showed how the balance sheet portfolio of capital changes in composition over the business cycle, and if we impose that the assets are complements, we obtain a procyclical time varying rate of depreciation. The effective rate turns out to be procyclical because the portfolio moves towards more short term assets on a positive shock and more long term assets on a negative shock. The demand function for each type of capital depends on the movements in marginal utility of consumption for the household. We show we can generate impulse responses nearly identical to the canonical RBC for all the conventional variables, and additionally for the new measure of capital and depreciation.

We also generate second moments from our model and compare it to second moments from the data and the canonical RBC model and find that our model does almost as well as the RBC model at matching data moments.

We will use the model developed here in chapters two and three to introduce financial frictions and analyse their impact in this model setting.

References

[1] Alice Albonico, Sarantis Kalyvitis, and Evi Pappa. Capital maintenance and depreciation over the business cycle. *Journal of Economic Dynamics and Control*, 39:273–286, 2014.

[2] Steve Ambler and Alain Paquet. Stochastic depreciation and the business cycle. *International Economic Review*, pages 101–116, 1994.

[3] John Baldwin, Guy Gellatly, Marc Tanguay, and André Patry. Estimating depreciation rates for the productivity accounts. In *OECD Workshop on Productivity Measurement, Madrid Spain, October*, pages 17–19, 2005.

[4] Susanto Basu. Procyclical productivity: increasing returns or cyclical utilization? *The Quarterly Journal of Economics*, 111(3):719–751, 1996.

[5] Susanto Basu. Procyclical productivity: increasing returns or cyclical utilization? *The Quarterly Journal of Economics*, 111(3):719–751, 1996.

[6] Paul Beaudry, Alban Moura, and Franck Portier. Reexamining the cyclical behavior of the relative price of investment. *Economics Letters*, 135:108–111, 2015.

[7] Jeffrey I. Bernstein and M. Ishaq Nadiri. No title. *Investment, depreciation, and capital utilization*, 1988.

[8] Mark Bils and Jang-Ok Cho. Cyclical factor utilization. *Journal of Monetary Economics*, 33(2):319–354, 1994.

[9] Raouf Boucekkine, Giorgio Fabbri, and Fausto Gozzi. Maintenance and investment: complements or substitutes? a reappraisal. *Journal of Economic Dynamics and Control*, 34(12):2420–2439, 2010.

[10] Raouf Boucekkine, Fernando Del Rio, and Blanca Martínez. Technological progress, obsolescence, and depreciation. *Oxford Economic Papers*, 61(3):440–466, 2009.

[11] Raouf Boucekkine and Ramón Ruiz-Tamarit. Capital maintenance and investment: complements or substitutes? *Journal of Economics*, 78(1):1–28, 2003.

[12] Craig Burnside, Martin Eichenbaum, and Sergio Rebelo. Capital utilization and returns to scale. *NBER macroeconomics annual*, 10:67–110, 1995.

[13] Varadarajan V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. Business cycle accounting. *Econometrica*, 75(3):781–836, 2007.

[14] Fabrice Collard and Tryphon Kollintzas. No title. *Maintenance, utilization, and depreciation along the business cycle*, 2000.

[15] W. Erwin Diewert. *Issues in the measurement of capital services, depreciation, asset price changes, and interest rates*, pages 479–556. Measuring capital in the new economy. University of Chicago Press, 2005.

[16] Paul Gomme, Finn E. Kydland, and Peter Rupert. Home production meets time to build. *Journal of Political Economy*, 109(5):1115–1131, 2001.

[17] Jeremy Greenwood, Zvi Hercowitz, and Gregory W. Huffman. Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417, 1988.

[18] Nir Jaimovich and Max Floetotto. Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics*, 55(7):1238–1252, 2008.

[19] Robert G King and Sergio T Rebelo. Resuscitating real business cycles. *Handbook of macroeconomics*, 1:927–1007, 1999.

[20] Wendy CY Li and Bronwyn H. Hall. Depreciation of business rd capital. *Review of Income and Wealth*, 66(1):161–180, 2020.

[21] Felipe Meza and Erwan Quintin. Factor utilization and the real impact of financial crises. *The BE Journal of Macroeconomics*, 7(1), 2007.

[22] Randy A. Nelson and Michael R. Caputo. Price changes, maintenance, and the rate of depreciation. *Review of Economics and Statistics*, 79(3):422–430, 1997.

[23] Julio J. Rotemberg and Michael Woodford. Markups and the business cycle. *NBER macroeconomics annual*, 6:63–129, 1991.

Appendix

1.A Second order approximations

Consider the following Taylor series expansion;

$$
\begin{aligned}
y &= \left[\int_0^1 x_i^{\frac{1}{p}}\ di\right]^p \\
&\approx \left[\int_0^1 \left(x_0^{\frac{1}{p}} + \frac{1}{p}x_0^{\frac{1-p}{p}}(x_i - x_0) + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}(x_i - x_0)^2\right) di\right]^p \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1}{p}x_0^{\frac{1-p}{p}}\left(\int x_i\ di - x_0\right) + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\left(\int x_i^2\ di - 2\int x_i\ di\ x_0 + x_0^2\right)\right)^p \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)^p \qquad \left(\text{Assuming} \quad x_0 = \int x_i\ di\right) \\
&\approx \left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)^p \frac{x_0}{x_0} \\
&\approx \left[\frac{1}{x_0^{\frac{1}{p}}}\left(x_0^{\frac{1}{p}} + \frac{1-p}{2p^2}x_0^{\frac{1-2p}{p}}\operatorname{var}_i[x_i]\right)\right]^p x_0 \\
&\approx x_0\left(1 + \frac{(1-p)}{2p^2}\frac{\operatorname{var}_i[x_i]}{x_0^2}\right)^p
\end{aligned}
\tag{1A.1}
$$

We will use the general result in (1A.1) in Appendix B to derive the system equations.

1.B Model approximations

We start by combining equations (1.7) and (1.11), and the solving for $K(i)$ as under;

$$K_{t+1}(i) = K_{t+1}\left(\frac{\mathbb{E}_t F_{k,t+1}}{\mathbb{E}_t R^k_{t+1}(i)}\right)^{\varepsilon} \tag{1B.1}$$

where

$$\mathbb{E}_t R^k_{t+1}(i) = \frac{1 - m_{t+1}(1-\delta(i))}{m_{t+1}} + \underbrace{\sigma_{\lambda,R^k(i)}}_{\sim 0}$$

Now using the result in (1A.1), we can approximate (1B.1) as under;

$$\bar{K}_{t+1} \equiv \int_0^1 K_{t+1}(i)\, di \approx K_{t+1}\left(\frac{\mathbb{E}_t F_{k,t+1}}{\mathbb{E}_t \bar{R}^k_{t+1}}\right)^{\varepsilon} \left\{1 + \frac{\varepsilon(\varepsilon+1)\sigma^2_{\delta}}{2(\mathbb{E}_t \bar{R}^k_{t+1})^2}\right\} \tag{1B.2}$$

where

$$\begin{aligned} \mathbb{E}_t \bar{R}^k_{t+1} &\approx \int_0^1 \left(\frac{1 - m_{t+1}(1-\delta(i))}{m_{t+1}}\right) di \\ &\approx \left(\frac{1 - m_{t+1}(1 - \int_0^1 \delta(i)\, di)}{m_{t+1}}\right) \\ &\approx \left(\frac{1 - m_{t+1}(1-\bar{\delta})}{m_{t+1}}\right) \end{aligned} \tag{1B.3}$$

Now let $\left(\frac{\varepsilon(\varepsilon+1)\sigma^2_{\delta}}{2(\mathbb{E}_t \bar{R}^k_{t+1})^2}\right) := \mathbb{E}_t X_{t+1}$.

To obtain everything in the form of wedges, we start with the constraint on the portfolio construction, (1.9). Substitute (1B.1) into (1.9) for $K(i)$, and use the result in (1.7) to approximate. We obtain the following;

$$1 = \left(\frac{\mathbb{E}_t \bar{R}^k_{t+1}}{\mathbb{E}_t F_{k,t+1}}\right)^{-\varepsilon} \left\{1 + \mathbb{E}_t X_{t+1}\frac{\varepsilon - 1}{\varepsilon + 1}\right\}^{\frac{\varepsilon}{\varepsilon - 1}} \tag{1B.4}$$

Substituting for $\bar{R}$ from (1B.3) and rearranging, we get the Capital Euler from (1.25), reproduced below;

$$\lambda_t = \mathbb{E}_t \lambda_{t+1}\left[F_{k,t+1}\left\{1 + X_{t+1}\frac{\varepsilon - 1}{\varepsilon + 1}\right\}^{\frac{1}{\varepsilon - 1}} + (1-\bar{\delta})\right] \tag{1B.5}$$

Comparing (1B.3) and (1B.5) yields that;

$$\bar{R}_t = F_{k,t}\left\{1 + \mathbb{E}_{t-1}X_t\frac{\varepsilon - 1}{\varepsilon + 1}\right\}^{\frac{1}{\varepsilon-1}} \tag{1B.6}$$

If we were to express (1B.6) in the form of the standard wedge expression as in the main text;

$$\tau_t^{\gamma} := \left\{1 + \frac{1}{2}\varepsilon(\varepsilon - 1)\sigma_\delta^2\left(\frac{m_{t+1}}{1 - m_{t+1}(1 - \bar{\delta})}\right)^2\right\}^{\frac{1}{\varepsilon-1}} \tag{1B.7}$$

Now substitute (1B.6) into (1B.2), which gives;

$$K_t = \bar{K}_t\left(\frac{\left\{1 + \mathbb{E}_t X_{t+1}\frac{\varepsilon-1}{\varepsilon+1}\right\}^{\frac{\varepsilon}{\varepsilon-1}}}{1 + \mathbb{E}_t X_{t+1}}\right) \tag{1B.8}$$

If we were to express (1B.8) in the standard form of $K_t = \bar{K}_t(1 - \tau_t^K)$, then it would give us;

$$\tau_t^K = 1 - \frac{\left(1 + \mathbb{E}_t X_{t+1}\frac{\varepsilon-1}{\varepsilon+1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{1 + \mathbb{E}_t X_{t+1}} \tag{1B.9}$$

As for the time varying depreciation rate D_{t+1}, we obtain it from the observation that $\int(1 - \delta(i))K_t(i)\, di \neq (1 - \bar{\delta})\bar{K}_t$. So,

$$\begin{aligned}(1 - D_t)\bar{K}_t &= \int(1 - \delta(i))K_t(i)\, di \\ &= \underbrace{(1 - \bar{\delta})K_t\left(\frac{\mathbb{E}_{t-1}\bar{R}_t^k}{\mathbb{E}_{t-1}F_{k,t}}\right)^{-\varepsilon}\left\{1 + \mathbb{E}_{t-1}X_t + \frac{\varepsilon\sigma_\delta^2}{\mathbb{E}_{t-1}\bar{R}_t^k(1 - \bar{\delta})}\right\}}_{\text{approximating using Appendix A}}\end{aligned} \tag{1B.10}$$

Using (1B.2) in (1B.10), and simplifying, gives;

$$D_t \equiv \bar{\delta} + \tau_{t-1}^D = 1 - (1 - \bar{\delta})\left\{\frac{1 + \mathbb{E}_{t-1}X_t + \frac{\varepsilon\sigma_\delta^2}{\mathbb{E}_{t-1}\bar{R}_t(1-\bar{\delta})}}{1 + \mathbb{E}_{t-1}X_t}\right\} \tag{1B.11}$$

This gives us (1.34), as under;

$$\tau_{t-1}^{D} = 1 - \left\{ \frac{1 + X_t + \frac{\varepsilon \sigma_{\delta}^2}{\mathbb{E}_{t-1} \bar{R}_t^k (1-\bar{\delta})}}{1 + X_t} \right\} + \bar{\delta} \left[\left\{ \frac{1 + X_t + \frac{\varepsilon \sigma_{\delta}^2}{\mathbb{E}_{t-1} \bar{R}_t^k (1-\bar{\delta})}}{1 + X_t} \right\} - 1 \right] \tag{1B.12}$$

This completes the derivations for the system of equations used.

1.C Assets as substitutes

We present results when we change the assumption regarding complementarity of assets, and make them more substitutable by setting $\varepsilon = 3$, where assets are highly substitutable in production as compared to our original calibration.

1.C.1 Reallocation

Figure 1.C1 shows the steady state distribution of assets, as well as the response to a positive and negative technology shock.

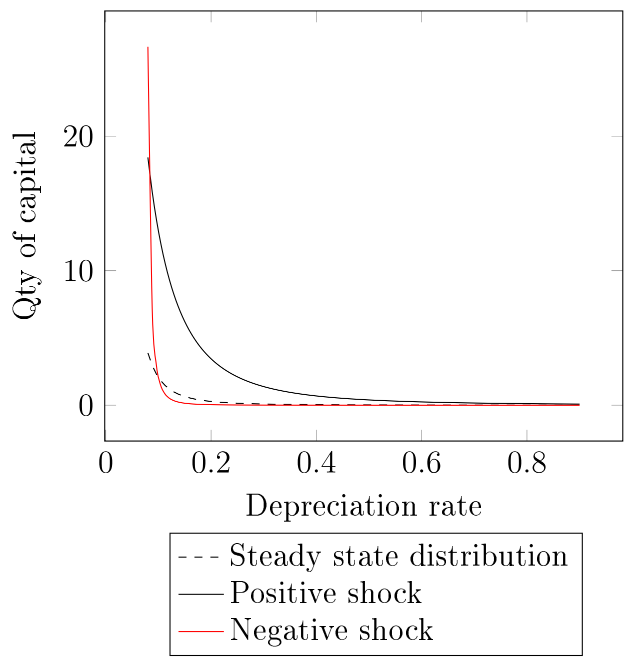


Figure 1.C1 Allocation when assets are substitutes

In case of the positive shock, we can see that the demand for long term and medium term assets rises by a lot, but the demand for the short term assets barely changes. This is in contrast to what we found in figure 1.2, where the demand for short term assets was higher on a positive shock. This change is driven by the fact that now assets are highly substitutable when constructing the production bundle and hence firms choose to have a large number of long term assets as substitutes for the short term assets which require recurring investment. As a result of this substitution the production bundle is actually smaller in terms of number of assets what in the steady state. The main difference between our original calibration and the current one is that the substitutability of assets allows firms to choose only a single type of asset in large quantities. Also, we can infer from this reallocation of assets in the portfolio that the rate of depreciation will fall on a positive shock.

Similarly, for a negative shock, the most movement is observed in the very long term assets, but falls for medium term assets and is mostly the same for short term assets. This causes the portfolio to be proportionally heavy in short term assets and hence the aggregate depreciation rate is again countercyclical.

1.C.2 Steady state values

Steady State Values

Variable	Description	Model	Canonical RBC
C/Y	Consumption-Output ratio	0.70	0.71
$\bar{K}/Y$	Portfolio Capital-Output ratio	5.50	4.76
K/Y	Production Capital-Output ratio	6.12	4.76
N	Labour	0.98	0.97
D	Time-varying aggregate depreciation	0.06	–
τ	Efficiency variable	1.04	–

Table 1.C.1 Alternate steady state values

Compared to table 1.2, we can see in 1.C.1 that the quantity of both kinds of capital is very high, and the effective rate of depreciation low. Also, $K > \bar{K}$ implying that the production capital is bigger than the balance sheet portfolio, which seems counter intuitive, but results from the increased substitutability of the assets. This also causes the efficiency variable τ to be greater than 1.

1.C.3 Impulse response functions

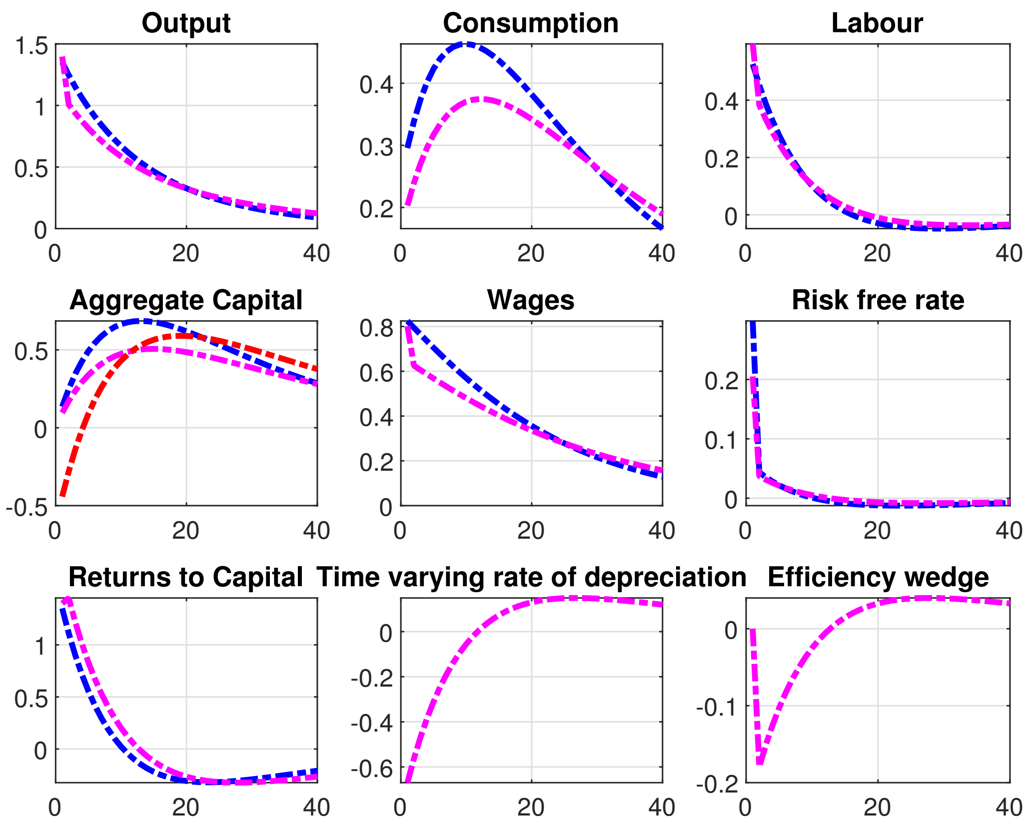


Figure 1.C2 Substitute assets and a positive technology shock

Assets are substitutes. Portfolio Model, Canonical RBC Model, Production capital in Portfolio model

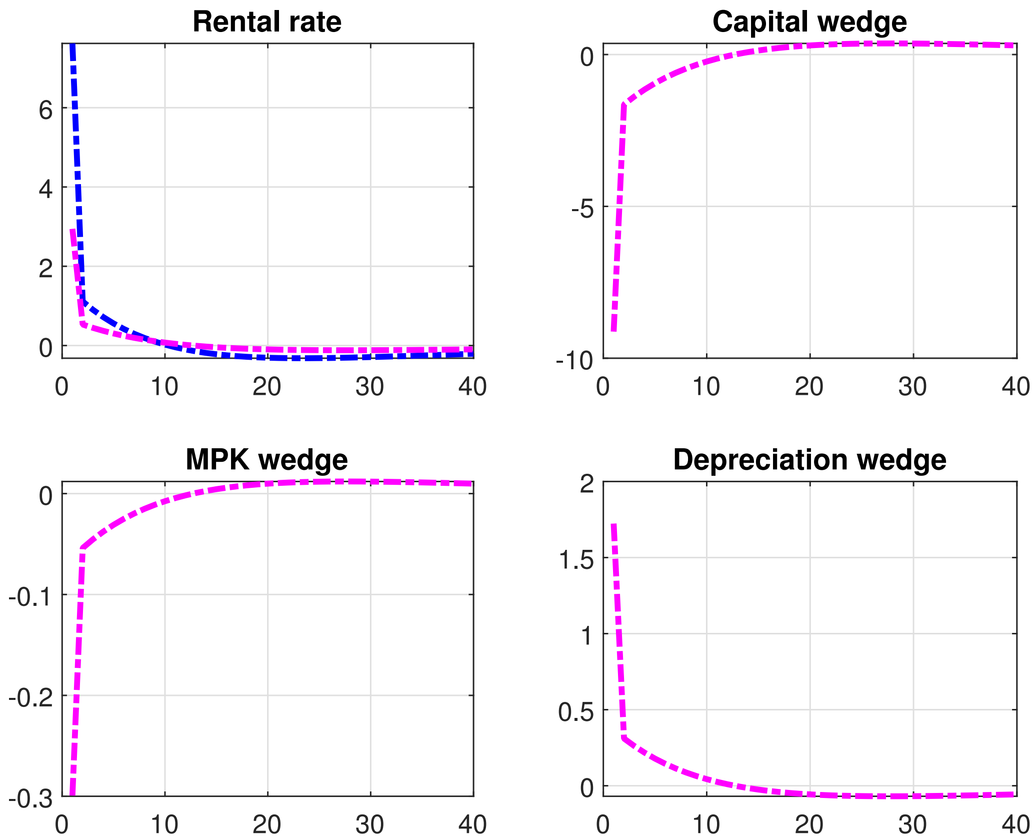


Figure 1.C3 Substitute assets and a positive technology shock (contd.)

Assets are substitutes. Portfolio Model, Canonical RBC Model

We can see in figures **??** and **??** that the response of output, labour, risk free rate, wages, and the marginal product of capital are very similar to that in the canonical RBC model. However, the response of capital is slightly strange. The balance sheet portfolio does not increase as much as in the canonical RBC case, and the production capital actually falls on impact of a positive technology shock. It seems like increasing the substitutability among capital assets has the effect that firms reduce the amount they invest in new capital on the balance sheet, and instead substitute the already available capital for the same. This also causes the production bundle to fall in size. Consumption, which is also already at a level higher than the canonical RBC in the steady state, rises by less. This results from the rise in investment at the normal level as in the canonical RBC and a fall in the production bundle which signals that output will not match that of the canonical model next period. To smooth consumption, it rises by less in the current period.

1.C.4 Rental rates

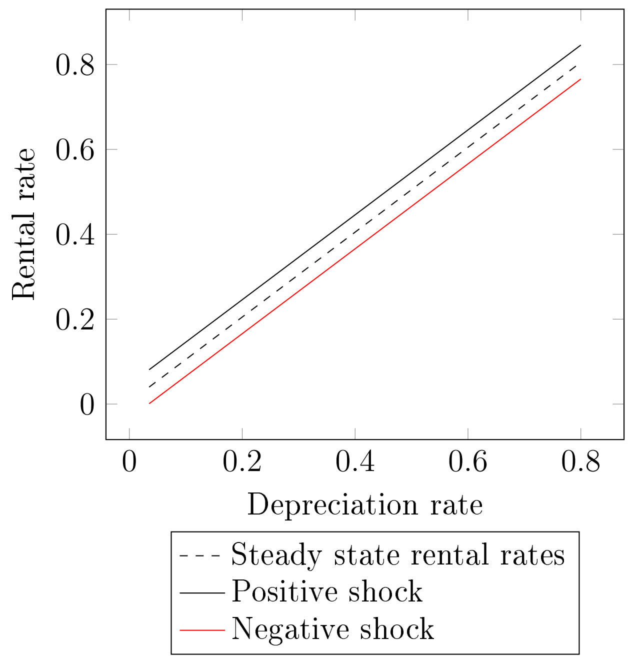


Figure 1.C4 Rental rates for individual assets

Figure 1.C4 shows what happens to the rental rates for all types of assets on impact of a positive or negative shock to technology. The steady state distribution shows that the rental rate is lowest for the long duration assets and highest for the short duration assets. This seems counter intuitive, because it would seem that the long duration assets would be the most expensive to rent. This anomaly is a result of the fact that we do not consider the prices of individual types of assets. In the presence of differential prices, the rental rates would also depend on these prices, and thus this anomaly would be addressed. For the current setting, we can see that on a positive shock, the rental rates rise uniformly and there is a parallel shift outward for the curve, and on a negative shock, the rental rates fall uniformly and there is a parallel shift inward for the curve.

Chapter Two

CAPITAL PORTFOLIO, COLLATERAL CONSTRAINT, AND PRODUCTIVITY

2.1 Introduction

We have so far modified the canonical RBC model and included a continuum of capital assets to generate a time varying effective rate of depreciation, and a measure of a wedge between the achievable steady state and the efficient steady state. We now move a step further to include loans through a collateral constraint which is the financial friction in the model. We also draw a straight line connecting the financial friction, change is the balance sheet portfolio, and a change in productivity through a time varying measure of the efficiency wedge. We focus on the efficiency wedge following the results of Chari et al. (2007) where they conduct a business cycle accounting exercise and find that the efficiency wedge is important is explaining the business cycle fluctuations. We carry out tests on U.S. data for fixed assets based on depreciations rates, and find that there is indeed a portfolio reallocation after the financial crisis.

We introduce a different kind of misallocation of factors compared to the existing literature. Most existing literature introduces heterogeneity of firms and idiosyncratic shocks or firm specific taxes on factors to generate misallocation. There is also literature which links financial friction to misallocation through the lack of access to funds forcing firms to have lower factors of production. The way these firms then overcome the constraint is by relying on retaining earnings and relying on savings. However, the ability to save and the rate of accumulating these funds depends on idiosyncratic shocks to productivity. So, a change in individual productivity affects access to finance, which causes misallocation of factors, and hence aggregate productivity is affected. The chnages in individual productivity are exacerbated by the financial friction, which reduces aggregate productivity. We rely on a very different set up and mechanism in our model. We introduce heterogeneity in asset types and have a representative aggregate firm. Also, our shocks actually originate in the financial sector and show up as a reduction in productivity because of the transmission mechanism of the model.

We find that our dynamic model can replicate some of the facts observed in data, and allows us to have a theoretical explanation reconciling a shock in the financial sector causing changes in the capital portfolio held by the aggregate firm which, in turn, is reflected as a lowering of productivity. We check the effects of alternative calibrations and preference specifications, and find that, except for minor differences, the results still hold.

We do make some simplifying assumptions, especially regarding prices of capital assets, and discuss how these affect the final results.

We discuss the data work carried out next, followed by a review of related literature before moving on to the model and results.

2.2 Empirical work

We present some facts from the data regarding the cyclical behaviour of productivity after a financial shock, as well as some insights into how firms in the U.S. have responded to financial crises with changes in the composition of their asset portfolios. Broadly, we can summarise the data work into following three stylised facts:

Stylised Fact 1 *TFP falls after a shock which originates in the financial sector*

Stylised Fact 2 *There is a reshuffling of the balance sheet portfolio towards more short term assets after the crisis in terms of percentage change in holdings*

Stylised Fact 3 *The fall in long term assets in dollar amount is much higher than that for short term assets, simple because of the difference in value between the asset classes. So, the overall dollar value of the balance sheet is lower after the crisis*

First we discuss the empirical work related to productivity, and then portfolio reallocation.

2.2.1 Productivity and financial shocks

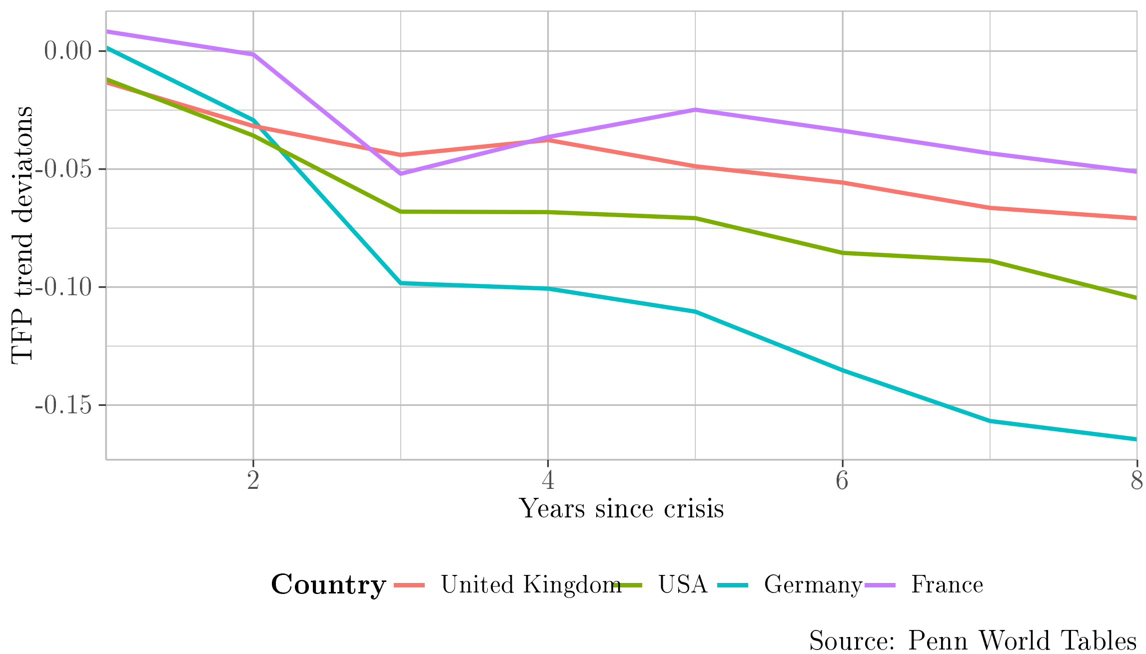
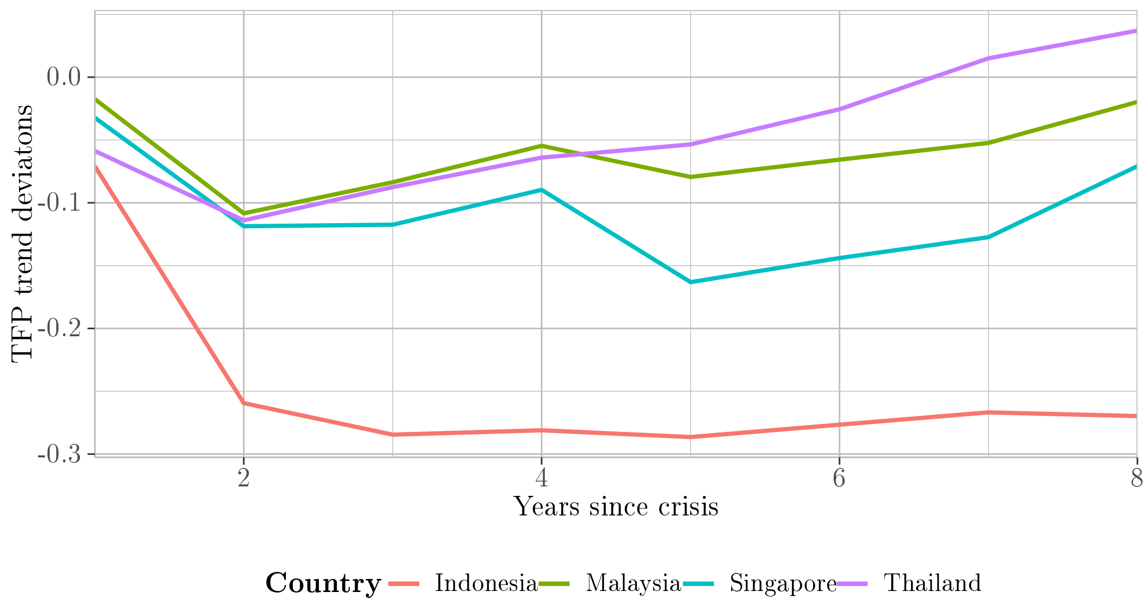


Figure 2.1 TFP for western countries post 2008 crisis

Figure 2.1 plots the linearly detrended productivity for UK, US, Germany, and France post the 2008 financial crisis. It is clear that after a financial shock, the TFP falls for a substantial period in these countries. Even after 8 years on the X axis, there is no clear upward trend. This might appear as a technology shock qualitatively, but it is just the financial shock manifesting itself through a fall in productivity. And this is not particular to the developed countries either.



Source: Penn World Tables

Figure 2.2 TFP for Asian countries post 2008 crisis

Figure 2.2 plots the linearly detrended productivity for Asian countries post the 1997 Asian financial crisis. Again, a similar fall in cyclical TFP is observed for all countries in the sample. Although both crises were very different in nature, one starting off in the banking sector while the other starting off as a sovereign debt crisis due to problems with the exchange peg, both had the impact of lowering asset prices which then caused bankruptcies and lowered access to financing for private sector due to an already high level of indebtedness.

The link between financial crises and productivity is not contentious, and there have been various studies drawing that link, as we will discuss in the literature section. However, we now move on to see whether there is a connection between the size of the balance sheet, the composition of the balance sheet, and the financial crisis.

2.2.2 Financial shocks and the balance sheet portfolio

We use data from the Bureau of Economic Analysis (BEA) for 100 different asset classes, ranging from the lowest to highest depreciation rates, based on average age of asset class. Age is calculated by the BEA using the permanent inventory method (PIM) which calculates age for any asset class at the end of the year based on how much was added in the current year, and how much depreciation was charged on that class[1]. Here we have used age calculated on the historical value of assets as against the current value of assets. Current value based age calculation gives more weight to current cost of old assets, and thus has more impact of price trends. We take the average of the age for each asset class from 1987 to arrive at an approximation of mean age over time for that asset class.

We present this information is in the form of bar plots for the treatment and control groups, where we can also see the direction of movement of the mean age difference. An increase in mean age implies no or little new investment was made in that asset class. Conversely, a fall in the mean age between the two time periods implies new investments were made in that asset class. We plot the percentage by which age drops further between the time periods under consideration, and so a negative bar implies asset class has aged further, and little or no investment was made. A positive bar implies that asset class is younger now because investment was made recently. Darker bars show significant changes and lighter ones insignificant changes. The X axis is rates of depreciation from lowest to highest, but it has been converted to a categorical scale, so the distance between bars is not truly indicative of the rate of depreciation. Put differently, the rightmost bars on a numerical scale would resemble the ones in histograms above, but appear closer because of scale transformation. A bar plot with numerical scale is available in appendix 2.B.3.

[1] Further information can be found at the BEA information page

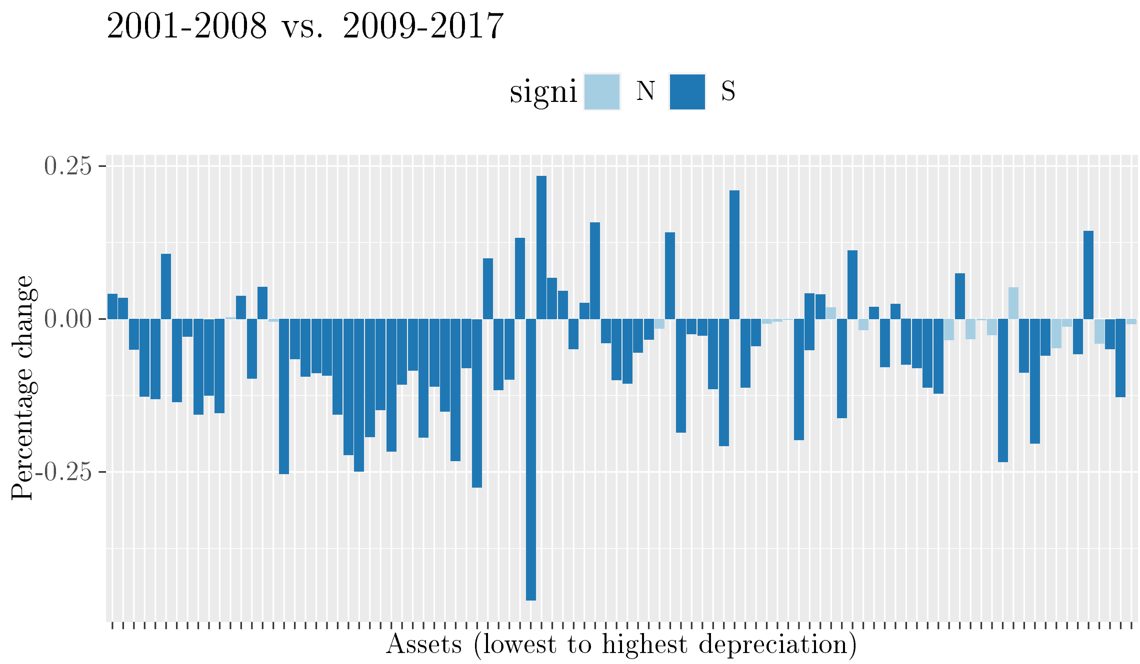


Figure 2.3 Change in means (2000-2008 vs. 2009-2017)

Source: BEA

Figure 2.3 shows that most significant change is concentrated in the first third of the scale, as seen previously in figure 2.B2. But now we can also see that the change age has fallen further, so investment in the long term assets has actually gone down over the crisis. The change towards the right end of the plot is mixed as to increase and decrease in ages of asset classes, indicating that the fall in investment for relatively short term assets was not as high as the long term ones. A control group plot for a different time period is available in the Appendix.

But this information is based solely on age of asset classes. We also want to know how the dollar values behave over an extended period of time, and so we now look at a plot of cyclical fluctuations in different asset groups in figure 2.4. This gives us a visual on how exactly the investment in asset classes has changed over time, which adds to already existing information about what happened ‘on average’ between two time periods for each asset class.

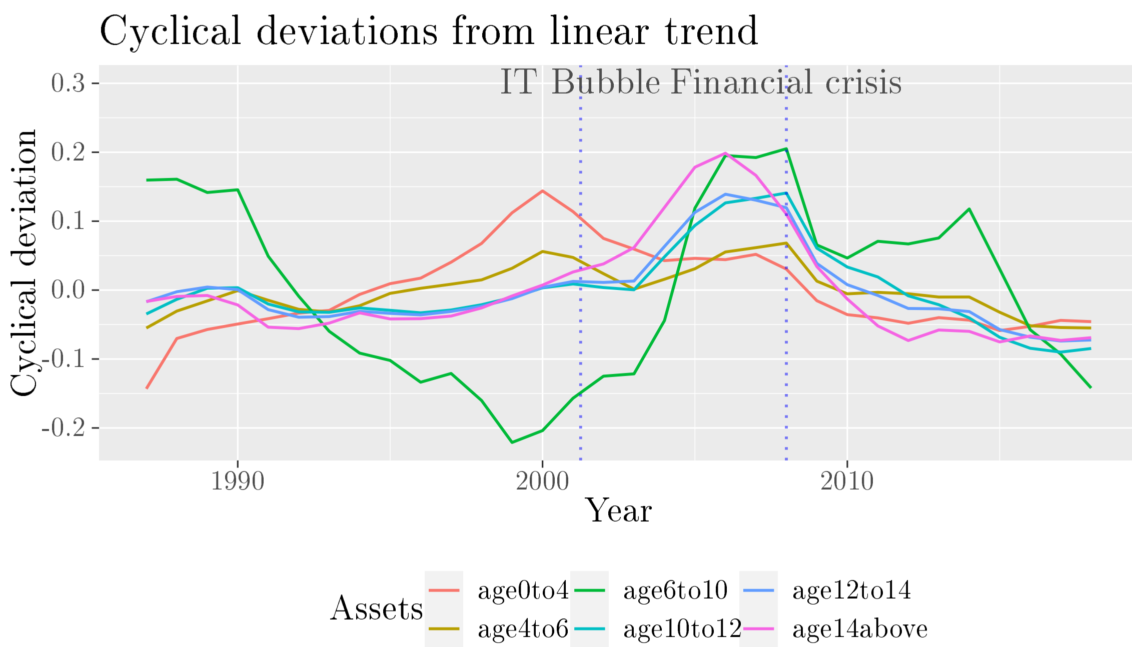


Figure 2.4 Change over time in asset classes

Source: BEA

Figure 2.4 shows that investment in all asset classes has fallen post the crisis, something we saw in figure 2.3. The assets class of age over 14 years shows the biggest fall in investment after the crisis. Asset class with age around 8 , although falls sharply, recovers quickly. The red line with the most short term asset class does not seem to fall as steeply as the others.

We now plot the same information after weighting assets by the share they claim in the total balance sheet. We see that smaller fluctuations in the longer term assets will have a bigger impact on the total balance sheet value than larger fluctuations in short term assets. This is evident in figure 2.5.

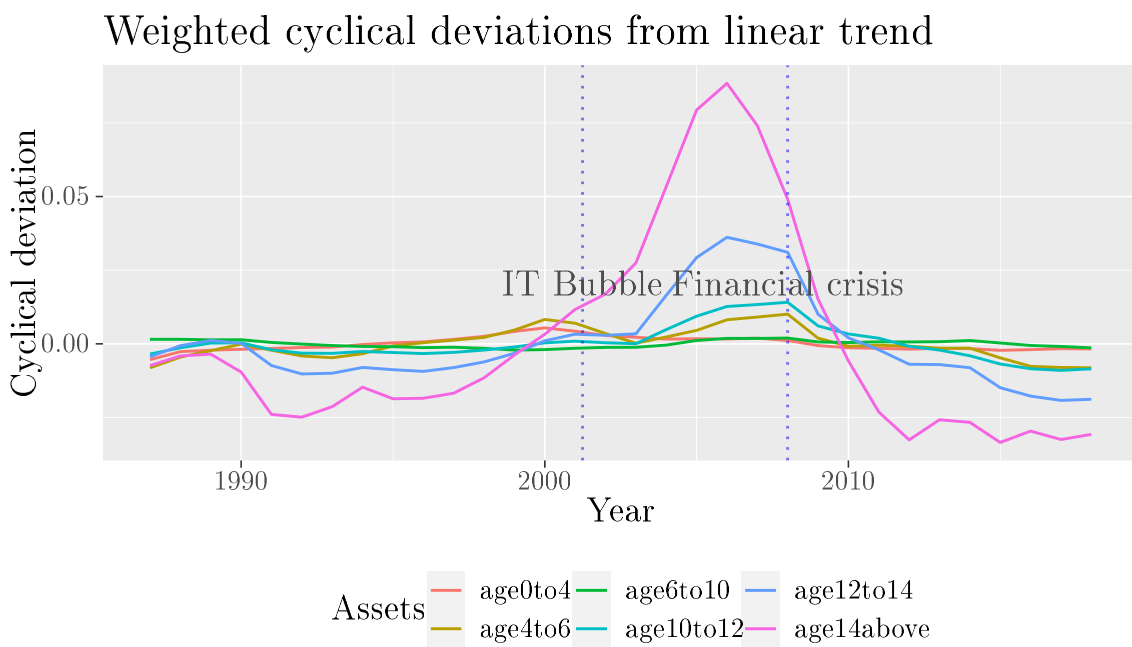


Figure 2.5 Change over time in weighted asset classes

Source: BEA

We plot the cyclical deviations from a linear trend for aggregated asset classes and we then weigh the fluctuations by the asset class' dollar share in the balance sheet. This allows us to visualise how the balance sheet as a whole would move around in dollar terms over the cycle. We see that the low depreciation assets cause the most dollar movement, whereas the impact of high depreciation short term assets is smaller. Compared to the low depreciation assets, it seems almost as if there were no change in short term assets. However, we know that to not be the case from figure 2.4.

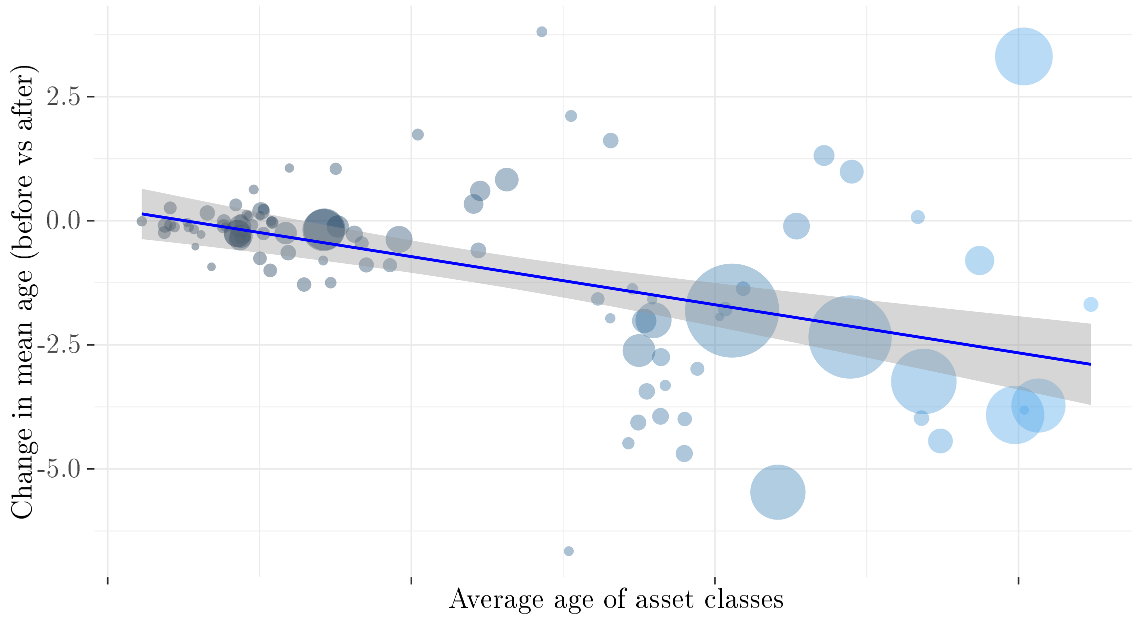


Figure 2.6 Scatter for change in means according to asset class

In figure 2.6, we plot the change in investment as per asset class over the crisis, measured, as before, by change in mean age of asset class. As before, a fall in mean age implies low or no investment. Placement of each 'bubble' represents amount of change in mean age on the Y axis, and size of each bubble indicates the weight carried in the balance sheet by that asset class in dollar terms. Again we see that investment has fallen most in the long term assets on the right most of the scale, whereas the short term assets register a lower drop. The difference between figures 2.6 and 2.3 is that the latter is in percentage change terms, whereas this is in absolute terms. Percentage change gives us a common base to compare the changes in investment, whereas looking at absolute values of change, although not directly comparable, gives us an insight into changes in the balance sheet as a whole without forcing a common base.

To summarise and reiterate, the data exercise above generates the following insights;

i. TFP falls after a shock which originates in the financial sector

ii. There is a reshuffling of the balance sheet portfolio towards more short term assets after the crisis in terms of percentage change in holdings

iii. The fall in long term assets in dollar amount is much higher than that for short term assets, simple because of the difference in value between the asset classes. So, the overall dollar value of the balance sheet is lower after the crisis

We will attempt to model the above findings and investigate the theoretical link between a crisis originating in the financial sector, causing a change in the balance sheet portfolio, and also reducing productivity, in the model section. Next, we discuss related literature which talks about misallocation of factors and its relation to financial frictions and the efficiency wedge in different settings.

2.3 Literature

Hsieh and Klenow (2009) show how misallocation of resources can lower TFP for manufacturing firms in India, China, and USA. The cause of misallocation in their model is differential tax rates on capital and output for different firms in each sector, which results in a sub optimal allocation of labour and capital between firms. Marginal revenue products are different across different firms in a sector depending on the size of wedges each of them faces, and each individual firms' total revenue productivity (TFPR) is, therefore, proportional to the size of these wedges. Aggregating over each individual firms' output to arrive at the industry level output using a CES aggregator function results in an expression (and their estimation equation) for industry level total factor productivity. This depends on each firms' TFP levels, as well as the wedges they face, and larger the wedges, more is the extent of misallocation of resources. The authors find that data from all three countries shows some amount of dispersion, and possible gains from reallocation of these resources. The wedges in their model are ad-hoc, however, and they are not affected by any optimising behaviour on the agents' part. The model we develop further does have some elements of optimising behaviour affecting the dispersion of TFP, in the sense that a firm that is constrained for financing chooses assets over the cycle optimally, given this constraint, and that shows up as an 'efficiency wedge'.

Banerjee and Munshi (2004) investigate the efficiency of allocation of capital in textile manufacturing firms in India, and find that allocation is not guided by marginal products as in the neoclassical framework. Even after controlling for experience and cohort effects, they find that the firms with lower productivity acquire more capital based on community identity of owners, and consistently perform poorly as compared to the 'outsider' firms. This misallocation results from differential access to politically provided inputs, to capital, to buyers, as well as a differential propensity to exit based on opportunity costs for outsiders. This result is in line with Hsieh and Klenow (2009), except that the ad-hoc tax wedges are actually benefits derived from community linkages in their analysis. Both papers, however, consider firm heterogeneity as a template to measure loss from misallocation of capital. We consider efficiency loss by modelling a representative firm that chooses between different asset types, depending on the financial market conditions.

Restuccia and Rogerson (2008) build a model with heterogeneous firms which have their own respective draws of productivity and output tax (or subsidy). Unlike Hsieh and Klenow (2009) who generate TFP distortions using taxes and subsidies, Restuccia and Rogerson (2008) model firms with different productivities, and the taxes (subsidies) cause output and productivity to be lower the more they are correlated with the individual firm productivities. If firms with lower productivity are subsidised, and those with higher productivity are taxed, the aggregate productivity drops as a result of misallocation of the factors. This ties in with the findings of Banerjee and Munshi (2004), in that they find firms with lower productivity getting easier financing and possessing more capital which affects aggregate TFP.

Building on Restuccia and Rogerson (2008), Bartelsman et al (2013) construct an alternative measure of misallocation based on the covariance of size of firms and their productivity. Similar to Banerjee and Munshi (2004), the authors suggest that the degree of covariance between size and productivity can be a policy induced distortion which differs across countries. Unlike Hsieh and Klenow (2009), whose marginal revenue products (and thus total revenue productivity) would be same across firms in the absence of distortions, Bartelsman et al (2013) introduce overhead labour costs and quasi-fixed capital which allow for dispersion even in the absence of distortions (as well as making dispersion of labour productivity greater than dispersion of TFP, another fact they find in their data). They find that even for their measure of dispersion, aggregate performance is negatively impacted. Guner et al (2008) also study the issue of how aggregate productivity is affected when the size of firms is taxed. In their model, the misallocation comes in the form of a distortion in how managerial ability is spread across different sized firms, with and without a penalty on firm size. The penalty is introduced in the form of taxes on factor inputs beyond a certain threshold, and they find that this results in a decrease in average establishment size, but an increase in the number of establishments, as well as a drop in overall productivity. Gabler and Poschke (2013) extend the analysis of productivity dependent distortions by allowing firms to choose their productivity by conducting experiments with varying degrees of risk and returns. This affects not just the allocation of resources, but also how firm productivity evolves. Firms might indeed choose to have a level of productivity which is below the kind of threshold that Guner et al (2008) mention for tax purposes, etc. This is the kind of distortion which will not be well explained by a misallocation story. They find that such a specification also leads to the outcome that firms are more numerous but smaller in size in equilibrium.

Although the literature discussed so far addresses theoretically, as well as tests empirically, misallocation and its effects on productivity, it does not take a stand on what kind of frictions would be responsible for such misallocation. At best, they discuss the sources as being ad-hoc taxes imposed on factors, or study effects of informal arrangements and networks. We are more interested on how allocation can become distorted as a result of financial frictions, specifically borrowing constraints. We now discuss literature which connects the friction to misallocation using different models.

Midrigan and Xu (2014) study misallocation as one of the two channels affecting productivity, the other being firm entry distortion, where both are a result of a financial friction which takes the form of constraint on amount of debt that can be issued by firms. In their model, producers who want to enter the capital intensive 'modern' sector from a more primitive 'traditional' sector need to pay up front costs which are non trivial, in addition to buying capital, and would necessitate some access to financing or internal funds. Depending on their draw of productivity, each new entrant optimally makes net worth accumulation decision and, in the presence of a borrowing constraint, this dispersion leads to misallocation and lowers TFP. The shadow cost of funds for each producer varies with their net worth and productivity, and those with low net worth save whereas those with a higher net worth dissave. The misallocation can further be classified as resulting from a difference in how long they have been in the modern market, and that resulting from an inadequate response to productivity shocks by constrained producers. Although they find the latter channel to be weak as compared to the former in their model, their data suggests both channels to be fairly weak. In the model, however, the loss from entry barriers caused by the friction are the biggest cause of aggregate productivity distortion. Unlike Hsieh and Klenow (2009), Midrigan and Xu (2014) focus solely on distortions arising from financial frictions, and find that they are not very large.

Buera et al (2011) also model an economy with two production sectors, services and manufacturing, and study how the presence of differential fixed costs as well as differential access to capital affects aggregate productivity by distorting capital allocation among heterogeneous units. They also have some elements of self financing like Midrigan and Xu (2014) wherein the services sector firms find it easier to undo misallocation from access to finance, mainly due to lower fixed costs and lower financing needs as compared to the manufacturing sector. Like Midrigan and Xu (2014), they too find that in addition to misallocation, the distortion is also a function of the number of production units in manufacturing being too low. Interestingly, they also find results similar to those mentioned by Banerjee and Munshi (2004), where inefficient firms stay in business and find easier access to finance, whereas the more productive firms find it difficult to enter the market due to limited capacity to finance investment. The authors also find that capital accumulation in the model is impacted negatively by the financial friction due to a rise in the price of investment, and, as in the data, the manufacturing sector is impacted most by the financial frictions through misallocation and also through entry distortions. Our model presents an alternative channel through which capital accumulation is affected by the financial friction, in that, how the representative firm chooses from a continuum of asset classes driven by limited access to finance gives rise to the distortion in efficiency and TFP.

Banerjee and Moll (2010) build a simple model with capital accumulation and credit constraints to investigate why misallocation, especially the kind discussed by Hsieh and

Klenow (2009) and Restuccia and Rogerson (2008), does not disappear on its own over time through savings to substitute for credit as well as accumulation resulting from precautionary savings motive of constrained firms. They find that as long as the production function is concave, distortions along the intensive margin disappear asymptotically as long as all agents are equally patient. However, distortions will persist in case the production function is convex over a portion of the domain. Such nonconvexity in the maximisation problem is introduced using fixed costs by Midrigan and Xu (2014), Bartelsman et al (2013), and Buera et al (2011). This also generates distortion along the extensive margin by restricting entry without adequate access to financing. They consider several explanations for the persistence of misallocation, like high TFP firms facing high taxes but low TFP firms being better connected politically (Banerjee and Munshi (2004)), explicit policy of discrimination against large firms (as happens in India), and shocks to profitability and the frequency of such shocks. They also suggest that the transition to a steady state with low distortions from a highly distorted initial point will be much slower in the presence of financial constraints, and hence the distortion will seem persistent.

López (2017) builds on the Ayagari (1994) model of heterogeneous agents such that each firm has its own managerial ability, and idiosyncratic shocks, and face individual borrowing constraints which affect factor allocation. As is the case with Midrigan and Xu (2014) and Buera et al (2011), poorer entrepreneurs save more to overcome the borrowing constraint. As in Hsieh and Klenow (2009), in the absence of the friction, factor allocation is efficient, and as long as there is friction, the constrained entrepreneurs will have lower than optimal allocation of capital and labour. Similar to Banerjee and Moll (2010), the misallocation persists in the steady state because of the heterogeneity of time preference among agents. The model predicts that beyond a threshold level private credit-to-GDP ratio, the relationship between access to credit and TFP is linear, and similar results are also found in data.

Moll (2014) studies the transition dynamics for an economy with financial friction in the form of collateral constraints. He finds that the persistence of the productivity shock is important in determining the magnitude of productivity losses as well as the duration of convergence to steady state. If shocks are persistent, it gives agents an opportunity to save and overcome the financial friction, but the convergence to steady state is slower, whereas transitory shocks lead to quick convergence but larger productivity losses in the long term. Methodologically, they aggregate over all agents with their wealth shares as weights and arrive at a representation of aggregate TFP which is endogenous. We also derive a comparable representation of endogenous TFP which is dependent on the financial friction directly, whereas in Moll (2014), it depends indirectly on the friction, in that, each agents response to the degree of persistence of the productivity shock *given the friction* is the basis of their result. Their results also touch upon the issue of size and productivity, mentioned by Bartelsman et al (2013) and Guner (2008), that bigger but less numerous firms increase aggregate productivity, but here that outcome is not a result of any tax wedge and depends on the quality of credit markets.

Caggese and Cuñat (2013) extend the limited access to finance story to check how it affects aggregate productivity in an open economy with firms choosing to export depending on their wealth and ability to pay fixed production costs up front. With costly bankruptcy, firms develop a precautionary motive and their decision to become exporters depends on the financial friction. Older firms with more wealth but low productivity dominate the export market, and gains from trade liberalisation are not fully exploited, lowering overall productivity. This misallocation story is similar to Banerjee and Moll (2010), Restuccia and Rogerson (2008), etc. who introduce a nonconvexity in terms of fixed costs leading to entry restrictions and TFP reduction.

Gilchrist et al (2013) develop a TFP accounting procedure which maps the differences in borrowing costs across firms into resource misallocation, although no explicit mention of any borrowing constraint is made in the model. Compared to Hsieh and Klenow (2009) who ascribe the dispersion of TFP across plants to differences in marginal revenue products but do not provide a concrete form of financial friction, Gilchrist et al (2013) can draw a link from differential borrowing costs to resource misallocation. Each firm in their model needs to borrow to fund capital as well as labour at its individual cost of borrowing which determines how much of the factors each firms employs. If all firms face the same cost of capital, there is no misallocation. Also, they use second order approximations to the aggregate over all firm productivities, and the aggregate over firm specific capital and labour 'wedges' from differential borrowing cost, which allows them to express these aggregates in terms of the variance of each wedge. Another interesting result of their theoretical model is that the distortions based on size matter only to the extent that they bring about a dispersion in input wedges, and not merely because they are size-based wedges. Comparing this to the model by Guner et al (2008) provides an additional insight into what exactly size-based wedges would be capturing in a model. They find that TFP loss from misallocation is relatively small as compared to Hsieh and Klenow (2009), and more in line with the results of Midrigan and Xu (2014).

Khan and Thomas (2013) add another friction in the form of investment irreversibility to a dynamic general equilibrium model of heterogeneous agents with collateral constraints to study how capital reallocation is distorted and lowers aggregate productivity. In the presence of such additional real friction, they find that a financial shock can produce a deep and prolonged recession in their model which is qualitatively similar to the 2007 U.S. recession, more so than what a negative shock to technology can produce. Our base model will not have any real frictions, but we also demonstrate how a financial shock can produce a deep recession through reallocation of the aggregate capital portfolio. Karabarbounis and Macnamara (2019) model the effect of financial frictions on firm level and aggregate TFP in the presence of other wedges like adjustment costs and costly equity payouts, and study how the frictions interact and impact misallocation and cause TFP reduction. The fact that it is difficult to find an empirical counterpart to the collateral multiplier motivates their specification of modelling investment decisions and identifying the transmission mechanism from constraint to misallocation. Similar to Gilchrist et al (2013), they model dispersion of of credit terms for firms (by modelling issue of long duration bonds) which causes constraints for financing, but additionally have capital adjustment costs, and equity payouts which introduce a wedge between the user cost and marginal product of capital. They find that most of the impact of lowering TFP comes from the channel of other frictions interacting with the credit constraint, which is slightly different from Hsieh and Klenow (2009) etc. who do not model any other channel apart from the dispersion of marginal revenue products across firms.

Matsuyama (2007) introduces heterogeneity among investment projects (instead of among firms) to a standard neoclassical setting in order to generate misallocation. In their model, investment opportunities are indexed from the least productive to the most, but there is a trade-off when it comes to pledgeability, in that, the most pledgeable investments are the least productive ones. Investments are dependent on access to financing through collateralisation, and hence a change in credit conditions forces investment into the low productivity projects. Although the focus of the paper is to investigate credit cycles and credit traps, this misallocation would no doubt lower aggregate TFP. Unlike Hsieh and Klenow (2009) (among others) above, the mechanism for misallocation does not depend on any kind of endogenous or exogenous wedges which distort allocation among firms. The returns to investment are already fixed and the misallocation arises from the inability to access finance to invest in high productivity projects. Our model also has a representative agent firm like Matsuyama (2007), but our firm chooses assets to invest in and use for production instead of projects, and this choice depends on availability of finance.

Duval et al (2017) carry out an empirical test of the relationship between financial frictions and fall in productivity over the financial crisis of 2008 using a difference-in-difference causal estimation. They define 'vulnerabilities' of firms as being the firms' leverage and pre-crisis share of debt maturing during the crisis, and find that they have an impact on TFP growth after the credit supply shock hit. The more interesting result which they find is that during the crisis, firms reallocated their investments to assets which were more collateralisable and reduced investment in intangibles. They, however, present no theory or a model to demonstrate how this might have happened. We find a similar result from our model as regards reallocation, but we do not consider intangibles in the model. We only model tangibles with differing durations but we can show how exactly such a mechanism might work in the presence of credit shocks.

Doerr (2018) tests the relationship between rising real estate prices and firm productivity through reallocation of factors of production. They find that unproductive firms grow faster than productive firms when the collateral constraint is relaxed, mostly because the unproductive firms are older and hold more real estate compared to the younger firms which are not yet at a stage to invest substantially in land. They also offer an explanation for the acyclicality of TFP leading to the 2008 crisis: misallocation to unproductive firms dampened productivity growth. This explanation, however, does not address the fact that borrowing constraints were rarely binding leading into the crisis when credit was freely available. It would be more insightful if there were some quantification of the misallocation in equilibrium like Hsieh and Klenow (2009) among many others, or of transition dynamics like Moll (2014).

Catherine et al (2017) estimate their model structurally to quantify the aggregate effect of financial frictions in the form of collateral constraints. They find that an increase in the value of firms' collateral leads to an increase in investment, similar to what Doerr (2017) finds. They find that removing the financial friction increases over all output and welfare, mostly from better allocation of resources, which contributes to a quarter of the gain. Higher capital accumulation and increased labour supply account for a half and a quarter of the gains respectively. However, a limitation of the study is that a relaxation of the collateral constraint does not feed back into real estate prices, and hence the measured effects might be under stated. They nevertheless provide some numerical estimates of the effects of persistence of productivity shocks reducing impact of financial frictions, as mentioned by Moll (2014).

Benmelech et al (2005) study the link between liquidation value of assets and debt contracts using regulation of zoning flexibility data, and find that the quality of collateral determines not just the amount of loan but the duration and the rate of interest charged on it as well. This is relevant to us in the sense that when firms restructure their asset holding after the financial shock hits in our model, it is this relationship between quality of collateral and amount of loan that determines the extent to which productivity is affected. Specifically, because firms in the model cannot access credit freely when the shock hits, they choose to hold cheaper, less valuable assets and this leads to a suboptimal aggregate capital portfolio composition.

Although not directly relevant, Jeong and Townsend (2007) attempt to decompose TFP from Thai data into its underlying sources and find that occupational choice and financial deepening are important in explaining TFP growth. They find that TFP dynamics is affected by what happens with factor prices in relation to wealth distribution, especially in the presence of credit constraints. The importance of this result for us is that even in the medium-to-long term, the connection between TFP growth and access to credit seems to be important. Our model only addresses the relationship over a business cycle frequency, but it is also important in explaining TFP growth over longer periods.

2.4 Model

The model consists of two sectors; production firms and households. We begin by stating the model predictions:

Model Prediction 2.1 *Demand for individual types of capital depends positively on the credit market conditions. Tighter conditions lead to lower demand for all types of assets.*

Model Prediction 2.2 *A shock originating in the financial sector can generate business cycle like dynamics, although the transmission mechanism of the shock is very different to a technology shock.*

Model Prediction 2.3 *A financial shock can also bring about a reduction in measured aggregate productivity and look like a negative technology shock. The mechanism is the reshuffling of the balance sheet portfolio on impact of a financial shock and tightening of credit market conditions.*

2.4.1 Production firms

Following the results from Altug and Labadie (2008), and the specification of Kiyotaki et al. (2016), we define the individual firm's dynamic optimisation problem as under;

$$V_t = \max_{k,k(i)_{+1},n,l_{+1}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} (1-\theta)\theta^{t+j} m_{t+j} \Lambda_{t+j} \right] \tag{2.1}$$

where $m_{t+j} := \beta^j \cdot \frac{\mathbb{E}_t \lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor, and the cash flow is

$$\Lambda_{t+j} = \left(k_{t+j}^{\alpha} n_{t+j}^{1-\alpha} - W_{t+j} n_{t+j} - \int k_{t+j+1}(i) \text{ di} + \int (1-\delta(i)) k_{t+j}(i) \text{ di} + l_{t+j+1} - R_{t+j} l_{t+j} \right) \quad \forall j = \{0, 1, 2, ..\}$$

We also introduce survival probability for individual firms, represented by θ. For each individual firm, the probability of existing at date $t+j$ is given by $(1-\theta)\theta^{t+j}$. The firm maximises its discounted cash flows Λ, which, as shown by Altug and Labadie (2008), is equivalent to maximising end of period value. The inflows for the firm consist of sales, loans, and the market value of nondepreciated capital stock, while the outflows are wages, new capital investment, and repayment of debt from previous period. Again, we have a continuum of capital assets with corresponding, constant rates of depreciation. The loans carry the risk free rate of interest. Lower case letters k, n, l represent individual firm capital, labour, loans whereas upper cases indicate aggregates.

We can cast (2.1) into a two period optimisation problem and state the Bellman equation as under;

$$V(k(i), k, l|\xi) = \max_{k(i),k,l,n} \Big[(1-\theta)\pi_t + \theta \mathbb{E}_t m_{t+1} V(k'(i), k', l'|\xi')\Big] \tag{2.2}$$

where $\pi_t := k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - \int k_{t+1}(i) \text{ di} + \int (1-\delta(i))k_t(i) \text{ di} + l_{t+1} - R_t l_t$

With probability $(1-\theta)$, the firm cannot survive to the next period, and is given the net liquidation proceeds by the sector. This transfer will be reflected in the aggregate firm flow of funds each period, as we will show later.

Each firm faces a collateral constraint which pins down the amount of borrowing the firm can avail.

$$R_{t+1} l_{t+1} = \xi_t \chi \int (1-\delta(i))k_{t+1}(i)\text{di} \qquad (\phi_t) \tag{2.3}$$

The amount each firm can borrow depends on the expected future risk free rate R_{+1}, a loan-to-value (LTV) parameter χ, a shock to the LTV ξ, and the non depreciated value of balance sheet capital. The assumption built in is that it takes time to liquidate all capital, and so nondepreciated value of capital is considered. ϕ is the shadow value of the constraint. The LTV negative shock can be likened to a loss of confidence in the debt market where banks suddenly demand a higher quality collateral, or are willing to further a loan even lower in amount compared to the collateral. This is something that was observed in the financial crisis, where banks were unwilling to lend as house prices collapsed and no one knew who was holding the bad assets.

Another constraint on optimisation is the production capital bundle construction.

$$k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{di}\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_{k,t}) \tag{2.4}$$

Again, the firm chooses assets from the balance sheet portfolio to form the production bundle. The degree of substitutability is given by ε. The shadow value of the production bundle F_k also happens to be the marginal product of capital, as before.

Solving the above optimisation problem yields the following optimality conditions for $k(i)$, l, n, and k respectively;

$$1 = \theta \mathbb{E}_t \left[F_{k,t+1} k_{t+1}(i)^{-\frac{1}{\varepsilon}} k_{t+1}^{\frac{1}{\varepsilon}} + (1-\delta(i))\right] + \phi_t \xi_t \chi (1-\delta(i)) \tag{2.5}$$

$$\phi_t = \frac{1}{\mathbb{E}_t R_{t+1}} - \theta \mathbb{E}_t[m_{t+1}] \tag{2.6}$$

$$W_t = (1-\alpha) k_t^{\alpha} n_t^{-\alpha} \tag{2.7}$$

$$F_{k,t} = \alpha k_t^{\alpha-1} n_t^{1-\alpha} \tag{2.8}$$

2.4.2 Demand function

(2.5) can be rearranged to obtain the demand function for each type of capital asset as under;

$$k_{t+1}(i) = k_{t+1} \left(\frac{1-(1-\delta(i))(\theta \mathbb{E}_t[m_{t+1}] + \phi_t \xi_t \chi)}{\theta \mathbb{E}_t[m_{t+1} F_{k,t+1}]}\right)^{-\varepsilon} \tag{2.9}$$

(2.9) implies that $\frac{\partial k(i)}{\partial \xi} > 0$, or that if there is a negative shock to LTV, capital accumulation will fall as demand for each type of capital will be lower.

We can also check what impact depreciation rates have on asset demand, given a negative LTV shock.

$$\frac{\partial^2 k(i)}{\partial \xi \partial \delta(i)} < 0$$

The negative LTV shock reduces capital demand, but the impact is higher for low depreciation assets and lower for high depreciation assets. This can be seen in a more intuitive way by setting $\delta(i) = 1$ in (2.9), so that the numerator in the brackets reduces to just 1 and there is no impact at all of the shock ξ. However, if $\delta(i) = 0$, the numerator reduces to $1 - (\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi)$, and if the movements in m and ϕ on a fall in ξ are in a way that $(\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi) > 1$, then the effect might change direction. So, if $\delta(i)$ is sufficiently low, we will have an increase in demand for the very low depreciation assets, while that for other assets falls, because as $\delta(i)$ increases, the impact of the shock falls from (2.9). As $\xi \downarrow$, the collateral constraint tightens $\phi \uparrow$, so the net effect of the term $\phi \xi \chi$ is ambiguous, but it can be argued that the direction of movement of this term will be dominated by the direction of movement of ξ because the movement in ϕ from (2.6) is only to the extent of $(1 - \theta)$ times the stochastic discount factor m movement. From the household optimisation (discussed in the households sub section), we have that $\frac{1}{\mathbb{E}_t R_{t+1}} = \mathbb{E}_t m_{t+1}$, and using this in (2.6) yields the desired result. Depending upon how the shock works its way though the economy (discussed in dynamic model results), we might have that for very low depreciation assets, the negative shock increases demand.

As for the steady state distribution of assets in the balance sheet, we have that $\frac{\partial k(i)}{\partial \delta(i)} < 0$, which implies a higher proportion of low depreciation assets, and as depreciation rate approaches 1, demand for that asset in steady state falls. This is intuitively apparent when we set $\delta(i) = 1$ and $\delta(i) = 0$, to obtain $(\theta \beta F_k)^{\varepsilon}$ and $\left(\frac{\theta \beta F_k}{1 - [\theta \beta + \phi \chi]}\right)^{\varepsilon}$ respectively, and compare them. It is clear that $\theta \beta F_k < \frac{\theta \beta F_k}{1 - [\theta \beta + \phi \chi]}$, delivering the same result mentioned earlier.

Another way to see how the portfolio reallocation happens on a shock, we start by taking the ratio of two capital assets, $k(i)$ and $k(j)$, where $\delta(i) > \delta(j)$;

$$\frac{k(i)}{k(j)} = \left(\frac{1-(1-\delta(i))[\theta m_{+1}+\phi\xi\chi]}{1-(1-\delta(j))[\theta m_{+1}+\phi\xi\chi]}\right)^{-\varepsilon}$$

Taking the derivative of the above ratio with respect to the shock ξ yields

$$\frac{\partial\frac{k(i)}{k(j)}}{\partial\xi} = \varepsilon\left(\frac{k(i)}{k(j)}\right)\phi\chi\left[\frac{1-\delta(i)}{1-(1-\delta(i))[\theta m_{+1}+\phi\xi\chi]} - \frac{1-\delta(j)}{1-(1-\delta(j))[\theta m_{+1}+\phi\xi\chi]}\right]$$

Consider $\delta(i) > \delta(j)$, so that, from the above, $\frac{\partial\frac{k(i)}{k(j)}}{\partial\xi} < 0$ which implies that when there is a negative shock, the ratio $\frac{k(i)}{k(j)}$ rises, and combining with the fact that $\frac{\partial k(i)}{\partial\xi} > 0 \;\; \forall i \in I$, it appears to be that the *fall* in demand for $k(i)$ is less than the *fall* in demand for $k(j)$. However, if we consider the left and right extremes of the scale, $\delta(i) = 0$ and $\delta(j) = 1$, we have noted before that the demand for very long term assets will possible rise, and that for very short term assets will be unaffected, which causes the ratio to *fall*.

So, the direction of response to a negative shock seems to be conditional on the rate of depreciation; for assets beyond a certain threshold, there will be lower fall in demand for short term assets than long term assets making it seem like demand is moving towards short term assets, but up until said threshold, there will be increase in investment in the very long term assets. If we compare the ratio of the highest depreciation asset over the lowest depreciation asset before and after the shock, we will find that the ratio has actually fallen. That should not be seen to indicate that the change is uniform across the depreciation scale, or that investment shifts to short term assets after the shock. There is a more intricate response taking place, where low depreciation assets go up, high depreciation assets do not fall too much, and in between these extremes, reallocation appears to happen from low depreciation to high depreciation assets.

Figure 2.7 presents the steady distribution implied by the demand function, and the response to a negative shock to LTV. We see that the assets at the lowest end of the X axis

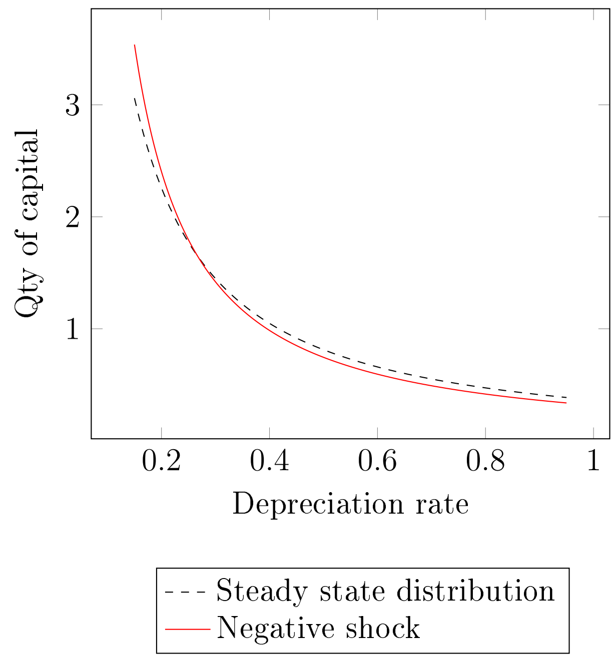


Figure 2.7 Steady state portfolio and LTV shocks

scale actually go up after the shock. As we go to the right along the X axis, we see that there is a fall in the amount of assets held. Overall, the balance sheet is smaller, and more heavy with low depreciation assets. Comparing this result to figure 2.3, we find some similarities, namely that the very low depreciation assets went up over after the crisis and that the size of the aggregate balance sheet was smaller. Also, we see qualitatively that the changes at the rightmost of the scale are not as pronounced as at around the middle of the depreciation scale. It does appear that change has been more towards higher depreciation assets *except* at the lowest end of the scale (which, admittedly, is much lower compared to the model output in figure 2.7). Also recall that this plot does not take into consideration the balance sheet dollar weights of the asset classes, and hence the change in high depreciation assets will appear higher than actual in dollar terms.

Figure 2.7 shows that demand for long term assets is high in steady state as well as on impact of shock. One possible reason, coming from our modelling choice, is that because we have assumed prices of all capital assets to be the same for simplicity, the firms are inclined to choose more long term assets which serve as better collateral and give access to higher loans. Especially in a crisis, firms pick even more of the very long term assets and reduce holdings of the short term assets. This is not something we observe in the data, and can be addressed by having some kind of 'cost' to choosing the low depreciation assets over high depreciation ones. We will address this in the final chapter, but for now we will continue with this specification and discuss the implications.

2.4.3 Comparison with chapter one model

We present a comparison graph which shows how the reshuffling of assets happens in our model with a collateral constraint versus in the model from the first chapter which has no financial friction. We would however like to point out that because the model from chapter one does not have a financial sector, the shock considered is a negative technology shock, whereas for the full model we are interested in a negative financial shock. The transmission mechanism of both shocks is vastly different, as we discuss further (in the section after impulse response plots), and a direct comparison of reallocation might not be one of comparing like for like. A negative technology shock is clearly a supply shock whereas a negative financial shock in our setting has a different transmission mechanism, and does not clearly present itself as either a demand or a supply shock. Nevertheless, to get an idea about how the shocks change firm incentives, we conduct the below exercise in figure 2.8.

Each point on either line represents the deviation as a proportion for that asset class compared to the steady state value which is normalised to 1. We can see that for the model without collateral constraint, the firms clearly change their allocation reducing the overall size of the portfolio drastically, but holding on to the long term assets in a slightly higher proportion that the short term assets. While longest term assets are around 55% of their steady state value, the shortest term assets are around 20% of their steady state value after the shock in the model without collateral. However, in the model with a collateral constraint,

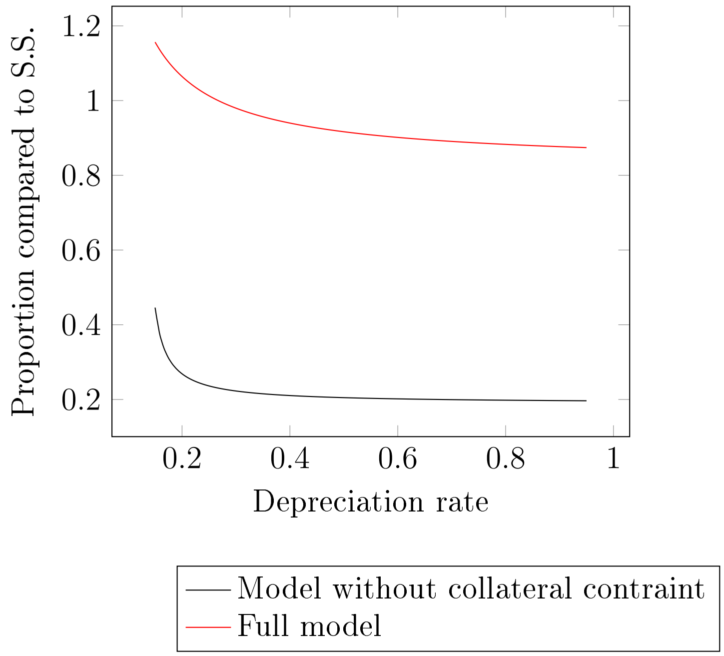


Figure 2.8 Steady state reshuffling compared

called the 'full model', the holding of longest term assets actually increases by up to almost 20% of their steady state value, whereas the holding of shortest term assets is around 90% of their steady state value.

Although the shocks are different fundamentally, it appears that firms hold on to far too much capital in the full model, whereas the optimal response seems to be to reduce holding of all assets even if the pattern of post-shock holding seems similar. As to reasons for what causes firms to hold on to excess capital, and what prevents a first best response on firms' part, it seems like the presence of a collateral constraint which requires assets to be of a longer duration preferably is causing said response by firms. In general, however, it seems like the full model does a decent job of replicating the pattern observed in data.

2.4.4 Equal prices for capital assets

So far, we have not actively addressed the modelling simplification of equal prices for all types of capital assets. In this sub section we will discuss the market structure that leads to this result. We will also provide more information in a text box about how that can be changed and what would then be the issues with that.

Assume that the firm now chooses the level of investment in each capital asset, so that the optimisation problem is as under;

$$\max_{k,k(i)_{+1},n,l,z} \mathbb{E}_t \sum_{j=0}^{\infty} (1-\theta)\theta^{t+j} m_{t+j} z_{t+j} \tag{2.10}$$

subject to the flow of funds constraint each period

$$z_t = k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - \int I_t(i)\mathrm{d}i + l_{t+1} - R_t l_t \qquad (\gamma) \tag{2.11}$$

The capital portfolio formation constraint

$$k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_k) \tag{2.12}$$

The collateral constraint

$$R_{t+1} l_{t+1} = \xi\chi \int Q_t(i)(1-\delta(i))k_{t+1}(i)\mathrm{d}i \qquad (\phi) \tag{2.13}$$

And a continuum of constraints for each type of capital asset

$$k_{t+1}(i) = I_t(i) + (1-\delta(i))k_t(i) \qquad (\mu(i)) \qquad \forall i \in [0,1] \tag{2.14}$$

Here, z in (2.10) is the cash flow each period which the firm directly chooses to maximise now, and γ is the shadow value of the flow of funds constraint. The collateral constraint now has the price of each type of capital asset, $Q(i)$, which determines the amount of loans. A new continuum of explicit constraints in the form of (2.14) are now considered. The difference from the original specification is that now the firm chooses the investment in each type of capital asset, and $\mu(i)$ is now the shadow value of adding an additional type of capital of type (i).

First order conditions for z, $I(i)$, $k_{+1}(i)$ yield the following equations, respectively (the other first order conditions for k, n and l are not discussed as they do not change);

$$\gamma_t = 1 \tag{2.15}$$

$$\gamma_t = \mu_t(i) \qquad \forall i \in [0,1] \tag{2.16}$$

$$\mu_t(i) = \theta \mathbb{E}_t[m_{t+1}\mu_{t+1}(i)](1-\delta(i)) + \chi\xi_t\phi_t Q_t(i)(1-\delta(i)) + \theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]k_{t+1}^{\frac{1}{\varepsilon}}k_{t+1}(i)^{-\frac{1}{\varepsilon}} \tag{2.17}$$

We also define the price of capital type (i) as $Q_t(i) = \frac{\mu_t(i)}{\gamma_t}$ where $\mu_t(i)$ is the marginal benefit of adding a single unit of capital type (i) and γ_t is the cost in terms of profit of spending on a single unit of capital type (i). The ratio of the two, $Q_t(i)$, gives the units of profit the firm is willing to sacrifice right now to gain an additional unit of utility from a given type of capital asset.

It is straightforward to see that $\gamma_t = 1 = \mu_t(i) = Q_t(i) \forall i \in [0,1]$. This is the result we use in our specification. The result is drawing on the fact that we do not have a differentiated cost of adjusting different types of capital in this setting. The firms can equally easily adjust all types of capital and reshuffle the balance sheet portfolio to their liking. A differentiated cost of adjustment for different types of capital would modify this behaviour. Although we do not provide a solution in the current piece of work, we analyse the problem in further detail in the text box that below.

Imposing the results we just obtained, and solving for the demand function for each type of capital asset, we arrive at the exact specification in (2.9). Hence we can proceed with our analysis based on those results.

Differentiated adjustment costs

If we were to consider, instead of (2.14), that investment in any type of capital entails differential investment adjustment costs as under;

$$k_{t+1}(i) = \left[1 - \frac{\omega_i}{2}\left(\frac{I_t(i)}{I_{t-1}(i)} - 1\right)^2\right] I_t(i) + (1-\delta(i))k_t(i) \qquad (\mu(i))$$

where ω_i is the adjustment cost parameter for asset type (i). This allows us to change the result that the price of each type of capital is the same. Taking the first order condition for $I(i)$ using (2.10) and the modified constraint above, we obtain;

$$\gamma_t = \theta\mathbb{E}_t\left[m_{t+1}\mu_{t+1}(i)\omega_i\left(\frac{I_{t+1}(i)}{I_t(i)} - 1\right)\left(\frac{I_{t+1}(i)}{I_t(i)}\right)^2\right]$$
$$+ \mu_t(i)\left[1 - \omega_i\left(\frac{I_t(i)}{I_{t-1}(i)} - 1\right)\frac{I_t(i)}{I_{t-1}(i)} - \frac{\omega_i}{2}\left(\frac{I_t(i)}{I_{t-1}(i)} - 1\right)^2\right]$$

Now if we have a continuum of differentiated adjustment costs ω_i, we can change the pace of adjustment of each type of asset. Additionally, $Q_t(i) = \frac{\mu_t(i)}{\gamma_t} \neq 1$ now as each $\mu_t(i)$ will depend on the parametrisation of ω_i. So the outcome is a differentiated cost of types of capital. Also note that the steady states for this specification and our previous specification are identical, and the steady state asset allocation will be identical.

However, this problem is not easy to solve using the second order approximation methods we use. Specifically, it will be impossible to analytically substitute $k(i)$ for $I(i)$ any more, so the approximation needs to be around the aggregate investment $I_t = \int I_t(i)\mathrm{d}i$ and would involve use of the variance of aggregate investment as a parameter. Moreover, this parameter will be time varying and presents the issue of what moments are to be matched when the moments themselves become inputs to the solution.

We can get around this issue by abandoning the continuum of asset types and having a finite number of asset classes, but clearly that is a different model and a different solution method. Although interesting and pertinent, we do not address that in the current model setting.

2.4.5 Second order approximations

As in the first chapter, we approximate the integral to the second order around the average rate of depreciation which facilitates solving the model using variance of rates of depreciation. Detailed derivations are available in appendix 2.A.3, while we present the final results below.

The aggregated demand function for entire balance sheet capital is as under;

$$\int k_{t+1}(i)\mathrm{di} := \bar{k}_{t+1} = k_{t+1}\left(\frac{\eta_t}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon} \cdot \tau_t^k \tag{2.18}$$

where

$$\eta_t := 1 - (1-\bar{\delta})[\theta\mathbb{E}_t m_{t+1} + \phi_t\xi_t\chi] \tag{2.19}$$

$$d\eta_t := [\theta\mathbb{E}_t m_{t+1} + \phi_t\xi_t\chi] \tag{2.20}$$

$$\tau_t^k := \left\{1 + \frac{1}{2}\varepsilon(\varepsilon+1)\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)^2\right\} \tag{2.21}$$

We also approximate the time varying effective rate of depreciation from $(1-D_t)\bar{k}_t = \int(1-\delta(i))k_t(i)\mathrm{di}$;

$$D_t = 1 - (1-\bar{\delta}) \cdot \frac{\tau_{t-1}^d}{\tau_{t-1}^k} \tag{2.22}$$

where

$$\tau_{t-1}^d := \left\{1 + \frac{1}{2}\varepsilon\sigma_\delta^2\left(\frac{d\eta_{t-1}}{\eta_{t-1}}\right)\left[\frac{2}{1-\bar{\delta}} + (\varepsilon+1)\frac{d\eta_{t-1}}{\eta_{t-1}}\right]\right\} \tag{2.23}$$

The capital Euler which results from approximating the production bundle construction (2.4) constraint is as under;

$$\lambda_t = \frac{\theta\beta\mathbb{E}_t\lambda_{t+1}\left[F_{k,t+1}\cdot(\tau_t^\gamma)^{\frac{1}{\varepsilon-1}} + (1-\bar{\delta})\right]}{(1-(1-\bar{\delta})\phi_t\xi_t\chi)} \tag{2.24}$$

where

$$\tau_t^{\gamma} := \left\{ 1 + \frac{1}{2}\varepsilon(\varepsilon - 1)\sigma_{\delta}^2 \left(\frac{d\eta_t}{\eta_t} \right)^2 \right\} \tag{2.25}$$

(2.25) is different from a standard capital Euler in two respects: τ^{γ} and the term in the denominator. τ^{γ} is the 'wedge' which results from approximating around the mean of the depreciation rates, and goes away if there is only one rate of depreciation, implying $\sigma_{\delta}^2 = 0$. In the standard RBC case, we also have that $\chi = 0$ when loans are not present, which, if imposed, makes the denominator 1 and (2.25) reduces to a standard capital Euler. However, the presence of the denominator influences the model dynamics and transmission mechanism significantly, as we will discuss in the dynamic model results.

2.4.6 Households

Households are standard utility maximising agents who solve the following optimisation problem;

$$\max_{C_{t+j}, N_{t+j}, L_{t+j}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma} - 1}{1 - \sigma} - \nu \frac{N_{t+j}^{1+\psi}}{1 + \psi} \right) \right] \tag{2.26}$$

subject to the period budget constraint

$$C_t + L_{t+1} = W_t N_t + R_t L_t + T_t \qquad \forall t \qquad (\lambda_t) \tag{2.27}$$

Above are all aggregate variables, because the households are identical and each represents the aggregate. T are net transfers received from the aggregate firm, discussed in detail in the next sub section. Solving the above routine gives the following optimality conditions;

$$C_t^{-\sigma} = \lambda_t \tag{2.28}$$

$$\nu N_t^{\psi} = \lambda_t W_t \tag{2.29}$$

$$\lambda_t = \beta \mathbb{E}_t[\lambda_{t+1} R_{t+1}] \tag{2.30}$$

These are standard conditions from the canonical RBC and need no further discussion. Alternative specification of the utility function in the form of GHH preferences is studied in detail in appendix 2.C.5.

2.4.7 Market clearing

The way we have set up the production sector, with firms exiting each period and being replaced by new firms, requires that each period firms that close be paid off their liquidation value and also seed capital for new firms be received from households. Unless the exiting firms are returned their net worth, the aggregate firm will accumulate capital indefinitely, and there will be no need for loans to finance further capital acquisition. This will make the collateral constraint redundant, as well as causing issues with market clearing and the National Income Accounting (NIA) identity, which will not hold.

The aggregate flow of funds constraint is as under;

$$K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - \int K_{t+1}(i)\mathrm{di} - R_t L_t + \int (1-\delta(i))K_t(i)\mathrm{di} + L_{t+1} - T_t = 0 \tag{2.31}$$

Similar to the individual firms, inflows are from aggregate sales, loans, and market value of undepreciated capital, whereas outflows are in the form of wages, new investment, loan repayments, and net transfers T. The transfers are arrived at as under;

$$T_t = \zeta \bar{K}_{ss} - (1-\theta)(\bar{K}_{t+1} - L_{t+1}) \tag{2.32}$$

The proportion of steady state balance sheet portfolio $\bar{K}$ which the households provide to the new firm as seed capital is ζ. The amount returned to all firms that close down is their capital assets net of loan obligations, $(\bar{K}_{t+1} - L_{t+1})$, and such proportion of firms are $(1-\theta)$ of the total number of firms by the law of large numbers considering that the probability of exiting is $(1-\theta)$ each period.

Combining (2.31) and (2.27) yields the NIA;

$$Y_t = C_t + \bar{K}_{t+1} - (1 - D_t)\bar{K}_t \tag{2.33}$$

where Y results from the production function;

$$Y_t = K_t^{\alpha} N_t^{1-\alpha} \tag{2.34}$$

Next we discuss how misallocation is relevant in our set up and what causes the efficiency wedge.

2.4.8 Misallocation and the efficiency wedge

As discussed previously, conventionally most misallocation literature takes as its base heterogeneous firms, and then either some kind of firm specific wedge is introduced in the form of taxes, or there are idiosyncratic productivity shocks which cause misallocation of factors between firms. Some papers also assume two types of agents, borrowers and lenders, and show how a heterogeneous agent model can generate misallocation , and how firms can overcome that over time with savings. They find that most times the shock to productivity needs to be large for misallocation to be persistent. In our model, however, we do not have a productivity shock. Instead, we have a financial shock which mimics a productivity shock. Also, we have heterogeneity along the asset types dimension, not along the firm dimension.

In our model, a shock originating in the financial sector changes the balance sheet portfolio composition of the aggregate firm by making the high depreciation assets look less attractive compared to the very low depreciation ones. As the balance sheet portfolio changes, the production bundle also is forced to change and these choices, which are not the first best but driven more by the fact that firms are constrained for loans, lead to the efficiency wedge. Put differently, the efficiency wedge results from the fact that production capital falls by more than balance sheet capital because it has to now choose from a portfolio which might not be the first best.

In the current set up of the model, however, the steady state is not the first best efficient one because of the presence of a wedge even in equilibrium, implying that as long as there exists a financial friction in the form of a collateral constraint, the efficient steady state will not be obtained. We will discuss more about the efficient steady state and the bias towards long term assets arising from assumption of equal price of all assets in detail in the final chapter.

So, in our setting, misallocation of factors happens in the sense that perhaps assets which would not have that much weight in the balance sheet portfolio if the friction weren't to exist end up being a higher proportion. We do not rely on any heterogeneity among firms, nor any idiosyncratic shocks, nor even an aggregate technology shock. We are able to generate similar implications using a much simpler specification which we approximate to the second order and solve by perturbation.

The efficiency wedge takes the following form;

$$1 - \tau_t = \left(\frac{K_t}{\bar{K}_t}\right)^{\alpha} \tag{2.35}$$

As (2.35) shows, the wedge today is actually decided a period earlier. This specification results from replacing K with $\bar{K}$ in the production function (2.34);

$$Y_t = (1 - \tau_t)\bar{K}_t^{\alpha} N_t^{1-\alpha} \tag{2.36}$$

The movement in the efficiency wedge results from optimising behaviour in response to a financial shock by the aggregate firm, and not from exogenous disturbances themselves as is the case in some of the literature discussed earlier.

Next we define the competitive equilibrium and present the system of equations.

2.4.9 System equations

We first define the competitive equilibrium. The states $\boldsymbol{s}$ are portfolio capital $\bar{K}$, production capital K, effective depreciation rate D, loans L, and the shock process ξ.

Definition 2 *Recursive Equilibrium: A recursive competitive equilibrium is defined as a set of functions for (1) households' policies $C^h(\boldsymbol{s})$, $N^h(\boldsymbol{s})$, and $L^l(\boldsymbol{s})$; (2) production firms' policies $K(\boldsymbol{s})$, $\bar{K}(\boldsymbol{s})$, $L(\boldsymbol{s})$, and $N(\boldsymbol{s})$; (3) aggregate prices $W(\boldsymbol{s}$ and $R(\boldsymbol{s})$; (4) law of motion for aggregate states $\boldsymbol{s}' = \boldsymbol{\Phi}(\boldsymbol{s})$; such that (i) households policies satisfy its first order conditions; (ii) firms' policies are optimal and $V(\boldsymbol{s})$ satisfies the Bellman equation; (iii) wage and interest rates clear the labour and capital markets, and $m(\boldsymbol{s}) = \beta^{U_c(C',N')}/_{U_c(C,N)}$; (iv) the law of motion for $\boldsymbol{\Phi}(\boldsymbol{s})$ is consistent with individual decisions and the stochastic process for ξ.*

Following is the full system of equations that describes the entire model. The system is linearised and solved using perturbation methods around a deterministic steady state.

$$C_t^{-\sigma} = \lambda_t \tag{2.37}$$

Marginal Utlity of Consumption

$$\nu N_t^{\psi} = \lambda_t W_t \tag{2.38}$$

Labour Supply

$$\lambda_t = \beta \mathbb{E}_t[\lambda_{t+1} R_{t+1}] \tag{2.39}$$

Loans Euler

$$W_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{2.40}$$

Wages

$$F_{k,t} = \alpha K_t^{\alpha-1} N_t^{1-\alpha} \tag{2.41}$$

Marginal Product of Capital

$$Y_t = (1-\tau_t) \bar{K}_t^{\alpha} N_t^{1-\alpha} \tag{2.42}$$

Production Function

$$Y_t = C_t + \bar{K}_{t+1} - (1-D_t)\bar{K}_t \tag{2.43}$$

National Income Accounting Identity

$$\lambda_t = \frac{\theta\beta\mathbb{E}_t\lambda_{t+1}\left[F_{k,t+1}\cdot(\tau_t^{\gamma})^{\frac{1}{\varepsilon-1}} + (1-\bar{\delta})\right]}{(1-(1-\bar{\delta})\phi_t\xi_t\chi)} \tag{2.44}$$

Capital Euler

$$\tau_t^{\gamma} := \left\{1 + \frac{1}{2}\varepsilon(\varepsilon-1)\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)^2\right\} \tag{2.45}$$

Euler wedge

$$\bar{K}_{t+1} = K_{t+1}\left(\frac{\eta_t}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}\cdot\tau_t^k \tag{2.46}$$

Portfolio & B/S Capital

$$\eta_t := 1 - (1-\bar{\delta})[\theta\mathbb{E}_t m_{t+1} + \phi_t\xi_t\chi] \tag{2.47}$$

Intermediate variable

$$d\eta_t := [\theta\mathbb{E}_t m_{t+1} + \phi_t\xi_t\chi] \tag{2.48}$$

$\frac{\partial\eta}{\partial\delta}$

$$\tau_t^k := \left\{1 + \frac{1}{2}\varepsilon(\varepsilon+1)\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)^2\right\} \tag{2.49}$$

Capital wedge

$$D_t = 1 - (1-\bar{\delta})\cdot\frac{\tau_{t-1}^k}{\tau_{t-1}^d} \tag{2.50}$$

Time varying depreciation rate

$$\tau_t^d = \left\{1 + \frac{1}{2}\varepsilon\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)\left[\frac{2}{1-\bar{\delta}} + (\varepsilon+1)\frac{d\eta_t}{\eta_t}\right]\right\} \tag{2.51}$$

Depreciation wedge

$$\phi_t = \frac{1}{\mathbb{E}_t R_{t+1}} - \theta \mathbb{E}_t m_{t+1} \tag{2.52}$$

Loans foc

$$R_{t+1} L_{t+1} = \chi \xi_t (1 - D_{t+1}) \bar{K}_{t+1} \tag{2.53}$$

Collateral constraint

$$\mathbb{E}_t m_{t+1} = \beta \left(\frac{\mathbb{E}_t \lambda_{t+1}}{\lambda_t} \right) \tag{2.54}$$

Stochastic discount factor

$$T_t = \zeta \bar{K}_{ss} - (1 - \theta)(\bar{K}_{t+1} - L_{t+1}) \tag{2.55}$$

Transfers

$$1 - \tau_t = \left(\frac{K_t}{\bar{K}_t} \right)^{\alpha} \tag{2.56}$$

New TFP Measure

$$\xi_t = \xi_{t-1}^{\rho} e^{v_t} \tag{2.57}$$

LTV shock

The above forms a system of 21 equations in the following 21 variables;

$$C \;\; \lambda \;\; N \;\; W \;\; M \;\; R \;\; L \;\; D \;\; \bar{K} \;\; K \;\; \phi \;\; Y \;\; F_k \;\; \eta \;\; d\eta \;\; \tau^k \;\; \tau^d \;\; \tau^{\gamma} \;\; \tau \;\; T \;\; \xi$$

2.4.10 Calibration

The parameters values used are as under;

Parameter Values

Parameter	Value	Description
β	0.995	Discount factor
α	$1/3$	Capital share
σ	1.01	Household intertemporal substitutability
ψ	1	Frisch elasticity parameter
ν	1	Disutility from labour
ε	1.2	Substitutability of capital assets
ζ	0.03	Transfers parameter
χ	0.9	LTV parameter
θ	0.9	Probability of firm survival
$\bar{\delta}$	0.08	Average rate of depreciation
σ_{δ}^2	0.01	Variance of depreciation rates
ρ	0.9	Shock persistence
σ^2	0.01	Variance of financial shock

Table 2.1 Parameters for extended model

The degree of asset substitutability $\varepsilon = 1.2$ is now higher compared to the basic model in chapter one, where it was < 1. We choose this value mainly for the reason that with this degree of substitutability we meet all three requirements of the model, namely a sensible measure of the efficiency wedge τ in the steady state, procyclical depreciation, and procyclical efficiency wedge. All three are very essential to the model, but we also present results with alternative calibration in appendix 2.C.5. In the approximations, ε appears in the variables τ^k, τ^d, and τ^γ which are important in pinning down $\bar{K}$, D, and λ respectively. In the first two, ε appears as $(\varepsilon + 1)$, so the fact whether it is greater and or smaller than 1 does not matter greatly. However, for τ^γ (from (2.25)), it appears as $(\varepsilon - 1)$, so if it is greater than (less than) 1 it will have a positive (negative) effect on the value of the variable and $\varepsilon \lesseqgtr 1 \Leftrightarrow \tau^\gamma \downarrow\uparrow$. Moreover, it is raised to the power $\frac{1}{\varepsilon-1}$, so either it is raised to a high positive power if $\varepsilon > 1$ or to a high negative power if $\varepsilon < 1$. So, if $\varepsilon > 1$ the net effect is positive, whereas if $\varepsilon < 1$ the net effect is negative on λ_t. We find, however, that this does not change much as regards response of $_t$ as the movement in other variables seems to dominate the changes in τ^γ, as can be seen in appendix 2.C.5.

The transfers parameter ζ is arrived at by finding the transfers from the firm to households in steady state that clear the market, and then backing out the implied parameter from (2.32) in the steady state. This will be explained further in the section on steady state analysis.

Survival probability θ for individual firms is set to 90%. An increase in survival probability reduces transfers from aggregate firm to households each period and if survival rate is 100%, then firms again over accumulate capital with households providing seed capital for new firms each period, and that makes the collateral constraint irrelevant, as we saw previously.

The Frisch elasticity parameter ψ is calibrated to the canonical RBC value. The Frisch elasticity of labour supply to change in wage rate is defined as $\frac{\partial N/N}{\partial W/W} = \frac{1}{\psi}$, so an increase in ψ actually reduces the slope of the labour supply curve, and hence the impact of a change in supply on wage rate. Changing this parameter changes the response of the model. We discuss a different value of ψ, as well as the impact of breaking the link between the wealth effect and labour supply using GHH preferences, in appendix 2.C.5.

All other parameters are calibrated as described in chapter one, and to reproduce a relevant point from the previous model calibration;

> The variance of depreciation rates σ_δ^2 is set at 0.01, although it might be argued that it can be anything up to 0.08 or thereabout. However, the role of this parameter is only to change the magnitude of response in the dynamics, and to

some extent, change slightly the values in steady state, so it does not seem to be critical to the extent of the value of ε.

Next we analyse the steady state and how it is solved.

2.4.11 Steady state analysis

Some variables are pinned down in the steady state. Among them, $R = \frac{1}{\beta}$, $m = \beta$ help pin down ϕ. This pins down η and $d\eta$, which in turn pins down τ^k, τ^d, τ^γ, D, and F_k.

This reduces the system in steady state to the following;

$$C^{-\sigma} = \lambda \tag{2.58}$$

$$\nu N^{\psi} = \lambda W \tag{2.59}$$

$$W = (1-\alpha)K^{\alpha}N^{-\alpha} \tag{2.60}$$

$$F_k = \alpha K^{\alpha-1}N^{1-\alpha} \tag{2.61}$$

$$\bar{K} = K \cdot \left(\frac{\eta}{\theta\beta\Gamma}\right)^{-\varepsilon} \cdot \tau^k \tag{2.62}$$

$$0 = C + L(1-R) - WN - T \tag{2.63}$$

$$Y = C + D\bar{K} \tag{2.64}$$

$$RL = \chi(1-D)\bar{K} \tag{2.65}$$

$$\tau = \left(\frac{K}{\bar{K}}\right)^{\alpha} \tag{2.66}$$

$$Y = K^{\alpha}N^{1-\alpha} \tag{2.67}$$

As can be seen from (2.63), we pin down the transfers in steady state using the aggregate firm flow of funds and the national income accounting identity. The NIA must hold in each period including in the equilibrium, and we use this to back out transfers as the balance from aggregate firm flow of funds. This numerical amount obtained for T is then used in

(2.32) on the left hand side, along with the steady state values of $\bar{K}$ and L to back out the value of parameter ζ.

The numerical steady state solution is as under;

Steady State Values

Variable	Description	Model
C/Y	Consumption-Output ratio	0.80
$\bar{K}/Y$	Portfolio Capital-Output ratio	6.91
K/Y	Production Capital-Output ratio	5.93
N	Labour	0.91
D	Time-varying aggregate depreciation	0.03
τ	Efficiency wedge	0.95

Table 2.2 Steady State Values for extended model

As mentioned previously, our current calibration gives us that $\tau = 0.95$, which indicates this steady state is not efficient because of the presence of a collateral constraint even in equilibrium. This will be affected by how we calibrate ε, and alternative calibrations are presented in appendix 2.C.5.

2.5 Results

We present results from the dynamic model in this section. First we show the impulse response functions from a shock to the LTV parameter, and then discuss them in detail.

2.5.1 Impulse response functions

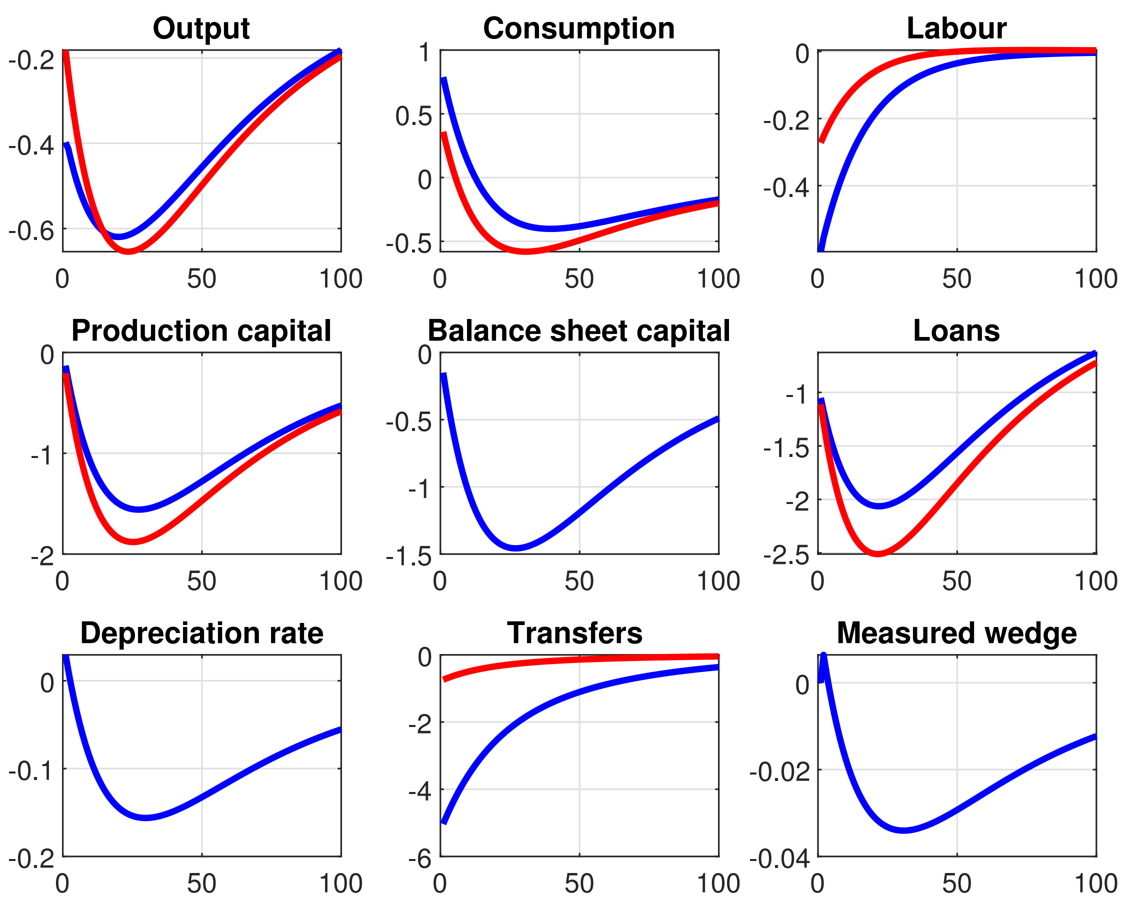


Figure 2.9 Response to negative financial shock

Portfolio Model, Without portfolio

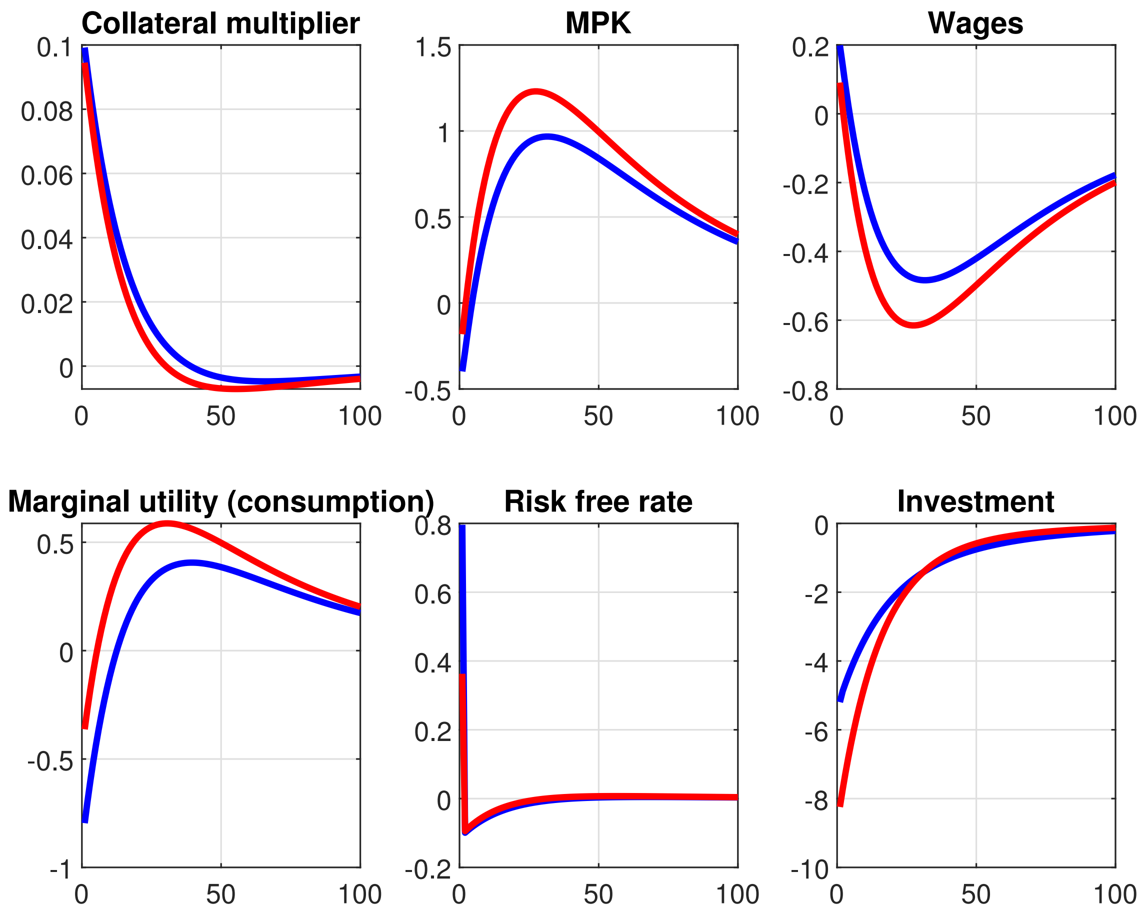


Figure 2.10 Response to negative financial shock (contd.)

Portfolio Model, Without portfolio

2.5.2 Discussion

The response of variables in the portfolio model (in blue) depends upon how the shock is transmitted through the economy, and it is different from a technology shock seen previously. Here, as market confidence dries up and the LTV falls suddenly, we have seen how demand for capital assets falls which causes the balance sheet portfolio to shrink and be reallocated as well, with very low depreciation assets holding an even higher share, and other assets being reallocated as seen previously. As the portfolio shrinks, loans fall as well. With a different balance sheet portfolio to choose from, the production bundle shrinks, and since assets are now in a very different proportion, the fall in production bundle is higher than the balance sheet portfolio, as mentioned previously. The existing capital seems excessive for the new environment, which causes marginal product of capital to fall as well. With a fall in capital accumulation, labour supply and demand falls.

However, wages have gone up! The reason is that, from (2.24), we know that the impact of a fall in the LTV parameter is to cause a fall in λ as well. Intuitively, this means that as loans fall with a fall in LTV, households now have more funds left over to consume, and so consumption rises, or marginal utility of consumption λ falls. That households are left with more funds also causes them to feel wealthier and thus increase leisure, or reduce labour supply. Demand for labour does not change on impact, but a fall in supply causes the wage rate to rise. This phenomenon depends on the slope of the supply curve, as well as the consumption-leisure substitutability and the wealth effect. If we make the supply curve flatter (by increasing ψ), or break the wealth effect link using GHH preferences, we can actually generate a more business cycle-like effect from wages. However, this has some unfavourable consequences as regards portfolio choice, discussed further in appendix 2.C.5.

Net transfers to households fall as firms lower the capital they accumulate as well as loans they avail. The depreciation rate falls because of the change in composition in balance sheet portfolio, seen in 2.7, with more low depreciation assets. The efficiency wedge measure also falls, implying a move further away from the efficient steady state, and is highly persistent. Even after 10 years, it has barely returned to half of the steady state level. This can explain why financial shocks can seem to lower productivity greatly and for a prolonged period, and appear as negative technology shocks.

The marginal product of capital also falls on impact due to a fall in labour. It gradually starts rising as capital continues to fall.

Finally, the shadow value of collateral goes up, implying that the constraint binds much harder after the shock. This, however, means little for our purposes as we solve the model in a very small neighbourhood of the steady state where the constraint is binding even outside of equilibrium.

The response from the model with a collateral constraint but only a single type of capital (in red) is qualitatively similar to the portfolio model but differs in magnitudes for some variables. The fall in transfers is much lower for the single capital model, as is the fall in labour supply. Loans fall almost the same amount in both models, however, the single capital model shows a deeper slump in subsequent periods. Details of the single capital model are available in Appendix 2.D.

2.5.3 Second moments for model with collateral constraint

We use the same data and follow the same methodology to extract data moments as explained in Second moments for model with time varying depreciation. The moments for the RBC model are also the same as in Second moments for model with time varying depreciation. The RBC+ column presents moments generated by the single capital model. There is only a single type of capital as in the RBC setting which affects dynamics slightly. Full column is the full model with collateral constraint and a capital portfolio.

Moments for model with collateral constraint

	σ_X			σ_X/σ_Y			$ACF(1)$			$corr(X,Y)$		
Variable	Data	RBC+	Full	Data	RBC+	Full	Data	RBC+	Full	Data	RBC+	Full
Y	0.02	0.003	0.005	1.0	1.0	1.0	0.78	0.86	0.70	1.0	1.0	1.0
I	0.04	0.10	0.06	2.4	33.3	12.0	0.88	0.64	0.64	0.75	0.64	0.93
C	0.01	0.005	0.01	0.6	1.6	2.0	0.63	0.70	0.65	0.79	-0.13	-0.81
N	0.02	0.003	0.01	1.1	1.0	1.8	0.81	0.64	0.64	0.89	0.62	0.92
W	0.01	0.003	0.003	0.6	1.0	0.6	0.65	0.87	0.74	0.02	0.46	-0.46

Table 2.3 Moments for model with collateral constraint

We can see that the standard deviation of output is much lower in both our models, although the portfolio model does a slightly better job. The portfolio model also does better at matching the standard deviation on Investment, whereas the single capital model is off the mark. The portfolio model also matches the moments for Consumption and Labour better than the single capital model. As for correlations, the portfolio model does better than the single capital model for Investment and Labour, but both models show a negative correlation of output and consumption because of the explanation provided above. Also, it is important to remember that this is a financial shock and not a productivity shock, so the transmission mechanism differs by quite a bit and hence the sign on the correlation of output and consumption.

2.6 Conclusion

We analysed firm balance sheet data from the U.S. and found a pattern of balance sheet portfolio reallocation over the financial crisis. Given the already known fact that financial crises reduce productivity, we attempt to build a model which can account for both these phenomena; namely, a model where a shock originating in the financial sector causes firms to change their optimal portfolio of assets and also reduces aggregate productivity. The financial shock in our model moves through the economy by first changing the demand for individual types of capital and bringing about a reallocation in the balance sheet portfolio, and this also shows up as a lowering of productivity given that the firm forms a production bundle from the portfolio using a CES function. The impact on the production bundle as well as the portfolio is also dependent on the substitutability of the assets; the more complementary they are, the larger the fall in portfolio and the production bundle. This results from the fact that complementary assets need to be be bought together in a set proportion, and hence cannot be substituted by other kinds of assets. We check how the model responds to different parametrisations, as well as for different preference specifications. The transmission mechanism remains the same, except for minor changes.

Two issues that need to be addressed in our set up are as under;

1. We assume away the differential prices of different asset types for the sake of simplicity. However, this is an important aspect which needs to be modelled. Our results which indicate an extremely high proportion of long term assets in the portfolio might, to some extent, be influenced by this assumption. Because long term assets provide better collateral to access loans, firms will be biased towards choosing a higher proportion of these assets. If we were to model for prices and accommodate that low depreciation assets will be more expensive, we might see a different pattern of movement in the portfolio.

2. We also have that there is an efficiency wedge in the steady state even without financial

friction. That is an issue which needs to be addressed in the model. We want to have a model where the wedge does not exist in equilibrium unless there is a financial friction.

These two issues can be addressed by either having a continuum of prices for each type of capital asset, or some kind of asset specific weights which can influence the demand function for each asset such that it corrects the imbalance in demand and removes the efficiency wedge. This will be the focus of further work.

References

[1] Tobias Adrian, Paolo Colla, and Hyun Song Shin. *Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007-9*. NBER Macroeconomics Annual 2012, Volume 27. National Bureau of Economic Research, Inc, 2012.

[2] Laura Alfaro, Andrew Charlton, and Fabio Kanczuk. *Plant-Size Distribution and Cross-Country Income Differences*, pages 243–272. In [31], April 2009.

[3] Sumru Altug and Pamela Labadie. *Asset pricing for dynamic economies*. Cambridge University Press, 2008.

[4] Abhijit Banerjee and Kaivan Munshi. How efficiently is capital allocated? evidence from the knitted garment industry in tirupur. *The Review of Economic Studies*, 71(1):19–42, Jan 1, 2004.

[5] Abhijit V. Banerjee and Esther Duflo. Do firms want to borrow more? testing credit constraints using a directed lending program. *Review of Economic Studies*, 81(2):572–607, 2014.

[6] Eric Bartelsman, John Haltiwanger, and Stefano Scarpetta. Cross-country differences in productivity: The role of allocation and selection. *The American Economic Review*, 103(1):305–334, Feb 1, 2013.

[7] Eric Bartelsman, John Haltiwanger, and Stefano Scarpetta. Cross-country differences in productivity: The role of allocation and selection. *The American Economic Review*, 103(1):305–334, Feb 1, 2013.

[8] Eric Bartelsman, John Haltiwanger, and Stefano Scarpetta. Cross-country differences in productivity: The role of allocation and selection. *The American Economic Review*, 103(1):305–334, Feb 1, 2013.

[9] Pierpaolo Benigno and Michael Woodford. Optimal monetary and fiscal policy: A linear-quadratic approach. *NBER macroeconomics annual*, 18:271–333, 2003.

[10] Efraim Benmelech, Mark J. Garmaise, and Tobias J. Moskowitz. Do liquidation values affect financial contracts? evidence from commercial loan contracts and zoning regulation. *The Quarterly Journal of Economics*, 120(3):1121–1154, 2005.

[11] Pedro Bento. Misallocation, establishment size, and productivity. *American economic journal*, 9(3):267–303, 2017.

[12] Pedro Bento and Diego Restuccia. Misallocation, establishment size, and productivity. *American Economic Journal: Macroeconomics*, 9(3):267–303, 2017.

[13] Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1:1341–1393, 1999.

[14] Francisco J. Buera, Joseph P. Kaboski, and Yongseok Shin. Finance and development: A tale of two sectors. *American economic review*, 101(5):1964–2002, 2011.

[15] Francisco J. Buera and Benjamin Moll. Aggregate implications of a credit crunch: The importance of heterogeneity. *American Economic Journal: Macroeconomics*, 7(3):1–42, 2015.

[16] Andrea Caggese and Vicente Cuñat. Financing constraints, firm dynamics, export decisions, and aggregate productivity. *Review of Economic Dynamics*, 16(1):177–193, 2013.

[17] John Y. Campbell. Mortgage market design. *Review of finance*, 17(1):1–33, 2012.

[18] Sylvain Catherine, Thomas Chaney, Zongbo Huang, David Sraer, and David Thesmar. Aggregate effects of collateral constraints. *NBER Working Paper*, 2017.

[19] Varadarajan V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. Business cycle accounting. *Econometrica*, 75(3):781–836, 2007.

[20] Ian Christensen, Paul Corrigan, Caterina Mendicino, and Shin-Ichi Nishiyama. Consumption, housing collateral, and the canadian business cycle. 2009.

[21] Thomas F. Cooley and Vincenzo Quadrini. Financial markets and firm dynamics. *American economic review*, 91(5):1286–1310, 2001.

[22] José-María Da-Rocha. *Firing costs, misallocation, and aggregate productivity.* National Bureau of Economic Research, Cambridge, MA, 2016.

[23] José-María Da-Rocha, Diego Restuccia, and Marina Mendes Tavares. Firing costs, misallocation, and aggregate productivity. *Journal of Economic Dynamics and Control*, 98:60–81, 2019.

[24] Rui Luís de Castro, Gian Luca Clementi, and Glenn Macdonald. Legal institutions, sectoral heterogeneity, and economic development. *Review of Economic Studies*, 76(2):529–561, 2009.

[25] Mikhail Dmitriev and Jonathan Hoddenbagh. Collateral constraints and state-contingent contracts. 2016.

[26] Sebastian Doerr. Collateral, reallocation, and aggregate productivity: Evidence from the us housing boom. *Available at SSRN 3184162*, 2018.

[27] Romain Duval, Gee Hee Hong, and Yannick Timmer. Financial frictions and the great productivity slowdown. *The Review of Financial Studies*, 33(2):475–503, 2020.

[28] Andrés Erosa. Financial intermediation and occupational choice in development. *Review of Economic Dynamics*, 4(2):303–334, 2001.

[29] Robert C Feenstra, Robert Inklaar, and Marcel P Timmer. The next generation of the penn world table. *American economic review*, 105(10):3150–82, 2015.

[30] Lucia Smith Foster, John Haltiwanger, and Chad Syverson. Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? Technical report, Aug 2005.

[31] Jeffrey Frankel and Christopher Pissarides. *NBER International Seminar on Macroeconomics 2008*. University of Chicago Press, 2009.

[32] Alain Gabler and Markus Poschke. Experimentation by firms, distortions, and aggregate productivity. *Review of Economic Dynamics*, 16(1):26–38, 2013.

[33] Daniel Garcia-Macia. *The financing of ideas and the great deviation*. International Monetary Fund, 2017.

[34] Carlos Garriga, Finn E. Kydland, and Roman Šustek. Monk: Mortgages in a new keynesian model. 2018.

[35] Mark Gertler and Peter Karadi. A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34, 2011.

[36] Mark Gertler and Nobuhiro Kiyotaki. *Financial intermediation and credit policy in business cycle analysis*, volume 3 of *Handbook of monetary economics*, pages 547–599. Elsevier, 2010.

[37] Mark Gertler and Nobuhiro Kiyotaki. Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, 105(7):2011–2043, 2015.

[38] Mark Gertler, Nobuhiro Kiyotaki, and Andrea Prestipino. Credit booms, financial crises, and macroprudential policy. *Review of Economic Dynamics*, 2020.

[39] Mark Gertler, Nobuhiro Kiyotaki, and Andrea Prestipino. A macroeconomic model with financial panics. *The Review of Economic Studies*, 87(1):240–288, 2020.

[40] Mark Gertler, Nobuhiro Kiyotaki, and Albert Queralto. Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics*, 59:S17–S34, 2012.

[41] Simon Gilchrist, Jae Sim, and Egon ZakrajÅ›ek. Uncertainty, financial frictions, and investment dynamics. 2010.

[42] Simon Gilchrist, Jae W. Sim, and Egon ZakrajÅ¡ek. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics*, 16(1):159–176, 2013.

[43] Simon Gilchrist and Egon Zakrajsek. Credit spreads and business cycle fluctuations. *American Economic Review*, 102(4):1692–1720, 2012.

[44] Grey Gordon and Pablo A. Guerron-Quintana. Dynamics of investment, debt, and default. *Review of Economic Dynamics*, 28:71–95, Apr 2018.

[45] Nezih Guner, Gustavo Ventura, and Yi Xu. Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, 11(4):721–744, 2008.

[46] Andreas Hornstein. Notes on collateral constraints in a simple model of housing, 2008.

[47] Nir Jaimovich and Max Floetotto. Firm dynamics, markup variations, and the business cycle. *Journal of Monetary Economics*, 55(7):1238–1252, 2008.

[48] Hyeok Jeong and Robert M. Townsend. Sources of tfp growth: occupational choice and financial deepening. *Economic Theory*, 32(1):179–221, 2007.

[49] Urban Jermann and Vincenzo Quadrini. Macroeconomic effects of financial shocks. *The American economic review*, 102(1):238–271, 2012.

[50] Marios Karabarbounis and Patrick Macnamara. Misallocation and credit market constraints: the role of long-term financing. 2019.

[51] Aubhik Khan and Julia K. Thomas. Collateral constraints, capital specificity and the distribution of production: the role of real and financial frictions in aggregate fluctuations. 2009.

[52] Aubhik Khan and Julia K. Thomas. Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107, 2013.

[53] José Joaquín López. Financial frictions and productivity: Evidence from mexico. *The Quarterly Review of Economics and Finance*, 66:294–301, 2017.

[54] Kiminori Matsuyama. Credit traps and credit cycles. *American Economic Review*, 97(1):503–516, 2007.

[55] Caterina Mendicino. Credit market and macroeconomic volatility. 2007.

[56] Caterina Mendicino. On the amplification role of collateral constraints. *Economics Letters*, 117(2):429–435, 2012.

[57] Felipe Meza and Erwan Quintin. Financial crises and total factor productivity. *Available at SSRN 695882*, 2005.

[58] Virgiliu Midrigan and Daniel Yi Xu. Finance and misallocation: Evidence from plant-level data. *American economic review*, 104(2):422–458, 2014.

[59] Benjamin Moll. Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221, 2014.

[60] Edward C. Prescott. "lawrence r. klein lecture" needed: A theory of total factor productivity. *International economic review (Philadelphia)*, 39(3):525, Aug 1, 1998.

[61] Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous plants. Technical report, Apr 2007.

[62] Diego Restuccia and Richard Rogerson. Misallocation and productivity. *Review of Economic Dynamics*, 16(1):1–10, Jan 2013.

[63] Julio J. Rotemberg and Michael Woodford. Markups and the business cycle. *NBER macroeconomics annual*, 6:63–129, 1991.

[64] Ping Wang and Burak Uras. Production flexibility, misallocation and total factor productivity. Technical report, National Bureau of Economic Research, Oct 1, 2017.

[65] Michael Woodford. *Interest and prices: Foundations of a theory of monetary policy.* princeton university press, 2011.

Appendix

2.A Approximations

We present below all approximations in detail.

2.A.1 Demand function approximation

The demand function from (2.9) is as under;

$$k_{t+1}(i) = k_{t+1}\left(\frac{\overbrace{1-(1-\delta(i))(\theta\mathbb{E}_t[m_{t+1}]+\phi_t\xi_t\chi)}^{\eta_t(i)}}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}$$

We start by approximating $\eta_t(i)^{-\varepsilon}$ around $\bar{\delta} = \int \delta(i)\mathrm{di}$, the average rate of depreciation.

$$\begin{aligned}
\eta_t(i)^{-\varepsilon} &\approx \eta_t^{-\varepsilon} - \varepsilon\eta_t^{-\varepsilon-1}d\eta_t(\delta(i)-\bar{\delta}) + \frac{1}{2}\left[\varepsilon(\varepsilon+1)\eta_t^{-\varepsilon-2}\left(d\eta_t\right)^2\right](\delta(i)-\bar{\delta})^2 + \|\mathcal{O}\|^3 \\
\Rightarrow \int \eta_t(i)^{-\varepsilon}\mathrm{di} &\approx \eta_t^{-\varepsilon} + 0 + \frac{1}{2}\varepsilon(\varepsilon+1)\eta_t^{-\varepsilon-2}d\eta_t^2\sigma_\delta^2 + \|\mathcal{O}\|^3 \\
&\approx \eta_t^{-\varepsilon}\left\{1+\frac{1}{2}\varepsilon(\varepsilon+1)\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)^2\right\}
\end{aligned} \tag{2A.1}$$

Substituting (2A.1) into (2.9) gives (2.18);

$$\int k_{t+1}(i)\mathrm{di} := \bar{k}_{t+1} = k_{t+1}\left(\frac{\eta_t}{\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}]}\right)^{-\varepsilon}\cdot\left\{1+\frac{1}{2}\varepsilon(\varepsilon+1)\sigma_\delta^2\left(\frac{d\eta_t}{\eta_t}\right)^2\right\}$$

2.A.2 Effective rate of depreciation approximation

We start with the identity

$$\begin{aligned}(1-D_t)\bar{k}_t &\equiv \int (1-\delta(i))k_t(i)\text{di} \\ &= \int (1-\delta(i))\eta_t(i)^{-\varepsilon}\text{di}\ k_t(\theta\mathbb{E}_t[m_{t+1}F_{k,t+1}])^{\varepsilon}\end{aligned} \tag{2A.2}$$

We now approximate $(1-\delta(i))\eta_{t-1}(i)^{-\varepsilon}\text{di}$;

$$\begin{aligned}(1-\delta(i))\eta_{t-1}(i)^{-\varepsilon} \approx (1-\bar{\delta})\eta_{t-1}^{-\varepsilon} &+ \left[-\eta^{-\varepsilon} - \varepsilon(1-\bar{\delta})\eta_{t-1}^{-\varepsilon-1}d\eta_{t-1}\right](\delta(i)-\bar{\delta}) \\ &+ \frac{1}{2}\left[2\varepsilon\eta_{t-1}^{-\varepsilon}\left(\frac{d\eta_{t-1}}{\eta_{t-1}}\right) + \varepsilon(\varepsilon+1)(1-\bar{\delta})\left(\frac{d\eta_{t-1}}{\eta_{t-1}}\right)^2\right](\delta(i)-\bar{\delta})^2 + \|\mathcal{O}\|^3\end{aligned}$$

$$\int (1-\delta(i))\eta_{t-1}(i)^{-\varepsilon}\text{di} \approx (1-\bar{\delta})\eta_{t-1}^{-\varepsilon}\left\{1 + \frac{1}{2}\varepsilon\sigma_{\delta}^2\frac{d\eta_{t-1}}{\eta_{t-1}}\left(\frac{2}{1-\delta} + (\varepsilon+1)\frac{d\eta_{t-1}}{\eta_{t-1}}\right)\right\} \tag{2A.3}$$

Combining (2A.1), (2A.2), and (2A.3) yields the following;

$$D_t = 1 - (1-\bar{\delta})\cdot\left\{\frac{\left\{1 + \frac{1}{2}\varepsilon\sigma_{\delta}^2\frac{d\eta_{t-1}}{\eta_{t-1}}\left(\frac{2}{1-\delta} + (\varepsilon+1)\frac{d\eta_{t-1}}{\eta_{t-1}}\right)\right\}}{\left\{1 + \frac{1}{2}\varepsilon(\varepsilon+1)\sigma_{\delta}^2\left(\frac{d\eta_{t-1}}{\eta_{t-1}}\right)^2\right\}}\right\} \tag{2A.4}$$

2.A.3 Capital Euler approximation

We start with the production bundle constraint (2.4)

$$k_t = \left[\int k_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\text{di}\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (F_{k,t})$$

Moving it forward one period, and substituting for $k(i)$ gives;

$$1 = \left[\int \eta_t(i)^{1-\varepsilon} \mathrm{di} \right]^{\frac{\varepsilon}{\varepsilon-1}} (\theta \mathbb{E}_t m_{t+1} F_{k,t+1})^{\varepsilon} \tag{2A.5}$$

We approximate $\int \eta_t(i)^{1-\varepsilon}\mathrm{di}$;

$$\begin{aligned}
\eta_t(i)^{1-\varepsilon} &\approx \eta_t^{1-\varepsilon} + \left[(1-\varepsilon)\eta_t^{-\varepsilon} d\eta_t\right](\delta(i) - \bar{\delta}) + \frac{1}{2}\left[\varepsilon(\varepsilon-1)\eta_t^{-\varepsilon-1}(d\eta_t)^2\right](\delta(i)-\bar{\delta})^2 + \|\mathcal{O}\|^3 \\
\int \eta_t(i)^{1-\varepsilon}\mathrm{di} &\approx \eta_t^{1-\varepsilon} + 0 + \frac{1}{2}\left[\varepsilon(\varepsilon-1)\eta_t^{-\varepsilon-1}(d\eta_t)^2\right]\sigma_\delta^2 \\
&\approx \eta_t^{1-\varepsilon} \underbrace{\left\{1 + \frac{1}{2}\varepsilon(\varepsilon-1)\sigma_\delta^2 \left(\frac{d\eta_t}{\eta_t}\right)^2\right\}}_{\tau^\gamma}
\end{aligned} \tag{2A.6}$$

Combining (2A.5) and (2A.6) and simplifying, we get;

$$1 = \frac{\eta_t}{\theta \mathbb{E}_t[m_{t+1}F_{k,t+1}]} \cdot \left\{1 + \frac{1}{2}\varepsilon(\varepsilon-1)\sigma_\delta^2 \left(\frac{d\eta_t}{\eta_t}\right)^2\right\}^{\frac{1}{1-\varepsilon}}$$

Substituting for $m = \beta\frac{\lambda_{+1}}{\lambda}$ in the above, as well as inside η (but not in τ^γ) gives;

$$\begin{aligned}
\theta\beta\mathbb{E}_t[\lambda_{t+1}F_{k,t+1}]\tau_t^\gamma &= \lambda_t\left[1 - (1 - \bar{\delta}[\theta\beta\frac{\mathbb{E}_t\lambda_{t+1}}{\lambda_t} + \phi_t\xi_t\chi])\right] \\
\Rightarrow \lambda_t &= \frac{\theta\beta\mathbb{E}_t\lambda_{t+1}[F_{k,t+1}\cdot\tau_t^\gamma + (1-\bar{\delta})]}{1-(1-\bar{\delta})\phi_t\xi_t\chi}
\end{aligned} \tag{2A.7}$$

2.B Supporting plots and graphs

This appendix contains supporting plots and graphs not included in the main text, but that provide added insight into the data analysis.

2.B.1 Histograms

We first present a visual representation of what the balance sheet of the aggregate firm in U.S. looks like in figure 2.B1 below. Figure 2.B1 shows us that the balance sheet has more number of short term to medium term assets, and low amount of very long term assets (by quantity, NOT by value).

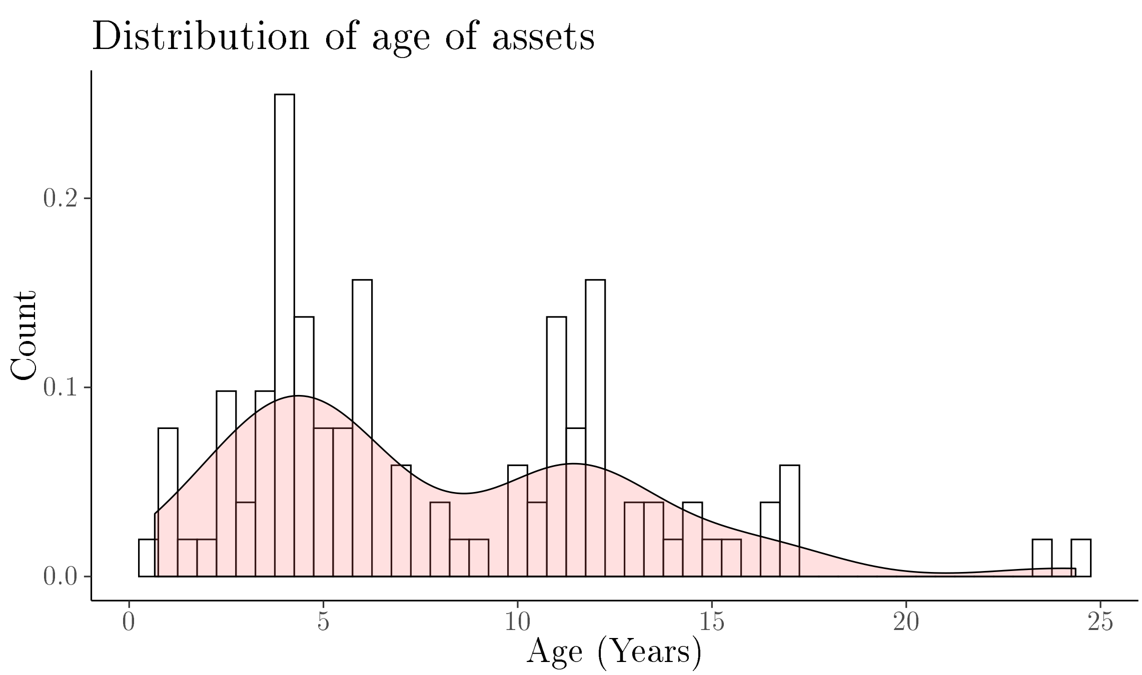


Figure 2.B1 Histogram for all assets

Source: BEA

Next, we plot a histogram of change in mean age of each asset class over the 2008 crisis. We divide the data into pre 2008 and post 2008, and calculate the mean age for each asset class for 2000-2008 and 2009-2017. Then, we check if the change in mean age for each asset class over the two time periods is statistically significant using a t-test. If the change is statistically significant, it shows that investment in the said asset class has changed during the crisis compared to previously. We again plot a histogram based on mean age of that asset class, where each bin shows how many asset classes have changes for that bin value, and the colour shows whether the changes are significant.

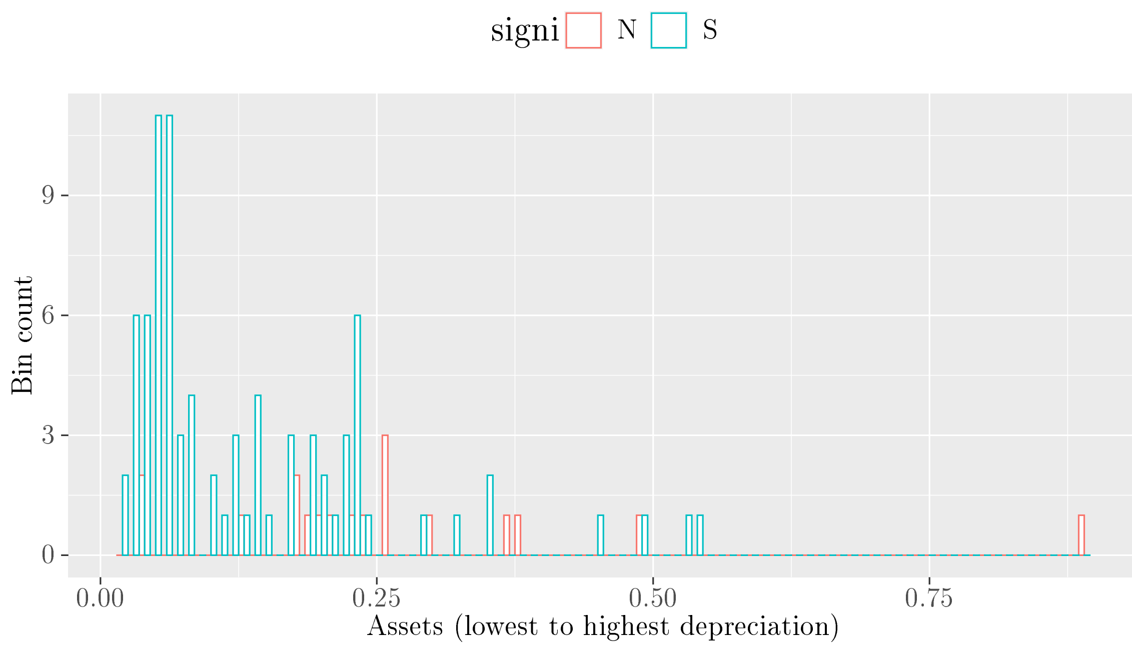


Figure 2.B2 Histogram (2000-2008 vs. 2009-2017)

Source: BEA

Figure 2.B2 shows that most change in asset investment based on mean historical age happens in the long term assets, and the change is significant at the 5% level. As can be seen, the most significant change happens in assets at the lower end of the depreciation scale, while towards the middle and the right, the changes are insignificant. This implies that over a crisis the firms have changed investment most in long and medium term assets, and least in short term assets. But how does it compare to a non crisis period? To answer that question, we carry out a similar exercise for a control period where there was a very small crisis arising from the bursting of an asset price bubble in 2001. We take the time periods 1990-2000 and

2001-2008 and plot the change in means as significant and insignificant in figure 2.B3.

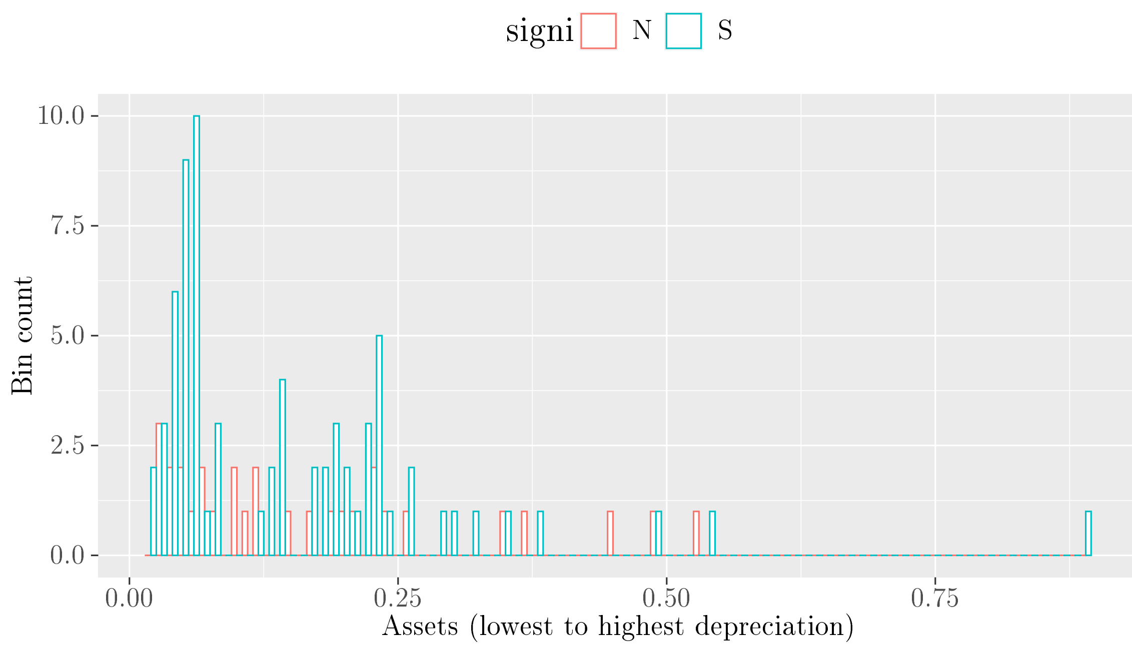


Figure 2.B3 Histogram (1990-2000 vs. 2001-2008)

Source: BEA

Although qualitatively similar as regards where the maximum change happens, the differences lie in the fact that now even some changes in long term assets are insignificant, but the changes at the right extreme in short term assets are now significant. This indicates that in a relatively stable period, the changes happen along the entire depreciation scale, but in periods of crisis they appear to be more towards the lower end of the scale.

We plot the weighted distribution of assets in 2.B4.

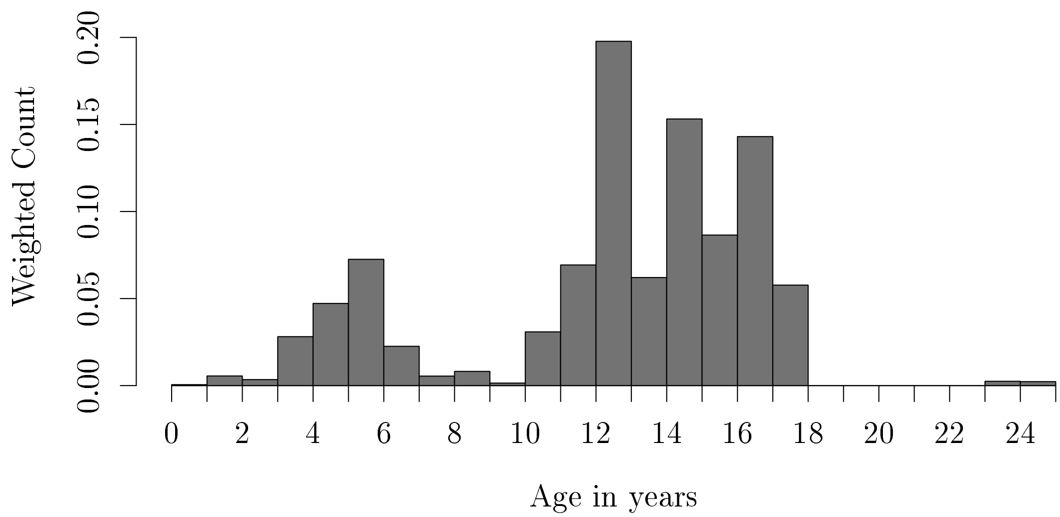


Figure 2.B4 Weighted histogram for all assets

Source: BEA

To construct 2.B4, we use the same age data that we used to construct 2.B1, but now we weigh each asset class with the proportion in dollar value that it represents of the total balance sheet. We now see that the mass has moved from the short term assets compared to 2.B1 and is now more at the medium-to-long term assets with age range 10-20 years.

2.B.2 Bar plots

As a control group, we present the bar plot for a different set of time periods: 1990-2000 and 2001-2008 in figure 2.B5.

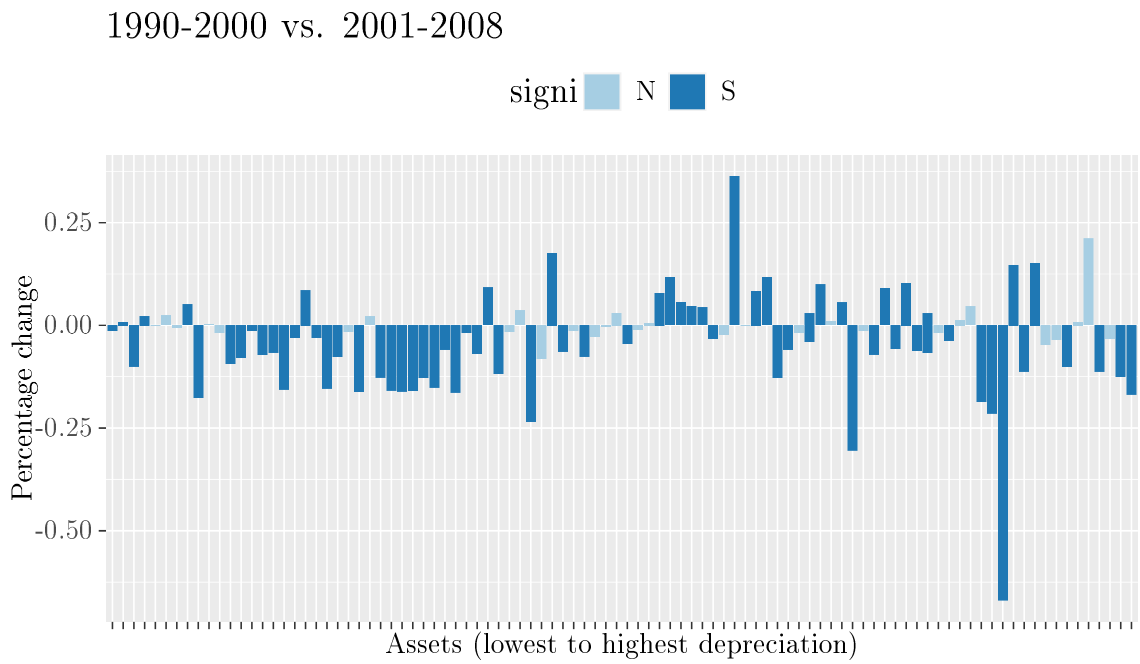


Figure 2.B5 Change in means (1990-2000 vs. 2001-2008)

Source: BEA

Now we see a different pattern of changes in asset classes. Most of the positive change in investment is in the second half of the scale, and some negative change as well. The magnitude of the negative change at the lower end of the scale is much lower than that at the higher end of the scale. This indicates that, based on just the quantity of assets, after the crisis there appears to have been a rise in investment in short term assets compared (except for one big fall) to the previous period. The investment in long term assets has not fallen to the same extent as in figure 2.3.

We present the same information for treatment and control groups on a numerical scale in below plots.

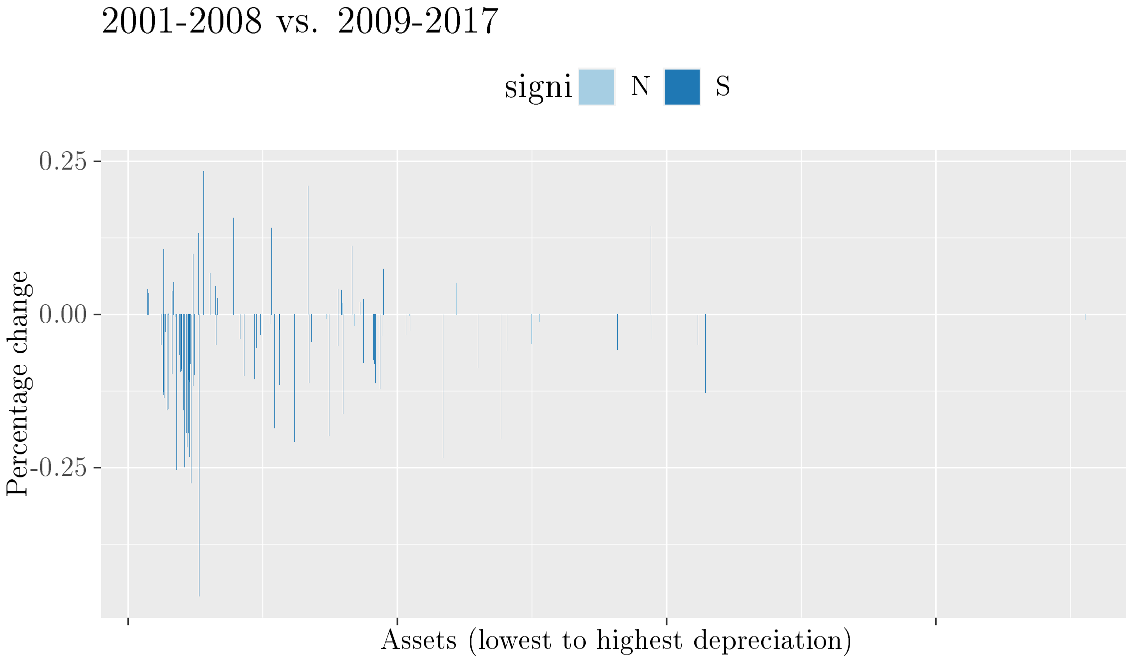


Figure 2.B6 Change in means (numerical) (2000-2008 vs. 2009-2017)

Source: BEA

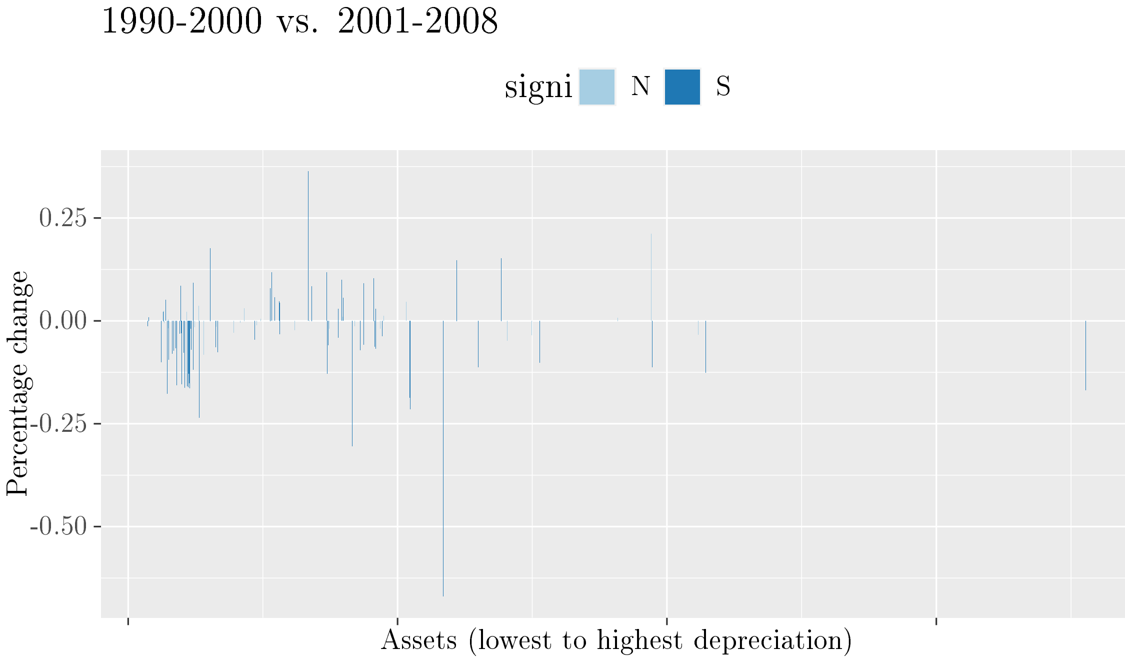


Figure 2.B7 Change in means (numerical) (1990-2000 vs. 2001-2008)

Source: BEA

The above figures 2.B6 and 2.B7 present the same information as in 2.3 and 2.B5,only difference being that X axis is now a numerical scale. The additional information provided by this is the distance between asset classes where changes happen.

2.B.3 Cyclical deviations, unweighted and weighted

Here we present first the unweighted cyclical deviations for all asset classes separately, and then with asset classes aggregated into slightly larger bins, but not as large as figure 2.4.

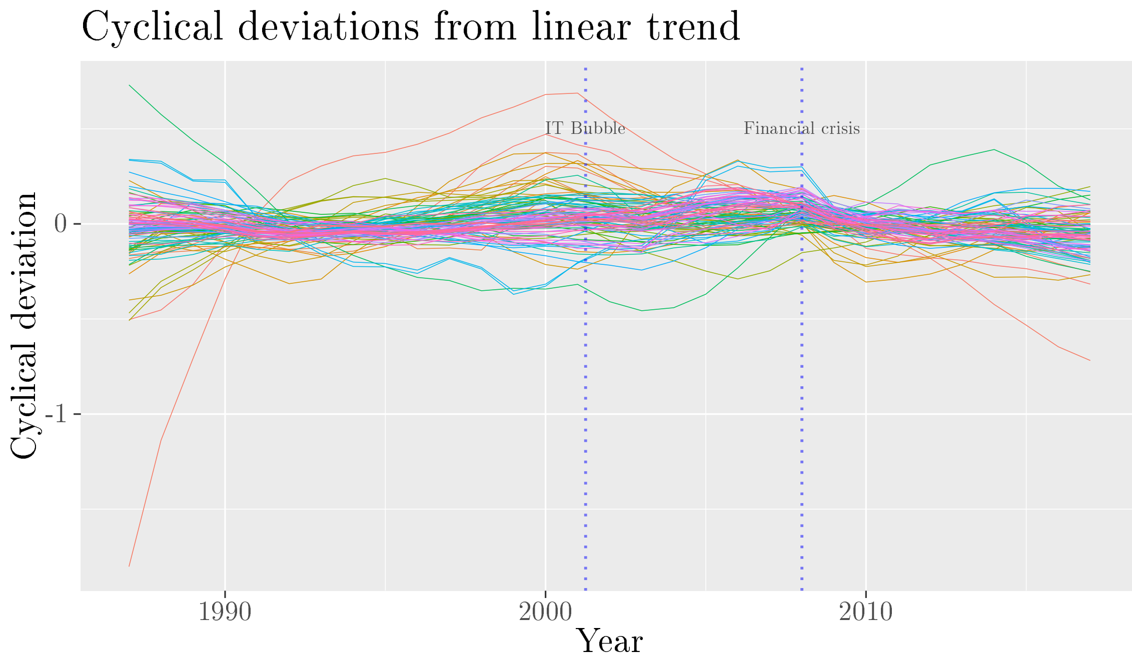


Figure 2.B8 Change over time in each asset class

Source: BEA

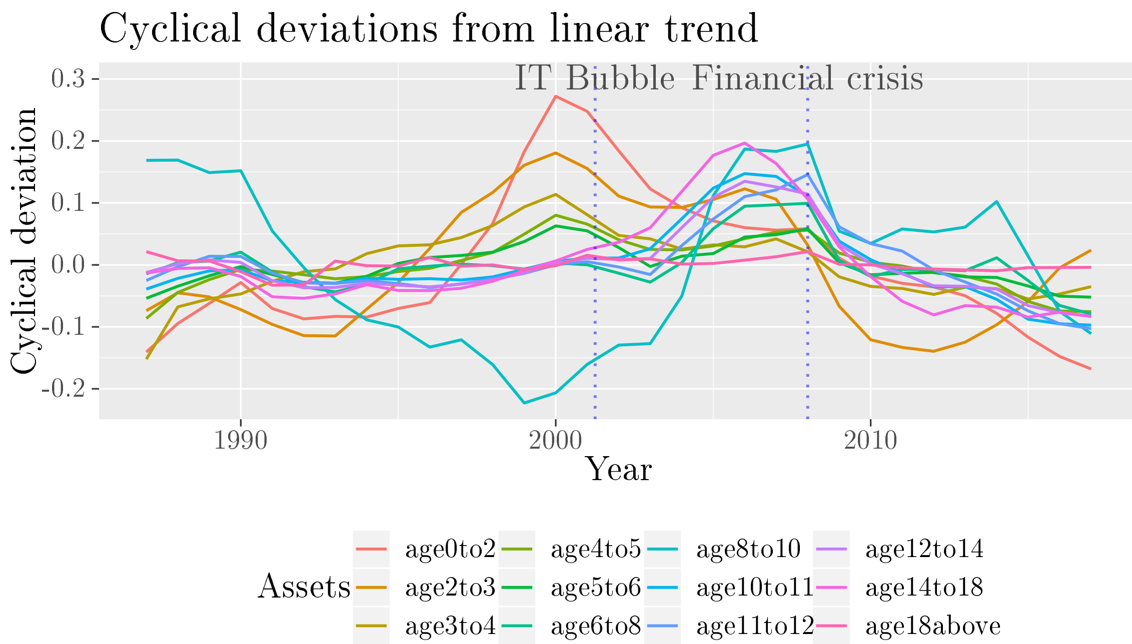


Figure 2.B9 Change over time in asset classes (bigger bins)

Source: BEA

The following is equivalent to figure 2.5 as regards data used, but provides a finer picture about what happens in all asset classes, as well in slightly bigger bins.

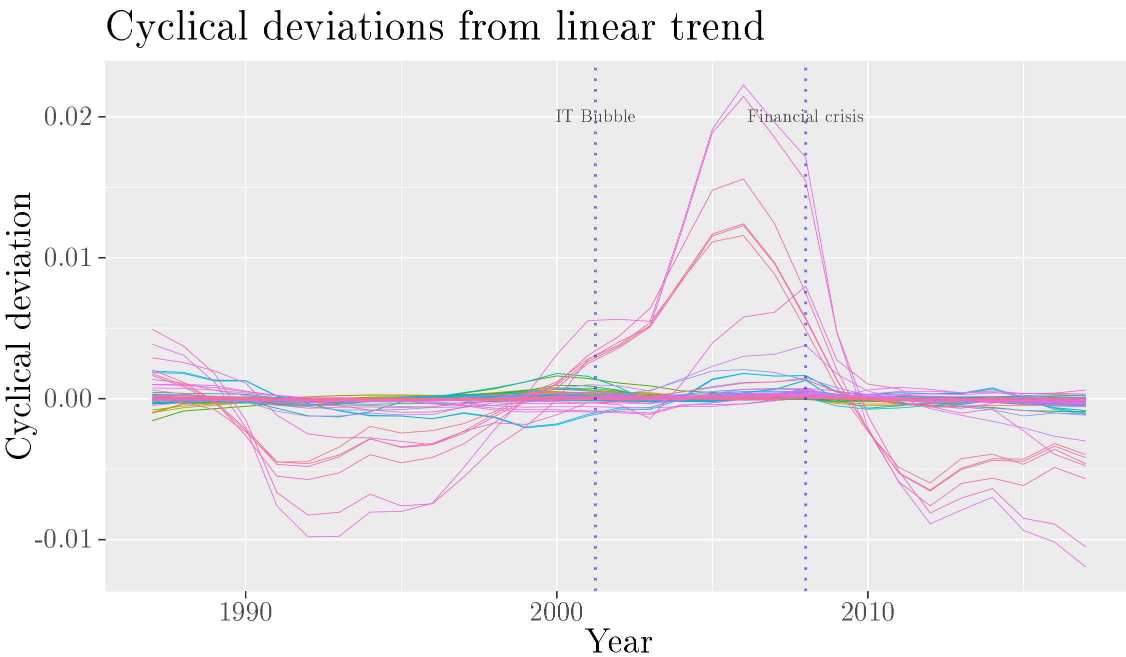


Figure 2.B10 Change over time in each weighted asset class

Source: BEA

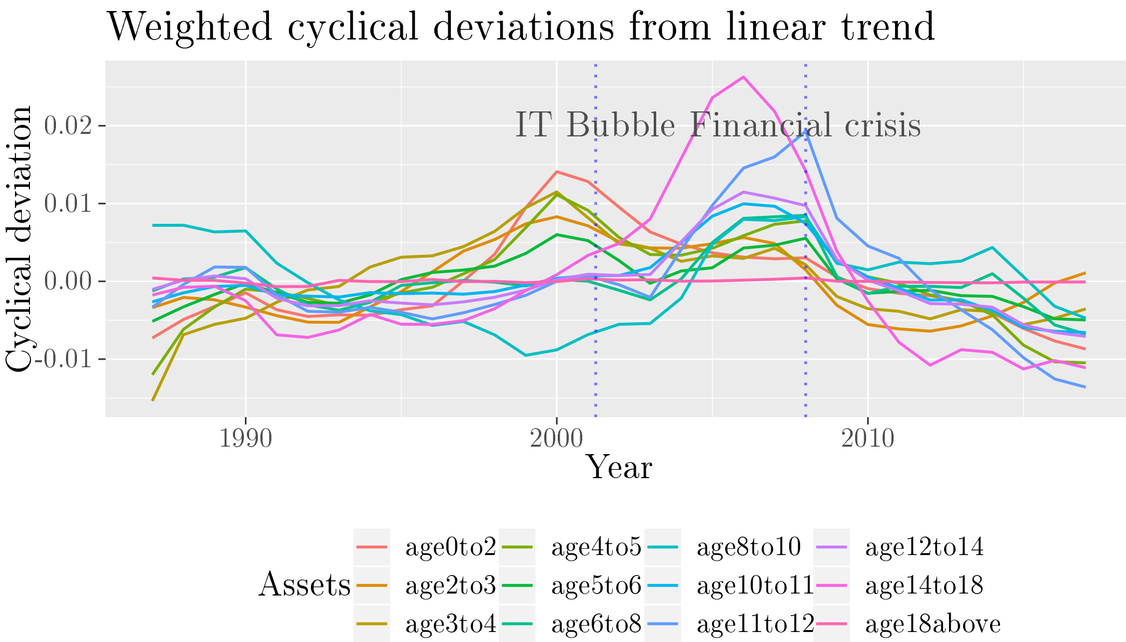


Figure 2.B11 Change over time in weighted asset classes (bigger bins)

Source: BEA

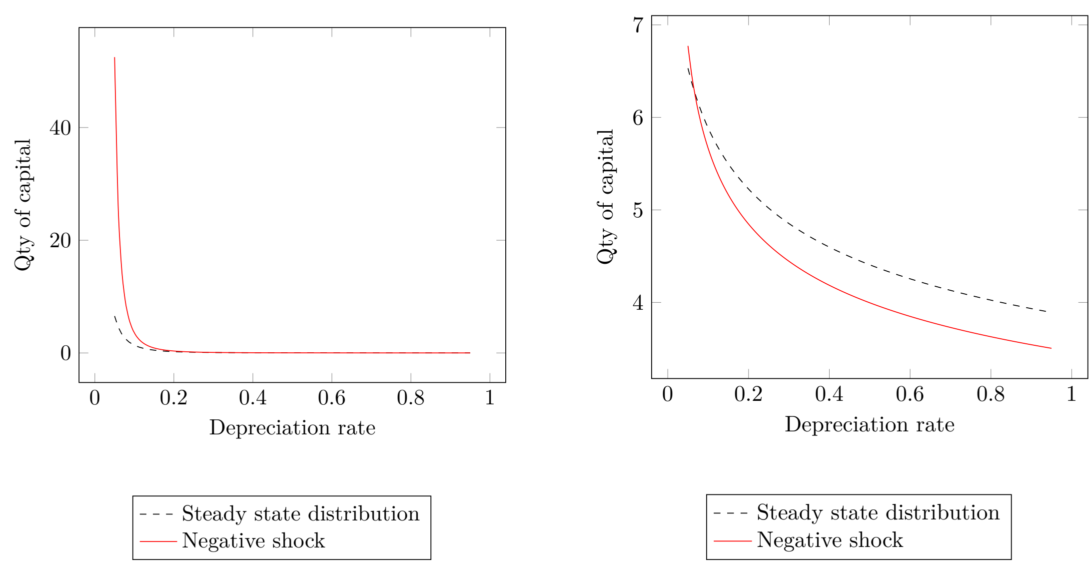
2.C Alternative calibrations

We present alternative calibration results in this section and discuss what the implications are for the model and its output. Unless explicitly mentioned, all other parameters for each exercise are the baseline calibration mentioned in table 2.1.

2.C.1 Complements vs Substitutes

We present results for the portfolio reallocation, steady state implications, and dynamic results of changing the parameter ε and setting it much closer to making the assets complementary, specifically $\varepsilon = 0.2$, as well as assets being closer substitutes with $\varepsilon = 3.0$.

The asset portfolio is restructured as under with this calibration;



(a) Assets are substitutes **(b)** Assets are complements

Figure 2.C1 Plots of response to negative shocks

Figure 2.C1a shows that the proportion of long term assets rises greatly and that of short term assets seems to fall. Qualitatively this is similar to figure 2.7, but the effect here is more pronounced at the lower end of the scale. When assets are substitutable, the firm chooses

the longer term assets more on impact of a negative shock as they not only provide better collateral, but also yield marginal product over a longer duration.

Figure 2.C1b shows a different pattern of asset reallocation as compared to figure 2.C1a. Although very long term assets increase slightly, the fall in medium and short term assets is seemingly quite pronounced. This is a result of the investment in long term assets not increasing as much as previously which creates the impression of a bigger fall in short term assets. The reason again is the same as in previous chapter: when assets are complements, the change is more pronounced in either direction because they have to be bought together to have any use in production, and hence no one asset class can increase to the extent observed when assets are substitutes.

The steady state values for this calibration are as under;

Steady State Values

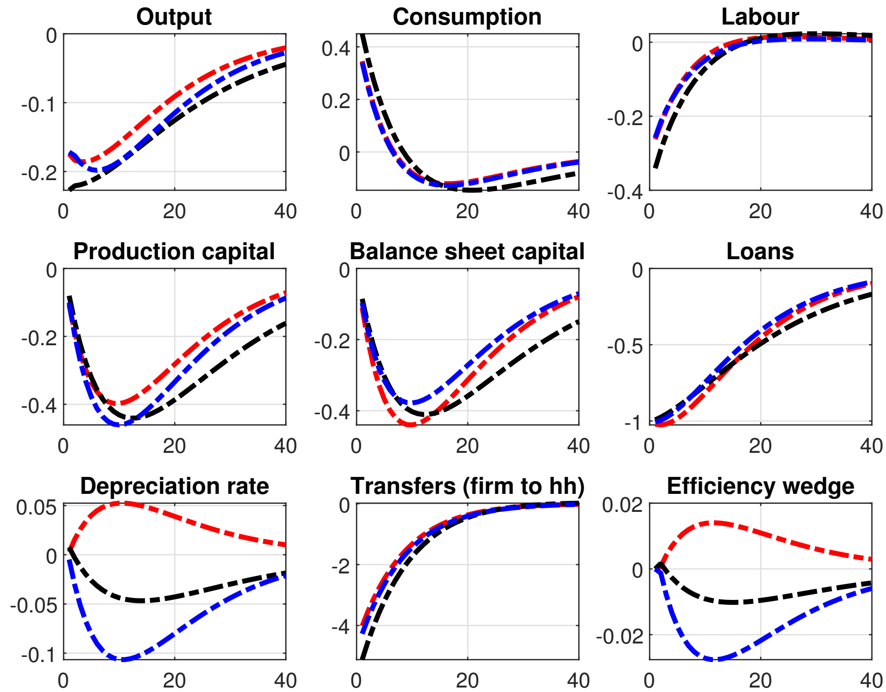
Variable	Description	Baseline	Substitutes	Complements
C/Y	Consumption-Output ratio	0.80	0.76	0.76
$\bar{K}/Y$	Portfolio Capital-Output ratio	6.91	4.46	3.96
K/Y	Production Capital-Output ratio	5.93	4.75	3.58
N	Labour	0.91	0.93	0.94
D	Time-varying aggregate depreciation	0.03	0.05†	0.06
τ	Efficiency wedge	0.95	1.02	0.97

†Depreciation rate $\bar{\delta} = 0.1$ in steady state

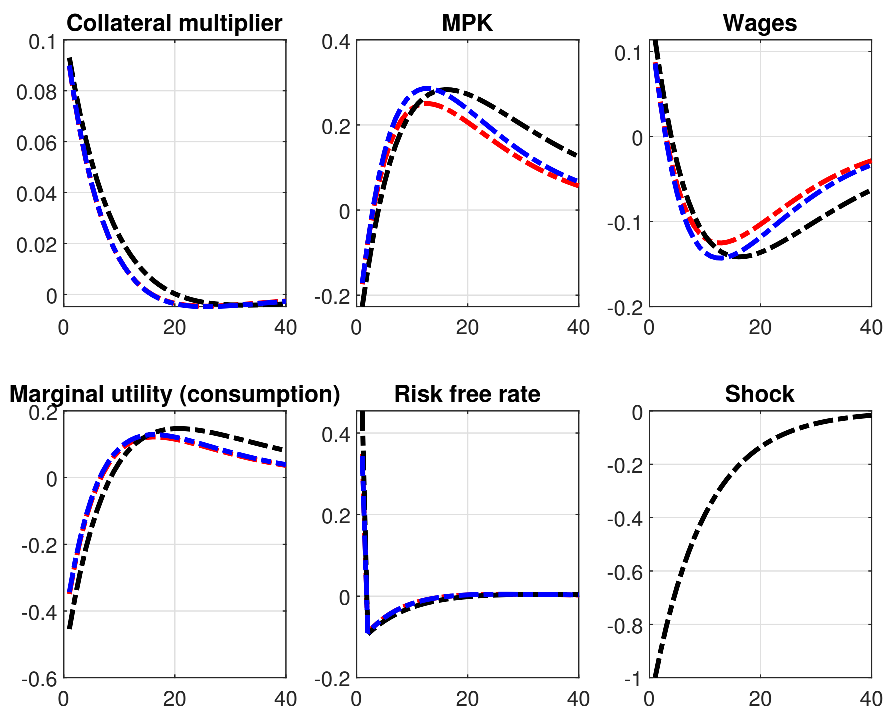
Table 2.C.1 Steady State Values (Substitutes vs Complements)

The steady state capital is lower when assets are substitutes in this case because we have used a higher rate of depreciation $\bar{\delta}$ than in the baseline case. This is necessitated by the fact that for a lower rate of depreciation the effective depreciation D in the steady state is almost zero, implying only one type of assets form a part of the balance sheet portfolio.

Lower steady state capital when assets are complements results from the fact that firms need to hold a relatively balanced portfolio when assets are complements which lowers investment in very high duration assets in steady state. This is also reflected in the higher effective rate of depreciation in the steady state D.



(a) Substitutes vs Complements



(b) Substitutes vs Complements contd.

Figure 2.C2 Plots of response to negative shocks (Substitutes vs Complements)

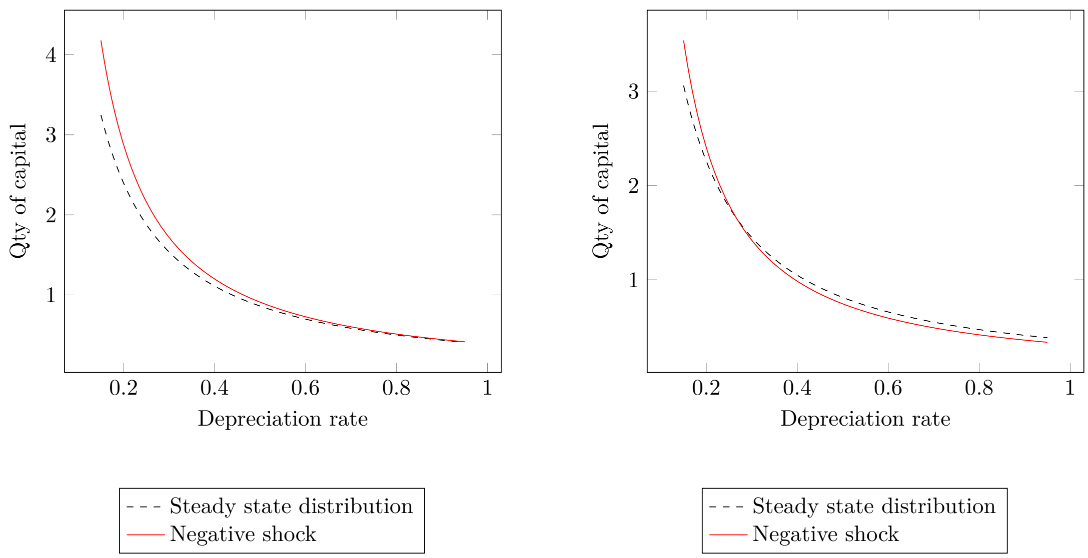
Baseline model, Substitutes model, Complements model

The impulse response functions look qualitatively similar for all three model, except for the effective rate of depreciation and for the efficiency variable. The effective rate of depreciation shows more pronounced response in the same direction as the baseline model when assets are substitutes, whereas the response is in the opposite direction when assets are complements. This ties in with our discussion of the response to shocks above, where we pointed out that when assets are substitutes the portfolio is heavier in long term assets after a negative shock which reduces the effective depreciation even further, whereas when assets are complements the increase in long term assets on impact of a negative shock is very marginal, and hence the effective rate rises due to the higher proportion of short term assets in the portfolio.

2.C.2 Lower Frisch elasticity

Here we present results for a lower value of the Frisch elasticity which makes labour supply flatter, and hence the impact on wages of a change in supply is lower. We set $\psi = 4$ in the exercise below.

Below is the change in portfolio composition after a negative shock when labour supply is flatter;



(a) Flatter labour supply **(b)** Baseline

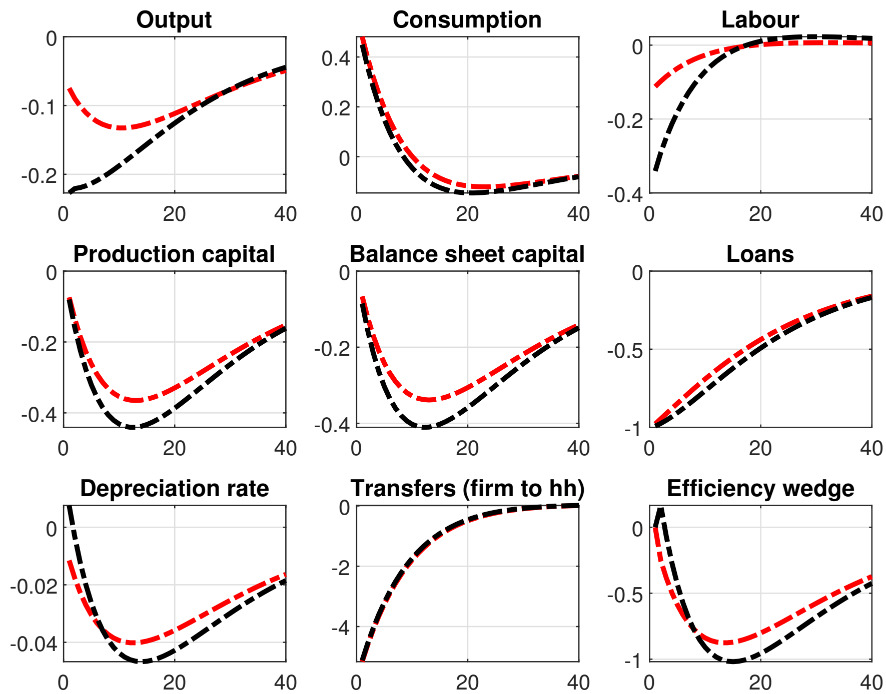
Figure 2.C3 Steady state portfolio and flatter L^S

Figure 2.C3a shows that the reallocation of assets changes for this calibration. Specifically, the drop in assets is much lower now, and the increase in proportion of low depreciation assets is higher. The change appears to result from the fact that wages do not increase by much now causing firms to have marginally more funds to invest in capital assets. As seen earlier, because of the assumption of equal prices for all assets, the natural bent is to invest more in low depreciation assets which act as better collateral for loans. With slightly more funds to invest, firms buy more low depreciation assets proportionally. So, the balance sheet shrinks, but is composed of more low depreciation assets than previously.

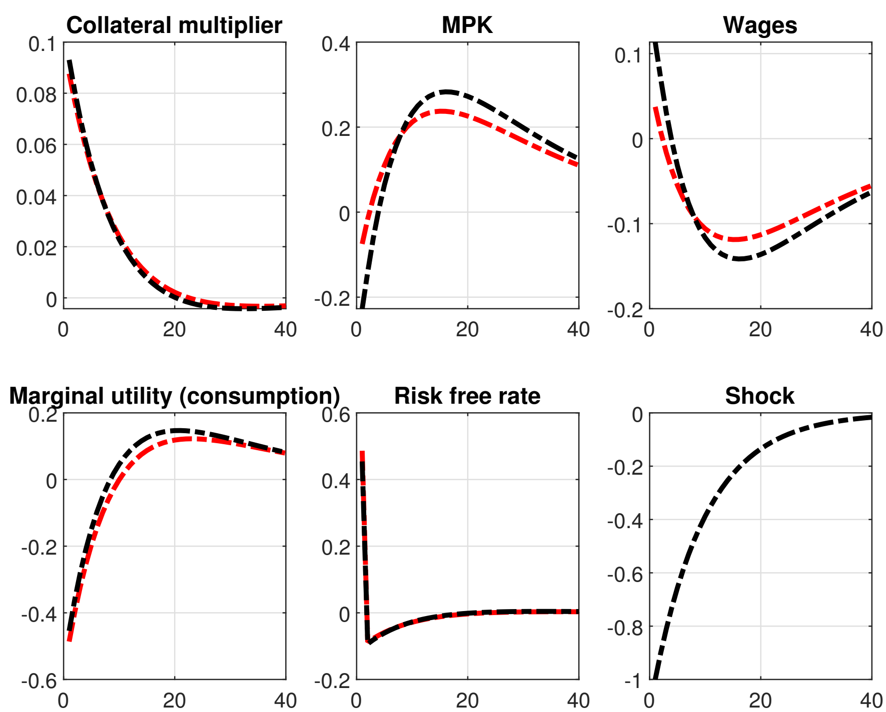
Steady State Values			
Variable	Description	Model	Baseline
C/Y	Consumption-Output ratio	0.80	0.80
$\bar{K}/Y$	Portfolio Capital-Output ratio	6.91	6.91
K/Y	Production Capital-Output ratio	5.93	5.93
N	Labour	0.96	0.96
D	Time-varying aggregate depreciation	0.03	0.03
τ	Efficiency wedge	0.95	0.95

Table 2.C.2 Steady State Values (flatter L^S)

Again, the steady state values in table 2.C.2 seem reasonable, allow us to proceed to the dynamic results, presented below.



(a) Flatter L^S



(b) Flatter L^S contd.

Figure 2.C4 Plots of response to negative shocks with flatter L^S

Baseline model, Lower Frisch elasticity

*Response of efficiency wedge scaled upwards for clarity

The impulse response functions confirm our analysis about the impact of a flatter labour supply. Wages show a far lower increase now than earlier, while the rest of variables have the same response qualitatively and almost the same response quantitatively. Marginal product of capital falls on impact from a fall in labour, but to a lesser extent.

2.C.3 GHH preferences

We now change the preference specification to the GHH one as under;

$$U(C_t, N_t) = \frac{\left(C_t - \nu \frac{N^{1+\psi}}{1+\psi}\right)^{1-\sigma} - 1}{1-\sigma} \tag{2C.1}$$

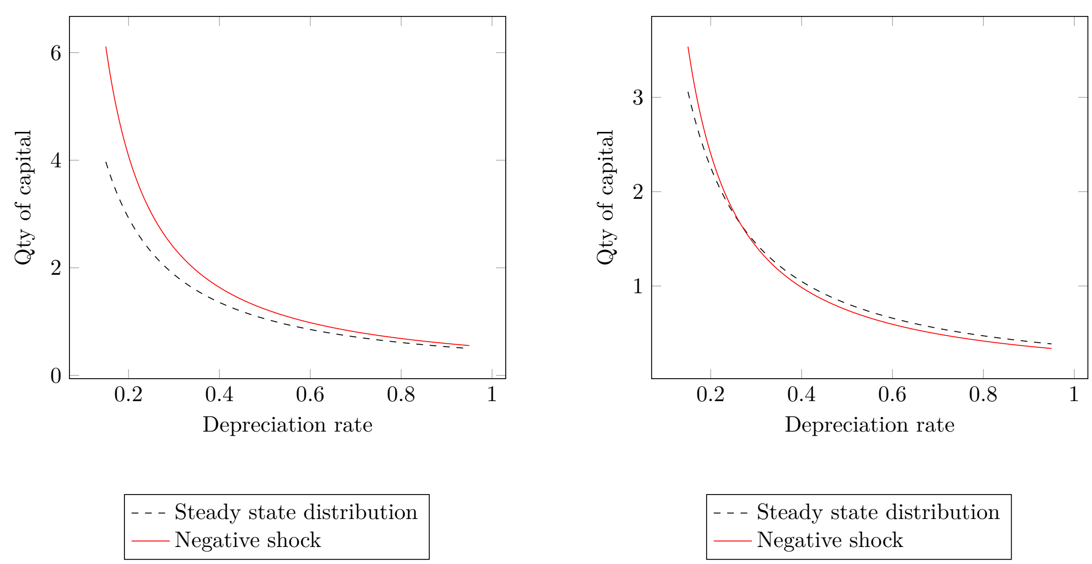
The changes that brings about in optimality conditions are as under;

$$\lambda_t = \left(C_t - \nu \frac{N^{1+\psi}}{1+\psi}\right)^{-\sigma} \tag{2C.2}$$

$$\nu N_t^{\psi} = W_t \tag{2C.3}$$

Now, the labour supply is independent of intratemporal substitution from the wealth effect, and any changes in marginal utility of consumption λ will have no direct impact on labour supply. So, in our case, the labour supply does not change on impact of shock.

The portfolio reallocation happens as under on impact of a negative shock.



(a) GHH Preferences **(b)** Baseline

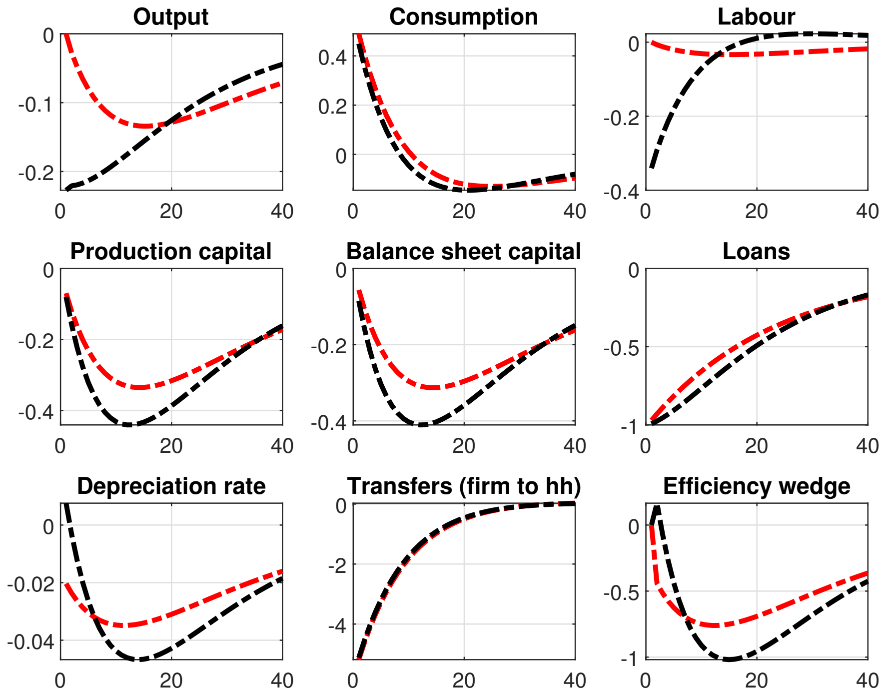
Figure 2.C5 Steady state portfolio and GHH preferences

With the link between the wealth effect and labour supply severed, the impact seen earlier with a flatter labour supply curve is now amplified, and firms seem to have more funds to invest which causes further investment in low depreciation assets compared to earlier. This is reflected in figure 2.C5a.

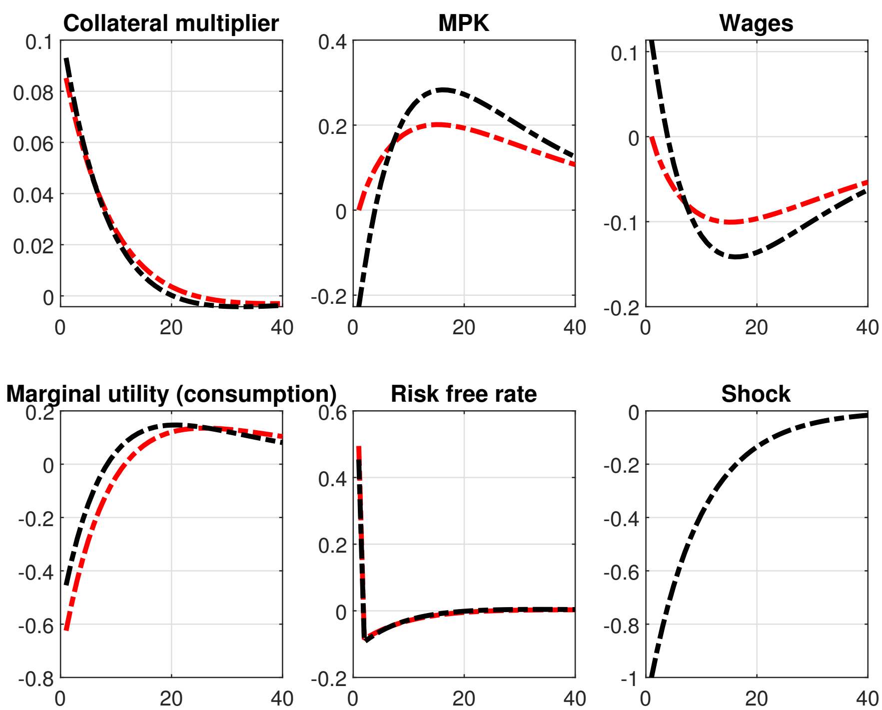
Steady State Values			
Variable	Description	Model	Baseline
C/Y	Consumption-Output ratio	0.80	0.80
$\bar{K}/Y$	Portfolio Capital-Output ratio	6.91	6.91
K/Y	Production Capital-Output ratio	5.93	5.93
N	Labour	0.96	0.91
D	Time-varying aggregate depreciation	0.03	0.03
τ	Efficiency wedge	0.95	0.95

Table 2.C.3 Steady State Values (GHH preferences)

The steady state values are again in order, and almost identical to the previous exercise with a flatter L^S.



(a) GHH Preferences



(b) GHH Preferences contd.

Figure 2.C6 Plots of response to negative shocks with GHH preferences

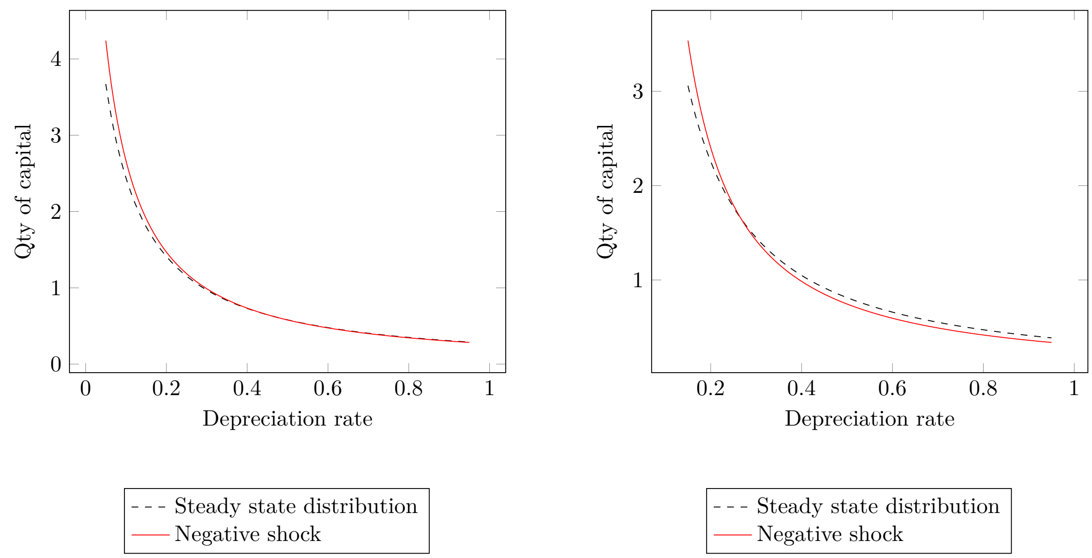
Baseline model, GHH preferences

*Response of efficiency wedge scaled upwards for clarity

The additional difference from the CRRA specification is the response of labour supply, and in turn, output. Because labour supply does not respond at all to the changes in marginal utility any more, it does not fall on impact. Output does not fall either, because K_{-1} is a state and labour hasn't responded. Also, the impact on wages is zero because now the L^S curve does not shift, and nor does the demand L^D. Similarly, the marginal product of capital does not change on impact for the same reasons. As capital falls, marginal product rises gradually. Wages fall over time as labour demand falls with falling capital.

2.C.4 Lower loan-to-value ratio

We present results when the loan-to-value parameter $\chi = 0.4$ is lower than the baseline case.



(a) Lower χ **(b)** Baseline

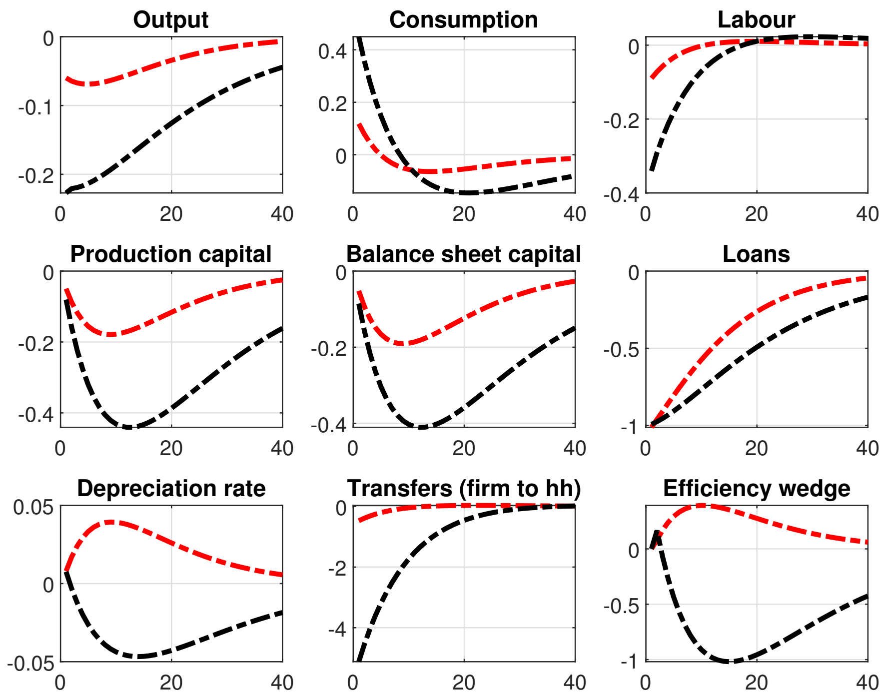
Figure 2.C7 Steady state portfolio and lower credit access

Qualitatively, the reallocation on impact of a negative shock looks similar to the baseline case in figure 2.C7a with very low depreciation assets increasing and replacing very short to short duration assets. This is not surprising as we have not changed anything that would affect this behaviour. What is affected, though, is the extent of capital accumulation due to a lowering of access to credit. With lower amount of loans on offer for the same amount of capital, firms find it difficult to invest to the extent they would want, and capital accumulation suffers. This also raises the steady state ratio of consumption to output, and because investment is lower, steady state labour required is also lower, as seen in table 2.C.4.

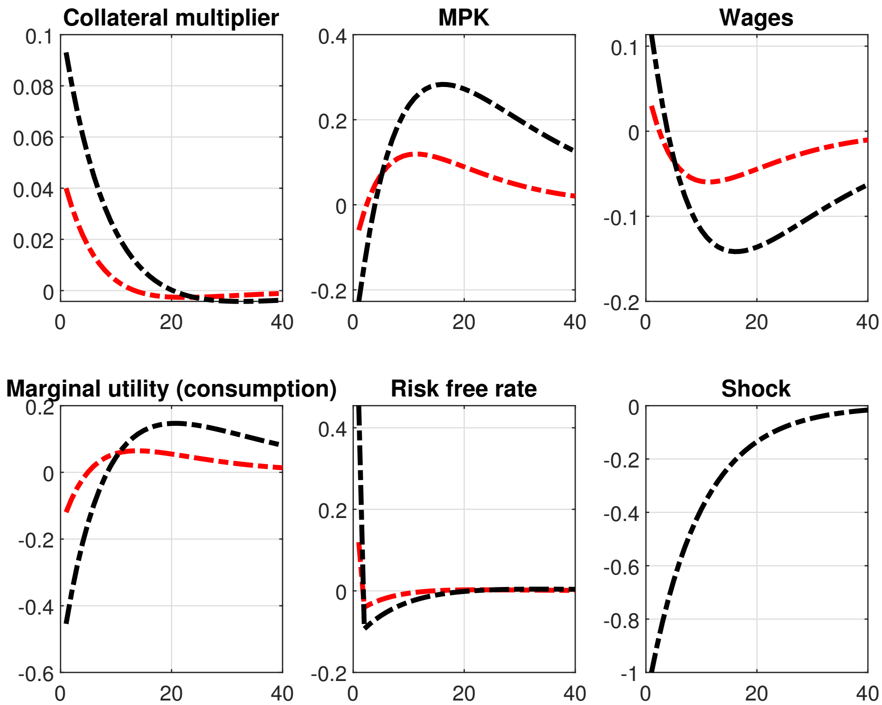
Steady State Values with $\chi = 0.4$			
Variable	Description	Model	Baseline
C/Y	Consumption-Output ratio	0.90	0.80
$\bar{K}/Y$	Portfolio Capital-Output ratio	3.23	6.91
K/Y	Production Capital-Output ratio	2.78	5.93
N	Labour	0.86	0.91
D	Time-varying aggregate depreciation	0.03	0.03
τ	Efficiency wedge	0.95	0.95

Table 2.C.4 Steady State Values ($\chi = 0.4$)

Next, we present results of the dynamic model.



(a) Lower loan-to-value



(b) Lower loan-to-value contd.

Figure 2.C8 Plots of response to negative shocks with $\chi = 0.4$

Baseline model, Modified model

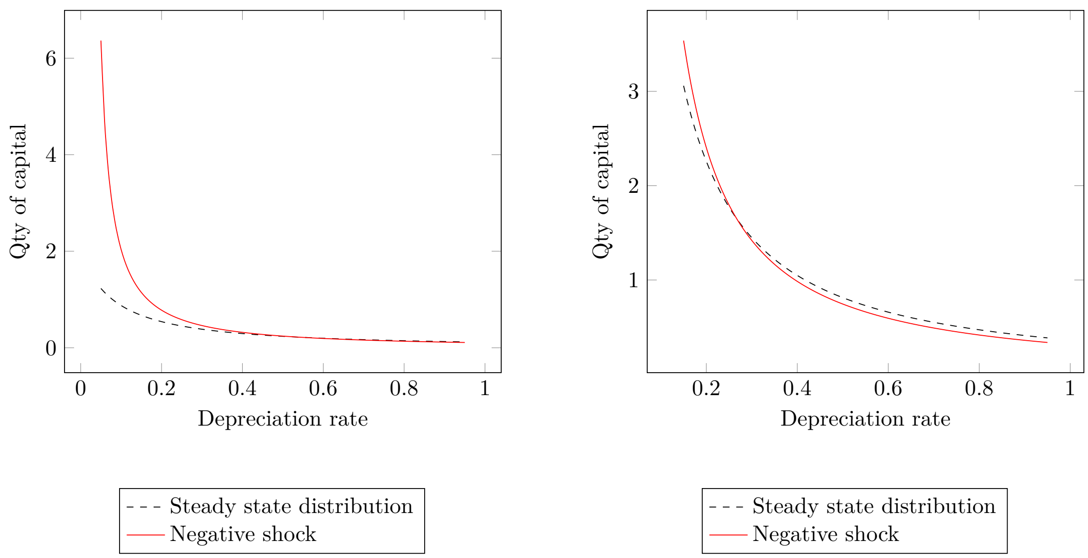
*Response of efficiency wedge scaled upwards for clarity

The response to a negative shock is similar qualitatively for the baseline as well current model, but magnitudes are very different. The model with lower loan-to-value shows a much more muted response as compared to the baseline. Output, labour, capital, and loans fall in both models, but more so in the baseline model. Consumption, on the other hand, falls in the current model whereas it had gone up in the baseline case. The reason it went up in the baseline case, as discussed previously, is that when loans fall, the household sees it as a relaxation of their budget constrain because they won't be lending much, which prompts them to increase consumption. However, in the current case, it seems like because loans were already fairly low in equilibrium, that loosening of the budget constraint does not have much of an impact and the households see the negative shock as a lowering of current and future consumption. Also, because the rise in wages is far less pronounced in the current case, this adds to the perception of households of a tightening budget constraint.

Another difference comes from how the effective rate of depreciation responds to the shock. Given the reshuffling of the portfolio in figure 2.C7a, it is clear that short duration assets do not fall by as much compared to figure 2.C7a. This causes the effective rate of depreciation to be countercyclical from the change in portfolio reallocation.

2.C.5 Lower survival probability

Figure 2.C10 shows the reallocation among assets on a negative financial shock when the survival probability of firms θ is much lower. Specifically, we now set survival probability to 50% instead of the baseline case of 90%.



(a) Lower firm survival

(b) Baseline

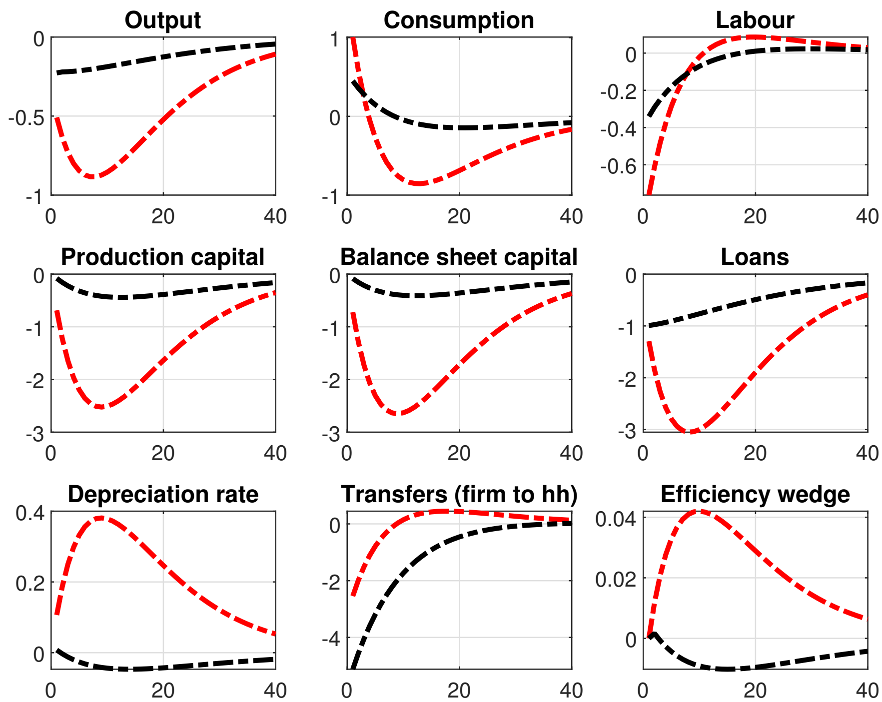
Figure 2.C9 Steady state portfolio and lower credit access

The reallocation plot looks very similar to the case when assets were assumed to be substitutes in figure 2.C1a. However, in figure 2.C9a the investment in long term assets goes up by quite a bit, and that in medium term assets also rises. With fewer firms surviving to the next period, investment in long term assets rises as firms seem less worried about the future and more concerned about maximising the holding of those assets which form the best collateral. The choices are driven more by the collateral constraint now than by future expectations of the path of the economy. In the extreme case, if no firs were to survive to the next period, the demand for assets would be infinity as firms try to maximise their current value.

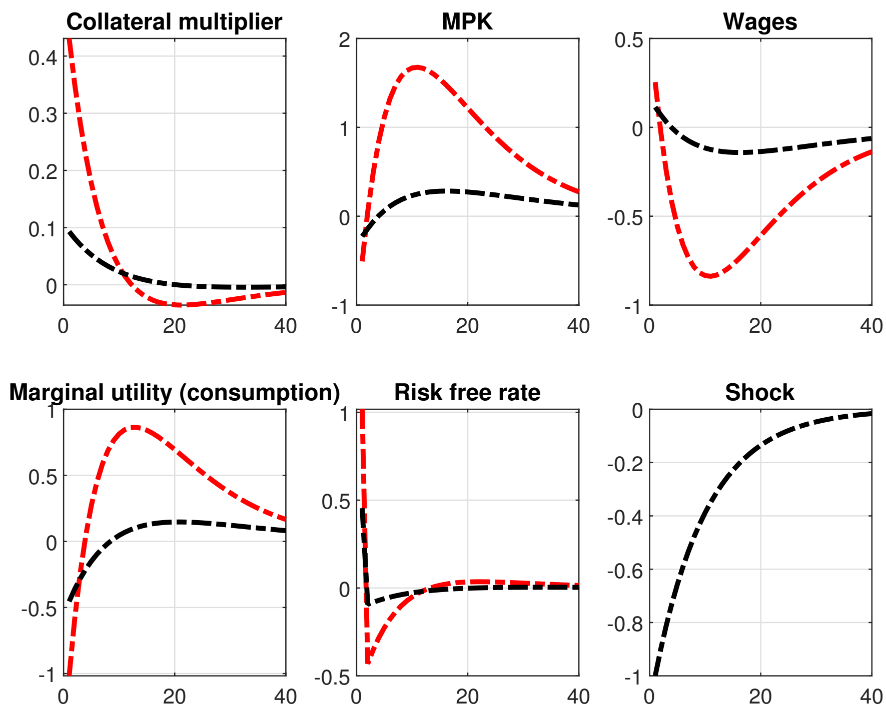
Steady State Values with $\theta = 0.5$			
Variable	Description	Model	Baseline
C/Y	Consumption-Output ratio	0.94	0.80
$\bar{K}/Y$	Portfolio Capital-Output ratio	2.04	6.91
K/Y	Production Capital-Output ratio	1.72	5.93
N	Labour	0.84	0.91
D	Time-varying aggregate depreciation	0.03	0.03
τ	Efficiency wedge	0.95	0.95

Table 2.C.5 Steady State Values (lower firm survival)

Table 2.C.5 shows that that consumption output ratio is higher with lower survival rate, implying higher consumption by households from increased transfers from firms on account of liquidation each period. The capital output ratio is much lower than the baseline case indicating lack of capital accumulation because of a large number of firms going out of business each period.



(a) Lower survival rate



(b) Lower survival rate contd.

Figure 2.C10 Plots of response to negative shocks with $\theta = 0.5$

Baseline model, Modified model

The fall in output, labour supply, capital, loans, and wages is much higher than in the baseline case, whereas rise in consumption is higher on impact. The subsequent fall in consumption is far greater, though, and tracks the response of wages. As investment in capital falls sharply due to less chance of survival, loans fall as well. A sharper fall in loans makes households feel richer initially due to a relaxing of the budget constraint and consumption goes up by more, but subsequently consumption falls sharply due to reduced capital accumulation and a reduction in wages.

2.D Model with single capital

We present below all approximations in detail.

2.D.1 Production

Firms maximise;

$$V(k,l|\xi) = \max_{k,l,n}\left[(1-\theta)\pi_t + \theta\mathbb{E}_t m_{t+1} V(k',l'|\xi')\right] \tag{2D.1}$$

where $\pi_t := k_t^{\alpha} n_t^{1-\alpha} - W_t n_t - k_{t+1} + \int(1-\delta)k_t + l_{t+1} - R_t l_t$

Each firm faces a collateral constraint which pins down the amount of borrowing the firm can avail.

$$R_{t+1} l_{t+1} = \xi_t \chi \int (1-\delta) k_{t+1} \qquad (\phi_t) \tag{2D.2}$$

Solving the above optimisation problem yields the following optimality conditions for k, n and l respectively;

$$1 = (1-\delta)[\theta\mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi] + \alpha\theta\mathbb{E}_t[m_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha}] \tag{2D.3}$$

$$W_t = (1-\alpha) k_t^{\alpha} n_t^{-\alpha} \tag{2D.4}$$

$$\phi_t = \frac{1}{\mathbb{E}_t R_{t+1}} - \theta\mathbb{E}_t[m_{t+1}] \tag{2D.5}$$

(2D.6)

2.D.2 Households

Households are standard utility maximising agents who solve the following optimisation problem;

$$\max_{C_{t+j},N_{t+j},L_{t+j}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma} - 1}{1-\sigma} - \nu \frac{N_{t+j}^{1+\psi}}{1+\psi} \right) \right] \tag{2D.7}$$

subject to the period budget constraint

$$C_t + L_{t+1} = W_t N_t + R_t L_t + T_t \qquad \forall t \qquad (\lambda_t) \tag{2D.8}$$

Above are all aggregate variables, because the households are identical and each represents the aggregate. T are net transfers received from the aggregate firm, discussed in detail in the next sub section. Solving the above routine gives the following optimality conditions;

$$C_t^{-\sigma} = \lambda_t \tag{2D.9}$$

$$\nu N_t^{\psi} = \lambda_t W_t \tag{2D.10}$$

$$\lambda_t = \beta \mathbb{E}_t[\lambda_{t+1} R_{t+1}] \tag{2D.11}$$

2.D.3 System equations

$$C_t^{-\sigma} = \lambda_t \tag{2D.12}$$

Marginal Utlity of Consumption

$$\nu N_t^{\psi} = \lambda_t W_t \tag{2D.13}$$

Labour Supply

$$\lambda_t = \beta \mathbb{E}_t[\lambda_{t+1} R_{t+1}] \tag{2D.14}$$

Loans Euler

$$m_{t+1} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \tag{2D.15}$$

SDF

$$W_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{2D.16}$$

Wages

$$F_{k,t} = \alpha K_t^{\alpha-1} N_t^{1-\alpha} \tag{2D.17}$$

Marginal Product of Capital

$$Y_t = K_t^{\alpha} N_t^{1-\alpha} \tag{2D.18}$$

Production Function

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{2D.19}$$

National Income Accounting Identity

$$1 = (1-\delta)[\theta \mathbb{E}_t m_{t+1} + \phi_t \xi_t \chi] + \alpha\theta \mathbb{E}_t[m_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha}] \tag{2D.20}$$

Capital Euler

$$\phi_t = \frac{1}{\mathbb{E}_t R_{t+1}} - \theta \mathbb{E}_t[m_{t+1}] \tag{2D.21}$$

Loans foc

$$T_t = \zeta K_{ss} - (1-\theta)(K_{t+1} - L_{t+1}) \tag{2D.22}$$

Transfers

$$L_{t+1} = \xi_t \chi (1-\delta) K_{t+1} \tag{2D.23}$$

Collateral constraint

The variables are;

$$C \;\; \lambda \;\; N \;\; W \;\; m \;\; R \;\; L \;\; K \;\; \phi \;\; Y \;\; F_k \;\; T$$

2.D.4 Steady state

In the steady state, R, m, ϕ, and F_k are pinned down by the parameters. So we solve the following system to obtain the values of variables;

$$C \;\; \lambda \;\; N \;\; W \;\; L \;\; K \;\; Y \;\; T$$

$$C^{-\sigma} = \lambda \tag{2D.24}$$

$$\nu N^{\psi} = \lambda W \tag{2D.25}$$

$$W = (1-\alpha)K^{^{-\alpha}} \tag{2D.26}$$

$$F_k = \alpha K^{\alpha-1} N^{1-\alpha} \tag{2D.27}$$

$$Y = K^{\alpha} N^{1-\alpha} \tag{2D.28}$$

$$Y = C + \delta K \tag{2D.29}$$

$$L = \frac{\chi(1-\delta)K}{R} \tag{2D.30}$$

$$T = K^{\alpha} N^{1-\alpha} - WN - \delta K + (1-R)L \tag{2D.31}$$

The numerical steady state solution is as under;

Steady State Values		
Variable	Description	Model
C/Y	Consumption-Output ratio	0.94
K/Y	Production Capital-Output ratio	2.51
N	Labour	0.84

Table 2.D.1 Steady State Values for single capital model

The dynamic model results are presented in Impulse response functions.

Chapter Three

WELFARE COSTS OF FINANCIAL SHOCKS

3.1 Introduction

We conduct welfare analysis for models in chapters one and two using the Lucas 1987 [8] methodology. Specifically, we ask the following two questions;

1. What is the cost of welfare in terms of consumption for financially constrained firms, as compared to unconstrained ones?
2. What is the cost in terms of consumption of a business cycle generated by a financial shock?

The two questions are different in required methodology: the first question demands a comparison between models where firms are constrained versus where they aren't constrained, and tries to quantify the loss, in terms of consumption, of a financial constraint. The second question does not involve comparison between two different cases; it merely asks whether a business cycle generated by a financial shock has a cost in consumption terms, and, if yes, what is the magnitude. Answering the first question would involve comparing lifetime discounted utility from two models and finding the additional consumption needed in the constrained case to make the agent just as well off as in the unconstrained case. Answering the second question merely requires comparison between the steady state lifetime discounted utility and the unconditional welfare over all possible realisations of the shock, and to quantify the impact of a shock by asking how much the agent has given up in terms of steady state consumption to attain the level of unconditional mean welfare.

We will provide more details about computation in the appropriate sections that follow. We address the questions in the same order mentioned above.

3.2 Welfare costs

We will detail the computation method, related algebra, and the results for each of the questions asked above, starting with model comparison.

3.2.1 Model comparison

We compare the models from chapters one and two to compute the welfare costs of having a financial constraint using the methodology outlined by Schmitt-Grohé and Uribe (2007) [9]. Specifically, we compare the conditional welfare of agents in both models and compute what proportion of steady state consumption ϑ^c is necessary to make them equally well off in both cases. Following the work of Kim et al. [6], [7], [5], and [4], as well as Woodford (2002) [10], who show why welfare approximations using the unconditional mean provide incorrect results, we use the conditional mean of welfare.

We define the conditional welfare from the model without collateral constraint from chapter one (called the reference model) as;

$$V_0^r = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^r, N_t^r) \tag{3.1}$$

and the conditional welfare from the model with collateral constraint (called the actual model) as;

$$V_0^a = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^a, N_t^a) \tag{3.2}$$

where $U(C_t, N_t) = \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \nu \frac{N_t^{1+\psi}}{1+\psi} \right)$, as previously. As in [9], we consider the deterministic steady state and hence consider conditional expectations to account for transition to the stochastic steady state. However, it is important to note that while [9] conduct their analysis where all models start from the *same* deterministic steady state, for us, our models start from *their respective* deterministic steady states.

We define the compensating variation ϑ^c as the additional consumption necessary in case of the financially constrained model to bring the agents to the same level of conditional welfare as the unconstrained model. ϑ^c is expressed as under[1];

$$\text{welfare cost} = \vartheta^c \times 100 = \left\{ \left(\left[(1-\beta)V_0^r + \nu \frac{(N^a)^{1+\psi}}{1+\psi} \right] (1-\sigma) + 1 \right)^{\frac{1}{1-\sigma}} \frac{1}{C^a} - 1 \right\} \times 100 \tag{3.3}$$

We carry out the welfare cost comparison for a range of values for the parameter of substitutability ε and the variance of rates of depreciation σ_δ^2.

Welfare costs of substitutability

We consider the elasticity of substitution $\varepsilon \in [1.05, 3.5]$ for both models and compare costs of welfare in figure 3.1.

[1] Detailed derivation in Appendix 3.C

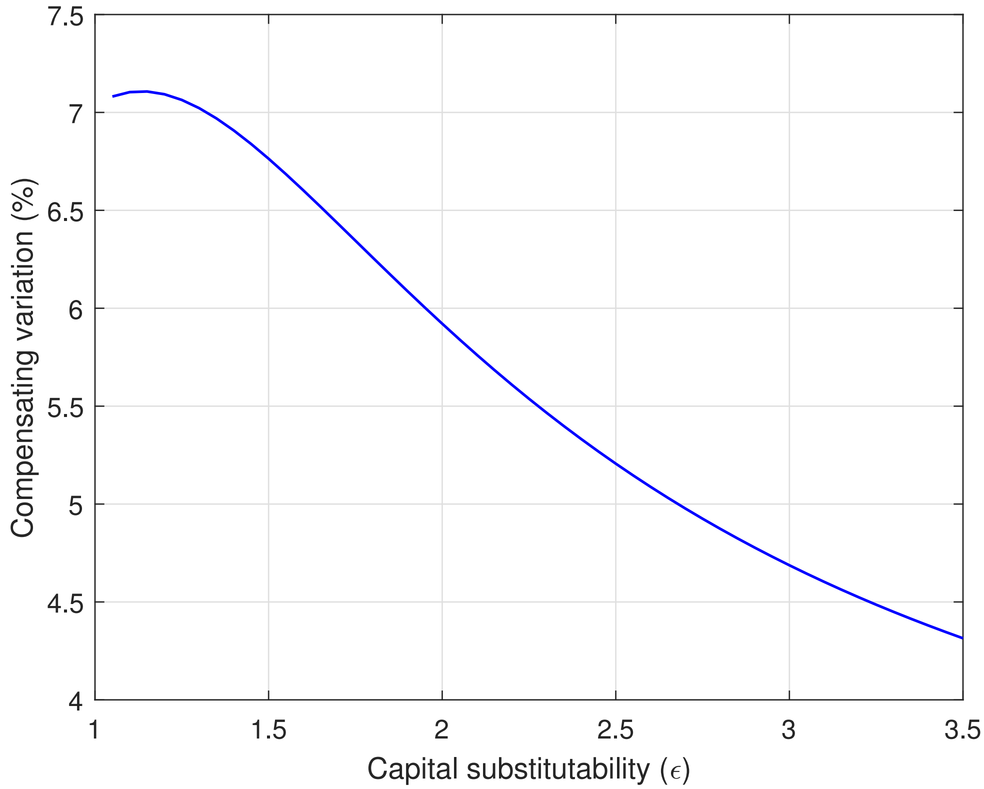
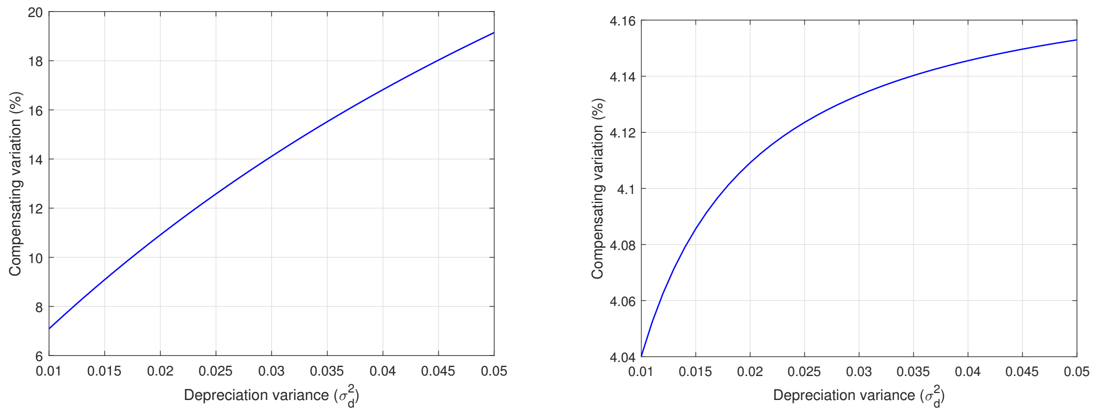


Figure 3.1 Model comparison based on substitutability

The welfare cost is highest when assets are least substitutable. This result is different from the one in 3.3 as that relates to the preference of firms regarding substitutability of assets, whereas this relates to the comparison between two firms where one is constrained and the other isn't, given substitutability. The fact that the constrained firm cannot choose assets freely when a negative financial shock hits, while the unconstrained firm can, is reflected in figure 3.1. As assets become more substitutable and constrained firms can choose to move to a different asset class when financial conditions deteriorate causes the welfare cost to be lower.

Welfare costs of depreciation variance

We vary the variance of depreciation rates which implies that there are either a large variety of assets available which extends the range of accounting depreciation rates, or that for the same assets the depreciation rates are spread out further. Figure 3.2b shows the welfare implications.



(a) $\varepsilon = 1.2$ **(b)** $\varepsilon = 4$

Figure 3.2 Welfare costs of depreciation variance

Welfare costs are, in general, higher in figure 3.2a compared to figure 3.2b, implying that higher is the substitutability of assets, lower is the cost of welfare for a wider spread of depreciation rates. Nevertheless, the costs for a higher variance of depreciation rates are higher, again driven by the fact that constrained firms will find it harder to choose the optimal portfolio if the assets are either too spread out or if there are a range of assets available with different rates of depreciation.

3.2.2 Cost of business cycle

So far we have answered the question pertaining to the welfare costs resulting from a financial constraint, *given the business cycle*. We compared two models and arrived at the compensating variation which makes consumers as well off. Now, we answer the question pertaining to the cost of *having a business cycle*. Here, we do not compare models; we merely ask how costly is a business cycle generated by a financial shock. We compute the unconditional mean welfare over a long time simulated series and compare that to the welfare in the deterministic steady state. The agents do not expect any shocks in the deterministic steady state, and the mean unconditional welfare computed over a simulated time series is lower than the steady state welfare for reasons of business cycle fluctuations. Hypothetically, if there never were any shocks[2], the time series would merely be the constant steady state at each simulation, and the mean welfare would equal the steady state welfare. However, this is not the case due to the existence of shocks. And the more variable the shock, the more the loss of welfare.

The cost of welfare ϑ is found from the following;

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U((1-\vartheta)C, N)$$

or,

$$\vartheta = 1 - \left(\left[(1-\beta)V_0 + \nu \frac{(N)^{1+\psi}}{1+\psi} \right] (1-\sigma) + 1 \right)^{\frac{1}{1-\sigma}} \frac{1}{C} \tag{3.4}$$

We compute the cost of the business cycle resulting from a financial shock in what follows. To compare, the costs from a business resulting from a productivity shock are presented in appendix 3.C.

Substitutability of capital assets

In figure 3.3, we compute the compensating variation for values of $\varepsilon \in [1.05, 3.5]$. We find that the welfare costs rise with a rise in substitutability between capital assets. All other parameter values are set at the baseline calibration mentioned in table 2.1.

[2]Not to be confused with there being no shocks but agents still expecting them, as in the stochastic steady state. Here, we mean if a shock process were never to exist, even in agents expectation, like in a deterministic steady state.

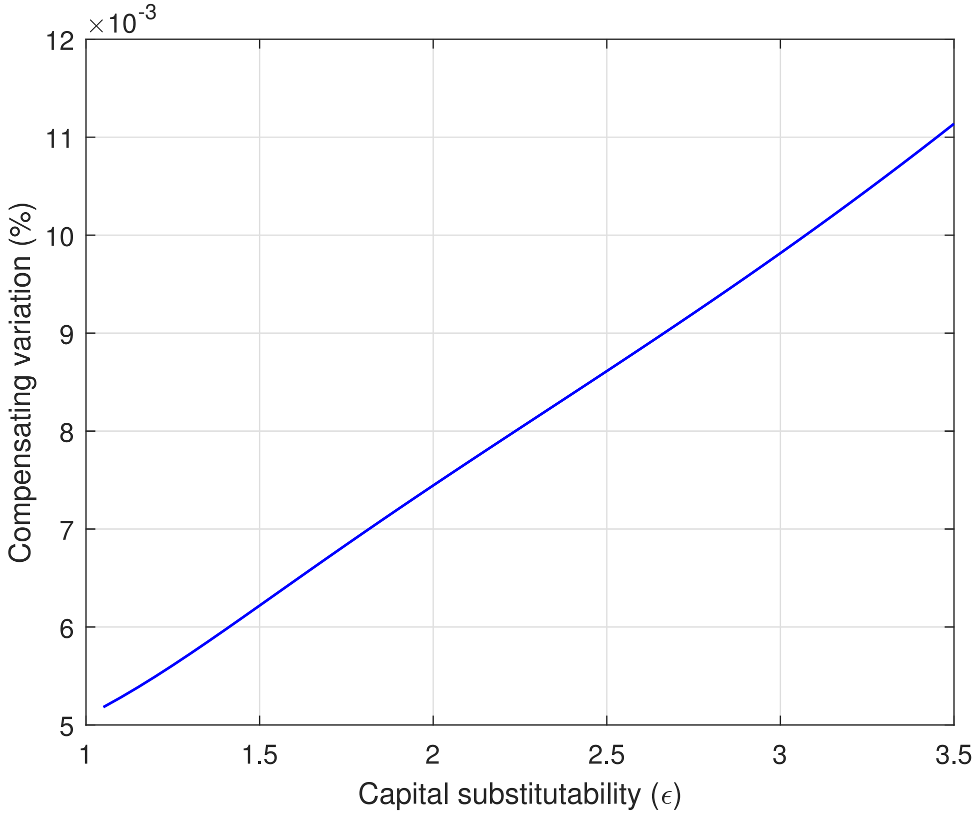


Figure 3.3 Welfare costs of capital substitutability

If assets are not substitutable, $\varepsilon = 0$, we have a Leontief production function where each asset has to be present in the same quantity as every other asset and there can be no movement from one asset type into another on impact of a negative financial shock. Although the balance sheet portfolio shrinks, it does so evenly across all asset types. Conversely, the closer substitutes assets are, the more is the movement from high depreciation assets to low depreciation ones on impact of a negative financial shock. This movement, as discussed previously, is driven by the collateral constraint which incentivises firms to load up on very low depreciation assets as they provide better collateral. However, low depreciation assets have very low marginal productivity and loading up on them causes output as well as wage income to be lower than it would have been if assets were complements, and this is reflected in the higher welfare costs towards the right end of the scale in figure 3.3. It needs to be clarified that the root of this movement from one asset class to another is driven, in the current model, by the lack of differential prices for asset classes, and the welfare costs would be substantially lower if low depreciation assets were priced appropriately relative to high depreciation assets.

Frisch elasticity parameter

Next, we consider the welfare costs of changing the inverse Frisch elasticity parameter, ψ. If ψ is low, the labour supply is highly responsive to the wage rate, and vice versa. In figure 3.4 we present the compensating variation necessary for values of $\psi \in [1, 5.9]$. We find that as labour supply becomes less responsive to wages, the welfare costs drop.

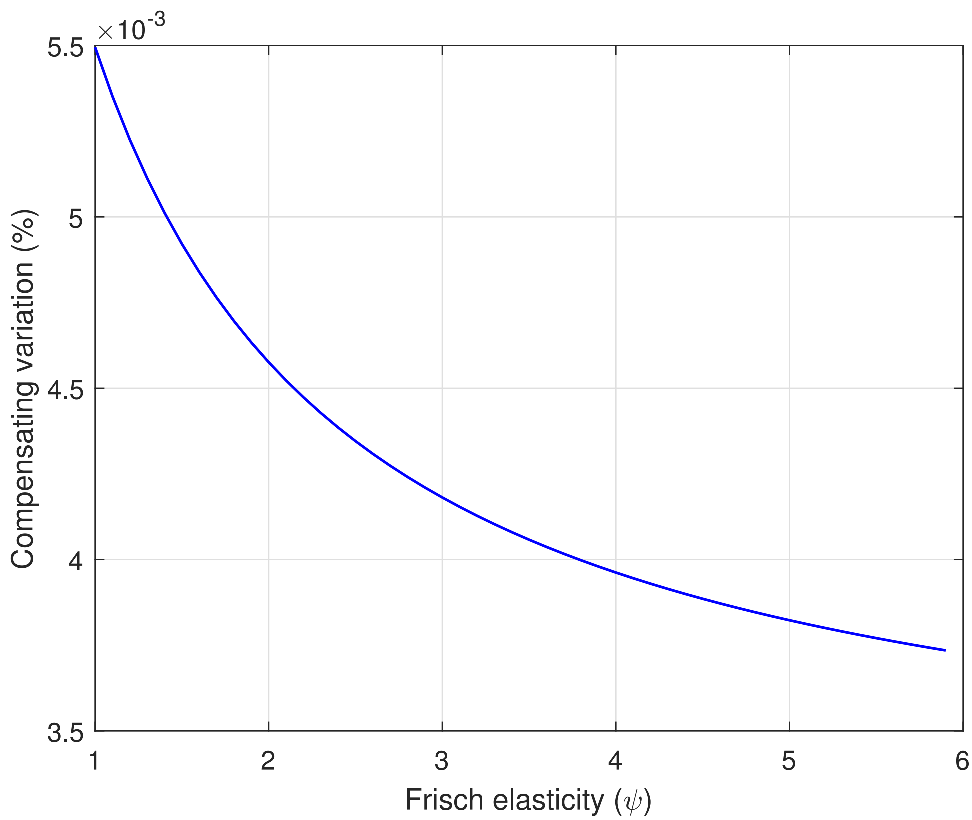


Figure 3.4 Welfare costs of labour elasticity

When labour supply is less elastic to the change in wages, the drop is lower on impact of a negative financial shock, and hence output falls by less and so does consumption. This implies that the cost of welfare is lower as the labour supply curve flattens, as demonstrated in figure 3.4.

3.3 Conclusion

We conducted welfare analysis for the model in chapter two using the unconditional felicity function of utility and found that welfare costs of a financial shock are higher if assets are more substitutable, or if labour supply is more elastic. While the result for substitutability comes from the fact that firms cannot optimise as regards asset composition when constrained financially, the result for labour supply elasticity doesn't seem directly connected to the financial constraint.

We also compare models from chapters one and two to quantify the welfare impact of having financial constraints. We find that a financially constrained firm results in a larger welfare loss when assets are not substitutable compared to the unconstrained firm which can choose the optimal portfolio freely. As the substitutability of assets increases, the welfare cost drops as the optimal choice becomes relatively easier. Also, having a widely dispersed depreciation rates schedule affects welfare to a larger extent. This is also conditional on the substitutability of assets, and more substitutability reduces the welfare cost of a higher variation in depreciation rates.

References

[1] Nicolas Coeurdacier, Helene Rey, and Pablo Winant. The risky steady state. *American Economic Review*, 101(3):398–401, 2011.

[2] Oliver De Groot. Computing the risky steady state of dsge models. *Economics Letters*, 120(3):566–569, 2013.

[3] Michel Juillard, Ondra Kamenik, et al. Solving sdge models: approximation about the stochastic steady state. Technical report, Society for Computational Economics, 2005.

[4] Jinill Kim and Sunghyun Kim. Conditional versus unconditional utility as welfare criterion: Two examples. *Computational Economics*, 51(3):719–730, 2018.

[5] Jinill Kim, Sunghyun Kim, Ernst Schaumburg, and Christopher A Sims. Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics and Control*, 32(11):3397–3414, 2008.

[6] Jinill Kim and Sunghyun Henry Kim. Spurious welfare reversals in international business cycle models. *journal of International Economics*, 60(2):471–500, 2003.

[7] Jinill Kim and Sunghyun Henry Kim. Welfare effects of tax policy in open economies: stabilization and cooperation. *Available at SSRN 478566*, 2003.

[8] Robbert E Lucas. Models of business cycles basil blackwell. *New York*, 1987.

[9] Stephanie Schmitt-Grohé and Martin Uribe. Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics*, 54(6):1702–1725, 2007.

[10] Michael Woodford. Inflation stabilization and welfare. *Contributions to macroeconomics*, 2(1):1009, 2002.

Appendix

3.A Welfare cost derivations

We present the derivations for welfare cost parameter ϑ^c in model comparison.

We have from (3.1) in steady state;

$$V_0^r = \frac{1}{1-\beta}\mathbb{E}_0[U(C^r, N^r)] \tag{3A.1}$$

We ask the question 'how much additional consumption is required in the model with credit constraints to obtain the same welfare as the unconstrained model?' The consumption in V_0^a needs to be higher than actual C^a for it to match up to the welfare level V_0^r, implying;

$$\begin{aligned} V_0^r &= \frac{1}{1-\beta}\mathbb{E}_0[U((1+\vartheta^c)C^a, N^a)] \\ &= \frac{1}{1-\beta}\left(\frac{(1+\vartheta^c)(C^a)^{1-\sigma}-1}{1-\sigma} - \nu\frac{(N^a)^{1+\psi}}{1+\psi}\right) \\ \Rightarrow \vartheta^c &= \left(\left[(1-\beta)V_0^r + \nu\frac{(N^a)^{1+\psi}}{1+\psi}\right](1-\sigma)+1\right)^{\frac{1}{1-\sigma}}\frac{1}{C^a} - 1 \end{aligned} \tag{3A.2}$$

Another way to arrive at the same answer is to ask 'how much consumption would agents have to give up in the unconstrained model to obtain the same welfare as the constrained model?'

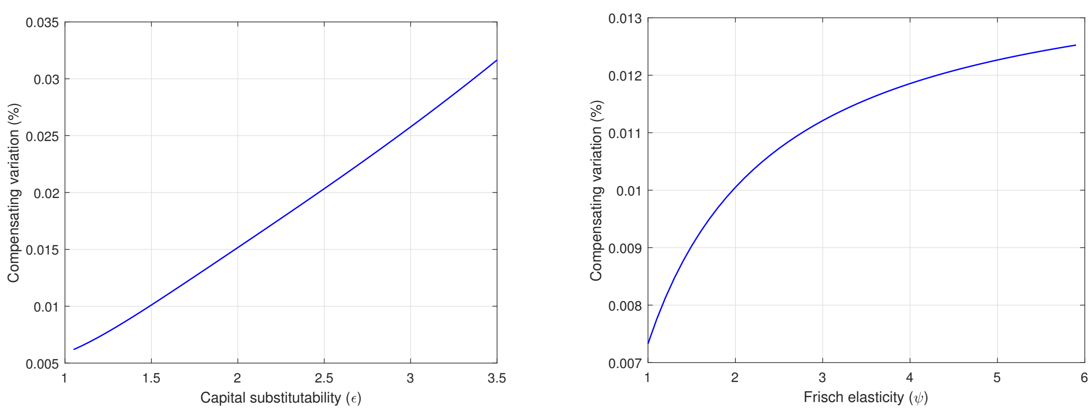
$$
\begin{aligned}
V_0^a &= \frac{1}{1-\beta}\mathbb{E}_0[U((1-\tilde{\vartheta}^c)C^r, N^r)] \\
&= \frac{1}{1-\beta}\left(\frac{(1-\tilde{\vartheta}^c)(C^r)^{1-\sigma}-1}{1-\sigma} - \nu\frac{(N^r)^{1+\psi}}{1+\psi}\right) \\
\Rightarrow \tilde{\vartheta}^c &= 1 - \left(\left[(1-\beta)V_0^a + \nu\frac{(N^r)^{1+\psi}}{1+\psi}\right](1-\sigma)+1\right)^{\frac{1}{1-\sigma}}\frac{1}{C^r}
\end{aligned}
\tag{3A.3}
$$

3.B Cost of business cycle

The cost of business cycles generated by financial shock is arrived at as under;

$$
\begin{aligned}
V_0 &= \frac{1}{1-\beta}\mathbb{E}_0[U((1-\vartheta)C, N)] \\
&= \frac{1}{1-\beta}\left(\frac{(1-\vartheta)(C)^{1-\sigma}-1}{1-\sigma} - \nu\frac{(N)^{1+\psi}}{1+\psi}\right) \\
\Rightarrow \vartheta &= 1 - \left(\left[(1-\beta)V_0 + \nu\frac{(N)^{1+\psi}}{1+\psi}\right](1-\sigma)+1\right)^{\frac{1}{1-\sigma}}\frac{1}{C}
\end{aligned}
\tag{3B.4}
$$

3.C Cost of a productivity shock



(a) ε changes **(b)** ψ changes

Figure 3.C1 Business cycle costs of technology shock

Figure 3.C1a is qualitatively similar to figure 3.3, although the magnitude is higher. Figure 3.C1b is qualitatively different from figure 3.4 due to the differing responses to financial shock versus productivity shock in the model. In case of the financial shock, the labour supply curve moves back, causing wages to rise and labour to fall. The flatter the curve, more is the rise in wages and lower is the welfare cost. Conversely, in case of a technology shock, the labour demand curve moves back, causing wages to fall. The flatter is the supply curve, higher is the impact on wages which affects household income as well as consumption. Hence, costs rise with a flatter labour supply curve.

As regards magnitude, it appears that a productivity shock generates a higher welfare cost compared to a financial shock of the same magnitude in our model.

CONCLUDING REMARKS

We analysed firm balance sheet data from the U.S. and found a pattern of balance sheet portfolio reallocation over the financial crisis. Given the already known fact that financial crises reduce productivity, we attempt to build a model which can account for both these phenomena; namely, a model where a shock originating in the financial sector causes firms to change their optimal portfolio of assets and also reduces aggregate productivity. Ours is a model where a continuum of capital assets with varying depreciation rates, and thus, varying importance as collateral are chosen to construct the balance sheet portfolio. Our model helps draw a straight line between financial crises, a change in the balance sheet portfolio, and a drop in productivity without having to rely on technology shocks. In other words, we can show how a shock originating in the financial sector works its way through the economy and makes optimal portfolio selection difficult for firms due to falling value of collateral, which in turn leads to selection of a sub optimal portfolio and a fall in efficiency due to misallocation, which qualitatively resembles a negative technology shock. Our work is related to seminal papers like Kiyotaki Moore (1997), Bernanke, Gertler, and Gilchrist (1999), and Chari, Kehoe, and McGrattan (2007), and we try to reconcile some of their insights such as the finding that efficiency wedges (along with labour wedges) are responsible for deepening and lengthening recessions, and that the presence of financial frictions can lead to prolonged downturns.

In our model, a shock to the loan-to-value parameter tightens the borrowing conditions and brings on a financial crisis by reducing access to credit for firms. This directly reduces demand for individual types of capital, but to varying degrees depending on their rate of depreciation. Specifically, high depreciation assets are less sensitive to the shock compared to low depreciation assets, with the caveat that for very low depreciation assets, demand will actually rise on a negative shock. This reflects the desire of firms to invest in the longest duration assets in bad times to be able to access credit and to acquire the asset which yields its marginal product for longer than short duration assets. The impact on the production bundle as well as the portfolio is also dependent on the substitutability of the assets; the more complementary they are, the larger the fall in portfolio and the production bundle. This results from the fact that complementary assets need to be be bought together in a set proportion, and hence cannot be substituted by other kinds of assets. We check how the model responds to different parametrisations, as well as for different preference specifications. The transmission mechanism remains the same, except for minor changes. A fall in demand for mid range assets on the depreciation scale is something we observe in data for the U.S. for the financial crisis, so to some extent the rise in very low depreciation assets, although very slightly, but the fall in high depreciation asset demand is not validated by data. Data indicates that demand for high depreciation assets goes up slightly. We also show how this leads to an efficiency wedge in as well as out of equilibrium: as long as there is a collateral constraint, the steady state is inefficient and the wedge is pro cyclical resembling a negative productivity shock.

The welfare analysis of chapter three indicates that higher asset substitutability can lead to lower welfare costs if firms are not financially constrained. Also, the variance of rates of depreciation being low helps improve welfare, conditional on asset substitutability. The parameter of substitutability can be estimated from data, whereas the variance of depreciation rates is available from accounting schedules. Although of low significance by itself, having an accurate estimate of substitutability will improve policy analysis and effectiveness in further work.

One important thing that we have not addressed is the model is the role of intangibles. Investment in intangibles, intuitively, would be procyclical: in good times firms would acquire copyrights and patents, and investing in research would make more sense, whereas in recessions firms would want to cut down on research costs and registration of patents would drop. Especially for financially constrained firms who rely on collateralised credit, investment in physical, tangible assets which can be used as collateral will be preferred over intangibles. However, including intangibles and their amortisation costs, as well as their contribution as a factor of production is an interesting aspect to model. The way intangibles have been described here would likely not change the implications from the model greatly, however there are several ways of modelling this and other specifications might affect results.

Another modelling change we wish to incorporate is to have prices for individual types of capital assets and such prices will be determined endogenously as in the standard model with a single type of capital. This will complicate the model significantly, but it seems like a more realistic setting which might be able to replace production bundle weights of assets used here. Adding differential costs of adjustment for asset types is also something that we intend to include in the model. Adjustment costs will be higher for low depreciation assets and lower for high depreciation assets which is intuitive and will generate more interesting dynamics. Further modifications in the model will be directed at generating policy implications by adding nominal rigidities like sticky prices or real rigidities like sticky wages, although how exactly it is to be done and the related algebra is not immediately clear.

We also need to try and get clearer information from the data collected on fixed assets and TFP. As of now, the data work involves using the age of assets and their dollar values to try and decipher the changes over the financial crisis in each asset type. Although insightful, there might be a better way of extracting information from the same data by using some form of structural causal modelling, where a series from data proxy for the movement in the collateral constraint can be used along with series for all asset types in a causal model to check if any causal relationship from changes in collateral constraint to changes in asset types over the crisis can be drawn. Also, including data on intangibles, as discussed above, can make the analysis richer and more interesting.